AN IDENTIFIER-TRACKING BASED MODEL REFERENCE ADAPTIVE CONTROL WITHOUT THE KNOWLEDGE OF RELATIVE DEGREE

by

XIAO HU

Presented to the Faculty of the Graduate School of

The University of Texas at Arlington in Partial Fulfillment

of the Requirements

for the Degree of

DOCTOR OF PHILOSOPHY

THE UNIVERSITY OF TEXAS AT ARLINGTON

May 2011

Copyright © by Xiao Hu 2011

All Rights Reserved

ACKNOWLEDGEMENTS

My greatest appreciation goes to my Supervising Professor, Kai S. Yeung. Throughout my doctoral work Dr. Yeung gave me valuable research guidance and encouraged me to develop independent thinking and research skills. Without his support, it would be impossible for me to get this work done.

Further, I would like to express my gratitude to Professors William E. Dillon, Khosrow Behbehani, Jonathan Bredow and Wei-Jen Lee for taking time to serve on my doctoral committee and for their help during the course of this work.

Last, but not least, I would like to express my heart-felt gratitude to my husband and my parents. None of this would have been possible without the love and unceasing support of my families.

April 4, 2011

ABSTRACT

AN IDENTIFIER-TRACKING BASED MODEL REFERENCE
ADAPTIVE CONTROL WITHOUT THE KNOWLEDGE
OF RELATIVE DEGREE

Xiao Hu, Ph.D.
The University of Texas at Arlington, 2011

Supervising Professor: Kai S. Yeung

The purpose of Model Reference Adaptive Control (MRAC) is to create a controller with adjustable parameters to obtain the desired response from a reference model. One of the basic assumptions in MRAC is that the relative degree of the plant is known exactly. However, this assumption is too restrictive for some practical plants, since the relative degree of the plant may not be specified in advance. The study of MRAC with unknown relative degree has been important from both theoretical and practical point of view. This dissertation focuses on a new design approach for the model reference adaptive control of a single-input single-output linear time-invariant plant to relax this crucial assumption.

The proposed method, called the "Model reference adaptive control without the knowledge of relative degree", does not require the knowledge of relative degree of the plant. This is achieved by the specific structure of reparameterization for control plant. The n-th order plant with unknown relative degree can have one identical structure for different relative degrees by employing the new method of reparameterization. For this reason, the structure of

proposed model reference adaptive controller does not change, even if the unknown relative degree varies from 1 to n.

The proposed method is based on a stacked identifier structure. The goal is to make the output of the plant asymptotically track the output of the first identifier, and then driving the output of the first identifier to track that of the second identifier, and so forth, up to the n-th identifier where n is the order of the plant. Lastly, the output of the n-th identifier is forced to converge to the desired response of the reference model.

This new MRAC scheme guarantees the signal boundness and zero tracking error. All the parameter update laws are derived based on Lyapunov stability theory. Simulation studies are illustrated to show the effectiveness of the proposed method.

TABLE OF CONTENTS

ACKNOWLEDGEMENTS……..... iii

ABSTRACT…… iv

LIST OF ILLUSTRATIONS….....….viii

Chapter Page

1. INTRODUCTION……...…….…… 1

2. AN IDENTIFIER-TRACKING BASED MRAC WITHOUT THE KNOWLEDGE OF RELATIVE DEGREE – SECOND ORDER PLANTS……..... 3

2.1 Introduction……...3

2.2 MRAC of plants with relative degree one or two…… 4

2.2.1 Reparameterization of the Unknown Plant…… 4

2.2.1.1 Plant…….……4

2.2.1.2 Control Scheme……. 5

2.2.1.3 Identifiers……5

2.2.1.4 Re-parameterization of the Plant and Identifiers…….. 6

2.2.2 Parameter Update Laws for the Identifiers……..... 9

2.2.3 Control Law u(t) …….. 19

2.2.4 Boundedness of All Signals in the Entired Feedback System…….……..... 23

2.2.5 Convergence of the Tracking Errors……....26

2.2.6 Simulation Studies…….....27

3. AN IDENTIFIER-TRACKING BASED MRAC WITHOUT THE KNOWLEDGE OF RELATIVE DEGREE – GENERAL CASE FOR N_TH ORDER PLANT……. 45

3.1 Introduction…….....45

3.2 MRAC of plants without the knowledge of relative degree…… 45

3.2.1 Reparameterization of the Unknown Plant……..45

3.2.1.1 Plant……..…… 45

3.2.1.2 Control Scheme…… .....….. 46

3.2.1.3 Identifiers…….....47

3.2.1.4 Re-parameterization of the Plant
and Identifiers…….…….....…48

3.2.2 Parameter Update Laws for the Identifiers……..52

3.2.3 Control Law u(t)……... 56

3.2.4 Boundedness of All Signals in the Entired
Feedback System……..60

3.2.5 Convergence of the Tracking Errors…… 63

3.2.6 Simulation Studies…….. 63

4. CONCLUSIONS……79

4.1 General Conclusion…….. 79

4.2 Future Research …….. 80

REFERENCES……...……...…...82

BIOGRAPHICAL INFORMATION……..90

LIST OF ILLUSTRATIONS

Figure Page

2.1 Schematic diagram for 2nd order plant of unknown relative degree 5

2.2 MRAC with uncertain relative degree for Identifier #1.....8

2.3 MRAC with uncertain relative degree Identifier #2. 9

2.4 MRAC with uncertain relative degree for Identifier #1 and Identifier #2.....18

2.5 MRAC with uncertain relative degree for 2nd order plant of relative degree 1or 2 22

2.6 2nd order stable plant, q=2. (a) error, (b) reference input signal,
(c) plant vs reference output and (d) control signal u 29

2.7 2nd order unstable plant, q=2. (a) error, (b) reference input signal,
(c) plant vs reference output and (d) control signal u 31

2.8 2nd order stable plant, q=2. (a) error, (b) reference input signal,
(c) plant vs reference output and (d) control signal u 33

2.9 2nd order unstable plant, q=2. (a) error, (b) reference input signal,
(c) plant vs reference output and (d) control signal u 35

2.10 2nd order stable plant, q=1. (a) error, (b) reference input signal,
(c) plant vs reference output and (d) control signal u 37

2.11 2nd order stable plant, q=1. (a) error, (b) reference input signal,
(c) plant vs reference output and (d) control signal u 39

2.12 1st order stable plant, q=1. (a) error, (b) reference input signal,
(c) plant vs reference output and (d) control signal u 41

2.13 1st order unstable plant, q=1. (a) error, (b) reference input signal,
(c) plant vs reference output and (d) control signal u 43

3.1 Schematic diagram for N_th order plant of unknown relative degree 46

3.2 MRAC with uncertain relative degree for Identifier #1.....51

3.3 MRAC with uncertain relative degree for Identifier # γ …….. 52

3.4 MRAC with uncertain relative degree for Identifier #1, …, Identifier #(γ-1) and Identifier #γ 55

3.5 MRAC with uncertain relative degree for n-th order plant 59

3.6 3rd order stable plant, q=3. (a) error, (b) reference input signal, (c) plant vs reference output and (d) control signal u 64

3.7 3rd order stable plant, q=3. (a) error, (b) reference input signal (c) plant vs reference output and (d) control signal u 66

3.8 3rd order unstable plant, q=3. (a) error, (b) reference input signal, (c) plant vs reference output and (d) control signal u 68

3.9 3rd order stable plant, q=3. (a) error, (b) reference input signal (c) plant vs reference output and (d) control signal u 70

3.10 3rd order stable plant, q=2. (a) error, (b) reference input signal, (c) plant vs reference output and (d) control signal u 72

3.11 3rd order unstable plant, q=2. (a) error, (b) reference input signal, (c) plant vs reference output and (d) control signal u 74

3.12 3rd order stable plant, q=1. (a) error, (b) reference input signal, (c) plant vs reference output and (d) control signal u 76

3.13 3rd order unstable plant, q=1. (a) error, (b) reference input signal, (c) plant vs reference output and (d) control signal u 78

CHAPTER 1

INTRODUCTION

In the design and analysis of Model Reference Adaptive Control, the control parameters are updated based on the error between output of the system and the desired response from the reference model. More specifically, the objective of this method is to force the output of a partially known linear time-invariant (LTI) plant to asymptotically track the output of a specified reference model for all reference model inputs in a pre-specified class, typically the set of all piecewise continuous and bounded functions. In order to fulfill the objective of the desired reference model, the plant does not to be known completely; however, the plant is assumed to satisfy certain conditions and some structural information is assumed to be available. Having the knowledge of relative degree is among the four classical assumptions [7] made about a single-input single-output (SISO) system. Recently there has been interest in examining the extent to the assumption of knowing the relative degree is necessary. Attempts to relaxing this assumption include [22]-[27]. The prior knowledge of relative degree can be restricted to the known values of either one or two. In other instances, by the development of new MRAC schemes the assumption can be relaxed to requiring only an upper bound of the relative degree. This is done at the expense of updating additional parameters in variable structure designs [23]-[24].

This paper presents an alternative way to solve the problem using a specific reparameterization scheme and a method which will be called the “An identifier-tracking based model reference adaptive control without the knowledge of the relative degree”. The idea is to integrate identifiers in the control scheme in order that the control structure comes closer to the structure of the plant. One identical plant structure is developed for n-th order plant. This plant

structure remains unchanged even if the unknown relative degree assumes different values. Thus the relative degree of the plant can be anywhere from 1 to n. No state measurement of the plant is required. The adaptive controller ensures uniformly bounded transients and asymptotic tracking for the system simultaneously. Simulation studies are given to show that the guaranteed transient response of the proposed scheme is obtained without knowing the relative degree.

The dissertation is arranged in such a manner that Chapter 1 gives an introduction to the area of traditional MRAC research. In Chapter 2, a new parameterization scheme is presented for MRAC of a second order plant, with unknown relative degree one or two, which is an essential milestone for extending the method to higher order plants with unknown relative degrees. Adaptive laws are designed to update the controller parameters. Stability analysis is presented in this chapter. Chapter 3 is devoted to the extension of the design and analysis to the general case, i.e., plants of arbitrary degrees and arbitrary unknown relative degrees. Chapter 4 gives a conclusion of this dissertation and possible areas for future research.

CHAPTER 2

AN IDENTIFIER-TRACKING BASED MRAC WITHOUT THE KNOWLEDGE OF RELATIVE DEGREE - SECOND ORDER PLANTS

2.1 Introduction

For most adaptive schemes, the ability to deal with second order plants is an essential milestone for extending the method to higher relative degrees. We consider second order plants, the relative degree could be anywhere from 1 to 2 and is assumed to be unknown. We consider second order plants, for which we will develop an adaptive scheme with a double-identifier structure. We shall divide the development of the new MRAC into four parts:

i). Plant,

ii). Control scheme,

iii). Identifiers, and

iv). Reparameterization of the identifiers.

The notation used in the adaptive control literature varies widely. In this paper, upper case letters are used to denote matrices, operators, or transfer functions and lower case letters are used for scalars or vectors. When $u(t)$ is a function of time, $u(s)$ denotes its Laplace transform; both u and $u(.)$ denote $u(t)$ or $u(s)$ according to the context. $P(s)$ is a plant transfer function or a plant transfer function operator with $s = \frac{d}{dt}$.

2.2 MRAC of plants with relative degree one or two

2.2.1. Reparameterization of the Unknown Plant

2.2.1.1. Plant

In order to bring out clearly the features of the method, a second-order plant is considered. Higher-order plants can be treated in a similar fashion as will be described in Section 3.

$$P(s) = \frac{b_1^{'}s + b_0^{'}}{s^2 + a_1^{'}s + a_0^{'}} \tag{2.1}$$

where $b_0^{'} > \varepsilon > 0$ with a known lower bound ε.

Note that first-order plants can be handled as second-order ones by multiplying the same linear factor to the numerator as to the denominator of the transfer function. As a result, the method introduced here can handle plants of up to the second order. Notice that the leading coefficient in the numerator may or may not be zero so that the relative degree q of the system (denominator degree minus the numerator degree) may be either one or two. It is assumed that the sign of $b_0^{'}$ is known. Without loss of generality, we shall assume the two cases: $b_1^{'} = 0$ and $b_1^{'} \neq 0$. This will be discussed in 2.2.1.3.

2.2.1.2. Control Scheme

Figure 2.1 shows the adaptive control scheme to be presented here.

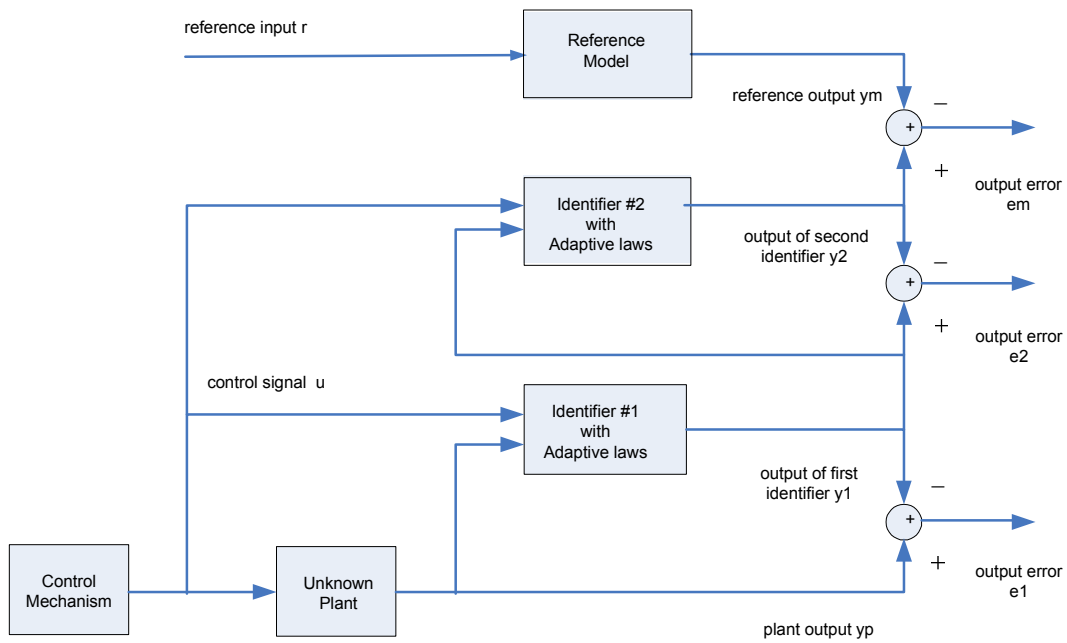


Figure 2.1: Schematic diagram for 2nd order plant with unknown relative degree

The control scheme consists of two identifiers, a reference model, and a control law.

(It will be seen later that the number of identifiers used is equal to the maximum possible relative degree of the plant)

2.2.1.3. Identifiers

The primary function of the identifiers is to let their outputs track the output of the plant. In the method introduced here, the identifiers are constructed based on the following third-order structure with constant coefficients;

$$D(s) = \frac{b_1 s + b_0}{s^3 + a_2 s^2 + a_1 s + a_0} \tag{2.2}$$

Where the coefficient $b_0 > \varepsilon > 0$. The structure in (2.2) can be seen to encompass the various

relative-degree cases of the second order plant in (2.1) as follows:

Case I: The leading coefficient $b_1^{'} = 0$

$$P_\mu(s) = \frac{b_0^{'}}{s^2 + a_1^{'}s + a_0^{'}} \cdot \frac{s+\mu}{s+\mu} = \frac{b_{1\mu}s + b_{0\mu}}{s^3 + a_{2\mu}s^2 + a_{1\mu}s + a_{0\mu}} \tag{2.3}$$

where μ is some positive constant.

Thus the identifier structure is equivalent to the plant (2.1).

Case II: The leading coefficient $b_1^{'} \neq 0$

$$P_\nu(s) = \frac{b_1^{'}s + b_0^{'}}{s^2 + a_1^{'}s + a_0^{'}} \cdot \frac{\nu}{s+\nu} = \frac{b_{1\nu}s + b_{0\nu}}{s^3 + a_{2\nu}s^2 + a_{1\nu}s + a_{0\nu}} \tag{2.4}$$

where ν is some arbitrarily large positive constant.

Even though the identifier structure (2.4) is not "equivalent" to the plant (2.1), there exists a large enough constant ν such that structure (2.4) will be arbitrarily close to structure (2.1). As a result, the output of the identifier can track the output of the plant as required above.

2.2.1.4. Re-parameterization of the Plant and Identifiers

The plant can be re-parameterized according to the identifier structure (2.2) as

$$y_p(t) = \frac{1}{s+\lambda}\left(\beta_1^*\tilde{u}_1(t) + \beta_2^*\tilde{u}_2(t) + \alpha_0^* y_p(t) + \alpha_1^*\tilde{y}_{p1}(t) + \alpha_2^*\tilde{y}_{p2}(t)\right) \tag{2.5}$$

Where

$$\tilde{u}_1 \triangleq \frac{1}{s+\lambda}u(t) \tag{2.6}$$

$$\tilde{u}_2 \triangleq \frac{1}{(s+\lambda)^2}u(t) \tag{2.7}$$

$$\tilde{y}_{p1} \triangleq \frac{1}{s+\lambda}y_p(t) \tag{2.8a}$$

$$\tilde{y}_{p2} \triangleq \frac{1}{(s+\lambda)^2}y_p(t) \tag{2.8b}$$

It can easily be shown that unknown constant coefficients in (2.5) are related to those in (2.2) by the following matrix equation:

$$\Lambda\, w = z\,,\ \Lambda = \left[\begin{array}{c|c} \Lambda_1 & 0 \\ \hline 0 & \Lambda_2 \end{array}\right] \tag{2.9}$$

where $w, z \in R^{5x1}$, $\Lambda \in R^{5*5}$, $\Lambda_1 \in R^{3x3}$ and $\Lambda_2 \in R^{2x2}$ are given by

$$w = \begin{bmatrix} \alpha_0^* & \alpha_1^* & \alpha_2^* & \beta_1^* & \beta_2^* \end{bmatrix}^T \tag{2.10}$$

$$z = \begin{bmatrix} 3\lambda - a_2 & 3\lambda^2 - a_1 & \lambda^3 - a_0 & b_1 & b_0 \end{bmatrix}^T \tag{2.11}$$

$$\Lambda_1 = \begin{bmatrix} 1 & 0 & 0 \\ 2\lambda & 1 & 0 \\ \lambda^2 & \lambda & 1 \end{bmatrix} \tag{2.12}$$

$$\Lambda_2 = \begin{bmatrix} 1 & 0 \\ \lambda & 1 \end{bmatrix} \tag{2.13}$$

In accordance with (4) the first identifier #1 is chosen to be:

$$y_1(t) = \frac{1}{s+\lambda}\left(\beta_{11}\tilde{u}_1(t) + \beta_{21}\tilde{u}_2(t) + \alpha_{01}y_p(t) + \alpha_{11}\tilde{y}_{p1}(t) + \alpha_{21}\tilde{y}_{p2}(t)\right) \tag{2.14}$$

Figure 2.2 shows a schematic diagram of Identifier #1.

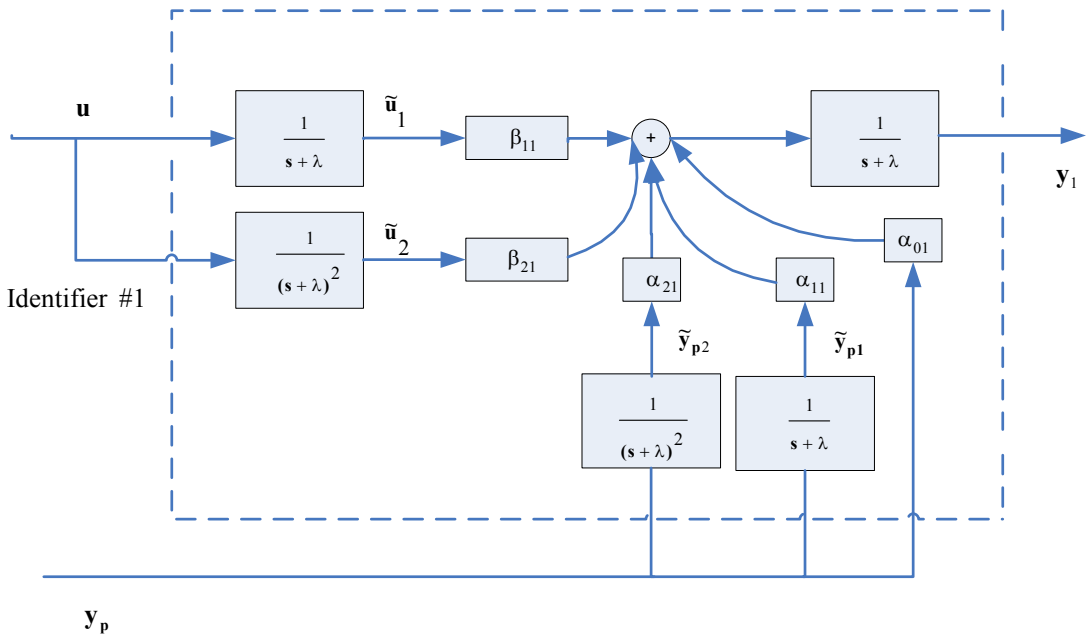


Figure 2.2: MRAC with uncertain relative degree for Identifier #1.

Similarly, the second identifier #2 is chosen as:

$$y_2(t) = \frac{1}{s+\lambda}\left(\beta_{12}\tilde{u}_1(t) + \beta_{22}\tilde{u}_2(t) + \alpha_{02}y_1(t) + \alpha_{12}\tilde{y}_{11}(t) + +\alpha_{22}\tilde{y}_{12}(t)\right) \tag{2.15}$$

where

$$\tilde{y}_{11} \triangleq \frac{1}{s+\lambda}y_1(t) \tag{2.16}$$

$$\tilde{y}_{12} \triangleq \frac{1}{(s+\lambda)^2}y_1(t) \tag{2.17}$$

Figure 2.3 shows a schematic diagram of Identifier #2.

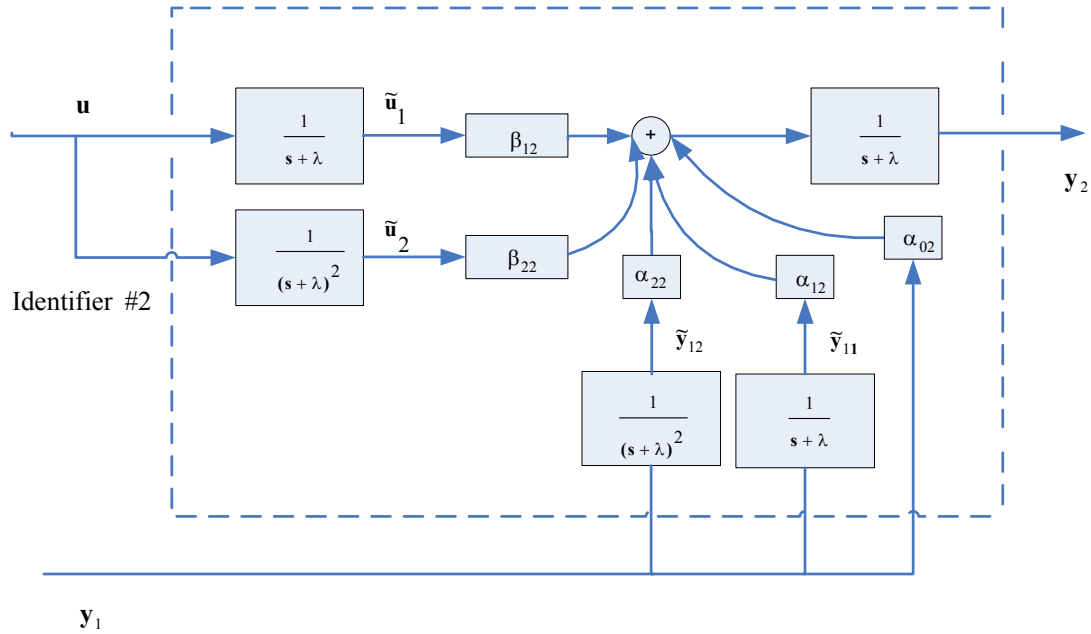


Figure 2.3 MRAC with uncertain relative degree for Identifier #2.

2.2.2. Parameter Update Laws for the Identifiers

Identifier #1

Define the tracking error between plant and the first identifier #1 as:

$$e_1(t) \triangleq y_p(t) - y_1(t) \tag{2.18}$$

$$\tilde{\beta}_1 \triangleq \beta_1^* - \beta_{11}$$

$$\tilde{\beta}_2 \triangleq \beta_2^* - \beta_{21}$$

$$\tilde{\alpha}_0 \triangleq \alpha_0^* - \alpha_{01}$$

$$\tilde{\alpha}_1 \triangleq \alpha_1^* - \alpha_{11}$$

$$\tilde{\alpha}_2 \triangleq \alpha_2^* - \alpha_{21} \tag{2.19}$$

From (2.5), (2.14) and (2.19), the error equation is given by

$$e_1(t) = y_p(t) - y_1(t)$$

$$= \frac{1}{s+\lambda}(\tilde{\beta}_1 \tilde{u}_1(t) + \tilde{\beta}_2 \tilde{u}_2(t) + \tilde{\alpha}_0 y_p(t) + \tilde{\alpha}_1 \tilde{y}_{p1}(t) + \tilde{\alpha}_2 \tilde{y}_{p2}(t)) \tag{2.20}$$

Multiplying both sides of (2.20) by the polynomial operator $s+\lambda$ and moving all terms to the R.H.S. except for the $\dot{e}_1$ term yields

$$\dot{e}_1(t) = -\lambda e_1(t) + \tilde{\beta}_1 \tilde{u}_1(t) + \tilde{\beta}_2 \tilde{u}_2(t) + \tilde{\alpha}_0 y_p(t) + \tilde{\alpha}_1 \tilde{y}_{p1}(t) + \tilde{\alpha}_2 \tilde{y}_{p2}(t) \tag{2.21}$$

To go through the stability analysis, we choose a Lyapunov function candidate

$$V = \frac{1}{2}\left[e_1^2 + \frac{1}{g}\left(\tilde{\alpha}_0^2 + \tilde{\alpha}_1^2 + \tilde{\alpha}_2^2 + \tilde{\beta}_1^2 + \tilde{\beta}_2^2\right)\right], \quad g > 0 \tag{2.22}$$

The derivative of V is given by

$$\dot{V} = e_1 \dot{e}_1 + \frac{1}{g}\left(\tilde{\alpha}_0 \dot{\tilde{\alpha}}_0 + \tilde{\alpha}_1 \dot{\tilde{\alpha}}_1 + \tilde{\alpha}_2 \dot{\tilde{\alpha}}_2 + \tilde{\beta}_1 \dot{\tilde{\beta}}_1 + \tilde{\beta}_2 \dot{\tilde{\beta}}_2 \right)$$

Substituting $\dot{e}_1$ from (2.21) gives

$$\dot{V} = -\lambda e_1^2 + \tilde{\alpha}_0\left(e_1 y_p + \frac{1}{g}\dot{\tilde{\alpha}}_0 \right) + \tilde{\alpha}_1\left(e_1 \tilde{y}_{p1} + \frac{1}{g}\dot{\tilde{\alpha}}_1 \right) + \tilde{\alpha}_2\left(e_1 \tilde{y}_{p2} + \frac{1}{g}\dot{\tilde{\alpha}}_2 \right)$$

$$+ \tilde{\beta}_1\left(e_1 \tilde{u}_1 + \frac{1}{g}\dot{\tilde{\beta}}_1 \right) + \tilde{\beta}_2\left(e_1 \tilde{u}_2 + \frac{1}{g}\dot{\tilde{\beta}}_2 \right) \tag{2.23}$$

Choosing the parameter update laws as

$$\dot{\alpha}_{01} = -\dot{\tilde{\alpha}}_0 = g e_1 y_p \tag{2.24}$$

$$\dot{\alpha}_{11} = -\dot{\tilde{\alpha}}_1 = g e_1 \tilde{y}_{p1} \tag{2.25}$$

$$\dot{\alpha}_{21} = -\dot{\tilde{\alpha}}_2 = g e_1 \tilde{y}_{p2} \tag{2.26}$$

$$\dot{\beta}_{11} = -\dot{\tilde{\beta}}_1 = \begin{cases} 0, & \text{if } g e_1 \tilde{u}_1 \le 0 \text{ and } \beta_{11} \le \varepsilon \\ g e_1 \tilde{u}_1, & \text{otherwise} \end{cases} \tag{2.27}$$

$$\dot{\beta}_{21} = -\dot{\tilde{\beta}}_2 = g e_1 \tilde{u}_2 \tag{2.28}$$

renders

$$\dot{V} = \begin{cases} -\lambda e_1^2 + \tilde{\beta}_1 e_1 \tilde{u}_1 \le 0, & \text{if } g e_1 \tilde{u}_1 \le 0 \text{ and } \beta_{11} \le \varepsilon \\ -\lambda e_1^2 \le 0, & \text{otherwise} \end{cases} \tag{2.29}$$

where $\varepsilon \in R$ and ε is arbitrarily small.

