# PREDICTIVE POPULATION MODEL FOR BIGHEADED CARPS

# IN MISSISSIPPI RIVER

by

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# ABSTRACT

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Bigheaded carps refer to two species of fish called Silver carp and Bigheaded carp which have invaded the Upper Mississippi River System and are causing nuisance to the native fish like catfish, paddlefish and gizzard Shad by encroaching their habitat. The exponential growth of their population due to their overwhelming reproduction capability and adaptation features calls for a careful study of their population dynamics. In this study, we offer a mathematical model with artificial excessive fishing effort to reduce the population of Bigheaded carps. We numerically observe the correlation among growth rate, fishing rate and the future population of the Bigheaded carps.

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CHAPTER 1

# INTRODUCTION

## 1.1 Bigheaded Carps in Upper and Middle Mississippi River

Upper Mississippi River starts at the confluence of Ohio river and extends 600 miles northward to Minneapolis, MN [Figure 1.1]. Middle Mississippi river starts at the confulence of Ohio River in Cairo, Illinois and extends 200 miles northward above Missouri river which constitutes the unimpounded reach[Figure 1.2]. In the past few decades Bigheaded carps are making their way in this river system. Bigheaded Carps typically refer to two species of Asian Carps known as Bighead Carp (Hypopthalmichthys nobilis) and Silver Carp (Hypopthalmichthys molitrix). Both of them are voracious eaters. .They consume upto 40 percent of their own body weight in plankton each day[1]. They consume plankton—algae and other microscopic organisms—stripping the food web of the key source of food for small and big fish. Asian carp can grow to large sizes: some as large as 110 pounds [6] though the average size is around 30-40 pounds. The diet of Asian carp overlaps with the diet of native fishes in the Mississippi and Illinois Rivers [4], which implies that the bigheaded carps compete directly with native fish for food. Therefore, in December 14,2010, the Asian Carp Prevention and Control Act, S.1421, passed by the Senate, has added the bighead carp species of Asian carp to a list of injurious species that are prohibited from being imported or shipped in the United States under the Lacey Act [11].


**Figure 1.1 Upper Mississippi River (UMR)**


**Figure 1.2 Pooled Reaches and Open River**

The reproduction of bigheaded carps in the UMR becomes perilous because it is connected to the Great Lakes by Chicago Waterway System through Illinois River. The carps could enter the Great Lakes through this waterway system. If these fish enter the Great Lakes, their probability of surviving and reproducing in the Great Lakes is high due to the natural and man-made connections and the widespread availability of suitable habitat [7]. There are evidences of non-linear growth of Carp population in this river system. Between 1991 and 2000, as scientists watched the Asian carp spread in the Mississippi and Illinois Rivers, Asian carp abundances surged exponentially [4]. Between 1994 and 1997, for instance, commercial catch of bighead carp in the Mississippi River increased from 5.5 tons to 55 tons between [4].

## 1.2 Potential Impact on the Great Lakes Food Web and Economy

Although these species have been widely introduced throughout the world for food, the native range of silver and bighead carp is eastern Asia [16]. In the early 1970s, these fishes were introduced into Arkansas for private aquaculture. By the 1980s, these species occurred in public waters, probably escapees from hatcheries and travelled north through the Mississippi river [14]. Successful reproduction of both species is occurring in the UMR and Missouri drainages [15].Population densities of silver carp increased dramatically in the early 2000s [4], greatly increasing the visibility of this species, which is notorious for breaching the surface of the water in large aggregations. If these species breach the electric barrier constructed by the U.S. Army Corps of Engineers in the Chicago Sanitary and Ship Canal, then they will threaten the Great Lakes as well [10].

The potential impact of Asian carps on the Great Lakes' sport, tribal, and commercial fisheries can be seen now along the Mississippi River basin, where in just a few short years following introduction of Asian carp into an area, many commercial fishing locations have been abandoned, as native fish have nearly disappeared from the catch, replaced by Asian carp. The growth of Asian carps could cause serius economic impact to the Great Lakes' commercial, tribal, and sport fisheries which is currently valued at $7 billion annually[8]. Reduced abundance of native fishes will result in less angling quality and effect those who rely on sport and commercial fisheries for livelihood.

Since the diet of Asian carp coincides with the diet of native fishes like ciscos, bloaters, and yellow perch, which in turn, are preyed upon by lake trout and walleye, it will give rise to competition for food [12]. Bighead carp would consume zooplankton in the Great Lakes and silver carp would prey heavily on phytoplankton. Asian carp appear to be highly opportunistic when it comes to feeding. For instance, bighead carp diet in the Mississippi River is more varied than in their native range, showing the carp take advantage of the food that is present. By feeding on plankton, the Asian carp feed on the "low end" of the food web, and few people doubt that the carp would have significant negative impacts on the food web [12].

Natural reproduction of Asian carp occurs in channels of larger river in swift current where velocities exceed 0.8m/sec. Flooding during the monsoon season is a requirement for spawning because the floodlands provide nursery areas for the larvae and juveniles [13, page 14]. Mississippi river provides a suitable spawning habitat for these species. The Great Lakes is also suitable because the Great Lakes basin contains numerious streams with suitable spawning habitat and large areas of vegetated shorelines, particularyly large bays, wide river mouths, connecting channels, wetlands and lentic areas [7,10].

### 1.3 Tentative Solution to the Bigheaded Carp Problem

The distribution of Bighead and Silver Carp in the river systems in the United States is shown in Figure 2 and Figure 3 respectively. Both species are well established in the United States, with bighead carp being present in 19 states, and silver carp occurring in at least 12 states [10].The population density of the Bigheaded Carps is found to be higher in Upper Mississippi and Middle Mississippi river. In some portion of the river,they constitute upto 95% of the biomass in the system[4]. Changes in productivity of bighead carp in the basin of the Amu dar'a River increased from 8,200 kg/yr in 1967-69 to 5 million kg/yr in 1973 and 1974[2].

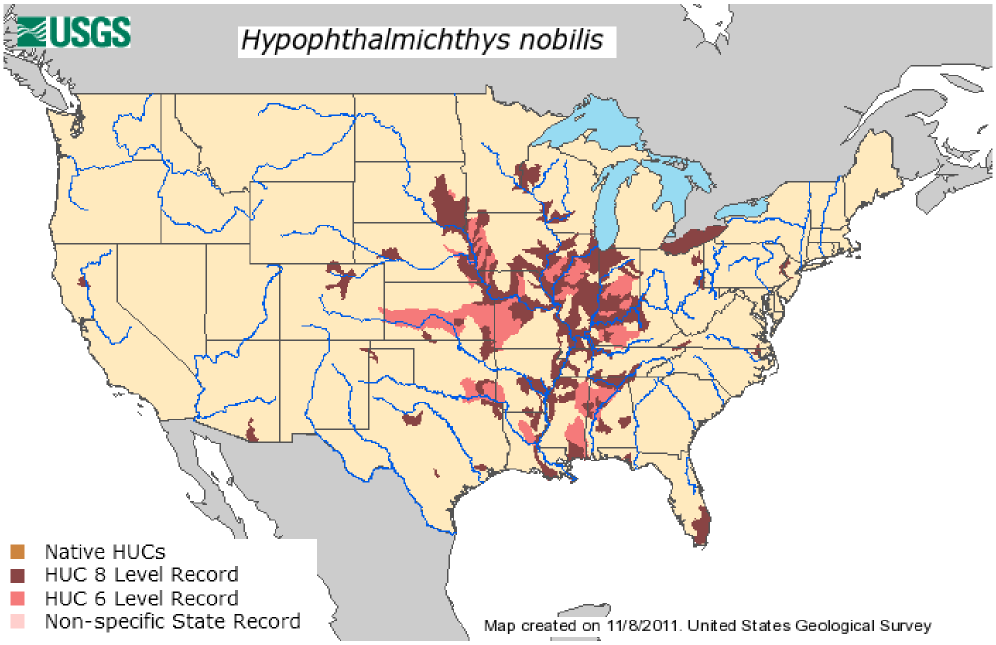


**Figure 1.3 Distribution of Bighead Carp in the United States**

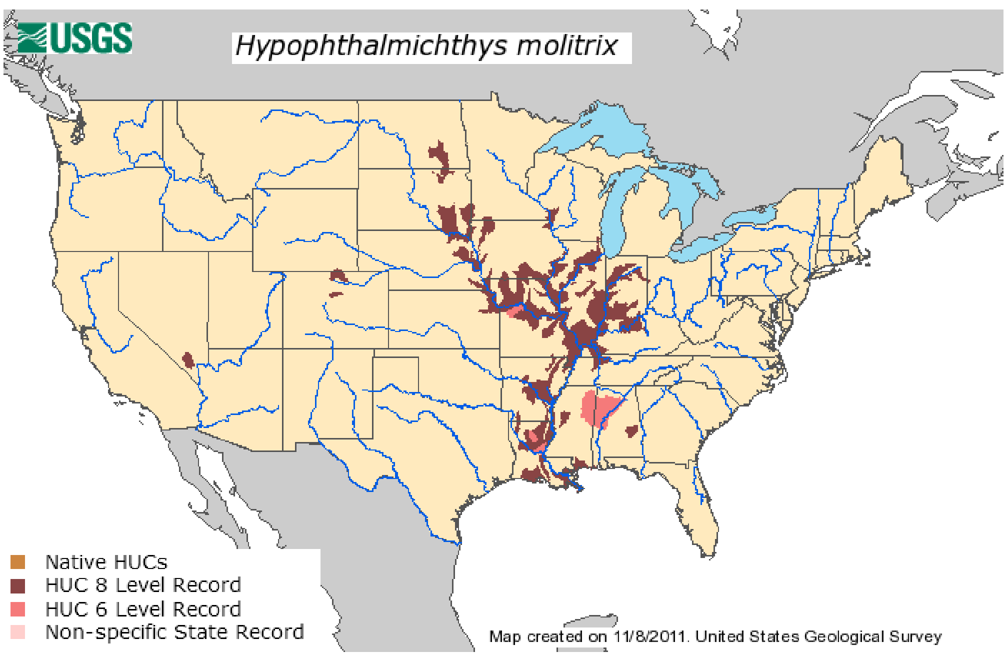


**Figure 1.4 Distribution of Silver Carp in the United States**

Although some fisherman have started fishing for Asian Carp given the easiness and volume of catch, the sales price of 14 cents a pound(approximately $4 a fish for an average size 15-25 pound fish) has not attracted too many fisherman towards it[3]. If we could open a new market like East Asia for this fish where it is loved, fisherman could get a better price due to growth in demand so that more fisherman would be attracted towards Carp fishing. Then we can allow the fishing to reach such a level so as to maintain just a sustainable population for Asian Carp.

