AN INVESTIGATION OF THE CONCEPTUAL UNDERSTANDING OF CONTINUITY AND DERIVATIVES IN CALCULUS OF EMERGING SCHOLARS VERSUS NON-EMERGING SCHOLARS PROGRAM STUDENTS

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ABSTRACT

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The Emerging Scholars Program (ESP) has been adapted at colleges and universities across the nation in efforts to increase student access to Science, Technology, Engineering and Mathematics (STEM) disciplines. This study uses a written assessment to gain insight regarding conceptual knowledge on continuity and derivatives for ESP students versus non-ESP students in the same lecture course in first semester calculus at a large urban university in the southwest. We analyze the assessment results of 22 ESP and 48 non-ESP students and discuss findings, particularly, those that indicate statistically significant differences regarding continuity over an interval.

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CHAPTER 1

INTRODUCTION

Understanding how students learn and how they build upon their conceptual understanding of mathematical concepts is often a challenge. Developing methods and programs to help students become more efficient with such actions may be even more so a challenge, since students learn in various ways and one certain method that aids one student may actually hinder the capabilities of another. Educators and researchers have often tried various techniques and experiments, some which produce significant results while others do not, in hopes of gaining a better understanding of studentsôconceptual knowledge. With each facet of discovery, educators can improve not only the curriculum, but can help shift learning in a way that is beneficial for the intended students.

At the University of Texas at Arlington, the Arlington Undergraduate Research-based Achievement for STEM (AURAS) was created as a retention program to help first-year students majoring in engineering, mathematics and science ease into the transition from high school to college. The ESP model, based upon Uri Treismanõs research on increasing African American studentsôsuccess in calculus at the University of California, has been adapted across the nation in various ways, and has been shown to positively impact retention rates in students pursuing a STEM-based major (Treisman, 1992; Bonsangue, 1994; Murphy, Stafford, & McCreary, 1998; Moreno & Muller, 1999; Adams & Lisy, 2007). For students who participate in an ESP for a specific mathematics or science course, their grades in the course tend to be higher than those of their peers who are not in ESP (Bonsangue, 1994; Moreno, Muller, Asera, Wyatt, & Epperson, 1999; Duncan & Dick, 2000; Drane, Smith, Light, Pinto, & Swarat, 2005; Adams & Lisy, 2007). However, no studies have attempted to determine the difference, other than grade outcomes, in depth of the mathematical or scientific knowledge gained by ESP students compared to non-ESP students.

In this study we investigate possible effects of the ESP model, not only on student grades, but also on the development of their conceptual understanding of calculus concepts. However, calculus covers a range of topics, and covering the bulk of all the calculus concepts would span more than one study alone. Thus, we direct the focus on the impact of the ESP model on the conceptual understanding of the concepts on continuity and the derivative. Thus, we pursue the following research question: After a semester-long intensive problem solving experience in Calculus I, do ESP students have a stronger conceptual understanding of the calculus concepts of continuity and derivative than Non-ESP students enrolled in the same lecture section?

Although we strive to gain insight on studentsôconceptual knowledge of continuity and derivative, another goal of this work is to compare the strengths of the concept images amongst the two groups of ESP and non-ESP students.

CHAPTER 2

LITERATURE REVIEW

According to Arcavi (2000), it is not easy to identify the ñinitial impetusesò that start a researcherøs drive in developing, shaping, and polishing their project. Through his experience and his observations, he describes the initial stages of problem-driven research in mathematics education, explaining three initial departure points: research studies motivated by interesting or puzzling behavior related to teaching and learning mathematics, research motivated by didactical opportunities, and others motivated by specific curriculum projects or classroom practice. While trying to understand how students learn mathematics, many researchers incorporate their inquiries into assessments or experimental courses that target studentsô motivation and studentsô conceptual understanding. To gain insight into what information the students retain and how they utilize what they know, it is helpful for those researchers to understand the studentsô concept images.

The idea of concept image is introduced in Tall & Vinner (1981) as the ñtotal cognitive structure,ò such as any mental pictures, properties and processes that are associated with the concept in question. This concept image is shaped and molded through experiences over the years. Tall & Vinner also refer to what they identify as the concept definition, which is described as the words, theorems, or formal statements that specify the concept. In light of this distinction, there have been many studies that have incorporated and concentrated on a studentøs concept image of various mathematical concepts in efforts to better comprehend student performance.

In that same article, Tall & Vinner conducted a study on the concepts of limits and the continuity of functions. In regards to the concept of limits, 70 students completed a questionnaire, and 22 of them participated in follow-up interviews. For the concept of the continuity of functions, a questionnaire was given to 41 students who maintained a grade of A and B in mathematics. As a result of their study, they saw that the participants exhibited a limited view of functions, because the students had trouble manipulating the definitions of limits and continuity. Students tend to have difficulties, because they are in situations where ñthey may have a strong mental picture yet the concept definition image is weakò (Tall &Vinner, 1981). Thus, they can understand statements of theorems, but have difficulty following the proofs.

In a study that focused on 271 college studentsô and 36 junior high school teachersô concept of function, Vinner & Dreyfus (1989) noticed the compartmentalization phenomena that many of the participants exhibited. More specifically, in completing the 7-item questionnaire, many of the students gave the Dirichlet-Bourbaki definition for the function concept, yet they were unable to apply what they knew about the definition to questions regarding functions, thus resulting in inconsistent behavior.

However, though this was shocking, the amount of cases of compartmentalization decreased as the studentsô exposure to higher mathematics increased (Vinner & Dreyfus, 1989). So the more exposure to mathematics that students receive, the better they perform because their experiences shape the image they have for concepts of functions. In gauging the intuitive functional concepts of image, preimage, growth, extrema, and slope, Dreyfus & Einsenberg (1982) conducted a study on student intuition, which they referred to as the ñmental representation of facts that appear self-evidentò. it was observed, in their study that targeted 24 classes between the grades of 6 and 9, that students at higher grades showed greater intuition in completing their questionnaire. Moreover, the students from better environments with less disadvantages performed better than those more disadvantaged and from poor environments (Dreyfus & Eisenberg, 1982).

In a study that focused on the concept of functions, a 25-item questionnaire was given to the following ñAò students: 30 from college algebra, 18 from second-semester calculus, and 14 graduate students who recently completed abstract algebra or complex analysis. In comparing undergraduate students with graduate students, it was also observed that more education and/or exposure aided to a better understanding of functions (Carlson, 1998).

The pace at which courses are taught, as well as the limited time that students are given per concept, causes many undergraduate students to develop a superficial understanding of concepts such as functions (Carlson, 1998). However, time can sometimes prove to be less of an inhibiting factor than one can assume. An experimental course in which first-year calculus students attended 1.5 extra hours of calculus lab per week to focus on more in-depth tasks, showed that the additional time was not sufficient enough to ñboost student performance beyond what was already done by a problem-solving-based calculus treatment as was employed during the studyò (Dawkins & Epperson, 2007). Even so, there are cases in which the present course that a student is enrolled in provides the quality exposure that they need for conceptual understanding, yet there are other reasons for poor performance. White & Mitchelmore (1996) showed, through their teachings in an experimental calculus course with 40 first-year mathematics students, that even though there were no issues in the way that the course was taught, the problem with studentsô concept of calculus was actually an underdeveloped concept of variables. Thus, mature function understanding is associated with strong conceptual underpinnings (Carlson, Oehrtman & Thompson, 2007).

A studentøs reasoning pattern is also essential in the development of their conceptual understanding. More specifically, a ñprocess view of functions is crucial for developing rich conceptual understanding of content in an introductory calculus courseò (Carlson, Oehrtman, & Thompson, 2007). This process view is one in which the student is able to dynamically transform quantities. They should be able to view an entire activity collectively as taking a group of objects and obtaining a new group of objects, as opposed to an action view, in which the student can only comprehend step-by-step procedures. One way to build on the process view of functions is the application of covariational reasoning (Carlson, Coe, Hsu, Jacobs, & Larson, 2002; Carlson, Oerhrtman, & Thompson, 2007).

It is these struggles in the concept of functions that also affect a studentøs concept of limits and continuity (Cottrill, Dubinsky, Nichols, Schwingendorf, Thomas, & Vidakovic, 1996; Tall & Vinner, 1981). In a study on the concept of limit, conducted through 25 interviews with students who were subject to an experimental calculus course that involved computer activities and classwork done in groups, it was seen that the insufficient development of a strong dynamic conception makes it difficult for students to move to a formal conception of limit (Cottrill, et al., 1996). Moreover, according to Bezuidenhout (2001), the mathematical knowledge that students have at the start of their calculus courses, õmay be deficient in many aspects and they may have very limited conceptions about limits and continuity.ö He concluded this as a result of his study that focused on the concept of limits and continuity of 170 engineering students and 523 first-year students from three universities subject to written tests, followed by 2-hour interviews with 15 of the students.

In efforts to improve studentøs conceptual underpinnings, many researchers have relied on visual methods to reinforce concepts. Though topics such as function transformations are difficult in terms of comprehension, when given the opportunity, students show a readiness to approach functions using visual means, which was exhibited by 16 students in a teaching experiment (Dreyfus & Einsenberg, 1994). In a study in which questions regarding the concept of limits were coupled with the concept of continuity of functions, a portion of the students connected the use of limits of functions with the continuity of functions. The sample included 21 students who learned continuity in a previous semester, and then applied it in the next semester, as well as 27 students with science and math majors that only recently learned continuity. As a result of the study, it was also realized that graphs have a major influence on studentsô opinion of continuity and would even cause many of them to answer questions incorrectly, though they knew facts that showed otherwise (Takaci, Pesic, & Tatar, 2006). Furthermore, from his experience while teaching continuity, Millspaugh (2006) has noticed that ñstudents are fairly comfortable with finding limits when they have a graph to work with.ò

With visual means and graphs proving to be a vital factor in studentsô conceptual learning, there have been studies moving beyond the topics of limits, in which computers were used to understand studentsô graphical understanding of functions and derivatives. Moving beyond limits is equally important, because studentsô inability to understand ñthe limit conceptôs role in calculus may é be due to inappropriate and weak mental links between knowledge of ôlimitô and knowledge of other calculus concepts such as ócontinuityõ, óderivativeô and óintegralô (Bezuidenhout, 2001).

As observed in one study on the concept of derivatives with 24 students in a regular section and 17 students in an experimental section, the instructional treatment that used computers to understand studentsô graphical understanding seemed to contribute to studentsô acquisition of a stronger conception of functions and derivatives than traditional methods (Asiala, Cottrill, Dubinsky, & Schwingendorf, 1997). In another study with graphical implementation in experimental calculus sections over two semesters with 56 students, it was initially observed that an algebraic representation of functions and derivatives dominated the thinking of students. However, later in the study, the students assimilated visual thinking and showed improvement in the concepts in favor of geometric interpretation (Habre & Abboud, 2006). Also on the concept of derivative, a study has shown that the studentsô concept image varied based on their departmental affiliation (Bingolbali & Monaghan, 2008). The sample in this study was comprised of 50 mechanical engineering students and 32 mathematics students, where each group was taught calculus in their own departments by professors within the department. The mechanical engineering students were showed to have performed better on ñrate of changeò problems, whereas the mathematics students performed better on ñtangent lineò problems. So not only do visual tools and strong underpinnings on the foundations of functions help studentsô conceptual development, it turns out that how professors present material and their preferred methods influence studentsô concept image as well.

Broadening the scope of concepts, some studies have been on the effect of calculus as a whole on studentsô skills and motivations. Walter & Hart (2009) analyzed an entire honors calculus course, by videotaping and documenting all sessions, to determine student motivations in problem solving tasks. The students were introduced to scenarios, sometimes ones in which they have not learned the necessary mathematical concepts that are needed to complete the tasks. Motivated by their own desire to learn how to solve the given problems, the students would try to teach each other ways to approach the concept.

Thus, the majority of the studies aforementioned targeted studentsô conceptual understanding. The concept images that the students have for each mathematical concept are shaped by their own experiences as well as what teachers and those around them expose them to. Though students could ñsometimes find limits, differentiate and integrate, they do not develop a relational understanding of the conceptual underpinnings of calculusò (Bezuidenhout, 2001). Thus, in order for the students to effectively use the concept images that they build upon, they must also have strong underpinnings about the topics prior to the new concept. A sturdy understanding of variables helps build an understanding of functions. This in turn aides in the formation of studentsô concept images of continuity and derivatives.

CHAPTER 3

METHODOLOGY

3.1 Sample

This study was conducted at a large urban university in the Southwest. The university enrolls 33,000 students, with about 25 percent of those pursuing graduate-level degrees. With almost 190 bachelor's, master's, and doctoral degrees being offered at the university, the school also has various additional programs available to students. Prospective students wishing to pursue a degree in the sciences, technology, engineering or mathematics disciplines have the option to apply to the Emerging Scholars Program that has been implemented by the AURAS program. In order to apply to the ESP, prospective students must: (1) intend to major in Engineering, Physics, Mathematics, or Chemistry disciplines, (2) have an approximate 3.0-3.5 GPA in high school, (3) matriculate as a first time, first semester freshman, and (4) not have calculus college credit upon matriculation. Upon acceptance, students in ESP are required to attend ESP-specific workshops per week corresponding to the mathematics or chemistry course they are currently enrolled in (http://www.uta.edu/auras/index.html).

The sample in this study consists of 70 undergraduate college students from two regular calculus lab sections, who were also concurrently enrolled in the same Calculus I course for the Fall 2010 semester. For the first lab section, the assessment was completed by 38 students total: 22 ESP students and 16 non-ESP students. For the second lab section, the assessment was completed by 32 non-ESP students.

The non-ESP students attended their lab section for fifty minutes, twice a week. These lab sections were sufficient to receive credit for the course, as put forth by the university. The ESP students also attended the same regular lab that the non-ESP students did, for fifty minutes twice a week. Furthermore, the ESP students were required to attend an additional lab section as part of their enrollment in the AURAS Emerging Scholars Program at the university. These extra lab sections met twice a week and lasted 110 minutes per meeting, but did not account for any additional lab credit. Thus, the ESP students were exposed to calculus concepts for 220 more minutes per week than the non-ESP students. The regular fifty-minute lab sections attended by both non-ESP and ESP students were taught by the same graduate teaching assistant. The additional 110-minute lab section that the ESP students attended was taught by a different graduate student.

3.2 Student Background

The students in the sample had various backgrounds, which were reported in a general questionnaire given by the AURAS Program. Of the 22 ESP students that completed the written assessment used in this study, all of them also submitted their initial AURAS questionnaire; and of the 48 non-ESP students that completed the written assessment used in this study, five of the students missed the initial AURAS questionnaire. The first seventeen questions of the AURAS questionnaire can be found in Appendix A.

From the reported information restricted to the sample, only one female was enrolled in the ESP program, compared to 21 females not enrolled in the ESP program, leaving 21 male ESP students and 22 male non-ESP students. Moreover, the studentsô ethnic composition, exhibited by Table 3.1, shows that both ESP and non-ESP groups had roughly the same amount of Hispanic students, while the non-ESP group contained more than half the amount of non-Hispanic students than the ESP group.

Table 3.1 The Sampleös Ethnic Composition

	Ethnicity		
Student Type	**Hispanic**	**Non-Hispanic**	**No Report/Blank**
ESP	9	12	1
Non-ESP	8	32	3

As seen in Table 3.2, all but one of the ESP students were freshmen, which is consistent with the ESP program specifications. The one sophomore in ESP was beginning his first semester in engineering. On the other hand, the non-ESP students consisted of freshmen and various upperclassmen, with one student as ñOther,ò that was a degreed freshman. Though the ESP students were mainly freshmen, their age group ranged from 17 to 26 years old, with a majority (sixteen) of the students being of age 18 at the time of the questionnaire. The non-ESP studentsô age group ranged from 18 to 29 years old, with a majority (fifteen) of the students being 18 years old.

Table 3.2 Student Sample Composition by Classification (College Year)

	Classification				
Student Type	**Freshmen**	**Sophomore**	**Junior**	**Senior**	**Other**
ESP	21	1	0	0	0
Non-ESP	20	12	3	7	1

In regards to high school education, Table 3.3 unfortunately shows that not every student in the sample has had pre-calculus or exposure to calculus beforehand. Of the five students in ESP that have not had a pre-calculus or calculus course, three had taken up to Trigonometry, one only up to Geometry, and the remaining student did not report their most advanced math class taken. Of the three non-ESP students that did not have exposure to pre-calculus or calculus, one had taken up to Algebra II, and the remaining two students had only taken up to Geometry.

Table 3.3 Student Sample Composition by Most Advanced Math Class Taken in High School

	Mathematics Course			
Student Type	**AP Calculus**	**Non-AP Calculus**	**Pre-Calculus**	**Other**
ESP	12	2	3	5
Non-ESP	15	7	18	3

According to Table 3.4 below, the majority (about 82%) of the ESP students listed Engineering as their intended major, while for the non-ESP students, 42% of them listed Engineering, and 37% listed one of the Chemical Sciences as their intended majors. Thus, according to the studentsôintended major, almost all of them are required to take Calculus I as part of their degree program.

Table 3.4 Student Sample Composition by Intended College Major

	Number of Students	
Major	**ESP**	**Non-ESP**
Aerospace Engineering	5	3
Bioengineering	0	1
Civil Engineering	2	3
Computer Engineering	2	2
Electrical Engineering	0	2
Mechanical Engineering	8	6
Software Engineering	1	1
Biological Chemistry	0	10
Chemistry/Biochemistry	0	6
Computer Science	2	0
Mathematics	1	2
Physics	1	1
Not Reported/Undecided	0	6
Total	22	43

3.3 Calculus Lecture Section

Enrollment in Calculus I at the university requires a passing grade of at least a C in Pre-Calculus II or a sufficient score on the Math Aptitude Test or sufficient SAT/ACT scores. The semester-long course also requires concurrent enrollment in a calculus lab section. Students typically spend three hours a week in lecture, while their professor covers the concepts of limit, continuity, differentiation and integration.

Calculus I students are subject to two departmental midterms: the first is administered roughly a month after the semester begins, and the second midterm is given about a month after the first midterm. The students are also required to take the departmental final exam at the end of the semester during a designated time. All of the departmental exams are comprehensive and are a mixture of multiple-choice and free-response problems. All previous midterms and some final exams are accessible to students online, as well as in person at a designated center. The university also has a Math Clinic, which is open to students 7 days a week.

3.4 Lab Sections

3.4.1 Non-ESP and ESP Regular Joint Lab Sections

As mentioned earlier, the lab sections that the non-ESP and ESP students both attended together were held twice a week. On one of those days, there would be a quiz worksheet or an online quiz. The quizzes would usually consist of two previously assigned homework problems, and students were given roughly ten to fifteen minutes to complete them. Sometimes, the quizzes were taken during the studentsôregular lecture session. On the same day, the students also worked on lab worksheets in groups. These groups were chosen by the students themselves, and they worked in the same group for the duration of the semester. The lab worksheets covered recent material discussed in the studentsôregular lecture session. The calculus professor and the graduate teaching assistant facilitated the lab section and would assist the groups when help was needed.

On the other lab day of the week, the graduate teaching assistant would work out recent homework problems that the students had questions over, while engaging the students in discussion. If the students did not have any questions regarding the homework, then the graduate teaching assistant would assign each group a problem from their current chapter. The students would work on the assigned problem and later present them to the class if they had time.

