TRAJECTORY UNCERTAINTY ANALYSIS FOR
THE DEVELOPMENT OF A QUEUING MODEL
FOR THE NATIONAL AIRSPACE SYSTEM

by

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The University of Texas at Arlington in Partial Fulfilment
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To my mother Shakunthala Prasad, father Nagendra Prasad and my brother Simha Chakravarthy for believing in me and without whose love and support, the completion of this work would not have been possible.
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April 15th, 2010
ABSTRACT

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MANJU NAG, M.S.
The University of Texas at Arlington, 2010

Supervising Professor: Kamesh Subbarao

In this research, we propose to support the development of a queuing model for the National Airspace System (NAS) by providing distributions for the various phases of flight, accounting for the uncertainties in flight trajectory which includes factors such as weight, power factor, lift co-efficient. In addition to this, a detailed study is done on the impact of weather on flight routes and two days in 2006 are compared to show the effect of weather on flight times.

The research develops from a good understanding of the NAS and its requirements to implement a system capable of dealing with issues regarding airspace capacity and air traffic. The need for this emerges from the Next Generation Air Transportation System (NGATS), a system when implemented should be able to handle air traffic and capacity ranging from 150% to 250% of the current traffic and capacity. NGATS proposes a few changes in the existing system to overcome the drawbacks and is studied in detail. Air traffic delays are identified and classified and a case study is done on the Dallas Fort Worth International airport.

The Base for Aircraft Data (BADA) database for aircraft performance is used to characterise flight phases in various phases of flight and is developed and maintained by the European Organization for the Safety of Air Navigation “Euro control Experimental Center”. Factors affecting time during various phases of flight such as climb, cruise and descent is identified and the sensitivity of these were investigated for variations in key operation related parameters such as weight at take off, aerodynamics and power. An aircraft performance calculator was developed on MATLAB, which, when the aircraft category and suitable inputs are specified, calculates the time taken by the aircraft.
in each of its phase. Monte Carlo experiments are conducted for these parameters and a distribution of time for different phases of flight is obtained.

Future ATM Concepts Evaluation Tool (FACET) is a simulation tool developed by NASA to analyse, implement and test new air traffic management and operational concepts. Cruise uncertainty was studied using FACET and the results obtained validated the previous uncertainty model developed using the aircraft performance calculator. Weather uncertainty was studied using the National Convective Weather Diagnostic (NCWD) data. The impact of weather on air traffic was analysed using Enhanced Traffic Management System (ETMS) data. ETMS data gives the radar positions of any aircraft detected in the NAS and based on this, the air traffic and flight routes were compared for a good weather day and a bad weather day.

Queuing models offer an efficient replacement to Monte Carlo simulations. To develop and implement a system which should be capable of handling 250% of the current capacity, the NAS has to be tested for flaws in the operational procedures and procedural, structural and operational changes have to be made to realise the objectives of NGATS. Queuing models offer this capability with development, analysis and implementation of traffic flow models. This research proposes to aid the realisation of NGATS by identifying distributions to develop the queuing model. Chapter 6 introduces us to topics which can be incorporated into the uncertainty model. Traffic flow patterns are identified and the impact of weather on traffic flow, service times are observed.
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CHAPTER 1
INTRODUCTION

This research works on the futuristic vision of the National Airspace System (NAS) towards increasing the operational capacity, safety and other areas identified in the Next Generation Air Transportation System (NGATS). In order to understand the NAS and its drawbacks and to understand the purpose of NGATS, we need a good understanding of the various components of NAS, its functionality and NGATS with its proposed areas of improvement. These topics are explained in detail in the following sections.

1.1 National Airspace System

The NAS consists of 21 centers within the US boundary as defined by FACET [1]. Each of these centers are further divided into sectors. Three levels of sectors are identified based on the operational altitudes. Low level sectors, high level sectors and super high level sectors. The NAS as of today contains the following [2]

- More than 750 Air Traffic Control (ATC) facilities
- Approximately 18,000 airports
- 4,500 air navigation facilities
- The NAS handles approximately 50,000 flights in a day over the US airspace
- 11.5 million take-offs and landings were reported in 2001

1.1.1 Components of the NAS

Air Traffic Control System Command Center (ATCSCC):

The ATCSCC is in charge of managing the air traffic flow within the NAS. The headquarters is situated in Herndon, Virginia. The ATCSCC takes action implementing ground delay programs, flight cancellations when unforeseen events occur which creates an imbalance between air traffic demand and capacity. These actions help reduce congestion and delays thus facilitating maximum use of the NAS.
Air Traffic Controlling Facilities (ATC):
The NAS has 21 Air Traffic Control Command Centers (ARTCC). Their primary concern is to separate and control the movement of aircraft within a specified airspace. The 21 ARTCC are located in the 21 centers in the US. Each of these centers employ 300 or more controllers, who guide aircraft towards their destination, reroute aircraft around bad weather and keep them safe from mid air accidents.

Terminal Radar Approach Control (TRACON):
These centers are located near airports and cover airspace of 50 mile radius or more. They might control the airspace of other airports as well if they are located within the 50 mile radius. The TRACON is responsible to the arrival sequencing at the airports. The ATM decision support tools have shown the capability to increase the arrival traffic throughput of TRACON facilities without impacting the air traffic controller work loads [3]. National Aeronautics and Space Administration (NASA) Ames Research Center and The Federal Aviation Administration (FAA) are developing...
decision supporting tools to aid the tactical control of TRACON departure traffic. Trajectory prediction algorithms are found in most of the core decision support tools [4]. The Expedite Departure Path (EDP) is a decision support tool that provides the TRACON traffic management coordinators with departure traffic loading, scheduling information and radar controllers with advisories for tactical management of terminal area departure traffic [5].

**Control Tower:**

Control towers manage the surface traffic (aircraft operations on the ground) and within a specified airspace around the airport. An important part of the functionality of the control tower is to manage proper spacing between aircraft taking off and landing.

1.2 Next Generation Air Transportation System (NGATS)

As a cornerstone of the U.S. economy, aviation is a key catalyst for economic growth and has a profound influence on our quality of life. The FAA and airline industry forecast that air traffic operations are expected to increase 150 to 250 percent over the next two decades [6]. There is not much of a change in the capacity over the last two decades although, there is a tremendous increase in the demand. The demand exceeds capacity in most of the European and American airports [7]. The demand for flight operations and the service offered to meet the demand has to be analysed. In the process, it becomes necessary to foresee the bottle necks in airport infrastructure and to hypothesize future scenarios to test if the infrastructure can support the growth [8]. It is necessary not only to automate the procedures to achieve adequate safety levels but also planning functions in order to increase the safety and the efficiency of the system [9].

Analysis of even the conservative growth estimate shows a significant lack of existing and planned capacity. With the current pace of increase in traffic we expect a grid lock in the sky. This might be a contribution of human factors, weather conditions and various other parameters causing the congestion. As a result the commission on Future of Aerospace in the United States has proposed a change in the air transportation system. The Joint Planning and Development Office (JPDO) is in charge of developing the vision for 2025 next generation air transportation [6].
1.2.1 NGATS Vision

The NGATS vision calls for a system-wide transformation leading to a new set of capabilities that will allow the system to respond to future needs of the nations air transportation. The main focus would be

- Communication and physical infrastructure
- The acceleration of automation and procedural changes based on 4-dimensional (4D) trajectory analyses to substantially increase capacity with safety and efficiency of the National Airspace System (NAS)
- Dynamic reconfiguration of airspace to be scalable to geographic and temporal demand.

1.3 Problem Definition and Motivation

Computer models and operations research methods are used to identify capacity limitations in the NAS. Alternative tools have to be explored and the focus has to shift from just one airport to the entire NAS. Parametric analysis is done to identify sensitive regions and certain flow restrictions acting on the NAS [10]. Considering the requirements of NGATS, a foundation has to be laid on which futuristic air traffic management procedures can be implemented and tested for delays.

Uncertainties in a flight trajectory contribute towards air traffic delays and they are cumulative. Chapters 2 and 3 give a range of flight time considering various uncertain factors. Trajectory uncertainties are due to aviation operations, precision of navigation and control. To be able to develop and implement the NGATS, traffic flow metrics are essential. Use of tools such as FACET enables us to implement and test various operational strategies and derive air traffic flow metrics. FACET on an average would require 8 hours to simulate air traffic for a single day. Computationally, it becomes expensive considering the number of simulations one would require in the process of conducting a Monte Carlo experiment.

An alternative approach would be to develop queuing models which can provide explicit relationships between traffic flow efficiency and trajectory uncertainties [11]. Traffic Flow Management (TFM) can be improved by understanding the accountability of uncertainty in the demand and capacity of NAS resources [12]. Ground delay programmes are simulated as single server queues with demand uncertainties in the form of cancellations, pop ups and aircraft arrival delays. Shirley and Sheldon [13] presents a finite capacity queue for Poisson arrivals which can be implemented for an airport. Gilbo [14] presented a model that considers runways, arrival and departure fixes as a
single resource system. The air traffic flow through the airport system is optimized by taking into account the runway capacity and the capacity of the fixes. Queuing models can be built on the specific requirements to model a section of the NAS.

The operational characteristics of a queuing model depends on two statistically important parameters namely the inter arrival times and the service time. For any system considered, the model can assume any distribution which has to be identified. This research proposes to present the distribution types for various phases of flight which can be used to develop the queuing model towards the realization of the NGATS.

The analysis tends to become complex by the inclusion of the uncertain parameters which contribute towards the service times and the inter arrival times. With no uncertainty, the service time distribution is straightforward based on the controlling inputs and the relation existing between the inputs and the service times. With uncertainty, the final service time distribution would be a combination of the various uncertain factors yielding a distribution whose type has to be identified and queuing models have to be derived based on these identified distributions. In order to account for uncertainty in flight time, we follow the steps explained below

- Identify the uncertain parameters and their distributions.
- Develop an error model that would consider the uncertain parameters varying within its specified range.
- Calculate the distribution for the service times and the inter arrival times.
- Identify the distribution, understand its characteristics and develop a queuing model.
- Further, use the queuing model parameters to implement and analyse operational strategies towards realization of the NGATS.

Chapter 2, 3 and 4 explain in detail the performance factors of aircraft that affect delays and development of a calculator that would calculate the service times. These models are modified to include the uncertain parameters and their contribution to the service times are obvious based on the results. Although the Figure 1.2 shows various uncertain parameters, this research focuses on aerodynamics, power plant and weight.

A section in chapter 6 deals with the affect of wind and weather on flight times and flight routes. The average time duration for a weather delay is over twice the average non weather delay. Over 70% of delays nowadays are due to weather [15]. Wind and weather are not incorporated in the calculator and the distribution for the service times does not have a factor of weather in
1.4 Air Traffic Delay Classification

Air traffic delays can be caused by various factors as a result of human errors or unforeseen circumstances. The air traffic network being so complicated, a delay introduced at one point in the NAS propagates through the system. Understanding the cause of the delay and taking the necessary steps to avoid or reduce such delays becomes very necessary. Given below are a few identified delays seen in the NAS [16]

A flight is considered delayed if it arrived at (or departed) the gate 15 minutes or more after the scheduled arrival (departure) time as reflected in the Computerized Reservation System (CRS) [16]. Delays occur when the demand at airports or airspace exceeds the available capacity. On an average, over 700 planes are delayed every day in the United States airspace [17]. There is a fine line between some delays coded as Weather (extreme weather) and others coded as NAS (non-extreme weather). The purpose of the two categories is to identify the party or organization in the best position to take corrective action. Delays or cancellations coded (extreme weather) cannot be reduced by corrective action. Delays or cancellations coded NAS are the type of weather delays that could be reduced with corrective action by the airports or the FAA.

**Air Carrier:** The cause of the cancellation or delay due to circumstances within the airline’s control (e.g. maintenance or crew problems, aircraft cleaning, baggage loading, fueling, etc.).

**Extreme Weather:** Significant meteorological conditions (actual or forecast) that, in the judge-
Table 1.1. History of Delays in NAS

<table>
<thead>
<tr>
<th>Delay</th>
<th>2003</th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
<th>2007</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air Carrier</td>
<td>26.3%</td>
<td>25.8%</td>
<td>28%</td>
<td>27.8%</td>
<td>28.0%</td>
</tr>
<tr>
<td>Aircraft Arriving Late</td>
<td>30.9%</td>
<td>33.6%</td>
<td>34.2%</td>
<td>37.0%</td>
<td>38.2%</td>
</tr>
<tr>
<td>Security</td>
<td>0.3%</td>
<td>0.3%</td>
<td>0.2%</td>
<td>0.3%</td>
<td>0.2%</td>
</tr>
<tr>
<td>NAS</td>
<td>36.5%</td>
<td>33.5%</td>
<td>31.4%</td>
<td>29.4%</td>
<td>27.6%</td>
</tr>
<tr>
<td>Extreme Weather</td>
<td>6.1%</td>
<td>6.9%</td>
<td>6.2%</td>
<td>5.6%</td>
<td>6.1%</td>
</tr>
</tbody>
</table>

ment of the carrier, delays or prevents the operation of a flight such as tornado, blizzard or hurricane.

**National Airspace System (NAS):** Delays and cancellations attributable to the NAS that refer to a broad set of conditions, such as non-extreme weather conditions, airport operations, heavy traffic volume, and air traffic control.

**Aircraft Arriving Late:** A previous flight with same aircraft arrived late, causing the present flight to depart late.

**Security:** Delays or cancellations caused by evacuation of a terminal or concourse, re-boarding of aircraft because of security breach, inoperative screening equipment and/or long lines in excess of 29 minutes at screening areas.

