

DETERMINANTS OF IMPLIED VOLATILITY
MOVEMENTS IN INDIVIDUAL
EQUITY OPTIONS

by

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ABSTRACT

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In this study, I introduce a parsimonious model that explains implied volatility time series for individual stock options. The current state of risk management for individual equity options still seems to lack the presence of pertinent exogenous variables. This study suggests a few easily observable variables that can be used to explain the changes in implied volatilities of stock options. These variables can be used in the risk models in order to more accurately manage option positions for individual stocks. The first chapter provides a motivation for the VIX as the primary explanatory variable for changes in implied volatility. It also examines the role of fundamental variables. The second chapter shows that the VIX as a good explanatory variable for explaining changes in implied volatility. It also examines the return of the underlying asset as an explanatory variable. Various techniques are used to determine the efficacy of the variables such as Fama-Macbeth cross-sectional regressions, Principal Component analysis, and individual regressions for each company in the sample. The final chapter examines risk premia in straddle returns and provides a practical application of volatility hedging.

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CHAPTER 1
FUNDAMENTAL VARIABLES

1.1 Background

For many years, options portfolio managers have been at the mercy of the Greeks and have had to take a proactive approach to hedging their exposure. They were able to hedge a certain portion of their risk to adverse changes in the underlying asset by trading the underlying asset. But, where do portfolio managers turn to hedge their implied volatility exposure? Nowhere. If a portfolio manager wants to bet that a stock pick a bottom, what does he or she do to participate in the potential upside of the underlying asset, but does not have to participate in further decline of the underlying asset price? The traditional strategy to accomplish this goal is the long call option. The problem is that implied volatility has been rising significantly for the underlying asset, on average, as the asset has fallen in value. This forces the manager to pay a much higher premium for the call which requires the stock to rebound almost violently for the manager to make any money. The second approach the manager can use is a vertical call spread whereby the manager offsets the cost of the purchase of one call option by selling another call option at a higher strike price. This mitigates the situation of paying high premiums for just one call option but this vertical call spread is also subject to shifts in implied volatility. In this paper, I introduce ways that an options investor can hedge their implied volatility exposure. This will enable the investor to participate more effectively in directional bets made on underlying assets.

It has been predicted by the Constant Elasticity of Variance model that there should be a relationship between the movements in the underlying asset and the implied volatility of the underlying asset. My work shows that there is another force that plays a very large role in terms of implied volatility. There is a tradable instrument that exists in this new age of financial

innovation that allows investors to hedge their volatility exposure. This instrument has become very liquid in recent years and some investors are taking advantage of this instrument to hedge their underlying long/short equity exposure. They have only scratched the surface as far as the potential benefits this instrument can provide. We are living in a time when volatility can be thought of as an asset that can be traded strategically to yield positive risk adjusted returns.

In this study, I introduce a parsimonious model that explains implied volatility time series for individual stock options. The current state of risk management for individual equity options still seems to lack the presence of pertinent exogenous variables. This study suggests a few easily observable variables that can be used to explain the changes in implied volatilities of stock options. These variables can be used in the risk models in order to more accurately manage option positions for individual stocks.

It has been shown by Sharpe, in his seminal paper introducing the Capital Asset Pricing Model (CAPM), that the market return is a significant explanatory variable for individual stock returns. The CAPM states that the market compensates investors for taking systematic risk by investing in stocks that are incorporated into a well diversified portfolio. Put another way, the changes in the market price in an individual stock are related to the changes in the market price of the market.

The most important explanatory variable introduced in this paper is the implied volatility of the market portfolio, namely the S&P 500 index which is measured by the VIX. The VIX is a weighted average of short term call and put implied volatilities for the S&P 500 index, and is maintained by the Chicago Board of Options Exchange (CBOE). On average, the VIX explains the variance of the implied volatility of the options on individual stock. So, in essence, the stochastic volatility model introduced in this paper is really just the CAPM for implied volatility or an Implied Volatility Asset Pricing Model (IVAPM). We will introduce a "beta" measure for the systematic risk in terms of implied volatility of the market portfolio.

I will show later that the relationship between implied volatilities of individual stocks and the VIX is quite strong. It can also be shown that changes of the S&P 500 can be used as an explanatory variable for changes in the VIX. If percent changes of the S&P 500 are used to explain percent changes in the VIX, the parameter estimate for the explanatory variable is about negative 2.63 and is statistically significant. This means that if the S&P 500 goes down 1 percent in a month, then the VIX should increase by 2.63 percent. I will also include percent changes in the underlying stock, in addition to the VIX, to explain the implied volatility of the underlying stock. This variable should also be significant and should complement the VIX in explaining implied volatility of individual stocks.

Option investors are essentially trading volatility through their Vega exposure. This is analogous to a portfolio manager being exposed to the systematic risk through the CAPM beta. Many risk management applications for an options portfolio include an aggregate Vega measure that describes the expected change of the portfolio value with respect to a change in implied volatility. This sounds good if we assume unit elasticity of individual implied volatilities with respect to the market portfolio implied volatility, but in reality most stock option-implied volatilities have different sensitivities to exogenous variables. I mainly focus on the relative applicability of the implied volatility risk premium of the market portfolio compared to the total risk (measured by volatility) of an individual stock. I argue that the market portfolio for implied volatility is the most practical and meaningful risk measure to describe the evolution of stock option-implied volatilities. The proposed methodology essentially argues that a “beta” for changes in implied volatility for an individual stock relative to the market portfolio implied volatility should be introduced into the risk model. This will enable option portfolio risk managers to combine this “beta” measure with the Vega for an individual stock option position to arrive at a more meaningful risk parameter for the aggregate option portfolio. This will put the portfolio manager in a better position to hedge his exposure to the market portfolio implied volatility risk, much like an equity portfolio manager can hedge his beta exposure by going long or short S&P

500 futures. For an options portfolio manager, he will engage in hedging his market implied volatility risk by going long or short VIX futures, which are now starting to be traded on a more liquid basis.

1.2 Motivation

I hypothesize that there must be a relationship between the implied volatility of the market portfolio and the implied volatility of individual stocks. My hypothesis stems from the Single Index Model for stock returns. If the beta's of individual stocks are well behaved, there should be a linear relationship between the changes of implied volatility of the market portfolio and changes in implied volatility of individual stocks. I show this through simulation. I assume that the beta of each stock is well behaved, on average, through time. I simulate 500 firm returns for 5 years that are generated from the single index model. I assume that the beta of each stock ranges from -5 to +5 for stocks. A change in the range of beta does not change the results. I assume that the market portfolio value follows a random walk with drift process. I also assume that the stock returns are simulated with systematic risk (from the single index model) as well as unsystematic risk which is stationary. I calculate the standard deviation of returns for each firm at the end of the initial simulation period and continue to do so as the simulation proceeds forward through time. I then difference the standard deviation series for each stock. I also difference the standard deviation for the market portfolio. Finally I run regressions for each company's changes in standard deviation using the changes in market portfolio standard deviation. The next figure is a scatter plot of a random company from the simulation. The horizontal axis represents the changes in standard deviation of the market portfolio and on the vertical axis represents the changes in standard deviation of an underlying security.

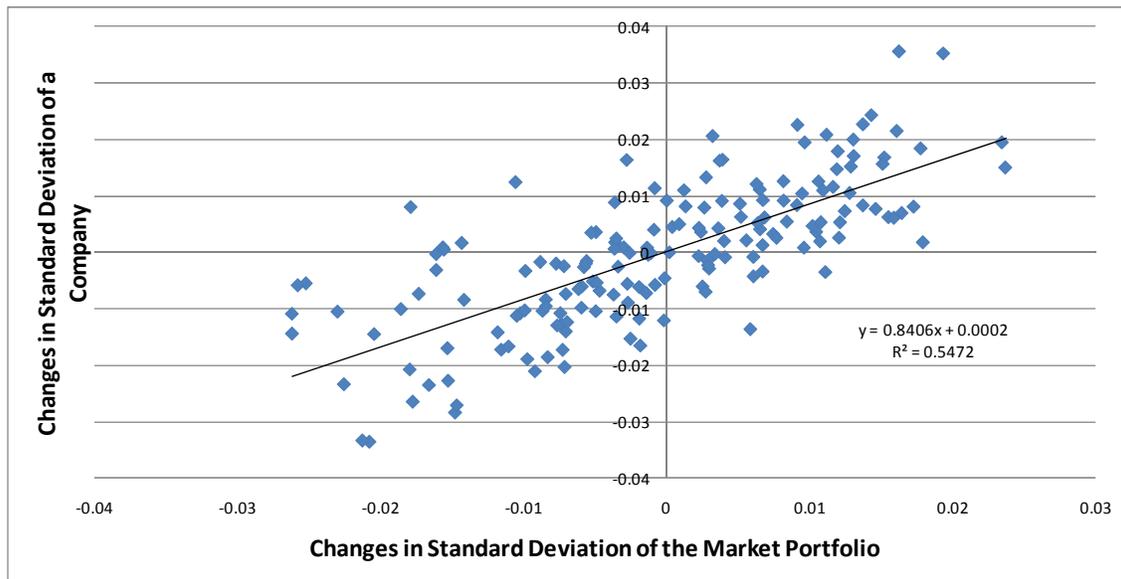


Figure 1.1 Scatter plot from a simulation of changes in standard deviation in the market portfolio vs. changes in standard deviation of a random company.

The figure shows a positive relationship between changes in the standard deviation of the market portfolio and changes in standard deviation for the company. The slope parameter is statistically significant at the 1% level and the intercept term is statistically indistinguishable from zero.

1.3 Data and Methodology

The data for this study comes from the Chicago Board of Options Exchange (CBOE). The CBOE has recently formed a joint venture with IVOLATILITY.COM who has created volatility indices for all stocks that are traded on the CBOE. They have essentially created a volatility index using similar methodologies that are used to construct the VIX for every stock and maturity. I will only use short term options for this specific study, namely 1 or 2 months to expiration, but these results can easily be expanded to other expiration periods. The test period is from November 2000 to April 2008. Monthly observations are used but not from month end to month end but rather from options expiration to the next options expiration. Equity options, as well as index options, expire on the third Saturday of every month. This method ensures that

there is an “apples to apples” comparison of implied volatility term structure. In terms of the term structure, I will use two definitions of changes in implied volatility. The first definition for change in implied volatility is simple the rolling change in 1 month implied volatility. The second definition is the change from 2 months implied volatility down to the 1 month volatility approximately 1 month later. This variable is created to proxy for a shift down the volatility term structure from one month to the next. For example, I will be looking at the difference between March’s current at the money implied volatility reading (which will have approximately 2 months to expiration in January) and the March’s at the money volatility reading one month later. The following figure looks at the first 12 months of the data and shows how the variable is created.

Ch60t30(Dec-2000)=(Dec-2000 1 Month Implied Volatility) - (Nov-2000 2 Month Implied Volatility)
 Ch60t30(Jan-2001)=(Jan-2001 1 Month Implied Volatility) - (Dec-2000 2 Month Implied Volatility)
 Ch60t30(Feb-2001)=(Feb-2001 1 Month Implied Volatility) - (Jan-2001 2 Month Implied Volatility)
 Ch60t30(Mar-2001)=(Mar-2001 1 Month Implied Volatility) - (Feb-2001 2 Month Implied Volatility)
 Ch60t30(Apr-2001)=(Apr-2001 1 Month Implied Volatility) - (Mar-2001 2 Month Implied Volatility)
 Ch60t30(May-2001)=(May-2001 1 Month Implied Volatility) - (Apr-2001 2 Month Implied Volatility)
 Ch60t30(Jun-2001)=(Jun-2001 1 Month Implied Volatility) - (May-2001 2 Month Implied Volatility)
 Ch60t30(Jul-2001)=(Jul-2001 1 Month Implied Volatility) - (Jun-2001 2 Month Implied Volatility)
 Ch60t30(Aug-2001)=(Aug-2001 1 Month Implied Volatility) - (Jul-2001 2 Month Implied Volatility)
 Ch60t30(Sep-2001)=(Sep-2001 1 Month Implied Volatility) - (Aug-2001 2 Month Implied Volatility)
 Ch60t30(Oct-2001)=(Oct-2001 1 Month Implied Volatility) - (Sep-2001 2 Month Implied Volatility)
 Ch60t30(Nov-2001)=(Nov-2001 1 Month Implied Volatility) - (Oct-2001 2 Month Implied Volatility)
 Ch60t30(Dec-2001)=(Dec-2001 1 Month Implied Volatility) - (Nov-2001 2 Month Implied Volatility)

This variable is created in order to hedge positions that start off with 2 months to expiration and are held for 1 expiration cycle. In total, there are about 4000 stocks with implied volatility data and roughly 2100 stocks that have enough data to conduct testing. Throughout this study, no regression will be run when the number of observations is less than 36. Thus, the sample will be very representative and robust for testing.