Another way to alleviate this problem would be to introduce a predator species for Asian Carp in the Mississippi River. In the 1950s and 1960s, the similar nuisance of invasive alewife (*Alosa pseudoharengus*) in Lake Michigan was not lessened until predatory Pacific salmon were stocked into the system[1]. Pike perch(Lucio perca) is known to be the predator of Asian Carps which can be introduced in the river system to assuage the problem[13].

Certain concentration of chemical called Rotenone has been proven to wipe out Asian Carps from the system[13]. But it affects all the gill-breathing organisms and the fish are suggested not to be consumable by human after the treatment of Rotenone.

In our model, we will consider artificial overfishing effort and its effect on the population of Bigheaded carps in the Upper Mississippi River System. Then we will simulate the behavior of population dynamics of Bigheaded carp numerically.

# CHAPTER 2

## REVIEW OF PREVIOUS STUDY

Asian Carps became a subject of interest not too long ago. Therefore, comprehensive population growth models for these species are not available. However a report submitted to the Department of Defense Technical Information Center in May 2007 by James E. Garvey, Kelly L. DeGrandchamp and Christopher J. Williamson describes the potential impact of selective removal of variable lengths of silver carp in a hypothetical sample population according to the statistics collected from the unimpounded reach of the Middle Mississippi river(MMR) and the pooled confluence area of the Illinois River(CONFL) between 1997 and 2005.

In this report they use the von Bertalanffy model to justify the lenth-age relationship for all populations.

$$L_t = L_\infty(1 - e^{-k(t-t_0)})$$

where *L* is the length in mm, *k* is a growth coefficient, $L_\infty$ is the theoretical maximum length, *t* is the age in years, and $t_0$ is the age when $L = 0$, which is usuall non-zero. They found that Silver carp from the MMR grew more rapidly, reaching a smaller maximum length than counterparts in the CONFL area [Table2. 1]. Using von Bertalanffy model, the growth constant was calculated.

**Table 2.1 Population Statistics for Asian Carps from Two Reaches in the Upper Mississippi River**

| Reach | Species | Maximum Age ($t$, years) | Maximum Length ($L_\infty$, mm TL) | Growth Rate ($k$) |
|---|---|---|---|---|
| MMR[2] | Silver carp | 5 | 778 | 0.63 |
| CONFL | Silver carp | 5 | 867 | 0.41 |
| | Bighead carp | 10 | 1242 | 0.24 |

[1] Parameters derive from age-length relationships incorporated into von Bertalanffy models. All models fit empirical data well ($R^2 > 0.9$)
[2] Williamson and Garvey (2005)

They also found the reproductive potential of both Asian carps to be very high relative to native river-specific species in the UMR system. In 2004, the average egg productions per silver carp and bighead carp were 269,388 and 118,485 respectively but in 2005, they rose to 1,478,331 and 777,154 respectively compared to averages of 70,000, 142,000 and 350,000 for catfish, paddlefish and gizzard shad respectively [17]. The change in year class strength varied drastically according to the environmental conditions such as precipitation and spring/summer flow.

They utilize the Beverton-Holt yield per recruit model to analyze the adult dynamics .

$$Y = \frac{F * N_t * e^{Zr} * W_\infty}{K} * [\beta(X, P, Q)] - [\beta(X_1, P, Q)]$$ , where

F = instantaneous fishing mortality;

$N_t$ = the number of recruits entering the fishery at some time t;

Z = instantaneous mortality rate;

r = time to recruitment (tr – t0);

$W_\infty$ = maximum theoretic length estimated from $L_\infty$ and the length weight regression;

K = the Brody growth constant from the von Bertalanffy model;

β = the incomplete beta function, $\beta(z,a,b) \equiv \int_0^z u^{a-1}(1-u)^{b-1}du$;

X = $e^{-Kr}$;

$X_1$ = $e^{-K(\text{Max Age} - t0)}$, Max Age is the maximum age from the sample;

P = Z/K;

Q = slope of the weight-length regression.

By the aid of Fishery Analysis and Simulation Tools (FAST) software, they estimate the effect of sizewise selective harvesting on the population dynamics of the species using a hypothetical initial population of 1,000 individuals. According to this model, the removal of individuals greater than 300 mm total length will cause the equivalent of about 50 percent annual mortality, which in turn will result in compensatory responses of the population and increase the population production of biomass [Figure 2.1] whereas reducing the vulnerable minimum total length below 300 mm will begin to reduce productive capacity of the population because small individuals will be removed before attaining their maximum growth capacity. This model suggests that even with high natural mortality rates experienced by this species [19], substantial loss of fish in addition through selective removal or exclusion from spawning areas is necessary to significantly affect population dynamics which leads to our model which focuses on planned overfishing to incur substantial loss of fish .

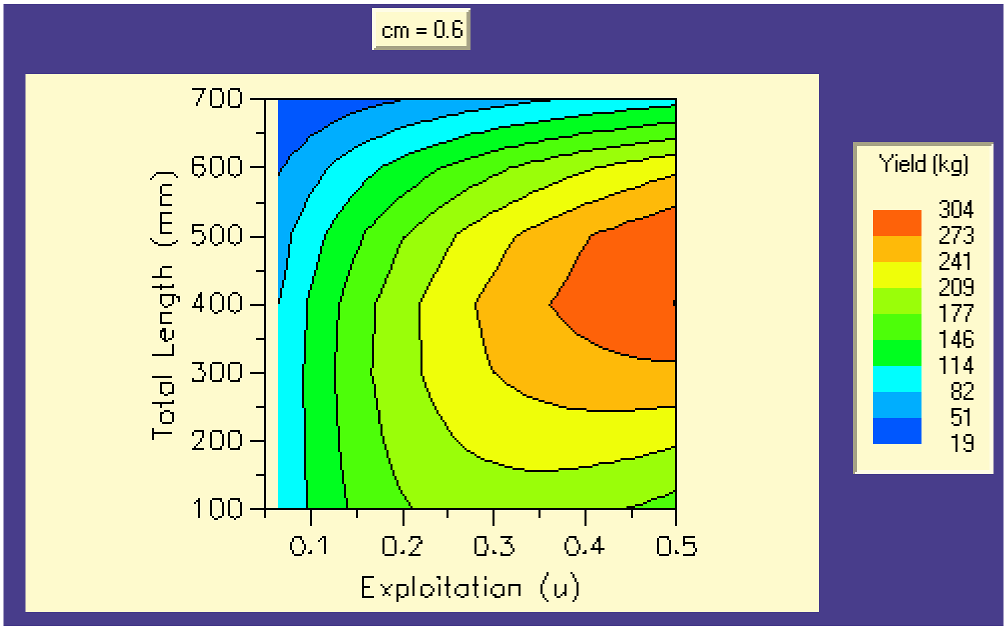


**Figure 2.1. Production of biomass per each new recruit in a hypothetical population of silver carp with 60 percent natural annual mortality and varying vulnerable minimum total lengths (y-axis) and rates of removal or exclusion (x-axis)**

# CHAPTER 3

# MATHEMATICAL MODEL

We propose a model to estimate the population growth of bigheaded carps.We consider the case of planned overfishing so as to reach the level of sustainable population and then numerically explore the effect of variation of fishing effort on the populattion dynamics of the bigheaded carps.

