FINITE ELEMENT ANALYSIS OF STRESSES

IN INVOLUTE SPUR & HELICAL GEAR

by

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ABSTRACT

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Developing an analytical approach and modeling procedure to evaluate stress distribution under velocity and moment would provide a useful tool to improve spur gear design with high efficiency & low cost. The purpose of this work is to analyze and validate the stress distribution in spur & helical gears using contemporary commercial FEM program ANSYS coupled with the Pro/E solid modeler.

Gear profiles are created in Pro Engineer using the Relation and Equation modeling procedure so as to make it a general model, dependant on certain key features like number of teeth, diametral pitch, and pressure and helix angle. These models were then analyzed for 3-D and 2-D bending stresses. Calculated results obtained when compared to standard AGMA stresses show good agreement.

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LIST OF SYMBOLS

d_1	Pinion Pitch Diameter
d_2	Gear Pitch Diameter
a	Addendum of gear tooth
b	Dedendum of gear tooth
E_i	Young's Modulus for Gear Material
ϕ	Pressure Angle
ψ	Helix Angle
ν_i	Poisson's ratio for gear Material
p_d	Diametral Pitch
Y	Lewis form factor
K_a	Application factor
K_s	Size factor
K_m	Load Distribution factor
K_v	Dynamic factor
Y_j	Geometry factor
W_t	Tangential component of normal force

CHAPTER 1

INTRODUCTION

1.1 Research Overview

Power transmission has always been of high importance. The efficiency of any machine depends on the amount of power loss in the process. One of the best methods of transmitting power between the shafts is gears. Gears are mostly used to transmit torque and angular velocity. There are also a wide variety of gear types to choose from. This thesis will deal with spur gear designed to operate on parallel shafts and having teeth parallel to the shaft axis, and helical gears designed to operate on parallel shafts. Other gear types such as bevel and worm can accommodate nonparallel shafts. Due to the compactness and high degree of reliability in transmitting power, it is possible that gears will dominate the future machines. Gears are standardized by AGMA (American Gear Manufacturers Association) depending on size and tooth shape. In this thesis AGMA methods and standards are taken into considerations for comparing the results obtained.

As the world moves on there is always new demands, which are to be fulfilled. People now prefer cars with low weight and low engine sound. This opens up a demand for quite power transmission. Automobile industries are one of the largest manufacturers of gears [1]. Higher reliability and lighter weight gears are necessary to make automobile light in weight as lighter automobiles continue to be in demand. The success in engine noise reduction promotes the production of quieter gear pairs for further noise reduction. Reduction in noise while transferring power is necessary and is also critical in today's rapidly growing field of automation. The only effective way to achieve gear noise reduction is to reduce the vibration associated with them.

Spur gears are the most commonly used gears to transmit power and rotational motion between parallel shafts. The tooth of gear is cut parallel to the shaft. The simplest motion of external spur gears can be seen by an example of two external rotating cylinders, if sufficient friction is present at the rolling interface. The main disadvantage of these rotating cylinders is the possibility of slip at interface, which is avoided by adding meshing teeth to rolling cylinders.

Designing highly loaded spur gears for power transmission systems that are both strong and quiet requires analysis methods that can easily be implemented and also provide information on contact and bending stresses. The finite element method is capable of providing this information, but the time needed to create such geometry is large. In order to reduce the modeling time, a preprocessor method that creates the geometry needed for a finite element analysis may be used. Modeling software's like Pro Engineer, Solid Works, Catia and many more are the best option available to create complex geometry for analysis. In this thesis Pro Engineer is used to create the Gear geometry and then it is imported in Ansys Workbench 12.0 for analysis.

Gear analyses in the past were performed using analytical methods, which required a number of assumptions and simplifications. In general, gear analyses are multidisciplinary, including calculations related to the tooth stresses and the failures. In this thesis, bending stress analyses are performed, with the main aim of designing spur and helical gears to resist bending failure.

Nowadays computers are becoming more and more powerful, and that is the reason why people tend to use numerical approach to develop theoretical models to predict the effects. Numerical methods can potentially provide more accurate solutions since they normally require much less restrictive assumptions. However the important thing is to choose the correct model and the solution methods to get the accurate results and also reasonable computational time.

In this thesis first the solid model of the spur gear is made with relations and equations modeling option in Pro Engineer. After the modeling of spur gear the assembly is created of two spur gears in contact. The contact is defined at the pitch circle radius with the appropriate center distance between the two gears. Then the whole assembly is imported in ANSYS

Workbench 12.0 for bending stress analysis. The results of ANSYS 12.0 are then compared with the AGMA standards for the specified gear set in contact. The purpose of this thesis is to develop a general model to study contact and bending stress of any spur gears in contact.

1.2 Literature Review

There has been great deal of research on gear analysis, and a large body of literature on gear modeling has been published. The gear stress analysis, the transmission errors, the prediction of gear dynamic loads, gear noise, and the optimal design for gear sets are always major concerns in gear design. In a study by Xu Rixin [2], analytical approach and modeling procedure to evaluate stress distributions and quenching process under applied velocity and moment was developed. Finite element simulation of spur gear was developed and used to predict distribution of stress and other material properties. The quenching result in the simulation proved the theory and ensured product quality.

In a study by Patchigolla [3] a finite element modeling approach was developed for determining the effect of gear rim thickness on tooth bending stresses in large spur gears.

Wei [4] defined stresses and deformations in involute spur gear by finite element method. He examined Hertz contact stresses using 2-D and 3-D FEM models. He also considered the variations of the whole gear body stiffness arising from the gear body rotation due to bending deflection, shearing displacement and contact deformation.

Saxena [5] analyzed the stress distribution in spur gear teeth using FEM program. He just studied a single teeth rather than whole spur gear. Spur gear profile was created in ANSYS. In his study he analyzed geometry factor for different type of gear profiles.

1.3 Objectives Of Research

There are number of investigations devoted to gear research and analysis. But still there remains, a general numerical approach capable of predicting the effects of variations in gear geometry, contact and bending stresses and torsional mesh stiffness. The objectives of this thesis are to use a numerical approach to develop theoretical models of the behavior of spur gears in mesh, to help to predict the effect of gear tooth stresses. The major work is summarized as follows.

- Applying Formulate of Lewis bending equation and Finite Element Method analysis of meshing spur gear and helical gear was conducted.
- Applying the relation equation in Pro/Engineer to develop the accurate three dimensional spur and helical gear models. It is easy to change the parameters of gear to arrive different model to analysis.
- Three-dimensional and two dimensional finite element models of spur gear system were established to investigate the root bending stress and contact stress distribution over the assumed operating speed with consideration of lubrication conditions.
- Results from the finite element analysis are then compared, with the results obtained according to AGMA standards. Both three-dimensional and two-dimensional results show good compliance with the AGMA stress values.

There are many types of gear failures but they can be classified into two general groups. One is failure of the root of the teeth because the bending strength is inadequate. The other is created on the surfaces of the gears. There are two theoretical formulas, which deal with these two failure mechanisms. One is the Hertzian equation, which can be used to calculate the contact stresses. The other is Lewis formula, which can be used to calculate the bending stresses. The surface pitting and scoring are the examples of failures, which resulted in the fatigue failure of tooth surface.

Pitting and scoring is a phenomenon in which small particles are removed from the surface of the tooth due to high contact stresses that are present between mating teeth. Pitting is actually the fatigue failure of the tooth surface. Hardness is the primary property of the gear tooth that provides resistance to pitting. In other words, pitting is a surface fatigue failure due to many repetitions of high contact stress, which occurs on gear tooth surfaces when a pair of teeth is transmitting power.

1.4 Layout Of Thesis

This thesis is consists of five chapters. Chapter 1 presents a general introduction, research overview, literature review and the objectives to be achieved. Chapter 2 covers the basics on spur and helical gears and characteristics of involute gears for different types of modeling. Chapter 3 describes the modeling procedure of spur and helical gear. The challenge in modeling involute gear is to create an involute tooth profile. The relation and equation modeling option in Pro Engineer fulfilled this challenge. The involute tooth profile was generated in the software by inserting the equations describing the tooth profile. The equations were dependant on some basic data like pressure angle, diametral pitch and values of addendum and dedendum. After the gear was generated in Pro/E, the two spur gears were assembled in Pro/E for the root bending stress and contact stress analysis in ANSYS 12.0. Chapter 4 focuses on the FEA model of spur gear assembly and also helical gear assembly and then presents the root bending stresses and contact stresses from 3-D models and 2-D models. The results are then compared with the theoretical values calculated by AGMA standards. Chapter 5 gives the conclusions of this thesis and number of recommendations for future work is listed.

CHAPTER 2

SPUR GEAR BASICS

Gears have a long history. The ancient Chinese South-Pointing Chariot, supposedly used to navigate across the Gobi desert in pre-Biblical times, contained gears. Leonardo DaVinci shows many gear arrangements in his drawings [1]. According to Norton[1], in old times gears were usually made of wood and some other easily found materials, teeth were also not accurately formed. It's this industrial revolution that big machines demanded and manufacturing techniques allowed the creation of gears with so much of accuracy and reliability. Gears have their own specialized terminology. The variables and terms used in this chapter are listed at the front of this thesis.

2.1 Gear Tooth Theory

When one thinks of transferring some kind of rotary motion between the parallel shafts, rolling cylinders is the simplest available option. They may be of two kinds an external set of rolling cylinders or an internal set. In order to make this mechanism a significant one, sufficient friction should be present at the rolling interface. If maximum available frictional force at the joint is exceeded by the demands of torque transfer there won't be any slip between the cylinders.

Possibility of slip and relatively low torque capability are two major drawbacks of the rolling cylinders. For timing purposes some drives require absolute phasing of the input and output shafts. This opens a requirement of adding some meshing teeth to the rolling cylinders, which are called gears and are together called a gear set. When two gears are placed in a mesh to form a gear set, it is conventional to refer to the smaller of the two gears as the pinion and to the other as the gear.

2.1.1 Fundamental Law of Gearing

Main purpose of adding teeth to rolling cylinders was to prevent gross slip. In old times water powered mills and windmills used wooden gears whose teeth were merely round wooden pegs, which were stuck into the rims of the cylinders. Due to the geometry of the so-called wooden pegs or gear teeth there was very low possibility of smooth velocity transmission. And it was so because the geometry of the tooth "pegs" violated the fundamental law of gearing, which states that the "angular velocity ratio between the gears of a gear set must remain constant throughout the mesh"[1]. The angular velocity ratio m_V equals to the ratio of the pitch radius of the input gear to that of the output gear.

$$m_V = \frac{\omega_{out}}{\omega_{in}} = \pm \frac{r_{in}}{r_{out}} \tag{1}$$

The pitch radii in the above mentioned equations are those of the rolling cylinders to which we are adding the teeth. The positive and negative sign is for internal or external cylinder sets. An external set reverses the direction of rotation between the cylinders and requires the negative sign. An internal gearset will have the same direction of rotation on input and output shafts and require the positive sign in equation [1]. The surfaces of the rolling cylinders will become the pitch circles, and their diameters the pitch diameters of the gears. The contact point between the two cylinders lies on the line of centers and is called the pitch point.

Gearset is essentially a device to exchange velocity for torque or vice versa. One of the best gearset applications will reduce the velocity and will increase the torque to drive heavy loads as in the automobile transmission. Other applications require an increase in velocity, for which a reduction torque in torque must be accepted. Either case, it is usually desirable to maintain a constant ratio between the gears as they rotate. For calculation purposes, the gear ratio m_G is taken as the magnitude of either the velocity ratio or the torque ratio whichever is > 1.

$$m_G = |m_V| \text{ Or } m_G = |m_A| \text{ for } m_G \geq 1 \qquad (2)$$

In order to comply with fundamental law of gearing, the gear tooth contours on mating teeth must be conjugates of one another. There are an infinite number of possible conjugate pairs that could be used, but only a few curves have seen practical application as gear teeth. The cycloid is still used as a tooth form in some watches and clocks, but most gears use the involute of a circle for their shape.

2.1.2 The Involute Tooth Form

The involute of a circle is a curve that can be generated by unwrapping a taut string from a cylinder. Some key points about this involute curve are listed below[1].

- The string is always tangent to the base circle.
- The center of the curvature of the involute is always at the point of tangency of the string with the base circle.
- A tangent to the involute is always normal to the string, which is the instantaneous radius of curvature of the involute curve.

