APPLICATIONS OF SENSITIVITY DATA TO EIGENSOLUTION REANALYSIS OF MODIFIED STRUCTURES

by

CHANDRASHEKHAR SURESHCHANDRA PATADIA

Presented to the Faculty of the Graduate School of

The University of Texas at Arlington in Partial Fulfillment

of the Requirements

for the Degree of

MASTER OF SCIENCE IN MECHANICAL ENGINEERING

THE UNIVERSITY OF TEXAS AT ARLINGTON

MAY 2010

ACKNOWLEDGEMENTS

First, I would like to express my deepest gratitude and admiration to my supervisor Dr. Bo Wang for his help throughout my Masters studies. Structural Dynamics and Structural Optimization courses offered by him were great. His excellent guidance and support made my learning experience a very special one.

I would also like to thank Dr. Nomura and Dr. Lawrence for their support and time in serving as my committee members.

Special thanks to family members, who have always been with me. Especially my mother for her love, continuous inspiration and support without which my Masters studies would not have completed.

Last but not least, I would like to thank god for showing me path, giving me strength and determination during difficult times.

April 19, 2010

ABSTRACT

APPLICATIONS OF SENSITIVITY DATA TO EIGENSOLUTION REANALYSIS OF MODIFIED STRUCTURES

Chandrashekhar Sureshchandra Patadia, M.S.

The University of Texas at Arlington, 2010

Supervising Professor: Bo Ping Wang

A new procedure for eigensolution reanalysis has been developed in this thesis. This new method is based on the sensitivity data. Methods for computing eigensolution sensitivity using both exact and approximate methods have been known for a long time. While eigenvalue sensitivity is used routinely in structural optimization with eigenvalue constraints, few applications of eigenvector derivatives are reported.

In this thesis, effective eigensolution reanalysis using first and second order eigenvector sensitivity data is presented. The proposed approach and other eigensolution reanalysis approaches are implemented in Matlab R2008a to get eignesolution of modified design. These results obtained by reanalysis techniques are compared with the exact analysis of modified eigensolution.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

Figure Page

LIST OF TABLES

Table Page

CHAPTER 1

INTRODUCTION

1.1 Introduction to Eigenproblem Reanalysis

While designing or optimizing a structure, structural dynamics analyses must be performed repeatedly for various configurations in order to make the design satisfy the constraints. These repeated analyses for each modified design are time consuming and expensive, especially if the structure is very large with thousands of degree of freedom.

The purpose of reanalysis techniques is to analyze the modified structure without performing full analysis and thus reduce the computation time and cost. Some reanalysis procedures are only for eigenvalue reanalysis [1-5]. Some procedures are for both eigenvalue and eigenvector reanalysis [6-11]. Wang and Caldwell used the Eigenvectors of initial design and static modes to generate a reduced eigenprobelm [8]. Kirsch introduced a CA (Combined Approximation) procedure which uses first few the terms of an infinite series as basis vectors in reduced eigenproblem [9]. Chen, Wu and Yang used epsilon algorithm table to approximate eigenvector and used this approximation to approximate eigenvalue using Raleigh's quotient [11]. In this thesis, a new approach based on sensitivity data has been developed for eigenproblem reanalysis.

Methods of computing eigensolution sensitivity have been known for a long time. Several methods are available for computing exact and approximate derivative [12-16]. In this thesis, Wang's approach for calculating approximate eigenvector derivative [14] has been extended for second derivative. While eigenvalue sensitivity is used routinely in structural optimization with eigenvalue constraints [17], few application of eigenvector derivativeses are reported in the literature. In this thesis, a new procedure is proposed for eigensolution reanalysis. The proposed new approach uses mode shapes and their derivatives as basis vectors for eigensolution of the modified system. It has been shown that sensitivity data produces better results with the proposed approach than that computed by perturbation method.

Numerical results using the proposed approach were obtained for different types of global modifications in various models such as 20 degree of freedom spring mass system, and a frame model. Comparison of numerical results with that obtained from several existing eigensolution reanalysis methods and direct analysis suggests that the proposed algorithms are very effective.

1.2 Outline of Thesis

In chapter 2, properties of eigenvector and eigennsolver algorithms already developed based on these properties are discussed.

In Chapter 3, various available reanalysis methods for global as well as local modifications are discussed.

In chapter 4, procedures for computing exact first order eigenvector derivative using modal method as well as algebraic method is illustrated. Then Wang's explicit and implicit methods for computing approximate first order eigenvector derivative are discussed. These procedures are extended for approximating second order eigenvector derivative.

The proposed approach using exact as well as approximate eigenvector derivatives is introduced and developed in Chapter 5.

In chapter 6, comparisons of results are shown for different models and different modifications. A 5 DOF spring mass system is modified locally whereas another 20 DOF spring mass system is modified globally. Local and global modifications for a plane frame structure are also investigated in this chapter. The global modifications include topology modification and change in cross section of members of frame.

Chapter 7 contains conclusion and future research.

CHAPTER 2

EIGENSOLVER ALGORITHMS FOR STRUCTURAL DYNAMICS

Eigenvectors in structural dynamics problem have some special properties. These properties are used in basic eigensolver techniques such as Inverse Iteration, Inverse Iteration with shift, Subspace Iteration etc. Most of the advanced algorithms are based on these basic techniques and some other properties of eigenvector. Some techniques for approximating eigenvector derivatives are also based on these properties. Therefore, properties of eigenvector are discussed here before discussing eigensolver algorithms.

2.1 Properties of Eigenvector

Important properties of eigenvector are summarized in this section.

2.1.1. Arbitrary Scaling- Normalization of Eigenvectors

This implies the Eigen vectors can be normalized arbitrarily.

If ϕ_r is an eigenvector corresponding to λ_r, then

$\bar{\phi}_r = c\phi_r, c \neq 0$ is also an eignevector.

Eigenvectors can be normalized any of the following ways:

- Making a selected element equal to 1,
- Making the largest element equal to 1.

- Making modal mass equal to 1,

$$\bar{\phi}_i = c\phi_i \text{ where } c = \frac{1}{\sqrt{GM_i}} \quad GM_i = \phi_i^T M \phi_i$$

2.1.2 M-Orthogonality and K-Orthogonality

$$\phi_r^T M \phi_s = 0 \quad (r \neq s)$$

$$\phi_r^T K \phi_s = 0 \quad (r \neq s)$$

Note that ϕ_1 and ϕ_2 are not orthogonal to each other. That is:
$\phi_1^T \phi_2 \neq 0$

2.1.3 Expansion Theorem

Any vector {y} of size (N×1) can be expressed as a linear combination of N Eigenvectors.

$$\{y\} = \sum_{i=1}^{N} c_i \phi_i$$

2.2 Eigensolver Algorithms

Practically a multi degree of freedom model may have thousands of degrees of freedom. Solving eigneproblem for structures usually involves determining some eigenvalues and corresponding eigenvectors.

$$K\Phi = M\Phi\Lambda \qquad (2.1)$$

The square roots of diagonal terms of Λ are natural frequencies of that structure. For structures with a few number of degree of freedom, it is possible to find eigenpairs using polynomial root finding method. For large structures, numerical algorithms are employed. These algorithms are based on different properties of eigenvector. Selection of eigensolver depends upon various factors such as size and sparsity of eigenproblem, eigenpairs that needs to be determined, the level of accuracy desired etc.

Simulation soft- wares such as ANSYS offers the user to choose from many of the available algorithms for solving structural dynamics eigenproblm of a large system. For example, undamped modal analysis for large structures in ANSYS can be done by employing any of the following methods [21]:

- Block Lanczos Method: uses Lanczos algorithm where the Lanczos recursion is performed with a block of vectors.
- PCG Lanczos Method: internally uses the Lanczos algorithms, combined with PCG iterative solve. This method should be used if only a few lower modes are required.
- Subspace Method: uses subspace iteration technique.
- Reduced Method: uses Householder-Bisection-Inverse iteration.

Here vector iteration methods such as Inverse Iteration- with and without shift, subspace iteration is discussed.

2.2.1. Inverse Iteration [20]

Then basic scheme in Inverse Iteration is given by:

$$K\bar{r}^{(s+1)} = Mr^{(s)} \tag{2.2}$$

Here, s+1 is iteration number. The first vector r_0 is any starting vector. For each iteration, r_s is the vector obtained from previous iteration and $\bar{r}_{s+1}$ is the resulting improved vector. The previous vector is normalized by either making maximum element equal to 1 or modal mass equal to 1.

$$r^{(s+1)} = \frac{\bar{r}^{(s+1)}}{\sqrt{\bar{r}^{(s+1)T} M \bar{r}^{(s+1)}}} \tag{2.3}$$

The corresponding eigenvalue for a particular iteration cycle is obtained from Raleigh's quotient:

$$\lambda^{(s+1)} = \frac{\bar{r}^{(s+1)T} Mr^{(s)}}{\bar{r}^{(s+1)T} M \bar{r}^{(s+1)}} \tag{2.4}$$

It is proved that the above procedure converges to the first eigenvalue and Eigenvector for any starting iteration vector as long as the starting iteration vector is not orthogonal to the first eigenvector [20]. To obtain mode shape other than first, it is necessary to remove the components of all preceding mode shapes from iteration vector, $r^{(s)}$.

2.2.2 Inverse Iteration with Shift [20]

Using Gram-Schmidt orthgonalization and associated matrix deflation removes components of previously determined eigenvectors and thus Inverse iteration can produce convergence to eigenvalue other than first. Another method to produce eigenvalue other than first is Inverse Iteration with spectrum shift. It is proved that using ρ as a shift in this method will converge to an eigenvalue closest to this shift.

The shifted stiffness matrix $\hat{K}$ can be computed as:

$$\hat{K} = K - \rho M \tag{2.5}$$

Let μ and ψ be the eigenvalue and eigenvector of the new eigenproblem which has shifted stiffness matrix.

$$\hat{K}\psi = \mu M \psi \tag{2.6}$$

Substituting equation (2.6) in to (2.5),

$$\left(K-\rho M\right)\psi = \mu M \psi$$

$$K\psi = \left(\mu+\rho\right) M \psi \tag{2.7}$$

Comparing the above equation with the original eigenproblem,

$$\phi = \psi \text{ and } \lambda = \left(\mu+\rho\right)$$

Therefore, iterative procedure similar to that shown in Inverse Iteration procedure accompanied with a proper shift will lead to mode shape other than fundamental mode.

2.2.3 Subspace Iteration [20]

Subspace Iteration method is more suitable for calculating a subset of eigenpairs of a large system as compared to Inverse iteration with or without shift. It requires solving a reduced eigenproblem of $N_m \times N_m$, where, m is the number of eigenpairs that needs to be determined. This method is explained below:

The eigenproblem in matrix form for only m eigenpairs:

$$K\Phi_m = M\Phi_m\Lambda_m \tag{2.8}$$

The iterative procedure is started with m trial vectors. Then, the new vectors are calculated by the following equation:

$$K\bar{R}^{(s+1)} = MR^{(s)} \tag{2.9}$$

Let $v_1,\ldots,v_m$ be an approximation of eigenvectors which is a linear combination of vectors.

$$v_i = \left[r_1,\ldots,r_m\right]\begin{Bmatrix}\psi_{1i}\\ \vdots \\ \psi_{mi}\end{Bmatrix} = \bar{R}^{(s+1)}\psi_i^{(s+1)} \tag{2.10}$$

Substituting the approximations of eigenvectors shown in equation (2.10) in (2.8) and pre-multiply by $\bar{R}^{(s+1)T}$

$$\bar{R}^{(s+1)T}K\bar{R}^{(s+1)}\Psi^{(s+1)} = \bar{R}^{(s+1)T}M\bar{R}^{(s+1)}\Psi^{(s+1)}\Lambda_m \tag{2.11}$$

Where, $\Psi^{(s+1)} = \left[\psi_1\ldots\psi_m\right]^{(s+1)}$

Let $\bar{R}^{(s+1)T} K \bar{R}^{(s+1)} = K_R$ and $\bar{R}^{(s+1)T} M \bar{R}^{(s+1)} = M_R$

From Equation (2.11),

$$K_R \Psi^{(s+1)} = M_R \Psi^{(s+1)} \Lambda_m \qquad (2.12)$$

In the above eigenproblem is reduced eigenproblem, matrices K_R and M_R are $N_m \times N_m$ size only and gives approximation of N_m eigen pairs.

Improved approximation for the eigenvectors is:

$$R^{(s+1)} = \bar{R}^{(s+1)} \Psi^{(s+1)} \qquad (2.13)$$

These vectors are used to find basis vectors for the next iteration from equation (2.9) and then equations (2.10) through (2.13) are followed to obtain better approximation of eigenvectors. This procedure is done recursively until the convergence is achieved.

CHAPTER 3

PROCEDURES FOR EIGENPROBLEM REANALYSIS

A Structural Dynamics analysis may have to be performed several times while designing or optimizing the structure. The modification in the design may be local or global. Usually, the modifications in the design are done in order to increase lower eigenvalues or increase the difference between two consecutive eigenvalues.

As shown in Chapter 2, direct eingesolution of modified design involves factorization of matrices which are changed for each modification. Therefore, these methods are time consuming for reanalysis. Some reanalysis techniques have been developed which uses the eigen solution of Initial design to approximate the eigen solution of modified design. Several methods have been suggested by researchers for reanalysis of eigenproblem [6-11]**.** In section 3.2, some of the reanalysis techniques for local as well as global modifications are discussed. In the next chapter, new approach based on sensitivity is developed. Some of the potential applications of a reanalysis technique include:

- Estimate the effect of structural modification
- Quick assessment of structural dynamics effects of structural damage
- Structural dynamics optimization

3.1 Existing Procedures for Eigenproblem Reanalysis

Assume that initial eigen problem is:

$$K_0\Phi_0 = M_0\Phi_0\Lambda_0 \tag{3.1}$$

Now, let the modifications in the stiffness matrix and mass matrix be ΔK and ΔM respectively. The modified stiffness and mass matrices are given by,

$$K_1 = K_0 + \Delta K \qquad M_1 = M_0 + \Delta M \tag{3.2}$$

Eigen problem of modified design is:

$$K_1\Phi_1 = M_1\Phi_1\Lambda_1 \tag{3.3}$$

From Equation (3.2),

$$(K_0 + \Delta K)\Phi_1 = (M_0 + \Delta M)\Phi_1\Lambda_1 \tag{3.4}$$

Practically, design has to be modified on trial and error basis. So, it needs to be checked for a large number of modifications. Solving the eigenproblem directly using modified stiffness and mass matrices for each modification causes large computation cost and time for large structures. So, the objective of reanalysis is to solve modified design eigenproblem without solving it directly.

Local modifications are limited to modifications that affect a few degrees of freedom and elements only. On the other hand, global modifications are the modifications which involve comparatively large number of degree of freedom and elements. Here, a local modification method is explained in section 3.1.1.

For global modifications, CA procedure, Wang-Caldwell method and Epsilon algorithm method is discussed in sections 3.1.2, 3.1.3 and 3.1.4 respectively.

3.1.1 Local Modification Method

Let $Z = K - \Omega^2 M$ = dynamic stiffness matrix

and, $R = Z^{-1}$ = receptance matrix

Then, using mode superposition method, modal representation of receptance matrix can be shown to be:

$$R = \Phi \begin{bmatrix} \ddots & & \\ & \frac{1}{\omega^2 - \Omega^2} & \\ & & \ddots \end{bmatrix} \Phi^T \qquad R_{IJ} = \sum_{r=1}^{N} \frac{\phi_{Ir}\phi_{Jr}}{\omega_r^{\,2} - \Omega^2}$$

ω = natural frequency of initial design

Ω = natural frequency of modified design

- Approximation of $R_{ij}(\Omega)$ due to Modal Truncation

Define $R_{iIJ}^{(n)} = \sum_{r=1}^{N} \frac{\phi_{Ir}\phi_{Jr}}{\omega_r^{\,2} - \Omega^2}$

and, $F_{IJ}^{(s,t)} = \sum_{r=s}^{t} \frac{\phi_{Ir}\phi_{Jr}}{\omega_r^{\,2}}$

Then, $R_{IJ}^{(L)}$ = L-mode approximation of R_{IJ}

Note that $R_{IJ} \cong R_{IJ}^{(L)} + F_{IJ}^{(L+1,N)}$

$R_{IJ} = R_{IJ}^{(L)} + F_{IJ}^{(L+1,N)}$

$$\hat{R}_{IJ} = R_{IJ}^{(L)} + F_{IJ}^{(1,N)} - F_{IJ}^{(1,L)}$$

In expanded form

$$R_{IJ} = \sum_{r=1}^{L} \frac{\phi_{Ir}\phi_{Jr}}{\omega_r^{\,2} - \Omega^2} + \left(K^{-1}\right)_{IJ} - \sum_{r=1}^{L} \frac{\phi_{Ir}\phi_{Jr}}{\omega_r^{\,2}} \tag{3.5}$$

Where, R_{IJ} is $(I,J)^{th}$ element of recentance matrix.

The following two cases of local modification are considered: (a) Addition of a spring and (b) Addition of a mass

3.1.1.1 Addition of a Spring

If a spring of stiffness k_a has been added between n^{th} DOF and ground, I = J = n, it can be shown that,

$$\mathrm{R_{IJ}} = -\frac{1}{\mathrm{k_a}} \tag{3.6}$$

Equating equations (3.5) and (3.6),

$$-\frac{1}{\mathrm{k_a}} = \sum_{r=1}^{L} \frac{\phi_{Ir}\phi_{Jr}}{\omega_r^{\,2} - \Omega^2} + \left(K^{-1}\right)_{IJ} - \sum_{r=1}^{L} \frac{\phi_{Ir}\phi_{Jr}}{\omega_r^{\,2}} \tag{3.7}$$

If only first mode and Eigen value are used in the above equation, L = 1.

$$-\frac{1}{\mathrm{k_a}} - \left(K^{-1}\right)_{IJ} + \frac{\phi_{I1}\phi_{J1}}{\omega_1^{\,2}} = \frac{\phi_{I1}\phi_{J1}}{\omega_1^{\,2} - \Omega^2}$$

So, $\Omega^2 = \omega_1^{\,2} - \frac{\phi_{I1}\phi_{J1}}{\mathrm{t}}$ (3.8)

$$\text{Where, t} = -\frac{1}{k_a} - \left(K^{-1}\right)_{IJ} + \frac{\phi_{I1}\phi_{J1}}{\omega_1^{\,2}}$$

If first two modes and eigen values are used in equation (3.7), L = 2. Then we have,

$$A(\Omega^2)^2 + B\Omega^2 + C = 0 \qquad (3.9)$$

$$\text{Where, A} = -\frac{1}{k_a} - \left(K^{-1}\right)_{IJ} + \left[\frac{p_1}{\omega_1^{\,2}} + \frac{p_2}{\omega_2^{\,2}}\right], \quad p_1 = \phi_{I1}\phi_{J1}, \quad p_2 = \phi_{I2}\phi_{J2}$$

$$B = \left(p_1 + p_2\right) - t\left(\omega_1^{\,2} + \omega_2^{\,2}\right)$$

$$C = -\left[p_1\omega_2^{\,2} + p_2\omega_1^{\,2} - \omega_1^{\,2}\omega_2^{\,2}\right]$$

Equation (3.8) is a quadratic equation for Ω^2 and the roots are as follows.

$$\Omega^2 = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

3.1.1.2 Addition of a Mass

When a mass m_a is added at degree of freedom I,

$$R_{IJ} = \frac{1}{m_a\Omega^2} \qquad (3.10)$$

From Equation (3.5),

$$\frac{1}{m_a\Omega^2} = \sum_{r=1}^{L}\frac{\phi_{Ir}\phi_{Jr}}{\omega_r^{\,2} - \Omega^2} + \left(K^{-1}\right)_{IJ} - \sum_{r=1}^{L}\frac{\phi_{Ir}\phi_{Jr}}{\omega_r^{\,2}}$$

After simplification, following quadratic equation is obtained.

$$A(\Omega^2)^2 + B\Omega^2 + C = 0 \qquad (3.11)$$

$A = m_a$

$$B = -\left[\frac{1+p_1 m_a}{X} + m_a \omega_1^2\right]$$

$$C = \frac{\omega_1^2}{X}$$

$$\text{Where, } X = \left(K^{-1}\right)_{IJ} - \frac{\phi_{I1}\phi_{J1}}{\omega_1^2} \text{ and } P_1 = \phi_{I1}\phi_{J1}$$

Note that the above method is for modification is for modification in a single degree of freedom only. Similarly, equation for eigenvalue of modified design with change in stiffness as well as mass matrices simultaneously can be derived.

The methods for reanalysis of global modifications are presented in the next section.

3.1.2 CA Procedure [9]

This procedure was introduced by Kirsch.

