WLAN BASED INDOOR LOCALIZATION

by

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ABSTRACT

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Localization based on received signal strength has been a topic of Interest due to its relative simplicity in measurement and minimal hardware requirements. Much of the research in this field has been on developing an indoor propagation model. A generic indoor propagation model has largely been unsuccessful due to the complex multipath environment present in indoor environments. The main focus of this study is to find the position of the target when the estimated distance from the measurement points are known. The straightforward approach to this problem is by using trilateration techniques. A novel weighted centroid based method is introduced and its performance is compared to the different trilateration techniques.
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CHAPTER 1
INTRODUCTION

1.1 Localization

The process of determining the position of an object in space is called localization. Location awareness is of prime importance in many commercial, public service and wireless networks. Localization is commonly used for intruder detection, blue force tracking, health care monitoring, asset tracking and emergency 911 services.

Localization can be considered as a two phase process. The initial phase is the measurement phase and the final phase is the location update phase [1]. During the measurement phase, the measuring units or the Access points (APs) measure the signal characteristics from the target or the Mobile station (MS). According to the type of signal characteristics used for position estimation, localization can be classified as Received Signal Strength (RSS), Time of Arrival (TOA), Time difference of Arrival (TDOA) and Phase of Arrival (POA) localization. In the measurement phase, the measurements are affected by noise, multipath, interference and other environmental factors. During the location update phase, the measurements are aggregated and used as inputs to the localization algorithm. Based on how these values are fed to the localization algorithm they can be further classified as follows:

a) Centralized (Remote Localization) and Disturbed (Self localization)

In the centralized algorithm, the position of all the targets or MS is determined by the central controller. The central processor gathers measurement from the APs as well as MS and computes the position of the target. The disadvantage of centralized algorithm is that they are usually not scalable and are impractical for large networks. In Distributed localization, there is no central controller and every agent
infers its own position based only on locally collected information. GPS is a classic example of this. Contrary to the Centralized algorithm, Distributed algorithms are scalable which makes them an attractive option for larger networks.

b) Absolute and Relative

Absolute Localization refers to the localization in a predetermined coordinate system. Relative localization refers to the localization based on the proximity to known reference points.

c) Non-cooperative and Cooperative

In a non-cooperative Localization, each access points will communicate directly with the central unit. There is no communication between the access points. The disadvantage of this method is that there should be large number of access points required for localization since some of the access points cannot directly communicated to the central unit. In cooperative localization the access points can communicate with each other thus not all access points need to communicate directly with the central unit. Figure 1 shows a cooperative localization scenario in which node2 cannot communicate with node5 and node4 cannot communicate with node1 directly. However since it is cooperative localization, node2 can communicate to node 5 through node 4 and node 4 can communicate to node1 through node2, thereby reducing the density of access nodes required for localization.

Figure 1.1 Cooperative Localization [1]
1.2 Need for Indoor Localization

Basic localization using scientific instruments have been around for a very long time. Astrolabe, sextant, compass and chronometer, are few of the commonly used instruments for localization[7][2]. The most popular instrument used for localization is the Global positioning system (GPS) which was developed by the U.S Department of defense in the 1970's. However it was not available for public use till the early 1980's. Lot of research has happened in this field since then and it has now reached a point where civilian GPS offers accuracy close to 10 meters. With the evolution of the cellular based localization concept or the E-911 in mobile communication, localization has gained more significance. The real challenge in this field has been to localize in indoor environment. In indoor environment, most of the above mentioned device either fails or do not have considerable accuracy. In the case of GPS, the signals are too weak to penetrate the buildings and in case of the E-911, the multipath environment in indoor gives a systematic overestimate of the distance between the base station and the user. In public safety and military applications, indoor localization are used to track inmates in prison and to navigate firefighters and soldiers to complete their mission inside buildings [2].

With the increased Wireless Local Area Network (WLAN) deployment in industrial environments, localization using WLAN has attracted much interest. Positioning based on 802.11 is an attractive solution due to its low cost setup and the availability of access points in the network[4]. With the advent of feature rich smart phones, reduction of device power consumption in localization systems have also become an active research area [5]. The chief difficulty in localization using WLAN is to predict the signals strength accurately. In indoor environment the non Gaussian noise resulting from the complex multipath effects makes the accurate prediction of RSS values very difficult. Other environmental factors such as building geometry, network traffic and presence of people also affect the RSS values[6]. Accuracies ranging from 3-30 meters have been achieved in the past for WLAN based localization. Bahl et al.[7][10] proposed an in building user location and tracking system-RADAR which uses the nearest neighbor technique. The RADAR system offers accuracy up to 3 meters with 50% probability. Horus system [8] used a probabilistic method for WLAN localization which gave an accuracy of 2.1 meter with 90% probability. Battiti et al. [9] proposed a location determination using neural network based classifier. An accuracy up to 3 meters was
achieved by this system. This thesis is based on signal strength based calibration free WLAN Localization.

1.3 Contribution of thesis

Most of the WLAN Localization is signal strength based and much of the research in this field has been to develop a generic indoor propagation model which fits all types of environments. This has largely been unsuccessful due to the complex multipath environment present in indoor environment. The main focus of the study is to estimate the position of the target (mobile station) given the estimated distance from the measurement points (access points or APs) and the position of the APs. The distance from the target to the access points are estimated using signal strength based approach. Different least square algorithms used for trilateration are studied and a novel weighted centroid based approach is proposed. The performance of these methods are compared using both numerical approach and using experimental data collected in various indoor environments. Therefore, the key contribution in this research is in improving the existing RSS based method for Indoor Localization.

1.4 Outline of thesis

In Chapter 2, an extensive overview of the existing Indoor localization techniques is studied. The first part of chapter 2 deals with different localization techniques based on measurement techniques. Different techniques such as RSS, TOA, TDOA and POA based techniques are studied. In the second part of chapter 2, a survey of the mathematical methods currently used in Indoor localization is studied.

Chapter 3 explains the robust dynamic propagation model method used for distance estimation. This method dynamically estimates the propagation model that best fits the environment using the RSS measurements obtained in real time.

Chapter 4 deals with the different position estimation techniques used. Different trilateration techniques such as linear, non linear and iterative reweighted least square algorithm is studied. A weighted centroid based method is proposed at the end of the chapter.
Chapter 5 deals with the simulation study and the experimental setup used for comparing the performance of the various position estimation techniques discussed in chapter 4. The experimental results are explained and a conclusion is derived from the study.
CHAPTER 2
LOCALIZATION – A BACKGROUND STUDY

An extensive survey of the different localization techniques and the mathematical algorithms applied to the localization problem is studied in this chapter. Localization systems are broadly classified according to the measuring principle applied to the localization problem. The advantages and disadvantages of each of these techniques are studied.

2.1 Types of Localization

According to the type of measurement used, localization techniques can be classified as Time Of Arrival (TOA), Time Difference Of Arrival (TDOA), Received Signal Strength (RSS), Phase Of Arrival (POA) and Angle Of Arrival (AOA)[10]. The following section discusses about these techniques.

2.1.1 Time of Arrival (TOA)

Time of Arrival is the time that the signal takes to reach the receiver. Figure 2.1 shows a typical time of arrival based system. Here the distance between the transmitter and receiver is estimated by multiplying the time delay between transmission and reception of signal with the propagation velocity. At least three measurements are required for estimating the position in 2D. The important requirements in TOA based estimation is that there should be time synchronization between the transmitter and receiver and also a time stamp should be labeled in the transmitting signal in order for the measuring unit to find the distance the signal has travelled. Different signaling techniques such as direct sequence spread spectrum (DSSS)[11] or Ultra wide band [12] are commonly used in TOA based measurements.

Additive noise and multipath effects are the major sources of error in TOA based estimation [2]. For a given bandwidth and signal to noise ratio (SNR), there is a lower bound on the achievable accuracy.
The CRB provides a lower bound on the variance of the TOA estimate in a multipath free channel. For a signal with bandwidth \(B\) where B is much lower than the center frequency \(F_c\) and when signal and noise power are constant over the signal bandwidth [13]. The CRB can be expressed as

\[
\text{var}(\text{TOA}) \geq \frac{1}{8\pi^2 BT S F_c^2 SNR} \quad (2.1)
\]

Where \(T_s\) is the signal duration time in seconds. Thus for sufficiently high SNR, the bound predicted by CRB in [2.1] can be achieved in multipath free channels. The error caused due to multipath is much greater than those caused by additive noise. Multipath signals arrive very soon after the Line of sight (LOS) signal and can cause false detection of peak which adversely affects the accuracy of the system. Multipath signals can also attenuate the LOS signals causing it to be lost in noise and missed completely. This can cause large positive error in TOA. In general TOA works well only in LOS conditions. Another disadvantage of TOA based method is that they require additional hardware and it is often energy consuming.
2.1.2 Time Difference of Arrival (TDOA)

In TDOA based methods, the position of the mobile transmitter is located using the difference in time at which signal arrives at multiple measuring units rather than the absolute time. For each TDOA based measurement, the transmitter will lie on a hyperboloid with constant range difference between two measuring units [10]. The equation of hyperbola can be expressed as

\[
R_{i,j} = \sqrt{(x_i - x)^2(y_i - y)^2 + (z_i - z)^2} - \sqrt{(x_j - x)^2(y_j - y)^2 + (z_j - z)^2} \quad (2.2)
\]

Where \((x_i, y_i, z_i)\) and \((x_j, y_j, z_j)\) represent the fixed receivers and \((x, y, z)\) represent the coordinates of the target. Eqn 2.2 can be solved using non linear regression or an easier solution is to linearise using Taylor series expansion and solve iteratively [14]. A minimum of two TDOA based measurements are required for position estimation in 2D. Figure 2.2 below shows the scenario where two hyperbolas are formed from measurements at three points \(A,B\) and \(C\). \(P\) is the target.