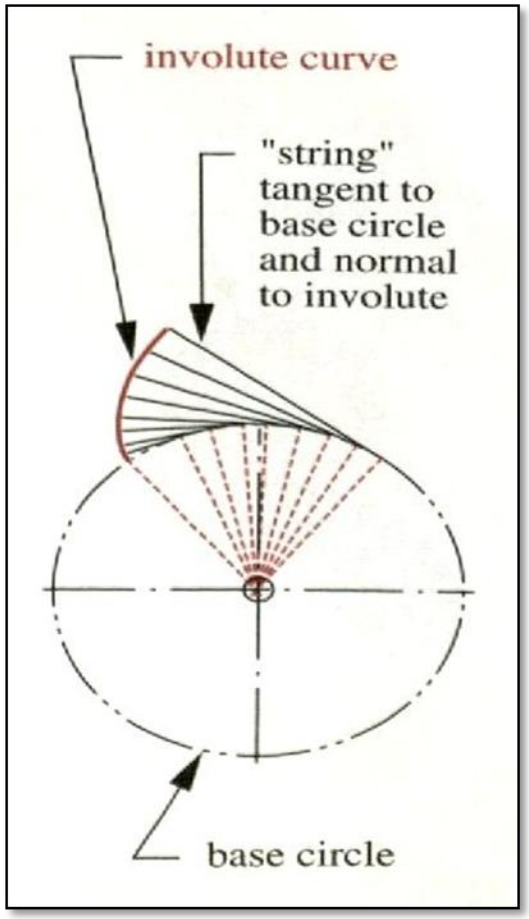


Figure 2.1 Involute Curve [1]

Figure above shows the way in which involute curve is formed. These represent gear teeth. The cylinders from which the strings are unwrapped are called the base circles of the respective gears. The base circles should always be smaller than the pitch circles, which are at radii of the original rolling cylinders. The gear tooth must project both below and above the rolling cylinder surface, and the involute only exists outside of the base circle. The amount of tooth that sticks out above the pitch circle is called addendum (a_p) and (a_g) for pinion and gear, respectively.

There always is a common tangent to both involute tooth curves at the contact point, and a common normal, perpendicular to the common tangent. The common normal is the line of action and always passes through the pitch point regardless of where in the mesh the two teeth comes in contact [1].

2.1.3 Pressure Angle

Pressure Angle is termed as angle between line of action and the direction of velocity in a gear-set. There are some standard values for pressure angle like 14.5°, 20° and 25°, out of which 20° is the most common. Gears with any other pressure angles except the standard values are very rare to find, as to make those kinds of gears a special cutter is to be made. The important point is gears, which are to be operated together, have to be from the same family of pressure angle.

2.2 Gear Mesh Geometry

Gear mesh geometry is the most complicated part of gears, because most of the loss in power transmission occurs at this point. The figure below shows a pair of involute tooth coming in contact and also leaving contact. From the figure we can see that common normal's for both the contact points passes from the same pitch point. We can see that the property of involute curve makes the gear tooth obey the fundamental law of gearing while meshing occurs.

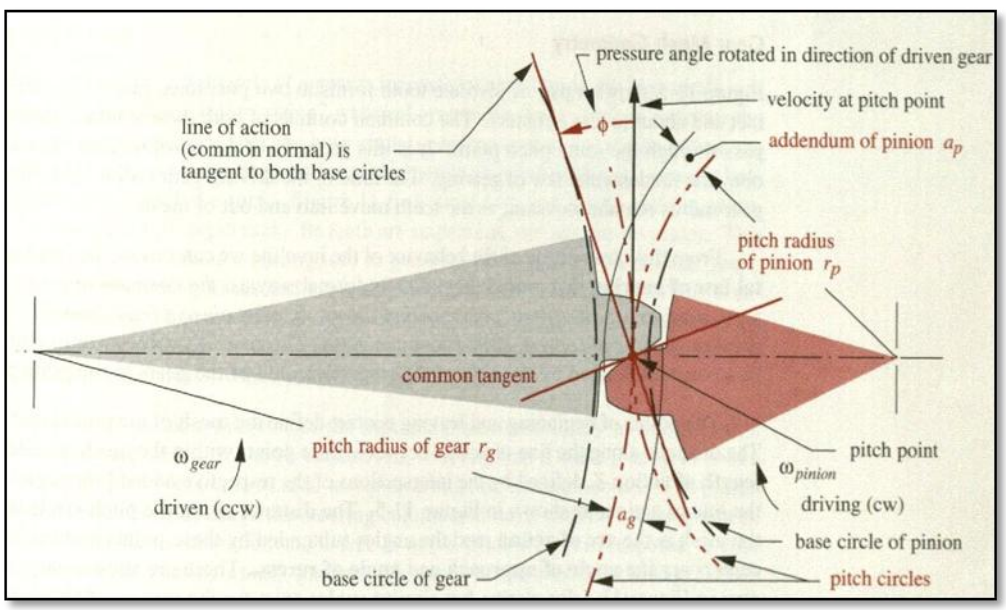


Figure 2.2 Gear Mesh Geometry [1]

Mesh between the gear and the pinion, is defined by the points of beginning and leaving contact. The distance along the line of action between the two contact points in mesh is called length of action (Z). The length of action (Z) can be calculated by the formulae written below.

$$Z = \sqrt{[(r_p + a_p)^2 - (r_p \cos\phi)^2]} + \sqrt{[(r_g + a_g)^2 - (r_g \cos\phi)^2]} - C \sin\phi \qquad (3)$$

Where:

r_p & r_g = Radius of Pitch Circle for pinion ad gear respectively

a_p & a_g = Addendum for pinion and gear respectively.

C= Center distance

ϕ= Pressure Angle.

2.3 Stresses In Involute Gears

There are two kinds of stresses in gear teeth, root bending stresses and tooth contact stresses. These two stresses results in the failure of gear teeth, root bending stress results in fatigue fracture and contact stresses results in pitting failure at the contact surface. So both these stresses are to be considered when designing gears. Usually heavily loaded gears are made of ferrous materials that have infinite life for bending loads. But it is impossible to design gears with infinite life against surface failure. In this thesis both the principal failure modes are studied based on the calculation of bending and contact stresses.

2.3.1 Bending Stress

Equation for calculation of bending stress in a gear tooth was developed by W.Lewis in 1892.[1]He derived an equation called 'Lewis Equation'. Lewis equation is no longer used in the way it was derived, but it serves as a basis for a modern version equation defined by AGMA. Some new factors were added in Lewis equation so as to make it more accurate and precise in calculating bending stress for gears.

The AGMA bending stress equation is valid for only certain assumptions these assumptions are listed below.

1. There is no interference between the tips and root fillets of mating teeth.

2. No teeth are pointed.
3. The contact ratio is between 1 and 2.
4. There is nonzero backlash.
5. The root fillets are standard, assumed smooth, and produced by a generating process.
6. The friction forces are neglected.

The AGMA bending stress equation differs slightly for U.S and S.I specification gears because of the relationship between module and diametral pitch. The equation below is the AGMA bending stress equation for U.S gears.

$$\sigma_b = \frac{W_t p_d K_a K_s K_m}{F Y_j K_v} \tag{4}$$

Where;

W_t = Normal Tangential Load

K_m = Load distribution factor

K_v = Dynamic Factor

F = Face Width

Y_j = Geometry Factor

p_d = Diametral Pitch

K_a = Application factor

K_S = Size factor

All the factors listed above in the equation can be obtained from machine design books like [1]. The bending stress equation considers only the component of the tangential force acting on the tooth, and does not considers the effect due to radial force. According to theory the maximum bending stress occurs when the contact point moves near to the pitch circle because at that moment only one tooth is in contact and this tooth pair takes the entire torque.

2.3.2 Contact Stress

Pitting is a surface fatigue failure resulting from repetitions of high contact stress. When loads are applied to bodies, their surfaces deform elastically near the point of contact; so that a small area of contact is formed. It is assumed that, as this small area of contact forms, points that come into contact are points on the two surfaces that originally were equal distances from the tangent plane [9]. Pitting commonly appears on operating surfaces of gear teeth, a fundamental cause is excessive loading that raised contact stresses beyond critical levels. In this thesis contact stresses has been studied for five different models. The results thus obtained are compared with the AGMA Contact Stress equation, to see whether they are under the limit or not. The AGMA contact stress equation is as follows.

$$\sigma_c = \sqrt{\frac{W_t C_a C_m C_s C_f}{F I d C_v}} \tag{5}$$

Where,

W: Load per unit width

F: Face width of pinion

I: Geometry Factor

d: Pitch Diameter

C_a, C_m, C_s, C_f, C_v: Same as K factors.

CHAPTER 3

MODELING OF INVOLUTE GEAR

First and the foremost step in this thesis is to model a spur gear assembly. The most complicated part in spur gear is the involute profile of its teeth. There are number of ways of creating involute profile of a spur gear. In this thesis the spur gear model was designed in Pro Engineer design modeler. Pro/E is a suite of programs, which are basically used in designing and manufacturing range of products. Its field of applications is really wide, its been used by many of the manufacturing companies. Nowadays Pro/E is also becoming famous in telecom industry due to its versatility in assembling components effectively. This thesis basically deals with the solid modeling feature of Pro/E.

Solid modeling means that the computer model one creates contains all the information that a real solid object would have. It has volume and so, if one provides a value for density of its material, it has mass and inertia. Solid model differs from surface model on the pact that if a hole or a cut is made in a solid model, a new surface is automatically created and the model knows which side of surface is solid material. One of the best features of solid modeling is that it is impossible to create a computer model that is ambiguous or physically non-realizable.

For an example the figure shown below appears to be a three-pronged tuning fork at the left end, but only has two square prongs coming off the handle at the right end. So with solid modeling one cannot create a model, which doesn't exist physically or in real world. This type of ambiguity is quite easy to do with just 2D, wireframe, or sometimes even surface modeling.

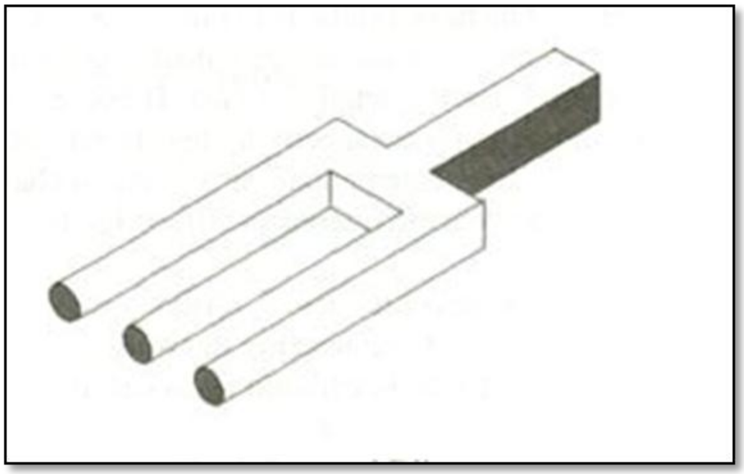


Figure 3.1 Incorrect Solid Model

In this thesis solid modeling is used to create a spur gear model. To create an involute profile of the gear, an option called as relation and equation modeling was adapted to make the model a general model. Relation and equation modeling is helpful if one wants to duplicate geometry depending on certain key factors. Like in this thesis an involute curve is created which has dependency on number of teeth, diametral pitch, pitch circle diameter and pressure angle. Below it is shown how this method helps to get the general involute spur gear model.

3.1 Relation And Equation Modeling

The word ‘Relation’ and ‘Equation’ itself gives the idea about relating the feature with the help of equations. Relations are used to express dependencies between the dimensions of a feature. Following are steps involved in this modeling procedure.

3.1.1 Defining Part Parameters

First step is to define the basic parameters on which the model has dependencies. This can be done by going to Tools / Parameters menu, and inserting the basic gear parameters. The table below lists all the basic parameters needed for modeling spur gear.

Table 3.1 Gear Design Parameters

Variable Name	Variable Type	Value	Description
N	Integer	23	No of Teeth
P	Real number	5.842"	Diametral Pitch
D	Real number	100 MM	Pitch Diameter
PHI	Real number	20	Pressure Angle

These parameters determine all the other parameters that define the gear tooth profile. The figure below shows the screen shot from Pro/E while defining parameters.

