

DESIGN OPTIMIZATION USING AUGMENTED LAGRANGIAN PARTICLE SWARM
OPTIMIZATION

by

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ABSTRACT

DESIGN OPTIMIZATION USING AUGMENTED LAGRANGIAN PARTICLE SWARM OPTIMIZATION

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Engineering optimization problems normally include multiple, non-trivial, non-linear constraints. Traditionally gradient based optimization methods were used to solve these types of problems because of their high computational efficiency. But they generally require continuity of the optimization problem and the computation of derivatives and tend to get trapped in local minima. Hence non gradient based, evolutionary algorithms (EA) which are based on heuristic and stochastic nature and do not require continuity of the design space of the optimization problem and increase the probability to find a global optimal solution, therefore are becoming a popular choice in recent years.

This work uses evolutionary algorithm based method called Augmented Lagrangian Particle Swarm Optimization (ALPSO). Note that Particle Swarm Optimization (PSO) technique was developed to solve unconstrained problem. An extended non-stationary penalty function approach, Augmented Lagrange Multiplier Method is used for constraint handling. The ALPSO algorithm is robust and reliable technique for solving Engineering optimization problem. The capability of ALPSO is demonstrated by solving extension-twist coupling problem in composite laminate design. Besides verification of previously known optimal design computed

by sequential quadratic programming method, ALPSO also generates new global optimal designs.

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CHAPTER 1

INTRODUCTION

1.1 Overview

Engineering optimization problems often require solving design problems involving number of disciplines. Nevertheless, including all the disciplines increases the number of design variables which in turn increases the complexity of the problem. This motivated the rise of multidisciplinary design optimization (MDO). MDO incorporates all relevant disciplines (all types of constraints) simultaneously and seeks superior global optimum as compared to other gradient based methods which and tend to get trapped in local minima.

Engineering optimization problems normally include multiple, non-trivial, non-linear constraints. These types of non linear problems can be formulated by

$$\underset{x}{\text{minimize}} f(x), \quad x \in ID \cap IF, \quad ID \subseteq IR^n, \quad (1.1)$$

Subject to linear and non linear constraints

$$g(x) = 0, \quad g : IR^n \rightarrow IR^{m_e}, \quad (1.2)$$

$$h(x) \leq 0, \quad h : IR^n \rightarrow IR^{m_i} \quad (1.3)$$

Where f is the nonlinear objective function, which is to be minimized with respect to design variables x . IF represents the feasible region, ID the search space which is additionally bounded by simple bounds

$$x_l \leq x \leq x_u. \quad (1.4)$$

The solution to above type of nonlinear problem can be addressed by gradient based optimization methods which are fast and have high computational efficiency. But they generally require continuity of the optimization problem and the computation of derivatives and tend to get

trapped in local minima. Hence non gradient based, evolutionary algorithms (EA) which are based on heuristic and stochastic nature are becoming a popular choice in recent years. This type of algorithm generally does not require continuity of the design space of the optimization problem and increase the probability to find a global optimal solution.

1.2 Scope of study

The focus of this thesis is the study of the use of simple structure of the basic Particle Swarm Optimization (PSO) technique which was introduced by Kennedy, Eberhart and Shi in 1995. And combines the PSO technique with an extended non-stationary penalty function approach, called Augmented Lagrange Multiplier Method (ALPSO), for constraint handling where ill conditioning is a far less harmful problem and the correct solution can be obtained even for finite penalty factors. The ALPSO algorithm is robust and reliable technique for solving Engineering optimization problem. After demonstrating the use of ALPSO to solve simple numerical constrained optimization examples, ALPSO is used to find the optimal ply angles of composite laminates with extension twist coupling.

Composite laminates can be tailored through arrangement of each ply's fiber orientation angle (referred to as ply angles) to have desired elastic properties. Through this arrangement desired coupling between in-plane and out-of-plane deformation modes can be achieved to meet performance requirements. Among the achievable coupling is Extension Twist coupling which has its applications in rotor blades to change the angle of attack with a change in rotor speed.

From previous work the optimal ply angles for Extension Twist coupling were obtained using Sequential Quadratic Programming (SQP). Since objective function and constraints are highly nonlinear and SQP is gradient based method, there exist multiple local global solutions that could be better design than those obtained from previous work. This work focuses on exploring other global optimum solution.

CHAPTER 2

LITERATURE REVIEW

The Particle Swarm Optimization was first studied and introduced by Kennedy and Eberhart in 1995 [1]. The basic Particle Swarm Optimization (PSO) is inspired by the collective intelligence of swarms of biological populations (swarm) like schools of fish or swarm of insects or bird flocks. These populations exhibit a collective co-ordinated behavior which adapts according to its environment. It combines self experiences with social experiences of the population that have previously been discovered. They implemented the PSO algorithm to find an effective optimization solution for non linear function.

A Modified Particle Swarm Optimizer was introduced by Eberhart and Shi [2]. Introduced a inertia weight parameter to the original particle swarm optimizer to enhance the performance of PSO and increase the chances to find the global optimum within the given set of iterations.

Kai Sedlaczek and Peter Eberhard [3] introduced Augmented Lagrangian Particle Swam Optimization to solve constrained optimization problems. Using Augmented Lagrangian method with PSO to convert constrained optimization problem into series of unconstrained optimization problems.

Eberhart and Shi [5] performed studies on the effect of the swarm size on the convergence behavior and quality of solution using PSO algorithm. They found the swarm size has very less influence on the PSO algorithm and determined the feasible range of swarm size to produce better optimal solution.

Bergh [7] derived a simple inequality condition for the inertia factor parameter by analyzing the dynamic behavior and stability of PSO algorithm that would result in guaranteed the convergence.

Winckler, S. J. [9] developed asymmetric stacking sequence of laminates that could ensure shape and stability with changes in temperature or moisture called Hygrothermal stability.

Weaver [10] developed a class of laminates that had minimum of seven plies that had asymmetric stacking sequence to ensure Hygrothermal stability by enforcing the coupling stiffness matrix to zero.

Chen [12] studied the necessary and sufficient conditions for a Hygrothermally stable laminate. He identified four conditions that correspond to making the two axial non-mechanical stress resultants equal and making the remaining non-mechanical stress and moment resultants zero to yield zero nonmechanical curvatures.

Haynes, Armanios [4] developed a new class of stacking sequence that ensures the Hygrothermal curvature stability of the laminates with Extension-twist Coupling.

Aditya, Wang [13] obtained optimal design of Hygrothermally Stable Laminates with Extension-Twist Coupling by Ant Colony Optimization.

CHAPTER 3

AUGMENTED LAGRANGIAN PARTICLE SWARM OPTIMIZATION

3.1 The Basic Particle Swarm Algorithm

The basic Particle Swarm Optimization (PSO) is inspired by the collective intelligence of swarms of biological populations (swarm) like schools of fish or swarm of insects or bird flocks. These populations exhibit a collective co-ordinated behavior which adapts according to its environment. It combines self experiences with social experiences of the population that have previously been discovered. Therefore, it is a population-based algorithm which is based on heuristic and stochastic nature.



Figure 3.1 Pictures of schools of fish and bird flocks

The population (a group of designs) is referred to as the swarm, which consists of individual particle (a design). Each particle has a position and a velocity associated within a particular design space. The position of each particle is determined by the value of respective design variable 'x'. The current position of the particle is updated by the velocity vector of that particle for every iteration. In the numerically implementation, the trajectory of the i – th particle at iteration $k + 1$ is updated as:

$$x_i^{k+1} = x_i^k + \Delta x_i^{k+1} \quad (3.1)$$

Where x_i^k is the current particle position, x_i^{k+1} is the updated particle position and Δx_i^{k+1} is the velocity update equation, which is given by

$$\Delta x_i^{k+1} = w \Delta x_i^k + c_1 r_{1,i}^k (x_i^{best,k} - x_i^k) + c_2 r_{2,i}^k (x_{swarm}^{best,k} - x_i^k) \quad (3.2)$$

Where $x_i^{best,k}$ is the best previously obtained position of the particle and $x_{swarm}^{best,k}$ is the best position in the entire swarm at the current iteration k . $r_{1,i}^k$ and $r_{2,i}^k$ are random numbers uniformly distributed between [0,1] and this represents the stochastic nature of the algorithm.

The term $c_1 r_{1,i}^k (x_i^{best,k} - x_i^k)$ represents best particle's self experience and the term $c_2 r_{2,i}^k (x_{swarm}^{best,k} - x_i^k)$ the social experience of the particles. Therefore the terms c_1 and c_2 represent cognitive experience factor and social experience factor. The term w represents inertia factor. Together all the terms simulate the behavior of entire swarm population. Fig 3.1 illustrates PSO particle position and velocity update.

It should be noted that equation (3.1) is similar to design variable updating in any gradient based method. However there is a big difference while in gradient based method, Δx_i

is always required to be in descending direction. This means at the new design, the objective function should be reduced. In PSO, Δx_i is given by equation (3.2), regardless whether it is in descending direction or not. This means the new solutions generated by PSO may not be always better than the old solution. This is the mechanism for PSO to avoid being trapped to a local minimum. The premium we pay for this desired outcome is more functional evaluations are required for PSO than gradient based method.

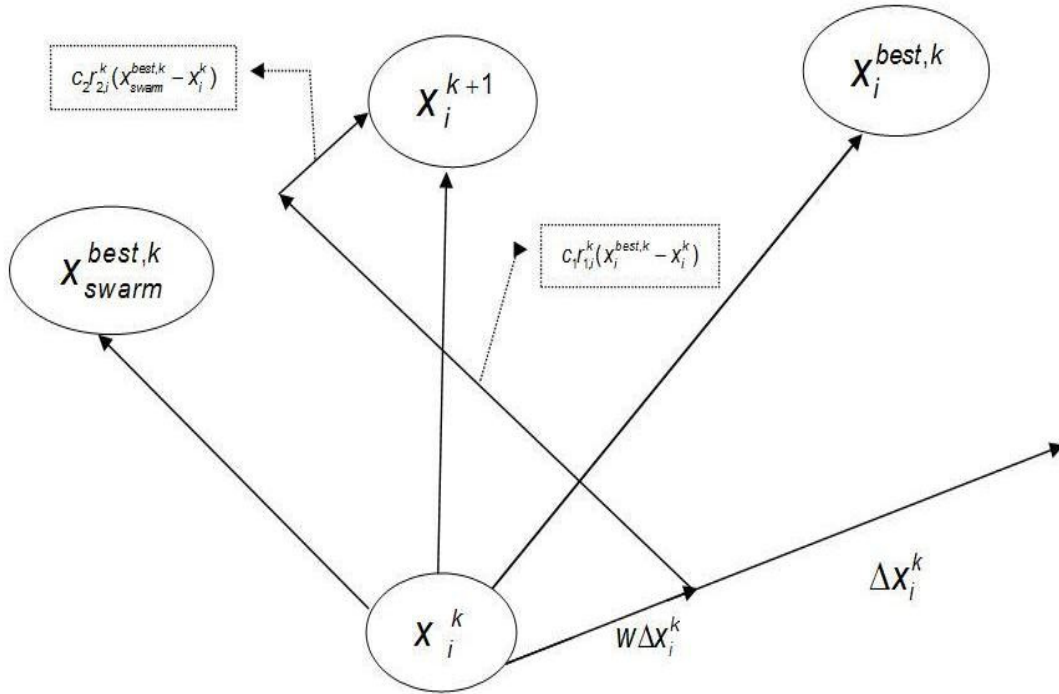


Figure 3.2 Schematic representation of PSO particle position and velocity update.

For ease of reference, all the terms used are defined as shown below

x_i^k value of current position of the particle

x_i^{k+1} updated value of the particle position

Δx_i^{k+1} is the velocity update equation given by equation (3.2)

W value of inertia factor parameter, during this work the value was taken a constant value of 0.9.

C_1, C_2 values of cognitive factor parameter, during this work the value were taken a constant value of 0.8.

$x_i^{best,k}$ value of best previously obtained position of the particle.

$x_{swarm}^{best,k}$ value of best position in the entire swarm at the current iteration.

Using equations (3.1) and (3.2) as explained above and applying it to a set of particles n , the PSO algorithm can outlined as followed:

[Step 1] Set iteration count $k = 0$. Initialize n particles with randomly chosen positions x_i^k and velocities Δx_i^k within the limits of design space.

[Step 2] Evaluate the corresponding objective values and determine $x_i^{best,0}$ and $x_{swarm}^{best,0}$.

[Step 3] Update velocity equation (3.2) and particle position equation (3.1). Evaluate the objective function values at each corresponding position. Update iteration count to $k + 1$.

Determine best particle of the iteration $x_i^{best,k}$ and best particle in the swarm $x_{swarm}^{best,k}$.

[Step 4] Compare the best particle in the swarm with respect to best particle generated through each iteration and update the best position for each particle.

[Step 5] If the termination criterion is satisfied, stop the algorithm with $x_{swarm}^{best,k}$ as solution. If the iteration count k crosses the specified limit k_{max} terminate the algorithm with failure.

[Step 6] Repeat the steps 3-5 until the specified convergence criterion is satisfied.

The accuracy of the solution of the algorithm mainly depends on cognitive, social and inertia parameters. The inertia weight parameter w is considered to important in determining the convergence behavior. Inertia weight parameter is used to control the impact on the current velocity from previous velocities and thus helping the tradeoff between global and the local exploration abilities of swarm. Assigning a large inertia weight supports exploration of global search behavior of the swarm. But using, a smaller value results in search of smaller areas of design space. Hence to obtain better results a proper value of inertia weight should be selected such that it provides balance between global and local areas search of the design space. Two approaches have been proposed for the proper selection of inertia factor. The first approach is called linear approach which was proposed by Eberhart and Shi [2], in this approach the inertia weight decreases linearly after each iteration depending on maximum number of iterations and maximum and minimum value of inertia parameter.

$$w_{k+1} = w_{max} - \frac{w_{max} - w_{min}}{k_{max}} k \quad (3.3)$$

The second approach is called dynamic approach, proposed by Groenwold and Venter Fourie [8]. In this approach the value inertia weight decreases by fraction f_w if there is no enhancement in the solution after certain number of iterations.

$$w_{k+1} = f_w w_k \quad (3.4)$$

Where the value of f_w varies between 0 and 1. According numerical experiments the dynamic approach has better convergence rate as compared to linear approach. But the optimal

value of convergence of inertia weight parameter is mainly dependent on the problem. In many cases premature convergence produces inferior local solution which is highly unwanted in solving multimodal optimization problem. Bergh [7] formulated a simple inequality condition which guaranteed convergence.

$$w > \frac{1}{2}(c_1 + c_2) - 1 \quad (3.5)$$

3.2 Methods for constraint handling

The PSO technique is very efficient in solving global unconstrained optimization problems. But to solve constrained problems using PSO different approaches have been proposed to cope with the constraints. One of the classic method used for solving constrained optimization problems is penalty method.

3.2.1 The Penalty Function Method

Penalty Method transforms the constrained problem into series of unconstrained problems. The objective function of the constrained problem is replaced by penalty function. The penalty function is formed by adding a penalty parameter and a measure of violation of the constraints to the original objective function. The design search space of constrained problems consists of two types of points. One is feasible points that satisfy all the constraints and the other is unfeasible points that violates atleast one of the constraints. The measure of violation of the constraints is non zero for unfeasible points and is zero for feasible points.

The penalty functions are mainly categorized into two types. First one is called stationary penalty function that uses fixed penalty values for the entire minimization process. The second type is called non-stationary penalty function, in which the penalty values are dynamically modified during the minimization process. The results obtained from the non-stationary penalty method are often to be exceptional as compared to the ones obtained from the stationary penalty method. For this type of penalty function very large penalty parameters are required for convergence. This could lead to numerical difficulty.

3.2.2 Augmented Lagrangian Multiplier Method

The Augmented Lagrangian Multiplier method circumvents the problem of large penalty parameters of penalty function. This method adds an additional term which is the product of Lagrange multiplier and constrained function, the objective function combined with the terms included Lagrange multiplier is called Lagrangian function. The Lagrange function is similar to penalty method which transforms the constrained optimization problem into unconstrained optimization problem. The Lagrangian function is given by

$$L(x, \lambda) = f(x) + \sum_{i=1}^{m_e} \lambda_i g_i(x) + \sum_{j=1}^{m_e} \lambda_{j+m_e} h_j(x) \quad (3.6)$$

Where λ is the Lagrange multiplier, $f(x)$ is the objective function, $g(x)$ and $h(x)$ are equality and inequality constraints violation respectively.