![Figure 2.2 TDOA based measurement [10]](image)

TDOA can be estimated using cross correlation techniques. If the transmitted signal is \(s(t)\) and the received signal at measuring unit \(i\) is \(x_i(t)\) then
Where \( n_i(t) \) is the noise and \( d_i \) is delay variable. Similarly the signal at measuring unit \( j \) is

\[
x_j(t) = s(t - d_j) + n_j(t)
\]

The cross correlation of these two signals is given by Eqn 2.5

\[
R_{x_i, x_j} = \frac{1}{T} \int_0^T x_i(t) x_j(t - \tau) dt
\]

The TDOA estimate is the value \( \tau \) that maximizes \( R_{x_i, x_j}(\tau) \). Like in TOA based estimation TDOA measurements requires time synchronization between the transmitter and receiver. They also requires additional hardware and the accuracy is greatly affected in NLOS situations.

2.1.3 Received Signal strength (RSS)

Received signal strength is the measure of the power of the signal at the receiver. In the previous two methods i.e. the TOA and TDOA, additional hardware is required for synchronization, however RSS method can be easily measured and requires no special hardware. This is one of the primary reason for using RSS based methods [15].

![Figure 2.3 RSSI vs. Distance-Logarithmic variation. [15]](image)
The RSS values can be translated to distance using Model based method or finger print based method. These methods are described below

a) Model based method : In this method, the signal attenuation or path loss from transmitter to receiver is taken into consideration. Using a theoretical or an empirical model, the RSS value is converted to distance. This is a very efficient method for outdoor localization since there are a number of accurate propagation models available due to the dominant LOS situation. Okumara, Hata, Walsh Ikegami are few of the commonly employed propagation models in Outdoor environment. However in indoor environment due to the multipath environment most of the propagation models are site specific. Several researchers have tried to develop generic propagation models with very little success. Once the distance are estimated form the RSS values, trilateration can be used for position estimation.

b) Finger print based method or signal pattern matching: The basic approach is to finger print each location in the space of interest with a vector of received signal (RSS measurements) of the various transmitters. A mobile device is then localized by matching the observed RSS readings against the database. An early system that used this system was the RADAR [7], which used a deterministic finger print for each location. Several other schemes have improved upon RADAR most notably Horus[16] which employs a stochastic approach of RSS map and uses a maximum likelihood approach. The biggest disadvantage of all these methods are that a considerable amount of manual effort is required to perform detailed measurements across the entire indoor space and to maintain the RF map over time.

2.1.4 Phase of Arrival (POA)

Phase of Arrival (POA) based method uses the phase difference to estimate the range [10].

![Figure 2.4 Positioning based on POA](image)

Figure 2.4 Positioning based on POA [10]
Figure 2.4 shows the transmitter and receiver arrangement for a POA based position estimation. Assuming that the transmitter placed at location A to D are transmitting a pure sinusoidal signal at same frequency and same offset. Each signal experience a finite delay when they reach the target. This delay can be expressed as \( \phi = \frac{2\pi fD}{c} \) where \( \phi \) denotes the phase difference, \( f \) the frequency and \( c \) is the speed of light. The only condition here is that the transmitted signal’s wavelength should be greater than the diagonal of the cubic building, i.e. \( 0 < \phi < 2\pi \). The range can be found using \( D_i = \frac{c\phi}{2\pi f} \). Once this is found, we can use any of the position algorithms used in TOA based estimation. Another method of position estimation involves the receiver measuring the phase difference between two signals transmitted by a pair of stations and using TDOA based position algorithms for position estimation. POA based method usually gives accurate position estimation only for LOS situations. Thus they are often used with TOA, TDOA or RSS based method to fine tune the position estimate.

2.1.5 Angle of Arrival (AOA)

In Angle of Arrival based ranging, the direction of each reference point with respect to sensor node in some reference frame is used to determine the position of the sensor nodes. The Angle at which the signal is received is used to estimate the distance at which the sender node is located, using simple geometric relationship. AOA is usually measured using directional antenna or antenna array.

![Figure 2.5 Positioning based on AOA [10]]
As shown in Figure 2.5 AOA method may use at least two known reference points (A, B) and two measured angles to derive the 2D location of the target P. The advantage of AOA based method is that only three measuring units are required for 3D position estimate and 2 measuring units for 2D. Also unlike the case of the POA, TOA or TDOA based measurements, there is no time synchronization required for the measuring units. The disadvantage for this method include relatively large and complex hardware requirements and location estimate degradation as the mobile target moves away from the measuring units. Also in case of Non Line of Sight (NLOS) situations, multipath reflections will lead to misleading directions. Thus the accuracy of the position estimate greatly suffers in NLOS conditions.

2.2 Mathematical algorithms in Indoor Localization

This section describes the mathematical method used for position estimation in indoor environments. These techniques can be grouped into four categories; geometry based, minimization of cost function, finger printing and Bayesian techniques [17]. A short mathematical background is given before explaining these techniques.

a) Mathematical Background

Consider a Mobile station (MS) in a normal indoor localization system which receives a set of signals from \( n \) base stations placed at known locations \( x_i \) \((i=1, 2...n)\) and obtains a set of measurements \( \{r_1, r_2,...r_n\} \). There is a linear relationship between these two variables as shown in Eqn 2.6 [14].

\[
r_i = f(x_i) + e
\]

(2.6)

where \( f \) is the function of the position of the base station \( (x_i) \) and \( e \) is the error in position estimate with a probability distribution function \( p_e(e) \). The functional dependency between \( f(x_i) \) and \( p_e(e) \) completely characterize the measurement process. The maximum likelihood estimate of the position of MS is the point that maximizes the probability

\[
\hat{x} = \arg\max \{ p(r / x) \}
\]

(2.7)
Theoretical bounds can be established for the ultimately attainable precision of any estimation methods by using the Fisher information matrix and the Cramer-Rao lower bound (CRLB). These bounds are computed in literature for most of the estimation techniques [18].

2.2.1 Geometry based methods

When the range, difference of range, or angle between the BS and MS are measured with relative small error, geometry based method can be used. Thus a set of simple algebraic equations can be evaluated to estimate the position. For example in case of TOA based estimation

\[ r_i^{TOA}(x) = \|x - x_i\| + e_i \]  \hspace{1cm} (2.8)

where \(e\) is assumed to be a zero-mean Gaussian distribution. In the case of TDOA and AOA based measurement a simple relationship can be obtained by geometrical consideration. For RSS based measurement, the equation can be obtained through a propagation model but the set of equations are non linear and in general is over determined and therefore do not have a closed form solution. Approximate closed form or iterative methods can be used for solving these equations. Another method is by linearizing Eqn 2.8 and the non linear terms can be grouped together into an additional variable. Thus the equation will be of the form

\[ R = AX \]  \hspace{1cm} (2.9)

or

\[
\begin{bmatrix}
    r_1^2 - \|x_1\|^2 \\
    \cdots \\
    r_n^2 - \|x_n\|^2
\end{bmatrix} =
\begin{bmatrix}
    x_1 & y_1 & 1 \\
    \cdots \\
    x_n & y_n & 1
\end{bmatrix}
\begin{bmatrix}
    -2x \\
    -2y \\
    \|x\|^2
\end{bmatrix},
\]  \hspace{1cm} (2.10)

A least square solution to Eqn 2.10 can be found by finding the pseudo inverse.

\[
\hat{x} = \left( A^T \Sigma_2^{-1} A \right)^{-1} A^T \Sigma_2^{-1} Z
\]  \hspace{1cm} (2.11)
Where $\sum_2$ the covariance matrix error of the set of measurements. This method gives a suboptimal solution since the third element along the X is not an independent variable. An iteratively reweighted least square (IRLS) method is used when the covariance matrix of the error is not known and assuming that we have many different measurements from different BS[19]. Another linearization technique involves subtracting the first of Eqn 2.10 from the remaining giving a linear set of equations of the form

$$R = \begin{bmatrix}
  r_2^2 - r_1^2 + \|x_1\|^2 - \|x_2\|^2 \\
  \vdots \\
  r_n^2 - r_1^2 + \|x_1\|^2 - \|x_n\|^2
\end{bmatrix},$$

(2.12)

$$A = \begin{bmatrix}
  -2(x_2 - x_1) & -2(y_2 - y_1) \\
  \vdots & \vdots \\
  -2(x_n - x_1) & -2(y_n - y_1)
\end{bmatrix} X = \begin{bmatrix}
  x \\
  y
\end{bmatrix}. $$

(2.13)

Since during this process of linearization some of the information is discarded, these equations do not give optimal solutions either. Other methods include casting the set of equations into a constrained minimization problem [20] or using multidimensional scaling [21] and subspace decomposition [22]. Equivalent closed form solution for localization based on TDOA[23][24] and AOA [25][26] also exist. Although these closed form solutions are convenient, they do not provide maximum likelihood position estimate and will be in general be biased, even when the error is unbiased Gaussian. Maximum likelihood position estimate can be obtained from non-linear equations by linearization using Taylor series expansion [27]. Taylor linearization works best for GPS positioning since the variation of pseudo ranges caused by the displacement of the user on the surface of the earth is small compared to the satellites.