1.4 Literature Review

Ever since Black and Scholes' seminal paper in 1973 on the pricing of options, many other studies have attempted to explain how the market arrives at option prices and how those prices diverge from the Black-Scholes option pricing model. The Black-Scholes option pricing model provided the bridge so researchers could extract the market's perception of future volatility from current option prices. The first study to introduce this method was Latane and Rendelman (1976) where they show that the implied standard deviation (volatility) can be found by assuming that the market price for an option is correct and solving for the implied volatility parameter that would make the price in the options market equal to the price calculated using the Black-Scholes model. They also show that since the volatility surface might not be flat, a weighted average of implied volatilities can be calculated. This is similar to how the VIX is calculated. Schmalensee and Trippi (1978) show that volatility is not constant over time and might be negatively serially correlated. They also show that changes in implied volatility are not related to changes in historical volatility.

Later studies have shown a relationship between changes in the underlying asset and the implied volatility of options on that asset. French, Schwert, and Stambaugh (1987) show that changes in implied volatility are negatively correlated with the changes in the market price of the underlying asset. Bakshi and Kapadia (2003) examine the volatility risk premium for individual equity options. They document the negative volatility risk premium in terms of implied and realized volatility both in index and individual equity options. However, they show that the difference between realized and implied volatilities is smaller for individual equity options compared to index options. They also show that idiosyncratic risk does not appear to be priced. However, market volatility risk seems to be priced in individual equity options. They show this by forming a delta-hedged portfolio of a short stock and a long call for 25 stocks. They show that the gains are negatively correlated with the market volatility. This implies that investors are risk

averse in terms of market volatility risk. Furthermore, Giot (2005) shows that intense spikes in implied volatility are profitable signals that precede an upward move in the underlying asset.

This study is different from the previous studies in various ways. In some ways, this paper is continuing the idea flow from the Bakshi and Kapadia (2003) study. However, we use a more direct approach. We regress the time series implied volatilities of individual stocks using the market index implied volatility as an explanatory variable, along with other independent variables. We include all optionable stocks rather than 25 stocks. The database consists of more recent and voluminous data from 2000 to 2008 compared to 1991 to 1995 used in the earlier study. This increases the sample size of the study considerably. This study will enable practitioners to include a few more meaningful parameters, which are easily obtained, into their risk management models to hedge more accurately. Additionally, it might be possible to set up trades that exploit deviations from the equilibrium relationship uncovered with this study.

1.5 The Role of Fundamental Variables

Since fundamental variables play very important role in stock returns, it is important to examine if these relationships bleed into the implied volatility dimension. Although there are many fundamental metrics that are related to stock returns, I will examine the most pertinent ones. These include the size effect (measured by the natural log of the market capitalization), the value effect (measured by BE/ME), and industry effects. I will use the Fama and Macbeth(1972) procedure to test whether or not there is a statistically significant cross-sectional parameter for each one of the fundamental variables. For industry effects, I will use the 2 digit SIC code to see if there is a statistically significant parameter across industries. I will calculate Fama and Macbeth parameter estimates using both the first definition of change in implied volatility (i.e. the change from 2 month volatility down to 1 month) and the rolling at the money implied volatility. The following table summarizes the results for the size effect.

Table 1.1 Size effect's role in changes in implied volatility in equity options

| Variable | Ch30 | Ch60t30 |
|-----------------|-------------|----------------|
| Estimate | -0.0013 | -0.0015 |
| t-stat | -0.7907 | -1.1004 |
| p-value | 0.43 | 0.27 |

The table shows that there is no statistically significant size effect based on the Fama Macbeth methodology for either definition for change in implied volatility. This means that size does not significantly affect the way implied volatility changes among stocks. The next table shows Fama Macbeth estimates for the value (or BE/ME) effect.

Table 1.2 The value effect's role in changes in implied volatility in equity options

| Variable | Ch30 | Ch60t30 |
|-----------------|-------------|----------------|
| Estimate | 0.0000 | 0.0002 |
| t-stat | 0.2168 | 1.9619 |
| p-value | 0.83 | 0.05 |

The table shows that there is a statistically significant value effect for changes in implied volatility for the second change in volatility measure but it does not seem to be economically significant. The annualized figure is a paltry 26 basis points per year. This implies that high BE/ME stock's implied volatility might move around more relative to low BE/ME stock's implied volatility; however, the parameter estimate is so small it would be difficult to profit from the relationship.

Finally, I will examine the industry effects in terms of the changes in implied volatility. This examination does not need to utilize Fama Macbeth estimates because most companies do not change their industry. I will simply run a pooled OLS regression with 70 dummy variables to represent each SIC code to see if there are any persistent industry effects for the changes in implied volatility. The following table presents the parameter estimates.

Table 1.3 Dummy variable regression based on industry SIC codes and changes in implied volatility in equity options

| Industry | ch30 | ch60t30 | Industry | ch30 | ch60t30 | Industry | ch30 | ch60t30 | Industry | ch30 | ch60t30 |
|--------------------------------|---------------------|-----------------------|------------|---------------------|----------------------|------------|---------------------|----------------------|------------|----------------------|----------------------|
| industry1 | 0.00345 (0.0542) | -0.0166 (0.0502) | industry21 | 0.0103 (0.0500) | -0.0116 (0.0462) | industry41 | 0.00958 (0.0498) | -0.0151 (0.0461) | industry61 | 0.0133 (0.0501) | -0.0108 (0.0464) |
| industry2 | 0.0171 (0.0585) | -0.00877 (0.0541) | industry22 | 0.0140 (0.0503) | -0.0144 (0.0465) | industry42 | 0.0113 (0.0509) | -0.00947 (0.0471) | industry62 | 0.0112 (0.0498) | -0.00946 (0.0461) |
| industry3 | 0.0196 (0.0575) | -0.0230 (0.0532) | industry23 | 0.0111 (0.0501) | -0.0184 (0.0463) | industry43 | 0.0100 (0.0498) | -0.0104 (0.0461) | industry63 | 0.0115 (0.0497) | -0.0128 (0.0460) |
| industry4 | 0.00876 (0.0498) | -0.0157 (0.0461) | industry24 | 0.0121 (0.0497) | -0.00989 (0.0460) | industry44 | 0.0174 (0.0505) | -0.0103 (0.0468) | industry64 | 0.0105 (0.0524) | -0.0166 (0.0485) |
| industry5 | 0.0185 (0.0503) | -0.00711 (0.0465) | industry25 | 0.0116 (0.0499) | -0.0116 (0.0462) | industry45 | 0.00979 (0.0500) | -0.00915 (0.0463) | industry65 | 0.0129 (0.0502) | -0.00202 (0.0464) |
| industry6 | 0.0108 (0.0496) | -0.0114 (0.0459) | industry26 | 0.00864 (0.0496) | -0.0141 (0.0459) | industry46 | 0.00628 (0.0498) | -0.0138 (0.0460) | industry66 | 0.0197 (0.0547) | -0.00148 (0.0506) |
| industry7 | 0.0189 (0.0506) | -0.00851 (0.0468) | industry27 | 0.00800 (0.0496) | -0.0140 (0.0459) | industry47 | 0.0155 (0.0501) | -0.00436 (0.0464) | industry67 | 0.0126 (0.0497) | -0.0135 (0.0459) |
| industry8 | 0.0184 (0.0499) | -0.00193 (0.0462) | industry28 | 0.0127 (0.0497) | -0.0103 (0.0460) | industry48 | 0.0119 (0.0498) | -0.00888 (0.0460) | industry68 | -0.00990 (0.0578) | -0.0466 (0.0535) |
| industry9 | 0.0184 (0.0501) | 0.00233 (0.0464) | industry29 | 0.00865 (0.0496) | -0.0136 (0.0459) | industry49 | 0.0112 (0.0497) | -0.0106 (0.0460) | Constant | 0.00553 (0.0496) | 0.0241 (0.0459) |
| industry10 | 0.0159 (0.0507) | -0.0142 (0.0469) | industry30 | 0.0114 (0.0500) | -0.00823 (0.0462) | industry50 | 0.0205 (0.0496) | -0.00285 (0.0459) | | | |
| industry11 | 0.00932 (0.0497) | -0.0177 (0.0460) | industry31 | 0.00557 (0.0503) | -0.0110 (0.0466) | industry51 | 0.0257 (0.0498) | 0.00260 (0.0461) | | | |
| industry12 | 0.00923 (0.0502) | -0.0156 (0.0465) | industry32 | -0.0134 (0.0631) | -0.0512 (0.0584) | industry52 | 0.0136 (0.0497) | -0.00773 (0.0460) | | | |
| industry13 | 0.0164 (0.0508) | -0.0103 (0.0470) | industry33 | 0.0149 (0.0500) | -0.00187 (0.0463) | industry53 | 0.0145 (0.0496) | -0.00744 (0.0459) | | | |
| industry14 | 0.0130 (0.0500) | -0.0137 (0.0463) | industry34 | 0.0146 (0.0500) | -0.0128 (0.0462) | industry54 | 0.0103 (0.0503) | -0.0106 (0.0465) | | | |
| industry15 | 0.0206 (0.0504) | 0.00482 (0.0467) | industry35 | 0.0221 (0.0499) | 0.000187 (0.0461) | industry55 | 0.0275 (0.0505) | 0.00580 (0.0467) | | | |
| industry16 | 0.0144 (0.0501) | -0.00518 (0.0463) | industry36 | 0.0204 (0.0523) | 0.00612 (0.0484) | industry56 | 0.0224 (0.0496) | -0.00923 (0.0459) | | | |
| industry17 | 0.0173 (0.0499) | -0.00739 (0.0461) | industry37 | 0.0206 (0.0508) | -0.00360 (0.0470) | industry57 | 0.0189 (0.0502) | -0.00484 (0.0464) | | | |
| industry18 | 0.0180 (0.0498) | -0.000982 (0.0461) | industry38 | 0.0130 (0.0496) | -0.00930 (0.0459) | industry58 | 0.0165 (0.0508) | 0.00119 (0.0470) | | | |
| industry19 | 0.0161 (0.0496) | -0.00977 (0.0459) | industry39 | 0.0127 (0.0496) | -0.0102 (0.0459) | industry59 | 0.0103 (0.0496) | -0.0116 (0.0459) | | | |
| industry20 | 0.0103 (0.0498) | -0.0141 (0.0461) | industry40 | 0.0103 (0.0497) | -0.0104 (0.0460) | industry60 | 0.0196 (0.0514) | -0.00954 (0.0476) | | | |
| Observations | 180541 | 180541 | | | | | | | | | |
| R-squared | 0.000 | 0.000 | | | | | | | | | |
| Standard errors in parentheses | | | | | | | | | | | |
| *** p<0.01, ** p<0.05, * p<0.1 | | | | | | | | | | | |

The table shows that there are no statistically significant industries for changes in implied volatility. This is not that surprising, but it would be imprudent to exclude this test because I do not want to induce an unobserved variable bias into the analysis.