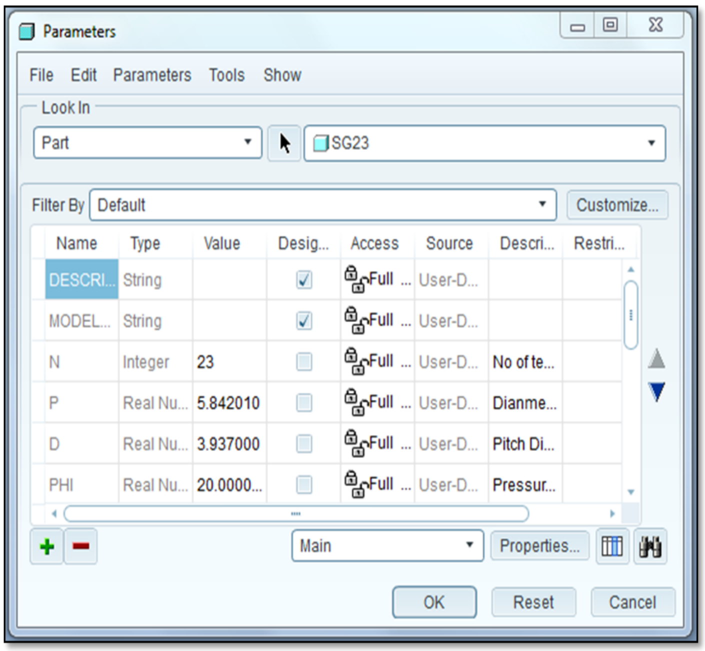


Figure 3.2 Gear Parameters

For the remaining parameters, which are to be related to basic parameters, relations can be defined in Tools / Relations menu. Formulas for the remaining parameters are written below.

Tools=> Relations

a=0.8/P

b=1/P

Ro=(D/2)+a

Ri=(D/2)*cos(PHI)

Rd=(D/2)-b

gamma_c=sqrt((D^2)/4-Ri^2)

theta0=(360/(4*N))-((gamma_c/Ri)*(180/pi))+atan(gamma_c/Ri)

After defining and relating all the gear parameters we can then as needed in sketched features. The figure below shows the screen shot from Pro/E after the completion of this step.

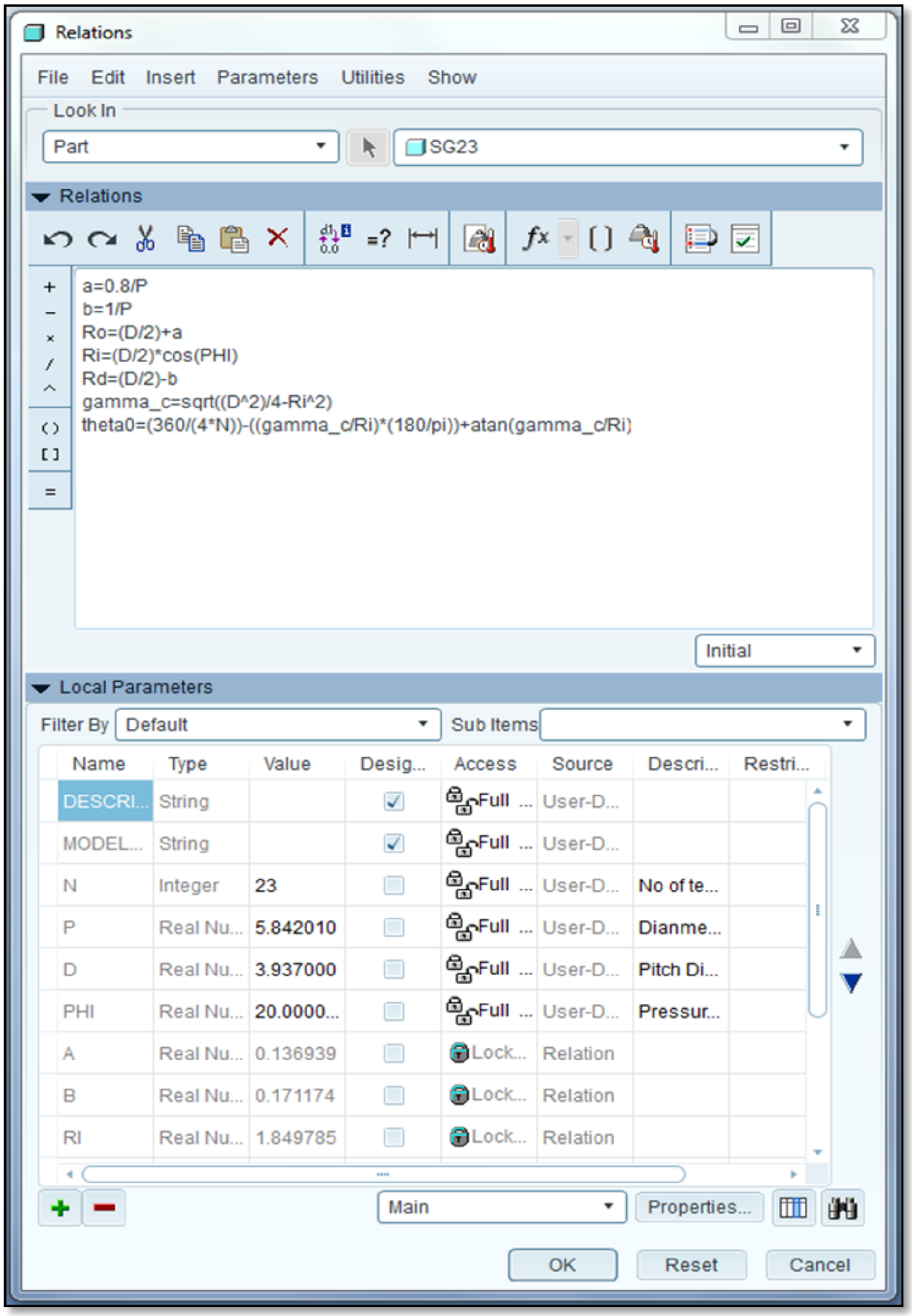


Figure 3.3 Relation and Equation for Gear model

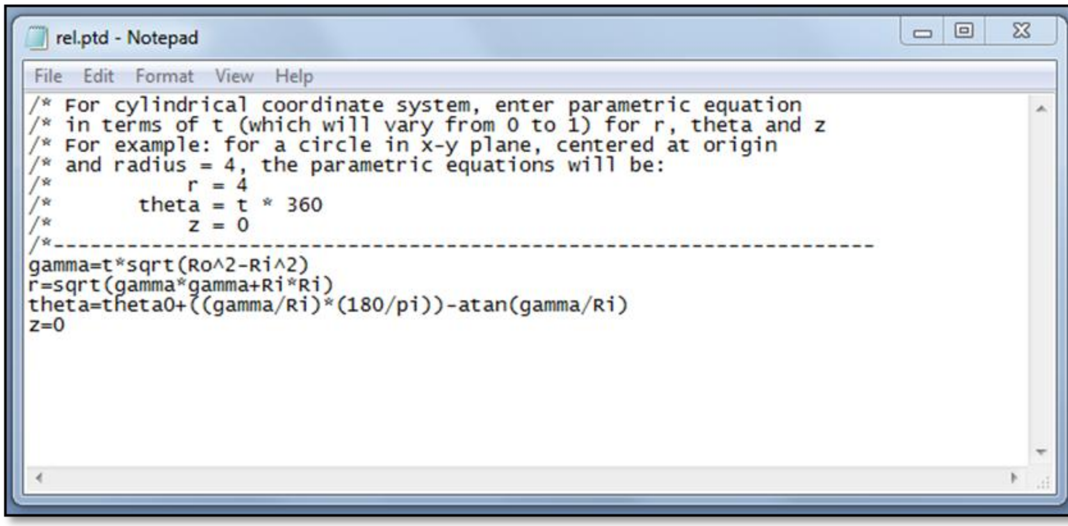
3.1.2 Extruding Base Feature

After defining the gear parameters, next step is to extrude circular disc having a diameter equal to addendum diameter and thickness equal to face width. To do this a circle is to be drawn with center on the sketch references and then go to Tools / Relations menu to define relations between the sketch dimensions and the part parameters. After defining these relations, the circle should have a diameter equal to the addendum diameter of the gear blank.

3.1.3 Generating Involute Curve

Next step is to define a datum curve showing the involute profile for radius $R_i \leq R \leq R_o$.Go to the Insert / Model Datum / Curve menu. Select From Equation, when prompted for a co-ordinate system for a part "PART_CSYS_DEF". Set the co-ordinate system to cylindrical system. At this point, a notepad window (shown below in the figure) will pop up where one can enter all the equations for the datum curve. Enter the equations listed above for R, θ and Z in terms of t (a parametric variable ranging from 0 to 1) and other part parameters. For the involute profile, the equations will be:

```
gamma=t*sqrt(Ro^2-Ri^2)
r=sqrt(gamma*gamma+Ri*Ri)
theta=theta0+((gamma/Ri)*(180/pi))-atan(gamma/Ri)
z=0
```



```
/* For cylindrical coordinate system, enter parametric equation
/* in terms of t (which will vary from 0 to 1) for r, theta and z
/* For example: for a circle in x-y plane, centered at origin
/* and radius = 4, the parametric equations will be:
/*          r = 4
/*      theta = t * 360
/*          z = 0
/*-------------------------------------------------------------------
gamma=t*sqrt(Ro^2-Ri^2)
r=sqrt(gamma*gamma+Ri*Ri)
theta=theta0+((gamma/Ri)*(180/pi))-atan(gamma/Ri)
z=0
```

Figure 3.4 Equations for Involute curve

When all the datum curve equations are entered, close notepad and click preview, a curve on the flat surface of the gear blank should be seen like the one shown in the figure below.

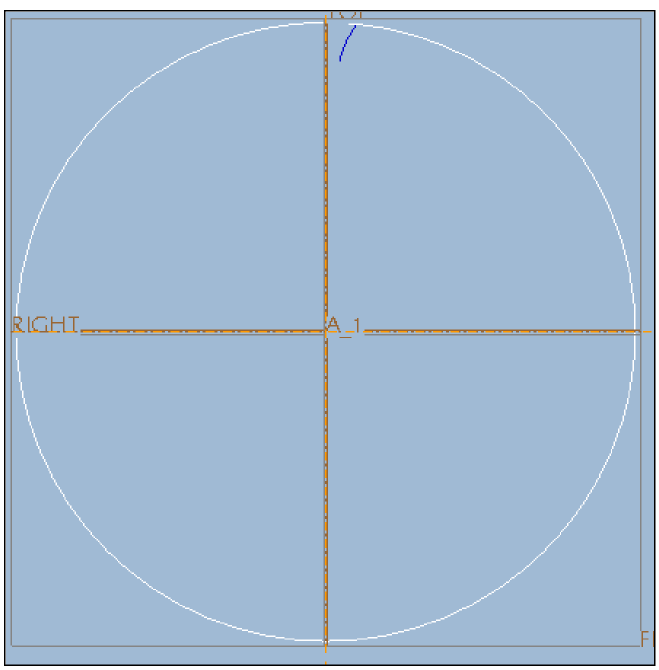


Figure 3.5 Generated Involute Profile

3.1.4 Sketching Tooth Space

Now as we have a datum curve outlining one side of an involute tooth space, the curve is used in a sketch to make and extruded cut through the gear blank. The skecth will consists of:

1. From $R_i \leq r \leq R_o$ the sketch will be coincident with the datum curve defined in Section 3.1.3.
2. At $r = R_d$, the sketch will be an arc of radius R_d.
3. From $R_d \leq r \leq R_i$, the sketch will be a radial line connecting the involute curve with the inner arc.

As the involute curves go all the way to the edge of the gear blank, it is not mandatory to make a sketch cut a closed profile. Start an extruded cut on the previous sketch plane, with depth all the way through the gear blank. Using datum curve as a guide:

1. Sketch a circle of radius smaller than the innermost point of the involute curve for the dedendum circle.
2. Use the edge option in the sketcher to make an edge coincident with the involute curve.
3. Draw a vertical centerline through the center of the gear blank.
4. Draw a line from the center of the gear blank to the innermost point of the involute curve.
5. Mirror the involute curve and the angled line across the vertical centerline.
6. Trim out all the excess line and arc segments, leaving a C-shaped profile made of two involute edges, two straight lines, and one arc.
7. Finally, go to Tools / Relations menu and set the radius of the inner arc to the value of parameter R_d.

After finishing the above-mentioned steps, the profile sketch should look like figure. Click the finish button to extrude the sketch.