3.1.2.1 CA Procedure for Static Reanalysis

Let K_0 and R_0 be the stiffness matrix and load vector of initial design with n number of degree of freedom respectively. The corresponding displacement vector is computed from equation below:

$$K_0 r_0 = R_0 \qquad (3.12)$$

If modifications in the stiffness matrix and load vector are ΔK and ΔR respectively,

$$K_1 r_1 = R_1 \qquad (3.13)$$

$$K_1 = K_0 + \Delta K,\ R_1 = R_0 + \Delta R$$

CA procedure used by Kirsch, which solves reduced problem for displacements of modified design are obtained by implementing the following procedure:

- Calculation of basis vectors:

$$r_1 = K_0^{-1} R$$

$$r_i = -\left(K_0^{-1} \Delta K_0\right) r_{i-1} \quad i = 2,3,..,q$$

$$r_B = [r_1, r_2, ..., r_q]$$

- Compute reduced matrices using the following equations:

$$K_R = r_B^T K r_B,\ R_R = r_B^T R$$

- Solve following equation for y.

$$K_R y = R_R$$

- Compute displacements of modified design by following equation:

$$r = r_B y$$

Here, number of basis vectors, q is much smaller that number of degree of freedom, n. So, K_R is reduced matrix (q × q) compared to K (n × n).

3.1.2.2 CA procedure for Eigenproblem Reanalysis

Let K_0 and M_0 be the stiffness matrix and mass matrices of initial design with n number of degree of freedom respectively. The corresponding eigen vector is computed from equation below:

$$K_0\phi_0 = \lambda_0 M_0\phi_0 \quad (3.14)$$

If modifications in the stiffness and mass matrices are ΔK and ΔM respectively, the eigen problem for the modified system is:

$$K\phi = \lambda M\phi \quad (3.15)$$

$$K = K_0 + \Delta K,\ M = M_0 + \Delta M \quad (3.16)$$

If number of degree of freedom is large, inverting modified stiffness matrix is time consuming and costly. CA procedure used by Kirsch, which solves reduced problem of modified design, is explained below:

- Calculation of basis vectors:

$$r_1 = K_0^{-1} M \phi_0 \quad (3.17)$$

$$r_i = -\left(K_0^{-1} \Delta K_0\right) r_{i-1} \quad i = 2,3,..,q \quad (3.18)$$

$$r_B = [r_1, r_2, ..., r_q] \quad (3.19)$$

- Compute reduced matrices using the following equations:

$$K_R = r_B^T K r_B \,,\; M_R = r_B^T M r_B \qquad (3.20)$$

- Solve following equation for y.

$$K_R y = M_R y \Lambda \qquad (3.21)$$

- Compute eigenvector of modified design by following equation:

$$r = r_B y \qquad (3.22)$$

The above procedure is for computing the first eigenpair. For other eigenpairs, Gram-Schmidt orthogonalization has to be used. Assuming that the first m eigenvalues and Eigenvectors are found from the procedure mentioned above, then $(m+1)^{th}$ eigenpair is better approximated by using additional procedure mentioned below:

- Calculate coefficients using each approximated Eigenvector:

$$\alpha_i = \phi_i^T M \bar{\phi}_{m+1} \qquad (3.23)$$

Where, i = 1, 2…m, $\bar{\phi}_{m+1}$ is a non orthogonal starting iteration vector.

- Calculate following improved basis vector and follow equations (3.18) through (3.22).

$$\phi_{m+1} = \bar{\phi}_{m+1} - \sum_{i=1}^{m} \alpha_i \phi_i \qquad (3.24)$$

CA procedure was also suggested to use inverse iteration and inverse iteration with shift for reanalysis.

The CA procedure using Gram-Schmidt Orthogonalization for mode other than first requires that all preceeding modes should first be approximated. For example, to approximate third eigenvalue, first and second Eigenvectors needs to be computed first.

3.1.3 Epsilon Algorithm [11]

Epsilon algorithm [23-25] is used to accelerate convergence of an infinite series. Epsilon algorithm can be used to approximate eigen solutions by generating a vector sequence. The vectors sequence can be generated using either Neumann series expansion or perturbation method.

Table 3.1 Epsilon Algorithm Table

ε_{-1}	0		0		0		0		0	…..
ε_0		s_0		s_1		s_2		s_3		s_4 …..
ε_1			$\varepsilon_1^{(0)}$		$\varepsilon_1^{(1)}$		$\varepsilon_1^{(2)}$		$\varepsilon_1^{(3)}$	…..
ε_2				$\varepsilon_2^{(0)}$		$\varepsilon_2^{(1)}$		$\varepsilon_2^{(2)}$	…..	
ε_3					$\varepsilon_3^{(0)}$		$\varepsilon_3^{(1)}$	…..		
ε_4						$\varepsilon_4^{(0)}$	…..			

In order to solve eigneproblem by this procedure, following procedure is used:

- First row vectors of epsilon table are null vectors.

$$\varepsilon_{-1}^{(j)} = \{0\},\ j = 0,1,2...n \tag{3.25}$$

- Calculate second row vectors of epsilon table.

  If basis vectors of infinite series are denoted by $u_0, u_1, ..., u_n$

  s_j 's members of a sequence. $s_j = \sum_{k=0}^{j} u_j$

$$\varepsilon_0^{(j)} = s_j \tag{3.26}$$

- The rest of elements of epsilon table can be obtained using the following equation:

$$\varepsilon_{k+1}^{(j)} = \varepsilon_{k-1}^{(j+1)} + \left[\varepsilon_k^{(j+1)} - \varepsilon_k^{(j)}\right]^{-1} \tag{3.27}$$

Where, j, k=0, 1, 2,...

Inverse of a real vector can be expressed as:

$$u^{-1} = \frac{u}{\sum_{i=1}^{n} |u_i|^2} \tag{3.28}$$

In case of undamped system, eigenvectors are real. Hence, the above equation can be used to get inverse of a vector while generating epsilon algorithm.

- The vectors of even row of epsilon table are approximations of eigenvector of modified design. Corresponding eigenvalue is approximated by Raleigh's quotient:

$$\lambda^i = \frac{u_i^T K u_i}{u_i^T M u_i} \tag{3.29}$$

The basis vectors for epsilon algorithm should be members of an infinite series which can be selected by one of the following ways:

- Matrix perturbation method for eigenproblems:

The eigenvalue and eigenvector of modified design can be expressed in terms of eigenpair of initial design, their first and second order perturbations by following equation:

$$\lambda^i = \lambda_0^i + \lambda_1^i + \lambda_2^i + ... \tag{3.30}$$

$$u^i = u_0^i + u_1^i + u_2^i + ... \tag{3.31}$$

The perturbations of eigenvalue and eigenvector can be obtained by following equations:

$$\lambda_1^i = u_0^{iT} \left(\Delta K - \lambda_0^i \Delta M \right) u_0^i$$

$$u_1^i = \sum_{j=1, j\neq i}^{n} \frac{1}{\left(\lambda_0^i - \lambda_0^j \right)} \left[u_0^{iT} \left(\Delta K - \lambda_0^i \Delta M \right) u_0^j \right] u_0^j - \frac{1}{2} \left[u_0^{iT} \Delta M u_0^i \right] u_0^i$$

$$\lambda_2^i = u_0^{iT}\Delta K u_1^i - \lambda_0^i u_0^{iT}\Delta M u_1^i - \lambda_1^i u_0^{iT} M_0 u_1^i - \lambda_1^i u_0^{iT}\Delta M u_0^i$$

$$u_2^i = \sum_{j=1, j\neq i}^{n} \frac{1}{\left(\lambda_0^i - \lambda_0^j\right)}\left[u_0^{jT}\Delta K u_1^i - \lambda_0^i u_0^{jT}\Delta M u_1^i - \lambda_1^j u_0^{iT} M_0 u_1^i - \lambda_1^i u_0^{jT}\Delta M u_0^i\right] u_0^j$$

$$-\frac{1}{2}\left[u_1^{iT} M_0 u_1^i + u_0^{iT}\Delta M u_1^i - u_1^{iT}\Delta M u_0^i\right] u_0^i$$

- Neumann series expansion method

Eigen-problem of initial and modified design are,

$$K_0\phi_0 = \lambda_0 M_0\phi_0 \tag{3.32}$$

$$K\phi = \lambda M\phi \tag{3.33}$$

K and M are stiffness and mass matrices of modified design respectively.

$$K = K_0 + \Delta K$$

$$M = M_0 + \Delta M$$

Substituting above in equation (3.2.33),

$$\left(K_0 + \Delta K\right)\phi = \lambda_0 M_0\phi_0 + \left(\lambda M\phi - \lambda_0 M_0\phi_0\right) \tag{3.34}$$

Approximating eigen value and eigen vector of modified design by that of initial design,

$$\left(K_0 + \Delta K\right)\phi = \lambda_0 M_0\phi_0 + \lambda_0\Delta M\phi_0 \tag{3.35}$$

$$\phi = \left(I + K_0^{-1}\Delta K\right)^{-1} K_0^{-1}\left(\lambda_0 M_0\phi_0 + \lambda_0\Delta M\phi_0\right)$$

$$= \left(I + B\right)^{-1} K_0^{-1}\left(\lambda_0 M_0\phi_0 + \lambda_0\Delta M\phi_0\right)$$

Using Neumann series expansion,

$$\phi \approx \left(\mathrm{I} - \mathrm{B} + \mathrm{B}^2 - \ldots\right)\mathrm{K}_0^{-1}\left(\lambda_0 \mathrm{M}_0 \phi_0 + \lambda_0 \Delta \mathrm{M} \phi_0\right) \tag{3.36}$$

Where, $\mathrm{B} = \mathrm{K}_0^{-1}\Delta\mathrm{K}$

Simplifying the terms of equation (3.36),

$$\mathrm{u}_0 = \mathrm{K}_0^{-1}\left(\lambda_0 \mathrm{M}_0 \phi_0 + \lambda_0 \Delta \mathrm{M} \phi_0\right) \tag{3.37}$$

$$\begin{aligned} \mathrm{u}_1 &= -\mathrm{K}_0^{-1}\Delta\mathrm{K}\left[\mathrm{K}_0^{-1}\left(\lambda_0 \mathrm{M}_0 \phi_0 + \lambda_0 \Delta \mathrm{M} \phi_0\right)\right] \\ &= -\mathrm{K}_0^{-1}\Delta\mathrm{K}\mathrm{u}_0 \end{aligned} \tag{3.38}$$

$$\mathrm{u}_2 = -\mathrm{K}_0^{-1}\Delta\mathrm{K}\mathrm{u}_1 \tag{3.39}$$

Thus, the series of equation (3.36) can be written as:

$$\phi \approx \mathrm{u}_0 + \mathrm{u}_1 + \mathrm{u}_2 + ... \tag{3.40}$$

The above terms can also be used as basis vectors to generate epsilon triangle. Here, the number of rows in epsilon table should be an even number and the solution is the vectors of even number row. Thus, the number of basis selected (n) should be an odd number.

3.1.4 Wang Caldwell Approach [8]

This method uses truncated modes of initial design and residual static modes as basis vectors for obtaining the following reduced eigenproblem.

$$\mathrm{K}_\mathrm{R} = \mathrm{T}^\mathrm{T}\mathrm{K}_1\mathrm{T},\ \mathrm{M}_\mathrm{R} = \mathrm{T}^\mathrm{T}\mathrm{M}_1\mathrm{T} \tag{3.41}$$

$$K_R \phi_{1r} = \lambda_{1r} M_R \phi_{1r} \tag{3.42}$$

$$T = \left[\Phi_{01-0r} \ \bar{\Psi}_{01-0r} \right] \tag{3.43}$$

Here, Φ_{01-0r} are lower modes of initial design and $\bar{\Psi}$ are residual static modes.

The eigen problem of modified design is:

$$\left([K]+[\Delta K]\right)\phi_r = \lambda_r \left([M]+[\Delta M]\right)\phi_r \tag{3.44}$$

Assuming ψ_r is the static mode due to load of $\lambda_r [\Delta M]\phi_r - [\Delta K]\phi_r$

$$[K]\psi_r = \lambda_r [\Delta M]\phi_r - [\Delta K]\phi_r \tag{3.45}$$

Assuming the static mode as a linear combination of eigenvectors of initial design,

$$\psi_r = \sum_{k=1}^{N} c_k \phi_{0k} \tag{3.46}$$

If first $\hat{N}$ eigenvectors are kept and the contribution of rest of eigenvectors is approximated by $\bar{\psi}_r$.

$$\bar{\psi}_r = \sum_{k=\hat{N}}^{N} c_k \phi_{0k} \tag{3.47}$$

From equation (3.46) and (3.47),

$$\psi_r = \sum_{k=1}^{\hat{N}} c_k \phi_{0k} + \bar{\psi}_r \tag{3.48}$$

In order to get c_j, where j = 1 to $\hat{N}$,

$$\phi_{0j}{}^{T}[M]\psi_r = \sum_{k=1}^{\hat{N}} c_k \phi_{0j}{}^{T}[M]\phi_{0k} + \phi_{0j}{}^{T}[M]\bar{\psi}_r \tag{3.49}$$

From equation (3.47) the last term in the above equation is zero as ϕ_{0j} is M orthogonal to all other eigenvectors except ϕ_{0j}.

$$c_j = \phi_{0j}{}^{T}[M]\psi_r \tag{3.50}$$

Once all c_j are calculated, $\bar{\psi}_r$ is calculated by using equation (3.48) to get r^{th} static mode. This procedure is performed to calculate 1 to $\hat{N}$ static modes and then equations (3.41) through (3.42) are followed. Note that the modal data of modes $\hat{N}$+1 to N is not required here.

CHAPTER 4

PROCEDURES FOR CALCULATING FIRST AND SECOND ORDER SENSITIVITY DATA

In this chapter, computation of exact and approximate first and second order derivative of eigenvector is illustrated using algebraic method and modal method.

In modal method, first order eigenvector derivative is expressed as a linear combination of all eigenvectors. However, full modal data is not available usually. Many approximation techniques for first eigenvector derivative using available modes are available based on modal method. Among them, truncated modal method is the simplest one in which the contribution of higher modes is truncated while calculating derivatives of lower modes. Liu [16] and Wang [13] suggested procedures for approximation of contribution of truncated higher modes. Wang's methods are: explicit and implicit methods. In this thesis, Wang's implicit and explicit methods have been extended to approximate second order derivative which considerably improved the accuracy of reanalysis results. These results will be used in the proposed new eigensolution reanalysis method in the next chapter.

4.1 Computation of Exact First Order Sensitivity Data

4.1.1 Modal Expansion Method

Assuming derivative of eigen vector as a linear combination of the eigenvectors of Initial design [12],

$$\phi_r' = \sum_{k=1}^{N} q_k \phi_k \qquad (4.1)$$

Where N is the number of degree of freedom.

Differentiating the eigen problem, $(K - \lambda_r M)\,\phi_r = 0$,

$$(K - \lambda_r M)\,\phi_r' = -(K' - \lambda_r' M - \lambda_r M')\phi_r \qquad (4.2)$$

Substituting equation (4.1) into equation (4.2),

$$(K - \lambda_r M) \sum_{k=1}^{N} q_k \phi_k = -(K' - \lambda_r' M - \lambda_r M')\phi_r = F_r \qquad (4.3)$$

In order to compute the co-efficients q_s, pre-multiply equation (4.3) by $\phi_s^T, s \neq r$

$$\phi_s^T (K - \lambda_r M) \sum_{k=1}^{N} q_k \phi_k = -\phi_s^T (K' - \lambda_r' M - \lambda_r M')\phi_r \qquad (4.4)$$

If $k \neq s$, $\phi_s^T (K - \lambda_r M) \sum_{k=1}^{N} q_k \phi_k = 0$, $\phi_s^T M \phi_r = 0$

If $k = s$, $\phi_s^T K \phi_k = \lambda_s$, $\phi_s^T \lambda_r M \phi_k = \lambda_r$

as the eigenvectors are M-orthogonal.

$$q_s = \frac{-\phi_s^T (K' - \lambda_r M')\phi_r}{\left(\lambda_s - \lambda_r\right)} \qquad (4.5)$$

For computing the co-efficient q_r, starting from mass normalization condition,

$$\phi_r^T M \phi_r = 1$$

Differentiating the above M-orthogonality condition,

$$\phi_r^T M \phi_r' = -0.5\, \phi_r^T M' \phi_r$$

For k = s, using equation (4.1) in the above equation,

$$\phi_r^T M \sum_{k=1}^{N} q_k \phi_k = -0.5\, \phi_r^T M' \phi_r$$

$$q_r = \frac{-0.5 \phi_r^T M' \phi_r}{\phi_r^T M \phi_r} \qquad (4.6)$$

Substitution of equation (4.5) and (4.6) into equation (4.1) gives the exact eigenvector derivative if all N modes are used.

4.1.2 Algebraic Method

The eigenproblem of the original design is:

$$(K - \lambda M)\phi = 0 \qquad (4.7)$$

Where, λ and ϕ are first eigenvalue and eigenvector of the initial design respectively.

Eigen problem of the modified design is:

$$(K_1 - \lambda_1 M_1)\,\phi_1 = 0 \tag{4.8}$$

Differentiating equation (4.7),

$$(K - \lambda M)\,\phi' + (K' - \lambda' M - \lambda M')\,\phi = 0 \tag{4.9}$$

$$(K - \lambda M)\,\phi' - \lambda' M\,\phi = -(K' - \lambda M')\phi \tag{4.10}$$

If the modes are M – orthogonal, normalization condition

$$\phi^T M\,\phi = 1 \tag{4.11}$$

Differentiating above equation,

$$\phi^T M\,\phi' = -0.5\,\phi^T M'\,\phi \tag{4.12}$$

Putting equations (4.10) and (4.12) in matrix form together,

$$\begin{bmatrix} (K - \lambda M) & -M\phi \\ -\phi^T M & 0 \end{bmatrix} \begin{Bmatrix} \phi' \\ \lambda' \end{Bmatrix} = \begin{Bmatrix} -(K' - \lambda M')\phi \\ \frac{1}{2}\phi^T M'\,\phi \end{Bmatrix} \tag{4.13}$$

In the above equation, K' = ΔK, and M' = ΔM

So, $Az = b$ (4.14)

Where, A = $\begin{bmatrix} (K - \lambda M) & -M\phi \\ -\phi^T M & 0 \end{bmatrix}$ and b = $\begin{Bmatrix} -(\Delta K - \lambda \Delta M)\phi \\ \frac{1}{2}\phi^T \Delta M\,\phi \end{Bmatrix}$, z = $\begin{Bmatrix} \phi' \\ \lambda' \end{Bmatrix}$

$z = A^{-1}\,b$

The matrix A does not depend upon change in the design variables. It can be computed using eigen problem data of initial design.

Differentiation of the set of equations in (4.13) gives second derivative of eigenvalue and eigenvector.

$$\begin{bmatrix} (K - \lambda M) & -M\phi \\ -\phi^T M & 0 \end{bmatrix} \begin{Bmatrix} \phi'' \\ \lambda'' \end{Bmatrix} = \begin{Bmatrix} 2\lambda' \Delta M\phi - 2\Delta K\phi' + 2\lambda \Delta M\phi' + 2\lambda' M\phi' \\ 2\phi^T \Delta M\, \phi' + \phi' M\phi' \end{Bmatrix} \tag{4.15}$$

4.2 Approximation of First Order Eigenvector Derivative Using Wang's Explicit and Implicit Methods [14]

In practice, it may be difficult to obtain all eigenvectors. Also, computation of exact eigenvector derivate leads to increased computation time. Therefore, approximate methods have been developed. Truncated modal method ignores the contribution of higher modes to the lower eigenvector derivatives. Wang [14] suggested a method in which an additional term for approximation for truncated higher order modes is added.

4.2.1 Wang's Explicit Method

Assuming the derivative of eigenvector as a linear combination of n eigenvectors,

$$\phi_r' = \sum_{k=1}^{N} c_k \phi_k \tag{4.16}$$

If the contribution of only first $\hat{N}$ available lower modes are computed, then equation (4.16) can be written as:

$$\phi_r' = \sum_{j=1}^{\hat{N}} c_j \phi_j + S_R \tag{4.17}$$

S_R is the contribution of higher modes $\hat{N}$+1 to N which is an approximation of contribution of higher order truncated eigenvectors.

The coefficients c_j's are computed as shown in section 4.2.1. Similarly,

$$S_R = \sum_{j=\hat{N}+1}^{\hat{N}} \frac{\phi_j^T F_r}{\left(\lambda_r - \lambda_j\right)} \phi_j$$

Where,

$$F_r = -\left(K' - \lambda_r M' - \lambda_r' M\right) \phi_r \tag{4.18}$$

If the number of lower modes used, $\hat{N}$ is considerably more than the number of eigenvector derivative r, then $\lambda_j >> \lambda_r$ in S_R

$$\left(\lambda_j - \lambda_r\right) \approx \lambda_j$$

$$S_R \approx S_{RA} = \sum_{j=\hat{N}+1}^{N} \frac{\phi_j^T F_r}{\lambda_j} \phi_j$$

$$= \sum_{j=1}^{N} \frac{\phi_j^T F_r}{\lambda_j} \phi_j - \sum_{j=1}^{\hat{N}} \frac{\phi_j^T F_r}{\lambda_j} \phi_j \tag{4.19}$$

Note that equation (4.19) depends on all eigenvectors. This dependence can be removed by the following procedure.

Note that, $\sum_{j=1}^{N} \frac{\phi_j^T F_r \phi_j}{\lambda_j} = K^{-1} F_r$

Let $y_r = \mathrm{K}^{-1}\mathrm{F}_r$ and $h_r = \sum_{j=1}^{\hat{\mathrm{N}}} \frac{\phi_j^T \mathrm{F}_r \phi_j}{\lambda_j}$

Substituting these in equation (4.19),

$$S_R \approx w = y_r - h_r \tag{4.20}$$

Here w is residual static mode as it is obtained by deducting the contribution of lower modes from static mode y_r

From equation (4.17), the approximation for eigenvector derivative becomes,

$$\phi_r' \approx \sum_{j=1}^{\hat{\mathrm{N}}} c_j \phi_j + w \tag{4.21}$$

In equation (4.21), w is computed using equation (4.20) and the coefficients are calculated using equation (4.5) and (4.6).