Remark:

(a) We see that in both cases, $\dot{V}$ is negative semi-definite. This implies that e_1, $\tilde{\alpha}_0$, $\tilde{\alpha}_1$, $\tilde{\alpha}_2$, $\tilde{\beta}_1$ and $\tilde{\beta}_2$ are bounded; and from (2.19), α_{01}, α_{11}, α_{21}, β_{11} and β_{21} are also bounded. The convergence to zero of e_1 will be shown later using Barbalat's Lemma after establishing the boundedness of all signals in the entire system.

(b) The division of the adaptation of β_{11} into two cases as given by (2.27) is to ensure that,

$$\beta_{11}(t) \ge \varepsilon > 0, \text{ for all } t \ge 0 \tag{2.30}$$

with the choice of initial condition $\beta_{11}(0) \ge \varepsilon$. This is needed in order to avoid division by zero later.

Identifier #2

Identifier #2 is chosen as (2.15)

$$y_2(t) = \frac{1}{s+\lambda}\left(\beta_{12}\tilde{u}_1(t) + \beta_{22}\tilde{u}_2(t) + \alpha_{02}y_1(t) + \alpha_{12}\tilde{y}_{11}(t) + +\alpha_{22}\tilde{y}_{12}(t)\right)$$

where

$$\tilde{y}_{11} \triangleq \frac{1}{s+\lambda} y_1(t)$$

$$\tilde{y}_{12} \triangleq \frac{1}{(s+\lambda)^2} y_1(t)$$

Let

$$e_2(t) \triangleq y_1(t) - y_2(t) \tag{2.31}$$

The purpose of the parameter update laws for Identifier #2 is to achieve the tracking error between identifier #1 and identifier #2 such that $e_2 = (y_1 - y_2) \to 0$ as $t \to \infty$.

They are chosen as

$$\dot{\alpha}_{02} = d(\alpha_{01} - \alpha_{02}) + g e_{\alpha 0} y_1 + \dot{\alpha}_{01} \tag{2.32}$$

$$\dot{\alpha}_{12} = d(\alpha_{11} - \alpha_{12}) + g e_{\alpha 1} \tilde{y}_{11} + \dot{\alpha}_{11} \tag{2.33}$$

$$\dot{\alpha}_{22} = d(\alpha_{21} - \alpha_{22}) + g e_{\alpha 2} \tilde{y}_{12} + \dot{\alpha}_{21} \tag{2.34}$$

$$\dot{\beta}_{12} = \begin{cases} 0, & \text{if } d(\beta_{11} - \beta_{12}) + g e_{\beta 1} \tilde{u}_1 + \dot{\beta}_{11} \le 0 \text{ and} \\ & \quad \beta_{12} \le \beta_{11},\ d > 0 \\ d(\beta_{11} - \beta_{12}) + g e_{\beta 1} \tilde{u}_1 + \dot{\beta}_{11}, & \text{otherwise} \end{cases} \tag{2.35}$$

$$\dot{\beta}_{22} = d(\beta_{21} - \beta_{22}) + g e_{\beta 2} \tilde{u}_2 + \dot{\beta}_{21} \tag{2.36}$$

where

$$e_{\alpha 0} \triangleq \frac{1}{s+\lambda}(\alpha_{01} - \alpha_{02}) y_1$$

$$e_{\alpha 1} \triangleq \frac{1}{s+\lambda}(\alpha_{11} - \alpha_{12}) \tilde{y}_{11}$$

$$e_{\alpha 2} \triangleq \frac{1}{s+\lambda}(\alpha_{21} - \alpha_{22}) \tilde{y}_{12}$$

$$e_{\beta 1} \triangleq \frac{1}{s+\lambda}(\beta_{11} - \beta_{12}) \tilde{u}_1$$

$$e_{\beta 2} \triangleq \frac{1}{s+\lambda}(\beta_{21} - \beta_{22}) \tilde{u}_2 \tag{2.37}$$

(The update laws are derived as a natural consequence of observing the $\dot{V}$ expression in the

Lyapunov analysis, which will be shown later.)

From (2.31), (2.14) and (2.15), the error equation is given by

$$e_2(t) = \frac{1}{s+\lambda}(\beta_{11} - \beta_{12})\tilde{u}_1 + \frac{1}{s+\lambda}(\beta_{21} - \beta_{22})\tilde{u}_2 + \frac{1}{s+\lambda}(\alpha_{01}y_p - \alpha_{02}y_1)$$

$$+ \frac{1}{s+\lambda}(\alpha_{11}\tilde{y}_{p1} - \alpha_{12}\tilde{y}_{11}) + \frac{1}{s+\lambda}(\alpha_{21}\tilde{y}_{p2} - \alpha_{22}\tilde{y}_{12}) \tag{2.38}$$

Based on this equation, it is noted that though a traditional treatment of Lyapunov's analysis is possible for the first and second terms, it is not possible for the third to fifth terms. The reason is due to the occurrence of a product of a parameter with a signal, such as $\alpha_{01}y_p$, instead of the product of a "*parameter deviation*" with a signal as in the first term. So, we resort to establishing the boundedness of "*all*" signals in the overall system first, and then assuring the convergence to zero of e_2 through the use of Barbalat's Lemma later. We would like to demonstrate the boundedness of e_2 when the update laws are used. This can be accomplished by requiring the similar for the rest of R.H.S. terms of (2.38).

(i). Boundedness of $\frac{1}{s+\lambda}(\beta_{11} - \beta_{12})\tilde{u}_1$

Consider

$$e_{\beta 1} = \frac{1}{s+\lambda}(\beta_{11} - \beta_{12})\tilde{u}_1$$

Multiplying $s+\lambda$ to both sides of the equation yields

$$\dot{e}_{\beta 1} = -\lambda e_{\beta 1} + (\beta_{11} - \beta_{12})\tilde{u}_1 \tag{2.39}$$

Choose a Lyapunov function candidate (to secure boundedness of $e_{\beta 1}$ and $\beta_{11} - \beta_{12}$) as

$$V = \frac{1}{2}\left[ge_{\beta 1}^2 + (\beta_{11} - \beta_{12})^2\right] > 0, \quad g > 0 \tag{2.40}$$

The derivative of V is given by

$$\dot{V} = g e_{\beta 1} \dot{e}_{\beta 1} + (\beta_{11} - \beta_{12})(\dot{\beta}_{11} - \dot{\beta}_{12}) \tag{2.41}$$

Substituting $\dot{e}_{\beta 1}$ from (2.39) yields

$$\dot{V} = -\lambda g e_{\beta 1}^2 + (\beta_{11} - \beta_{12})\left[(g e_{\beta 1} \tilde{u}_1 + \dot{\beta}_{11}) - \dot{\beta}_{12}\right] \tag{2.42}$$

It is seen that the application of the parameter update laws as given in (2.35) will render

$$\dot{V} = \begin{cases} -\lambda g e_{\beta 1}^2 + (\beta_{11} - \beta_{12})(g e_{\beta 1} \tilde{u}_1 + \dot{\beta}_{11}) \le -\lambda g e_{\beta 1}^2 - d(\beta_{11} - \beta_{12})^2, \\ \qquad\qquad \text{if } d(\beta_{11} - \beta_{12}) + g e_{\beta 1} \tilde{u}_1 + \dot{\beta}_{11} \le 0 \text{ and } \beta_{12} \le \beta_{11} \\ -\lambda g e_{\beta 1}^2 - d(\beta_{11} - \beta_{12})^2, \quad \text{otherwise} \end{cases} \tag{2.43}$$

(The inequality in the first case follows as a result of the condition imposed for this case.) Thus, $\dot{V}$ is negative definite, implying that the equilibrium state is globally asymptotically stable. Hence, both $e_{\beta 1}$ and $\beta_{11} - \beta_{12}$ are bounded, and

$$e_{\beta 1} \to 0, \ \beta_{11} - \beta_{12} \to 0 \tag{2.44}$$

The division of the adaptation of β_{12} into two cases as given in (18d) and (18e) is to ensure that

$$\beta_{12}(t) \ge \beta_{11}(t), \text{ for all } t \ge 0 \tag{2.45}$$

This will be achieved as long as the initial conditions are chosen such that $\beta_{12}(0) \ge \beta_{11}(0)$. Condition (2.45) is needed in order to avoid division by zero later.

(ii). Boundedness of $\frac{1}{s+\lambda}(\beta_{21} - \beta_{22})\tilde{u}_2$

Consider

$$e_{\beta 2} = \frac{1}{s+\lambda}\left(\beta_{21} - \beta_{22}\right)\tilde{u}_2$$

Multiplying $s+\lambda$ to both sides of the equation yields

$$\dot{e}_{\beta 2} = -\lambda e_{\beta 2} + \left(\beta_{21} - \beta_{22}\right)\tilde{u}_2 \tag{2.46}$$

Choose a Lyapunov function candidate (to secure boundedness of $e_{\beta 2}$ and $\beta_{21} - \beta_{22}$) as

$$V = \frac{1}{2}\left[ge_{\beta 2}^2 + \left(\beta_{21} - \beta_{22}\right)^2\right] > 0\,, \quad g > 0 \tag{2.47}$$

The derivative of V is given by

$$\dot{V} = ge_{\beta 2}\dot{e}_{\beta 2} + \left(\beta_{21} - \beta_{22}\right)\left(\dot{\beta}_{21} - \dot{\beta}_{22}\right) \tag{2.48}$$

Substituting $\dot{e}_{\beta 2}$ from (2.46) yields

$$\dot{V} = -\lambda ge_{\beta 2}^2 + \left(\beta_{21} - \beta_{22}\right)\left[\left(ge_{\beta 2}\tilde{u}_2 + \dot{\beta}_{21}\right) - \dot{\beta}_{22}\right] \tag{2.49}$$

It is seen that the application of the parameter update law as given in (2.36) will render

$$\dot{V} = -\lambda ge_{\beta 2}^2 - d(\beta_{21} - \beta_{22})^2 < 0 \tag{2.50}$$

Thus, $\dot{V}$ is negative definite, implying that the equilibrium state is globally asymptotically stable.

Hence, both $e_{\beta 2}$ and $\beta_{21} - \beta_{22}$ are bounded, and

$$e_{\beta 2} \to 0\,,\ \beta_{21} - \beta_{22} \to 0 \tag{2.51}$$

(iii). Boundedness of $\frac{1}{s+\lambda}\left(\alpha_{01}y_p - \alpha_{02}y_1\right)$

Rearrange $\frac{1}{s+\lambda}\left(\alpha_{01}y_p - \alpha_{02}y_1\right)$ as

$$\frac{1}{s+\lambda}\left(\alpha_{01}y_p - \alpha_{02}y_1\right) = \frac{1}{s+\lambda}\alpha_{01}\left(y_p - y_1\right) + \frac{1}{s+\lambda}\left(\alpha_{01} - \alpha_{02}\right)y_1 \tag{2.52}$$

We shall treat the boundedness of the two terms on the R.H.S. separately.

In accordance with (2.18), the first term $\frac{1}{s+\lambda}\alpha_{01}\left(y_p - y_1\right)$ is bounded because α_{01} and $e_1 = y_p - y_1$ are bounded. We now turn to the second term and let

$$e_{\alpha 0} = \frac{1}{s+\lambda}\left(\alpha_{01} - \alpha_{02}\right)y_1 \tag{2.53}$$

Multiplying $s+\lambda$ to both sides of the equation yields

$$\dot{e}_{\alpha 0} = -\lambda e_{\alpha 0} + \left(\alpha_{01} - \alpha_{02}\right)y_1 \tag{2.54}$$

Choose a Lyapunov function candidate (to secure boundedness of $e_{\alpha 0}$ and $\alpha_{01} - \alpha_{02}$)

$$V = \frac{1}{2}\left[ge_{\alpha 0}^2 + \left(\alpha_{01} - \alpha_{02}\right)^2\right] > 0 \tag{2.55}$$

The derivative of V is given by

$$\dot{V} = ge_{\alpha 0}\dot{e}_{\alpha 0} + \left(\alpha_{01} - \alpha_{02}\right)(\dot{\alpha}_{01} - \dot{\alpha}_{02}) \tag{2.56}$$

Substituting $\dot{e}_{\alpha 0}$ from (2.54), we have

$$\dot{V} = -\lambda ge_{\alpha 0}^2 + \left(\alpha_{01} - \alpha_{02}\right)\left[\left(ge_{\alpha 0}y_1 + \dot{\alpha}_{01}\right) - \dot{\alpha}_{02}\right] \tag{2.57}$$

Applying the parameter update law in (18.d), renders

$$\dot{V} = -\lambda e_{\alpha 0}^2 - d\left(\alpha_{01} - \alpha_{02}\right)^2 \tag{2.58}$$

which is negative definite. This implies that the equilibrium state is globally asymptotically stable. Hence, $e_{\alpha 0}$ and $\alpha_{01} - \alpha_{02}$ are bounded, and

$$e_{\alpha 0} = \frac{1}{s+\lambda}\left(\alpha_{01} - \alpha_{02}\right)y_1 \to 0 \text{ as } t \to \infty \tag{2.59}$$

Consequently, from (2.52), with both terms on the R.H.S. being bounded, $\frac{1}{s+\lambda}\left(\alpha_{01}y_p - \alpha_{02}y_1\right)$ is also bounded.

(iv). Boundedness of $\frac{1}{s+\lambda}\left(\alpha_{11}\tilde{y}_{p1}-\alpha_{12}\tilde{y}_{11}\right)$

The treatment of $\frac{1}{s+\lambda}\left(\alpha_{11}\tilde{y}_{p1}-\alpha_{12}\tilde{y}_{11}\right)$ follows the same pattern as that of $\frac{1}{s+\lambda}\left(\alpha_{01}y_p-\alpha_{02}y_1\right)$ above, and is therefore omitted.

(v). Boundedness of $\frac{1}{s+\lambda}\left(\alpha_{21}\tilde{y}_{p2}-\alpha_{22}\tilde{y}_{12}\right)$

The treatment of $\frac{1}{s+\lambda}\left(\alpha_{21}\tilde{y}_{p2}-\alpha_{22}\tilde{y}_{12}\right)$ follows the same pattern as that of $\frac{1}{s+\lambda}\left(\alpha_{01}y_p-\alpha_{02}y_1\right)$ above, and is therefore omitted.

Summarizing the results of (i)-(v) in this section, it follows from (2.38) that e_2 is bounded. (The convergence $e_2 \to 0$ will be shown later)

Figure 2.4 shows a schematic diagram of Identifier #1 and Identifier #2.

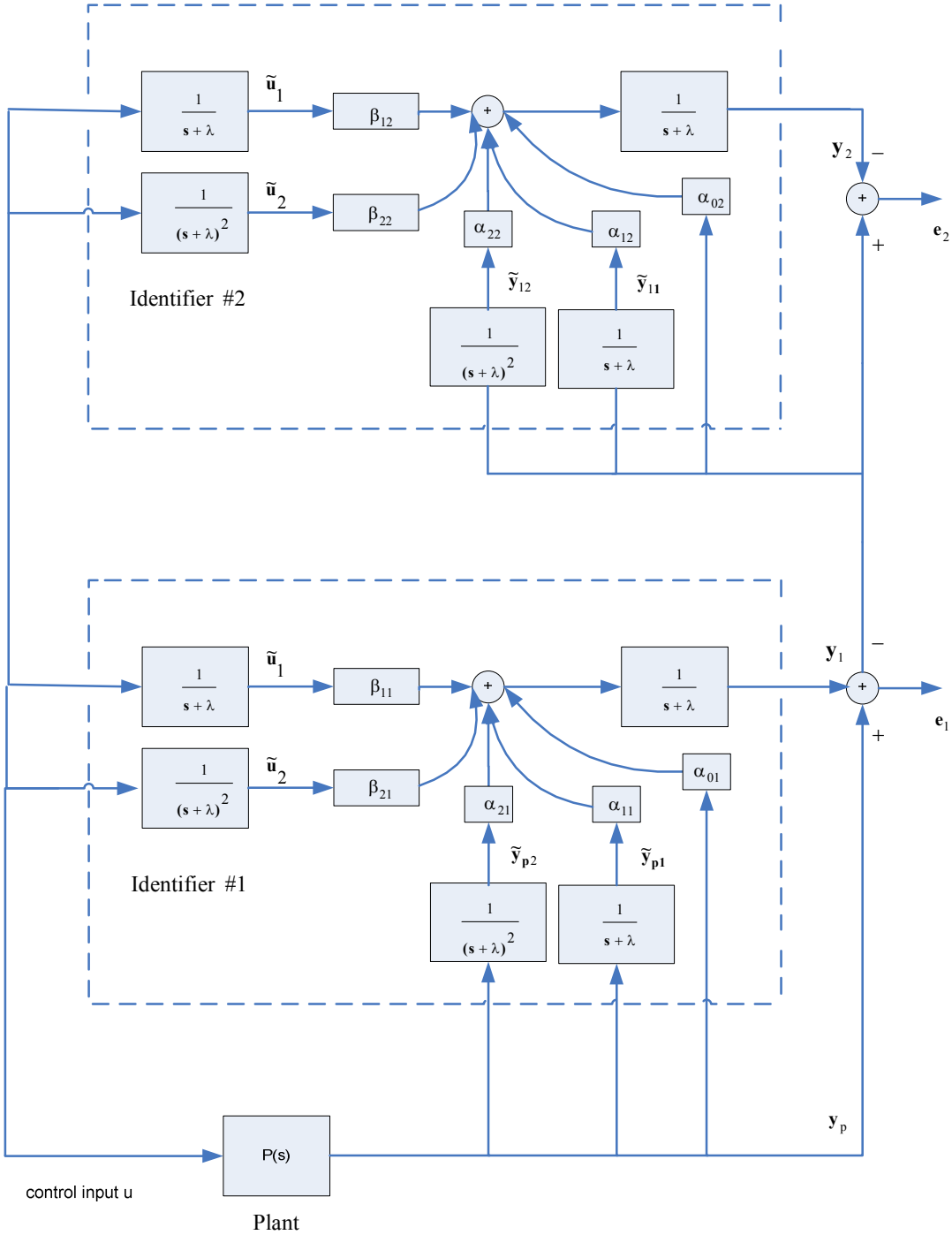


Figure 2.4: MRAC with uncertain relative degree for Identifier #1 and Identifier #2.

2.2.3. Control Law u(t)

Let the reference model with the input-output pair $\{r(.), y_m(.)\}$ be

$$\frac{y_m(s)}{r(s)} = M(s) = \frac{k_m}{(s+a_{m1})(s+a_{m0})} \tag{2.60}$$

where k_m, a_{m1} and a_{m0} are positive design parameters, $r(t)$ is a bounded, piecewise continuous function of time for $t \geq 0$. The purpose here is to derive a control law such that y_2 asymptotically tracks y_m.

Define tracking error e as

$$e \triangleq (s+a_{m0})e_{m2} \tag{2.61}$$

where

$$e_{m2} \triangleq y_2 - y_m \tag{2.62}$$

(Note that if $e \to 0$, then $e_{m2} \to 0$ and $y_2 \to y_m$)

From (2.61) and (2.62), we have

$$e = (s+a_{m0})(y_2 - y_m) = (\dot{y}_2 + a_{m0}y_2) - k_m r_x \tag{2.63}$$

where

$$r_x \triangleq \frac{r}{s+a_{m1}} \tag{2.64}$$

Choose a Lyapunov function candidate

$$V = \frac{1}{2}e^2 > 0 \tag{2.65}$$

A control law is now to be devised in order to make

$$\dot{V} = e\dot{e} = -ke^2, \quad k > 0 \tag{2.66}$$

negative definite. This will be done by making

$$\dot{e} = -ke \tag{2.67}$$

through an appropriate control law to be derived as follows:

From (2.63), the derivative of e is given by

$$\dot{e} = (\ddot{y}_2 + a_{m0}\dot{y}_2) - k_m\dot{r}_x \tag{2.68}$$

Multiplying $s + \lambda$ to both sides of (2.15) yields

$$\dot{y}_2 = \beta_{12}\tilde{u}_1 + \beta_{22}\tilde{u}_2 + \alpha_{02}y_1 + \alpha_{12}\tilde{y}_{11} + +\alpha_{22}\tilde{y}_{12} - \lambda y_2 \tag{2.69}$$

The second derivative is given by

$$\ddot{y}_2 = \beta_{12}\dot{\tilde{u}}_1 + m \tag{2.70}$$

where

$$m \triangleq \dot{\beta}_{12}\tilde{u}_1 + \beta_{22}\dot{\tilde{u}}_2 + +\dot{\beta}_{22}\tilde{u}_2 + \alpha_{02}\dot{y}_1 + +\dot{\alpha}_{02}y_1 + \alpha_{12}\dot{\tilde{y}}_{11} + \dot{\alpha}_{12}\tilde{y}_{11}$$

$$+ \alpha_{22}\dot{\tilde{y}}_{12} + \dot{\alpha}_{22}\tilde{y}_{12} - \lambda\dot{y}_2 \tag{2.71}$$

Substituting (2.70) into (2.68) yields

$$\dot{e} = ((\beta_{12}\dot{\tilde{u}}_1 + m) + a_{m0}\dot{y}_2) - k_m\dot{r}_x \tag{2.72}$$

Next we substitute $\dot{\tilde{u}}_1$ with $-\lambda\tilde{u}_1 + u$ from (2.6) and $\dot{e} = -ke$ from (2.67). The result is

$$-ke = \beta_{12}u + \left(-\lambda\beta_{12}\tilde{u}_1 + m + a_{m0}\dot{y}_2 - k_m\dot{r}_x\right) \tag{2.73}$$

Since our objective is to design a differentiator-free controller, replacing $\dot{y}_2$ and $\dot{r}_x$ in (2.73) with (2.69) and (2.64), respectively, gives the control law

$$u(t) = \frac{1}{\beta_{12}}\left(\lambda\beta_{12}\tilde{u}_1 - m - a_{m0}\left(\beta_{12}\tilde{u}_1 + \beta_{22}\tilde{u}_2 + \alpha_{02}y_1 + \alpha_{12}\tilde{y}_{11} + +\alpha_{22}\tilde{y}_{12} - \lambda y_2\right)\right.$$

$$\left. + k_m\left(r - a_{m1}r_x\right) - ke\right), \quad \beta_{12} > 0,\ k > 0 \tag{2.74}$$

Remarks:

(a). Note that the term m above as defined by (2.71) still contains the derivatives of some

adaptive coefficients. They can be replaced by their respective adaptive laws in (2.32) to(2.36). Also the other derivative terms $\dot{y}_1$, $\dot{y}_2$, $\dot{\tilde{y}}_{11}$, $\dot{\tilde{y}}_{12}$ and $\dot{\tilde{u}}_2$ can be substituted by their expressions in (2.14), (2.15) and (2.7), respectively, so as to dispense with the need of differentiations.

(b). Note from (2.30) and (2.45) that $\beta_{12} \geq \beta_{11} \geq \varepsilon > 0$ so that division by zero in the control law would not occur.

Thus, with $\dot{V} = -2kV$ in (2.66) being negative definite, the equilibrium state $e = 0$ is globally asymptotically stable, i.e. e is bounded and $e \to 0$ as $t \to \infty$.

Consequently, from (2.61),

$$e_{m2} \to 0 \text{ and } y_2 \to y_m \tag{2.75}$$

Figure 2.5 shows a schematic diagram of overall system.

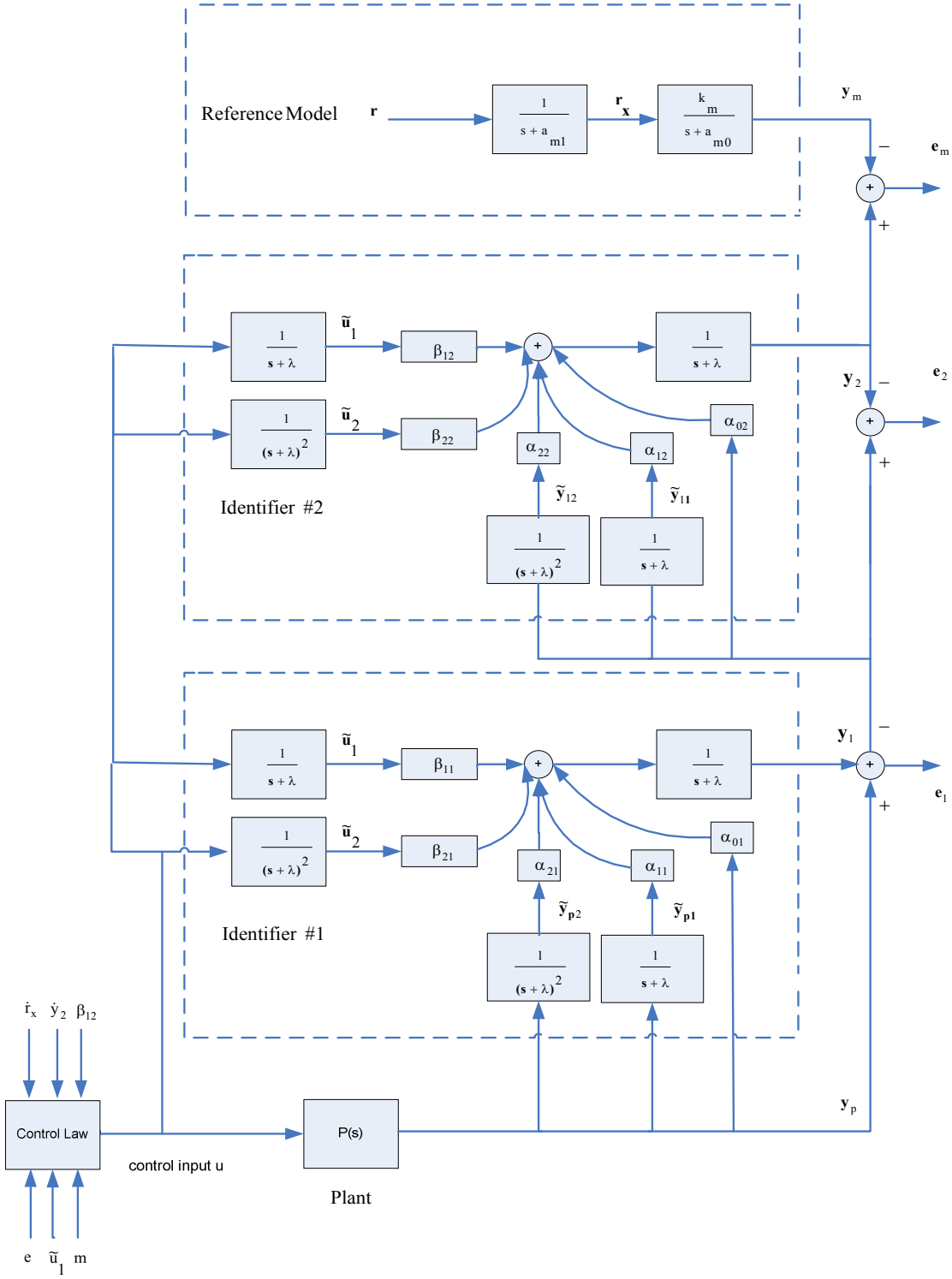


Figure 2.5: MRAC with uncertain relative degree for 2nd order plant of relative degree 1 or 2.