The solution x^* to constrained problem with the correct set of multipliers λ^* is a stationary point of L . But x^* is not necessarily a minimum of Lagrange function L . To convert the solution x^* from stationary point to minimum, the Lagrange function is augmented using quadratic extension.

$$L_A(x, \lambda, r_p) = f(x) + \sum_{i=1}^{m_e+m_i} \lambda_i \theta_i(x) + \sum_{i=1}^{m_e+m_i} r_{p,i} \theta_i^2(x) \quad (3.7)$$

with

$$\theta_i = \begin{cases} g_i(x), & i = 1(1)m_e, \\ \max \left\{ h_{i-m_e}(x), \frac{-\lambda_i}{2r_{p,i}} \right\}, & i = m_e + 1(1)m_e + m_i. \end{cases} \quad (3.8)$$

From the gradient based optimization problems, the term $\frac{-\lambda_i}{2r_{p,i}}$ in equation (3.8) is selected to have continuous derivatives $\frac{\partial L_A}{\partial \mathbf{x}}$ at $\hat{\mathbf{x}}$, where $h_{i-m_e}(\hat{\mathbf{x}}) = -\lambda_i / 2r_{p,i}$. The Lagrange function differs from penalty function due to addition of Lagrangian multipliers. In this method, each constraint infeasibility is penalized separately by the use of a vector of positive penalty factors \mathbf{r}_p . It can be shown that there exist finite constraint penalty factors \mathbf{r}_p in the solution \mathbf{x}^* of the Lagrange function and thus of the original constrained problem. But the proper values of Lagrange multipliers λ_i and penalty factor parameter \mathbf{r}_p are unknown and are always problem dependent and thus the solution of the Lagrange function cannot be computed by single unconstrained minimization of equation (3.7) but a sequence of unconstrained sub problems with subsequent updates of λ_i and r_p . The update scheme of Lagrange multipliers based on the solution \mathbf{x}^{*v} of the stationary condition of the v -th sub-problem. It holds for $\mathbf{x}^v \approx \mathbf{x}^{*v}$.

$$\left[\frac{\partial f(\mathbf{x})}{\partial \mathbf{x}} + \sum_{i=1}^{m_e+m_i} \lambda_i^v \frac{\partial \theta_i(\mathbf{x})}{\partial \mathbf{x}} + \sum_{i=1}^{m_e+m_i} 2r_{p,i}^v \theta_i(\mathbf{x}) \frac{\partial \theta_i(\mathbf{x})}{\partial \mathbf{x}} \right]_{\mathbf{x}=\mathbf{x}^v} = \varepsilon^v \approx 0. \quad (3.9)$$

The Lagrange multiplier can be formulate by comparing equations (3.9) and (3.6)

$$\lambda_i^{v+1} = \lambda_i^v + 2r_{p,i}^v \theta_i(\mathbf{x}) \quad (3.10)$$

3.3 Augmented Lagrangian Particle Swarm Optimization

By combining the Augmented Lagrangian Multiplier method as illustrated in section (3.2.2) with particle swarm optimization as explained in section (3.1). Since the appropriate Lagrange multipliers λ^* and penalty factor r_p are unknown and a sequence of unconstrained problems as shown in equation (3.7) must be solved sequentially. This method involves solving of unconstrained minimization with respect to x before enforcing any update of the Lagrange multipliers and penalty factors. The Augmented Lagrangian Particle Swarm Optimization algorithm is outlined as follows:

[Step 1] Set iteration count $k = 0$, $\nu = 0$. Initialize particles with randomly chosen positions and velocities within the limits of design space.

[Step 2] Initialize Lagrange multiplier parameter $\lambda^0 = 0$ and penalty factor parameter $r_p^0 = r_{p0}$. Evaluate the equation (3.7) using the initialized particle positions.

[Step 3] solve unconstrained problem using PSO algorithm steps from 2 to 6 as shown in the section (3.1) for a given set of iterations, k_{\max} . If the termination criterion is satisfied, stop the algorithm with $X^* = X^\nu = X_{\text{swarm},\nu}^{\text{best}}$, $\lambda^* = \lambda^\nu$. If $\nu > \nu_{\max}$, the algorithm stops with failure.

[Step 4] Update parameters r_p and λ according to equation (3.10).

[Step 5] Repeat the steps 3 to 5 until the termination criterion is satisfied stop the algorithm with as solution. If the iteration count crosses the specified limit terminate the algorithm with failure.

The penalty factor is updated using heuristic update scheme to get stationary and minimum point at X^* . The value of penalty factor $r_{p,i}$ is increased if the intermediate solution of the ν -th sub-problem is not closer to the i -th constraint than the previous solution $X^{\nu-1}$ of

$\nu - 1$ th problem. On the contrary if the $i - th$ constraint condition is satisfied with respect user defined tolerance limit. It is formulated according the following conditions given below

$$r_{p,i}^{\nu+1} = \begin{cases} 2r_{p,i}^{\nu} & \text{if } |g_i(x^{\nu})| > |g_i(x^{\nu-1})| \wedge |g_i(x^{\nu})| > \epsilon_g, \\ \frac{1}{2}r_{p,i}^{\nu} & \text{if } |g_i(x^{\nu})| > \epsilon_g, \\ r_{p,i}^{\nu} & \text{else,} \end{cases}$$

$$i = 1(1)m_e,$$

$$r_{p,j+m_e}^{\nu+1} = \begin{cases} 2r_{p,j+m_e}^{\nu} & \text{if } |h_j(x^{\nu})| > |h_j(x^{\nu-1})| \wedge |h_j(x^{\nu})| > \epsilon_h, \\ \frac{1}{2}r_{p,j+m_e}^{\nu} & \text{if } |h_j(x^{\nu})| > \epsilon_h, \\ r_{p,j+m_e}^{\nu} & \text{else,} \end{cases} \quad (3.11)$$

$$j = 1(1)m_i,$$

Where ϵ_h and ϵ_g user defined tolerances for constraints violation.

Generally the large values of do not change the stationary conditions of equation (3.7) but decreasing the value of the corresponding penalty factors is important in estimation of accurate and convergence of Lagrange multipliers within limited number of iterations. The lower bound on the penalty factors results in improved convergence of Lagrange multipliers. To ensure that their magnitude is effective in creating a measurable change, this lower bound is calculated using the following

$$r_{p,i} = \frac{1}{2} \sqrt{\frac{|\lambda_i|}{\epsilon_{g,h}}} \quad (3.12)$$

The other parameter that is problem dependent and is unknown is swarm size. If the swarm size is small then it increases the probability of premature convergence to a local minimum and decreases the search in the global design space. On contrary if the swarm size is

large, it increases the computational time and cost by increasing the number of function evaluations. Since the problem is solved as sequence of unconstrained problems it increases in computation time and cost. Eberhart and Shi performed studies on the effect of the swarm size on the convergence behavior and quality of solution using PSO algorithm [5]. It was determined from their study that the swarm size has very less influence on the PSO algorithm if the swarm size falls in reasonable range. The optimal swarms range if from 30 to 40 particle range.

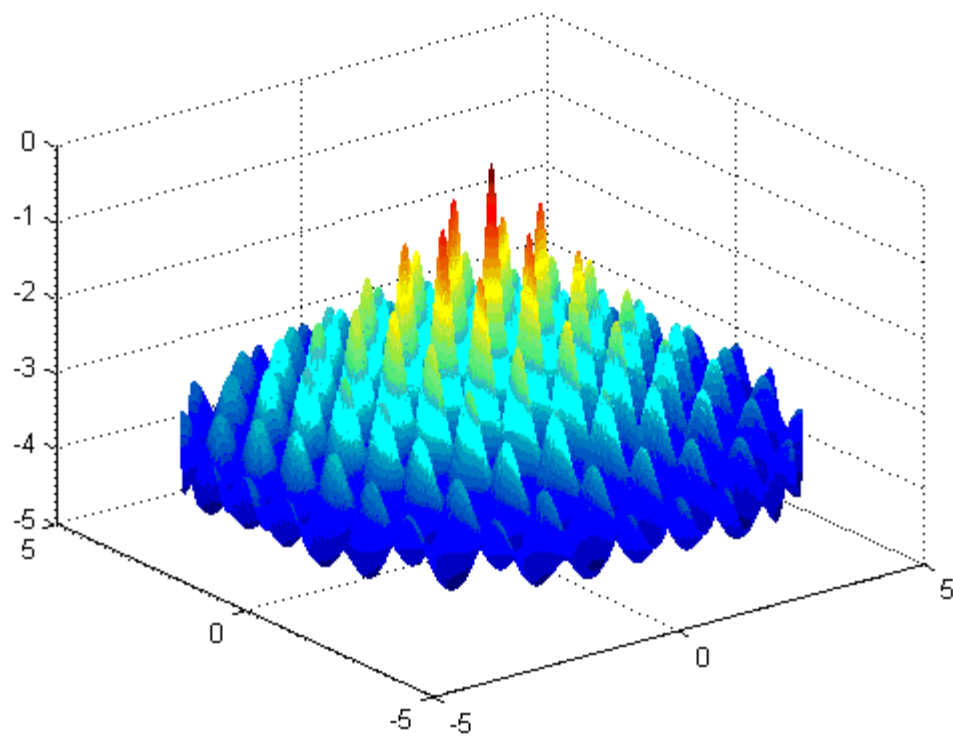


Figure 3.3 Plot representing Local and global extrema

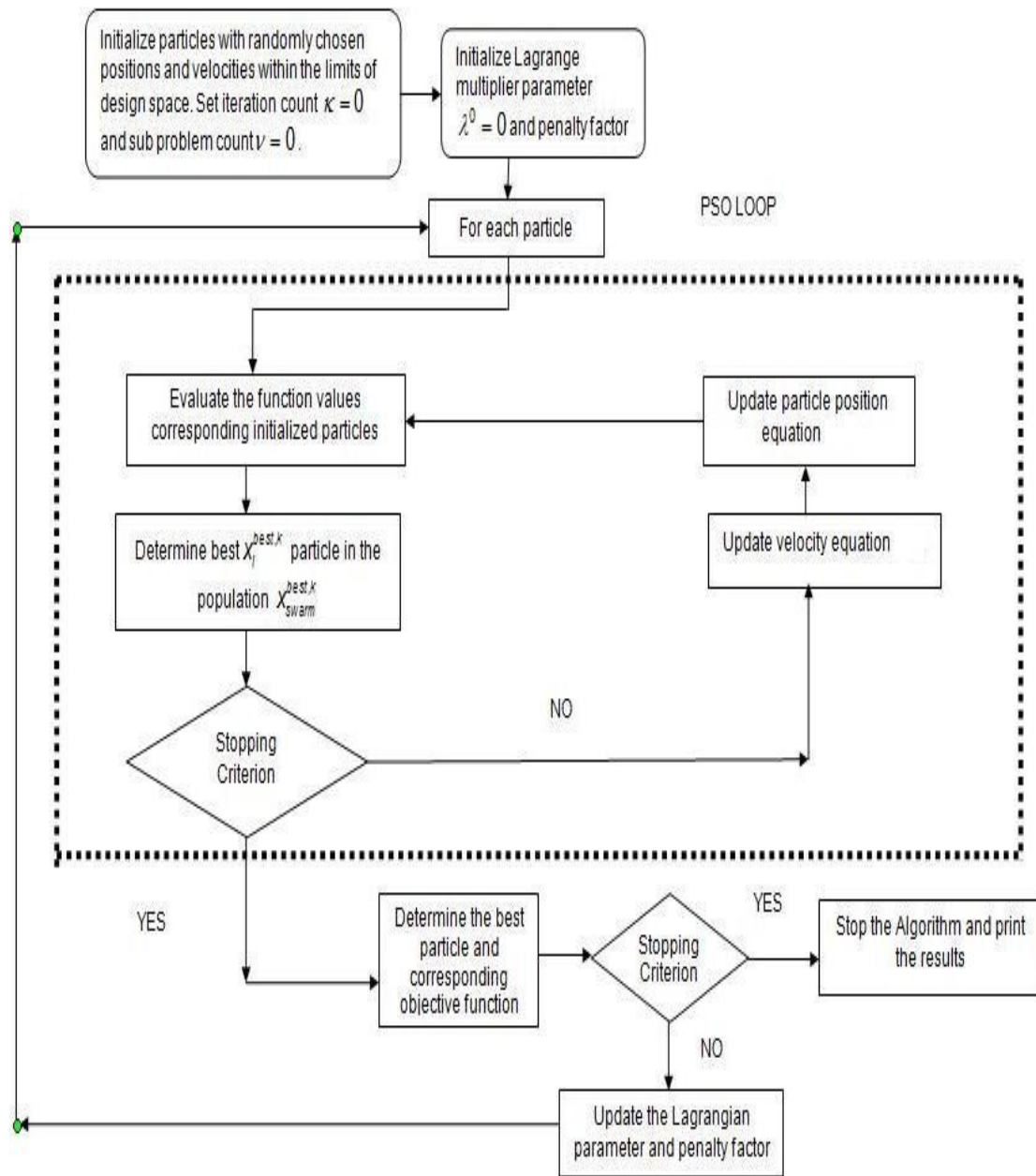


Figure 3.4 Plot of flow chart representing ALPSO algorithm

3.4 Numerical Examples

To illustrate the use of ALPSO to solve constrained optimization seven constrained benchmark problems [3] are solved in this section. The problems are defined in the appendix A and B. Table 3.1 summarizes the results of these problems in Appendix A. For comparison, problems 3 to problem8 are chosen from [11], where evolutionary strategy was used to solve the constrained problems. The number of particles, dimension search space and optimal function value are tabulated from column 2 through 4. For all the benchmark problems the inertia factor was a constant value of $= 0.9$ and the cognitive and social factor parameter were set a constant value of 0.8. The constraint tolerance was set to . All the problems were solved using ALPSO algorithm described in the section 3.3. Matlab was used to implement the algorithm.

Table 3.2 Results of benchmark problems from Appendix A

Problem	Number of Particles	Dimension of the search space	Optimal Value of the Objective function
1	30	2	13
2	30	2	0.00173
3	30	2	-0.096
4	30	2	-6962
5	30	2	0.75
6	30	3	-1
7	100	5	-30530

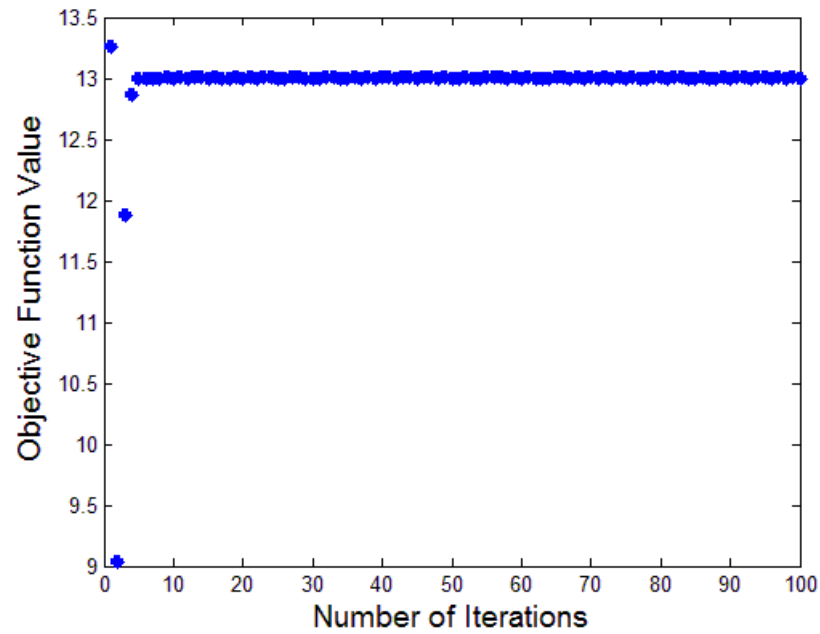


Figure 3.5 Plot of Objective function vs. iterations of Problem 1 Appendix A

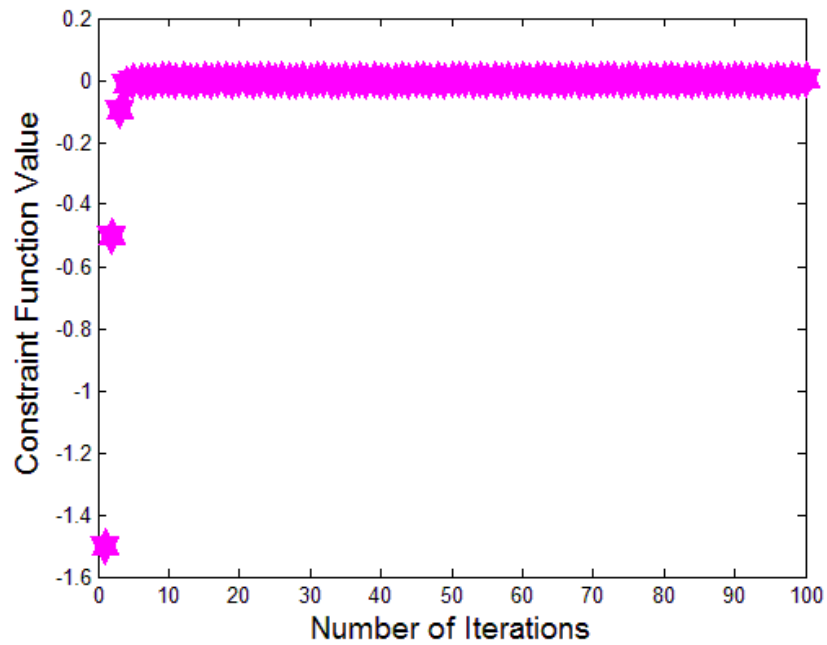


Figure 3.6 Plot of Constraint function vs. number of iterations of Problem 1 Appendix A