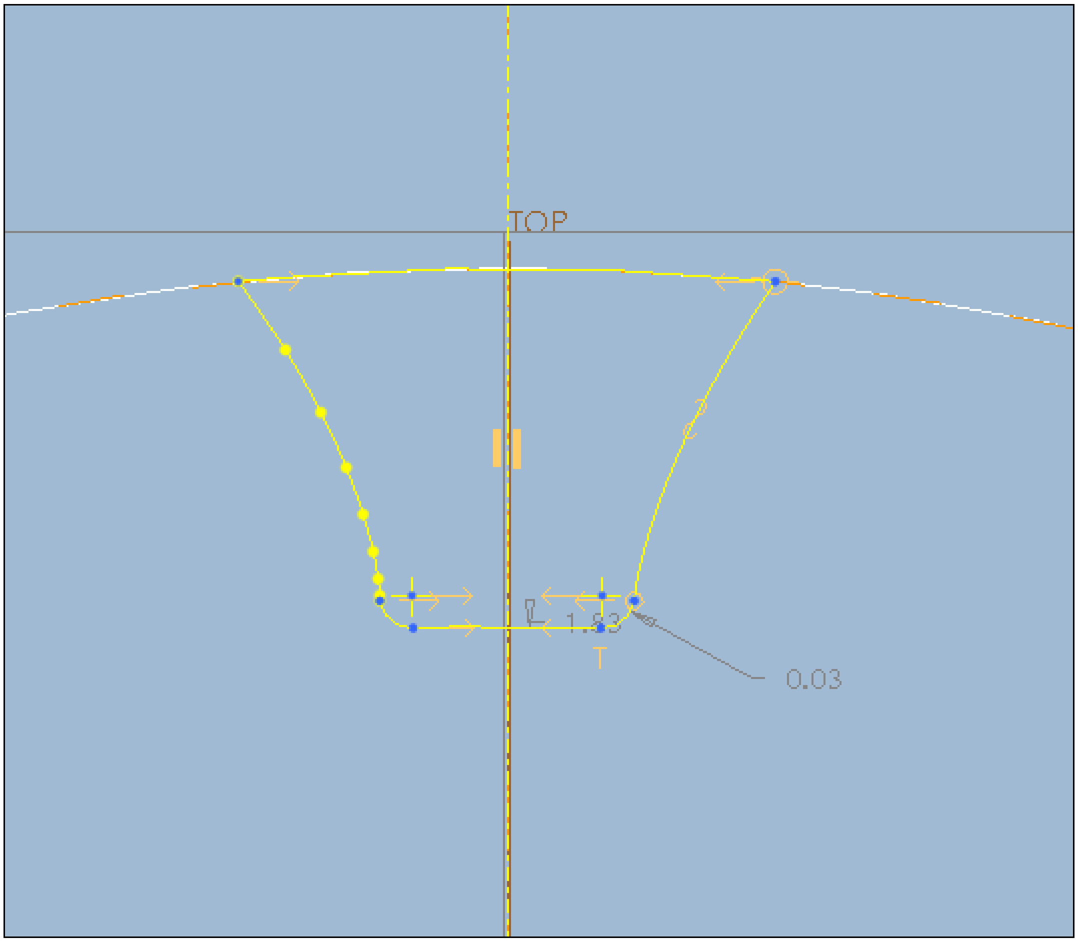


Figure 3.6 Gear Tooth space

3.1.5 Patterning Tooth Space

After a single tooth space is generated it has to be patterned along the centre axis of the gear blank. Using the pattern option in Pro/E and by selecting the following option one can easily pattern the tooth space, as needed depending on the number of teeth.

- Pattern Type: Axis Pattern
- Axis for pattern: Axis, which goes through the center of the gear blank.
- Number of copies: 23

Figure below shows the preview and the spur gear model created after patterning the tooth space.

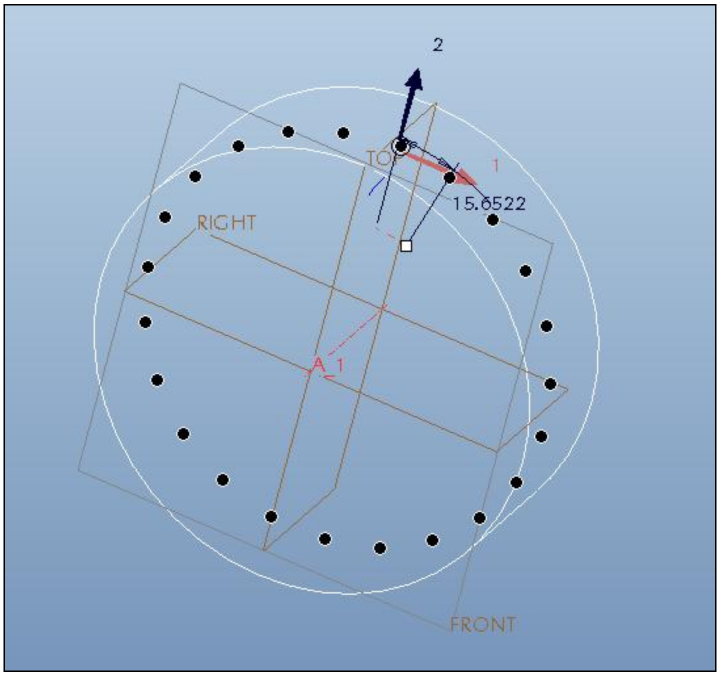


Figure 3.7 Tooth Pattern

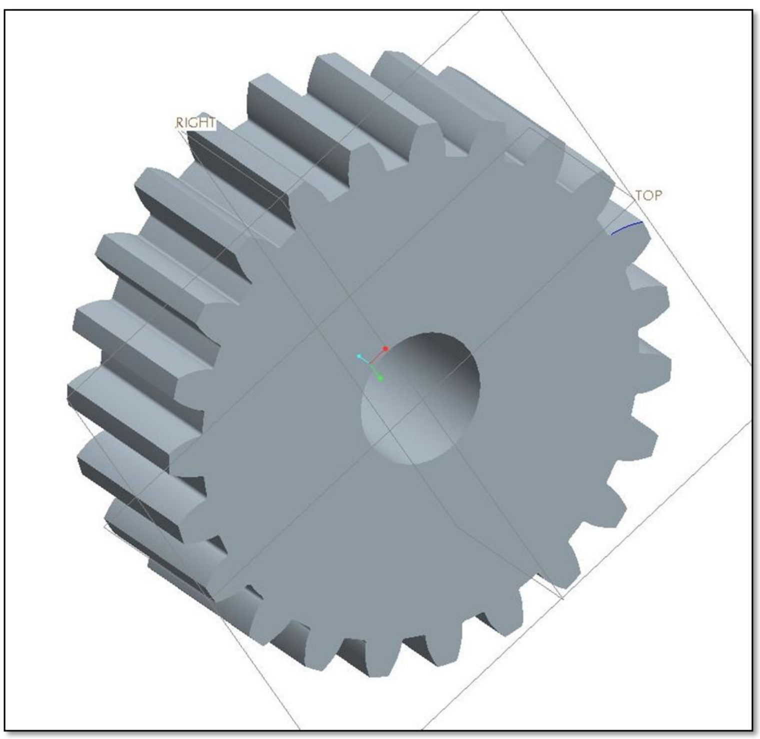


Figure 3.8 Spur Gear Model

3.2 Assembly Of Spur Gear In Mesh

To make the analysis precise and less time consuming, only gear with one tooth is taken into consideration. The figure below show the gear with only one tooth, this can easily done by removing the extra teeth.

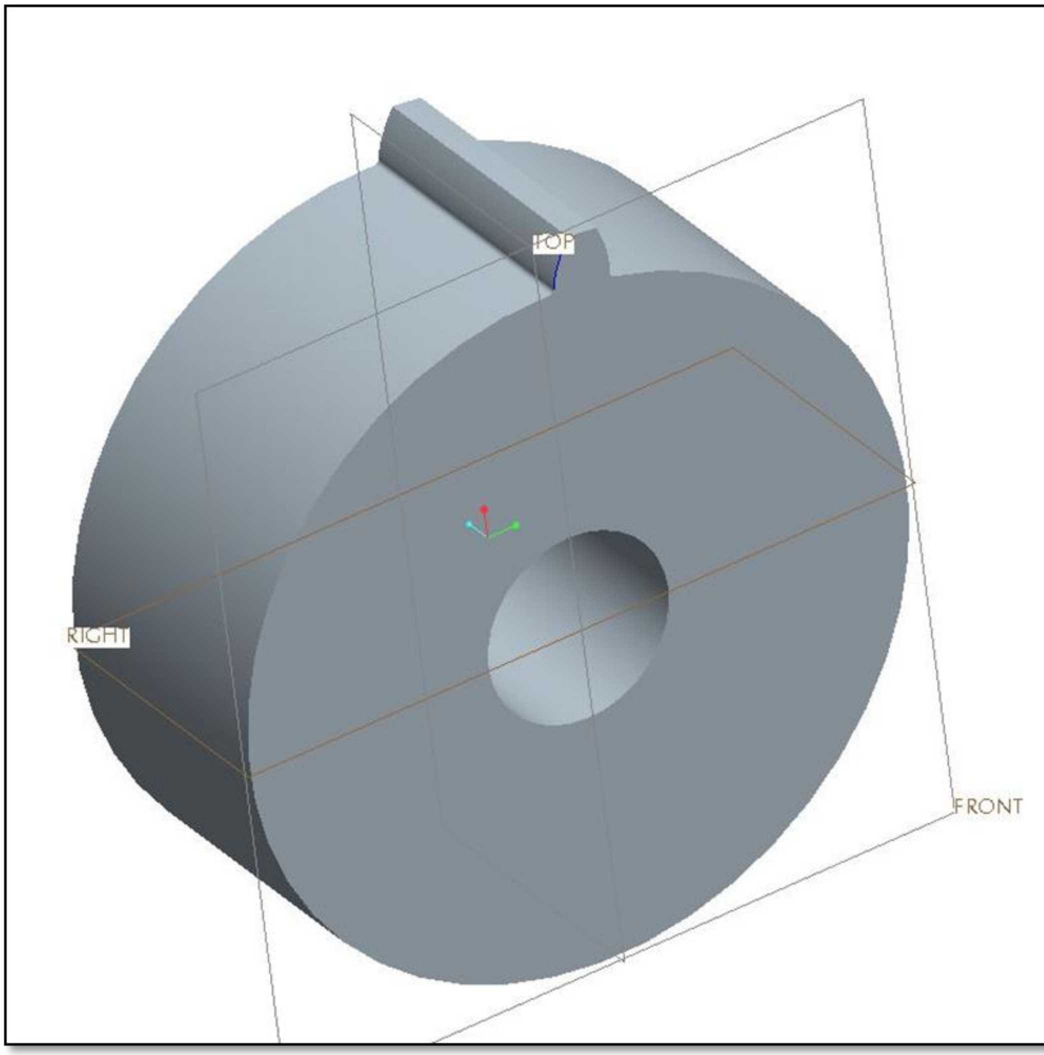


Figure 3.9 Spur Gear with one tooth

After the gear with one tooth is ready, it has to be opened in assembly option in Pro/E. The two gears are then assembled by defining the tangential contact between the teeth and by constraining them to calculated center distance. The image below from Pro/E shows two gears meshed at their pitch circle radius.