1.5 Delay Case Study

It is widely recognized that the capacity of the nation’s air traffic control system is now grossly inadequate, and that the major limitation to the system is the rate at which airports can land aircraft [18]. Few Key variables are identified whose alteration makes a huge difference in the service rate see at any airport. They are identified as given below

- Gate separation
- Time separation
- The aircraft velocity

Chapter 2 discusses in detail the performance of aircraft affecting delays. A detailed study is done on landing and departing aircraft which have to be scheduled according to their performances. A case study was performed to understand the factors that affect the flight planning vis-A-vis the aforementioned delay classification. This study gives us a better idea of the causes and effects of certain controllable and uncontrollable parameters. The DFW International airport was considered
Table 1.2. NAS Delay at Dallas Ft Worth International, January to July, 2007

<table>
<thead>
<tr>
<th></th>
<th>Number of Operations</th>
<th>Operations</th>
<th>Delayed Minutes</th>
<th>Minutes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weather</td>
<td>12,571</td>
<td>77.81%</td>
<td>676,857</td>
<td>85.82%</td>
</tr>
<tr>
<td>Volume</td>
<td>1,517</td>
<td>9.39%</td>
<td>45,458</td>
<td>5.76%</td>
</tr>
<tr>
<td>Equipment</td>
<td>16</td>
<td>0.10%</td>
<td>570</td>
<td>0.0</td>
</tr>
<tr>
<td>Closed Runway</td>
<td>1,369</td>
<td>8.47%</td>
<td>41,592</td>
<td>5.27%</td>
</tr>
<tr>
<td>Other</td>
<td>684</td>
<td>4.23%</td>
<td>24,250</td>
<td>3.07%</td>
</tr>
<tr>
<td>Total Operations</td>
<td>16,157</td>
<td>100.00%</td>
<td>788,727</td>
<td>100.00%</td>
</tr>
</tbody>
</table>

Table 1.3. On Time Arrival Performance, DFW International, January to July, 2007

<table>
<thead>
<tr>
<th></th>
<th>Number of Operations</th>
<th>Operations</th>
<th>Delayed Minutes</th>
<th>Minutes</th>
</tr>
</thead>
<tbody>
<tr>
<td>On Time</td>
<td>119,390</td>
<td>68.79%</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>Air Carrier Delay</td>
<td>8,574</td>
<td>4.94%</td>
<td>623,977</td>
<td>21.97%</td>
</tr>
<tr>
<td>Weather</td>
<td>2,974</td>
<td>1.71%</td>
<td>238,230</td>
<td>8.39%</td>
</tr>
<tr>
<td>NAS</td>
<td>16,157</td>
<td>9.31%</td>
<td>788,727</td>
<td>27.78%</td>
</tr>
<tr>
<td>Security</td>
<td>60</td>
<td>0.03%</td>
<td>2,162</td>
<td>0.08%</td>
</tr>
<tr>
<td>Aircraft</td>
<td>17,635</td>
<td>10.16%</td>
<td>1,186,505</td>
<td>41.78%</td>
</tr>
<tr>
<td>Arriving Late</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cancelled</td>
<td>7,287</td>
<td>4.20%</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>Diverted</td>
<td>1,484</td>
<td>0.86%</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>Total Operations</td>
<td>173,561</td>
<td>100.00%</td>
<td>2,839,601</td>
<td>100.00%</td>
</tr>
</tbody>
</table>

for the study and the delays encountered therein were classified. *(All data presented are obtained from the Bureau of Transportation Statistics [16])*

Air Traffic delays in general occur when the demand exceeds the airport or the airspace capacity. Traffic flow management deals with trying to balance the two. Increasing the capacity is an important solution with options of major changes in operational procedures, facility constructions and improvements in navigational equipments. For short term changes or for immediate solutions, tactical optimisation models are used [19]. These models become complex with the involvement of uncertainties. The uncertain parameters involved in flight are identified in the following chapters. Their sensitivities are analysed leading to uncertainty analysis. Finally, the practical performance of the aircraft performance calculator is compared with ETMS data.
CHAPTER 2
AIRCRAFT PERFORMANCE

2.1 Introduction

The mission profile of a typical flight is shown in Figure 2.1. Every aircraft flies a similar mission profile under ATM [20], airline planning and aircraft performance constraints. The performance of an aircraft in each phase of its flight mission, especially during climb and cruise, is vital. Since the trajectories of all aircraft within the same airspace interfere with each other’s trajectories causing congestion, it is normal to have delays at each phase of flight for every aircraft. Statistical data indicates that the fastest aircraft are 395% faster than the slower ones. The slower aircraft incur a delay six times more than the faster ones (statistics obtained from [20]). The costs incurred due to delays have been quantified by Eurocontrol

- 5.73 billion Euros/year produced by air traffic control delays (1999)
- $2 billion/year produced by longer trajectories due to the fixed airways network (in Europe)
- $10 billion/year due to air traffic control actions that generate deviations from optimal trajectories

The objective here is to study the trajectory uncertainty models that will augment the queuing models for the NAS. We use the European Organization for the Safety of Air Navigation, Eurocontrol Experimental Centres BADA data [21] to study the performance of aircraft in various phases of flight.

Figure 2.1. A Typical Flight Profile.
2.1.1 Aircraft Performance Parameters and Evaluation

Along with prevailing conditions such as security delays, taxi out delay, etc. which add to the cause, aircraft performance plays a major role in contributing to the overall delays [22]. We try to relate the performance characteristics with the delays encountered. (All the data used for the calculations are taken from BADA performance characteristic table). Further explanatory notes on the data presented in the performance tables are given below:

- Cruise data is only specified for flight levels greater than or equal to 30.
- Performance data is specified up to a maximum Flight Level (FL) of 400 or to highest level for which a positive rate of climb can be achieved at the low mass.
- True Air Speed for climb, cruise and descent is determined based on the speed schedules specified in Sections 4.1, 4.2 and 4.3 respectively of the BADA User Manual.
- Rates of climb are calculated at each flight level assuming the energy share factors associated with constant Calibrated Air Speed (CAS) or constant Mach speed laws.
• The fuel consumption in climb is independent of the aircraft mass and thus only one value is given. There are three different climb rates however corresponding to low, nominal and high mass conditions.

• The rate of descent and fuel consumption in descent is calculated assuming the nominal mass. Values for other mass conditions are not given.

Discontinuities in climb rate can occur for the following reasons:

• Change in speed between flight levels (e.g. removal of 250 knot restriction above FL100)

• Transition from constant CAS to constant Mach (typically around FL300)

• Transition through the tropopause (FL360 for ISA)

Discontinuities in descent rate can occur for the following reasons:

• Transition through tropopause (FL360 for ISA)

• Transition from constant Mach to constant CAS

• Change in assumed descent thrust (specified by the BADA h parameter)

• Change in speed between flight levels (e.g. application of 250 knot limit below FL100)

2.1.1.1 Aircraft Performance Parameters

Climb and descent are two important phases in a flight profile. Depending on the category of aircraft, the climb time can vary from 15 minutes to 45 minutes for the aircraft considered in this study. The engine power and the acceleration determine the rate of climb. Given below is the equation for the rate of climb \[21\]

\[r = \frac{(T - D) V}{W - \frac{V}{g} \frac{dV}{dt}}\]  

(2.1)

Where,

\(r\) : rate of climb/descent (ft/min)

\(T\) : thrust[Newtons]

\(D\) : drag force[Newtons]

\(V\) : air speed[Knots]

\(W\) : weight of the aircraft[Kg]

• The rate of climb is a function of the weight. The weight varies as the aircraft progresses into its flight route. This adds to the uncertainty in the projected time duration.
Table 2.1. Nominal Mass Levels (nominal loads) for Take-off and Climb

<table>
<thead>
<tr>
<th></th>
<th>Mass Levels(Kg)</th>
<th>Max Altitude(Feet)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A340</td>
<td>200,000</td>
<td>41,000</td>
</tr>
<tr>
<td>B737</td>
<td>60,000</td>
<td>41,000</td>
</tr>
<tr>
<td>MD82</td>
<td>55,000</td>
<td>37,000</td>
</tr>
</tbody>
</table>

- The rate of climb and the cruise speed are also a function of the altitude as seen from the BADA data.
- Study and prediction of delays in this effort, incorporates the uncertainties such as weight variation along the trajectory.

In addition to the above, performance of a specific aircraft affects other aircraft in the operating scenario which has to be accounted for in the delay analysis.

2.2 Departure and Arrival Analysis

Controlling the flow of air traffic strategically such that the demand meets the operational capacity and does not exceed it is one of the major goals of ATM [23]. Varying methods are seen to optimise the airport capacity. Gilbo [23] presents a technique where arrival and departure operations are treated as interdependent processes and the airport capacity is strategically allocated between arrivals and departures. Before take-off, an aircraft taxis to the runway and in most occasions, the take-off of an aircraft is delayed by a few minutes due to the operational capacity of the runway [18]. There are a number of factors that contribute to the runway capacity. In this example, we analyse a queue of three aircraft scheduled to take off simultaneously. The performance characteristics of one aircraft will affect the performance of the other.

- For the current scenario, we consider an A343, B737 and a MD82
- The choice of the above was to include a heavy aircraft, a light aircraft and an aircraft used most in number.
- The American airlines constitutes to a majority of the US airspace and almost 50% of their fleet consists of MD80/83s

As seen in Equation 2.1, the weight of an aircraft is inversely proportional to the rate of climb. A343 weighing the highest has the least rate of climb followed by the MD82 and the B737. Another significant factor determining aircraft separation during take-off and climb is the *wake turbulence*. 
Wake turbulence is caused by a pair of counter rotating vortices trailing from the wing tips of an aircraft. It is a function of the weight, airspeed and wing design of the aircraft. The vortices from larger aircraft pose problems for the following aircraft. Turbulence generated within the vortices can damage aircraft if encountered at close range and the separation distance is shown in the Table 2.2.

### Table 2.2. Wake Separation Distance

<table>
<thead>
<tr>
<th>Weight(Kg)</th>
<th>Distance ( S ) (nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( W_i &gt; 136,000 ) and ( W_{i+1} &gt; 136000 )</td>
<td>( S_{i+1} = 4 )</td>
</tr>
<tr>
<td>( W_i &gt; 136,000 ) and ( 7000 &lt; W_{i+1} \leq 136000 )</td>
<td>( S_{i+1} = 5 )</td>
</tr>
<tr>
<td>( W_i &gt; 136,000 ) and ( W_{i+1} &lt; 7000 )</td>
<td>( S_{i+1} = 6 )</td>
</tr>
<tr>
<td>( 7000 &lt; W_i \leq 136000 ) and ( W_{i+1} &gt; 136000 )</td>
<td>( S_{i+1} = 3 )</td>
</tr>
<tr>
<td>( 7000 &lt; W_i \leq 136000 ) and ( 7000 &lt; W_{i+1} \leq 136000 )</td>
<td>( S_{i+1} = 5 )</td>
</tr>
<tr>
<td>( W_i &lt; 7000 ) and ( W_{i+1} &lt; 7000 )</td>
<td>( S_{i+1} = 3 )</td>
</tr>
</tbody>
</table>

Figure 2.3. Rate of Climb and Speed Comparison.

2.2.1 Performance Comparison

The rate of climb and the cruise velocity for these three aircraft were computed and compared. The Figure 2.2.1 describes the comparison

- For each aircraft, BADA Aircraft performance summary tables specify the speed and rate of climb and cruise at various flight levels.
- It is assumed that the rate of climb decreases linearly with altitude between two succeeding flights.
Table 2.3. Departure Delay

<table>
<thead>
<tr>
<th>Aircraft</th>
<th>Estimated departure delay (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A340</td>
<td>00</td>
</tr>
<tr>
<td>B737</td>
<td>116</td>
</tr>
<tr>
<td>MD83</td>
<td>74</td>
</tr>
</tbody>
</table>

- The wake turbulence separation is obtained from the table and from the BADA performance data, the take off–air speed is used to calculate the time separation required.

The time separation between the two aircraft is

\[ T_{i+1} = \frac{S_{i,i+1}}{V_{oi}} \]  

Where,

- \( S_{i,i+1} \): wake turbulence separation between \( i^{th} \) and the \((i + 1)^{st}\) aircraft (nm)
- \( V_{oi} \): Climb speed at runway elevation altitude of the \( i^{th} \) aircraft (knots)

2.2.2 Departure Delay

Applying the above to our queue of aircraft, for a B737 following an A340, \( S = 5 \text{ nm} \) and \( V = 158 \text{ knots} \). Using the time separation equation we get the values listed in Table 2.3.

A B737 takes off 116 seconds after an A340 takes off. Similarly an MD83 cannot take–off for 74 seconds after a B737 has taken off. Total delay for 3 flights is 5 minutes.

2.2.3 Arrival Delay

We try to relate the performance characteristics with the arrival delays with the following assumptions in the model [18].

- Aircraft land in the order in which they arrive at the entry gate (i.e., no aircraft may be passed after it has crossed the gate).
- Aircraft arrive at the gate independently and in random sequence.
- Aircraft must maintain a minimum distance separation at the gate and a minimum time separation at the runway.
- The runway is used only for landing and is operating to capacity, i.e., aircraft operate as close to each other as minimum separation permits.
• Aircraft maintain constant velocity from the time they enter the gate until they reach the runway.
• All flights scheduled to land at the same time and only one operational runway. Ideal Instrument Flight Rule (IFR) conditions assumed
• Time separation: 60 sec
• Gate separation: 3 miles
• Distance btw gate and the airport: 10 miles

The time separation at the airport is given by

\[ T = \frac{n}{V_2} - \frac{m}{V_1} \]  

(2.3)

Where,

- \( V_1 \): Velocity of aircraft 1 (knots)
- \( V_2 \): Velocity of aircraft 2 (knots)
- \( M \): Distance between the gate and the airport (miles)
- \( n : m + \) Gate separation (miles)

According to IFR conditions, \( m = 10 \) miles and \( n = 13 \) miles. We define a velocity \( V^* \) such that, during the course of landing, the gate separation is altered so \( T \) is maintained a constant

\[ V^* = \frac{nV_1}{V_1T + m} \]  

(2.4)

If \( V^* > V_2 \), time separation increases and its satisfied by the equation (2.3). If \( V^* < V_2 \), time separation is still 1 min (60 sec), the aircraft flies at a reduced speed to achieve the 1 minute separation.