3.4.2 ESP Lab Sections

As mentioned before, the additional lab sections that the ESP students attended were workshops, held twice weekly for 110 minutes each session. Students would work in groups of three, and on occasion, the groups were assigned to encourage variation and enable the students to get to know their classmates better. The students were given worksheets that had difficult and thought-provoking problems that were related the material covered in their regular calculus lecture sessions. In these groups, the students shared their ideas on how to approach a solution for each problem. If the students within each group agreed with one another, they would work on the problem with that same approach. If a student did not agree, then he or she would work on the problem using their own preferred approach. The students would then compare their progress and solutions with each other in their groups. Students would ask each other questions when they encountered problems and would take time to explore their ideas. If the problem was not resolved, the group would then ask for assistance from the graduate teaching assistant that was assigned to the ESP lab sessions.

During the lab sessions, group members were encouraged to actively participate in any discussions. Moreover, groups were encouraged to work at the chalk board when they wanted a change in setting, or when the students wanted to start from scratch on a particular problem.

Following the lab sessions, the graduate teaching assistant would stay behind for roughly 20 minutes. During this time, students were able to stay and ask questions, whether the questions were related to the worksheets, a particular concept, or a problem similar to assigned homework problems. The graduate teaching assistant would provide more direct answers to questions that were unable to be addressed in their large lab sessions. Moreover, in her explanations, the graduate teaching assistant would generally reference a worksheet problem, so as to help the students develop connections amongst the calculus concepts.

In addition to the worksheets during a workshop, review sessions were also held. On the first workshops during the week of an exam, generally 45 minutes were set aside for review. During this time, the students would work to solve a portion of exam questions from the previous year on their own. The following lab session, the students would get their results and a portion of the dayꞌs session was devoted to working the same problems simultaneously in groups at the chalk board.

3.5 Written Assessment

To address the research question in this study, a written assessment was created with a variety of problems addressing both the concepts of continuity and derivatives. The assessment, provided in Appendix B, consisted of seven items total. The first three items addressed the concept of continuity. Item #1 consisted of four of the same functions presented by Tall & Vinner (1981), as well as an additional function, all with their corresponding graphs provided. The five functions deal with function continuity on their respective domains. Item #2 consisted of five functions, similar to those in Item #1, that deal with function continuity on a given interval. Item #3 consisted of five True/False statements that address ideas of function continuity.

The remaining four items in the assessment addressed the concept of the derivative. Item #4 was similar to an item from Asiala, Cottrill, Dubinsky, & Schwingendorf (1997), consisting of two parts that require answers derived from a provided graph. The formats of Items #5, #6, and #7 were similar to each other, each addressing the concept of the derivative from a different perspective. All of Items #2, #3, #5, #6, and #7 were self-created for the purposes of this study.

The assessment was administered during the studentsô normal lab section for 24 minutes. It was initially planned to be 20 minutes long, but time was extended a few minutes to account for some of the students who were tardy to the first lab. The second lab section was given the same amount of time to complete the assessment, regardless of the late arrival of some students. Some of the studentsôtardiness may account for the blank responses on some of their tests, because they did not have ample time to attempt or complete all of the items on the assessment.

3.6 Scoring

3.6.1 Initial Scoring for Result Analysis

Two different rubrics were developed to score the various items from the assessment: the Accuracy Rubric (AR) and the Conceptual Understanding Rubric (CUR), each which are provided in Appendix C. The Accuracy Rubric, which determines a quantitative score based on whether or not the student answered the items correctly, is an adaptation of the accuracy portion of Mathematics Problem Solving Official Scoring Guide used by the Oregon Department of Education Office of Assessment and Evaluation (2008). The Conceptual Understanding Rubric, which determines a qualitative score based on the measure of the studentsô explanations for the particular assessment items, is an adaptation of the conceptual understanding column of the Mathematics Problem Solving Official Scoring Guide from the Oregon Department of Education and the QUASAR General Rubric (Lane, 1993).

Each part of Items #1, #2, #4, #5, #6, and #7 was scored with both the AR and the CUR, while each part of Item #3 was scored with only the AR. Thus, each attempted part of the items, aside from those of Item #4, received an AR score of 1 or 3. The two parts of Item #4 that were attempted could receive a possible AR score of 1, 2, or 3. Furthermore, each attempted part of the items, aside from those of Item #3, received a CUR score of 1, 2, 3, 4 or 5. A summary of the scoring can be seen in Table 3.5. For the purposes of analysis, responses to items that received a CUR of 4 or 5 were considered to be quality response.

Table 3.5 Summary of the Possible Scores for Attempted Assessment Items

Items	Possible AR Scores	Possible CUR Scores
#1 (A-E)	1 or 3	1, 2, 3, 4, or 5
#2 (A-E)	1 or 3	1, 2, 3, 4, or 5
#3 (A-E)	1 or 3	N/A
#4 (A & B)	1, 2, or 3	1, 2, 3, 4, or 5
#5	1 or 3	1, 2, 3, 4, or 5
#6	1 or 3	1, 2, 3, 4, or 5
#7	1 or 3	1, 2, 3, 4, or 5

It should be noted that in cases where a student did not provide an explicit answer necessary for the accuracy score, their answer was inferred by their detailed response justifications. If the written justification was not indicative of where the studentsôresponse was leaning to, then the item was assumed to be incorrect. Furthermore, students that did not attempt items or their parts, did not receive a numerical score and instead were given the score of ñbñ(or ñBlankòon the figures). This was utilized to account for the scenario in which a student did not have enough time, and may or may not have answered the item correctly with a quality response.

The scores were used to analyze the data descriptively and to determine the range of conceptual knowledge exhibited by the students of the sample. Results from both groups of students were compared. Furthermore any changes or conflicts in responses from problem to problem were noted and are described later.

3.6.2 Rescoring for Further Statistical Analysis

Table 3.6 Overview of the Rescoring Method for Items #1, #2, #4, #5, #6, and #7 for Further Statistical Analysis

Description	**Original Score**	**Score Categorization**
Responses that were correct, with quality justification	AR: 3 and CUR: 4 or 5	4
Responses that were partially or not correct, with quality justification	AR: 1 or 2 and CUR: 4 or 5	3
Responses that were correct, without quality justification or no justification	AR: 3 and CUR: 1, 2, or 3	2
Responses that were partially or not correct, without quality justification or no justification	AR: 1 or 3 and CUR: 1, 2, or 3	1

For further analysis, the non-blank scores of the parts of Items #1, #2, #4, #5, #6, and #7 were later categorized as shown in Table 3.6, while the blank responses were omitted. The score categorizations resulted in new scores for each item that were intended to be used to test for any statistical inferences from the studentsôbackground variables. Thus, the overall purpose of rescoring was to be able to easily use the data, by combining both AR and CUR scores, to determine if any of the background variables had any influence in the performance of the students. Since the background variables were limited to only the students that completed the initial AURAS questionnaire, this data set contained 22 ESP students and 43 non-ESP students.

All parts of the items were summed to represent the studentsônew composite score so that each problem only had one score associated with it. Thus, both of Items #1 and #2, with five parts (A-E) each, had maximum possible composite scores of 20. Item #4, with two parts (A & B), could receive a maximum composite score of 8, and each of Items #5, #6, and #7 had a maximum score of 4. In the case of Item #3, each part (A-E) received a score of 1 if correct and 0 if incorrect. The parts were then summed to represent the new composite score for Item #3, which had a maximum score of 5.

Select questions from the AURAS questionnaire were chosen as potential points of influence. The questions dealt with a variety of parameters, such as ethnicity, classification (college year), citizenship, parentsôeducation, Advanced Placement (AP) Calculus exposure, and their indicated majors. The studentsôresponses to each of these questions were grouped to help with the statistical analysis. However, after observing the range of responses, the only question that showed sufficient variety amongst the sample was Question #10, the one that dealt with the studentsôexposure to AP Calculus (AB and/or BC). Thus, this set was used with the studentsônew score with all the problems to test for statistical inferences. In order to accomplish this part, an associate professor of the university, with extensive experience in statistics, was sought out.

The professor wrote the necessary algorithms and ran the data sets in the Statistical Analysis Software. After acquiring the summary statistics for each of the items, the data was checked for normality, and then ranked if needed. The scores for the ESP and non-ESP students were stratified with the AP Calculus data, and tested using two-sample t-tests and ANOVA. The tests that showed evidence of interaction were analyzed further, by examining different combinations with the data sets. Furthermore, the scores for Items #5, #6, and #7 were tested with the studentsôintended major using chi-square goodness of fit tests. With the help of the professor, the resulting numbers were analyzed and significant findings were noted, which are presented later in the paper.

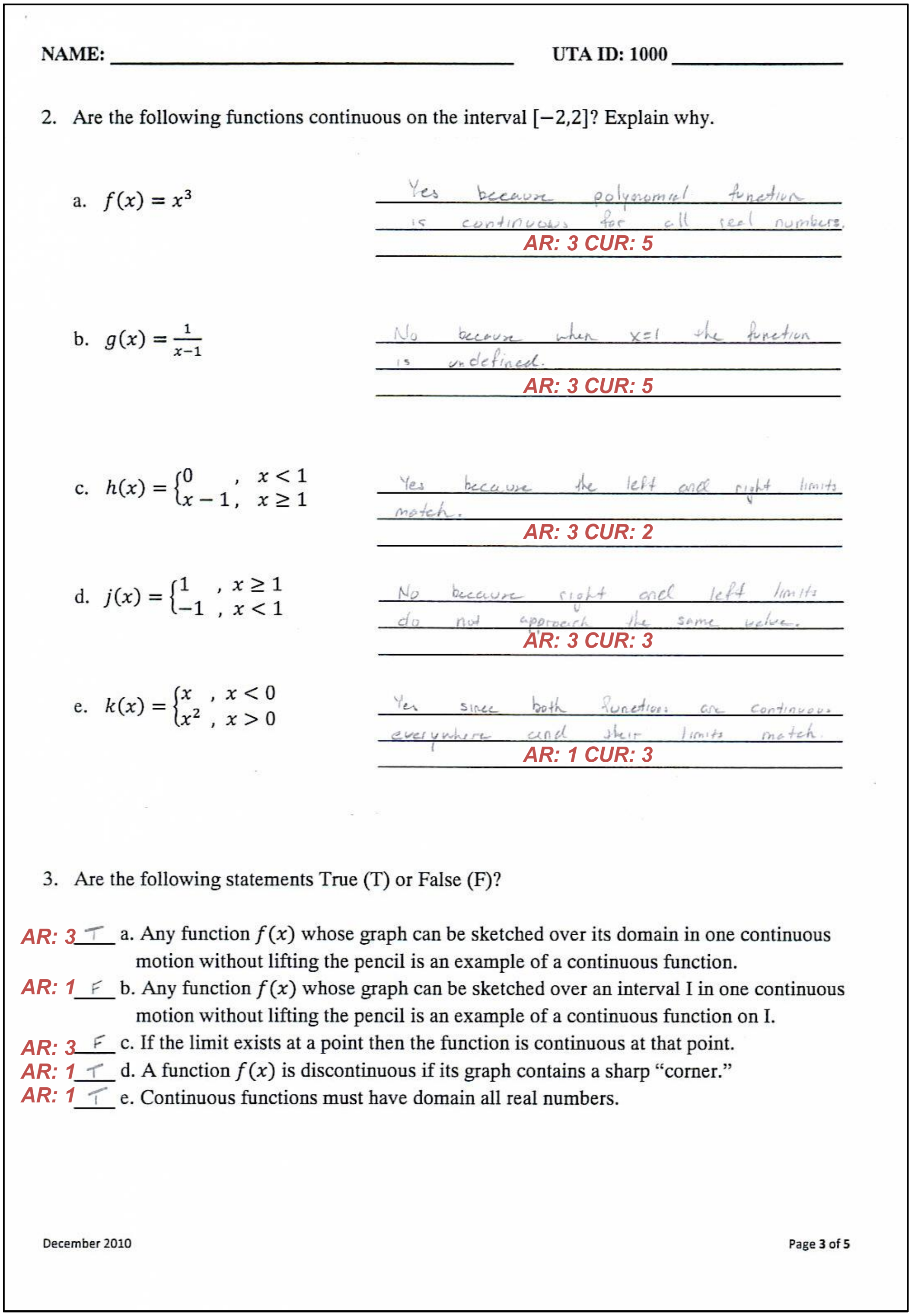
CHAPTER 4

RESULTS

This chapter reports studentsô scores based on their written responses to select items on the assessment. All items of the written assessment can be found in Appendix A. The responses were scored with the Accuracy Rubric (AR) and the Conceptual Understanding Rubric (CUR), both of which can be found in Appendix C. Recall that the possible scores are outlined in Table 3.4.1 in the Methodology section of this paper. Since this portion of the results deals with only the scores on the assessment, all of the studentsô responses are available. Thus, any values or percentages that are presented are out of 22 ESP students and 48 non-ESP students, unless otherwise noted.

For the purposes of presenting the data in graphs, the groups are labeled with an AR score and a CUR score as described here. The AR scores are presented as õAR: 1ö, õAR: 2ö or õAR: 3ö, dependent on the received AR score. The quality responses (those with a CUR of 4 or 5) are grouped as õCUR: 4+ö, whereas the responses that are not considered quality (those with a CUR of 1, 2 or 3) are grouped as õCUR: 3-ö. Thus, for example, a studentsô response that received an AR score of 1 and a CUR score of 5, would be found in the õAR: 1 CUR: 4+ö portion of a given graph, while a student that received an AR score of 3 and a CUR score of 2, would be found in the õAR: 3 CUR: 3-ö portion of a given graph.

Figures 4.1 and 4.2 are sample pages taken from the written assignment that the students completed, showing a few of the types of responses that were given. The scores that each response received are denoted in red in the figures.



NAME: ______________________ UTA ID: 1000 ______________

2. Are the following functions continuous on the interval $[-2,2]$? Explain why.

a. $f(x) = x^3$

Yes because polynomial function is continuous for all real numbers.

AR: 3 CUR: 5

b. $g(x) = \frac{1}{x-1}$

No because when x=1 the function is undefined.

AR: 3 CUR: 5

c. $h(x) = \begin{cases} 0 & , \ x < 1 \\ x-1, & x \geq 1 \end{cases}$

Yes because the left and right limits match.

AR: 3 CUR: 2

d. $j(x) = \begin{cases} 1 & , x \geq 1 \\ -1 & , x < 1 \end{cases}$

No because right and left limits do not approach the same value.

AR: 3 CUR: 3

e. $k(x) = \begin{cases} x & , x < 0 \\ x^2 & , x > 0 \end{cases}$

Yes since both functions are continuous everywhere and their limits match.

AR: 1 CUR: 3

3. Are the following statements True (T) or False (F)?

AR: 3 T a. Any function $f(x)$ whose graph can be sketched over its domain in one continuous motion without lifting the pencil is an example of a continuous function.

AR: 1 F b. Any function $f(x)$ whose graph can be sketched over an interval I in one continuous motion without lifting the pencil is an example of a continuous function on I.

AR: 3 F c. If the limit exists at a point then the function is continuous at that point.

AR: 1 T d. A function $f(x)$ is discontinuous if its graph contains a sharp "corner."

AR: 1 T e. Continuous functions must have domain all real numbers.

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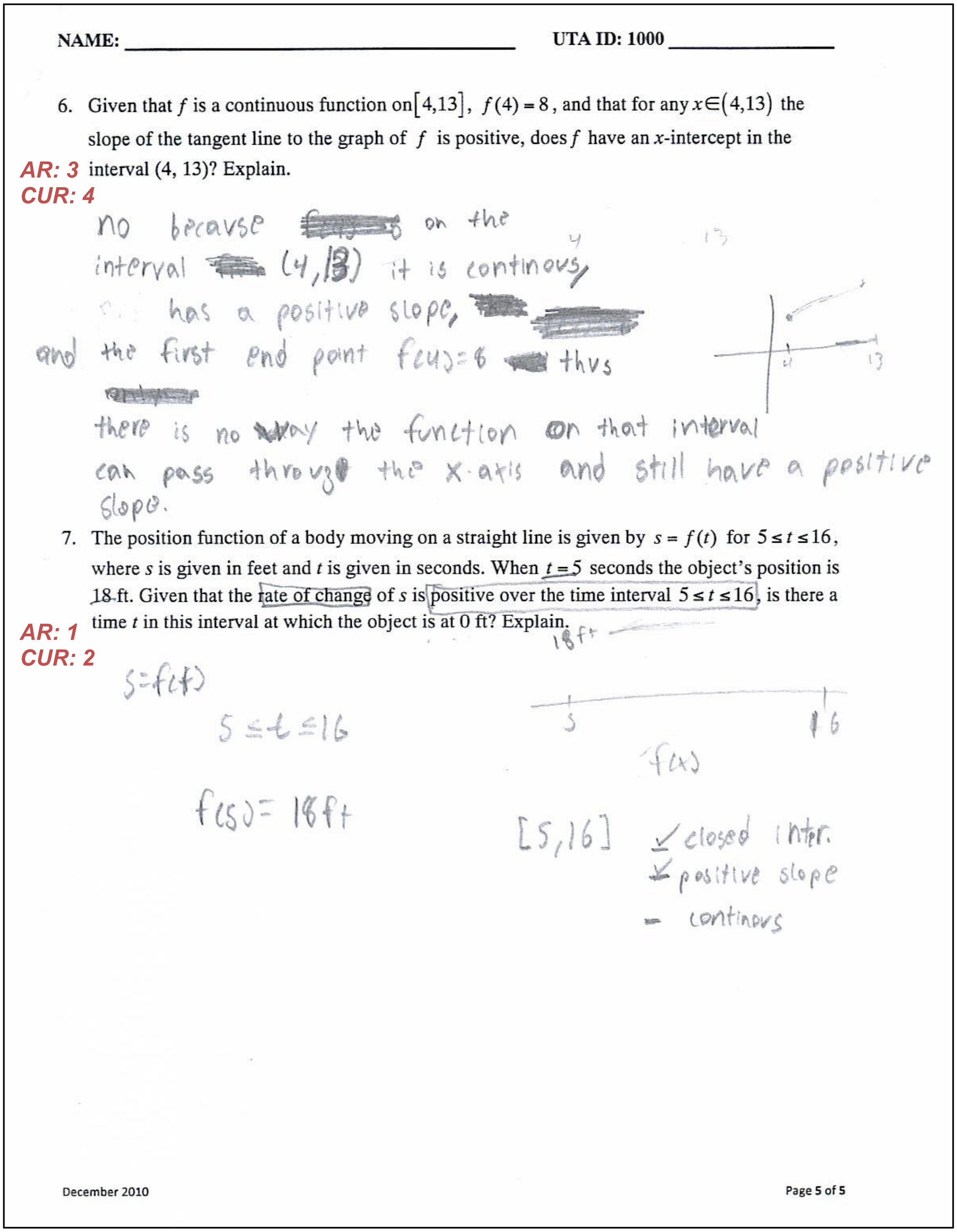
Figure 4.1 An example of a non-ESP studentsôassessment showing responses to Items #2 and #3 and corresponding scores in red.



NAME: ______________________ UTA ID: 1000 ____________

6. Given that f is a continuous function on $[4,13]$, $f(4)=8$, and that for any $x \in (4,13)$ the slope of the tangent line to the graph of f is positive, does f have an x-intercept in the interval (4, 13)? Explain.

AR: 3
CUR: 4

no because on the interval (4,13) it is continous, has a positive slope, and the first end point f(4)=8 thus there is no way the function on that interval can pass through the x-axis and still have a positive slope.