For indoor cases, Taylor linearization is prone to convergence error or initial conditions. In case of the linear equation $r = h(x)$, we see that there is a functional dependence with the arrangement of base stations placed at position $\{x\}$. In determinate cases this causes problem to compute pseudo inverse in eq 2.11 or Jacobian in equation and thus the error in position estimate increases relative to the error.
in the estimation of the measured quantity. This ratio in literature is referred to as DOP[29] and have been extensively studied particularly for GPS. Thus the base stations must be deployed in such a way that the DOP is minimized.

Geometry based localization methods are computationally efficient however they are not well suited for detecting outlier data caused due to NLOS conditions. Hypothesis testing can be used for detection of NLOS conditions where null Hypothesis $H_0$ indicates LOS and alternative $H_1$ indicative of NLOS. An increase of variance of time records of measured ranges can also indicate NLOS [28]. Another technique is the residue measurement method. This method takes advantage of the redundancy and self consistency of set of measurement data. Here the difference between the empirical measurements $r$ and those computed back from the position estimate $(\hat{\mathbf{x}})$. Measurements with large residue would indicate NLOS situation and thus these observations can be excluded and the process repeated for improved estimation. Another technique uses a statistical method for outlier detection. Here multiple position estimate can be produced by exhaustively using small subsets of $n_{\text{min}}$ elements and a robust estimator like a median is used for final position estimation [30]. Here robustness is achieved at the expense of computational time since the number of independent position estimate grows quickly with the position estimate.

2.2.2 Minimization of the cost function

Another method used is the minimization of the cost function approach. This approach is derived from the idea of maximizing $p(r|x)$ which eventually would come down to minimizing the following cost function

$$ V(x) = \log p_{\mathbf{e}}(r - h(x)) \quad (2.14) $$

The advantage of this approach is that it is applicable to arbitrary pdfs and provides maximum likelihood position estimate [31]. In case of zero mean Gaussian noise the covariance matrix reduces to Eqn 2.15.

$$ V(x) = (r - h(x))^T \sum^{-1} (r - h(x)) \quad (2.15) $$
Commonly used methods for minimization are Gauss Newton and Levenberg Marquardt method. As in the case of Taylor linearization, convergence problem arise for bad initialization.

In the case of indoor localization the signal strength can usually be represented by the empirical relation.

$$ r_i^{RSS}(x, \alpha_i) = RSS_0 - 10\alpha_i \log \|x - x_i\| + e_i $$

(2.16)

Where $RSS_0$ represents the power transmitted by the BS and ‘n’ is the path loss exponent. Thus an improved position estimate can be produced by minimizing the cost function

$$ V(x, \alpha_1, \ldots, \alpha_n) = \sum_{i=1}^{m} \| r_i - r_i^{RSS}(x, \alpha_i) \| $$

(2.17)

Here the only constraint is that $m$ should be larger than the sum of number of base stations and coordinates of the position to be estimated. Some of the researches also include several other attenuating factors such as presence of the wall which gives a better estimate of the position. However as studied by some of the researchers[32] it is observed that complicating Eqn 2.16 does not give significant improvement in accuracy. Protection against NLOS outliers can be achieved by converting these equations into constrained minimization problems. This technique is found to be effective as is studied in [33].

As mentioned before, the biggest challenge in Indoor localization is the elimination of Outlier/NLOS conditions. In not too complex environments such as UWB based LPS systems, error ($p(e)$) appears as isolated instances, the methods mentioned in the last two sections are efficient. However in most of RF based Localization systems it is better to consider NLOS as part of the estimation problem and that the $p(e)$ can be as large as $h(x)$ itself. The next two methods can be used under those circumstances.
2.2.3 Finger print based method

Finger print based method is based on received signal strength. It basically consists of two phases, the calibration stage and the localization stage. During the calibration stage, an extensive set of measurements are taken by moving the MS over a sufficiently dense set of points \( x_j \) that covers the indoor environment. During the localization stage, the system reads a set of signals which is compared to the previously recorded data and the position that closely matches is chosen as estimate of the location. Machine learning techniques are used in finger print based localization. Since finger print based method makes no distinction between LOS and NLOS, they are inherently robust to NLOS situations.

The simplest choice for position estimate in finger print based method is the euclidean distance method; here the position estimate is the point \( x_j \) that minimizes the euclidean distance in the measurement space i.e.

\[
\hat{x} = \arg\min \left\{ z_j^2 \right\}, \text{ with } z_j^2 = \sum_{i=1}^{n} (r_i - \rho_i(x_j))^2
\] (2.18)

If we assume that the measured \( r_i \) follows Gaussian distribution then \( z_j^2 \) is distributed with \( n \) degrees of freedom, and it is central if \( x_j \) is the true position and non central for remaining points in the grid. Theoretically the non central parameter can be used for detection.

\[
\hat{x}_{MLE} = \arg\max \left\{ p(r_1, \ldots, r_n | x_j) \right\}
\] (2.19)

However when the variance of distribution is high it becomes difficult to detect between central and non central distribution. Thus for finer accuracy the grid spacing should be close to each other. The k-nearest neighbor is another method that is commonly employed. Here the position estimate is produced by averaging the position of the k position estimate with lower distance \( Z_j \) in the signal space.

Bayesian interference method can also be used in finger print based method, where the maximum likelihood position estimate is given as. By Bayes’ theorem Eqn 2.19 is equivalent to finding the position $x_i$ which maximizes

$$p(x_j | r_1, ..., r_n) = \frac{p(r_1, ..., r_n | x_j) p(x_j)}{p(r_1, ..., r_n)}$$ (2.20)

Assuming that all positions are equally likely and by using the conditional independence of measurement from all base stations, Eqn 2.20 can be modified as

$$p(r_1, ..., r_n | x_j) = p(r_1 | x_j) \cdot p(r_2 | x_j) \cdot ... \cdot p(r_n | x_j)$$ (2.21)

Also by assuming normal pdf for $p(r_j | x_j)$, the position estimate can be given as an area of confidence.

Other statistical learning techniques that are used in Finger print techniques are neural networks[30], decision trees[31] and support vector machines[32].

Finger print based method produce the most accurate position estimation. However its biggest limitation is its use in real time situations. The calibration phase is time consuming and the position estimate can be done in areas only where recorded data exists. In the event of modification of the indoor environment their accuracy is greatly affected.

2.2.4 Bayesian methods

Bayesian method of position estimation is an extension of Bayesian Interference method [35] mentioned in the previous section. In this method, the position of the MS at any time $t$ is modeled as a probability distribution $p(x_t / r_1, r_2, r_3, ..., r_t)$ based on set of all past measurement $r_1, r_2, r_3, ..., r_t$. The position estimation is a two stage process, the first stage is the prediction step and the second stage is called the correction step. In the prediction stage the position of the MS is estimated from previous data without actually taking the measurement.

$$p^-(x_t) = \int p(x_t | x_{t-1}) p(x_{t-1}) dx$$ (2.22)
$p(x_t|x_{t-1})$ is the motion model which is the estimate of the position of the user in the next time interval from the current estimated position. The velocity of the movement of the MS or the sensor readings of the MS can be used for finding the motion model. In the final step called the correction step, the error in position estimate is reduced by matching the computed estimate of the position with the set of sensor measurements collected in the time interval from $t-1$ to $t$.

Using Baye's rule the equation becomes

$$p(x_t) = a_t p(r_t|x_{t-1}) p^{-}(x_{t-1})$$  \hspace{1cm} (2.23)

where $a_t$ is the normalization constant, $p(x_{t-1})$ is the prior probability and $p(r / x)$ is the observation model. The observation model $p(r / x)$ is the probability of receiving measurement $r$ when the user is standing at position $x$. Like the calibration stage, in Finger print based method, the observation model is developed by extensive set of measurements obtained at different locations in the displacement area. The observation model is developed from histogram of empirical measurements which are then fitted with the best fit probability distribution function[34]. Eqns 2.22 and 2.23 are applied each time a new measurement is available to refine the current estimate of the position.