It should be noted here than equation (4.21) uses the static solution and modes 1 to $\hat{\mathrm{N}}$ only, and does not depend on higher modes $\hat{\mathrm{N}}+1$ to N.

4.2.2 Wang's Implicit Method

This method is similar to Wang's Explicit Method. The only difference is that here the residual static mode w_r is multiplied to a coefficient, d_r

$$\phi_r' \approx \sum_{j=1}^{\hat{\mathrm{N}}} c_j \phi_j + d_r w_r = \mathrm{T}q \tag{4.22}$$

$$\mathrm{T} = [\phi_1 \ \cdots \phi_{\hat{\mathrm{N}}} \vdots \ w_r] \tag{4.23}$$

$$\left(K - \lambda_r M\right)\ \phi_r' = -\left(K' - \lambda_r' M - \lambda_r M'\right)\phi_r = F_r \qquad (4.24)$$

Substituting equation (4.22) in to equation (4.24) and pre-multiplying by T^T

$$T^T\left(K - \lambda_r M\right)Tq = T^T F_r$$

The term on left hand side is a diagonal matrix. Co-efficients c_j's are same as that of Wang's Explicit method. The co-efficient for residual static mode can be obtained from the following equation as it is not orthogonal to K and M matrices.

$$d_r = \frac{w_r{}^T F_r}{w_r{}^T K w_r - \lambda_r w_r{}^T M w_r} \qquad (4.25)$$

4.3 Extension of Wang's Explicit and Implicit Methods for Second Order Derivative of Eigenvector

While using exact eigenvector derivatives in the reanalysis, it was observed that addition of second derivative as basis vector considerably improves the results. Therefore, in this thesis, Wang's approximation method is extended for approximating second derivative of eigenvector.

Taking the derivative of the eigenvalue problem leads to:

$$\left(K - \lambda_r M\right)\ \phi_r' + \left(K' - \lambda_r M' - \lambda_r' M\right)\phi_r = 0 \qquad (4.26)$$

Differentiating above equation again,

$$\left(K - \lambda_r M\right)\ \phi_r'' + \left(K' - \lambda_r M' - \lambda_r' M\right)\phi_r' = -\left(K' - \lambda_r M' - \lambda_r' M\right)\phi_r' - \left(K'' - \lambda_r' M' - \lambda_r M'' - \lambda_r'' M - \lambda_r' M'\right)\phi_r$$

or,

$$\left(K - \lambda_r M\right)\ \phi_r'' = -2\left(K' - \lambda_r M' - \lambda_r' M\right)\phi_r' + \left(2\lambda_r' M' + \lambda_r'' M\right)\phi_r \qquad (4.27)$$

Let ϕ_r'' be a linear combination of all eigenvectors.

$$\phi_r'' = \sum_{k=1}^{n} b_k \phi_k$$

Substitute this into equation (4.27) and then pre-multiply by $\phi_l^T, l \neq r$

$$\phi_l^T \left(K - \lambda_r M\right) \sum_{k=1}^{n} b_k \phi_k = -2\phi_l^T \left(K' - \lambda_r M' - \lambda_r' M\right)\phi_r' + \phi_l^T \left(2\lambda_r' M' + \lambda_r'' M\right)\phi_r \qquad (4.28)$$

where,

$$\lambda_r' = \phi_r^T \left(K' - \lambda_r M'\right)\phi_r \qquad (4.29)$$

$$\lambda_r'' = \phi_r'^T \left(K' - \lambda_r M'\right)\phi_r + \phi_r^T \left(-\lambda_r' M'\right)\phi_r + \phi_r^T \left(K' - \lambda_r M'\right)\phi_r' \qquad (4.30)$$

For $l \neq r$,

$$b_l = -2\frac{\phi_l^T \left(K' - \lambda_r M' - \lambda_r' M\right)\phi_r'}{\left(\lambda_l - \lambda_r\right)} + \frac{\phi_l^T \left(2\lambda_r' M' + \lambda_r'' M\right)\phi_r}{\left(\lambda_l - \lambda_r\right)} \qquad (4.31)$$

For $l = r$,

$$b_r = -\phi_r'^T M \phi_r' - 2\phi_r^T M' \phi_r' \qquad (4.32)$$

Thus, the second derivative of eigen vector as a linear combination of eigen vectors is,

$$\phi_r'' = \sum_{k=1}^{N} b_k \phi_k$$

Now, we want to compute an approximation of second derivative of eigenvector by extending Wang's method. This is achieved by assuming,

$$\phi_r'' = \sum_{k=1}^{\hat{N}} b_k \phi_k + S_R \tag{4.33}$$

Where, $S_R = \sum_{j=\hat{N}+1}^{N} b_j \phi_j$

$$= \sum_{j=\hat{N}+1}^{N} \left[-2\frac{\phi_j^T \left(K' - \lambda_r M' - \lambda_r' M\right)\phi_r'}{\left(\lambda_j - \lambda_r\right)} + \frac{\phi_j^T \left(2\lambda_r' M' + \lambda_r'' M\right)\phi_r}{\left(\lambda_j - \lambda_r\right)} \right] \phi_j \tag{4.34}$$

Assuming $\lambda_j >> \lambda_r$

$$S_R \approx \sum_{j=\hat{N}+1}^{N} \left[-2\frac{\phi_j^T F_{r1}}{\lambda_j} + \frac{\phi_j^T F_{r2}}{\lambda_j} \right] \phi_j$$

$$= \sum_{j=1}^{N} \left[-2\frac{\phi_j^T F_{r1}}{\lambda_j} + \frac{\phi_j^T F_{j2}}{\lambda_j} \right] \phi_j - \sum_{j=1}^{\hat{N}} \left[-2\frac{\phi_j^T F_{j1}}{\lambda_j} + \frac{\phi_j^T F_{r2}}{\lambda_j} \right] \phi_j$$

$$= -\left[2\sum_{j=1}^{N} \frac{\phi_j^T F_{r1}}{\lambda_j} \phi_j - \sum_{j=1}^{\hat{N}} \frac{\phi_j^T F_{r2}}{\lambda_j} \phi_j \right] + \left[\sum_{j=1}^{N} \frac{\phi_j^T F_{r1}}{\lambda_j} \phi_j - \sum_{j=1}^{\hat{N}} \frac{\phi_j^T F_{r2}}{\lambda_j} \phi_j \right]$$

$$= \left[-2K^{-1} F_{r1} + K^{-1} F_{j2} \right] - \left[-2\sum_{j=1}^{\hat{N}} \frac{\phi_j^T F_{r1}}{\lambda_j} \phi_j + \sum_{j=1}^{\hat{N}} \frac{\phi_j^T F_{r2}}{\lambda_j} \phi_j \right]$$

$$\text{Let } y_r = \left[-2K^{-1}F_{r1} + K^{-1}F_{j2} \right], \tag{4.35}$$

$$\text{and, } h_r = \left[-2\sum_{j=1}^{\hat{N}} \frac{\phi_j^T F_{r1}}{\lambda_j}\phi_j + \sum_{j=1}^{\hat{N}} \frac{\phi_j^T F_{r2}}{\lambda_j}\phi_j \right] \tag{4.36}$$

$$\text{Then, } S_R \approx y_r - h_r = w \tag{4.37}$$

Where,

$$F_{r1} = \left(K' - \lambda_r M' - \lambda_r' M \right) \phi_r' \tag{4.38}$$

$$F_{r2} = \left(2\lambda_r' M' + \lambda_r'' M \right) \phi_r \tag{4.39}$$

Method of approximation for second derivative is similar for both explicit and implicit methods except in implicit method, approximation of contribution of higher mode is multiplied by a factor d_r given by following equation.

$$d_r = \frac{w_r^T \left(-2F_{r1} + F_{r2} \right)}{w_r^T K w_r - \lambda_r w_r^T M w_r} \tag{4.2.19}$$

Substitution of co-efficients and S_R into equation (4.33) gives an approximation for the second derivative of eigenvector.

Note that in the above calculations, second derivative of stiffness and mass matrices are null matrices.

CHAPTER 5

THE PROPOSED APPROACH USING SENSITIVITY DATA

Various methods for calculating the sensitivity data were discussed in the previous chapter. In this chapter, the basis of the proposed approach is discussed which is generalized later to use various combinations of eigenvector and its derivatives as basis vectors for eigensolution data desired.

5.1. Basis of The Proposed Approach

Assuming eigenvector of modified design as a linear combination of initial design eigenvector, its first and second order derivatives,

$$\begin{aligned}\phi_1 &= a\phi + b\phi' + c\phi'' \\ &= \begin{bmatrix}\phi & \phi' & \phi''\end{bmatrix}\begin{bmatrix}a\\ b\\ c\end{bmatrix} \\ &= T\hat{q}\end{aligned} \tag{5.1}$$

Raleigh's quotient is defined by the following equation

$$\begin{aligned}R(q) &= \frac{\phi_1^T K_1 \phi_1}{\phi_1^T M_1 \phi_1} = \omega^2 = \lambda \\ &= \frac{\left[T\hat{q}\right]^T\left[K_1\right]\left[T\hat{q}\right]}{\left[T\hat{q}\right]^T\left[M_1\right]\left[T\hat{q}\right]}\end{aligned}$$

Or, $R(q) = \dfrac{\hat{q}^T \hat{K}_1 \hat{q}}{\hat{q}^T \hat{M}_1 \hat{q}}$ Here, $\hat{K}_1 = T^T K_1 T$ and $\hat{M}_1 = T^T M_1 T$

$$= \frac{\sum_{i=1}^{\hat{N}} \sum_{j=1}^{\hat{N}} \hat{q}_i \hat{q}_j \hat{k}_{ij}}{\sum_{i=1}^{\hat{N}} \sum_{j=1}^{\hat{N}} \hat{q}_i \hat{q}_j \hat{m}_{ij}} = \frac{N(q)}{D(q)} \qquad (5.2)$$

The co-efficients a, b and c should be chosen to make R(q) stationary.

$$\frac{\partial R(q)}{\partial \hat{q}_i} = 0$$

This leads to,

$$N(q)\frac{\partial D(q)}{\partial \hat{q}_i} - D(q)\frac{\partial N(q)}{\partial \hat{q}_i} = 0 \qquad (5.3)$$

From Equation (5.2),

$$\frac{\partial N(q)}{\partial \hat{q}_i} = \frac{\partial \sum_{i=1}^{\hat{N}} \sum_{j=1}^{\hat{N}} \hat{q}_i \hat{q}_j \hat{k}_{ij}}{\partial \hat{q}_i} = 2\sum_{i=1}^{\hat{N}} \hat{q}_j \hat{k}_{ij}$$

Similarly, $\dfrac{\partial D(q)}{\partial \hat{q}_i} = 2\sum_{j=1}^{\hat{N}} \hat{q}_j \hat{m}_{ij}$

Substituting these in equation (5.1) and then simplifying,

$$\sum_{j=1}^{\hat{N}} \left(\hat{k}_{ij} - \hat{\lambda} \hat{m}_{ij} \right) \hat{q}_i = 0 \quad \text{for i = 1,2,3}$$

In matrix form, this is the reduced eigenproblem.

$$\left[\hat{K} - \hat{\lambda} \hat{M} \right] \hat{q} = 0$$

Once $\hat{q}$ is obtained by solving the eigen problem $\hat{K}\hat{q} = \hat{\lambda}\hat{M}\hat{q}$, approximation of mode shape can be obtained from equation (5.1).

It was observed that using the sensitivity data in perturbation method may produce acceptable results for very small amount of modifications. For even moderate modifications, the results are not acceptable and make the reanalysis meaningless. However, using the same sensitivity data in the proposed approach gives excellent accuracy for both eigenvalue as well as eigenvector.

For example, consider initial design of the spring mass system shown below.

Figure 5.1 Initial Design of 5 DOF Spring Mass System

A spring of stiffness K_a has been added to second nodal mass. The other end of the spring is fixed as shown in figure.

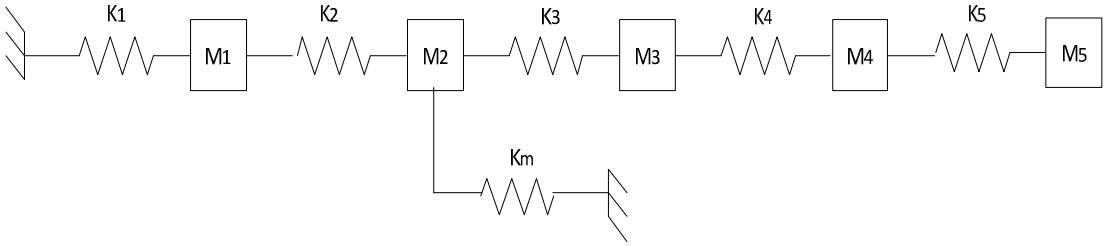

Figure 5.2 Addition of a Spring in 5 DOF Spring Mass System

The spring stiffness K_a is varied from 100N/m to 2000 N/m in a step of 100. The first and second order sensitivity data were calculated and approximation of first eigenvalue was obtained using,

1. Standard perturbation method using the following equation:

$$\lambda_1 = \lambda_0 + \frac{d\lambda}{dx}\Delta x + \frac{1}{2}\frac{d^2\lambda}{dx^2}\Delta x^2 \qquad (5.4)$$

2. The proposed sensitivity approach

The magnitude of error increases for both methods. The perturbation method gives approximately 10% error even if the stiffness of additional spring is 100 N/m. However, the proposed approach produces almost similar results as that produced by exact analysis even for large modification.

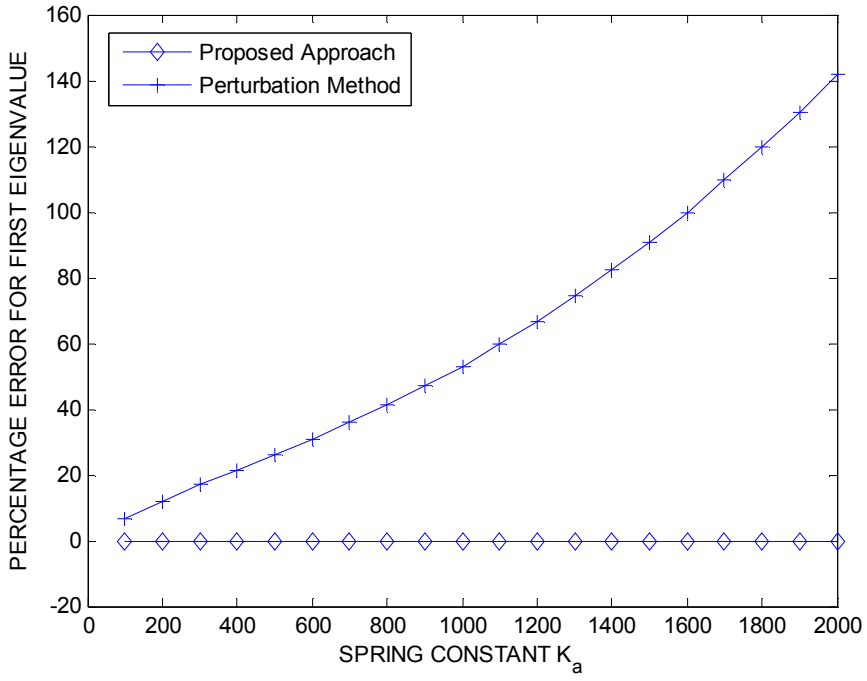

Figure 5.3 Comparison of Results produced by Perturbation Method and Proposed Approach

5.2 Combinations of Exact Eigenvector Derivatives as Basis Vectors

5.2.1 Combinations of Basis Vectors for Fundamental Mode

Let T be the combinations of basis vectors used in reduced eigenproblem. If only first derivative is used,

$$T = [\phi \; \phi'] \tag{5.5}$$

The reduced Eigen problem is:

$$K_R q = M_R q \Lambda_R \tag{5.6}$$

$$K_R = T^T K_1 T \, , \, M_R = T^T M_1 T \tag{5.7}$$

Solving the reduced problem, Λ_R is a diagonal matrix which gives eigenvalue of modified design. Eigenvector of modified design can be found using equation below.

$$\phi_1 = T q \tag{5.8}$$

If the second derivative of eigenvector is also used as a basis in reduced eigenproblem,

$$T = [\phi \; \phi' \; \phi''] \tag{5.9}$$

Then, rest procedure is same as shown in equations (5.5) to (5.8).

The results obtained by using both first and second eigenvector derivative as basis give better results than that obtained by using only first eigenvector derivative.

5.2.2 Combinations of Basis Vectors for Other Mode

The procedure mentioned in the above section is for fundamental eigenvalue and eigenvector. For r^{th} eigenvalue and eigenvector of modified design, let ϕ_r = r^{th} eigenvector of the initial design. Eigenvector derivatives can be used in either of the following ways:

- Combination S-1 uses 1 to r initial design eigenvectors, their first and second derivatives.

$$T = [\phi_1 \cdots \phi_r \ \phi_1' \cdots \phi_r' \ \phi_1'' \cdots \phi_r''] \tag{5.10}$$

- Combination S-2 uses only r initial design eigenvector, its first derivative and second derivative.

$$T = [\phi_r \ \phi_r' \ \phi_r''] \tag{5.11}$$

- Combination S-3 uses 1 to r initial design eigenvectors and their first derivatives.

$$T = [\phi_1 \cdots \phi_r \ \phi_1' \cdots \phi_r'] \tag{5.12}$$

S-2 involves solving for (N+1) × (N+1) matrix. On the other hand, for S-1 and S-3, it needs to solve r different (N+1) × (N+1) matrices.

5.3 Combinations of Approximate Eigenvector Derivatives as Basis Vectors

Suppose r^{th} eigenvalue and mode of modified design needs to be found. Let ϕ_r = r^{th} eigenvector of the initial design.

- Combination S-E-2 uses only r initial design eigenvector, its first and second derivative obtained by Wang's explicit method.

$$T = [\phi_r \ \phi_r' \ \phi_r''] \tag{5.14}$$

- Combination S-E-3 uses 1 to r initial design eigenvectors and their first derivatives obtained by Wang's explicit method.

$$T = [\phi_1 \cdots \phi_r \ \phi_1' \cdots \phi_r'] \tag{5.13}$$

- Combination S-I-1 uses only 1 to r initial design eigenvector, its first and second derivative obtained by Wang's implicit method.

$$T = [\phi_1 \cdots \phi_r \ \phi_1' \cdots \phi_r' \ \phi_1'' \cdots \phi_r''] \tag{5.14}$$

- Combination S-I-2 uses only r initial design eigenvector, its first derivative and second derivatives obtained by Wang's implicit method.

$$T = [\phi_r \ \phi_r' \ \phi_r''] \tag{5.16}$$

- Combination S-I-3 uses 1 to r initial design eigenvectors and their first derivatives obtained by Wang's implicit method.

$$T = [\phi_1 \cdots \phi_r \ \phi_1' \cdots \phi_r'] \tag{5.15}$$

Once T matrix is obtained, equations (5.5) to (5.8) are followed to get approximate eigenvalue and eigenvector.

CHAPTER 6

COMPARISON OF RESULTS

In this chapter, the results obtained by different methods are compared for same modification. Eeigenvalue and eigenvector of modified design are obtained by various reanalysis methods using the data of Initial design. These results are compared with each other as well as with exact full reanalysis obtained by directly solving eigenproblem. The exact eigensolution is obtained using eig function of Matlab. In section 6.1, a 5 DOF spring mass system is modified locally whereas in sections 6.2 to 6.6 frame structure is modified in various ways. Global modifications for frame model are: (1) change in width of frame members, (2) Topology Modification.

The following error definitions are used in the comparison.

- Error In EigenValue: Percentage error in eigenvalue obtained by reanalysis is:

$$\in = \left(\frac{\lambda_e - \lambda_r}{\lambda_e} \right) \times 100\% \qquad (6.1)$$

λ_r = Eigenvalue computed by reanalysis procedure.

λ_e = Eigenvalue computed by Exact Procedure.

- Error in Eigenvector: Eigenvector has to be first normalized in order to compare. Here, eigenvector is normalized by either making the first element equal to 1 or making the maximum element equal to 1. Then, eigenvectors obtained by different reanalysis methods as well as exact eigenvector are plot for comparison.

For ease of reference, various combinations used in global modification examples are summarized in Table 6.1:

Table 6.1 Summary of Combinations Used in Reanalysis as Basis Vectors

Sr. No.	Procedure	Basis vectors	No. of Basis vectors
1	Sensitivity Approach S-1	$\left[\phi_1 \cdots \phi_r \;\; \phi_1' \cdots \phi_r' \; \phi_1'' \cdots \phi_r''\right]$	3r
2	Sensitivity Approach S-2	$\left[\phi_r \quad \phi_r' \quad \phi_r''\right]$	3
3	Sensitivity Approach S-3	$\left[\phi_1 \cdots \phi_r \;\; \phi_1' \cdots \phi_r'\right]$	2r
4	Sensitivity Approach S-I-2	$\left[\phi_{ra} \quad \phi_{ra}' \quad \phi_{ra}''\right]$	3
5	Wang-Caldwell Approach	$\left[\phi_1 \cdots \phi_r \;\; \bar{\psi}_1' \cdots \bar{\psi}_r'\right]$	2r
6	CA Procedure	$\left[r_1 \; r_2 \cdots r_n\right]$	n=3
7	Sensitivity Approach S-I-1	$\left[\phi_1 \cdots \phi_r \;\; \phi_{1a}' \cdots \phi_{ra}' \; \phi_{1a}'' \cdots \phi_{ra}''\right]$	3r

Where, r = number of mode to be evaluated.

