# LES FOR WING TIP VORTEX AROUND AN AIRFOIL

by

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### ABSTRACT

### LES FOR WING TIP VORTEX AROUND AN AIRFOIL

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The wing tip vortex is very important because of its effects on the noise generation, blade/vortex interactions on helicopter blades, propeller cavitations on ships, and other fields. The objective of this work is to use the numerical simulation with high order accuracy and high resolution to investigate the formation and the near field evolution of a wing tip vortex at high Reynolds number.

The computational domain includes a rectangular half-wing with a NACA 0012 airfoil section, a rounded wing tip and the surrounding boundaries. The wing has an aspect ratio of 0.75. The angle of attack is 10 degrees. The Reynolds number based on

free-stream velocity and the chord length is  $4.6 \times 10^6$ . In the simulations, the free-stream Mach number is set to 0.2.

The flow field is solved by a fully-implicit time-marching N-S solver with 6th order compact scheme and 8th order filter. Non-reflecting boundary conditions with buffer are also utilized in the code to avoid non-physical reflection. The code uses MPI parallel computation whose performance scales almost linearly over a large number of processors.

The numerical simulation showed that the turbulent shear layer and the interaction between the primary and the secondary vortices are the major sources of turbulent activity in the vortex core. The information about the phenomena provided by the simulation can be used in various engineering applications at high Reynolds numbers.

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### CHAPTER 1

### INTRODUCTION

Tip vortices are generated at the tip of lifting surfaces where fluid flows from the high-pressure side to the low-pressure side, then rolls up and travels to the wing wake. The tip vortex system originated from the complex three-dimensional separated flow is highly unsteady and turbulent. The interactions between the primary and the secondary vortices, the vortices and the separated shear layer, and the vortices and the wakes occur simultaneously in the flow field.

The wing tip vortex is of importance because of its effects on many practical problems such as the landing distances for aircraft, the blade/vortex interactions on helicopter blades, propeller cavitations on ships, and other fields. For example, tip vortices contribute to the induced drag of the generating surface, a situation that is exacerbated for low aspect ratio surfaces such as marine propellers. The pressure driven flow about the tip of the lifting surface also decreases the efficiency of compressor and turbine blades. When vortices shed from the helicopter rotor blade interact with a following blade, the resulting unsteady forces may contribute to the premature blade failure. Blade vortex interaction also had been identified as the major source of noise generated by a rotor. In the airport, the trailing wing tip vortices produced by the large transport aircrafts pose a hazard to smaller following aircrafts. Therefore, in order to

minimize the separation time of aircraft during takeoff, as well as to reduce the tipvortex-induced drag and noise, it is absolutely necessary to study and predict or control the wing tip vortex, so as to allow the most efficient use airport facilities and improve the performance of blade and propellers.

Numerous experimental investigations have been conducted to understand the tip vortex structure and its dissipation or persistence. Shekarriz et al (1993) studied the evolution of the wingtip vortex by mapping its instantaneous lateral velocity at several consecutive axial locations. They also measured the axial velocity distribution. Lavi Zuhai et al (2001) carried out experimental investigations on wing tip vortex. They found that the wing sheds multiple vortices. The interaction of the vortices gives rise to the unsteady motion of the wing tip vortex. Andreas Vogt et al (1996) presented their PIV measurement results on the tip vortex of a NACA0012 wing. The velocity profiles indicate a non linear increase of the azimuthal velocity with growing distance from the vortex center as it is implied by the solid body rotation model of the vortex core. Devenport et al (1996) performed experiments on the tip vortex generated from a rectangular NACA 0012 half-wing. A very important conclusion of their work is that the vortex is laminar. Chow, Zilliac and Bradshaw (1997) took measurements of a wing tip vortex. They found that the turbulence in the tip vortex decayed quickly along stream-wise direction. Spalart (1998) gave a review on the wingtip vortex. Gregory Zilliac (1998) studied the skin friction distribution on a wingtip. D. Birch at el (2004) carried out the experimental investigation on the induced drag of a tip vortex in the near wake of a rectangular, square-tipped NACA0015 airfoil. K. J. Desabrais and H. Johari