Bayesian methods are robust to NLOS situations as they can be included directly in the observation model.

$$p_e(e) = p_{LOS}N(0,\sigma_r^2) + (1 - p_{LOS}) p_e^{NLOS}(e)$$  \hspace{1cm} (2.24)

This is similar to the hypothesis testing mentioned before. Thus using the probability of occurrence of LOS and NLOS and pdfs of error in each instance, the effect of NLOS situations can be effectively mitigated[35]. Due to the iterative nature of the Bayesian methods, an improved position estimate is obtained by processing many imprecise measurements. Bayesian localization can also be used with measurements of different nature like RSS and TDOA by simple multiplication of their respective observation. There are many different implementation of the Bayesian localization method. The simplest
of them is the Kalman filter method in which the position error follows a normal distribution. An extension of Kalman filter method is the Multihypothesis tracking, in which several different hypothesis MS are considered. Some researchers [36] have also made use of Voronoi graph where the displacement area is topologically represented as nodes and edges. Another efficient implementation of the Bayesian technique is the particle filter method [37]. In particle filter method while estimating the position the focus in on area with higher position probability. Here, the probability distribution of the location is approximated by a set of points which are called the particles.

\[ p(x_t) \approx \left\{ \left( x_t^i, w_t^i \right) \right\}, i = 1, \ldots, n_p \quad (2.25) \]

Where \( x_t^i \) is the location of the \( i \)-th particle at time \( t \) and \( w_t^i \) is the positive weight given by the probability density at position \( x \). The two steps (Eqn 2.22, 2.23) are computed over these particles and at each stage, resampling is done so that more weightage is given to regions which have higher probability. Simple observation models can be developed for path loss models like Eqn 2.16.

More accurate models can be obtained with Gaussian processes (GP’s) [38]. GP’s are a non-parametric probabilistic approach assuming that the measurement follows a normal distribution.

\[ p(r_i | x) = N\left( \rho_{r_i}(x), \sigma^2_{r_i}(x) \right) \quad (2.26) \]

Where mean and variance are dependent on the position \( x \). Linear regression estimate for this distribution are produced from a training set of locations and measured signal strength \( \{x, r_j\} \). The advantage of this method is its ability to handle the non-linear functional dependency of signal strength and position using relatively simple mathematics and giving sensible estimate of \( p(r|x) \) even when the calibration data is unavailable. Gaussian process have been successfully used in Wi-Fi based Indoor and GSM based Outdoor localization [38][39] showing accuracy comparable to fingerprint based methods.
CHAPTER 3

INDOOR PROPAGATION MODELLING

As discussed in the previous chapters, the positioning method based on received signal strength (RSS) maps the RSS values to the position of the mobile station (MS) to be localized. Due to the complex multipath channel in indoor environment, the most challenging aspect is to model the Indoor channel using the signal strength values. Fingerprint based methods are the most accurate methods however an extensive set of measurements (calibration) are required in the environment of interest which makes it infeasible for real time applications. In this thesis, the dynamic propagation model method mentioned in [40] is used for estimating the distance from the MS to the AP. The following sections of this chapter explain this method in great detail.

3.1 Mathematical background

Consider an indoor localization environment in which there are \textit{M} access points (AP$_1$, AP$_2$, AP$_3$, \ldots AP$_M$) which measures the RSS values from the mobile station. Let $t_1$, $t_2$, \ldots $t_N$ be the $N$ time instants in which the access points (AP$_1$) record the RSS values ($P_{Ri}(t_j)$) measured at time $t_j$. In this study, the Mobile station (MS) and the Access points (AP) are assumed to be static. It is to be noted here that, the presence of moving objects between the MS and the AP can affect the value of RSS measurements [40].

In general the attenuation of a signal [41] through a mobile radio channel is caused by three nearly independent factors: (i) path loss, (ii) multipath fading and (iii) shadowing. Path loss factor or path loss exponent($n$) characterize the rate at which signal power decays as the distance $d$ from the mobile station increases. In free space the signal power is proportional to $d^{-2}$. A path loss exponent greater than 2 is observed when the signal is subjected to reflection and deflection from surrounding objects, such as floors, walls and foliage. Multipath fading or fast fading is the rapid fluctuation of the complex envelope of the received signal, caused by reception of multiple copies of a transmitted signal.
through multipath propagation. The amplitude distribution is often described by Rayleigh or Rician distribution\cite{42}, depending on whether there exists a dominant component among the multiple copies. When there is no distinguishable component, the real and imaginary components of the complex envelope of the received signal can be viewed as a sum of numerous small random variables. Thus both the real (R) and the imaginary (I) can be modeled as a Gaussian random variable. Since the in phase and quadrature phase component of a band pass signal are uncorrelated, R and I are also uncorrelated. Hence they are independent Gaussian variables. When R and I have zero means and common variance, the amplitude $\sqrt{R^2 + I^2}$ can be shown to be a Rayleigh variable. When R or I (or both) have non-zero mean, the resulting distribution is Rician. Shadowing also referred to as slow fading, represents a slow variation in the received signal strength, due to the obstacles in the propagation paths. Experimental observations reveal that the type of fading is log normal. Mathematical proof for this is also well explained in various literature\cite{43,44,45}.

Considering all these factors, the RSS values, \(Pr\) can be modeled by means of the following expression\cite{41}

$$Pr = \frac{G_T \cdot G_R}{4 \cdot \pi} \cdot P_T \cdot \frac{g^2 \cdot \gamma}{d^n}$$

(3.1)

Where \(Pr\) is the transmitted power, \(G_T\) and \(G_R\) are the transmitter and receiver gains, respectively. \(d\) is the distance between the emitter and the receiver gains respectively, \(n\) is the path loss exponent, and \(g\) and \(\gamma\) are the parameters that conform the Rayleigh/Rician and log-normal distributions, respectively. Averaging over time, the fast fading term is eliminated\cite{46,47}. Taking logarithm in (3.1) and following the mathematical derivation in\cite{48}, the RSSI values can be expressed by the following equation

$$Pr = \alpha - 10 \cdot n \cdot \log 10(d) + X$$

(3.2)
Where $X$ denotes zero mean Gaussian random variable caused by shadowing [49][50]. The term $\alpha$ is a constant which depends on several factors such as averaged fast and slow fading, gains $G_T$, $G_R$ and transmitted power $P_T$, therefore this value can be known beforehand.[41]. Equation (3.2) is a widely used expression and forms the basis of the popular propagation models such as Okumara-Hata, Egli model etc. There are two key assumptions while making use of equation (3.2) for localization. They are as follows

1. The $\alpha$ parameter remains constant when the antenna gains $G_T$, $G_R$ and transmitted power $P_T$ remain constant. This is often the case for IEEE 802.11 WLANs. Therefore the value of $\alpha$ can be found by measurements taken in a generic environment similar to the environment of interest.

2. Although the value of path loss exponent is environment specific, it is assumed that its value remains constant for a short period of time [41]

Thus if $P_{Ri}(t_i)$......$P_{Ri}(t_N)$ are the RSSI values obtained at the $i$th Access point in the time interval $t_1$ to $t_N$ such that the path loss exponent remains constant, then the equation (3.2) can be modified as

$$P_{Ri}(t_j) - [\alpha - 10.\log_{10}d_i(t_j)] \approx N(0, \sigma)$$

(3.3)

3.2 Maximum likelihood distance estimator

This section describes the Maximum likelihood estimate and the Cramer rao bound for the estimation of distance between the APs and the MS when using equation (3.2) and (3.3). At first, the case of a single RSS value is discussed and later the result is extended for the case of several RSS values.

a) Case of Single RSSI values

As shown before and experimentally validated in [40], the RSS values $P_{Ri}(t_i)$ obtained by MS at any time instant $t_j$ can be modeled as a Gaussian random variable that is
\[ P R_i(tj) \approx N(\alpha - 10.ni.log10di(tj), \sigma) \] (3.4)

The probability density function (pdf) of \( P_{R_i} \), conditioned on the distance \( d \) between the MS and the AP can be expressed as

\[
fd(P_{R_i}) = \frac{1}{\sigma_{ij}} \exp \left[ -\frac{(P_{R_i}(tj) - (\alpha - 10.ni.log10di(tj))^2}{2\sigma^2} \right]
\] (3.5)

Thus the maximum likelihood estimate of \( d \) is

\[
d = \text{arg max} \quad fd \quad (P_{R_i})
\] (3.6)

To find the maximum, the differential of the likelihood function is taken. Thus we have the following cost function

\[
\frac{\partial}{\partial d} \ln \left( fd \left( P_{R_i} \right) \right) = \left( \frac{P_{R_i}(tj) - (\alpha - 10.ni.log10(d))}{\sigma_i(tj)^2} \right) \cdot 10.ni.log10 e
\] (3.7)

For finding the maximum, the differential of the cost function is equated to zero. Therefore we have

\[
\frac{\partial}{\partial d} \ln \left( fd \left( P_{R_i} \right) \right) = \frac{P_{R_i}(tj) - (\alpha - 10.ni.log10(d))}{\sigma_i(tj)^2} \cdot 10.ni.log10 e = 0
\] (3.8)

Rearranging equation (3.8) we have

\[
\hat{d} = 10 \left( \frac{\alpha - P_{R_i}(tj)}{10.ni} \right)
\] (3.9)