7. The position function of a body moving on a straight line is given by $s=f(t)$ for $5 \le t \le 16$, where s is given in feet and t is given in seconds. When $t=5$ seconds the object's position is 18 ft. Given that the rate of change of s is positive over the time interval $5 \le t \le 16$, is there a time t in this interval at which the object is at 0 ft? Explain.

AR: 1
CUR: 2

s=f(t)

5 ≤ t ≤ 16

f(5)= 18ft

18ft

f(x)

[5,16] ✓ closed intr.
✓ positive slope
– continous

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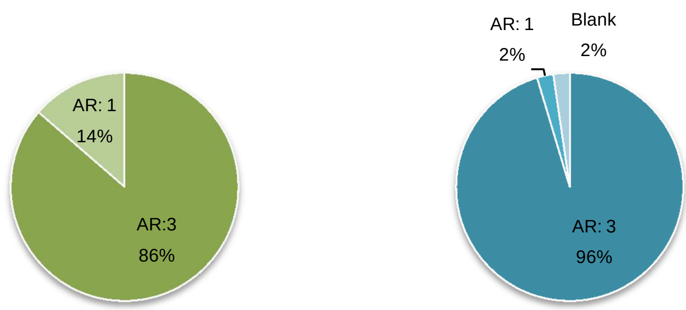
Figure 4.2 An example of an ESP studentsôassessment showing responses to Items #6 and #7 and corresponding scores in red.

4.1 Assessment Items #3-A, #1-B, #1-E, and #3-E

All parts of Assessment Item #3 were various T/F statements regarding the concept of continuity. Item #3-A addressed the idea of sketching a continuous function's graph over its domain in one continuous motion without lifting the pencil. According to the results in Figure 4.3 below, 86% of the ESP students believed that it was true, compared to 96% of the non-ESP students who also thought it was possible. Thus, the non-ESP students were more accurate on this item than the ESP students.



(a) (b)

Figure 4.3 Score distribution for Item #3-A for (a) ESP students and (b) non-ESP students.

Assessment Items #1-B and #1-E both had functions with graphs whose continuity had to be determined. Though all parts of Item #1 addressed the concept of continuity over a domain, these two specifically brought about cases in which the functions' domain had to carefully be considered, before answering the problem. The graph of the function in Item #1-B contained an asymptote, while the graph of the function in Item #1-E contained an "open point", both of which were outside of their respective functions' domain. According to the results in Figure 4.4 59% of the ESP students and only 29% of the non-ESP students answered Item #1-B correctly, (i.e. received a score of AR: 3). Moreover, only 14% of the ESP students and 23% of the non-ESP students answered Item #1-E correctly, as shown in Figure 4.5. Comparing the accuracy percentages of Items #1-B and #1-E to the percentages of Item #3-A, both groups of students showed a higher frequency of correct answers on the T/F statement than they did on the application of the statement.

Figure 4.4 Score distribution for Item #1-B for (a) ESP students and (b) non-ESP students.

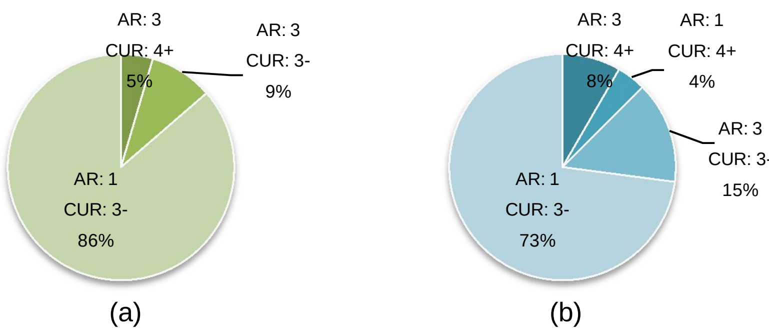


Figure 4.5 Score distribution for Item #1-E for (a) ESP students and (b) non-ESP students.

Looking at Item #1-B alone, a greater percentage of ESP students answered the item correctly than the non-ESP students. Conceptually, 32% of the ESP students provided a quality response (i.e. received a score of CUR: 4+) for Item #1-B, compared to only 14% of the non-ESP students with such a score. Thus, while both percentages were low, a greater percentage of the ESP students provided a quality response than the non-ESP students. On the other hand, for Item #1-E, a greater percentage of non-ESP students answered the item correctly than the ESP students. Conceptually, 12% of the non-ESP students provided a quality response, as opposed to only 5% of the ESP students with a comparative score. Thus, in the case of Item #1-

E, the non-ESP students had higher percentages of correct and quality responses than the ESP students. Table 4.1 summarizes these percentages.

Table 4.1 Summary Percentages for Correct and Quality Responses for Items #3-A, #1-B, and #1-E

	Item #3-A	Item #1-B		Item #1-E	
Student	**Correct (i.e. AR: 3)**	**Correct**	**Quality Response (i.e. CUR: 4+)**	**Correct**	**Quality Response**
ESP	86%	59%	32%	14%	5%
Non-ESP	96%	29%	14%	23%	12%

Considering the students that answered Items #1-B and #1-E incorrectly, several examples of their responses are shown in Figures 4.6 and 4.7, respectively. Of the incorrect responses to Item #1-B, 13 students (7 of which were ESP students), stated that the function was discontinuous because of the presence of an asymptote within the graph of the function. Moreover, 8 students (3 of which were ESP students) used limits in their responses to argue that the function was discontinuous.

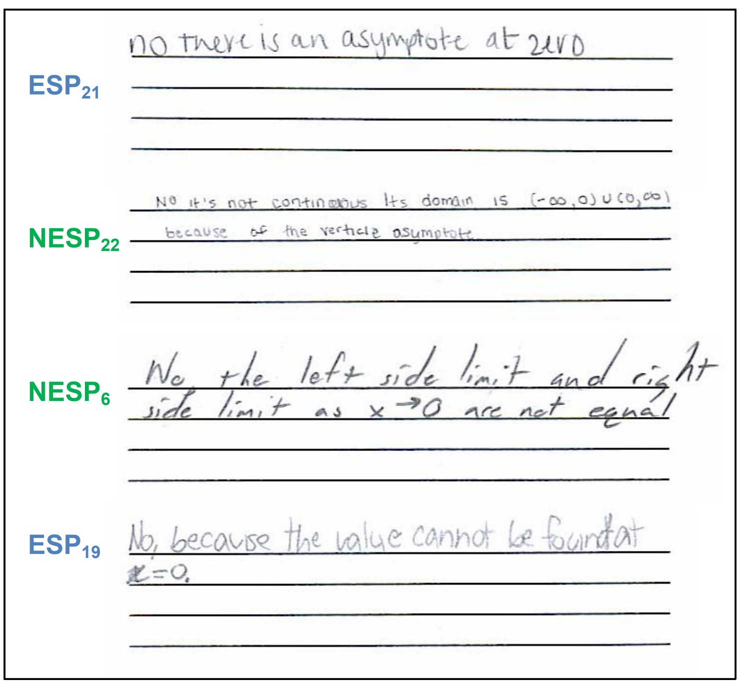


Figure 4.6 Examples of responses to Item #1-B from ESP students 21 & 19 and non-ESP students 22 & 6.

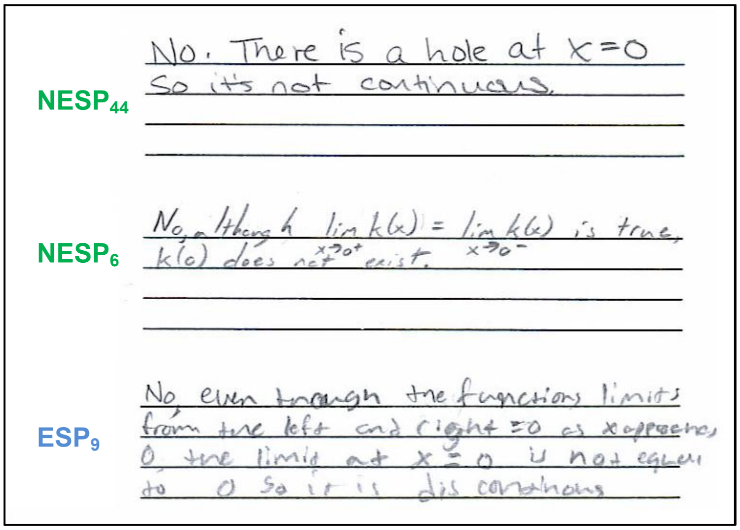


Figure 4.7 Examples of responses to Item #1-E from ESP student 9 and non-ESP students 44 & 6.

Of the incorrect responses to Item #1-E shown in Figure 4.7, 10 non-ESP and 9 ESP students believed that the hole in the graph accounted for the function's discontinuity, while 5 non-ESP and 4 ESP students thought that since the function was undefined at a point on the graph, the function was discontinuous.

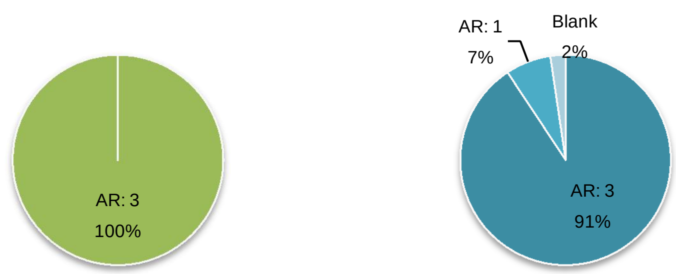
Assessment Items #1-B and #1-E were also related to Assessment Item #3-E, which was a T/F statement that addressed whether it was a necessity for continuous functions to have domain all real numbers. Figure 4.8 shows that 32% of the ESP students and 42% of the non-ESP students answered Item #3-E correctly with an answer of “F” for false. Thus, these students knew that continuous functions need not have domain all real numbers. In the case of Item #3-E, a greater percentage of non-ESP students answered it correctly than ESP students. Moreover, the non-ESP students exhibited a higher percentage of correct answers on Item #3-E than they did on Items #1-B and #1-E. The ESP students, on the other hand, had a greater percentage of correct problems on Item #1-B than they did on #3-E, but a higher frequency of correct answers on #3-E than they did on #1-E.

(a) (b)

Figure 4.8 Score distribution for Item #3-E for (a) ESP students and (b) non-ESP students.

4.2 Assessment Items #3-B and #2-C

Assessment Item #3-B addressed the ability to sketch a continuous function's graph over a given interval in one continuous motion without lifting the pencil. According to the results displayed in Figure 4.9, all (100%) of the ESP students and 91% of the non-ESP students answered Item #3-B correctly. Thus, ESP performance was only slightly better, in terms of accuracy, than the non-ESP students.



(a) (b)

Figure 4.9 Score distribution for Item #3-B for (a) ESP students and (b) non-ESP students.

Assessment Item #2-C presented a piece-wise function without a graph, and was related to Item #3-B, in that the students had to determine whether the function was continuous over a given interval. As shown in Figure 4.10, 82% of the ESP students answered Item #2-C accurately, but only 18% of the group provided a quality justification. In the case of the non-ESP students, 77% answered accurately, but only 25% provided a quality response. So while a greater percentage of ESP students answered Item #2-C accurately, the non-ESP students provided a higher percentage of quality responses. However, in comparing these percentages with those of Item #3-B, both groups were more accurate with Item #3-B than Item #2-C.

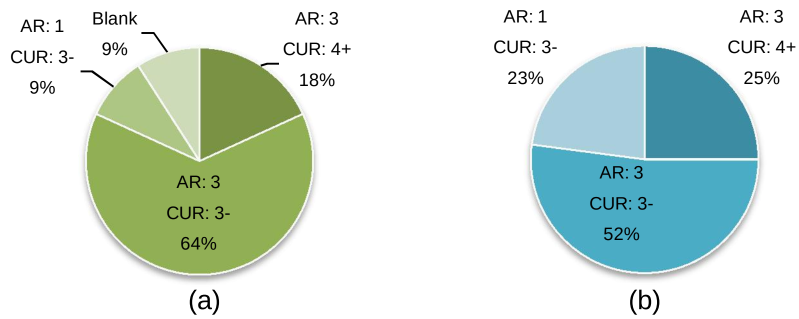


Figure 4.10 Score distribution for Item #2-C for (a) ESP students and (b) non-ESP students.

Of the responses to Item #2-C, some of the correct responses explicitly used the pencil method to determine continuity. The incorrect responses, however, showed a wide variety of response, three of which are displayed as the bottom two responses in Figure 4.11.

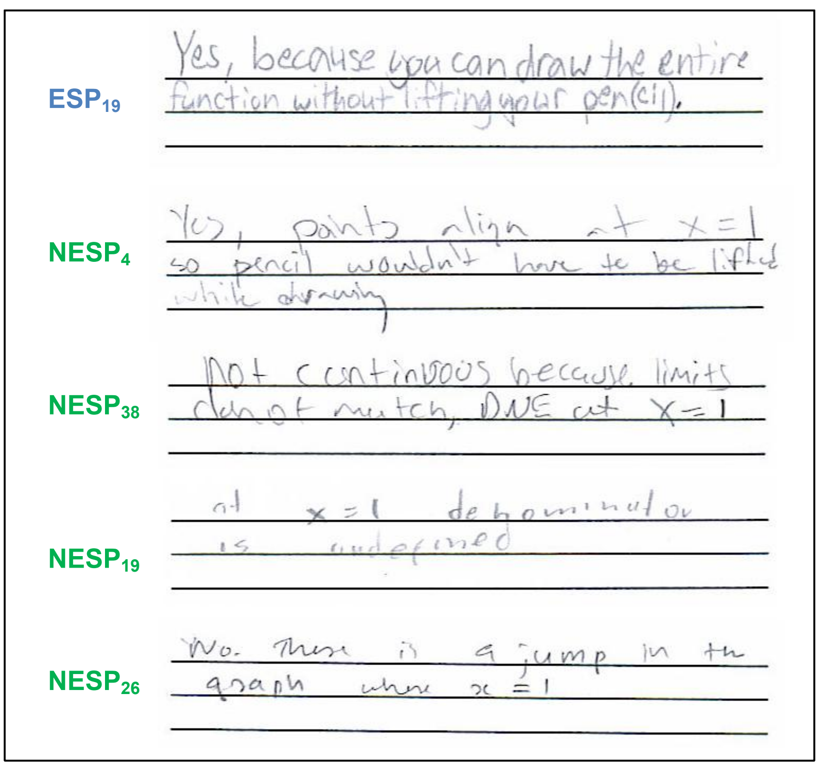


Figure 4.11 Examples of responses to Item #2-C from ESP student 19 and non-ESP students 4, 38, 19 & 26.

4.3 Assessment Items #3-C and #2-E

Assessment Item #3-C addressed the existence of a limit at a point as the defining characteristic that determines continuity at that point. Figure 4.12 displays the studentsô performance, showing that 59% of the ESP students and 51% of the non-ESP students answered Item #3-C correctly. Thus, performance amongst the two groups was roughly the same.

(a) (b)

Figure 4.12 Score distribution for Item #3-C for (a) ESP students and (b) non-ESP students.

Item #2-E presented a piece-wise function without a graph, and was related to Item #3-C, in that the function was not defined at a point, yet the limit did exist. In this case, the function was not continuous on the given interval, because the point at which the function was not defined was within the interval. According to Figure 4.13, 64% of the ESP students and 48% of the non-ESP students answered Item #2-E correctly. Moreover, 59% of the ESP students and 38% of the non-ESP students provided a quality response. Thus, not all students that answered accurately had a quality justification. In both cases of the ñAR: 3ò and ñCUR: 4+ò scores, the ESP students had a higher percentage than the non-ESP students. Comparing the accuracy of Item #2-E with that of Item #3-C, the ESP students performed slightly better on Item #2-E than #3-C, but the non-ESP students did slightly better on #3-C than #2-E.

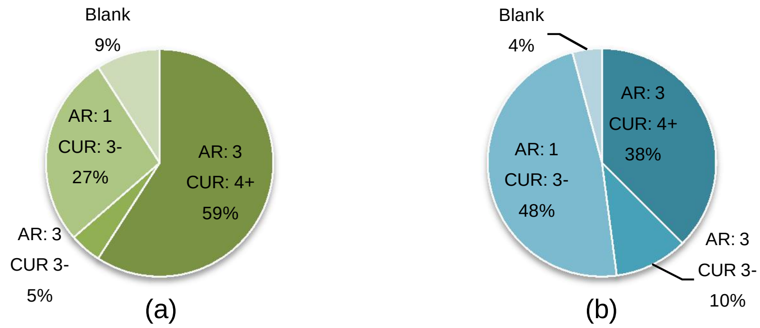


Figure 4.13 Score distribution for Item #2-E for (a) ESP students and (b) non-ESP students.

The last example, ESP_8, in Figure 4.14 shows a studentsôcorrect response to Item #2-E, and how it was related to Item #3-C. The first four examples, all of which are incorrect, show the limited view that students have regarding limits and continuity. In summary, there were more than twenty different types of incorrect answers provided for Item #2-E.

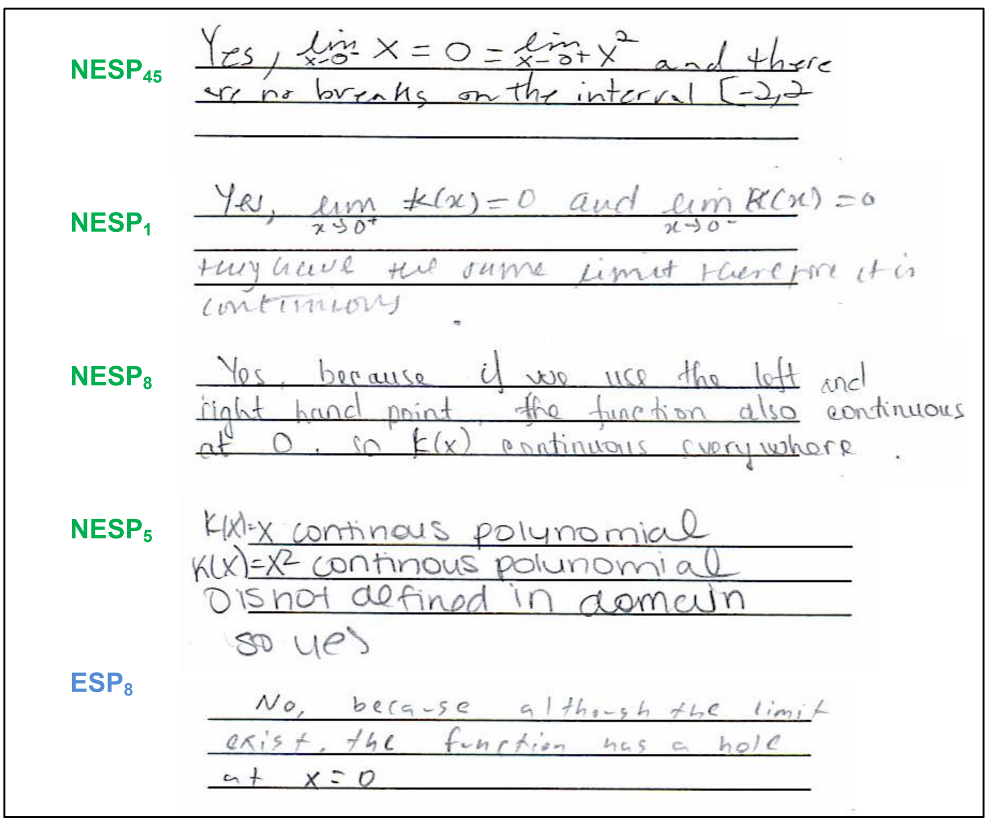
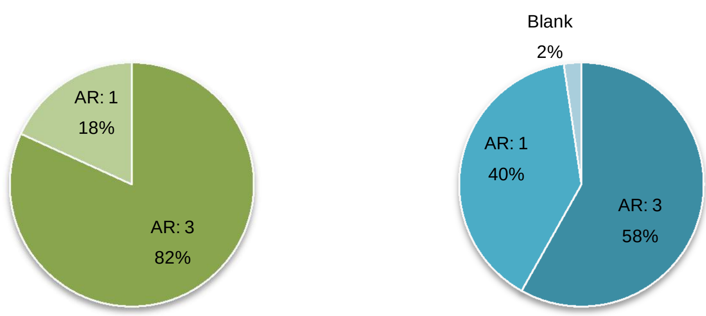


Figure 4.14 Examples of reponses to Item #2-E from ESP student 8
and non-ESP students 45, 1, 8, & 5.

