TIME DEPENDENT QUEUING MODELS OF THE NATIONAL AIRSPCE SYSTEM

by

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To my father Weerasak, mother Sermboon Roongrat, and everyone in my family.

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ABSTRACT

TIME DEPENDENT QUEUING MODELS OF THE NATIONAL AIRSPACE SYSTEM

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Air transportation in the US system has dramatically changed in the past few decades. The National Airspace System (NAS) has increasingly become congested. A high volume of air traffic demand is one of the major challenges of the NAS. However, air traffic is very difficult to study due to many uncertainties involved. It is important that we be able to understand the relationship under uncertainties due to aviation operations, precision of navigation and control, and traffic flow efficiency. Many queuing models have been studied to better understand and quantify these relationships. In the past decade, most queuing network models assume that inter-arrival times and service times are exponentially distributed and stationary, which may not be suitable for all scenarios. These queuing models are time invariant and have several drawbacks. In particular, they do not account for increases and decreases in demand that are

commonly observed in the NAS throughout a day. Previously, the NAS has been studied and analyzed by using traditional Makovian queues. However, observations from simulations of real traffic data reveal that the inter-arrival time and service time probability distributions cannot be represented by exponential probability density functions. The Coxian distribution is a phase-type distribution that has gained special importance in the research on queuing networks. In this study, several methods of fitting Coxian distribution to data together with different time dependent queuing models of the NAS are developed and discussed.

In the past few decades, Coxian distributions have become increasingly more popular. The probability distribution functions for inter-arrival times/service times of airspace systems cannot be represented by traditional probability distribution functions. In the first part of this dissertation, we describe different algorithms to fit Coxian distributions to the service times of major Air Traffic Centers. Several fitting methods are developed and discussed. Finally, we compare and evaluate those methods by using the mean square error (MSE) and the number of phases in the distribution.

In the second part of this dissertation, we discuss a practical approach for modeling the NAS with time-dependent Coxian queues. Time-dependent $C_{m(t)}(t)/C_k/s$ queuing models of the National Airspace are developed in which the inter-arrival distribution is a time-dependent piece-wise constant Coxian random variable, and the service time distribution is a Coxian random variable. We describe an algorithm for calibrating a $C_{m(t)}(t)/C_k/s$ queuing model from simulated data of an Air Route Traffic

Control Center and an algorithmic approach to determine average measures of the queues. Finally, we give future directions for studying such queuing models.

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CHAPTER 1

INTRODUCTION

1.1 Introduction

Air transportation in the US system has dramatically changed in the past few decades. The National Airspace System (NAS) has increasingly become congested. A heavy volume of air traffic demand is one major challenge of the NAS. NASA Ames reported [72], "The current air traffic demand on the US national airspace frequently exceeds its available capacity, due to a large number of factors including bad weather, over-scheduling by the airlines, national security, and air traffic control equipment outages." Over the next ten years, projections reveal an increase in air traffic demand [30]. Moreover, travel and business in the aviation industry are expected to grow. By the year 2015, the number of passengers that United States (U.S.) Airlines carry will double [36].

The Federal Aviation Authority [37] noted, "According to the General Aviation Manufacturer's Association (GAMA) statistics for 2000, operations at general aviation (GA) airports with [Federal Aviation Administration (FAA)] control towers totaled approximately 50,000 aircraft operations per day. Aircraft landings and departures have increased steadily with more than 11.5 million reported for 2001." The Air Traffic Control System has dramatically changed from 1935-2000 as shown in Figure 1.1. The amount of user demand on the NAS is quickly exceeding the resources. For the period of January through June 2000, delays increased by 13.6 percent from the same time period in 1999 [36]. In June alone, delays increased 20 percent. From May through July 2000, delays increased 6.8 percent from 1991 and totaled more than 86,684 [21]. An increasing demand for air travel is one of the most challenging problems in US air transportation. In the next 15 years, the annual air traffic in the US is expected to grow about 3-5% [30]. Figure 1.2 shows the Airspace Concept Evaluation System (ACES) simulation of U.S. traffic demand in 2022. Today the aerospace industry is faced with rapidly growing demand the leads to heavy air traffic. Within the U.S. airspace, more than 40,000 commercial flights operate in a typical day [17].

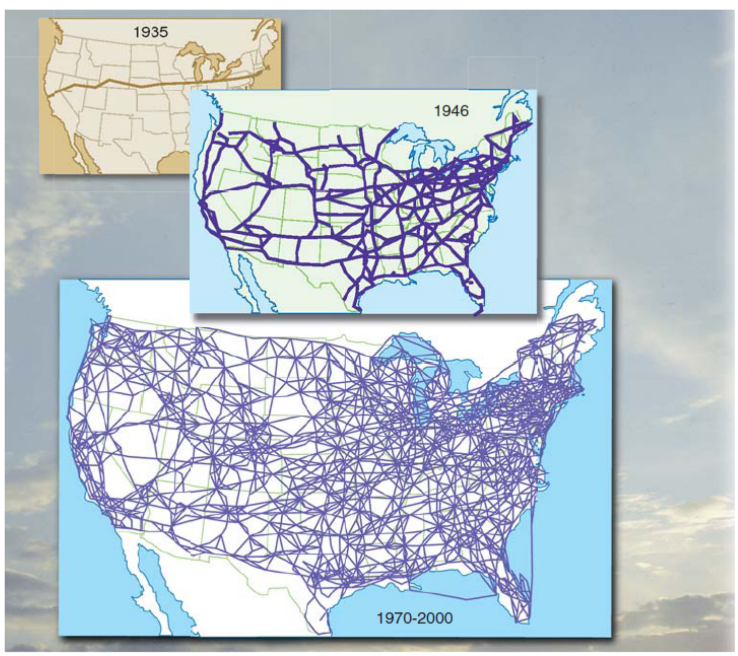

Figure 1.1: Air Traffic Control System Expansion

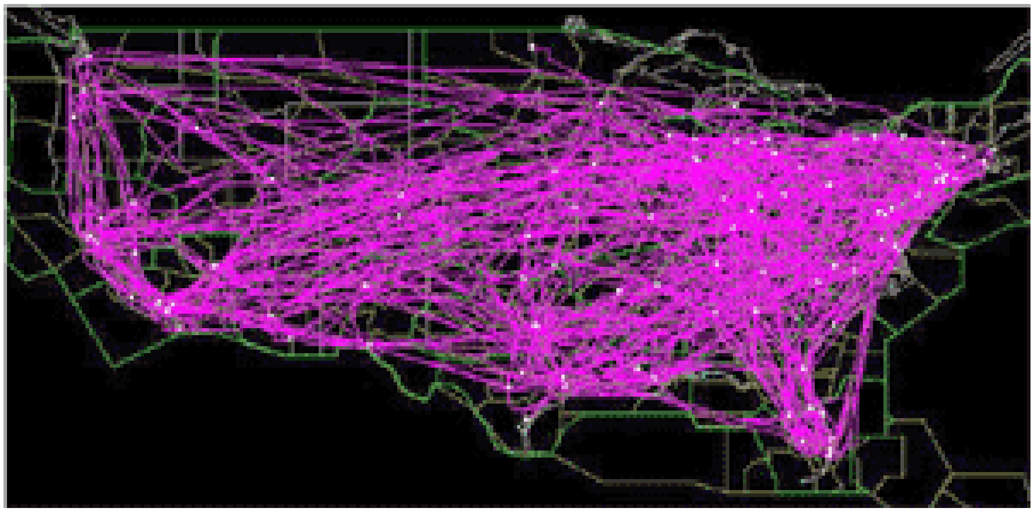

Figure 1.2: Simulated U.S. Traffic Demand in 2022 using the Airspace Concept Evaluation System (ACES)
Source: http://vams.arc.nasa.gov/activities/aces.html

Therefore, an efficient and effective air traffic management system is vital to the U.S. transportation infrastructure. The inflation adjusted gross domestic product (GDP) has increased by 62 percent due to deregulation in the airline industry since 1978 [50]. The US Department of Transportation [29] said, "In this same time period, total scheduled passenger air transportation has increased by 190 percent, and total airfreight ton miles have increased by 289 percent. Since 1997, flight delays have been significantly increased. These trends are expected to continue. In 1998, airline delays in the U.S. cost industry and passengers $4.5 billion."

However, air traffic is very difficult to study due to many uncertainties involved. It is important that we be able to understand the relationship under uncertainty. Many queuing models have been studied to improve delay problems. In the past decade most queuing network models assume that inter-arrival times and service times are exponentially distributed and stationary, which may not be suitable for all scenarios. These queuing models are time invariant and have several drawbacks. In particular, they do not account for increases and decreases in demand that are commonly observed in the National Airspace System (NAS) throughout a day. Previously, the NAS has been studied and analyzed by using traditional Makovian queues. The observations from simulations of real traffic data, however, show that the inter-arrival and service time distributions do not follow an exponential probability density function. In the past few decades, Coxian distributions have become increasingly more popular. The Coxian distribution is a phase-type distribution that has gained special importance in research on queuing networks. In this study, methods of fitting a Coxian distribution to air traffic data are developed. The Coxian parameters obtained by the developed fitting methods are used to model $C_{m(t)}(t)/C_k/s$ queues of the NAS.

Figure 1.3 illustrates the $C_{m(t)}(t)/C_k/s$ queue procedure. In this study, we developed MATLAB code to extract inter-arrival time and service time distributions from the FACET simulation. The inter-arrival times are fitted with our developed fitted Coxian distribution method called the *Random Sample Method* or *RSM* within each time period. In addition, service time distributions are also fitted with RSM. These steps, called *calibration*, calibrate a model from data. Once queuing parameters are obtained, the queuing models will be appropriately revised based upon what if scenarios. This procedure will only used by NASA for studying precision and navigation effects and other uncertainties such as weather. Then, the next procedure is to enumerate states and determine the state probability vector. The algorithm to determine the state probabilities is developed and will be discussed more in specific details in chapter 4. These steps are called *queuing analysis*. Lastly, we have to validate our model to check whether or not the model is accurate. In this study, we validate our developed queuing model with the FACET simulation.

From Figure 1.3, notice that the red boxes indicate major contributions of this study. The several methods of fitting data to Coxian distribution to data are discussed in detail in chapter 3. Moreover, we also develop an algorithm to determine probability state vectors, which is discussed in chapter 4.

1.2 Research Overview/Contributions

Although air traffic queuing models have been extensively studied over a decade, most research on queuing models focuses on steady state Markovian queues with time invariant inter-arrival and service time distributions. However, real world queuing systems vary with time. There are many time-dependent phenomena, such as rush hour or periodicity. Moreover, traditional Markovian queues, which assume inter-arrival times and service times follow the exponential distribution, do not seem to fit with real data. In our research, the objective is to develop more accurate queuing models by implementing time dependent Coxian queuing models of the National Airspace System.

Chapter 3 describes different algorithms to fit Coxian distributions to the service time data of major Air Traffic Centers. We develop several fitting methods. Fitting a Coxian phase-type distribution to data can be done by maximizing the log likelihood.

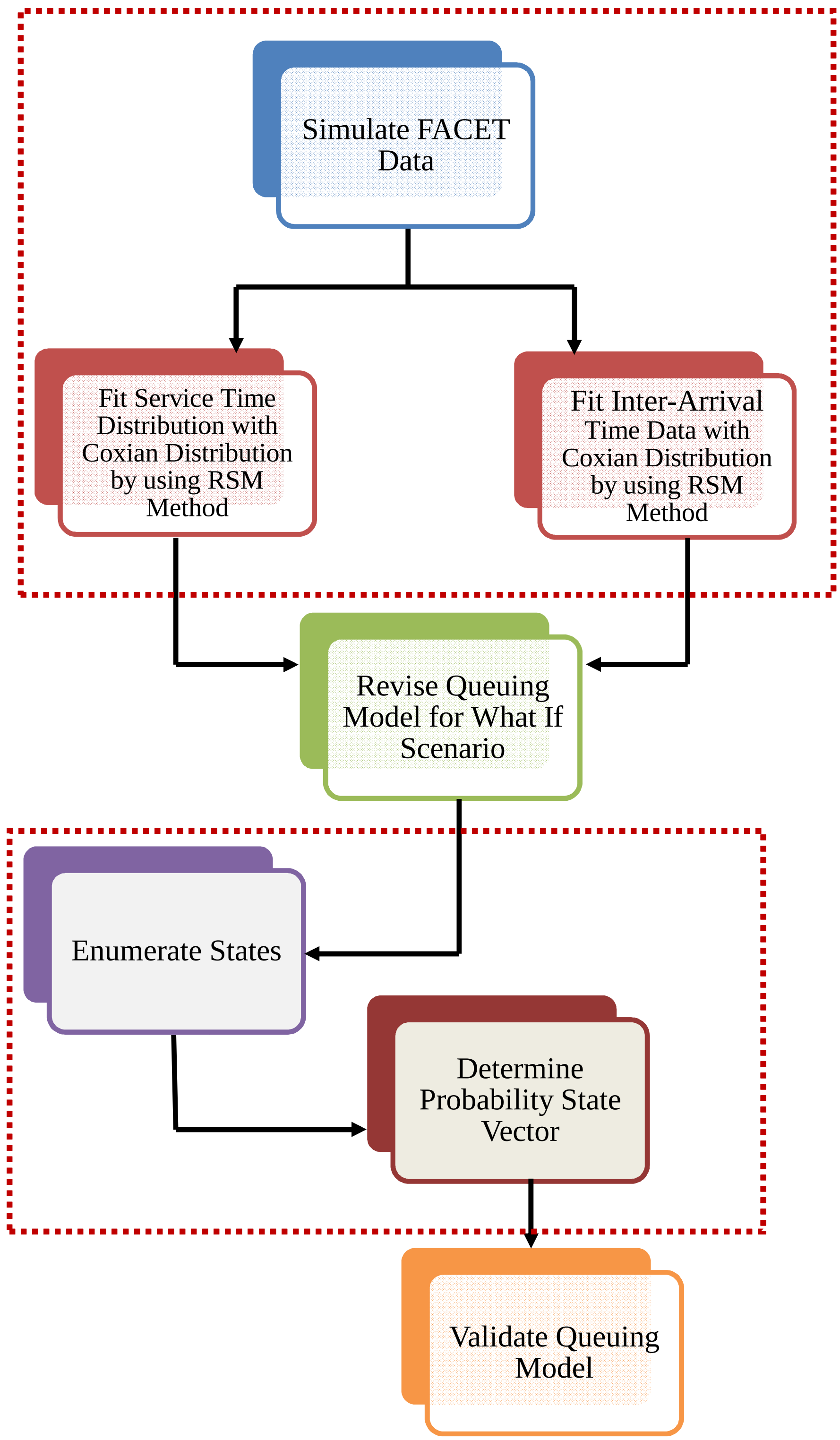

Figure 1.3 $C_{m(t)}(t)/C_k/s$ Queue Procedure

This method has been extensively studied in the literature [2, 61, 62, 63, 64]. Furthermore, a fitting procedure for phase-type distributions has been developed by using an expectation maximization algorithm [76].