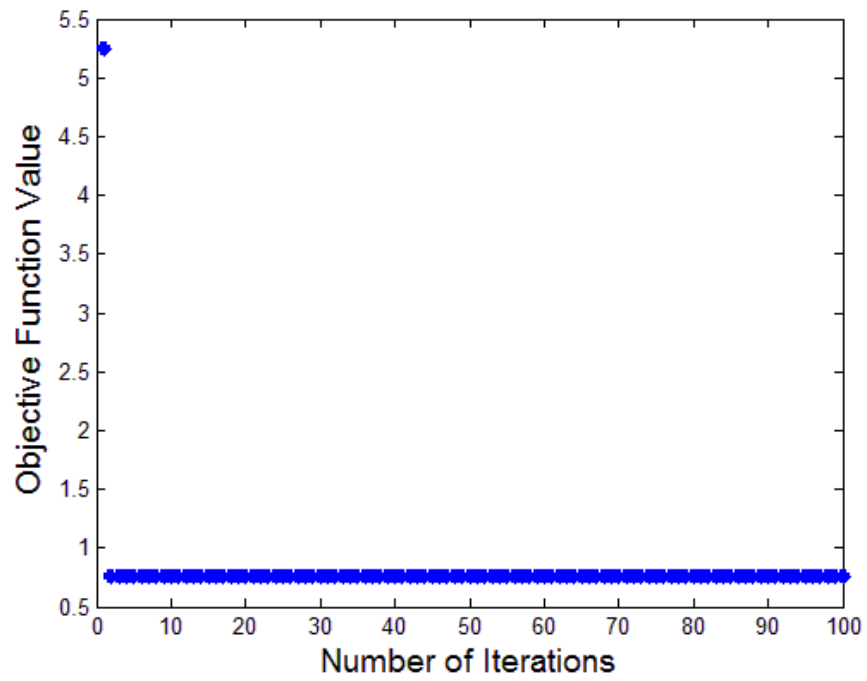


Figure 3.7 Plot of Objective function vs. number of iterations of Problem 5 Appendix A

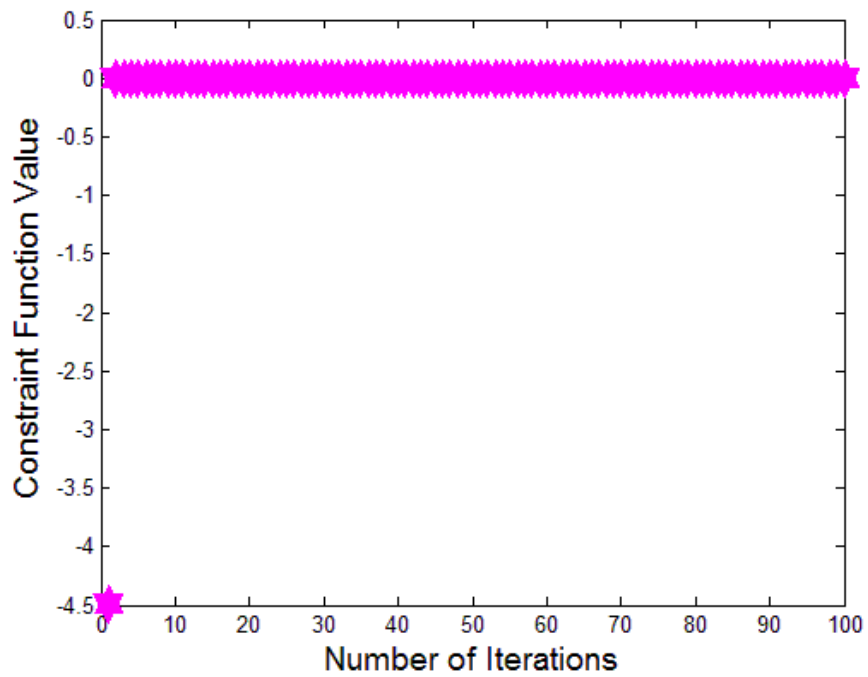


Figure 3.8 Plot of Objective function vs. number of iterations of Problem 5 Appendix A

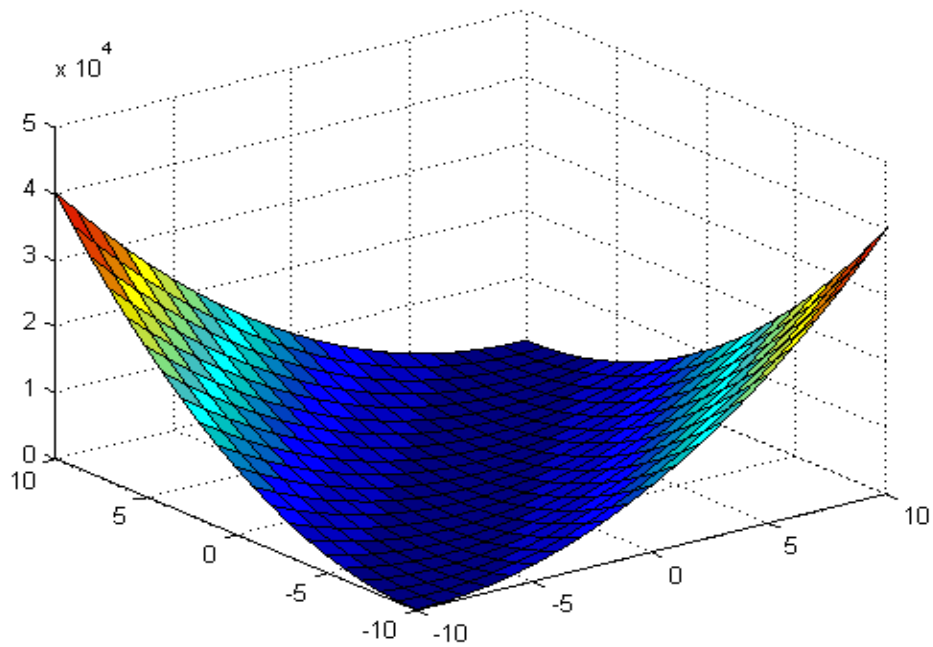


Figure 3.9 Plot of Rosenbrock function R over a design space, Appendix A

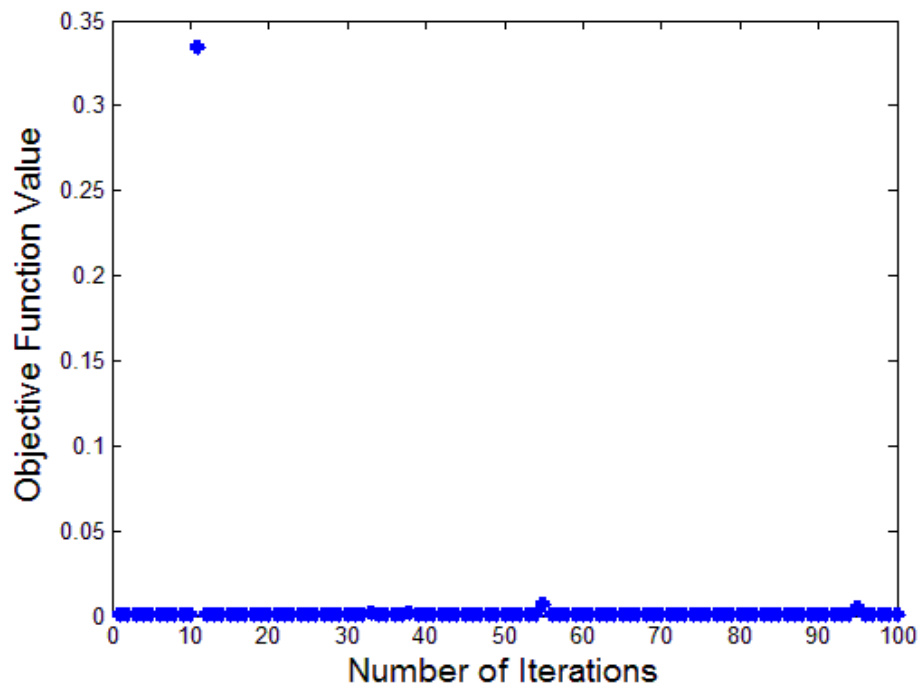


Figure 3.10 Plot of optimal value of unconstrained Rosenbrock function R , Appendix A

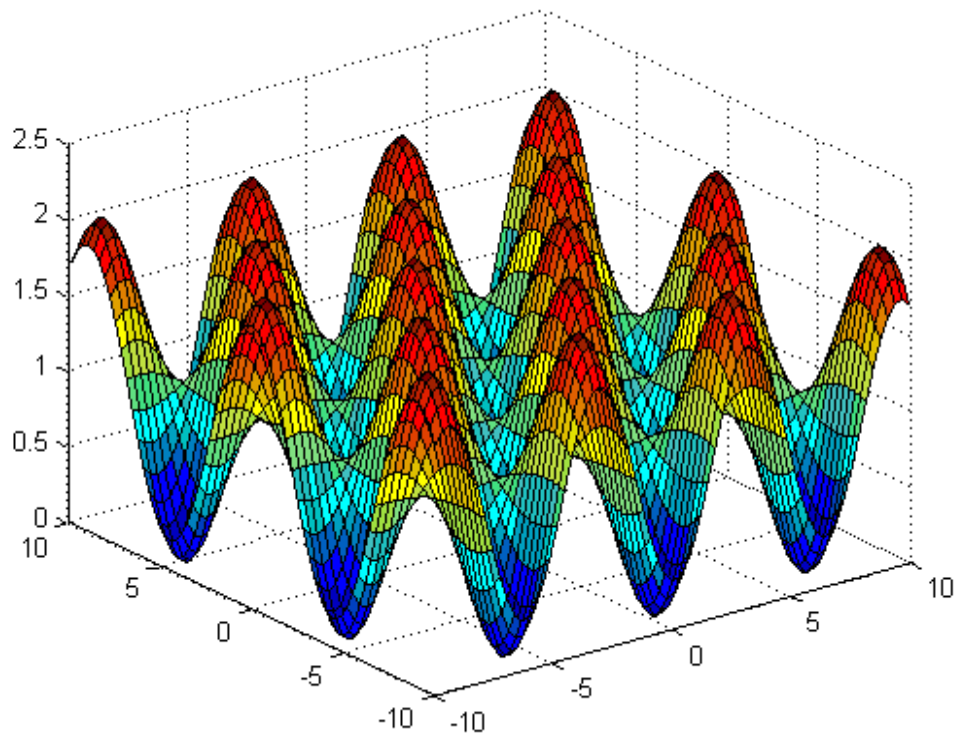


Figure 3.11 Plot of Griewank function G over a design space, Appendix A

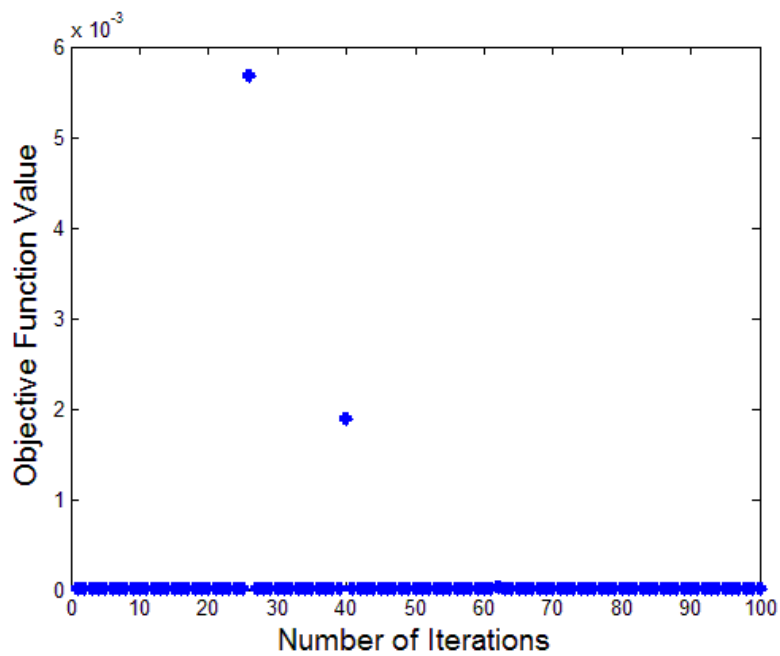


Figure 3.12 Plot of optimal value of unconstrained Griewank function G , Appendix A

Using ALPSO benchmark problem from [17] and [18] were solved and results are tabulated in table 3.2. The number of particles, dimension search space and optimal function value are tabulated from column 2 through 4. For all the benchmark problems the inertia factor was a constant value of $W = 0.9$ and the cognitive and social factor parameter c_1, c_2 were set a constant value of 0.8. The constraint tolerance $\epsilon_{g,h}$ was set to 10^{-4} . All the problems were solved using ALPSO algorithm described in the section 3.3. Matlab was used to implement the algorithm. Similar results as ALPSO are reported in the references [17] and [18].

Table 3.3 Results of benchmark problems from Appendix B

Problem	Number of Particles	Dimension of the search space	Optimal Value of the Objective function
1	100	7	680.7
2	30	3	0.0127
3	30	3	2.236

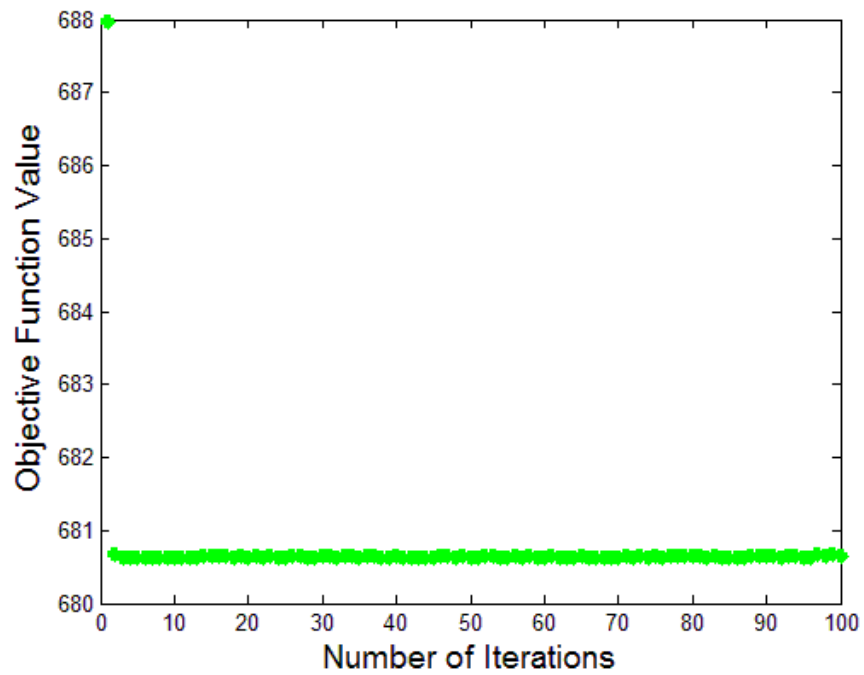


Figure 3.13 Plot of Objective function vs. number of iterations of Problem 1 Appendix B