Then the Fisher Information matrix is
\[ I_d = E_d \left[ \frac{\partial}{\partial d} \ln \left( f_d \left( P_{R_i} \right) \right) \right]^2 \]

\[ = \left( \frac{10n_i \log_{10} e}{\sigma_i(t_j)^4} \right)^2 \cdot d^2 \cdot E_d \left[ P_{R_i} \left( t_j \right) - (\alpha - 10n_i \log_{10}(d)) \right]^2 \]

\[ = \left( \frac{10n_i \log_{10} e}{\sigma_i(t_i) d} \right)^2 \] (3.10)

and since \( \text{Var}(\hat{d}) \geq \frac{1}{I_d} \), the Cramer Rao lower bound becomes

\[ \sqrt{\text{Var}(\hat{d})} \geq \frac{\sigma_i(t_i) d}{10n_i \log_{10} e} \] (3.11)

b) Case of several RSSI

Consider the case that MS obtains RSS values in time instants \( t_1, \ldots, t_N \), where \( t_N - t_1 \) a time interval in the order of few is seconds such that the distance and the environment between the MS and the AP do not change. The distance between the MS and the AP in this case can be estimated in the following way

Let \( P_{R_1(t_1)}, \ldots, P_{R_2(t_N)} \) represent the received power at time instants \( t_1, \ldots, t_N \). The joint pdfs of \( (P_{R_1(t_1)}, \ldots, P_{R_2(t_N)}) \) conditioned on distance \( d \) between the MS and the AP is

\[ f_d \left( P_{R_1(t_1)}, \ldots, P_{R_N(t_N)} \right) = \prod_{j=1}^{N} \frac{1}{\sigma_i(t_j)} \exp \left[ - \frac{(P_{R_j} - (\alpha - 10n_i \log_{10} d))^2}{2\sigma_i(t_j)^2} \right] \] (3.12)

Following the similar method as before
\[
\frac{\partial}{\partial \alpha} \ln \left( f_d \left( P_R \left( t_1 \right), \ldots, P_R \left( t_N \right) \right) \right) = \\
= \sum_{j=1}^{N} \left( P_R \left( t_j \right) - \left( \alpha - 10n_i \log_{10} d \right) \right) \cdot 10n_i \log_{10} e \cdot \sigma_i \left( t_j \right)^2 \cdot d
\]  

(3.13)

Since the time interval \( t_1, \ldots, t_N \) is short such that the distance between the MS and the AP remains constant, it is logical to assume that \( \sigma_i = \sigma_i(t_j) = \sigma_i(t_k), \text{ for all } j \neq k \), therefore

\[
\frac{\partial}{\partial \alpha} \ln \left( f_d \left( P_R \left( t_1 \right), \ldots, P_R \left( t_N \right) \right) \right) = 0 \Leftrightarrow \\
\Leftrightarrow \sum_{j=1}^{N} \left[ P_R \left( t_j \right) - \left( \alpha - 10n_i \log_{10} d \right) \right] = 0 \Leftrightarrow \\
\sum_{j=1}^{N} P_R \left( t_j \right) = \frac{\alpha - 10n_i \log_{10} d}{N}
\]

Thus the MLE of the distance is

\[
\hat{d} = 10^{\left( \alpha - \bar{P}_R \right)/10n_i}
\]

(3.15)

Following the steps similar to the previous section, the Fisher information and the CRLB can be found as follows

\[
I_d = E_d \left[ \frac{\partial}{\partial \alpha} \ln \left( f_d \left( P_R \left( t_1 \right), \ldots, P_R \left( t_N \right) \right) \right) \right]^2 = \\
= \left( \frac{10n_i \log_{10} e}{\sigma_i d^2} \right)^2 E_d \left[ \sum_{j=1}^{N} P_R \left( t_j \right) - \left( \alpha - 10n_i \log_{10} d \right) \right]^2 = \\
= \left( \frac{10n_i \log_{10} e}{\sigma_i d} \right)^2 \cdot N \sigma_i^2 = \left( \frac{10n_i \log_{10} e}{\sigma_i d} \right)^2 \cdot N
\]

(3.16)

\[
\sqrt{\text{var} \left( \hat{d} \right)} \geq \frac{\sigma_i \cdot d}{\sqrt{N} 10n_i \log_{10} e}
\]

(3.17)
3.3 Path loss exponent determination

To estimate the distance using (3.9) and (3.15) we need to know the values of $\alpha$ and the path loss exponent $n$. As mentioned before, the value of $\alpha$ can be found by taking a few measurements in the region of interest and can be used as a general value for a specific experimental setup. The challenge here is to estimate the path loss exponent accurately. A widely used simplification is to assume that all the path loss exponent that model propagation between MS and APs are equal [41]. However, this assumption is often an oversimplification, since the propagation environment in different directions are often different in indoor localization. Results in Chapter 5 also agree to this.

a) Compatibility of distance estimate

Using the Maximum likelihood estimate of the distance in equation (3.9) and (3.15) with values of $\alpha$ and $n$ we can find the estimate of the distance. This section describes a method to quantify the compatibility of the distance estimate. Using the compatibility of the estimate we can find the measure of goodness of distance estimate.

Let $\hat{d}_i$, $i=1...M$ be the $M$ estimate of the distance between the MS and M APs which are on known positions $(a_i, b_i)$, $i=1...M$. When all the estimated centers $(a_i, b_i)$ and radius $\hat{d}_i$ will intersect at single point as shown in the Figure 3.1

Figure 3.1 Error free distance estimates [40]
Thus we can measure the compatibility of a group of $M$ distance estimate by measuring the extent to which the $M$ circles, each having estimated distance as radius, cut each other on a single point. The compatibility is quantified by the method based on the radical axes of all the pairs of circles. The radical axis of two circles is the locus of the points at which tangents drawn to both circles have equal length or in otherwise, it is the locus of points having equal geometric power from both circles [51]. When the two circles are not concentric, the radical axis is a straight line perpendicular to the line connecting the centers of the circle with center closer to the center of the circle with smaller radius. When the two circles have equal radius, radical axis coincides with the perpendicular bisector of the line joining two circles. Figure 3.2 shows several examples of radical axis of circle pairs.

![Figure 3.2 Radical axes for three pairs of circles](image)

The geometric power of a point with respect to a circle is a real number that reflects the relative distance of the point from the circle. Thus the geometric power of a point with regard to a circle is zero if the point belongs to a circle, it is negative if it lies inside the circle and it is positive if the point is outside the circle. $M$ circles would cut each other on a single point if and only if the $M(M-1)/2$ radical axis obtained with different pairs of circles cut each other on a single point and this point belongs to all the $M$ circles. This intersection point will have zero geometric power with regard to all the circles.
Let

\[
A \begin{pmatrix} x \\ y \end{pmatrix} = b
\]

(3.18)

Represent the system of linear equation formed by the intersection of \( \frac{M(M-1)}{2} \) radical axes. Here \( A \) is a matrix with \( \frac{M(M-1)}{2} \) rows and two columns and \( b \) is a vector with \( \frac{M(M-1)}{2} \) element. Figure (3) shows the instance where four circles do not intersect at a point.

![Figure 3.3 Least square solution of six radical axes [40]](image)

When the radical axis cut each other on a single point, the system of linear equation in equation (3.18) has a unique solution. In general case the system of linear equation is an over determined and the least square solution is found. The least square solution is the point \( (\hat{x}, \hat{y}) \) that minimizes the sum of squared distance to the different radical axis.

In general the least square solution can be expressed by the following equation

\[
\begin{bmatrix} \hat{x} \\ \hat{y} \end{bmatrix} = (A^t A)^{-1} A^t b
\]

(3.19)
If the M circles cut each other on a single point then it should satisfy the following equation

\[(\hat{x} - a_i)^2 + (\hat{y} - b_i)^2 - \hat{d}_i^2 = 0, i = 1, \ldots, M\]  \hspace{1cm} (3.20)

More these values are different from zero the further the M circles would cut each other at a point. The compatibility is defined based on this concept. The compatibility of M estimates of distance \(\hat{d}_1, \hat{d}_2, \ldots, \hat{d}_M\) is defined as

\[C(\hat{d}_1, \hat{d}_2, \ldots, \hat{d}_M) = -\sum (\frac{(\hat{x} - a_i)^2 + (\hat{y} - b_i)^2}{\hat{d}_i^2} - 1)^2\]  \hspace{1cm} (3.21)

Here the function is weighted with square sum of radius of each circle to give more relevance to smaller circle and the minus sign indicate that the sum of square and compatibility are inversely related.

b) Least square optimization

During time interval \([t_1, t_N]\) using the known APs and RSSI values in eqn (3.21), the compatibility of distance estimate depends only on the path loss exponents \(n_1, n_2, \ldots, n_M\) used to estimate distances from RSSI values. Thus Equation 3.21 can be reformulated as

\[C(n_1, n_2, n_3, \ldots, n_M) = \sum_{j=1}^{M} C(\hat{d}_1, \hat{d}_2, \ldots, \hat{d}_M) = -\sum_{j=1}^{N} \sum_{i=1}^{M} \left(\frac{(\hat{x} - a_i)^2 + (\hat{y} - b_i)^2}{\hat{d}_i(t)^2} - 1\right)^2\]  \hspace{1cm} (3.22)

