STATISTICAL MODELING APPROACH TO AIRLINE REVENUE MANAGEMENT WITH OVERBOOKING

by

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Presented to the Faculty of the Graduate School of
The University of Texas at Arlington in Partial Fulfillment
of the Requirements for the Degree of

DOCTOR OF PHILOSOPHY

THE UNIVERSITY OF TEXAS AT ARLINGTON

August 2006

ACKNOWLEDGEMENTS

I would like to thank my supervising professors Dr. Victoria Chen and Dr. Jay Rosenberger for their invaluable advice during the course of my doctoral studies. They were always available to help me with my questions. They were very supportive, understanding, and patient. There need not be any more motivation, than to know that we are doing our research under their guidance. Their classes were also very impressive and research oriented, which always kept me motivated. I wish to thank my committee members Dr. Bill Corley, Dr. Jamie Rogers, and Dr. D. L. Hawkins, for their interest in my research and for taking time to serve in my dissertation committee. If not for Dr. Corley, I would not have even thought of joining the doctoral program. It was he who influenced me so much during my MS program that I decided to do a PhD. His classes were very creative and gave room for the students to think. I would also like to thank Dr. Rogers and Dr. Chen, for the knowledge and experience I gained working as a graduate teaching associate. It was amazing to find how well organized and planned Dr. Rogers was in her work. I also started implementing it in my day to day work and it helped me a lot. Also, many thanks to Dr. Hawkins. The class I took under him was excellent, and I still relish those moments. It no doubt kept me very busy and occupied all the time, but the knowledge and confidence I gained in that subject is just inestimable. Thanks to Dr. Don Liles and Dr. Chen for the financial support I received during my doctoral program as a teaching associate and research assistant. My thanks to Dr. Dirk Günther for helping me with my questions, data and code.

I would like to thank Dr. Ventaka Pilla whose invaluable help, concern and suggestions towards my research and personal life made a great difference in my stay in the

United States. If not for he him, I would not have had a healthy stay. It was really great discussing my work with him. Also, thanks to all my friends in the COSMOS lab, Duraikannan Sundaramoorthi, Aihong Wen, Prattana Punnakitikashem (Sandy), Prashant Kumar Tarun, Heesu Hwang (Peter), Tai-kuan Sung, Dachuan Shih (Thomas), Siriwat Visoldilokpun (Pop), Panita Suebvisai, Jawahar Veera, and Huiyuan Fan for their help in making the COSMOS lab a wonderful place for research. It was their presence in the lab that made it so lively. Thanks to my friends who joined me in playing badminton. I really enjoyed playing with them, which made my day awesome.

Finally, I would like to express my deep gratitude to my parents Mr. Siddappa Khandappa and Mrs. Bhageerathy Bheemappa for giving me an opportunity to pursue my doctoral program and for their invaluable support all through my masters and doctoral programs. They supported me in all respects at all times. There were times when I called them late at night and in early morning expressing the amount of frustration I had and how difficult it was living with my roommates. It was their immense moral support, advise and motivation that kept me strong during those difficult times. I can never forget to mention the sacrifice my lovely brother Mr. Raghavendra made for me during my stay in the United States. It is impossible to imagine my stay in the United States far from my family without his support. He was always present and able to sense the need for me to talk to my parents. He made sure I was able to chat with my parents and kept me happy by sending me emails, the family pictures, birthday party videos, etc. My day always started with reading his email. No matter how much I express my thanks to him, it will still not be enough. I bow down to my parents and brother for their immense patience and the support they gave me to do my master's and doctoral programs. Also, many many thanks to my sister Mrs. Mala and my brother-in-law Dr. Bhaskar Sawanth for being very helpful. Last but not the least thanks to my little niece, Neha Sawanth who brought more life into our family and keeps every one on their toes.

ABSTRACT

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Publication No. ______

SHEELA SIDDAPPA, Ph.D.

The University of Texas at Arlington, 2006

Supervising Professor: Victoria C. P. Chen, Jay M. Rosenberger

Revenue Management (RM) in the airline industry plays a very important role in maximizing revenue under various uncertainty issues, like customer demand, the number of seats to be maintained in inventory, the number of seats to be overbooked, etc. In this dissertation, a Markov decision process (MDP) based approach using statistical modeling is presented. Prior versions of this statistical modeling approach have employed remaining seat capacity ranges from zero to the capacity of the aircraft. In reality, actual remaining capacities are near capacity when the booking process begins and near zero when the flights depart. Thus, our modified version uses realistic ranges to enable a more accurate statistical model, leading to a better RM policy. We also consider overbooking, no-shows and cancellations and estimate the optimal number of seats to be overbooked using a hybrid approach that combines Newton's and steepest ascent method. The extended statistical modeling approach in this dissertation consists of three modules: (1) the revised statistical modeling module, (2) the overbooking module, and (3) the availability processor module. The first two modules are conducted off-line to

identify optimal overbooking pads and derive a policy for accepting/rejecting customer booking requests. The last module occurs on-line to conduct the actual decisions as the booking requests arrive. To enable a computationally-tractable solution method, the revised statistical modeling module, under an assumed maximum overbooking pad of 20%, consists of three components: (1) simulation of the deterministic bid price approach to identify the realistic ranges of remaining seat capacity at different points in time; (2) solutions to deterministic and stochastic linear programming problems that provide upper and lower bounds, respectively, on the MDP value function; and (3) estimation of the upper and lower bound value functions using statistical modeling. Next, the overbooking module identifies the optimal number of seats to be overbooked. Finally, the value function approximations are used with the optimal overbooking pads to determine the RM accept/reject policy.

TABLE OF CONTENTS

ACKNOWLEDGEMENTS ii
ABSTRACT iv
LIST OF FIGURES viii
LIST OF TABLES ix
Chapter
1. INTRODUCTION 1
1.1 RM Background and Issues 1
1.2 Modeling the Booking Process in Airline Industry 2
1.3 Summary of some of the problems faced by RM 3
1.4 Problem Definition 3
1.5 Seat inventory control 4
1.6 Contribution 6
2. RM METHODOLOGIES 7
2.1 RM Literature and Background 7
2.1.1 Dynamic programming 8
2.1.2 Bid pricing 9
2.1.3 Discrete Choice Models 13
2.1.4 Hybrid Approaches to solve the RM problem 14
2.1.5 Statistical Modeling Approach to RM 15
2.2 Background on the Statistical Modeling Approach to RM 15
2.2.1 Markov Decision Problem Formulation 15
2.2.2 Statistical Modeling Module 16

2.2.3 Availability Processor Module 17
2.3 Statistical Background 18
2.3.1 Orthogonal Array (OA) 19
2.3.2 Multivariate Adaptive Regression Splines (MARS). 21
2.3.3 Derivative of MARS Function 23
2.4 Overbooking 25
3. EXTENDED STATISTICAL MODELING APPROACH TO RM 28
3.1 Revised Statistical Modeling Module 29
3.1.1 Generation of Realistic State Space 29
3.1.2 Estimation of Number of Scenarios 31
3.1.3 Approximation of Value Functions 31
3.1.4 Solving the Stochastic Network Optimization Model 32
3.2 Overbooking Module 35
3.2.1 Newton's Method 36
3.2.2 Steepest Ascent 36
3.2.3 Hybrid Approach to Estimate the Optimal Overbooking Pad . . . 39
3.2.4 Derivation of Cost Function 40
3.3 Availability Processor Module 44
4. COMPUTATIONAL RESULTS FOR THE EXAMPLE PROBLEM 45
5. FUTURE WORK 51
REFERENCES 53
BIOGRAPHICAL STATEMENT 58

LIST OF FIGURES

Figure		Page
2.1	Flow Chart representing general bid price approach	12
2.2	Flow chart representing statistical modeling method	18
2.3	Flow chart representing availability processor	19
2.4	Continuous derivative MARS function	22
2.5	Flow chart representing the possible results of overbooking	26
3.1	Flow chart representing the generation of the reduced/realistic state space	30
3.2	Plot illustrating the minimum value in any given region	33
3.3	Profit curve	36
3.4	Flow chart representing the off-line module for estimating the optimal overbooking pad	37
3.5	Iterative descent for minimizing a function Z, where c is the cost reduced at every iteration	38
3.6	Failure of the successive stepsize reduction rule for a one-dimensional function	40
3.7	Line search using the Armijo Rule	41

LIST OF TABLES

Table Page

2.1 Example: Threshold Bid Price values set by the airlines. 10

2.2 Example of a MARS approximation. 25

4.1 Average revenues from 2000 simulations of the 31-leg hub using three methods: DET, DET STAT, and DET REV STAT 46

4.2 Average revenues from 2000 simulations of the 31-leg hub using four methods: DET, STOCH, STAT, and REV STAT. 48

4.3 Average revenue from 2000 simulations of the 31-leg hub using two methods considering overbooking: DET and REV STAT. 49

4.4 Optimal overbooking pad (OBP) for different load factors. 50

CHAPTER 1

INTRODUCTION

1.1 RM Background and Issues

Before deregulation in 1979, airlines had been regulated by the CAB (Civil Aeronautics Board) since 1938 [1]. The CAB decided on the routes to be taken and the fares to be charged to the customers. The costs were passed on to the passengers with guaranteed profit levels. Carriers simply accepted the passengers on a first come, first serve basis. There were limited booking classes, and there was little control over revenues, other than to sell more seats. After deregulation in 1979, there was tremendous growth in the number of certified airlines, and high pressure on pricing. Airline industries began to explore ways to compete effectively, and different approaches in revenue management (RM) evolved. Since then airlines have expanded their efforts in RM to increase their revenue. Revenue Management, also known as Yield Management, is defined as, "Selling the right seat at the right time to the right passenger for the right price" [2]. RM is being applied in various transportation sectors such as auto rental, ferries, rail, tour operators, cargo, and cruises. Other areas, like hotels/resorts, extended stay hotels, healthcare, manufacturing apparel, and companies that produce perishable goods etc., also use RM [3].

Since deregulation airlines have developed a complex and diverse fare structure. They offer variety of fares to meet different classes of customers. Airlines use restrictions, such as the purchase tickets 21 days in advance, to establish different class of services. Seats in the same fare class are sold at different fares to different customers over time. There is competition among the airlines to expand and explore RM faster and better than the others, so as to improve their revenue. American Airlines, for example, reported an increase in revenue of 5% due to improved RM methods in 1992, which translated to $1.4 billion over a 3-year period [4].

1.2 Modeling the Booking Process in Airline Industry

In an airline reservation system, customers request a particular *itinerary.* If a flight travels directly from the origin to the destination without stopping, it is called a *leg.* An itinerary consists of one or more legs. Information on all available itineraries will be provided to the customer. The booking process starts three months prior to the date of departure. In RM models, the customer makes a *booking request* by bidding a price for the itinerary/leg desired, and once the request is placed, an airline representative uses a computer reservation system (CRS) to decide if the request is to be accepted or rejected. In an airline's *RM policy*, the customer's price is compared with the threshold or bid price of the airline, and if the threshold value is less than the customer's bid, then the request is accepted; otherwise the request is rejected. Demand is observed to increase gradually then rapidly and finally decrease as it gets closer to the date of departure.

It is very difficult to match supply and demand in any transportation industry. The critical decision is the number of seats to be reserved for each fare class. If too many low fare seats are reserved, then airlines might lose the potential high fare passengers. If too few low fare seats are reserved, then airlines will loose the large low fare demand. Booking requests rejected for any fare class due to non-availability/filled seats is called *spill* of demand.

Some passengers would like to cancel their ticket before the day of departure or at the time of departure; this is called a *cancellation.* Certain passengers on the other hand, buy the ticket, but do not show up on the day of departure; these are called *no-shows.* Thus, due to cancellations and no-shows some seats fly empty on the day of departure, even if the number of seats sold equals the flight capacity. Airlines willing to utilize their resources efficiently, sell more seats than the capacity of the flight. This excess booking, above the capacity of the flight is called *overbooking.* Despite overbooking, if seats fly empty, it is called *spoilage.*

1.3 Summary of some of the problems faced by RM

Demand forecasting is one of the major problems faced by RM. It is practically impossible to predict exactly the demand for any given flight. Demand is very erratic. Hence, it is very difficult to match supply and demand. Airlines must distinguish between time-sensitive and price-sensitive types of customers, know their requirements, and develop different marketing strategies to accommodate all types of customers. Airlines try to hold some inventory in order to capture high fare passengers at the last minute. Unfortunately, if the demand for high fare passengers becomes less, some of the seats fly empty, which is revenue lost to the company. It is less costly to have a discounted passenger than to fly the seat empty. Thus, minimizing inventory spoilage is a major concern of the airline industry.

Ticket cancellations and/or customer no-shows cause some of the seats to fly empty. In order to efficiently utilize the capacity and increase revenue, airlines sell more seats than the capacity of the flight. Ideally, the number of seats to be overbooked should be such that, at the time of departure there are exactly as many passengers as the capacity of the flight. A mismatch in the number would result in a loss of revenue.