Many researchers have performed the method of moments to fit the Coxian distribution. According to Schmickler and Johnson [54, 55], the method of moments matching has been developed to match the moments of a mixture of Erlang distributions. Moreover, differences between the moments matching parameter method with the maximum likelihood method were discussed in Lang et al [9]. The third group of algorithms is based on least squares estimation. The least squares method was to estimate the model parameters in Faddy [60]. In this research, we compare and evaluate fitting methods by using the goodness of fit based on the mean square error (MSE) and the number of phases in the distribution.

In chapter 4 of this dissertation, we discuss a practical approach for modeling the NAS with time-dependent Coxian queues. Various studies of time dependent queues are on the Markovian queue $M(t)/M/s$. For example, Lee [10] studied design of time-dependent telecommunication systems with infinite servers. Runnenburg [45, 46] developed a renewal theory for Markov-dependent random variables to study the waiting-time of a single-server queue with Markov-dependent inter-arrival times. Green et al. [51, 52] studied effects of non stationary on multi-server Markovian queuing systems. Furthermore, Jennings et al. [66] observed time-dependent systems when the arrival rate changes very rapidly relative to the service times.

Moreover, the Coxian queues in most literature, are time invariant. For example, Bertsimas [22] used the Coxian distribution and generalization method of stages introduced by Erlang to achieve the solution of queuing system. Neuts [57] studied a special class of probability distributions with rational Laplace transforms. Also, numerical investigations of queuing systems involving this special class of distributions can be found in Neuts [59]. Moreover, Marie [75] studied the queue-length probability distribution of a single server queue with a Coxian service distribution and exponentially distributed inter-arrival times with a state-dependent arrival rate. Herzog et al. [81] obtained numerical results for a single server queue with state-dependent arrival and service rates, and assumed that the inter-arrival times as well as the service times follow a Coxian distribution.

Therefore, in this research we propose time-dependent $C_{m(t)}(t)/C_k/s$ queuing models of the National Airspace. The time-dependent Coxian queues are developed in which the inter-arrival distribution is a time-dependent piece-wise constant Coxian random variable, and the service time distribution is Coxian random variable. We describe an algorithm for calibrating a $C_{m(t)}(t)/C_k/s$ queuing model from simulated data of an Air Route Traffic Control Center and an algorithmic approach to determine average measures of the queues. The $C_{m(t)}(t)/C_k/s$ queue has not yet been studied in any literature. Finally, in chapter 5, we discuss conclusions and give future extensions of our research.

CHAPTER 2

BACKGROUND AND RELATED LITERATURE

2.1 Overview of the National Airspace System

The FAA [36] reports, "The National Airspace System (NAS) of the United States is one of the most complex aviation systems in the world and consists of thousands of people, procedures, facilities, aircraft, control facilities, procedures, navigation and surveillance equipment, analysis equipment, decision support tools that enables safe air travel in the United States." The US airspace is divided into 21 large areas called *centers*, each of which is divided into *sectors* [72] as shown in Figure 2.1 below.

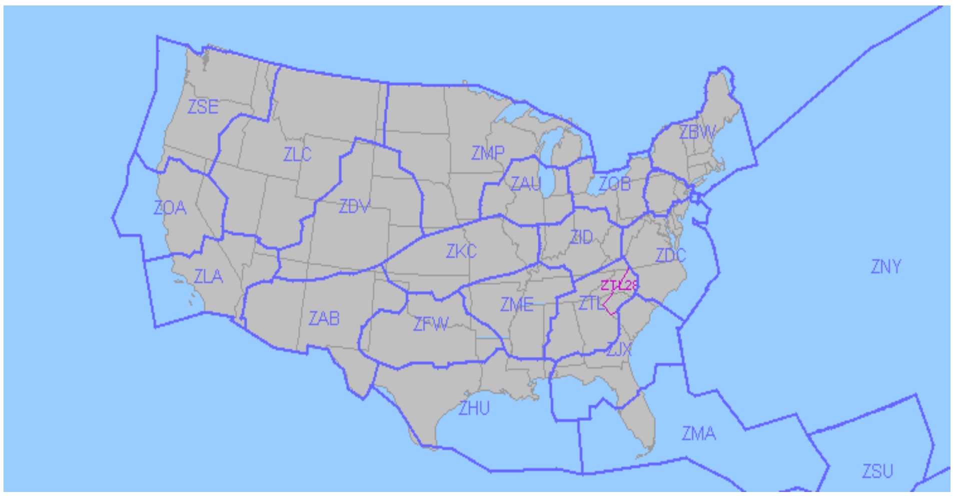

Figure 2.1 Topology for a Center Level Queuing Network Model for the NAS

The National Airspace System (NAS) Plan was announced in January 1982 by the Federal Aviation Administration (FAA) [36]. The plan improved aviation operations tremendously such as better flight service stations, more advanced systems for Air Traffic Control (ATC), and faster communication. Furthermore, better computers and software were developed that reduced the number of flights. According to the FAA [36], "The NAS requires 14,500 air traffic controllers, 4,500 aviation safety inspectors, and 5,800 technicians to operate and maintain services. It has more than 19,000 airports and 600 air traffic control facilities. In all, there are 41,000 NAS operational facilities. In addition, there are over 71,000 pieces of equipment, ranging from radar systems to communication relay stations." Each day, about 50,000 flights use NAS services on average [36].

Many mathematical models of air traffic flow have been studied by the National Aeronautics and Space Administration (NASA) and the FAA [72, 73]. NASA attempts to develop the analysis methodologies to improve air traffic flow. In addition, the design of the NAS has been studied in Beach and Connolly [53], Caldwell [12], Hinds and Kiesler [68], Orasanu and Salas [43], Rasmussen et al [44], Robertson et al [77], and Smith and Geddes [70]. Those studied include a design of decision support making in aviation environment. Wickens and Hopkins [21, 82] studied human factors in air traffic control. Moreover, Kerns et al [50], Billings [18], Lacher and Klein [8] primarily focused on the design of the traffic flow management system within the NAS.

2.1.1 NAS System

Air Traffic Control System Command Center (ATCSCC):

The Air Traffic Control System Command Center (ATCSCC), operational since 1994 and headquartered in Herndon, Virginia, is in charge of managing the air traffic flow within the NAS [37]. The ATCSCC takes action implementing ground delay programs, fight cancellations when unforeseen events occur that create an imbalance between air traffic demand and capacity. These actions help reduce congestion and delays thus facilitating maximum use of the NAS.

Air Traffic Controlling Facilities (ATC):

The primary concern of the 21 Air Traffic Control Command Centers (ARTCC) in the NAS is to separate and control the movement of aircraft within a specified airspace. Each of the 21 ARTCC located in the 21 centers in the U.S. employs 300 or more controllers, who guide aircraft towards their destination, reroute aircraft around bad weather, and keep them safe from mid-air accidents.

Terminal Radar Approach Control (TRACON):

These centers are located near airports and cover airspace of a 50-mile radius or more. If they are located within the 50-mile radius, they might control the airspace of other airports as well. The Terminal Radar Approach Control (TRACON) is responsible for the arrival sequencing at the airports. Air Traffic Management (ATM) decision support tools have shown the capability to increase the arrival traffic throughput of TRACON facilities without impacting the air traffic controller workloads [89]. NASA Ames Research Center and the FAA are developing decision support tools, such as trajectory prediction algorithms, to aid the tactical control of TRACON departure traffic [20]. Jung [88] states, “The Expedite Departure Path (EDP) is a decision support tool that provides the TRACON traffic management coordinators with departure traffic loading, scheduling information and radar controllers with advisories for tactical management of terminal area departure traffic.”

Control Tower:

The surface traffic is managed by control towers (aircraft operations on the ground) and within a specified airspace around the airport. The control tower plays an important role in managing proper spacing between aircraft taking off and landing.

2.1.2 Future ATM Concepts Evaluation Tool (FACET)

At NASA Ames Research Center, Moffett Field, California, the Future Air traffic management Concepts Evaluation Tool (FACET) was developed [48]. By using actual air traffic data from the FAA, FACET analyzes the flight plan route and predicts trajectories for the climb, cruise and descent phases of flight for each aircraft type [71]. NASA Ames Research Center developed the FACET simulation for modeling ATM. The purpose of FACET is to provide better evaluation of advanced ATM concepts and enable users to keep track of flight plans.

Developed as an accurate complex model of air traffic flow problem, FACET is a simulation model of the US airspace [49]. FACET simulates air traffic based on flight plans and allows the user to analyze congestion patterns of different sectors and centers by propagating the trajectories of proposed flights forward in time [37]. Extensively used by the FAA, NASA and industry (over 40 organizations and 5000 users), FACET can be used to both simulate and display air traffic or provide rapid statistics on recorded data [1].

In our study, we developed approaches for extracting inter-arrival and service time distributions using the FACET simulation of the air traffic data. Figures 2.2 and 2.3 show FACET snapshots of air traffic over the United States on Jan 15, 2005 at 12:19 a.m. UTC and on July 10, 2006 at 2:45 p.m. EST, respectively.

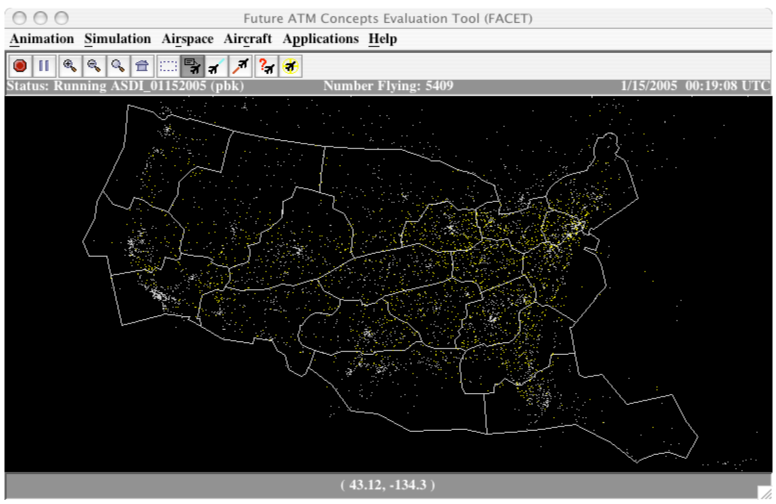

Figure 2.2 A FACET snapshot of air traffic over the United States on Jan 15, 2005 at 12:19 a.m. UTC
Source: http://www.nasa.gov/vision/earth/improvingflight/FACETSOY.html

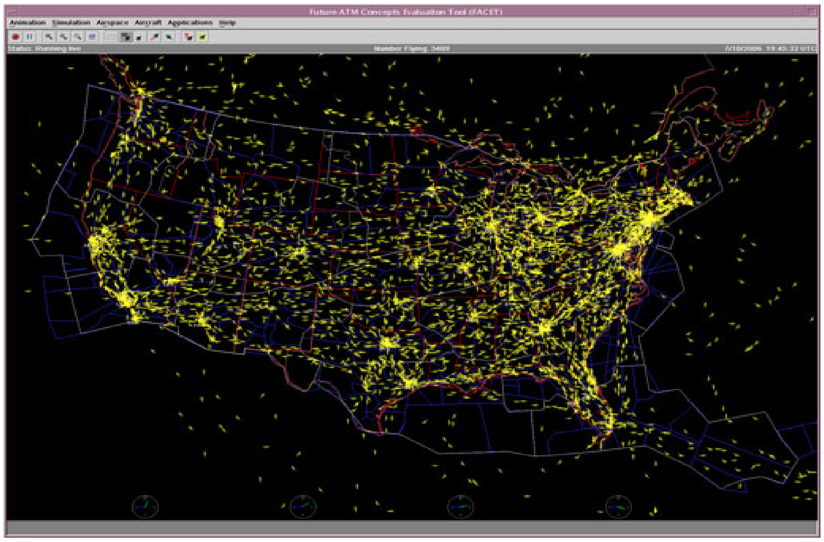

Figure 2.3 A FACET snapshot of air traffic over the United States on July 10, 2006 at 2:45 p.m. EST/ 11:45 a.m. PDT
Source: http://www.nasa.gov/vision/earth/improvingflight/FACETSOY.html

2.2 Overview of Queuing Models

Queuing models have been studied over the past several decades. Queues and queuing systems have been widely studied in research since the first telephone systems were introduced. Queuing theory was known with the work of A. K. Erlang of the Copenhagen Telephone Company in 1900s [38]. Erlang developed important knowledge in tele-traffic engineering. Queuing applications have rapidly grown in many branches including telecommunications, computer science, air traffic control, logistics, and manufacturing.

Queuing networks are used widely to analyze computer, communication, manufacturing and transportation systems [80]. Smith et al [40] says, “In 1957, Jackson published an analysis of a multiple device system where in each device contained one or more parallel servers and jobs could enter or exit the system anywhere.” Furthermore, Jackson further studied open and closed systems with dependent service rates in 1963 [42]. The special case of closed queuing systems has also been simplified [83]. Different queuing models and non-Poisson service distributions have been developed [28]. Moreover, the product-form solutions of queuing networks application have been studied the design and modeling of facilities with over four decades before [10**,** 40, 65].

Many authors have studied Makovian queues with finite capacity. Moreover, many practical situations to understand the transient behavior of queuing system were studied by Cohen, Kabayashi and Duda [4, 33, 47]. Kimura et al. [78, 79] derived diffusion approximations for various queue characteristics in a *GI/G/*1*/N* system. By using a diffusion process with a reflecting boundary for the heavy traffic case, the transient approximations were investigated for the single server case [33, 38, 39]. Moreover, Choi and Shin [13] studied *M/G/m* system with transient diffusion approximation with infinite capacity.

2.2.1 Time-Dependent Queues

The evolution of a real-world queuing system varies with time. However, most queuing models in the past decade have been dedicated to time-homogeneous models. In practice, there are many time-dependent phenomena, such as rush hour or periodicity, which they fail to capture.