The significance of the above is that, a lower value of \( V_2 \) gives a time separation greater than the set time \( T \). A value of \( V_2 \) greater than \( V_1 \) signifies that the aircraft should slow down from its normal velocity to achieve a constant time \( T \). In such cases the gate separation should be varied to suit the velocity of the aircraft.

**Sequence of Service:** A340-B737-MD83 Using the time and velocity equations, we obtain the velocity for the specific aircraft from BADA data

• Estimated Arrival delay for B737 = 89 seconds.
• The estimated arrival delay for the MD83 = 149 seconds.
• The MD83 would be serviced after a 1 minute separation from when the B737 was serviced.

**Sequence of Service:** MD83-B737
• This is a specific case where a low powered aircraft was following a relatively higher powered aircraft.

• This would be the initial case of $V^* > V_2$ and the aircraft experienced a delay of more than a minute. To be precise, B737 was delayed by 73 seconds.

Airline flight schedules are particularly sensitive to individual flight delays because of the manner in which operating resources are linked together. Among the connective resources affected by delayed flight operations are crews, aircraft, passengers and gate space. Due to the application of separation minima in consideration of collision risk and wake turbulence, the trajectories of all aircraft in flight within the airspace are constrained by the trajectories of others.

The assumptions we use to analyse the arrival delays, actually add to the uncertainty. In reality the velocity of the aircraft vary as it approaches landing. Although individually it might be a negligible change and does not amount to much, such changes are cumulative and propagate to make a significant difference in the expected and real time operations.

A generalized study has shown that a 2 mile distance between gate and airport runway would give a time separation of approx 30 to 65 seconds [18]. If the velocity of the aircraft that are landing is around 80 to 140 knots, the number of landings are known to reduce by 22% while the number of landings are expected to increase with the gate at 10 miles from the runway and the aircraft velocity 100 to 120 knots. Under typical IFR conditions, a runway can expect 30 to 40 landings per hour if the distance between the gate and the airport is 10 miles and the separation between two aircraft is approximately 3 miles.
CHAPTER 3
SENSITIVITY ANALYSIS

The aircraft performance characteristics play a significant role in air traffic delays and we evaluate the functional dependence of time-to-climb, time-to-cruise and the time-to-descend on parameters such as the weight, velocity and power factor. The study reveals key information as to which of the aviation operations parameters are most significant in the uncertainty analysis. Also, since a functional dependence (an empirical fit) is determined, distributions of the various time passages are simple transformations of whatever the distributions of the underlying parameters are assumed to be. Monte Carlo experiment was conducted to determine the sensitivities. Based on the simulation results, we were able to identify the important parameters that affect the flight times in the various phases of flight.

Design of experiments method was chosen to reduce the computation time and the number of runs required to conduct the Monte Carlo experiment reduced from 10,000 to 600. To ensure a systemic sampling of the uncertain distributions, Latin Hypercube Sampling (LHS) technique was used to sample data. Once these stochastic models are developed on the basis of the trajectory uncertainty, they will be related to queuing network models to model the US airspace. These models will be further used to ascertain the traffic flow and its efficiency.

3.1 Aircraft Performance Calculator

Eurocontrol’s BADA data was used to study the performance of aircraft in various phases of flight. To match the results and maintain consistency, the actual time to climb, cruise and descent equations were programmed based on the BADA models, in essence duplicating the BADA calculator on MATLAB.

3.1.1 Total Energy Model

Although trajectory predictions in the air traffic system are based on aircraft type [21], performance of individual aircraft is uncertain due to variations in weight, power plant, and aerodynamics. This study would reveal key information as to which of the aviation operations parameters is most
significant in the uncertainty analysis. With the calculator, any physical quantity that affects the flight of an aircraft could be varied and the performance of the aircraft can be studied. The total energy model was used to develop the calculator. It equates the work done by forces acting in the aircraft to the rate of increase in potential and kinetic energy.

\[(T - D)V_{TAS} = mg \frac{dh}{dt} + mV_{TAS} \frac{dV_{TAS}}{dt}\]  

(3.1)

where

- \( T \): Thrust acting along the aircraft velocity vector [Newtons]
- \( D \): Aerodynamic drag [N]
- \( m \): Aircraft mass [Kg]
- \( h \): Altitude [m]
- \( g \): Acceleration due to gravity [m/s\(^2\)]
- \( V_{TAS} \): True airspeed [m/s]

The BADA user manual gives the aircraft equations of motion defining the thrust, drag, fuel consumption and other required parameters. Complex relations exist between the rate of climb/descent with the mass of the aircraft. The climb and descent regulations (specified by BADA) explained in Chapter 2 are incorporated in the calculator. A detailed explanation on the energy model, power-velocity relations, cruise and descent performances can be found in the Appendix A. A few of the important parameters whose variation can be tracked with the calculator are

- Weight of the aircraft,
- Velocity of the aircraft and altitude, fuel consumption, rate of climb, rate of descent, power output of the engines
- Altitude depended temperature, pressure, density of air etc.

3.1.2 Validation of the Calculator with BADA

The BADA aircraft performance tables were compared against the calculator results for the validation of the calculator. We considered the following:

**Aircraft:** MD82

**Weight of the aircraft:** 55000 Kg

**Cruise altitude:** 32000 feet
Figure 3.1. Validation of the Calculator Against BADA.

Figure 3.1 shows the comparison and is observed that the calculator follows the BADA table very closely. The time taken to climb to 32,000 feet according to the calculator was 17.7 minutes as compared to 16.8 minutes from the BADA performance table. Our implementation of the BADA calculator shows a variation (from the BADA calculations) of approximately two seconds for every 1000 feet gain in altitude.

3.2 Sensitivity Analysis

Sensitivity analysis was done to identify those parameters which have significant impact on the time of an aircraft in various phases of flight. To start with, we identified three parameters which might play a significant role.

Weight: The rate of climb and rate of descent are closely related to the weight of an aircraft which is an uncertain factor. Almost 30,000 Kgs of fuel goes into an aircraft such as an MD82 which
accounts to almost 50 percent of the total weight of the aircraft when completely loaded. Based on
the flight plan and the distance to cover, fuel is appropriately loaded. An aircraft probably weighs
only 70% of its initial weight when it is in its approach to land in most cases.

**Lift co-efficient:** An aircraft changes its physical configuration frequently based on its flight plan
or depending on the phase of flight. The elevators, the ailerons change the lift acting on the aircraft.
Apart from this, the lift co-efficient changes based on the air pressure and density. A Monte Carlo
experiment was conducted to gauge the impact of varying lift co-efficient on the time taken to climb,
cruise or descent.

**Power factor:** Although in most cases, the aircraft is required to take off from the runway at its
maximum thrust to avail the maximum rate of climb thus easing departures, the airlines usually take
off with a reduced power climb to increase the life span of the engines [21]. This is not a documented
event and this changes the time to climb.

3.2.1 Weight Sensitivity Analysis

The objective is to analyse the variation in time taken to climb with the variation in the
weight, which can be calculated for different aircraft by specifying certain parameters such as the
take off weight and the altitude. An MD82 was considered for the case study.

Certain assumptions were made on the take-off weight of the aircraft. The calculator is
built to input weight as sampled from any distribution and in this case a non-central-chi-squared
distribution was assumed for the weight. The curve for the time to climb was seen to follow a first
order polynomial fit. The method of linear least squares estimation was used and the coefficients
were calculated and were used to predict the time to climb for other weight configurations of an
MD82. Figure 3.2 gives the graphs for this analysis.

3.2.1.1 Prediction of the Performance Calculator

The calculator was run for specific cases for an MD82, A343 and a B737. The time to climb
was calculated for varying weights, lift coefficient and the power factor. The change in the time
to climb was noted and a linear 2\textsuperscript{nd} order polynomial fit was used to predict the data for the
three aircraft under consideration and the plots for the same are shown below. The aircraft were
to climb to a cruise altitude of 32,000 feet. The co-efficients for the 2nd order polynomial fit are
in the ascending order of the powers of weight. Figure 3.3 shows the fit for all the three aircraft
and Equation 3.2 gives the quadratic equation for the time to climb. The polynomial fit yielded $a_0 = \{619.6, 516.3, 254.2\}$, $a_1 = \{0.015, -0.002, -0.001\}$ for an MD82, A343 and B737 respectively.

$$t_c = a_0 + a_1 W_{TO} + a_2 W_{TO}^2$$ (3.2)

Where,

$t_c$ : Time to climb for the specific aircraft (min)

$W_{TO}$ : Weight at take-off (Kg)

Figures 3.3 clearly indicate the similarity in the time to climb trends as a function of the weights alone despite differences in the power, aerodynamics and other aviation operations param-
Figure 3.3. Distribution and Polynomial Fit for an A343, B737 and MD82 respectively.
eters for the different aircraft. In any case this is an expected result and a simple quadratic fit was deemed sufficient.

3.2.2 Thrust and Lift Coefficient Sensitivity Analysis

The BADA calculator coded in MATLAB gives the time to climb for ideal conditions seen in the flight manuals. But in reality, the airlines operate on reduced power to increase the engine life. The sensitivity analysis was done by varying the power from 80% to 100% and figure 3.4 shows the same for different weight configurations.

Similarly the lift co-efficient was varied from 80% to 120% of its value and the time to climb for these specific cases were calculated. Figure 3.5 gives the graph for time to climb to 32,000 feet with varying lift-coefficients for various weight configurations.

The velocity of the aircraft depends on lift coefficient and so does the induced drag which translates to varying time to climb. It is to be mentioned however that the variation in the time to climb is not as significant as the time to climb dependency on weight and for a 25% deviation from nominal lift coefficient, the values are within 2 minutes. The figure 3.6 reveal this trend.

Similar to the analysis on weight sensitivity, a 2nd degree polynomial fit was obtained for the time to climb curves with varying power. The 2nd order polynomial fit for time to climb yielded
Figure 3.5. Lift Co-efficient Sensitivity for Time to Climb.

Table 3.1. Polynomial Fit - Lift Co-efficient

<table>
<thead>
<tr>
<th>Aircraft</th>
<th>$a_0$</th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$a_3$</th>
<th>$a_4$</th>
<th>$a_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>MD82</td>
<td>8078</td>
<td>-25423</td>
<td>36763</td>
<td>-26630</td>
<td>9728</td>
<td>-1420</td>
</tr>
<tr>
<td>A343</td>
<td>14078</td>
<td>-48878</td>
<td>74160</td>
<td>-56469</td>
<td>21616</td>
<td>-3309</td>
</tr>
<tr>
<td>B737</td>
<td>8587</td>
<td>-30084</td>
<td>46484</td>
<td>-35977</td>
<td>13968</td>
<td>-2167</td>
</tr>
</tbody>
</table>

$a_0 = \{3477.3, 3880.7, 3071.6\}$, $a_1 = \{-3742.7, -4243.3, -4818.4\}$ and $a_2 = \{1363.6, 1564.7, 2567.0\}$ for an MD82, A343 and B737 respectively.

A 2$^{nd}$ order polynomial fit would be insufficient for the time to climb with varying lift-coefficient. Hence a 5$^{th}$ order polynomial fit was tried and the prediction was accurate to follow the actual curve. The co-efficient values are shown in Table 3.1

3.3 Observations and Conclusions

**Weight Sensitivity**

The weight has a significant impact on time to climb which varies over a range of 10-12 minutes for different weights. The time to climb for a heavy MD82 is almost twice the duration for an MD82 lightly loaded on fuel. The time to climb is sensitive to weight and the determination of the exact weight is important for an accurate prediction of time to climb.
Figure 3.6. Polynomial Fit for an A343, B737 and MD82 Respectively.
**Power Plant**

The power plant determines the rate of climb and any variation in the power plant triggers a change in several other parameters which contribute to the change in the time to climb. It can be seen that, power reduced to 80% of the actual power produces a delay of about 3-5 minutes for the 3 aircraft considered. Unlike weight, the time to climb is not too sensitive to power factor considering that weight uncertainty induces variation in time to climb by almost 50%.

**Lift Co-efficient**

We notice that the times to climb are close together for values of $C_{LTO}$ which range from 90% to 110% of the actual value. A significant difference is noticed only when the $C_{LTO}$ drops to 80% and below of the true lift co-efficient where the time to climb increases by approximately 20 to 50 seconds (less than a minute). In effect, this is expected as the maximum benefit of clean aerodynamics is obtained during the cruise phase through fuel optimality.
CHAPTER 4
TRAJECTORY UNCERTAINTY ANALYSIS

The uncertain parameters mentioned in chapter 3 were used to calculate the uncertainty in time to climb, cruise and descend. We know the individual distributions for the uncertain parameters. But the question is, when all these three parameters are involved, what would the resulting time distribution look like? This analysis becomes important from the point of view of developing queuing models that includes uncertainties. Previous study has shown that the departure delay is better modelled with a Poisson distribution [24]. This chapter discusses the distribution fit for time to climb, descent and cruise distributions.

4.1 Monte Carlo Experiment

A Monte Carlo experiment with 10,000 runs was an initial start towards the analysis. The number of samples required to capture the behaviour of the distribution was a guess. Further, this was optimized using the confidence interval methods and Latin Hypercube Sampling (LHS) techniques to reduce the number of runs and thus save on the computation power required.

4.1.1 Confidence Intervals

Confidence intervals are used to determine the reliability of the estimate. For the current case, we do a matching of the means. That is, we define an interval around the mean which would house the mean for a calculated number of samples (new) with a predefined confidence level [25]. Equation 4.1 gives the relationship between sample size, the distribution standard deviation and the error range.

\[ n = \left( \frac{Z \sigma}{E} \right)^2 \]  

(4.1)

Where

- \( n \): Sample size (minimum number of runs required to obtain a statistically valid result through Monte Carlo)
- \( Z \): Area under the curve for a normal distribution
$\sigma$: standard deviation of the parameter for a known population.

$E$: confidence interval: Offers a $+/- E$ for the mean of the calculated population.