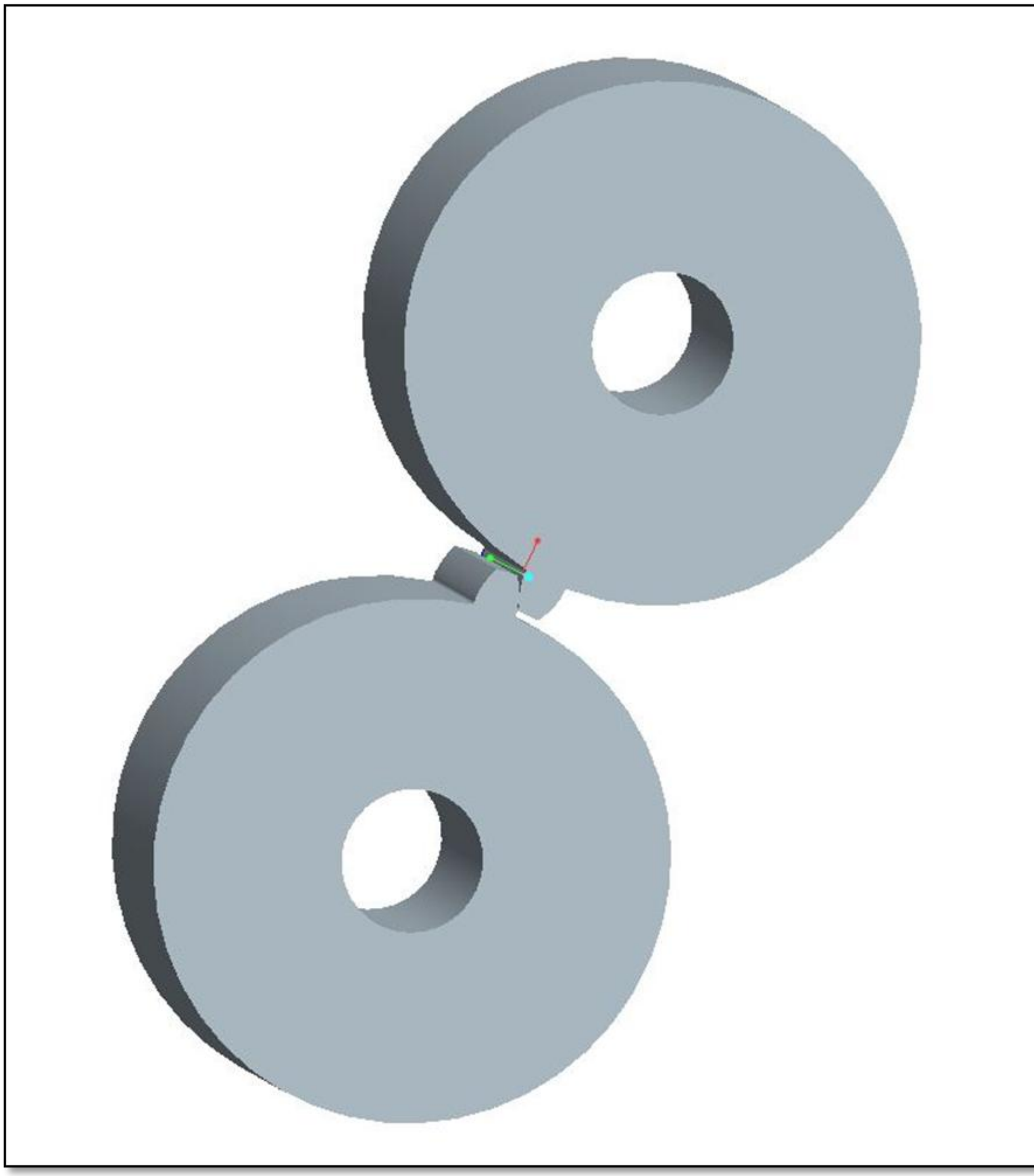


Figure 3.10 Spur Gear Assembly

3.3 Helical Gear Modeling

The modeling of the helical gear near about same as the spur gear. The tooth profile is same as spur gear i.e involute. So in order to model helical gear in Pro/E one will need helix angle along with all the parameters required for spur gears. Intially steps in helical gear is same as of spur gear. The only difference is that the tooth follow the helical path instead of the straight path followed by spur gear.

Follow the steps from spur gear till 3.1.4 which is generating single tooth space. For helical gear model this tooth space is to be sketched and then a dependant copy of this tooth space is to be done. The whole feature of the tooth space has to selected and then it has to be rotated dependng on the value of the helix angle. Once the tooth profile is copied and rotated, next step will be just to extrude it and then pattern it depending on the number of teeth. The figure shows the Helical designed for analysis.

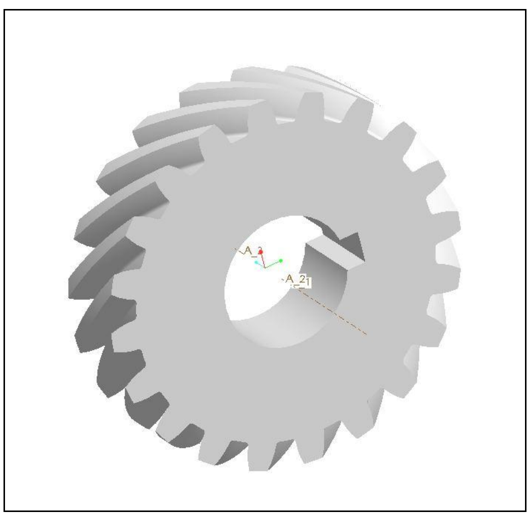


Figure 3.11 Helical Gear

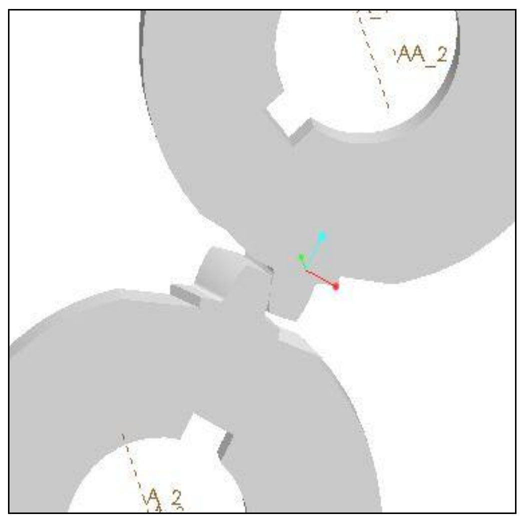


Figure 3.12 Helical Gear Assembly

CHAPTER 4

FINITE ELEMENT ANALYSIS OF AN INVOLUTE GEAR

4.1 Procedure

Using the method analyzed in previous chapter five different models of spur gear were created. Using the assembly option in Pro/E five assemblies were created corresponding to the models. The assembly, which was created in Pro/E, was imported in ANSYS Workbench 12 for further analysis. The other way of importing the assembly is by importing the IGES or STEP file of the assembly. In this chapter the spur gear and helical gear assembly is subjected to assume driving condition and are analyzed for the bending and contact stresses. The results thus obtained from ANSYS are then compared with AGMA theoretical stress values. The table below lists the dimension used for the spur and helical gears.

Table 4.1 Spur Gear parameters

1	Modulus of Elasticity	200 GPa
2	Poisson's Ratio	0.3
3	Type of Gear	Standard Involute, Full depth
4	Module	4 mm
5	Pressure Angle	20°
6	Face Width (F)	43 mm
7	No of teeth (N)	23
8	Pitch Diameter	100 mm
9	Transmitted Load (W)	3000 lbs
10	Revolution Per Minute (RPM)	3000

Table 4.2 Helical Gear Parameters

1	Modulus of Elasticity	200 GPa
2	Poisson's Ratio	0.3
3	Type of Gear	Standard Involute, Helical
4	Module	4 mm
5	Pressure Angle	20°
6	Helix Angle	45°
7	Face Width (F)	5.08 mm
8	No of teeth (N)	20
9	Pitch Diameter	85 mm
10	Transmitted Load (W)	3000 lbs
11	Revolution Per Minute (RPM)	3000

After the assembly is imported in ANSYS Workbench 12, assembly is subjected to the boundary conditions. In this thesis it is assumed that the one gear is fixed and the other gears is given torque along its axis. As both the teeth are already in contact, the main purpose is to study the root bending stress and the contact stresses due to the applied torque. Following are steps followed for the Finite Element Analysis.

4.2 Three Dimensional Analysis Of Spur Gear

ANSYS has many type of analysis, so it is necessary to select the correct type of analysis from the menu bar. As the imported geometry is 3-Dimensional, select 3-D and Static Structural Analysis from menu and connect the geometry to the analysis tab. Then the next step is to enter the Young's Modulus and Poisson's ratio of the material. This can be done by selecting the Engineering Data from the analysis tab and inserting the corresponding values.

4.2.1 Defining Contact Region

Once the geometry is attached with Static Structural analysis tab, next thing is to define the contact between the two involute teeth. ANSYS has an inbuilt option, which automatically reads the attached geometry for any predefined contacts or other boundary definitions. The contact between the two teeth is assumed to be frictionless; the figure below shows the contact being defined as frictionless. One of the most important things is to change the 'Interface Treatment' to 'Adjust to touch'. This option defines the kind of contact between the selected bodies. The figure below shows the image from ANSYS showing the contact defined for the two spur gear teeth in mesh.

Scope	
Scoping Method	Geometry Selection
Contact	1 Face
Target	1 Face
Contact Bodies	SG23[39]
Target Bodies	SG23[46]
Definition	
Type	Frictionless
Scope Mode	Automatic
Behavior	Symmetric
Suppressed	No
Advanced	
Formulation	Augmented Lagrange
Interface Treatment	Adjust to Touch
Normal Stiffness	Program Controlled
Update Stiffness	Never
Pinball Region	Program Controlled
Time Step Controls	None

Figure 4.1 Defining Contact

4.2.2 Mesh Generation

The mesh with the default settings is not adequate to get the accurate results. In this analysis both the gear were finely meshed with 'Sizing' option in menu. The element size was chosen to be 0.001 and it was then refined at the bending surfaces to get the finer mesh and continuous stress values. The image below shows the meshed assembly according to the size written above. The mesh looks fine enough for the analysis.

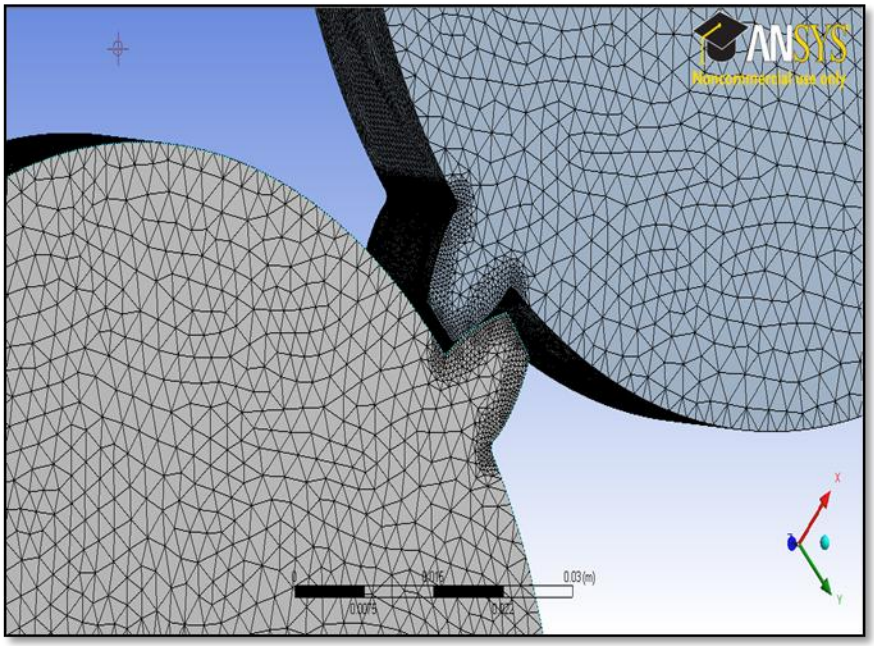


Figure 4.2 Three Dimensional Model with Mesh

4.2.3 Supports and Loads

The lower gear is given a fixed support and the top gear is given frictionless support. The top gear is also given a toque or a moment in clockwise direction. The image below shows how the supports and loads were applied to the gear model.

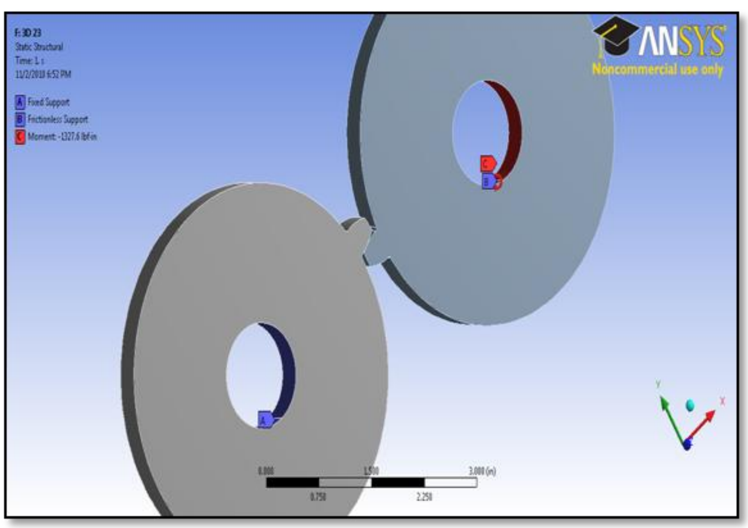


Figure 4.3 Applied Boundary Condition

4.3 Two Dimensional Analysis Of Spur Gear

3-D assembly of spur gear is converted into 2-D assembly by using the inbuilt design modeler of ANSYS 12. The figure below shows the meshed 2-D geometry of two spur gears in contact.