Most real-world queuing systems exhibit some sort of time-dependent behavior, including time-varying arrival and service rates. Kenneth [74] observed, "However, analysis of the time-dependent behavior of even the simple $M(t)/M(t)/\infty$ queuing system requires numerical integration of an infinite number of differential-difference equations for general, real-valued and integral arrival/service-rate functions." An analysis of mobile cellular telecommunication system design has extensively used time-dependent queuing networks with infinite-servers [10]. Stewart and Marie [87] discussed a review of time-dependent queuing networks with infinite-servers. Furthermore, Runnenburg [45, 46] studied the waiting-time process of a single-server queue with Markov-dependent inter-arrival times and negative exponential services.

Jennings et al [66] observed that the time-dependent nature of a system performs well when the arrival rate changes very rapidly relative to the service times. Some non-stationary effects on multi-server Markovian queuing systems are given in Green et al. [51,52], which assumed that at each time point steady state is achieved, and used the instantaneous arrival rate $\lambda(t)$, for the mean arrival rate at time t. In addition, there has recently been much interest in the solution of time-dependent queuing problems by numerical integration [15, 31, 69].

2.2.2 Queuing Network by Enumeration of States

Haverkort and Knottenbelt [16, 84] performed approaches that search an exhaustive state space. Those approaches can be used to utilize mass storage and/or distributed hardware in an efficient way to represent the state space [32, 67]. The Champman-Kolmogorov equation based state enumeration will be require for the solution process if the inter-arrival and service time distributions are not exponential [34].

2.3 Overview of the Coxian Distribution

The Erlang distribution was introduced from the study of phases in telephone systems by Erlang [7]. Then, Cox [23, 24] generalized the class of Erlang distributions and studied probability distributions as mixtures of Erlang distributions. A Coxian distribution was first introduced by Cox [23, 24]. This distribution is widely used in many science and engineering applications.

Neuts [59] stated that the time to absorption of a finite Markov chain in continuous time that represents in a stochastic model can be represented as the Coxian phase-type distribution. There is one absorbing state or phase, and the stochastic process starts in transient state. Coxian distributions have become more important because of their universality: the Coxian distribution can approximate any distribution function closely [90]. The Coxian distribution is the most general form of a phase-type distribution that results from a system of one or more sequences, or phases.

2.3.1 Fitting Coxian Distribution to Data

It is very difficult to fit distributions or real data to the Coxian phase-type distribution according to many reports in the literature [26]. One of the major problems of fitting phase type distributions occurs because the functions are nonlinear [9]. Because it is not possible to find an exact solution, a numerical algorithm is required [58]. Methods of estimation, methods of maximum likelihood, methods of moments (moment matching) and the least squares method are methods that have been studied earlier. These three methods have been widely used for fitting data to the phase-type distribution. The maximum likelihood method has been developed for many years.

Fitting a Coxian phase-type distribution to data can be done by maximizing the log likelihood. This method has been extensively studied in much literature. For example, Bobbio and Cumani [2] applied the maximum likelihood method to maximize a combining linear program function. The minimum likelihood was used in Faddy [61, 62, 63] and Faddy and McClean [64] to study Nelder Mead algorithm. Furthermore, a phase-type distribution fitting procedure was developed by using an expectation maximization algorithm [76].

Many researchers have performed the method of moments to fit Coxian distributions. According to Schmickler and Johnson [54, 55] the method of moments matching is developed to match the moments of a mixture of Erlang distributions. Moreover, the comparison between the moments matching parameter method with the maximum likelihood method is discussed in Lang and Arthur [9]. The third group of algorithms is based on least squares estimation. The least squares method has been used to estimate the model parameters in Faddy [60]. Even though, many attempts have been performed by researchers, these approaches have limitations. In practice, the method of moments is considered the least accurate method when the original distribution is unknown [11].

2.3.2 Coxian Queuing Model

The Coxian distribution is a special case of a phase-type distribution. It has gained special importance in the research on queuing networks. BCMP methods often require distribution functions of random quantities that have rational Laplace transforms [28]. This queuing network method was name after the authors [41]. These quantities might be the inter-arrival times or service times of a queuing network. Moreover, Coxian distributions have been widely used in queuing applications and other applied probability models. According to Cox [25], “Coxian phase-type distribution is a more specific method that represents real air traffic data as a special type of Markov model.” An approach adopted for deriving queuing results using more general inter-arrival and service time distributions is to approximate these processes by Erlang or Coxian distribution [85, 86].

Various studies of queuing models with Coxian distributions have been reported in the literature. In particular, Herzog et al. [81] obtained numerical results for a single server queue with state dependent arrival and service rates, assuming that the inter-arrival times as well as the service times follow a Coxian distribution. Marie [75] studied the queue-length probability distribution of a single server queue with a Coxian service distribution and a Markovian time-dependent arrival rate. This approach was obtained using a recursive technique, which was based on the notion of the conditional throughput. Marie's model was extended to several servers as described in Stewart and Marie [87] using numerical techniques. Bertsimas [22] used the Coxian distribution and generalization method of stages introduced by Erlang to achieve the solution of a queuing system. Finally, a special class of probability distributions with rational Laplace transforms, which are related to finite Markov chains, were considered by Neuts [57]. Also, a special class of distributions can be found in Neuts [59] with queuing system numerical investigations.

CHAPTER 3

FITTING COXIAN DISTRIBUTION TO AIR TRAFFIC DATA

3.1 Introduction

As mentioned previously, simulations of real traffic data reveal that the inter-arrival time and service time distributions cannot be represented by exponential distribution random variables. The Coxian distribution has gained more attention to many researchers due to its universality: any distributions can be approximated closely by Coxian distribution. Literature on fitting the Coxian distribution to data was described in the previous chapter.

In this chapter, we describe different algorithms to fit Coxian distributions to the service time data of major Air Traffic Centers. We develop several fitting methods. We compare and evaluate fitting methods by using the mean square error (MSE) and the number of phases in the distribution. We begin the chapter with the description of the Coxian distribution in section 3.2. In section 3.3, the methods of fitting a Coxian distribution to data are presented. Finally, the experimental results are discussed in section 3.4.

3.2 Coxian Distribution

The description of the Coxian distribution here is from Menon et al. [94]. The phases of the Coxian distribution are characterized by the following parameters. Let μ_1,

$\mu_2 \ldots \mu_n$ be the service rates of an n-phase Coxian distribution. The transition probabilities for transitioning to the next phase are denoted by a_1, $a_2 \ldots a_{n-1}$ as shown in Figure 3.1.

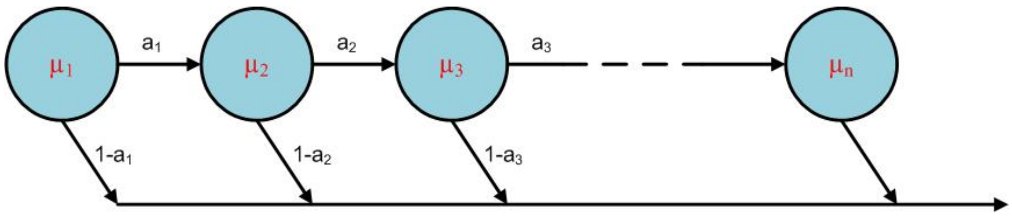

Figure 3.1 The Coxian Distribution

Here Q is a $(n + 1) \times (n + 1)$ matrix that represents the transition rate matrix for an n-phase Coxian distribution, which can be given by

$$Q = \begin{bmatrix} T & T_0 \\ 0 & 1 \end{bmatrix} \tag{3.1}$$

Let α be a state probability vector of the Coxian distribution. Let n be the number of exponential phases associate with states 1,…,n, and let state n +1 be the absorption state. T is a $n \times n$ matrix corresponding to the n exponential phases and T_o is the $n \times 1$ column vector corresponding to the absorption state.

$$T = \begin{bmatrix} -\mu_1 & a_1\mu_1 & & & & \\ & -\mu_2 & a_2\mu_2 & & & \\ & & -\mu_3 & a_3\mu_3 & & \\ & & & \cdots & \cdots & \\ & & & & \cdots & a_{k-1}\mu_{k-1} \\ & & & & & -\mu_k \end{bmatrix} \qquad T_0 = \begin{bmatrix} (1-a_1)\mu_1 \\ (1-a_2)\mu_2 \\ (1-a_3)\mu_3 \\ \cdots \\ (1-a_{k-1})\mu_{k-1} \\ \mu_k \end{bmatrix} \tag{3.2}$$

Hence, the moments of the distribution can be calculated by

$$E(X^m) = (-1^m)m!\,(\alpha T^{-m} e), m = 1,2,.. \tag{3.3}$$

The density function of the Coxian distribution of the Laplace transform can be given by the following equation [3.4]:

$$F(s) = \sum_{n=1}^{N} A_n\, b_n \prod_{i=1}^{n} \frac{\mu_i}{\mu_i + s} \tag{3.4}$$

where,

$$A_n = a_0.a_1 \dots a_{n-1} \tag{3.5}$$

$$b_n = 1 - a_n \tag{3.6}$$

3.3 Methods of Fitting a Coxian Distribution to Data

Previously, we fitted an *n*-phase Erlang distribution to the inter-arrival and service time data obtained from the FACET simulation. However, the *n*-phase Erlang distribution fitted poorly to the inter-arrival and service time data at some centers. Consequently, we develop different fitting methods that improve the fit to data. In this section, we describe several methods to fit a Coxian distribution to the service time data of major air traffic centers. We then compare and evaluate these methods by using the mean square error (MSE) and the number of phases in the distribution. Nonlinear Optimization, Fitting Histogram, and Random Sample Method are discussed in sections 3.3.1, 3.3.2, and 3.3.3, respectively.

3.3.1 Nonlinear Optimization Method

Fitting a Coxian distribution to data involves identifying the service rates μ and the continuation probabilities a. In this section, we describe a *nonlinear optimization* method by matching the moments of the distribution with the data. The *nonlinear optimization* method is our first approach to fit data with the Coxian distribution. In our implementation of the nonlinear optimization method, we used FMINCON, which is an optimization routine in MATLAB to find a minimum of constrained non-linear functions.

3.3.1.1 Nonlinear Optimization Algorithm

The following steps were used to fit data by using the nonlinear optimization method.

1. We began the fit with *n* initial phases of Erlang distribution.
2. We weighted the objective. The differences in lower moments were more penalized than those of higher moments because the lower moments are more descriptive of a probability distribution than higher moments. The revised objective essentially created a sequential goal program.
3. Subsequent Coxian distribution parameters were initialized by those of the previous distribution and the continuation probability of the final phase at close to zero.
4. The error in the fit was the *MSE*.

Table 3.1 Comparison of Mean Square Error (MSE) and Number of Phases (k) between Erlang and Coxian Distribution fit for each center

Center		Fitted Method			
		Nonlinear Optimization		**Erlang**	
ID	Name	**MSE**	**k**	**MSE**	**K**
1	'Albuquerque'	**0.025331**	**6**	**0.037093**	**2**
2	'Atlanta'	**0.013714**	**6**	**0.013714**	**1**
3	'Boston'	**0.021343**	**2**	**0.021343**	**2**
4	'Chicago'	**0.016379**	**3**	**0.016379**	**2**
5	'Cleveland'	**0.015167**	**5**	**0.015167**	**2**
6	'Denver'	**0.015422**	**7**	**0.011788**	**1**
7	'Fort Worth'	**0.009603**	**5**	**0.009603**	**2**
8	'Houston'	**0.048014**	**4**	**0.003407**	**1**
9	'Indianapolis'	**0.017635**	**3**	**0.017635**	**2**
10	'Jacksonville'	**0.014748**	**7**	**0.014748**	**2**
11	'Kansas City'	**0.002361**	**4**	**0.002361**	**2**
12	'Los Angeles'	**0.015301**	**8**	**0.015301**	**1**
13	'Memphis'	**0.00902**	**7**	**0.00902**	**2**
14	'Miami'	**0.031226**	**3**	**0.031226**	**2**
15	'Minneapolis'	**0.007984**	**2**	**0.007984**	**1**
16	'New York'	**0.009733**	**4**	**0.009733**	**2**
17	'Oakland'	**0.02913**	**5**	**0.02913**	**2**
18	'Salt Lake City'	**0.013672**	**7**	**0.013672**	**1**
19	'Seattle'	**0.009755**	**3**	**0.009755**	**2**
20	'Washington DC'	**0.021013**	**4**	**0.021013**	**2**

From Table 3.1 we observed that the nonlinear optimization method does not improve MSE compared to fitting the data to the Erlang distribution at most centers. We believe this occurs because of the initialization of the nonlinear optimization method with n initial Erlang phases. We also observed that at the Albuquerque center, the nonlinear optimization method with $k = 6$ gives a lower MSE value compared to fitting the data with the Erlang distribution with k = 2. Notice that the accuracy of the nonlinear optimization fit can be further improved by increasing the number of phases.

3.3.2 Fitting Histogram Method

The nonlinear optimization method does not give us a better fit compared to fitting the data to an Erlang distribution. In this section, we introduce a fitting histogram method. This method has been previously studied in the literature [90]. In this method we fit a Coxian distribution to the individual bin of a generated histogram from the data.

3.3.2.1 Fitting Histogram Method Algorithm

1. Generate a histogram from the given data.
2. For each bin of the histogram, fit it with a k-phased Erlang distribution.
3. Set the transition probabilities of the k-phases within each bin to be 1.
4. Set the transition probability from one bin to the next bin to be as follows:

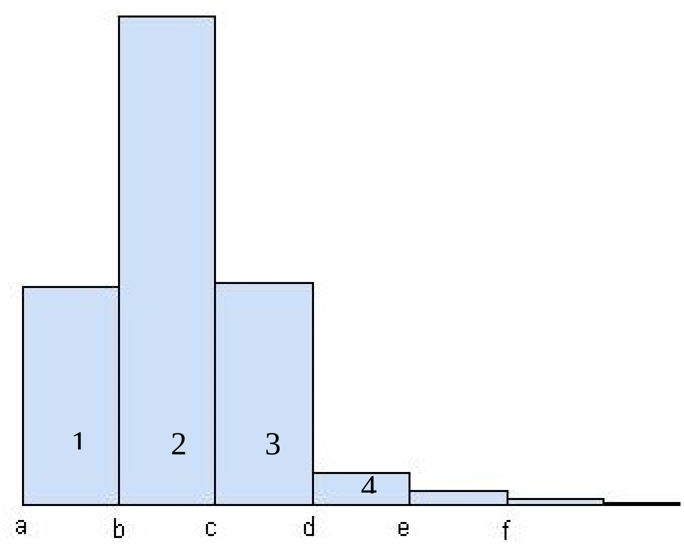

Figure 3.2 Example of Data Histogram

The mean phase length of bin 1 can be calculated as

$$\mu_1 = \frac{(2 * k)}{a + b} \tag{3.7}$$

where k = the number of phases in each bin

a = the minimum value of bin 1

b = the maximum value of bin 1

The probability of transitioning from bin 1 to bin 2 can be calculated as

$$A_{1 \to 2} = (1 - h_1) \tag{3.8}$$

where h_1 = the height (frequency) of bin 1.