It should be noted that in this thesis, CA procedure is used for approximating only the fundamental eigenvector and eigevalue. For the 33 member frame structure, ten eigenvectors were used to approximate eigenvector derivatives using Implicit and explicit methods.

6.1 5 Degree of Freedom Spring Mass System-Addition of a Spring

Consider an initial design of 5 degree of freedom spring mass system as shown in figure 6.1. Boundary conditions are fixed-free.

Figure 6.1 5 DOF Spring Mass System-Initial Design

M_1 = 5 Kg, K_1 = 1000 N/m

M_2 = 3 Kg, K_2 = 1000 N/m

M_3 = 2 Kg, K_3 = 1000 N/m

M_4 = 1 Kg, K_4 = 1000 N/m

M_5 = 2 Kg, K_5 = 1000 N/m

This design is modified by adding a spring which is connected to second mass and ground as shown in figure 6.2.

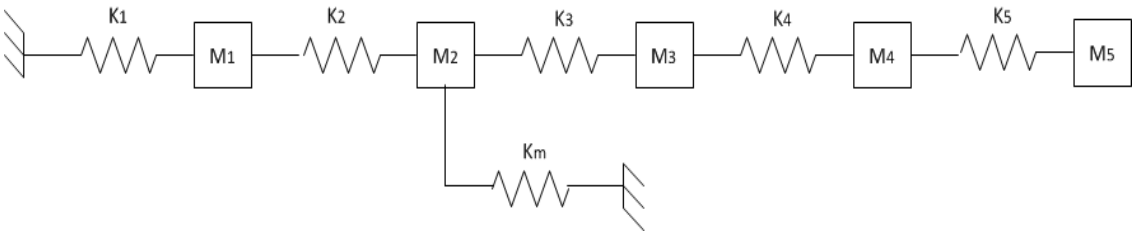

Figure 6.2 5 DOF Spring Mass System-Modified Design

The first eigenvalue of this modified design is calculated by the following procedures while varying the stiffness of this additional spring, K_m from 100 to 2000 in a step of 100: (a) Local Modification, L=1, (b) Local Modification, L=2, (c) Wang-Caldwell Approach, (d) Sensitivity Approach, S-3 and (e) CA Procedure.

The percentage error in first eigenvalue for each approach is calculated using this data and plot as shown in figure 6.3.

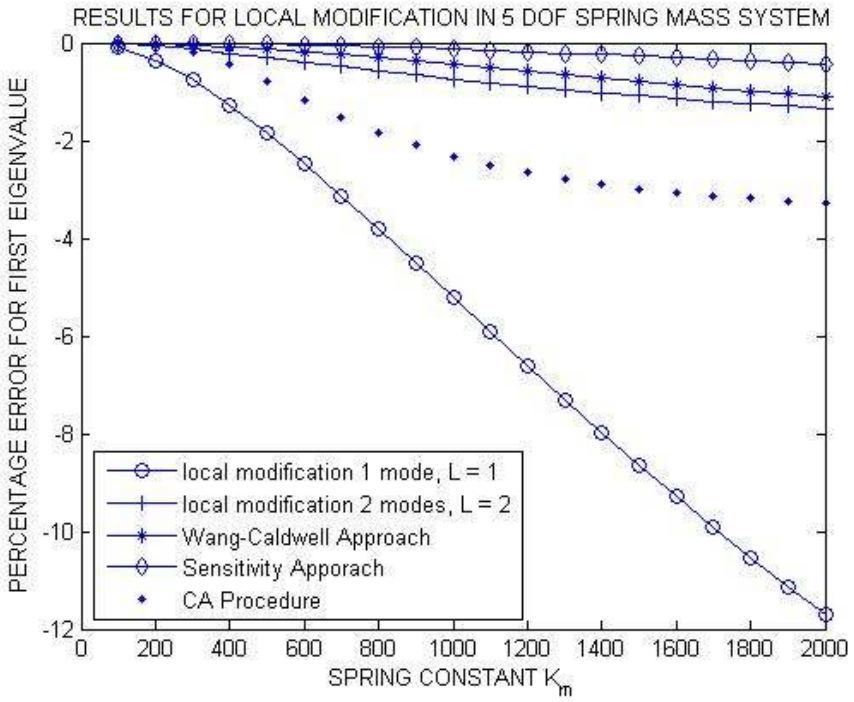

Figure 6.3 Comparison of Results for Local Modification

As it is clear from the above figure, Wang-Caldwell Approach and Sensitivity Approach gives better results even for local modification too. In fact, the largest error while using Sensitivity Approach is as small as 0.43%.

6.2 20 Degree of Freedom Spring Mass System

A 20 Degree of freedom spring mass system with boundary conditions fixed-free is as shown in the figure 6.4:

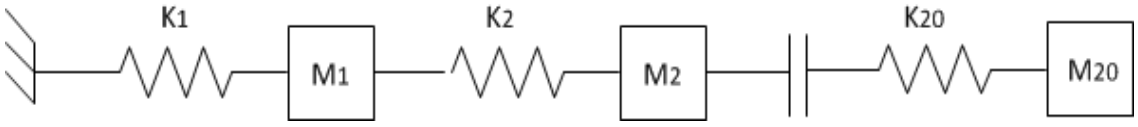

Figure 6.4 20 DOF Spring Mass System

Table 6.2 Comparison of Initial Design and Modified Design for 20 DOF Spring Mass System

Ele-ment No.	Initial Design			Modified Design		
	Mass (Kg)	Spring Stiffness (N/m)	Eigen-value	Mass (Kg)	Spring Stiffness (N/m)	Eigenvalue
1	5	950	1.870652	6	1150	2.176614
2	3	1200	16.1704	3	1300	18.27391
3	2	900	54.20709	5	1490	44.4074
4	1	1000	71.07304	3	1200	78.97088
5	2	1600	144.6704	2	1500	126.2775
6	3	800	158.7756	4	1350	201.4716
7	1	600	225.2654	3	800	231.9876
8	6	700	480.732	7	1000	388.1066
9	5	500	500.3639	2	500	392.9219
10	2	800	762.5931	6	1100	551.5609
11	1	900	825.5778	1	1000	679.1302
12	8	200	905.167	5	800	875.2149
13	2	1200	1257.122	5	1100	963.8726
14	2	700	1438.915	2	800	1268.899
15	5	1500	1496.88	5	1700	1417.845
16	1	900	1644.542	0.5	700	1555.343
17	1	500	1668.058	3	800	1967.007
18	1	1400	2510.71	1	1300	1988.533
19	1	1500	3400.134	1.5	1300	3164.926
20	1	900	4698.838	2	1100	3526.194

The original design as well as modified design are defined in Table 6.2. As it can be seen in table 6.2. masses as well as stiffnesses of spring elements associated with almost all degree of freedom are changed in modified design. Thus, this design has been modified globally.

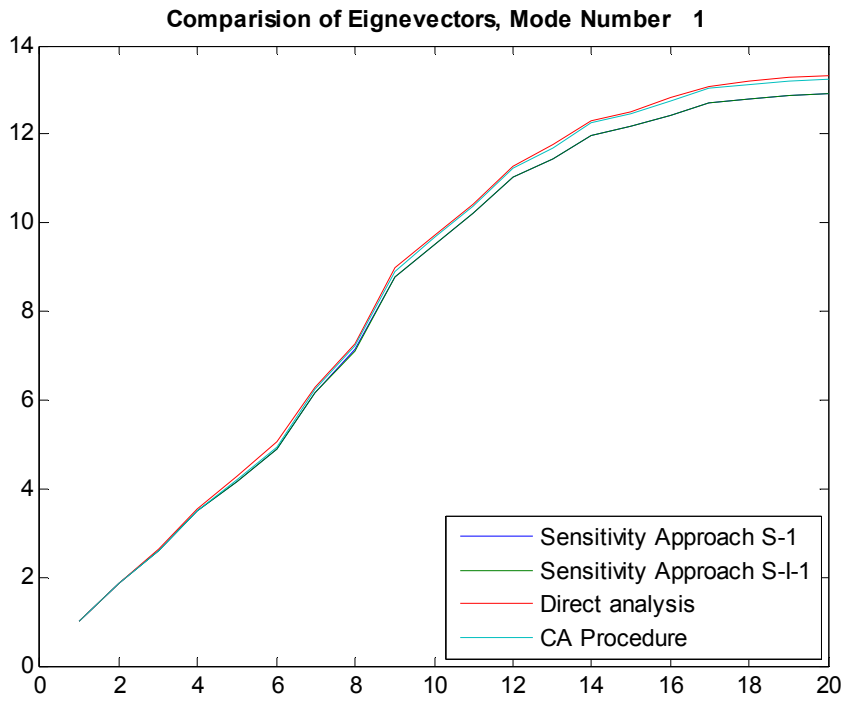

Figure 6.5 Comparison of Eigenvector 1 Computed by S-1and S-I-1, Case 6.2

In Figures 6.5 through 6.13, the first, second and fourth eigenvectors computed by various combinations are compared. These eigenvectors are normalized by making the first element equal to 1. Solutions using combinations S-1 and S-I-1 produce the eigenvectors almost identical to the exact one. The combinations S-1 produces better results than those produced by S-3 as it uses more number of basis vectors. The results for combinations S-2 and S-I-2 are not as good as S-1 or S-I-1because it uses only 3 basis vectors. Figure 6.14 summarizes error in first four eigenvalues computed by various combinations. S-1 and S-I-1 gives very good approximations of eignevalues with maximum absolute percentage error even less than 0.2%.