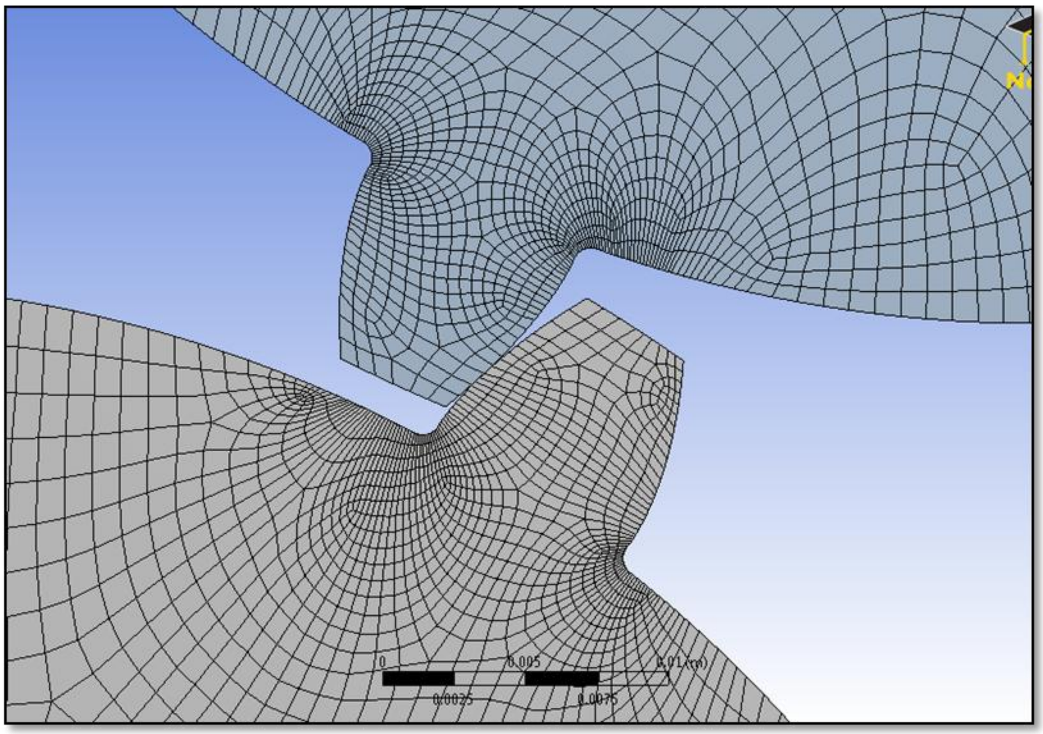


Figure 4.4 Two-Dimensional Model with Mesh

After the 2-D model is generated the same procedure as the 3-D model is to be followed. The only difference is that this will be 2-D analysis.

4.4 Root Bending Stress Of Spur Gear

Using ANSYS, two and three dimensional root bending stresses are obtained, which are then compared with the AGMA theoretical stress. The AGMA bending stress equation was mentioned in previous chapter. The images below shows the stress distribution in 3-D and 2-D models respectively. The table followed lists the results for five different models along with the percentage of error between 3-D and AGMA Stress values.

AGMA Bending Stress for spur gear model:

$$\sigma_b = \frac{W_t p_d K_a K_s K_m}{F Y_j K_v} => \frac{674.426 * 5.84 * 1 * 1.6}{1.7 * 0.36 * 0.8} = 12871.4 PSI = 88.74 MPA$$

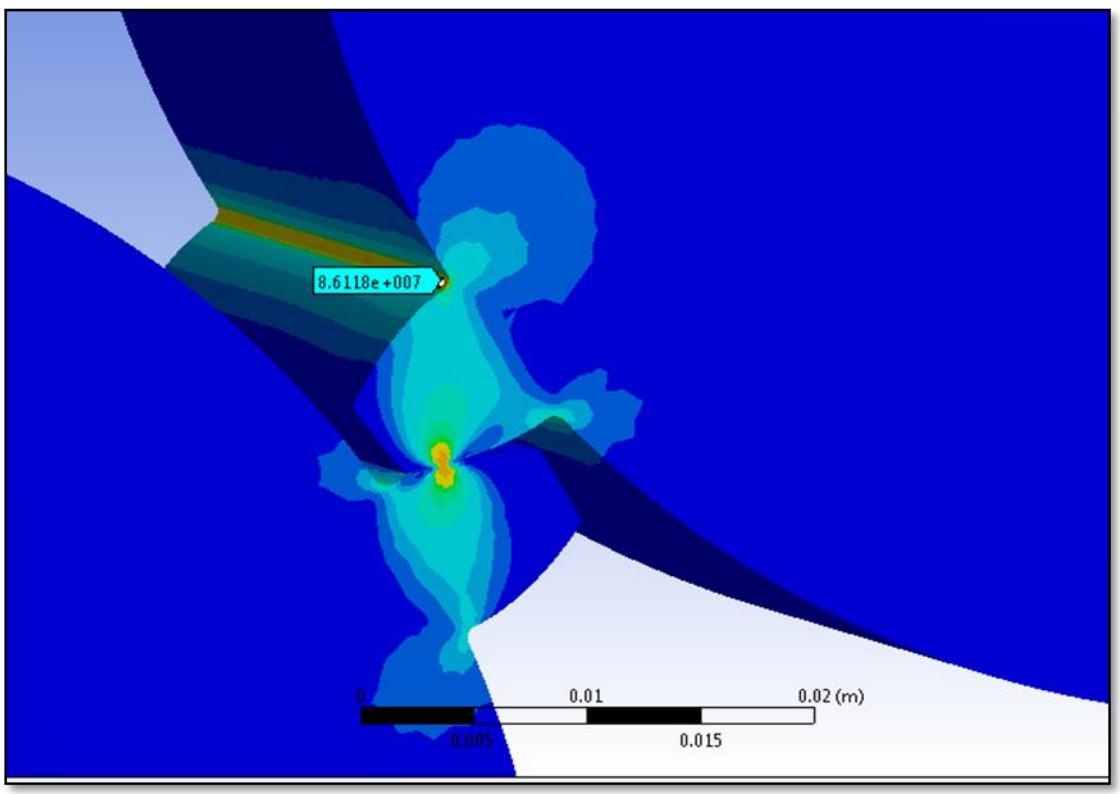


Figure 4.5 Three Dimensional Von Mises Stress

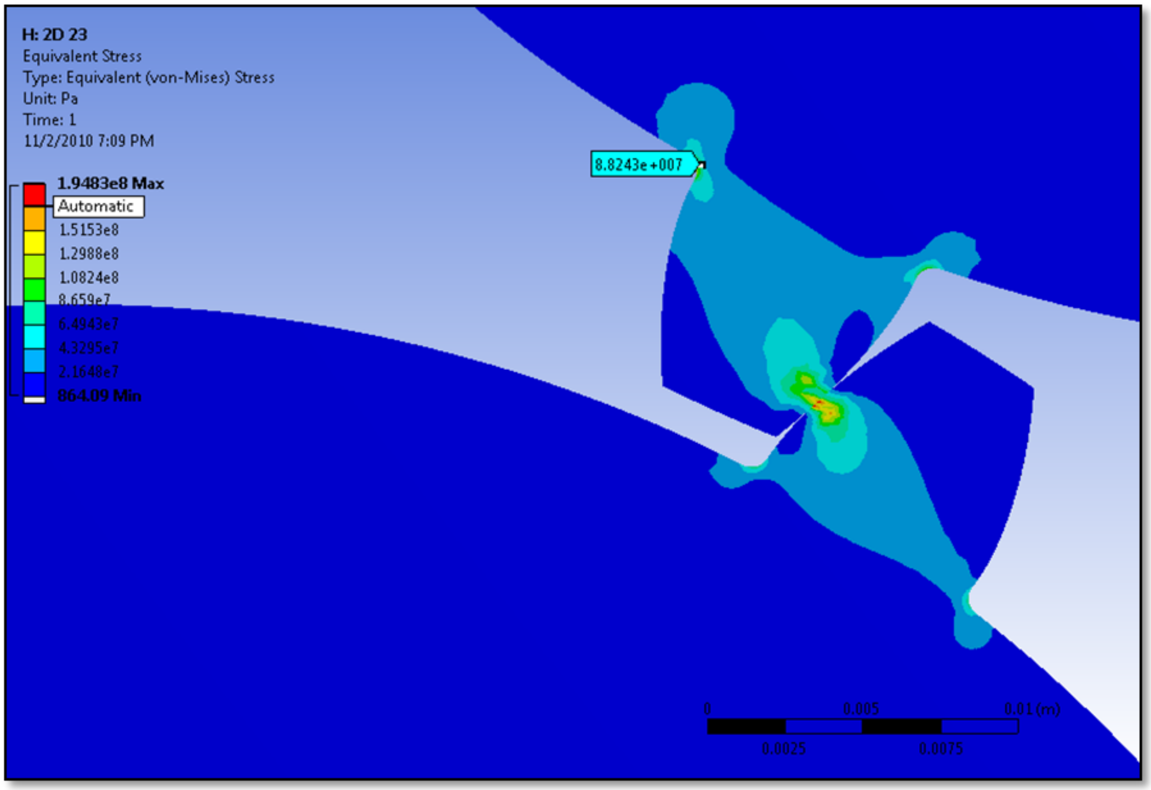


Figure 4.6 Two Dimensional Von Mises Stress

Table 4.3 Von Mises (Bending) Stresses for Spur gear Models

No of Teeth (N)	AGMA Stresses (MPA)	3D Stress (ANSYS) (MPA)	2D Stress (ANSYS) (MPA)	Difference Error (%)
23	88.74	86.18	88.24	2.92
25	92.27	94.23	94.87	2.10
28	103.34	100.64	100.99	2.64
31	114.41	116.65	116.27	1.94
34	125.5	123.28	126.91	1.78

4.5 Root Bending Stress Of Helical Gear

Helical gear assembly was imported in ANSYS 12 and the same boundary conditions were applied as the spur gear model. The model was analyzed for the root bending stress for the applied moment. In helical gear only 3-D analyses was performed because of the helical profile of its teeth. The table below The figure below from ANSYS shows the stress distribution plot along the tooth.

AGMA Bending Stress for helical gear model:

$$\sigma_b = \frac{W_t p_d K_a K_s K_m K_B K_I}{F Y_j K_v} => \frac{674.426*6*1*1.6*1*1}{0.2*0.36*0.8} = 112404.4PSI = 775MPA$$

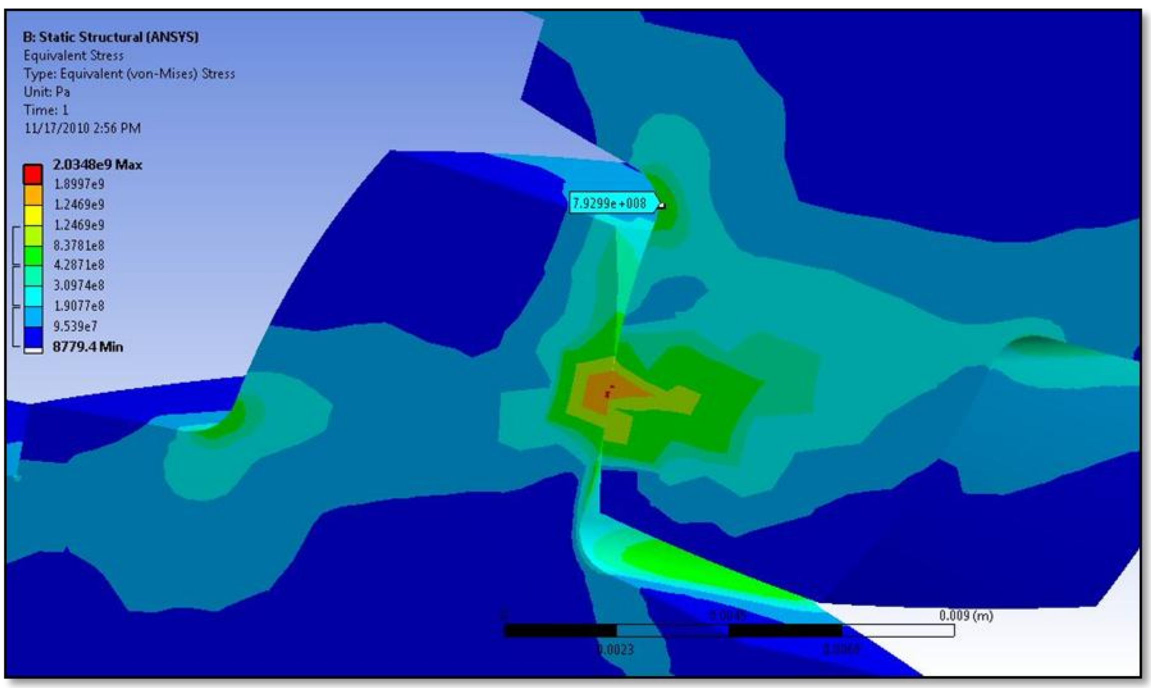


Figure 4.7 Three Dimensional Von Mises (Bending) Stress of Helical Gear

The results obtained from ANSYS 12 for Root Bending stress is $\sigma_b = 793MPA$. While results obtained from AGMA bending stress equation is $\sigma_b = 775MPA$. The results show good compliance with an error difference of 2.3%.

4.6 Von Mises Contact Stresses Of Helical And Spur Gear

As already mentioned high contact stresses results in pitting failure of the gear tooth, it is necessary to keep contact stresses under limit. Contact stresses were studied in the same manner as bending stresses were calculated. In this thesis Von Mises Contact stresses are obtained at the contact region. The image below from ANSYS shows the stress at the contact. The table below lists the contact stresses from all the Spur and Helical gear models.