1.6 Remarks

In an effort to exercise prudent econometric analysis as well as provide a comprehensive picture of the way in which fundamental variables are related to the changes in implied volatility, I presented Fama Macbeth estimates as well as pooled OLS estimates to identify relationships between fundamental variables and the changes in implied volatility. The only variable that is statistically significant is BE/ME ratio. Since the value premium is so important in stock returns, it might be important in terms of changes in implied volatility. Even though the BE/ME ratio is significant, it is not economically significant because of its very small parameter estimate. Now that I have explored the extent to which fundamental variables play a role in the changes in implied volatility, I can move on to more pertinent topics such as using the VIX as the common factor in changes in implied volatility.

CHAPTER 2

FACTOR BASED MODELS

2.1 Summary Statistics for the Variables

Before we delve into the prospective models that could explain changes in implied volatility for equity options, let us first examine the descriptive statistics for the variables. Table 2.1 contains these statistics.

Table 2.1 Summary Statistics

| | $\Delta IV(\text{Stock}_{i,t})$ | ΔVIX |
|--------------------|---------------------------------|--------------|
| Mean | -0.0024 | -0.0023 |
| Standard Deviation | 0.1743 | 0.2182 |
| Skewness | 0.5017 | 0.5553 |
| Kurtosis | 4.9294 | 0.2430 |
| Range | 4.1021 | 1.0383 |
| Min | -2.3676 | -0.4564 |
| Max | 1.7345 | 0.5819 |
| Count | 180541 | 89 |

2.2 Market Implied Volatilities and Individual Implied Volatilities

In this section, I first discuss the relationship between the implied volatility of the index, measured by the VIX, and the individual stock implied volatilities. Let us first run a simple linear regression between the level of each stock's implied volatility and the level of the VIX (as the explanatory variable). Other model structures and transformations did not result in any significant increase of explanatory power. The model is as follows:

$$IV(\text{Stock}_i)_t = b_0 + b_1 VIX_t \quad (1)$$

Due to the massive amount of data in the form of about 2100 time series of individual stocks, the most understandable way to present the findings is to show descriptive statistics as well as histograms for the parameter estimates for equation 1. The results are quite direct and easily understood. For example, on average, the stock's implied volatility is 32% higher than the

VIX. This makes intuitive sense because an individual stock should be more risky, in terms of volatility, than the market portfolio. More importantly, the VIX, on average, explained about 45% of the variance in the implied volatility of the individual stocks. Table 2.2 provides descriptive statistics for the parameter estimates. Figure 2.1 shows the histogram of the parameter estimates.

Table 2.2 Descriptive Statistics for parameter estimates in equation 1

| | b1 | b0 | R-Square |
|---------------------------|-----------|-----------|-----------------|
| Mean | 1.3245 | 0.1931 | 0.4502 |
| Median | 1.1474 | 0.1623 | 0.4753 |
| Standard Deviation | 0.8682 | 0.1715 | 0.2159 |
| Kurtosis | 5.9908 | 5.2996 | -0.5785 |
| Skew | 1.4781 | 1.5662 | -0.2832 |
| Range | 10.8891 | 1.7148 | 0.9878 |
| Min | -2.8686 | -0.3305 | 0.0000 |
| Max | 8.0204 | 1.3843 | 0.9878 |
| Count | 2152 | 2152 | 2152 |

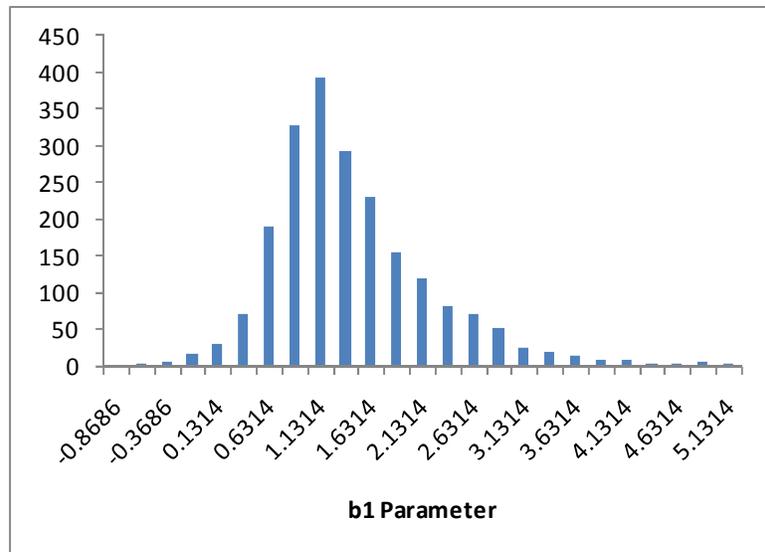


Figure 2.1 Parameter estimates for b1 parameter in equation 1. This figure shows a distribution or the b1 parameter.

The VIX was not a significant explanatory variable for every company. There were a few companies with very low coefficient of determination statistics. In all of these cases (approximately 1% of the sample), this resulted in negative slope coefficients, but the rest of the distribution was quite healthy.

Next, we show how changes in implied volatilities for individual stocks are related to changes in the implied volatility of the market index, the VIX. Specifically, we measure the percent changes of implied volatilities for all stocks and the VIX from the third Friday of every month to the third Friday of the following month, instead of one month end to the next month's end. We do this in order to synchronize with the expiration cycle. We estimate the following equation for each of the 2100 stocks:

$$\Delta IV(\text{Stock}_i)_t = b_0 + b_1 \Delta VIX_t \quad (2)$$

Again, explanatory power is present for the changes in the implied volatilities of individual stocks by using changes in the VIX as an explanatory variable. On average, when the VIX goes up 1%, this translates to a percent change in the implied volatility of the average stock of about 0.33 percent. This b_1 parameter is analogous to the beta in the CAPM because it measures the sensitivity of the implied volatility of a stock to the market portfolio implied volatility. Table 2.3 explains the moments, as well as other descriptive statistics, of each parameter estimate in equation 2. Figure 2.2 shows the histogram of parameter estimates for equation 2.

Table 2.3 Panel regression for equation 2 using the first definition

F test that all $u_i=0$: $F(2109, 150250) = 0.33$ Prob > F = 1.0000

. xtreg iv vix, fe

Fixed-effects (within) regression
 Group variable (i): firm
 Number of obs = 152362
 Number of groups = 2110

R-sq: within = 0.1554
 between = 0.0985
 overall = 0.1550
 Obs per group: min = 36
 avg = 72.2
 max = 89

corr($u_i, \delta b$) = 0.0030
 $F(1, 150251) = 27636.30$
 Prob > F = 0.0000

| iv | Coef. | Std. Err. | t | P> t | [95% Conf. Interval] |
|---------|-----------|--------------------------------------|--------|-------|----------------------|
| vix | .3353672 | .0020173 | 166.24 | 0.000 | .3314132 .3393212 |
| _cons | .0077869 | .0004847 | 16.07 | 0.000 | .0068369 .0087368 |
| sigma_u | .01334824 | | | | |
| sigma_e | .18827702 | | | | |
| rho | .00500122 | <fraction of variance due to u_i > | | | |

F test that all $u_i=0$: $F(2109, 150251) = 0.31$ Prob > F = 1.0000

Since changes in the variables are used in the regression, the average coefficient of determination should drop relative to the regression of levels in volatility shown above. To depict the statistical significance of the slope coefficient, figure 2.3 includes the distribution of absolute t-statistics for the slope coefficient.

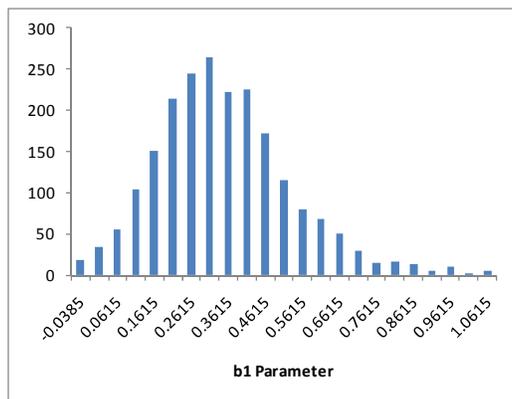


Figure 2.2 Histogram for b1 parameter in equation 2

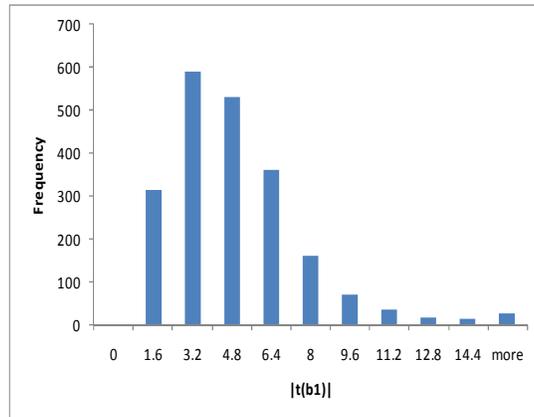


Figure 2.3 Absolute t-stats for b1 in equation 2

Figures 2.2 and 2.3 show the distribution of the sensitivity of the change in implied volatility for individual stocks to a change in VIX. Figure 3 shows the absolute value of the t-statistics for the regression expressed by equation 2. The 10% critical value for the t-statistic given the number of data points in the regression is 1.66. In the sample, 1775 companies, or 84%, had statistically significant t-values for the b1 parameter. This provides evidence for the relevance of the VIX in explaining implied volatilities for individual stocks. Finally, the average p-value, which adjusts for the number of observations, is, for the VIX, about 0.07 which is comforting.

Now that I have presented estimates for the first definition for the change in implied volatility, I will now present the version of table 2.3 using the other definition. Table 2.4 shows the results.

Table 2.4 Panel regression for equation 2 using the second definition

| | | | | |
|--|-------------------------|-------------------------|-------------------|-----------------|
| Fixed-effects (within) regression | | Number of obs | = | 145478 |
| Group variable (i): firm | | Number of groups | = | 2128 |
| R-sq: | within = 0.1504 | Obs per group: | min = 25 | |
| | between = 0.0048 | | avg = 68.4 | |
| | overall = 0.1492 | | max = 85 | |
| corr(u_i, Xb) | = -0.0025 | F(1,143349) | = | 25366.61 |
| | | Prob > F | = | 0.0000 |

| vol | Coef. | Std. Err. | t | P> t | [95% Conf. Interval] | |
|----------------|------------------|--|---------------|-----------------|-----------------------------|------------------|
| vix | .3416446 | .0021451 | 159.27 | 0.000 | .3374403 | .345849 |
| _cons | -.0095944 | .0004256 | 22.54 | 0.000 | -.0087602 | -.0104286 |
| sigma_u | .01572725 | | | | | |
| sigma_e | .15751271 | | | | | |
| rho | .00987111 | <fraction of variance due to u_i> | | | | |

| | | | | |
|-------------------------------|--------------------------|-------------|----------------------|---------------|
| F test that all u_i=0: | F(2127, 143349) = | 0.61 | Prob > F = | 1.0000 |
|-------------------------------|--------------------------|-------------|----------------------|---------------|

This table shows that the average factor loading is very similar to those in table 3. Namely, if the VIX goes up by 1%, the average stock implied volatility will change by 0.34%

2.3 Individual Stock Return Effects

The stochastic volatility model described above is in and of itself a very robust and feasible model for changes in and levels of implied volatilities of individual stocks. However, more variables can be added to the model to further explain changes in implied volatility for individual stocks. Another possible explanatory variable is the changes in price of the underlying asset. This has been proposed theoretically by Cox (1996). He proposes that the volatility is related to the level of the underlying asset's price. This model is one model that explains the behavior of the volatility curve on equity and index options. The implication from the theoretical prediction is that changes in implied volatility should be related to changes in the underlying stock price.

As a starting point, we estimate a regression for the changes in implied volatility of the S&P 500, the VIX. We use the monthly percent changes in the S&P 500 as the explanatory variable. Table 2.5 shows parameter estimates for the regression.