The $Z$ value is assumed to be from a normal distribution and this is valid because, although some parameters like the weight are assumed to be non-central-chi-squared variable, according to central limit theorem, the mean of all identically distributed independent random variables approaches normal distribution [26]. With this assumption, we go ahead with the above mentioned method to obtain the number of runs required.

For all the current cases, we define the confidence level of 95% and the confidence interval is dependent on the parameter under consideration. From a normal distribution table, for a value of $\alpha = 0.05$, the table gives us 1.96 for $Z_{\alpha/2}$

The three uncertain parameters will have a specific number for minimum runs depending on the possible error and the standard deviation of the unknown distribution. The following sections explains how we arrive at a sample size needed to conduct the Monte Carlo experiment.

**Sample Size for Weight**

For an MD82, a non-central-chi-squared variable, its weight varies from 53 tons to 56 tons. For 10,000 samples, the standard deviation was found to be 470 Kg which was the same as the standard deviation for a population of one million. The error around the mean has an acceptable range of 50 Kg.

$\sigma$: 470 Kg

$E$: 38 Kg

Minimum number of samples required: 590 $\sim$ 600 runs.

**Sample Size for Power Factor and Lift Co-efficient**

Both of these parameters are just the multiplying factor of the actual values and not the values itself. For a normal distribution, the following values were considered

$\sigma$: 0.2

$E$: 0.02

$\mu$: 1

Minimum number of samples required: 272 $\sim$ 300 runs.

In order to be able to choose a good number of samples to satisfy both the parameters, we choose the highest of the sample size and thus, the minimum number of runs required to conduct a Monte Carlo experiment is 600 runs.
4.1.2 Latin Hypercube Sampling (LHS)

Latin Hypercube Sampling is a technique employed to reduce the number of samples required to conduct a Monte Carlo experiment. LHS is a systematic way of sampling the distribution so that the characteristics of the distribution are retained but with a reduced number of samples [27].

The distribution is divided into equi-probable segments based on the number of samples required [28]. A sample is picked from each segment and the methodology of picking a sample from the segment varies. A random process might be used or the midpoint can be chosen. Considering the case shown below for the weight distribution of an A343, the distribution is divided into 20 equi-probable segments. Further, a single sample is picked from each of these segments thus arriving at 20 samples as seen in Figure 4.1.2. The processes of selecting a sample from the many samples available in the chosen segment are many. In the current study, the midpoint of each segment is chosen and the weight corresponding to the chosen probability is used as an input to the calculator. This method proved to save on computation by reducing the required number of samples to be able to capture the entire distribution.

**Latin Hypercube Sampling Validation**

LHS technique of sampling should reduce the number of samples required by a process of systematic sampling of the chosen distribution. The initial Monte Carlo experiments were done with 10,000 samples. With the use of LHS and the confidence interval method, the number of
samples were reduced to 600. The characteristics of the distribution remained unchanged with the use of LHS. Table 4.1 shows the validation for two distributions namely Rayleigh (Wind) and Non-Central-Chi-Squared (Weight).

It can be seen from Table 4.1 that the two results have a negligible difference which can be attributed to the reduction in the number of samples. It can be concluded that the code developed to produce Latin hypercube samples retains the characteristics of the actual distribution meanwhile reducing the number of samples required to obtain at the same results.

### 4.2 Uncertainty Analysis on Climb

The uncertainty analysis was done on the three aircraft under consideration, i.e. MD82, A343 and a B737. The uncertainty was seen in the weights, power factor and the lift co-efficient. The weight of an aircraft at take-off was assumed to follow a non-central-chi-squared distribution. Most of the airlines follow a reduced power climb to increase the life span of the engines. The unavailability of statistical data made us assume a Gaussian distribution for the power uncertainty. A similar Gaussian distribution assumed for the lift co-efficient.

The probability density function of the time to climb was obtained and the distribution type had to be identified. The parameters used to fit the distribution were identified and an initial guess was made on the values. We used a non-linear least squares optimization to optimize the cost function and calculate the distribution parameters.
Figure 4.2. Uncertainty Analysis on MD82, B737 and A343.

The inputs to the aircraft performance calculator were samples from individual distributions for weight, lift coefficient and the power factor. The calculator was run the required number of times and the distribution for the time to climb and the uncertain parameters are shown in Figures 4.2.

4.2.1 Distribution Type

The time to climb, as seen in the histogram in Figure 4.2, shows a steep peak and a relatively thin and a long tail. A specific case of the time to climb for an MD82 was considered for the distribution identification. A visual analysis of the climb distribution led us to consider Erlang, Gamma, F type, Weibull, Rayleigh, Inverse Gaussian and Beta distributions. PDF matching method
Table 4.2. Distribution Parameters

<table>
<thead>
<tr>
<th></th>
<th>Initial guess</th>
<th>Optimised value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Erlang</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\lambda$</td>
<td>1.77</td>
<td>1.94</td>
</tr>
<tr>
<td>$k$</td>
<td>3</td>
<td>30</td>
</tr>
<tr>
<td><strong>Gamma</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$k$</td>
<td>20.522</td>
<td>20.6</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.74</td>
<td>0.75</td>
</tr>
<tr>
<td><strong>Inverse Gaussian</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu$</td>
<td>16</td>
<td>15.74</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>331</td>
<td>339.27</td>
</tr>
</tbody>
</table>

was used to identify the distribution and the equations for Erlang, Gamma and Inverse Gaussian are presented in this section.

**Erlang Distribution**

\[
f(x; k, \lambda) = \frac{\lambda^k x^{k-1} e^{-\lambda x}}{(k-1)!}
\]  

(4.2)

Where $x > 0$

$x$ is the time to climb and $\lambda$ is the rate parameter and $k$, the shape parameter. We started with an initial value $\lambda = 5$ and $k = 1$ and the optimized values obtained are shown in Table 4.2.

**Gamma Distribution**

\[
f(x; k, \theta) = x^{(k-1)} \frac{e^{-\frac{x}{\theta}}}{\theta^k \Gamma(k)}
\]  

(4.3)

Where $x, k, \theta > 0$.

$k$ is the shaping parameter and $\theta$ is the scaling parameter.

**Inverse Gaussian Distribution**

\[
f(x; k, \lambda) = \left( \frac{\lambda}{2\pi x^3} \right)^{\frac{1}{2}} e^{-\frac{-\lambda (x - \mu)^2}{2\mu^2 x}}
\]  

(4.4)

Where $x, \lambda, \mu > 0$. $\lambda$ is the shaping parameter and $\mu$ is the mean.

It was seen that the Erlang, Gamma and the Inverse Gaussian distributions produced the best fit with the least cost functions. The Monte Carlo simulations were rerun for a reduced number of samples using the LHS technique and the results are shown in Figure 4.3 where a comparison between the distribution fit for an MD82 is also seen.

In Table 4.3, although the Erlang distribution fits with the least cost function, the variance is off the true value. The Gamma Distribution improves on the cost function but the mean and the variance shows a bad approximation. The Inverse Gaussian has a better cost function than the Gamma function and also a better approximation on the mean and the variance. Amongst all the distributions considered, the Inverse Gaussian fits best.
Sensitivity analysis in Chapter 3 proved that the time to climb is most sensitive to weight which is a non-central-chi-squared parameter which is basically a derivative of a multivariate Gaussian distribution. The non-central-chi-squared distribution is the distance from the origin of a random vector, in this case, the distribution of weight with a non zero mean. Time to climb is directly proportional to weight and from Chapter 3, the contribution of lift co-efficient and power factor (varied within 0.8 to 1) is not significant towards the uncertainty in climb time. Thus the time to climb can be assumed to be a function mostly dependent on weight of the aircraft. The optimised value of $\lambda$ for the time to climb distribution was 339 which is greater than 0. As $\lambda$ tends to infinity, the Inverse Gaussian assumes a Normal (Gaussian) distribution and in this case, weight, a
non-central-chi-squared parameter which is a derivative of a multivariate Gaussian distribution thus explaining the reason for Inverse Gaussian producing the best fit. The above analysis was extended for A343 and B737. The results for these two categories of aircraft are shown in Table 4.4.

### 4.2.2 Coxian Distribution

The Coxian distribution is the most general form of phase distributions. Any distribution function can be approximated arbitrarily closely by a Coxian distribution [29]. Its characteristic function can be written as shown in Equation 4.5
Where \(a_i\), \(b_i\) and \(c_i\) are the function parameters by which the distribution properties are determined. The Coxian distribution is basically a series of step functions, whose amplitude and the number of steps have to be suitably chosen so as to approximate it to the target distribution.

The Coxian distribution with two phases is commonly used for approximating a simple distribution, since its parameter calculation procedure is straightforward and the analytical closed-form solution for the Coxian function parameters are available. Here the PDF matching method of function approximation is used. The PDF matching will approximate a target PDF on distributed collocation points by adjusting the Coxian distribution parameters.

We make use of the shaping parameter \(k\) to match the Coxian PDF with the target PDF and \(k\) represents the number of phases used to approximate the distribution. The time to climb has a distribution with a thin peak and a long and slender tail. We can obtain a better approximation by increasing the number of phases. This is shown in the two plots below. The number of phases is increased until the cost function is reduced suitably. Since the time to climb ranges over a wide period, we get a better approximation if we increase the number of phases. This is seen in Figure 4.4

The Coxian distribution can be used to approximate the Inverse Gaussian distribution \([30]\) and our previous studies have shown that the Inverse Gaussian produces the best fit. The Coxian fit (error = 0.0479) has an improved error over the Inverse Gaussian.
Table 4.5. \( \mu \) for Coxian Distribution

<table>
<thead>
<tr>
<th>Phase</th>
<th>( \mu ) (min)</th>
<th>Phase</th>
<th>( \mu ) (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-10</td>
<td>1.1626</td>
<td>15</td>
<td>1.3305</td>
</tr>
<tr>
<td>11</td>
<td>1.647</td>
<td>16</td>
<td>1.3583</td>
</tr>
<tr>
<td>12</td>
<td>1.1858</td>
<td>17</td>
<td>1.3625</td>
</tr>
<tr>
<td>13</td>
<td>1.083</td>
<td>18</td>
<td>1.3854</td>
</tr>
<tr>
<td>14</td>
<td>1.2283</td>
<td>19</td>
<td>1.3990</td>
</tr>
</tbody>
</table>

Figure 4.5. Distribution Fit for Time to Descend.

4.3 Uncertainty Analysis on Descent

Descent is the one of the final phases in a flight plan when the aircraft starts the descent phase from its cruise. Once it reaches 8000 feet, it switches into the approach phase. Below 3000 feet, it is considered to be landing. An MD82 was considered for this study of descent. We assumed a take-off weight of 55,000 Kg and as it approaches descent, the aircraft would weigh just about 70% of its take-off weight considering the fuel burnt during climb and cruise. Thus, we assumed that the MD82 would weight around 39000 Kg at the start of descent. A non-central chi square distribution centered around 39000 Kg was used to conduct the Monte Carlo experiment. The time to descend distribution and the distribution fit is shown in Figure 4.5.

The time taken to descend is spread over a margin of half a minute or more. During the descent phase, the aircraft uses minimal fuel which is around 0.3 Kg/sec in this case, and since the descent lasts no more than 22 minutes, the total weight loss is around 400 Kg. The descent of an aircraft follows a known flight path with scheduled true and calibrated air speeds and comparing the
weight loss during descent to the aircraft take-off weight, the uncertainty during the descent phase is minimal and can be neglected for the current uncertainty analysis.

The Inverse Gaussian is the only fit that produced results compared to the other distributions considered with the cost = 0.015, the distribution sum = 0.99945, mean (min) = 21.09 and the variance (min^2) = 1.29.

### 4.4 Uncertainty Analysis for Cruise

90% of the flight duration would be in the cruise phase. The uncertainties in the cruise account to most of the delays. As was done in the climb analysis, Monte-Carlo simulations were performed to obtain the distribution of the time of first passage of 100 nm for a specific cruise velocity distribution (Gaussian). The general purpose function allows for specification of distribution in heading and elevation angles (lateral and longitudinal deviations) as well as certain key autopilot gain parameters. This setup allows us to evaluate the effect of navigation precision as well as controller performance on the service times (i.e. time to cruise). Kinematic equations of motion for a point-mass aircraft model in 3-D configuration space are shown in Equation 4.6

\[
\begin{align*}
\dot{x} &= V \cos \gamma \cos \chi \\
\dot{y} &= V \cos \gamma \sin \chi \\
\dot{h} &= V \sin \gamma
\end{align*}
\] (4.6)

where

- \(V\): Aircraft speed (knots)
- \(\gamma\): Flight angle
- \(\chi\): Heading angle

Figure 4.6 shows the coordinate system used to define the kinematic equations. Assuming that an aircraft moves along x-axis which is the desired path, and a feedback control system maintains the lateral deviation \(y\) and the altitude \(h\) close to zeros, the desired system state behaviour for \(y\) and \(h\) is expressed by Equation 4.7

\[
\begin{align*}
\dot{y} &= -k_y y \\
\dot{h} &= -k_h h
\end{align*}
\] (4.7)
For \( k_y > 0 \) and \( k_h > 0 \), the system states, \( y \) and \( h \), are guaranteed to be stable. From Equations 4.6 and 4.7 we have Equation 4.8. The effective speed can be expressed by

\[
\dot{x} = \frac{dx}{dt} = \sqrt{V^2 - k_y^2 y^2 - k_h^2 h^2}
\]  

(4.9)

### 4.4.1 Service Time Evaluation

The service time can be obtained by integrating the reciprocal of the expression in Equation 4.9 by changing the independent variable from \( t \) to \( x \).

\[
t_f = \int_0^{\xi_f} \frac{1}{x} dx = \int_0^{\xi_f} \frac{dx}{\sqrt{V^2 - k_y^2 y^2 - k_h^2 h^2}}
\]  

(4.10)