## 3.1 Logistic Equation

In this model we assume that the carp population's growth is proportional to the population at that instant so we can form the differential equation

$$\frac{dy}{dt} = ry, \qquad (1)$$

where *y* is the population of carp at time *t* and *r* is the growth rate constant. We implicitly include the natural mortality rate $m_n$ in *r*. From the study reviewed earlier, the reproductive potential of bigheaded carps varies from year to year according to the environmental conditions. However, we regard this to be steady in our model. We can impose fluctuation of reproductive potential in our model during simulation by placing applicable value of *r*. We can solve this differential equation assuming $y_0$ as the population at t=0 as

$$y = y_0 e^{rt}, \qquad (2)$$

According to the solution, the population grows exponentially for t>0. However, the limiting factors e.g. food supply, oxygen level, predators etc affects the extent to which the population can grow. So we modify equation (1) as

$$\frac{dy}{dt} = ry\left(1 - \frac{y}{K}\right), \qquad (3)$$

where K is the carrying capacity for the system. This is also called the Verhulst model or the logistic model. The left term is positive as *y*<*K* but when *y*>*K* the left term becomes negative. When *y*=*K*, $\frac{dy}{dt}$ becomes steady and it is said to be in an equilibrium state. Any deviation from the equilibrium population *K* will generate negative potential that will try to stabilize by restoring the population back to K. The solution of equation (3) is given by

$$y = \frac{y_0 K}{y_0 + (K - y_0)e^{-rt}}, \text{ for } y_0 > 0, K > 0 \qquad (4)$$

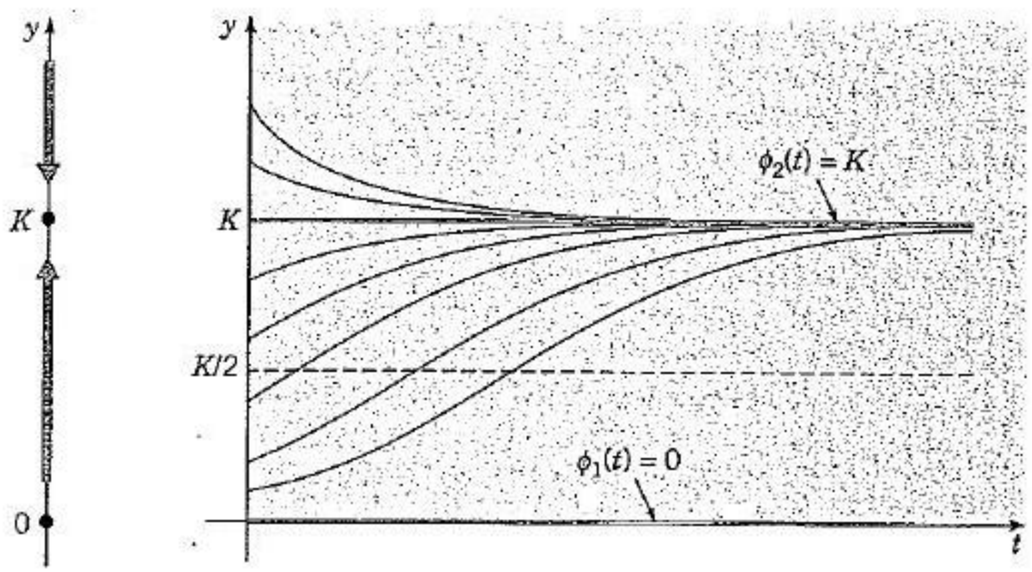


**Figure 3.1[21] Family of solutions for equation(3). The actual path depends on the initial value, $y_0$ as well as the rate of growth constant,*r,* and limiting population, *K***

The figure above shows that irrespective of the initial value of $y_0 > 0$, the value of y approaches *K* as *t* increases. In ecosystem, there is a minimum sustainable population for species such that if the population decreases below it, the species dies out. Adding this threshold population $T$ into equation (4) we get,

$$\frac{dy}{dt} = -ry\left(1-\frac{y}{T}\right)\left(1-\frac{y}{K}\right), \quad \text{for } y > 0, 0 < T < K \tag{5}$$

The graph of the family of solutions for this differenctial equation is given by

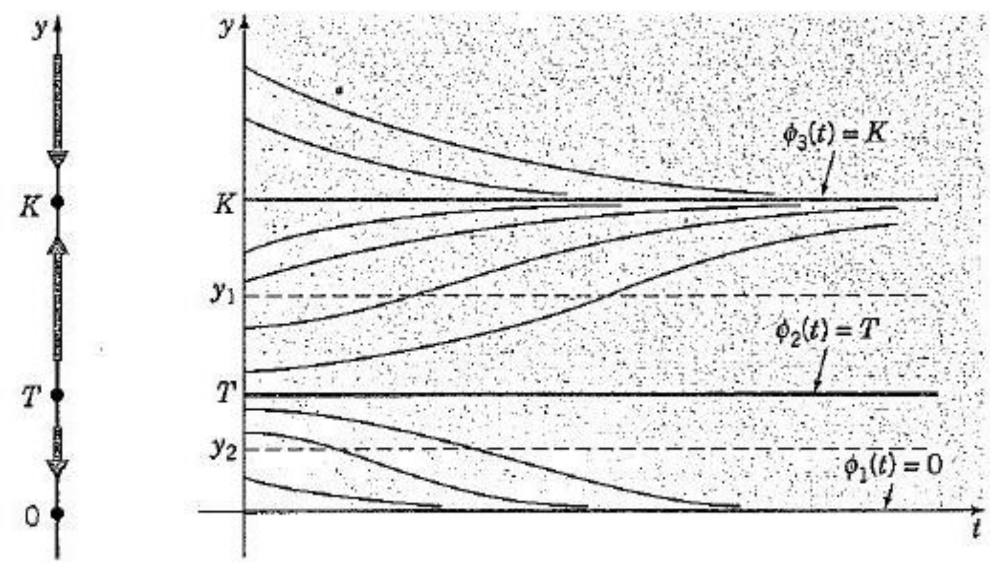


**Figure 3.2 [21] The graph of family of solution for equation (5)**

In the figure, we can see that if the population reaches below the threshold level $T$, it eventually goes to zero.

## 3.2 Excessive Human Fishing

Now, we include human fishing equation (5) to get,

$$\frac{dy}{dt} = -ry\left(1-\frac{y}{T}\right)\left(1-\frac{y}{K}\right) - Ey, \quad \text{for } y > 0, 0 < T < K \qquad (6.1)$$

where $E$ is the rate of human fishing. When the spread between K and T is very large which is the actual case in Bighead carps population, $y$ can assume wide range of values and we can run into situations where y is very large compared to T. When this happens, the term $\left(1-\frac{y}{T}\right)$ will distort the behavior of the function. Hence, we introduce a step function to alleviate this situation. We modify equation (6) as:

$$\frac{dy}{dt} = ry\Gamma(y,T)\left(1-\frac{y}{K}\right) - Ey, \quad \text{for } y > 0, 0 < T < K \qquad (6.2)$$

where,

$$\Gamma(y,T) = \begin{cases} -\left(1-\frac{y}{T}\right), & if \ \frac{y}{T} \le \varepsilon \\ 1, & if \ \frac{y}{T} > \varepsilon \end{cases} \quad \text{, for } y > 0, 0 < T < K \qquad (6.3)$$

Here, the value of $\varepsilon$ can be effected by the rate of extraction effort $E$ by humans. Desirable value of $\varepsilon$ is less than $2$ in equation (6.3). We call it function behavior constant. Moreover, since we are not concerned with the threshold population of Bigheaded carps given the current case of overpopulation, we can drop the parameter T from equation (6) to get,

$$\frac{dy}{dt} = r\left[\left(1-\frac{y}{K}\right) - E\right]y, \quad \text{for } y > 0, K > 0 \qquad (7)$$

Now reverting to equation (6.1), in order to calculate the state of equilibrium we let $\frac{dy}{dt}=0$,

and we get,

$$0=-ry\left(1-\frac{y}{T}\right)\left(1-\frac{y}{K}\right)-Ey \text{, for } y>0, 0<T<K \tag{8.1}$$

or, $$\left(\frac{r}{TK}\right)y^2-r\left(\frac{1}{K}+\frac{1}{T}\right)y+(r+E)=0 \tag{8.2}$$

The above equation is in quadratic form which has two roots for $y>0, 0<T<K$ i.e.

$$y_K=\frac{KT\left[r\left(\frac{1}{K}+\frac{1}{T}\right)+\sqrt{r^2\left(\frac{1}{K}+\frac{1}{T}\right)^2-\frac{4r(r+E)}{KT}}\right]}{2r} \text{, and} \tag{9.1}$$

$$y_T=\frac{KT\left[r\left(\frac{1}{K}+\frac{1}{T}\right)-\sqrt{r^2\left(\frac{1}{K}+\frac{1}{T}\right)^2-\frac{4r(r+E)}{KT}}\right]}{2r} \tag{9.2}$$

It is obvious that both $y_K$ and $y_T$ rely on $E$. To get the absolute sustainable rate of fishing for a specific value of $E$, we can multipuly $y_K$ by $E$. Then we get,

$$Y_E=\frac{EKT\left[r\left(\frac{1}{K}+\frac{1}{T}\right)+\sqrt{r^2\left(\frac{1}{K}+\frac{1}{T}\right)^2-\frac{4r(r+E)}{KT}}\right]}{2r} \tag{10}$$

Now, we can find the maximum $Y_E$ by taking the first derivative of $Y_E$ with respect to E and letting it equal to 0. We get,

$$\frac{dY_E}{dE} = -\frac{E}{\sqrt{r^2\left(\frac{1}{K}+\frac{1}{T}\right)^2 - \frac{4r(r+E)}{KT}}} + \frac{KT\left[r\left(\frac{1}{K}+\frac{1}{T}\right)+\sqrt{r^2\left(\frac{1}{K}+\frac{1}{T}\right)^2 - \frac{4r(r+E)}{KT}}\right]}{2r} \quad (11)$$

Letting $\frac{dY_E}{dE} = 0$ and solving for E we get,

$$E_m = \frac{K^2 r - 4KTr + rT^2 + r\sqrt{K^4 + K^3 T + KT^3 + T^4}}{9KT} \quad (12)$$

This $E$ denotes the optimum fishing rate and we denote it by $E_m$. We can find the optimum harvest by multiplying $E_m$ by $y_K$. i.e.

$$Y_m = E_m y_K \quad (13)$$

In equation (9.x) we see that the term $\sqrt{r^2\left(\frac{1}{K}+\frac{1}{T}\right)^2 - \frac{4r(r+E)}{KT}}$ (13)

decreases as $E$ increases. So we can increase $E$ to such a level that equation (13) is zero. Using this in equation (9) we get $y_K = y_T$ meaning they merge into one steady value $y_s$ which is in a semi stable equilibrium state. It is stable from above but not from below. We denote this $E$ by $E_s$ and we can calculate it by setting equation (13) to zero and solving for $E$,i.e.

$$E_s = \frac{r\left(K^2 - 2KT + T^2\right)}{4KT} \tag{14}$$

With this we can find the absolute maximum fishing rate that will still maintain the minimum sustainable fish population, i.e.

$$Y_s = E_s y_K \tag{15}$$

Any fishing effort surpassing this rate will amount to overfishing and will result in the overall decline of the population. Using this model, we will simulate the population dynamics of bigheaded carps.