Thus we can estimate the path loss exponents \(n_1, n_2, \ldots, n_M\) as the path loss exponents that maximizes compatibility expressed in equation 22.
Thus the path loss exponent estimation problem is formulated into a nonlinear least square problem. This can be solved using robust technique like Levenberg–Marquardt algorithm [52][53].

c) Path loss exponent constraints

Compatibility of distance estimate is a necessary but not sufficient condition to achieve precision in distance estimate. Therefore in order to obtain optimum path loss exponents, instead of maximizing the compatibility in a global fashion, the path loss exponent belonging to a feasible set of solution is found

\[
\hat{n}_1 \hat{n}_2 ... \hat{n}_M = \arg \max_{(n_1,n_2,...,n_M)} C(n_1,n_2,n_3,...,n_m) = \arg \min_{(n_1,n_2,...,n_M)} \sum_{j=1}^{N} \sum_{i=1}^{M} \left( \frac{(\hat{x} - a_i)^2 + (\hat{y} - b_i)^2}{\hat{d}(t_i)^2} - 1 \right)^2
\]

(3.24)

To determine an appropriate feasible set, heuristic reasoning can be used. For example it is clear that the path loss exponent will lie between a maximum and a minimum value which can be known a priori. Extensive measurements have been taken in Indoor environments by various researches [20] and it is found that the path loss exponent will lie in the range of 1 to 4. Path loss exponent less than 2 in Indoor environment is due to the wave guide effect found in Indoor environment [54][55].

Therefore we have

\[ n_{\min} < n_i < n_{\max} \]

Other constraints can be added to make the feasible set more restrictive. Let \( \{ P_{Ri}(t_j) \} \) be the set of RSSI values obtained in time instants \( t_1, t_2 ... t_N \) for the M APs. Let \( AP_1, AP_2 ... AP_M \) be the M Access points sorted according to the RSSI values obtained in this time interval, then
\[ \overline{P}_{R1} \geq \overline{P}_{R2} \geq \ldots \ldots \overline{P}_{RM} \quad (3.25) \]

And let \( \hat{d}_1, \hat{d}_2, \ldots, \hat{d}_M \) be the corresponding distance estimate. In a usual homogenous deployment of APs it is reasonable to assume that the distance from the first AP, \( \hat{d}_1 \) is less than a constant, \( \hat{d}_1 \leq D1 \).

Also fraction of the known distance between AP\(_1\) and AP\(_M\) can also be used. If \( d_{1,M} \) is the distance between AP\(_1\) and AP\(_M\), then it follows that

\[ \hat{d}_M + \hat{d}_1 \leq d_{1,M} \Rightarrow \hat{d}_M \geq d_{1,M} - \hat{d}_1 \quad (3.26) \]

Here \( \hat{d}_1 \) is the minimum distance in the set \( \{ \hat{d}_1, \hat{d}_2, \ldots, \hat{d}_M \} \) which is the AP receiving the maximum average signal strength and \( \hat{d}_M \) is the maximum distance in the set \( \{ \hat{d}_1, \hat{d}_2, \ldots, \hat{d}_M \} \) which is the AP receiving the minimum average signal strength. (a) and (b) encloses these distance estimates.

Applying (3.9) and (3.15) to (a) and (b), the distance constraint can be translated to path loss exponent constraints. Therefore

\[ \hat{d}_1 \leq D1 \Leftrightarrow 10^{\frac{(\alpha - \overline{P}_{R1})}{10n1}} \leq D1 \Leftrightarrow n1 \geq \frac{\alpha - \overline{P}_{R1}}{10\log_{10}D1} \]

\[ d_{1,M} - \hat{d}_1 \leq 10^{\frac{(\alpha - \overline{P}_{RM})}{10nM}} \geq d_{1,M} - D1 \Leftrightarrow \]

\[ \Leftrightarrow nM \leq \frac{\alpha - \overline{P}_{RM}}{10\log_{10}(d_{1,M} - D1)} \quad (3.27) \]

Another type of constraint can be imposed by relation between different distances. Even though \( PR_i \geq PR_j \), \( i < j \), this fact does not imply that \( d_i \leq d_j \). However, if the difference in average signal strength is great enough then it is logical to assume that the distance difference exists. Therefore,

\[ \overline{P}_{R_i} - \overline{P}_{R_j} > C1 \Rightarrow \hat{d}_i \leq \hat{d}_j \quad (3.28) \]
The difference in distance depends on how great the difference in average signal strength is. Therefore, if \( P_{Ri} - P_{Rj} > C_1 \Rightarrow \hat{d}_i \leq K \hat{d}_j \) where \( K \) is a value which decreases when the difference between the RSS increases. In the case when \( ni = nj \) the equation becomes

\[
\hat{d}_i = 10^{\frac{(P_{Ri} - P_{Rj})}{10ni \hat{d}_j}} \quad (3.29)
\]

And therefore we can assume

\[
\hat{d}_i \leq 10^{\frac{(P_{Ri} - P_{Rj})}{10ni C_2 \hat{d}_j}} \quad (3.30)
\]

Where \( C_2 \) is a number slightly higher than 1 and describes the relevance of the difference in average signal strength in terms of distance difference.

As done before the distance constraint can be converted to path loss exponent constraint

\[
\hat{d}_i \leq 10^{\frac{(P_{Ri} - P_{Rj})}{10ni C_2 \hat{d}_j}} \Leftrightarrow \frac{\alpha - P_{Ri}}{10ni} \leq \frac{P_{Rj} - P_{Ri}}{10ni C^2} + \frac{\alpha - P_{Rj}}{10nj} \Rightarrow ni(\alpha - P_{Ri}) + nj\left(\frac{P_{Rj} - P_{Ri}}{C^2} - (\alpha - P_{Ri})\right) \geq 0
\]

\[(3.31)\]

Therefore a feasible set of solution \( \Lambda \) is
\[ \Lambda = \{ (n_1, \ldots, n_m) \} : \]
\[ n_{\min} \leq n_i \leq n_{\max}, \quad \frac{\alpha - \overline{P}_{Ri}}{10 \log 10D_i} \leq n_i \]
\[ n_M \leq \frac{\alpha - \overline{P}_{RM}}{10 \log 10(d_1, M - D_1)} \]
\[ 0 \leq n_i(\alpha - \overline{P}_{Ri}) + n_j\left(\frac{\overline{P}_{Rj} - \overline{P}_{Ri}}{C_2} - (\alpha - \overline{P}_{Ri})\right) \]
\[ \text{if } \overline{P}_{Ri} - \overline{P}_{Rj} > C_1 \]

(3.32)

\( \Lambda \) is a convex set and it is a polyhedral set, therefore the variants of the Levenberg Marquardt Algorithm [22] can be used for solving this. A good initial starting point for the algorithm is \( n_i = 2 \).

Thus the method described in this chapter can be used for estimating the distance between the MS and the APs. Once the distance is estimated, the position of MS can be found using various trilateration techniques. These techniques are described in the next chapter.
CHAPTER 4

POSITION ESTIMATION TECHNIQUES

This chapter deals with the mathematical methods used to calculate the position of the mobile station using the estimated distance from the access points and the position of the access points. The assumption used here is that the access points and the mobile station are at the same height. Three different trilateration techniques are discussed and the centroid based positioning technique used in [19] is also detailed. A weighted centroid algorithm is proposed at the end of the chapter.

4.1 Problem formulation

Let the position of mobile station (MS) be \( P(x, y) \) and let the \( n \) access points be located at \( T(x_i, y_i) \) where \( i = 1, 2, 3 \ldots n \) as shown in Figure 4.1.
The obvious approach to formulate the positioning problem is to treat the coordinates of the mobile station as the point of intersection of several circles whose centers are the location of the access points. Let \( r_i \) where \( i=1,2,3...n \), be the exact distance between the access points and the mobile station. Then the equation of any of these circles will be of the form

\[
(x - x_i)^2 + (y - y_i)^2 = r_i^2
\]  

(4.1)

The point of intersection of these \( n \) circles is obtained by letting \( i = 1,2,3...,n \) and solving the resulting nonlinear equations simultaneously. The solution technique is not feasible as it produces a non linear equation of high degree. Also since the equation is quadratic many case of signs also would have to be considered. Linearizing the system of equation geometrically converts the problem into one of finding the point of intersection of several lines. When the exact distances from three access points are available, the solution of linear system of equation is completely determined. There are two equations, two unknowns and exactly one solution. However when approximate distances are used, the position that is obtained by direct solution is no longer acceptable and the problem has to be solved using least square techniques [19][56][57]. Figure 3.1 and 3.3 shows the situation of exact and approximate distances respectively. In the following sections of this chapter, three statistical approaches to trilateration problem is discussed.