(1998) measured the circulation distribution of a wingtip vortex directly by the ultrasound technique. The development of the wingtip vortex is also influenced by the wing end-cap geometry. By way of PIV (Particle Image Velocimetry), Souza el al (2001) investigated the wingtip vortex generated by a rectangular NACA0012 halfwing. Wings with different tip shape (Square, semi-circular and semi-elliptical with a 2:1 ratio) were considered. In their studies, for the different wing tips, there were not significant differences between the results such as the ratio of the circulation to the theoretical bound circulation. Elgin Anderson at el (2003) found the relationship between the vortex strength and axial velocity in a trailing vortex. However, their results indicated that the magnitude of the axial velocity is sensitive to the two end-cap configurations (flat and rounded). E.A. Anderson at el (2000) studied the influence of wing end-cap geometry and flow parameters on the wingtip vortex. There are also some experiments on controlling wing tip vortex. Charles S. Matthewson et al (1998) carried out an experimental study on the effects of the destabilizing the tip vortices from a model rotor blade using perturbations introduced by discrete jets located at the tips. Joshua et al (1999) performed the aerodynamic analysis on the potential of wing tip sails to control the wing tip vortex and increase the aerodynamic efficiency.

There are also many numerical simulations for wingtip vortex based on the Reynolds averaged Navier-Stokes equations (RANS, see Pankajakshan *et al.* 2001). Dacles-Mariani, *et al.* (1995) gave a RANS investigation on wingtip vortex based on the experimental boundary data. The computational domain in their study includes a rectangular wing with a NACA0012 airfoil section and a rounded wing tip. A modified

Baldwin-Barth turbulence model was used in their simulation of the wingtip vortex. They also addressed the dissipation effect due to the numerical methods and turbulence model. Their work shows that it is possible to predict the mean flow of the tip vortex near field by RANS code. Dacles-Mariani et al (1996) compared effects of different turbulence models in the computation. They found that both the Baldwin-Barth model and Spalart-Allmaras model are not adequate for accurate tip vortex flow prediction. Dacles-Mariani et al (1997) explored the validity of modeling the near- and intermediate wake region behind wings with a simplified Navier-Stokes equations. E. A. Anderson et al (2001) carried out the numerical investigation into vortex structures formed by wing with flat end-caps. Robert E. Spall (2001) addressed the excessive diffusion of wingtip vortex in the RANS. His work focused on the use of computational grids that provide very high resolution of the tip vortex without excessive placement of grid points in regions where velocity gradients are small. G. Lombardi and F. Cannizzo carried out the numerical investigation on the tip vortex by using the Euler solver. They found that the flow behavior is related to the viscous effects and the inviscid methods are unable to adequately represent tip flow typology. Chow et al (1997)'s experiment indicated that the isotropic-eddy-viscosity-based prediction model could not fully model the turbulence in the vortex. Furthermore because the generation of the wingtip vortex is strongly time-dependent, the capacity of RANS to study the mechanism of wingtip vortex is still questionable.

The alternative approaches include direct numerical simulation (DNS) and large eddy simulation (LES). There are some reports regarding DNS and LES for near field turbulent wake (Mittal *et al.* 1996; Xia *et al.* 1997), but only for 2-D circular cylinder at a Reynolds number of 3900. For high Reynolds number flow, there are some reports from CTR (Wang 1997, 1999, 2001) for trailing edge or circular cylinder.

Large Eddy Simulation (LES) is a technique that has been successfully applied to the study of turbulent flows, and has became a very important method of numerical simulation, especially for the simulations of flows with high Reynolds numbers. In LES, only the large energy-carrying scales are resolved, while the influence of small or sub-grid scales must be modeled appropriately. A filtering process is usually used to separate the large- and small-scale motions. Large-scale structures are resolved by the filtered equations, but a subgrid model is employed to formulate the contributions from the sub-grid scale fluctuations, such as the sub-grid scale stress and heat flux terms. LES is able to solve problems within complex geometry (Ducros et al., 1996). Furthermore, compared with DNS, LES is more favorable in simulations of high Reynolds number flows, since it needs much fewer grid points.

T. Gerz et al (1998) discussed the decay of the wingtip vortices and the distribution of the turbine exhaust under the influence of turbulence in a stratified atmosphere by means of LES. Youssef et al (1998) used LES to investigate the mechanism of turbulence generation of wingtip vortex. They devised a simple model of the near wake of a wing with finite span. That model was used to initialize the flow field for temporal LES of the interaction between a nominally 2D wake and the rolling trailing vortex sheet. The modified MacCormack scheme with second order in time and fourth order in space for the convective terms was used to solve the large eddy

equations. Smagorinsky model was used to represent the SGS stresses. Fleig *et al* (2004) use LES to investigate the wingtip vortex generated by a finite blade with NACA0012 section at Reynolds numbers of  $4.06 \times 10^5$ . Smagorinsky eddy viscosity model and up to 300 million grid points were used in their simulations. The solution was advanced in time using a second-order Beam-Warming approach and a three-point backward differencing for the time derivative. The spatial derivatives were discretized using a third order finite-difference upwind scheme.