Table 2.5 Statistical output (monthly) for $\Delta VIX_t = \text{intercept} + \text{spx } \Delta \text{SPX}_t$

| | <i>Coefficients</i> | <i>Standard Error</i> | <i>t Stat</i> | <i>P-value</i> |
|-----------|---------------------|-----------------------|---------------|----------------|
| Intercept | 3.2472349 | 0.926748387 | 3.503901 | 0.00055774 |
| Spx | -2.632409402 | 0.231449848 | -11.3736 | 0.00000000 |

Table 2.5 shows that changes in implied volatility of the S&P 500 index is negatively related to changes in the S&P 500 index. The slope coefficient is negative and statistically significant. Now, we can use the same logic as we did with the VIX and S&P 500 on individual stocks. We can include percent changes in the underlying securities to examine their role in the changes in implied volatility of those securities. This test will reveal to what degree changes in the underlying stock (idiosyncratic risk) is incorporated into the pricing of individual stock options. The model we estimate is as follows:

$$\Delta IV(\text{Stock}_i)_t = b_0 + b_1 \Delta \text{Stock}_{i,t} \quad (3)$$

We find that percent changes in the underlying stock prices can be used as an explanatory variable to explain changes in implied volatility. Table 2.6 reports the results. It shows a negative relationship between changes in the underlying stock and changes in the implied volatility of that stock. Namely, for a 1 percent decrease in the underlying stock, the corresponding implied volatility should go up about 0.78 percent. This is quite small in comparison with the S&P 500 regression presented in table 2.3. Recall, that the corresponding b_1 parameter for the S&P 500 index was -2.63.

Table 2.6 Panel Regression for equation 3 using the first definition

| | | | | |
|--|-----------|--------------------|---|----------|
| Fixed-effects (within) regression | | Number of obs | = | 152362 |
| Group variable (i): firm | | Number of groups | = | 2110 |
| R-sq: within | = 0.1073 | Obs per group: min | = | 36 |
| between | = 0.0270 | avg | = | 72.2 |
| overall | = 0.1063 | max | = | 89 |
| corr(u _i , X _b) | = -0.0420 | F(1,150251) | = | 18064.26 |
| | | Prob > F | = | 0.0000 |

| iv | Coef. | Std. Err. | t | P> t | [95% Conf. Interval] |
|---------|-----------|---|---------|-------|----------------------|
| stock | -.5204231 | .0038721 | -134.40 | 0.000 | -.5280123 - .5128339 |
| _cons | .0199516 | .0004969 | 40.15 | 0.000 | .0189777 .0209254 |
| sigma_u | .01490125 | | | | |
| sigma_e | .19355663 | | | | |
| rho | .005892 | <fraction of variance due to u _i > | | | |

| | | | | |
|------------------------------------|-------------------|------|------------|--------|
| F test that all u _i =0: | F(2109, 150251) = | 0.36 | Prob > F = | 1.0000 |
|------------------------------------|-------------------|------|------------|--------|

It is prudent to study the absolute value for t-statistics for the slope coefficients to explore the relationship more closely. Figure 2.4 shows the histogram for the absolute value of t-statistics for the slope coefficient in equation 3.

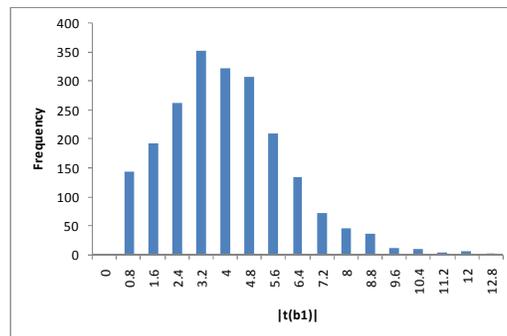


Figure 2.4 Histogram for absolute values of t-statistics on the b1 parameter in equation 3

Eighty-three percent of the companies have statistically significant slope coefficients at the 10% level. This is an indication of preliminary evidence that idiosyncratic effects are priced in the options. This makes intuitive sense because it follows the same relationship between the VIX and the S&P 500. On the other hand, more companies' changes in implied volatilities were explained more accurately by the VIX model, because there were more statistically significant slope coefficients on the VIX model. Perhaps the 2 explanatory variables are proxies for another variable, or are possibly being affected by another variable. Could it be the market portfolio

itself? We must be able to account for this missing variable problem. We will now explore the relative applicability of the two models.

Now let's see how the underlying stock changes affect changes in implied volatility using the second definition of changes in implied volatility. Table 2.7 presents these results.

Table 2.7 Panel Regression for equation 3 using the second definition

| | | | | | | | |
|--|------------------|--|----------------|-----------------------------|------------------------------|------------------|-------------|
| Fixed-effects (within) regression | | | | Number of obs | = | 145478 | |
| Group variable (i): firm | | | | Number of groups | = | 2128 | |
| R-sq: | within | = | 0.0851 | Obs per group: | min | = | 25 |
| | between | = | 0.0118 | | avg | = | 68.4 |
| | overall | = | 0.0840 | | max | = | 85 |
| corr(u_i, Xb) | = | -0.0265 | | F(1,143349) | = | 13330.47 | |
| | | | | Prob > F | = | 0.0000 | |
| vol | Coef. | Std. Err. | t | P> t | [95% Conf. Intervall] | | |
| ret | -.2842065 | .0024616 | -115.46 | 0.000 | -.2890311 | -.2793818 | |
| _cons | -.0041414 | .0004292 | -9.65 | 0.000 | -.0049825 | -.0033003 | |
| sigma_u | .01616546 | | | | | | |
| sigma_e | .16345085 | | | | | | |
| rho | .00968667 | <fraction of variance due to u_i> | | | | | |
| F test that all u_i=0: | | F(2127, 143349) = | 0.60 | Prob > F = 1.0000 | | | |

There still is a negative relationship between changes in implied volatility and underlying stock return, but the average parameter estimate is smaller in absolute value compared to the first definition of changes in implied volatility.

2.4 Comparing the Two Variables

There are two models that are being tested for their relative applicability in terms of explaining changes in implied volatilities for underlying stocks, and it is still unclear what model better explains changes in implied volatility for individual stocks. The first of which is the IVAPM, where the evolution of implied volatilities of individual stocks is expressed in terms of the evolution of the implied volatility of the "market portfolio," measured by the VIX. The second model uses changes in the underlying stock to explain changes in implied volatility of the corresponding stock. This is referred to as the idiosyncratic model. We have winsorized the data in order to prevent outliers from creating the illusion of statistical significance. In this case, we

replace the top and bottom 5% of observations with the corresponding 5% percentile value, for both tails. This is allowed for a few reasons. The first is that changes in implied volatilities have very fat tails and have a few outliers that can create an illusion of statistical significance. The other reason is because for all of the variables that are studied, they will all move up or down during extreme events, such as 9/11 and data points like that will create the illusion of statistical significance in the IVAPM. Finally, in terms of idiosyncratic risk, negative news during earnings announcements or other news can produce extreme moves in both the underlying stock and, most importantly, the implied volatility of the underlying stock. These extreme events will inflate the t-statistics for the idiosyncratic model. So, to be fair to both models, every variable is winsorized. Winsorizing the data does not significantly change the inference.

The models are combined in a multiple regression to see if one factor subsumes the other factor. Again, the variables utilized have been winsorized. The model estimated is as follows:

$$\Delta IV(\text{Stock}_i)_t = b_0 + b_1 \Delta \text{Stock}_{i,t} + b_2 \Delta \text{VIX}_t \quad (4)$$

Table 8 shows the parameter estimates for equation 4. There is still a negative relationship between changes in implied volatilities and changes in the stock price and a positive relationship between changes in the VIX and changes in the individual implied volatility.

Table 2.8 Panel Regression for equation 4 using the first definition

| | | | |
|-----------------------------------|--------------------|---|-----------------|
| Fixed-effects (within) regression | Number of obs | = | 152362 |
| Group variable (i): firm | Number of groups | = | 2110 |
| R-sq: within = 0.1946 | Obs per group: min | = | 36 |
| between = 0.0780 | avg | = | 72.2 |
| overall = 0.1940 | max | = | 89 |
| corr(u_i, Xb) = -0.0092 | F(2,150250) | = | 18151.00 |
| | Prob > F | = | 0.0000 |

| iv | Coef. | Std. Err. | t | P> t | [95% Conf. Interval] |
|---------|------------------|--|---------------|--------------|-------------------------------|
| vix | .269739 | .002114 | 127.60 | 0.000 | -.2655955 .2738824 |
| stock | -.3376817 | .003947 | -85.55 | 0.000 | -.3454177 -.3299457 |
| _cons | .0121002 | .000476 | 25.42 | 0.000 | .0111673 .0130331 |
| sigma_u | .0135546 | | | | |
| sigma_e | .18385263 | | | | |
| rho | .00540604 | <fraction of variance due to u_i> | | | |

| | | | | |
|------------------------|-------------------|-------------|------------|---------------|
| F test that all u_i=0: | F(2109, 150250) = | 0.33 | Prob > F = | 1.0000 |
|------------------------|-------------------|-------------|------------|---------------|

The VIX coefficient is 0.27 meaning if the VIX increases by 1%, the average stock implied volatility will increase by 0.27%. Also, when a stock increases by 1%, the corresponding implied volatility for the stock will decrease by 0.33%. The estimation shows that both the movements in the VIX as well as the individual stock movements can be combined in the same model to help explain movements in implied volatility for individual stocks.

Table 2.9 shows the same output for table 7 but for the second definition of changes in implied volatility.

Table 2.9 Panel Regression for equation 4 using the second definition

| | | | |
|-----------------------------------|--------------------|---|-----------------|
| Fixed-effects (within) regression | Number of obs | = | 145478 |
| Group variable (i): firm | Number of groups | = | 2128 |
| R-sq: within = 0.1899 | Obs per group: min | = | 25 |
| between = 0.0179 | avg | = | 68.4 |
| overall = 0.1884 | max | = | 85 |
| corr(u_i, Xb) = -0.0082 | F(2,143348) | = | 16801.93 |
| | Prob > F | = | 0.0000 |

| vol | Coef. | Std. Err. | t | P> t | [95% Conf. Interval] |
|---------|------------------|--|---------------|--------------|-------------------------------|
| vix | .2951403 | .0021671 | 136.19 | 0.000 | -.2908929 .2993877 |
| ret | -.2004864 | .0023965 | -83.66 | 0.000 | -.2051834 -.1957894 |
| _cons | .0092401 | .0004156 | 22.23 | 0.000 | .0084255 .0100547 |
| sigma_u | .01574548 | | | | |
| sigma_e | .1538033 | | | | |
| rho | .01037176 | <fraction of variance due to u_i> | | | |

| | | | | |
|------------------------|-------------------|-------------|------------|---------------|
| F test that all u_i=0: | F(2127, 143348) = | 0.65 | Prob > F = | 1.0000 |
|------------------------|-------------------|-------------|------------|---------------|

The factor loading for the stock (b1) essentially stays the same as the first definition of implied volatility. The factor loading for the VIX parameter (b2) is smaller in magnitude. I also present a column which shows the average adjusted r-squared for the regressions.

2.5 Principal Component Analysis

Now that we have found that changes in the VIX is a better explanatory variable for describing changes in implied volatilities for individual stocks, we can look at the relationship in terms of principal component analysis. Principal component analysis can decompose a large matrix consisting of the time series of implied volatility asset returns (represented by percent changes in implied volatilities for individual stocks) into a few orthogonal factors that have a high degree of explanatory power on the constituent matrix. The actual observations of the principal components are not very meaningful but the correlation of those components to specific variables is very important. We want to conduct principal component analysis on the changes in implied volatilities for stocks in our sample to form a few principal components to see if they are correlated with the VIX. If they are highly correlated, then the VIX might be a priced factor and should be used to price individual equity options. Table 2.10 shows the output from the principal component analysis.