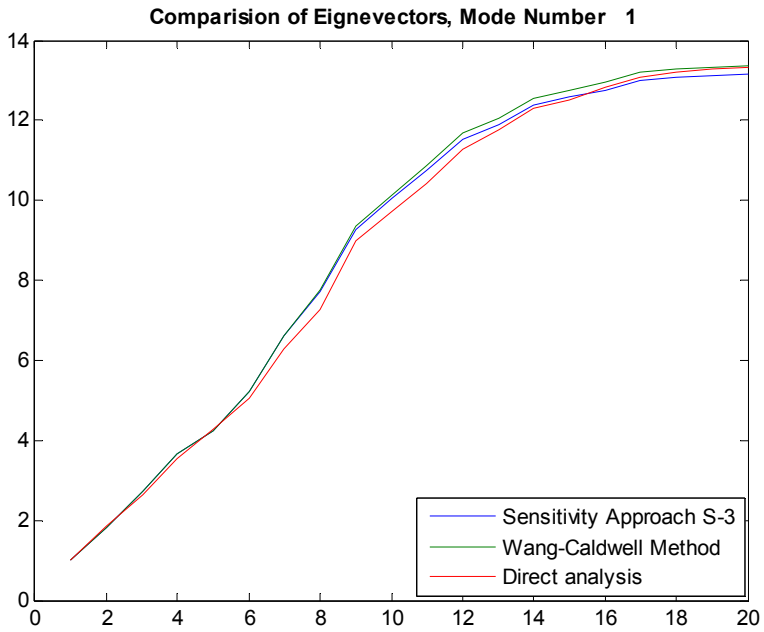

Figure 6.6 Comparison of Eigenvector 1 Computed by S-3 and W-C, Case 6.2

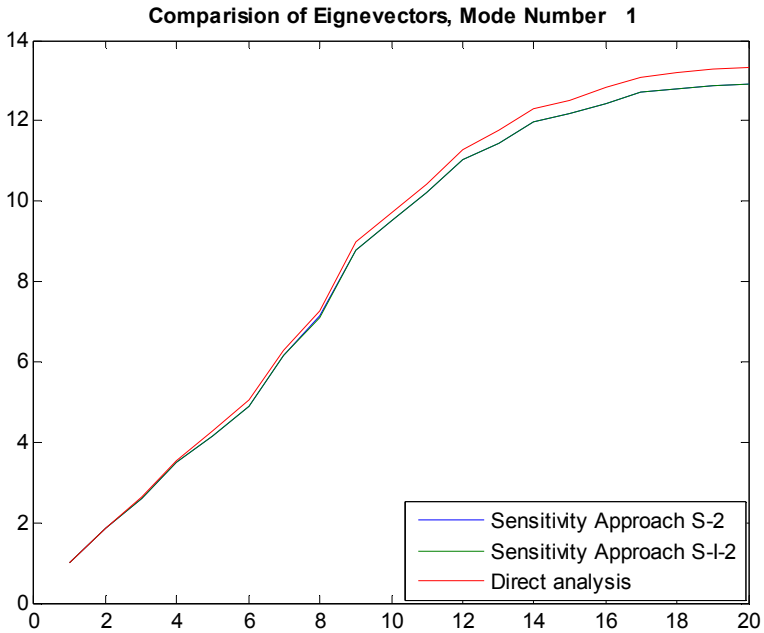

Figure 6.7 Comparison of Eigenvector 1 Computed by S-2 and S-I-2, Case 6.2

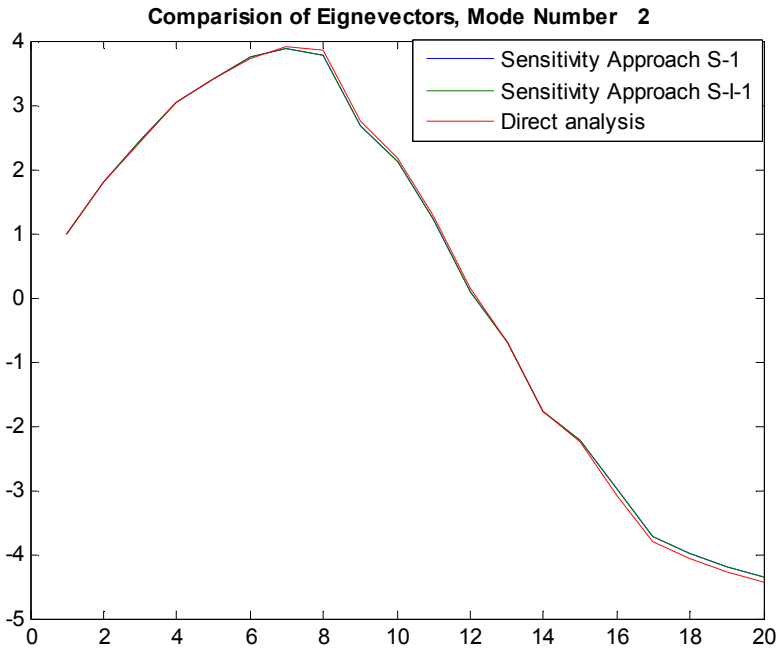

Figure 6.8 Comparison of Eigenvector 2 Computed by S-1and S-I-1, Case 6.2

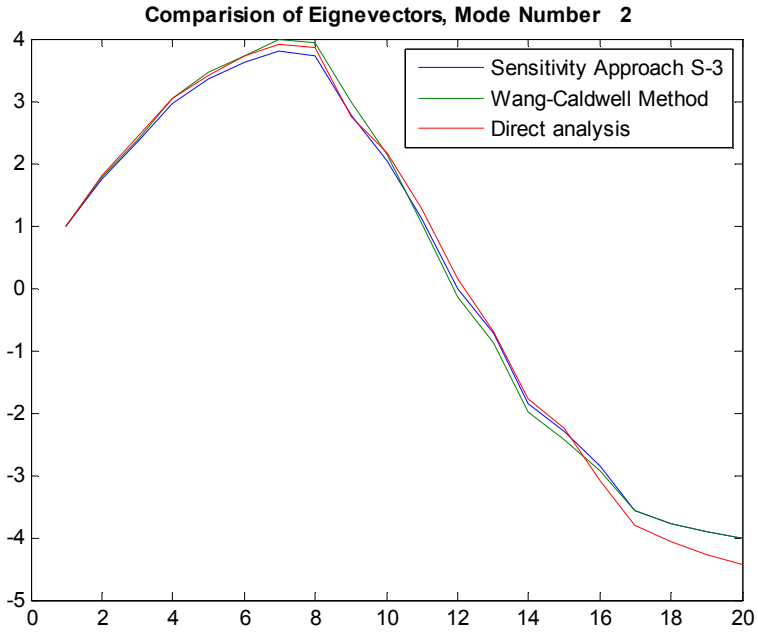

Figure 6.9 Comparison of Eigenvector 2 Computed by S-3 and W-C, Case 6.2

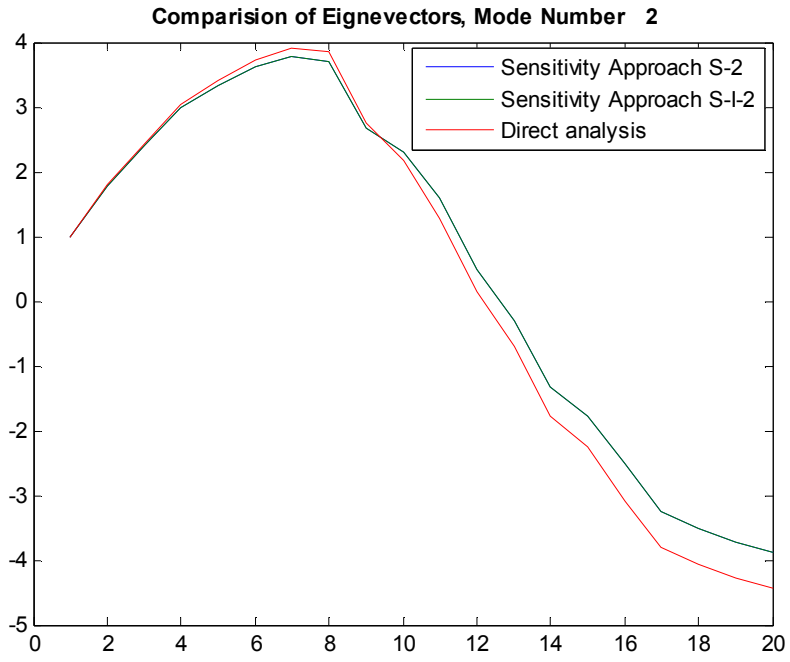

Figure 6.10 Comparison of Eigenvector 2 Computed by S-2 and S-I-2, Case 6.2

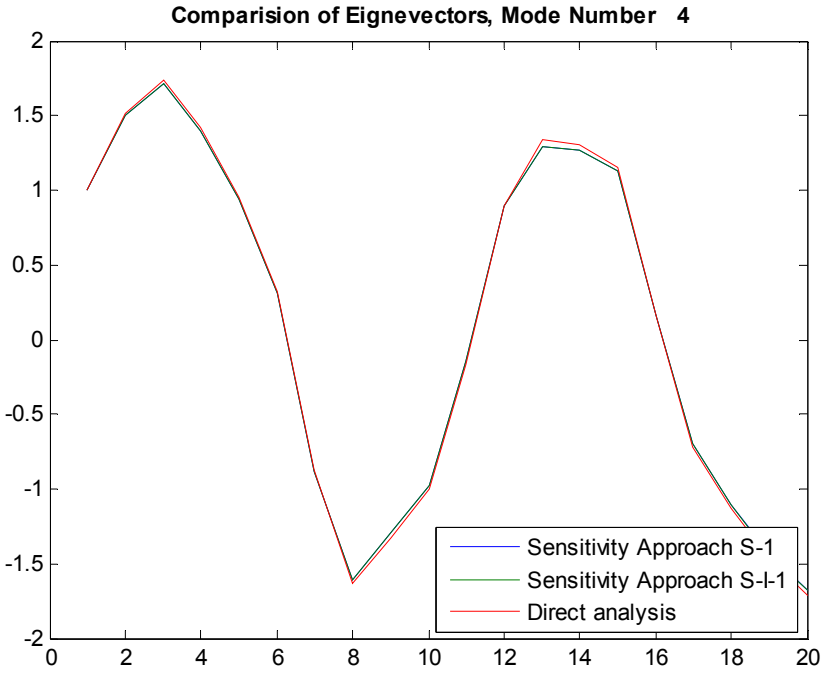

Figure 6.11 Comparison of Eigenvector 4 Computed by S-1and S-I-1, Case 6.2

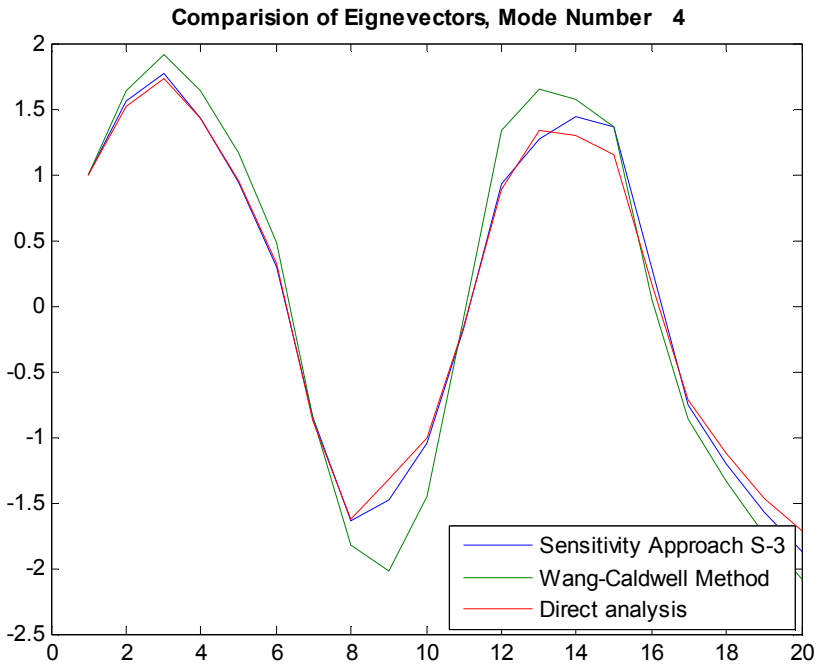

Figure 6.12 Comparison of Eigenvector 4 Computed by S-3 and W-C, Case 6.2

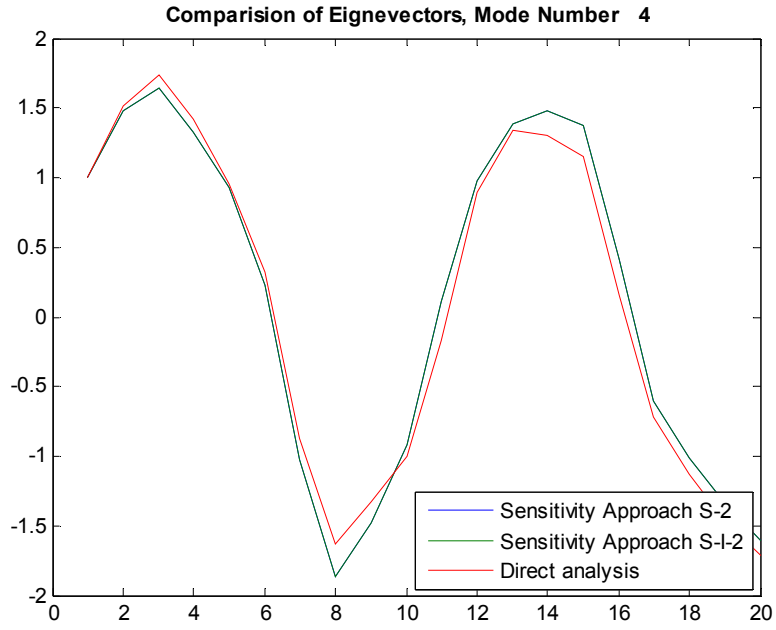

Figure 6.13 Comparison of Eigenvector 4 Computed by S-2 and S-I-2, Case 6.2

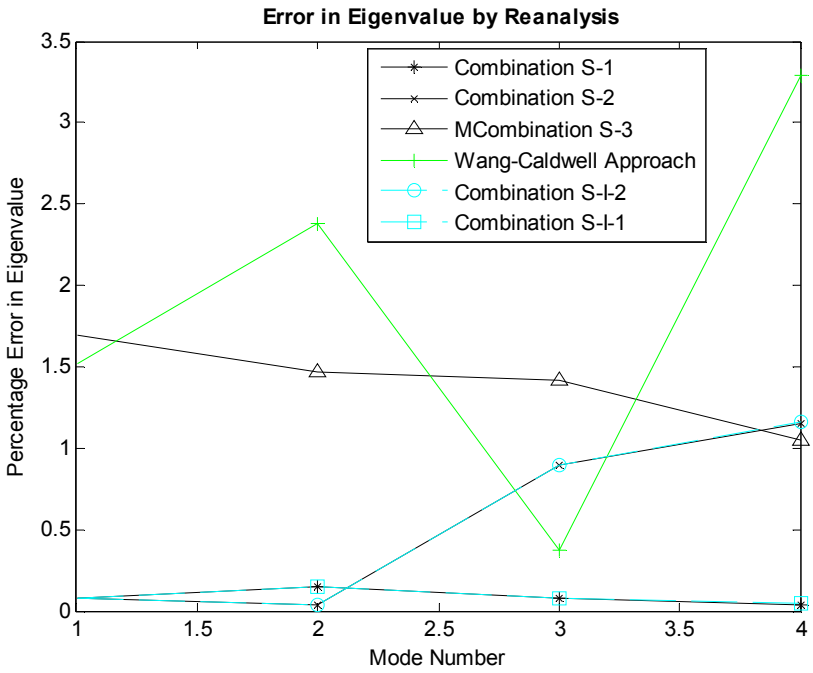

Figure 6.14 Comparison of Absolute Error in Eigenvalues Computed by Reanalysis Procedures, Case 6.2

6.3 33 Member Frame Structure- Local Modification

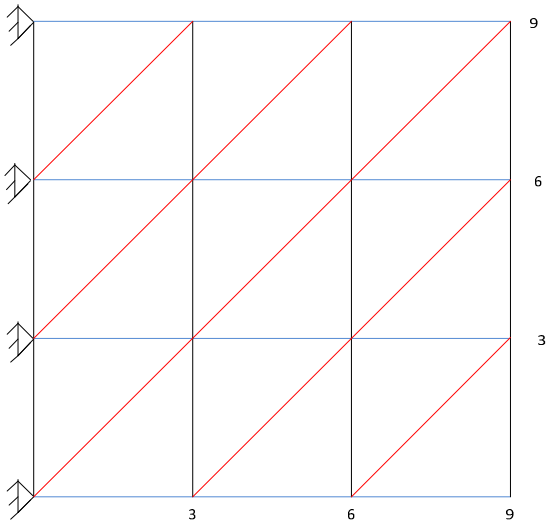

Figure 6.15 33 Member Frame Structure – Initial Design

For a two dimensional frame structure, each node has three degree of freedom: Translation in X axis, Translation in Y axis and Rotation about Z axis. The Initial design of a 33 member frame structure is as shown in figure 6.9. This initial design is modified in cases 6.3 through 6.6.

As shown in the figure, four nodes of the structure are fixed and hence it has 36 degrees of freedom. The members of this structure are grouped as horizontal members, vertical member and inclined members which are marked in blue black and red color respectively. The members have a square cross section. The data of Initial design is as follows:

E = 200 GPa, ρ = 7800 Kg/m^3

The cross section of all members is 1 cm × 1 cm.

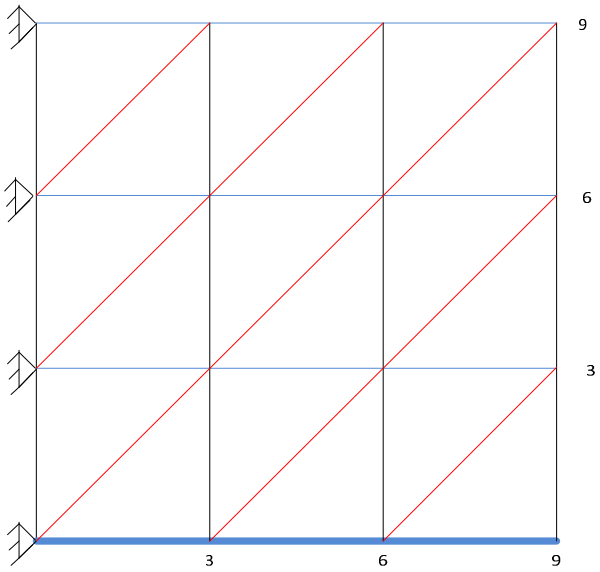

Figure 6.16 33 Member frame structure- Local Modification

In this case, the design is modified locally by increasing the cross section of three horizontal members to 2 cm × 2 cm shown in the figure 6.16.

Table 6.3 Eigenvalues of Initial and Modified Design, Case 6.3

Mode No.	Eigenvalue of Initial Design	Eigenvalue of Modified Design
1	355.86	364.13
2	381.67	409.61
3	437.15	470.61
4	439.48	480.04

In Figures 6.17 through 6.25, the first, second and fourth eigenvectors computed by various combinations are compared. These eigenvectors are normalized by making the magnitude of largest element equal to 1. Combination S-1 and S-I-1 produces the eigenvectors better results than CA procedure. The combinations S-1 produces better results than that produced by S-3 as it uses more number of basis vectors. S-2 and S-I-2 does not produce acceptable results for higher modes as it uses only 3 basis vectors. Figure 6.26 summarizes error in first four eigenvalues computed by various combinations. S-1 and S-I-1 gives very good approximations of eignevalues with maximum absolute percentage error less than 0.5%. The results for combinations S-2 and S-I-2 are same as S-1 and S-I-1 for the first mode.