2.2.4. Boundedness of All Signals in the Entired Feedback System

With reference to the entire system in Fig. 2.1, the following signals have been shown to be bounded:

From the analysis of Identifier #1: e_1, α_{01}, α_{11}, α_{21}, β_{11} and β_{21}

From the analysis of Identifier #2: e_2, α_{02}, α_{12}, α_{22}, β_{12} and β_{22}

From the analysis of the control law: e and e_{m2}

From the reference model: r and y_m

The signals that remain to be shown bounded are as follows:

Boundedness of y_p, y_1 and y_2:

Since y_m, e_{m2}, e_2 and e_1 are bounded, it follows from (2.31) and (2.18) that y_2, y_1 and y_p are bounded.

Boundedness of $\tilde{y}_{p1}$, $\tilde{y}_{p2}$, $\tilde{y}_{11}$, $\tilde{y}_{12}$, $\dot{\tilde{y}}_{p1}$, $\dot{\tilde{y}}_{p2}$, $\dot{\tilde{y}}_{11}$, $\dot{\tilde{y}}_{12}$, $\dot{r}_x$, $\dot{y}_m$, $\ddot{y}_m$, $\dot{e}$, $\ddot{e}$, $\dot{e}_{m2}$, $\ddot{e}_{m2}$, $\dot{y}_2$ and $\ddot{y}_2$:

The signals $\tilde{y}_{p1}$, $\tilde{y}_{p2}$, $\tilde{y}_{11}$, $\tilde{y}_{12}$, $\dot{\tilde{y}}_{p1}$, $\dot{\tilde{y}}_{p2}$, $\dot{\tilde{y}}_{11}$, $\dot{\tilde{y}}_{12}$, $\dot{r}_x$, $\dot{y}_m$ and $\ddot{y}_m$ are outputs of "proper" stable transfer functions with bounded inputs. Hence they are bounded. Also, from (2.67), we see that $\dot{e}$ and $\ddot{e}$ are bounded. It follows from (2.61) that the same is true of $\dot{e}_{m2}$ and $\ddot{e}_{m2}$. Consequently, from (2.62), $\dot{y}_2$ and $\ddot{y}_2$ are bounded.

Boundedness of $\tilde{u}_2$, $\tilde{u}_1$, $\dot{e}_1$, $\dot{e}_2$, $\dot{y}_1$ and $\dot{y}_p$:

The boundedness of $\tilde{u}_2$ is derived from (2.7) with substituting $u = (\frac{s^2 + a_1 s + a_0}{b_1 s + b_0}) y_p$ in (2.1)

as it is the output of “proper” stable transfer functions with bounded inputs.

$$\tilde{u}_2 = \frac{1}{(s+\lambda)^2} u(t) = \frac{1}{(s+\lambda)^2} \left(\frac{s^2 + a_1 s + a_0}{b_1 s + b_0}\right) y_p \tag{2.76}$$

With bounded $\tilde{u}_2$, the boundedness of $\tilde{u}_1$ is derived from (2.69). The signal $\dot{e}_1$ in (2.20) is bounded because the signals $\tilde{u}_1$, $\tilde{u}_2$, y_p, $\tilde{y}_{p1}$ and $\tilde{y}_{p2}$ are bounded. In a similar fashion, the boundedness of $\dot{e}_2$ can be established. Finally, $\dot{y}_1$ and $\dot{y}_p$ are also bounded due to the boundedness of $\dot{y}_2$, $\dot{e}_2$ and $\dot{e}_1$.

Boundedness of $\dot{\alpha}_{01}$, $\dot{\alpha}_{11}$, $\dot{\alpha}_{21}$, $\dot{\beta}_{11}$, $\dot{\beta}_{21}$, $\dot{\alpha}_{02}$, $\dot{\alpha}_{12}$, $\dot{\alpha}_{22}$, $\dot{\beta}_{12}$ and $\dot{\beta}_{22}$:

The boundedness of the variables $\dot{\alpha}_{01}$, $\dot{\alpha}_{11}$, $\dot{\alpha}_{21}$, $\dot{\beta}_{11}$, $\dot{\beta}_{21}$, $\dot{\alpha}_{02}$, $\dot{\alpha}_{12}$, $\dot{\alpha}_{22}$, $\dot{\beta}_{12}$ and $\dot{\beta}_{22}$ as given in (2.24)~(2.28) and (2.32)~(2.36) can be seen through the substitution of all occurring derivative terms by their respective adaptive laws. For example, $\dot{\alpha}_{02}$ in (2.32) has the derivative term $\dot{\alpha}_{01}$. It can be substituted with $\dot{\alpha}_{01}$ from (2.24), which is the adaptive law in Identifier #1.

Boundedness of $\ddot{y}_p$, u, $\dot{\tilde{u}}_1$ and m:

The boundedness of u is established from (2.1) if one can show the boundedness of y_p, $\dot{y}_p$ and $\ddot{y}_p$. This is demonstrated as follows. Substituting $\tilde{u}_1$ and $\tilde{u}_2$ from (2.6), (2.7) into (2.69) gives

$$\dot{y}_2 = \beta_{12} \frac{1}{s+\lambda}\left(\left(\frac{s^2 + a_1 s + a_0}{b_1 s + b_0}\right) y_p\right) + \beta_{22} \frac{1}{(s+\lambda)^2}\left(\left(\frac{s^2 + a_1 s + a_0}{b_1 s + b_0}\right) y_p\right) + \alpha_{02} y_1$$
$$+ \alpha_{12}\tilde{y}_{11} + \alpha_{22}\tilde{y}_{12} - \lambda y_2 \tag{2.77}$$

It is assumed that the sign of b_0 is known. Without loss of generality, we shall assume the two cases:

(1). The leading coefficient $b_1 = 0$

$$\dot{y}_2 = b_0^{-1}\left[\beta_{12}\left(\dot{y}_p + (a_1-\lambda)y_p + \frac{a_0 - a_1\lambda + \lambda^2}{s+\lambda}y_p\right) + \beta_{22}\left(y_p + \frac{(a_1-2\lambda)s + a_0 - \lambda^2}{s^2+2\lambda s+\lambda^2}y_p\right)\right]$$
$$+\alpha_{02}y_1 + \alpha_{12}\tilde{y}_{11} + \alpha_{22}\tilde{y}_{12} - \lambda y_2 \qquad (2.78)$$

Differentiating (2.78) once gives

$$\ddot{y}_2 = b_0^{-1}\left[\dot{\beta}_{12}\left(\dot{y}_p + (a_1-\lambda)y_p + \frac{a_0 - a_1\lambda + \lambda^2}{s+\lambda}y_p\right) + \dot{\beta}_{22}\left(y_p + \frac{(a_1-2\lambda)s + a_0 - \lambda^2}{s^2+2\lambda s+\lambda^2}y_p\right)\right.$$
$$\left.+\beta_{12}\left(\ddot{y}_p + (a_1-\lambda)\dot{y}_p + \frac{a_0 - a_1\lambda + \lambda^2}{s+\lambda}\dot{y}_p\right) + \beta_{22}\left(\dot{y}_p + \frac{(a_1-2\lambda)s + a_0 - \lambda^2}{s^2+2\lambda s+\lambda^2}\dot{y}_p\right)\right]$$
$$+\dot{\alpha}_{02}y_1 + \dot{\alpha}_{12}\tilde{y}_{11} + \dot{\alpha}_{22}\tilde{y}_{12} + \alpha_{02}\dot{y}_1 + \alpha_{12}\dot{\tilde{y}}_{11} + \alpha_{22}\dot{\tilde{y}}_{12} - \lambda\dot{y}_2 \qquad (2.79)$$

Since $\dot{y}_2$, $\ddot{y}_2$, y_1, $\tilde{y}_{11}$, $\tilde{y}_{12}$, $\dot{y}_1$, $\dot{\tilde{y}}_{11}$, $\dot{\tilde{y}}_{12}$, y_p, $\dot{y}_p$ and the derivate of all adaptive variables have been shown to be bounded, $\ddot{y}_p$ is bounded. Thus, the boundedness of u, $\dot{\tilde{u}}_1$, m is assured from (2.1), (2.6) and (2.71) respectively.

(2). The leading coefficient $b_1 \neq 0$

The treatment of $b_1 \neq 0$ follows the same pattern as above, and is therefore omitted. Thus, we have shown the boundedness of all signals in the entire control system. Next, we would like to demonstrate the convergence of the tracking errors.

2.2.5. Convergence of the Tracking Errors

With reference to the entire system in Fig. 2.5, our purpose is to demonstrate that $y_p \to y_m$ as $t \to 0$. This is accomplished by showing the same for the signals e_{m2}, e_1 and e_2, which is given as follows:

(i) *Convergence of* e_{m2}:

This has been shown in (2.75).

(ii) *Convergence of* e_1:

We have shown in Section (2.2.4) that e_1, $\dot{e}_1$, $\dot{\tilde{\beta}}_1$ and $\dot{\tilde{u}}_1$ are bounded. Thus,

from (15),

$$\ddot{V} = \begin{cases} -2\lambda\dot{e}_1 e_1 + \dot{\tilde{\beta}}_1 e_1 \tilde{u}_1 + \tilde{\beta}_1\left(\dot{e}_1\tilde{u}_1 + e_1\dot{\tilde{u}}_1\right), & \text{if } g e_1\tilde{u}_1 \le 0 \text{ and } \beta_{11} \le \varepsilon \\ -2\lambda\dot{e}_1 e_1, & \text{otherwise} \end{cases}$$

is bounded. According to Barbalat's Lemma, $\dot{V} \to 0$, which means from (15) that $e_1 \to 0$ as $t \to \infty$.

(iii) *Convergence of* e_2:

Consider e_2 in (2.38),

$$e_2(t) = \frac{1}{s+\lambda}\left(\beta_{11} - \beta_{12}\right)\tilde{u}_1 + \frac{1}{s+\lambda}\left(\beta_{21} - \beta_{22}\right)\tilde{u}_2 + \frac{1}{s+\lambda}\left(\alpha_{01} y_p - \alpha_{02} y_1\right)$$

$$+ \frac{1}{s+\lambda}\left(\alpha_{11}\tilde{y}_{p1} - \alpha_{12}\tilde{y}_{11}\right) + \frac{1}{s+\lambda}\left(\alpha_{21}\tilde{y}_{p2} - \alpha_{22}\tilde{y}_{12}\right)$$

Convergence of e_2 will follows from the convergence of each individual term.

Convergence of $\frac{1}{s+\lambda}(\beta_{11}-\beta_{12})\tilde{u}_1$ and $\frac{1}{s+\lambda}(\beta_{21}-\beta_{22})\tilde{u}_2$ **:**

This has been shown in (2.44) and (2.51).

Convergence of $\frac{1}{s+\lambda}(\alpha_{01}y_p-\alpha_{02}y_1)$:

Consider $\frac{1}{s+\lambda}(\alpha_{01}y_p-\alpha_{02}y_1)=\frac{1}{s+\lambda}\alpha_{01}(y_p-y_1)+\frac{1}{s+\lambda}(\alpha_{01}-\alpha_{02})y_1$ from (2.52), the convergence of the first term on the R.H.S. $\frac{1}{s+\lambda}\alpha_{01}(y_p-y_1)$ follows from the convergence of $e_1=y_p-y_1$. The convergence of the second term $\frac{1}{s+\lambda}(\alpha_{01}-\alpha_{02})y_1$ is assured by (2.59).

Therefore, the convergence of $\frac{1}{s+\lambda}(\alpha_{01}y_p-\alpha_{02}y_1)$ is established.

Convergence of $\frac{1}{s+\lambda}(\alpha_{11}\tilde{y}_{p1}-\alpha_{12}\tilde{y}_{11})$ and $\frac{1}{s+\lambda}(\alpha_{21}\tilde{y}_{p2}-\alpha_{22}\tilde{y}_{12})$:

The discussion of the convergence of $\frac{1}{s+\lambda}(\alpha_{11}\tilde{y}_{p1}-\alpha_{12}\tilde{y}_{11})$ and $\frac{1}{s+\lambda}(\alpha_{21}\tilde{y}_{p2}-\alpha_{22}\tilde{y}_{12})$ is similar to that of $\frac{1}{s+\lambda}(\alpha_{01}y_p-\alpha_{02}y_1)$ and is here omitted.

Summarizing, with the convergence of all three R.H.S.-terms in (2.38), the convergence $e_2\to 0$ as $t\to\infty$ is assured.

2.2.6. Simulation Studies

The simulation studies presented in this section are to show the effectiveness of the proposed adaptive scheme. This is done for the case of second order plants, unknown relative degree q

equal to one or two.

Simulation 2.2.6.1: 2nd order Identifier Tracking based MRAC

The data for the simulation are as follows: 2nd order stable plant, q=2

$$P(s) = \frac{1}{s^2 + 4s + 3}, \; M(s) = \frac{1}{s^2 + s + 0.25}$$

$$r(t) = \begin{cases} 0 & t \in [0, 10] \\ 1 & t \in [10, 60] \,\&\, t \in [140, \infty] \\ -1 & t \in [60, 140] \end{cases}$$

$\lambda = 1$, $k = g = 1$, $e_p = y_p - y_m$

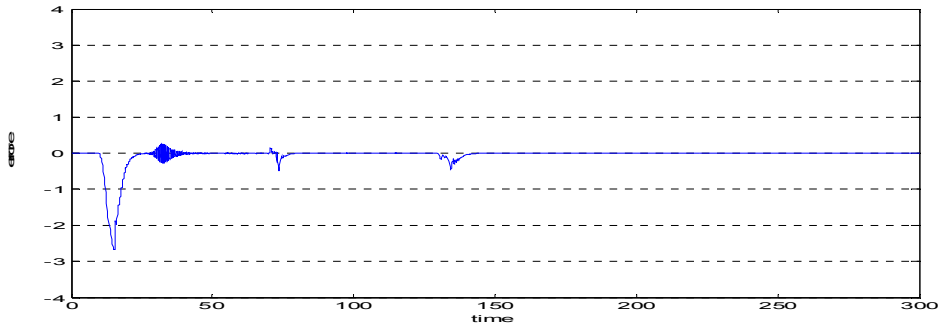
The initial conditions for the adaptive parameters are chosen in accordance with (2.30) and (2.45), in this case:

$\beta_{11}(0) = 1 \geq k_{lower}$, $\beta_{21}(0) = 1$ (k_{lower} is taken to be 0.01),

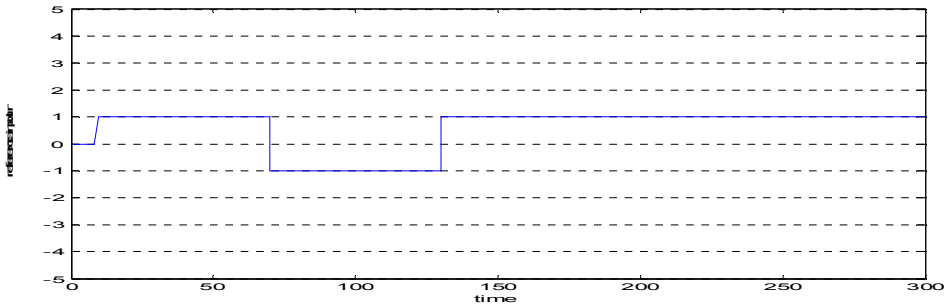
$\beta_{12}(0) = 1$, $\beta_{22}(0) = 1$,

$\alpha_{01}(0) = \alpha_{11}(0) = \alpha_{21}(0) = \alpha_{02}(0) = \alpha_{12}(0) = \alpha_{22}(0) = 1$

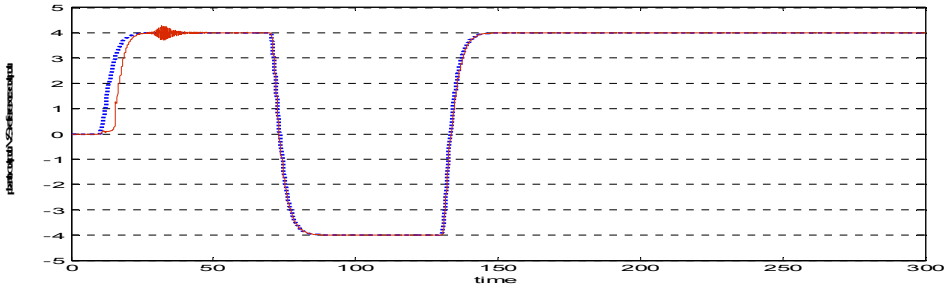
Simulation results are shown as figure 2.6.



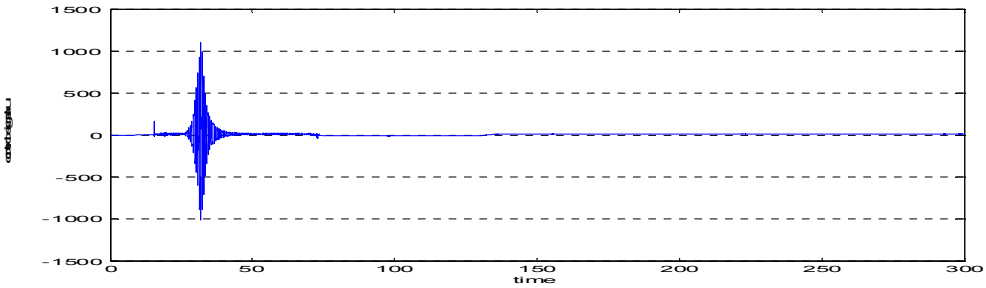
(a)



(b)



(c)



(d)

Figure 2.6: 2nd order stable plant, q=2. (a) tracking error, (b) reference input signal, (c) plant vs reference output and (d) control signal u

Simulation 2.2.6.2: 2nd order Identifier Tracking MRAC

The data for the simulation are as follows: 2nd order unstable plant, q=2

$$P(s) = \frac{1}{s^2 - 4s + 3},\ M(s) = \frac{1}{s\ \ + 2s + 1}$$

$$r(t) = \begin{cases} 0 & t \in [0,10] \\ 1 & t \in [10,160] \,\&\, t \in [310,460] \\ -1 & t \in [160,310] \,\&\, t \in [460,\infty] \end{cases}$$

$\lambda = 1$, $k = g = 1$, $e_p = y_p - y_m$

The initial conditions for the adaptive parameters are chosen in accordance with (2.30) and (2.45), in this case:

$\beta_{11}(0) = 0.5 \geq k_{lower}$, $\beta_{21}(0) = 2$ (k_{lower} is taken to be 0.01),

$\beta_{12}(0) = 0,\ \beta_{22}(0) = 0$,

$\alpha_{01}(0) = \alpha_{11}(0) = \alpha_{21}(0) = \alpha_{02}(0) = \alpha_{12}(0) = \alpha_{22}(0) = 0$

Simulation results are shown as figure 2.7.

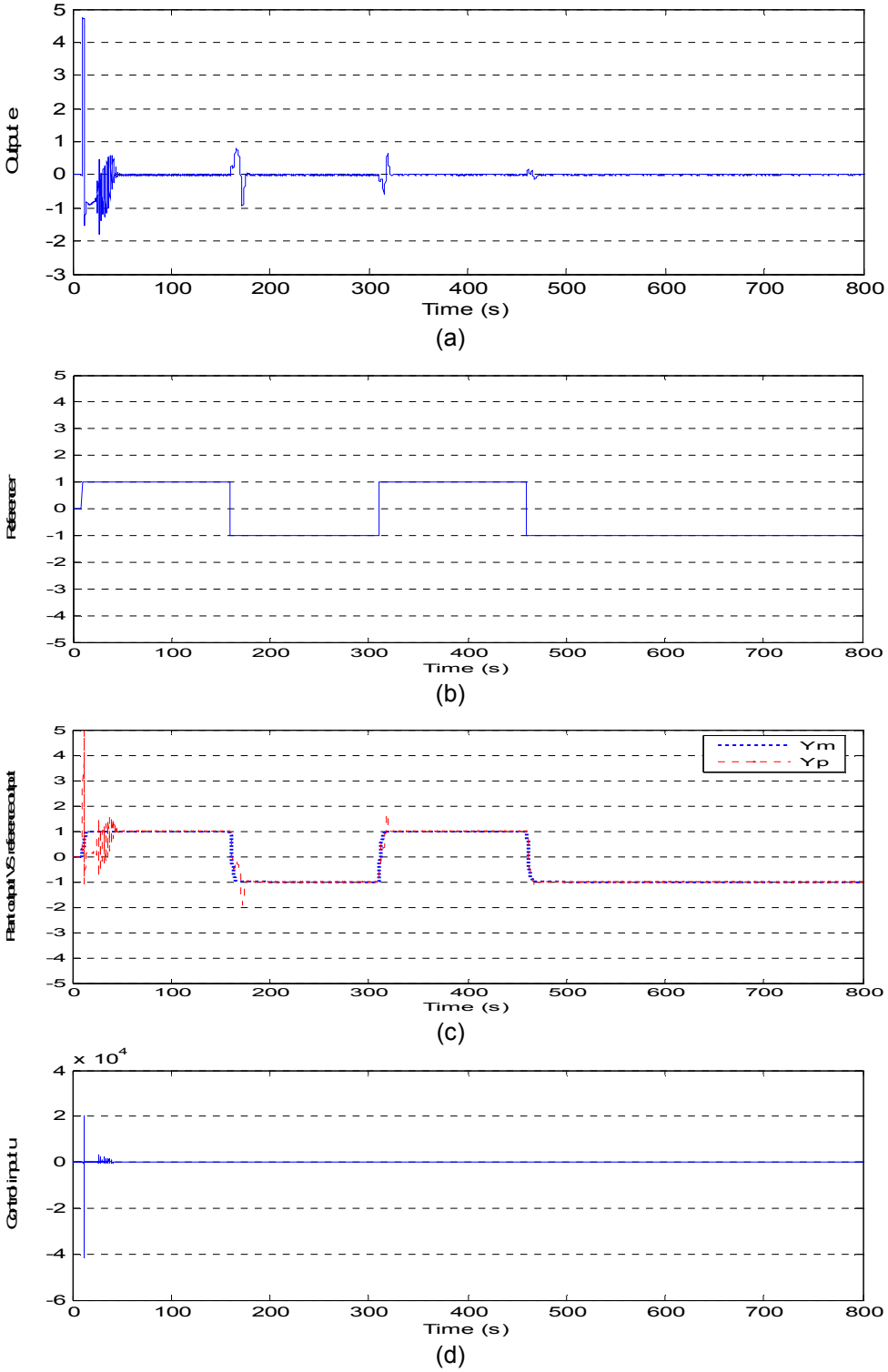


Figure 2.7: 2nd order unstable plant, q=2. (a) tracking error, (b) reference input signal, (c) plant vs reference output and (d) control signal u

Simulation 2.2.6.3: 2nd order Identifier Tracking MRAC

The data for the simulation are as follows: 2nd order stable plant, q=2

$$P(s) = \frac{1}{s^2 + 4s + 3},\ M(s) = \frac{1}{s\ \ + 2s + 1}$$

$$r(t) = \begin{cases} 0 & t \in [0,10] \\ 1 & t \in [10,160] \,\&\, t \in [310,460] \\ -1 & t \in [160,310] \,\&\, t \in [460,\infty] \end{cases}$$

$\lambda = 1$, $k = g = 1$, $e_p = y_p - y_m$

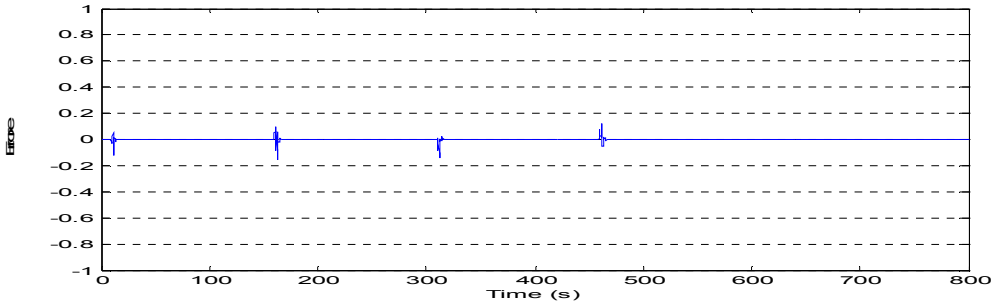
The initial conditions for the adaptive parameters are chosen in accordance with (2.30) and (2.45), in this case:

$\beta_{11}(0) = 0.5 \geq k_{lower}$, $\beta_{21}(0) = 2$ (k_{lower} is taken to be 0.01),

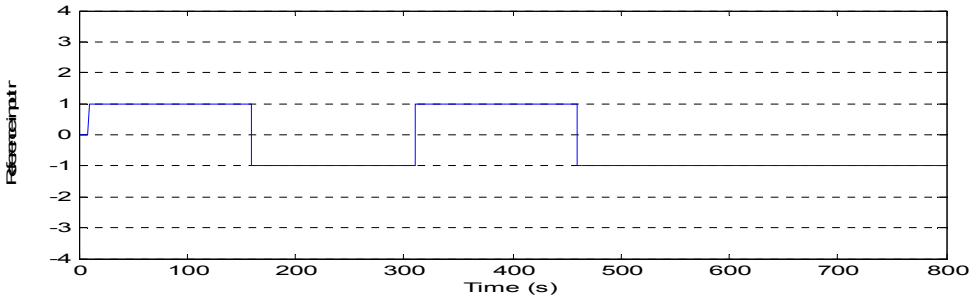
$\beta_{12}(0) = 0,\ \beta_{22}(0) = 0$,

$\alpha_{01}(0) = \alpha_{11}(0) = \alpha_{21}(0) = \alpha_{02}(0) = \alpha_{12}(0) = \alpha_{22}(0) = 0$,

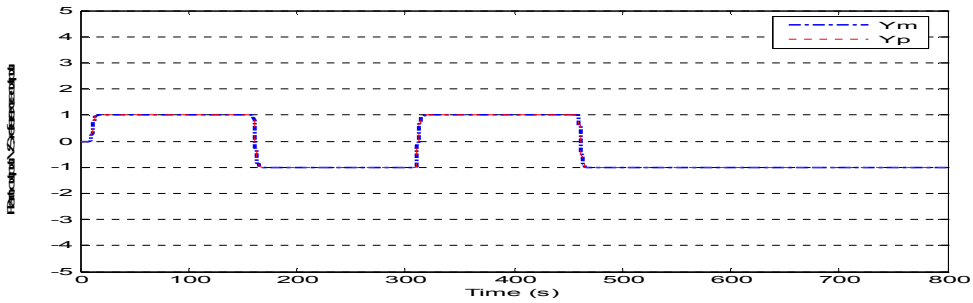
Simulation results are shown as figure 2.8.