4.4 Assessment Items #3-D and #1-C

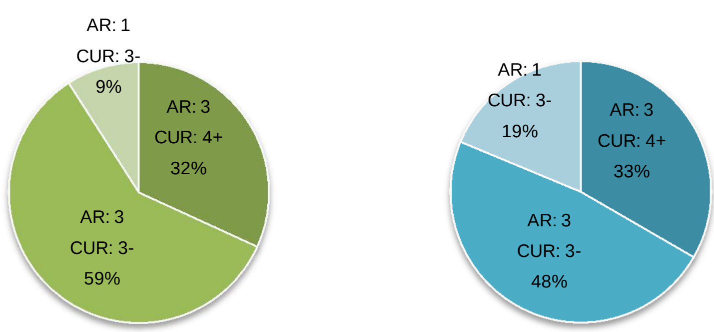
Assessment Item #3-D addressed continuity in the scenario in which a function's graph contains a sharp "corner". As interpreted through Figure 4.15, 18% of the ESP students and at a percentage nearly double this, 40% of the non-ESP students believed that the presence of a sharp "corner" renders a function discontinuous, resulting in them receiving an AR score of 1.



(a) (b)

Figure 4.15 Score distribution for Item #3-D for (a) ESP students and (b) non-ESP students.

Being linked to Item #3-D, Item #1-C presented a piece-wise function whose provided graph showed the existence of a sharp "corner". According to Figure 4.16, 91% of the ESP students and 81% of the non-ESP students agreed that the function was continuous, and thus received an AR score of 3. However, with roughly the same percentages from both groups, only 32% of the ESP students and 33% of the non-ESP students were able to justify their answer with a quality response, resulting in CUR scores of 4 or 5.



(a) (b)

Figure 4.16 Score distribution for Item #1-C for (a) ESP students and (b) non-ESP students.

Of the students that incorrectly stated that the function was discontinuous, 5 non-ESP and 2 ESP students explicitly used the existence of the sharp corner in the graph of the function as grounds for determining the function's continuity. Three other non-ESP students referred to the sharp corner as a "kink," "angle," or "sudden change." Figure 4.17 below shows an example of some responses to Item #1-C.

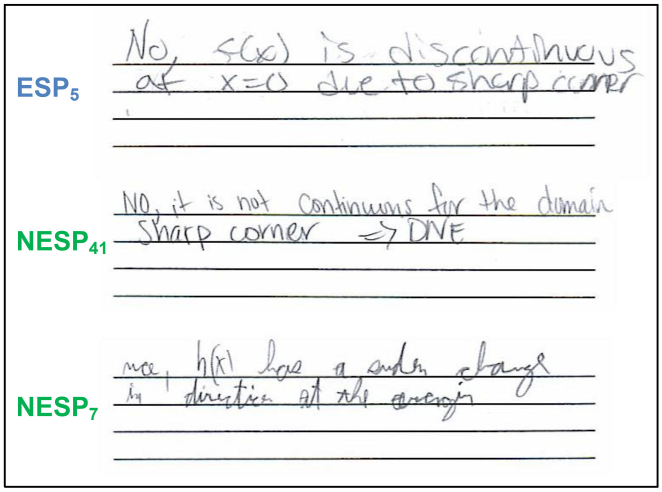
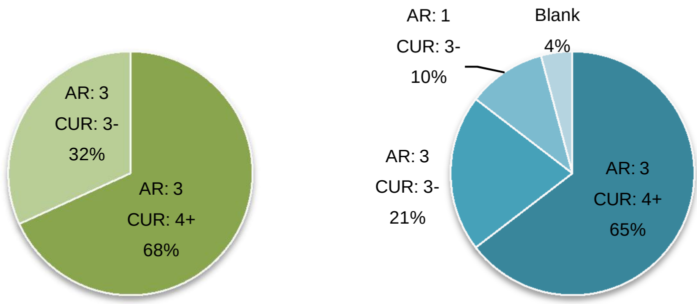


Figure 4.17 Example responses to Item #1-C from ESP student 5 and non-ESP students 41 & 7.

4.5 Assessment Item #4

Assessment Item #4 was the first problem on the concept of the derivative, aided by a labeled graph of a function. More specifically, though, Item #4-A did not require prior knowledge to the concept, as it was an introductory part that simply asked the students to determine the value of a function evaluated at a given x, using the graph, a concept which is commonly learned in Algebra II. Figure 4.18 shows that though all the ESP students evaluated the function correctly, only 68% of the students provided a quality justification for their answer. On the other hand, 86% of the non-ESP students evaluated the function correctly, with 65% of the non-ESP group that accounted for a quality response. Thus, ESP students did better in terms of providing the correct numerical value, but in regards to the quality of their responses, the two groups performed roughly the same.



(a) (b)

Figure 4.18 Score distribution for Item #4-A for (a) ESP students and (b) non-ESP students.

In Figure 4.19 are common examples of the responses for Item #4-A. Two ESP and 2 non-ESP students derived an equation for line l as shown by ESP student #6 (ESP_6). Eight non-ESP and 4 ESP students submitted responses similar to non-ESP student #2 ($NESP_2$) as their justification. The remaining two examples are responses that returned no numerical answer and only had an explanation of a way to determine the value.

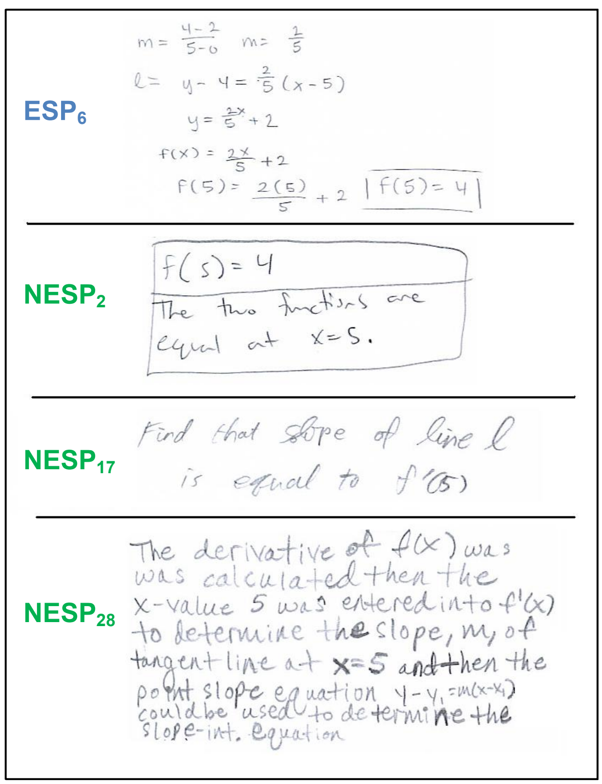


Figure 4.19 Examples of responses to Item #4-A from ESP student 6 and non-ESP students 2, 17, & 28.

Assessment Item #4-B was the part of Item #4 that did require some knowledge of the concept of the derivative. For this item, students were asked to determine the value of the derivative of the function at a given x, using the information provided in the graph. Of the two groups, 45% of the ESP students and 35% of the non-ESP students provided the correct numerical value, shown in Figure 4.20. One of the non-ESP students, which accounted for 2% of the group, received an AR score of 2, since their answer showed minor calculation error. Thus, ESP students were better at evaluating the value of the derivative of the function at the desired point. In regards to their answer justifications, 41% of the ESP students, as well as 41% of the non-ESP students, provided quality reasoning for their submitted numerical answer.

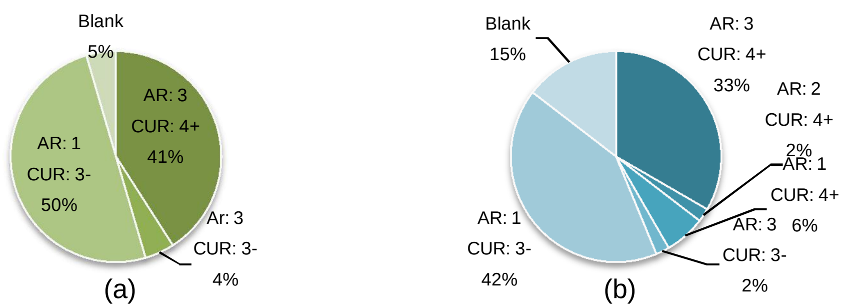


Figure 4.20 Score distribution for Item #4-B for (a) ESP students and (b) non-ESP students.

Comparing the score distributions for Item #4-A and #4-B, both groups of students showed higher percentages in accuracy for Item #4-A than #4-B. Moreover, both groups also showed higher percentages in responses that received scores indicating quality justifications for Item #4-A than #4-B.

Figure 4.21 on the next page shows a variety of student responses to Item #4-B. Of the responses that received AR scores of 1, eight ESP and twelve non-ESP students provided a numerical answer of 4, three ESP and six non-ESP students provided other numerical values, and six non-ESP students merely provided an explanation with no numerical answer. Of the students that provided the answer of 4 to Item #4-B, two ESP and five non-ESP students justified their answer as done by ESP student #3 in Figure 4.21. On the other hand, four ESP students and three non-ESP students justified their numbers, much like ESP student #18ôs reasoning in Figure 4.21, that the two lines touched at that point. Four other students (two from each group) arrived at the answer 4, after using line l directly as their function. Two students (one from each group) provided the correct answer of $\frac{2}{5}$ thus receiving an AR score of 3, but provided either a poor justification or none at all.

ESP$_3$

$f'(5)=4$ because point is given on graph

ESP$_{18}$

$f'(5):4$ $\quad f'(5)=f(5)$ b/c @ that point they are tangent & touch each other

NESP$_{17}$

Find the function of the derivative of $f(x)$ and plug in 5 to get $f'(5)$.

NESP$_{22}$

$f'(5)=0$

The tangent line is the derivative of $f(x)$.

ESP$_{10}$

$f'(5)=2$ because it is decreasing to 0 because $f(x)$ is approaching a max

ESP$_6$

$$f(x)=\frac{2x}{5}+2$$

$$f'(x)=\frac{2x(0)-(5)(2)}{5^2}+2$$

$$f'(5)=\frac{10}{25}+\frac{50}{25}$$

$$\boxed{f'(5)=\frac{60}{25}}$$

NESP$_2$

$$f'(5)=\frac{y_2-y_1}{x_2-x_1}=\frac{4-2}{5-0}=\boxed{\frac{2}{5}}$$

The mean value theorem states that on the interval

Figure 4.21 Examples of responses to Item #4-B from ESP student 3, 18, 10, & 6 and non-ESP student 17, 22, & 2.

4.6 Assessment Items #5, #6, and #7

These three assessment items were all related to one another in that they each presented the same ĩtypeòof problem on the concept of the derivative, in which the student was to understand the Intermediate Value Theorem in order to efficiently answer the question. The only differences between the items were that the given values were altered for each problem and each item was worded in a way as to present problem in a different format. Item #5 presented the problem algebraically, Item #6 presented it geometrically, and Item #7 presented the concept in a physical, applications-based method.

The results for the three items for both ESP and non-ESP groups shown in Figure 4.22, are summarized in Table 4.2. In regards to the AR scores for the two groups, the ESP students had a higher percentage that answered correctly for Items #5 and #6 than the non-ESP students. For Item #7, both groups performed the same in terms of accuracy. For all three items, a greater percentage of ESP students provided quality justifications for their responses than the non-ESP students.

In analyzing the three items collectively, both ESP and non-ESP groups performed their best, conceptually, on Item #6 which presented the problem in a geometric format (shown by the orange-shaded cells in the table). In terms of the accuracy of their Yes/No response, the ESP students also did their best on Item #6, whereas the non-ESP students answered best on Item #7, which was the physical, applications-based format (shown by the blue-shaded cells).

Table 4.2 Summary Percentages for Correct and Quality Responses for Items #5, #6, and #7

	Item #5		**Item #6**		**Item #7**	
Student	**Correct (AR: 3)**	**Quality Response (QR) (CUR: 4+)**	**Correct**	**QR**	**Correct**	**QR**
ESP	68%	41%	77%	54%	68%	36%
Non-ESP	52%	31%	58%	37%	68%	31%

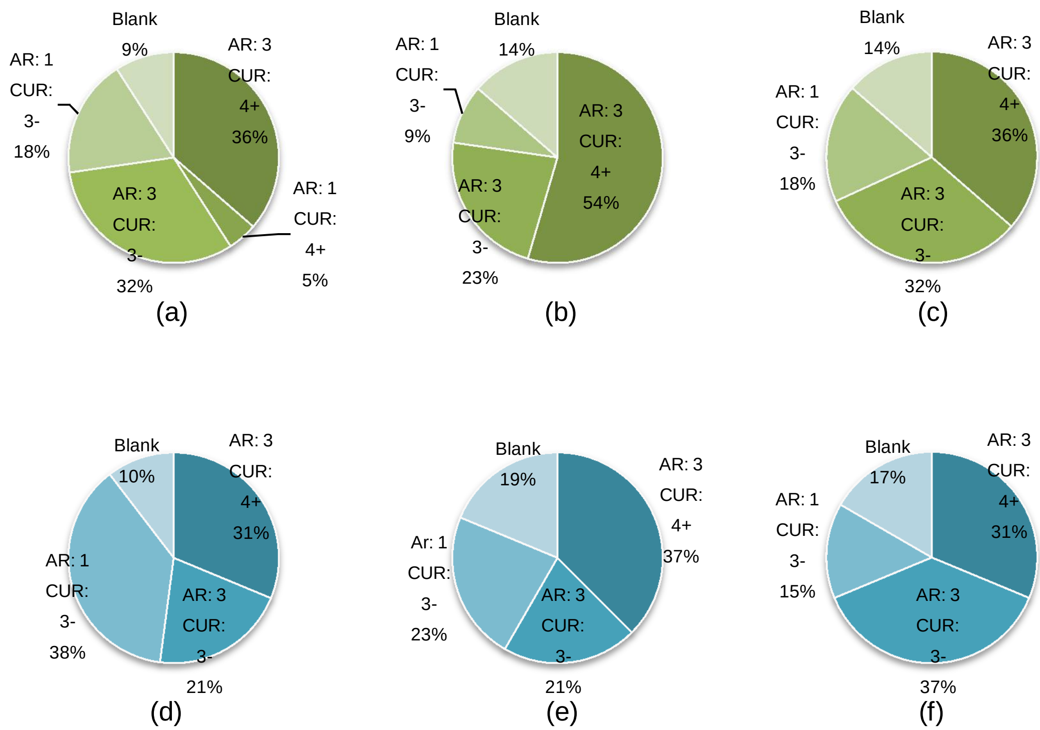


Figure 4.22 Score distribution for ESP students for (a) Item #5, (b) Item #6, (c) Item #7, and non-ESP students for (d) Item #5, (e) Item #6, and (f) Item #7.

Observing the performance of the ESP group alone, the studentsô accuracy percentages for Item #5 and Item #7 are the same, though their conceptual reasoning percentages were not equivalent, as the quality of their justifications were weaker for Item #5 than Item #7. On the other hand, the non-ESP studentsôconceptual reasoning faired the same for Item #5 and #7, in terms of percentages, though their accuracy did not.

In the case of all three items, there was roughly 9-19% of each group that left the problems blank. These blanks consisted of the students who did not have enough time to answer the problems and of students who did not attempt the problem. In terms of the students whose responses received AR scores of 1, almost all of them showed poor conceptual understanding, and thus furthermore received CUR scores of 1, 2 or 3.

In Figure 4.23 are examples of the types of responses given by the students with AR scores of 1 for Item #5. Five of the eighteen non-ESP students used the Intermediate Value

Theorem as shown by non-ESP student #46 in the figure. Three other non-ESP students used the Mean Value Theorem as their justification, shown by non-ESP student #40, and two non-ESP students used continuity, shown by non-ESP student #10. Six non-ESP students and one ESP student either provided no justification for their answer or only showed a slight attempt at solving the problem. The remaining students from each group provided various answers, one of which is shown by ESP student #9 in Figure 4.23.

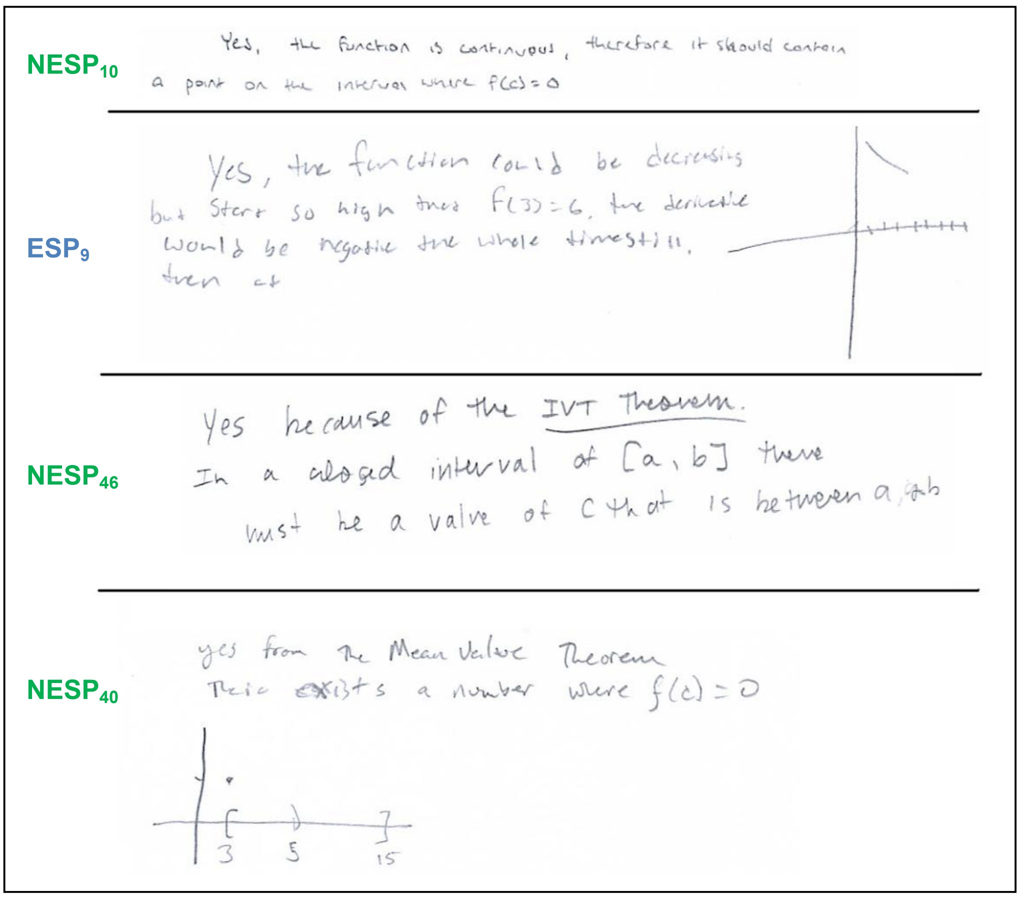


Figure 4.23 Examples of responses to Item #5 from ESP student 9 and non-ESP students 10, 46, & 40.