1.4 Problem Definition

Revenue management in the airline industry deals with managing inventory for each of the itinerary fare classes offered, so as to maximize revenue. Revenue management focuses more towards the inventory rather than the fare structure. Hence, we will concentrate on the seat inventory control problem, also called the yield management problem. Given the flight capacity and schedule, can we accept the request placed by the customer at any time τ? This question is answered by people in different ways using different approaches. A literature review in Chapter 2 gives a brief explanation of some approaches. The approach in this dissertation seeks to achieve a better RM policy by using statistical methods to approximate the value functions of a Markov decision process (MDP) and optimize overbooking.

1.5 Seat inventory control

Seat inventory control is the process of determining the right mix of seats to make available at different fares on a flight leg in order to maximize revenue [5]. By offering too many discounted fare seats, airlines can attract and capture most of the demand, and increase revenue. This might not be the maximum revenue, because offering too many low fare seats will displace most of the high fare passengers. On the other hand, offering fewer low fare seats misplaces most of the demand. This causes some of the seats to fly empty, which is revenue lost forever. Hence, the number of seats reserved for each of the fare classes makes a large difference in the revenue the airlines can generate. The challenge behind seat inventory control is how and when to make a trade off between cost of an empty seat, loss of the discount fare, and cost of turning away the full fare passenger [5].

The concept of *differential pricing*, charging different customers different prices for the same seat/product, helps control the seat inventory problem considerably. Customers are discriminated based on the fares they are willing to pay. Passengers willing to pay the lowest fare must be able to plan well ahead of time (e.g., 21 days prior to departure), should be ready to travel any day of the week and any time of the day. No refund will be given on the cancellation of the ticket. Full fare passengers, on the other hand can make last minute travel decisions. They are promised a refund on cancellation, etc. Thus, the quality of service changes with the amount the passenger is willing to pay. Example: Given an assumed demand, if the revenue generated for a single fare class and multiple fare class are \$ x and \$ y, respectively, then it is shown that y is always greater than x. Williamson [5] gives a detailed explanation of this concept.

In air transportation, it is very difficult to match supply and demand, which is the root cause for the seat inventory control problem. Due to various constraints, such as limited flights with fixed flight capacity, fixed routes, weather conditions, etc., it is difficult to schedule the right flight for every departure. Demand fluctuations can be handled by offering discounted fares when demand is lower, and higher fares when demand is higher.

The seat inventory control problem can be approached using (1) an individual flight leg, (2) an entire network of a carrier, or (3) separate subsets of the network. The flight leg approach is simplest to implement. Example: Consider a two-leg flight, Dallas-Houston-Galveston. If demand for the low fare class of Dallas-Houston is greater than the high/low fare class of the Dallas-Houston-Galveston itinerary, then there should be a trade off between the high/low fare passengers of the itinerary and the low fare high demand local (Dallas-Houston) passengers. A carrier can operate more than 1000 flights per day; thus, the flight leg approach becomes more complex to handle.

Belobaba [6] developed the Expected Marginal Seat Revenue (EMSR) model as a decision framework for maximizing flight leg revenue that can be applied to multiple fare class inventories. This method also provides an overview of the mathematical concepts, models, and solution methods for the seat inventory control problem. Williamson [5] addresses the seat inventory control problem using the network approach, taking into account the interaction of flight legs and the flow of traffic across the network. Wollmer

[7] developed an analytical approach to solve the multifare, single-leg nested case when lower-fare passengers book first. The booking request for a particular fare level is accepted if the number of empty seats is greater than their critical value (a decreasing function of the fare price), and rejected otherwise. De Boer [8] proposed a derivative of the EMSR booking limits calculation method that takes into account the effect of future capacity changes, which can lead to significant revenue gains.

1.6 Contribution

In this dissertation the RM problem is solved using a statistical modeling approach. The optimization procedures applied are similar to that of Günther [9]. This dissertation extends Günther's approach to identify a more realistic state space for the experimental design, take into account the overbooking concept. Some reasonable assumptions are made to estimate the number of seats oversold (identified only after the departure of the flight) and the associated cost. This somewhat simplifies the complex task of estimating the optimal number of seats to be booked in order to generate the maximum revenue.

A literature review and background on the different approaches employed to solve the RM problem is discussed in Chapter 2. Chapter 3 gives a detailed description of the statistical modeling approach used in this dissertation, followed by computational results in Chapter 4 and future work in Chapter 5.

CHAPTER 2

RM METHODOLOGIES

2.1 RM Literature and Background

Littlewood [10] was the first to address the RM problem of computing booking limits for a single leg with two fare classes. His rule: “Sell the discount seats as long as the revenue from the low fare passengers is greater than or equal to the product of marginal revenue from full fare and probability that full fare demand does not exceed the remaining capacity.” Belobaba [6] extended this rule to multiple fare classes. He introduced the term EMSR (Expected Marginal Seat Revenue). The EMSR method produces optimal booking limits only for the two-fare class problem, and is easy to implement.

Glover et al. [11] formulated the network RM problem. Two sets of arcs (forward arcs to model seat capacity and backward arcs to model deterministic passenger demand) were used in the network. Wollmer [7] came up with a binary decision problem with network structure to model uncertain demand. Belobaba [6] in his survey paper stated,“The practices indicate that seat inventory control is dependent on human judgment rather than systematic analysis.” Dror and Ladany [12] presented a network that accounted for cancellations and no-shows.

Belobaba [13] describes the implementation of a computerized system for making the tradeoff between booking requests and setting booking limits at Western Airlines. The EMSR model he developed for this application takes into account the uncertainty associated with the estimates of future demand and the nested structure of booking limits in the airline reservation system. Curry [14] developed a method to determine optimal booking limits for nested controls using fare nets. Wollmer [7] presented a model that addresses when to reject a booking request, so as to save the seat for a potential request at higher fare level. He developed an algorithm that maximizes mean revenue by establishing a critical value for each of the fare classes. Brumelle and McGill [15] showed that a fixed booking policy that maximizes revenue can be characterized by a simple set of conditions appropriate for either discrete or continuous demand distributions.

Robinson [16] studied when to reject the low fare passenger by relating the probability of filling the plane under the optimal policy to the ratio of the current to highest remaining fare classes. He demonstrated that Monte Carlo integration is easy to apply for this and can get approximately close to the optimal policy. Chatwin [17] determined that airlines should accept a reservation request if the current number of reservations is less than the booking limit, and decline the request otherwise. When the fare is constant over time or decreases towards flight-time, the optimal booking limit decreases towards flight-time. Olinick and Rosenberger [18] presented an RM model to optimize revenue under uncertainty using a super-gradient algorithm.

2.1.1 Dynamic programming

Hersh and Ladany [19], and Ladany and Bedi [20] developed dynamic programming formulations to allocate seats for a two-segment flight. Probabilities of booking requests were based on historical booking data and were assumed constant. They discuss overbooking and cancellations for flights with one intermediate stop. Ladany and Bedi [20] simplified the approach by removing all conditioning on current bookings. Rothstein [21] formulated the RM problem as a non-homogeneous Markovian sequential decision process considering overbooking. Bertsimas and de Boer [22] calculated booking limits that take into account the stochastic and dynamic nature of the demand and nested character of booking-limit control in a network. Their methodology combines a stochastic gradient algorithm and approximates dynamic programming ideas to improve the initial booking limits.

Lee and Hersh [23] developed a discrete-time dynamic model to find an optimal booking policy. They did not make any assumptions about the arrival pattern of the various booking classes. Their analysis showed that for problems with more than two booking classes and no multiple seat booking, the optimal booking policy can be reduced to two sets of critical values: (1) booking capacity and (2) decision periods. Lautenbacher and Stidham [24] solved the single-leg problem without overbooking using a discrete-time Markov decision process. Subramanian et al. [25] have taken overbooking, cancellations and no-shows into consideration while solving for the seat allocation problem using the Markov decision process for a single flight leg with multiple fare classes. They showed that: (1) booking limits need not be monotonic in the time remaining until departure, (2) it would be optimal to accept a low-fare class and reject a high = fare class because of differing cancellation refunds, and (3) the optimal policy depends both on the total capacity and remaining capacity of the flight. Zhang and Cooper [26] formulated a simultaneous seat-inventory control problem of a set of parallel flights between a common origin and destination with dynamic customer choice among the flights as an extension of the classic multiperiod, single-flight "block demand" revenue management model. They proposed a simulation-based technique for solving the stochastic optimization problem.

2.1.2 Bid pricing

Bid pricing is being practiced by most of the airlines. In the bid price approach a threshold or bid price is assigned to each flight leg and if a customer's booking request is greater than the sum of the bid prices along the desired itinerary, the request is accepted; otherwise, it is rejected. See Figure 2.1. Consider the following two examples. Example 1: For demand class f and leg i, suppose the customer bids \$200 while the threshold value is \$175. Since \$200 is greater than \$175, the request is accepted. Example 2: Suppose a customer bids \$1220 for the itinerary he wishes to travel. Table 2.1 gives the threshold values set by the airlines for each of the legs on the itinerary. Since the total price bid by the customer (\$1220) is greater than the sum of the bid prices for the legs \$1109, the request is accepted.

Table 2.1. Example: Threshold Bid Price values set by the airlines.

Flight Leg	Airline Bid Price (\$)
2	125
4	300
23	124
45	560
Total	1109

Bid pricing is easy to implement and requires storage of only a single bid price for each flight leg. It gives a nested itinerary and fare class specific control policy. It is easy to manage the inventory. Despite all these advantages it has the disadvantage that it is difficult to estimate/determine good bid prices. Frequent revision is required with re-optimization and re-forecasting. Some of the methods used to estimate bid prices are discussed below.

2.1.2.1 Deterministic Bid Price Approach

Define the following notation:

- T = the total number of reading dates, indexed by t, where $t = T$ represents the first reading date.
- $\boldsymbol{r}$ = the vector of fares associated with each demand class.
- $\boldsymbol{u}$ = a seat allocation decision vector for all demand classes.

- $\boldsymbol{x}_t$ = a vector of remaining seat capacities at reading date t.
- $\boldsymbol{d}_t$ = a random vector of remaining demand at reading date t.
- A = a 0-1 itinerary-leg matrix, with one if that itinerary includes the leg.

Given the number and position of reading dates, the flight schedule and capacities, a deterministic linear programming problem (DET) is solved at the reading date t. Gallego and van Ryzin [27] used a network model to compute bid prices. It is modeled as below.

$$\text{(DET)}\max \quad \boldsymbol{r}\boldsymbol{u} \tag{2.1}$$

$$\text{s.t. } A\boldsymbol{u} \ \leq \boldsymbol{x}_t \tag{2.2}$$

$$0 \leq \boldsymbol{u} \leq E[\boldsymbol{d}_t]. \tag{2.3}$$

The dual solution of (DET) will provide bid prices for each flight leg. Every time a request is accepted, remaining seat capacity is updated, and at reading dates $t < T$ updated remaining capacity and expected demand values are used to solve (DET) to generate new bid prices. Results show that revenue increases by increasing the number of the reading dates. The higher the number of reading dates, the higher the accuracy of the bid prices and, hence, the more the revenue.

2.1.2.2 Stochastic Bid Price Approach

In addition to the notation above, define the following:

- u_{ft} = a seat allocation decision vector for demand class f at reading date t, where $\boldsymbol{u}_t$ is the corresponding vector for all demand classes.
- d_{ft} = a random variable of the demand for demand class f on reading date t.