Table 2.10 Principal Component Analysis for the first definition

| # | Eigenvalue | Difference | Proportion | Cumulative |
|---|------------|------------|------------|------------|
| 1 | 235.8803 | 201.1260 | 0.3531 | 0.3531 |
| 2 | 34.7543 | 11.4234 | 0.0520 | 0.4051 |
| 3 | 23.3308 | 8.4094 | 0.0349 | 0.4401 |
| 4 | 14.9214 | 1.8621 | 0.0223 | 0.4624 |
| 5 | 13.0593 | 0.2982 | 0.0195 | 0.4820 |
| 6 | 12.7611 | 1.5948 | 0.0191 | 0.5011 |

Table 2.11 Principal Component Analysis for the second definition

| | Eigenvalue | Difference | Proportion | Cumulative |
|----------|-------------------|-------------------|-------------------|-------------------|
| 1 | 233.9375 | 209.7087 | 0.3690 | 0.3690 |
| 2 | 24.2288 | 6.0794 | 0.0382 | 0.4072 |
| 3 | 18.1494 | 2.3259 | 0.0286 | 0.4358 |
| 4 | 15.8235 | 3.4976 | 0.0250 | 0.4608 |
| 5 | 12.3258 | 0.7531 | 0.0194 | 0.4802 |
| 6 | 11.5727 | 0.2215 | 0.0183 | 0.4985 |
| 7 | 11.3513 | 1.5920 | 0.0179 | 0.5164 |
| 8 | 9.7593 | 0.5978 | 0.0154 | 0.5318 |

We see that the first principal component explains about 35% of the variance of the sample. Adding one more factor results in a cumulative explanatory power of 41% and 6 principal components yields a 50% explanatory power. The general rule of thumb in terms of principal components to include is when the marginal variance explained is greater than 5%; therefore, we will look at the first 2 principal components in relation to the VIX to see if the VIX explains them. Table 2.9 shows the eigenvalues for the correlation matrix for the second definition for changes in implied volatility. This table shows only one significant principal component. Figure 2.7 shows a graphical representation of the VIX relative to principal components 1 and 2.

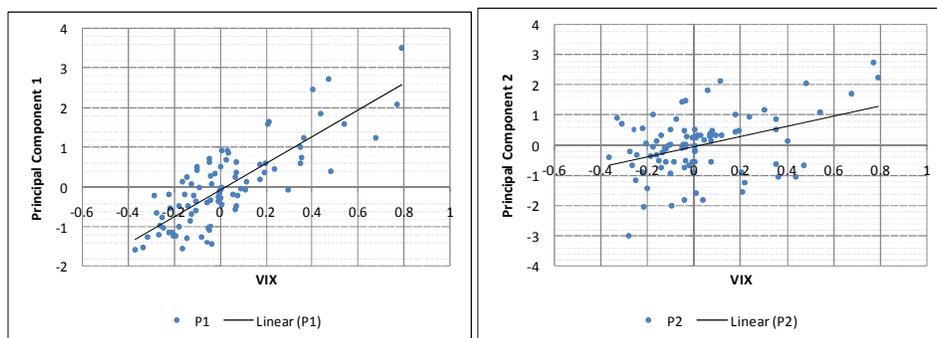


Figure 2.5 Scatter Plots for Principal Components 1 and 2 and the VIX

The scatter plots indicate a positive simple relationship between principal component 1 and the VIX that passes through the origin. The scatter plot for principal component 2 looks

more tenuous and noisy but there still seems to be some positive relationship. Table 12 shows regression output for each principal component on the VIX (as the explanatory variable).

Table 2.12 Regression for Principal Components 1 and 2 on the VIX

| Principal Component 1 | | | | Principal Component 2 | | | |
|------------------------------|---------------------|---------------|----------------|------------------------------|---------------------|---------------|----------------|
| <i>Regression Statistics</i> | | | | <i>Regression Statistics</i> | | | |
| Multiple R | 0.80842 | | | Multiple R | 0.40410 | | |
| R Square | 0.65355 | | | R Square | 0.16330 | | |
| Adjusted R Square | 0.64956 | | | Adjusted R Square | 0.15368 | | |
| Standard Error | 0.59198 | | | Standard Error | 0.91996 | | |
| Observations | 89 | | | Observations | 89 | | |
| | <i>Coefficients</i> | <i>t Stat</i> | <i>P-value</i> | | <i>Coefficients</i> | <i>t Stat</i> | <i>P-value</i> |
| Intercept | -0.07567 | -1.20063 | 0.23315 | Intercept | -0.03783 | -0.38619 | 0.70030 |
| Vix | 3.38244 | 12.81076 | 0.00000 | Vix | 1.69076 | 4.12063 | 0.00009 |

Table 2.12 shows a strong positive relationship between the VIX and both principal component factors. The VIX is highly correlated (0.81) to principal component 1 and has a statistically significant slope coefficient at the 1% level. The regression line seems to pass through the origin and is signified from the fact that the intercept is not statistically significant. We see similar results from the second regression but not to the same degree. These results show the power of the VIX in explaining the evolution of implied volatility for individual equity options.

I will now repeat this analysis for the second definition for changes in implied volatility. The scatter plot also shows a positive and simple relationship between the first principal component and changes in the VIX.

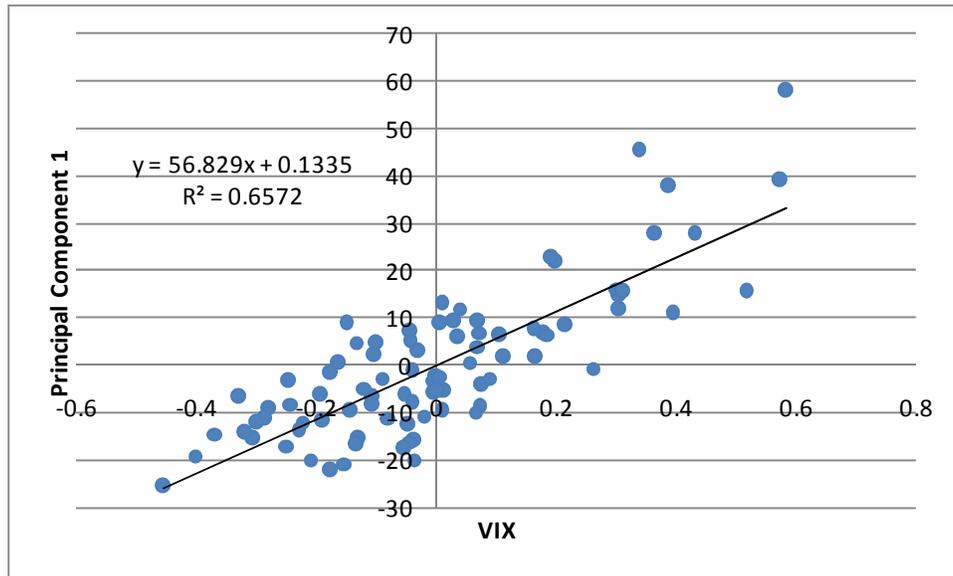


Figure 2.6 Scatter Plots for Principal Component 1 for the second definition and the VIX

Table 2.13 Regression for Principal Component for the first definition and the VIX

| SUMMARY OUTPUT | | | | | | |
|------------------------------|---------------------|-----------------------|---------------|----------------|-----------------------|------------------|
| <i>Regression Statistics</i> | | | | | | |
| Multiple R | 0.810705625 | | | | | |
| R Square | 0.65724361 | | | | | |
| Adjusted R Square | 0.653303881 | | | | | |
| Standard Error | 9.005844047 | | | | | |
| Observations | 89 | | | | | |
| <i>ANOVA</i> | | | | | | |
| | <i>df</i> | <i>SS</i> | <i>MS</i> | <i>F</i> | <i>Significance F</i> | |
| Regression | 1 | 13530.34619 | 13530.35 | 166.8246 | 6.18631E-22 | |
| Residual | 87 | 7056.154749 | 81.10523 | | | |
| Total | 88 | 20586.50094 | | | | |
| | <i>Coefficients</i> | <i>Standard Error</i> | <i>t Stat</i> | <i>P-value</i> | <i>Lower 95%</i> | <i>Upper 95%</i> |
| Intercept | 0.133475487 | 0.954673493 | 0.139813 | 0.889131 | -1.764041415 | 2.030992389 |
| VIX | 56.82917446 | 4.399884894 | 12.91606 | 6.19E-22 | 48.08392699 | 65.57442193 |

Table 2.13 shows that the VIX is very significant in explaining the first principal component for the second definition for changes in implied volatility. The changes in VIX is also very highly correlated to the first principal component. The correlation coefficient is 0.81 which is similar to the correlation coefficient in the regression for the first principal component for the first definition for the changes in implied volatility.

In summary, principal component analysis reveals common factors for changes in implied volatility on individual equity options. The meaningful principal components, which together explain about 40% of the variance of individual equity option volatility returns, are explained, at a very high level of statistical significance, by the VIX. These results are robust to both definitions for changes in implied volatility. These results help solidify its explanatory power and further support the inclusion of the VIX in equity option volatility asset pricing.

2.6 Cross-Sectional Tests

We have found that the VIX is an important variable in explaining the time series of implied volatility returns. Through principal component analysis we found that common factors can be extracted from the time series matrix and can be explained by the VIX. Now, it is time to see if the VIX is priced in the cross-section. If the slope coefficient, on average, is statistically different from zero, that implies, for our sample, the volatility risk of the market portfolio is priced. This is the same as beta being priced in the cross-section of average expected stock returns when the CAPM was tested. This will also justify the importance of the Implied Volatility Asset Pricing Model.

We use a very similar methodology as Black, Jensen, and Scholes (1972) whereby we form 30 portfolios based on pre-ranking sensitivity of individual implied volatility to the market portfolio implied volatility. The first 3 years of the data set is used to calculate the sensitivities of each stocks implied volatility changes with respect to the VIX. We partition the rankings into 30 portfolios that are re-balanced at the beginning of each year (2004 to 2008) based on sensitivities calculated at the end of the previous year using the last 3 years of data. The average beta and an equal weighted return for each of the 30 portfolios are calculated for the remaining portion (almost 5 years of monthly returns) of the data for time series risk/return tests. We regress returns on the 30 portfolios against the percent changes for the VIX. Once portfolio sensitivities and average returns are calculated, a cross-sectional regression is estimated to calculate the risk premium for market portfolio implied volatility. Table 2.14 shows the results.

Table 2.14 Regression for Principal Component for the first definition and the VIX

| | | | |
|-------------------|---------------------|---------------|----------------|
| Multiple R | 0.5381 | | |
| R Square | 0.2895 | | |
| Adjusted R Square | 0.2642 | | |
| Standard Error | 0.0019 | | |
| Observations | 30 | | |
| | <i>Coefficients</i> | <i>t Stat</i> | <i>P-value</i> |
| Intercept | 0.0179 | 10.8744 | 0.0000 |
| Slope | 0.0169 | 3.3779 | 0.0022 |

These results provide evidence in terms of the importance of the IVAPM. We see that changes in the implied volatility of the market portfolio is a priced factor in the cross-section of implied volatility returns for about 2000 companies from 2000 to 2008. We find that the “risk premium” associated with the market portfolio implied volatility is about 1.7% per month. This premium using portfolios matches Fama-Macbeth estimates we estimate using individual stocks. In our Fama-Macbeth tests, we used the prior 3 years to estimate sensitivities as well. We also find significance for the intercept parameter, about 1.8% per month. This is a fairly sizable (but volatile) premium, but when it comes to options, we already know that there is substantial risk involved and that is what investors demand to take risk in the implied volatility dimension.

2.7 Fama Macbeth Estimates

We have found that the VIX is an important variable in explaining the time series of implied volatility returns. Through principal component analysis we found that common factors can be extracted from the time series matrix and can be explained by the VIX. Now, it is time to see if the VIX is priced in cross-section. If the slope coefficient, on average, is statistically different from zero, that implies, for our sample, the volatility risk of the market portfolio is priced. This is the same as beta being priced in the cross-section of average expected stock returns when the CAPM was tested. This will also justify the importance of the Implied Volatility Asset Pricing Model.

We use a very well known methodology, from the Fama Macbeth (1972) paper, which has been used many times to measure risk premia and to determine if the risk premium is

priced. We will test the two explanatory variables which we think might be priced (i.e. the changes in the VIX and the underlying stock return). We will look at univariate cross-sectional regressions for each variable to see if either variable is priced, and if they are both priced, we will combine these variables into a multivariate framework and run the Fama Macbeth estimation to see if one variable subsumes the other. Let us first look at the changes in the VIX. The next table provides the parameter estimates as well as their significance levels. We present Newey-West standard errors as other papers have done when presenting Fama Macbeth output. We also break up the whole data set into two equal subperiods for robustness.