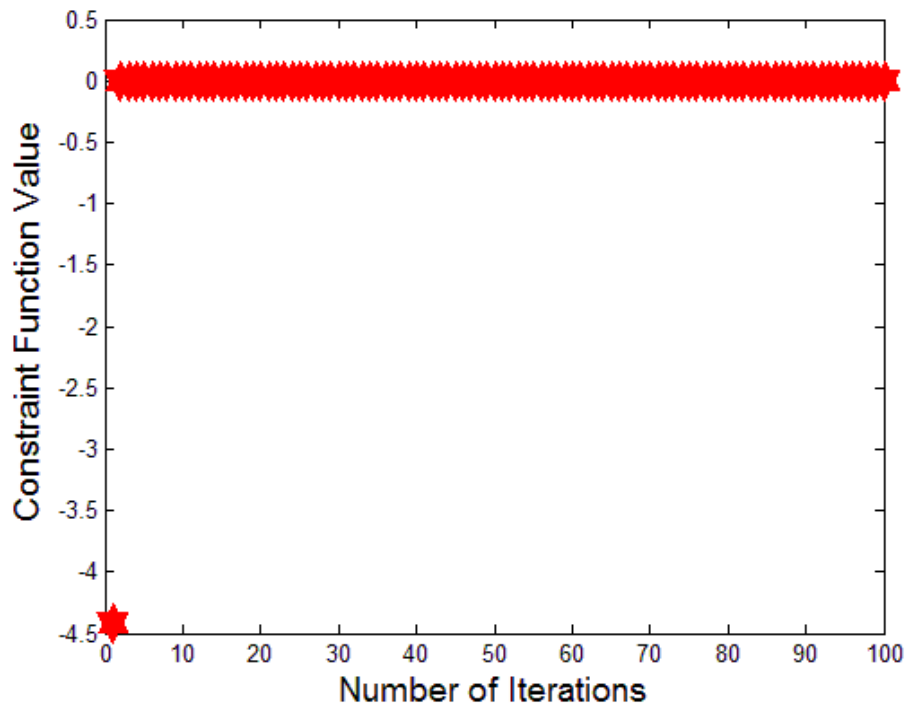


Figure 3.14 Plot of Objective function vs. number of iterations of Problem 1 Appendix B

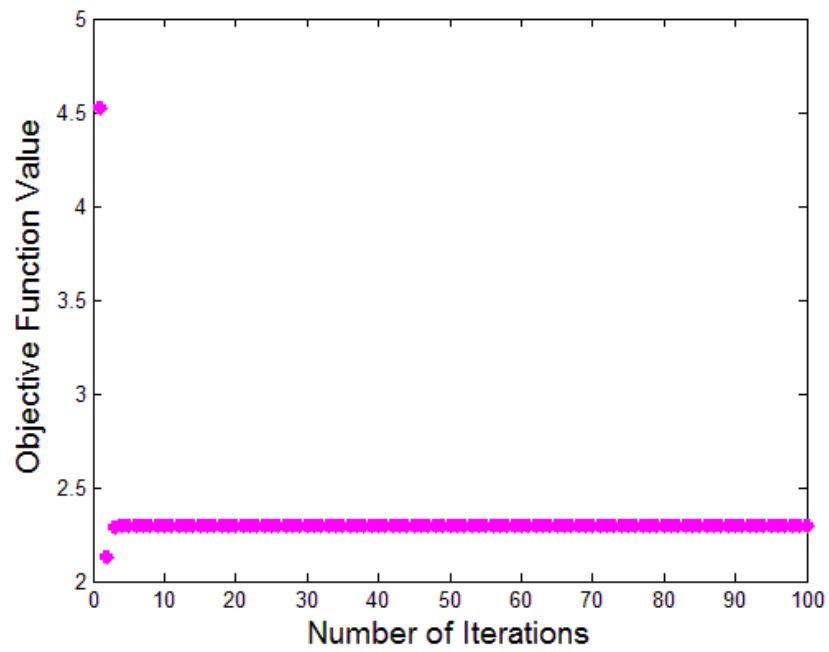


Figure 3.15 Plot of Objective function vs. number of iterations of Problem 3 Appendix B

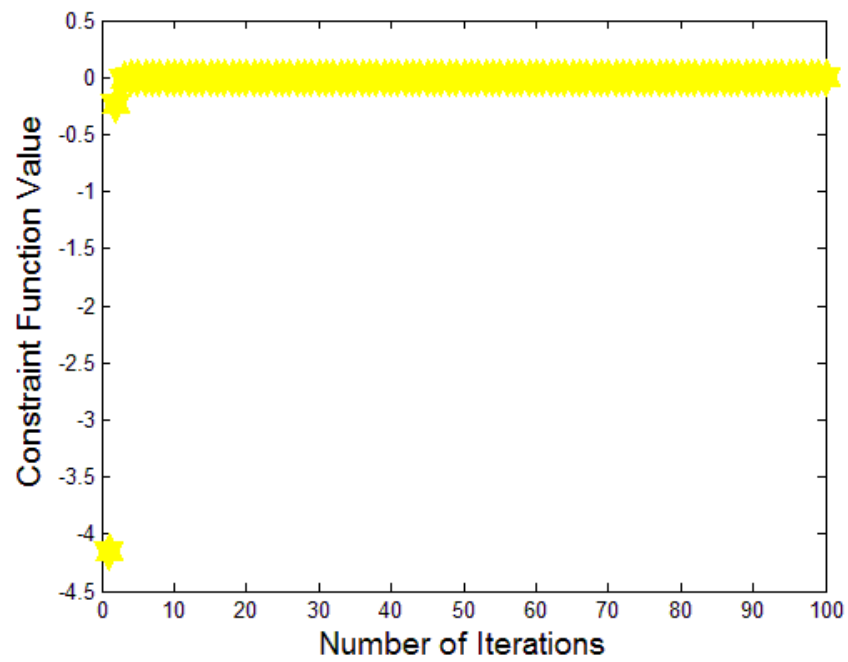


Figure 3.16 Plot of Objective function vs. number of iterations of Problem 3 Appendix B

CHAPTER 4

APPLICATION OF ALPSO FOR SOLVING OPTIMAL DESIGN PROBLEM

Composite laminates can be tailored through arrangement of each ply's fiber orientation angle (referred to as ply angles) to have desired elastic properties. Through this arrangement desired coupling between in-plane and out-of-plane deformation modes can be achieved to meet performance requirements. Among the achievable coupling is Extension Twist coupling which has its applications in rotor blades. For example, extension twist coupling has its applications in wind turbine blades by adjusting the twist distribution to achieve angle of attack that maintains desired optimal speed. This can also be used in tilt rotor blades to adjust twist distribution which can be used to increase efficiency of vertical and forward flight.

The stacking sequence determines the stability of the laminates. Symmetric stacking sequence ensures the Hygrothermal curvature stability of the laminates. Hygrothermal stability ensures structural stability of the laminates against any out-plane deformations because of environmental conditions. Bending twist coupling is achievable in both symmetric and asymmetric layups and has its application in swept forward wing aircraft which increases the divergence speed.

4.1 Hygrothermal Stability conditions

According to Classical Laminate Theory (CLT) [6] the constitutive equation describes the stiffness matrix of a laminate plate. The resultant forces and moments are functions of the in-plane strains and curvatures and are shown below as follows

$$\begin{Bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \\ M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} & B_{11} & B_{12} & B_{16} \\ A_{21} & A_{22} & A_{23} & B_{12} & B_{22} & B_{26} \\ A_{31} & A_{32} & A_{33} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \\ \kappa_{xx} \\ \kappa_{yy} \\ \kappa_{xy} \end{Bmatrix} + \begin{Bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \\ M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix}^{(T,H)} \quad (4.1)$$

Where $N_{xx}, N_{yy}, N_{xy}, M_{xx}, M_{yy}$ and M_{xy} are the stress resultants and

$\varepsilon_{xx}, \varepsilon_{yy}, \gamma_{xy}, \kappa_{xx}, \kappa_{yy}$ and κ_{xy} are the strains and curvatures. The stiffness coefficient are given as

$$\begin{aligned} A_{ij} &= \sum_{k=1}^n \bar{Q}_{ij(k)} (h_k - h_{k-1}) \\ B_{ij} &= \frac{1}{2} \sum_{k=1}^n \bar{Q}_{ij(k)} (h_k^2 - h_{k-1}^2) \\ D_{ij} &= \frac{1}{3} \sum_{k=1}^n \bar{Q}_{ij(k)} (h_k^3 - h_{k-1}^3) \end{aligned} \quad (4.2)$$

Where $\bar{Q}_{ij(k)}$ and h_k represent the transformed reduced stiffness and height relative to the laminate midplane for the k^{th} ply respectively. The non mechanical stress resultants are shown as follows

$$\begin{Bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \end{Bmatrix}^{(T,H)} = (\Delta T, \Delta H) \sum_{k=1}^n [\overline{Q}]_k \{\overline{\alpha}, \overline{\beta}\}_k (h_k - h_{k-1}) \quad (4.3)$$

$$\begin{Bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix}^{(T,H)} = \frac{1}{2} (\Delta T, \Delta H) \sum_{k=1}^n [\overline{Q}]_k \{\overline{\alpha}, \overline{\beta}\}_k (h_k^2 - h_{k-1}^2)$$

Where $\{\overline{\alpha}, \overline{\beta}\}_k$ are the transformed in-plane thermal and moisture expansion coefficient for the k^{th} ply. H represents hygral quantities and T represents the thermal quantities, the number of plies in the laminate is n . The stiffness and thermal coefficient transformations for specially orthotropic lamina are given by

$$[\overline{Q}] = [T_\sigma]^{-1} [Q] [T_\epsilon] = [T_\sigma]^{-1} \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} [T_\epsilon] \quad (4.4)$$

$$\{\overline{\alpha}, \overline{\beta}\}_k = [T_\epsilon]^{-1} \{\alpha, \beta\} = [T_\epsilon]^{-1} \begin{Bmatrix} \alpha_{11} \\ \alpha_{22} \\ 0 \end{Bmatrix}$$

Where the engineering strain and stress transformation matrices are given by

$$[T_\epsilon] = \begin{bmatrix} c^2 & s^2 & cs \\ s^2 & c^2 & -cs \\ -2cs & 2cs & c^2 - s^2 \end{bmatrix}$$

$$[T_{\sigma}] = \begin{bmatrix} c^2 & s^2 & 2cs \\ s^2 & c^2 & -2cs \\ -cs & cs & c^2 - s^2 \end{bmatrix} \quad (4.5)$$

Where $s = \sin \theta$, $c = \cos \theta$ and θ is the fiber orientation angle.

Substituting equations (4.4) and (4.5) into equation (4.3) and simplifying the equations we get the following expression for the non mechanical in plane forces and shears and out-of-plane moments and curvatures.

$$\begin{Bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix}^{(T,H)} = T_1 \sum_{k=1}^n \begin{Bmatrix} \cos 2\theta_k \\ -\cos 2\theta_k \\ \sin 2\theta_k \end{Bmatrix} (2k - n - 1) \quad (4.6)$$

$$\begin{Bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \end{Bmatrix}^{(T,H)} = T_2 \begin{Bmatrix} 1 \\ 1 \\ 0 \end{Bmatrix} + T_3 \sum_{k=1}^n \begin{Bmatrix} \cos 2\theta_k \\ -\cos 2\theta_k \\ \sin 2\theta_k \end{Bmatrix} \quad (4.7)$$

Where

$$T_1 = \frac{t^2(\Delta T, \Delta H)}{4} [(\alpha_{11}, \beta_{11})(Q_{11} - Q_{12}) + (\alpha_{22}, \beta_{22})(Q_{12} - Q_{22})] \quad (4.8)$$

$$T_2 = \frac{nt(\Delta T, \Delta H)}{2} [(\alpha_{11}, \beta_{11})(Q_{11} + Q_{12}) + (\alpha_{22}, \beta_{22})(Q_{12} + Q_{22})] \quad (4.9)$$

$$T_3 = \frac{t(\Delta T, \Delta H)}{2} [(\alpha_{11}, \beta_{11})(Q_{11} - Q_{12}) + (\alpha_{22}, \beta_{22})(Q_{12} - Q_{22})] = \frac{2T_1}{t} \quad (4.10)$$

4.2 Derivation of Hygrothermal conditions

Hygrothermal stability can be achieved by having the out-of-plane curvatures

K_{xx} , K_{yy} , and K_{xy} equal to zero for any change in temperature and moisture and neglecting

mechanical loads. This is represented as shown below

$$\begin{Bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \\ M_{xx} \\ -M_{xx} \\ M_{xy} \end{Bmatrix}^{(T,H)} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \\ B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{66} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} \quad (4.11)$$

Using (4), the necessary and sufficient conditions to ensure hygrothermal stability conditions are as shown below.

Condition A

Equal non-mechanical stress resultants and zero non-mechanical shear and moment resultants,

$$\begin{aligned} N_{xx}^{(T,H)} &= N_{yy}^{(T,H)} \\ N_{xy}^{(T,H)} &= M_{xx}^{(T,H)} = M_{yy}^{(T,H)} = M_{xy}^{(T,H)} = 0 \end{aligned} \quad (4.12)$$

The conditions of equation (4.12) are satisfied when

$$\begin{aligned} \sum_{k=1}^n (2k-n-1) \cos 2\theta_k &= 0 \\ \sum_{k=1}^n (2k-n-1) \sin 2\theta_k &= 0 \\ \sum_{k=1}^n \sin 2\theta_k &= 0 \\ \sum_{k=1}^n \cos 2\theta_k &= 0 \end{aligned} \quad (4.13)$$

Condition B

The coupling stiffness matrix is zero, i.e.

$$B_{ij} = 0 \quad (4.14)$$

The conditions of equation (4.12) are satisfied when

$$\begin{aligned} \sum_{k=1}^n (2k - n - 1) \cos 2\theta_k &= 0 \\ \sum_{k=1}^n (2k - n - 1) \sin 2\theta_k &= 0 \\ \sum_{k=1}^n (2k - n - 1) \cos 4\theta_k &= 0 \\ \sum_{k=1}^n (2k - n - 1) \sin 4\theta_k &= 0 \end{aligned} \quad (4.15)$$

The Hygrothermal stability conditions obtained are material independent. This gives various advantages like providing material independence in the preliminary design and material independence provides robustness against the variability in elastic constants and coefficient of thermal and moisture expansion.

4.3 Extension Twist Coupling of Hygrothermally stable laminates

From [4], Extension Twist Coupling is a type of deformation behavior of laminates under loading. It is produced by an axial in plane load induces a change in twist curvature of the laminate structure.

Actually the extension twist coupling occurs due to axial stress that causes shearing strains in off axis plies that are positioned at some non zero distance from the mid plane. The shearing forces are induced in these plies because they are constrained by bonding of plies above and below them. Due to the shear force acting through the mid plane, induces a moment about laminates axis therefore causing twist.

In this section, a constrained optimization is carried out to determine the stacking sequence of Hygrothermally stable laminate with extension twist coupling with a set number of plies. The results obtained are compared with the previous determined optimal solutions.

The optimization process objective function is formulated by expressing the twist rate as a function of applied nominal stress. The objective function is evaluated for given set of iterations and each time the twist rate is calculated using CLT.

Solving equation (4.1), the curvatures and mid plane strains can be calculated as

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \\ \kappa_{xx} \\ \kappa_{yy} \\ \kappa_{xy} \end{Bmatrix} = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} & \beta_{11} & \beta_{12} & \beta_{16} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} & \beta_{12} & \beta_{22} & \beta_{26} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} & \beta_{16} & \beta_{26} & \beta_{66} \\ \beta_{11} & \beta_{12} & \beta_{16} & \delta_{11} & \delta_{12} & \delta_{16} \\ \beta_{12} & \beta_{22} & \beta_{26} & \delta_{12} & \delta_{22} & \delta_{26} \\ \beta_{16} & \beta_{26} & \beta_{66} & \delta_{16} & \delta_{26} & \delta_{66} \end{bmatrix} \begin{Bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \\ M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix} + \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \\ \kappa_{xx} \\ \kappa_{yy} \\ \kappa_{xy} \end{Bmatrix}^{(T,H)} \quad (4.16)$$

Where the non mechanical deformations are calculated as

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \\ \kappa_{xx} \\ \kappa_{yy} \\ \kappa_{xy} \end{Bmatrix} = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} & \beta_{11} & \beta_{12} & \beta_{16} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} & \beta_{12} & \beta_{22} & \beta_{26} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} & \beta_{16} & \beta_{26} & \beta_{66} \\ \beta_{11} & \beta_{12} & \beta_{16} & \delta_{11} & \delta_{12} & \delta_{16} \\ \beta_{12} & \beta_{22} & \beta_{26} & \delta_{12} & \delta_{22} & \delta_{26} \\ \beta_{16} & \beta_{26} & \beta_{66} & \delta_{16} & \delta_{26} & \delta_{66} \end{bmatrix} \begin{Bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \\ M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix}^{(T,H)} \quad (4.17)$$

The twist rate in laminated composite strip, φ , due to a nominal axial stress, σ_0 , alone can be calculated as

$$\varphi = \frac{1}{2}(nt\beta_{16}\sigma_0 + \kappa_{xy}^{(T,H)}) \quad (4.18)$$

Where t and n denote the thickness and number of plies respectively

Since the optimizer search stable laminates, the non mechanical curvature in equation (4.18) is zero, recommending the objective function to be minimized as

$$g(\{\theta_k : k = 1 \dots n\}) = -\beta_{16}^2 \quad (4.19)$$

4.4 Implementation and Results for Extension Twist Coupling

Augmented Lagrangian Particle Swarm Optimization (ALPSO) was used to obtain the optimal stacking sequence. The equations (4.12) and (4.13) are used fulfill the Hygrothermal stability, since equations (4.14) and (4.15) exclude any extension twist coupling. MATLAB 7.6.0 was used for implementing the ALPSO algorithm.