The mean phase length of bin 2 can be calculated as

$$\mu_2 = \frac{(2 * k)}{b + c} \tag{3.9}$$

where c = the maximum value of bin 2.

The probability of transitioning from bin 2 to bin 3 can be calculated as

$$A_{2 \to 3} = (1 - \frac{h_2}{1 - h_1}) \tag{3.10}$$

where h_2 = The height (frequency) of bin 2.

Therefore, according to above calculation we can obtain the Coxian parameters to fit the data to a Coxian distribution as follows:

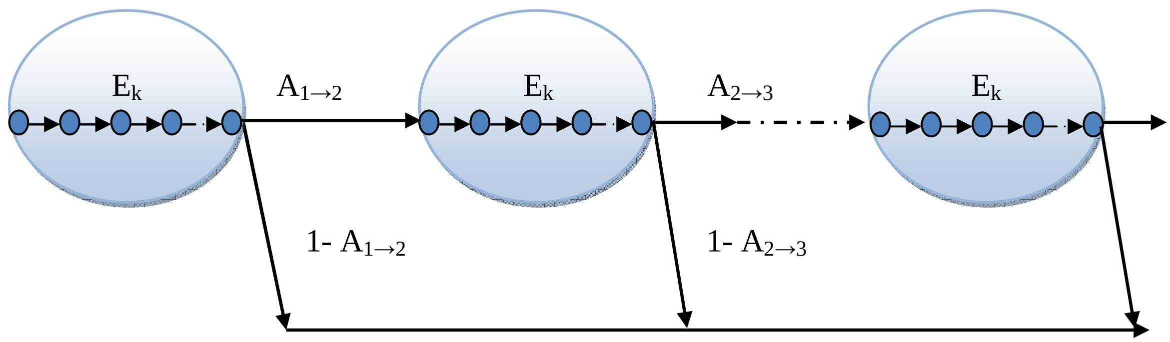

Figure 3.3 Coxian distribution by using the fitting histogram method

From Figure 3.3, each node represents a histogram bin. We then fit data with the Coxian distribution. However, if we zoom in to each bin block, then we can see that each bin consists of k phases. Using the Erlang distribution, our mean value is:

$$\mu_1 = \mu_2 = \cdots \ldots = \mu_k = \frac{k}{\lambda} \tag{3.11}$$

where λ = service time.

The transition probability within each bin is 1.

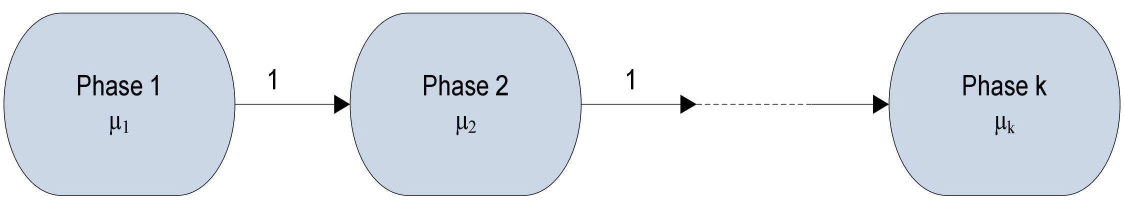

Figure 3.4 k-phased Erlang Distribution

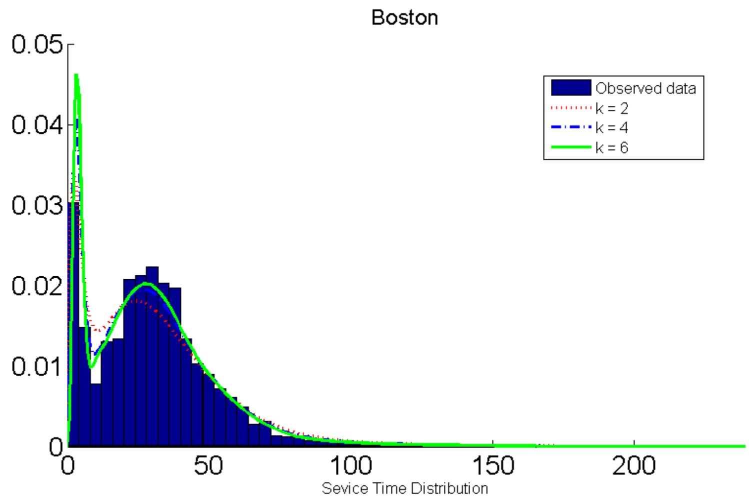

Figure 3.5 Comparison of fitting histogram method by using different *k* values at Boston center

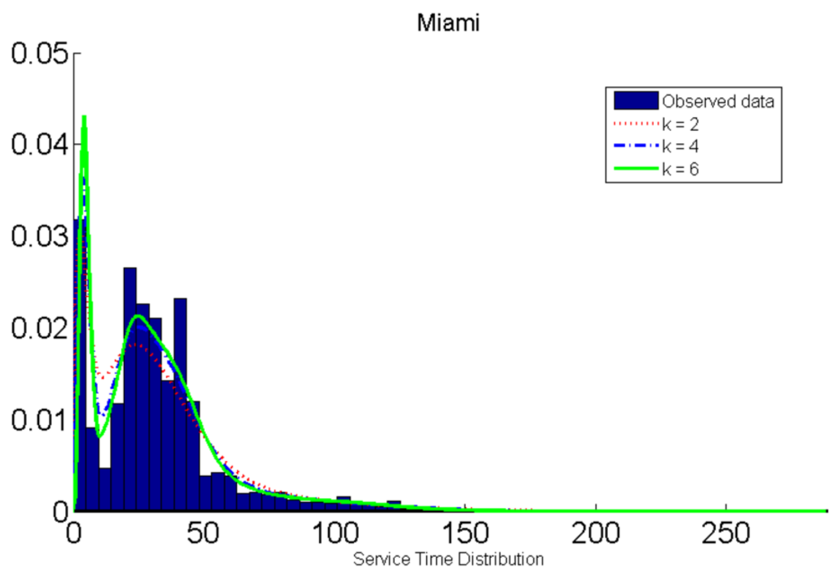

Figure 3.6 Comparison of fitting histogram method by using different *k* values at Miami Center

Figures 3.5 and 3.6 illustrate the Coxian distributions fitted to service time distributions obtained from FACET. The red dotted line represents a Coxian distribution fit to data with k = 2 (the total number of Coxian phases in this experiment equals to 30 bins × 2 phases for each bin = 60 phases). The blue dashed line represents a Coxian distribution fit to data with $k = 4$ (total number of Coxian phases in this experiment equals to 30 bins histogram × 4 phases for each bin = 80 phases). Finally, the green solid line represents a Coxian distribution fit to data with $k = 6$ (total number of Coxian phases in this experiment equals to 30 bins histogram × 6 phases for each bin = 180 phases).

From the above figures, we observed that if we increase the number of phases in the bin, then the better we fit the Coxian distribution to the data. Even though the fitting histogram method fit data almost perfectly to the histogram, the total number of phases is way too large for use in practice.

3.3.3 Random Sample Method (RSM)

Two different fitting methods were discussed in the previous sections. The nonlinear optimization method does not give lower MSE values compared to fitting the data to an Erlang distribution. On the other hand, the fitting histogram method gives us a good data fit with low MSE but with a very large number of phases compared to other methods.

In this section, a random sample method (RSM) will be introduced. RSM is a quick and powerful method that gives a very good fit to the data with both low MSE and few phases. The RSM algorithm steps and an example will be discussed in sections 3.3.3.1 and 3.3.3.2, respectively.

3.3.3.1 RSM Algorithm

1. Start with iteration $j = 1$
2. Let X^j be the data set at iteration j
3. Fit data set X^j to an Erlang (k, μ) distribution
4. From data set X^j subtract each data point with a random sample from an exponential distribution with mean parameter μ.
5. Remove negative data points from X^j.
6. Set $\mu_j = \mu$ and a_j to be the size of data set X^j divided by the size of data set X^{j-1}.
7. Set $j = j+1$
8. If the size of data set $X^j < \varepsilon$, then stop otherwise go back to step 3.

3.3.3.2 Random Sample Method Example

In this section we demonstrate a small example of fitting data with RSM. Consider data given in Table 3.2 to be the original data set in this example. For iteration 1, the first step is to fit the data given in Table 3.2 with an Erlang distribution with μ parameter equal to 0.048 as shown in Figure 3.7. Then we subtract random samples from an exponential distribution with mean parameter μ = 0.048 from each data point. The new data set can be shown in Table 3.3. The last step of this method is to set the Coxian parameter μ_1 to equal to 0.048 and the transition probability a_1 to the size of the new data set of iteration 2 as given in Table 3.3 divided by the size of the original data set of iteration 1 as given in Table 3.2. In this case, a_1 is equal to 30/30 = 1. The Coxian parameters after the first iteration can be shown in Figure 3.8.

Table 3.2 Data set for iteration 1

Index	Value	Index	Value
1	10.5	16	24.5
2	34.5	17	4.5
3	19.5	18	3.5
4	197.5	19	36
5	225	20	28.5
6	35.5	21	3.5
7	12	22	10
8	28.5	23	4
9	32.5	24	21.5
10	56.5	25	32
11	58.5	26	41.5
12	80	27	26.5
13	54.5	28	17
14	36	29	44.5
15	14	30	45.5

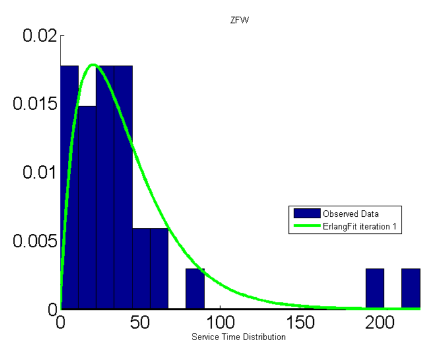

Figure 3.7 Fitted Erlang distribution with μ = 0.048 for iteration 1

Table 3.3 Data set after remove negative data point at iteration 1

Index	**Value**	**Index**	**Value**
1	8.568735	16	22.56873
2	32.56873	17	2.568735
3	17.56873	18	1.568735
4	195.5687	19	34.06873
5	223.0687	20	26.56873
6	33.56873	21	1.568735
7	10.06873	22	8.068735
8	26.56873	23	2.068735
9	30.56873	24	19.56873
10	54.56873	25	30.06873
11	56.56873	26	39.56873
12	78.06873	27	24.56873
13	52.56873	28	15.06873
14	34.06873	29	42.56873
15	12.06873	30	43.56873

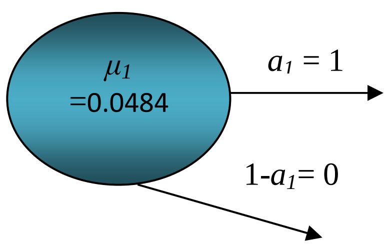

Figure 3.8 The Coxian parameters for iteration 1

Now let the data set given in Table 3.3 be the data set for this iteration 2. The basic concept is to fit this data set to an Erlang distribution with μ parameter equal to 0.0507 as shown in Figure 3.9. Then we subtract random samples from the exponential distribution with mean parameter μ = 0.0507 from each data point. The new data set can be shown in Table 3.4. Notice that, some of the data points are negative values, which are removed out from the data set. Therefore, our new data set for the next iteration is given as Table 3.5.

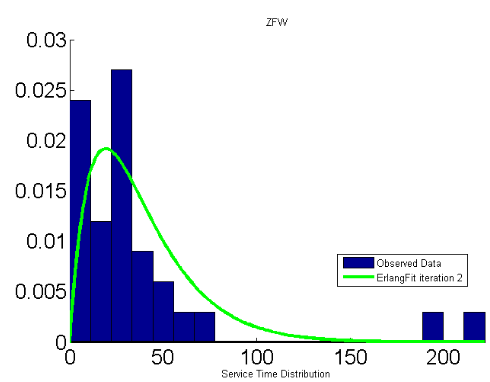

Figure 3.9 Fitted Erlang distribution with μ = 0.0507 for iteration 2

Table 3.4 Data set after subtracting exponential random variable at iteration 2

Index	Value	Index	Value
1	-24.9566	16	-10.9566
2	-0.95662	17	-30.9566
3	-15.9566	18	-31.9566
4	162.0434	19	0.54338
5	189.5434	20	-6.95662
6	0.04338	21	-31.9566
7	-23.4566	22	-25.4566
8	-6.95662	23	-31.4566
9	-2.95662	24	-13.9566
10	21.04338	25	-3.45662
11	23.04338	26	6.04338
12	44.54338	27	-8.95662
13	19.04338	28	-18.4566
14	0.54338	29	9.04338
15	-21.4566	30	10.04338

Table 3.5 New data set for iteration 3

Index	**Value**
1	162.0434
2	189.5434
3	0.04338
4	21.04338
5	23.04338
6	44.54338
7	19.04338
8	0.54338
9	0.54338
10	6.04338
11	9.04338
12	10.04338

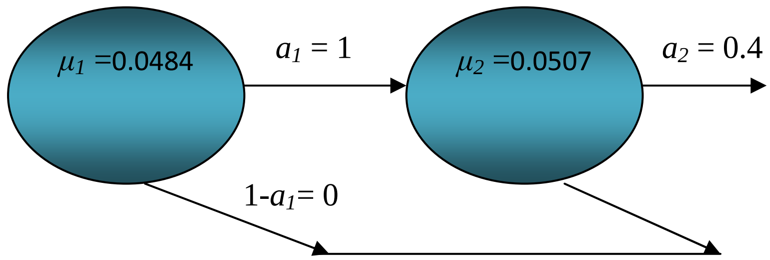

Figure 3.10 The Coxian parameters for iteration 1 and 2

The final step of this iteration is to set the Coxian parameter μ_2 to equal to 0.0507, and the transition probability a_2 can be calculated as 12/30 = 0.4. The Coxian parameters after the first and second iterations can be shown in Figure 3.10. We continue doing these same steps until the size of the data is really small; then we stop. The final Coxian parameters in this example can be shown in Figure 3.11.