Table 2.15 Fama-Macbeth slopes for the implied volatility risk premium

| | Period 1 | Period 2 | Whole |
|-----------------------|-----------------|-----------------|--------------|
| Estimate | -0.0351 | -0.0299 | -0.0325 |
| Standard Error | 0.0168 | 0.0132 | 0.0136 |
| p-Value | 0.0463 | 0.0322 | 0.0206 |

The table shows that the monthly risk premium for changes in the VIX in terms of changes in implied volatility for stocks is priced. The parameter estimates are statistically significant for the whole period as well as the two subperiods at the 5% level of significance. The first time period is from December 2003 to January 2006. We allowed for 36 months of changes in implied volatility to assign the pre-ranking “beta’s” for the stocks. What we find in the table is very interesting. Namely, we find a negative risk premium or, to put it another way, a risk discount. This means that future changes in implied volatility load in the opposite way in which the pre-ranking factor loading implied. To be more specific, the highest (lowest) factor loading had the smallest (largest) change in implied volatility approximately one month later. In terms of the annualized figures, the whole period shows a -38.9% risk premium. The reasons for this negative risk premium are not understood fully and would be a very interesting area for further study.

Now that we have presented the risk premium for the changes in the VIX, let us now turn our attention to the potential risk premium associated with the return for the underlying

asset. Again we use the same methodology as before and we split up the data into the same subperiods.

Table 2.16 Fama-Macbeth slopes for the implied volatility risk premium.

| | Period 1 | Period 2 | Whole |
|-----------------------|-----------------|-----------------|--------------|
| Estimate | 0.0089 | 0.0021 | 0.0054 |
| Standard Error | 0.0049 | 0.0060 | 0.0039 |
| p-Value | 0.0819 | 0.7331 | 0.1687 |

The table shows that the underlying price changes in the stock are not priced using the Fama Macbeth approach. Even though we have shown that univariate regressions show a significant relationship, we cannot confirm this in the Fama Macbeth methodology. This implies that even though the parameter estimates might be significant, they do not carry any priced risk premium.

These Fama Macbeth results provide evidence in terms of the importance of the IVAPM. We see that changes in the implied volatility of the market portfolio is a priced factor in the cross-section of implied volatility returns for about 2000 companies from 2000 to 2008. We find that the “risk premium” associated with the market portfolio implied volatility is about -3.25% per month. This is a fairly sizable premium, but when it comes to options, we already know that there is substantial risk involved and that is what investors demand to take risk in the implied volatility dimension.

2.8 Remarks

We introduce a model using data from 2000 to 2008 for 4000 companies that explains the evolution of implied volatilities in individual stocks, on average. The implied volatility asset pricing model (IVAPM) where changes in the implied volatility of the market portfolio are used to explain changes in implied volatility of individual stock option implied volatility seems to be a dominant factor, which subsumes the idiosyncratic risk measured by percent changes in the stocks, and should be used in the risk management of an individual equity option portfolio. The tests conducted in this study show a strong relationship between the two variables and imply that option investors try to hold an efficient portfolio of options that is related to the market

option portfolio. We have tested this model using similar methods that were used to test the CAPM. We used principal component analysis to extract common factors from the constituent matrix and found that the VIX was highly correlated to common factors. We also tested the IVAPM cross-sectionally to see if the market portfolio implied volatility risk was priced and found that it was. This model closely resembles the CAPM in that there is an equilibrium relationship in terms of options investors holding an efficient portfolio that balances return and implied volatility risk. The explanatory variable, the VIX, is easily observed and can be seamlessly included into the risk management models for underlying stock option portfolios. Any stock that offers options can be utilized in the IVAPM framework. Just as the CAPM can be applied to individual securities, so can the IVAPM be applied. Since the VIX is shown to be a common factor in changes in implied volatility, it should be easier to for a options portfolio manager, who has options from many different stocks in one portfolio, to manage his risk more precisely. This is a highly robust and easy to use model.

CHAPTER 3

STRADDLE RETURN RISK PREMIA

Now that I have explored the possibility of considering volatility as an asset class through risk premia and common factor analysis, I will now explore how trading volatility in stocks is conducted and how the prior analysis can be applied to trade volatility more precisely. Let us first consider the position an option trader might execute if he or she has an opinion about volatility on a certain stock. The option strategy that is most exposed to changes in implied volatility is the straddle. From the Black Scholes model, we know that Vega is the change in the option price with respect to a change in implied volatility of the underlying asset. Figure 3.1 shows the value of Vega at different levels of moneyness and time to maturity.

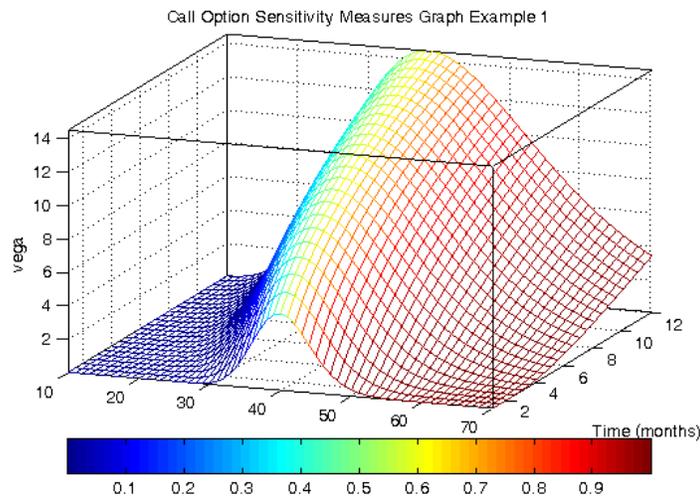


Figure 3.1 Vega in terms of time to maturity and moneyness

There are a few interesting characteristics about Vega. The first of which is that it is highest when the strike price equals the stock price or at-the-money. The second is that Vega is the same for both calls and puts. Therefore, if one buys an at-the-money straddle (buy a call and put at the same strike and expiration), the person can achieve the highest Vega with respect to all other option strategies. This is because the total Vega of the position is the sum of the Vega's for the put and the call, which are both at their maximum at-the-money. Since I am limited to 8 years of data and want to have as many observations as possible, I will be examining the returns generated from systematically trading one month to expiration at-the-money straddles for all stocks in the database (about 4000). I will calculate the returns for every stock from 2000 to 2008. I will then examine the average returns generated from the strategy from systematically purchasing straddles on every available stock. Volatility seems to be overpriced, on average, so I suspect there to be a negative return. The average return is not the most important issue. The issues are is there a common factor for straddle returns and risk premium to straddle investing exist.

3.1 Data and Methodology

I will use the IVOLATILITY.COM database which calculates the implied volatility of each stock whose options trade in the CBOE. I will re-create the option price for at the money call and at-the-money put. I will then pull the price data from CRSP for the stocks to calculate the return for the straddle. I will calculate the price of the call and put for each stock as if each stock was trading at 100 dollars and use the returns from the underlying stock to translate the ending value for stock price based on the fact that it was 100 dollars last period. For example, if a stock was 20 dollars when I purchased the straddle and moved up to 22 dollars at expiration, the corresponding underlying prices for the options I calculate will be 100 and 110 (because the stock moved up 10%). I am doing this for very specific reasons. First, it will allow me to make an apples to apples comparison of straddle returns across all stocks. Second, and more

importantly, it will enable me to specifically compare realized volatility to implied volatility. Knowing which options are cheap and which options are expensive are directly related to the relationship between realized and implied volatility. Additionally, if there is a risk premium to straddle investing relative to a common factor, that common factor can be used as the rubric to determine the extent to which options are over-priced or under-priced. Finally, the analysis that I showed in the last chapter implies that one can hedge their volatility exposure. Therefore, straddle investing and volatility hedging can be combined to yield superior risk adjusted returns.

3.2 Average Straddle Returns

In this section, I will calculate average straddle returns for all stocks that have traded on the CBOE from November 2000 to April 2008. I will examine returns from buying straddles (that have one month to expiration) on all available stocks on the Friday before expiration and close out the straddle on the day before the expiration date. Since expiration dates are the 3rd Saturday every month, most trades will be conducted on the 3rd Friday of each month. I first want to look at all returns that contain cross-sectional and time series returns. Then I want to break the returns down into time series returns. Finally, I want to look at cross-sectional returns.

Table 3.1 descriptive statistics of all straddle returns

| | Straddle Return_{i,t} |
|----------------------|--------------------------------------|
| Mean | -0.0120 |
| Median | -0.0198 |
| Std Deviation | 0.0702 |
| Kurtosis | 1.4414 |
| Skew | 0.6875 |
| Range | 0.5511 |
| Min | -0.2273 |
| Max | 0.3239 |
| Count | 162680 |

Table 3.1 shows that, on average, straddle investing does not make money. The table is showing monthly returns; hence, annualizing the mean return translates to a -14.4% annual

return. The mean is not statistically significant. This means that being long volatility (or short volatility) in a systematic fashion is not a good idea. Another very important interpretation of these results is that realized volatility is usually less than implied volatility. This does not mean that one should never be long volatility. The maximum statistics show that there are some straddle returns that are quite impressive. Now, let us now look at time series straddle returns for companies. I will show this in the form of a histogram for the 4000 companies. Notice the shape of the histogram.

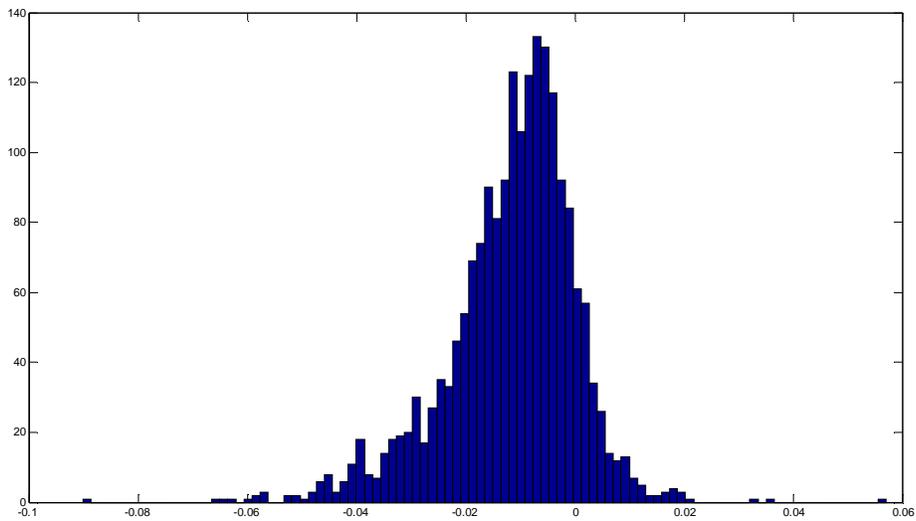


Figure 3.2 histogram of straddle returns by company

Figure 3.2 shows a fairly normal distribution of straddle returns by company. There left tail is slightly fatter than the right and there are very few companies in this sample which exhibit positive mean straddle returns.

The most interesting part of this analysis is to examine the cross-sectional returns. The following chart shows the average return for straddles across all stocks from 2000 to 2008. Notice how there are periods where straddle investing is very profitable and periods where buying straddles are a bad idea.