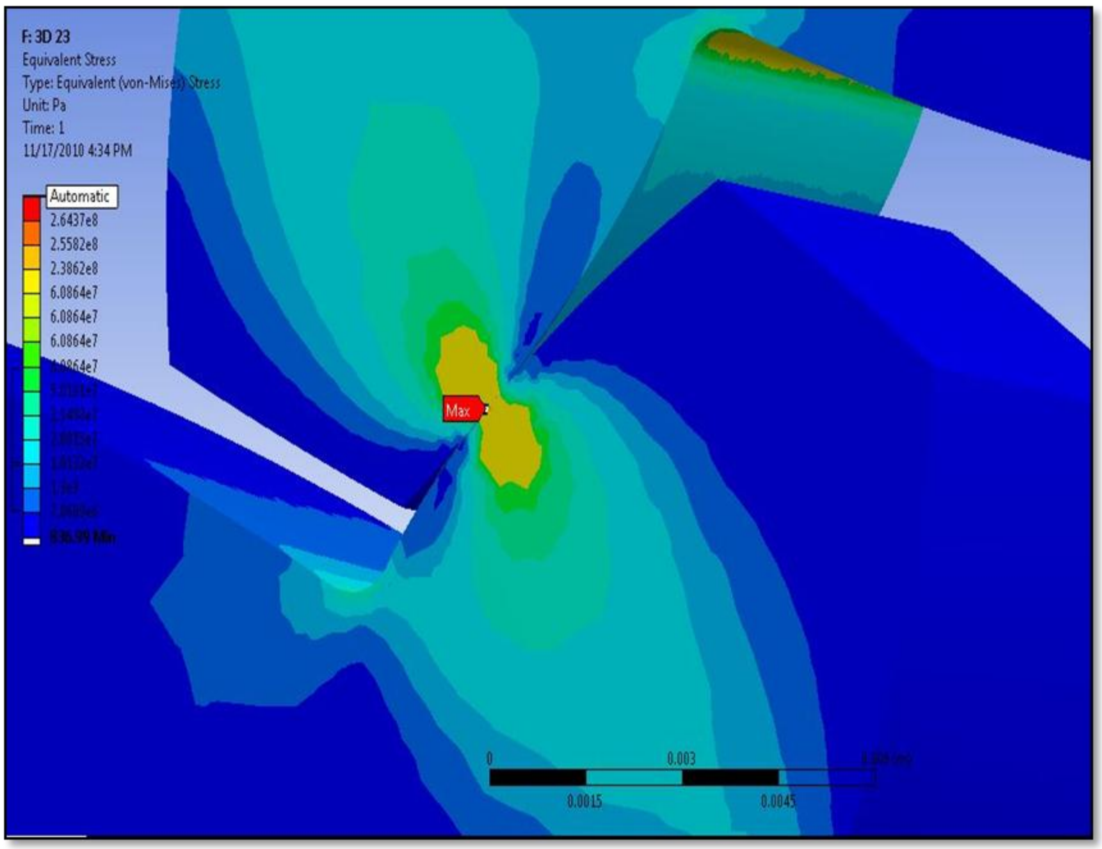


Figure 4.8 Three Dimensional Von Mises (Contact) Stress for Spur Gear

Table 4.4 Contact Stresses for Gear Models

Gear Type	No of Teeth (N)	Von Mises Stress (3D) (MPA)	Von Mises Stress (2D) (MPA)
Spur	23	282	275
Spur	25	239	212
Spur	28	394	285
Spur	31	286	299
Spur	34	230	217
Helical	20	2034	--

Stress values obtained from AGMA contact stress are Maximum value of contact stress. For Spur Gears $\sigma_h(Max) = 663MPa$ and for helical gears $\sigma_h(Max) = 2782MPa$.

The Von Mises Contact stress values from ANSYS when compared to Hertz Stress shows that the stresses all well under the maximum value, and hence the model designed is safe from pitting failure.

In order to find the contact stresses accurately one might need to analyze only contact stresses by creating a contact element at the meshing point. This thing can be done in ANSYS classic software. The key point here is to define the type of contact.

It is known from [7] that the peak value of the equivalent stress using the Von Mises criterion, the maximum shear stress, and the maximum orthogonal shear stress can be calculated from the maximum Hertz Stress by using the equation mentioned below.

$$\sigma_{VonMises} = 0.57\sigma_H$$

$$\tau_{MaxShear} = 0.30\sigma_H$$

$$\tau_{OrthoShear} = 0.25\sigma_H$$

Where σ_H is the maximum Hertz Stress.[7]

So using this Von Mises criterion contact stresses can be compared with the theoretical Hertz Contact stresses.

CHAPTER 5

CONCLUSION & FUTURE WORK

In this study a three dimensional deformable-body model of spur and helical gear was developed. The results obtained were then compared with the AGMA theoretical stress values. The results are in good congruence with the theoretical values, which implies that the model designed is correct. According to the results obtained following conclusions can be drawn.

1. By using a relational equation modeling in Pro Engineer, one can accurately design complicated parts like involute tooth gears. This process can be very helpful in contact problems as it needs model with high accuracy It also decreases the lead times and improves overall engineering productivity.
2. A discrete model of involute gear was proposed. The Finite Element results matched well with the numerical results. Thus this parametric model turns out to be a fast and accurate method of computing stress problem of the involute tooth gear system.
3. It was seen that numerically obtained bending stress values by AGMA standards were in good agreement with the Finite Element 3-D and 2-D stresses, and the Von Mises Contact stresses were also under maximum value. Thus this study provides a tool to help understand the research and development for involute gear design.

At the end of this work, it is concluded that solving gear stress problems in design is fairly easy by use of commercial FEM and designing packages available today.

This study can be extended for calculation of accurate contact stresses by using the contact element in ANSYS classic and then it can be analyzed for different types of contact depending on the lubrication used.

