TENNIS, GEOMETRIC PROGRESSION, PROBABILITY AND BASKETBALL

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Tennis, Geometric Progression, Probability and Basketball

The following problem about a tennis match is well-known. See Halmos [1, 2].

Consider $2^n$ tennis players playing a single elimination match. Ask the question: what are the number of games played? The answer can be obtained in two ways. First using the geometric progression

$$2^{n-1} + 2^{n-2} + \cdots + 2 + 1$$

we find that the answer is $2^n - 1$. We can also explain the answer as follows: for each game played there is a loser. Thus the total number of games played is equal to the number of losers. Since there is only one winner the total number of games played is equal to $2^n - 1$, the number of losers.

Halmos in [1] (also see [2]) discussed a problem closely related to the above problem in his article titled "mathematics as a creative art".

A natural question is: how can one interpret a similar geometric progression with fractional terms? In the game above we can ask for the probability of losing. This assumes that each round is independent of previous ones. We also assume that each player has an equal chance of winning. Thus, for example, my cousin who is the champion of a well-established tennis club in Fresno is not one of the players. The probability of winning such a game is $\frac{1}{2^n}$ and hence the probability of losing is equal to $1 - \frac{1}{2^n}$. This answer can be obtained from a geometric progression as follows. There is $\frac{1}{2}$ chance of losing the first round. If you win the first round and lose the second round there is $\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$ chance (using independence). Continuing the process we obtain the total probability of

$$\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots + \frac{1}{2^n} = 1 - \frac{1}{2^n}.$$

The next natural question is: what if we change from 2 to 3? The answer popped into the author's head while playing a game of 21 on a basketball court. In a game of 21 a few
players play on a single basket, each trying to win for himself or herself while the other
players try to get the ball and score for themselves. Consider $3^n$ players divided in
groups of 3 playing on $3^{n-1}$ baskets. Then consider the winners of these $3^{n-1}$ games
and divide them into $3^{n-2}$ groups of 3 and continue the process. Then the total number
of games played is equal to

$$3^{n-1} + 3^{n-2} + \cdots + 3 + 1 = \frac{3^n - 1}{2}.$$  

For each game played there are two losers. Thus the number of games played is equal to
half the number of losers which explains the answer $\frac{3^n - 1}{2}$. One can also ask for the
probability of losing which is equal to $1 - \frac{1}{3^n}$ which can also be obtained as the sum of a
geometric progression (exercise). (By "exercise", of course we do not mean to play tennis
or basketball.)

All of the above, especially the probability interpretation is not a mystery. After all
instead of sweating on a tennis or basketball court on a hot summer day, one can simulate
the game with a fair or weighted fictional coin in a cool office, to decide what to play and
to decide who will win. Your turn.