4.2 Linear least square method

This method involves linearizing the set of equations represented by (4.1). The \( j^{th} \) constraint is used as a linearizing tool. Adding and subtracting \( x \) and \( y \) in (4.1) yields

\[
(x - x_j + x_j - x_i)^2 + (y - y_j + y_j - y_i)^2 = r_i^2
\]

(4.2)

Expanding and reordering the terms gives a linear system of the form \( Ax = s \) of (n-1) equations.

where
\[ A = \begin{bmatrix} x_2 - x_1 & y_2 - y_1 \\ x_3 - x_1 & y_3 - y_1 \\ x_n - x_1 & y_n - y_1 \end{bmatrix} \]  

(4.3)

and

\[ \hat{x} = [x - x_1, y - y_1]^{-1} \]

\[ s_{ij} = \frac{1}{2} \left[ r_{ij}^2 - r_i^2 + d_{ij}^2 \right] \]

\[ s = \begin{bmatrix} s_{11} & s_{12} & \cdots & s_{1n} \\ s_{21} & s_{22} & \cdots & s_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ s_{n1} & s_{n2} & \cdots & s_{nn} \end{bmatrix}^{-1} \]

\[ d_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \]  

(4.4)

The system described above has \((n-1)\) equations with two unknowns. Therefore only three APs are needed to determine the position of MS. The coordinates of the position obtained by applying the linear least square method to the linear system derived above are more accurate than the coordinates obtained by solving the three equations from the linearized system directly. Since the measured distances are only approximate, the problem requires determination of \(\hat{x}\) such that \(Ax \approx \hat{s}\). Minimizing the square of residuals

\[ \hat{r}^T \hat{r} = (s - Ax)^T (s - Ax) \]  

(4.5)

This leads to standard equation

\[ A^T Ax = A^T s \]  

(4.6)

If \(A^T A\) is non–singular and well conditioned then:

\[ x = (A^T A)^{-1} A^T s \]  

(4.7)
If $A^T A$ is singular, then the normalized QR decomposition of $A$ is generally used. In this method $A = QR$ where $Q$ is an orthonormal matrix and $R$ is an upper triangular matrix. The solution for $\hat{x}$ in the normalized QR -decomposition is found using

$$R\hat{x} = Q^T \hat{x}$$ (4.8)

and then by back substitution when $A$ is full rank. If the matrix $A^T A$ is close to singular then QR decomposition can be used else singular value decomposition (SVD) can be used to solve the linear least square method. However when the variances of the $ri$ are not identical, this estimator is not necessarily optimal [19]. In principal, one can attempt to improve on the performance of the above technique using the iteratively least square technique in which variance of the $ri$ are estimated in an iterative fashion.

4.3 Iteratively reweighted least square method

When the variances of $ri$ are unequal, the ordinary least square method can be improved upon. Iterative least square method is the best linear unbiased estimator in this case. Iterative method is used to find the point of least error [58]. Let the reference point and the corresponding distance be denoted by $(x, y)$ and $ri$ respectively. A trivial initial estimate is considered $(x_e, y_e)$.

The error in the estimated distance is calculated as

$$|f_i| = |r_i - \sqrt{(x_i - x_e)^2 + (y_i - y_e)^2}|$$ (4.9)

Applying the first degree of Taylor series approximation the adjustment $(\Delta x, \Delta y)$ used in the iteration can be found using

$$\Delta = (B^T \Sigma^{-1} B)^{-1} B^T \Sigma^{-1} f$$ (4.10)

Where the variances of $ri$ compose the element of the diagonal matrix $\Sigma$ and
where $n$ is a small fraction depending on the number of iterations. The iteration is continued unless the error converges.

4.4 Non linear least square method

When approximate distances are involved non linear least square method gives the most accurate estimate of position[19] [56]. This method is restricted to the case where the mobile station is inside the perimeter of the access points and below the common plane of the access points. The accuracy of this technique decreases as the elevation of mobile station increases and the mobile station moves outside the perimeter of the access points. In this method the sum of the square of the error in distance estimate is minimized using least square method. This problem is formulated as follows

$$ F(x, y) = \sum_{i=1}^{n} (r_i - f_i)^2 = \sum_{i=1}^{n} f_i(x, y)^2 $$  

\[4.12\]
Where

\[
f_i(x,y) = \bar{r}_i - r_i = \sqrt{(x-x_i)^2 + (y-y_i)^2} - r_i
\]  \hspace{1cm} (4.13)

\(\bar{r}_i\) is the exact distance. Different iterative techniques are used to minimize the function in \(f(x,y)\). The most commonly used technique is Newton ralphson method and is used for finding the optimal solution \(p(x,y)\).

A good initial starting point for the iterative method would be to use the result of the linear least square method\[7\]. Differentiating (4.13) with respect to \(x\) and \(y\) gives

\[
\frac{\partial F}{\partial x} = 2 \sum_{i=1}^{n} f_i \frac{\partial f_i}{\partial x}
\]

\[
\frac{\partial F}{\partial y} = 2 \sum_{i=1}^{n} f_i \frac{\partial f_i}{\partial y}
\]

(4.14)

Introducing the vectors \(\hat{h}\) and \(\hat{f}\)

\[
\hat{h} = 2J^T \hat{f}
\]

\[
f = [f_1 f_2 f_n]^{-1}
\]

\[
\hat{r} = \left[ \frac{\partial F}{\partial x} \frac{\partial F}{\partial y} \right]^{-1}
\]

(4.15)

Where \(J\) is the Jacobian of the matrix

Using the newtons method, the estimated position \(\hat{p}\) can be found iteratively using

\[
\hat{p}_{k+1} = \hat{p}_k - \left(J_k^T J_k\right)^{-1} J_k^T \hat{f}_k
\]  \hspace{1cm} (4.16)

Where
\[
J^T f_k = \left[ \sum_{i=1}^{n} \frac{(x - x_i)f_i}{f_i + r_i} \sum_{i=1}^{n} \frac{(y - y_i)f_i}{f_i + r_i} \right]^{-1}
\]  

(4.17)

\( \hat{P}_k \) is the \( k \)th approximate solution

This technique works faster especially when the matrix \( J^T J \) is augmented by a diagonal matrix which effectively biases the search direction towards that of steepest descent

4.5 Weighted centroid method

This technique is a novel weighted centroid based approach. The first step involves finding the area of intersection of the circles drawn with distance between Access points and Mobile station as radius and each Access points as center. This step is similar to the Centroid based method used in [59]. Figure 4.2 shows this scenario.

![Figure 4.2 Centroid method [59]](image)

The three access points are at \( A(xa, ya), B(xb, yb), C(xc, yc) \) and \( ra, rb, rc \) is the distance between each Access point and Mobile station. The first step is estimating the points E, F and G.

E is the point that satisfies the following equation
\[ (xe - xa)^2 + (ye - ya)^2 < ra^2 \]
\[ (xe - xb)^2 + (ye - yb)^2 = rb^2 \]
\[ (xe - xc)^2 + (ye - yc)^2 = rc^2 \] (4.18)

Similar method can be used for finding points F and G. Once the points E, F, G are found, in the centroid method mentioned in [59] the position of Mobile station can be estimated using

\[
x_m = \frac{(xe + xf + xg)}{3}
\]
\[
y_m = \frac{(ye + yf + yg)}{3}
\] (4.19)

However in the proposed weighted centroid algorithm, a weighting centroid is used for estimating the Mobile station. Since the variance of the range estimate is proportional to the actual range for lognormal model [60]. The access points that are closer to the Mobile station will have smaller error. Thus the points are weighted according to the estimated distance.

\[
x_m = \frac{(X_e \times w_{1} + X_f \times w_{2} + X_g \times w_{3})}{(w_{1} + w_{2} + w_{3})}
\]
\[
y_m = \frac{(Y_e \times w_{1} + Y_f \times w_{2} + Y_g \times w_{3})}{(w_{1} + w_{2} + w_{3})}
\] (4.20)

where w1, w2 and w3 are the reciprocal of range estimate of Access points AP1, AP2 and AP3 respectively. i.e.

\[
w_1 = \frac{1}{\hat{d}_a^2}; \\
w_2 = \frac{1}{\hat{d}_b^2}; \\
w_3 = \frac{1}{\hat{d}_c^2};\]
The performance of each of these techniques is compared using a simulation approach and experimentally in the next chapter.
5.1 Simulation setup

This section presents the simulation setup for the performance comparison of the various position estimation techniques mentioned in Chapter 4. Figure 5.1 shows the simulation setup.

The path loss exponent and the distance from the MS to the AP are estimated using the method proposed by Santiago Mazuelas et al. in [40] which was explained in detail in Chapter 3. For the simulation, the Access points (AP) are placed on the circumference of a circle of radius 40 meters as
shown in the Figure 5.1. The Mobile station (MS) can take any random position inside the circumference of the circle. A group of 1000 RSSI values are generated for each of the six access points using the Eqn (3.2) with uniform path loss exponents. Specifically, \( n_1, n_2 \in U(1.3,1.7), n_3 \in U(1.3,2.25) \)
\( n_4, n_5 \in U(1.3,3.25), n_6 \in U(1.3,4.25) \) where \((n_1, n_2, n_3, n_4, n_5, n_6)\) are the six different path loss exponents that characterize the propagation channel from the MS to the APs. The standard deviation of shadow fading was simulated as a random value between 2.85 dBm and 3.45 dBm. The simulation is repeated 1000 times randomly varying the position of the MS inside the circle.