A stochastic model, also called the probabilistic nonlinear programming model (PNLP) is intended to provide a better representation of the random variable for demand. This

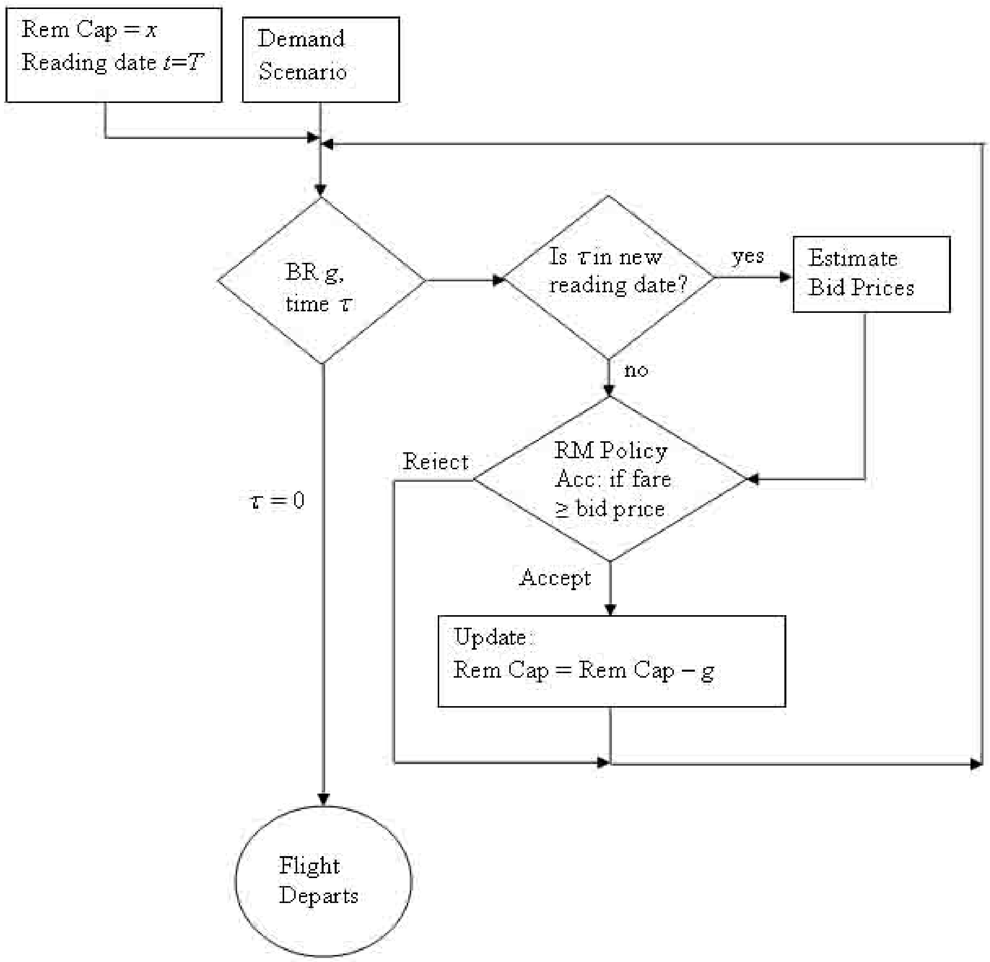

Figure 2.1. Flow Chart representing general bid price approach.

model is also referred to as the *stochastic network model* (STOCH) and provides lower bound on revenue [9]. The model is as given below.

$$\text{(STOCH)}\ \max \sum_{\tau=1}^{t} \sum_{f=1}^{m} r_f E[\min(d_{f\tau}, u_{f\tau})] \tag{2.4}$$

$$\text{s.t. } A\left(\sum_{\tau=1}^{t} \boldsymbol{u}_\tau\right) \leq \boldsymbol{x}_t \tag{2.5}$$

$$u_{ft} \geq 0. \tag{2.6}$$

Again, the dual will provide bid prices for each flight leg. Van Ryzin et al. [28] demonstrate that the dual solution of the (DET) model gives better bid prices than the dual solution of the (STOCH) model. Talluri and van Ryzin [29] have analyzed the randomized version of deterministic linear programming to compute network bid prices. Their method is more difficult to implement than the (DET) method. It consists of simulating the itinerary demand with a sequence of realizations, and solving (DET) to allocate capacities to itineraries for each realization. The dual prices from the sequence are averaged to form a bid price approximation.

Higle and Sen [19] presented a two-stage stochastic programming model to overcome the shortcomings of the (DET) and (STOCH) models. The first stage allocates capacity to all the fare classes, and the second stage models capacity utilization. Their simulation results show that this provides better revenue improvements than a linear programming approach. They also prove that their approach is prone to less error than those resulting from the linear programming method.

2.1.3 Discrete Choice Models

Research work in the field of discrete choice models for RM is of considerable interest these days. The network capacity control problem is treated as a setting where customers choose from the different alternatives (very frequent flights with the same origin and destination; flights to the destination can be direct, or have one or two stops; different fare classes with different fares, etc.) provided by the airlines. Customers make their choices among the various alternatives provided and rank them in their order of importance. Example: Order list for customer type 1: low price, time of departure, and destination airport. Order list for customer type 2: time of departure, and itinerary. Van Ryzin and Vulcano [30] formulated a continuous demand and capacity approximation which allows for the partial acceptance of requests. The model efficiently calculates the sample path gradient of the network revenue function. The gradient is then used to construct a stochastic steepest ascent algorithm. They showed that the algorithm converges to a stationary point of the expected revenue function under mild conditions. Van Ryzin and Vulcano [31] analyzed a continuous model of the discrete choice problem that retains most of the desirable features of the Bertsimas and de Boer [22] method but avoids many of its pitfalls. Because their model is continuous, they are able to compute gradients exactly using a simple and efficient recursion. Their gradient estimates are often an order of magnitude faster to compute than first-difference estimates, which is an important practical feature given that simulation-based optimization is computationally intensive. Talluri and van Ryzin [32] analyzed the revenue management problem on a single flight leg with buyer's choice of fare classes modeled explicitly. They modeled the "buy up" (buying a higher fare when lower fares are closed) and "buy down" (substituting a lower fare for a higher fare when discounts are open) behavior of the customers, which specifies the probability of purchasing each fare product as a function of the set of available fare products. The model includes nearly every choice model of practical interest.

2.1.4 Hybrid Approaches to solve the RM problem

Curry [14] combined both the EMSR and mathematical programming approach. The EMSR approach accounts for CRS nesting, but only controls seat inventory, by controlling leg bookings. Mathematical programming handles realistically large problems and accounts for multiple origin-destination (OD) itineraries and side constraints. Curry developed equations to solve the RM problem, when fare classes are nested on an OD itinerary, and inventory is not shared among the ODs.

Cooper and Melo [33] worked on policies that combine both mathematical programming and MDP methods. Their idea was to employ a simple allocation policy when far from time of departure and develop a detailed decision rule close to departure. They used sampling-based stochastic optimization methods to solve the formulation. The solution was capable of using deterministic optimization techniques. They employed an MDP solution for a portion of the booking process rather than approximations of MDP value functions. Their results showed that the hybrid policies perform well for two-leg problems, but their approach cannot be used for larger networks.

2.1.5 Statistical Modeling Approach to RM

The research in this dissertation is based on the statistical modeling approach of Chen et al. [34]. They formulated the RM model as an MDP, similar to that of Lautenbacher [35]. Traditionally MDP was solved using SDP and can provide a superior RM policy, but SDP is computationally intensive. Hence, Chen et al. [34] developed a new Statistical Modeling approach motivated by the OA (Orthogonal Array) and MARS (Multivariate Adaptive Regression Splines) SDP method of Chen et al. [36], to estimate upper and lower bounds of the MDP value functions.

2.2 Background on the Statistical Modeling Approach to RM

The following subsections give a detailed description of the statistical modeling approach adapted by Günther [9]. An improvement in the statistical modeling approach is considered in this dissertation.

2.2.1 Markov Decision Problem Formulation

The MDP formulation for the RM problem divides the booking period into t_{MDP} time intervals, with at most one booking request per interval. These intervals are indexed in decreasing order, $i = t_{\mathrm{MDP}}, \ldots, 1, 0$, where $i = 1$ denotes the first interval immediately preceding departure, and $i = 0$ is at departure. The reading periods can have multiple booking requests while MDP intervals can have only one booking request. The state vector x_i contains the remaining leg capacities at the beginning of time interval i. Let $p_i^f(g)$ denote the probability that a request for g seats, for itinerary-fare class f occurs in time interval i; $p_i(0)$ denotes the probability of no booking requests in time interval i.

Consider state x at the beginning of time interval i. If a booking request for g seats that arrives during time interval i is accepted, then a new state x' is reached at the beginning of time interval $i-1$, where x' subtracts g seats from the legs involved in the requested itinerary. $F_i(x)$ denotes the optimal value function, the maximum expected revenue collected over time intervals i through departure when the system is at state x at the beginning of time interval i. $F_0(x) = 0$ for all x. Thus, the MDP for RM can be written as:

$$F_i(x) = \sum_{f=1}^{m} \sum_{g=1}^{G_f} p_i^f(g) \begin{cases} \max\{gr_f + F_{i-1}(x'), F_{i-1}(x)\}, & \text{if } (x' \geq 0) \\ F_{i-1}(x), & \text{otherwise.} \end{cases}$$

Günther [9] developed an MDP based OA/MARS approach to RM. In this approach, the RM problem is solved in two parts, off-line and on-line. The off-line part or the *statistical modeling module*, derives the accept/reject policy while the on-line part or *availability processor*, conducts the actual decisions. His model assumes:

1. The booking process starts ninety days before the day of departure.
2. Flight capacities and schedule are known.
3. There is no overbooking or cancellation.

2.2.2 Statistical Modeling Module

The steps involved in this module are given below.

1. The reading dates are chosen and remaining seat capacity is set equal to the flight capacity for all the flight legs.

2. A design of experiments (DoE) method, specifically an OA design, is constructed to provide discretized coverage of the state space. The state space ranges from zero to the plane capacities of the flight in the network.
3. For each of the discretization points, the (DET) model (refer equation 2.1 - 2.2) and the (STOCH) model (refer equation 2.4 - 2.6) are solved. The bound obtained by solving the (DET) model is proven to provide an upper bound, denoted by $F_t^U(x)$ (see van Ryzin and Talluri [28]), while the bound obtained by solving the (STOCH) model is proven to provide a lower bound, denoted by $F_t^L(x)$ (see Günther [9]). This loop is repeated at all reading dates.
4. For each reading date, a MARS approximation is fit separately to estimate the (DET) and (STOCH) revenues over the entire state space. Thus a total of $2T$ different MARS approximations are generated.

Figure: 2.2 describes the procedure followed in the statistical modeling module using a flow chart. The essential statistical models $\hat{F}_t^U$ and $\hat{F}_t^L$ are made available for the on-line module.

2.2.3 Availability Processor Module

Let $g_{f\tau}$ be a booking request of group size g for fare class f at time τ. Its fair market value (FMV) is estimated using

$$\text{Pessimistic} = \hat{F}_\tau^L(\boldsymbol{x}) - \hat{F}_\tau^U(\boldsymbol{x}'), \tag{2.7}$$

$$\text{Optimistic} = \hat{F}_\tau^U(\boldsymbol{x}) - \hat{F}_\tau^L(\boldsymbol{x}'), \tag{2.8}$$

$$\text{FMV} = \frac{\text{Pessimistic} + \text{Optimistic}}{2}. \tag{2.9}$$

A flow chart representation of the availability processor moduleis shown in Figure: 2.3. The RM policy is defined as, "accept the booking request only if the requested fare is greater than the FMV."

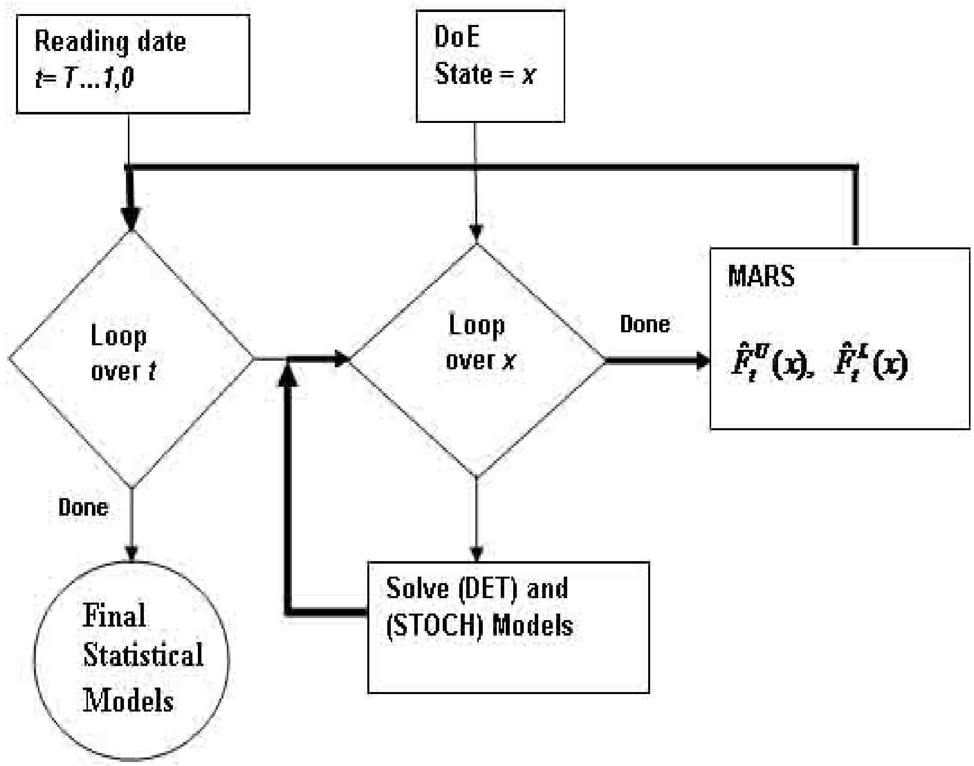

Figure 2.2. Flow chart representing statistical modeling method.