Despite those numerous studies, the formation and early development of wing tip vortex is still not fully understood. For instance, the trailing vortices remain a long way downstream while the turbulence decays quickly. The physics is still not well understood.

### CHAPTER 2

## GOVERNING EQUATIONS AND PHYSISCS MODEL

### 2.1 Governing Equations

The governing equations solved are the conservation form of three-dimensional compressible Navier-Stokes equations in body-fitted coordinates. For the Navier-Stokes equations, the Favre-filtering operation is used. The resolved velocity and temperature fields, written in terms of the Favre-filtered quantities, can be defined as

$$\widetilde{F} = \frac{\rho F}{\overline{\rho}}$$

Where the "— " denotes the spatial filtering. The non-dimensional Favrefiltered governing equations of continuity, momentum, and temperature are described in conservative forms as follows:

$$\frac{\partial \overline{\rho}}{\partial t} + \frac{\partial}{\partial x_k} (\overline{\rho} \widetilde{u}_k) = 0$$
(2.1)

$$\frac{\partial \overline{\rho} \widetilde{u}_{k}}{\partial t} + \frac{\partial}{\partial x_{l}} (\overline{\rho} \widetilde{u}_{k} \widetilde{u}_{l}) = -\frac{\partial \overline{p}}{\partial x_{k}} + \frac{1}{\operatorname{Re}} \frac{\partial \widetilde{\sigma}_{kl}}{\partial x_{l}} + \frac{\partial \tau_{kl}}{\partial x_{l}}$$

$$\frac{\partial \overline{\rho} \widetilde{T}}{\partial t} + \frac{\partial}{\partial x_{k}} (\overline{\rho} \widetilde{u}_{k} \widetilde{T}) = -\gamma (\gamma - 1) M_{\infty}^{2} \overline{p} \frac{\partial \widetilde{u}_{k}}{\partial x_{k}} + \frac{\gamma (\gamma - 1) M_{\infty}^{2}}{\operatorname{Re}} \widetilde{\sigma}_{kl} \frac{\partial \widetilde{u}_{k}}{\partial x_{l}} + \frac{\partial}{\partial x_{l}}$$

$$+ \frac{\partial}{\partial x_{k}} (\frac{\gamma \widetilde{\mu}}{\operatorname{Pr} \operatorname{Re}} \frac{\partial \widetilde{T}}{\partial x_{k}}) + \frac{\partial q_{k}}{\partial x_{k}}$$
(2.2)

where  $\rho$  is the density,  $u_k$  the velocity component in the kth direction, p the pressure, and T the temperature. The viscous stress is

$$\widetilde{\sigma}_{kl} = \widetilde{\mu} \left[ \left( \frac{\partial \widetilde{u}_k}{\partial x_l} + \frac{\partial \widetilde{u}_l}{\partial x_k} \right) - \frac{2}{3} \frac{\partial \widetilde{u}_m}{\partial x_m} \delta_{kl} \right]$$
(2.4)

In the non-dimensionalization, the reference values for length, density, velocity, and temperature are  $\delta_{in}$ ,  $\rho_{\infty}$ ,  $U_{\infty}$ , and  $T_{\infty}$ , respectively.  $\delta_{in}$  is the displacement thickness of inflow. The Mach number, the Reynolds number, the Prandtl number, and the ratio of specific heats, are defined respectively as follows:

$$M_{\infty} = \frac{U_{\infty}}{\sqrt{\gamma R T_{\infty}}}, \qquad \text{Re} = \frac{\rho_{\infty} U_{\infty} \delta_{in}}{\mu_{\infty}}, \qquad \text{Pr} = \frac{C_p \mu_{\infty}}{\kappa_{\infty}}, \qquad \gamma = \frac{C_p}{C_v}$$

where R is the ideal gas constant,  $C_p$  and  $C_v$  are the specific heats at constant pressure and constant volume, Through this paper, Pr=0.7, and  $\gamma$ =1.4. The viscosity is determined according to Surtherland's law, in dimensionless form