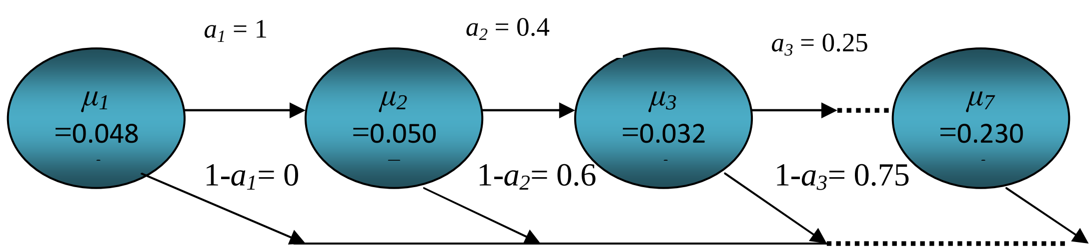

Figure 3.11 The final Coxian distribution

3.4 Experimental Results

Several fitting methods were developed and discussed earlier. In this section, we compare and evaluate a variety of fitting methods by using the mean square error (MSE) and the number of phases in the distribution. The results of fitting different methods to service time data at the Miami and Albuquerque centers are given in Figures 1 and 2, respectively.

Figures 3.12 and 3.13 represent the comparison of MSE and the total number of phases (k) for each of the fitting methods. The results show that RSM finds a good fit to the service time data at the Albuquerque and Miami centers. By using this method, we get low MSE values and fewer phases (k) compared to the nonlinear optimization method and the fitted histogram method at both centers. Even though, the fitted histogram method gives the lowest MSE value, it requires 300 phases, which is not really practical.

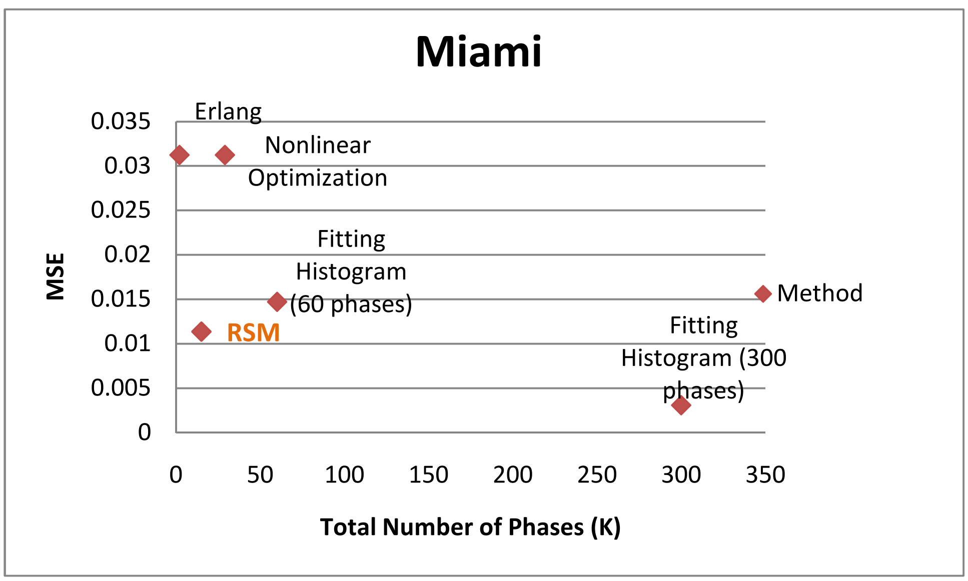

Figure 3.12 Comparison of different fitting Coxian distribution methods to service time data at the Miami Center

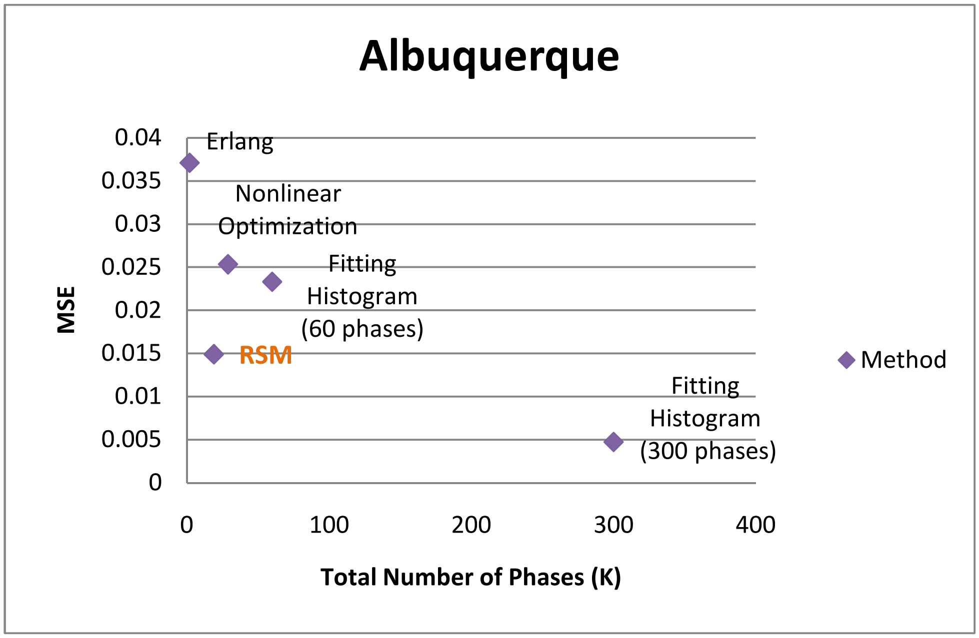

Figure 3.13 Comparison of different fitting Coxian distribution methods to service time data at the Albuquerque Center

Notice that at the Miami center, fitting service time data with the nonlinear optimization method does not improve the MSE value compared to fitting data to an Erlang distribution. As discussed earlier, we believe this occurs because of the initialization of the nonlinear optimization method with an initial *n*-phase Erlang.

From our experimental results, we can conclude that the RSM is a quick and powerful method that yields a lower MSE and fewer phases compared to other methods. This method will be used for fitting time dependent inter-arrival time $C_{m(t)}(t)/C_k/s$ queuing models, which will be discussed in chapter 4.

CHAPTER 4

TIME-DEPENDANT QUEUING MODELS OF THE NATIONAL AIRSPACE SYSTEM

4.1 Introduction

The real world queuing models have arrival rate and service rate vary with time. In this chapter we present time dependant queuing models of the national airspace system. In section 4.2, we describe an algorithm for calibrating $C_{m(t)}(t)/C_k/s$ queuing model. The state enumeration technique is discussed in section 4.3. Furthermore, we present an algorithmic approach to determine average measures of the queues in section 4.4. Finally, we validate time dependant queuing models with FACET simulation.

4.1.1. Problem Description

In our study, we developed approaches for extracting inter-arrival and service time distributions using the FACET simulation for June 1-7, 2007 of the air traffic data. We make the following assumptions:

Assumption 1: It is possible to un-truncate the arrival data.

Assumption 2: The set of times representing fundamental changes in the inter-arrival distribution is 24 one-hour time periods.

Assumption 3: The service time distribution does not change and remains stable for the entire time horizon.

The first step to calibrate a $C_{m(t)}(t)/C_k/s$ queuing model was to extract inter-arrival and service time data from the FACET simulation. We then fitted 24 one-hour inter-arrival time periods and service time data to Coxian distribution by using RSM as discussed in chapter 3.

Furthermore, we applied the state enumeration technique for the queuing solution. The steady state solution can be obtained by integrating the Markov process equation $\dot{x} = Qx$ until steady state is reached. More specific details will be discussed in the next section. Finally, an algorithmic approach is used to determine average measures of the queues. The average measures can be obtained by calculating the probability state vector for each time period.

4.2 Calibrating a Time Dependant Queuing Model

In this section, we describe an algorithm for calibrating a $C_{m(t)}(t)/C_k/s$ queuing model in which the service distribution is a Coxian random variable, and the inter-arrival distribution is a time-dependent piece-wise constant Coxian random variable from FACET data.

4.2.1. Calibrating Queuing Parameters

4.2.1.1 Estimating Arrival Rates

Using seven days of FACET data, we developed MATLAB code to extract network arrivals that include both external sources and aircraft arriving from other centers within the network called *network arrivals* directly from FACET. Let $X_1,...,X_l$ be a set of independent Coxian random variables of the inter-arrival time at different periods throughout the day. In this study, we assume *l* equals to 24 one- hour time period. Then, the inter-arrival time is *a time-dependent piece-wise constant Coxian random variable* given by equation (4.1)

$$X(t) = \begin{cases} X_1 & t_1 > t \geq t_0 \\ X_2 & t_2 > t \geq t_1 \\ \vdots & \vdots \\ X_l & t_l > t \geq t_{l-1} \end{cases} \tag{4.1}$$

For each time period, we then fitted inter-arrival times to a Coxian distribution. However, fitting a Coxian distribution to the data involves identifying the rates μ and the continuation probabilities a. We discussed RSM to fit a Coxian distribution with data in chapter 3.

4.2.1.2 Estimating Service Rates

Like estimating arrival rates, we developed MATLAB code to extract the time that an aircraft takes to cross a specific center, which is referred to as *service time.* The service time distributions are also fitted by RSM.

<u>4.3 Queuing Network by Enumeration of States</u>

The Champman-Kolmogorov equation based state enumeration will be require for the solution process if the inter-arrival and service time distributions are not exponential [34]. The continuous time long run behavior of a Markov process can be described as

$$\dot{x}(t) = Qx(t) \tag{4.2}$$

where $x(t)$ is a probability vector [92]. The value $x_i(t)$ represents the probability of the system being in state i where $i = 1,\ldots,n$. Let $Q(t)$ be the $n \times n$ matrix. This matrix is called the *infinitesimal generator matrix* or the *transition rate matrix* for a Markov process. Consider a queue with arrival rate λ and service rate μ for all nodes in the queuing system. The Markov process in steady-state, we simple have

$$xQ = 0 \tag{4.3}$$

Here x is the stationary probability vector where x_i is the steady-state probability that the system is in state *i*. The vector x can also be obtained from equation (4.3). The normalizing condition can be calculated as follow

$$\sum_{i=1}^{n} x(i) = 1 \tag{4.4}$$

The steady state solution can be obtained by integrating Equation (4.2) until steady state is reached.

As discussed above, by integrating $\dot{x} = Qx$, the steady state solution to the Markov process can be reached. The transient solutions can also be obtained by using this method. Using the matrix exponent e_t^Q , the solution of time in variant system solution can also be obtained.

4.3.1 State Enumeration

As discussed in Sengupta and Tandale [91], the analysis of a queue with a Coxian arrival process distribution and a Coxian service time distribution with *s* servers can be represented in Figure 1.7. Let μ_{G1}, $\mu_{G2,,}$…, μ_{Gm} be the service parameters with an *m*-phase Coxian distribution, and let a_{G1}, a_{G2},…., a_{Gm} be the transition probabilities of the arrival process. We refer to this as a *generato*r. Furthermore, the C_m/C_k/s queue service node can be represented by a k-phase Coxian, with service parameters μ_{N1}, $\mu_{N2}, \ldots, \mu_{Nk}$, and transition probabilities $a_{N1}, a_{N2}, \ldots, a_{Nk}$. In this study, we assume that the service node has s identical servers, each of which has the same set of parameters. The $C_m/C_k/s$ queuing system can be represented by Figure 4.1.

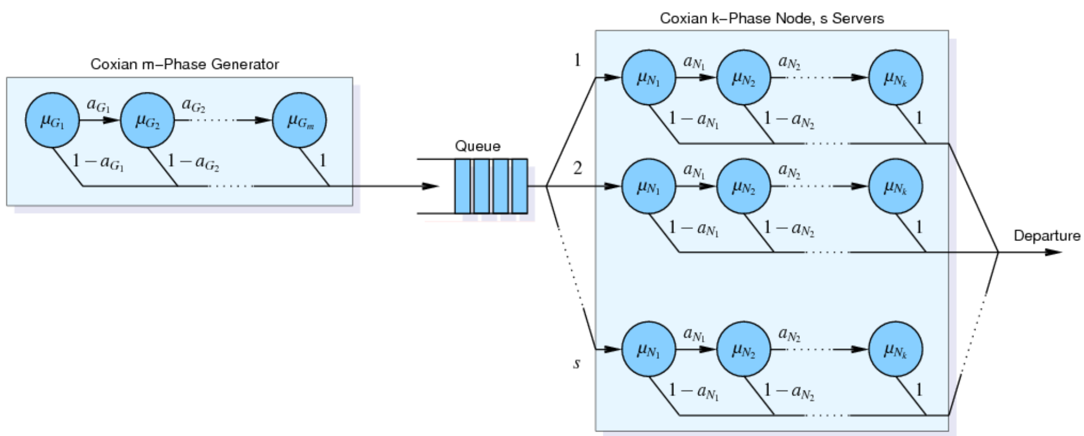

Figure 4.1 $C_m/C_k/s$ Queue

In this study, the $C_m/C_k/s$ queuing system state can be defined by (1) the phase of the arrival process or generator, (2) the number of items in service node, and (3) the number of servers in the same phase of service node. Let a be the phase of the generator, let b be the number of items in service, and let $c_1, c_2, \ldots, c_k$ be the number of servers in phases 1,2,…, k, Therefore, the state of the system can be represented by the sequence $a : b(c_1, c_2, \ldots, c_k)$. The total number of servers in phases 1,2,…, k at a service node can be given as [91]:

$$\sum_{i=1}^{k} c_i = n, \qquad 0 \le n \le s \;, n = 0,1,2, \ldots$$

$$\sum_{i=1}^{k} c_i = s, \qquad n \ge s \tag{4.5}$$

Here we want to illustrate the queuing state system with 2 generator phases, 3 servers, each with 3 service phases. In this example, the generator is in phase 2, and the maximum number of items in service is 4. Table 4.1 shows the possible states of such queue.

Table 4.1 Enumerated States for $C_2/C_3/3$ Queue with Generator in Phase 2 and $n \le 4$ Items in Service [91]

Index	State	Index	State	Index	State	Index	State
0	2:0(0,0,0)	8	2:2(1,10)	16	2:3(1,2,0)	24	2:4(1,0,2)
1	2:1(0,0,1)	9	2:2(2,0,0)	17	2:3(2,0,1)	25	2:4(1,1,1)
2	2:1(0,1,0)	10	2:3(0,03)	18	2:3(2,1,0)	26	2:4(1,2,0)
3	2:1(1,0,0)	11	2:3(0,1,2)	19	2:3(3,0,0)	27	2:4(2,0,1)
4	2:2(0,0,2)	12	2:3(0,2,1)	20	2:4(0,0,3)	28	2:4(2,1,0)
5	2:2(0,1,1)	13	2:3(0,3,0)	21	2:4(0,1,2)	29	2:4(3,0,0)
6	2:2(0,2,0)	14	2:3(1,0,2)	22	2:4(0,2,1)	30	1:0(0,0,0)
7	2:2(1,0,1)	15	2:3(1,1,1)	23	2:4(0,3,0)	.	.