APPENDIX A

VON MISES (BENDING) STRESS PLOTS FOR OTHER GEAR MODELS

Root Bending Stress Plots For Other Gear Models

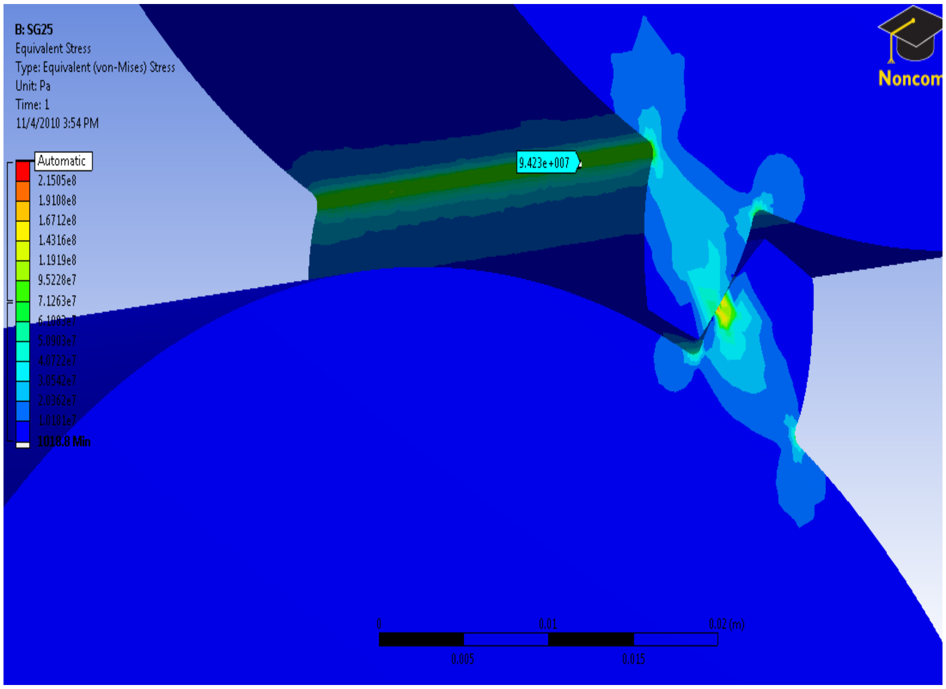


Figure A.1 3-D Von Mises Stress for Gear with 25 Teeth

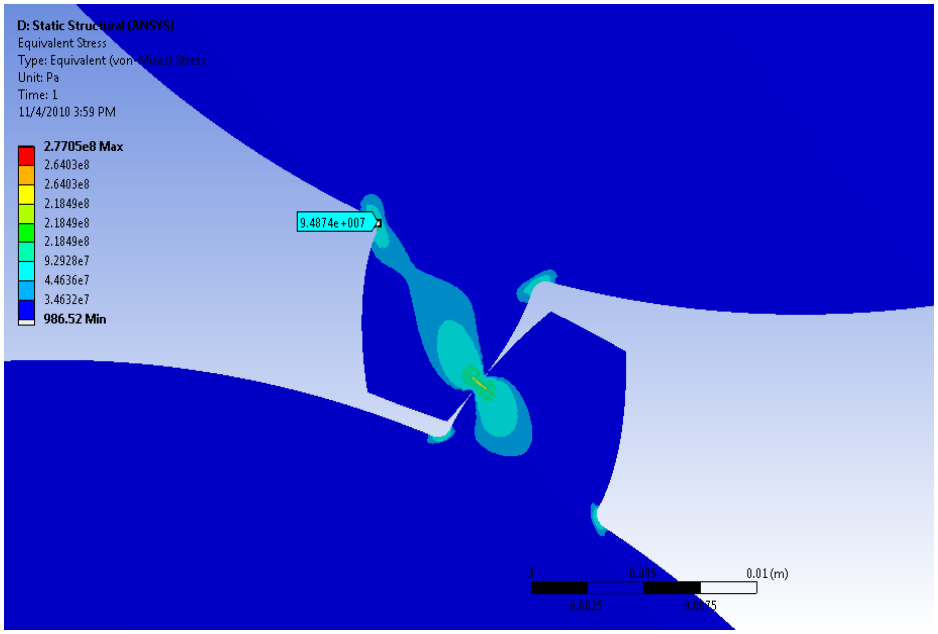


Figure A.2 2-D Von Mises Stress for Gear with 25 Teeth

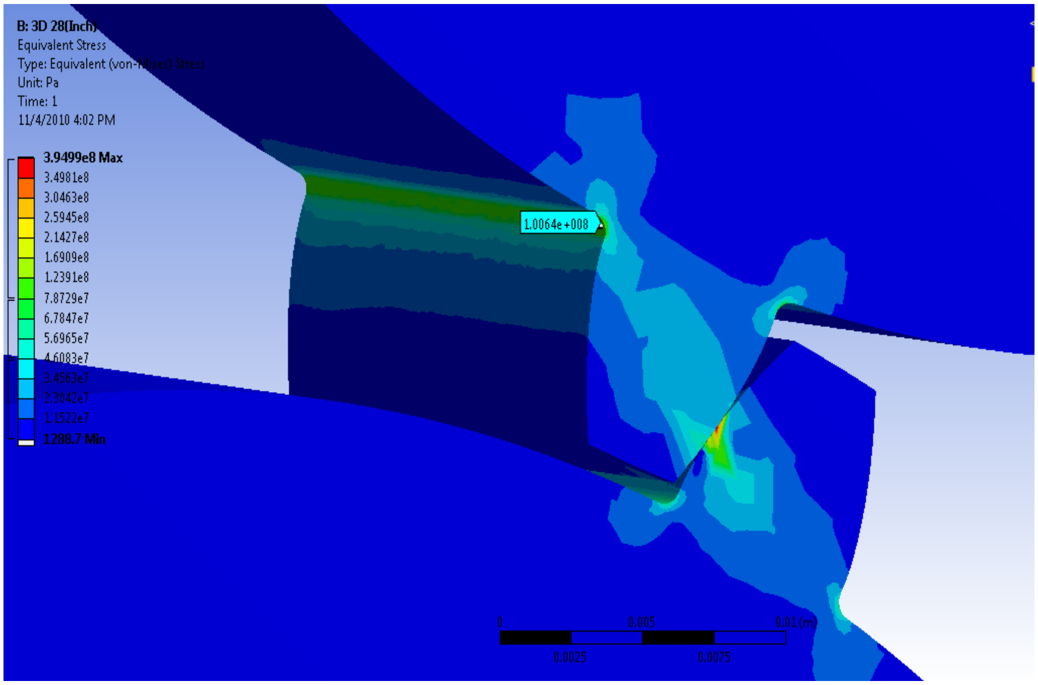


Figure A.3 3-D Von Mises for Gear with 28 Teeth

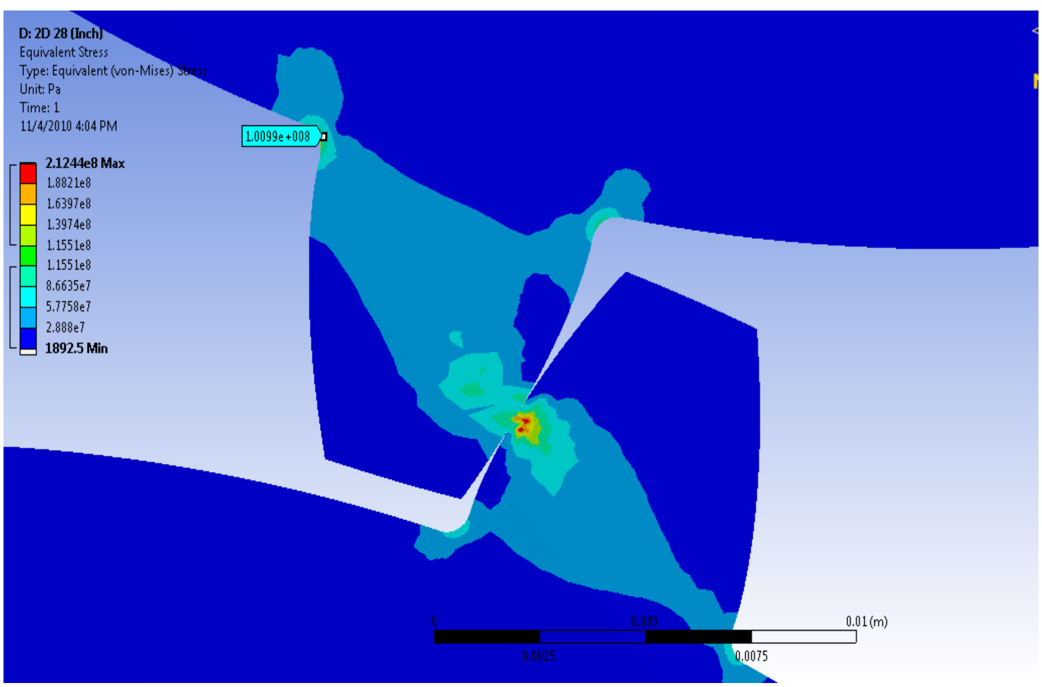


Figure A.4 2-D Von Mises for Gear with 28 Teeth

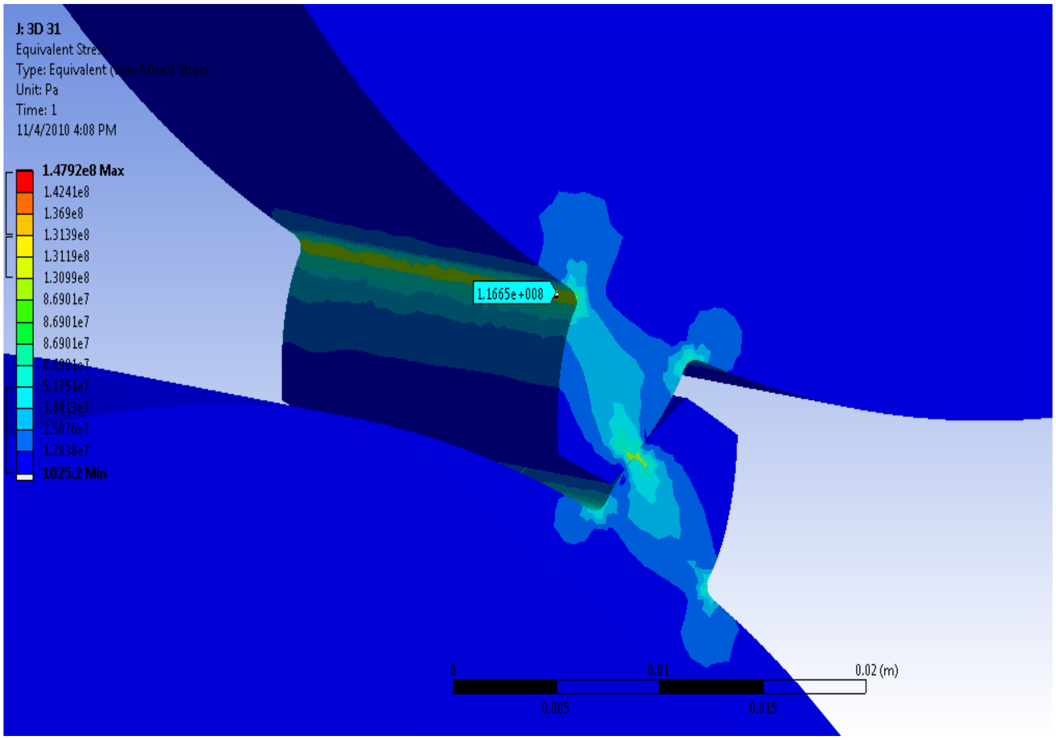


Figure A.5 3-D Von Mises for Gear with 31 Teeth

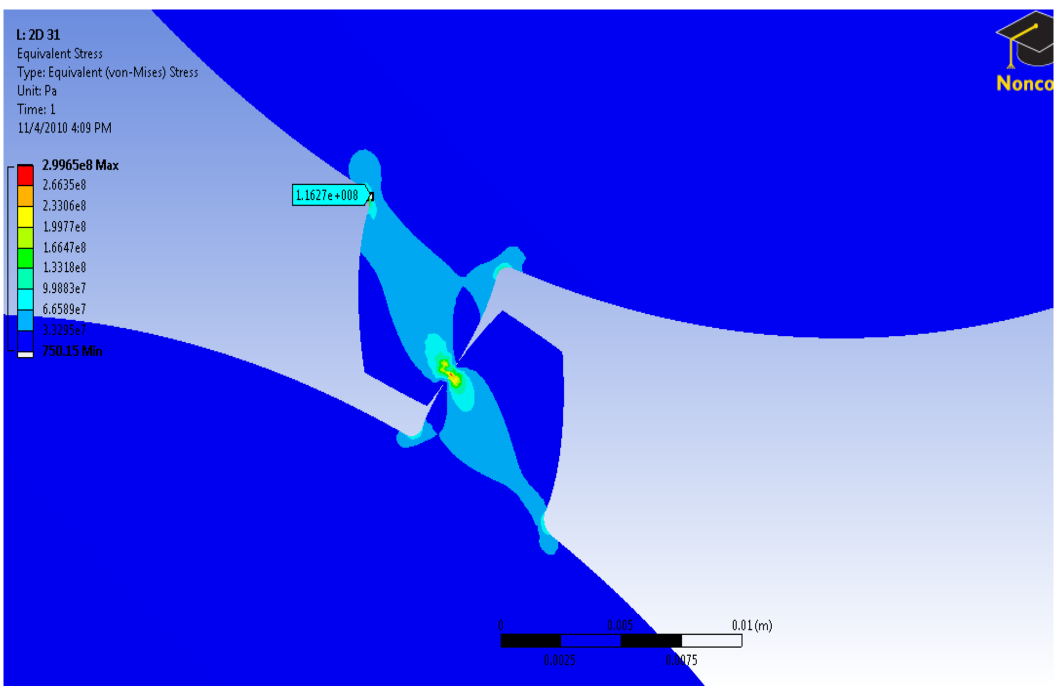


Figure A.6 2-D Von Mises for Gear with 31 Teeth

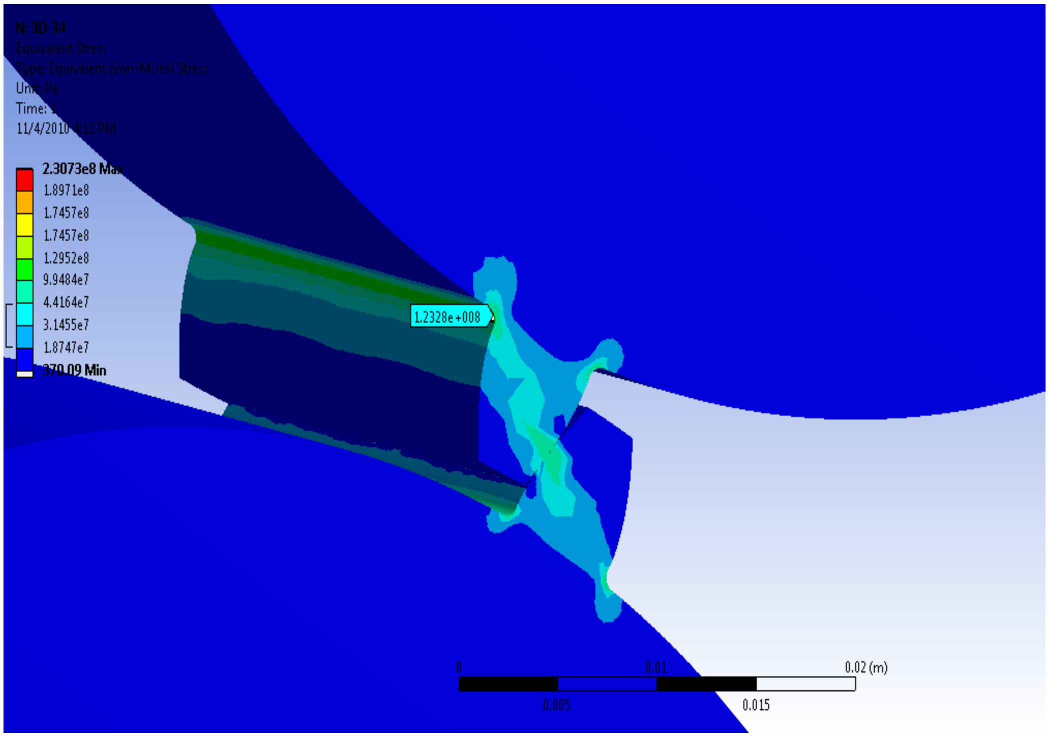


Figure A.7 3-D Von Mises for Gear with 34 Teeth

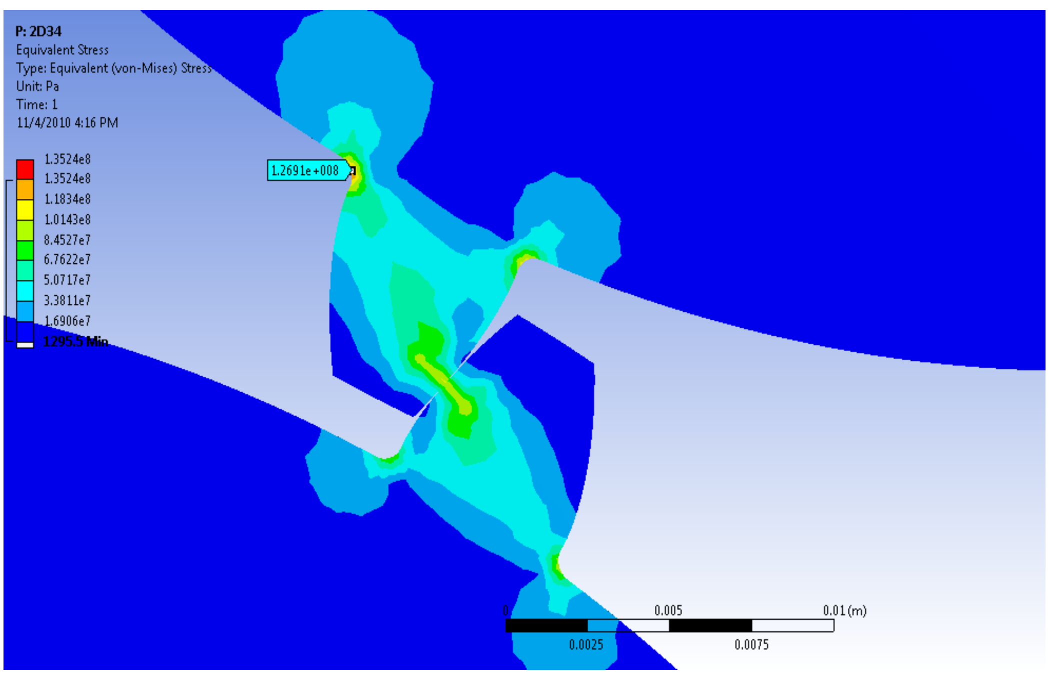


Figure A.8 2-D Von Mises for Gear with 34 Teeth

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BIOGRAPHICAL INFORMATION

Shreyash Patel received his Master of Science Master of Science in Mechanical Engineering degree from The University of Texas at Arlington in December 2010. Author completed his bachelor's degree in Mechanical Engineering from KITS Ramtek in July 2008. His research interests include finite element analysis, solid modeling in Pro/E and structural dynamics.