For deterministic systems, the evaluation of the above expression is generally straightforward. However, the variables \( V, y, \) and \( h \) in the integrand in the above equation are random variables, and the distribution of \( t_f \) needs to be calculated through stochastic integration of the function with these random variables, whose analytical evaluation is not available. Exhaustive approaches based on Monte-Carlo integration techniques can be employed to obtain numerical results for the service time distribution. However, such numerical results provide little information on how the distribution of \( t_f \) is related to navigation uncertainty parameters, and thus the distribution may need to be calculated for every different combination of uncertainty parameters. Although the associated computational cost may be moderate, reasonably accurate approximate analytical expressions are desirable.
Table 4.6. Cruise Evaluation Parameters

<table>
<thead>
<tr>
<th>Simulation Parameter</th>
<th>Parameter value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_0$</td>
<td>400 (nm/hr)</td>
</tr>
<tr>
<td>$\sigma_v$</td>
<td>80 (nm/hr)</td>
</tr>
<tr>
<td>$k_y$</td>
<td>0.2 $sec^{-1}$</td>
</tr>
<tr>
<td>$\sigma_y$</td>
<td>500 ft</td>
</tr>
<tr>
<td>$k_h$</td>
<td>0.5 $sec^{-1}$</td>
</tr>
<tr>
<td>$\sigma_h$</td>
<td>200 ft</td>
</tr>
</tbody>
</table>

4.4.1.1 Gaussian Approximation of the Effective Speed

Suppose that $V$, $y$, and $h$ follow normal distributions. That is $V \sim N(V_o, \sigma_v^2)$, $y \sim N(0, \sigma_y^2)$, and $h \sim N(0, \sigma_h^2)$. The speed $V$ is the sum of the nominal speed $V_0$ and the zero-mean Gaussian perturbation $v$, and hence Equation 4.9 can be rewritten as follows

$$\dot{x} = \sqrt{(V_0 + v)^2 - k_y^2 y^2 - k_h^2 h^2}$$ (4.11)

Non-linear transformation of the PDF with a single random variable is generally straightforward, however the linear summation of multiple distributions requires convolution integral, and thus the analytical expression for the summed probabilistic distribution may not be available except for some special cases such as a sum of Gaussian (Normal) distributions. In Equation 4.11, the first term in the square root follows a non-central-chi-square distribution, and the second and third terms have central-chi-square distributions. It is not straight forward to get an analytical expression for the probabilistic distribution associated with all of these three non-Gaussian random variables. Even if an analytical expression is available, it would be too complicated to proceed any further toward the evaluation of the service time which involves stochastic integration.

The time to cruise is simulated for various distances of 20, 50 and 100 nautical miles. The velocity was assumed to be a non-central-chi-squared variable and the lateral and vertical deviations were assumed to be central-chi-square functions. Table 4.6 gives the variance and the means considered for the simulation.

The horizontal and the vertical deviations are shown in feet. The simulation converts them to nautical miles just to maintain the consistency in the units. The velocity is assumed to be a non-central-chi-squared variable with 10 degrees of freedom. The lateral and the vertical deviations were of a single degree of freedom.
The nominal service time can be calculated by dividing flight distance by the nominal aircraft speed. However, the actual service time varies due to the presence of navigation errors and other uncertainties. Aircraft are controlled based on navigation sensor information provided to the autopilot or human-in-the-loop feedback control systems, and navigation errors will cause variations in speed, heading and flight path angles. Speed variation due to navigation uncertainty can either increase or reduce the service time. However, heading/flight angle deviations always increase the service time since the effective flight path is stretched by any deviations from the straight-line or great-circle air route. The initial set of Monte Carlo experiment was conducted for 2500 runs.
### Table 4.7. Cruise Time Distribution Parameters

<table>
<thead>
<tr>
<th></th>
<th>MC Experiment 2500 runs</th>
<th>MC Experiment 400 runs (LHS)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Gamma Fit cost</strong></td>
<td>0.0257</td>
<td>0.0277</td>
</tr>
<tr>
<td><strong>Mean (min)</strong></td>
<td>13.44</td>
<td>13.19</td>
</tr>
<tr>
<td><strong>Variance (sq min)</strong></td>
<td>1.588</td>
<td>1.43</td>
</tr>
<tr>
<td><strong>Inverse Gaussian Fit cost</strong></td>
<td>0.0219</td>
<td>0.0243</td>
</tr>
<tr>
<td><strong>Mean (min)</strong></td>
<td>13.488</td>
<td>13.24</td>
</tr>
<tr>
<td><strong>Variance (sq min)</strong></td>
<td>1.61</td>
<td>1.45</td>
</tr>
</tbody>
</table>

#### 4.4.1.2 Sample Size Determination

The method of confidence intervals and the LHS technique was used to determine and minimize the number of samples required. A confidence interval of 95% yielded the following minimum number of runs:

**Speed**: the Speed is considered to be a non-central-chi-squared variable with a standard deviation of 40 nm/hr. Let us assume an error of 4 nm/hr i.e.

\[ E: 4 \text{ nm/hr} \]

\[ n = \text{Number of samples required: } 384 \sim 400. \]

**Vertical Deviation:**

\[ \sigma: 500 \text{ feet} \]

\[ E: 60 \text{ feet} \]

\[ n = \text{Number of samples required: } 266 \sim 300. \]

**Horizontal Deviation:**

\[ \sigma: 200 \text{ feet} \]

\[ E: 25 \text{ feet} \]

\[ n = \text{Number of samples required: } 245 \sim 250. \]

The minimum number of runs required was chosen to be 400. To make this study more efficient, we shall also make use of the LHS method to sample the distributions. Figure 4.8 shows the distributions and the fit for varying sample size. Table 4.7 gives the distribution parameters. The Monte Carlo experiment for 2500 runs and 400 runs produced the same result. The Inverse Gaussian produces the best fit to cover a distance of 100 nm with a mean time of 13 minutes and variance of over a minute and a half.
4.5 Uncertainty Analysis Observations and Conclusions

Time to climb is most sensitive to weight, variation in lift co-efficient and power factor produces very minute changes in the time to climb distribution. The Monte Carlo simulation over time to climb resulted in the Inverse Gaussian distribution producing the best fit. The descent distribution spans a little over two minutes, thus the uncertainty in the two minute period does not contribute much to the uncertainty over the aircraft trajectory. Cruise uncertainty is basically characterized by the lateral and vertical deviations which are influenced by various configurations of wind, thus affecting the cruise velocity. A Monte Carlo experiment on the three phases of flight resulted in the Inverse Gaussian distribution fitting the best. The computation power required was further reduced by the use of confidence interval method and the process of sampling distributions to run Monte Carlo simulations was refined by the use of LHS technique. These stochastic models
can be related to queuing parameters to model the national airspace and can be used to study the traffic flow efficiency and related parametric behaviour.
CHAPTER 5

UNCERTAINTY ANALYSIS IN FACET AND VALIDATION FROM ETMS DATA

Future ATM Concepts Evaluation Tool (FACET) is a simulation tool developed by NASA to analyse, implement and test new air traffic management and operational concepts which uses a simulation package to simulate the US airspace. FACET is also used in conjunction with Carat#, which is a software developed to allow access to the FACET functionality using scriptable interfaces. FACET was used to model the uncertainty in trajectory and the results obtained validated the previous uncertainty model developed using the aircraft performance calculator. FACET can model 5000 aircraft on a desktop on any operating system [1] and it uses 4 dimensional aircraft trajectories in the presence of winds around earth kinematic equations. FACET has the simulation parameters saved in a data look up table built within itself and these tables and parameters are accessed during FACET simulations which can predict the location of the aircraft. As-flown trajectories are obtained from FACET fast time simulations based on initial flight plans stored in the same TRX files.

5.1 Cruise Uncertainty in FACET

In the process of the simulation, FACET allows us to identify any aircraft flying in the NAS and track certain performance parameters. Modifying some of these parameters is also possible. FACET incorporates these modifications into simulation and updates it accordingly. There are a certain constraints which restrict us from performing uncertainty in climb and descent in FACET. Once the default climb or descent rate is modified, FACET releases control over these phases of flight and assumes that the user will define rate of climb, velocity, cruise altitude and other related parameters. In effect, the time to climb and time to descent will be a straight forward linear integration of the cruise altitude and the rate of climb as set by the user.

Due to the above restrictions, cruise analysis alone was implemented on FACET. Considering the uncertain parameters studied earlier, FACET does not allow us to change the weight or the lift co-efficient. But we know that a strong relation exists between weights and velocity. The lift co-efficient is also a function of the velocity. By back tracking, the variations in weight and lift co-efficients were mapped onto variations in weight. An unknown distribution of weight was
derived from a known distribution of the chosen uncertain parameter. This was used to calculate a
distribution of time which was identified with the most probable distribution fit.

5.1.1 Speed Uncertainty

An algorithm in FACET is developed to identify an aircraft of a particular category. In order to reduce computation problems with FACET, a short cruise distance of 10 nautical miles is chosen. For our study, we considered an A320. An algorithm identifies an A320 in the US airspace and Carat# supports suitable functions to change specific parameters. The velocity of the aircraft was to vary from 80% to 120% of its actual velocity. The simulation is run for a specific amount of time until the aircraft covers a known distance, and the time for first passage is recorded. A time distribution is obtained for a corresponding velocity distribution by conducting a Monte Carlo experiment for 2500 runs.

The cruise time distribution had to be identified for an A320 whose velocity varies from 150 knots to 650 knots. The bulk of the time taken was between 50 and 150 seconds with a few cases above 150 seconds. The methodology used previously was used again to identify the distribution. We arrive at the distributions and the fit shown in Figure 5.1 and in Table 5.1.

Although the Erlang fit produces the best fit with the least cost and the best approximation of the distribution sum, we have to consider other metrics like the variance and the mean. Considering these two, we can conclude that the Inverse Gaussian is a better fit than the Erlang in the approximation of the mean and the variance.
Table 5.1. Service Time Distribution for Uncertainty in Speed

<table>
<thead>
<tr>
<th>Distributions</th>
<th>Sum</th>
<th>Mean (sec)</th>
<th>Variance (sq sec)</th>
<th>cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original</td>
<td>0.9986</td>
<td>88.99</td>
<td>448.53</td>
<td>NA</td>
</tr>
<tr>
<td>Erlang</td>
<td>0.989</td>
<td>84.01</td>
<td>225.96</td>
<td>0.0156</td>
</tr>
<tr>
<td>Gamma</td>
<td>0.974</td>
<td>84.195</td>
<td>318.25</td>
<td>0.0173</td>
</tr>
<tr>
<td>Inverse Gaussian</td>
<td>0.977</td>
<td>85.48</td>
<td>384.388</td>
<td>0.018</td>
</tr>
</tbody>
</table>

Table 5.2. Cruise Uncertainty Analysis and Monte Carlo Experiment in FACET

<table>
<thead>
<tr>
<th>Inverse Gaussian Distribution variants</th>
<th>MC Experiment-2500 runs</th>
<th>MC Experiment FACET</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean (sec)</td>
<td>86</td>
<td>85</td>
</tr>
<tr>
<td>Variance (sq sec)</td>
<td>409</td>
<td>384</td>
</tr>
</tbody>
</table>

Our initial cruise analysis was done for a cruise distance of 100 nautical miles. To validate that analysis, the analysis was done for 10 nautical miles and the results were validated with FACET. The velocity of the aircraft was assumed to be 450 knots and the cruise distance was 10 nautical miles. The distribution parameters comparing Monte Carlo experiment and simulation in FACET is compared in Table 5.2.

5.1.2 Weight Uncertainty

FACET does not support changes in weight. Changes in weight produce changes in velocity. This is used to map a weight distribution onto a distribution in velocity.

\[ V = V_{ref} \sqrt{\frac{m}{m_{ref}}} \]  \hspace{1cm} (5.1)

We obtain the reference velocity \( V_{ref} \) and the reference mass \( m_{ref} \) from the aircraft data. The speed of the aircraft is derived from Equation 5.1 are used to run the simulation again as explained earlier. The reference velocity and the reference mass were obtained from the BADA cruise calculator.

The maximum take-off weight for an A320: 77000 Kg
Nominal weight: 64000 Kg at cruise.
Altitude: 31000 feet
The cruise speed: 445 knots
Number of runs: 2000
Figure 5.2. Time to Cruise Distribution Fit for Weight Uncertainty on FACET.

Table 5.3. Service Time Distribution for Uncertainty in Weight

<table>
<thead>
<tr>
<th>Distributions</th>
<th>Sum</th>
<th>Mean(sec)</th>
<th>Variance(sq sec)</th>
<th>cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original</td>
<td>0.9987</td>
<td>82.8823</td>
<td>81.08</td>
<td>NA</td>
</tr>
<tr>
<td>Erlang</td>
<td>0.9931</td>
<td>80.78</td>
<td>49.82</td>
<td>0.0313</td>
</tr>
<tr>
<td>Gamma</td>
<td>0.9792</td>
<td>80.301</td>
<td>73.00</td>
<td>0.0302</td>
</tr>
<tr>
<td>Inverse Gaussian</td>
<td>0.9842</td>
<td>80.81</td>
<td>73.34</td>
<td>0.0282</td>
</tr>
</tbody>
</table>

For a nominal weight of 64000 Kg considered, it produced a velocity distribution ranging from 270 knots to 570 knots. The corresponding time passage to cover a distance of 10 miles is shown in Figure 5.2 along with the distribution fits. The Inverse Gaussian produced the best fit with the least cost function.

5.1.3 Lift Co-efficient Uncertainty

A similar study was done on the lift coefficient uncertainty. An A320 was considered for the study and the lift coefficient was made to vary from 0.2 to 0.8 centered at 0.45. The altitude specific lift coefficient for an A320 at an altitude of 31000 feet was 0.45. The distribution obtained in the lift coefficient was mapped to a distribution in velocity.

\[
V_{gnd\text{speed}} = \sqrt{\frac{2W_{new}g}{\rho C_L S}} \tag{5.2}
\]

Where,

\( V_{gnd\text{speed}} \): Aircraft ground speed (knots)
\( W_{new} \): Weight of aircraft from the new distribution (Kg)
\( C_L \): Lift Co-efficient
Figure 5.3. Time to Cruise Distribution Fit for Lift Co-efficient Uncertainty on FACET.