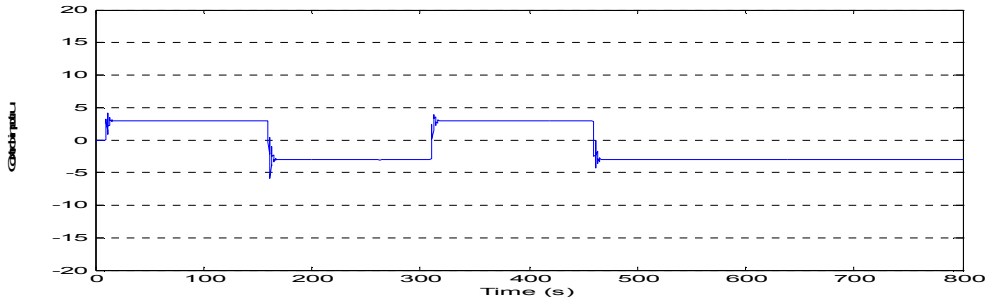
(a)



(b)



(c)



(d)

Figure 2.8: 2nd order stable plant, q=2. (a) tracking error, (b) reference input signal, (c) plant vs reference output and (d) control signal u

Simulation 2.2.6.4: 2nd order Identifier Tracking MRAC

The data for the simulation are as follows: 2nd order unstable plant, q=2

$$P(s) = \frac{1}{s^2 - 4s + 3}, \; M(s) = \frac{1}{s^2 + 1.2s + 0.36}$$

$$r(t) = \begin{cases} 0 & t \in [0, 10] \\ 1 & t \in [10, 40] \,\&\, t \in [70, \infty] \\ -1 & t \in [40, 70] \end{cases}$$

$\lambda = 1$, $k = g = 1$, $e_p = y_p - y_m$

The initial conditions for the adaptive parameters are chosen in accordance with (2.30) and (2.45), in this case:

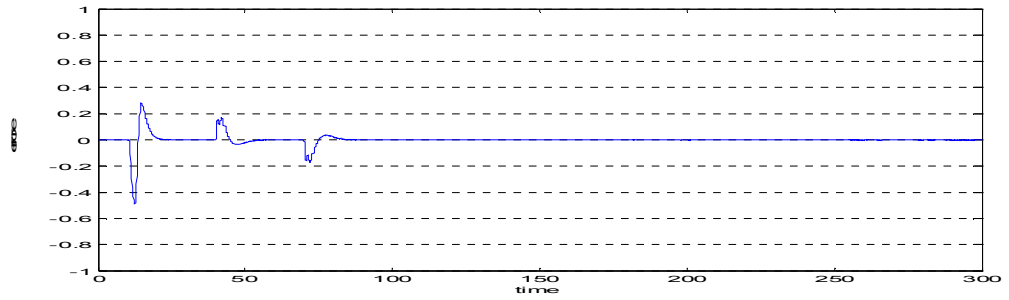
$\beta_{11}(0) = 1 \geq k_{lower}$, $\beta_{21}(0) = 1$ (k_{lower} is taken to be 0.01),

$\beta_{12}(0) = 1$, $\beta_{22}(0) = 1$,

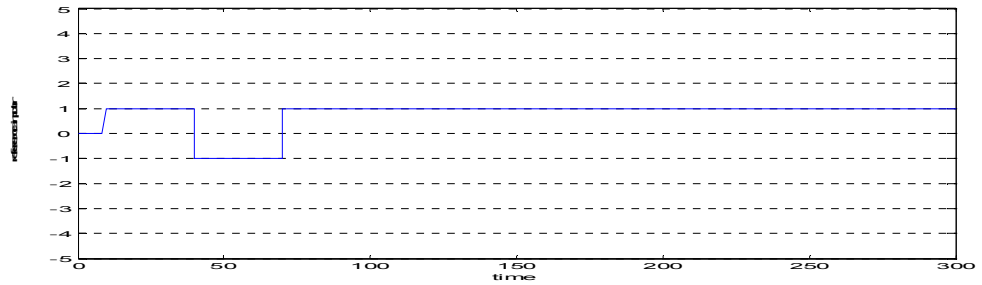
$\alpha_{01}(0) = \alpha_{11}(0) = \alpha_{21}(0) = \alpha_{02}(0) = \alpha_{12}(0) = \alpha_{22}(0) = 1$

Simulation results are shown as figure 2.9.

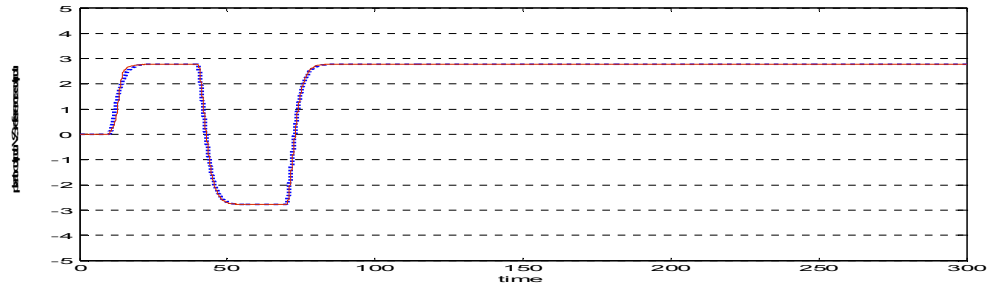
,



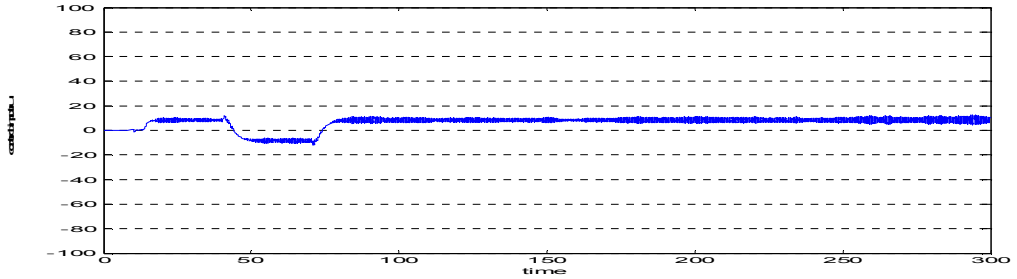
(a)



(b)



(c)



(d)

Figure 2.9: 2nd order unstable plant, q=2. (a) tracking error, (b) reference input signal, (c) plant vs reference output and (d) control signal u

Simulation 2.2.6.5: 2nd order Identifier Tracking MRAC

The data for the simulation are as follows: 2nd order stable plant, q=1

$$P(s) = \frac{\mathbf{s}+2}{\mathbf{s}^2+4\mathbf{s}+3},\ M(s) = \frac{1}{\mathbf{s}^2+\mathbf{s}+0.25}$$

$$r(t) = \begin{cases} 0 & t \in [0,10] \\ 1 & t \in [10,60] \,\&\, t \in [140,\infty] \\ -1 & t \in [60,140] \end{cases}$$

$\lambda = 1$, $k = g = 1$, $e_p = y_p - y_m$

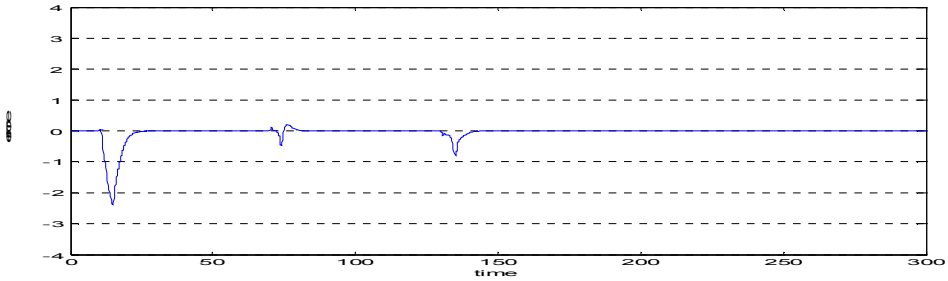
The initial conditions for the adaptive parameters are chosen in accordance with (2.30) and (2.45), in this case:

$\beta_{11}(0) = 1 \geq k_{lower}$, $\beta_{21}(0) = 1$ (k_{lower} is taken to be 0.01),

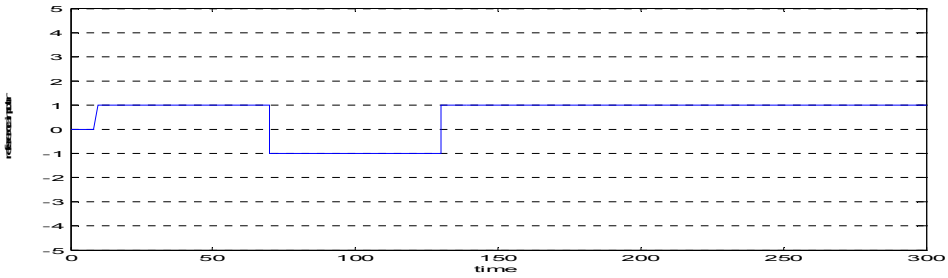
$\beta_{12}(0) = 1$, $\beta_{22}(0) = 1$,

$\alpha_{01}(0) = \alpha_{11}(0) = \alpha_{21}(0) = \alpha_{02}(0) = \alpha_{12}(0) = \alpha_{22}(0) = 1$

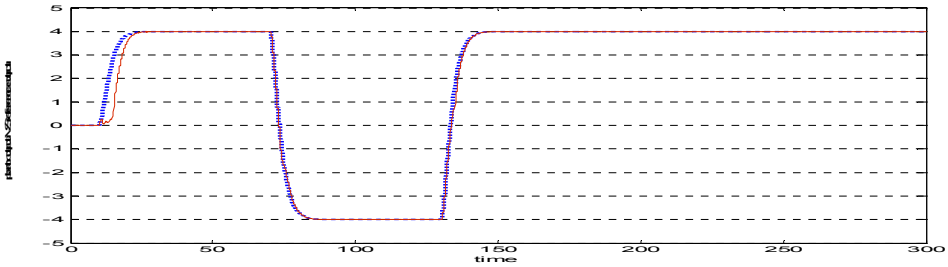
Simulation results are shown as figure 2.10.



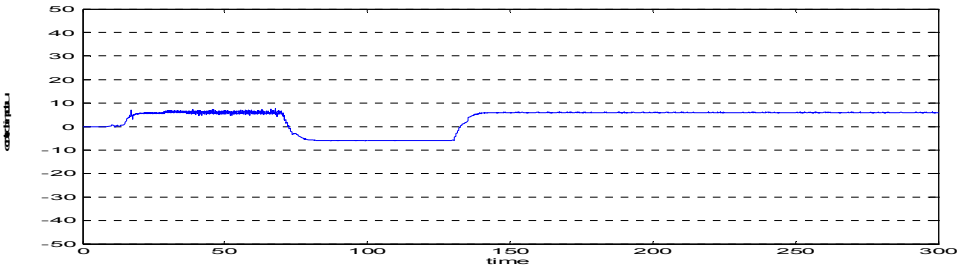
(a)



(b)



(c)



(d)

Figure 2.10: 2nd order stable plant, q=1. (a) tracking error, (b) reference input signal, (c) plant vs reference output and (d) control signal u

Simulation 2.2.6.6: 2nd order Identifier Tracking MRAC

The data for the simulation are as follows: unstable plant, n=2, q=1

$$P(s) = \frac{\mathbf{s}+2}{\mathbf{s}^2 - 4\mathbf{s} + 3},\ M(s) = \frac{1}{\mathbf{s}^2 + 1.2\mathbf{s} + 0.36}$$

$$r(t) = \begin{cases} 0 & t \in [0,10] \\ 1 & t \in [10,40] \,\&\, t \in [70,\infty] \\ -1 & t \in [40,70] \end{cases}$$

$\lambda = 1$, $k = g = 1$, $e_p = y_p - y_m$

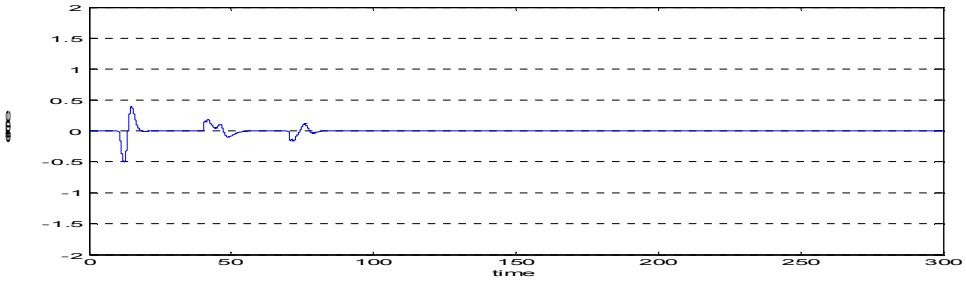
The initial conditions for the adaptive parameters are chosen in accordance with (2.30) and (2.45), in this case:

$\beta_{11}(0) = 1 \geq k_{lower}$, $\beta_{21}(0) = 1$ (k_{lower} is taken to be 0.01),

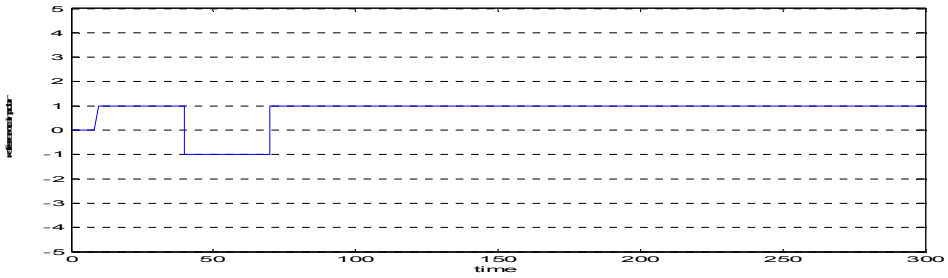
$\beta_{12}(0) = 1$, $\beta_{22}(0) = 1$,

$\alpha_{01}(0) = \alpha_{11}(0) = \alpha_{21}(0) = \alpha_{02}(0) = \alpha_{12}(0) = \alpha_{22}(0) = 1$

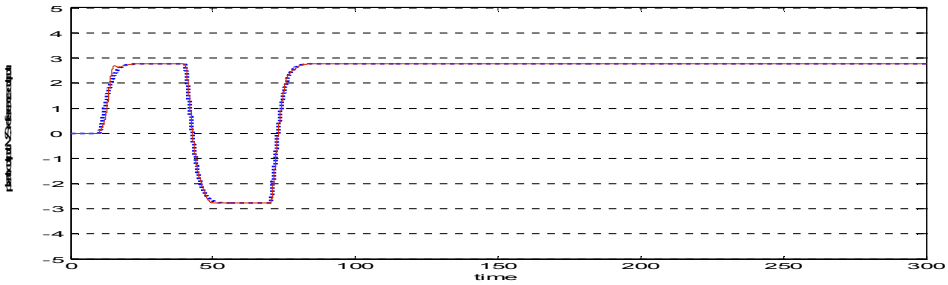
Simulation results are shown as figure 2.11.



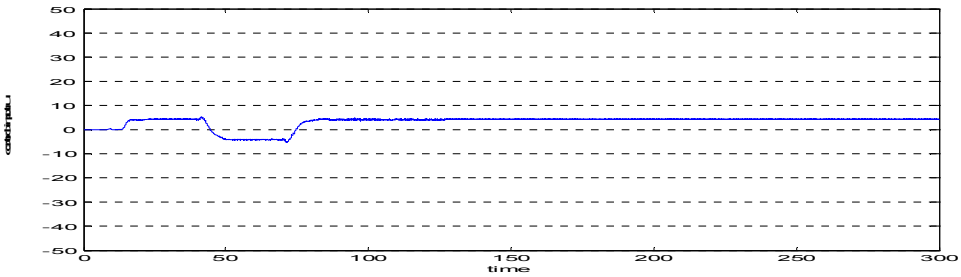
(a)



(b)



(c)



(d)

Figure 2.11: 2nd order unstable plant, q=1. (a) tracking error, (b) reference input signal, (c) plant vs reference output and (d) control signal u

Simulation 2.2.6.7: 1st order Identifier Tracking MRAC

The data for the simulation are as follows: stable plant, n=1, q=1

$$P(s) = \frac{1}{\mathbf{s}+2}, \; M(s) = \frac{1}{\mathbf{s}^2 + 2\mathbf{s} + 1}$$

$$r(t) = \begin{cases} 0 & t \in [0,10] \\ 1 & t \in [10,40] \,\&\, t \in [80,\infty] \\ -1 & t \in [40,80] \end{cases}$$

$\lambda = 1$, $k = g = 1$, $e_p = y_p - y_m$

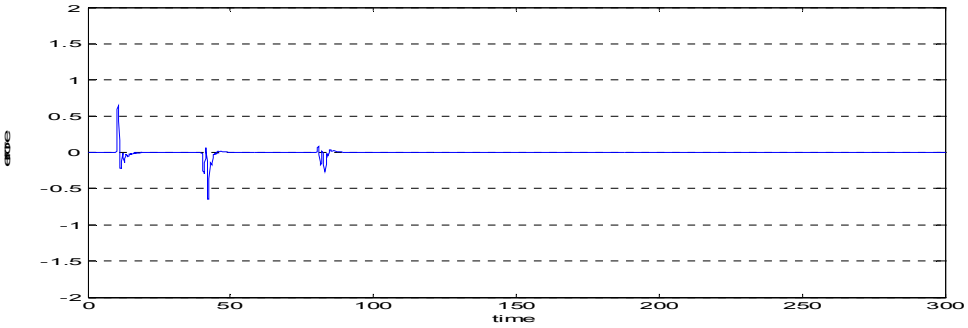
The initial conditions for the adaptive parameters are chosen in accordance with (2.30) and (2.45), in this case:

$\beta_{11}(0) = 1 \geq \varepsilon$, $\beta_{21}(0) = 1$ (ε is taken to be 0.01),

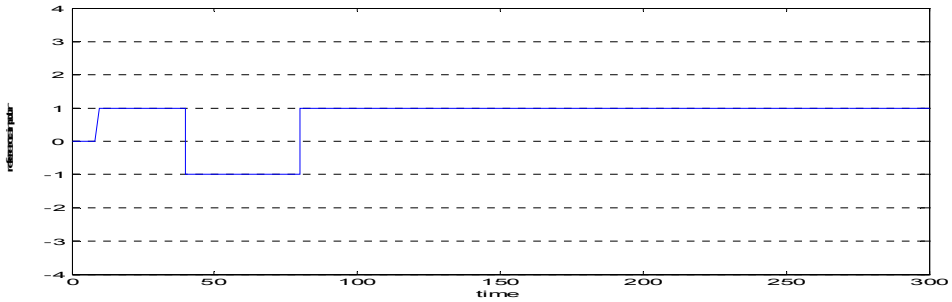
$\beta_{12}(0) = 1$, $\beta_{22}(0) = 1$,

$\alpha_{01}(0) = \alpha_{11}(0) = \alpha_{21}(0) = \alpha_{02}(0) = \alpha_{12}(0) = \alpha_{22}(0) = 1$,

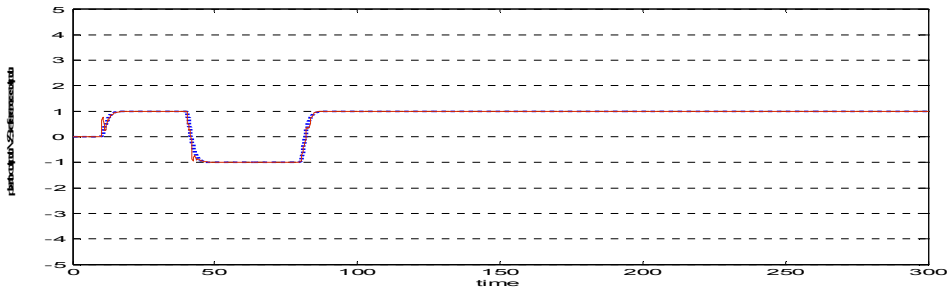
Simulation results are shown as figure 2.12.



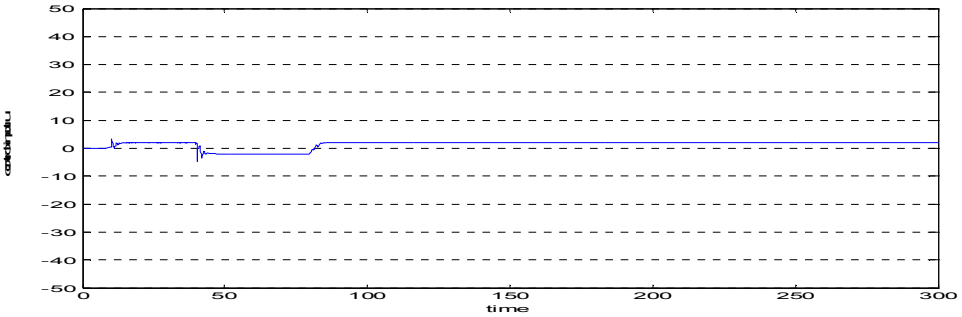
(a)



(b)



(c)



(d)

Figure 2.12: 1st order stable plant, q=1. (a) tracking error, (b) reference input signal, (c) plant vs reference output and (d) control signal u

Simulation 2.2.6.8: 1st order Identifier Tracking MRAC

The data for the simulation are as follows: unstable plant, n=1, q=1

$$P(s) = \frac{1}{\mathbf{s} - 2}, \; M(s) = \frac{1}{\mathbf{s}^2 + 2\mathbf{s} + 1}$$

$$r(t) = \begin{cases} 0 & t \in [0, 10] \\ 1 & t \in [10, 40] \,\&\, t \in [80, \infty] \\ -1 & t \in [40, 80] \end{cases}$$

$\lambda = 1$, $k = g = 1$, $e_p = y_p - y_m$

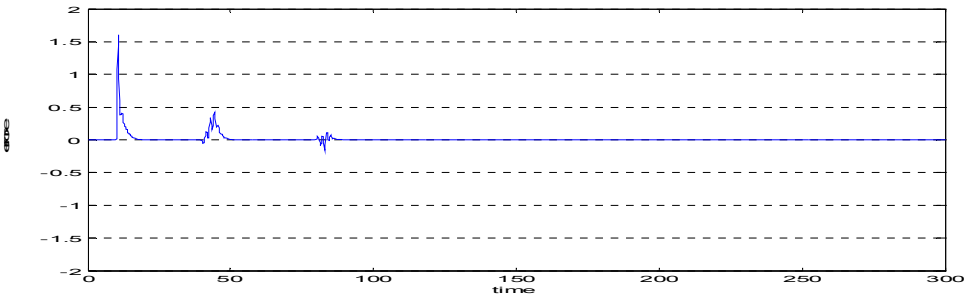
The initial conditions for the adaptive parameters are chosen in accordance with (2.30) and (2.45), in this case:

$\beta_{11}(0) = 1 \geq \varepsilon$, $\beta_{21}(0) = 1$ (ε is taken to be 0.01),

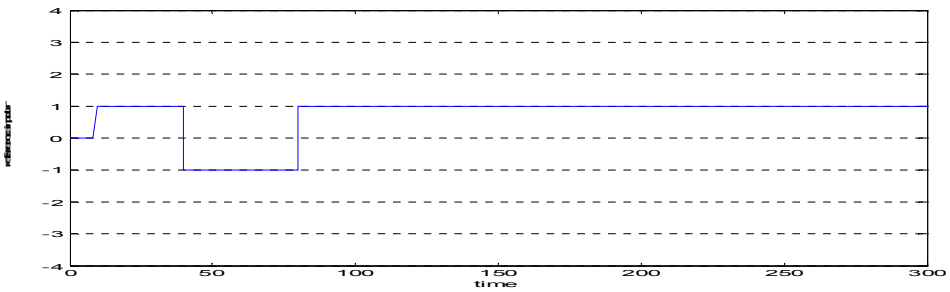
$\beta_{12}(0) = 1$, $\beta_{22}(0) = 1$,

$\alpha_{01}(0) = \alpha_{11}(0) = \alpha_{21}(0) = \alpha_{02}(0) = \alpha_{12}(0) = \alpha_{22}(0) = 1$

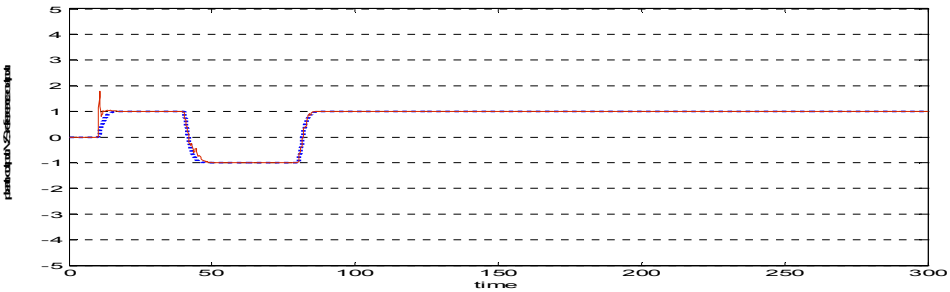
Simulation results are shown as figure 2.13.



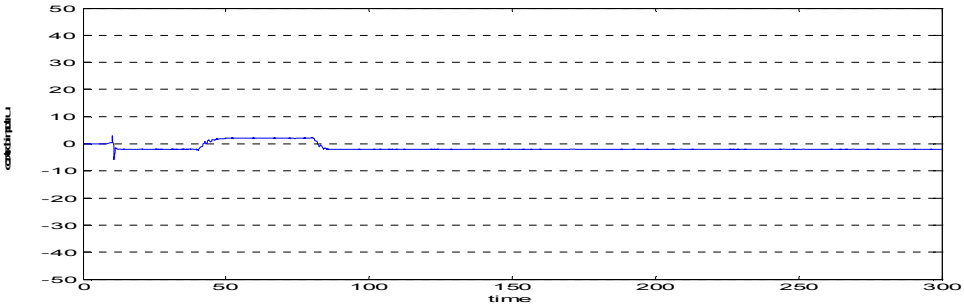
(a)



(b)



(c)



(d)

Figure 2.13: 1st order unstable plant, q=1. (a) tracking error, (b) reference input signal, (c) plant vs reference output and (d) control signal u

Discussion

Simulations were performed for 1st and 2nd order plants using the proposed MRAC scheme with different initial conditions. The simulation results are shown in Figure 2.6~2.13. The results indicated that the new method of MRAC gives bounded solutions and asymptotic zero tracking error.

CHAPTER 3

AN IDENTIFIER-TRACKING BASED MRAC WITHOUT

THE KNOWLEDGE OF RELATIVE DEGREE

– GENERAL CASE FOR N_TH ORDER PLANT

3.1 Introduction

The primary objective of this chapter is to develop the general case for the model reference adaptive control of a single-input single-output linear time-invariant plant without the prior knowledge of relative degree.

3.2 MRAC of plants without the knowledge of relative degree

3.2.1. Reparameterization of the Unknown Plant

In this section, we extend the Stacked Identifiers MRAC to the general case.