2.3 Statistical Background

In the statistical modeling approach, methods from design and analysis of computer experiments (DACE) are used to approximate the MDP revenue value function. In general, DACE is used to study an unknown function $f(\cdot)$, defined by a computer experiment for a complex system (see Chen et al. [37], [38]). Specifically,

1. A DoE is used to select points within the input space of the computer experiment.
2. The computer experiment is executed at the design points and corresponding responses for $f(\cdot)$ are output.
3. A statistical model is fit over these data to obtain an estimate $\hat{f}(\cdot)$

The most basic DoE is a full factorial design, and the most basic statistical model is a linear regression model. The statistical modeling approach of Günther [9] and Chen et

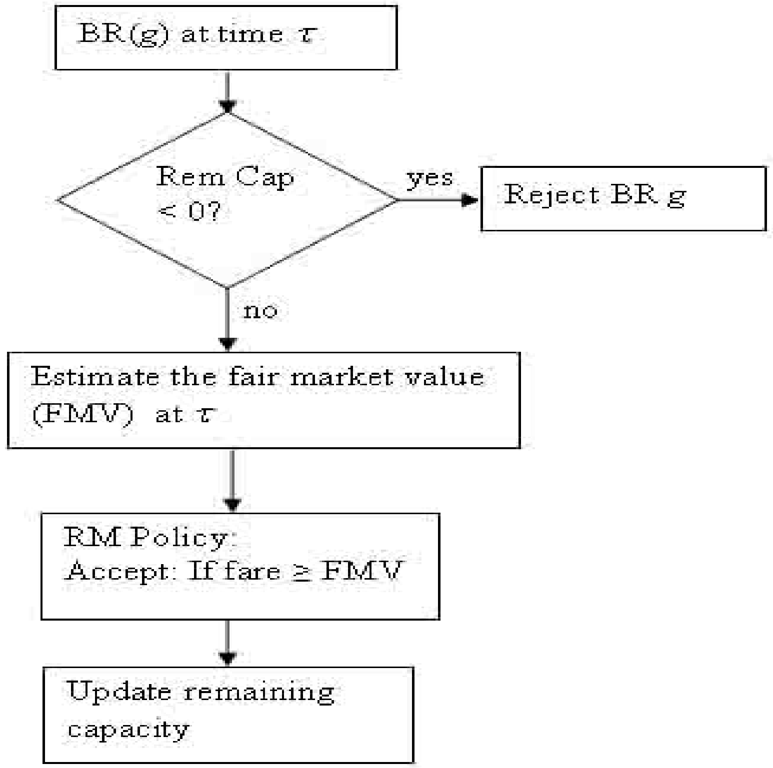

Figure 2.3. Flow chart representing availability processor.

al. [34] uses an OA/ MARS method. In a *full factorial design*, every level of every factor appears with every level of every other factor. Thus, it consists of the full grid.

In the following subsections, experimental designs and multivariate adaptive regression splines are described. These two specific approaches were employed by Günther [9] and are utilized in this dissertation. They are described below.

2.3.1 Orthogonal Array (OA)

A *fractional factorial design* is a carefully chosen fraction of the full factorial design. An orthogonal array is a special form of fractional factorial design aimed at saving time and money required by the experiment. An OA is represented by OA(N, n, q, d), where

$N = \lambda q^d$ is the number of runs, n is the number of predictor varaibles, q is the number of levels (prime or a power of a prime), and d is the strength of the design (in practice $d \leq 3$ is sufficient). Thus, only a subset $N{=}\lambda q^d$ of the complete grid of points q^n over the entire state space is chosen. An OA of strength d for n variables ($d < n$), each at q levels, contains all possible factor level combinations in any subset of d factors, with the same frequency λ. Therefore, when projected down onto any d dimensions, a complete factorial grid of q^d points replicated λ times is represented (see Chen [39]). Bose and Bush [40] showed that for an OA(q^d, n, q, d) of index unity, the number of constraints n satisfies the inequality

$$n \leq q + d - 1 \text{ , when } q \text{ is even} \tag{2.10}$$

$$n \leq q + d - 2 \text{ , when } q \text{ is odd.} \tag{2.11}$$

As an example consider an OA with four runs, three predictor variables with two levels each, and of strength two. It can be represented by OA(4,3,2,2). In the table below, rows represent the runs, columns represent the variables, and the numbers 0 and 1 represent the two factor levels. It is said to be of strength two because, if we consider any two columns, all possible factor level combinations can be observed, in the same or different order. It can be seen that, the greater the desired strength, the harder it is to construct the array; hence, usually strength two or three is used.

0	0	0
0	1	1
1	0	1
1	1	0

Some of the properties of OAs are:

1. Two columns of an array are orthogonal if all possible level combinations of the two columns appear equally often in an array.
2. In the context of experimental designs, the columns of an OA correspond to different variables whose effects are being analyzed. The entries in an array specify the levels at which variables are to be applied.
3. Any OA of strength d is also an OA of strength $d^{'}, 0 \leq d^{'} < d$. The index of the array when considered as an array of strength $d^{'}$ is $\lambda q^{d-d^{'}}$, where λ denotes the index of the array when considered to have strength d.
4. Two OAs are said to be isomorphic if one can be obtained from the other by a sequence of permutations of the columns, the rows, and the levels of each factor.

2.3.2 Multivariate Adaptive Regression Splines (MARS).

MARS was introduced by Friedman [41] as a statistical modeling method for estimating a completely unknown relationship between a single response and several input variables. MARS is essentially a linear combination of simple basis functions with a forward stepwise algorithm to select basis function terms followed by a backward procedure to prune the model. MARS is flexible and easy to implement, and the computational effort depends on the number of basis functions added to the model. The strategy adopted, is to deliberately overfit the data with an excessively large model and then trim it back with a backward stepwise strategy. This procedure is explained below.

The jth MARS basis function added to the model is a product of L_j truncated linear functions:

$$B_j(x) = \prod_{l=1}^{L_j} [S_{l,j}.(x_{v(l,j)} - K_{l,j})]_+ \tag{2.12}$$

where $x_{v(l,j)}$ is the predictor variable corresponding to the lth truncated linear function in the jth basis function, $K_{l,j}$ are knot locations at which the basis function bends, and $S_{l,j}$ is $+1$ or -1. The MARS model is of the form

$$\hat{g_M}(x) = a_0 + \sum_{j=1}^{M} a_j \prod_{l=1}^{L_j} [S_{l,j}.(x_{v(l,j)} - K_{l,j})]_+ \tag{2.13}$$

where a_0 is the coefficient for the constant basis function, M is the number of linearly independent basis functions, a_j is the coefficient of j^{th} basis function $B_j(x)$, and $x_{v(l,j)}$ is the predictor variable. To enable a continuous first and second derivative, Chen et al.

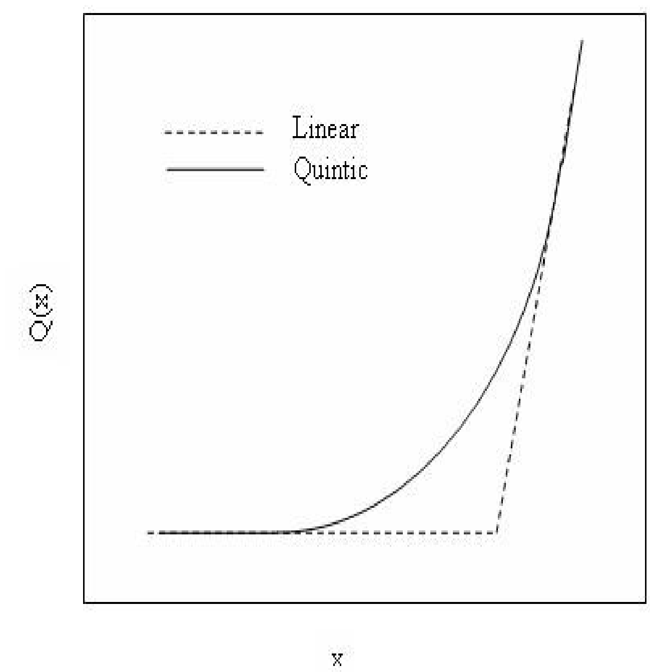

Figure 2.4. Continuous derivative MARS function.

[36] replaced the truncated linear functions with quintic functions as shown in Figure: 2.4. For sign S and K_-, K, K_+ knots

$$Q(x|S=1,K_-,K,K_+) = \begin{cases} 0, & x \leq K_- \\ \alpha_+(x-K_-)^3 + \beta_+(x-K_-)^4 + \gamma_+(x-K_-)^5, & K_- < x < K_+ \\ x-K, & x \geq K_+, \end{cases} \tag{2.14}$$

and

$$Q(x|S=-1,K_-,K,K_+) = \begin{cases} K-x, & x \leq K_- \\ \alpha_-(x-K_+)^3 + \beta_-(x-K_-)^4 + \gamma_+(x-K_-)^5, & K_- < x < K_+ \\ 0, & x \geq K_+. \end{cases} \tag{2.15}$$

where,

$$\alpha_+ = \frac{6K_+ - 10K + 4K_-}{(K_+ - K_-)^3}, \tag{2.16}$$

$$\beta_+ = \frac{-8K_+ + 15K - 7K_-}{(K_+ - K_-)^4}, \tag{2.17}$$

$$\gamma_+ = \frac{-3K_+ - 6K + 3K_-}{(K_+ - K_-)^5}, \tag{2.18}$$

$$\alpha_- = \frac{(-1)(6K_- - 10K + 4K_+)}{(K_- - K_+)^3}, \tag{2.19}$$

$$\beta_- = \frac{(-1)(-8K_- + 15K - 7K_+)}{(K_- - K_+)^4}, \tag{2.20}$$

$$\gamma_- = \frac{(-1)(3K_+ - 6K + 3K_-)}{(K_- - K_+)^5}. \tag{2.21}$$

2.3.3 Derivative of MARS Function

The derivative of a function is defined as an instantaneous rate of change of the function. For example the derivative of ω with respect to x is $\frac{d\omega}{dx}$. A C program was written to find the gradient of the quintic MARS function. Consider a univariate quintic MARS approximation represented as below.

$$\hat{g_M}(x) = a_0 + \sum_{j=1}^{M} a_j Q_j. \tag{2.22}$$

The first derivative is given by

$$d(\hat{g_M}(x))/dx = \sum_{j=1}^{M} a_j Q_j^{'}, \tag{2.23}$$

where,

$$Q_j^{'} = \begin{cases} 0, & x \leq K_- \\ 3\alpha_+(x-K_-)^2 + 4\beta_+(x-K_-)^3 + 5\gamma_+(x-K_-)^4, & K_- < x < K_+ \\ 1, & x \geq K_+, \end{cases} \tag{2.24}$$

or

$$Q_j^{'} = \begin{cases} -1, & x \leq K_- \\ 3\alpha_-(x-K_+)^2 + 4\beta_-(x-K_-)^3 + 5\gamma_+(x-K_-)^4, & K_- < x < K_+ \\ 0, & x \geq K_+. \end{cases} \tag{2.25}$$

for $S = +1$ or -1 respectively. For a two factor interaction,

$$\hat{g_M}(x) = a_0 + \sum_{j=1}^{M} a_j Q_j, \tag{2.26}$$

$$d(\hat{g_M}(x))/dx = \sum_{j=1}^{M} a_j Q_j^{'}, \tag{2.27}$$

where, $Q_j^{'} = Q_{j1}^{'} Q_{j2} + Q_{j1} Q_{j2}^{'}$.

Consider an example with two variables, three basis functions with the third basis function as an interaction between the two variables.

$$\hat{g_M}(x) = a_0 + a_1[Q_1] + a_2[Q_2] + a_3[Q_{31}Q_{32}] \tag{2.28}$$

$$d(\hat{g_M}(x))/dx = a_1[Q_1^{'}] + a_2[Q_2^{'}] + a_3[Q_{31}^{'}Q_{32}] + a_3[Q_{31}Q_{32}^{'}]. \tag{2.29}$$

Sign	Var
-1	x1
1	x2
1	x1
-1	x2

Table 2.2. Example of a MARS approximation.

2.4 Overbooking

Selling more seats than the capacity of the flight, to compensate for potential cancellations and no-shows, is called overbooking. If the actual number of seats sold is within the capacity of the flight (overbooking − cancellations − no-shows < 0) the airline will still incur a loss, but this loss would be comparatively less than the loss from not having overbooked. On the other hand, if more passengers show up than the capacity of the flight, the ticket holders are *bumped* (forbidden to fly) involuntarily. Compensation must be provided for the bumped passengers. Figure: 2.5 shows the possible outcomes of overbooking. If the airline is able to accommodate the bumped customer in its own airline in the next flight, then the airline is said to have *recaptured* the customer. It is observed that as the overbooking level increases, revenue also increases to a maximum, and then slowly starts to decrease. However, this maximum revenue is very difficult to attain in reality.