$$\mu = \frac{T^{3/2}(1+S)}{T+S}, \quad S = \frac{110.3K}{T_{\infty}}$$

In Equation (2. 3) and (2. 4), the sub grid model scale stress and heat flux are denoted by

$$\tau_{kl} = \overline{\rho} \left( \widetilde{u_k u_l} - \widetilde{u}_k \widetilde{u}_l \right) \tag{2.5}$$

$$q_k = -\overline{\rho} \left( \widetilde{u_k T} - \widetilde{u}_k \widetilde{T} \right)$$
(2.6)

Those are needed to be modeled.

Finally, the equations can be written in the conservative forms,

$$\frac{1}{J}\frac{\partial Q}{\partial t} + \frac{\partial (E - E_{\nu})}{\partial \xi} + \frac{\partial (F - F_{\nu})}{\partial \eta} + \frac{\partial (G - G_{\nu})}{\partial \zeta} = 0$$
(2.7)

The vector of conserved quantities Q, inviscid flux vector (E, F, G), and viscous flux vector  $(E_v, F_v, G_v)$  are defined as

$$Q = \begin{pmatrix} \overline{\rho} \\ \overline{\rho}\tilde{u} \\ \overline{\rho}\tilde{v} \\ \overline{\rho}\tilde{w} \\ \overline{e} \end{pmatrix}, \quad E = \frac{1}{J} \begin{pmatrix} \overline{\rho}\tilde{U} \\ \overline{\rho}\tilde{U}\tilde{u} + \tilde{p}\xi_x \\ \overline{\rho}\tilde{U}\tilde{v} + \tilde{p}\xi_y \\ \overline{\rho}\tilde{U}\tilde{w} + \tilde{p}\xi_z \\ \tilde{U}(\tilde{e} + \tilde{p}) \end{pmatrix}, \quad F = \frac{1}{J} \begin{pmatrix} \overline{\rho}\tilde{V} \\ \overline{\rho}\tilde{V}\tilde{u} + \tilde{p}\eta_x \\ \overline{\rho}\tilde{V}\tilde{v} + \tilde{p}\eta_y \\ \overline{\rho}\tilde{V}\tilde{w} + \tilde{p}\eta_z \\ \overline{\rho}\tilde{V}\tilde{w} + \tilde{p}\eta_z \\ \tilde{V}(\tilde{e} + \tilde{p}) \end{pmatrix}, \quad G = \frac{1}{J} \begin{pmatrix} \overline{\rho}\tilde{W} \\ \overline{\rho}\tilde{W}\tilde{u} + \tilde{p}\zeta_x \\ \overline{\rho}\tilde{W}\tilde{v} + \tilde{p}\zeta_z \\ \overline{\rho}\tilde{W}\tilde{w} + \tilde{p}\zeta_z \\ \tilde{W}(\tilde{e} + \tilde{p}) \end{pmatrix}$$

$$E_{v} = \frac{1}{J} \begin{pmatrix} 0 \\ \tau_{xx}\xi_{x} + \tau_{yx}\xi_{y} + \tau_{zx}\xi_{z} \\ \tau_{xy}\xi_{x} + \tau_{yy}\xi_{y} + \tau_{zy}\xi_{z} \\ \tau_{xz}\xi_{x} + \tau_{yz}\xi_{y} + \tau_{zz}\xi_{z} \\ q_{x}\xi_{x} + q_{y}\xi_{y} + q_{z}\xi_{z} \end{pmatrix}, \quad F_{v} = \frac{1}{J} \begin{pmatrix} 0 \\ \tau_{xx}\eta_{x} + \tau_{yx}\eta_{y} + \tau_{zx}\eta_{z} \\ \tau_{xy}\eta_{x} + \tau_{yy}\eta_{y} + \tau_{zy}\eta_{z} \\ \tau_{xz}\eta_{x} + \tau_{yz}\eta_{y} + \tau_{zz}\eta_{z} \\ q_{x}\eta_{x} + q_{y}\eta_{y} + q_{z}\eta_{z} \end{pmatrix}, \quad G_{v} = \frac{1}{J} \begin{pmatrix} 0 \\ \tau_{xx}\zeta_{x} + \tau_{yx}\zeta_{y} + \tau_{zx}\zeta_{z} \\ \tau_{xy}\zeta_{x} + \tau_{yy}\zeta_{y} + \tau_{zy}\zeta_{z} \\ \tau_{xz}\eta_{x} + \tau_{yz}\eta_{y} + \tau_{zz}\eta_{z} \\ q_{x}\eta_{x} + q_{y}\eta_{y} + q_{z}\eta_{z} \end{pmatrix}, \quad G_{v} = \frac{1}{J} \begin{pmatrix} 0 \\ \tau_{xx}\zeta_{x} + \tau_{yx}\zeta_{y} + \tau_{zx}\zeta_{z} \\ \tau_{xy}\zeta_{x} + \tau_{yy}\zeta_{y} + \tau_{zy}\zeta_{z} \\ \tau_{xz}\zeta_{x} + \tau_{yz}\zeta_{y} + \tau_{zz}\zeta_{z} \\ q_{x}\zeta_{x} + q_{y}\zeta_{y} + q_{z}\zeta_{z} \end{pmatrix}$$