Although the Hygrothermal stability conditions are material independent but the extension twist coupling is material dependent. The T300/976 graphite/epoxy material system [4] was used. The optimization was performed for laminates with five through ten plies. The optimized stacking sequence for laminates with extension twist is tabulated in Table (4.1)

Table 4.1 Extension Twist coupling of constrained laminates

Number of Plies n	Stacking sequence Obtained through ACO	Stacking sequence Obtained through ALPSO	Stacking sequence Obtained through SQP
5-Ply	[-1.1564 /0.0671 /0.6255/ 1.2275 /- 0.6878]	[0.2339 /-1.0905 /1.699 /-2.0548 /- 0.2056]	[-58.7 /11.4 /45 /78.6 /-31.3]
6-Ply	[0.4013 /-1.0639 /- 0.8576 /0.821 /1.2397 /-0.2942]	[1.2863 /-0.3812 /- 0.6187 /0.8086 /0.5628 /-1.1023]	[21.2 /-63.8/- 48.7 /48.7 /63.8 /-21.2]
8-Ply	[-0.3803 /1.3048 /1.2306 /1.1094 /- 1.2001 /0.6697 /- 1.4152 /0.3965]	[1.3842 /-0.4341 /- 0.5928 /4.3011 /0.9485 /-1.2638 /1.7878 /0.4199]	[-21.5 /72.1 /57.9/-29.6 /29.6 /-57.9/- 72.1 /21.5]
10-Ply	[-0.3520 /1.2962 /1.2316 /-0.6559 /1.0142 /-1.0189 /- 1.1471 /0.4908 /0.3956 /-1.3298]	[1.2393 /1.2064 /- 3.5557 /-0.5605 /- 0.771 /0.9176 /- 2.3502 /8.1411 /- 1.3605 /0.5018]	[16.2 /-69.0 /- 65.3 /31.8 /42.1 /-42.1/-31.8 /65.3 /69.0 /- 16.2]

The new designs obtained through ALPSO algorithm are compared with previous designs and robustness of all the design solutions was computed by perturbing the ply angles from -2 to 2 degrees. The histogram of the perturbed designs of objective function and sum of squared constraints functions are as shown below. The left side of the page contains histograms from the previous designs and the right side of the page contains histograms from

the new designs generated from ALPSO method. It can observed that both solutions are robust.

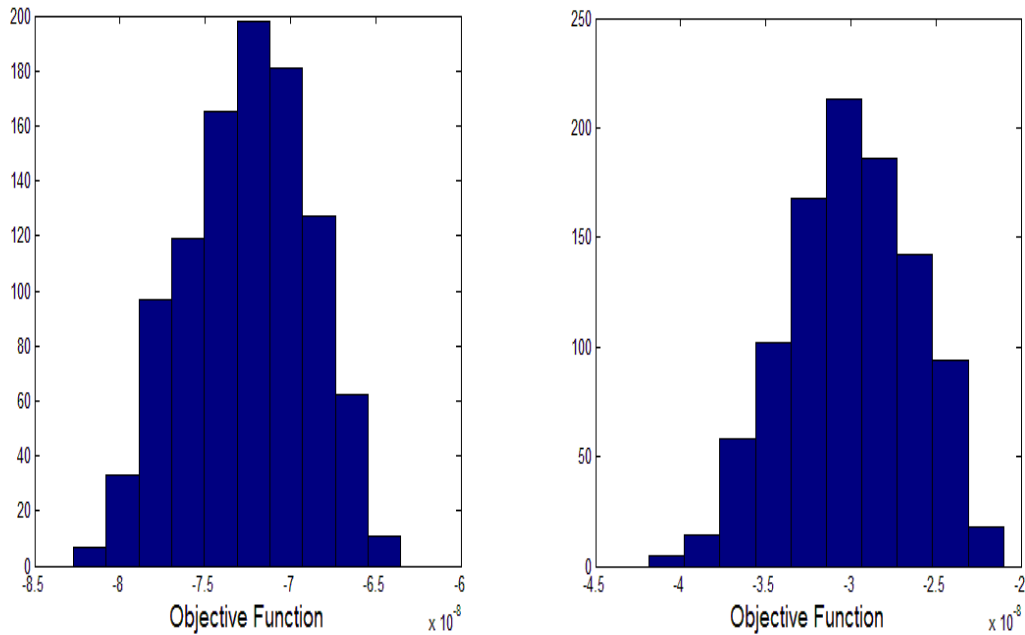


Figure 4.1 Robustness of Objective function of 5-Ply E-T coupling Laminate

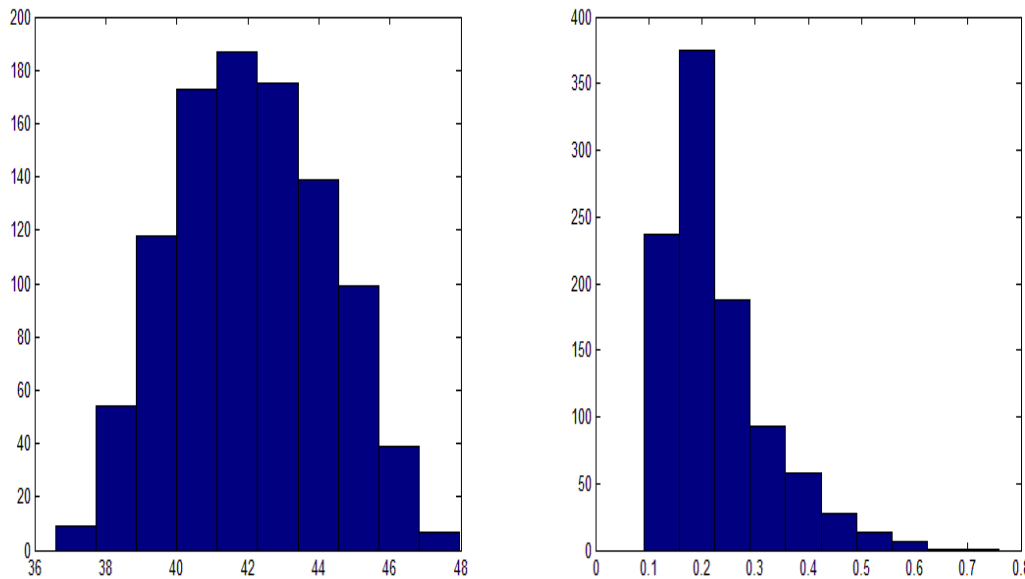


Figure 4.2 Robustness of sum of squared constraints functions of 5-Ply E-T coupling Laminate

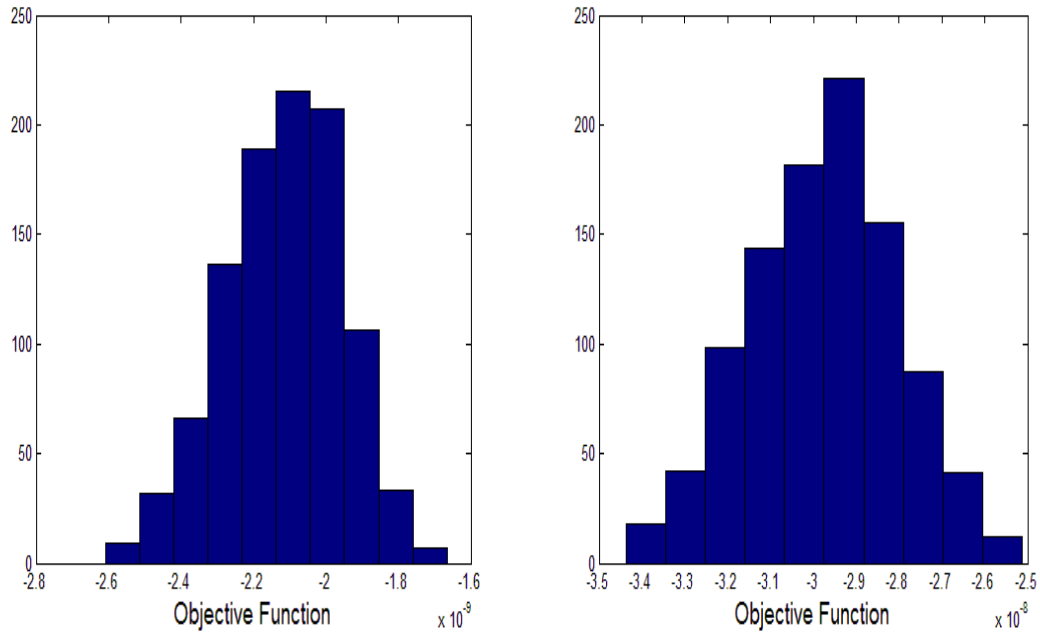


Figure 4.3 Robustness of Objective function of 6-Ply E-T coupling Laminate

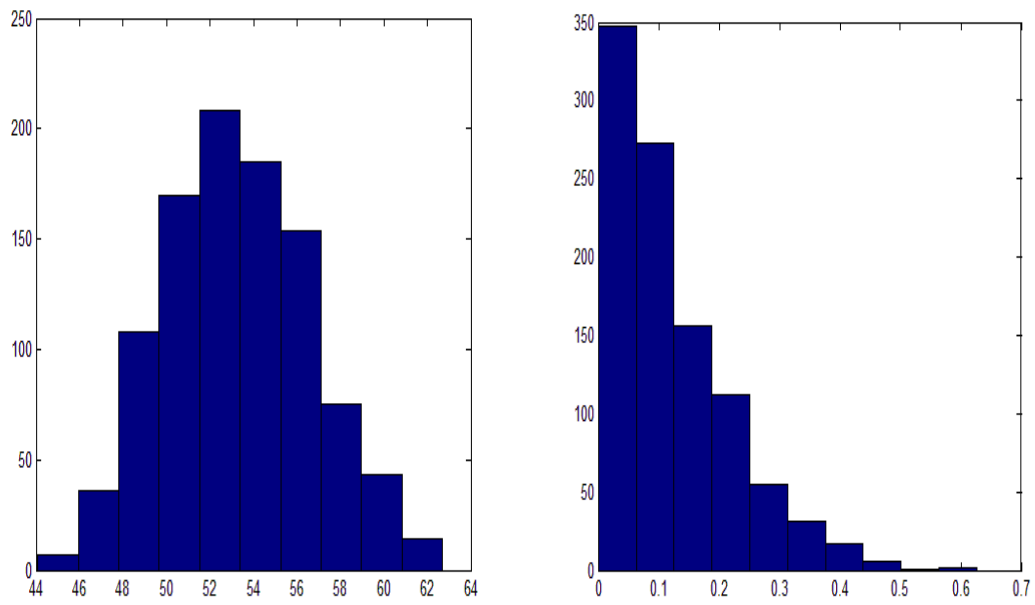


Figure 4.4 Robustness of sum of squared constraints functions of 6-Ply E-T coupling Laminate

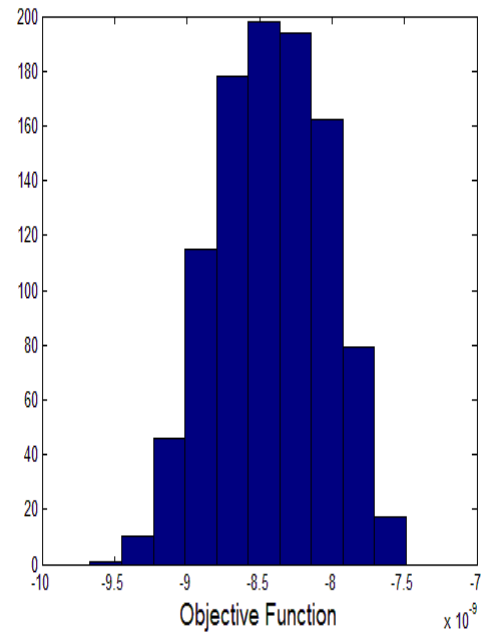
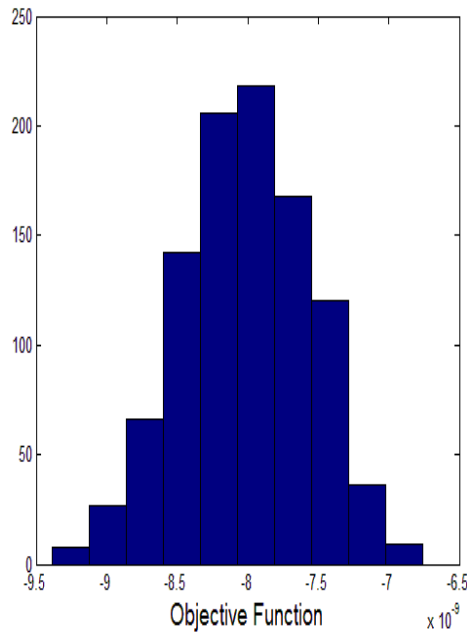


Figure 4.5 Robustness of Objective function of 8-Ply E-T coupling Laminate

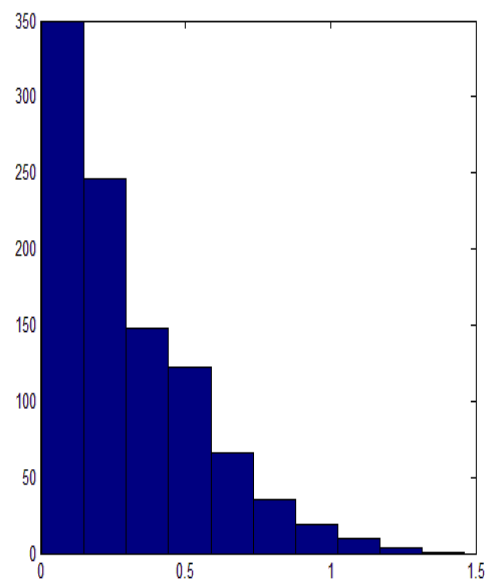
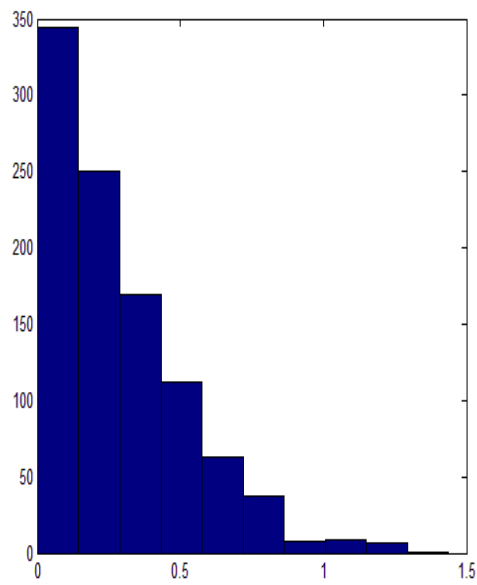


Figure 4.6 Robustness of sum of squared constraints functions of 8-Ply E-T coupling Laminate