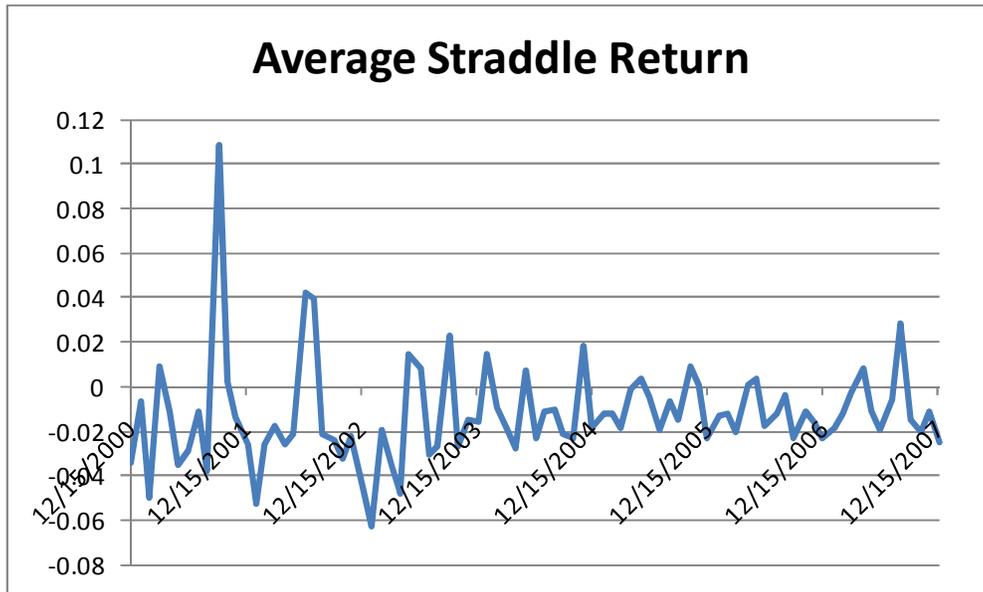


Figure 3.3 Average straddle returns over time

Figure 3.3 graphically shows the returns one would have realized if he or she systematically bought straddles on all stocks in an equally weighted portfolio from 2000 to 2007. Notice that most months are negative meaning that volatility was overpriced, on average, during that period. However, some monthly returns are positive, the highest being the returns during the 9/11 attacks. This was an unanticipated event and would not be priced in the options before the event, which is why the return was so great.

Using the data from figure 2, I would like to see if there is a statistically significant straddle return. I will simply look at the average return for the average company in each period and look at the standard deviation of that time series to form a t-statistic. There are 89 observations, so that is enough for a statistical inference. Table 3.2 shows these results.

Table 3.2 descriptive statistics of all straddle returns

| | Cross-sectional Estimate |
|---------------------------|---------------------------------|
| Mean | -0.0124 |
| Standard Deviation | 0.0227 |
| t-stat | -5.0386 |
| p-value | 0.0000 |

Table 3.2 shows that the t-statistic is statistically significant and negative. This means that there exists some ranking mechanism by which to select firms whose volatility is expensive and inexpensive. The next section explores what common factor might be driving this Cross-sectional estimate.

3.3 A Common Factor for Straddle Returns

Up to this point, I have showed that systematic buying of straddles does not yield any statistically significant returns. In this section, I will introduce a common factor that might help explain straddle returns and thus might lead to a potential risk premium for straddle returns. Let us turn back to the VIX to see if that might explain straddle returns. I will run regressions for straddle returns for all companies against the VIX to see if it has any explanatory power. I will run the following regression:

$$\text{StraddleReturn}(\text{Stock}_i)_t = b_0 + b_1 \Delta \text{VIX}_t \quad (1)$$

Table 3.3 Panel regression for equation 1

| | | | |
|--------------------------------------|----------------------|---|----------|
| Fixed-effects (within) IV regression | Number of obs | = | 137139 |
| Group variable: firm | Number of groups | = | 1955 |
| R-sq: within = | Obs per group: min = | | 36 |
| between = 0.0001 | avg = | | 70.1 |
| overall = 0.0047 | max = | | 85 |
| corr(u_i, Xb) = -0.0006 | Wald chi2(1) | = | 26505.70 |
| | Prob > chi2 | = | 0.0000 |

| s | Coef. | Std. Err. | z | P> z | [95% Conf. Interval] |
|---------|-----------|-----------------------------------|---------|-------|----------------------|
| vix | -.900531 | .0487969 | -18.45 | 0.000 | -.9961712 -.8048907 |
| _cons | -.5037945 | .0031025 | -162.39 | 0.000 | -.5098753 -.4977138 |
| sigma_u | .17416493 | | | | |
| sigma_e | 1.1479108 | | | | |
| rho | .02250199 | (fraction of variance due to u_i) | | | |

| | | | | |
|------------------------|------------------|------|------------|--------|
| F test that all u_i=0: | F(1954,135183) = | 1.49 | Prob > F = | 0.0000 |
|------------------------|------------------|------|------------|--------|

| | |
|---------------|------|
| Instrumented: | vix |
| Instruments: | lvix |

Table 3.3 shows that the parameter estimate for equation 1 is about -0.9. This means that for every percent change in the VIX, the average straddle return decreases by 0.9 percent. This regression is a fixed effect regression with an instrumental variable for the VIX using the lagged changed of the VIX as the instrument. This is done to resolve a simultaneity bias. This bias exists because I am measuring both variables at the same time. There is much variability with these parameter estimates as shown by the standard deviation figure in table 3. The most important question is that do the factor loadings result in the correct ranking of returns in the next period. The next section addresses that issue.

3.4 Straddle Return Risk Premium

I will now present Fama Macbeth (1972) estimates for the risk premium for the VIX with respect to straddle returns. I will break this risk premium up into two equal subperiods. The risk premia and corresponding statistics are reported in table 3.4.

Table 3.4 Fama Macbeth Estimates for risk premium for straddle returns on the VIX

| | Period 1 | Period 2 | Whole |
|-----------------------|-----------------|-----------------|--------------|
| Estimate | -0.0430 | -0.0280 | -0.0353 |
| Standard Error | 0.0161 | 0.0162 | 0.0113 |
| p-Value | 0.0127 | 0.0953 | 0.0030 |

Table 3.4 shows that there is a statistically significant and negative risk premium for straddle investing. The annualized risk premium for the whole period is -42.2% per annum. Interestingly enough, the magnitude of this risk premium is very close to risk premium estimates for the VIX for implied volatility returns. It could be that straddle returns and changes in implied volatility are proxying for the same thing. Since straddle returns are delta hedged with the underlying asset by design, the matching of risk premia from this chapter and from the prior chapter is very comforting. The reason why this risk premium is negative is not fully understood. It could be that volatility is over-priced on average. This is a postulation and should be studied further.

3.5 How to use the VIX to Hedge Volatility Exposure

Hedging your volatility exposure with the VIX is fairly simple. It only requires one extra trade in addition to the options trade you have already put on. Let's say you would like to participate in a large move in an underlying stock in either direction because you think that volatility is "cheap." Because you think volatility is cheap for this particular name, you would probably buy a straddle (the purchase of a call and put at the same strike price and expiration). Let's say you buy an at-the-money straddle for the stock with 2 months until expiration. Here is a matrix that contains profit and losses for all possible combinations of stock and implied volatility movement. I will call this the unhedged volatility play. For this example, I assume the stock has an implied volatility of 40% and you exit the trade 1 month later. Therefore, you hold the straddle for 1 month (or one expiration cycle). The following figure shows the returns.

Underlying Asset Price Change

| | -30.00% | -27.00% | -24.00% | -21.00% | -18.00% | -15.00% | -12.00% | -9.00% | -6.00% | -3.00% | 0.00% | 3.00% | 6.00% | 9.00% | 12.00% | 15.00% | 18.00% | 21.00% | 24.00% | 27.00% | 30.00% |
|------|---------|---------|---------|---------|---------|---------|---------|--------|--------|--------|-------|-------|-------|-------|--------|--------|--------|--------|--------|--------|--------|
| 0.28 | 15.53 | 12.53 | 9.53 | 6.53 | 3.55 | 0.63 | -2.16 | -4.67 | -6.68 | -7.99 | -8.44 | -8.00 | -6.75 | -4.85 | -2.49 | 0.16 | 2.99 | 5.91 | 8.87 | 11.85 | 14.85 |
| 0.29 | 15.53 | 12.53 | 9.53 | 6.53 | 3.56 | 0.64 | -2.12 | -4.59 | -6.57 | -7.85 | -8.28 | -7.85 | -6.62 | -4.75 | -2.42 | 0.21 | 3.01 | 5.92 | 8.88 | 11.86 | 14.85 |
| 0.30 | 15.53 | 12.53 | 9.53 | 6.53 | 3.57 | 0.65 | -2.08 | -4.52 | -6.46 | -7.70 | -8.13 | -7.69 | -6.49 | -4.65 | -2.35 | 0.25 | 3.04 | 5.93 | 8.88 | 11.86 | 14.85 |
| 0.31 | 15.53 | 12.53 | 9.53 | 6.54 | 3.57 | 0.68 | -2.04 | -4.45 | -6.35 | -7.57 | -7.97 | -7.55 | -6.36 | -4.55 | -2.28 | 0.29 | 3.07 | 5.95 | 8.89 | 11.86 | 14.85 |
| 0.32 | 15.53 | 12.53 | 9.53 | 6.54 | 3.58 | 0.71 | -1.99 | -4.37 | -6.24 | -7.43 | -7.82 | -7.40 | -6.23 | -4.45 | -2.21 | 0.34 | 3.09 | 5.96 | 8.90 | 11.87 | 14.86 |
| 0.33 | 15.53 | 12.53 | 9.53 | 6.54 | 3.59 | 0.73 | -1.95 | -4.29 | -6.13 | -7.29 | -7.68 | -7.26 | -6.11 | -4.35 | -2.14 | 0.39 | 3.12 | 5.98 | 8.91 | 11.87 | 14.86 |
| 0.34 | 15.53 | 12.53 | 9.53 | 6.55 | 3.60 | 0.75 | -1.90 | -4.22 | -6.02 | -7.16 | -7.53 | -7.12 | -5.98 | -4.25 | -2.07 | 0.44 | 3.16 | 6.00 | 8.92 | 11.88 | 14.86 |
| 0.35 | 15.53 | 12.53 | 9.53 | 6.55 | 3.61 | 0.78 | -1.85 | -4.14 | -5.92 | -7.03 | -7.39 | -6.98 | -5.86 | -4.16 | -1.99 | 0.49 | 3.19 | 6.02 | 8.93 | 11.89 | 14.86 |
| 0.36 | 15.53 | 12.53 | 9.53 | 6.56 | 3.63 | 0.80 | -1.81 | -4.07 | -5.81 | -6.90 | -7.25 | -6.85 | -5.74 | -4.06 | -1.92 | 0.54 | 3.22 | 6.04 | 8.94 | 11.89 | 14.87 |
| 0.37 | 15.53 | 12.53 | 9.53 | 6.56 | 3.64 | 0.83 | -1.76 | -3.99 | -5.71 | -6.78 | -7.12 | -6.71 | -5.62 | -3.96 | -1.85 | 0.60 | 3.26 | 6.06 | 8.96 | 11.90 | 14.87 |
| 0.38 | 15.53 | 12.53 | 9.54 | 6.57 | 3.65 | 0.86 | -1.71 | -3.91 | -5.60 | -6.65 | -6.98 | -6.58 | -5.51 | -3.86 | -1.77 | 0.65 | 3.30 | 6.09 | 8.97 | 11.91 | 14.88 |
| 0.39 | 15.53 | 12.53 | 9.54 | 6.57 | 3.67 | 0.89 | -1.66 | -3.84 | -5.50 | -6.53 | -6.85 | -6.45 | -5.39 | -3.77 | -1.70 | 0.70 | 3.33 | 6.11 | 8.99 | 11.92 | 14.88 |
| 0.40 | 15.53 | 12.53 | 9.54 | 6.58 | 3.68 | 0.92 | -1.60 | -3.76 | -5.40 | -6.41 | -6.72 | -6.33 | -5.28 | -3.67 | -1.62 | 0.76 | 3.37 | 6.14 | 9.00 | 11.93 | 14.89 |
| 0.41 | 15.53 | 12.53 | 9.54 | 6.59 | 3.70 | 0.95 | -1.55 | -3.68 | -5.30 | -6.29 | -6.59 | -6.20 | -5.16 | -3.57 | -1.54 | 0.82 | 3.41 | 6.17 | 9.02 | 11.94 | 14.89 |
| 0.42 | 15.53 | 12.53 | 9.55 | 6.60 | 3.72 | 0.98 | -1.50 | -3.61 | -5.20 | -6.18 | -6.47 | -6.08 | -5.05 | -3.48 | -1.47 | 0.87 | 3.45 | 6.19 | 9.04 | 11.95 | 14.90 |
| 0.43 | 15.53 | 12.53 | 9.55 | 6.60 | 3.73 | 1.02 | -1.45 | -3.53 | -5.10 | -6.06 | -6.35 | -5.96 | -4.94 | -3.38 | -1.39 | 0.93 | 3.49 | 6.22 | 9.06 | 11.96 | 14.91 |
| 0.44 | 15.53 | 12.53 | 9.55 | 6.61 | 3.75 | 1.05 | -1.39 | -3.45 | -5.00 | -5.95 | -6.22 | -5.84 | -4.83 | -3.29 | -1.32 | 0.99 | 3.54 | 6.25 | 9.08 | 11.97 | 14.92 |
| 0.45 | 15.53 | 12.53 | 9.55 | 6.62 | 3.77 | 1.08 | -1.34 | -3.38 | -4.91 | -5.83 | -6.10 | -5.72 | -4.72 | -3.20 | -1.24 | 1.05 | 3.58 | 6.28 | 9.10 | 11.99 | 14.92 |
| 0.46 | 15.53 | 12.54 | 9.56 | 6.63 | 3.79 | 1.12 | -1.29 | -3.30 | -4.81 | -5.72 | -5.98 | -5.60 | -4.61 | -3.10 | -1.16 | 1.11 | 3.62 | 6.31 | 9.12 | 12.00 | 14.93 |
| 0.47 | 15.53 | 12.54 | 9.57 | 6.64 | 3.81 | 1.15 | -1.23 | -3.22 | -4.71 | -5.61 | -5.87 | -5.48 | -4.51 | -3.01 | -1.09 | 1.17 | 3.67 | 6.34 | 9.14 | 12.02 | 14.94 |
| 0.48 | 15.53 | 12.54 | 9.57 | 6.65 | 3.83 | 1.19 | -1.18 | -3.15 | -4.62 | -5.50 | -5.75 | -5.37 | -4.40 | -2.92 | -1.01 | 1.23 | 3.71 | 6.38 | 9.16 | 12.03 | 14.95 |