In the case of Item #6, two ESP students and ten non-ESP students received an AR score of 1 and their justifications received low CUR scores. Three non-ESP students justified their answer as non-ESP student #38 did in Figure 4.24, using continuity as their only reason.

Two students, one ESP and one non-ESP, did not provide any justification for their answers. The remaining examples in Figure 4.24 show the types of various responses from the other students.

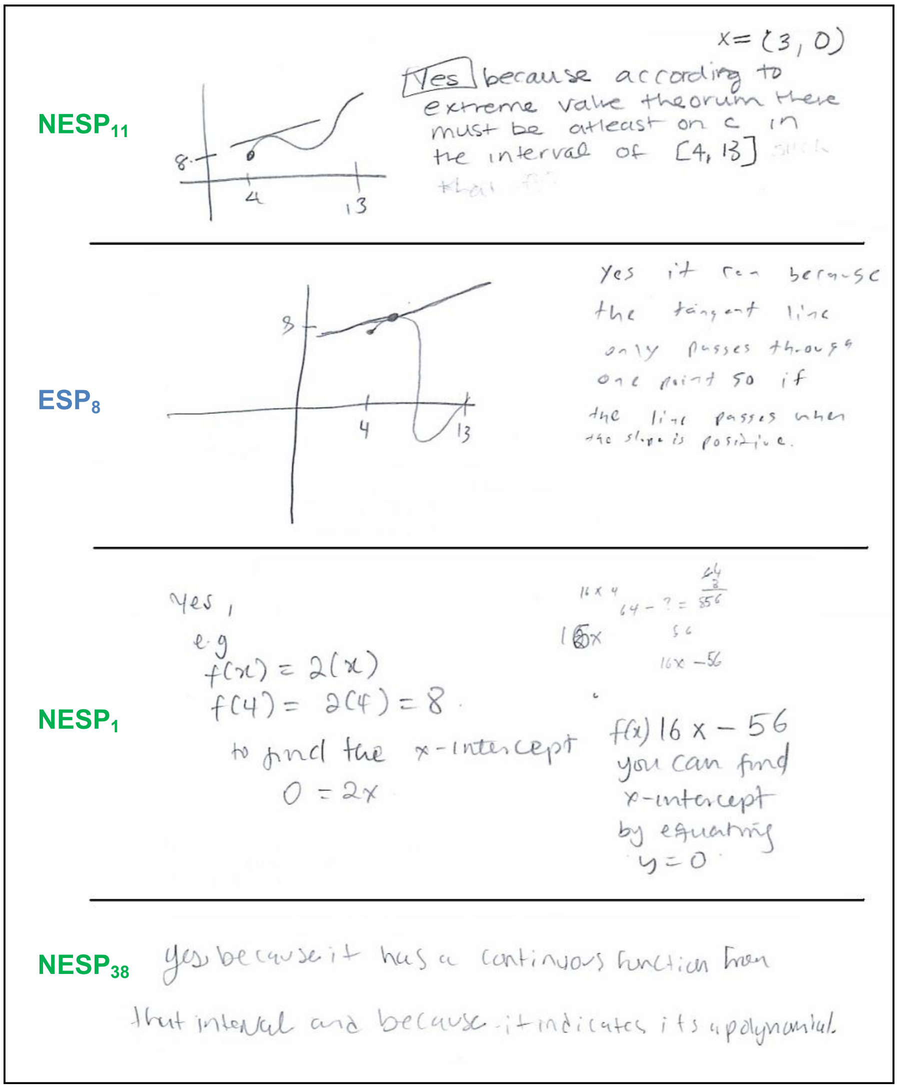


Figure 4.24 Examples of responses to Item #6 from ESP student 8 and non-ESP students 11, 1, & 38.

Of the small percentage of students that received an AR score of 1 for Item #7, seven were from the non-ESP group and four were from the ESP group. Three students total (with one being an ESP student) gave a specific point to answer the situation, as shown by non-ESP student #9 in Figure 4.25. Three other students, one ESP and two not, simply used the ñrate of change was positiveò as their justification. The remaining students with an AR score of 1 for Item #7 either replied with a ñyesò and no justification, or showed minor attempt to answer the problem with no implication of whether the answer was a yes or a no. Figure 4.25 also shows two other examples in which case a student received an AR score of 3, but their justifications were weak, thus giving them CUR scores of 1, 2, or 3.

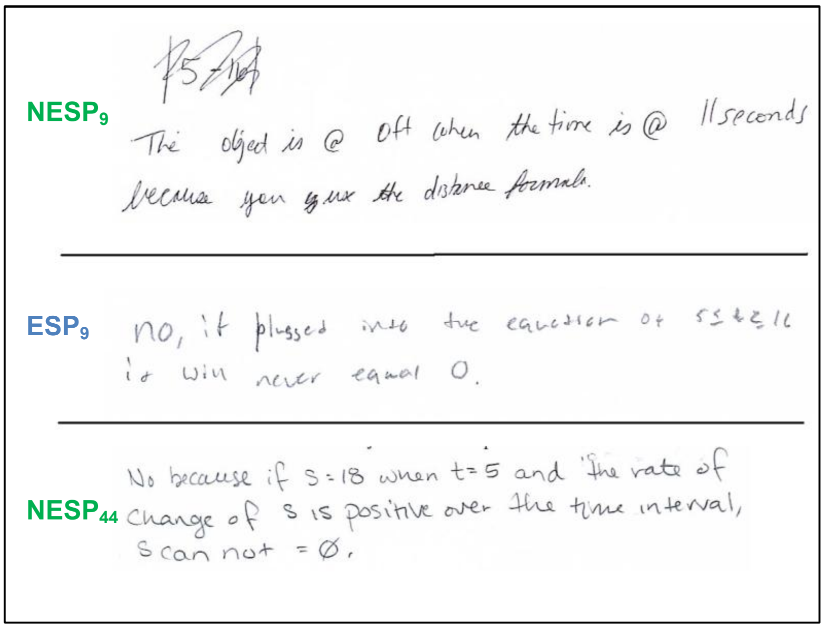


Figure 4.25 Examples of responses to Item #7 from ESP student 9 and non-ESP students 9 & 44.

<u>4.7 Consideration of the StudentsôBackground Information</u>

In the following sections, the results of the studentsôscores are projected with some of the corresponding data that they provided in the AURAS questionnaires. These sections, like all the previous sections of Chapter 4 up to this point, are meant for descriptive purposes. Recall that though all 22 ESP students submitted their initial questionnaire, 5 non-ESP studentsô questionnaires are not available, and thus only 45 of the 48 non-ESP studentsô scores are presented with the background information.

4.7.1 Most Advanced Math Class Taken in High School

The following, Figure 4.26, shows the most advanced math class that the students took in high school to get an idea of the type of math background that the students had upon enrolling in Calculus I at the university. Though the majority of the students had either calculus or pre-calculus before, there were also a few students from both groups that had limited math experience. Note that in the figure, the students under the label of ñCalculusò are those whom have taken any form of calculus, whether it was an AP course or not.

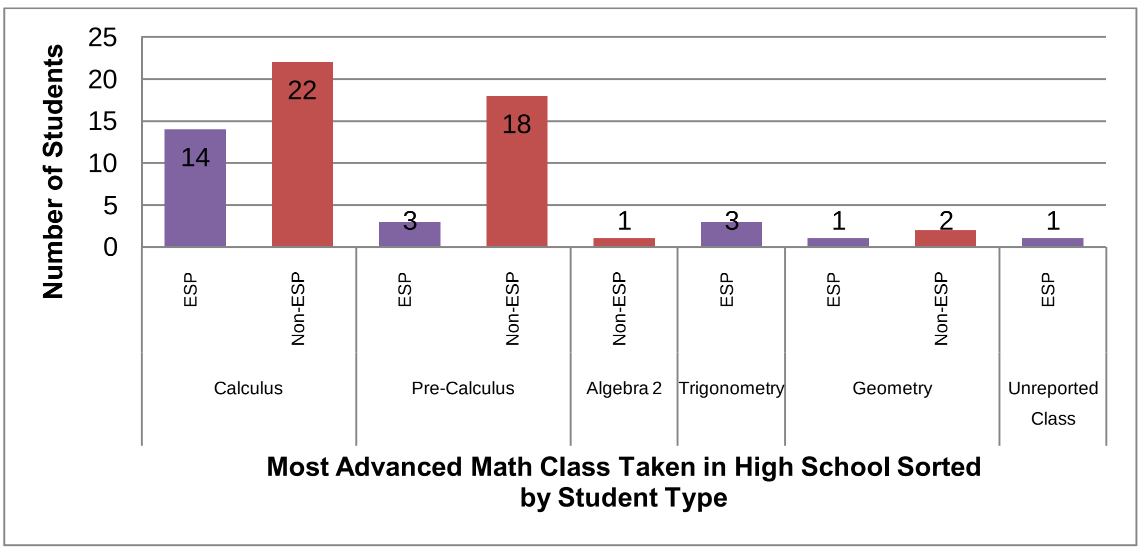


Figure 4.26 Summary of the most advanced math class taken in high school sorted by student type.

4.7.1.1 Corresponding Performance on Item #4-A

As mentioned earlier, Item #4-A required students to determine the value of a function using a provided graph and a given x, a type of problem most commonly introduced in Algebra II courses. Taking the ESP studentsô scores from #4-A and sorting them by their most advanced math class taken, Figure 4.27 shows that all the students provided a correct numerical value regardless of their background. Moreover, the quality responses were not limited to the students that have taken higher math classes.

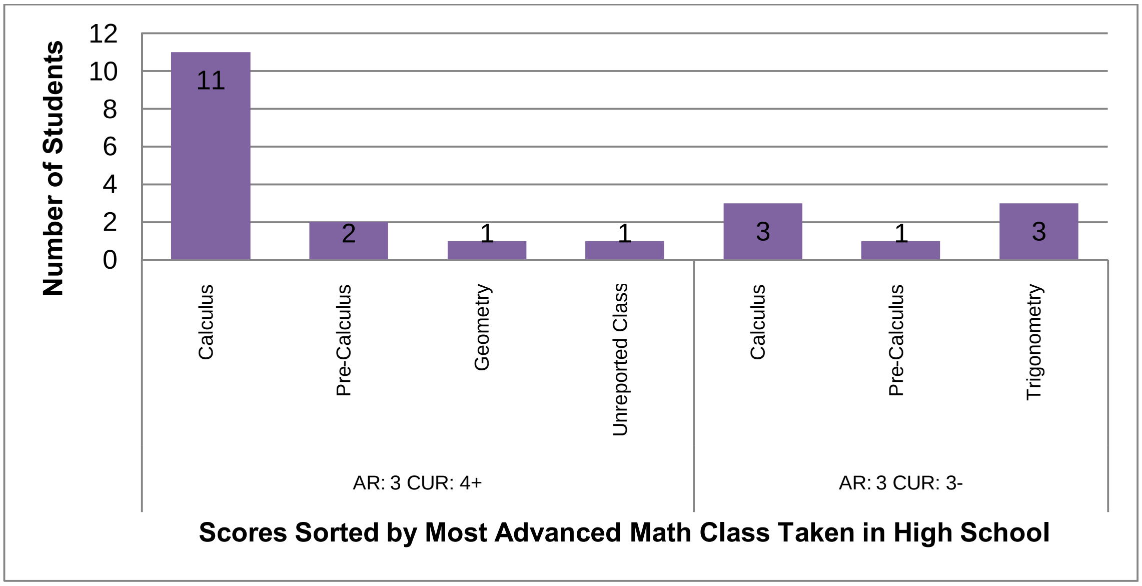


Figure 4.27 ESP studentsôperformance on Item #4-A sorted by most advanced math class taken in high school.

In Figure 4.28, which shows the non-ESP studentsôscores, the students that answered accurately had various backgrounds. In regards to the students that received an AR score of 1, they actually had either calculus or pre-calculus in high school.

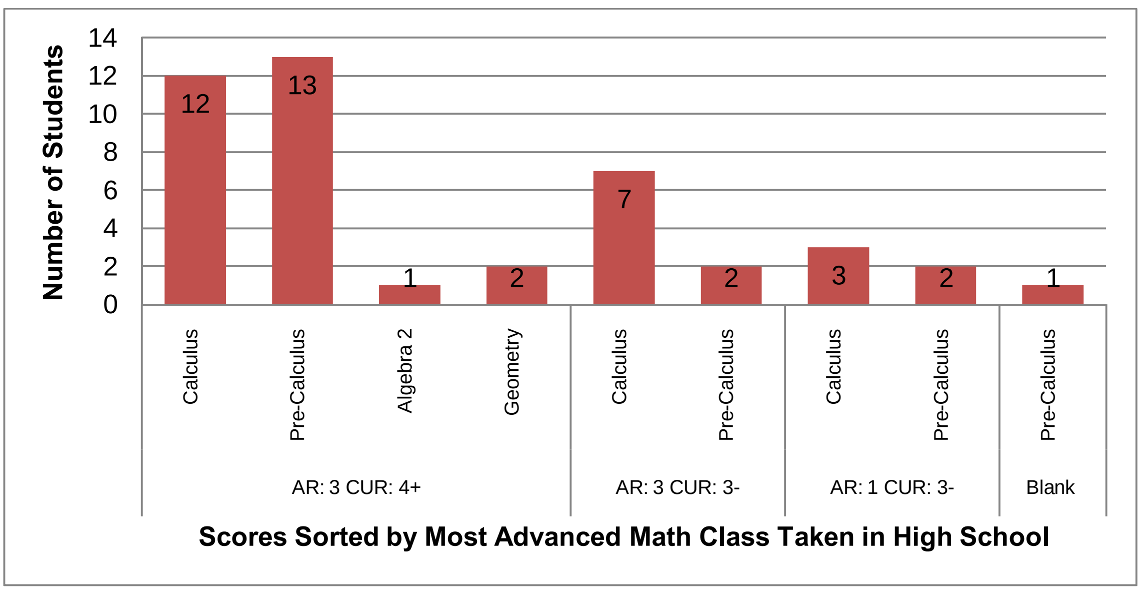


Figure 4.28 Non-ESP studentsôperformance on Item #4-A sorted by most advanced math class taken in high school.

4.7.1.2 Corresponding Performance on Item #4-B

Item #4-B, on the other hand, required some knowledge of calculus, because students were asked to determine the derivative of the function at the given x, using the information from the provided graph. As shown in Figure 4.29, 7 ESP students who have had calculus before provided an incorrect numerical answer than the 6 with calculus experience who were correct. Furthermore, there were 3 ESP students without high school calculus experience, who did fairly well both in accuracy and their conceptual reasoning.

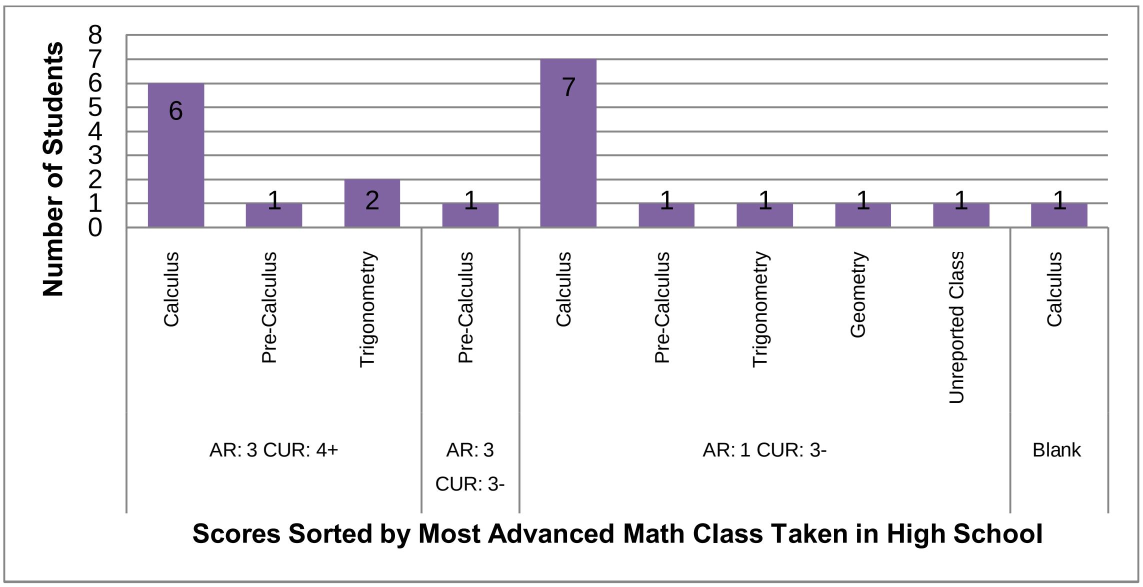


Figure 4.29 ESP studentsôperformance on Item #4-B sorted by most advanced math class taken in high school.

In the case of non-ESP students, Figure 4.30 shows that none of the students, whose most advanced math class was below the level of pre-calculus, did well in terms of accuracy nor did they provide quality justifications for their responses. However, there were also a few non-ESP students who did have pre-calculus and calculus before, who also performed on that level. Also, all of the students who received an AR of 3 and a CUR score of 4 or 5 had pre-calculus or calculus experience in high school.

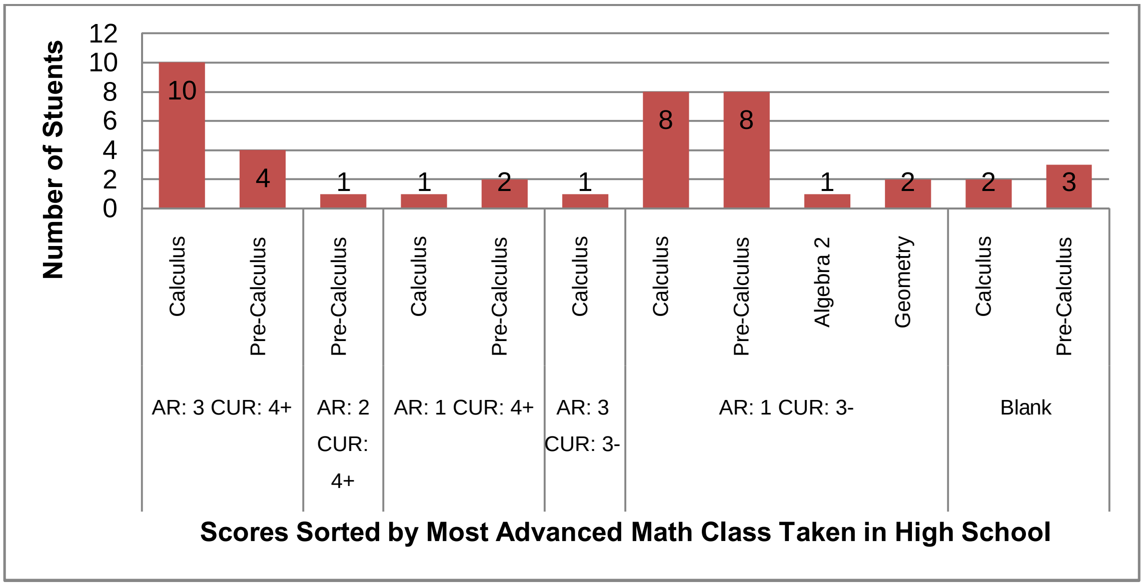


Figure 4.30 Non-ESP studentsô performance on Item #4-B sorted by most advanced math class taken in high school.