4.3.1.1 State Transitions

The transitions of a $C_m/C_k/s$ queue which we described as sequence

a: b $(c_1, c_2, \ldots, c_k)$ above can be defined as following events:

1. A phase change in the arrival node or generator
2. An aircraft arrival into the service node or departure from the generator
3. An aircraft departure from the service node.

Each transition is described in detail.

- Phase Change in Arrival node (Generator)

Let μG_i be the service rate in the process of generating an arrival with an *m*-phase Coxian distribution, and let aG_i be the transition probability in phase *i* where $i=1,\ldots m$. The transition rate can be given by $\mu G_i aG_i$ when the generator changes from the phase *i* to phase $(i+1)$. The state transition of a phase change in the arrival node can be given by

$$a \leftarrow a+1 \tag{4.6}$$

- Arrival into Service Node

An *arrival* event occurs when an aircraft departing from the generator moves to be served at a service node. The transition rate is $\mu G_i\ (1 - aG_i)$ where $i = 1,\ldots,m$ represents the phase of the arrival node from which the aircraft departs. An event occurs in which an aircraft departs from phase *i* and returns back to phase 1. This event is given by:

$$a \leftarrow 1 \tag{4.7}$$

The number of aircraft in service is incremented by one, which is given by

$$b \leftarrow b+1 \tag{4.8}$$

If the number of aircraft in service is less than the number of available server in current state, then the arriving aircraft is immediately served by an idle server, which is given by

$$c_1 \leftarrow c_1+1 \tag{4.9}$$

If the number of aircraft in service is greater than or equal to the number of available servers, then aircraft has to wait in the queue, and so only *b* is incremented as in (4.8).

- Departure from the Service Node

When an aircraft leaves the server from a service node, the *departure* event occurs. Let μN_j be the service rate of server in service node in *j*th phase where j = 1,…,*k*. Let aN_j be the transition probability at service node. Then the transition rate for the *j*th phase can be simply given by $\mu N_j(1\text{-}aN_j)$, when the aircraft depart from service node.

$$c_j \leftarrow c_j - 1 \tag{4.10}$$

The number of aircraft in service can be also given by

$$b \leftarrow b - 1 \tag{4.11}$$

4.3.1.2 Total Number of States

Let *N*(*n*) be the total number of states in which there are at most *n* aircraft in the system. Then is *N*(*n*) as described in Sengupta and Tandale [91]:

$$N(n) = \begin{cases} m\sum_{i=0}^{n} P(i,k) & n = 0,\ldots,s \\ m\left[\sum_{i=0}^{s} P(i,k) + (n-s)P(s,k)\right] & n = s,\ldots \end{cases} \tag{4.12}$$

where *P(i, k)* is the number of ways that *k* nonnegative integers sum to *i*, which is given by

$$\begin{aligned} & p_1 + p_2 + \ldots\ldots + p_k = i \\ & p_{1\ldots\ldots k} \geq 0 \end{aligned} \tag{4.13}$$

Then, *P(i, k)* can be written as

$$P(i,k) = \binom{i+k-1}{k-1} \tag{4.14}$$

Hence, we have

$$\sum_{i=0}^{n} P(i,k) = \binom{n+k}{k} \tag{4.15}$$

The total number of states is obtained by substituting (4.15) in (4.12)

$$N(n) = \begin{cases} m\binom{n+k}{k} & n = 0,.....,s \\ m\left[(n-s) + \dfrac{(s+k)}{k}\right]\binom{s+k-1}{k\text{-}1} & n \ = \ s,....... \end{cases} \tag{4.16}$$

4.4 Determining Probability State Vector

The research in this section was previously described in Menon et al. [93]. We described how enumeration of states can be used within a $C_{m(t)}(t)/C_k/s$ queuing network. The enumeration of states methodology solves for the steady state value of x, where $x(i)$ is the probability that the system is in state i. Furthermore, the approach to determine average measures of the queues is discussed. The transition matrix $Q(t)$ can be written as given in equation (4.16) due to the piece-wise time dependent inter-arrival time of a $C_{m(t)}(t)/C_K/s$ queuing model.

$$Q(t) = \begin{cases} Q_1 & t_1 > t \geq t_0 \\ Q_2 & t_2 > t \geq t_1 \\ \vdots & \vdots \\ Q_l & t_l > t \geq t_{l-1} \end{cases} \tag{4.17}$$

This shows that the transition rate matrix varies, which can lead to the steady state probabilities not existing. However, we still would like to calculate certain average measures based upon the probabilities of each state over time horizon using equation (4.18), where $M(x)$ is a measure based upon the state probability vector x, and $\overline{M}$ is the average of the measure over time.

$$\overline{M} = \frac{\int_{t_0}^{t_l} M(x(t))dt}{t_l - t_0} \tag{4.18}$$

Equation (4.18) can be rewritten to determine the average state probability vector $\overline{x}_i$ for each time period $\forall i = 1,...,l$ and an average measure $\overline{M}$ for each time period using equation (4.19) due to the linearity of many standard queuing model measures with respect to the state probability vector.

$$\overline{M} = \frac{\sum_{i=1}^{l} (t_i - t_{i-1}) M_i \overline{x}_i}{t_l - t_0} \tag{4.19}$$

How to calculate $\overline{x_i}$ for each time period t_i, $\forall i = 1,.....l$ will be discussed. By integrating numerically using the ODE 45 in MATLAB, equation (4.20) shows how to calculate the state probability vector over time as following

$$x(t) = e^{Q_i(t-t_{i-1})} x(t_{i-1}), \quad \forall t \in \left[t_{i-1}, t_i\right), \forall i = 1,...,l \tag{4.20}$$

For a sufficiently small $\varepsilon > 0, x(t_i - \varepsilon)$ and $x(t_i)$ represent probability state vectors of two different state spaces at a given time t_i. Because this can lead to a significant calculation complication, $x(t_i), \forall i = 1,.....,l$ will be redefined by two different vectors. Let $x^{+}(t_i)$ and $x^{-}(t_i)$ be the state probability vectors at time t_i in state spaces associated with periods $[t_i, t_{i+1})$ and $[t_{i-1}, t_i)$, respectively for each time t_i, $\forall i = 1, \ldots, l-1$. Variables $x(t_0)$ and $x(t_l)$ are redefined as $x^{+}(t_0)$ and $x^{-}(t_l)$, as shown in equation (4.20), to further specify notation.

$$x^{-}(t_i) = e^{Q_i(t_i - t_{i-1})} x^{+}(t_{i-1}), \quad \forall i = 1, \ldots, l \tag{4.21}$$

An algorithm to calculate the probability vector for each time period $\bar{x}_i, \forall i = 1, \ldots, l$ is given by the following steps.

Step 1: Let $i = 1$ and assume $x^{+}(t_0)$ is given.

Step 2: Find $x^{-}(t_i)$ and $\bar{x}_i$ using integration.

Step 3: If $i < l$, then project $x^{-}(t_i)$ into the state space of period $[t_i, t_{i+1})$ to find $x^{+}(t_i)$ and goto step 2.

Even though state vector $x^{+}(t_0)$ may not have existed, the algorithm assumes that it is given in Step 1. The time very early in the morning before most aircraft have entered the airspace, t_0, and some time very late in the evening after most aircraft have already left the airspace, t_l, are implicit values. It is possible that the time t_l can be set to exactly one day after time t_0. State vector $x^{-}(t_l)$ can be assumed to be the steady-state probability associated with Q_l because the airspace is generally emptying in the time period $[t_{l-1}, t_l)$. The steady-state probability can be determined by solving $Q_l x = 0$

and $1x = 1$. Using the same projection method in step 3, we can project this steady state probability into the state space of period $[t_0, t_1)$ to find $x^+(t_0)$.

4.4.1 Projection Algorithm

An approach for projecting the probability vector $x^-(t_i)$ to vector $x^+(t_i)$ as in step 3 is shown in this section. Vectors $x^-(t_i)$ and $x^+(t_i)$ will be rewritten as x^- and x^+ to simplify notation. The sets of the states of the inter-arrival distribution the Coxian queue associated with time periods $[t_{i-1}, t_i)$ and $[t_i, t_{i+1})$ will be defined as A^- and A^+ respectively, and the set of states of the customers in service in the Coxian queues will be defined as S. Let $x_{ij}^- (x_{ij}^+)$ be the associated component of vector $x^-(x^+)$ for each state $i \in A^-$ or $i \in A^+$ and each state $j \in S$.

The projection algorithm is described below.

Step 1: *Determine the probability state vector of the inter-arrival distribution* α^-. For each state, $i \in A^-$ set $\alpha_i^- = \sum_{j \in S} x_{ij}^-$.

Step 2: *Determine the probability state vector of the inter-arrival distribution* α^+. Solve the goal programming problem in (4.21) in which the parameters $0 < w_1 < w_2 < \ldots$..

$$\begin{aligned}
&\min \sum_{i=1}^{\infty} d_i + c_i \\
&\text{s.t. } i!(\alpha^+ (T^+)^{-i} 1) - i!(\alpha^- (T^-)^{-i} 1) + w_i d_i - w_i c_i = 0 \qquad \forall i = 1,2,\ldots. \\
&\quad \sum_{i \in A^+} \alpha_i^+ = 1 \\
&\quad \alpha_i^+ \geq 0 \qquad \forall i \in A^+ \\
&\quad d_i, c_i \geq 0 \qquad \forall i = 1,2,\ldots..
\end{aligned} \tag{4.22}$$

Step 3: *Determine the probability state vector* x^+. For each state $i \in A^+$ and each state, $j \in S$ set $x_{ij}^+ = \alpha_i^+ \sum_{i \in A^-} x_{ij}^-$.

$$\dot{x} = Q_{24} x$$

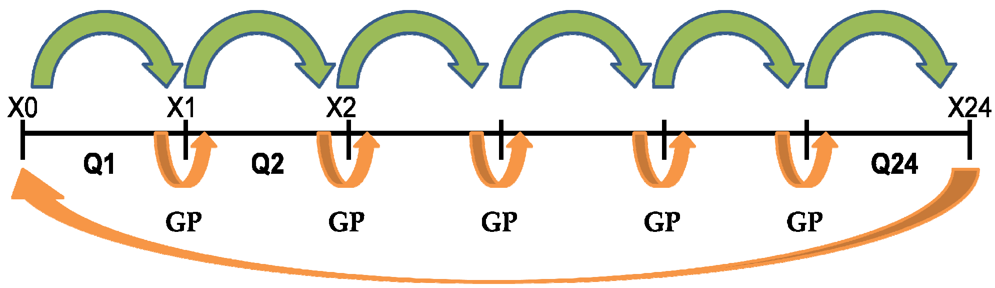

Figure 4.2 Steps for determining probability state vector at each time period

Figure 4.2 shows the steps for determining the probability state vector. The vector x_{24} is the steady-state probability associated with Q_{24} that can be determined by solving $Q_{24}x = 0$ and $1x = 1$. In order to find x_0, we can project this steady state probability into the state space of initial time period t_0 using the projection method discussed in step 3. We then continue to integrate and project for each time period.

4.5 Validating Time-Dependent Queuing Models with FACET

We attempt to develop the queuing model with Coxian inter-arrival time distributions and Coxian service time distributions. However, the model suffered from an enormous state space. Consequently, in this study we employ a time-dependent Coxian distribution for the inter-arrival time distribution, and a Markovian distribution for the service time distribution. Using the Coxian distribution inter-arrival distribution has a limited increase in the state space.

4.5.1 Center Level Study

The $E_{m(t)}(t)/M/s$ queuing models were studied earlier in Menon et al. [93]. In this section, we compare results of the $E_{m(t)}(t)/M/s$ and $C_{m(t)}(t)/M/s$ with those of the FACET simulation. Results for the expected number of aircraft in the NAS based upon the major center-level network are given in Figures 4.3 through 4.8, which show the average number of aircraft in the system for each of the two queuing models. The average numbers of aircraft are obtained from integrating $\dot{x}(t) = Qx(t)$ equation at each time period until steady-state reached. The red dotted line, labeled ErlangFit, represents a queuing model with time-dependent Erlang inter-arrival time distributions and a Markovian service time distribution. The blue dashed line, labeled CoxianFit, represents a queuing model with time-dependent Coxian inter-arrival time distributions and a Makovian service time distribution. The purple line represents the averaged FACET simulation for June 1-7, 2007. We simply calculate the average number of aircraft from FACET by counting the number of aircraft at a particular center every 30 second time step, and then we take the average of the total number of aircraft for every 1 hour time period.

The results show that the both $E_{m(t)}(t)/M/s$ and $C_{m(t)}(t)/M/s$ models perform well until the 13th time period with the exception of the first period of the day. This can be attributed to the fact that the models assume that the number of aircraft are periodic, which is not necessarily apparent in FACET. We believe that this is due to initialization within FACET. Finally, we also observe that from 14th time period, the results of the $C_{m(t)}(t)/M/s$ models estimate the FACET simulation better than the results of the

$E_{m(t)}(t)/M/s$ model. Therefore, we can conclude that the Coxian distribution gives a more accurate national airspace queuing model than Erlang distribution.