CHAPTER 4

NUMERICAL SIMULATION AND ANALYSIS

We use matlab software to simulate the behavior of our mathematical model. We use fourth order Runga-Kutta method to implement the simulation. First, we start with a small growth constant and no harvesting $(r = 0.04, E = 0.0)$. Then, we introduce minimal harvesting and we increase the harvesting rate and observe equilibrium state along the way. Later, we attempt to use a realistic growth rate deduced from the reproductive potential and natural mortality rate as indicated in the study reviewed earlier. Then we will vary the fishing effort and observe the behavior of our model and its stability.

The following table lists the parameters and their value range used during the simulation.

**Table 4.1 Model parameters with descriptions and values**

| Parameter | Description | Value Range |
|---|---|---|
| $K$ | Maximum Carrying Capacity | 10000/ mile |
| $T$ | Threshold Capacity | 1000/mile |
| $r$ | Growth Rate | 0.05 – 0.4 |
| $E$ | Human Effort of Harvesting( induced mortality) | 0.0 – 0.7 |
| $E_m$ | Harvesting Rate for Optimum Yield | 0.0737 – 0.5531 |

Table 4.1 - continued

| | | |
|---|---|---|
| $E_M$ | Harvesting Rate for Maximum Yield | 0.0810 – 0.6075 |
| $\varepsilon$ | Functional Behavior Constant | 1 – 10 |
| Pop | Initial Population Vector | 500 – 12000 |
| $t$ | Time in Weeks | 520 ( 10 years) |

4.1 Low Growth Rate

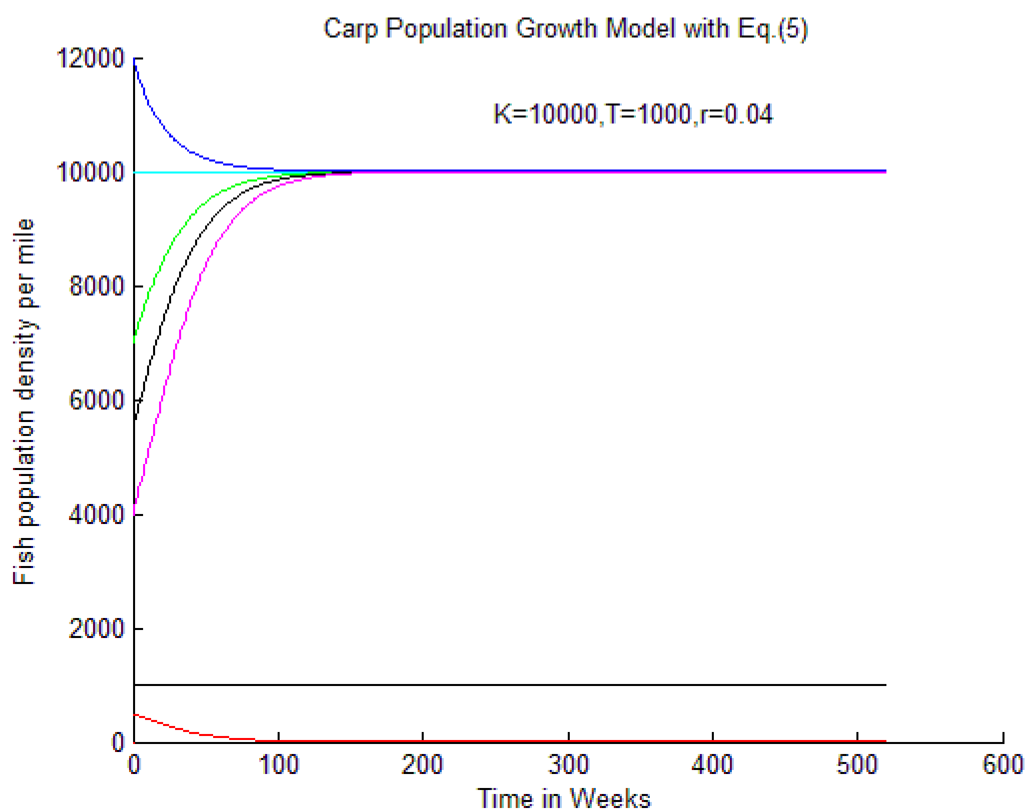


**Figure 4.1 Simulation of Eq.(5), with no human interaction**

In this simulation initial population vector = [12000 10000 7000 5500 4000 1000 500]’ has been used. As we can see the graph of the function with initial population of 500 dies out, the one with 1000 remains steady and all the others converge to 10000 which is the carrying capacity of the system. So we have two equilibrium state – one at 1000 and the other at 10000. We talk more about the stability after further simulations.

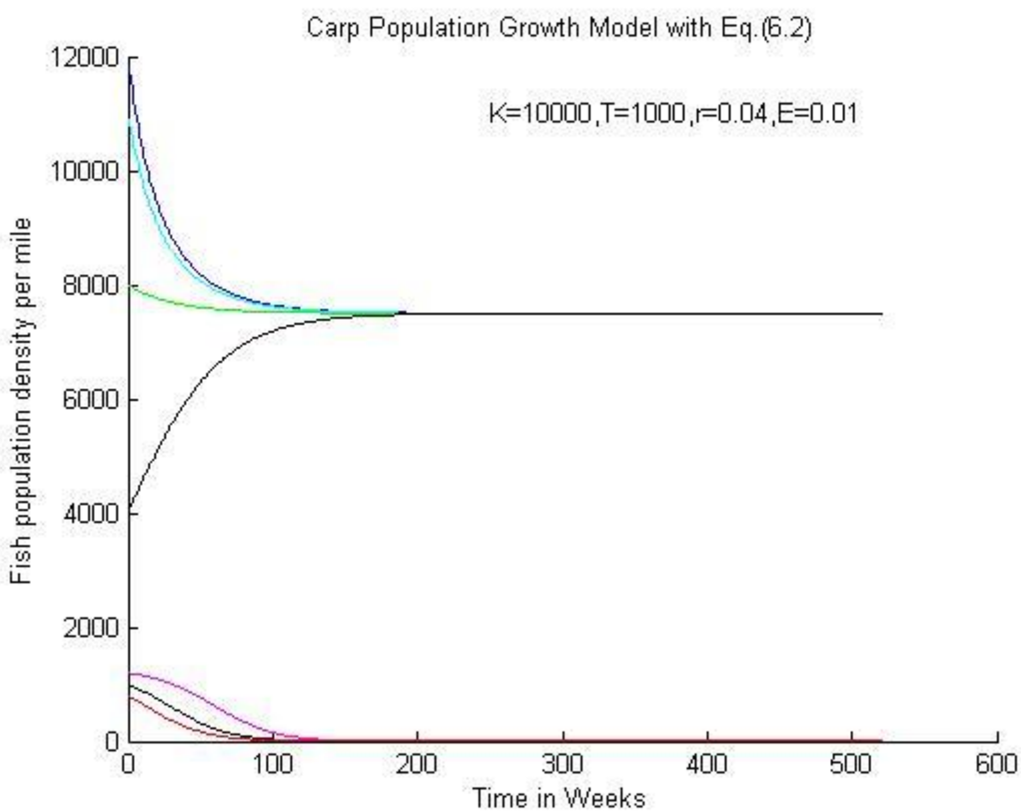


**Figure 4.2 Simulation of Eq.(6.2) with human effort E = 0.01**

In this simulation, we can see that the graph of the function with initial population of 1500 dies out, while others converge close to 8000.