4.7.2 Students' Calculus Exposure in High School

This section will now focus on only the students that have taken some type of calculus class in high school. Table 4.3 summarizes which students have had calculus and which have not, out of the 65 students that have completed the written assessment, as well as the initial AURAS questionnaire. So in regards to this section, 14 ESP students and 22 non-ESP studentsô performance will be examined.

Table 4.3 Amount of Students with and without Calculus Exposure

Student	Calculus	No Calculus	Total
ESP	14	8	22
Non-ESP	22	21	43

4.7.2.1 Corresponding Performance on Items #1 and #2

Restricting the data to the students who have had calculus class in high school, the responses to the parts of Items #1 and #2 that received an AR score of 3 and a CUR score of 4 or 5 were tabulated in Table 4.4. To clarify the table format, ñ*Aò under the merged column ñItem

#1" refers to Item #1-A, "*A" under the merged column "Item #2" refers to Item #2-A, and so forth. In the case of Item #1, the students from both groups were "balanced" in the sense that a greater percentage of ESP students received the designated score on the first two parts than their non-ESP counterparts and vice-versa for two of the other parts of Item #1, all of which are shown by the table's green-shaded cells. In regards to Item #2, however, a larger percentage of the non-ESP students received the specified score on the first four parts than the ESP students, exhibited by the blue-shaded cells. By this comparison, the non-ESP students that have had calculus class in high school, showed a higher percentage of "AR: 3 CUR: 4+" scores on the Items of #1 and #2, when taken collectively, than the ESP students that had prior calculus exposure.

Table 4.4 Percentages of Responses that Scored AR: 3 CUR: 4+ for the Parts of Items #1 and #2 Restricted to Students Who Had Prior Calculus Exposure

	Item #1					Item #2				
Student	***A**	***B**	***C**	***D**	***E**	***A**	***B**	***C**	***D**	***E**
ESP	71%	36%	29%	64%	0%	29%	64%	14%	29%	43%
Non-ESP	64%	18%	32%	64%	9%	82%	82%	32%	82%	32%

4.7.2.2 Corresponding Performance on Items #5, #6 and #7

The results of Items #5, #6, and #7 were restricted to the sample of students who have had a high school calculus course and are displayed in Table 4.5. The scores for the responses were grouped by (1) the responses that received an AR score of 3, (2) the responses that received a CUR score of a 4 or 5, and (3) the responses that received both an AR score of 3, and a CUR score of either a 4 or 5. According to the table, the ESP students performed better than the non-ESP students on Item #6 and vice-versa for Item #7. In regards to Item #5, though a greater percentage of non-ESP students received an AR score of 3, a larger portion of the ESP students provided a response with quality justification. However, both groups were equivalent in the percentage of students that answered both accurately and with a quality response.

Table 4.5 Percentages of Responses with Specified Scores for Items #5, #6, and #7 Restricted to Students Who Had Prior Calculus Exposure

	Item #5		Item #6		Item #7	
Score Specification	**ESP**	**Non-ESP**	**ESP**	**Non-ESP**	**ESP**	**Non-ESP**
AR: 3	57%	68%	79%	63%	65%	68%
CUR: 4+	57%	50%	65%	45%	36%	41%
AR: 3 & CUR: 4+	50%	50%	65%	45%	36%	41%

4.8 Statistical Inferences

This section includes the data that was acquired with the help of the statistics professor as mentioned in the Methodology. Recall that for this portion, the responses were categorized and rescored so that each problem received just one composite score, which was a summation of the scores for all of its parts. Moreover, from the AURAS questionnaire responses, the one that showed the most sufficient variety of data for statistical purposes was the question that dealt with the studentsôAP Calculus exposure.

4.8.1 Statistical Analysis from the AP Calculus Exposure Data

The studentsô AP Calculus exposure data is summarized in 4.31. This data set was used, along with all of the studentsônew composite scores, to determine whether the studentsô performance was in any way influenced by their background in AP Calculus.

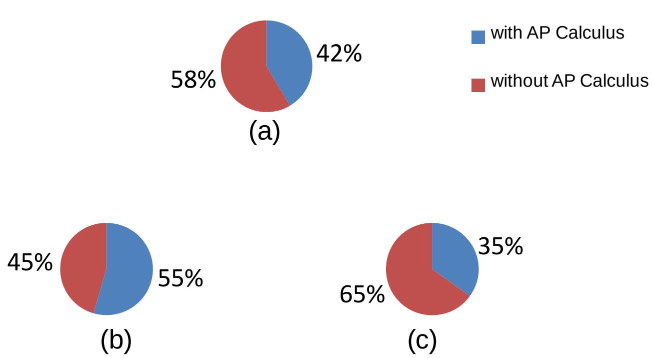


Figure 4.31 Summary of the students with and without AP Calculus, where (a) is the entire sample, (b) is the ESP group only, and (c) is the non-ESP group only.

Testing the scores of all the items, for both ESP and non-ESP students, with their AP Calculus data resulted in some sort of interaction with the scores for Item #2. To investigate further, the students that had taken AP Calculus were separated from the students that did not take the class. From here, the AP and non-AP Calculus students were examined separately with the scores from Item #2 for ESP and non-ESP students.

Figure 4.32 shows the results of the two-sample t-test based on ranked scores for Item #2 with the AP Calculus group. This data has an indicated p-value, for the two-sided test, of 0.0140. This p-value, being less than 0.05, indicates that the difference in means of the two groups (ESP and non-ESP students) is statistically significant.

The TTEST Procedure

Statistics

Variable	Group	N	Lower CL Mean	Mean	Upper CL Mean	Lower CL Std Dev	Std Dev	Upper CL Std Dev	Std Err	Minimum	Maximum
rankscore2	ESP	10	12.014	26.15	40.286	13.592	19.761	36.076	6.249	7.5	61.5
rankscore2	Non	15	34.243	41.533	48.824	9.638	13.164	20.762	3.399	19	61.5
rankscore2	Diff (1-2)		-28.96	-15.38	-1.811	12.491	16.072	22.544	6.5612		

T-Tests

Variable	Method	Variances	DF	t Value	Pr > |t|
rankscore2	Pooled	Equal	23	-2.34	0.0280
rankscore2	Satterthwaite	Unequal	14.3	-2.16	0.0480

Equality of Variances

Variable	Method	Num DF	Den DF	F Value	Pr > F
rankscore2	Folded F	9	14	2.25	0.1674

Figure 4.32 Results of the two-sample t-test for Item #2 (ranked) scores with the AP Calculus group.

Figure 4.33 shows the results of the two-sample t-test for the ranked scores for Item #2 with the non-AP Calculus group. The p-value for the two-sided test is 0.0030. The results here indicate that the difference in means for the ESP and non-ESP students is statistically significant.

The TTEST Procedure

Statistics

Variable	Group	N	Lower CL Mean	Mean	Upper CL Mean	Lower CL Std Dev	Std Dev	Upper CL Std Dev	Std Err	Minimum	Maximum
rankscore2	ESP	10	33.382	43.15	52.918	9.3921	13.655	24.928	4.318	19	61.5
rankscore2	Non	28	18.092	25	31.908	14.086	17.816	24.251	3.367	1	61.5
rankscore2	Diff (1-2)		5.5439	18.15	30.756	13.721	16.872	21.917	6.2157		

T-Tests

Variable	Method	Variances	DF	t Value	Pr > |t|
rankscore2	Pooled	Equal	36	2.92	0.0060
rankscore2	Satterthwaite	Unequal	20.7	3.31	0.0033

Equality of Variances

Variable	Method	Num DF	Den DF	F Value	Pr > F
rankscore2	Folded F	27	9	1.70	0.4085

Figure 4.33 Results of the two-sample t-test for Item #2 (ranked) scores with the non-AP Calculus group.

With these results, t-tests were then done on the scores of Item #2 without any background variables, to determine if there was anything significant between the groups, before taking into account the studentsôAP Calculus exposure. Recall that the maximum composite score possible for Item #2 was a 20. The initial t-test, with results in Figure 4.34, involved the entire sample to test the hypothesis of whether the mean of the scores was greater than 13.4. The two-sided testȹs p-value was 0.0407, thus indicating that the sampleȹs mean was statistically significantly different from the test value of 13.4.

The sampleȹs scores for Item #2 were then separated, and t-tests were done to test the hypothesis of whether the mean of the scores of each group was higher than 13.6. Figure 4.35 shows that with a p-value of .04835 (half the value of .0967), the ESP students had a statisically significant different mean than the test value of 13.6. On the other hand, with a p-value of 0.3158, the non-ESP students did not have a statistically significant different mean than the test value of 13.6.

The TTEST Procedure

Statistics										
Variable	N	Lower CL Mean	Mean	Upper CL Mean	Lower CL Std Dev	Std Dev	Upper CL Std Dev	Std Err	Minimum	Maximum
Score2	63	13.303	14.159	15.015	2.8921	3.3993	4.1239	0.4283	2	20

T-Tests			
Variable	DF	t Value	Pr > |t|
Score2	62	1.77	0.0814

Figure 4.34 Results of the one-sample t-test for all studentsôItem #2 scores with the hypothesis of the mean greater than 13.4.

The TTEST Procedure

(a)

Group=ESP

Statistics										
Variable	N	Lower CL Mean	Mean	Upper CL Mean	Lower CL Std Dev	Std Dev	Upper CL Std Dev	Std Err	Minimum	Maximum
Score2	20	13.363	14.8	16.237	2.3355	3.0711	4.4855	0.6867	10	20

T-Tests			
Variable	DF	t Value	Pr > |t|
Score2	19	1.75	0.0967

(b)

Group=Non

Statistics										
Variable	N	Lower CL Mean	Mean	Upper CL Mean	Lower CL Std Dev	Std Dev	Upper CL Std Dev	Std Err	Minimum	Maximum
Score2	43	12.772	13.86	14.949	2.9156	3.5361	4.4944	0.5392	2	20

T-Tests			
Variable	DF	t Value	Pr > |t|
Score2	42	0.48	0.6316

Figure 4.35 Results of the one-sample t-test for Item #2 scores of the (a) ESP group and (b) non-ESP group, with the hypothesis of the mean greater than 13.6.

As stated before, of all the scores of the items and background variables available, the only data sets that showed any statistical interactions or inferences was Item #2 and AP Calculus. The implications of this and the descriptive results of the other sections of this chapter are further discussed in the following chapter.

CHAPTER 5

DISCUSSION AND CONCLUSIONS

In this chapter, the implications from the results outlined in Chapter 4 are discussed and connected back to the results and comments from other research material.

5.1 Compartmentalization

Recall the compartmentalization phenomena mentioned by Vinner & Dreyfus (1989), in which the participants exhibited inconsistent behavior when they were unable to apply the definition that they were familiar with. In regards to the results from this study, comparing the percentages shown for Item #3-D with the accuracy percentages for Item #1-C, there does not seem to be much difference, aside from slight change in the non-ESP studentsô percentages. However, taking a closer look at each studentsô response to both items, 4 of the ESP students answered ñTrueò for Item #3-D, that a function is discontinuous if its graph contains a sharp corner. Of those four students, 2 of them had explicitly used the presence of a sharp corner as their justification for why they believed Item #1-C was discontinuous. Thus while these two students stuck with their sharp corner theory, the other two showed inconsistency. Within the non-ESP group, 19 students answered Item #3-D incorrectly, and of those, 9 of them stated that the presence of a ñsharp cornerò, ñkinkò or ñangleò in the functionôs graph rendered the function discontinuous. Thus, the other ten students exhibited inconsistency in their answers for the two problems, while these nine students seemed to have developed a misconception within their concept image of continuity.

Compartmentalization is also apparent in the case of Item #3-A, #1-B, and #1-E. The discrepancy is not so much present for the ESP students in regards to Items #3-A and #1-B, but instead between the Items #3-A and #1-E. The non-ESP students, on the other hand, exhibited a difference in accuracy in regards to Item #3-A and both of Items #1-B and 1-E. Looking more closely at the raw scores, many of the students that answered Item #3-A correctly, had answered either of Items #1-B and #1-E incorrectly or both, as shown in Table 5.1.

Table 5.1 Comparison of Items #3-A, #1-B, and #1-E

Student	**All of #3-A, #1-B, #1-E Correct**	**Both #3-A and #1-E Correct, and #1-B Incorrect**	**Both #3-A and #1-B Correct, and #1-E Incorrect**	**#3-A Correct, and Both #1-B and #1-E Incorrect**	**#3-A Incorrect or Blank**
ESP (n=22)	1	2	10	6	3
Non-ESP (n=48)	5	6	9	26	2

Thus, 18 ESP and 15 non-ESP students showed some sort of inconsistency between their answers for the three items. However, some of the students justified their answers with responses that suggest that they observed the continuity of the function over an interval instead of the function's domain, as stated in the problem. The existence of a vertical asymptote in Item #1-B and an undefined point in Item #1-E was sufficient for the students to claim that the function was discontinuous on the domain. If so, then perhaps these students were more accustomed to dealing with continuity on a given interval, rather than on the function's domain. If that is not the case, then it could be that the students have a weak understanding of the distinction between an interval and the function's domain. This is highly plausible since many of the research literature have shown that weak underpinnings or underdeveloped concepts hinder the performance of students.

5.2 Evidence of Weak Underpinnings

More specifically, though, in the case of Item #1-E and #2-E, there were a few students (mostly non-ESP students) that showed a weak understanding of continuity over an interval vs. continuity over a function's domain. In Figure 5.1, the top problem (Item #1-E) shows that the student responded as though they were determining whether the function was continuous over an interval, while for the bottom problem (Item #2-E), the student responded as though they were determining the function's continuity over its domain. Thus, in this student's case, their concepts of continuity over an interval and continuity over the function's domain are switched.

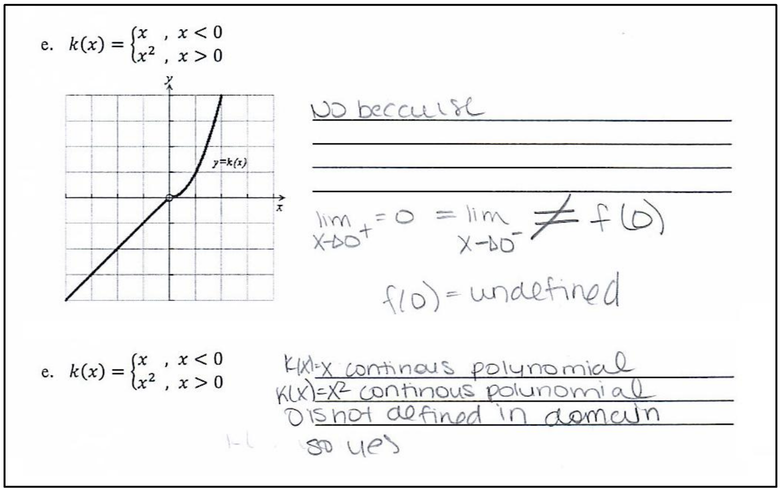


Figure 5.1 An example of a student's weak understanding of intervals versus a function's domain.

Also, in Figure 5.2 below, the student's response to both the top problem (Item #1-E) and the bottom problem (Item #2-E) included the word "domain." While their answer to Item #1-E was correct, their response to Item #2-E was flawed, in that they believed the "hole" was within the domain. In this case, the student exhibited a weak understanding of an interval vs. the domain of a function.

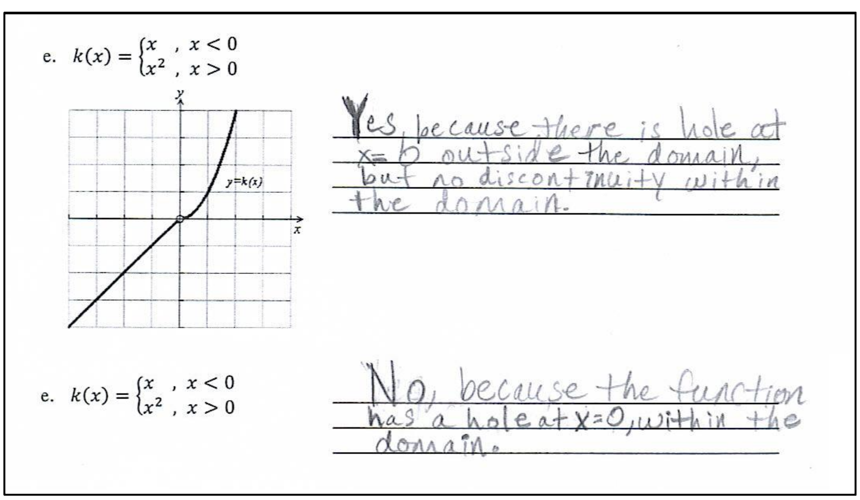


Figure 5.2 A second example of a student's weak understanding of intervals versus a function's domain.

Moreover, in regards to the insufficient development of strong dynamic conception, the concept of functions has a tendency to affect a studentøs concept of limits and continuity (Cottrill, et al., 1996). In the case of Items #1 and #2, there were a couple of responses from students (mainly non-ESP) who stated that a given function was discontinuous because it either was not a function or that the function did not exist, as shown in Figure 5.3.

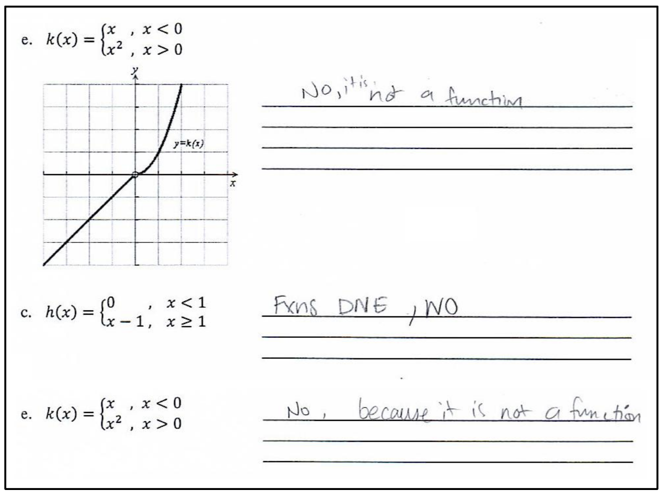


Figure 5.3 Examples of responses showing underdeveloped concept of function in regards to Items #1-C, #2-C, and #2-E (top to bottom).

On the same notion of the existence of weak underpinnings, there were responses to Item #4-A from two ESP and two non-ESP students, which suggested that the students had an underdeveloped concept of variables, as was also exhibited in White & Mitchelmore (1996). In Figure 5.4 below, although the given graph states that $y = f(x)$, the student used y within their derivation of an equation for line l. From there, they incorrectly equated the equation for line l with $f(x)$. Though they did get the correct numerical value, coincidentally because that point also lies on that line, the process that they took involved the improper use of the variables available. If the studentøs concept of variables had been stronger, they would have understood clearly what each variable was defined as and would not have taken this approach to solving the problem.