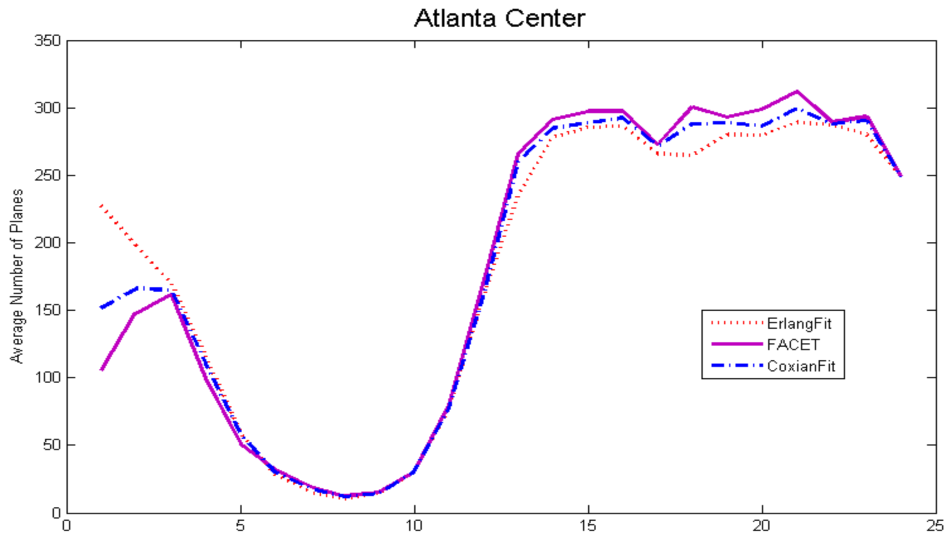

Figure 4.3 Time-dependent queuing models versus FACET averaged for June 1-7, 2007 at the Atlanta Center

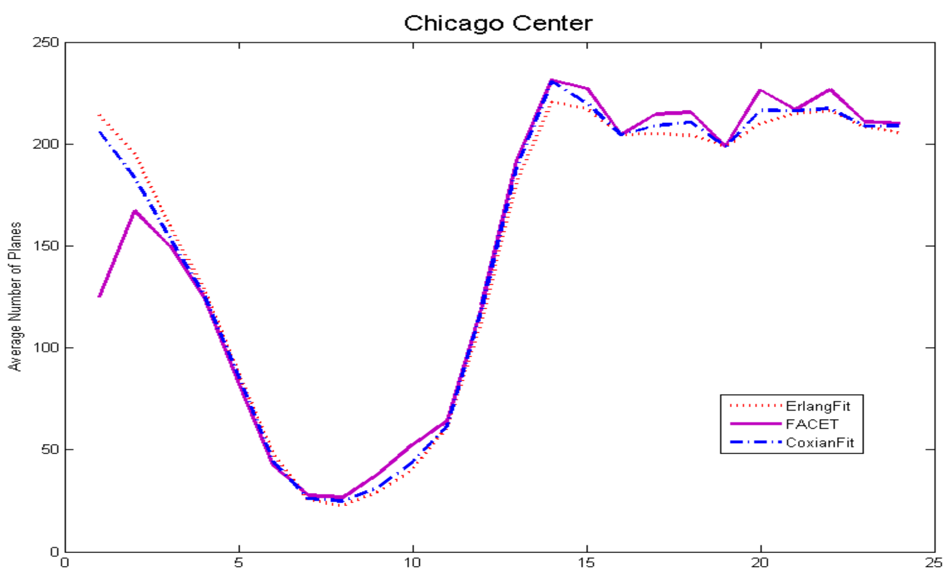

Figure 4.4 Time-dependent queuing models versus FACET averaged for June 1-7, 2007 at the Chicago Center

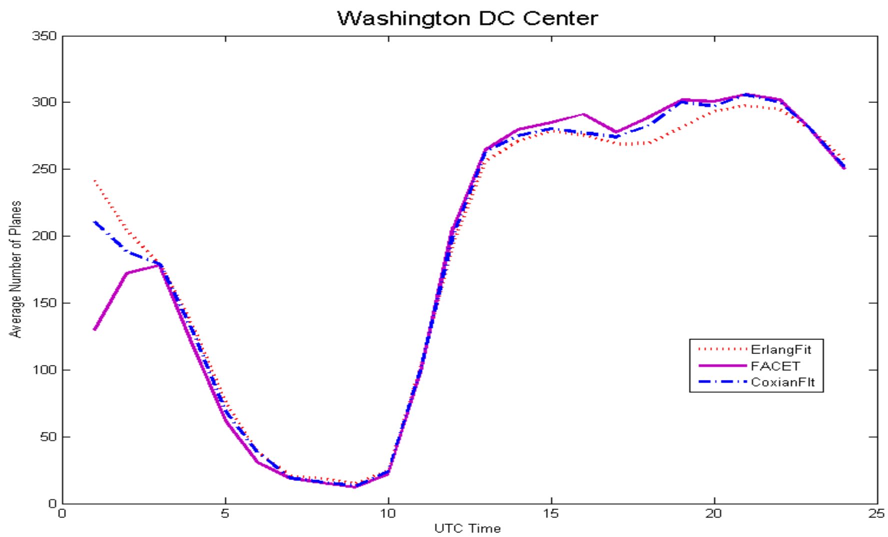

Figure 4.5 Time-dependent queuing models versus FACET averaged for June 1-7, 2007 at the Washington DC Center

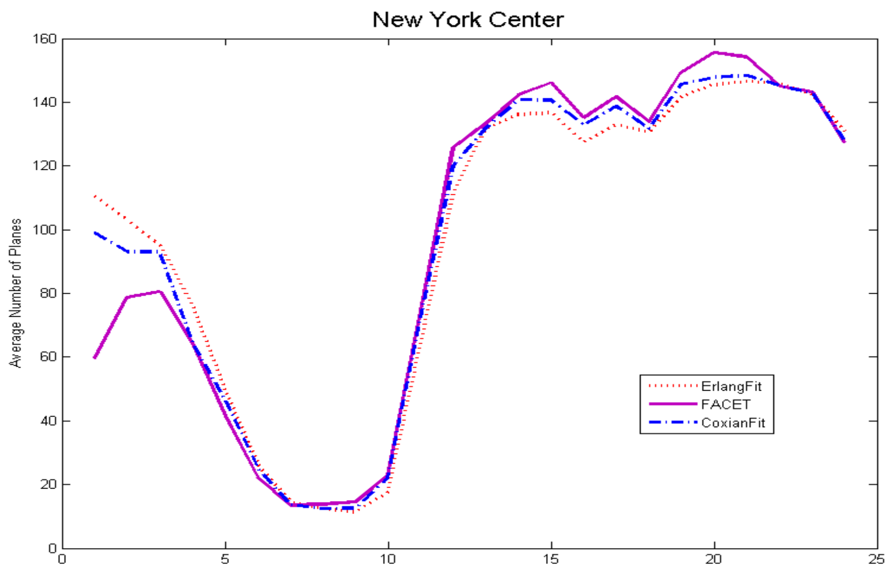

Figure 4.6 Time-dependent queuing models versus FACET averaged for June 1-7, 2007 at the New York Center

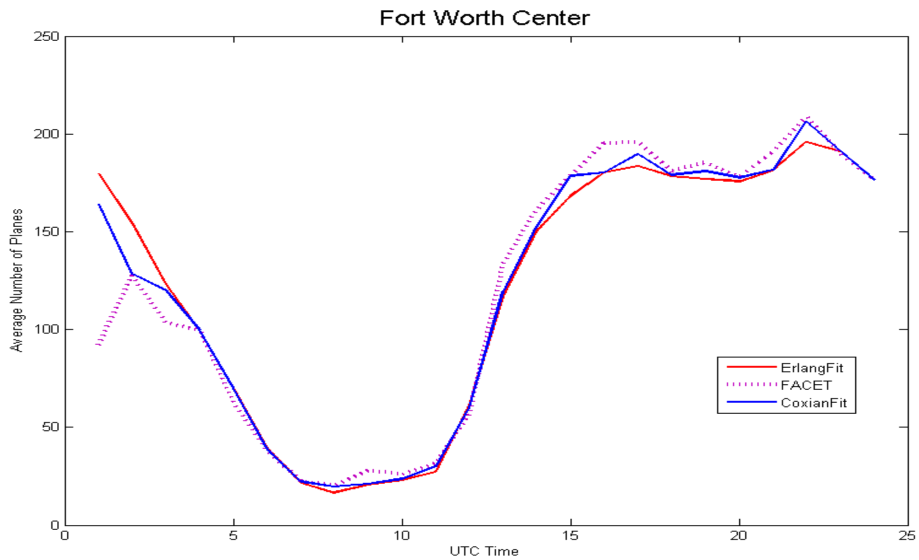

Figure 4.7 Time-dependent queuing models versus FACET averaged for June 1-7, 2007 at the Fort Worth Center

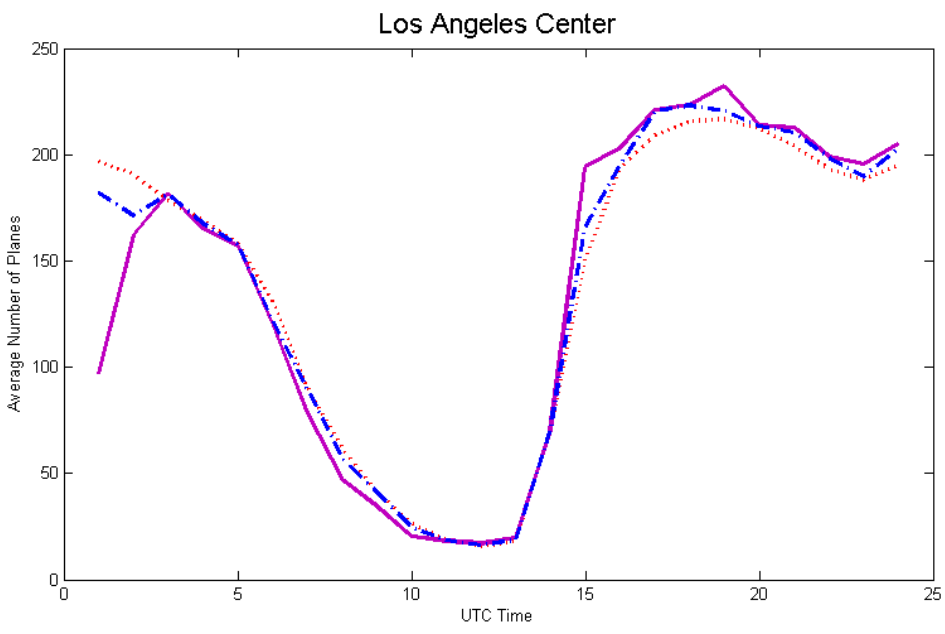

Figure 4.8 Time-dependent queuing models versus FACET averaged for June 1-7, 2007 at the Los Angeles Center

4.5.2 Cell Level Study

We define the US. boundaries by specifying latitude and longitude with 1.5-degree-by-1.5-degree as a *cell*. Moreover, we assumed that the dimension of the cell is squared. We then compare results for models of five cells that included the major airports ATL, DFW, JFK, LAX, and ORD. The results are given in Figures 4.9 through 4.13, which show the average number of aircraft in the system for each of the two queuing models. The dark red solid line represents cell capacity which is the number of server in the queue. MATLAB code is developed to extract cell level information from FACET including capacity.

We observe that the results of the $E_{m(t)}(t)/M/s$ model, and the $C_{m(t)}(t)/M/s$ model fit the FACET simulations very well in most airports until time period 13 with the exception of the first period of the day as discussed above. These results are consistent with those in previous section. Notice that, at time period 19 at LAX, the average number of aircraft of the $E_{m(t)}(t)/M/s$ model is over LAX's capacity and the FACET simulation results. On the other hands, using the $C_{m(t)}(t)/M/s$ model was actually more accurate. Moreover, we also again observe that the overall results $C_{m(t)}(t)/M/s$ model are more accurate than the results of the $E_{m(t)}(t)/M/s$ model compared with FACET. Again, consistent with those in previous section, we can conclude that the Coxian distribution gives more accurate queuing model than the Erlang distribution.

The summary of results is shown in Table 4.1. The table compared each distribution fitted with the MSE value for major centers and cells level. From those tables the averaged MSE values of Coxian fitted are obtained by take the average of

MSE for every time period. Notice that, MSE value is much lower for the Coxian distribution than the Erlang distribution at both center and cell level. Hence, the results of the compared MSE value for each distritbution are consistent with the results in the previous section. Therefore, we can conclude that fitting air traffic data to the Coxian distribution improves the accuracy of queuing model of the NAS.

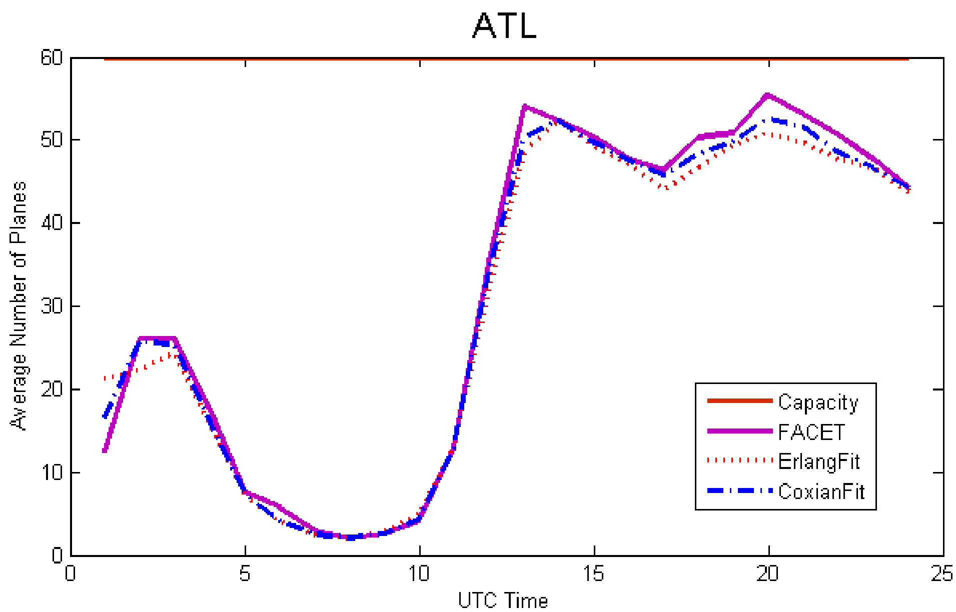

Figure 4.9 Time-dependent queuing models versus FACET averaged for June 1-7, 2007 at ATL.

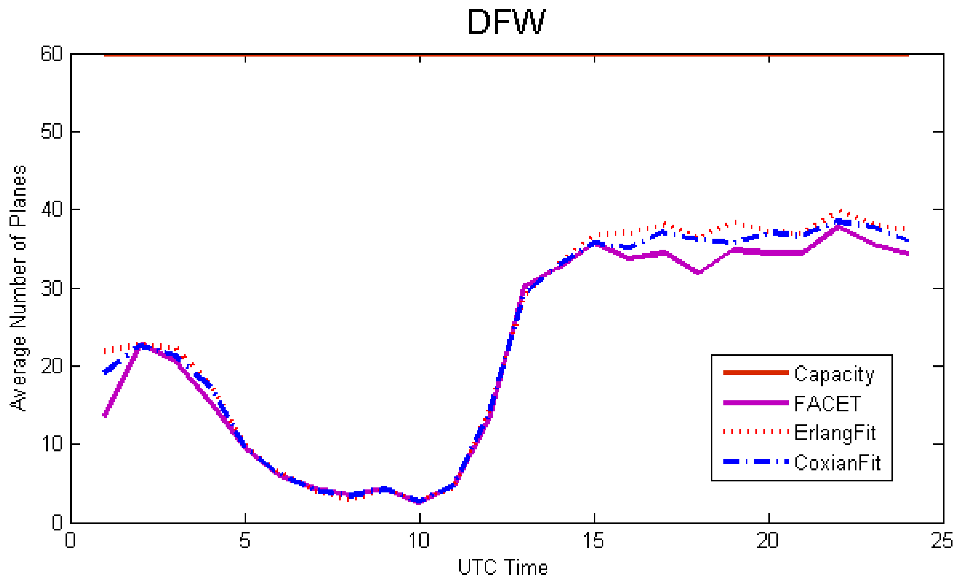

Figure 4.10 Time-dependent queuing models versus FACET averaged for June 1-7, 2007 at DFW.