Table 5.4. Service Time Distribution for Uncertainty in Lift Co-efficient

<table>
<thead>
<tr>
<th>Distributions</th>
<th>Sum</th>
<th>Mean (sec)</th>
<th>Variance (sq sec)</th>
<th>cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original</td>
<td>0.9920</td>
<td>80.749</td>
<td>64.185</td>
<td>NA</td>
</tr>
<tr>
<td>Erlang</td>
<td>0.9968</td>
<td>81.43</td>
<td>57.706</td>
<td>0.0266</td>
</tr>
<tr>
<td>Gamma</td>
<td>0.9954</td>
<td>81.26</td>
<td>62.87</td>
<td>0.0275</td>
</tr>
<tr>
<td>Inverse Gaussian</td>
<td>0.9935</td>
<td>81.26</td>
<td>62.95</td>
<td>0.0289</td>
</tr>
</tbody>
</table>

$\rho$: Density of air (Kg/m$^3$)

$S$: Surface area (m$^2$)

Thus from the observation of Figure 5.3 and Table 5.4, the Inverse Gaussian produces the best fit amongst all the three. The velocity, lift co-efficient of the aircraft and the weight have the same proportionality with the time passage. Our previous study has shown us that the time is very sensitive to weight as compared to lift-coefficient. Weight being a non-central-chi-squared variable, it is no surprise that the time distributions assumes an Inverse Gaussian. This was the case with time to climb distribution as explained in Chapter 4.

5.2 Trajectory Uncertainty Validation using ETMS Data

The aircraft performance calculator introduces uncertainty into various parameters in all phases of flight such as Climb, cruise and descent. The calculator was built on the basis of the BADA performance tables and was tested for three aircraft namely MD82, B737 and A343. Cruise performance was validated in FACET but climb/descent information was not available in FACET.
To validate the performance calculator, Enhanced Traffic Management System (ETMS) data was used [Data Source: Metron Aviation, VA].

5.2.1 Enhanced Traffic Management System (ETMS) Data

The ETMS data consisted of radar hits for all aircraft with one minute intervals. The data set for all aircraft in the United States airspace for one day consisted of latitudes, longitudes, time, altitude and the flight ID. Tracking the altitude gives us the phase of flight. The flight ID generated is unique to the aircraft. With the information of the sector and center boundaries, based on the latitudes and longitudes, the position of the aircraft was computed. 6 aircraft were chosen for the validation. All of the 6 aircraft passed through the ZOB center on June 15th, 2006. June 15th 2006 is a clear weather day over the ZOB center and was specifically considered as any weather would mean re-routing of the aircraft and this would be a deviation from its optimal filed route. This would introduce variation in the service times due to weather. Since the performance calculator does not include weather uncertainties, June 15th was chosen as the best day for the validation of the calculator.

The flight profile of an aircraft does not really follow the BADA performance tables exactly. This is due to the amendments in the flight plans that are implemented during the flight. The climb and the descent undergo changes as per the airspace requirements and the constraints to accommodate other aircraft.

BADA models descent as a straight forward descent and we cannot specify the rate of descent or the rate of climb. But the ETMS data shows that the climb and the cruise phases deviate to a significant extent from the BADA performance tables. In order to incorporate these variations into the validation, the following steps were taken.

- The calculator was used to calculate the time taken to reach the cruise altitude. The flight angles calculated were different from the ETMS data. This can be attributed to the fact that we do not know the actual flight path of the aircraft. The validation uses an A343 and the weight difference will contribute to the uncertainty in time to climb.
- Since the flight angles varied, the horizontal distance travelled varied as well. The distance travelled by an A343 is computed as below.

\[
\gamma = \sin^{-1} \frac{\dot{h}}{V} \tag{5.3}
\]
where
\(\gamma\): Climb/Descent angle (degree/rad)
\(\dot{h}\): Rate of climb (ft/min)
\(V\): True air speed (knots)

- The horizontal distance travelled during climb and descent was computed from the above relations. The ETMS data gives the total distance travelled from which, the cruise distance was calculated.
- The cruise analysis introduces uncertainties in the flight angle, lateral and horizontal deviations. The cruise velocity was calculated from the ETMS data based on the distance travelled and the time taken.
- The altitude of the aircraft gives the phase of flight. The performance calculator was used to follow the flown data to replicate the flight profile. The aircraft switches over to different phases as seen in the ETMS data.

Using simple trigonometric relations, the horizontal distance travelled was calculated for every 1000 feet as shown in Equation 5.4

\[
d = \frac{h}{\tan \theta}
\]  

(5.4)

Where,
\(d\): Horizontal distance (Km)
\(h\): Altitude (Km)
\(\theta\): Flight angle

5.2.2 Procedure for the Validation

1. The performance calculator is used to calculate the time taken to climb to the specified cruise altitude. The horizontal distance travelled during the climb is computed and stored.
2. In order to accommodate the difference in the climb and the descent profiles between the flown data and the calculator profiles, instead of cruise, the aircraft is made to descend to the last recorded altitude as per the ETMS data and the horizontal distance travelled is computed and saved for further analysis.
3. Based on the information of the total distance travelled from the ETMS data and the distance travelled during climb and descent, the cruise distance is computed.
5. Based on the cruise distance and the time spent by the aircraft during cruise from the ETMS data, the velocity of the aircraft is computed to calculate the cruise time. An uncertainty on the velocity in a manner as done in our previous study is added to the current model.

5. Finally, a distribution is obtained for each of the six aircraft for the total time for the flight profile and a box plot is obtained to show the upper and lower bounds of the simulation.

5.2.3 Analysis

The climb altitudes for all the six different aircraft were identified and a Monte Carlo experiment was conducted using Latin Hypercube sampling. Based on our previous analysis on the number of runs required, we choose 600 runs as the minimum number of runs to accommodate all the uncertainties. Table 5.5 gives the ETMS data for the chosen aircraft.
Figure 5.5. Trajectory Uncertainty Validation with ETMS data.
Table 5.6. Simulation Results for Aircraft 3

<table>
<thead>
<tr>
<th>Flight ID</th>
<th>481955746</th>
</tr>
</thead>
<tbody>
<tr>
<td>Origin Airport code</td>
<td>EWR</td>
</tr>
<tr>
<td>Destination Airport code</td>
<td>DSM</td>
</tr>
<tr>
<td>Total Distance (Kilometers)</td>
<td>1636</td>
</tr>
<tr>
<td>Cruise Altitude (feet)</td>
<td>32000</td>
</tr>
<tr>
<td>Climb Distance (Kilometers)</td>
<td>280.18</td>
</tr>
<tr>
<td>Cruise Distance (Kilometers)</td>
<td>1172</td>
</tr>
<tr>
<td>Descent Distance (Kilometers)</td>
<td>183.14</td>
</tr>
<tr>
<td>Minimum Time for Flight envelope (minutes)</td>
<td>115.08</td>
</tr>
<tr>
<td>Maximum Time for Flight envelope (minutes)</td>
<td>161.48</td>
</tr>
<tr>
<td>Average Time for Flight envelope (minutes)</td>
<td>129.5</td>
</tr>
<tr>
<td>Actual Flying Time-ETMS data</td>
<td>128</td>
</tr>
</tbody>
</table>

Figures 5.5 shows the flight profiles for all the six aircraft along with the flight profile for the flown data. As seen in Figure 5.5, we arrive at a range of values for the time taken for the entire flight. Some of the time durations are located towards the extremes of the distributions. This can be attributed to the fact that we do not have adequate information of the initial take off weight or the category of aircraft. The calculator results match very well with aircraft number 3. The results for that are in Table 5.6

From the Table 5.6 and the graphs in Figure 5.5, the actual flying time according to the ETMS data falls well within the upper and the lower bounds of the simulation. There is a very good match with the mean flying time as well for aircraft number 3. Figure 5.6 gives a box plot for all the 6 aircraft. The box plot is characterised by the upper bounds and the lower bounds. The plot in red is the ETMS as flown time duration for the flight envelope.

5.3 Conclusion

The aircraft performance calculator functions in accordance with the ETMS data. The results from the calculator follow the actual flying time specifically for aircraft number 3 flying from EWR to DST on June 15th 2006. With more information of the aircraft, the calculator can be configured to calculate the flying time for the specific aircraft.

The flight time for the other 5 aircraft, although within the range of the simulation results, the actual flying time does not fall within the 25th and the 75th percentile. With the simulation, we introduce the uncertainty keeping in mind the range of values the uncertain parameter can take.
Depending on the flying conditions on June 15\textsuperscript{th} and the flight parameters set to a specific value, we believe that the aircraft parametric values fall in the range of values used in the simulation.

With access to more of flown under similar flying conditions flight data, the aircraft performance calculator developed on MATLAB should be able to predict the range of time for the total duration during each of the phases of flight accurately. The calculator is robust and the uncertain parameters distributions can be changed with ease. With access to airline information, we can have an accurate estimate of the weight distribution and the power uncertainty distribution.
CHAPTER 6
TRAFFIC FLOW AND WEATHER UNCERTAINTY

The US airspace supports a very high volume of aircraft. The study of which becomes necessary while dealing with problems like increasing the capacity and trajectory optimization. The study of traffic flow helps us arrive at the required air traffic metrics necessary towards the realization of the NGATS. As per the prediction for the next two decades, significant operational and procedural changes have to be made to be able to accommodate the massive increase in demand. Precision in navigation and trajectory uncertainties leading to prediction in position and time have to improve in accuracy to ensure that the NAS meets the futuristic demand.

We have identified 20 centers and almost 60 sectors in the US airspace. The centers are of high importance with regard to the traffic handled. A few pacing airports are identified where there in a very high traffic volume and this volume usually exceeds the airport capacity [31]. These airports have a huge impact on the traffic throughout the US airspace wherein the delays generated at these hot spots will propagate and the affect might be seen in another airport many centers away. The volume of air traffic varies in the centers and the aircraft count in the center/sector is a key metric in air traffic control[32]. Peak traffic is seen at specific times during the day and these time points can me red marked and can be regarded as the points at which the traffic changes its rate and flow characteristics. The identification of such break points and the pattern of flow becomes important in the process of obtaining the traffic flow metrics.

6.1 Traffic Flow Evaluation

Traffic flow is measured as the number of aircraft per unit time. The traffic Flow (p) is given by the following relation:

\[ p = \lambda v \]  

(6.1)

where

\( \lambda \): number of aircraft/unit distance

or \( \lambda = \frac{1}{\text{Wake-Separation}} \)

\( v = 200 \) to 400 knots
Using the above mentioned criteria, a study was done to observe the kind of distribution obtained for air traffic. A Gaussian distribution for the velocities and an Inverse Gaussian distribution for the concentration was assumed where the concentration is the aircraft/unit distance. Considering the wake and a jam packed situation (highest traffic such that the separation between 2 aircraft is just the wake) we can have 1 aircraft for a distance of approximately 8 nautical miles. The following histograms were obtained as seen in Figure 6.1. The flow can be definitely approximated by an Inverse Gaussian. The flow is a product of the velocity and the concentration where the concentration is an Inverse Gaussian.

The above calculation is not really accurate. We can use many other distributions to obtain the traffic flow. The velocity of an aircraft depends on the traffic where an aircraft would fly at its optimal cruise speed when the traffic is minimal. And similarly it would fly at a speed just above the stall speed when the traffic is maximum. Considering these conditions, we have to obtain distributions for the above mentioned parameters. To understand better, the concentration is the reciprocal of the mean spacing and the flow is the reciprocal of the mean headway. Headway and spacing are basically the functions of the velocities and the velocities vary based on the traffic.

In the course of the cruise, let us assume that the aircraft will be cruising at a velocity $x$. But due the interference from other aircraft and traffic, it will not able to maintain the same cruise speed. So let the over all average speed of the aircraft be denoted by $y$. we now have a function $y = y(x, \lambda)$,
Figure 6.2. Distribution Fit for Traffic Flow.

Table 6.1. Air Traffic Flow Distribution Fit

<table>
<thead>
<tr>
<th>Distributions</th>
<th>Sum</th>
<th>Mean</th>
<th>Variance</th>
<th>cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original</td>
<td>0.9989</td>
<td>40.38</td>
<td>234.81</td>
<td>NA</td>
</tr>
<tr>
<td>Inverse Gaussian</td>
<td>0.9953</td>
<td>40.32</td>
<td>231.56</td>
<td>0.0138</td>
</tr>
<tr>
<td>F Distribution</td>
<td>0.1956</td>
<td>14.12</td>
<td>1413.109</td>
<td>0.0712</td>
</tr>
</tbody>
</table>

the nature of this function is not easy to determine. We have a few boundary conditions that the above mentioned function should satisfy. At a free uninterrupted flight, the aircraft would fly at the optimal speed $x$. that is $y(x, 0) = x$ and when the traffic is at its maximum, the aircraft would fly at a speed above its stall speed for that altitude and the flight configuration. Hence $y(x, \lambda) = x_{\text{stall}}$.

A distribution was identified for the traffic flow. The initial analysis on the fit turned true with the best fit being an Inverse Gaussian. Figure 6.2 and Table 6.1 shows the distribution and the parameters values.

6.1.1 Traffic Flow Pattern

With the 20 centers identified, we had to collect certain data from all the centers/sectors. This data included the number of aircraft landed, departed from the airport, number of aircraft flying or the total number in the center and these metrics would answer a few questions. The traffic is time varying and follows a set pattern. A dynamic programming model approach was used and
the specific break points where the flow changes were identified. A FACET simulation was run with 24 hour data from a particular day of the year. We obtain the graphs shown in Figure 6.3 for the number of aircraft at the center, landed and the number of aircraft departed.