Define the following notation used in overbooking:

- x = Total capacity of the flight
- n_c = Number of cancellations and no-shows
- y = Total number of seats overbooked, overbooking pad.

Chatwin [17] stated, "experience shows that nearly 15% of the seats fly empty if overbooking is not considered." Thus, in order to efficiently utilize the resources and increase revenue, airlines wish to sell more seats than the capacity of the flight. Bodily

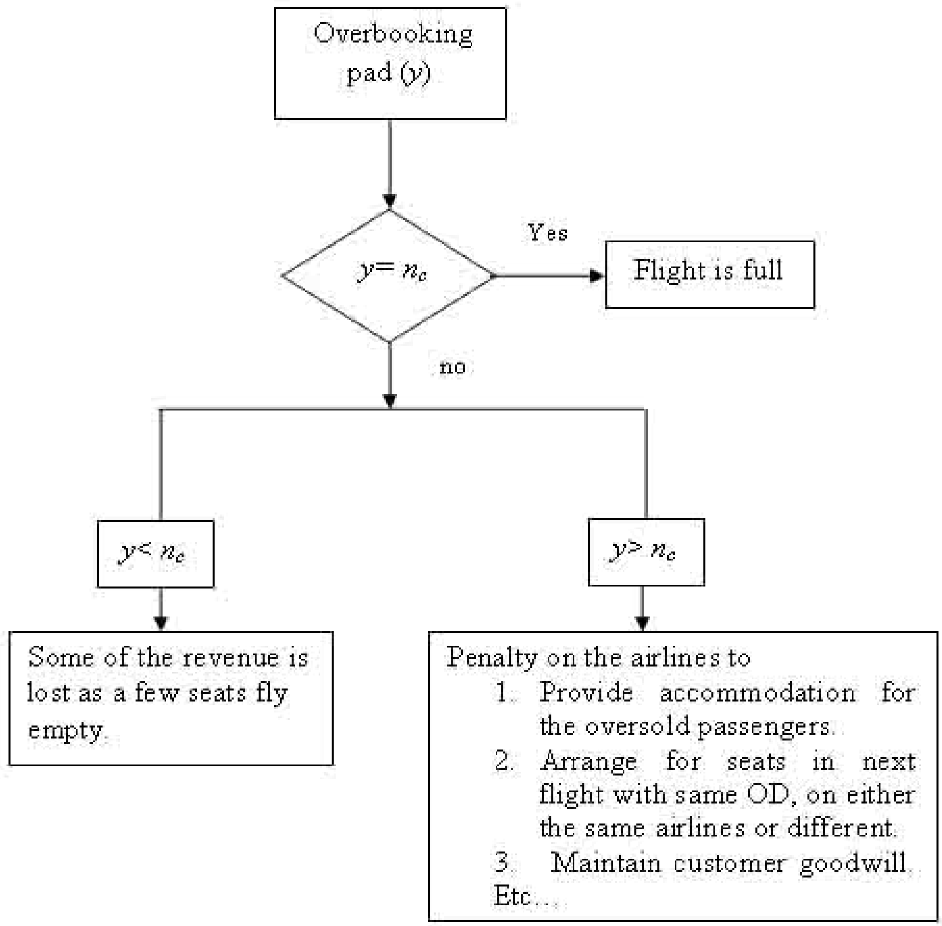

Figure 2.5. Flow chart representing the possible results of overbooking.

and Pfeifer [3] considered the probability of customer cancellation, and determined that it depends on (1) when the reservation was made and (2) unknown events that might occur before departure. The drawback of this approach was that it did not consider the dynamic nature inherent in the reservation process.

Kosten [42] developed a continuous time stochastic model. His model was proved impractical by McGill [43] because it required solutions to many differential equations. Rothstein [21] was the first to formulate airline overbooking as a dynamic programming problem. This approach was computationally intractable due to the curse of dimensionality. Chatwin [17] mentioned two ways to overcome the above problem of computation: (1) approximate the states by aggregating them or (2) develop a theory of the structure of optimal solution, so as to facilitate more efficient computation. The first approach was followed by Alstrup et al. [44]. They developed a dynamic programming approach to solve the overbooking problem for two fare classes. They assumed that customers requested and cancelled reservations in groups of five, thereby reducing the size of the state space by a factor of 25. Their dynamic yield management model is being implemented worldwide. Chatwin implemented the second idea in his dissertation. He considered overbooking models with (1) discrete time and discrete state spaces and (2) a continuous time birth and death process.

Subramanian et al. [25] formulated the overbooking RM problem as a finite-horizon, discrete-time Markov decision process (MDP). It was an extension to the model developed by Lee and Hersh [23]. Chatwin [45] solves the multiperiod overbooking problem that relates to a single flight leg and service class. The conditions on fares, refunds, distributions of passenger demand for reservations and cancellations in each period, and the bumping penalty function are given, to ensure that a booking-limit policy is optimal. In other words, Chatwin states"*in each period the airline accepts reservation requests up to a booking limit if the number of initial reservations is less than that booking limit; it declines reservation requests otherwise.*" The model is applied to the discount allocation problem in which lower fare classes book prior to higher fare classes. Rothstein [46] analyzes the problems that motivate overbooking, discusses the practices of the airlines, and describes significant contributions and implementations of operations research. Karaesmen and van Ryzin [47] modeled the overbooking problem as a two-period optimization problem. In the first period, given the probabilistic knowledge of cancellations, reservations are accepted. In the second period, cancellations are realized and surviving customers are assigned to the various inventory classes to minimize penalties.

CHAPTER 3

EXTENDED STATISTICAL MODELING APPROACH TO RM

This dissertation develops an extension to Günther's MDP/OA-MARS statistical modeling approach. The goals are to:

- Identify appropriate ranges for remaining seat capacities in the MDP, so as to reduce the modeling domain and enable more accurate MARS approximations.
- Account for overbooking and determine an optimized overbooking level.

The off-line and on-line modules described in Sections 2.2.2 and 2.2.3 are modified as follows: The off-line phase consists of two major modules:

1. A revised statistical modeling module that conducts a preprocessing simulation to identify realistic ranges of the remaining seat capacity state variables and then builds statistical models of the (DET) and (STOCH) revenue functions to estimate bounds on the value function of the MDP.
2. An overbooking module that utilizes Newton's and steepest ascent method to estimate the total number of seats to be overbooked, i.e., the *overbooking pad* for each flight leg.

The on-line phase consists of the availability processor module that uses the statistical models from the off-line phase in the RM policy to make the booking decisions. Some of the assumptions made in this approach are:

1. The flight capacity and the schedule are fixed and known.
2. No charge is incurred by the customers for cancellation of a ticket.
3. There is no refund on cancellation of any type (demand class) of ticket.

3.1 Revised Statistical Modeling Module

This is a revised version of the original statistical modeling module in Section 2.2.2. Specifically, realistic ranges of the remaining capacity state variables are generated instead of assuming the same ranges, from zero to capacity, throughout the booking period. Intuitively, these ranges should be closer to the capacity at the beginning of the booking period and move closer to zero toward departure.

3.1.1 Generation of Realistic State Space

In the statistical modeling module developed by Günther [9], the state space remains the same for all the reading dates. Hence, the design points are spread out over a wider region than required. In practice it is difficult to find a flight with zero seats booked on the day of the departure and all the seats booked ninety days prior to the day of departure. In order to be more realistic, the possible/realistic ranges for each reading date are estimated. These are called *trust regions.*

The demand scenarios are generated based on real data. Flight capacity is initialized to $(1+\delta)\times$(actual flight capacity), where $\delta = 0.2$, handles the overbooking aspect. The (DET) optimization, as described in Section 2.1.2.1 is employed only at the reading dates. The RM policy which states, "accept booking request only if the fare is greater than the fair market value," is used to make decisions on accepting/rejecting the request. Upon accepting the request, the remaining seat capacity is updated to the remaining capacity minus group size g for the requested legs. At each reading date, a (DET) model is solved to obtain updated bid prices, and the process repeats until the flight departs. Demand scenarios are simulated many times, and at the end of each reading date, remaining seat capacities are noted. Figure 3.1 shows the generation of the trust regions. The decision to accept or reject a booking request (BR) is made based on the deterministic bid price approach. Remaining seat capacities obtained at each reading date over

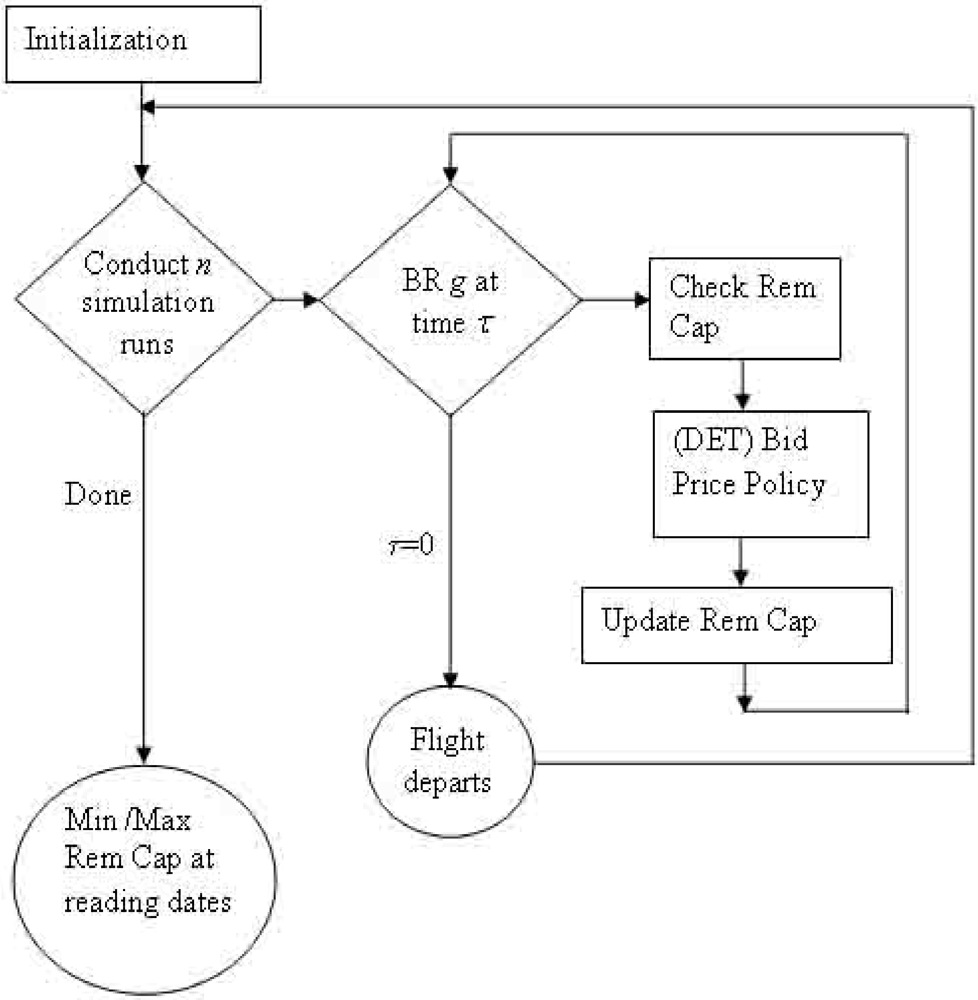

Figure 3.1. Flow chart representing the generation of the reduced/realistic state space.

the entire simulation are used to determine the maximum and minimum capacities at those reading dates.

3.1.2 Estimation of Number of Scenarios

A simulation for a rough sample size of N=30 is run. The resulting data is used to estimate the standard deviation of remaining capacity, σ. The desired sample size is then estimated using,

$$\text{Sample size} \quad N \geq \left(\frac{2 z_{\alpha/2} \sigma}{E} \right)^2, \tag{3.1}$$

where E is 5%×(Expected value of the sample ± confidence co-efficient ×standard error), and $z_{\alpha/2}$ is the $\alpha/2$ upper tail percentile of the standard normal distribution. A total of 125 simulation runs were conducted, which was well above the estimated sample size of 85.

3.1.3 Approximation of Value Functions

The remaining seat capacity is initialized to $(1+\delta)\times$(flight capacity), where δ=0.2. An OA experimental design is employed to find the discretization points. At each reading date, the range within which the discretization points should exist is set equal to their corresponding realistic region. At each reading date, and for each design point, both (DET) and (STOCH) models are solved (refer Figure:2.2.)

As in the method of Chen et al. [34], the (DET) model is used to provide an upper bound on the MDP value function and the (STOCH) model is used to provide a lower bound. Solving the deterministic model is a straightforward LP, but there are different approaches for solving the stochastic model. The next section describes the approach used in this dissertation. The software ILOG CPLEX 9.0 was used to solve the LP. The MARS approximation is fit over all the (DET) and (STOCH) value functions at each reading date as explained in Chapter 2. Thus, by the end of this stage, there will be a total of $2T$ MARS approximations, which are fed to the on-line phase.