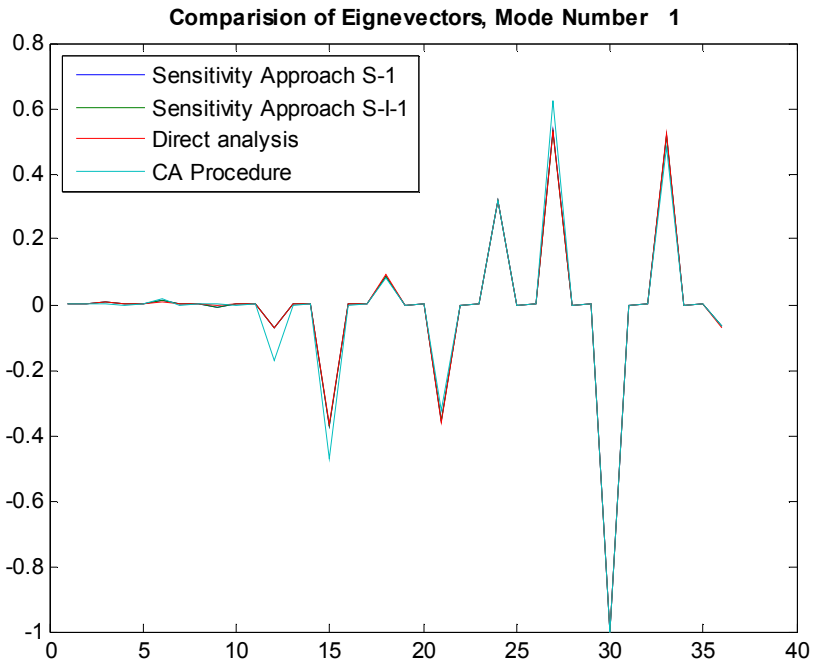

Figure 6.17 Comparison of Eigenvector 1 Computed by S-1and S-I-1, Case 6.3

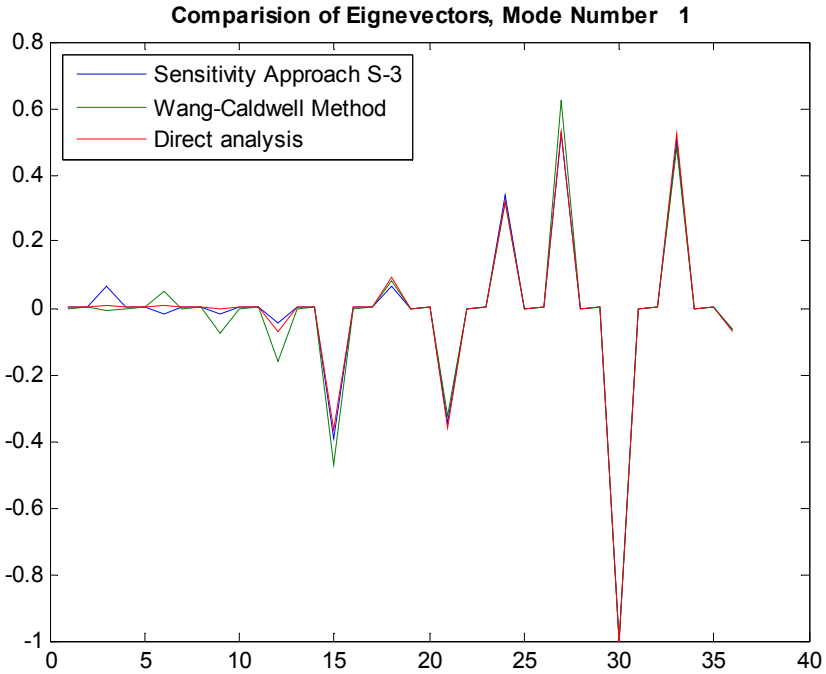

Figure 6.18 Comparison of Eigenvector 1 Computed by S-3 and W-C, Case 6.3

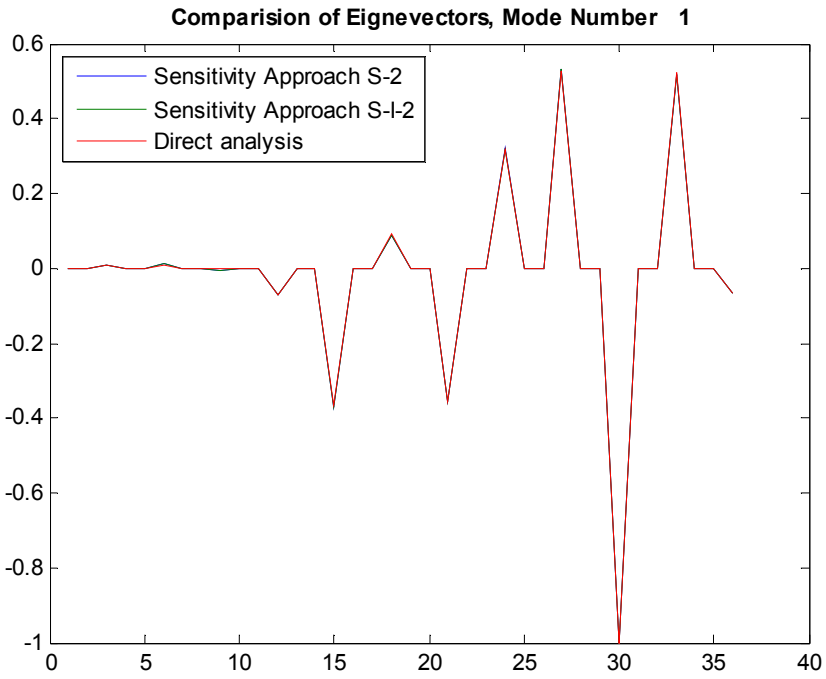

Figure 6.19 Comparison of Eigenvector 1 Computed by S-2 and S-I-2, Case 6.3

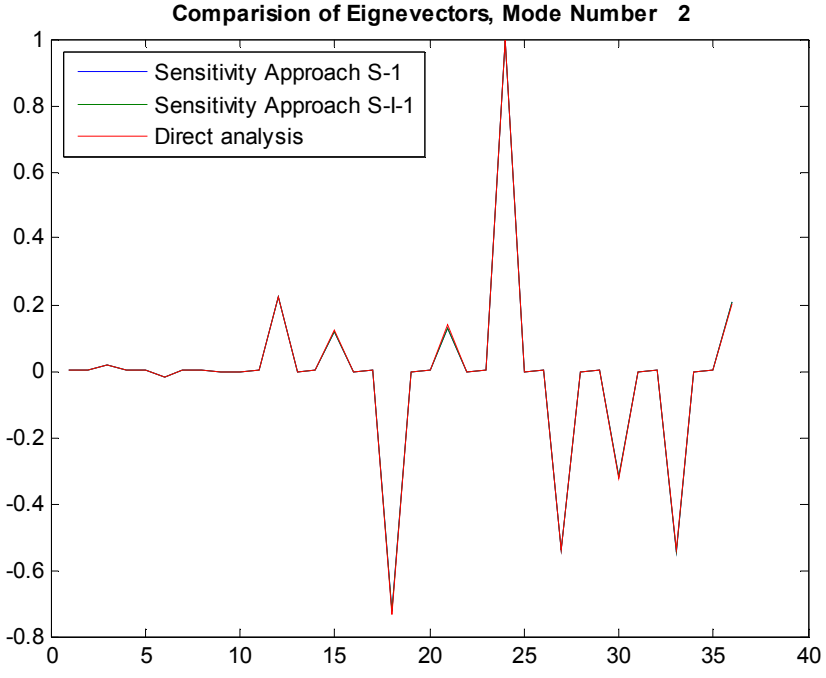

Figure 6.20 Comparison of Eigenvector 2 Computed by S-1and S-I-1, Case 6.3

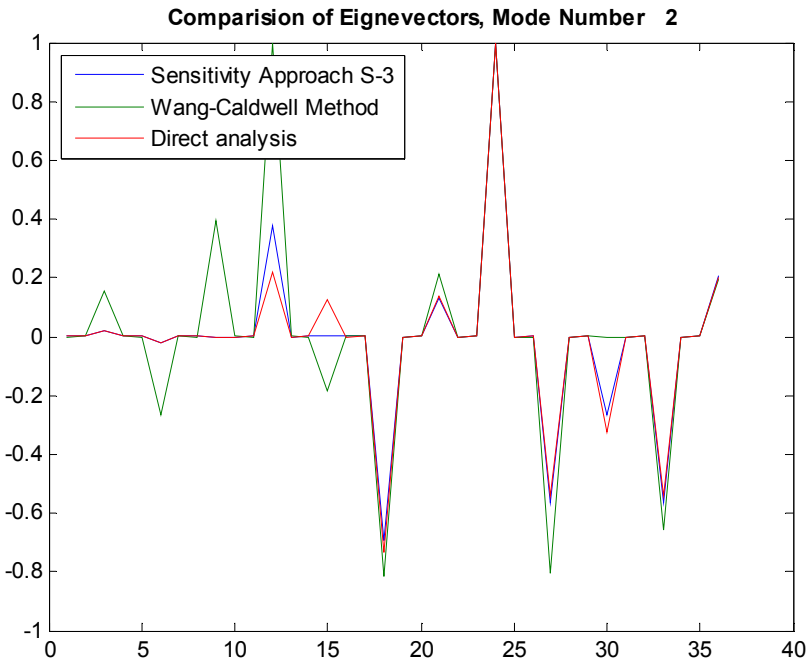

Figure 6.21 Comparison of Eigenvector 2 Computed by S-3 and W-C, Case 6.3

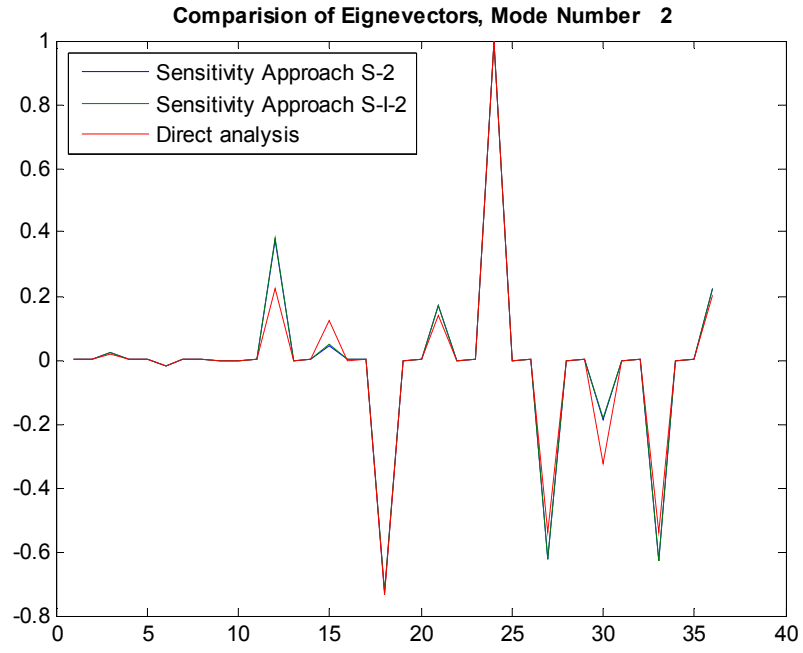

Figure 6.22 Comparison of Eigenvector 2 Computed by S-2 and S-I-2, Case 6.3

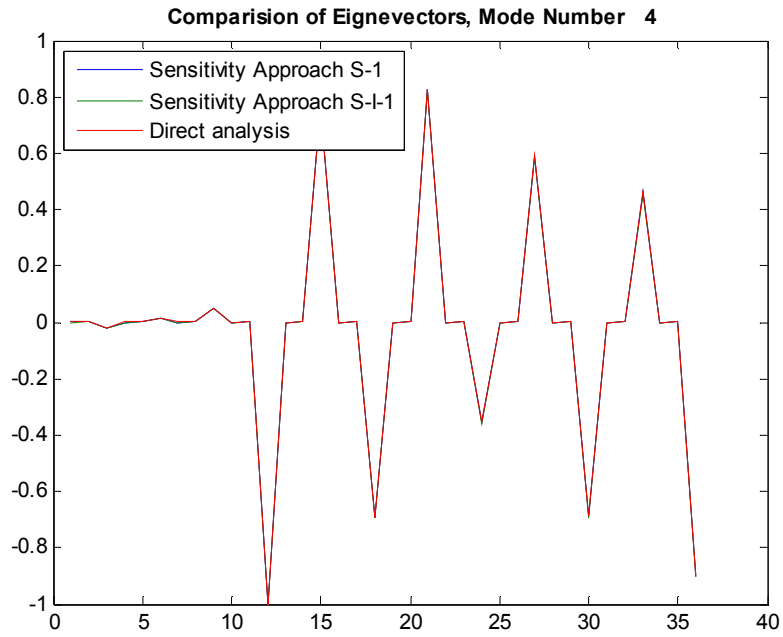

Figure 6.23 Comparison of Eigenvector 4 Computed by S-1and S-I-1, Case 6.3

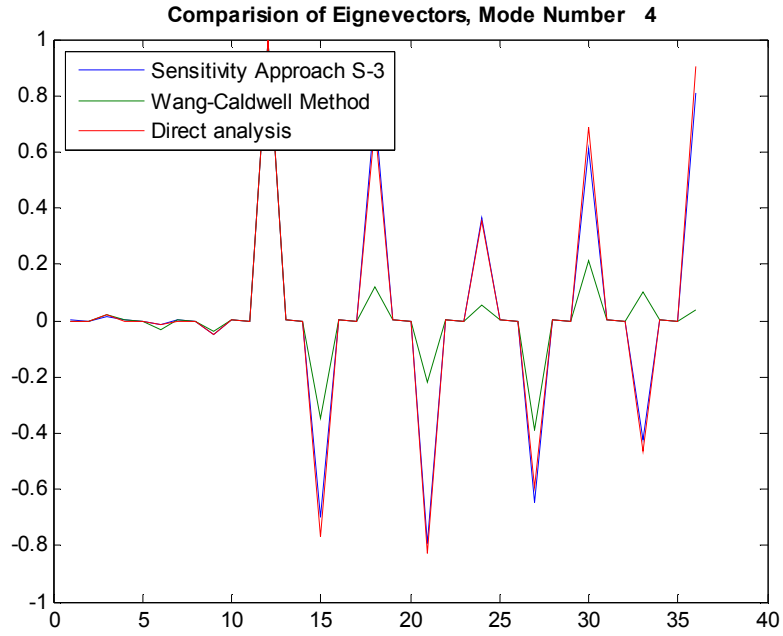

Figure 6.24 Comparison of Eigenvector 4 Computed by S-3 and W-C, Case 6.3

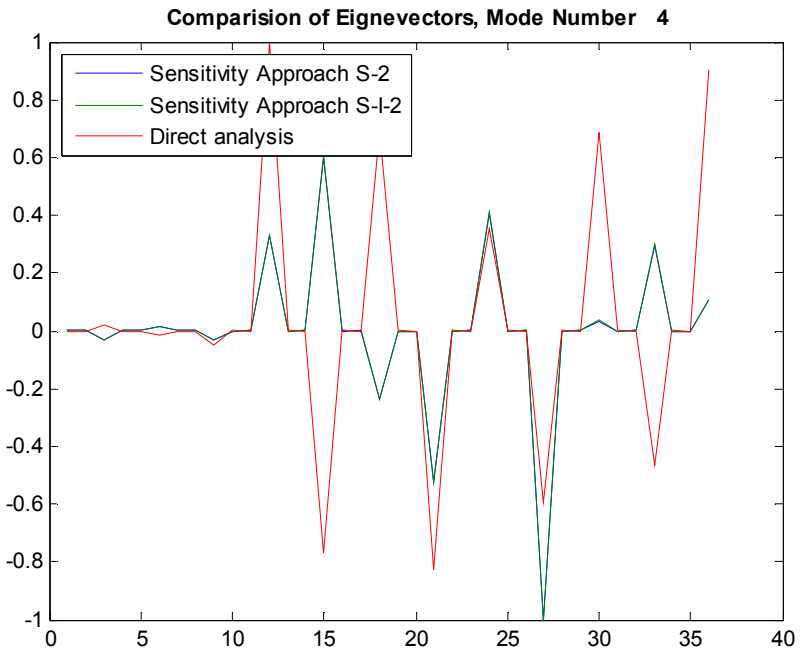

Figure 6.25 Comparison of Eigenvector 4 Computed by S-2 and S-I-2, Case 6.3

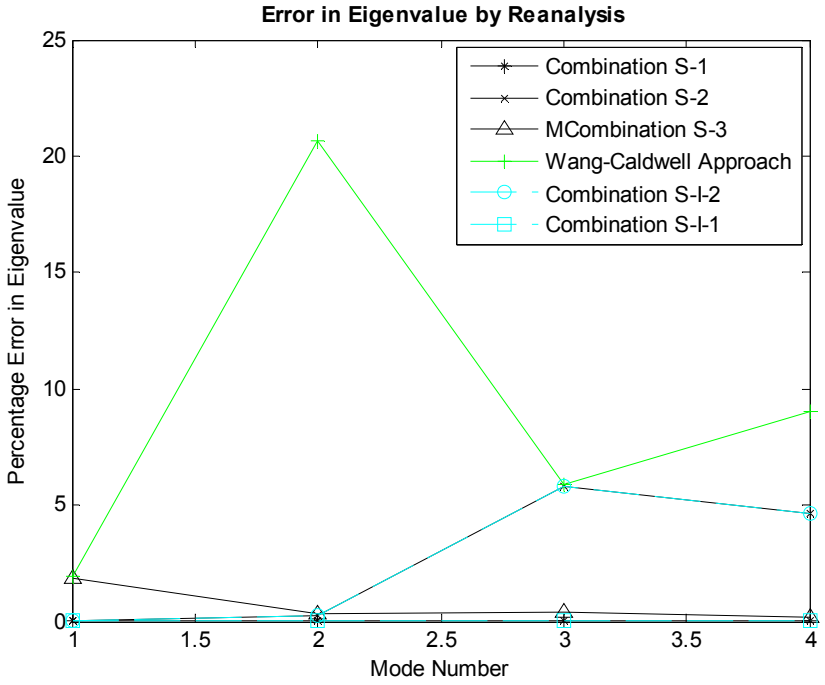

Figure 6.26 Comparison of Absolute Error in Eigenvalues Computed by Reanalysis Procedures, Case 6.3

6.4 33 Member Frame Structure- Cross Section Dimension Change

In this case, the cross sections of all horizontal members were increased to 1.05 cm × 1.05 cm whereas sections of all vertical members were decreased to 0.95 cm × 0.95 cm.

Table 6.4 Cross Section Dimensions - Initial and Modified Designs, Case 6.4

Members	Initial Design	Modified Design
Horizontal	1cm × 1cm	1.05 cm× 1.05 cm
Vertical	1cm × 1cm	0.95 cm × 0.95 cm
Inclined	1cm × 1cm	1cm × 1cm

In Figures 6.27 through 6.32, the first and fourth eigenvectors computed by various combinations are compared. These eigenvectors are normalized by making the magnitude of largest element equal to 1. The combinations S-1 and S-I-1 produces best results for eigenvectors. Figure 6.33 summarizes error in first four eigenvalues computed by various combinations. S-1 and S-I-1 gives very good approximations of eignevalues with maximum absolute percentage error less than 0.2%. The first and fourth Eigenvalue of Initial and modified design computed by exact analysis are shown in the following table:

Table 6.5 Eigenvalues of Initial and Modified Design, Case 6.4

Mode No.	Eigenvalue of Initial Design	Eigenvalue of Modified Design
1	355.87	386.10
4	439.48	483.84

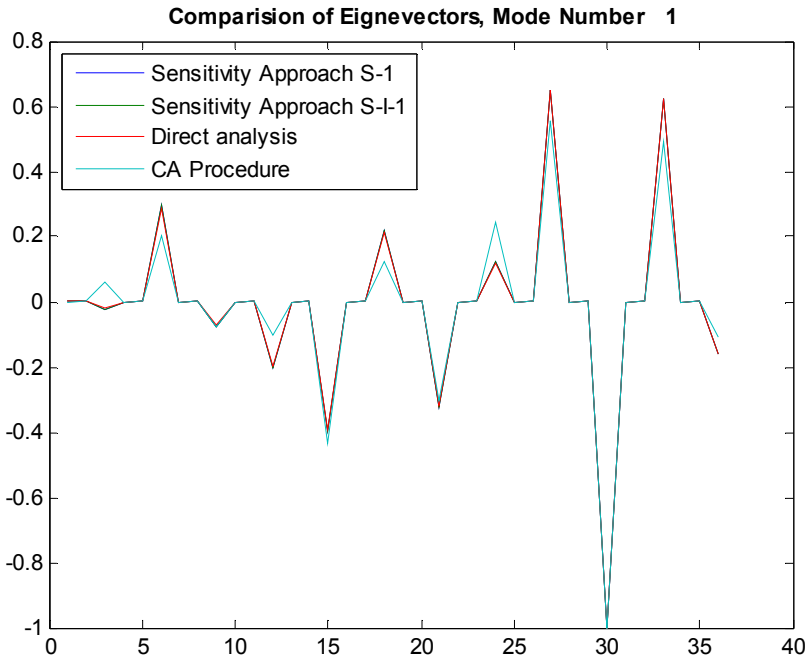

Figure 6.27 Comparison of Eigenvector 1 Computed by S-1and S-I-1, Case 6.4

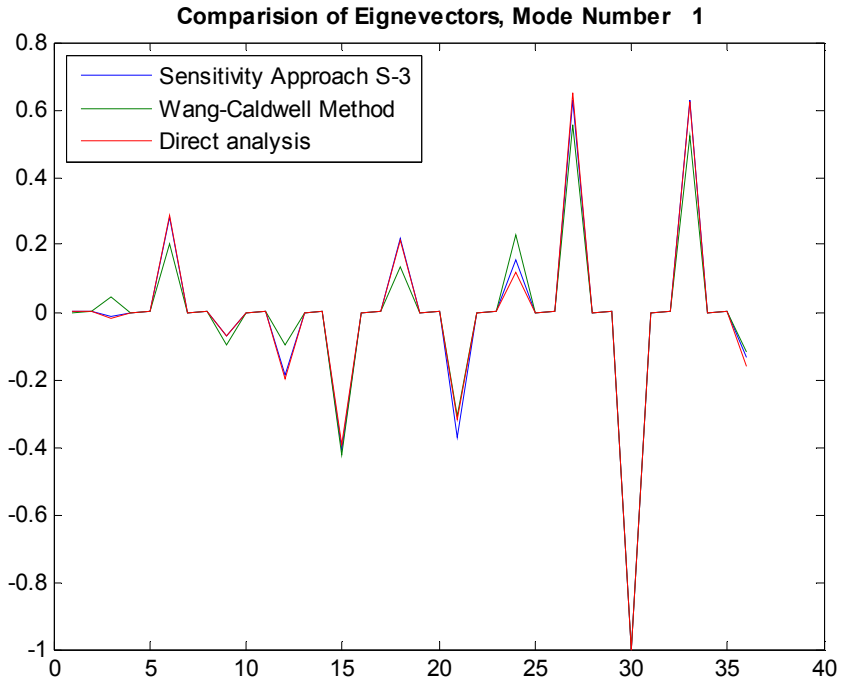

Figure 6.28 Comparison of Eigenvector 1 Computed by S-3 and W-C, Case 6.4

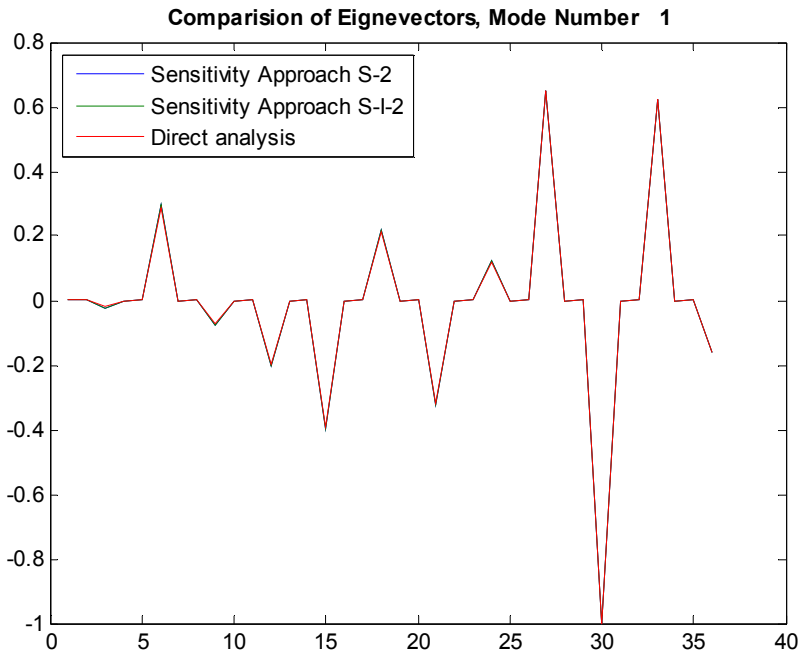

Figure 6.29 Comparison of Eigenvector 1 Computed by S-2 and S-I-2, Case 6.4

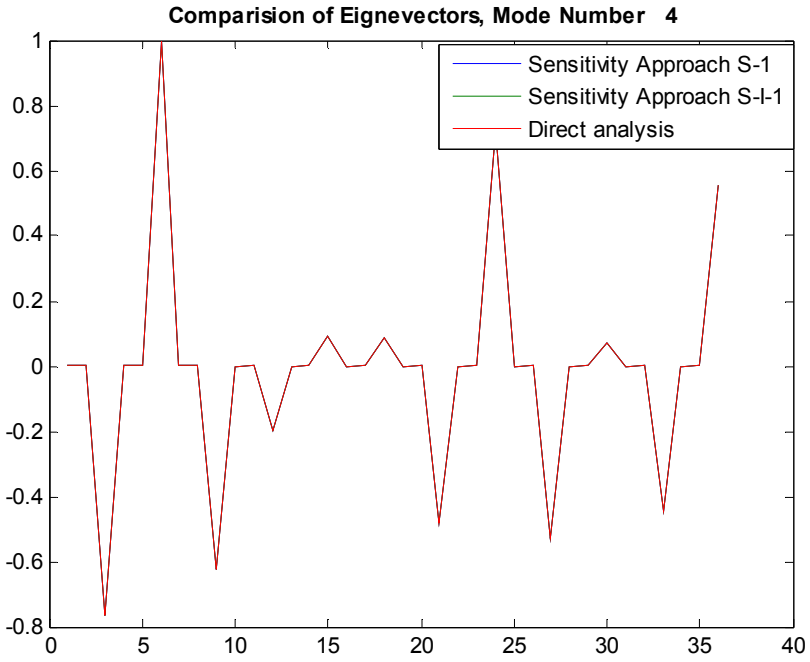

Figure 6.30 Comparison of Eigenvector 4 Computed by S-1and S-I-1, Case 6.4

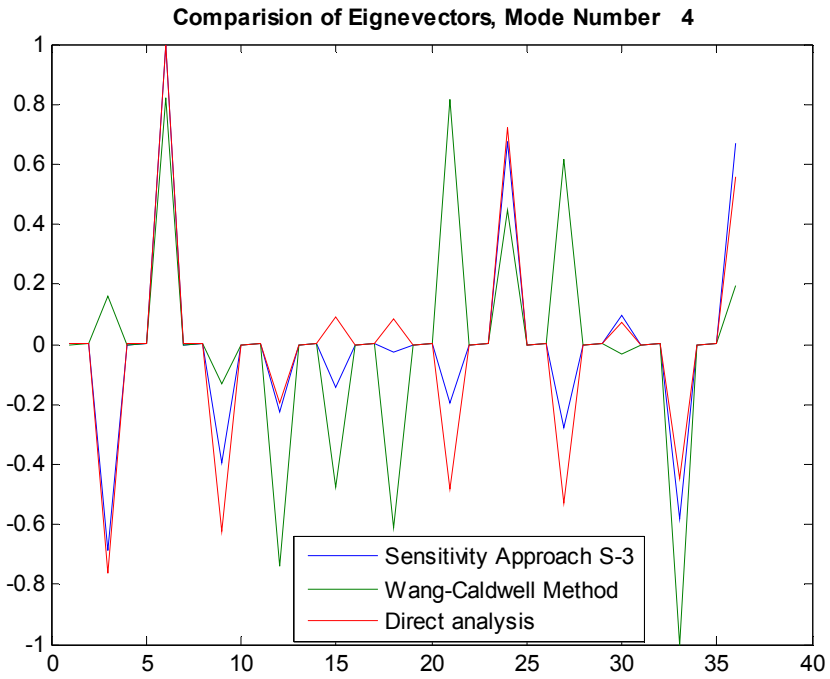

Figure 6.31 Comparison of Eigenvector 4 Computed by S-3 and W-C, Case 6.4

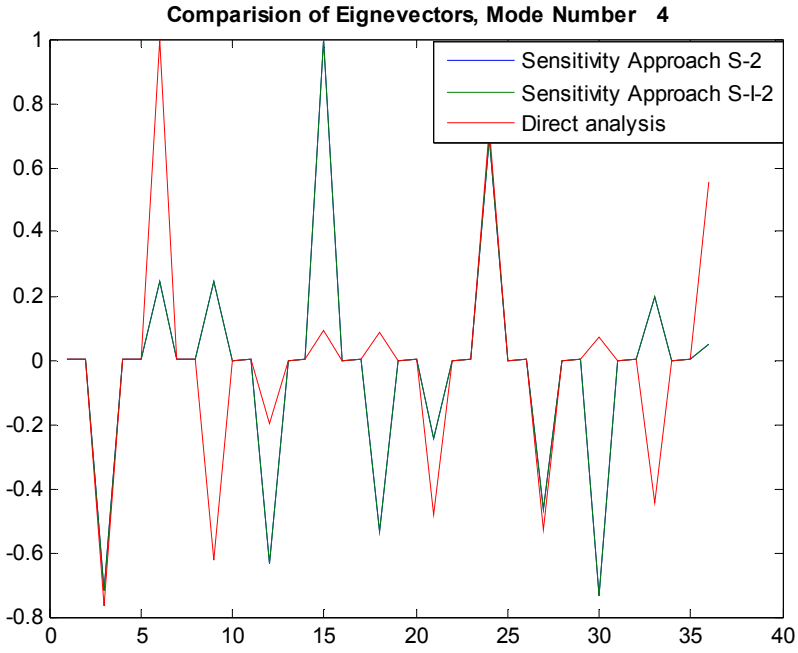

Figure 6.32 Comparison of Eigenvector 4 Computed by S-2 and S-I-2, Case 6.4

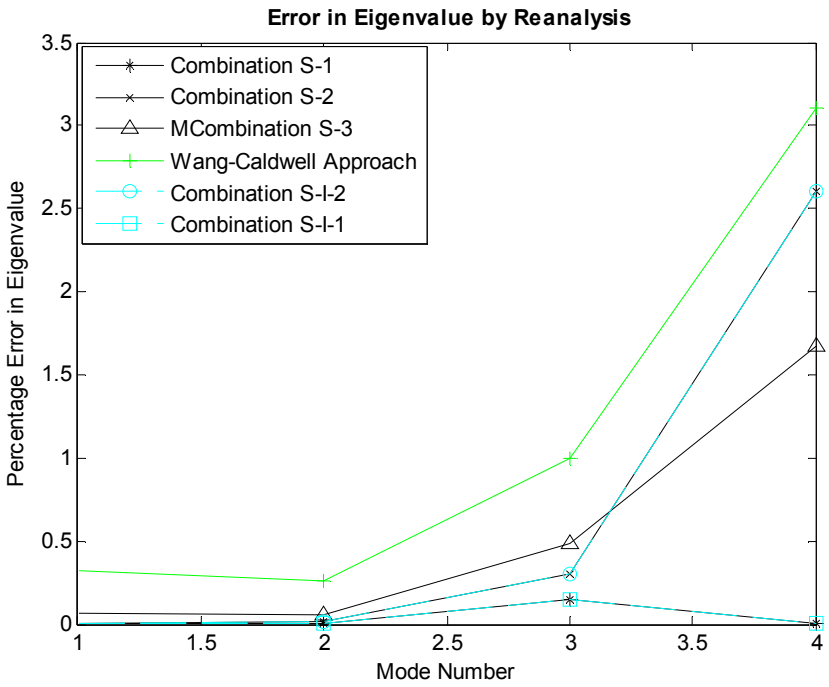

Figure 6.33 Comparison of Absolute Error in Eigenvalues Computed by Reanalysis Procedures, Case 6.4

6.5 33 Member Frame Structure- Topological Modification

In this case, the data for initial design is same as that in the above section. The modified design is as shown in the figure 6.34. Thus, this will change the lengths of some members of all groups and therefore both stiffness and mass matrices. The first and third Eigenvalue of Initial and modified design are shown in the table 6.6.