Where  $J \equiv \frac{\partial(\xi, \eta, \zeta)}{\partial(x, y, z)}$  is Jacobian of the coordinate transformation between the

curvilinear  $(\xi, \eta, \zeta)$  and cartesian (x, y, z) frames, and  $\xi_x, \xi_y, \xi_z, \eta_x, \eta_y, \eta_z, \zeta_x, \zeta_y, \zeta_z$ are coordinate transformation metrics.

The contravariant velocity components U, V, W are defined as

$$U = u\xi_x + v\xi_y + w\xi_z, \quad V = u\eta_x + v\eta_y + w\eta_z, \quad W = u\zeta_x + v\zeta_y + w\zeta_z,$$

Where  $\tilde{e}$  denotes the total energy.

#### 2.2 Filtered Structure Function Model

Although our object is to study the incompressible flow field, the current LES code still use compressible equations at the low Mach number flows, where flow could be considered as incompressible. The filtered structure-function model developed by Ducros et al.(1996) was used in our simulation. The subgrid scale shear stress and heat flux can be modeled as

$$\tau_{kl} = \overline{\rho} v_l \left[ \left( \frac{\partial \widetilde{u}_k}{\partial x_l} + \frac{\partial \widetilde{u}_l}{\partial x_k} \right) - \frac{2}{3} \frac{\partial \widetilde{u}_m}{\partial x_m} \delta_{kl} \right]$$
(2.8)

$$q_{k} = \frac{\gamma \overline{\rho} v_{t}}{\Pr_{t}} \frac{\partial \tilde{T}}{\partial x_{k}}$$
(2.9)

Here,  $Pr_t$  is the turbulent Prandtl number taken equal to 0.6 as in isotropic turbulence,  $v_t$  is the turbulent kinetic viscousity defined as

$$\nu_t(x,t) = 0.0014 C_k^{-3/2} \Delta \left[ \tilde{F}_2^{(3)}(x,t) \right]^{1/2}$$
(2.10)

where  $F_2^{(3)}$  is the filtered structure function. In this case,  $\tilde{F}_2^{(3)}$  takes the fourth-neighbor formulation proposed by Normand & Lesieur (1992).

$$\begin{split} \tilde{F}_{2}^{(3)} &= \frac{1}{4} \bigg[ \left\| \tilde{u}_{i+1,j,k}^{(3)} - \tilde{u}_{i,j,k}^{(3)} \right\|^{2} + \left\| \tilde{u}_{i-1,j,k}^{(3)} - \tilde{u}_{i,j,k}^{(3)} \right\|^{2} + \left\| \tilde{u}_{i,j+1,k}^{(3)} - \tilde{u}_{i,j,k}^{(3)} \right\|^{2} + \left\| \tilde{u}_{i,j-1,k,}^{(3)} - \tilde{u}_{i,j,k}^{(3)} \right\|^{2} \\ &+ \left\| \tilde{u}_{i,j,k+1}^{(3)} - \tilde{u}_{i,j,k}^{(3)} \right\|^{2} + \left\| \tilde{u}_{i,j,k-1}^{(3)} - \tilde{u}_{i,j,k}^{(3)} \right\|^{2} \bigg] \\ & \text{where} \quad \tilde{u}_{i,j,k}^{(3)} &= HP^{(3)}(\tilde{u}_{i,j,k}) \end{split}$$
(2.11)