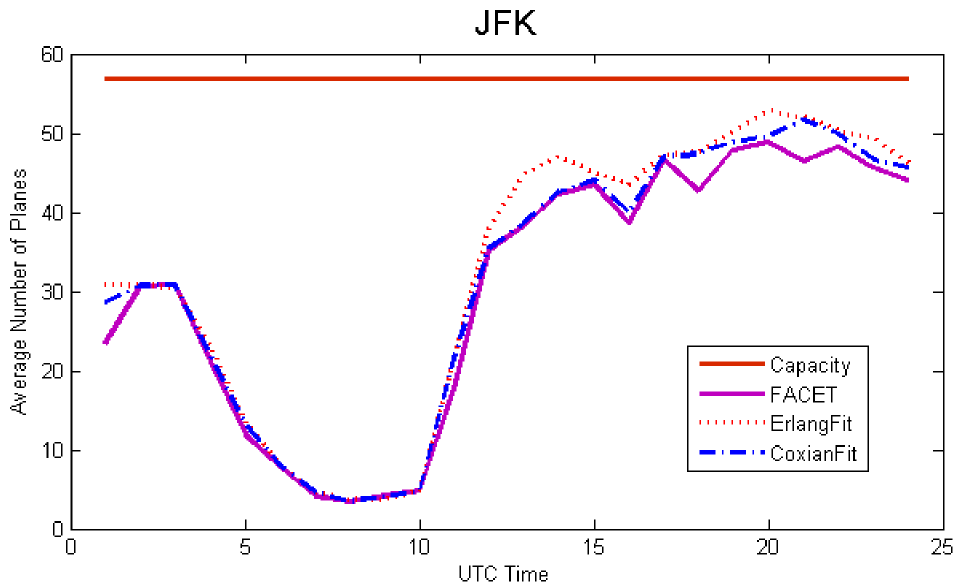

Figure 4.11 Time-dependent queuing models versus FACET averaged for June 1- 7, 2007 at JFK.

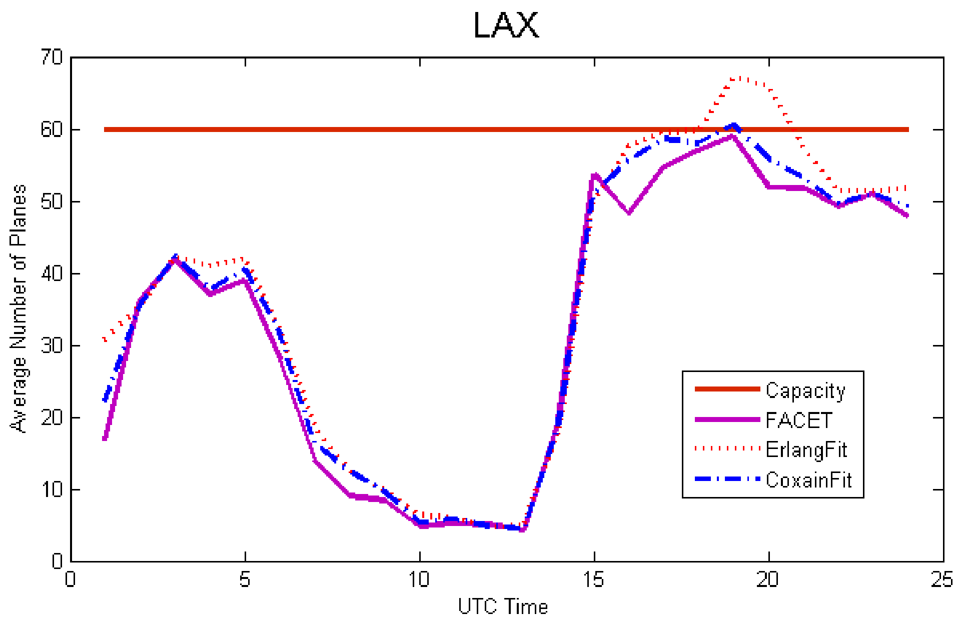

Figure 4.12 Time-dependent queuing models versus FACET averaged for June 1-7, 2007 at LAX.

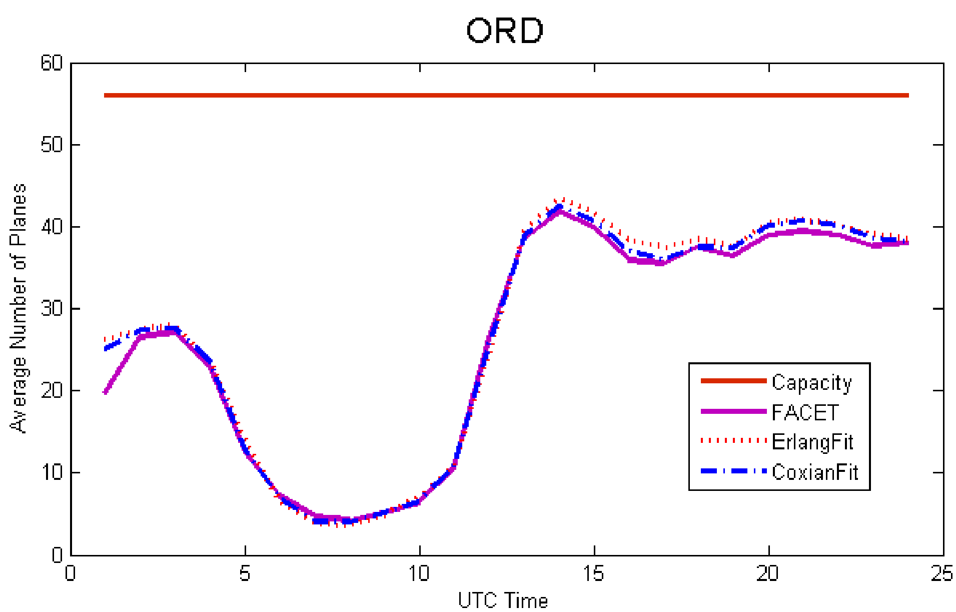

Figure 4.13 Time-dependent queuing models versus FACET averaged for June 1-7, 2007 at ORD.

Table 4.2 Average MSE values between Coxian and Erlang distribution at cell and center level

Center	MSE	
	Erlang	Coxian
ZFW	0.121475	0.0553
DC	0.10424	0.036759
Atlanta	0.16055	0.041591
NY	0.151116	0.047106
ZLA	0.133475	0.052113
chicago	0.088319	0.035221

Airport	MSE	
	Erlang	Coxian
DFW	0.036284	0.0154
JFK	0.035781	0.018289
LAX	0.050239	0.017288
ORD	0.021728	0.009293
ATL	0.027674	0.013103

4.5.3 Period Length Analysis

The analysis of $E_{m(t)}(t)/M/s$ and $C_{m(t)}(t)/M/s$ in sections 4.5.1 and 4.5.2 was performed with 1 hour 24 time periods. However, Menon et al. [93] discussed using periods that were not necessarily one hour and equal in length. By using the Markovian steady-state analysis within each time period, this section discusses an analysis of different period lengths of *Markovian Time-Dependent queue (M(t)/M/s)*. Figure 4.14 illustrates the compared results of Markovian steady state and Markovian time-dependent queuing models in which the time periods had durations of one hour (denoted 1 segment), two hours (denoted 2 segment), four hours (denoted 4 segment), and six hours (denoted 6 segment).

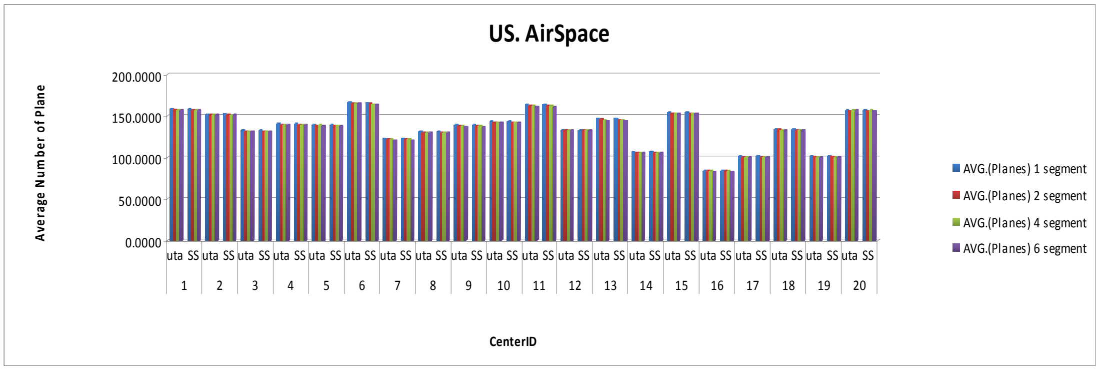

Figure 4.14 Results for the Queuing Models for Different Time Period Lengths at Each Center.

Notice that, the average number of aircraft increased with shorter time periods in most cases. The largest difference occurs in Center 13 in which the average number of aircraft varied by 2.8 in the steady-state model and 2.6 in the time-dependent analysis.

4.5.4 Long-Run Convergence Analysis

This section was described previously in Menon et al. [93]. In the analysis of the Coxian and Erlang queues, the time-dependent models are only integrated over a 24 one hour periods. The initial starting vector of the first period is a projection of the steady-state vector of the final period. However, the initial starting vector may not be representative of an appropriate starting vector in long-run conditions if the final period does not reach steady state. Furthermore, a potential misrepresentation in the starting vector may trickle into subsequent periods. This section analyzes the queues over 96 one-hour periods and compares them to those over the first 24 hours. Figure 4.15 displays the average number of aircraft when considering 24 versus 96 hours.

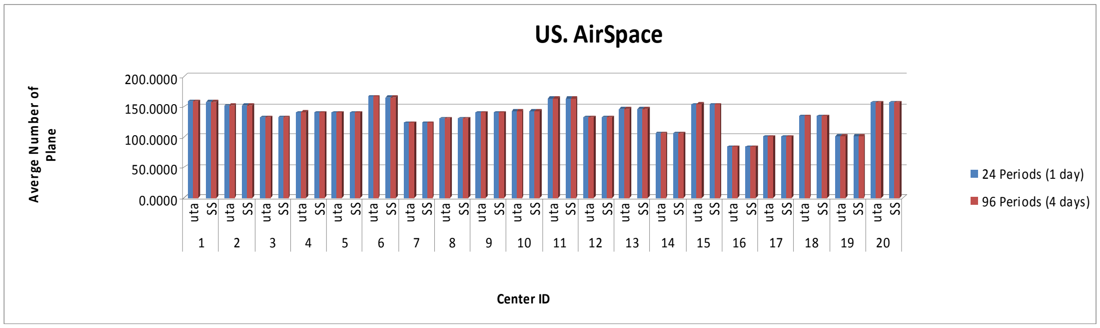

Figure 4.15 Results for queuing models over 24 hours versus 96 hours for each center.

The results reveal that performance in the first 24 hours is very similar to the entire 96 hours. Therefore, we can conclude that performing a long run queuing analysis is unnecessary. The analysis of 24 hr time periods give as accurate queuing model as the long run analysis. However, in this section the long-run convergence analysis is performed only with Markovian queues. The analysis of long run period convergence with a Coxian queue would be a very interesting future research topic.

CHAPTER 5

CONCLUSION AND FUTURE RESEARCH

In this dissertation, we described several fitting Coxian distribution to data methods, which are 1) Nonlinear Optimization 2) Fitting Histogram and 3) Random Sample. The nonlinear optimization method, which used the method of moment technique, did not give us a better fit compared to fitting the data to an Erlang distribution. Therefore, we introduced a fitting histogram method [90]. In this method, we fitted a Coxian distribution to the individual bin of a generated histogram from the data. Even though the fitting histogram method fit data almost perfectly to the histogram, the total number of phases was way too large for use in practice. Last but not least, we developed a random sample method (RSM), which is a quick and powerful method. RSM gave a very good fit to the data with both low MSE and few phases. This method was used for fitting time dependent inter-arrival time $C_{m(t)}(t)/C_k/s$ queuing models in chapter 4. We discussed a practical approach for modeling the NAS with time-dependent Coxian queues. Time-dependent $C_{m(t)}(t)/C_k/s$ queuing models of the National Airspace were developed in which the inter-arrival distribution is a time-dependent piece-wise constant Coxian random variable, and the service time distribution is a Coxian random variable. We described an algorithm for calibrating a $C_{m(t)}(t)/C_k/s$ queuing model from simulated data of an Air Route Traffic Control Center and an algorithmic approach to determine average measures of the queues. We attempted to develop $C_{m(t)}(t)/C_k/s$ however, $C_{m(t)}(t)/M/s$ was used in this study due to the uncontrollable state space when we used Coxian service time distributions. We compared our $C_{m(t)}(t)/M/s$ queues with $E_{m(t)}(t)/M/s$ models, which were previously studied. Furthermore, we also validated both of our queuing models with the FACET simulation. We observed that the overall results of the $C_{m(t)}(t)/M/s$ models were more accurate than the results of the $E_{m(t)}(t)/M/s$ models when compared with FACET. Therefore, we concluded that the Coxian distribution gives more accurate queuing models than the Erlang distribution.

Finally, the $C_{m\,(t)}(t)C_{k(t)}/s$ queuing model in which the inter-arrival distribution is a time-dependent piece-wise constant Coxian random variable, and the service time distribution is a time-dependent piece-wise constant Coxian random variable might be one of the interesting topic of future research. The more sophisticated $C_{m(t)}(t)C_{k(t)}/s$ queuing model would likely be a more accurate representation of the real world systems with uncertainties like weather.

As discussed earlier in section 4.5.4, a long-run convergence analysis with the Coxian queuing model is another topic of our future research. The results of a long-run convergence analysis with the Markovian queue compared 96 time periods with 24 time periods and gave similar results. In this study we only used the default capacity in FACET. However, there are lots of different models for the capacity of a cell. This would be our future work to study the effects of capacity on the approach.

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