3.1.4 Solving the Stochastic Network Optimization Model

In the (STOCH) model, consider $E[\min(d_{ft}, u_{ft})]$ of the objective function. Olinick and Rosenberger [18] showed that this function is concave. Following the approach of Kelley's cutting plane method [48], the objective function is expanded using a Taylor series about a constant $u_0 \geq 0$:

$$E[\min(d_{ft}, u_{ft})] = E[\min(d_{ft}, u_0)] + (u_{ft} - u_0)\nabla E[\min(d_{ft}, u_0)] + o(||u_{ft} - u_0||^2). \quad (3.2)$$

From the definition of expected value we know that, for any discrete variable X, $E[X] = \sum Xp(X)$. Let, $g(d_{ft}) = \min(d_{ft}, u_0)$, where d_{ft} is a discrete random variable for the demand for demand class f at reading date t. For simplicity, let $d_{ft} = \theta$ for the purpose of derivation. Then we can write the expected value of $g(\theta)$ as:

$$\begin{aligned} E[g(\theta)] &= E[\min(\theta, u_0)] \\ &= \sum_{\theta} g(\theta)p(\theta) \\ &= \sum_{\theta=0}^{u_0} \min(\theta, u_0)p(\theta) + \sum_{\theta=u_0+1}^{\max} \min(\theta, u_0)p(\theta). \end{aligned} \quad (3.3)$$

It can be observed from the Figure:3.2 that $\min(\theta, u_0) = \theta$, in the range of $\theta = 0$ to u_0 and $\min(\theta, u_0) = u_0$, beyond u_0. Hence,

$$E[\min(\theta, u_0)] = \sum_{\theta=0}^{u_0} \theta p(\theta) + \sum_{\theta=u_0}^{\infty} u_0 p(\theta), \quad (3.4)$$

where $p(\theta)$ is the probability of demand θ. Demand is assumed to follow a compound Poisson process with arrival rate λ. Let H_W be the cumulative distribution function

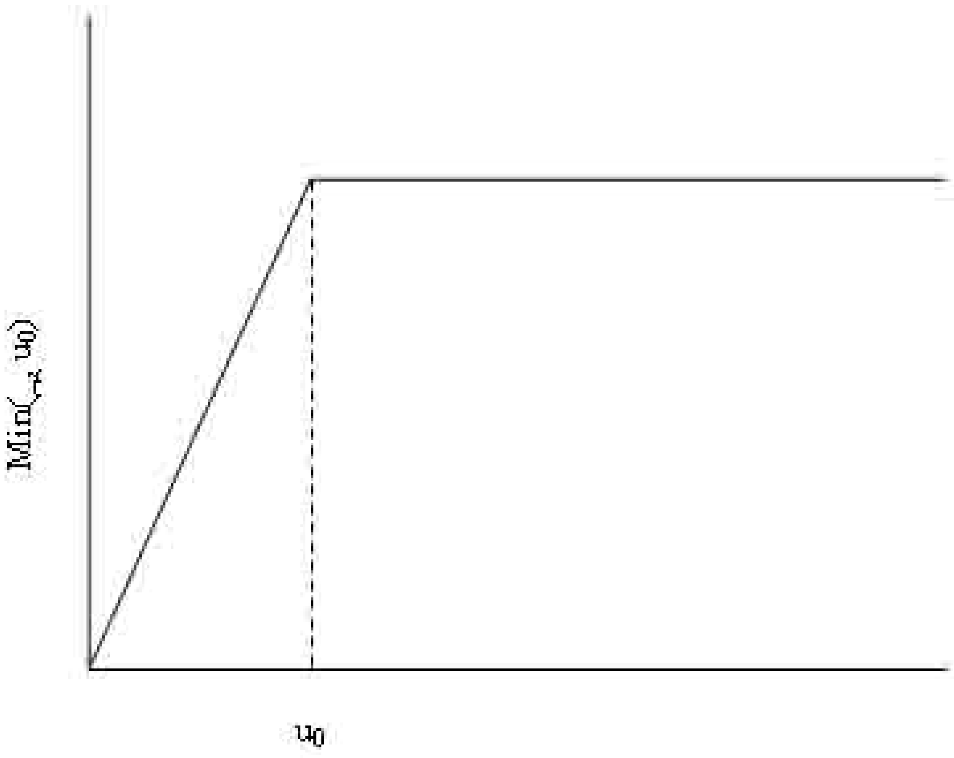

Figure 3.2. Plot illustrating the minimum value in any given region.

for the Poisson distribution and h_W be the probability mass function for the Poisson distribution. Hence,

$$E[\min(\theta, u_0)] = \sum_{\theta=0}^{u_0} \theta e^{-\lambda}\lambda^{\theta}/\theta! + \ u_0 \sum_{\theta=u_0+1}^{\infty} e^{-\lambda}\lambda^{\theta}/\theta! \tag{3.5}$$

$$= \lambda \sum_{\theta=1}^{u_0} e^{-\lambda}\lambda^{\theta-1}/(\theta-1)! + \ u_0 \sum_{\theta=u_0+1}^{\infty} e^{-\lambda}\lambda^{\theta}/\theta! \tag{3.6}$$

$$= \lambda \sum_{\theta=0}^{u_0-1} e^{-\lambda}\lambda^{\theta}/\theta! + \ u_0[1 - \sum_{\theta=0}^{u_0} e^{-\lambda}\lambda^{\theta}/\theta!] \tag{3.7}$$

$$= \lambda H_W(u_o - 1) + \ u_0[1 - H_W(u_o)], \tag{3.8}$$

Using finite differences, $\nabla E[\min(\theta, u_0)]$ is estimated by

$$\nabla E[\min(\theta, u_0)] = [(u_0+1) - (u_0+1)H_W(u_0+1) + \lambda H_W(u_0)] - [u_0 - u_0 H_W(u_0) + \lambda H_W(u_0-1)] \tag{3.9}$$

$$= \lambda h_W(u_0) - u_0 h_W(u_0+1) - H_W(u_0+1) + 1. \tag{3.10}$$

Define η_{ft} such that

$$\eta_{ft} \leq E[\min(d_{ft}, u_{ft})]. \tag{3.11}$$

Substituting for all the above in the original stochastic formulation, the final stochastic model obtained is as below.

$$\max \sum_{\tau=1}^{t} \sum_{f=1}^{m} r_f \eta_{f\tau} \tag{3.12}$$

$$\text{s.t } A\left(\sum_{\tau=1}^{t} \boldsymbol{u}_\tau\right) \leq \boldsymbol{x}_t \tag{3.13}$$

$$\eta_{f\tau} - u_{f\tau}[\lambda h_W(u_0) - u_0 h_W(u_0+1) - H_W(u_0+1) + 1] \leq \lambda H_W(u_0-1) - u_0[\lambda h_W(u_0) - u_0 h_W(u_0+1) - H_W(u_0+1) + H_W(u_0)] \quad \forall f = 1, 2, ...m,\ \forall \tau = 0, 1, ...t,\ \forall u_0 \in \Re^+ \tag{3.14}$$

$$u_{ft} \geq \eta_{ft} \geq 0. \quad \forall f = 1, 2, ...m,\ \forall \tau = 0, 1, ...t \tag{3.15}$$

The set (3.14) has an infinite number of constraints, so it is computationally intractable to solve this linear programming problem exactly as stated. However, this linear programming problem can be approximated by replacing the infinite set $\Re^+$ by a finite set in which $u_0 = 1, 2, \ldots, \overline{u}$, where $\overline{u}$ is a practical upper bound on the values $u_{f\tau}$, for all $f = 1, \ldots, m$ and $\tau = 0, \ldots, t$. In our computational experiments in Chapter 4, $\overline{u}$ is set to be 50.

3.2 Overbooking Module

Consider the following notation in addition to those in Section 2.4.

- Y = a random variable for the actual number of seats overbooked or oversold, realized on the day of departure.
- c_{pen} = a random variable for the cost/penalty incurred due to Y.
- W = a random variable for the total number of passengers that show up on the day of departure.
- x_T = the remaining capacity at reading date T, equaling the flight capacity.
- x_k = the sum of flight capacity and the overbooking pad, $(x + y)$.
- c = a cost equaling three times the bid price.

Let $O(y)$ be the overbooking cost function, where cost is assumed to increase, as y increases. Let $R(y)$ be the revenue function. As the number of seats booked are finite, revenue always increases as y increases. Thus, the profit function is

$$Z(y) = R(y) - O(y). \tag{3.16}$$

Figure 3.3 presents a conceptual diagram of the cost, revenue, and profit function, with respect to y. The objective of this model is $\max_{y \geq 0} Z(y)$. A simplified cost estimate is used and assumes: (1) the number of passengers that show up are independent and identically distributed, (2) the probability that any customer shows up is a constant, and (3) the cost per seat oversold is three times the bid price of the leg/itinerary.

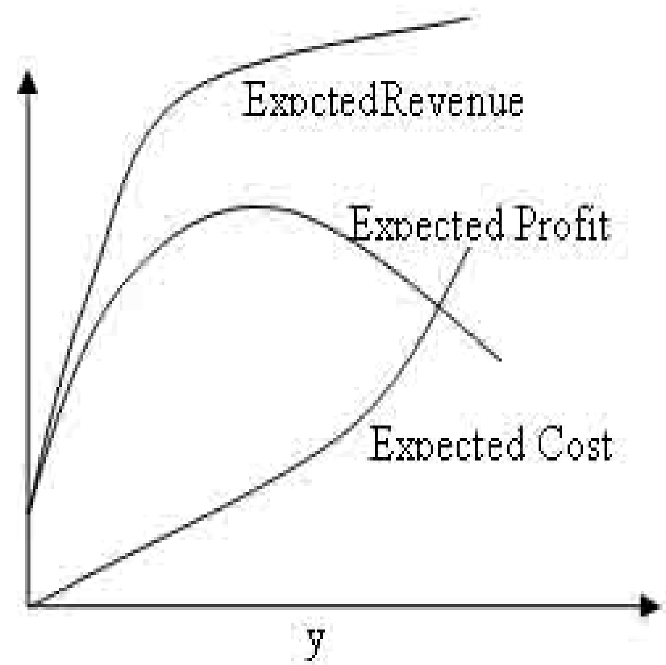

Figure 3.3. Profit curve.

3.2.1 Newton's Method

All gradient methods are used for either minimization or maximizing of a function. All methods use an iterative formula that contains the gradient of the function to find the minimum or maximum, hence, the name *gradient methods.* Newton's method is said to be the fastest of all the gradient methods [49]. In this dissertation it is employed to find the maximum profit point on the profit curve, along with the steepest ascent method (refer Figure: 3.4). Newton's method is explained in Algorithm 1.

3.2.2 Steepest Ascent

In this and the following two subsections, steepest descent is explained, but the same idea can also be used for maximization of a function. In this dissertation, attempts are made to reach the maximum point of the profit function. Consider the problem of

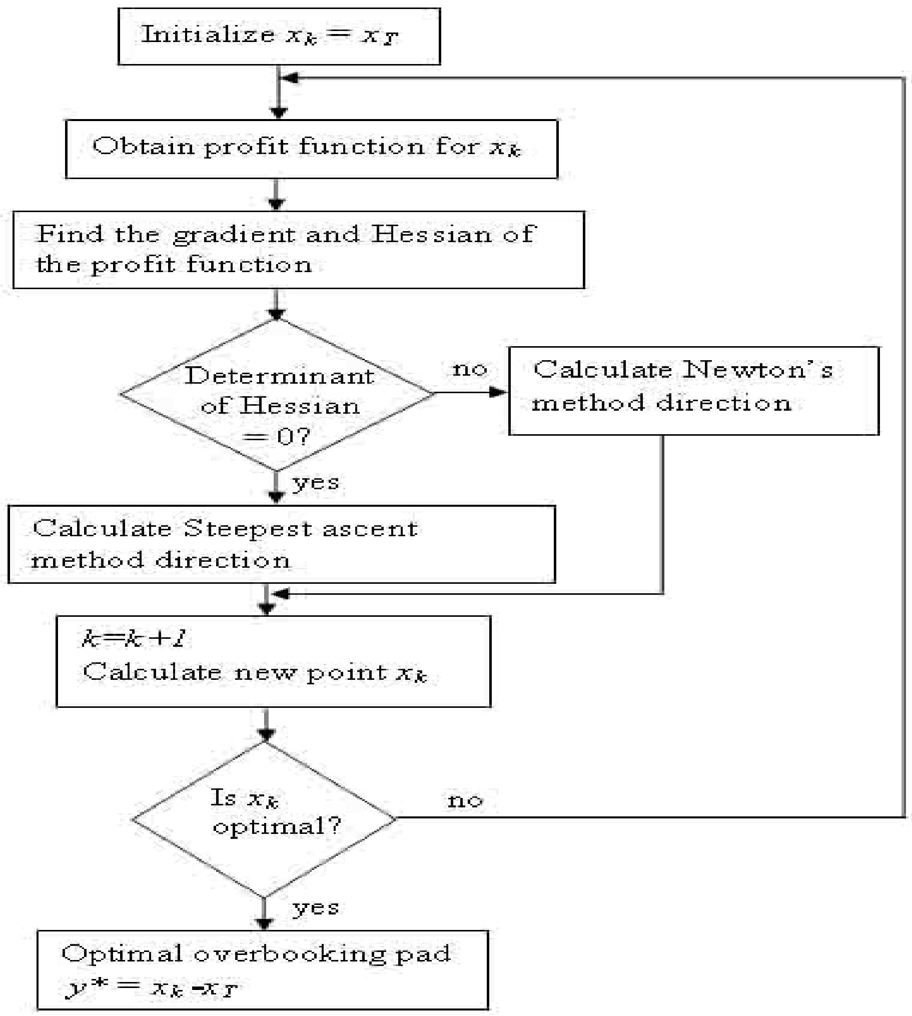

Figure 3.4. Flow chart representing the off-line module for estimating the optimal overbooking pad.

unconstrained minimization of a continuously differentiable profit function $Z : \Re^n \mapsto \Re$. The algorithms for this problem rely on an important idea, called iterative descent, that works as follows: We start at some point x_0 (an initial guess) and successively generate vector $x_1, x_2, ...$, such that Z is decreased at each iteration, that is $Z(x_{k+1}) < Z(x_k), k = 0, 1, ...$, (see Figure 3.5). In doing so, we successively improve our current solution estimate and Z decreases all the way to its minimum.