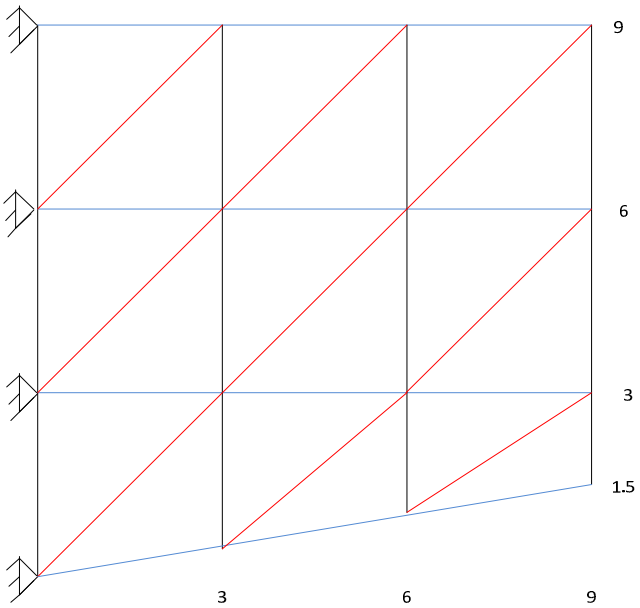

Figure 6.34 33 Member frame structure- Topological Modification, Case 6.5

Table 6.6 Eigenvalues of Initial and Modified Design, Case 6.5

Mode No.	Eigenvalue of Initial Design	Eigenvalue of Modified Design
1	355.87	368.58
4	437.15	477.25

In Figures 6.35 through 6.40, the first and fourth eigenvectors computed by various combinations are compared. These eigenvectors are normalized by making the magnitude of largest element equal to 1. The combinations S-1 and S-I-1 produces best results for eigenvectors. Figure 6.41 summarizes error in first four eigenvalues computed by various combinations. S-1 and S-I-1 gives very good approximations of eignevalues and the percentage error is very small.

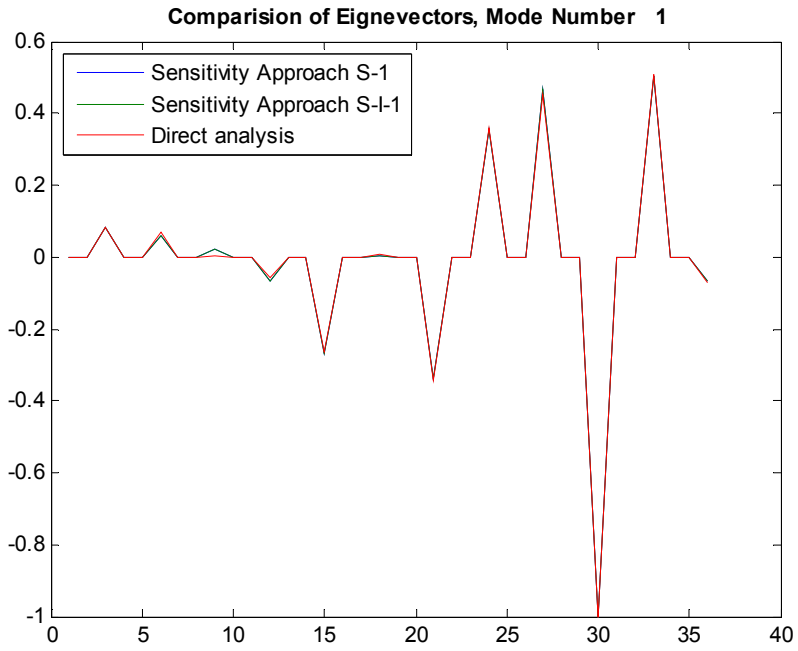

Figure 6.35 Comparison of Eigenvector 1 Computed by S-1and S-I-1, Case 6.5

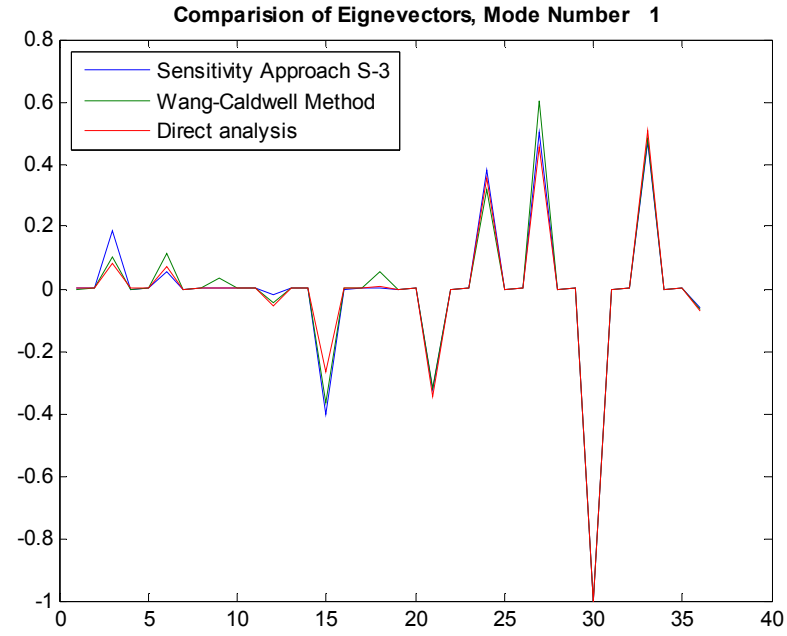

Figure 6.36 Comparison of Eigenvector 1 Computed by S-3 and W-C, Case 6.5

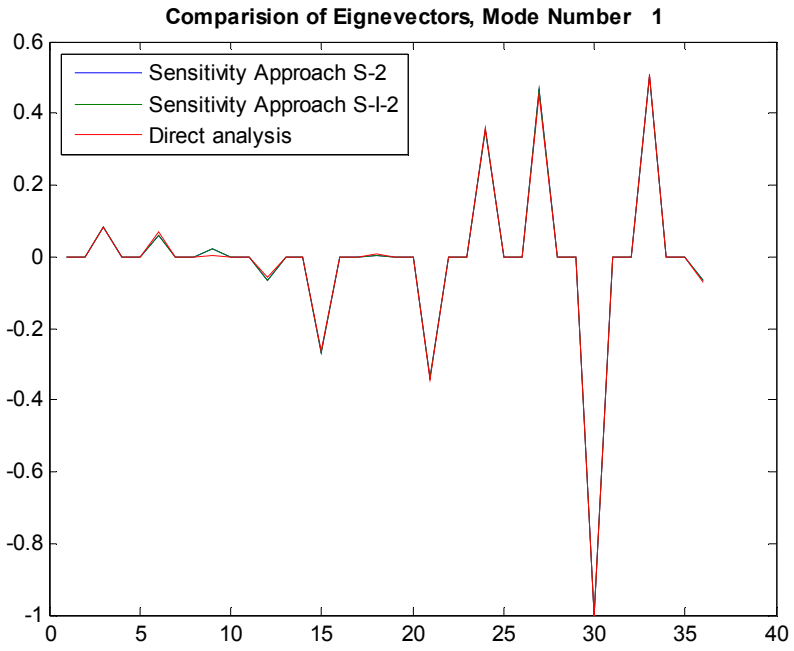

Figure 6.37 Comparison of Eigenvector 1 Computed by S-2 and S-I-2, Case 6.5

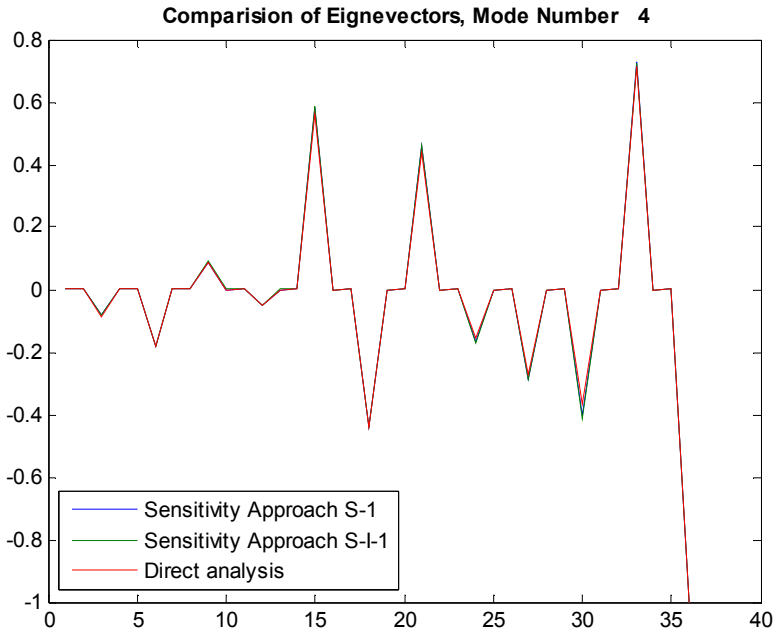

Figure 6.38 Comparison of Eigenvector 4 Computed by S-1and S-I-1, Case 6.5

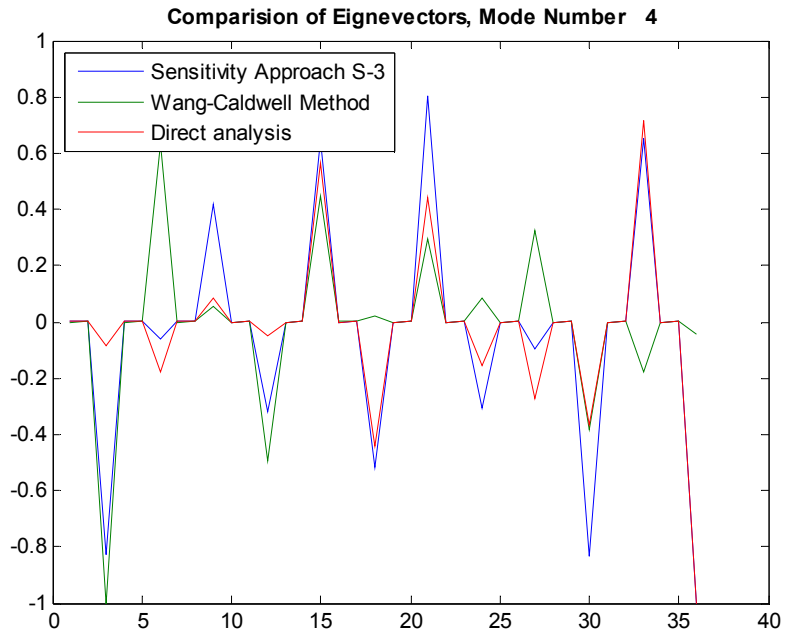

Figure 6.39 Comparison of Eigenvector 4 Computed by S-3 and W-C, Case 6.5

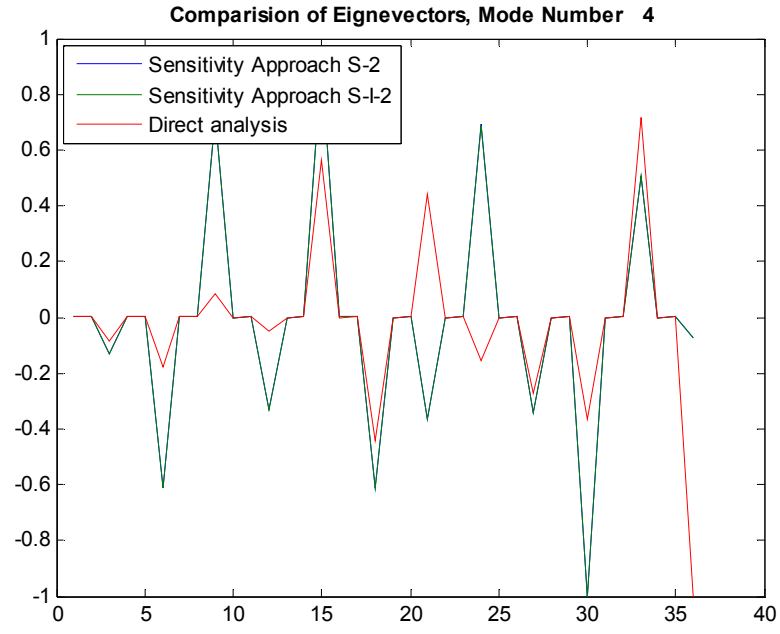

Figure 6.40 Comparison of Eigenvector 4 Computed by S-2 and S-I-2, Case 6.5

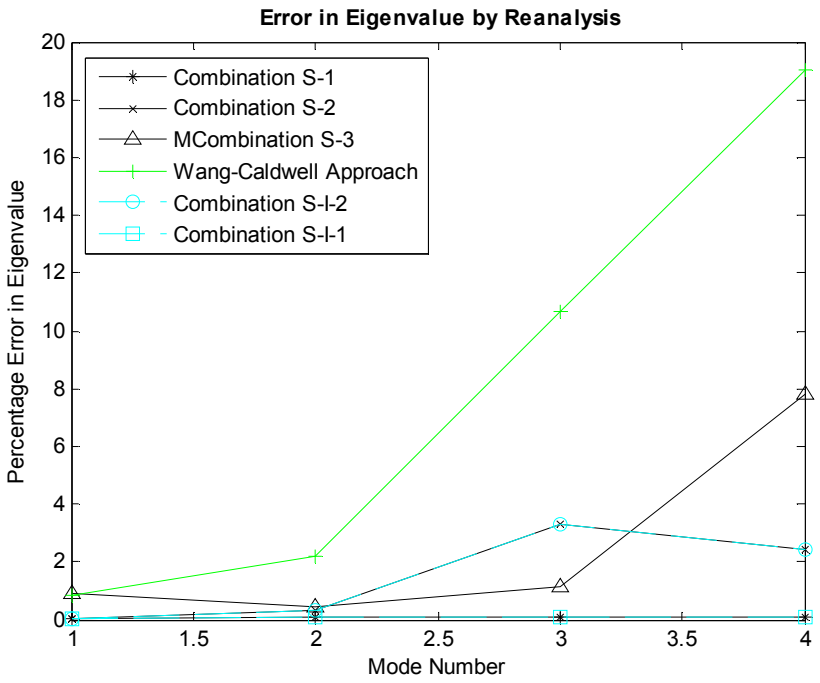

Figure 6.41 Comparison of Absolute Error in Eigenvalues Computed by Reanalysis Procedures, Case 6.5

6.6 33 Member Frame Structure- Cross Section Dimension Change in Steps

The section dimensions of the horizontal and vertical members are changed from 1 cm × 1 cm to 1.3 cm × 1.3 cm for horizontal members and 0.7 cm × 0.7 cm for vertical members respectively in a step of 0.002 cm × 0.002 cm. Combination S-1 is used for reanalysis using three, six and nine basis vectors.

As mentioned earlier, S-1 combination uses of all preceeding eigenvectors and their derivatives. For example, first eigenvector and its derivatives are used in addition to second eigenvector and its derivatives in

order to compute second eigenpair. Therefore, the number of basis vectors used is 6. This produces approximation for second as well as first eigenpair. Similarly, solving for third eigenpair, which uses 9 basis vectors produces approximation for first and second eigenpair also. In this section, the S-1 combination was used to solve for first, second and third eigenpairs. The first eigenpair obtained using three, six and nine basis vectors were compared. Similarly, second eigenpair obtained using six and nine basis vectors were compared.

Table 6.7 Combinations of Basis Vectors Used, Case 6.6

Number of basis	Basis vectors
3	$\left[\phi_1 \ \phi_1' \ \phi_1''\right]$
6	$\left[\phi_1 \ \phi_2 \ \phi_1' \ \phi_2' \ \phi_1'' \ \phi_2''\right]$
9	$\left[\phi_1 \ \phi_2 \ \phi_3 \ \phi_1' \ \phi_2' \ \phi_3' \ \phi_1'' \ \phi_2'' \ \phi_3''\right]$

As seen in figure 6.42, the proposed procedure produces acceptable results for first eigenvalue using three basis vectors at least up to first fifty steps. Increasing the number of basis vectors, ie, adding proceeding eigenvectors and their derivatives further improves the accuracy of reanalysis results for first eigenvalue and makes the reanalysis procedure reliable for larger modifications. However, that requires more calculations. The percentage

error in first, second and third eigenvalue for each modification is shown in the figures 6.42 through 6.44.

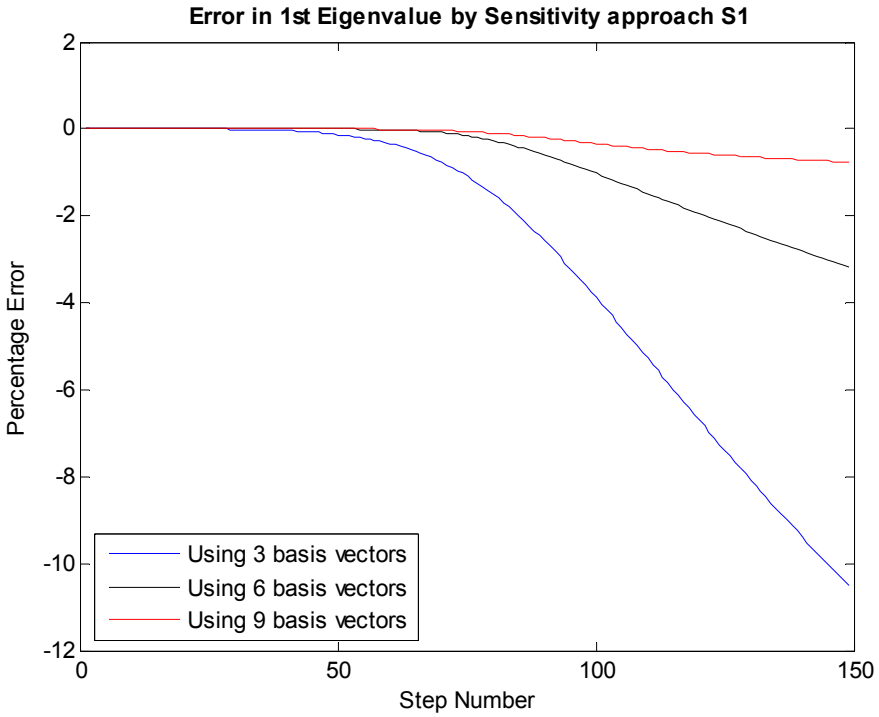

Figure 6.42 Comparison of Percentage Error in First Eigenvalue, Case 6.6

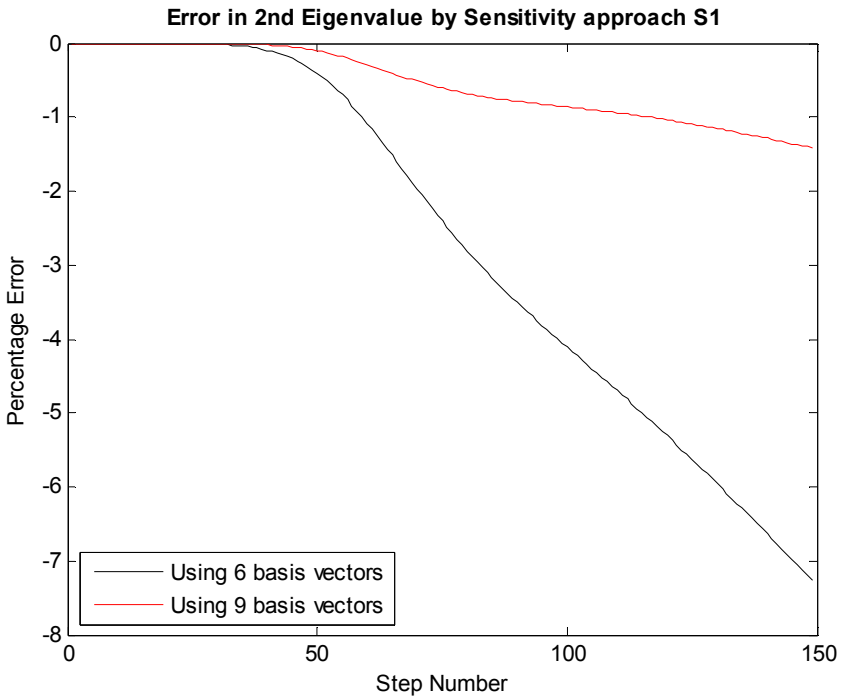

Figure 6.43 Comparison of Percentage Error in Second Eigenvalue, Case 6.6

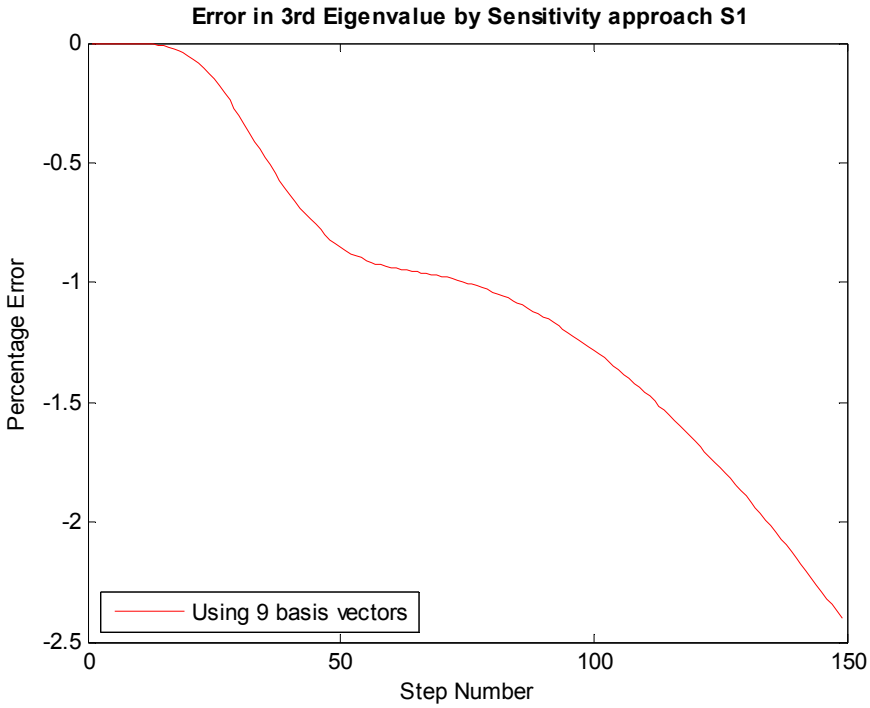

Figure 6.44 Comparison of Percentage Error in Third Eigenvalue, Case 6.6

6.7 340 Member Frame Structure- Cross Section Dimension Change

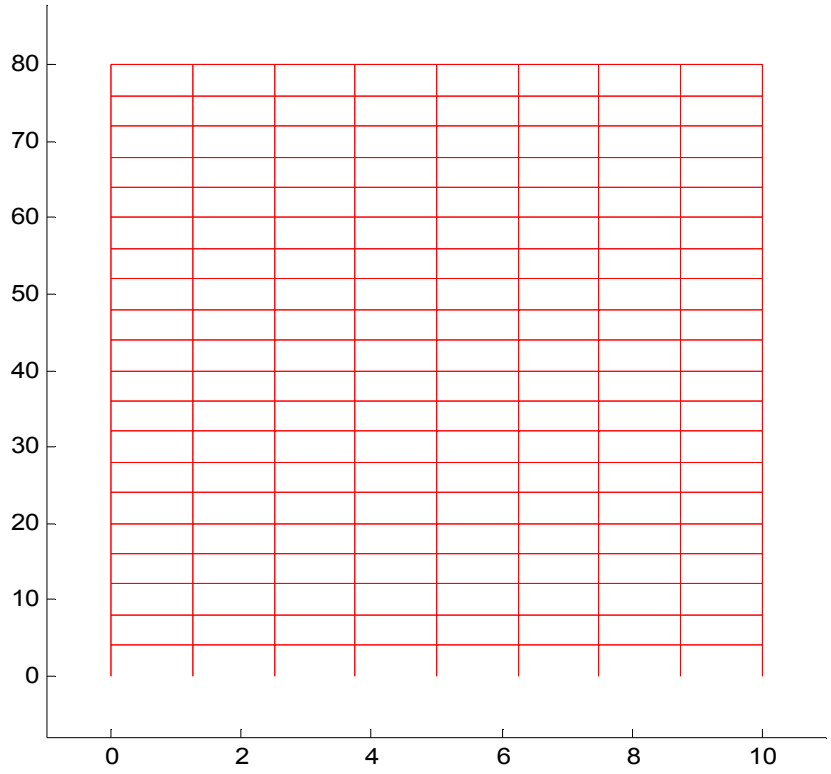

Figure 6.45 340 Member Frame Structure - Initial Design

Initial design of a large plane frame structure is as shown in figure 6.45. The lengths are in meters. The material and section properties are as follows.