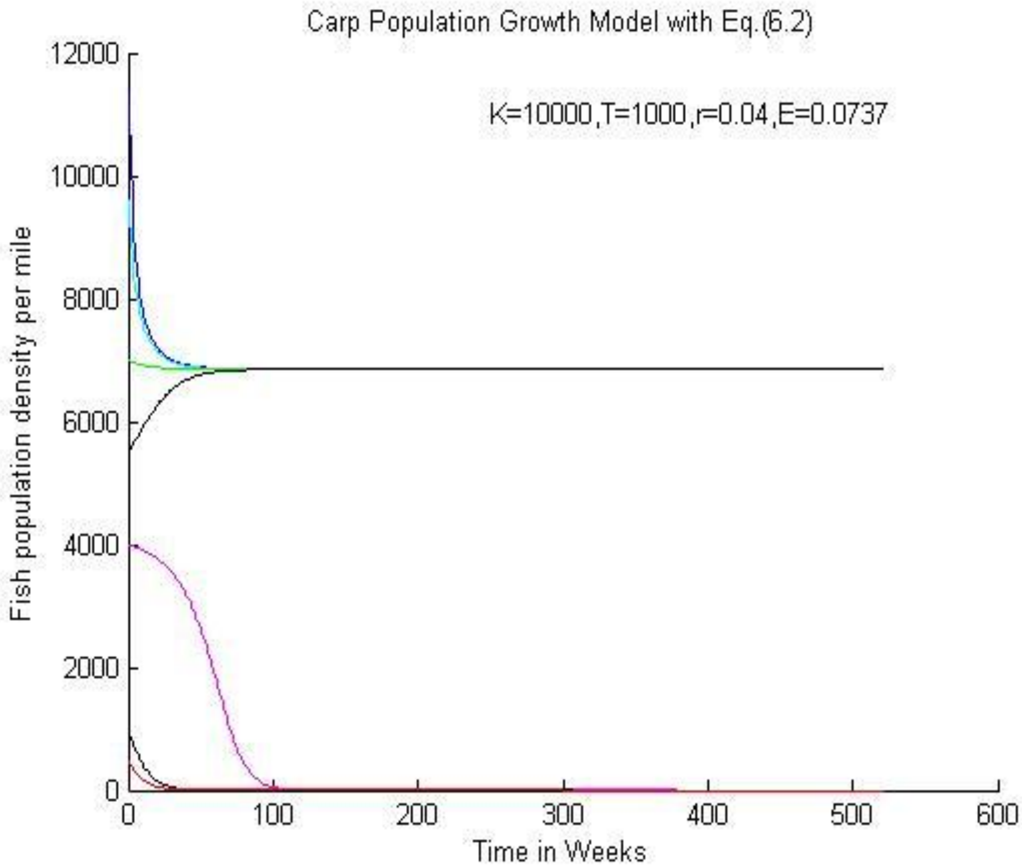


**Figure 4.3 Simulation of Eq.(6.2) with human effort E = 0.0737**

In this figure, we can see that the graph of the function with initial population of 4000 has died out as well. So, as we increase the fishing effort, fish with large initial population will be able to sustain its population. Furthermore, the equilibrium state decreases as we increase the fishing effort. The equilibrium state has decreased from approx. 8000 in previous figure to approx. 7000 in this figure.

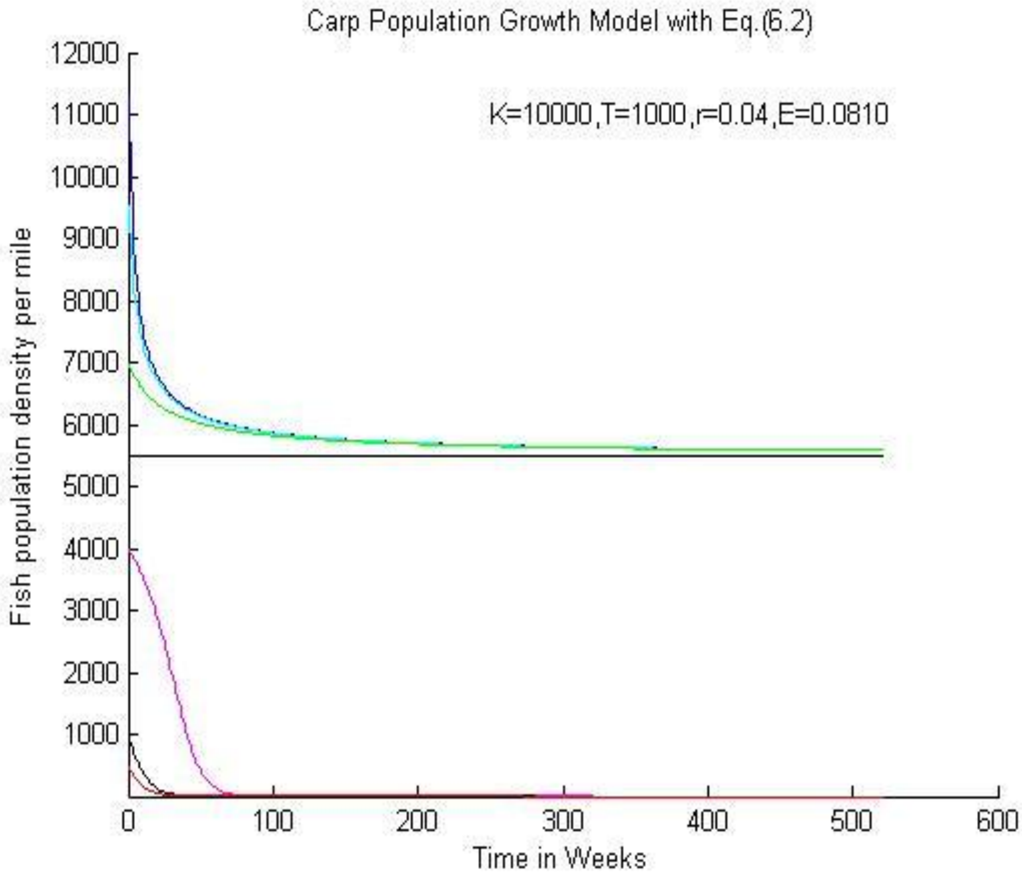


**Figure 4.4 Simulation of Eq.(6.2) with human effort E = 0.0810**

In this figure, the graph of the function with initial population of 12000, 10000 and 7000 converge to 5500 and then remain steady. The fishing effort $E = 0.0810$ was calculated using Eq.(14):

$$E_s = \frac{r\left(K^2 - 2KT + T^2\right)}{4KT}$$

which gives the maximum fishing rate that will still maintain sustainable fish population in the system. As we can see, this model is only stable from above and not from below.

## 4.2 High Growth Rate

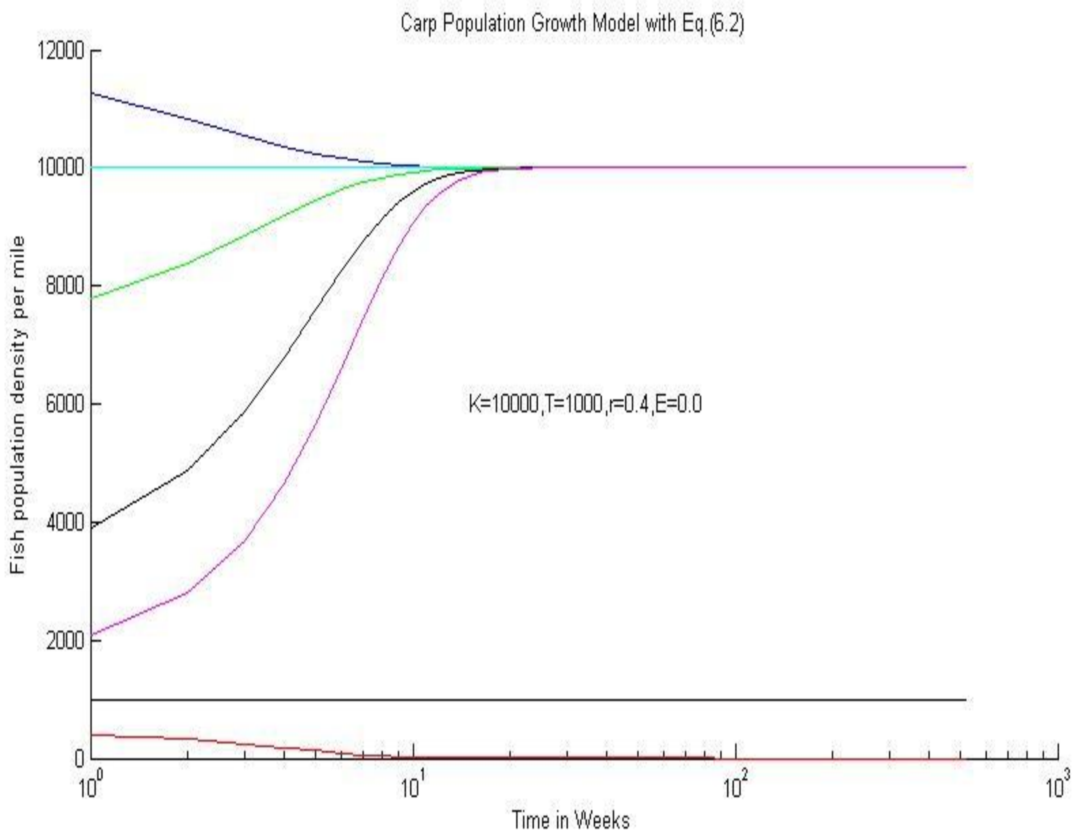


**Figure 4.5 Simulation of Eq.(6.2) with growth rate r = 0.4 and human effort E = 0.0**

In this figure, we can see that all the graph of the functions except the ones with initial population less than or equal to 1000, converge to 10000. It differs with Figure 4.1 in the sense that the convergence is very fast in this model because of the high growth rate constant. Hence logarithmic X-scale has been used to alleviate the steepness of the graph.