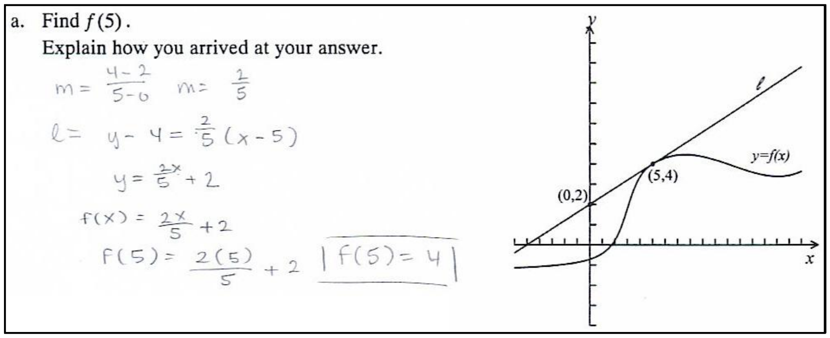


Figure 5.4 Example of a studentôs underdeveloped concept of variables for Item #4-A.

5.3 Implications from the Use of Visual Methods

5.3.1 Visuals and Performance on Item #4

A majority of the students determined the numerical answer for Item #4-A by directly using the provided graph, implying that the graph was both an easy and accessible tool for them. On the other hand, a few of the students, namely 2 ESP students and 2 non-ESP students attempted to justify their answer, by deriving an equation for the function in the graph, implying that perhaps they are more comfortable with the algebraic means of solving a problem.

In the case of Item #4-B, though, it was apparent in about 63% of the non-ESP students and about 50% of the ESP students that understanding the graph seemed to be an issue. Many of the students, as shown in Chapter 4, believed that the answer to Item #4-B was the same was the answer to Item #4-A, because either that was the point that the two lines had in common, or that it was simply given on the graph. This implies that they perhaps relied too heavily on the visual means, rather than realizing the necessity of needing to find a computational approach to deriving the answer. Moreover, the results show that the non-ESP students appear to have a weaker understanding of the connection between a tangent line and the functionôs derivative.

5.3.2 Visuals and Performance on Item #2

All the parts of Item #2 were presented without graphs. Since this was the case, there were instances in the studentsô responses that suggest a more algebraic-based thinking than geometric for them. In the case of Item #2-B, 3 students referred to the asymptote as a ŏhole,ò suggesting that they knew a discontinuity existed in the function, but they could not visualize which type. On the other hand, after examining the studentsô assessments, 5 ESP and 13 non-ESP students drew sketches of at least one of the parts of Item #2 to assist in their understanding of what the function looked like. However, this is not indicative of their success in answering the problem, as shown in Figure 5.5, where a studentsô incorrect sketch of Item #2-C led to the error in their response. This coincides with a past study, in which it was realized that graphs have a major influence on studentsô opinion of continuity and as a result of this, the students would sometimes answer questions incorrectly, though they knew facts that showed otherwise (Takaci, Pesic, & Tatar, 2006). The sketches do, however, show a readiness of some students to resort to visual methods, as in Dreyfus & Eisenberg (1994), when their students showed a readiness to approach function transformations using visual means.

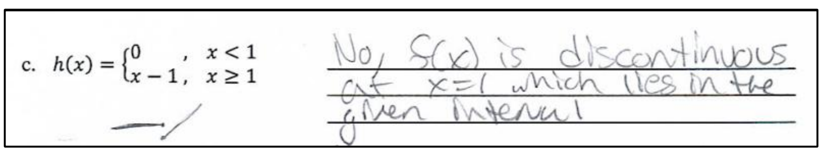


Figure 5.5 An example of a studentøs sketch leading to an incorrect response.

5.3.3 Visuals with Item #5, #6, and #7

The readiness to approach complicated problems is also apparent on the last two pages of the studentsô assessments. Ten of the ESP students and 14 of the non-ESP students sketched graphs to help visualize the situations put forth in the items. It should be noted that in the case of the three items, more students used graphs on Items #5 and #6, than they did on Item #7. This is probably because the format of Item #5 and #6 were more algebraic and geometrically based than Item #7. Unfortunately, not all of the students that used such a method were successful in their responses, but at least they attempted to utilize what they knew about the concept of derivatives graphically.

<u>5.4 More Comments on Items #5, #6, and #7</u>

Recall that Item #5 was presented in an algebraic format, Item #6 in a geometric format, and Item #7 in a more physical, applications-based method. From the responses to Item #5, the students more readily referred to theorems such as the Intermediate Value Theorem, Mean Value Theorem, and Squeeze Theorem, as part of their justifications for their responses to the problem. In regards to the other two items, only one student used a theorem in their response. The responses to Item #7 were the only ones that had any indication of a student attempting to find an actual point at which to satisfy the problem.

<u>5.5 The Statistical Analyses from Item #2</u>

As mentioned before, four parts of Item #1 were used in the assessment from Tall & Vinner (1981), with one additional part. Item #2 was then an adaptation of Item #1. However, unlike Tall & Vinner, the study in this paper targeted two groups, the ESP and non-ESP students, and their performance on the concepts of continuity and derivative. Moreover, the studentsôbackground variables were available for analysis, thus allowing the ability to test for statistical inferences, rather than just presenting the studentsôfrequency in types of responses.

Recall the various figures depicting the t-tests done with the composite score for Item #2 and the data containing the studentsôAP Calculus exposure and with the scores for Item #2 alone. In the initial t-tests, it was shown that in the case where the student had taken AP Calculus, the non-ESP students did statistically significantly better than the ESP students on Item #2. For the students that did not have any AP calculus, the ESP students did statistically significantly better than the non-ESP students on the same item. Thus, without the AP Calculus as a background variable, it appears that ESP did help the students to perform better on the parts of Item #2 than their non-ESP counterparts.

Because of the interaction that the scores had with AP Calculus, Item #2 was looked at more in depth. From the results of the t-tests, it was shown that the ESP students exhibited mean composite score that was statistically significantly higher than the test value of 13.6, where as the non-ESP students did not. Thus, in regards to Item #2, the ESP students performed statistically significantly better than the non-ESP students.

Thus, it can be assumed that there is something within Item #2 that should be considered more closely in the future, although it is unclear as to what it is at this point. Perhaps it was easier to see a difference in the two groupsôperformance of Item #2 because of the format of the problems. Though they were similar to those in Item #1, they had no graphs and only dealt with continuity over and interval instead of the functionõs domain, which is typically a harder concept.

5.6 Conclusions

For many of the items of the assessment, as shown in the Results in Chapter 4, the percentages of accuracy and quality responses were generally only slightly different, favoring the ESP students in most cases. In terms of the statistical inferences, though, the only significant difference was that which was present in examining the composite scores of Item #2 of the two groups.

Recall that the additional lab sections were intended to boost the ESP studentsôoverall performance by covering various calculus topics over the semester. Perhaps in order to see a more significant difference in performance on the concepts of continuity and derivative between the two groups of students, time spend on those two topics would have to be increased in the ESP lab sections.

Moreover, time allotted to complete the written assessment should be increased. This would allow for more students to attempt and complete all of the assessment items, more specifically, Items #5, #6, and #7. It could also be more efficient in the future to conduct follow-up interviews with the students following the assessment, in order to analyze and gain more insight on the studentsôreasons for their responses, as well as their train of thought when working through the problems. Furthermore, analysis of the work and testing done in the course itself could help to see how those items impact the studentsômathematical understanding.

APPENDIX A

AURAS BASELINE QUESTIONNAIRE

Baseline Questionnaire – Emerging Scholars Program

Name: ______________________________ UTA ID: 1000__________

I. BACKGROUND AND DEMOGRAPHICS

1. As of August 2010, are you a: (**CIRCLE ONE RESPONSE ONLY**)

1	Freshman, first semester	3	Sophomore, first semester
2	Freshman, second semester	4	Sophomore, second semester
5	Other (**SPECIFY**): ______________________		

2. What is your age ________ and birthdate MM/DD/YY____________________

3. What is your gender? (**CIRCLE ONE RESPONSE ONLY**)

1 Male 2 Female

4a. What is your ethnicity? (**CIRCLE ONE RESPONSE ONLY**):

1 Hispanic, Latino
2 Non-Hispanic
3 Do not wish to report ethnicity

4b. What is your racial heritage? (**CIRCLE ALL THAT APPLY**)

1	Black or African-American	4	Native American or Alaska Native
2	White	5	Native Hawaiian or Other Pacific Islander
3	Asian	6	Do not wish to report racial heritage

4c. Did you apply for financial aid with the FAFSA? Circle: Yes / No / Do not wish to report

4d. Your citizenship is:

1	US citizen	3	International citizen with valid US visa
2	US permanent resident	4	Other / Do not wish to report

4e. High school name, city, state: ______________________________________

5. In what month and year did you graduate from high school? _____ (month)/____ (year)

5a. In what month and year did you enroll at UTA? _____ (month)/____ (year)

5b. What did you do after you graduated from high school and before you started UTA? (**CIRCLE ALL THAT APPLY**)

1 Worked full-time or part-time
2 Served in the military
3 Attended community college
4 Attended another 4-year college/university
5 Cared for a family member(s)
6 Did volunteer service in the community
7 Traveled
8 Other (**SPECIFY**): ______________________________
9 Nothing

6. What are the highest education levels that your parents attained? (**CHECK THE APPROPRIATE ROW FOR YOUR FATHER AND YOUR MOTHER):**

Education Level	Mother	Father
1. Less than high school		
2. Some high school		
3. High school graduate		
4. Community college or technical/vocational school		
5. Some college		
6. College graduate		
7. Graduate school – MA/MS		
8. Graduate school – Ph.D., MD, Law degree		
9. Other (**SPECIFY**):		

7. Do you have any siblings who are in college or old enough to have gone to college (i.e. 18 or older)?

1 Yes
2 No (**SKIP TO Q.8**)

7a. Is this sibling/(Are these siblings) in college or graduated from college? (**CIRCLE ALL THAT APPLY**)

1 Yes, in college
2 Yes, college graduate
3 None of my siblings currently attends or went to college
4 No, attended college but did not graduate

II. HIGH SCHOOL EXPERIENCES

8. What is the most advanced math class that you took in high school? (**CIRCLE ONE RESPONSE ONLY**)

1 Calculus
2 Pre-Calculus
3 Algebra 2
4 Algebra 1
5 Trigonometry
6 Geometry
7 Other (**SPECIFY**) ______________________________

9. How many classes did you take in high school in: (**RECORD THE APPROPRIATE NUMBER OF CLASSES FOR EACH SUBJECT AREA**)

Subject Areas	Number of Classes
Mathematics	
Physics	
Chemistry	
Computer Science	
Statistics	
Engineering	
Other math, science or technology classes	

Baseline Questionnaire – Emerging Scholars Program

10. Did you take any Advanced Placement (AP), International Baccalaureate (IB) or dual credit classes in high school: **(CIRCLE "Y" OR "N" FOR EACH AREA)**

AP Classes Taken	Yes	No	IB Classes Taken	Yes	No
AP Biology	Y	N	IB Biology	Y	N
AP Chemistry	Y	N	IB Chemistry	Y	N
AP Physics B	Y	N	IB Physics	Y	N
AP Physics C	Y	N	IB Math SL II	Y	N
AP Calculus AB	Y	N	IB Math HL I	Y	N
AP Calculus BC	Y	N	IB Math HL II	Y	N
AP Computer Science A	Y	N	IB Computer Science	Y	N
Other AP Science Courses	Y	N	Other IB Sciences	Y	N
Dual credit math, science or engineering college classes: **(LIST)**:					

11. How well did your high school prepare you for college in the following areas? **(CIRCLE ONE NUMBER FOR EACH ROW)**

Areas	Very Well	Well	Somewhat	Poorly	Not At All
Study skills	1	2	3	4	5
Writing skills	1	2	3	4	5
Oral presentation skills	1	2	3	4	5
Interpersonal communications	1	2	3	4	5
Laboratory skills	1	2	3	4	5
Computer literacy (MSWord, Excel)	1	2	3	4	5
Computer programming, advanced	1	2	3	4	5
Mathematics	1	2	3	4	5
Sciences	1	2	3	4	5
Engineering	1	2	3	4	5

III. COLLEGE EXPECTATIONS AND PLANS

12. Think back to high school; which one of the following statements best describes your high school experience? **(CIRCLE ONE RESPONSE ONLY)**

1 It was very easy for me to get the grades I wanted in all my classes
2 With a few exceptions, it was easy for me to get the grades I wanted in my classes
3 I had to work some, but not at all hard to get the grades I wanted in my classes
4 I had to work hard to get the grades I wanted in my classes

12a. What grade point average (GPA) did you want to get in high school? **(CIRCLE ONE LETTER ONLY)**

A B C D

Baseline Questionnaire – Emerging Scholars Program

13. As a first year college student, how hard do you expect to work in college to get the grades you want? Do you expect to: **(CIRCLE ONE RESPONSE ONLY)**

1 Work less than you did in high school to get the grades you want
2 Work the same as you did in high school to get the grades you want
3 Work harder than you did in high school to get the grades you want

13a. What grade point average (GPA) do you strive to get in college? **(CIRCLE ONE LETTER ONLY)**

A B C D

14. What is your intended major? **(CIRCLE ONE RESPONSE ONLY)**

1	Aerospace Engineering	8	Electrical engineering
2	Bioengineering	9	Industrial Engineering
3	Biological Chemistry	10	Mathematics
4	Chemistry/Biochemistry	11	Mechanical Engineering
5	Civil Engineering	12	Physics
6	Computer Engineering	13	Software Engineering
7	Computer Science		

15. How confident are you that you will keep this major through college? **(CIRCLE ONE RESPONSE ONLY)**

1 Very confident
2 Confident
3 50% confident
4 Not confident
5 Not at all confident

16. What sources of information did you use to decide what major to pursue in college? **(CIRCLE ALL THAT APPLY)**

1 University advisors
2 University classes
3 University "open house" or campus visit days
4 Other university activities
5 National ranking data on the college or department
6 High school teacher
7 High school counselor
8 Suggestion(s) from peers
9 Parents' advice
10 Suggestion(s) from sibling, family member or family friend
11 Employer
12 Future employment prospects
13 Other **(SPECIFY)** ______________________________

17. How supportive are your parents/guardians of your decision to study the major you specified above? **(CIRCLE ONE RESPONSE ONLY)**

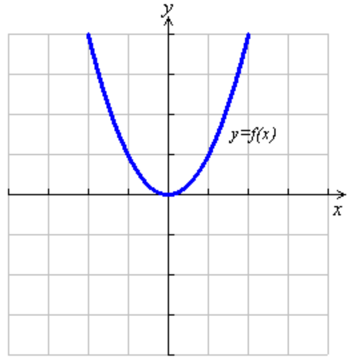
1 Very supportive
2 Supportive
3 Somewhat supportive
4 Neutral
5 Not supportive
6 Against my choice of a major
7 Did not discuss choice with them

APPENDIX B

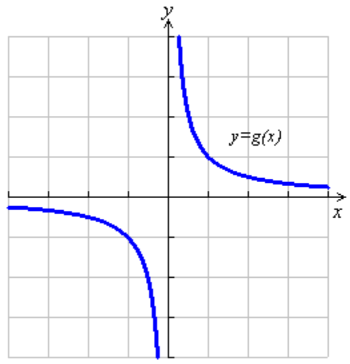
THE WRITTEN ASSESSMENT

1. Are the following functions continuous on their domain? Explain why.

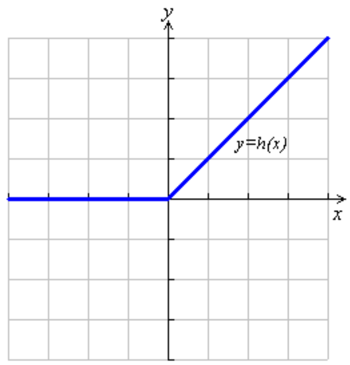
a. $f(x) = x^2$



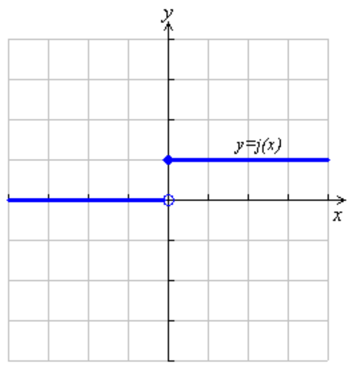
b. $g(x) = \dfrac{1}{x}, x \neq 0$



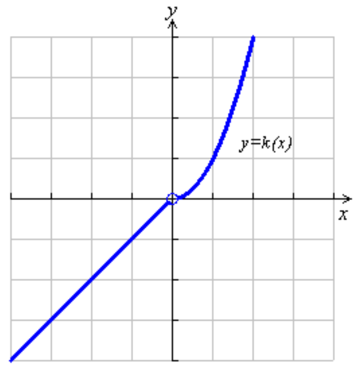
c. $h(x) = \begin{cases} 0, & x < 0 \\ x, & x \geq 0 \end{cases}$



d. $j(x) = \begin{cases} 0, & x < 0 \\ 1, & x \geq 0 \end{cases}$



e. $k(x) = \begin{cases} x, & x < 0 \\ x^2, & x > 0 \end{cases}$



2. Are the following functions continuous on the interval $[-2, 2]$? Explain why.

a. $f(x) = x^3$

b. $g(x) = \dfrac{1}{x-1}$

c. $h(x) = \begin{cases} 0, & x < 1 \\ x-1, & x \geq 1 \end{cases}$

d. $j(x) = \begin{cases} 1, & x \geq 1 \\ -1, & x < 1 \end{cases}$

e. $k(x) = \begin{cases} x, & x < 0 \\ x^2, & x > 0 \end{cases}$

3. Are the following statements True (T) or False (F)?

____ a. Any function $f(x)$ whose graph can be sketched over its domain in one continuous motion without lifting the pencil is an example of a continuous function.

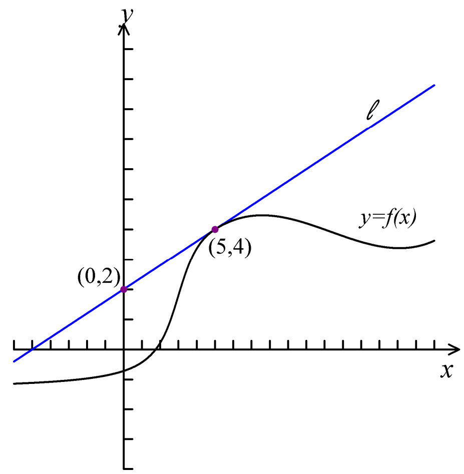
____ b. Any function $f(x)$ whose graph can be sketched over an interval I in one continuous motion without lifting the pencil is an example of a continuous function on I.

____ c. If the limit exists at a point then the function is continuous at that point.

____ d. A function $f(x)$ is discontinuous if its graph contains a sharp ñcorner.ò

____ e. Continuous functions must have domain all real numbers.

4. Suppose that the line l is tangent to the graph of the function f at the point $(5,4)$ as indicated in the figure at the right below.



   a. Find $f(5)$.
   Explain how you arrived at your answer.

   b. Find $f'(5)$.
   Explain how you arrived at your answer.

5. Given that f is a continuous function on $[3,15]$, $f'(x) > 0$ for $x \in (3,5)$, and $f(3) = 6$, is there a point $c \in (3,5)$ such that $f(c) = 0$? Explain.

6. Given that f is a continuous function on $[4,13]$, $f(4) = 8$, and that for any $x \in (4,13)$ the slope of the tangent line to the graph of f is positive, does f have an x-intercept in the interval (4, 13)? Explain.