5.1.1 Simulation results

Figure 5.2 gives the error in the estimated pathloss exponent. The path loss exponent estimate gave a mean error of 0.2013 and standard deviation of 0.14. Low pathloss error indicates the suitability of the dynamic propagation model for distance estimation. The distance between the MS and APs are estimated using the Eqn 3.15. The position of Mobile station can be estimated using the position estimation techniques described in Chapter 4. Figure 5.3 compares the performance of the different position estimation techniques. Cumulative probability function of the error is plotted for these techniques. Simulation results shows that the Weighted centroid method outperforms all other position estimation techniques.
Figure 5.2 Histogram of pathloss exponent error
Figure 5.3 Simulation results for the performance of the various position estimation techniques (Linear Trilateration (Normal), Iterative reweighted least square trilateration (IRWL), Weighted Centroid method (WCL), Centroid method (CL), Nonlinear Trilateration (Nonlinear).
5.2 Hardware Overview

The experiment was conducted in the Atrium at Nedderman Hall and the Hallway at Engineering Lab building at the University of Texas at Arlington. The experiment was done using a National Instruments PXI which was interfaced to a computer through a Labview program. The following section gives a brief overview of the PXI system.

5.2.1 National Instruments PXI

The framework of NI software/hardware used for the experiment is depicted in Figure 5.4.

A brief overview of each of these devices is described below.

![Block diagram of National Instruments PXI](image)

Figure 5.4 Block diagram of National Instruments PXI[61]

a) Hardware overview of PXI

PXI systems are composed of three basic components — chassis, system controller, and peripheral modules [61].
b) PXI Chassis

The Chassis provide the rugged and the modular packaging for the system. Chassis generally are available in 4, 6, 8, and 16 slots. These slots are for accommodating the peripheral modules.

Figure 5.5 Front view of National Instruments PXI

The PXI system used for the experiment has six slots. One slot is used for the system controller. Four slots are used for the peripheral devices which are the transmitter and the receiver unit.

c) System Controller

The system controller enables the device to be controlled from a desktop using the National Instruments Lab VIEW interface.
d) Peripheral devices - Transmitter unit

The transmit unit consists of two units— an Up converter (NI PXI 5610) and an Arbitrary waveform generator (PXI-5441.) When used together the unit is called the PXI-5670 Signal Generator. The arbitrary waveform generator (ARB) runs at a maximum sampling rate of 100M Samples/s. When used in conjunction with the Up converter, the Arbitrary waveform generator takes the discrete I-Q waveform created in Lab VIEW and then creates a continuous waveform at an IF of 25MHz which it sends to the Up converter. The Up converter then creates and transmits the desired RF signal. The Up converter can transmit at a maximum frequency of 2.7 GHz.

e) Peripheral devices - Receiver unit

The receiver also consists of two units. The first component is the down converter (PXI-5600) and the second is the high speed digitizer (PXI-5142.) The down converter also has a maximum frequency of 2.7 GHz and it down converts the received signal to an IF of 15 MHz It can receive a maximum bandwidth of 20 MHz. The digitizer operates at a maximum sampling frequency of 64M Samples/s. The digitizer is equipped with a digital down converter chip (DDC) that can carry out digital down conversion from IF when the bandwidth of the signal is less than 1.25 MHz. Otherwise if the bandwidth of the signal is greater than 1.25 MHz then the down conversion occurs in software, which is considerably slower.

5.2.2 Antenna

A 2.4 GHz Omni directional 7dBi indoor antenna was used for the experiment. Figure 5.6 shows the pictorial view of the antenna.
5.3 Experimental Setup

The first experiment was done in an Atrium in Nedderman Hall and the second one on the hallway at Engineering Lab building at University of Texas at Arlington. The following sections describe the experimental setup that was used for these experiments.

5.3.1 Experiment 1 - Nedderman Hall

For the experiment, the mobile Station (MS) or the target is an active transmitter, transmitting a continuous sine wave at 2.4 MHz. The power of the transmitter is kept at 10dBm. The Access points are the receivers. Only three access points are needed for position estimation. Figure 5.7 is the floor layout showing the relative positions of the AP and the MS for the experiment conducted in Nedderman Hall. Mobile station is kept at the center of the Hall and the signal strength is recorded at the various access points. The sampling rate of the system is kept at 1 MS/s. All the measurements are done in Line of Sight conditions.

Figure 5.6 Indoor Omni directional Antenna (2.4-2.5GHz), 7dBi Antenna
Figure 5.7 Floor layout of Nedderman Hall showing the position of MS and APs
For finding the value of $\alpha$ in Eqn (3.2), the transmitter and receiver are kept at a distance of 1 feet apart and the received power is recorded. There are five APs and the received power at each AP is recorded. Mobile station is localized using the measurements from any of the three Access points. Different position estimation algorithms are used for position estimation. Table 5.7 shows the position of the Access points (APs) and the estimated position of the target (MS). The distance is measured with Mobile station as the reference, thus the true position of Mobile station is the origin (0,0). The distance is expressed in feet.
5.3.2 Experiment II-Engineering Lab building

The second experiment was conducted in a hallway in Engineering Lab building at the University of Texas at Arlington. Due to the more complicated environment in the Hallway, localization problem is more challenging. Unlike the experiment conducted in Nedderman Hall, two of the access points are kept in Non Line of Sight conditions (Access point 2 and 4 in Figure 5.10).

Figure 5.9 Experimental arrangements in Hallway in Engineering Lab building
Figure 5.10 Nedderman Hall-Floor layout
### Table 5.1 Results of the experiment conducted in ELB

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>AP1</th>
<th>AP2</th>
<th>AP3</th>
<th>MS</th>
<th>ESTIMATED POSITION</th>
<th>ERROR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal</td>
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<td>[40 4]</td>
<td>[-15 0]</td>
<td>0 0</td>
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<td>22.9235</td>
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<td></td>
<td></td>
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<td>19.7836</td>
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<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td>(17.9781, 7.6246)</td>
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<td></td>
<td></td>
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</table>

Table 5.1 and Table 5.2 shows the results of the experiment conducted in Engineering Lab building and Nedderman Hall respectively. The columns 2, 3, 4 represent the position of the three access points used for Localization. Column 5 represents the true position of the mobile station. Column 6 gives the estimated position using various algorithms mentioned in Column 1.
Table 5.2 Results of the Experiment conducted in Nedderman Hall

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>AP1</th>
<th>AP2</th>
<th>AP3</th>
<th>MS</th>
<th>ESTIMATED POSITION</th>
<th>MEAN ERROR</th>
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<td>[25 20]</td>
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<td>[25 20]</td>
<td>[0 -50]</td>
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<td>[25 20]</td>
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Figure 5.11, Figure 5.12 and Figure 5.13 shows the estimated position and the true position of the Mobile station in a 2D plot for the experiment conducted in ELB.
Figure 5.11 2D plot of the estimated position when the Access points are at (20, 0), (40, 4) and (-15, 0)

Figure 5.12 2D plot of the estimated position when the Access points are at (20, 0), (-15, 0) and (68, 0)
Figure 5.13 2D plot of the estimated position when the Access points are at (20,0), (8,2) and (-15,0)

Figure 5.14, Figure 5.15, Figure 5.16 and Fig 5.17 shows the estimated position and the true position of the Mobile station in a 2D plot for the experiment conducted in ELB.

Figure 5.14 2D plot of the estimated position when the Access points are at (0,40), (12,40) and (25,20)
Figure 5.15 2D plot of the estimated position when the Access points are at \((0,40),(25,0)\) and \((0,-50)\)

Figure 5.16 2D plot of the estimated position when the Access points are at \((0,40),(25,20)\) and \((0,-50)\)
Figure 5.17 2D plot of the estimated position when the Access points are at (12,40), (25,20) and (25,0).

Figure 5.18 and 5.19 shows the bar plot of the mean error for the two experiments.

Figure 5.18 Performance comparison for the experiment in Nedderman Hall.
Figure 5.19 Performance comparison for the experiment in ELB

Figure 5.20 Summary of experimental results
5.5 Conclusion and Future Work

In this work, an improved position estimation algorithm for localization using RSSI is proposed. The proposed weighted centroid method performs better than other trilateration techniques. Performance comparison is done both numerically and experimentally. Numerical approach shows that the linear trilateration technique performs the worst and the weighted centroid, the best. From the experimental analysis it is observed that the linear least square gave the least accurate results. The accuracy of non-linear, iterative least square and centroid methods are comparable. In all cases, the weighted centroid method gave the most accurate estimate of the position.

This research work can be extended by studying the performance of these position estimation techniques in more composite environment. For such cases, the attenuation caused by wall and floor can be included in the propagation model. It could be worthwhile to study the performance of using other weighting schemes in the proposed weighted method. Weighting the measurements based on signal statistics is an interesting area that can be explored.
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BIOGRAPHICAL INFORMATION

Sreejith Sisupalan Lathikumari has been involved in graduate research at the Wave Scattering Research Center, University of Texas at Arlington. His research focused mainly on target localization in indoor environments. He completed his Bachelor of Science from University of Kerala, India in 2005. He has worked as a software developer for Perot systems and as telecom engineer for Tata Tele Services. He is currently working as a radio frequency engineer for T-Mobile, USA involved in network design and optimization of UMTS networks.