3.2.1.1. Plant

Consider a plant with an input-output pair $\{u(.), y_p(.)\}$ described by a transfer function

$$P(s) = \frac{N(s)}{D(s)} = \frac{b'_{n-1}s^{n-1} + \cdots + b'_1 s + b'_0}{s^n + a'_{n-1}s^{n-1} + a'_{n-2}s^{n-2} + \cdots + a'_1 s + a'_0}, \quad n \geq 1 \tag{3.1}$$

where $b'_{n-1}s^{n-1} + \cdots + b'_1 s + b'_0$ is a Hurwitz polynomial in s. The sign of b'_0 is assumed to be positive, with a known lower bound $b'_0 > \varepsilon > 0$.

Notice that the leading coefficient in the numerator may or may not be zero so that the relative degree of the system (denominator degree minus the numerator degree) is unknown but set the range from 1 to n. It is assumed that the sign of b_0 is known. Without loss of generality, we

shall assume the three cases. Those cases will be discussed in Section 3.2.1.3.

3.2.1.2. Control Scheme

Figure 3.1 shows the adaptive control scheme to be presented here.

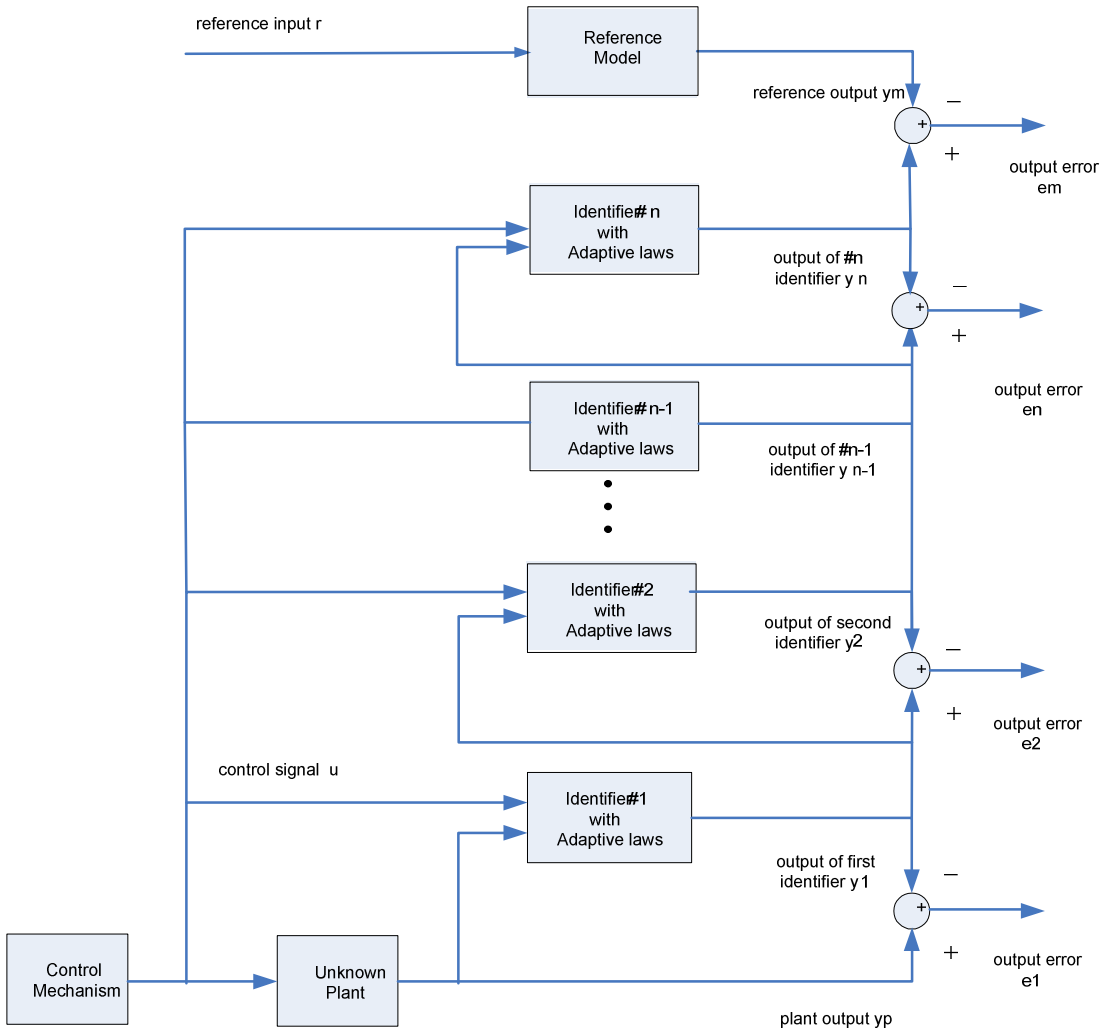


Figure 3.1: Schematic diagram for n_th order plant with unknown relative degree

The control scheme consists of n identifiers, a reference model, and a control law. The number of identifiers used is equal to the maximum possible relative degree of the plant which is n.

3.2.1.3. Identifiers

The primary function of the identifiers is still to let their outputs track the output of the n_th order plant. For general case, the identifiers are constructed based on the following n_th order structure with constant coefficients;

$$D(s) = \frac{b_{n-1}s^{n-1} + \cdots + b_1 s + b_0}{s^{2n-1} + a_{2n-2}s^{2n-2} + \cdots + a_1 s + a_0} \tag{3.2}$$

Where the coefficient $b_0 > \varepsilon > 0$. The structure in (3.2) can be seen to encompass the various relative-degree cases of the n_th order plant in (3.1) as follows:

(1). The leading coefficient $b_{n-1},\ b_{n-2},\ \cdots,\ b_1 = 0$

$$P'_{\delta}(s) = \frac{b'_0}{s^n + a'_{n-1}s^{n-1} + a'_{n-2}s^{n-2} + \cdots + a'_1 s + a'_0} \cdot \left(\frac{s+\delta}{s+\delta}\right)^{n-1}$$

$$= \frac{b_{(n-1)\delta}s^{n-1} + \cdots + b_{1\delta}s + b_{0\delta}}{s^{2n-1} + a_{(2n-2)\delta}s^{2n-2} + \cdots + a_{1\delta}s + a_{0\delta}} \tag{3.3}$$

(2). The leading coefficient $b_{n-1},\ b_{n-2},\ \cdots,\ b_{n-k} = 0,\quad b_{n-k-1}\ \cdots,\ b_1 \neq 0$

$$P'_{\delta\omega}(s) = \frac{b'_{n-k-1}s^{n-k-1} + \cdots + b'_1 s + b'_0}{s^n + a'_{n-1}s^{n-1} + a'_{n-2}s^{n-2} + \cdots + a'_1 s + a'_0} \cdot \left(\frac{s+\delta}{s+\delta}\right)^{k} \cdot \left(\frac{\omega}{s+\omega}\right)^{n-k-1}$$

$$= \frac{b_{(n-1)\delta\omega}s^{n-1} + \cdots + b_{1\delta\omega}s + b_{0\delta\omega}}{s^{2n-1} + a_{(2n-2)\delta\omega}s^{2n-2} + \cdots + a_{1\delta\omega}s + a_{0\delta\omega}},\quad k = 1\cdots,\ n-2 \tag{3.4}$$

(3). The leading coefficient $b_{n-1},\ b_{n-2},\ \cdots,\ b_1 \neq 0$

$$P'_{\omega}(s) = \frac{b'_{n-1}s^{n-1} + \cdots + b'_1 s + b'_0}{s^n + a'_{n-1}s^{n-1} + a'_{n-2}s^{n-2} + \cdots + a'_1 s + a'_0} \cdot \left(\frac{\omega}{s+\omega}\right)^{n-1}$$

$$= \frac{b_{(n-1)\omega} s^{n-1} + \cdots + b_{1\omega} s + b_{0\omega}}{s^{2n-1} + a_{(2n-2)\omega} s^{2n-2} + \cdots + a_{1\omega} s + a_{0\omega}} \tag{3.5}$$

where δ is positive constant and ω is some arbitrarily large positive constant

Even though the identifier structure (3.4) and (3.5) are not "equivalent" to the plant (3.1), there exists a large enough constant such that structure will be arbitrarily close to structure (3.1). As a result, the output of the identifier can track the output of the plant as required above.

3.2.1.4. Re-parameterization of the Plant and Identifiers

We will reparametrize the plant into a form suitable for the derivation of the identifier and the parameter update laws.

According to identifier structure (3.2), let the plant be parametrized by

$$y_p(t) = \frac{1}{s+\lambda}\left(\beta_1^* \tilde{u}_{(n-1)}(t) + \beta_2^* \tilde{u}_n(t) + \cdots + \beta_n^* \tilde{u}_{(2n-2)} + \alpha_0^* y_p(t) + \cdots + \alpha_{2n-2}^* \tilde{y}_{p(2n-2)}(t)\right) \tag{3.6}$$

where λ is a positive constant and

$$\tilde{u}_\varsigma \triangleq \frac{1}{(s+\lambda)^\varsigma} u \ , \ \varsigma = n-1, \cdots, 2n-2 \tag{3.7}$$

$$\tilde{y}_{p\sigma} \triangleq \frac{1}{(s+\lambda)^\sigma} y_p \ , \ \sigma = 1, \cdots, 2n-2 \tag{3.8}$$

Coefficient matching of terms of like powers in s in (3.2) and (3.6) gives the relationship between the parametrized coefficients w and the original plant coefficients z.

$$\Lambda w = z \ , \ \Lambda = \left[\begin{array}{c|c} \Lambda_1 & 0 \\ \hline 0 & \Lambda_2 \end{array}\right] \tag{3.9}$$

where $w, z \in R^{3n-1}$, $\Lambda \in R^{(3n-1)*(3n-1)}$, $\Lambda_1 \in R^{(2n-1)*(2n-1)}$ and $\Lambda_2 \in R^{nxn}$ are given by

$$w = \left[\begin{array}{ccccc:ccccc} \alpha_0^* & \alpha_1^* & \cdots & \alpha_{2n-3}^* & \alpha_{2n-2}^* & \beta_1^* & \beta_2^* & \cdots & \beta_{n-1}^* & \beta_n^* \end{array} \right]^T \tag{3.10}$$

$$z = \left[\begin{array}{ccccc:ccccc} \binom{2n-1}{1}\lambda - a_{2n-2} & \binom{2n-2}{2}\lambda^2 - a_{2n-3} & \cdots & \binom{2n-1}{2n-2}\lambda^{2n-2} a_1 & \binom{2n-1}{2n-1}\lambda^{2n-1} - a_0 & b_{n-1} & b_{n-2} & \cdots & b_1 & b_0 \end{array} \right]^T \tag{3.11}$$

$$\Lambda_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & \cdots & 0 \\ \binom{2n-2}{1}\lambda & 1 & 0 & 0 & \cdots & 0 \\ \binom{2n-2}{2}\lambda^2 & \binom{2n-3}{1}\lambda & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ \binom{2n-2}{2n-3}\lambda^{2n-3} & \binom{2n-3}{2n-4}\lambda^{2n-4} & \cdots & \cdots & 1 & 0 \\ \binom{2n-2}{2n-2}\lambda^{2n-2} & \binom{2n-3}{2n-3}\lambda^{2n-3} & \binom{2n-4}{2n-4}\lambda^{2n-4} & \cdots & \binom{1}{1}\lambda & 1 \end{bmatrix} \tag{3.12}$$

$$\Lambda_2 = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ \binom{n-1}{1}\lambda & 1 & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \binom{n-1}{n-2}\lambda^{n-2} & \binom{n-2}{n-3}\lambda^{n-3} & \cdots & 1 & 0 \\ \binom{n-1}{n-1}\lambda^{n-1} & \binom{n-2}{n-2}\lambda^{n-2} & \cdots & \lambda & 1 \end{bmatrix} \tag{3.13}$$

$\binom{\mu}{\nu}$ are combination symbols employed in the expansion of

$$(s+\lambda)^{\mu} = s^{\mu} + \sum_{\nu=1}^{\mu} \left(\binom{\mu}{\nu} \lambda^{\nu} \right) s^{\mu-\nu}, \quad \mu = 1, \cdots, n-1, \ \nu = 1, 2, \cdots, n \tag{3.14}$$

which is used in the derivation of (3.7).

To re-write (3.6) in a more compact form, let

$$\varphi^* \triangleq \begin{bmatrix} \beta_1^* & \beta^{*T} & \alpha_0^* & \alpha^{*T} \end{bmatrix}^T \tag{3.15}$$

$$\overline{w} \triangleq \begin{bmatrix} \tilde{u}_{(n-1)} & \tilde{u} & y_p & \tilde{y}_p \end{bmatrix}^T \tag{3.16}$$

$$\beta^* \triangleq \begin{bmatrix} \beta_2^* \ldots & \beta_n^* \end{bmatrix}^T \tag{3.17}$$

$$\alpha^* \triangleq \begin{bmatrix} \alpha_1^* \ldots & \alpha_{(2n-2)}^* \end{bmatrix}^T \tag{3.18}$$

$$\tilde{u} \triangleq \begin{bmatrix} \tilde{u}_n \ldots & \tilde{u}_{(2n-2)} \end{bmatrix}^T \tag{3.19}$$

$$\tilde{y}_p \triangleq \begin{bmatrix} \tilde{y}_{p1} \ldots & \tilde{y}_{p(2n-2)} \end{bmatrix}^T \tag{3.20}$$

Then y_p in (3.6) can be expressed as

$$\begin{aligned} y_p &= \frac{1}{s+\lambda}\left(\beta_1^* \tilde{u}_{(n-1)} + \beta^{*T}\tilde{u} + \alpha_0^* y_p + \alpha^{*T}\tilde{y}_p\right) \\ &= \frac{1}{s+\lambda}\varphi^{*T}\overline{w} \end{aligned} \tag{3.21}$$

For a nth order system, we need a total of n identifiers.

In accordance with the form of (3.21), Identifier #1 is constructed as

$$\begin{aligned} y_1 &= \frac{1}{s+\lambda}\left(\beta_{11}\tilde{u}_{(n-1)} + \beta_1^T\tilde{u} + \alpha_{01} y_p + \alpha_1^T\tilde{y}_p\right) \\ &= \frac{1}{s+\lambda}\varphi_1^T\overline{w} \end{aligned} \tag{3.22}$$

where

$$\varphi_1 \triangleq \begin{bmatrix} \beta_{11} & \beta_1^T & \alpha_{01} & \alpha_1^T \end{bmatrix}^T \tag{3.23}$$

$$\alpha_1 \triangleq \begin{bmatrix} \alpha_{11} \ldots & \alpha_{(2n-2)1} \end{bmatrix}^T \tag{3.24}$$

$$\beta_1 \triangleq \left[\beta_{21} \cdots \quad \beta_{n1}\right]^T \tag{3.25}$$

Figure 3.2 shows a schematic diagram of Identifier #1.

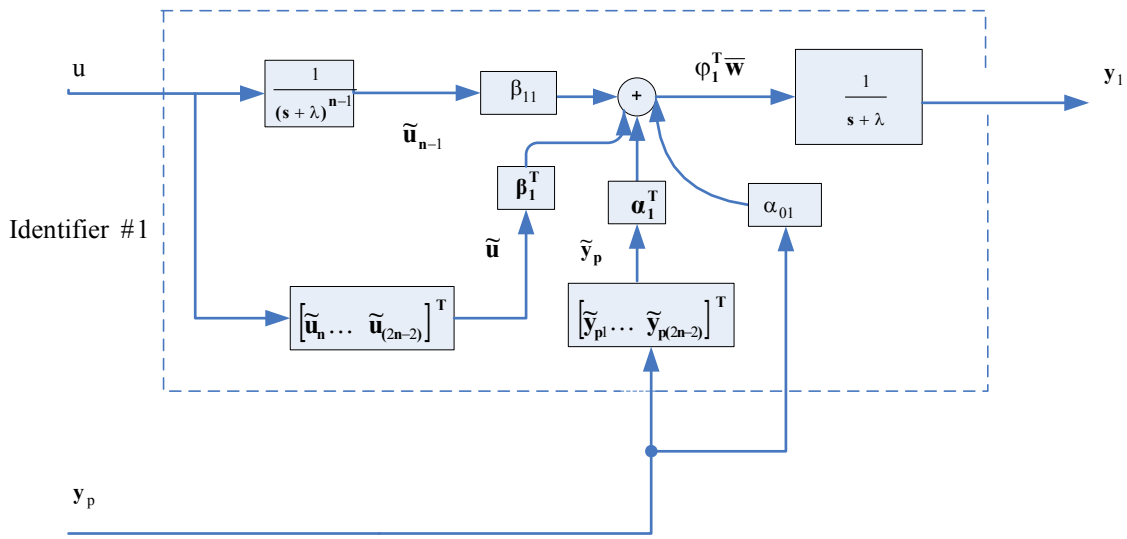


Figure 3.2: MRAC with uncertain relative degree for Identifier #1.

As for the rest of the identifiers, Identifier #γ ($\gamma = 2, 3, \cdots, n$) is constructed as

$$y_\gamma = \frac{1}{s+\lambda}\left(\beta_{1\gamma}\tilde{u}_{(n-1)} + \beta_\gamma^T \tilde{u} + \alpha_{0\gamma} y_{(\gamma-1)} + \alpha_\gamma^T \tilde{y}_{(\gamma-1)}\right)$$

$$= \frac{1}{s+\lambda}\varphi_\gamma^T \overline{w}_\gamma \tag{3.26}$$

where

$$\varphi_\gamma \triangleq \left[\beta_{1\gamma} \quad \beta_\gamma^T \quad \alpha_{0\gamma} \quad \alpha_\gamma^T\right]^T \tag{3.27}$$

$$\overline{w}_\gamma \triangleq \left[\tilde{u}_{(n-1)} \quad \tilde{u} \quad y_{(\gamma-1)} \quad \tilde{y}_{(\gamma-1)}\right]^T \tag{3.28}$$

$$\alpha_\gamma \triangleq \left[\alpha_{1\gamma} \cdots \quad \alpha_{(2n-2)\gamma}\right]^T \tag{3.29}$$

$$\beta_\gamma \triangleq \left[\beta_{2\gamma} \cdots \quad \beta_{n\gamma}\right]^T \tag{3.30}$$

$$\tilde{y}_{(\gamma-1)} \triangleq \left[\tilde{y}_{(\gamma-1)1} \cdots \quad \tilde{y}_{(\gamma-1)(2\gamma-2)}\right]^T \tag{3.31}$$

$$\tilde{y}_{(\gamma\text{-}1)\sigma} \triangleq \frac{1}{(s+\lambda)^\sigma} y_{(\gamma\text{-}1)}, \quad \sigma = 1, \cdots, 2\gamma - 2 \tag{3.32}$$

Figure 3.3 shows a schematic diagram of Identifier $\#\gamma$.

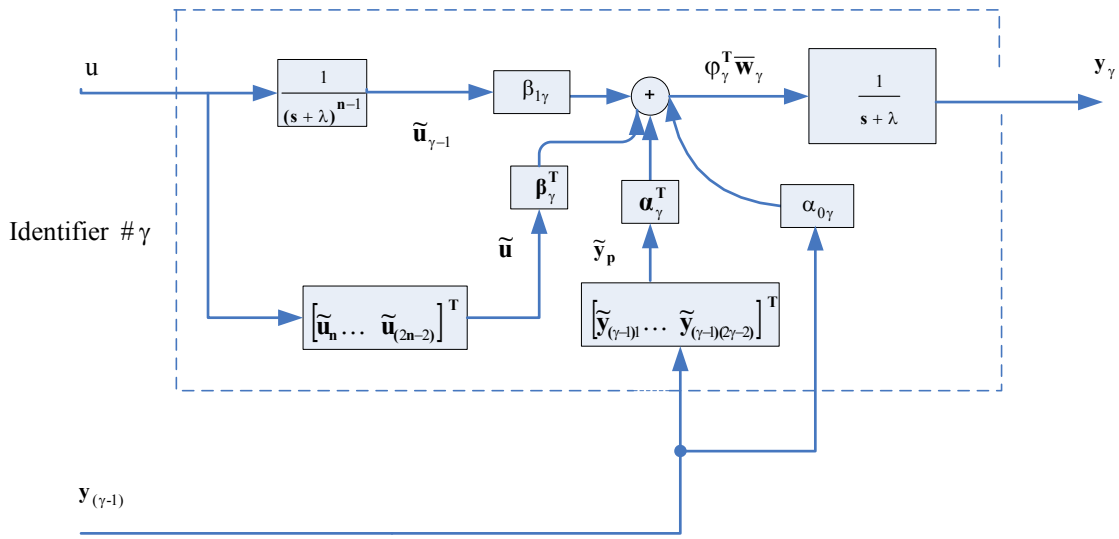


Figure 3.3: MRAC with uncertain relative degree for Identifier $\#\gamma$.

3.2.2. Parameter Update Laws for the Identifiers

Identifier #1

Define tracking error e_1 as

$$e_1 \triangleq y_p - y_1$$

$$= \frac{1}{s+\lambda}\left[\left(\beta_1^* - \beta_{11}\right)\tilde{u}_{(n-1)} + \left(\beta^{*T} - \beta_1^T\right)\tilde{u} + \left(\alpha_0^* - \alpha_{01}\right)y_p + \left(\alpha^{*T} - \alpha_1^T\right)\tilde{y}_p\right] \tag{3.33}$$

Comparing (3.33) with (2.20) and following the same Lyapunov analysis leads to the following parameter update laws

$$\dot{\alpha}_{01} = -\dot{\tilde{\alpha}}_0 = ge_1 y_p \tag{3.34}$$

$$\dot{\alpha}_1 = -\dot{\tilde{\alpha}} = ge_1 \tilde{y}_p \tag{3.35}$$

$$\dot{\beta}_{11} = -\dot{\tilde{\beta}}_1 = \begin{cases} 0, & \text{if } ge_1\tilde{u}_{(n-1)} \le 0 \text{ and } \beta_{11} \le \varepsilon \\ ge_1\tilde{u}_{(n-1)}, & \text{otherwise} \end{cases} \tag{3.36}$$

$$\dot{\beta}_1 = -\dot{\tilde{\beta}} = ge_1\tilde{u} \tag{3.37}$$

where

$$\tilde{\alpha}^T = \alpha^{*T} - \alpha_1^T$$

$$\tilde{\beta}^T = \beta^{*T} - \beta_1^T$$

Equations (3.34)~(3.37) are similar to (2.24)~(2.28).

Choosing an initial condition for the adaptive parameter

$\beta_{11}(0) \ge \varepsilon$ will ensure

$$\beta_{11}(t) \ge \varepsilon > 0, \quad t \ge 0 \tag{3.38}$$

Identifier # γ ($\gamma = 2, 3, \cdots, n$)

Define tracking error e_γ as

$$\begin{aligned} e_\gamma &\triangleq y_{(\gamma-1)} - y_\gamma \\ &= \frac{1}{s+\lambda}\Big[\left(\beta_{1(\gamma-1)} - \beta_{1\gamma}\right)\tilde{u}_{(n-1)} + \left(\beta_{(\gamma-1)}^T - \beta_\gamma^T\right)\tilde{u} + \left(\alpha_{0(\gamma-1)} y_{(\gamma-2)} - \alpha_{0\gamma} y_{(\gamma-1)}\right) \\ &\quad + \left(\alpha_{(\gamma-1)}^T \tilde{y}_{(\gamma-2)} - \alpha_\gamma^T \tilde{y}_{(\gamma-1)}\right)\Big] \end{aligned} \tag{3.39}$$

$$y_0 \triangleq y_p \tag{3.40}$$

$$\tilde{y}_0 \triangleq \left[\tilde{y}_{p1} \cdots \quad \tilde{y}_{p(\gamma-1)}\right]^T \tag{3.41}$$

Comparing (3.39) with (2.38) and following the same Lyapunov analysis leads to the following parameter update laws:

$$\dot{\alpha}_{0\gamma} = d\left(\alpha_{0(\gamma\text{-}1)} - \alpha_{0\gamma}\right) + ge_{\alpha 0} y_{(\gamma-1)} + \dot{\alpha}_{0(\gamma-1)} \tag{3.42}$$

$$\dot{\alpha}_{\gamma} = d\left(\alpha_{(\gamma\text{-}1)} - \alpha_{\gamma}\right) + ge_{\alpha(\gamma-1)} \tilde{y}_{(\gamma-1)} + \dot{\alpha}_{(\gamma-1)} \tag{3.43}$$

$$\dot{\beta}_{1\gamma} = \begin{cases} 0, & \text{if } d\left(\beta_{1(\gamma\text{-}1)} - \beta_{1\gamma}\right) + ge_{\beta(\gamma-1)} \tilde{u}_{(\gamma-1)} + \dot{\beta}_{1(\gamma-1)} \le 0 \text{ and} \\ & \beta_{1\gamma} \le \beta_{1(\gamma\text{-}1)}, \ d > 0 \\ d\left(\beta_{1(\gamma\text{-}1)} - \beta_{1\gamma}\right) + ge_{\beta(\gamma-1)} \tilde{u}_{(\gamma-1)} + \dot{\beta}_{1(\gamma-1)}, & \text{otherwise} \end{cases} \tag{3.44}$$

$$\dot{\beta}_{\gamma} = d\left(\beta_{(\gamma\text{-}1)} - \beta_{\gamma}\right) + ge_{\beta\gamma} \tilde{u}_{\gamma} + \dot{\beta}_{(\gamma-1)} \tag{3.45}$$

where

$$e_{\alpha 0\gamma} \triangleq \frac{1}{s+\lambda}\left(\alpha_{0(\gamma-1)} - \alpha_{0\gamma}\right) y_{(\gamma-1)}$$

$$e_{\alpha\gamma} \triangleq \frac{1}{s+\lambda}\left(\alpha_{(\gamma-1)} - \alpha_{\gamma}\right) \tilde{y}_{(\gamma-1)}$$

$$e_{\beta 1\gamma} \triangleq \frac{1}{s+\lambda}\left(\beta_{1(\gamma\text{-}1)} - \beta_{1\gamma}\right) \tilde{u}_{(n-1)}$$

$$e_{\beta\gamma} \triangleq \frac{1}{s+\lambda}\left(\beta_{(\gamma\text{-}1)} - \beta_{\gamma}\right) \tilde{u}_{\gamma} \tag{3.46}$$

The choice of an initial condition for the adaptive parameter $\beta_{1\gamma}(0) \ge \beta_{11}(0)$ will ensure

$$\beta_{1n}(t) \ge \beta_{1(n-1)} \ge \cdots \ge \beta_{11}(t) > 0, \quad t \ge 0 \tag{3.47}$$

Figure 3.4 shows a schematic diagram of Identifier #1, …, Identifier #(γ-1) and Identifier #γ.