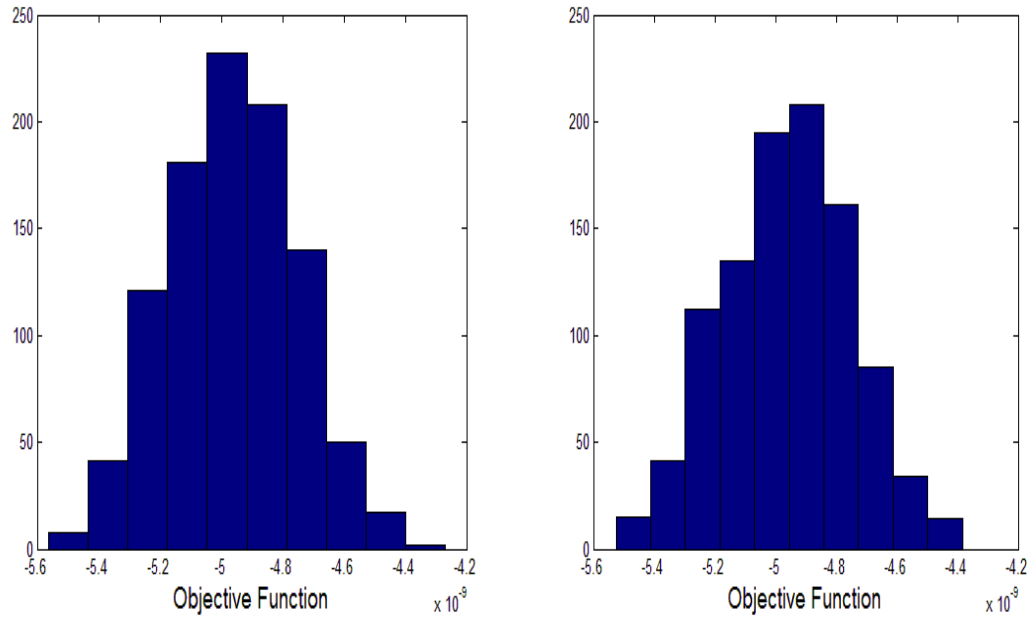


Figure 4.7 Robustness of Objective function of 10-Ply E-T coupling Laminate

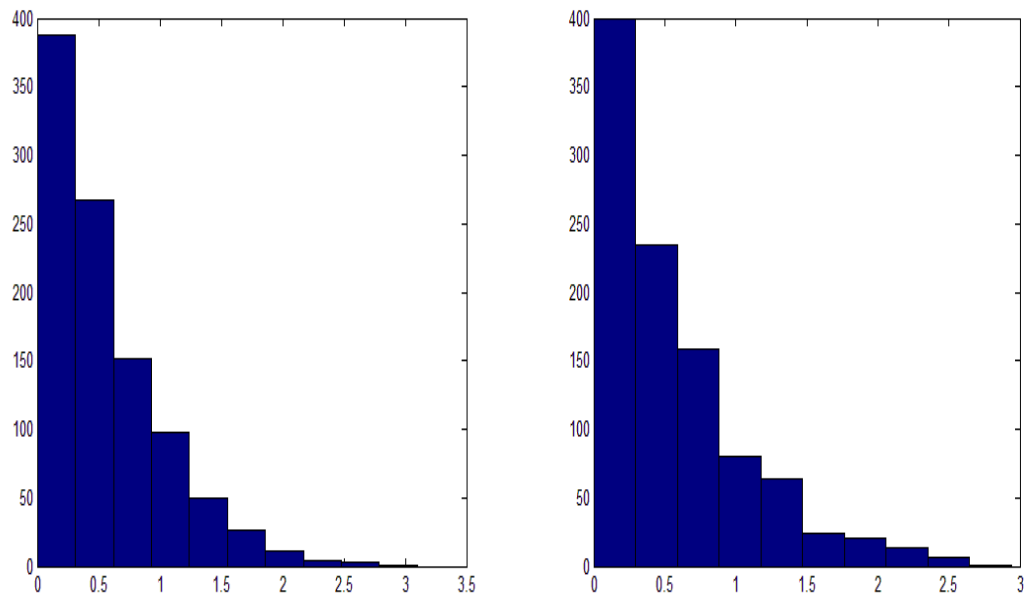


Figure 4.8 Robustness of sum of squared constraints functions of 10-Ply E-T coupling Laminate

4.5 Bending Twist Coupling of Hygrothermally stable laminates

From [4], Bending twist is a type of deformation behavior of laminates which results in a proportional twist caused by bending moment induced on the structure. It is created when the resultant, M_{xx} , produces an equipollent distributed axial stress through the thickness of the laminate, which in turn produces an average axial load in each lamina. The shear action is created in orthotropic laminas, when the axial force is applied. A twisting moment is created in the laminate by the resultant shear forces acting in each ply at some non zero distance from the midplane. The resultant moment is positive, if the plies above the midplane are in compression and while the plies below the midplane are in tension, meaning that a generally orthotropic lamina above the midplane will produce the opposite shear effect as if it is positioned below the midplane. Hence, the unidirectional off-axis laminates will produce a significant level of bend-twist coupling.

In this section, a constrained optimization is carried out to determine the stacking sequence of Hygrothermally stable laminate with bending twist coupling with a set number of plies.

The optimization process objective function is formulated by expressing the twist rate as a function of applied moment resultant. Same CLT assumption is used as extension twist coupling that the resulting laminate will be flat and that useful deformation range of these laminates validates using a linear theory. Using equations (4.16) and (4.17) the twist rate can be calculated as

$$\varphi = \frac{1}{2}(\varphi_{16}M_{xx} + \kappa_{xy}^{(T,H)}) \quad (4.20)$$

The optimizer searches the Hygrothermally stable laminates, the non-mechanical curvature in equation (4.20) is zero, resulting in objective function given by

$$g(\{\theta_k : k = 1 \dots n\}) = -\delta_{16}^2 \quad (4.21)$$

4.6 Implementation and Results for Bending Twist Coupling

Augmented Lagrangian Particle Swarm Optimization (ALPSO) was used to obtain the optimal stacking sequence. Four sets of conditions were used consecutively during the optimization of a given laminate: 1) no constraint on hygrothermal stability, 2) the constraints of condition A of equations (4.12) and (4.13), 3) the constraints of condition B of equations (4.14) and (4.15), 4) a constraint to a unidirectional laminate. The T300/976 graphite/epoxy material system [4] was used. The optimization was performed for laminates with two through ten plies. The optimized stacking sequence for laminates with bending twist is tabulated in Table (4.3)

Table 4.2 Bending Twist coupling of constrained laminates

Number of Plies <i>n</i>	Stacking sequence Obtained through ACO	Stacking sequence Obtained through ALPSO	Stacking sequence Obtained through SQP
4-Ply symmetric	[-50.53/ 39.45/ - 50.53/ 39.45]	[-50.53/ 39.44/- 50.53/ 39.44]	[-24.3/ 65.7/ - 24.3/ 65.7]
5-Ply symmetric	[-54.20/ 50.17/ - 1.9194/ -54.20/ 50.17]	[-42.75/ 28.90 /- 96.58/ -42.75/ 28.90]	[27.8/ -76.7/ 24.4/ 27.8/ -76.7]
6-Ply	[-21.05 / 61.14 / 52.22 -51.89/ -53.46 / 26.55]	[50.04/ -27.20/ - 40.17/ -219.16/- 76.19/ 5.35]	[-87.0/ -18.0 / 3.2 /51.1 /72.4 /- 38.6]
7-Ply symmetric	[61.15/ -59.22/ 0.77 /0.75 /61.15 /-59.22 /0.77]	[-44.91/ 15.16/ 75.12/ -104.87 /-44.91/ 15.16/ 75.12]	[33.6/ -88.5/ 33.6/ 88.5/ 33.6/ -88.5/ 33.6]
8-Ply	[49.26/ -51.30/ - 43.29/ 33.13/ -28.77/ 57.25/ 45.29/ -48.69]	[13.45/ 85.12/ -30.41/ 316.50/ -128.12/- 110.91/ -46.30/ -3.16]	[25.6/ 23.0/ - 67.0/-64.4/ 25.6/ 23.0/-67.0/ -64.4]
9-Ply symmetric	[42.84/ -28.51/ - 70.37/ 35.55/ -70.36/ 42.84/ -28.51/ -70.37/ 35.55]	[-79.58/ -4.40/ 43.73/ 129.35/ -136.26/ - 79.58/ -4.40/ 43.73/ 129.35]	[26.7 / 26.1 / - 74.6/-66.9 / 23.2 / 26.7 /26.1 /74.6 /-66.9]
10-Ply symmetric	[-53.05/ 51.58/ 2.56/ -83.94/ 2.5783/ -53.0/ 51.58/ 2.56/ -83.94/ 2.57]	[-3.38/ 225.09/ -59.6/ 270.59/ -419.66/ - 3.38/ 225.09/ -59.6/ 270.59/-419.6]	[-28.2 /-27.8 /78.0/75.0 /24.3 /-28.2 /-27.8/78.0 /75.0 /24.3]

The new designs obtained through ALPSO algorithm are compared with previous designs and robustness of all the design solutions was computed by perturbing the ply angles from -2 to 2 degrees. The histogram of the perturbed designs of objective function and sum of squared constraints functions are as shown below. The left side of the page contains histograms from the previous designs and the right side of the page contains histograms from the new designs generated from ALPSO method.

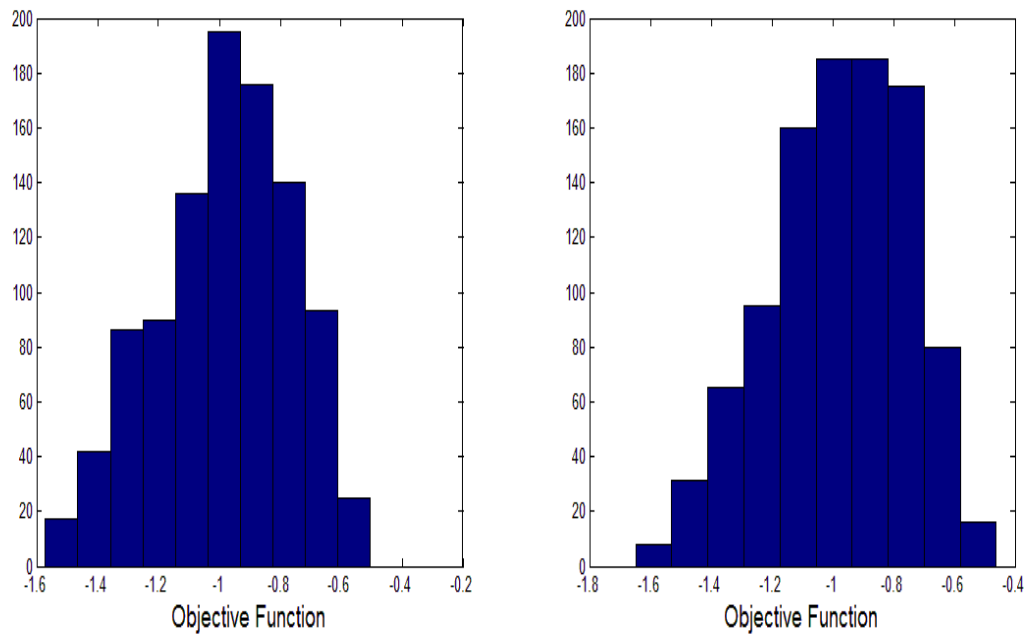


Figure 4.9 Robustness of Objective function of 4-Symmetric Ply B-T coupling Laminate.

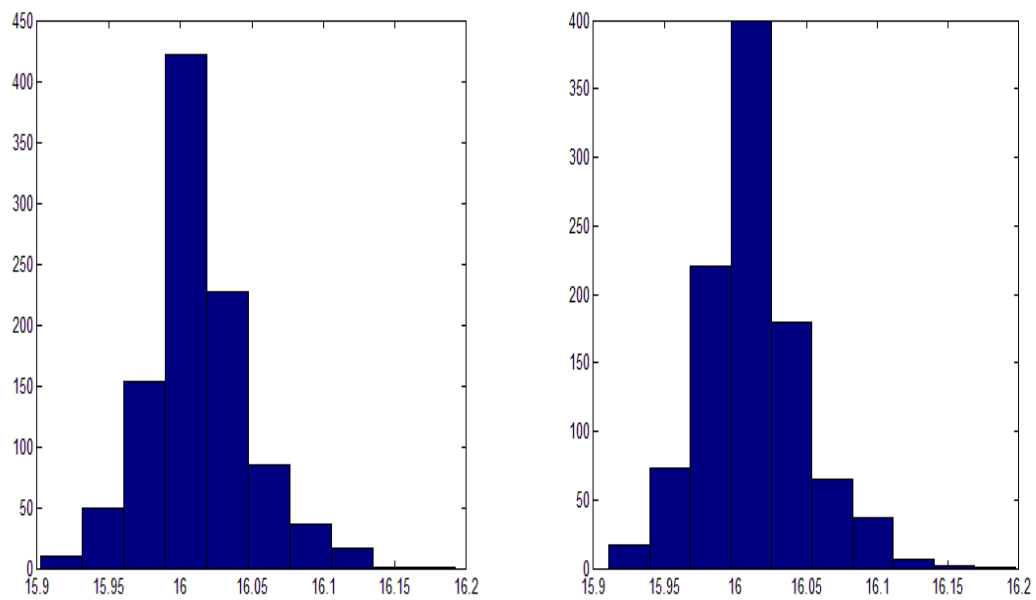


Figure 4.10 Robustness of sum of squared constraints functions of 4-Symmetric Ply B-T coupling Laminate

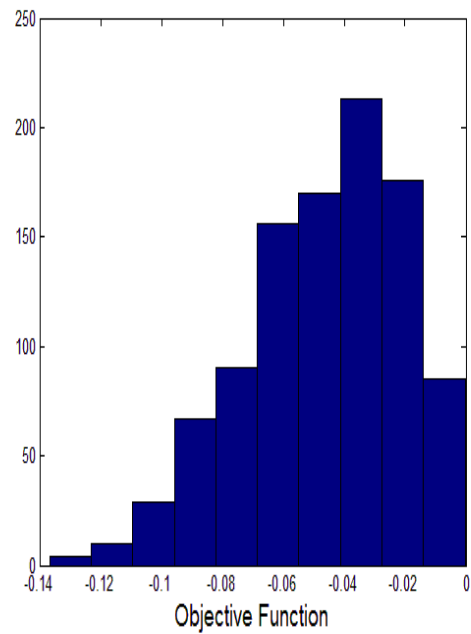
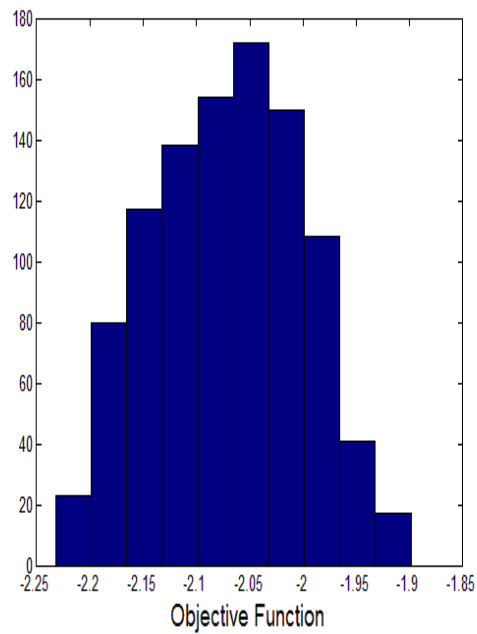


Figure 4.11 Robustness of Objective function of 5-Symmetric Ply B-T coupling Laminate.