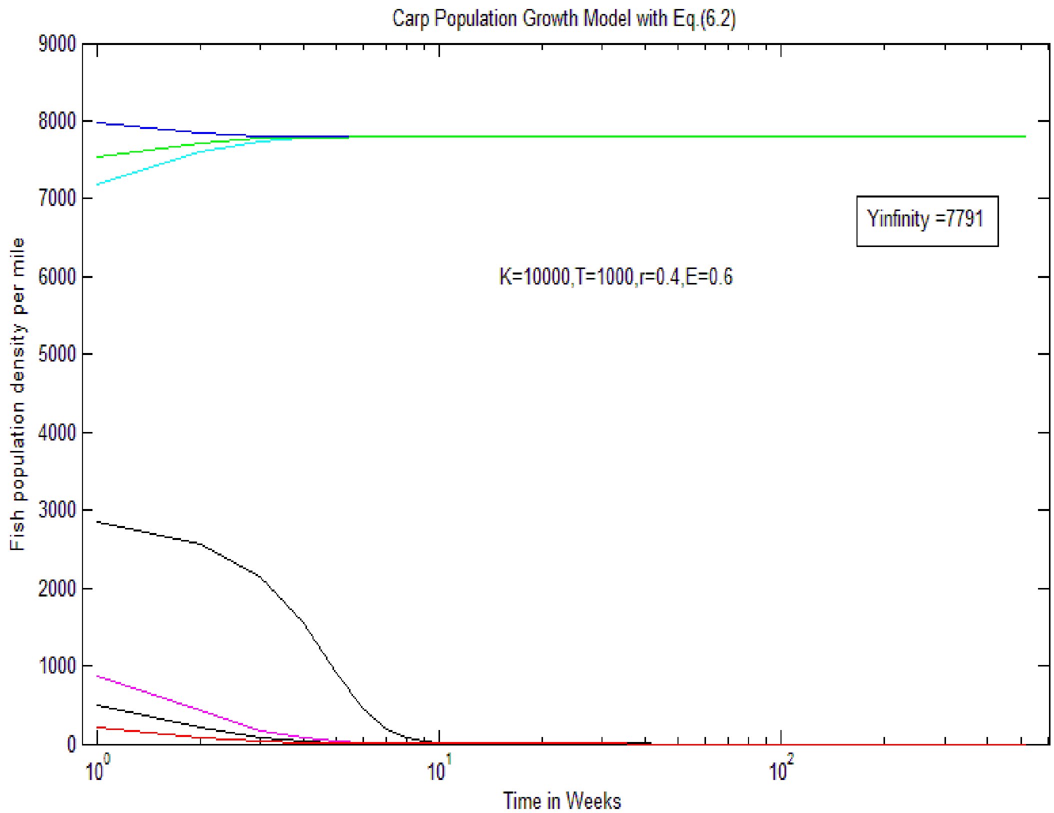


**Figure 4.6 Simulation of Eq.(6.2) with growth rate r = 0.4 and human effort E = 0.6**

This figure has the same characteristics as Figure 4.3. The graph of the functions with initial population of 4000 and less die out within the first 10 weeks and the others converge to 7791 in the first few weeks. This is because of the high growth rate used during the simulation. Now we decrease the growth rate constant to r = 0.3 and observe the behavior.

# 4.3 Realistic Growth Rate

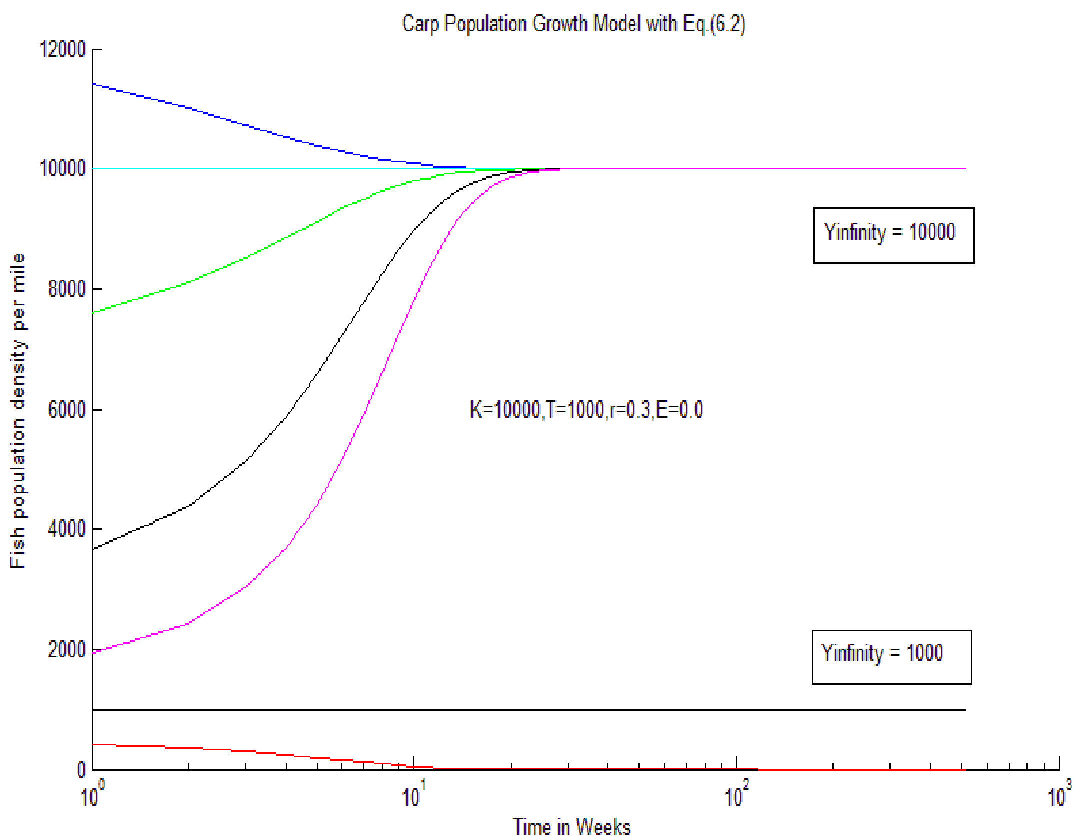


**Figure 4.7 Simulation of Eq.(6.2) with growth rate r = 0.3 and human effort E = 0.0**

In this figure, the graph of the functions with initial population greater than 1000 converge to 10000 within 20 to 30 weeks. This is more realistic than the previous model with r = 0.4.

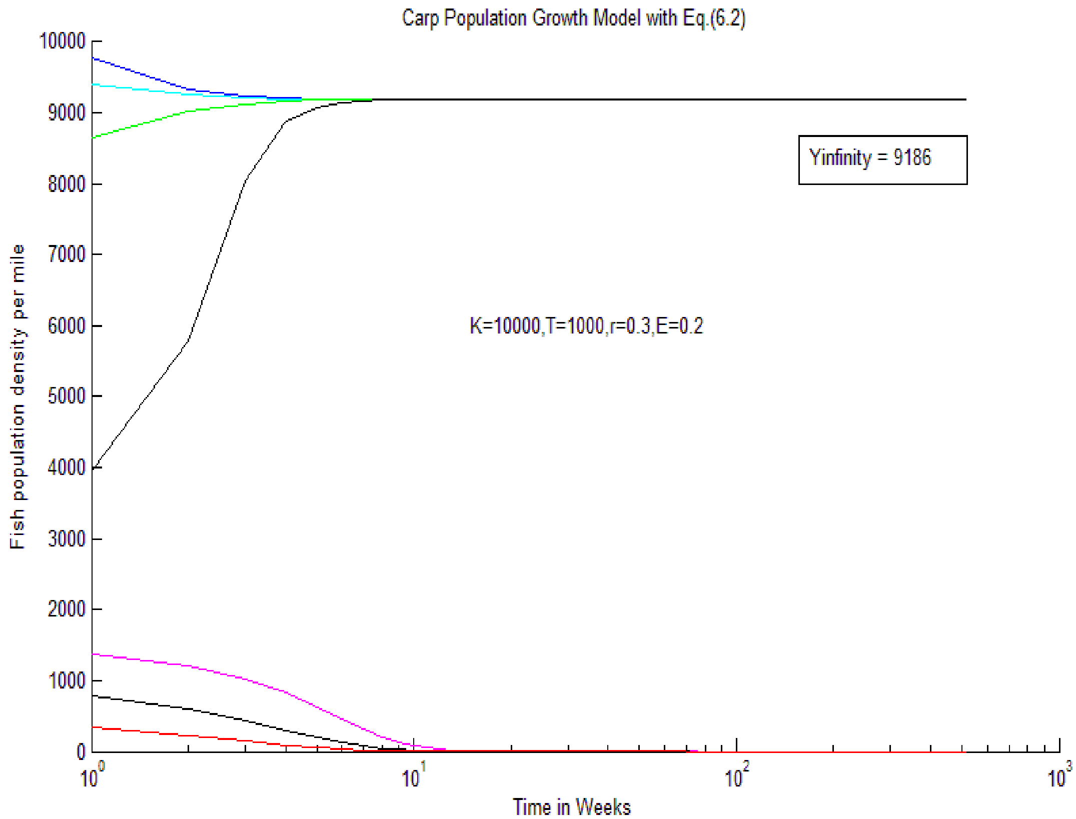


**Figure 4.8 Simulation of Eq.(6.2) with growth rate r = 0.3 and human effort E = 0.2**

In this figure, the graph of the function with initial population of 500, 1000 and 1500 die out while others converge to a steady population of 9186. The equilibrium state has dropped from 10000 to 9186 due to human effort. We can see that human fishing effort affects both $y_K$ and $y_T$ as given in Eq. (9.1 and 9.2).