 $\Delta = \sqrt{\Delta_x \Delta_y \Delta_z}$  is used to characterize the grid size.  $C_k$  is the Kolmogorov constant taking the value of 1.4.  $HP^{(3)}$  is a discrete Laplacian filter iterated 3 times, which is served as a high-pass filter before computing the filtered structure function. The first iteration of the Laplacian filter  $HP^{(1)}$  is defined by

$$\widetilde{u}_{i,j,k}^{(1)} = HP^{(1)}(\widetilde{u}_{i,j,k}) = \widetilde{u}_{i+1,j,k} - 2\widetilde{u}_{i,j,k} + \widetilde{u}_{i-1,j,k} 
+ \widetilde{u}_{i,j+1,k} - 2\widetilde{u}_{i,j,k} + \widetilde{u}_{i,j-1,k} + \widetilde{u}_{i,j,k+1} - 2\widetilde{u}_{i,j,k} + \widetilde{u}_{i,j,k-1}$$
(2.12)

### CHAPTER 3

### NUMERICAL METHODS

The LU-SGS implicit solver (Yoon, 1992) scheme based on the second-order backward Euler algorithm is used for time advancing. Spatial derivatives are calculated using the sixth-order centered compact differencing (Lele, 1992) in the interior of the domain, together with fourth-order closures at points immediately adjacent to the boundary and third-order closures at the boundary itself. Coordinate transformation metrics are also evaluated using the same compact scheme by an approach that satisfies the geometric conservation law numerically (Gaitonde & Visbal, 1999). High order compact filtering (Lele, 1992) is used at regular intervals to suppress the numerical oscillation associated with the high-order compact scheme. Parallel computation based on the Message Passing Interface (MPI) has been utilized to improve the performance of the code. The parallel computation is combined with the domain decomposition method. The computational domain is divided into *n* equal-sized subdomains along the  $\xi$  direction. Readers may refer to our early work (Shan *et al.*, 2000) for more details of the parallel algorithm. The details of the numerical method and the numerical verification and applications of the computer code can be found in our previous work (Jiang et al., 1999a, Shan et al, 2000, Shan et al, 2005).

#### <u>3.1 Compact Finite Difference Scheme</u>

Compact schemes have been widely used in the simulation of complex flows, especially in the direct numerical simulation of turbulent flows (Jiang *et al.*, 1999; Shan *et al.*, 1999; Visbal *et al.*, 1998). Standard finite difference schemes have explicit forms and need to be at least one point wider than the desired approximation order. It is also difficult to find suitable and stable boundary closure for high order schemes. Compared to the standard finite difference approximations, the compact schemes can achieve higher order accuracy without increasing the stencil width. As the compact schemes have implicit forms and involve derivative values of neighboring grid points, additional free parameters can be used not only to improve the accuracy but also to optimize the other properties such as resolution and stability.

The resolution is the largest wave number that can be accurately represented by the scheme. Many complex flows possess a large range of time and space scales. The resolution characteristic of the scheme is essentially important in complex flow simulations. A family of centered compact schemes proposed by Lele (1992) has been proved to have spectral-like resolution.

The general form of compact finite difference schemes can be written as follows:

$$\beta_{-}f_{j-2}^{'} + \alpha_{-}f_{j-1}^{'} + f_{j}^{'} + \alpha_{+}f_{j+1}^{'} + \beta_{+}f_{j+2}^{'} = \frac{1}{h}(b_{-}f_{j-2} + a_{-}f_{j-1} + cf_{j} + a_{+}f_{j+1} + b_{+}f_{j+2})$$
(3.1)  
where  $f_{i}^{'}$  is the derivative at point  $j$ .

In this work, the sixth order compact scheme is used to calculate the spatial derivatives in streamwise ( $\xi$ ), spanwise ( $\eta$ ), and wall-normal ( $\zeta$ ) directions. In the

spanwise direction, the spectral method can also be used in the place of the compact scheme. The coefficients cooresponding to sixth order compact scheme are as following

$$\beta_{-} = 0, \quad \alpha_{-} = \frac{1}{3}, \quad \alpha_{+} = \frac{1}{3}, \quad \beta_{+} = 0,$$

$$b_{-} = -\frac{1}{36}, \quad a_{-} = -\frac{1}{9}, \quad a_{+} = \frac{1}{9}, \quad b_{+} = \frac{1}{36}.$$
(3.2)