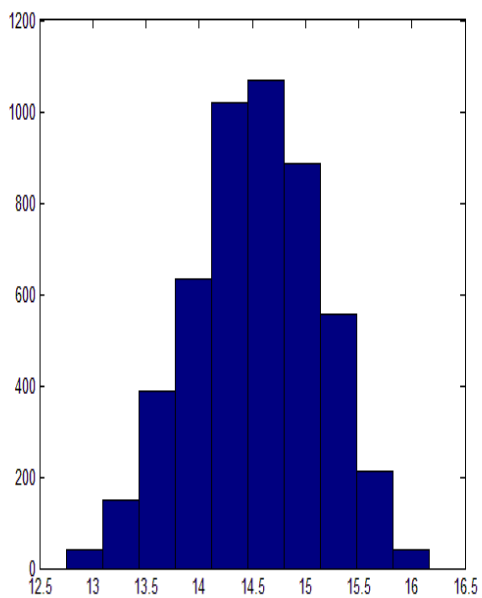
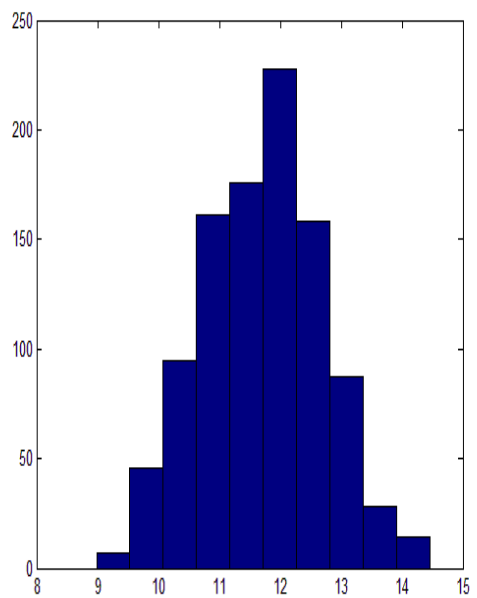


Figure 4.12 Robustness of sum of squared constraints functions of 5-Symmetric Ply B-T coupling Laminate

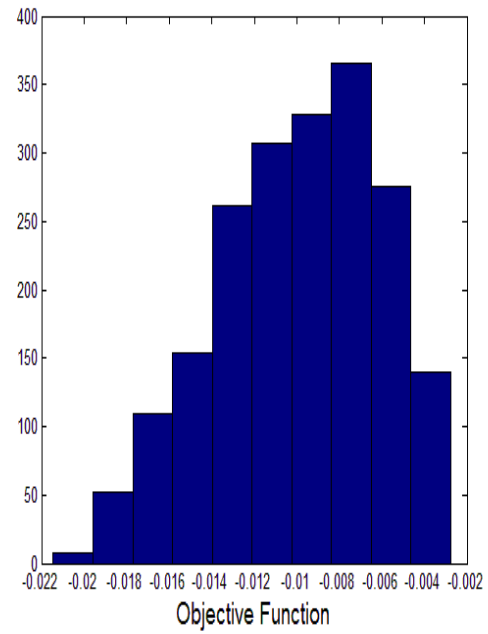
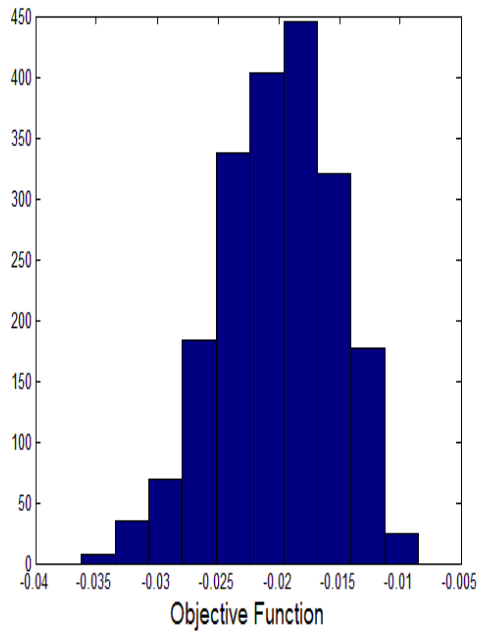


Figure 4.13 Robustness of Objective function of 6-Symmetric Ply B-T coupling Laminate.

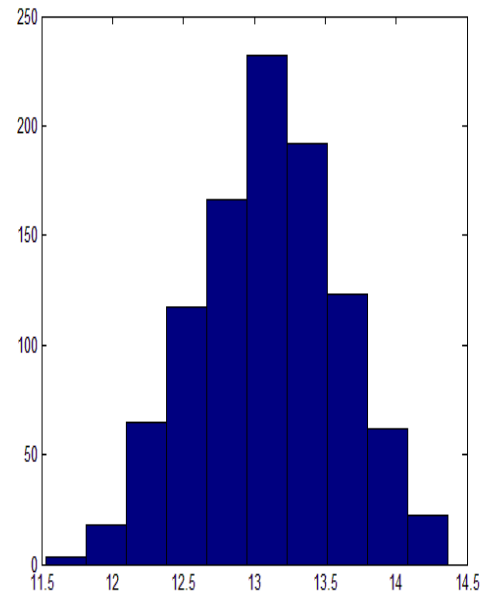
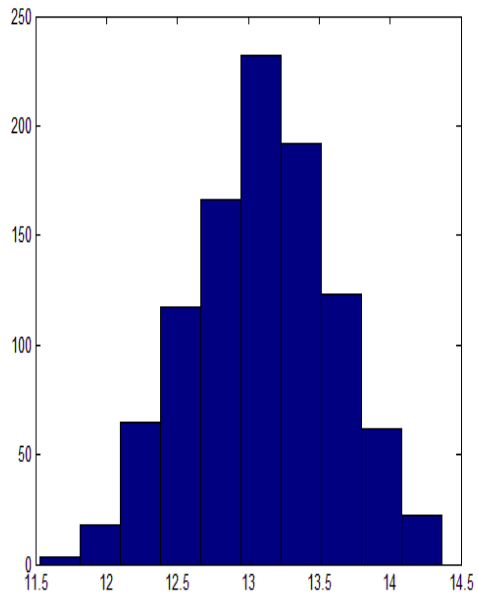


Figure 4.14 Robustness of sum of squared constraints functions of 6-Symmetric Ply B-T coupling Laminate

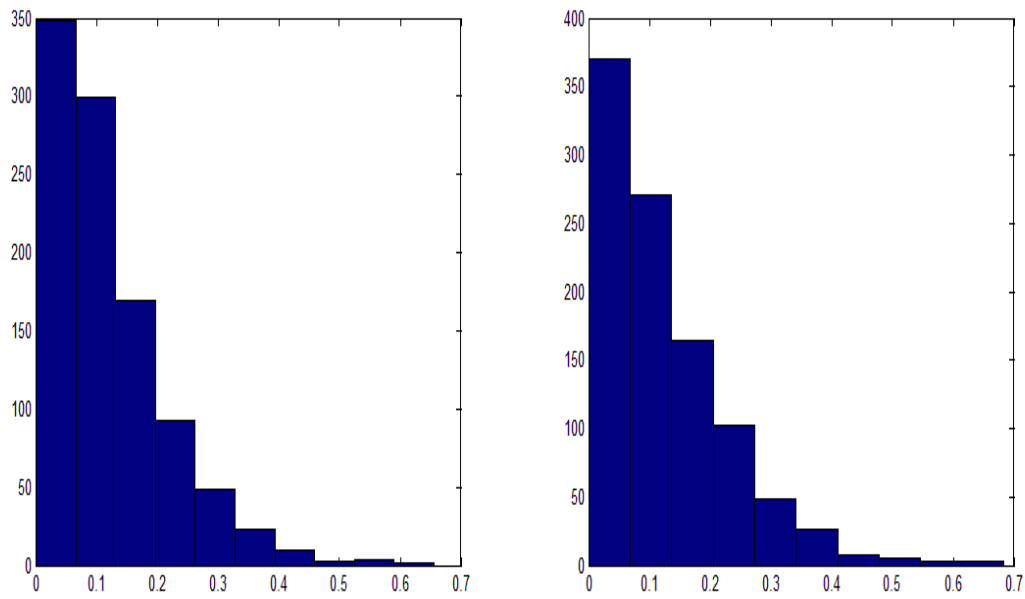


Figure 4.15 Robustness of Objective function of 6 Ply B-T coupling Laminate.

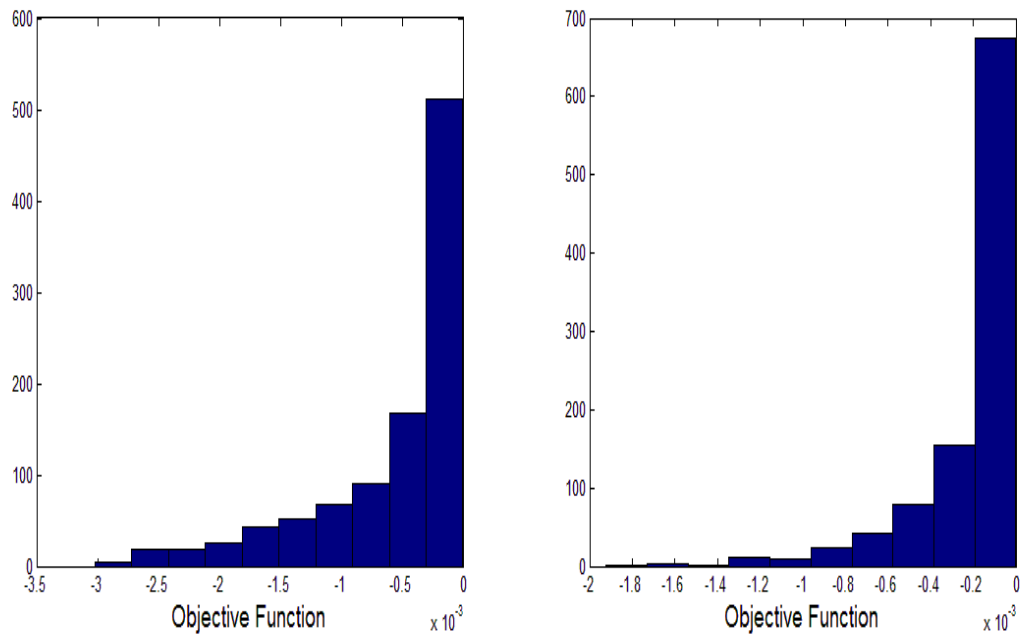


Figure 4.16 Robustness of sum of squared constraints functions of 6 Ply B-T coupling Laminate.

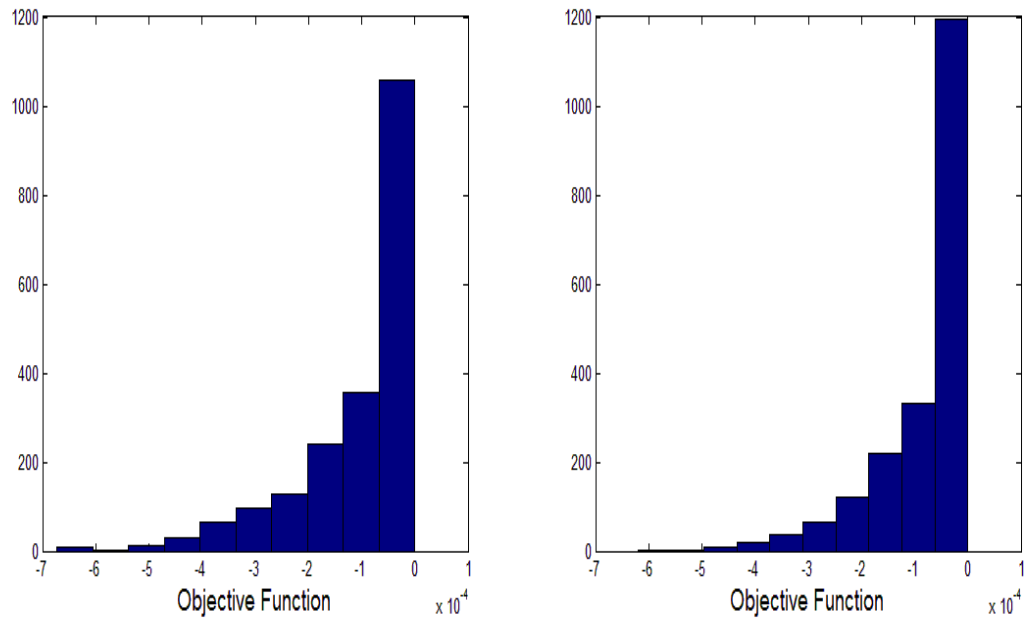


Figure 4.17 Robustness of Objective function of 7-Symmetric Ply B-T coupling Laminate.

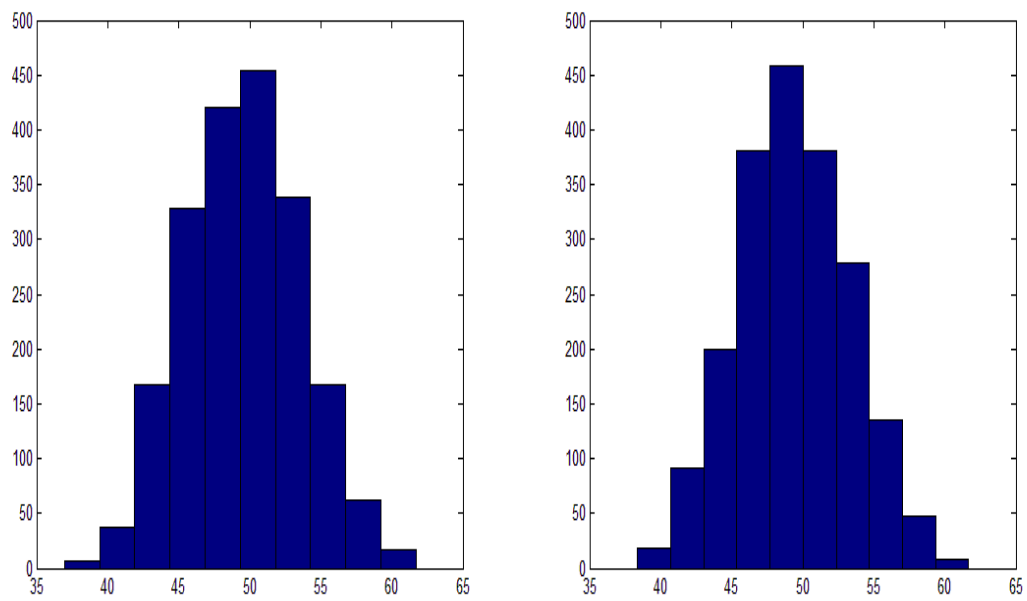


Figure 4.18 Robustness of sum of squared constraints functions of 7-Symmetric Ply B-T coupling Laminate.

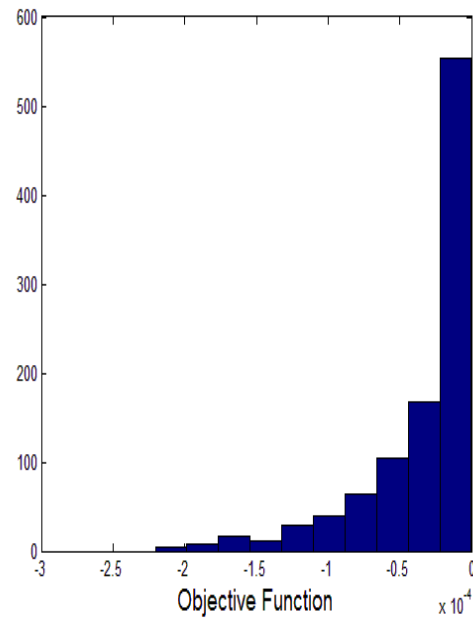
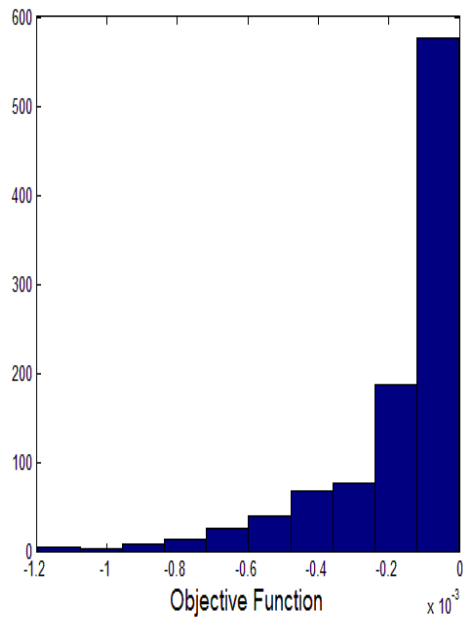


Figure 4.19 Robustness of Objective function of 8-Symmetric Ply B-T coupling Laminate.