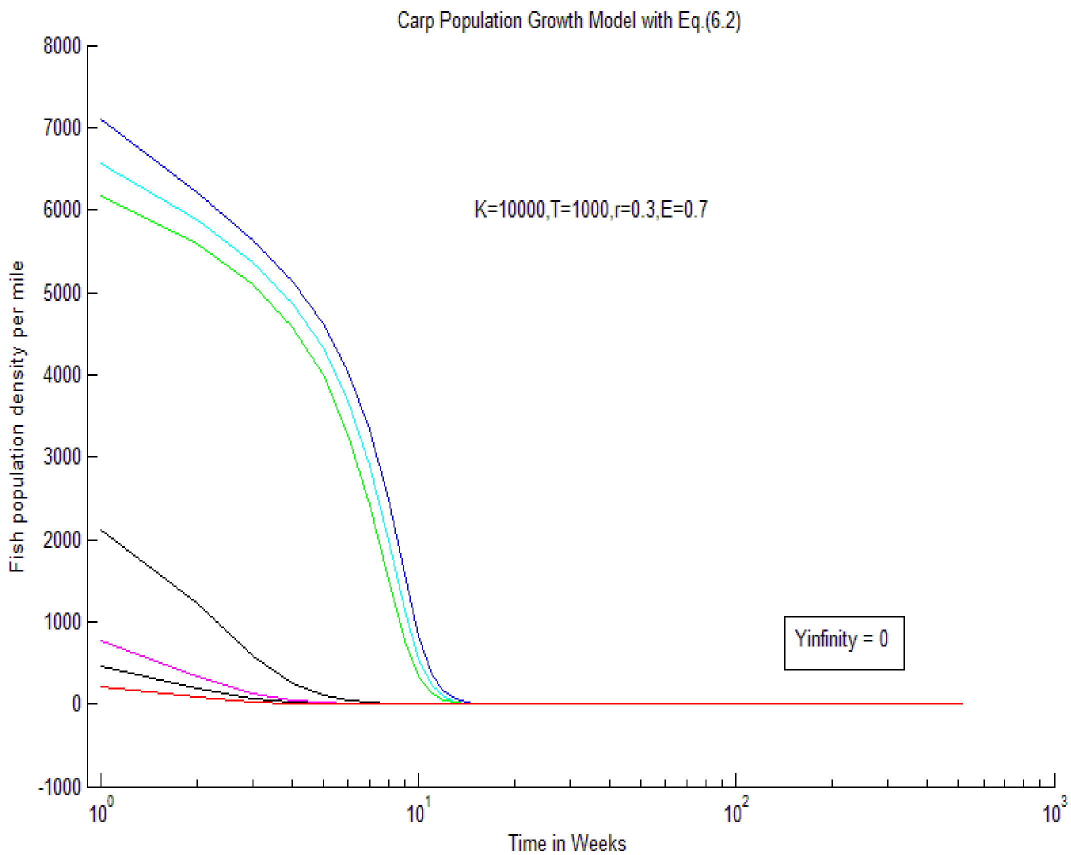


**Figure 4.9 Simulation of Eq.(6.2) with growth rate r = 0.3 and human effort E = 0.7**

In this figure, we drastically increase the human fishing effort to E = 0.7 from previous E=0.3 and we find that all the graphs of the functions die out. There is no state of equilibrium. Hence, this rate must be larger than both $E_m$ and $E_M$, the optimum fishing rate and maximum fishing rate respectively, that still sustains the ecosystem.

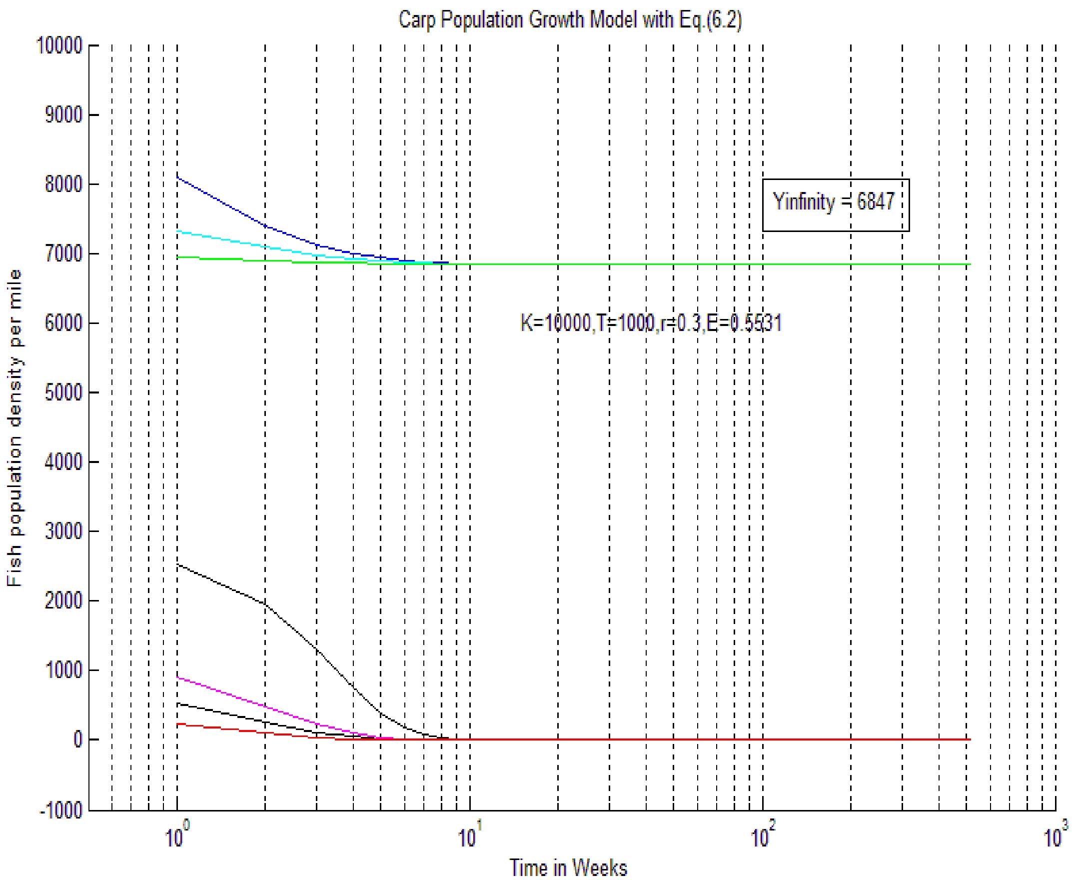


**Figure 4.10 Simulation of Eq.(6.2) with growth rate r = 0.3 and human effort E = 0.5531**

In this figure, the graph of the functions with initial population of 500,1000,1500 and 3000 die out but the others converge to a steady population of 6847. In this model, an initial population of 6847 or greater would achieve this stability and lesser would die out. Hence, this model is stable from above but unstable from below.

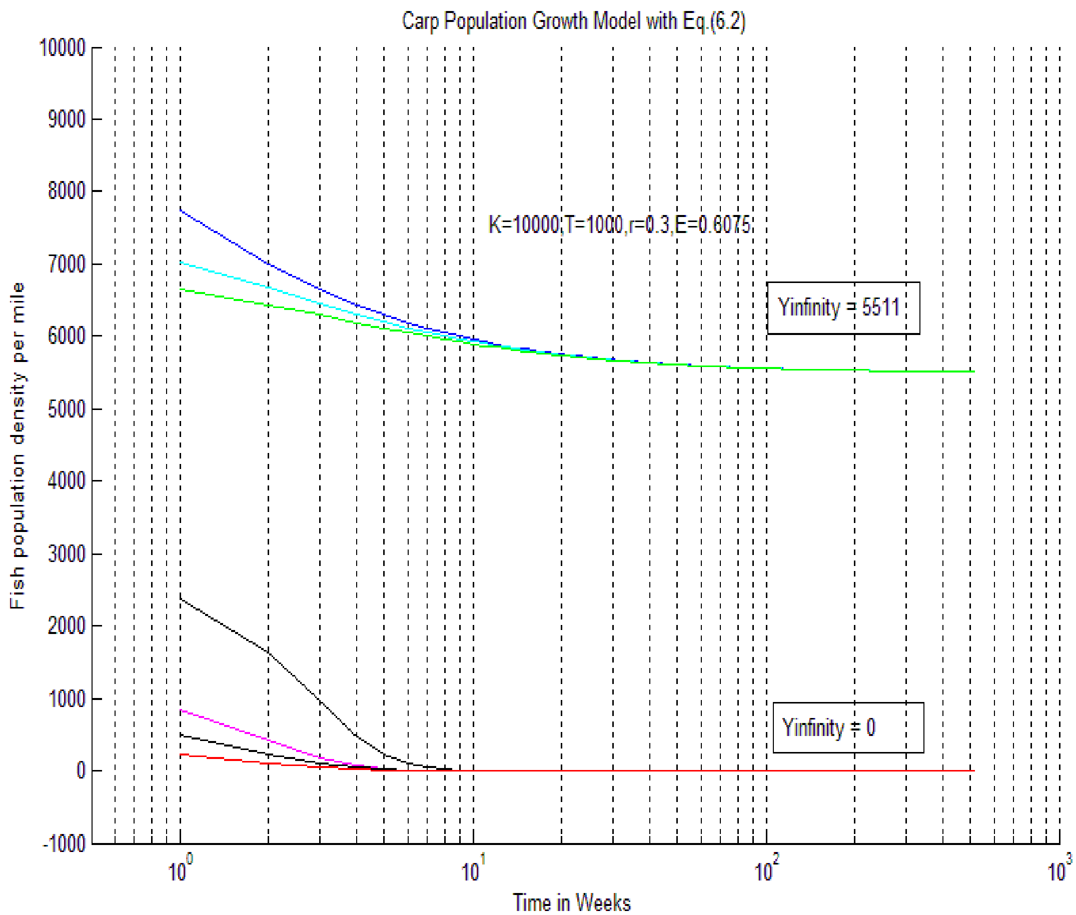


**Figure 4.11 Simulation of Eq.(6.2) with growth rate r = 0.3 and human effort E = 0.6075**

This model is stable from above since the graph of the function with initial population of 5511 or above converge to a steady population of 5511. The equilibrium state in this model is slightly lower than the previous model because of the increase in human effort.