Algorithm 1 Newton's Method.

Initialize $x_k = x_T, k = 0$
Step 1: Find Z_k, $\nabla_x Z_k$ and $\nabla^2_{xx} Z_k$
Step 2:
if norm of $\nabla_x Z_k = 0.0$ **then**
  Stop. Maximum profit is obtained at x_k.
else
  $k = k + 1$
  Estimate $x_k = x_{k-1} - \nabla_x Z_{k-1}(\nabla^2_{xx}(Z_{k-1}))^{-1}$ and $\nabla_x Z_k$
  if norm of $\nabla_x Z_k = 0.0$ **then**
    Stop Maximum profit is obtained at x_k.
  else
    Go to Step1
  end if
end if

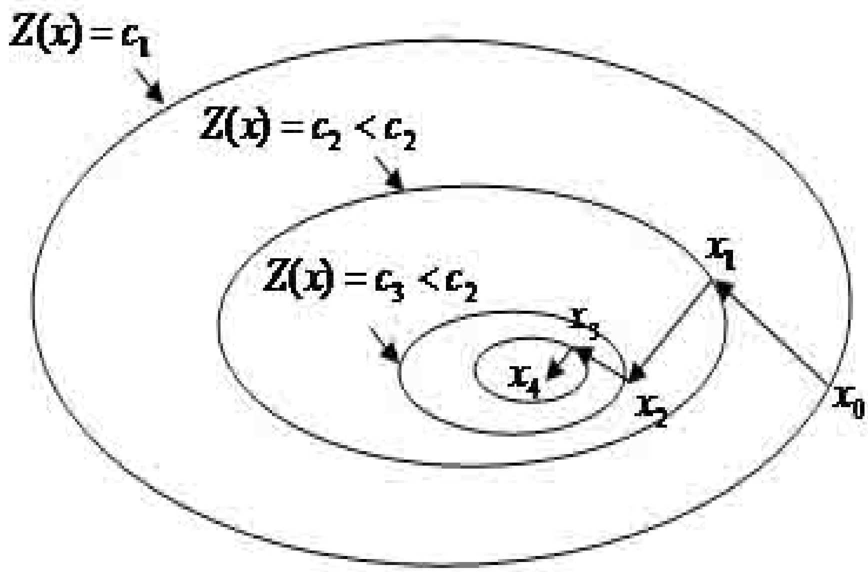

Figure 3.5. Iterative descent for minimizing a function Z, where c is the cost reduced at every iteration.

3.2.2.1 Selection of Descent Direction

The gradient methods are specified in the form

$$x_{k+1} = x_k - \gamma_k D_k \nabla Z(x_k), \tag{3.17}$$

where D_k is a positive definite symmetric matrix and γ_k is the stepsize. For the steepest descent approach $D_k = I$, $k = 0, 1, ...$, where I is an $n \times n$ identity matrix. This is the simplest choice, but it often leads to slow convergence.

3.2.2.2 Stepsize Selection

There are a number of rules for choosing the stepsize γ_k in a gradient method. To avoid the often considerable computation associated with line minimization rules, it is natural to consider rules based on successive stepsize reduction, such as the *Armijo Rule*. In this rule an initial stepsize γ is chosen, and if the corresponding vector $x_k + \gamma d_k$ does not yield an improved value of Z, the stepsize is reduced, perhaps repeatedly, by a certain factor, until the value of Z is improved. The Armijo Rule is essentially the successive reduction rule, suitably modified to eliminate the theoretical convergence difficulty shown in Figure 3.6. The fixed scalars γ, β, and $\sigma > 0$, with $\beta \in (0, 1)$, and $\sigma \in (0, 1)$ are chosen, and γ_k is set equal to $\beta_{m_k}\gamma$, where m_k is the first nonnegative integer m for which $Z(x_k) - Z(x_k+)\beta_{m_k}\gamma d_k \geq -\sigma\beta_m\gamma\nabla Z(x_k)' d_k$. In other words, the stepsize $\beta_m\gamma, m = 0, 1, ...$, are tried successively until the above inequality is satisfied for $m = m_k$. Figure 3.7 illustrates the rule.

3.2.3 Hybrid Approach to Estimate the Optimal Overbooking Pad

On implementing Newton's method to determine the maximum point, it was observed that the MARS function is flat across most of the surface, and, consequently, the determinant of the Hessian matrix of the profit function was often equal to zero. Hence,

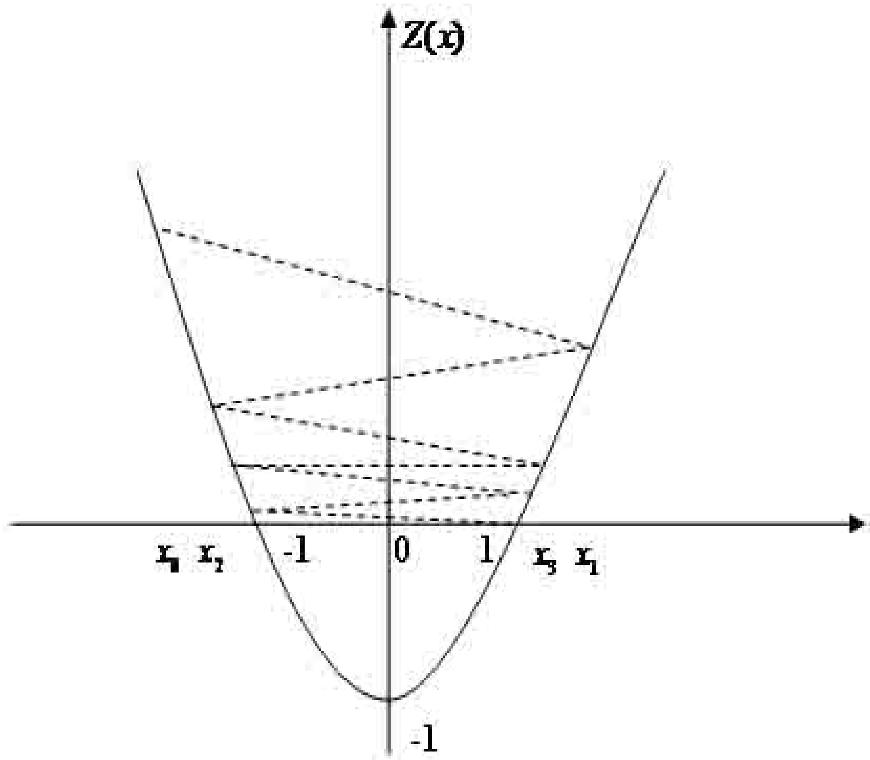

Figure 3.6. Failure of the successive stepsize reduction rule for a one-dimensional function.

steepest ascent was used in addition to Newton's method to overcome the above disadvantage. Algorithm 2 gives the procedure adopted in estimating the optimal overbooking pad (also, refer Figure 3.4.)

3.2.4 Derivation of Cost Function

The number of customers that show up, W, is assumed to be binomially distributed; hence, from the definition of the binomial distribution, we have

$$P(W = w) = \binom{x_k}{w} \alpha^w (1-\alpha)^{x_k - w}. \tag{3.18}$$

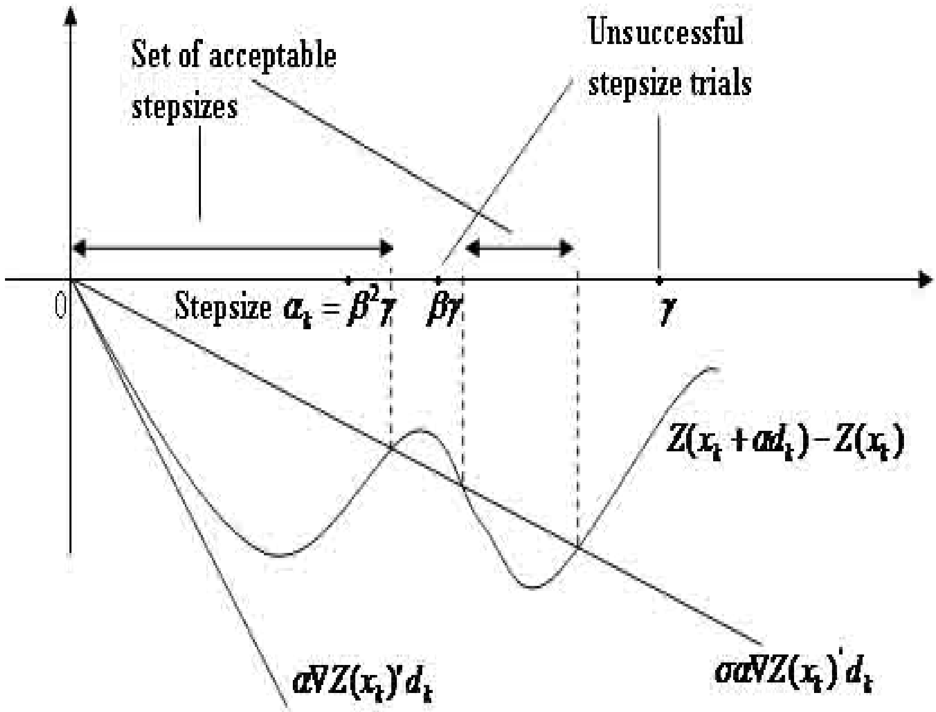

Figure 3.7. Line search using the Armijo Rule.

From the definition of Y, we have

$$\begin{aligned} Y &= [W - x_T]^+ \\ E[Y] &= E[W - x_T]^+ \\ &= E[(W - x_T)^+ | W > x_T] P[W > x_T] + E[(W - x_T)^+ | W \leq x_T] P[W \leq x_T] \\ &= E[(W - x_T)^+ | W > x_T] P[W > x_T]. \end{aligned}$$

Thus, the cost function is

$$\begin{aligned} c_{pen} &= cE[Y] \\ &= c \sum_{w=x_T+1}^{x_T+y} (w - x_T) P(W = w). \end{aligned}$$

Algorithm 2 Hybrid Newton's and steepest ascent method.