E = 200 GPa, ρ = 7800 Kg/m^3

Table 6.8 Cross Section Dimensions - Initial and Modified Design, Case 6.7

Members	Marking	Initial Design	Modified Design
Vertical	Blue	1.2 cm × 1.2 cm	1.1 cm× 1.1 cm
Horizontal - Lower	Red	0.6 cm × 0.6 cm	0.6 cm × 0.6 cm
Horizontal- Upper	Green	0.6 cm × 0.6 cm	0.5 cm × 0.5 cm

The frame structure is fixed at the ground. Each member is divided into two elements. Therefore, the total number of degree of freedom for this structure is 1560.

In the figure 6.46, the members marked as red are not modified. The section dimension of vertical members marked blue is decreased. The section dimension of upper horizontal members marked green is increased mentioned in table 6.8. The first four Eigenvalues of Initial and modified design computed by exact analysis are shown in the following table:

Table 6 9 Eigenvalues of Initial and Modified Design, Case 6.7

Mode No.	Eigenvalue of Initial Design	Eigenvalue of Modified Design
1	0.0134	0.0130
2	0.1237	0.0943
3	0.3608	0.3022
4	0.7569	0.6113

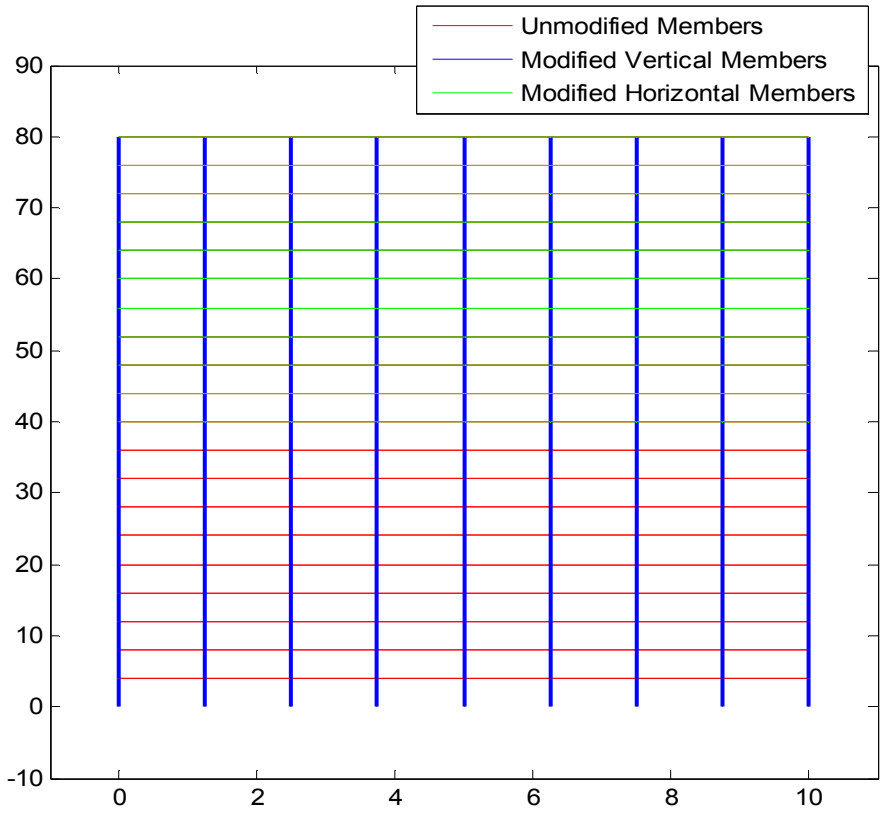

Figure 6.46 340 Member Frame Structure - Modified Design

Since the results of S-1 and S-I-1 combinations are best, here the first four mode shapes obtained by these combinations are compared with the exact mode shape of modified design in figures to . It should be noted that only ten eigenvectors were used to approximate eigenvector derivatives using Implicit and explicit methods.Figure 6.33 summarizes error in first four eigenvalues computed by various combinations. S-1 and S-I-1 gives very good approximations of eignevalues with maximum absolute percentage error very small.

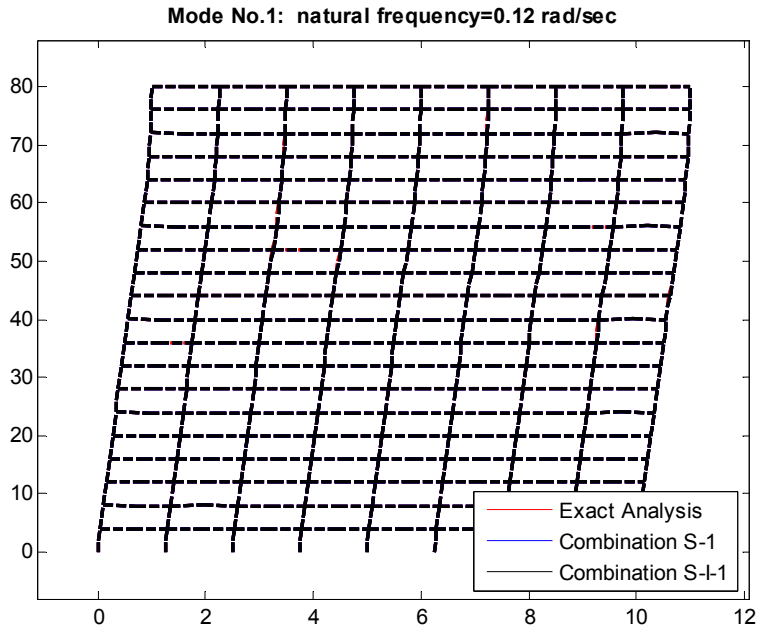

Figure 6.47 Comparison of Mode Shape 1 Computed by S-1and S-I-1, Case 6.7

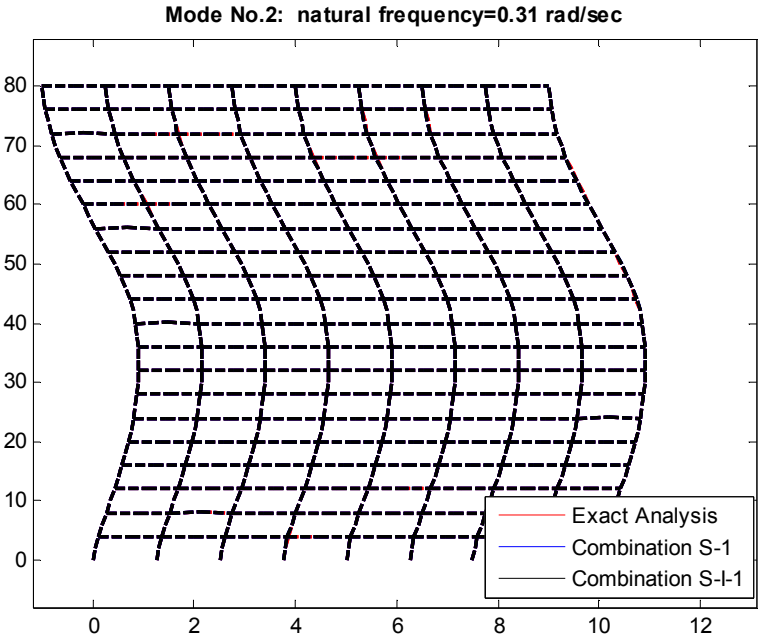

Figure 6.48 Comparison of Mode Shape 2 Computed by S-1and S-I-1, Case 6.7

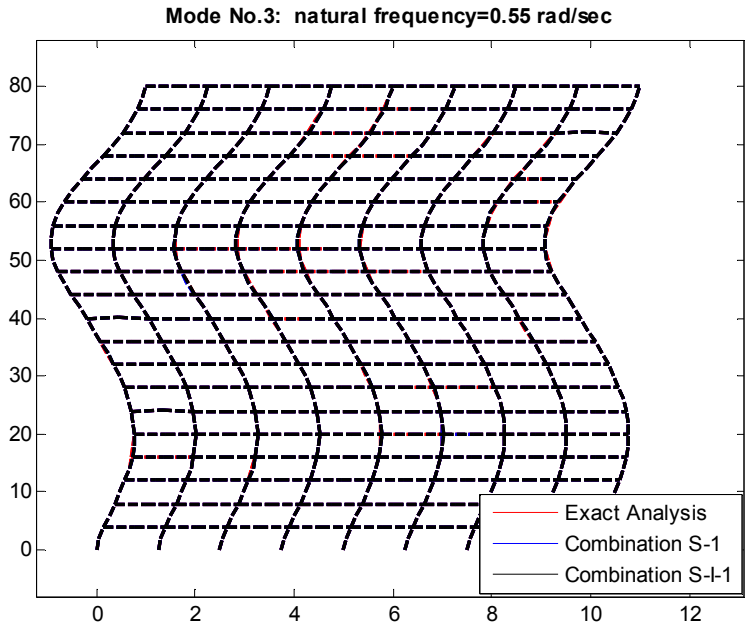

Figure 6.49 Comparison of Mode Shape 3 Computed by S-1and S-I-1, Case 6.7

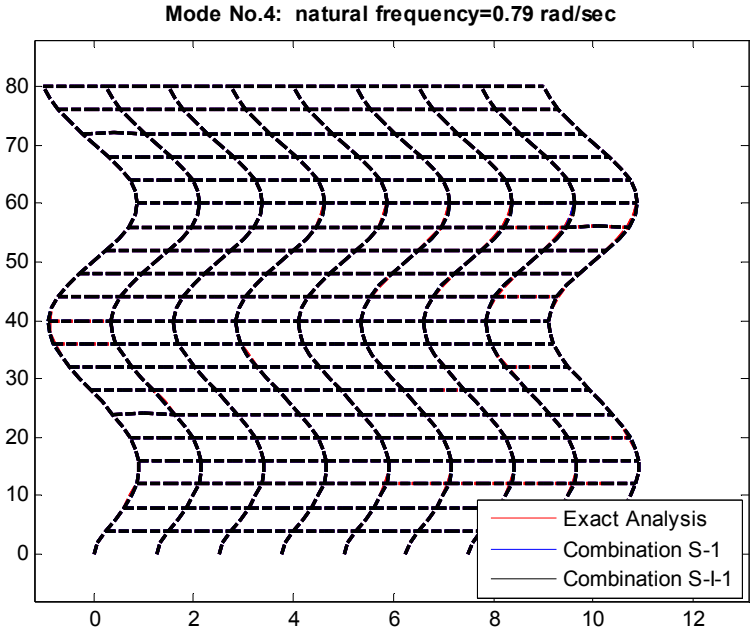

Figure 6.50 Comparison of Mode Shape 4 Computed by S-1and S-I-1, Case 6.7

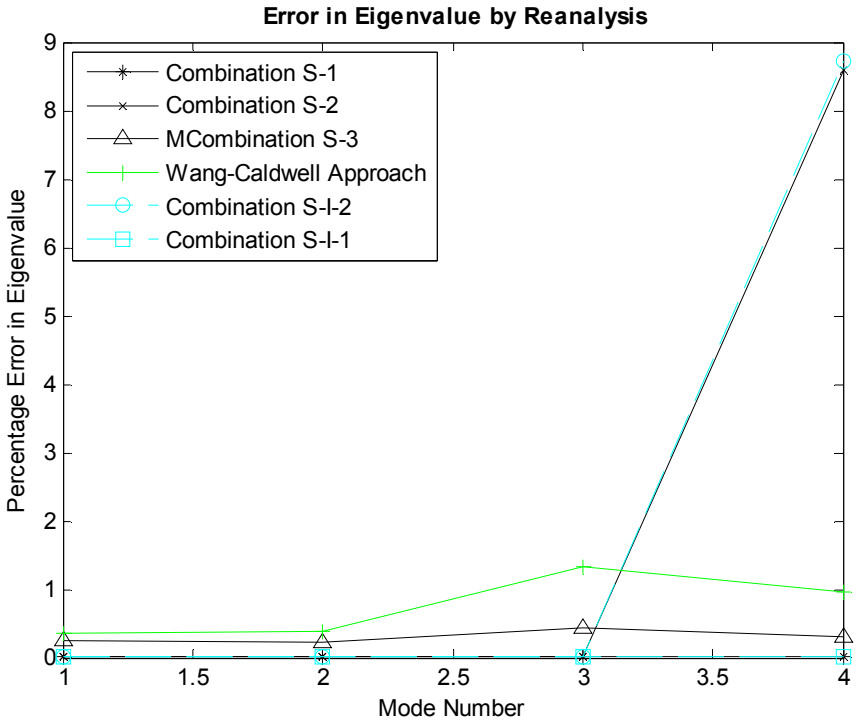

Figure 6.51 Comparison of Absolute Error in Eigenvalues Computed by Reanalysis Procedures, Case 6.7

CHAPTER 7

CONCLUSION AND FUTURE RESEARCH

The results are improved using Gram-Schmidt orthogonalization in case of CA procedure. For that, all preceding modes are to be computed using reanalysis procedure first. This makes calculations complicated. However, the results are still not as good as that obtained by sensitivity approach S-1.

As it is clear from results, sensitivity approaches are efficient methods for eigenproblem reanalysis for various types of modifications. For small modifications, S-2, S-E-2 and S-I-2 procedures gives sufficiently accurate results for first few modes with moderate modification. For moderate to large modifications or in case if a few consecutive eigenvalues and eigenvectors are to be reanalyzed, S-1 method is more suitable as it gives the best results for all desired eigenpairs.

The results of reanalyzed eigenvector obtained by approximate eigenvector sensitivity data and exact eigenvector sensitivity data as basis vectors are very much similar in case if the number of basis vectors is same.

The proposed sensitivity approach has applications in the efficient eigensolution reanalysis procedure in structural dynamics optimization. The reanalysis method S-1 which uses r basis vectors and their derivatives is very

efficient and gives almost accurate results even for larger modifications as shown in results. So, this can be used for structural optimization. A method can be developed which gives suggestions whether the results of reanalysis are converging or not. This will lead to make use of methods S-I-2 and S-E-2 which uses only 3 basis vectors in optimization more reliable.

The Forced response of a structure can be obtained from eigenvectors using modal superposition method. Usually, for large structures, forced response is calculated by using only some initial eigenvectors. So, these initial eigenvectors of modified design can be obtained by using reanalysis techniques which potentially can give efficient approximations of forced response. Therefore, the sensitivity data can also be used in automated optimization for constraints of forced response.

REFERENCES

1. Wang B. P., Pilkey W. D. Eigenvalue reanalysis of locally modified structures using a generalized Raleigh's method. AIAA Journal 1986; 24, 983-990

2. Bickford W.B. An improved computational technique for perturbations of the generalized symmetric linear algebraic eigenvalue problem. International Journal of Numerical Methods in Engineering. 1987; 24, 529–541

3. Chen S.H., Song D.T., Ma A.J. Eigensolution reanalysis of modified structures using perturbations and Raleigh quotients. Communication in Numerical Methods in Engineering. 1994; 10, 111–119

4. Wu B.S., Piao S.X. Applications of Pade' approximation to mechanics. Mechanics & Practice 1996: 18, 27–29

5. Chen S. H, Yang XW. Extended Kirsch combined method for eigenvalue reanalysis. AIAA Journal 2000; 38: 927-930

6. Wang B. P, Pilkey W. D., Palazzoto A. B. Reanalysis, modal synthesis and dynamic design. In State-of-the-Art Surveys on Finite Element Technology, Noor AK, Pilkey WD, Chapter 8. ASME: New York, 1983

7. Chuarong Z, Yimin B. Structural modification and vibration reanalysis. *Computer Methods in Applied Mechanics and Engineering* 1990; 83:99–108.

8. Wang B. P., Caldwell S. P. Improved eigensolution reanalysis procedures in structural dynamics. Finite Elements in Analysis and Design 1992; 11:191-200.

9. Kirsch Uri, Bogomolni M. Procedures for approximate eigenproblem reanalysis of structures. International Journal for Numerical Methods in Engineering. 2004. Volume 60 Issue 12.1969-1986

10. Kirsch Uri , Bogomolni M. Error evaluation in approximate reanalysis of structures. Structural and Multidisciplinary Optimization. Volume 28, Numbers 2-3, September 2004

11. Chen Su Huan, Xiao Ming Wu and Zhi Jun Yang. Eigensolution reanalysis of modified structures using epsilon-algorithm. International Journal for Numerical Methods in Engineering Jan 2006; 66:2115–2130.

12. Fox R. L. and Kapoor M. P. Rates of change of eigenvalues and eigenvectors. AIAA Journal 1968; 6, 2426-2429.

13. Nelson R. B, Simplified calculation of eigenvector derivatives. AIAA Journal 1976; 14, 1201-1205.

14. Wang B. P. Improved approximate method for computing eigenvector derivatives in structural dynamics. AIAA Journal 1991; 29, 1018-1020.

15. Yu Min, Liu Zhong-Sheng, Wang Da-Jun. Comparison of several approximate modal methods for computing mode shape derivatives. Computers and Structures 1996; 62, 381-393.

16. You-Quan Zhao, Zhong-Sheng Liu, Su-Huan Chen, Gui-yu Zhang. An accurate modal truncation method for eigenvector derivative. Computers and Structures 1999; 73, 609-614.

17. An Efficient Algebraic Method for the Calculation of Natural Frequency and Mode Shape Sensitivities-Part I. Distinct Natural Frequencies. Lee In-Won, Jung Gil-Ho. Computers and Structures. Volume 62. pp. 429-435, 1997

18. Friswell MI. Calculation of Second and Higher Order Eigenvector Derivatives. Journal of Guidance, Control, and Dynamics. VOL. 18, NO. 4: ENGINEERING NOTES 919

19. Grandhi R. Structural optimization with frequency constraints-a review. AIAA Journal 1993; 31:2296-2303.

20. Roy R Craig Jr., Andrew J. Kurdila. Fundamentals of Structural Dynamics. John Wiley & Sons, INC: 2006

21. http://www.ansys.com/solutions/solid-mechanics-modal.asp

22. Nataša Trišović. Eigenvalue sensitivity analysis In structural dynamics. FME Transactions. 2007; 35, 149-156

23. Graves-Morris PR, Roberts DE, Salam A. The epsilon-algorithm and related topics. Journal of Computationaland Applied Mathematics 2000; 122:51–80.

24. Sidi A. Extension and completion of Wynn's theory on convergence of columns of the epsilon table. Journal of Approximation Theory 1996; 86:21–40

25. Brezinski C. Convergence acceleration of some sequence by epsilon-algorithm. Numerische Mathematik 1978; 29:173–177.

BIOGRAPHICAL INFORMATION

The author pursued his Bachelor of Engineering degree in Mechanical Engineering from India in August 2007. Then he started his studies for Masters of Science in August 2008. His areas of interest are Structural Design, Finite Element Methods and Optimization.