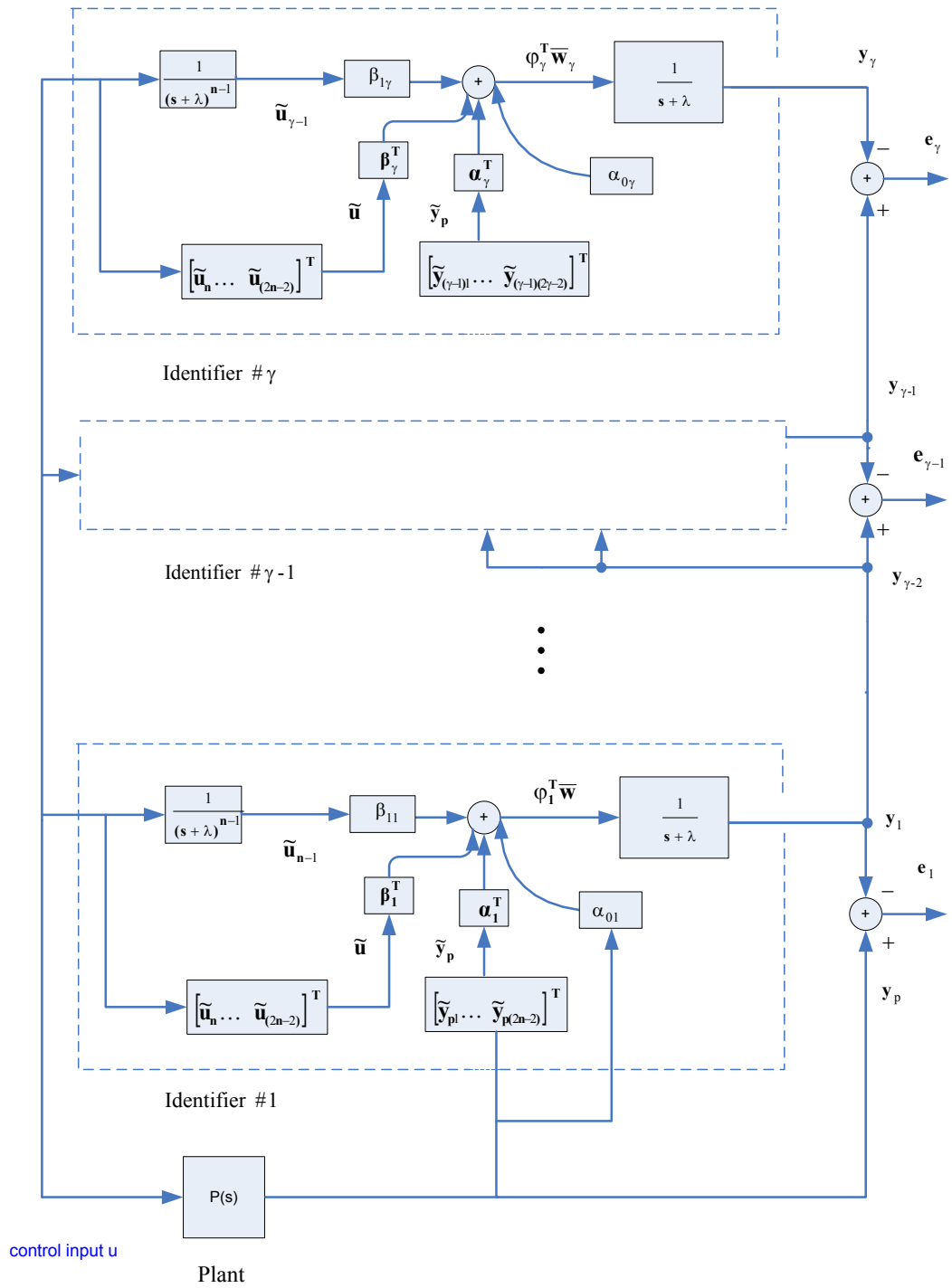


Figure 3.4 MRAC with uncertain relative degree for Identifier #1, …, Identifier #(γ-1) and Identifier #γ.

3.2.3. Control Law u(t)

The reference model has an input-output pair $\{r(.), y_m(.)\}$ and a transfer function $M(s)$ given by

$$\frac{y_m}{r} = M(s) = \frac{1}{s + a_{m(n-1)}} \cdot \frac{k_m}{s^{n-1} + a_{m(n-2)}s^{n-2} + \cdots a_{m1}s + a_{m0}} \tag{3.48}$$

The above transfer function consists of two blocks in series as shown in Fig. 3.1.

Let the output of the first block be

$$r_x \triangleq \frac{r}{s + a_{m(n-1)}} \tag{3.49}$$

Then, (3.48) can be rewritten as

$$\frac{y_m}{r_x} = \frac{k_m}{s^{n-1} + a_{m(n-2)}s^{n-2} + \cdots a_{m1}s + a_{m0}} \tag{3.50}$$

or, in time domain,

$$k_m r_x = y_m^{(n-1)} + a_{m(n-2)}y_m^{(n-2)} + \cdots a_{m1}\dot{y}_m + a_{m0}y_m \tag{3.51}$$

Define tracking error e as

$$e \triangleq \left(s^{n-1} + a_{m(n-2)}s^{n-2} + \cdots a_{m1}s + a_{m0}\right)e_{mn} \tag{3.52}$$

where

$$e_{mn} \triangleq y_n - y_m \tag{3.53}$$

(Note that if $e \to 0$, then $e_{mn} \to 0$ and $y_n \to y_m$)

Substituting (3.53) and (3.51) into(3,52) yields

$$e = \left[y_n^{(n-1)} + a_{m(n-2)}y_n^{(n-2)} + \cdots a_{m1}\dot{y}_n + a_{m0}y_n\right] - k_m r_x \tag{3.54}$$

Choose a Lyapunov function candidate

$$V = \frac{1}{2}e^2 > 0 \tag{3.55}$$

A control law is now to be devised in order to make

$$\dot{V} = e\dot{e} = -ke^2 \tag{3.56}$$

negative definite. This will be achieved by making

$$\dot{e} = -ke\,, \quad k > 0 \tag{3.57}$$

through the use of an appropriate control law to be derived as follows:

From (3.54), the derivative of e is given by

$$\dot{e} = \left[y_n^{(n)} + a_{m(n-2)} y_n^{(n-1)} + \cdots a_{m1}\ddot{y}_n + a_{m0}\dot{y}_n\right] - k_m \dot{r}_x \tag{3.58}$$

The next step is to find an expression for $y_n^{(n)}$.

From (3.22), displaying the $\tilde{u}_{(n-1)}$ term explicitly, we have

$$\dot{y}_n = \beta_{1n}\tilde{u}_{(n-1)} + r_{xn} \tag{3.59}$$

where

$$r_{xn} \underline{\underline{\Delta}}\ \beta_{2n}\tilde{u}_n + \cdots + \beta_{nn}\tilde{u}_{(2n-2)} + \alpha_{0n} y_{(n-1)} + \cdots + \alpha_{(2n-2)n}\tilde{y}_{(n-1)(2n-2)} - \lambda y_n \tag{3.60}$$

Successively differentiating (3.59), gives the νth derivatives of y_n

$$y_n^{(\nu)} = \sum_{i=0}^{\nu-1}\binom{\nu-1}{i}\left(\beta_{1n}{}^{(\nu-i-1)}\tilde{u}_{(n-1)}^{(i)}\right) + r_{xn}^{(\nu-1)}, \quad \nu = 2,\cdots,n \tag{3.61}$$

Letting $\nu = n$ and utilizing $\tilde{u}_{(n-1)}^{(n-1)} = -\left(\left(\sum_{\nu=1}^{n-1}\left(\binom{n-1}{\nu}\lambda^{\nu}\right)s^{(n-1)-\nu}\right)\tilde{u}_{(n-1)}\right) + u$ from (3.7) yields

$$y_n^{(n)} = \beta_{1n}u + \hat{y}_n^{(n)} \tag{3.62}$$

where

$$\hat{y}_n^{(n)} \underline{\underline{\Delta}} \sum_{i=0}^{n-2}\binom{n-2}{i}\left(\beta_{1n}{}^{(q-i-1)}\tilde{u}_{(n-2)}^{(i)}\right) + r_{xn}^{(n-1)} - \beta_{1n}\left(\left(\sum_{\nu=1}^{n-1}\left(\binom{n-1}{\nu}\lambda^{\nu}\right)s^{(n-1)-\nu}\right)\tilde{u}_{(n-1)}\right) \tag{3.63}$$

After $y_n^{(n)}$ is found as in (3.62), the expression for $\dot{e}$ in (3.58) becomes

$$\dot{e} = \left[\beta_{1n} u + \hat{y}_n^{(n)} + a_{m(n-2)} y_n^{(n-1)} + \cdots a_{m1} \ddot{y}_n + a_{m0} \dot{y}_n\right] - k_m \dot{r}_x \tag{3.64}$$

Next we substitute $\dot{e} = -ke$ from (3.57). The result is

$$-ke = \left[\beta_{1n} u + \hat{y}_n^{(n)} + a_{m(n-2)} y_n^{(n-1)} + \cdots a_{m1} \ddot{y}_n + a_{m0} \dot{y}_n\right] - k_m \dot{r}_x \tag{3.65}$$

Since our objective is to design a differentiator-free controller, replacing the derivative terms $\hat{y}_n^{(n)}$, $y_n^{(n-1)}, \cdots, \dot{y}_n$ and $\dot{r}_x$ with (3.62)~(3.63) and (3.49), respectively, gives the control law

$$u(t) = -\frac{m}{\beta_{1n}} - ke\,, \quad \beta_{1n} > 0\,,\ k > 0 \tag{3.66}$$

where

$$m \triangleq \sum_{i=0}^{n-2} \binom{n-2}{i} \left(\beta_{1n}{}^{(q-i-1)} \tilde{u}_{(n-2)}^{(i)}\right) + r_{xn}^{(n-1)} - \beta_{1n}\left(\left(\sum_{\nu=1}^{n-1}\left(\binom{n-1}{\nu}\lambda^{\nu}\right) s^{(n-1)-\nu}\right)\tilde{u}_{(n-1)}\right) +$$

$$\cdots + a_{m0}\left(\beta_{1n}\tilde{u}_{(n-1)} + r_{xn}\right) - k_m\left(r - a_{m(n-1)} r_x\right) \tag{3.67}$$

Note that division by zero in (3.66) will not occur because (3.47) and (3.38) guarantee that $\beta_{1n} \geq \beta_{1(n-1)} \geq \cdots \geq \beta_{11} \geq \varepsilon > 0$. Also note that the signals $\tilde{u}_{(n-2)}^{(n-2)}, \cdots, \tilde{u}_{(n-2)}$ and $r_{xn}^{(n-1)}$ can be obtained without actual differentiation because they are outputs of proper stable transfer functions with bounded inputs as shown in (3.7) and (3.60). As for the derivatives of the adaptive parameters, they can be replaced by their respective adaptive laws, thus dispensing of the need of differentiations.

Thus, with $\dot{V}$ in (3.56) being negative definite, the equilibrium state $e = 0$ is globally asymptotically stable, i.e. e is bounded and $e \to 0$ as $t \to \infty$.

Consequently, from (3.52)~(3.53),

$$e_{mn} \to 0 \text{ and } y_n \to y_m \tag{3.68}$$

Figure 3.5 shows a schematic diagram of overall system.

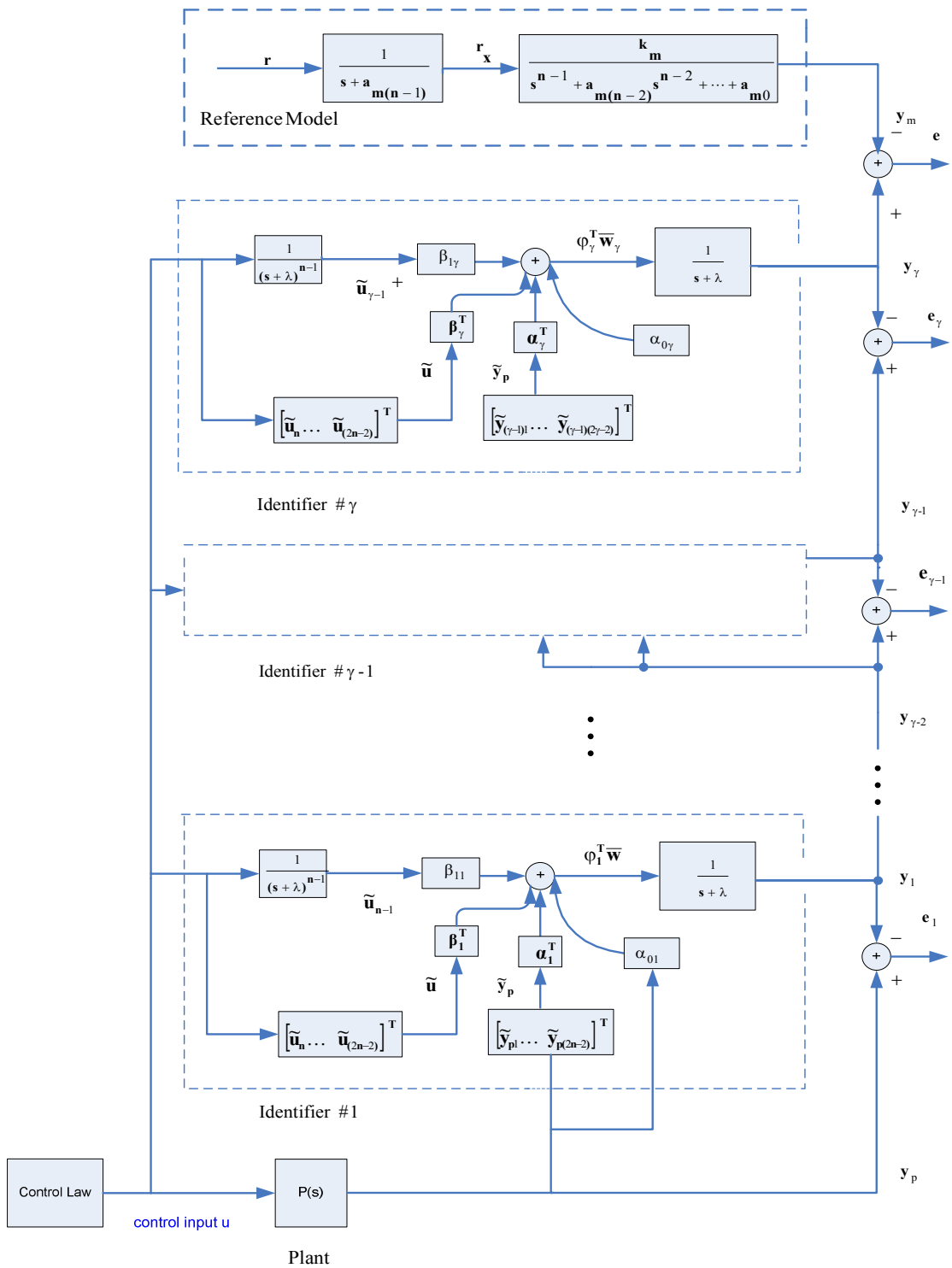


Figure 3.5: MRAC with uncertain relative degree for n-th order plant.

3.2.4. Boundedness of All Signals in the Entired Feedback System

With reference to the entire adaptive control system in Fig. 3.1, the following signals have been shown to be bounded:

From the analysis of Identifier $\#1,\cdots,$ Identifier $\#n$: $e_1,\cdots,e_n$, $\alpha_{01},\cdots,\alpha_{0n}$, $\alpha_1^T,\cdots,\alpha_n^T$, $\beta_{11},\cdots,\beta_{1n}$ and $\beta_1^T,\cdots,\beta_n^T$,

From the analysis of the control law: e and e_{mn}

From the reference model: r and y_m

The signals that remain to be shown bounded are, in appropriate groups:

Boundedness of y_p , $y_1,\cdots,y_{(n-1)}$ and y_n:

Since y_m , e_{mn} , e_n and $e_{(n-1)},\cdots,e_1$ are bounded, it follows from (71) and (3.33) that $y_n,\cdots,y_1$ and y_p are bounded.

Boundedness of $\tilde{y}_p$, $\tilde{y}_1,\cdots,\tilde{y}_{(n-1)}$, $\dot{\tilde{y}}_p$, $\dot{\tilde{y}}_1,\cdots,\dot{\tilde{y}}_{(n-1)}$, $\dot{r}_x$, $\dot{y}_m,\cdots y_m^{(n)}$, $\dot{e}$, $\ddot{e}$,$\cdots,e^{(n)}$, $\dot{e}_{mn},\ddot{e}_{mn},\cdots,e_{mn}^{(n)}$ and $\dot{y}_n,\ddot{y}_n,\cdots,y_n^{(n)}$:

The signals $\tilde{y}_p$, $\tilde{y}_1,\cdots,\tilde{y}_{(n-1)}$, $\dot{\tilde{y}}_p$, $\dot{\tilde{y}}_1,\cdots,\dot{\tilde{y}}_{(n-1)}$, $\dot{r}_x$ and $\dot{y}_m,\cdots y_m^{(n)}$ are outputs of "proper" stable transfer functions with bounded inputs. Hence they are bounded. Also, from (3.57), we see that $\dot{e}$, $\ddot{e}$,$\cdots,e^{(n)}$ are bounded. It follows from (3.52) that the same is true of $\dot{e}_{mn},\ddot{e}_{mn},\cdots,e_{mn}^{(n)}$. Consequently, from (3.53), $\dot{y}_n,\ddot{y}_n,\cdots,y_n^{(n)}$ are bounded.

Boundedness of r_{xn} , $\tilde{u}_{(n-1)}$, $\tilde{u}_{(n)},\cdots,\tilde{u}_{(2n-2)}$, $\dot{\tilde{u}}_{(n)},\cdots,\dot{\tilde{u}}_{(2n-2)}$, $\dot{e}_1,\cdots,\dot{e}_n$, $\dot{y}_1,\cdots\dot{y}_{(n-1)}$ and $\dot{y}_p$:

With bounded $\dot{y}_n$ from (3.59), if we can show the boundedness of r_{xn} that gives the

boundedness of $\tilde{u}_{(n-1)}$. Besides, the numerator of the plant $P(s)$ needs to be a Hurwitz polynomial in s.

From (3.60), substitute $u = \dfrac{s^n + a_{n-1}s^{n-1} + a_{n-2}s^{n-2} + \cdots + a_1 s + a_0}{b_{n-1}s^{n-1} + \cdots + b_1 s + b_0} y_p$ from (3.2) and (3.7) gives

$$r_{xn} = \beta_{2n} \frac{1}{(s+\lambda)^n}\left(\frac{s^n + a_{n-1}s^{n-1} + a_{n-2}s^{n-2} + \cdots + a_1 s + a_0}{b_{n-1}s^{n-1} + \cdots + b_1 s + b_0} y_p\right) + \cdots$$

$$+ \beta_{nn} \frac{1}{(s+\lambda)^{2n-2}}\left(\frac{s^n + a_{n-1}s^{n-1} + a_{n-2}s^{n-2} + \cdots + a_1 s + a_0}{b_{n-1}s^{n-1} + \cdots + b_1 s + b_0} y_p\right)$$

$$+ \alpha_{0n} y_{(n-1)} + \cdots + \alpha_{(2n-2)n}\tilde{y}_{(n-1)(2n-2)} - \lambda y_n$$

the boundedness of r_{xn}. After we establish the boundedness of $\tilde{u}_{(n-1)}$, the boundedness of $\tilde{u}_{(n)}, \cdots, \tilde{u}_{(2n-2)}$ and $\dot{\tilde{u}}_n, \cdots, \dot{\tilde{u}}_{(2n-2)}$ follows. The signal $\dot{e}_1$ in (3.33) is bounded because $\tilde{u}_{(n-1)}$, $\tilde{u}$, y_p and $\tilde{y}_p$ are bounded. In a similar fashion, the boundedness of $\dot{e}_2, \cdots, \dot{e}_n$ can be established. Thus, $\dot{y}_{(n-1)}, \cdots \dot{y}_1$, $\dot{y}_p$ are also bounded due to the boundedness of $\dot{y}_n$.

Boundedness of $\dot{\alpha}_{01}, \cdots, \dot{\alpha}_{0n}$, $\dot{\alpha}_1^T, \cdots, \dot{\alpha}_n^T$, $\dot{\beta}_{11}, \cdots, \dot{\beta}_{1n}$, $\dot{\beta}_1^T, \cdots, \dot{\beta}_n^T$ and $\dot{r}_{xn}$:

The boundedness of the variables $\dot{\alpha}_{01}, \cdots, \dot{\alpha}_{0n}$, $\dot{\alpha}_1^T, \cdots, \dot{\alpha}_n^T$, $\dot{\beta}_{11}, \cdots, \dot{\beta}_{1n}$ and $\dot{\beta}_1^T, \cdots, \dot{\beta}_n^T$ as given in (69) and (72) can be seen through the substitution of all occurring derivative terms by their respective adaptive laws. Consequently, $\dot{r}_{xn}$ (as obtained from 83b) is also bounded.

Boundedness of $\ddot{y}_p, \cdots, y_p^{(n)}$ u and m:

The boundedness of u is established from (3.2) if one can show the boundedness of

$y_p, \dot{y}_p, \ddot{y}_p, \cdots, y_p^{(n)}$. This is demonstrated as follows. Substituting (3.7) and (3.2) into (3.59) to give

$$\dot{y}_n = \beta_{1n} \frac{1}{(s+\lambda)^{n-1}} \left(\frac{s^n + a_{n-1}s^{n-1} + a_{n-2}s^{n-2} + \cdots + a_1 s + a_0}{b_{n-1}s^{n-1} + \cdots + b_1 s + b_0} y_p \right) + r_{xn} \tag{3.69}$$

It is assumed that the sign of b_0 is known. Without loss of generality, we shall assume the n cases:

(a). The leading coefficient $b_{n-1},\ b_{n-2,}\ \cdots,\ b_1 = 0$

$$\dot{y}_n = b_0^{-1}\beta_{1n}\left[\dot{y}_p + \xi_{n-1}y_p + \frac{\zeta_{n-2}s^{n-2} + \zeta_{n-3}s^{n-3} + \cdots \zeta_0}{(s+\lambda)^{n-1}} y_p \right] + r_{xn} \tag{3.70}$$

Differentiating (3.70) once gives

$$\ddot{y}_n = b_0^{-1}\left[\beta_{1n}(\ddot{y}_p + \xi_{n-1}\dot{y}_p + \frac{\zeta_{n-2}s^{n-2} + \zeta_{n-3}s^{n-3} + \cdots \zeta_0}{(s+\lambda)^{n-1}} \dot{y}_p) \right.$$

$$\left. + \dot{\beta}_{1n}(\dot{y}_p + \xi_{n-1}y_p + \frac{\zeta_{n-2}s^{n-2} + \zeta_{n-3}s^{n-3} + \cdots \zeta_0}{(s+\lambda)^{n-1}} y_p) \right] + \dot{r}_{xn}$$

Since $\ddot{y}_n$, $\dot{\beta}_{1n}$, y_p, $\dot{y}_p$ and $\dot{r}_{xn}$ have been shown to be bounded, $\ddot{y}_p$ is bounded. Using the same approach, differentiating (3.70) twice will leads to the boundedness of $\dddot{y}_p$. Continuing on in this fashion would lead to the boundedness of $y_p^{(4)}, \cdots, y_p^{(n)}$.

(b). The treatment of other (n-1) cases follows the same pattern as above, and is therefore omitted.

With the boundedness of u, the boundedness of m is assured from (3.67). Thus, all systems in the overall system are bounded.

3.2.5. Convergence of the Tracking Errors

The discussion of the convergence is exactly the same as that in Section 2.2.5. and is omitted.

3.2.6. Simulation Studies

We include a simulation for the case of $n = 3$ which has not been done in the literature.

Simulation 3.2.6.1: 3rd order Identifier Tracking MRAC

The data for the simulation are as follows: 3rd order stable plant, q=3

$$P(s) = \frac{1}{s^3 + 3.5s^2 + 3.5s + 1}, \quad M(s) = \frac{1}{s^3 + 2s^2 + 2s + 1}$$

$$r(t) = \begin{cases} 0 & t \in [0,10] \\ 1 & t \in [10,160] \,\&\, t \in [310,460] \\ -1 & t \in [160,310] \,\&\, t \in [460,\infty] \end{cases}$$

$\lambda = 1$, $k = g = 1$, $e_p = y_p - y_m$

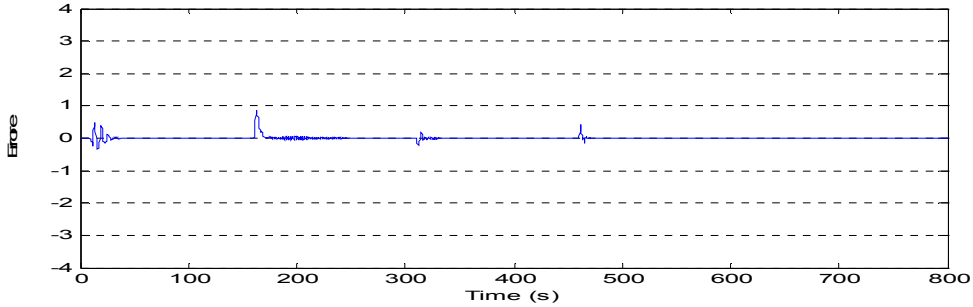
The initial conditions for the adaptive parameters are chosen in accordance with (3.38) and (3.47), in this case:

$\beta_{11}(0) = 0.5 \geq k_{lower}$, $\beta_{21}(0) = 1$ (k_{lower} is taken to be 0.01),

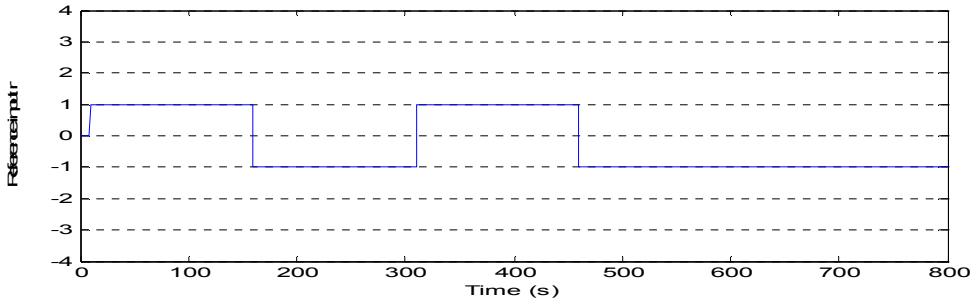
$\beta_{12}(0) = 0$, $\beta_{22}(0) = 0$,

$\alpha_{01}(0) = \alpha_{11}(0) = \alpha_{21}(0) = \alpha_{02}(0) = \alpha_{12}(0) = \alpha_{22}(0) = 0$

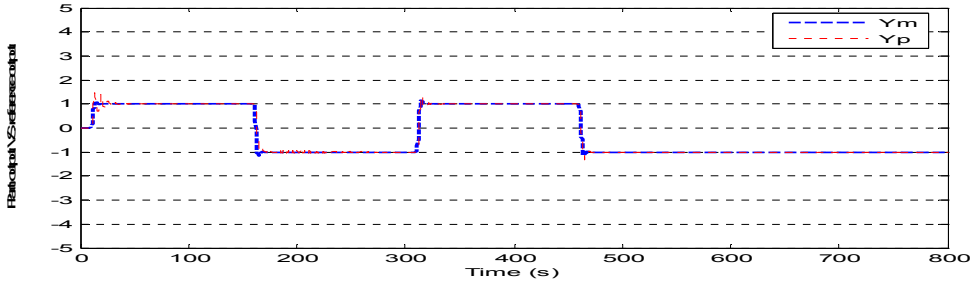
Simulation results are shown as figure 3.6.