Initialize $x_k = x_T, k = 0, \gamma = 1$.
Step 1: Find Z_k, $\nabla_x Z_k$ and $\nabla^2_{xx} Z_k$.
Step 2:
if norm of $\nabla_x Z_k = 0.0$ **then**
  Stop. Maximum profit is obtained at x_k.
else
  Step 3: Find determinant (det) of $\nabla^2_{xx} Z_k$.
  if det =0 **then**
    Use steepest ascent method to find the next point.
    $x_k = x_{k-1} - \gamma \nabla_x Z_{k-1}$;
    $\gamma = *0.95\gamma$;
    Go to Step 1.
  else
    Use Newton's method.
    $x_k = x_{k-1} - \nabla_x Z_{k-1} (\nabla^2_{xx}(Z_{k-1}))^{-1}$;
    Go to Step 1.
  end if
end if

In order to find the derivative of the cost function (required by Algorithm 2), the derivative of the cost function needs to be continuous, as gradient methods need at least first derivative information to find the direction of progress towards optimality. Since the binomial is a discrete distribution, for gradient-based optimization, it needs to be approximated by some continuous distribution. Given a continuous density function $h_W(w)$, we would re-write the cost function as:

$$\begin{aligned} c_{pen} &= c \int_{w=x_T+1}^{x_T+y} h_W(w)(w - x_T)dw \\ &= c \int_{w=x_T+1}^{x_T+y} w h_W(w)dw - x_T \int_{w=x_T+1}^{x_T+y} h_W(w)dw. \end{aligned} \tag{3.19}$$

The normal approximation to the binomial is considered here. Thus, from the definition of normal distribution, $h_W(w) = (1/\sigma\sqrt{2\Pi})e^{(w-\mu)^2/2\sigma^2}$, and hence,

$$c_{pen} = (c/\sigma\sqrt{2\Pi}) \int_{w=x_T+1}^{x_T+y} we^{(w-\mu)^2/2\sigma^2} dw - x_T \int_{w=x_T+1}^{x_T+y} h_W(w) dw \tag{3.20}$$

To compute the integral, the function int *gsl integration qags* (const *gsl function* * c_{pen}, double $x_T + 1$, double $x_T + y$, double *epsabs*, double *epsrel*, size *sz* limit, *gsl integration workspace* * *workspace*, double * *res*, double * *abserr*) from the GNU Scientific Library (GSL) was used. This function applies the Gauss-Kronrod 21-point integration rule adaptively until an estimate of the integral of c_{pen} over $(x_T + 1, x_T + y)$ is achieved within desired absolute and relative error limits, *epsabs* and *epsrel*. The results are extrapolated using the epsilon-algorithm, which accelerates the convergence of the integral in the presence of discontinuities and integrable singularities. The final approximation is obtained from the extrapolation, *res*, and an estimate of the absolute error, *abserr*. The subintervals and their results are stored in the memory provided by workspace. The maximum number of subintervals is given by the limit, which may not exceed the allocated size of the workspace [50]. Differentiating c_{pen} with respect to y,

$$\begin{aligned} \frac{dc_{pen}}{dy} &= (cy/\sigma\sqrt{2\Pi})e^{-(x_T+y-\mu)^2/2\sigma^2} \\ \frac{d^2c_{pen}}{dy^2} &= (c/\sigma\sqrt{2\Pi})e^{-(x_T+y-\mu)^2/2\sigma^2}[y(\mu - x_T - y)/\sigma^2 + 1]. \end{aligned} \tag{3.21}$$

We know that profit equals revenue minus cost. Hence, both the cost and revenue functions need to have continuous derivatives. We saw that the cost function has a continuous derivative. The revenue function is estimated using the MARS approximation derived by setting the range of remaining capacity to a maximum of 1.2×(flight capacity) and a minimum of zero at reading date T. As explained in Section 2.3.2, MARS can be smoothed to have continuous first and second derivatives; see Section 2.3.3.

3.3 Availability Processor Module

In this module, the RM policy discussed in Section 2.2.2 is employed to make the accept/reject decisions. The optimal overbooking pad Y^* obtained from the off-line phase is added to the actual capacity of the flight. This represents the total number of seats to be sold. Figure: 2.3 shows a flow chart representing the availability processor module.

In order to estimate the value function at any time τ during the 90-day booking period, interpolation is performed. Thus, $\hat{F}_\tau(x)$ is determined using the method of interpolation between $\hat{F}_{t1}(x)$ and $\hat{F}_{t2}(x)$. The formula used to estimate the FMV is the same as equations (2.7) - (2.9). To estimate the cost due to overbooking Y^* seats, a show up rate of α=0.8 was considered:

$$c_{pen} = c \sum_{w=x+1}^{x+Y^*} p(W = w) \tag{3.22}$$

where,

$$p(W = w) = \begin{pmatrix} x_T + Y^* \\ w \end{pmatrix} \alpha^w (1 - \alpha)^{x_T + Y^* - w}. \tag{3.23}$$

The expected profit generated by overbooking is given by the expected revenue generated by booking $(x_T + Y^*)$ seats minus the cost due to Y^*.

CHAPTER 4

COMPUTATIONAL RESULTS FOR THE EXAMPLE PROBLEM

A real airline hub with 31 different legs was used to test this methodology. Fifteen reading dates were selected and spaced as per the airline's requirements. The number of itineraries was 123, and the maximum demand was 50. Data on flight capacities and demand distribution parameters were provided by the airline.

An OA experimental design, was used to generate 31^2 design/discretization points. Flight capacities were increased to 1.2 times the original capacity to enable overbooking. The demand scenarios were generated based on the data given. Using all this information the revised statistical modeling module was executed, and the resulting RM policy was employed in a simulated booking process using the availability processor module. The results obtained from 2000 simulation runs for different methods and approaches are as given in Tables 4.1 - 4.3. In all tables, "Load" is the nominal load factor, defined to be the quotient of total requested capacity over available capacity. Airlines use nominal load factors of up to 150 percent. "CV" is the coefficient of variation. The upper bound column provides the maximum optimal revenue that can be generated and is obtained by solving the (DET) model 90 days before the day of departure with maximum capacities. The standard errors are given in parentheses and the percentage increase in expected revenue with respect to the (DET) bid price approach is also given.

As a first study, in Table 4.1 a comparison is given using only the (DET) model. The different approaches are (DET) bid pricing, the statistical modeling approach of Günther [9] (DET STAT), and this dissertation's approach using the revised statistical modeling module (DET REV STAT). The FMV for the statistical modeling approaches was estimated using $\hat{F}^U_\tau(x) - \hat{F}^U_\tau(x')$. The demand was assumed to follow a simple Possion distribution. Hence, the coefficient of variation is not considered in this table. For all methods, the RM policy is derived specifically for each load factor. It can be seen that DET STAT is better than the DET and DET REV STAT is better than DET STAT.

Table 4.1. Average revenues from 2000 simulations of the 31-leg hub using three methods: DET = Deterministic Bid Price, DET STAT = Statistical Modeling Approach using only (DET) model, DET REV STAT = Revised Statistical Modeling Module using only (DET) model. Standard errors are given in parentheses, and percent increase in average revenue from DET is shown.

Load	DET	DET STAT	DET REV STAT	Upper Bound
75	666187.6(279.92)	666219.9 (254.6) 0.004%	666472.5(288.8) 0.04%	675090.94
120	932469.5(276.01)	934393(252.3) (0.20%)	951969.1(253.6) (2.09%)	1103413
150	1045860(280.48)	1066304 (283.7) (1.95%)	1067141(254.5) (2.03%)	1275526

The results in Table 4.2 are obtained by assuming that there is no overbooking. The different approaches are (DET) bid pricing, (STOCH) bid pricing, the statistical modeling approach of Günther [9] (STAT), and this dissertation's approach using the revised statistical modeling module (REV STAT). Load factors of 75%, 120%, and 150% and various coefficient of variation values are tested. For all methods, an RM policy is derived specifically for each load factor and then tested across the various coefficients of variation. It is seen that STAT performs better than both DET and STOCH, and REV STAT shows improvement over the original STAT. It is also observed that STOCH performs better than DET in a few instances.

The results in Table 4.3 consider overbooking. A comparison is made between the (DET) bid price approach and this dissertation's statistical modeling approach using the revised statistical modeling module (REV STAT). For each leg, the capacity was set equal to the sum of maximum flight capacity and the optimal overbooking pad. For REV STAT, following the execution of the revised statistical modeling module, the overbooking module estimates optimal overbooking pads for each flight leg. Then the information from both of these modules is fed into the availability processor module to conduct the booking process. In Table 4.4, optimal overbooking pads are shown for each load factor. For both methods, an RM policy is derived specifically for each load factor and then tested across the various coefficients of variation. Again, it is observed that REV STAT performs better. Overall, it was seen that REV STAT performs better than the other approaches at all instances and is promising even when the variances are high.

Table 4.2. Average revenues from 2000 simulations of the 31-leg hub using four methods: DET = Deterministic Bid Price, STOCH = Stochastic Bid Price, STAT = Statistical Modeling Approach, REV STAT = Revised Statistical Modeling Module. Standard errors are given in parentheses, and percent increase in average revenue from DET is shown.

Load (%)	CV	DET	STOCH	STAT	REV STAT
75	0.4	790427.2 (586)	789450.1 (533.6) -0.12%	790536.7 (503.5) 0.01%	790975.7 (486.5) 0.07%
75	0.56	718729.4 (680.4)	720331.5 (762.6) 0.22%	725763.9 (623) 0.82%	728458.5 (645.3) 1.35%
75	0.63	662294.1 (769.2)	662522.5 (858.7) 0.03%	665756.5 (845.4) 0.52%	668229.7 (764.4) 0.89%
120	0.32	807252.8 (586)	801192.2 (508) -0.75%	818754.6 (455.2) 2.7%	820075 (467.5) 2.86%
120	0.45	779258.5 (603.2)	775292.9 (548.4) -0.51%	808538.4 (457.7) 3.76%	817417.1 (428.4) 4.90%
120	0.6	699243.0 (702.6)	709387.4 (774.0) 1.45%	720865.3 (736.8) 3.09%	739938.6 (632.7) 5.82%
150	0.48	787231.3 (591.1)	784911.87 (500.6) -0.29%	790875.8 (524.6) 0.46%	799574.9 (425.65) 1.57%
150	0.56	766360.9 (609.4)	724377.4 (770.1) -6.90%	789653.5 (643.9) 3.04%	794364.7 (700.6) 3.65%
150	0.7	721903.3 (679.0)	607282.8 (779.3) -15.88%	749655.7 (756.9) 3.84%	753297.7 (737.8) 4.35%

Table 4.3. Average revenue from 2000 simulations of the 31-leg hub using two methods considering overbooking: DET = Deterministic Bid Price, REV STAT = Revised Statistical Modeling Module. Standard errors are given in parentheses, and percent increase in average revenue from DET is shown.

Load	CV	DET	REV STAT
75	0.4	803494.4 (674.3)	804327.5 (456.5) 0.10%
75	0.56	700592.2 (716.9)	730582.6 (745.4) 1.39%
75	0.63	674676.6 (795.8)	679326.4 (854.6) 0.69%
120	0.32	816504.3 (590.7)	836917.1 (345.4) 2.5%
120	0.45	794350.7 (612.2)	829462.9 (536.8) 4.42%
120	0.6	710616.4 (717.3)	753143. (747.5) 5.89%
150	0.48	809851 (595.2)	825714.5 (625.7) 1.96%
150	0.56	786716.1 (623.2)	818648.7 (725.5) 4.06%
150	0.7	738337.4 (691.2)	773779.5 (764.2) 4.8%

Table 4.4. Optimal overbooking pad (OBP) for different load factors.

Flight Leg	Capacity	OBP for LF 75%	OBP for LF 120%	OBP LF 150%
1	113	6	8	9
2	152	6	7	7
3	164	2	3	5
4	208	9	11	12
5	151	1	4	5
6	107	6	8	9
7	152	2	4	5
8	154	1	2	4
9	153	4	6	7
10	179	0	2	4
11	155	7	8	8
12	166	0	0	2
13	168	0	1	2
14	167	6	8	8
15	150	1	2	2
16	156	6	9	10
17	221	10	13	13
18	159	0	3	5
19	168	13	13	14
20	166	11	16	22
21	160	11	12	12
22	201	0	1	5
23	155	0	0	3
24	188	12	14	14
25	157	1	2	5
26	156	15	18	20
27	165	5	8	12
28	150	4	8	10
29	155	1	2	5
30	149	17	19	24
31	193	15	19	20

CHAPTER 5

FUTURE WORK

The extended statistical modeling approach developed in this dissertation provides a good methodology to estimate the FMV, used in the RM policy. Hence, higher revenue is promised at all instances. Though this approach tries to be as close to reality as possible and updates the bid prices after every transaction, which is an ideal case, it makes certain assumptions. An improvement in this approach can be made by nullifying some of the assumptions and being more realistic on others. Below are some of the improvements that can be made in the overbooking module.

- The number of customers that show up was assumed to be binomially distributed, which does not represent reality. Hence, a more realistic distribution can be considered for the customer show up pattern.
- A cancellation fee can be included in the revenue model.
- Some of the passengers, specifically the high fare passengers, should be given a refund on cancellation of a ticket. Depending on when the cancellation was made and the type of ticket the customer has, a certain percentage of the price should be refunded.
- Attempts can be made to estimate a more accurate cost due to the bumping of oversold customers.
- Cost due to the loss of good-will of oversold passengers can also be considered. Bayesian methods may be used to identify this cost.

In the availability processor module, the following improvements can be considered.

- The FMV is estimated as the average of the pessimistic and optimistic value. A weighted average for the pessimistic and the optimistic may give better revenue.
- The number of reading dates can be varied, and the resulting expected revenues can be compared. It is known that the higher the number of reading dates, the more accurate the RM policy and the higher the revenue generated; however, the optimal number of reading dates should balance accuracy and computational time.

The fact that most companies are interested in discrete choice models for RM, application of a statistical approach to discrete choice models can be considered. The “buy-up” and “buy-down” behavior discussed in Section 2.1.3 can be estimated using statistical models.

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