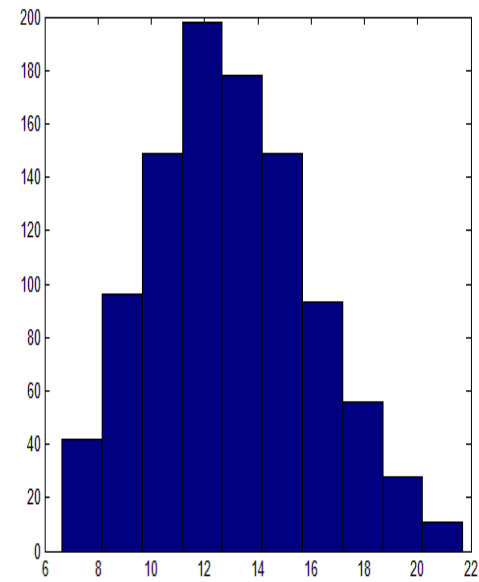
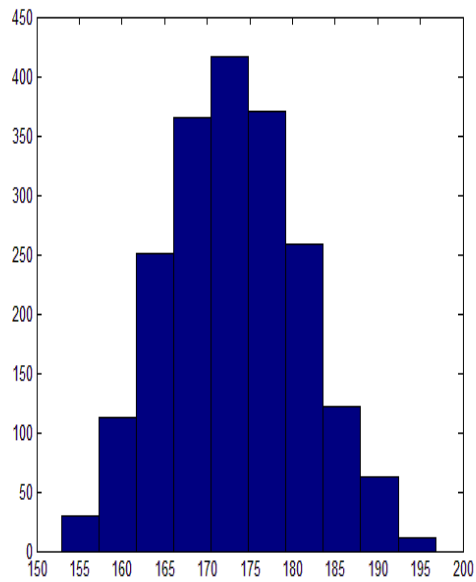


Figure 4.20 Robustness of sum of squared constraints functions of 8-Symmetric Ply B-T coupling Laminate

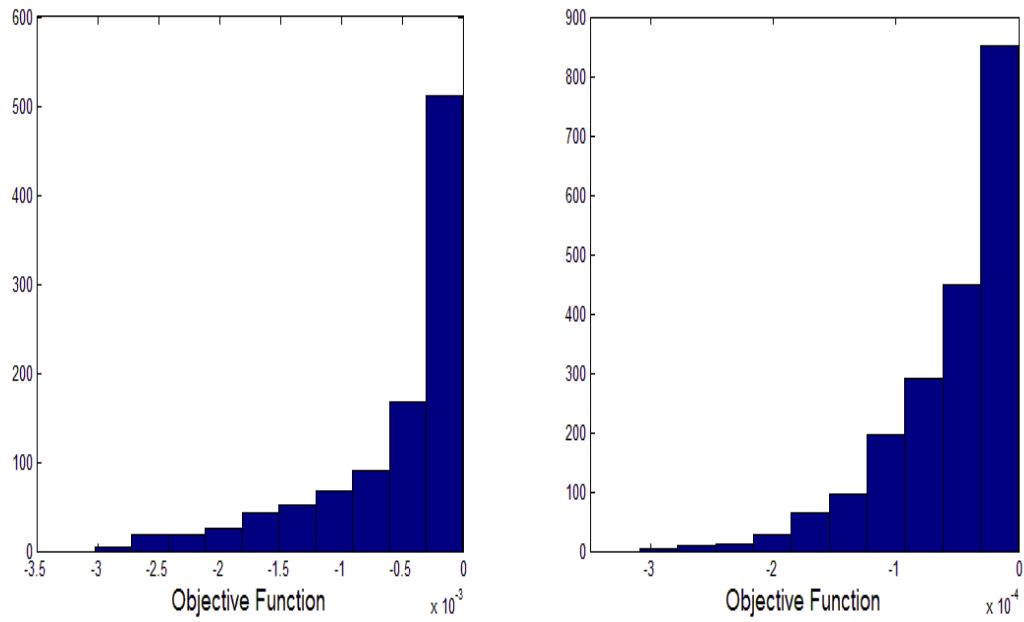


Figure 4.21 Robustness of Objective function of 9-Symmetric Ply B-T coupling Laminates.

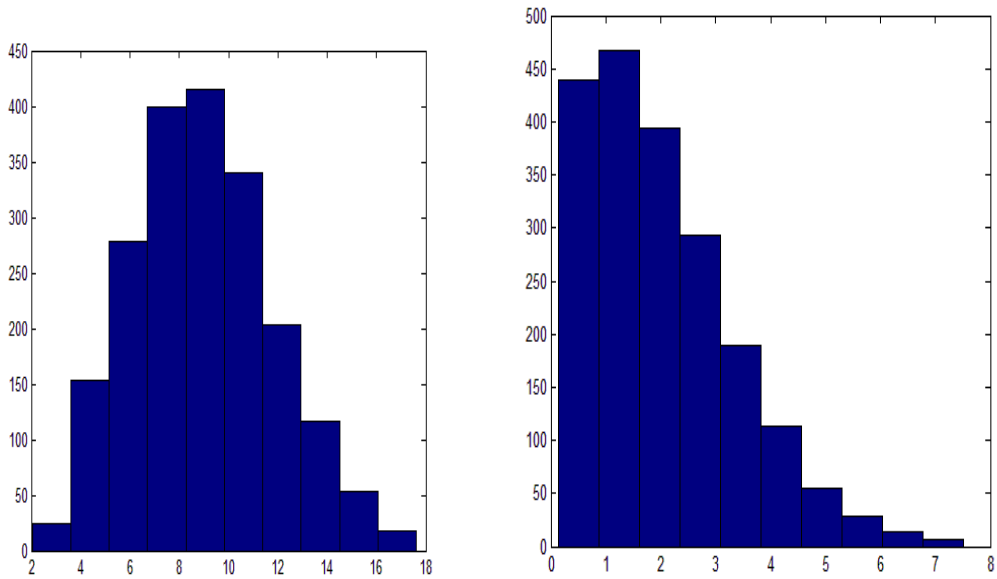


Figure 4.22 Robustness of sum of squared constraints functions of 9-Symmetric Ply B-T coupling Laminates.

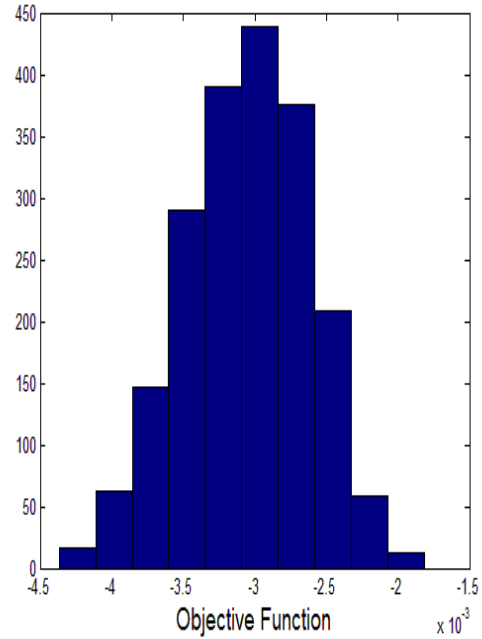
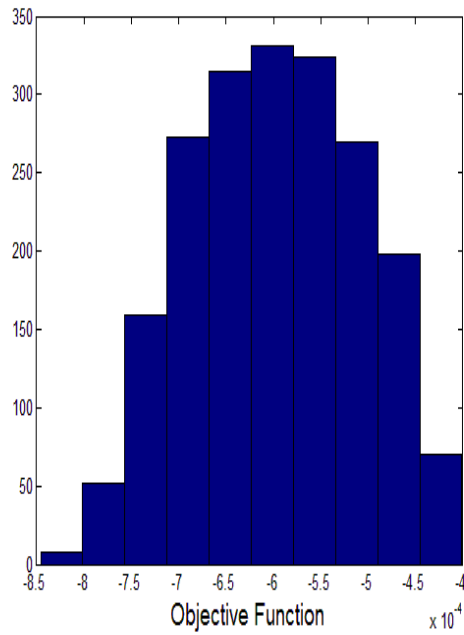


Figure 4.23 Robustness of Objective function of 10-Symmetric Ply B-T coupling Laminate.

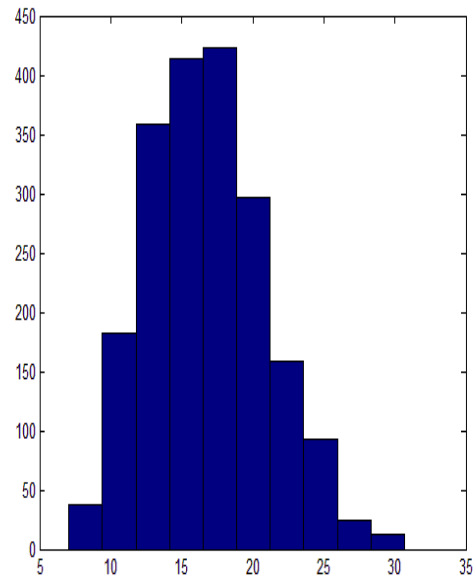
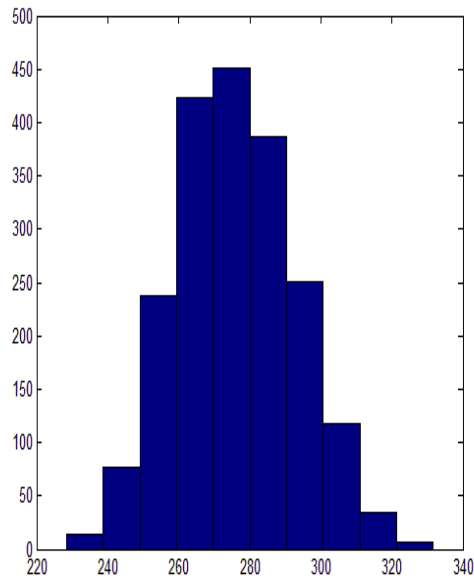


Figure 4.24 Robustness of sum of squared constraints functions of 10-Symmetric Ply B-T coupling Laminate

CHAPTER 5

CONCLUSION AND FUTURE WORK

In Design optimization, the objective function and constraints are usually highly nonlinear and non-convex leads to the possibility of existence of multiple local global solutions. Since the Sequential Quadratic Programming (SQP) is a gradient based method the chances of converged solution getting trapped into local minima is high. To get global optimal solution, one can use SQP with multiple starting point and select the best solution. This approach is used in previously used Extension Twist couplings design to get near global optimal. In this thesis, the ALPSO algorithm discovered new global optimal design which satisfied terms of objective function and constraints. The ALPSO algorithm was found to be robust and reliable technique for solving Engineering optimization problem. ALPSO method is one of the most effective method for solving constrained optimization problem due to its capacity to enforce constraints as compared to other methods like penalty method.

The ALPSO algorithm efficiency can be improved in many ways. The search behavior of the ALPSO algorithm is very sensitive to inertia weight and social parameters. During this work the inertia factor was selected to be a constant value and studies have shown that for a better efficiency and convergence of the solution towards the global value the inertia factor has to be dynamically or linearly varied adjusting according to the algorithm. One of the major disadvantage of PSO algorithm is the computational cost as it evaluates the function during each given set of iterations. Hence selecting the swarm size plays an important role in determining the computation time. In this work the population size of the swarm was selected to be 30, the optimal range of swarm size for efficient convergence rate is 20 to 60 and depends on problem.

One of the ways to reduce the computing time is to initialize the particles to be in the feasible region and randomly initialized particles are not always in the feasible region. In this work randomly initialized particles was used for initial generation. Hence initializing particles over a uniform design space within the feasible region may decrease the computation time and increase the rate of convergence of the optimal solution. Reducing the design search space after each iteration can also be implemented to increase the rate of convergence convergence. This work implements Augmented Lagrangian Method (ALM) with Particle Swarm Optimization (PSO) to solve constrained optimization problem. The ALM method can also be implemented with other Evolutionary Algorithms (EA) to find the global optimum.

APPENDIX A
EXAMPLES OF PROBLEMS1

The Rosen Brook function R, the Griewank function as well as test problems 1 to 7 used in chapter 3 are given here for completeness. For further information can be referred from [7], [11]

$$Rf(x) = 100(x_2 - x_1)^2 + (1 - x_1)^2$$

$$-10 \leq x_i \leq 10, i = 1, 2$$

$$Gf(x) = \frac{1}{4000}(x_1^2 + x_2^2) - \cos\left(\frac{x_1}{\sqrt{1}}\right)\cos\left(\frac{x_2}{\sqrt{1}}\right) + 1$$

$$-10 \leq x_i \leq 10, i = 1, 2$$

Problem 1 $f(x) = (x_1^2 + x_2^2)$

$$g_1(x) = x_1 - 3 = 0$$

$$h_1(x) = 2 - x_2 \leq 0$$

$$-10 \leq x_i \leq 10, i = 1, 2$$

Problem 2 $f(x) = \frac{1}{4000}(x_1^2 + x_2^2) - \cos\left(\frac{x_1}{\sqrt{1}}\right)\cos\left(\frac{x_2}{\sqrt{1}}\right) + 1$

$$g_1(x) = x_1 - 3 = 0$$

$$h_1(x) = 2 - x_2 \leq 0$$

$$-10 \leq x_i \leq 10, i = 1, 2$$

Problem 3 $f(x) = \frac{-\sin(2\pi x_1)^3 \sin(2\pi x_2)}{x_1^3(x_1 + x_2)}$

$$h_1(x) = x_1^2 - x_2 + 1 \leq 0$$

$$h_2(x) = 1 - x_1 + (x_2 - 4)^2 \leq 0$$

$$0.1 \leq x_1 \leq 10, 0 \leq x_2 \leq 10$$

Problem 4 $f(x) = (x_1 - 10)^3 + (x_2 - 20)^3$

$$h_1(x) = -(x_1 - 5)^2 - (x_2 - 5)^2 + 100 \leq 0$$

$$h_2(x) = (x_1 - 6)^2 + (x_2 - 5)^2 - 82.81 \leq 0$$

$$13 \leq x_1 \leq 100, 0 \leq x_2 \leq 100$$

Problem 5 $f(x) = x_1^2 + (x_2 - 1)^2$

$$g_1(x) = x_2 - x_1^2 = 0$$

$$-1 \leq x_i \leq 1, i = 1, 2$$

Problem 6 $f(x) = \frac{-(100 - (x_1 - 5)^2 - (x_2 - 5)^2 - (x_3 - 5)^2)}{100}$

$$h_j(x) = (x_1 - p)^2 + (x_2 - q)^2 + (x_3 - r)^2 - 0.0625 \leq 0$$

$$0 \leq x_i \leq 10, i = 1, 2, 3$$

$$p, q, r = 1, 2, \dots, 9, j = 1, 2, \dots, 9^3$$

Problem 7 $f(x) = 5.3578547x_3^2 + 0.8356891x_1x_5 + 37.293239x_1 - 40792.141$

$$h_1(x) = 85.334407 + 0.0056858x_2x_5 + 0.0006262x_1x_4 - 0.0022053x_3x_5 - 92 \leq 0$$

$$h_2(x) = -85.334407 - 0.0056858x_2x_5 - 0.0006262x_1x_4 + 0.0022053x_3x_5 \leq 0$$

$$h_3(x) = 80.51249 + 0.0071317x_2x_5 + 0.0029955x_1x_2 + 0.0021813x_3^2 - 110 \leq 0$$

$$h_4(x) = -80.51249 - 0.0071317x_2x_5 - 0.0029955x_1x_2 - 0.0021813x_3^2 + 90 \leq 0$$

$$h_5(x) = 9.300961 + 0.0047026x_3x_5 + 0.0012547x_1x_3 + 0.0019085x_3x_4 - 25 \leq 0$$

$$h_6(x) = -9.300961 - 0.0047026x_3x_5 - 0.0012547x_1x_3 - 0.0019085x_3x_4 + 20 \leq 0$$

$$78 \leq x_1 \leq 100, 33 \leq x_2 \leq 45, 27 \leq x_i \leq 45 \quad i = 3, 4, 5$$

APPENDIX B
EXAMPLES OF PROBLEMS 2

Benchmark constrained problem from [11] and [12]

$$\begin{aligned} \text{Problem 1} \quad f(x) &= (x_1 - 10)^2 + 5(x_2 - 12)^2 + x_3^4 + 3(x_4 - 11)^2 + 10x_5^6 + 7x_6^2 \\ &\quad + x_7^4 - 4x_6x_7 - 10x_6 - 8x_7 \end{aligned}$$

$$g_1(x) = -127 + 2x_1^2 + 3x_2^4 + x_3 + 4x_4^2 + 5x_5 \leq 0,$$

$$g_2(x) = -282 + 7x_1 + 3x_2 + 10x_3^2 + x_4 - x_5 \leq 0,$$

$$g_3(x) = -196 + 23x_1 + x_2^2 + 6x_6^2 - 8x_7 \leq 0,$$

$$g_4(x) = 4x_1^2 + x_2^2 - 3x_1x_2 + 2x_3^2 + 5x_6 - 11x_7 \leq 0$$

$$\text{Where } -10 \leq x_i \leq 10 \ (i = 1, \dots, 7)$$

$$\text{Problem 2} \quad f(x) = (x_3 + 2)x_2x_1^2$$

$$g_1(x) = 1 - \frac{x_2^3x_3}{71785x_1^4} \leq 0,$$

$$g_2(x) = \frac{4x_2^2 - x_1x_2}{12566(x_2x_1^3 - x_1^4)} + \frac{1}{5108x_1^2} - 1 \leq 0$$

$$g_3(x) = 1 - \frac{140.45x_1}{x_2^2x_3} \leq 0,$$

$$g_4(x) = \frac{x_1 + x_2}{1.5} - 1 \leq 0,$$

$$0.05 \leq x_1 \leq 2, 0.25 \leq x_2 \leq 1.3, 2 \leq x_3 \leq 15.$$

Problem 3 $f(x_1, x_2, y) = -y + 2x_1 + x_2$

s.t.

$$x_1 - 2\exp(-x_2) = 0$$

$$-x_1 + x_2 + y \leq 0$$

$$0.5 \leq x_1 \leq 1.4$$

$$y \in \{0, 1\}$$

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BIOGRAPHICAL INFORMATION

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