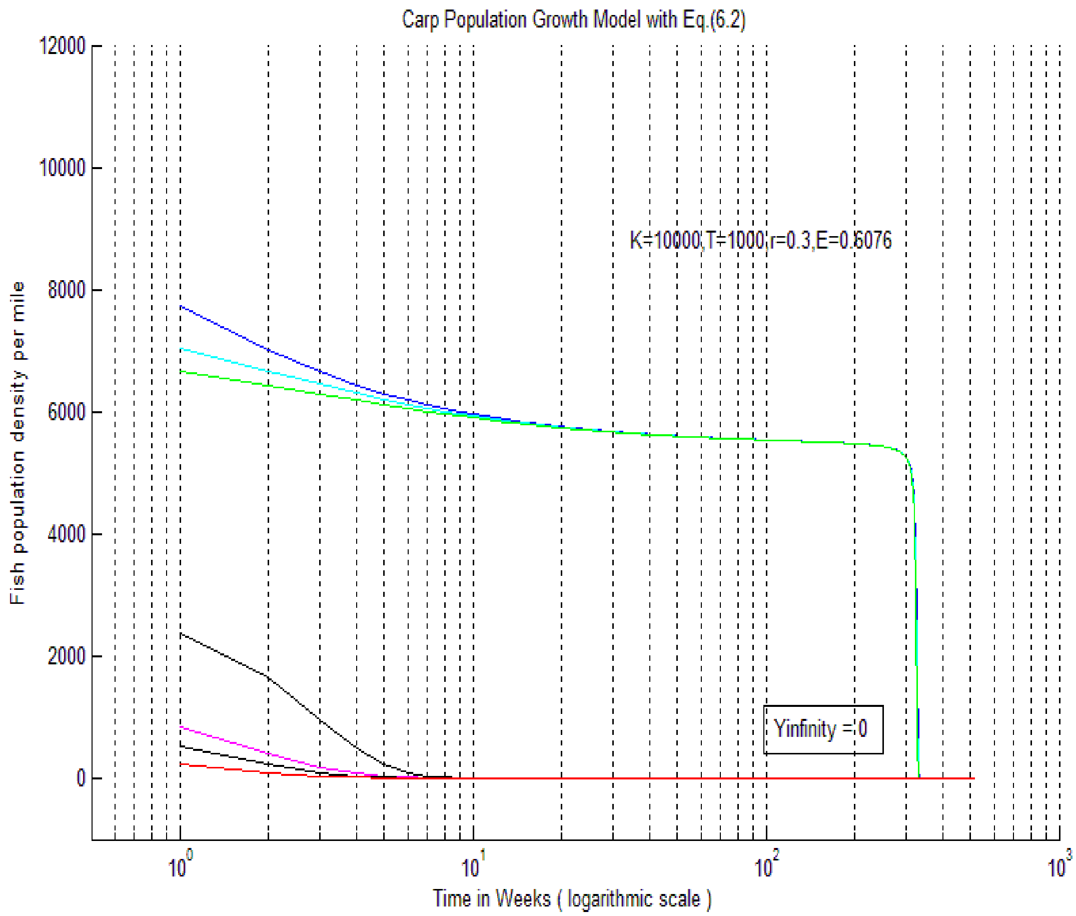


**Figure 4.12 Simulation of Eq.(6.2) with growth rate r = 0.3 and human effort E = 0.6076**

In this model, we increase the human fishing effort from E = 0.6075 to E = 0.6076 and we can see that there is an extended period of sustenance until 220 weeks, however, all the graphs eventually die out. Therefore, we know that $E_s$ = 0.6075 was the maximum human effort sustainable by the system.

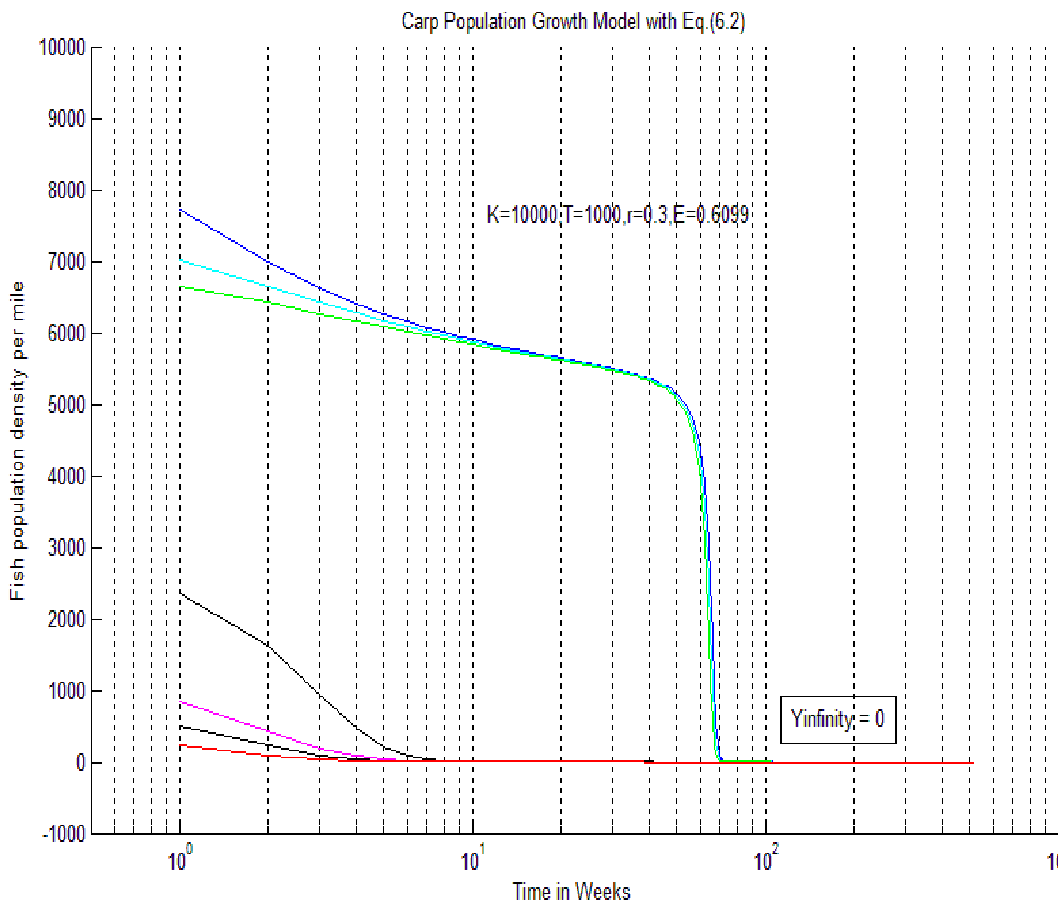


**Figure 4.13 Simulation of Eq.(6.2) with growth rate r = 0.3 and human effort E = 0.6099**

In this model, we increase the human fishing effort from E = 0.6066 to E = 0.6099 and we can see that though there is an extended period of sustainability until 140 weeks, all the graph of the function die out. We find that as we increase human fishing effort $E$ away from $E_s$, the sustainability of the system decreases.

# CHAPTER 5

## DISCUSSION AND CONCLUSIONS

In this study we have assumed Upper Mississippi River System to be linear. That means we assumed the width and depth of the river to be constant. It facilitated our study such that we can represent the population density of the bigheaded carp in terms of length of the river rather than area or volume. As you have noticed during the simulation, we denoted the carrying capacity of the river as 10000 fish/ mile and the threshold capacity as 1000 fish/mile. In doing so we can discretize the river into 50-100 mile segments and run the model separately with different parameter settings according to the population density and growth rate constant in that segment because Bigheaded carps are known to have drastically different growth rate constant depending upon region and environment. This also addresses the spawning requirement of the Bigheaded carps which requires at least a 30-mile river with a current of at least 0.8m/sec.

We started our fish population growth model without human interaction and we had two equilibrium state one at K and one at T. K was stable from above as well below as long as Pop > T while T was not stable. Slight perturbation of Pop away from T would result in divergence. For Pop not equal to T, the graph would either die out or converge to K. Then we incorporated human fishing effort into our model and we saw that both equilibrium state $Y_K$ and $Y_T$ are affected by it. The most interesting result was obtained when we took growth rate constant r = 0.3 and changed the human fishing effort from E = 0.0 to E = 0.7. We observed how realistic the behavior of the function was. We noticed how a slight increase in human effort from $E = 0.6075$ to $E = 0.6076$ would destabilize the system and declared $E = 0.6075$ as the maximum human fishing effort to allow sustainability of the system.

So we can implement different level of human effort to deal with the problem created by carp. In segments, where 95% of the biomass is constituted by carp, we can start $E > E_s$ for the first few months and when it reaches its extended period of stabilty, we can drop E between $E_m$ and $E_{s.}$ This coupled with introduction of a predator like Pike Perch who feed on adult carp and barrier implementation along every 20 miles of river to stop larva from freely floating will significantly reduce the population growth by decreasing the spawning potential of the Bigheaded carps.

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