The fourth order compact scheme is used at points j=2, N-1, and the third order one-sided compact scheme is used at the boundary points j=1, N.

$$j = 1 \qquad \alpha_{+} = 2, \quad b_{+} = \frac{1}{2}, \quad a_{+} = 2, \quad c = -\frac{5}{2};$$

$$j = 2/N - 1 \quad \alpha_{-} = \frac{1}{4}, \quad \alpha_{+} = \frac{1}{4}, \quad a_{-} = -\frac{3}{4}, \quad a_{+} = \frac{3}{4}, \quad c = 0;$$

$$j = N \qquad \alpha_{-} = 2, \quad b_{-} = -\frac{1}{2}, \quad a_{-} = -2, \quad b_{+} = \frac{5}{2}.$$
(3.3)

Those coefficients which are not listed are set to zero.

To demonstrate the high order accuracy of the compact scheme used in this work, the following one-dimensional convection equation is solved numerically using the six-order compact scheme. The fourth order Runge-Kutta scheme (Shu, 1988) is used for time integration.

$$u_t + u_x = 0, \quad -1 \le x \le 1$$
  

$$u(x,0) = u_0(x), \qquad \text{periodic with a person of } 2.$$
(3.4)

The initial function is given by  $u_0(x) = \sin(\pi x)$ .  $L_1$  and  $L_{\infty}$  errors are listed in Table 3.1. N is the number of grid points. This data shows that the six-order accuracy is achieved.

Ν	$L_{\infty}$ error	$L_{\infty}$ order	$L_{\infty}$ error	$L_{\infty}$ order
20	1.48E-5		9.46E-6	
40	2.26E-7	6.03	1.44E-7	6.02
80	3.57E-9	5.98	2.27E-9	5.98
160	5.88E-11	5.92	3.74E-11	5.92
320	1.07E-12	5.78	5.73E-13	6.02

Table 3.1 Errors of the numerical solution

In order to eliminate the spurious numerical oscillations caused by central difference schemes, spatial filtering is used instead of artificial dissipation. Implicit sixth-order compact scheme for space filtering (Lele, 1992) is applied for primitive variables u, v, w,  $\rho$ , P after a specified number of time steps. The general form of compact filters can be written as follows

$$\beta \hat{f}_{j-2} + \alpha \hat{f}_{j-1} + \hat{f}_{j+1} + \alpha \hat{f}_{j+1} + \beta \hat{f}_{j+2} = af_j + \frac{d}{2} (f_{j+3} + f_{j-3}) + \frac{c}{2} (f_{j+2} + f_{j-2}) + \frac{b}{2} (f_{j+1} + f_{j-1})$$
(3.5)

The coefficients for sixth-order filter are

$$\alpha = \frac{5}{8}, \quad \beta = \frac{3 - 2\alpha}{10}, \quad a = \frac{2 + 3\alpha}{4}, \quad b = \frac{6 + 7\alpha}{8}, \quad c = \frac{6 + \alpha}{20}, \quad d = \frac{2 - 3\alpha}{40}.$$
 (3.6)

The following explicit fourth-order filters are used near/at the boundaries.

$$\hat{f}_{1} = \frac{15}{16}f_{1} + \frac{1}{16}(4f_{2} - 6f_{3} + 4f_{4} - f_{5}),$$

$$\hat{f}_{2} = \frac{3}{4}f_{2} + \frac{1}{16}(f_{1} + 6f_{3} - 4f_{4} + f_{5}),$$

$$\hat{f}_{3} = \frac{5}{8}f_{3} + \frac{1}{16}(-f_{1} + 4f_{2} + 4f_{4} - f_{5}),$$
(3.7)

### 3.2 Implicit Time Integration

A second order Euler Backward scheme is used for time derivatives, and the fully implicit form of the discretized equations is given by

$$\frac{3Q^{n+1} - 4Q^n + Q^{n-1}}{2J\Delta t} + \frac{\partial \left(E^{n+1} - E_v^{n+1}\right)}{\partial \xi} + \frac{\partial \left(F^{n+1} - F_v^{n+1}\right)}{\partial \eta} + \frac{\partial \left(G^{n+1} - G_v^{n+1}\right)}{\partial \zeta} = 0$$
(3.8)