We considered 4 major centers with a high volume of traffic namely Dallas Fort Worth, Los Angeles, Washington DC and New York center. The total number of aircraft present in the center is seen to be at the highest at certain instances during the day. And a similar pattern is seen with all the centers. For example, Chicago O’Hare airport is extremely congested in the late afternoon and early evenings [33]. The simulation starts from midnight. But, we see that in all the plots, number of aircraft starts populating after the first hour. That is because, to start off with, we do not have any initial condition on the simulation. Hence we start with a zero and run the simulation for 24 hours.
The pattern of a reduced activity is seen in the first 10 to 12 hours of the day. An increase is seen after 12 hours. Eventually, as the day dawns to an end, the count in centers decrease. Accordingly, the number of aircraft taking off from the airports reduces drastically and most of the aircraft land at the respective destinations.

Another point to note would be, the increase in traffic does not occur at the same time duration for all the centers. It can be seen that there is a delay in the increase in traffic amongst the centers. This can be attributed to the fact that the centers under consideration are not from the same time zone. But since the simulation uses a standard clock, it assumes all the centers to be operating at one specified time according to which, statistics as seen above is produced. Taking into consideration the time zones, Washington and New York (Eastern Standard Time) fall on the same time zone which is an hour ahead of DFW (Central Standard Time). Los Angeles (Pacific Standard Time) is two hours behind DFW. Considering a time shift accordingly to line all the centers onto the Central Standard Time, we get the graph in Figure 6.3 for the count in centers.

The busy airports considered reach the peak traffic at the same local time. This behaviour is understood by the movement of planes based on the demand. Starting from midnight, we see the first peak at around 3 hours. The second massive peak is seen at around noon during the day. These changes are important in the view of developing a queuing model for the NAS. Based on the local time, the rate of arrivals, departures change and this has to be considered in the queuing model. To extract the specific time instances at which the behaviour of the airspace changes, we used a dynamic programming model to account for the changes. Using FACET, we extracted the arrivals at each of the centers for the 24 hour duration. This was used and a cumulative arrival was obtained for each center. One such cumulative arrival for the Washington center is shown in Figure 6.4.

The total arrivals at Washington were extracted from FACET and break points were calculated. The change in the slope of the cumulative arrival signifies a change in the arrival rate. This change is noted and the time instant at which the change occurs is treated as a break-point. The duration between two breakpoints gives the duration for which the airport had a constant or almost constant arrival rate.

With the break point model, the numbers of break-points (knots) required were chosen and the break-points were computed for all centers. The study was done with 4 segments i.e. 3 breakpoints for each center. This yielded a total of 60 break points for the NAS. Considering the fact that many of the centers lie on the same time zone, we were able to combine the break-points. Adding
to this, the behaviour of arrivals in many of the centers followed a similar pattern thereby filtering out multiple break-points for the same time instant, we identified 10 break-points for all of NAS. These time instances signify a change in the arrival pattern at the centers. The break-points are seen in Figure 6.4 and it can be noticed that the arrival rate almost remains a constant for 12 hours between 11 and 23 hours. This information on rate changes can be incorporated in queuing models to optimize the prediction and performance.

6.2 Development of Time Dependent Model

We keep a track of the movement of aircraft in the airspace by calculating the number of aircraft that cross over to the neighbouring centers and the ones that come into the center. Similarly we count the number of aircraft that landed and the ones that departed from the airport. The state equation for the count in the center is derived based on the method described in [34], [35] and [36]

• We break a 24 hour period into a specified number of periods.
• At the end of every period, we calculate the above mentioned parameters. Form the information collected, we try to develop a probability transition matrix (A matrix) or a state propagation matrix, which when multiplied with an initial condition, and a specified time, should be able to obtain the current number of aircraft in the center (state).
• At any point of time, it should satisfy the conservation of aircraft given by:

\[
\text{Count of a/c currently in center} = \text{number of a/c in transition from other centers} + \text{number of a/c that departed in the center} + \text{external arrivals}
\]
Figure 6.5. Aircraft Flow in the Two Centers.

The number of aircraft in the \(i^{th}\) center at \(k^{th}\) time-step is given by \(C_i(k)\) and represents the state of the Eulerian model whose pictorial representation is seen in Figure 6.5. Let's define \(\alpha_{i,j}(k)\) as the number of aircraft that moved from the \(j^{th}\) center to the \(i^{th}\) center between the time-step \(k\) and \(k+1\). Similarly, \(E_{aj}\) is defined as the external inputs to \(j^{th}\) center. External arrivals are defined as arrivals into the center from centers outside the network under consideration (e.g., International arrivals). \(D_i(k)\) defines the number of aircraft departed in the \(i^{th}\) center between the time steps \(k\) and \(k+1\). The output of the Eulerian model \(y_i(k)\) is defined as the number of aircraft that lands in the \(i^{th}\) center. Similarly, \(O_i(k)\) is defined as the number of aircraft that exits the center between the \(k^{th}\) and \(k+1\) time-step. Considering the conservation of aircraft, we have the following equation

\[
C_i(k+1) = C_i(k) - [\alpha_{ji}(k) + y_i(k) + O_i(k)] + [\alpha_{ij}(k) + D_i(k) + E_{ai}(k)]
\]  

(6.2)

The second term on the right hand side signifies the number of aircraft leaving \(i^{th}\) center and the third term signifies the number of aircraft entering the \(i^{th}\) center. The first two terms however is the number of aircraft that continued to stay in the center \(i\) between time-steps \(k\) and \(k+1\). This can be referred to as \(\alpha_{ii}\) represented by Equation 6.3

\[
\alpha_{ii} = C_i(k) - [\alpha_{ji}(k) + y_i(k) + O_i(k)]
\]  

(6.3)
The entries of the state propagation matrix will have transition probabilities defined by Equation 6.5

\[
A_{ii}(k) = \frac{\alpha_{ii}(k)}{C_i(k)} \tag{6.4}
\]

\[
A_{ij}(k) = \frac{\alpha_{ij}(k)}{C_j(k)} \tag{6.5}
\]

Substituting Equation 6.3 and 6.5 in 6.2 we have

\[
C_i(k+1) = A_{ii}(k)C_i(k) + A_{ij}(k)C_j(k) + D_i(k) + E_{ai}(k) \tag{6.6}
\]

This can be extended for centers \(i\) and \(j\) as seen in Equation 6.7, combining them we have the Eulerian model as seen in Equation 6.8

\[
\begin{bmatrix}
C_i(k+1) \\
C_j(k+1)
\end{bmatrix}
= \begin{bmatrix}
A_{ii}(k) & A_{ij}(k) \\
A_{ji}(k) & A_{jj}(k)
\end{bmatrix}
\begin{bmatrix}
C_i(k) \\
C_j(k)
\end{bmatrix}
+ \begin{bmatrix}
D_i(k) \\
D_j(k)
\end{bmatrix}
+ \begin{bmatrix}
E_{ai}(k) \\
E_{aj}(k)
\end{bmatrix} \tag{6.7}
\]

\[
C(k+1) = A_kC(k) + D(k) + E_a(k) \tag{6.8}
\]

Where,

\(C(k)\) : System state - aircraft count in the center at \(k^{th}\) time instant

\(A_k\) : Probability state transition matrix at time instant \(k\)

\(D(k)\) : Departed aircraft in the center

\(E_a(k)\) : All external arrivals to the center

### 6.2.1 The Probability Transition Matrix (A Matrix)

The \((i,j)\) entries in the \(A\) matrix is basically the percentage of aircraft that will flow into the \(i^{th}\) center from the \(j^{th}\) center. The diagonal elements would be the number of aircraft that remained in the same center all through the time period. Equation 6.9 gives the entries of the \(A\) matrix.

\[
A = \begin{bmatrix}
A_{1,1} & A_{1,2} & \ldots & A_{1,20} \\
A_{2,1} & \ldots & \ldots & \ldots \\
\ldots & \ldots & \ldots & \ldots \\
A_{20,1} & \ldots & A_{20,19} & A_{20,20}
\end{bmatrix} \tag{6.9}
\]
**Calculation of $A$ matrix** All the entries correspond to the aircraft coming into the center. Effectively we will need to find out

- Number of aircraft coming into the center from various centers
- Number of aircraft that took off during the current time duration
- Number of aircraft that landed over the time duration

Carat# does not give us this data directly and we need to manipulate and modify the data we get from it. We are able to obtain the number of aircraft that are flying in each center at each instant of time. Based on this information, we use the concepts of set theory to extract the required information

- Number of aircraft that took off: current list of a/c-previous list a/c
- Number of aircraft that landed: previous list of a/c-current list of a/c
- This list of landed and departed aircraft seen above is for the entire airspace and not the center specifically.

**Aircraft that moved from $j^{th}$ center to $i^{th}$ center:**

- Intersection of (A/c at $i^{th}$ center at current instant, A/c at $j^{th}$ center at previous time instant)
- The above is done for every center with every other center. We effectively have 400 computations to fill in the 400 entries in the $A$ matrix.

**To calculate the landed and the departed planes at each center:**

- Departed list for the center = Intersection of (Current list of a/c, Departed list for US airspace)
- Landed list for the center = Intersection of (Previous list of a/c, Landed list for US airspace)
- The number of aircraft in the same center = Intersection of (current list of a/c, previous list of a/c)
- The information of the departed and the incoming was put into the diagonal elements of the $A$ matrix such that, we needed just the initial number in the system to predict the number of aircraft actually present in the system.

The system was set up to do the above and the $A$ matrix was obtained. A validation was done on the Washington center seen in Figure 6.6. It was observed that the count in the center varies with time thus the entries of the $A$ matrix is time variant. In the process of obtaining the $A$ matrix, we had 24 matrices for each of the 24 hours simulated. Thus every element in $A$ matrix is a set of
24 such elements to account for the 24 hour run. We could fit a curve to these elements and obtain each of the entry as a function of time. Thus, just specifying the time would give us the current number of aircraft in the airspace. This would computationally save resources since FACET being a complicated tool requires 2 to 8 hours based on the user requirements to simulate NAS operations for one day.

6.3 Weather Uncertainty

A great deal of effort is put in by airlines to develop flight plans. A flight plan would consist of the originating city, departure time, destination and arrival time for the flight and they are created every two to three months [37]. Weather is an important factor that determines the flight route and since its forecast is accurate only within two hours of the prediction, the flight plans are amended many times. The National Convective Weather Diagnostic (NCWD) tool is used to predict weather and the program is sponsored by the FAA Aviation Weather Research (AWR) program as a part of the Convective Weather Product Development team. The NCWD target units are usually airlines dispatchers, general aviation and traffic management units. This section of weather uncertainty was researched in Metron Aviation under their guidance.
6.3.1 National Convective Diagnostic (NCWD) Tool

The NCWD predicts the weather with hazard levels whose intensity varies from 1 to 6, 6 being the worst possible case. Airspace with NCWD level 3, would be blocked for air traffic of any kind and they would be re-routed around the bad weather segment towards its destination considering other factors such as traffic intensity and capacity over new flight route. The NCWD levels are specified for a 4 square Kilometer area and based on the knowledge of the area of the center/sector, we can calculate the weather coverage which is the ratio of area covered by bad weather to the area of the center/sector.

Based on an analysis of historical weather data, we chose June 22, 2006 (Thursday) as the convective weather day. On this day convective weather coverage of intensity equal to or higher than National Weather Service (NWS) Level 3 reaches about 45% of the area of sector ZOB49. This leads to significant loss of available capacity for this sector as compared to clear weather day. At the same, on clear weather days ZOB49 has a 0% weather coverage. In order to compare re-routing strategies in response to en-route convective weather we compared flight plans and their evolution on June 22, 2006 (our convective weather day) with the flights plans for the same flights occurring on the same day on the week (Thursday) but a week earlier, i.e., on June 15, 2006. Figure 6.7 shows convective activity, on June 22, 2006 in the ZOB center. We used ETMS and extracted flight track data for both analysed days. We identified all flights traversing the sequence of sectors ZOB47-ZOB49-ZOB59 between 08:00 AM on June 15, 2006 and 08:00 AM on June 16, 2006 and compared this list to the flights which traversed the same sequence of sectors between 08:00 AM on June 22, 2006 and 08:00 AM on June 23, 2006.

6.3.2 Observations

June 15th, 2006 had 78 planes identified flying through ZOB49. These 78 planes took off from various points in the US airspace and landed elsewhere meanwhile passing through ZOB49. The origin and the destination airports were noted along with the Aircraft ID (ACID) and the Flight ID (FID). The FID is a number unique to every aircraft that is generated locally within the system to identify any aircraft irrespective of its time of flight. The ACID is an ID specified when the flight plan is filed to identify the aircraft by its route also based on the origin and destination airports. Off the 78 planes that flew though this route, the same ETMS query found only 18 flights passing through these sectors on June 22nd, 2006.
Figure 6.7. Weather Coverage over ZOB47 and ZOB49 for June 22.
The ACID, origin and destination airport data were used to identify the remaining 60 flights which did not pass through sectors. After querying ETMS, 59 of these flights were found rerouted and one was cancelled. Figure 6.8 shows in red the original 78 tracks passing through the sequence of sectors on clear weather day. It also displays in blue the re-routes used by 59 flights on bad weather day. Figure 6.8 clearly shows differences in re-routing the set of flights on bad weather day. Some flights were rerouted only locally, not too far from the constrained sectors. However, there were also a relatively large number of flights which were rerouted far to the north or far to the south, resulting in significantly longer travel time, and hence delays and increased fuel burn. The following observations were made from this analysis

- 78 planes were identified on a good weather day with NCWD levels of 0 for June 15th 2006.
- 18 planes were identified on a bad weather day with weather coverage reaching 60% for June 22nd 2006.
- 59 planes from the set of 78 were rerouted on June 22nd, 1 was cancelled.
- The re-routing increased the average service time by approximately 18 minutes
• It also increased the average flying distance for the chosen airport pairs by approximately 208 Kilometers

6.3.3 Conclusion

The above study shows that the weather has a significant impact on the flight routes and the service times. It is important to include weather uncertainty within the queuing model to account for such huge variations. An area of improvement would be to improve the weather prediction time horizon by using suitable statistical models. With the inputs from the weather models, network flow algorithms can be used to solve the re routing problems. The aircraft routing problem is formulated as a time dependent network flow problem and is proved to be NP hard [38]. Dimitris and Sarah [39] addresses the problem of determining how to re route aircraft in the air traffic control system when faced with dynamically changing weather. NGATS has a futuristic vision focussing on the air traffic demands and the needs for 2025 to be able to manage and handle a system that can operate to 250% capacity of the existing system by automation and procedural changes of the trajectory analyses to increase capacity. In this process, weather uncertainty is an area one cannot afford to ignore considering the impact it has on flight routes and flight time.
CHAPTER 7
SUMMARY, CONCLUSIONS AND FUTURE WORK

7.1 Summary

The research develops from a good understanding of the NAS, its drawbacks and the proposed improvements specified by the NGATS. NGATS focusses on a futuristic vision of a secure and high capacity air traffic navigation. In the process of realising this, NAS will undergo procedural and operational changes to reduce air traffic delays and to increase the safety and efficiency. The causes for delays were identified and classified. A case study was done on air traffic delays at Dallas Fort-Worth International Airport.