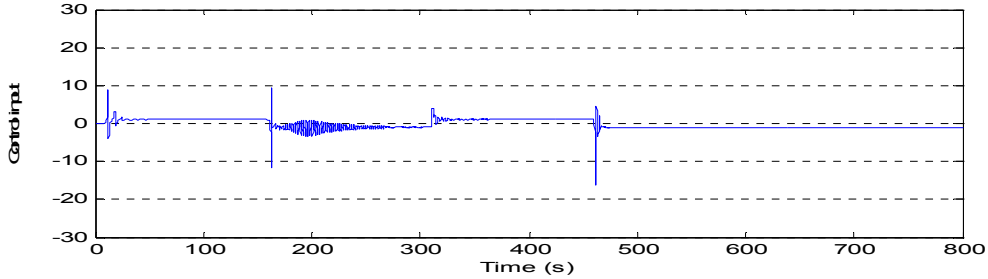
(a)



(b)



(c)



(d)

Figure 3.6: 3rd order stable plant, q=3. (a) tracking error, (b) reference input signal, (c) plant vs reference output and (d) control signal u

Simulation 3.2.6.2: 3rd order Identifier Tracking MRAC

The data for the simulation are as follows: 3rd order stable plant, q=3

$$P(s) = \frac{1}{s^3 + 4s^2 + 3s}, \; M(s) = \frac{1}{s^3 + 2s^2 + 2s + 1}$$

$$r(t) = \begin{cases} 0 & t \in [0,10] \\ 1 & t \in [10,160] \,\&\, t \in [310,\infty] \\ -1 & t \in [160,310] \end{cases}$$

$\lambda = 1$, $k = g = 1$, $e_p = y_p - y_m$

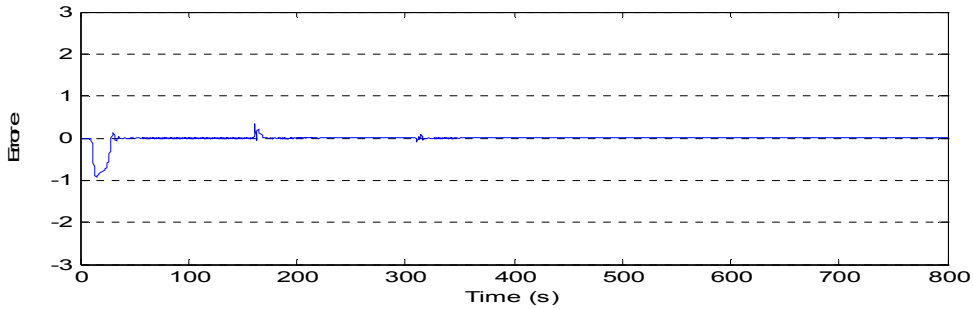
The initial conditions for the adaptive parameters are chosen in accordance with (3.38) and (3.47), in this case:

$\beta_{11}(0) = 1 \geq k_{lower}$, $\beta_{21}(0) = 2$ (k_{lower} is taken to be 0.01),

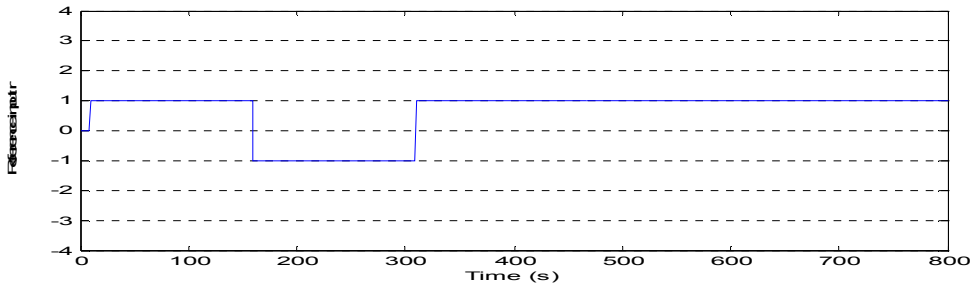
$\beta_{12}(0) = 1$, $\beta_{22}(0) = 1$,

$\alpha_{01}(0) = \alpha_{11}(0) = \alpha_{21}(0) = \alpha_{02}(0) = \alpha_{12}(0) = \alpha_{22}(0) = 1$

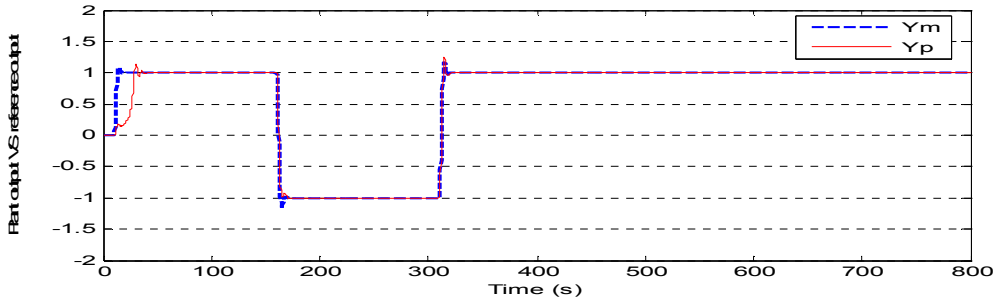
Simulation results are shown as figure 3.7.



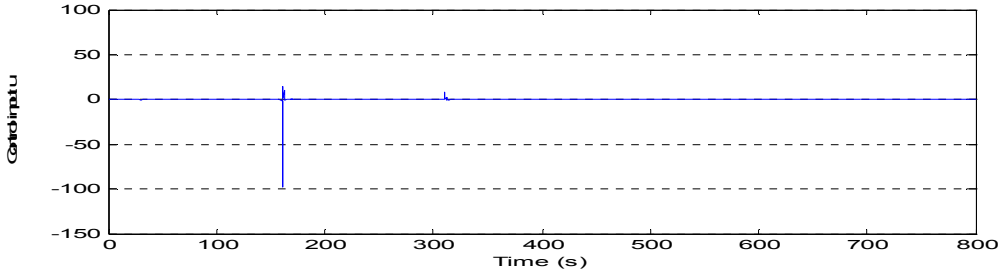
(a)



(b)



(c)



(d)

Figure 3.7: 3rd order stable plant, q=3. (a) tracking error, (b) reference input signal, (c) plant vs reference output and (d) control signal u

Simulation 3.2.6.3: 3rd order Identifier Tracking MRAC

The data for the simulation are as follows: 3rd order unstable plant, q=3

$$P(s) = \frac{1}{s^3 + 4s^2 - 3s}, \; M(s) = \frac{1}{s^3 + 2s^2 + 2s + 1}$$

$$r(t) = \begin{cases} 0 & t \in [0,10] \\ 1 & t \in [10,160] \,\&\, t \in [310,\infty] \\ -1 & t \in [160,310] \end{cases}$$

$\lambda = 1$, $k = g = 1$, $e_p = y_p - y_m$

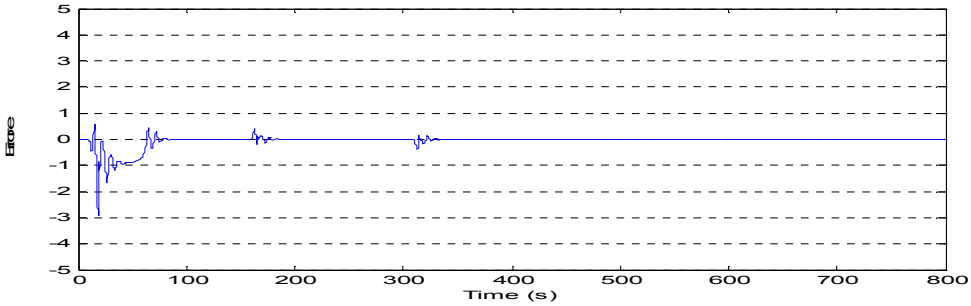
The initial conditions for the adaptive parameters are chosen in accordance with (3.38) and (3.47), in this case:

$\beta_{11}(0) = 1 \geq k_{lower}$, $\beta_{21}(0) = 2$ (k_{lower} is taken to be 0.01),

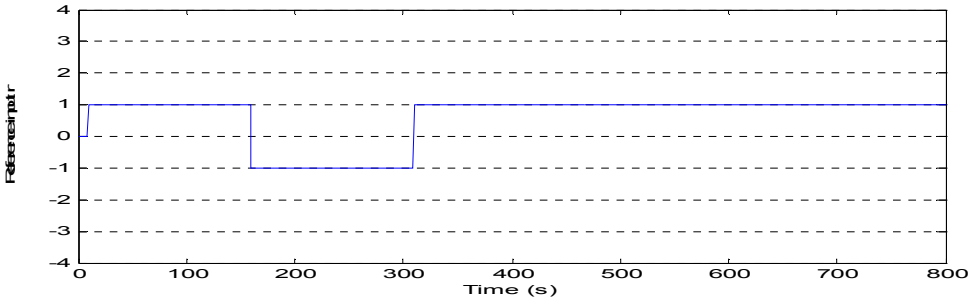
$\beta_{12}(0) = 1$, $\beta_{22}(0) = 1$,

$\alpha_{01}(0) = \alpha_{11}(0) = \alpha_{21}(0) = \alpha_{02}(0) = \alpha_{12}(0) = \alpha_{22}(0) = 1$

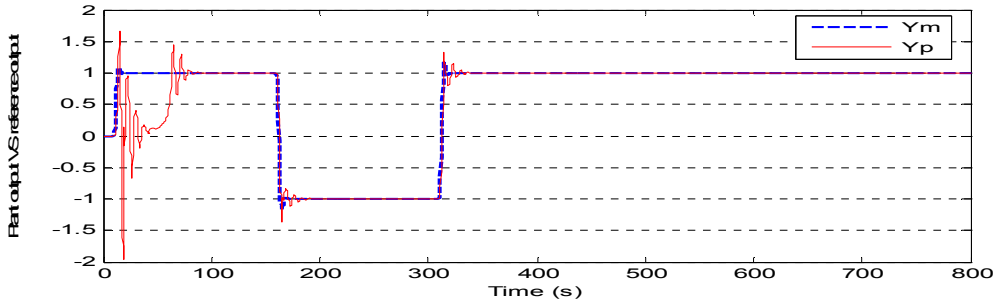
Simulation results are shown as figure 3.8.



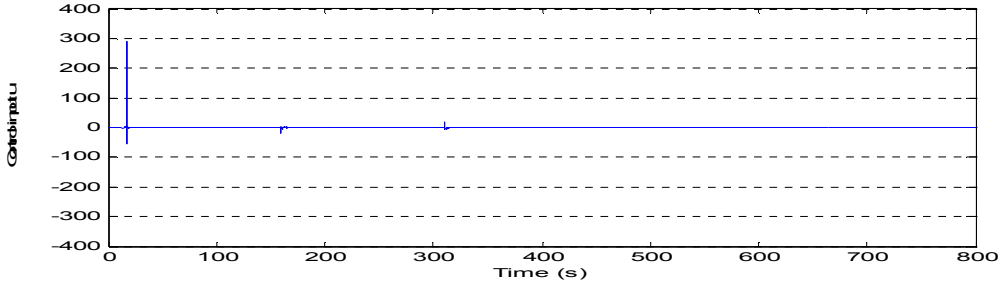
(a)



(b)



(c)



(d)

Figure 3.8: 3rd order unstable plant, q=3. (a) tracking error, (b) reference input signal, (c) plant vs reference output and (d) control signal u

Simulation 3.2.6.4: 3rd order Identifier Tracking MRAC

The data for the simulation are as follows: 3rd order stable plant, q=3

$$P(s) = \frac{1}{s^3 + 4s^2 + 3s}, \; M(s) = \frac{1}{s^3 + 2s^2 + 2s + 1}$$

$$r(t) = 2\sin(0.2t)$$

$$\lambda = 1, \; k = g = 1, \; e_p = y_p - y_m$$

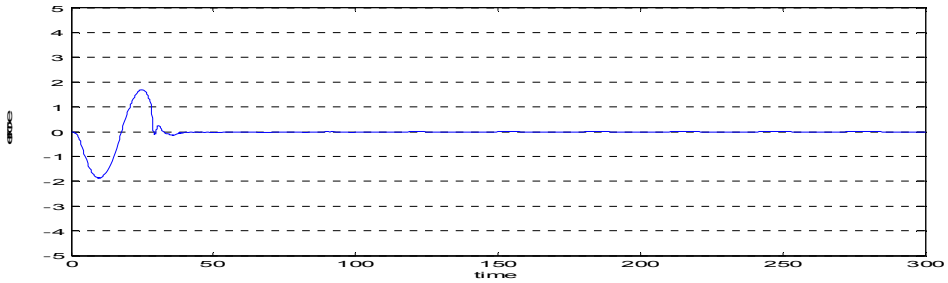
The initial conditions for the adaptive parameters are chosen in accordance with (3.38) and (3.47), in this case:

$\beta_{11}(0) = 1 \geq k_{lower}$, $\beta_{21}(0) = 1$ (k_{lower} is taken to be 0.01),

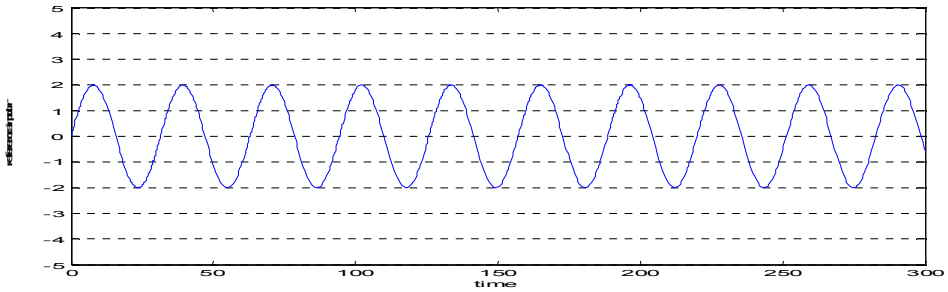
$\beta_{12}(0) = 1$, $\beta_{22}(0) = 1$,

$$\alpha_{01}(0) = \alpha_{11}(0) = \alpha_{21}(0) = \alpha_{02}(0) = \alpha_{12}(0) = \alpha_{22}(0) = 1$$

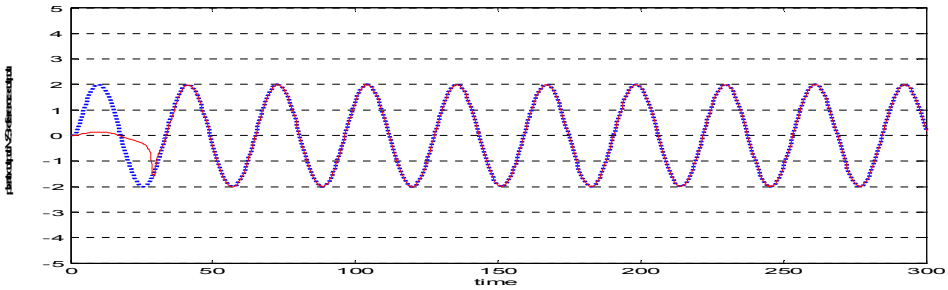
Simulation results are shown as figure 3.9.



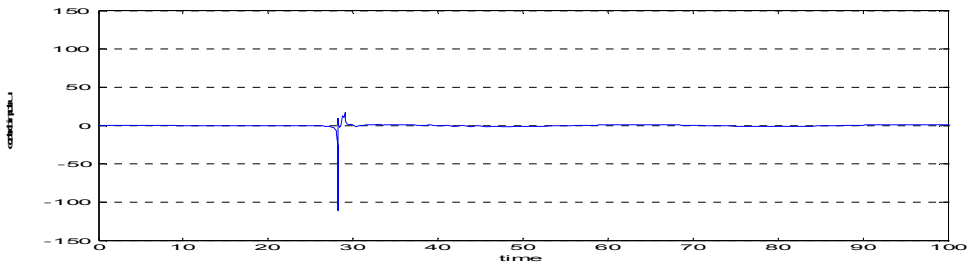
(a)



(b)



(c)



(d)

Figure 3.9: 3rd order stable plant, q=3. (a) tracking error, (b) reference input signal, (c) plant vs reference output and (d) control signal u

Simulation 3.2.6.5: 3rd order Identifier Tracking MRAC

The data for the simulation are as follows: 3rd order stable plant, q=2

$$P(s) = \frac{s+3}{s^3 + 3.5s^2 + 3.5s + 1},\ M(s) = \frac{1}{s^3 + 2s^2 + 2s + 1}$$

$$r(t) = \begin{cases} 0 & t \in [0,10] \\ 1 & t \in [10,160] \,\&\, t \in [310,\infty] \\ -1 & t \in [160,310] \end{cases}$$

$\lambda = 1$, $k = g = 1$, $e_p = y_p - y_m$

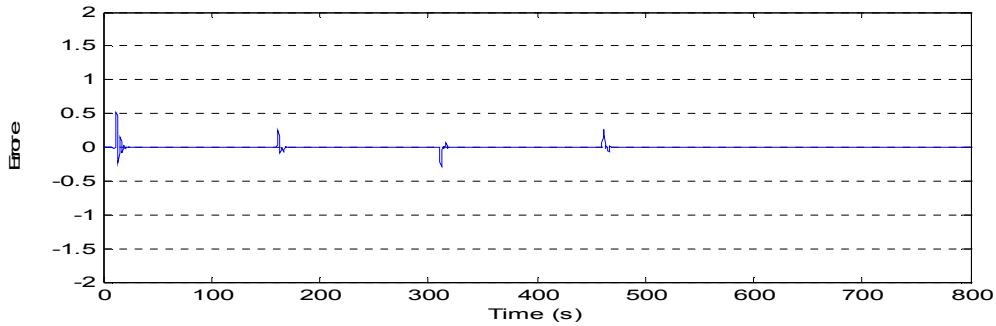
The initial conditions for the adaptive parameters are chosen in accordance with (3.38) and (3.47), in this case:

$\beta_{11}(0) = 0.5 \geq k_{lower}$, $\beta_{21}(0) = 1$ (k_{lower} is taken to be 0.01),

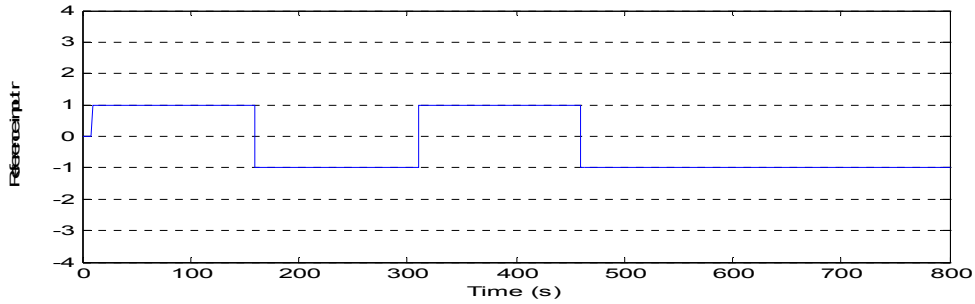
$\beta_{12}(0) = 0$, $\beta_{22}(0) = 0$,

$\alpha_{01}(0) = \alpha_{11}(0) = \alpha_{21}(0) = \alpha_{02}(0) = \alpha_{12}(0) = \alpha_{22}(0) = 0$,

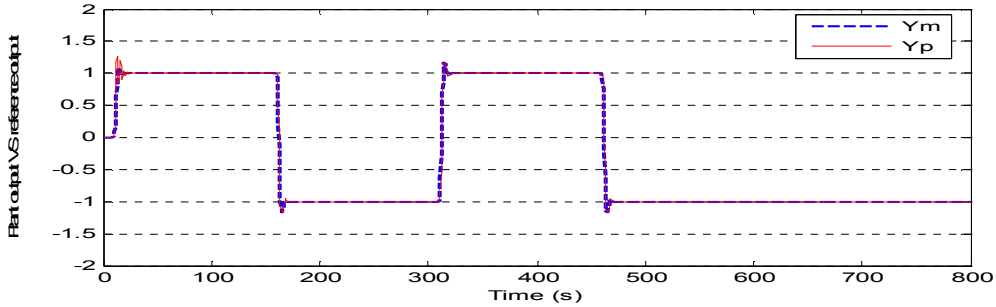
Simulation results are shown as figure 3.10.



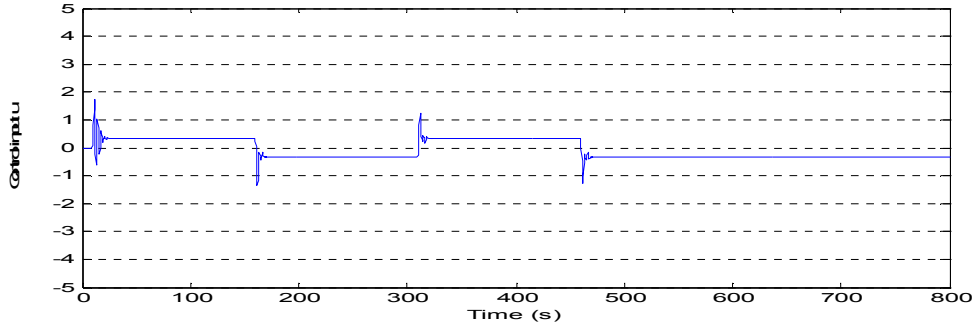
(a)



(b)



(c)



(d)

Figure 3.10: 3rd order stable plant, q=2. (a) tracking error, (b) reference input signal, (c) plant vs reference output and (d) control signal u

Simulation 3.2.6.6: 3rd order Identifier Tracking MRAC

The data for the simulation are as follows: 3rd order unstable plant, q=2

$$P(s) = \frac{s + 0.5}{s^3 + 4s^2 - 3s}, \; M(s) = \frac{1}{s^3 + 2s^2 + 2s + 1}$$

$$r(t) = \begin{cases} 0 & t \in [0, 10] \\ 1 & t \in [10, 40] \,\&\, t \in [70, \infty] \\ -1 & t \in [40, 70] \end{cases}$$

$\lambda = 1$, $k = g = 1$, $e_p = y_p - y_m$

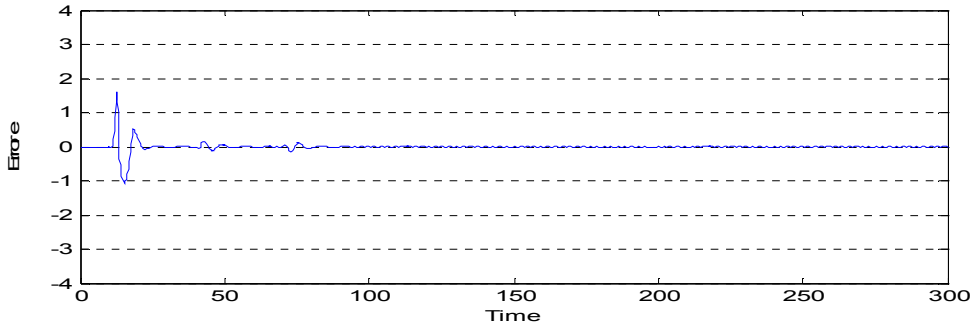
The initial conditions for the adaptive parameters are chosen in accordance with (3.38) and (3.47), in this case:

$\beta_{11}(0) = 1 \geq k_{lower}$, $\beta_{21}(0) = 2$ (k_{lower} is taken to be 0.01),

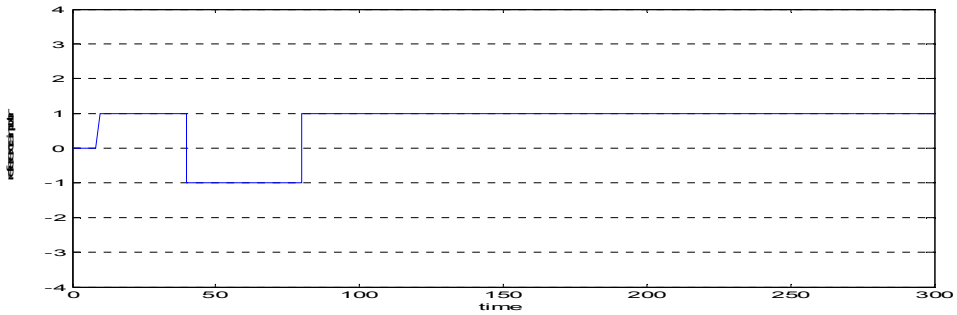
$\beta_{12}(0) = 0$, $\beta_{22}(0) = 0$,

$\alpha_{01}(0) = \alpha_{11}(0) = \alpha_{21}(0) = \alpha_{02}(0) = \alpha_{12}(0) = \alpha_{22}(0) = 0$,

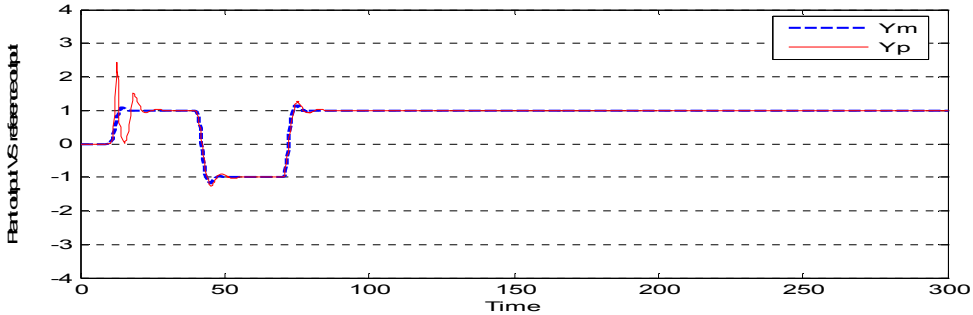
Simulation results are shown as figure 3.11.