7. The position function of a body moving on a straight line is given by $s = f(t)$ for $5 \le t \le 16$, where s is given in feet and t is given in seconds. When $t = 5$ seconds the objectôs position is 18 ft. Given that the rate of change of s is positive over the time interval $5 \le t \le 16$, is there a time t in this interval at which the object is at 0 ft? Explain.

APPENDIX C

GRADING RUBRICS WITH EXAMPLES

Accuracy Rubric

Score	Criteria
3	If the problem requires a YES/NO or True/False response, then the studentøs given answer is correct. If the problem requires a numerical response, then the studentøs provided value is correct.
2	If the problem requires a numerical response, then the studentøs provided value is incorrect because of minor calculation error.
1	If the problem requires a YES/NO or True/False response, then the studentøs given answer is incorrect. If the problem requires a numerical response, then the studentøs provided value is incorrect.

Conceptual Understanding Rubric

Score	Criteria
5	Studentøs explanation shows complete understanding of appropriate mathematical concepts and principles relative to the problem; use of mathematical terminology and notations are appropriate and complete.
4	Studentøs explanation shows nearly complete understanding of appropriate mathematical concepts and principles relative to the problem; use of mathematical terminology and notations are appropriate and nearly complete.
3	Studentøs explanation shows partially complete understanding of mathematical concepts and principles relative to the problem; use of mathematical terminology and notations is partially displayed.
2	Studentøs explanation shows limited or underdeveloped understanding of appropriate mathematical concepts and principles relative to the problem; inappropriate, sketchy, or nonexistent use of mathematical terminology and notations
1	Studentøs explanation shows completely inappropriate or no understanding of mathematical concepts and principles relative to the problem; inappropriate or nonexistent use of mathematical terminology and notations

Examples for CUR scoring

Item #1-A

5	➢ Polynomials (or a quadratic in this case) are continuous everywhere ➢ Mentions all of the following: a) $f(x_0)$ is defined, so that x_0 is in the domain of f b) $\lim_{x \to x_0} f(x)$ exists for x in the domain of f c) $\lim_{x \to x_0} f(x) = f(x_0)$
4	➢ The function can be drawn without lifting a pencil ➢ Identifies the function, using a term other than polynomial, such as ñparabolaò as means for justification
3	➢ In regards to a, b, c from above, the student either mentions b or c alone, or any combination of 2 of a, b, or c. ➢ Restricts a, b, c to only one specific point ➢ States that $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x)$ (uses reference to left and right hand limits, but only in regards to the ñproblem pointò)
2	➢ Uses factual criteria about the function to show that the function is not continuous ➢ Uses the functionôs differentiability in their justification ➢ Uses a vague reference to left and right hand limits (i.e. ñé meets l/r hand ruleò) ➢ States that $f(x)$ is defined for all x in the domain of f ➢ States that the function includes all real numbers
1	➢ States that the function is continuous on the interval without any further justification ➢ Uses nonfactual criteria or simply states that the function is not continuous

Item #1-B

5	➢ Mentions all of the following: a) $g(x_0)$ is defined, so that x_0 is in the domain of g b) $\lim_{x \to x_0} g(x)$ exists for x in the domain of g c) $\lim_{x \to x_0} g(x) = g(x_0)$ ➢ Argues that the function is ñundefined at $x = 0$, but domain is (-Ð,0) ∪ (0,Ð) so itôs continuousò
4	➢ Summarizes the above (a, b, c) with minor error ➢ Identifies that the only discontinuity is outside of the given domain (i.e. ñcontinuous on domain except at 0 because l/r are not the sameò)
3	➢ In regards to a, b, c from above, the student either mentions b or c alone, or any combination of 2 of a, b, or c ➢ Restricts a, b, c to only one specific point ➢ Only states that the function specifically does not include 0, that the domain is never 0, or that ñ $x \neq 0$ ò ➢ Identifies the piecewise function as ñtwo partsò that are continuous on their own domain
2	➢ Uses a vague reference to left and right hand limits ➢ Uses the functionôs differentiability in their justification ➢ Uses factual criteria about the function to show that the function is not continuous
1	➢ States that the function is continuous on the interval without any further justification ➢ Uses nonfactual criteria or simply states that the function is not continuous

Item #1-C

5	➢ Identifies that is the sum of two polynomials, which are each continuous everywhere ➢ Mentions all of the following: a) $h(x_0)$ is defined, so that x_0 is in the domain of h b) $\lim_{x \to x_0} h(x)$ exists for x in the domain of h c) $\lim_{x \to x_0} h(x) = h(x_0)$
4	➢ The function can be drawn without lifting a pencil ➢ Identifies that the function has no gaps/breaks/holes/discontinuities ➢ States that both [functions] of the piecewise are continuous, and checks the point at which the function definition switches
3	➢ In regards to a, b, c from above, the student either mentions b or c alone, or any combination of 2 of a, b, or c ➢ Restricts a, b, c from above to only a particular point ➢ States that $\lim_{x \to 0^-} h(x) = \lim_{x \to 0^+} h(x)$ (uses reference to left and right hand limits but only in regards to the ñproblem pointò)
2	➢ Uses a vague reference to left and right hand limits ➢ Uses the functionôs differentiability in their justification ➢ States that the domain of the function is at all real numbers ➢ States that $h(x)$ is defined for all x in the domain of h ➢ Uses factual criteria about the function to show that the function is not continuous
1	➢ Mentions the presence of a sudden change or a sharp corner ➢ Only mentions the nature of the point at which the function definition switches (i.e. ñsame value when function changesò) ➢ States that the function is continuous on the interval without further justification ➢ Uses nonfactual criteria or simply states that the function is not continuous

Item #1-D

5	➢ From the following: a) $j(x_0)$ is defined, so that x_0 is in the domain of j b) $\lim_{x \to x_0} j(x)$ exists for x in the domain of j c) $\lim_{x \to x_0} j(x) = j(x_0)$ shows that (b) does not hold because $\lim_{x \to 0} j(x)$ does not exist there
4	➢ Mentions a discontinuity at $x = 0$ ➢ States that $\lim_{x \to 0^-} j(x) = \lim_{x \to 0^+} j(x)$
3	➢ Mentions a discontinuity on the interval, not specifying where ➢ Mentions that L/R hand limits are not equal, not specifying where
2	➢ Uses a vague reference to limits (i.e. ñlimit do not equal each otherò with no L/R specification or location) ➢ States that the y-values are different or that the y-values jump ➢ Uses factual criteria about the function to show that the function is continuous
1	➢ Uses the wrong point as the ñproblem pointò ➢ States that the function is not continuous on the interval without justification ➢ Uses nonfactual criteria or simply states that the function is continuous everywhere/on the interval

Item #1-E

5	➢ Identifies that $k(x)$ is the sum of two polynomials, which are each continuous everywhere ➢ Mentions all of the following: a) $k(x_0)$ is defined, so that x_0 is in the domain of k b) $\lim_{x \to x_0} k(x)$ exists for x in the domain of k c) $\lim_{x \to x_0} k(x) = k(x_0)$
4	➢ Identifies that the function has no breaks/holes/jumps/discontinuities in the domain ➢ States that both [functions] of the piecewise are continuous, and checks the point at which the function definition switches ➢ Identifies a discontinuity at $x = 0$, which is not in the domain
3	➢ In regards to a, b, c from above, the student either mentions b or c alone, or any combination of 2 of a, b, or c ➢ Restricts a, b, c from above to only a particular point ➢ Or ➢ States that $\lim_{x \to 0^-} k(x) = \lim_{x \to 0^+} k(x)$ (uses reference to left and right hand limits but only in regards to the ñproblem pointò)
2	➢ Uses a vague reference to L/R limits ➢ States that $f(0)$ can be found using limits ➢ Uses factual criteria about the function to show that the function is not continuous ➢ Uses the functionôs differentiability in their justification
1	➢ Only mentions the nature of the point at which the function definition switches (i.e. ñvalues are equal when the function changesò) ➢ States that the function is continuous on the interval without any further justification ➢ Uses nonfactual criteria or simply states that the function is not continuous

Item #2-A

5	➢ Polynomials are continuous everywhere ➢ Mentions all of the following: d) $f(x_0)$ is defined, so that x_0 is in the domain of f e) $\lim_{x \to x_0} f(x)$ exists for x in the domain of f a) $\lim_{x \to x_0} f(x) = f(x_0)$
4	➢ The function can be drawn without lifting a pencil ➢ Identifies that the function has no breaks/holes/jumps/discontinuities ➢ Identifies $f(x)$ as something other than a polynomial, such as a ñparametricò
3	➢ In regards to a, b, c from above, the student either mentions b or c alone, or any combination of 2 of a, b, or c ➢ Restricts a, b, c from above to only a particular point
2	➢ States that $f(x)$ is defined for all x in the domain of f ➢ Uses a vague reference to L/R limits ➢ States that the function (or domain) contains all real numbers ➢ Uses factual criteria about the function to show that the function is not continuous
1	➢ States that the function is continuous on the interval without any further justification ➢ Uses nonfactual criteria or simply states that the function is not continuous

Item #2-B

5	➢ From the following: a) $g(x_0)$ is defined, so that x_0 is in the domain of g b) $\lim_{x \to x_0} g(x)$ exists for x in the domain of g c) $\lim_{x \to x_0} g(x) = g(x_0)$ ➢ shows that (a) does not hold because $g(1)$ is not defined on the interval ➢ shows that (b) does not hold because $\lim_{x \to 1} g(x)$ DNE
4	➢ Identifies an asymptote or a discontinuity at $x = 1$ ➢ States that $x = 1$ causes the denominator to be 0 ➢ Mentions that the $\lim_{x \to 1^-} g(x) = \lim_{x \to 1^+} g(x)$
3	➢ Mentions a discontinuity on the interval, with no specification ➢ States that there is a ñhole when $x = 1$ ò or a ñhole within the functionò ➢ Mentions that the ñdomain is (-Ð,1) ∪ (1,Ð)ò or simply that ñ$x \neq 1$ ò ➢ Uses the correct point as well as an incorrect point as the ñproblem pointsò
2	➢ Uses factual criteria about the function to show that the function is continuous
1	➢ Uses the wrong point as the ñproblem pointò ➢ States that the function is not continuous on the interval without any further justification ➢ Uses nonfactual criteria or simply states that the function is continuous everywhere/on the interval

Item #2-C

5	➢ Mentions all of the following: a) $h(x_0)$ is defined, so that x_0 is in the domain of h b) $\lim_{x \to x_0} h(x)$ exists for x in the domain of h c) $\lim_{x \to x_0} h(x) = h(x_0)$ ➢ Identifies that both parts of the piecewise are continuous and verifies that the point at which the function definition switches is also continuous
4	➢ The function can be drawn without lifting a pencil ➢ Identifies that the function has no breaks/holes/jumps/discontinuities ➢ Identifies that both parts of the piecewise are continuous, and attempts to check the point at which the function definition switches
3	➢ In regards to a, b, c from above, the student either mentions b or c alone, or any combination of 2 of a, b, or c ➢ Restricts a, b, c from above to only a particular point ➢ States that $\lim_{x \to 1^-} h(x) = \lim_{x \to 1^+} h(x)$ same (uses reference to left and right hand limits but only in regards to the ñproblem pointò)
2	➢ Uses vague reference to L/R hand limits ➢ States that the function is defined for all reals, or is never undefined ➢ Uses factual criteria about the function to show that the function is not continuous ➢ Uses the functionôs differentiability in their justification
1	➢ Only mentions the nature of the point at which the function definition switches (i.e. ñvalues are equal when the function changesò) ➢ States that the function is continuous on the interval without any justification ➢ Uses nonfactual criteria or simply states that the function is not continuous everywhere/on the other interval

Item #2-D

5	➢ From the following: a) $j(x_0)$ is defined, so that x_0 is in the domain of j b) $\lim_{x \to x_0} j(x)$ exists for x in the domain of j c) $\lim_{x \to x_0} j(x) = j(x_0)$ shows that (b) does not hold because $\lim_{x \to 1} j(x)$ does not exist there or shows that (c) does not hold because $\lim_{x \to 1} j(x) \neq j(1)$
4	➢ The function cannot be drawn without lifting a pencil ➢ Mentions a discontinuity at $x = 1$ ➢ States that L/R hand limits at $x = 1$ are not equal/the same
3	➢ Mentions a discontinuity on the interval ➢ States that the y-values jump
2	➢ Uses vague reference to L/R hand limits ➢ States that $x = 1$ is undefined ➢ Uses factual criteria about the function to show that the function is continuous
1	➢ Uses the wrong point as the ñproblem pointò ➢ States that the left and right hand values for $j(1)$ are not the same for $x = 1$ ➢ States that the function is not continuous on the interval without any justification ➢ Uses nonfactual criteria or simply states that the function is continuous everywhere/on the interval

Item #2-E

5	➢ From the following: a) $k(x_0)$ is defined, so that x_0 is in the domain of k b) $\lim_{x \to x_0} k(x)$ exists for □ in the domain of k c) $\lim_{x \to x_0} k(x) = k(x_0)$ shows that (a) does not hold because $k(0)$ is not defined on the interval
4	➢ Identifies a discontinuity at $x = 0$ ➢ Examples: a) ñno way $x = 0$ on functionò b) ñzero not in domainò c) ñ $x = 0$ DNE on graphò d) ñ0 isnót solved forò
3	➢ Mentions a discontinuity in the interval ➢ Only states that the parts of the piecewise function are ñboth polynomialsò
2	➢ States that the two lines do not meet ➢ Uses factual criteria about the function to show that the function is continuous
1	➢ Uses the wrong point as the ñproblem pointò ➢ States that there is a sudden change in direction ➢ Uses nonfactual criteria or simply states that the function is continuous everywhere/on the interval ➢ States that the function is not continuous on the interval without any further justification ➢ Examples: a) ñfunctions equal each other when switchò b) ñboth continuous when combinedò c) ñdefined on all realsò

Item #4-A

5	➢ Student states something along the lines of ñ $y = f(x)$, value of y when $x = 5$, $f(5) = 4$ ò ➢ Uses function f to find $f(5)$
4	➢ Only states that (5,4) is given on the graph or in the problem.
3	➢ Student derived a formula for line l, and plugs in $x = 5$ to their formula for l to find $f(5)$.
2	➢ States that the point (5,4) is where the two lines meet, thus $f(5) = 4$.
1	➢ Uses wrong concepts, such as trying to find the slope of l ➢ Random work with no result.

Item #4-B

5	➢ Mentions something along the lines that the derivative at the point is the slope of the tangent line at that point.
4	➢ Knows to find the slope of the tangent line to f, but justifies their calculation by stating ñslope of the lineò or ñline l is the slopeò or ñusing $\frac{y_2 - y_1}{x_2 - x_1}$ ò
3	➢ Derives the value 2/5, but goes on to use it in the calculation of a formula for line l, plugging in $x = 5$, and returning an answer other than 2/5. ➢ Uses $\frac{y_2 - y_1}{x_2 - x_1}$ to obtain 2/5 but states something else that is unrelated to the problem.
2	➢ Relates the tangent line to the derivative, but does not mention ñslopeò ➢ Student derives the value 2/5, but chooses (5,4) because that is where the two lines meet
1	➢ States that the point (5,4) is the point in common or that it is where the two lines meet, thus $f'(5) = 4$. ➢ States that (5,4) is given. ➢ No justification or random work with no result

Item #5

5	➢ $f'(x) > 0$ for x in (3,5), indicating that f is increasing on the given interval. Since $f(3) = 6$ and the function is increasing, then all y-values corresponding to x-values in (3,5) are greater than 6. Thus $f(c) = 0$ cannot be attained for c in (3,5). (i.e. mentions a) the ñstarting pointò b) ñ $f'(x) > 0$ ò c) $f(x)$ increasing/not decreasing)
4	➢ In regards to a, b, c from above, mentions only 2 of the 3 ➢ Similar to score 5, but mentions a ñconstant rateò as opposed to increasing/increasing constantly ➢ Mentions a, b, c, but states that the function is positive as opposed to increasing
3	➢ Mentions something along the lines of: i. ñThe function has to decrease in order to attain $f(c) = 0$ ò ii. ñ $f'(x)$ is positive, so the function goes upò ➢ Student uses ñ $f'(x) > 0$ ò directly to say that $f(c) = 0$ cannot be attained.
2	➢ Student makes note that ñ $f'(x) > 0$ ò has something to do with the answer, but is unable to pinpoint what that means exactly ➢ Uses factual information that unfortunately, does not apply to the specific values expressed in the problem
1	➢ Student provides an explanation as to why it would be possible ➢ Student shows little or no attempt at justification

Item #6

5	➢ The slope of the tangent line to the graph of f is positive for any x in (4,13), indicating that f is increasing on the given interval. Since $f(4)=8$, and the function is increasing, then all y-values corresponding to x-values in (4,13) are greater than 8. Thus, an x-intercept in (4,13) cannot be attained. (i.e. mentions a) the ñstarting pointò b) ñthe slope of the tangent line to the graph of f is positiveò c) $f(x)$ increasing/not decreasing)
4	➢ Mentions something along the lines of: i. The slope of the tangent line to the graph of f is positive, so the function is positive/the function does not go down ➢ Similar to score 5, but mentions a ñconstant rateò as opposed to increasing/increasing constantly
3	➢ Mentions something along the lines of: i. The function has to decrease in order to attain an x-intercept ii. The rate of change is positive, so the function goes up ➢ Student uses ñthe slope of the tangent line to the graph of f is positiveò directly to say that an x-intercept cannot be attained.
2	➢ Student makes note that ñthe slope of the tangent line to the graph of f is positiveò has something to do with the answer, but is unable to pinpoint what that means. ➢ States that an intercept cannot be attained with the presence of a positive slope (with no mention of the starting point)
1	➢ Student provides an explanation as to why it would be possible ➢ Student shows little or no attempt at justification

Item #7

5	➢ The rate of change of s is positive for t in (5,16), indicating that $s = f(t)$ is increasing on the given interval. Since $f(5) = 18$, and the function is increasing, then all positions corresponding to time (t) in (5,16) are greater than 18. Thus, a position of 0ft in (5,16) cannot be attained. (i.e. mentions a) the ñstarting pointò b) ñthe rate of change of s is positiveò c) $f(t)$ increasing/not decreasing or that the object is moving forward)
4	➢ Mentions something along the lines of ñthe rate of change of s is positive, so $f(t)$ is positiveò ➢ Similar to score 5, but mentions a ñconstant rateò as opposed to increasing/increasing constantly
3	➢ Mentions something along the lines of: i. ñThe function has to decrease in order to attain a position of 0ftò ii. ñThe rate of change is positive, so the function goes upò (or the function goes forward) ➢ Student uses ñthe rate of change of s is positiveò directly to say that a position of 0ft cannot be attained
2	➢ Student makes note that ñthe rate of change of s is positiveò has something to do with the answer, but is unable to pinpoint what that means exactly.
1	➢ Student provides a time t at which the position is at 0ft ➢ Student provides an explanation as to why it would be possible ➢ Student shows little or no attempt at justification

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BIOGRAPHICAL INFORMATION

Susan Chan graduated from Baylor University in Waco, TX in May 2008, with a Bachelor of Science degree in Mathematics with minors in both Biology and Chemistry. In the spring of 2009, she decided to return to school to continue her studies in the mathematical field and applied to graduate schools within the state of Texas. At the time of writing this thesis, she was pursuing a Master of Science degree in Mathematics at the University of Texas at Arlington, in Arlington, TX, planning to graduate in May 2011.