Aircraft performance of one aircraft affecting the performance of others was studied in detail by hypothesising the departure and landing scenarios considering a variety of aircraft. Further, we used the BADA performance tables to develop an aircraft performance calculator in MATLAB and was validated with the BADA performance table. Sensitivity analysis was done to identify key operational parameters which has an impact on various service times, further leading to an uncertainty analysis on flight trajectory. Service time distributions in various phases of flight were calculated and the distribution type was identified. Monte Carlo Experimental method was used through out using Confidence Interval and LHS methods employed to reduce the number of iterations.

FACET was explored and uncertainty in cruise was implemented in FACET which validated our previous results from the uncertainty analysis using the performance calculator. The uncertainty analysis over climb, cruise an descent was validated with ETMS data for June 15, 2006. We were able to identify air traffic flow patterns in the NAS for a chosen set of centers and a state space model was built and tested for accurate prediction of the states. Further, the impact of weather on air traffic delays and routes was studied by comparing flight routes on June 15, 2006 with routes on June 22, 2006 over the Chicago center. NCWD weather database was used to identify regions of NAS affected by weather. Querying the ETMS data gave us the re-routes and the extent to which that affected the service time.
7.2 Conclusions

The aircraft model is presented in Appendix A based on which the aircraft performance calculator was built in MATLAB. The calculator was used extensively to conduct Monte Carlo experiments in the process of uncertainty analysis. The calculator was validated with the BADA performance tables and our calculator shows a variation of two seconds for every 1000 feet gain in altitude. The fuel consumption, the true air speeds and rate of climb etc. followed the BADA tables very closely.

Sensitivity analysis resulted in identifying time to climb being most sensitive to weight. The uncertainty in time to climb due to weight ranges from 8-15 minutes for an MD82. The time to climb is almost twice as much for a heavy MD82 as compared to an MD82 lightly loaded on fuel. Time to climb is not as sensitive to lift co-efficient and power factor as it is to weight. A climb at 80% thrust would produce a delay of 3-5 minutes and a lift co-efficient variation of 40% around its mean produces a delay of 20-50 seconds. Comparing these time delays to the duration of the entire flight which would last over 4 hours, the uncertainty induced by lift co-efficient and power factor is insignificant. Weight alone characterises the climb phase. The Inverse Gaussian produces the best for time to climb with an average climb time of 15 min for an MD82 with a 12 min\(^2\) variance signifying that climb segment is characterised by uncertainty and weight is a key parameter that determines the climb time.

90% of the flight time is spent in the cruise phase. Cruise time is effected by wind, which can increase or decrease the flight time, lateral and horizontal deviations will only increase the flight time. Cruise is a phase where the speed of the aircraft is altitude dependent, thus, weight of an aircraft does not have any impact on the cruise time. Uncertainty analysis on cruise resulted in the average time passage, to cover a distance of 100 nautical miles was 13.5 minutes with the variance of 1.5 min\(^2\). The contribution of cruise uncertainty towards the uncertainty in flight trajectory is minimal.

Descent phase is characterised by a known flight path with scheduled true and calibrated air speeds. An MD82 on an average needs 22 minutes to descend from 32,000 feet with variance 1.2 min\(^2\). The aircraft uses minimal fuel during descent and loses about 400 Kg of fuel on an average. Thus, descent phase is independent of weight and the uncertainty is minimal and can be ignored for uncertainty analysis for trajectory of the flight.
Air traffic flow is characterised by demand and capacity which is time dependent subject to various constraints existing in the NAS. Understanding of the traffic flow pattern is necessary to develop a queuing model. The study of traffic behaviour in various centers showed that the demand reaches its peaks during certain time instances during the day and these demand changes induce a change in arrival rate and other traffic flow metrics. The comparison for June 15, 2006 and June 22, 2006 yielded an average increase in flight distance by 208 Kilometers and an average increase in service time by 18 minutes due to extreme weather conditions over the flight route. Incorporating weather uncertainty into the trajectory uncertainty model is important.

7.3 Future Work

Queuing models provide explicit relationships between traffic flow efficiencies and trajectory uncertainties and they are developed based on probability density distributions. Identification of the distribution is important for this reason. The performance calculator is programmed in a way that the input distribution can be easily chosen. But the characteristics of the input distribution is not well known. The future work in this area towards the realisation of the objectives and goals of NGATS would be:

- Identify the distribution type for weight, lift co-efficient and power factor by referring to historical data or airline data
- Include other areas of uncertainty namely pilot errors, navigation uncertainty and wind etc.
- Research re-routing algorithms to obtain the statistics for variations in flight times and flight routes.
- Improving the process of weather prediction and incorporate weather uncertainty into the uncertainty model.
- Implement a queuing model with uncertainty to model NAS.
APPENDIX A

AIRCRAFT MODEL
The aircraft performance calculator was developed based on the BADA performance tables. The BADA performance tables gives the rate of climb/descent, speed, fuel consumption and other such factors necessary to gauge the performance of the aircraft.

### A.1 Aircraft Energy Model

The Total-Energy Model equates the rate of work done by forces acting on the aircraft to the rate of increase in potential and kinetic energy

\[(T - D)V_{TAS} = mg \frac{dh}{dt} + mV_{TAS} \frac{dV_{TAS}}{dt}\]  \hspace{3cm} (A.1)

Where,
- \(T\): Thrust acting parallel to the aircraft velocity vector [Newtons]
- \(D\): Aerodynamic drag [Newtons]
- \(m\): Aircraft mass [kilograms]
- \(h\): Altitude [m]
- \(g\): Gravitational Acceleration \([m/s^2]\)
- \(V_{TAS}\): True airspeed \([m/s]\)
- \(\frac{dh}{dt}\): Time derivative of the altitude

by rearranging equation A.1 to isolate the rate of climb or descent, we get

\[\frac{dh}{dt} = \frac{(T - D)V_{TAS}}{mg} \left[ 1 + \left( \frac{V_{TAS}}{g} \right) \left( \frac{dV_{TAS}}{dh} \right) \right]^{-1}\]  \hspace{3cm} (A.2)

\[\frac{dh}{dt} = \frac{(T - D)V_{TAS}}{mg} f(M)\]  \hspace{3cm} (A.3)

\[f(M) = \left[ 1 + \left( \frac{V_{TAS}}{g} \right) \left( \frac{dV_{TAS}}{dh} \right) \right]^{-1}\]  \hspace{3cm} (A.4)

This energy share factor \(f(M)\) specifies how much of the available power is allocated to climb as opposed to acceleration while following a selected speed profile during climb. For a Constant Mach number below tropopause we have

\[f(M) = \left[ 1 + \frac{\gamma K_T}{2g} M^2 \right]\]  \hspace{3cm} (A.5)

Where,
- \(g\): is the gravitational acceleration, \(g = 9.81 \text{ m/s}^2\)
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Climb</th>
<th>Descent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acceleration (f(M))</td>
<td>0.3</td>
<td>1.7</td>
</tr>
<tr>
<td>Deceleration (f(M))</td>
<td>1.7</td>
<td>0.3</td>
</tr>
</tbody>
</table>

Table A.1. Mach Function Values

\(K_T\) : is the ISA temperature gradient with altitude below the tropopause, \(K_T = -0.0065\ \text{K/m}\)

\(M\) : is the Mach number

\(\gamma\) : is the isentropic expansion coefficient for air, \(\gamma = 1.4\)

This number is greater than 1 because below the tropopause, the temperature and thus, speed of sound decreases with altitude. Maintaining a constant Mach number during climb thus means that the true air speed decreases with altitude

For a Constant Calibrated Airspeed (CAS) below tropopause

\[
f(M) = \left[ 1 + \frac{\gamma K_T}{2g} M^2 + \left( 1 + \frac{\gamma - 1}{2} M^2 \right) \frac{\gamma}{\gamma - 1} \left[ 1 + \frac{\gamma - 1}{2} M^2 \right] \frac{\gamma}{\gamma - 1} \right]^{-1}
\]  
(A.6)

This number is less than 1 because as density decreases with altitude, maintaining a constant CAS during climb requires maintaining a continual increase in true air speed. In cases where neither constant Mach number nor constant CAS is maintained, Table A.1 gives the values for the mach function

A.2 True Air Speed

The true air speed is given by

\[
V_{TAS} = M \sqrt{\gamma RT}
\]  
(A.7)

where, \(M\) is the Mach number, \(T\) is the local temperature at altitude, \(R\) is the universal gas constant for air and \(\gamma\) is the isentropic expansion coefficient for air.

A.2.1 Variation of Speed with Mass of the Aircraft

Aircraft operating speeds vary with the aircraft mass. This variation is calculated as below

\[
V = V_{ref} \sqrt{\frac{m}{m_{ref}}}
\]  
(A.8)

Here, the aircraft reference speed \(V_{ref}\) is given for the reference mass \(m_{ref}\). The speed at another mass, \(m\), is then calculated as \(V\).
A.3 Power Plant Variations

The normal cruise thrust is by definition set equal to drag ($T = D$). However, the maximum amount of thrust available in cruise ($T_{\text{CruiseMax}}$) is limited. The maximum cruise thrust is calculated as a ratio of the maximum climb thrust ($T_{\text{MaxClimb}}$)

\[ T_{\text{CruiseMax}} = C_{Tcr} T_{\text{MaxClimb}} \]  \hspace{1cm} (A.9)

The coefficient $C_{Tcr}$ is currently uniformly set to 0.95 for all aircraft.

A.3.1 Take-Off Thrust

The take-off thrust ($T_{\text{takeoff}}$) is also computed as a fraction of the maximum climb thrust

\[ T_{\text{takeoff}} = C_{\text{takeoff}} T_{\text{maxcl}} \]  \hspace{1cm} (A.10)

Here the coefficient $C_{\text{takeoff}}$ is currently uniformly set to 1.2 for all aircraft.

A.3.2 Descent Thrust

The descent thrust is calculated similarly as cruise thrust with different correction factors used for high ($T_{\text{deshigh}}$) and low ($T_{\text{deslow}}$) altitude and approach and landing configurations, that is

For altitudes above 8,000 ft
if $h \geq h_{\text{des}}$

\[ T_{\text{deshigh}} = C_{T \text{deshigh}} T_{\text{maxcl}} \]  \hspace{1cm} (A.11)

if $h \leq h_{\text{des}}$

\[ T_{\text{desLow}} = C_{T \text{desLow}} T_{\text{maxcl}} \]  \hspace{1cm} (A.12)

Once the aircraft has descended below 8,000 ft it changes configuration to approach ($T_{\text{desApp}}$) as soon as the airspeed falls below a certain threshold
if $h \leq 8000$ ft and $V \leq V_{\text{minCruise}} + 10$ kts

\[ T_{\text{desApp}} = C_{T \text{desApp}} T_{\text{maxcl}} \]  \hspace{1cm} (A.13)

if $h \geq 3000$ ft and $V \leq V_{\text{minApproach}} + 10$ kts

\[ T_{\text{desLn}} = C_{T \text{desLn}} T_{\text{maxcl}} \]  \hspace{1cm} (A.14)
$T_{desL_n}$ is the thrust during the landing phase. The factors $C_T$ used throughout this section is the factor of the maximum climb thrust available during the other phases of flight.

A.3.3 Reduced Climb Power

Many aircraft use a reduced setting during climb ($C_{pow,red}$) in order to extend engine life and save cost. The correction factors ($C_{red}$) that are used to calculate the reduction in power have been obtained in an empirical way and have been validated with the help of air traffic controllers.

$$C_{Pow,red} = 1 - C_{red} \left( \frac{m_{max} - m_{act}}{m_{max} - m_{min}} \right)$$  \hspace{1cm} (A.15)

The value of $C_{red}$ is a function of the aircraft type. The usually used values for $C_{red}$ are: Turboprops: 0.25, Piston: 0, Jet: 0.15. $m$ is the mass of the aircraft. $min, max, act$ represent the minimum, maximum and the actual mass respectively.

The reduced climb power ($P_{red}$) is given as,

$$P_{red} = (T - D)V C_{red,pow}$$  \hspace{1cm} (A.16)

For all altitudes $\geq 0.8$ of $h_{max}$, $C_{red,pow} = 1$. The power reduction is to be applied in the Initial Climb and Climb phases.
REFERENCES


BIOGRAPHICAL STATEMENT

Manju Nag was born in Bangalore, India, in 1985. He received his bachelors degree from Vishveshvaraiah Technological University in Electronics and Communication in May, 2007. He came to the US to pursue a masters degree in Aerospace Engineering at The University of Texas at Arlington in September 2007. Working as a graduate research assistant, he specialised in the area of air traffic and uncertainties. As an intern, he worked for Metron Aviation for a duration of one semester before his graduation in May 2010. His current research interests are trajectory uncertainty and optimisation. He is a member of the AIAA student chapter.