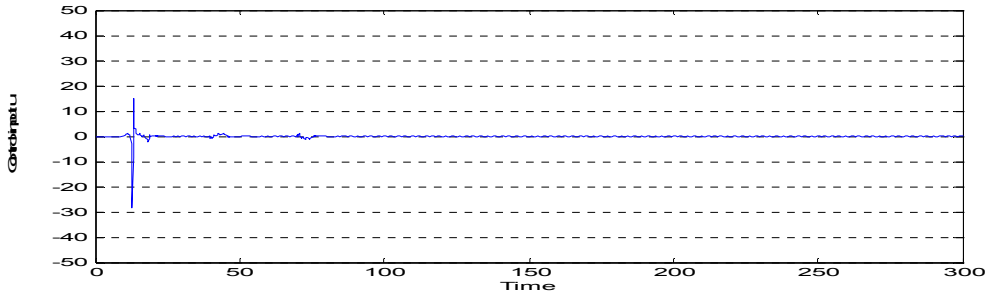
(a)



(b)



(c)



(d)

Figure 3.11: 3rd order unstable plant, q=2. (a) tracking error, (b) reference input signal, (c) plant vs reference output and (d) control signal u

Simulation 3.2.6.7: 3rd order Identifier Tracking MRAC

The data for the simulation are as follows: 3rd order stable plant, q=1

$$P(s) = \frac{s^2 + s + 1}{s^3 + 3.5s^2 + 3.5s + 1},\ M(s) = \frac{1}{s^3 + 2s^2 + 2s + 1}$$

$$r(t) = \begin{cases} 0 & t \in [0,10] \\ 1 & t \in [10,40] \,\&\, t \in [70,\infty] \\ -1 & t \in [40,70] \end{cases}$$

$$\lambda = 1,\ k = g = 1,\ e_p = y_p - y_m$$

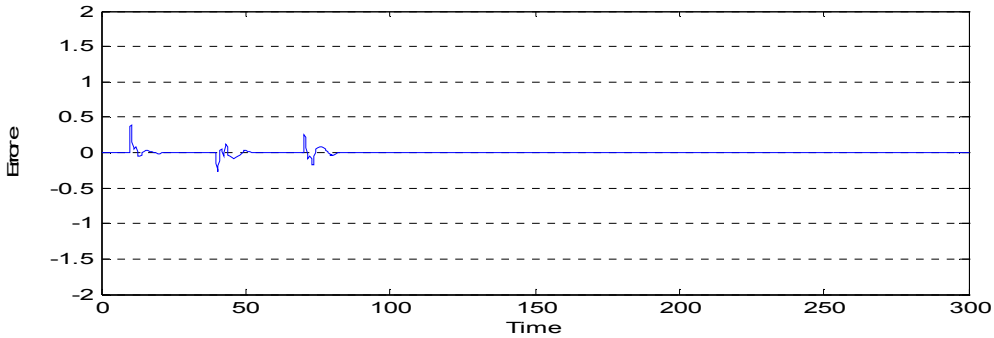
The initial conditions for the adaptive parameters are chosen in accordance with (3.38) and (3.47), in this case:

$\beta_{11}(0) = 0.5 \geq k_{lower},\ \beta_{21}(0) = 1$ (k_{lower} is taken to be 0.01),

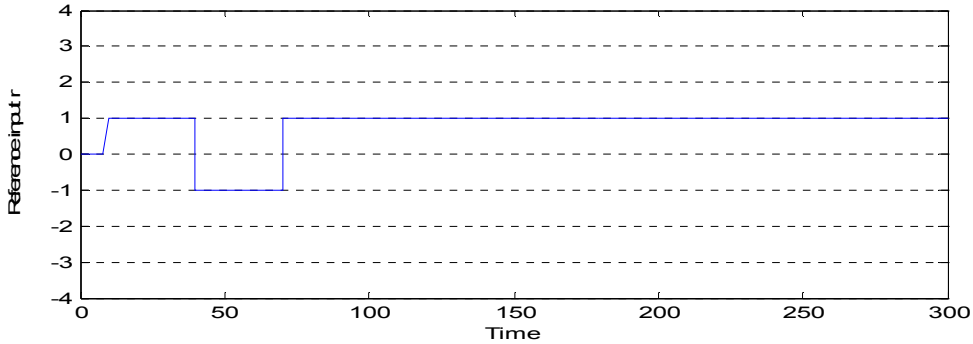
$\beta_{12}(0) = 0,\ \beta_{22}(0) = 0$,

$\alpha_{01}(0) = \alpha_{11}(0) = \alpha_{21}(0) = \alpha_{02}(0) = \alpha_{12}(0) = \alpha_{22}(0) = 0$,

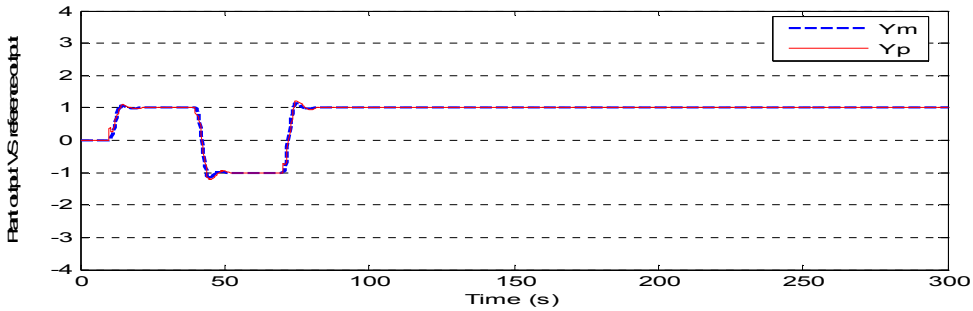
Simulation results are shown as figure 3.12.



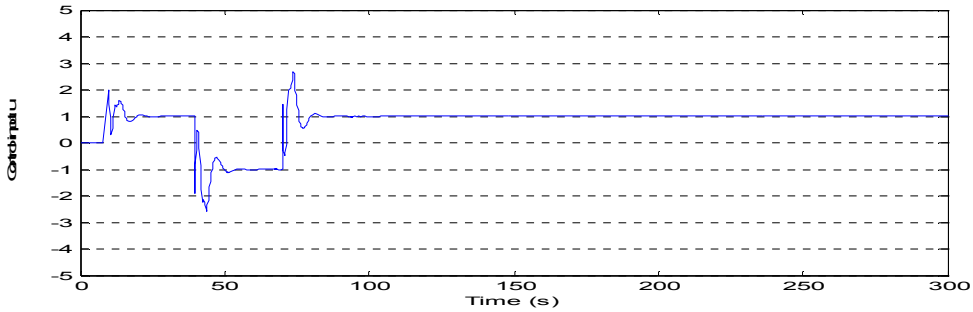
(a)



(b)



(c)



(d)

Figure 3.12: 3rd order stable plant, q=1. (a) tracking error, (b) reference input signal, (c) plant vs reference output and (d) control signal u

Simulation 3.2.6.8: 3rd order Identifier Tracking MRAC

The data for the simulation are as follows: 3rd order unstable plant, q=1

$$P(s) = \frac{s^2 + s + 1}{s^3 + 4s^2 - 3s},\ M(s) = \frac{1}{\mathbf{s}^3 + 2\mathbf{s}^2 + 2\mathbf{s} + 1}$$

$$r(t) = \begin{cases} 0 & t \in [0,10] \\ 1 & t \in [10,40] \,\&\, t \in [70,\infty] \\ -1 & t \in [40,70] \end{cases}$$

$\lambda = 1$, $k = g = 1$, $e_p = y_p - y_m$

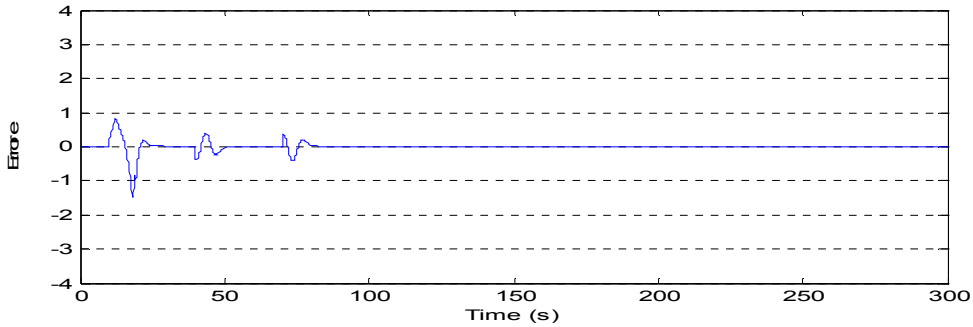
The initial conditions for the adaptive parameters are chosen in accordance with (3.38) and (3.47), in this case:

$\beta_{11}(0) = 0.5 \geq k_{lower}$, $\beta_{21}(0) = 1$ (k_{lower} is taken to be 0.01),

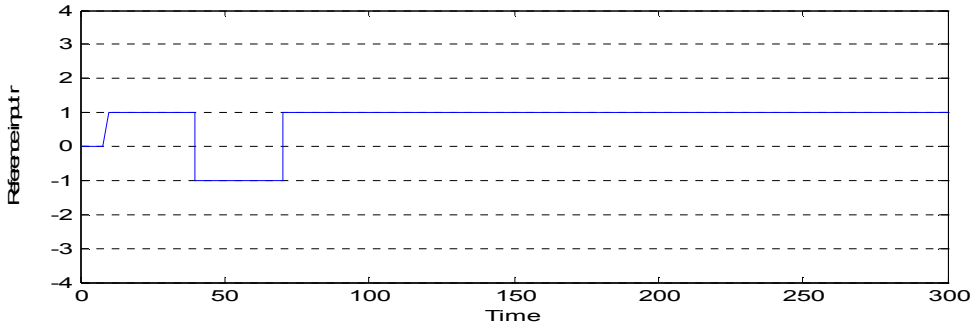
$\beta_{12}(0) = 0$, $\beta_{22}(0) = 0$,

$\alpha_{01}(0) = \alpha_{11}(0) = \alpha_{21}(0) = \alpha_{02}(0) = \alpha_{12}(0) = \alpha_{22}(0) = 0$,

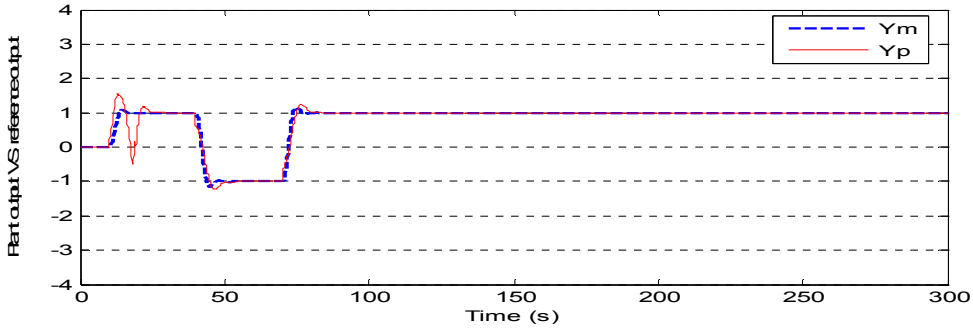
Simulation results are shown as figure 3.13.



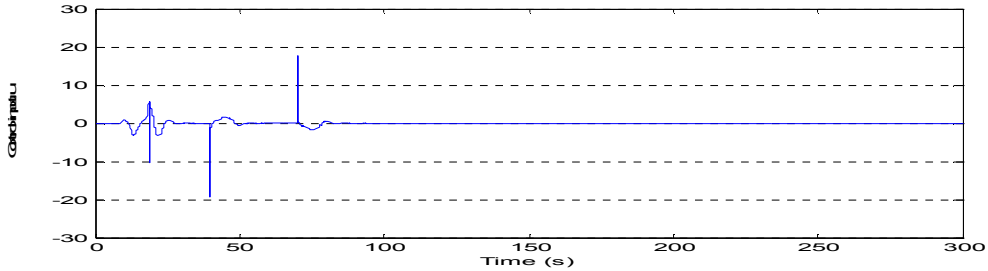
(a)



(b)



(c)



(d)

Figure 3.13: 3rd order unstable plant, q=1. (a) tracking error, (b) reference input signal, (c) plant vs reference output and (d) control signal u

CHAPTER 4

CONCLUSIONS

4.1 General Conclusions

In this dissertation, a new design method of MRAC for LTI systems with unknown degree is proposed. The scheme integrates a total of n layers of identifiers in the control where n is the order of plant. The method is first introduced for 2-nd order plants with unknown relative degrees. Then it is extend to the case of n-th order plants. The key points of this method are 1) a new parameterization scheme is developed to create one structure for plants with different unknown relative degrees. This structure can be seen to encompass the various relative-degree cases of an n-th order plant. The proposed control system is based on this structure. 2) A stacked-identifier based structure is implemented. The structure of each layer of identifiers is constructed according to the structure of the plant. Thus, the deviation between the designed identifier output and the plant output is expected to be small.

The following steps are implemented to achieve this goal:

1. Appropriately reparametrize the plant with uncertain relative degrees according to the order of the plant so that identifiers can be constructed based on this reparameterization.
2. Set up the first identifier which mimics the structure of the plant directly. Parameter adaptive laws are established to make the plant output asymptotically track the identifier output.
3. Identifiers 2 up to N are constructed. Input of each layer of identifier is the output of previous one. Adaptive laws are established to make output of the lower layer asymptotically track the output of upper layer.

4. Design a control law to make the identifier output asymptotically track the reference model output. That means output of the plant will track that of the reference model asymptotically.
5. Show all the signals are bounded by using Lyapunov stability theory.
6. Show that all tracking errors are converged to zero.

The new method allows relaxation of one of the crucial assumptions of MRAC. Simulations for the cases of $n = 2$ and $n = 3$ are given to demonstrate the effectiveness of this new method. In conclusion, an adaptive system is designed based on a new parameterization scheme. It successfully achieves the goal to design a MRAC system without the knowledge of its relative degree.

4.2 Future Research

We have developed a fundamental theory for identifier-tracking MRAC of linear time-invariant plants. Some future works are as follows:

A. For linear plants:

- Extension of the continuous time schemes to the discrete time case.
- Extension of the single-input single-output plants to multi-input multi-output plants.
- Development a new parameter identification algorithm.
- Robustness in the presence of unmodeled dynamics, time-varying parameters, and other perturbations.

B. For nonlinear plants:

Existing adaptive techniques for nonlinear systems generally require a linear parameterization of the plant dynamics. The two main approaches for constructing adaptive controllers are model-reference adaptive control (MRAC) and self-tuning (ST). The difference between MRAC

and ST arise from different perspectives. The MRAC, being updated the parameters as to minimize the tracking errors between the plant output and reference output. In ST, parameters update in another way to minimize the data-fitting error in input-output measurements. The existing theory for nonlinear adaptive controls requires the following conditions:

(a).The nonlinear plant can be linearly parameterized.

(b).The full state is measurable.

(c).Nonlinearities can be canceled stably by the control input if the parameters are known.

The new method of reparameterization developed in this dissertation may find application in the adaptive control of nonlinear systems.

REFERENCES

[1] Youping Zhang, Pei-Yaun Peng, and Zhong-Ping Jiang "Stable neural controller design for unknown nonlinear systems using backstepping", IEEE Transactions on Neural Networks, 11: 1347-1360, 2000.

[2] Chih-Min Lin, Ya-Fu Peng, and Chun-Fei Hsu "Robust cerebellar model articulation controller design for unknown nonlinear systems", IEEE Transactions on Circuits and Systems II, 51: 354-358, 2004.

[3] Hung, J.Y., Gao, W., and Hung, J.C. "Variable structure control: a survey", IEEE Transactions on Industrial Electronics, 40: 2-22, 1993.

[4] Slotine, J.E., and Li, W. *Applied Nonlinear Control*. Englewood Cliffs, NJ: Prentice-Hall Inc., 1991.

[5] Chun-Ming Huang, and Jen-Yang Chen, "An Adaptive Fuzzy PI Controller for Nonlinear Systems", IMACS Multiconference on Computational Engineering in Systems Applications, 1: 524-529, 2006.

[6] Astrom, K.J., and Wittenmark, B. *Adaptive Control*. Second edition. Addison-Wesley Inc., 1995.

[7] Narendra, K.S., and Annaswamy, A.M. *Stable Adaptive Systems*. Englewood Cliffs, NJ: Prentice-Hall Inc., 1989.

[8] Narendra, K.S., and Balakrishnan, J. " Adaptive control using multiple models" IEEE Transactions on Automatic Control, 42:171-187, Feb. 1997.

[9] Ming-Li Chiang, and Li-Chen Fu "Variable structure based switching adaptive control for a class of unknown switched linear systems" American Control Conference, 2009.

[10] Lozano-Leal, R.; Collado, J.; Mondie, S.; , "Model reference robust adaptive control without a priori knowledge of the high frequency gain," Automatic Control, IEEE Transactions on , vol.35, no.1, pp.71-78, Jan 1990

[11] Lin Yan; Hsu, L.; Costa, R.R.; Lizarralde, F.; , "Variable structure model reference adaptive control for systems with unknown high frequency gain," Decision and Control, 2003. Proceedings. 42nd IEEE Conference on , vol.4, no., pp. 3525- 3530 vol.4, 9-12 Dec. 2003

[12] El Rifai, K.; El Rifai, O.; Youcef-Toumi, K.; , "On robust adaptive switched control," American Control Conference, 2005. Proceedings of the 2005 , vol., no., pp. 18- 23 vol. 1, 8-10 June 2005

[13] Wu, A.C.; Fu, L. C.; Hsu, C. F.; , "Robust MRAC for Plants with Arbitrary Relative Degree Using a Variable Structure Design," American Control Conference, 1992 , vol., no., pp.2735-2739, 24-26 June 1992

[14] Fernandes, M.V.A.; Dias, S.M.; Araujo, A.D.; Oliveira, J.B.; Queiroz, K.I.M.P.; , "Shunt Indirect Variable Structure Model Reference Adaptive Controller for plants with arbitrary relative degree," Variable Structure Systems (VSS), 2010 11th International Workshop on , vol., no., pp.283-288, 26-28 June 2010

[15] Ming-Li Chiang; Li-Chen Fu;, "Variable structure model reference adaptivecontrol of unknown switched linear systems with relative degree greater than one," Decision and Control, 2009 held jointly with the 2009 28th Chinese Control Conference. CDC/CCC 2009. Proceedings of the 48th IEEE Conference on , vol., no., pp.4240-4245, 15-18 Dec. 2009

[16] Yeung, K.S., Chung, W.D. and Yu, H., "Identifier-tracking model reference adaptive control", International Journal of Control, 81:6, 976-988, 2008.

[17] Ming-Li Chiang; Li-Chen Fu; , "Variable structure based switching adaptive control for a class of unknown switched linear systems," American Control Conference, 2009. ACC '09. , vol., no., pp.3019-3024, 10-12 June 2009

[18] Nouri, A.S.; Hamerlain, M.; Mira, C.; Lopez, P.; , "Variable structure model reference adaptive control using only input and output measurements for two real one-link manipulators," Systems, Man and Cybernetics, 1993. 'Systems Engineering in the Service of Humans', Conference Proceedings., International Conference on , vol., no., pp.465-472 vol.4, 17-20 Oct 1993

[19] Chiang-Ju Chien; Li-Chen Fu; , "An adaptive variable structure control for fast time-varying unknown plants," Decision and Control, 1993., Proceedings of the 32nd IEEE Conference on , vol., no., pp.1428-1433 vol.2, 15-17 Dec 1993

[20] Narendra, K.S.; Balakrishnan, J.; , "Improving transient response of adaptive control systems using multiple models and switching," Automatic Control, IEEE Transactions on , vol.39, no.9, pp.1861-1866, Sep 1994

[21] Narendra, K.S.; Boskovic, J.D.; , "A combined direct, indirect, and variable structure method for robust adaptive control," Automatic Control, IEEE Transactions on , vol.37, no.2, pp.262-268, Feb 1992

[22] Miyasato, Y.; , "Model reference adaptive control for nonlinear systems with unknown degrees and with unknown relative degrees 1 or 2," Decision and Control, 1993., Proceedings of the 32nd IEEE Conference on , vol., no., pp.2641-2646 vol.3, 15-17 Dec 1993

[23] Miyasato, Y.; , "Model reference adaptive control for a class of nonlinear systems with unknown degrees and uncertain relative degrees," American Control Conference, 1999. Proceedings of the 1999 , vol.1, no., pp.571-575 vol.1, 1999

[24] Tao, G.; Ioannou, P.A.; , "Model reference adaptive control for plants with unknown relative degree," Automatic Control, IEEE Transactions on , vol.38, no.6, pp.976-982, Jun 1993

[25] Fu, Li-Chen; Chien, Chiang-Ju; , "A New Approach for solving Model Reference Adaptive Control Problem for Fast Time-Varying Unknown Plants with Relative Degree One," American Control Conference, 1990 , vol., no., pp.1058-1063, 23-25 May 1990

[26] McLain, R.B.; Henson, M.A.; Pottmann, M.; , "Direct adaptive control of partially known nonlinear systems," Neural Networks, IEEE Transactions on , vol.10, no.3, pp.714-721, May 1999

[27] Hsu, L.; Oliveira, T.R.; Peixoto, A.J.; , "Sliding Mode Control of Uncertain Nonlinear Systems with Arbitrary Relative Degree and Unknown Control Direction," Variable Structure Systems, 2006. VSS'06. International Workshop on , vol., no., pp.178-183, 5-7 June 2006

[28] Miller, D.E.; Davison, E.J.; , "An adaptive controller which provides an arbitrarily good transient and steady-state response," Automatic Control, IEEE Transactions on , vol.36, no.1, pp.68-81, Jan 1991

[29] Miller, D.E.; , "A new approach to model reference adaptive control," Automatic Control, IEEE Transactions on , vol.48, no.5, pp. 743- 757, May 2003

[30] Miller, D.E.; Mansouri, N.; , "Model Reference Adaptive Control Using Simultaneous Probing, Estimation, and Control," Automatic Control, IEEE Transactions on , vol.55, no.9, pp.2014-2029, Sept. 2010

[31] Chengyu Cao; Hovakimyan, N.; , "Novel Neural Network Adaptive Control Architecture With Guaranteed Transient Performance," Neural Networks, IEEE Transactions on , vol.18, no.4, pp.1160-1171, July 2007Narendra, K.S., and Annaswamy, A.M. *Stable Adaptive Systems.* Englewood Cliffs, NJ: Prentice-Hall Inc., 1989.

[32] Monopoli, R. V. " Model reference adaptive control with an augmented error signal." IEEE Transactions on Automatic Control, 19:474-484, 1974.

[33] Narendra, K.S., and Boškovic′, J.D. " A combined direct, indirect, and variable structure method for robust adaptive control." IEEE Transactions on Automatic Control, 37:262-268, Feb. 1992.

[34] Tso, S.K., and Shum, H.Y. " Robust Model reference adaptive control algorithm for a class of plants with bounded input disturbances" Electronics Letters, 26:1220-1221, July 1990.

[35] Narendra, K.S., and Balakrishnan, J. " Adaptive control using multiple models" IEEE Transactions on Automatic Control, 42:171-187, Feb. 1997.

[36] Kreisselmeier, G.., and Smith, M.C. " Stable adaptive regulation of arbitrary n^{th}-order plants." IEEE Transactions on Automatic Control, 31:299-305, 1986.

[37] Landau, I.D. " Adaptive control. A perspective." Adaptive Systems for Signal Processing, Communications, and Control Symposium 2000. AS-SPCC. The IEEE 2000.

[38] Narendra, K.S. " Parameter adaptive control - the end…or the beginning?" Decision and Control, 1994., Proceedings of the 33rd IEEE Conference on Volume 3, Dec. 1994.

[39] Ioannou, P.A., and Sun, J. *Robust Adaptive Control.* Englewood Cliffs, NJ: Prentice-Hall, Inc., 1995.

[40] Astrom, K.J., and Wittenmark, B. *Adaptive Control.* Second edition. AddisonWesley Inc., 1995.

[41] Slotine, J.E., and Li, W. *Applied Nonlinear Control.* Englewood Cliffs, NJ: Prentice-Hall Inc., 1991.

[42] Rohrs, C., Valavani, L., Athans, M.,and Stein, G. " Robustness of continuous-time adaptive control algorithms in the presence of unmodeled dynamics." IEEE Transactions on Automatic Control, 30:881-889, 1985.

[43] Narendra, K.S., and Annaswamy, A.M. " Robust adaptive control in the presence

of Bounded disturbances." IEEE Transactions on Automatic Control, 31:306-315, April 1986.

[44] Ioannou, P.A., and Tsakalis, K. " A robust direct adaptive controller." IEEE Transactions on Automatic Control, 31:1033-1043, Nov. 1986.

[45] Kreisselmeier, G.., and Narendra, K.S. " Stable model reference adaptive control in the presence of bounded disturbance." IEEE Transactions on Automatic Control, 27:1169-1175, Dec. 1982.

[46] Kreisselmeier, G.., and Anderson, B.D.O. " Robust model reference Adaptive control." IEEE Transactions on Automatic Control, 31:127-133, 1986.

[47] Landau, I.D. " Adaptive control. A perspective." Adaptive Systems for Signal Processing, Communications, and Control Symposium 2000. AS-SPCC. The IEEE 2000.

[48] Narendra, K.S. " Parameter adaptive control - the end…or the beginning?" Decision and Control, 1994., Proceedings of the 33rd IEEE Conference on Volume 3, Dec. 1994.

[49] Vidyasagar, M. *Nonlinear System Analysis*. Englewood Cliffs, N.J., Prentice-Hall Inc., 1978.

[50] Sastry, S., and Bodson, M., *Adaptive Control: Stability, Convergence, and Robustness.* Englewood Cliffs, N.J., Prentice-Hall Inc., 1989.

[51] Peterson, B.B., and Narendra, K.S. " Bounded error adaptive control." IEEE Transactions on Automatic Control, 27:1161-1168, Dec, 1982.

[52] Narendra, K.S., Lin, Y.H., and Valavani, L.S. " Stable adaptive controller design-Part II: proof of stability." IEEE Transactions on Automatic Control, 25:440-448, June, 1980.

[53] Narendra, K.S., Annaswamy, A.M., and Singh, R.P. " A general approach to the stability analysis of the adaptive systems." International Journal of Control, 41: 193-216, 1985.

[54] Narendra, K.S., and Valavani, L.S. " Stable adaptive controller design – direct control." IEEE Transactions on Automatic Control, 23:570-583, August, 1978.

[55] Narendra, K.S., and Valavani, L.S. " Stable adaptive observers and controllers." Proceedings of the IEEE, 64:1198-1208, August, 1976.

[56] Narendra, K.S., and Monopoli, R.V, *Applications of Adaptive Control*. New York: Academic Press, 1980.

[57] Narendra, K.S., Balakrishnan, J. and Ciliz, M. K. " Adaptation and Learning Using Multiple Models, Switching, and Tuning." IEEE Control Systems Magazine, 25: 440-448, 1980.

[58] Narendra, K.S., and Balakrishnan, J. " Improving transient-response of adaptive-control systems using multiple models and switching." IEEE Transactions on Automatic Control, 39:1861-1866, 1994.

[59] Morgan, A.P. and Narendra, K.S., " On the uniform asymptotic stability of certain linear nonautonomous differential equations." SIAM Journal on Control and Optimization, 15:5-24, Jan, 1977.

[60] Landau, I.D. *Adaptive Control*. New York, NY: Marcel Dekker, Inc., 1979.

[61] Hsu, L., Costa, R.R. and Lizarralde, F.C. " Output feedback adaptive variable structure control systems: the state of art." IEEE Decision and Control, 2:1408-1415, 1993.

[62] Kudva, P., and Narendra, K.S. " Synthesis of an adaptive observer using Lyapunov's direct method. " International Journal of Control, 18:1201-1210, 1973.

[63] Kailath, T. *Linear System*. Englewood Cliffs, N.J., Prentice-Hall Inc., 1980.

[64] Chen, C. T. *Introduction to Linear System Theory*. New York, NY: Holt, Rinehart and Winston, Inc., 1970.

[65] Goodwin, G. C. and Sun, K. S. *Adaptive Filtering, Prediction and Control.* Prentice Hall, New York, 1984.

[66] Naik, S.M., Kumar, P. R. and Ydstie, B.E. " Robust Continuous-Time Adaptive Control by Parameter Projection." IEEE Transactions on Automatic Control, 37:182-197, February, 1992.

[67] Giri, F., M'Saad, M., Dugard, L. and Dion, J.M. " Robust Adaptive Regulation with Minimal Prior Knowledge." IEEE Transactions on Automatic Control, 37:305-315, March, 1992.

[68] Duarte, M. A. and Narendra, K.S. " Combined Direct and Indirect Approach to Adaptive Control." IEEE Transactions on Automatic Control, 34:1071-1075, October, 1989.

[69] Masten, M. K. and Cohen, H. E. " An Advanced Showcase of Adaptive Controller Designs." Proceedings of the 1989 American Control Conference, Invited Session, pp. 424-429, June, 1989.

[70] Qu, Z. and Dawson, D. M. " Model Reference Robust Control of A Class of SISO Systems." Proceedings of the 1992 American Control Conference, Chicago, Illinois, pp. 1182-1186, June, 1992.

[71] Morse, A. S. " Global stability of parameter-adaptive control systems." IEEE Transactions on Automatic Control, 25:433-439, 1980.

[72] Morse, A. S. " Towards a unified theory of parameter adaptive control: tunability." IEEE Transactions on Automatic Control, 35:1002-1012, 1990.

BIOGRAPHICAL INFORMATION

Xiao Hu received her Bachelor's degree in Electrical Engineering from the University of Electronic Science and Technology of China in 1997, and Master's degree in Electrical Engineering from the University of Texas at Arlington in 2004. Prior to the master program, she worked for China Mobile in Chengdu during 1997-2002. She is working toward the Ph.D degree in Electrical Engineering at the University of Texas at Arlington, focusing on the Control System Design and Adaptive Systems.