Figure 3.4 Straddle Returns in Terms of Volatility and Underlying

As you can see, it would take a large move (about 12%) for this trade. Additionally, if implied volatility continues to contract, you will earn a more negative return.

Now, let's take the same example but include a volatility hedge. Just like a stock portfolio manager would hedge his or her systematic risk by shorting S&P 500 futures, so too would you short VIX futures to hedge your volatility risk. However, there is another step we need to take. You need to calculate the Vega of the straddle and multiply it by the sensitivity of the stock's implied volatility versus the VIX. So you would run the regression for equation 2 in chapter 2 discussed earlier. In this specific example, we will assume a sensitivity of 0.5. The next figure shows volatility hedged returns for the original straddle position.

Underlying Asset Price Change

| VIX | Stock Vol | -30.00% | -27.00% | -24.00% | -21.00% | -18.00% | -15.00% | -12.00% | -9.00% | -6.00% | -3.00% | 0.00% | 3.00% | 6.00% | 9.00% | 12.00% | 15.00% | 18.00% | 21.00% | 24.00% | 27.00% | 30.00% |
|-------|-----------|---------|---------|---------|---------|---------|---------|---------|--------|--------|--------|-------|-------|-------|-------|--------|--------|--------|--------|--------|--------|--------|
| 0.1 | 0.28 | 22.46 | 19.46 | 16.46 | 13.46 | 10.48 | 7.56 | 4.77 | 2.26 | 0.25 | -1.06 | -1.51 | -1.07 | 0.19 | 2.09 | 4.44 | 7.10 | 9.92 | 12.84 | 15.80 | 18.79 | 21.78 |
| 0.105 | 0.29 | 22.11 | 19.11 | 16.11 | 13.12 | 10.14 | 7.23 | 4.46 | 1.99 | 0.02 | -1.26 | -1.70 | -1.26 | -0.03 | 1.84 | 4.16 | 6.79 | 9.60 | 12.50 | 15.46 | 18.44 | 21.44 |
| 0.11 | 0.30 | 21.76 | 18.76 | 15.77 | 12.77 | 9.80 | 6.90 | 4.16 | 1.72 | -0.22 | -1.47 | -1.89 | -1.45 | -0.25 | 1.59 | 3.89 | 6.49 | 9.28 | 12.17 | 15.12 | 18.10 | 21.09 |
| 0.115 | 0.30 | 21.42 | 18.42 | 15.42 | 12.43 | 9.47 | 6.58 | 3.85 | 1.45 | -0.46 | -1.67 | -2.08 | -1.65 | -0.47 | 1.34 | 3.61 | 6.19 | 8.96 | 11.84 | 14.78 | 17.76 | 20.75 |
| 0.12 | 0.31 | 21.07 | 18.07 | 15.07 | 12.09 | 9.13 | 6.25 | 3.55 | 1.18 | -0.69 | -1.88 | -2.28 | -1.85 | -0.69 | 1.10 | 3.33 | 5.89 | 8.64 | 11.51 | 14.45 | 17.41 | 20.40 |
| 0.125 | 0.32 | 20.72 | 17.72 | 14.73 | 11.74 | 8.79 | 5.93 | 3.25 | 0.90 | -0.93 | -2.09 | -2.48 | -2.06 | -0.91 | 0.85 | 3.06 | 5.59 | 8.32 | 11.18 | 14.11 | 17.07 | 20.06 |
| 0.13 | 0.32 | 20.38 | 17.38 | 14.38 | 11.40 | 8.46 | 5.61 | 2.95 | 0.63 | -1.17 | -2.31 | -2.68 | -2.27 | -1.13 | 0.60 | 2.79 | 5.29 | 8.01 | 10.85 | 13.77 | 16.73 | 19.71 |
| 0.135 | 0.33 | 20.03 | 17.03 | 14.04 | 11.06 | 8.12 | 5.28 | 2.65 | 0.36 | -1.41 | -2.53 | -2.88 | -2.47 | -1.36 | 0.35 | 2.51 | 5.00 | 7.70 | 10.53 | 13.44 | 16.39 | 19.37 |
| 0.14 | 0.33 | 19.68 | 16.69 | 13.69 | 10.72 | 7.79 | 4.96 | 2.35 | 0.09 | -1.65 | -2.74 | -3.09 | -2.69 | -1.58 | 0.10 | 2.24 | 4.70 | 7.38 | 10.20 | 13.10 | 16.05 | 19.03 |
| 0.145 | 0.34 | 19.34 | 16.34 | 13.35 | 10.37 | 7.45 | 4.65 | 2.06 | -0.18 | -1.90 | -2.97 | -3.30 | -2.90 | -1.81 | -0.45 | 1.97 | 4.41 | 7.07 | 9.88 | 12.77 | 15.71 | 18.68 |
| 0.15 | 0.35 | 18.99 | 15.99 | 13.00 | 10.03 | 7.12 | 4.33 | 1.76 | -0.45 | -2.14 | -3.19 | -3.52 | -3.12 | -2.04 | -0.40 | 1.70 | 4.12 | 6.76 | 9.55 | 12.44 | 15.38 | 18.34 |
| 0.155 | 0.35 | 18.65 | 15.65 | 12.66 | 9.69 | 6.79 | 4.01 | 1.46 | -0.72 | -2.38 | -3.41 | -3.73 | -3.33 | -2.27 | -0.65 | 1.42 | 3.82 | 6.45 | 9.23 | 12.11 | 15.04 | 18.00 |
| 0.16 | 0.36 | 18.30 | 15.30 | 12.31 | 9.35 | 6.46 | 3.69 | 1.17 | -0.99 | -2.63 | -3.64 | -3.95 | -3.55 | -2.50 | -0.90 | 1.15 | 3.53 | 6.34 | 9.11 | 11.78 | 14.70 | 17.66 |
| 0.165 | 0.36 | 17.95 | 14.96 | 11.97 | 9.01 | 6.13 | 3.38 | 0.87 | -1.26 | -2.87 | -3.87 | -4.17 | -3.77 | -2.74 | -1.15 | 0.88 | 3.24 | 5.84 | 8.59 | 11.45 | 14.36 | 17.32 |
| 0.17 | 0.37 | 17.61 | 14.61 | 11.63 | 8.68 | 5.80 | 3.06 | 0.58 | -1.53 | -3.12 | -4.10 | -4.39 | -4.00 | -2.97 | -1.40 | 0.61 | 2.95 | 5.53 | 8.27 | 11.12 | 14.03 | 16.98 |
| 0.175 | 0.37 | 17.26 | 14.27 | 11.28 | 8.34 | 5.47 | 2.75 | 0.29 | -1.80 | -3.37 | -4.33 | -4.61 | -4.22 | -3.21 | -1.65 | 0.34 | 2.66 | 5.23 | 7.95 | 10.79 | 13.69 | 16.64 |
| 0.18 | 0.38 | 16.91 | 13.92 | 10.94 | 8.00 | 5.14 | 2.44 | -0.01 | -2.07 | -3.32 | -4.26 | -4.54 | -4.15 | -3.14 | -1.60 | 0.07 | 2.38 | 4.92 | 7.64 | 10.46 | 13.36 | 16.30 |
| 0.185 | 0.38 | 16.57 | 13.57 | 10.60 | 7.66 | 4.81 | 2.12 | -0.30 | -2.34 | -3.57 | -4.49 | -4.76 | -4.37 | -3.36 | -1.80 | 0.20 | 2.09 | 4.62 | 7.32 | 10.14 | 13.03 | 15.96 |
| 0.19 | 0.39 | 16.22 | 13.23 | 10.26 | 7.32 | 4.49 | 1.81 | -0.59 | -2.61 | -3.82 | -4.71 | -4.97 | -4.58 | -3.57 | -2.01 | 0.47 | 1.80 | 4.32 | 7.00 | 9.81 | 12.69 | 15.63 |
| 0.195 | 0.39 | 15.88 | 12.88 | 9.91 | 6.99 | 4.16 | 1.50 | -0.88 | -2.88 | -4.37 | -5.26 | -5.52 | -5.14 | -4.16 | -2.66 | 0.74 | 1.51 | 4.01 | 6.69 | 9.49 | 12.36 | 15.29 |
| 0.2 | 0.40 | 15.53 | 12.54 | 9.57 | 6.65 | 3.83 | 1.19 | -1.18 | -3.15 | -4.62 | -5.50 | -5.75 | -5.37 | -4.40 | -2.92 | -1.01 | 1.23 | 3.71 | 6.38 | 9.16 | 12.03 | 14.95 |

Figure 3.5 Hedged Straddle Returns in Terms of Volatility and Underlying

As you can see, the implied move for you to make money has become smaller. Before, the stock had to move at least 12% for you to make any money given no movement in implied volatility. Now, the stock only has to move 9% for you to make money. Additionally, if you are wrong about volatility going up, you will not be punished when your volatility exposure is hedged.

So far I have only discussed one example of how to hedge volatility exposure for a single stock. This strategy also works if you think implied volatility is too high for a certain stock. This would entice you to sell a straddle but you would be buying VIX futures to hedge your negative vega. This volatility hedging technique can also be used on a portfolio of options. All one would have to do is calculate the vega for each option in your portfolio and calculate the sensitivity of the changes in implied volatility for each underlying company and multiply the two figures and add those products up to arrive at an aggregate portfolio vega that takes into account the fact that some stocks might be more or less affected by changes in volatility of the market portfolio, measured by the VIX.

3.6 Remarks

Since I was able to find a common factor (the VIX) that carried a risk premium in straddle investing, the VIX can be used as a rubric to assess the extent to which option implied volatility is overpriced or underpriced. The existence of a risk premium for straddle investing using the VIX also implies that one can hedge their volatility exposure with the VIX. These chapters contribute to a literature a large puzzle piece in terms of volatility pricing. The VIX is a very important instrument in its own right but these chapters have illustrated the ability for one to hedge options portfolios with the VIX.

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