 $Q^{n+1}$  is estimated iteratively as:

$$Q^{n+1} = Q^p + \delta Q^p \tag{3.9}$$

where,

$$\delta Q^{p+1} = Q^{p+1} - Q^p \tag{3.10}$$

At step p = 0,  $Q^p = Q^n$ ; as  $\delta Q^p$  is driven to zero,  $Q^p$  approaches  $Q^{n+1}$ . The flux vectors are linearized as follows:

$$E^{n+1} \approx E^{p} + A^{p} \delta Q^{p}$$

$$F^{n+1} \approx F^{p} + B^{p} \delta Q^{p}$$

$$G^{n+1} \approx G^{p} + C^{p} \delta Q^{p}$$
(3.11)

So that the equation (3. 8) can be written as:

$$\left[\frac{3}{2}I + \Delta t J (D_{\xi}A + D_{\eta}B + D_{\zeta}C)\right] \delta Q^{p} = R$$
(3.12)

where *R* is the residual:

$$R = -\left(\frac{3}{2}Q^{p} - 2Q^{n} + \frac{1}{2}Q^{n-1}\right) - \Delta t J \left[D_{\xi}(E - E_{\nu}) + D_{\eta}(F - F_{\nu}) + D_{\zeta}(G - G_{\nu})\right]^{p}$$
(3.13)

The right hand side of Equation (3. 12) is discretized using sixth-order compact scheme (Lele, 1992) for spatial derivatives, and the left hand side of the equation is discretized following LU-SGS method (Yoon & Kwak, 1992). In this method, the Jacobian matrices of flux vectors are split as:

$$A = \frac{\partial E}{\partial Q}, \quad B = \frac{\partial F}{\partial Q}, \quad C = \frac{\partial G}{\partial Q}$$
(3.14)

The right hand side of Equation (3. 12) is discretized using sixth-order compact scheme (Lele, 1992) for spatial derivatives, and the left hand side of the equation is discretized following LU-SGS method (Yoon & Kwak, 1992).

In this method, the Jacobian matrices of flux vectors are split as:

$$A = A^{+} + A^{-}, \quad B = B^{+} + B^{-}, \quad C = C^{+} + C^{-}$$
 (3.15)

where,

$$A^{\pm} = \frac{1}{2} [A \pm r_A I], \quad B^{\pm} = \frac{1}{2} [B \pm r_B I], \quad C^{\pm} = \frac{1}{2} [C \pm r_C I]$$
(3.16)  
and

 $r_A = \kappa \max\left[\left|\lambda(A)\right|\right] + \tilde{\nu}, \quad r_B = \kappa \max\left[\left|\lambda(B)\right|\right] + \tilde{\nu}, \quad r_C = \kappa \max\left[\left|\lambda(B)\right|\right] + \tilde{\nu},$ 

(3.17)

where  $\lambda(A), \lambda(B), \lambda(C)$  are eigenvalues of A, B, C respectively,  $\kappa$  is a constant greater than 1.  $\tilde{\nu}$  is taken into account for the effects of viscous terms, and the following expression is used:

$$\tilde{\nu} = \max\left[\frac{\mu}{(\gamma - 1)M_r^2 \operatorname{Re}\operatorname{Pr}}, \frac{3}{4}\frac{\mu}{\operatorname{Re}}\right]$$
(3.18)

The first-order upwind finite difference scheme is used for the split flux terms on the left hand side of Equation (3. 12). This does not affect the accuracy of the scheme. As the left hand side is driven to zero, the discretization error will also be driven to zero. The finite difference representation of Equation (3. 12) can be written as:

$$\begin{bmatrix} \frac{3}{2}I + \Delta t J \left(r_A + r_B + r_C\right)I \end{bmatrix} \delta Q_{i,j,k}^p = R_{i,j,k}^p$$

$$-\Delta t J \begin{bmatrix} A^- \delta Q_{i+1,j,k}^p - A^+ \delta Q_{i-1,j,k}^p \\ + B^- \delta Q_{i,j+1,k}^p - B^+ \delta Q_{i,j-1,k}^p \\ + C^- \delta Q_{i,j,k+1}^p - C^+ \delta Q_{i,j,k-1}^p \end{bmatrix}$$
(3. 19)

In LU-SGS scheme, Equation (3. 19) is solved by three steps. First initialize  $\delta Q^0$  using

$$\delta Q_{i,j,k}^{0} = \left[\frac{3}{2}I + \Delta t J \left(r_{A} + r_{B} + r_{C}\right)I\right]^{-1} R_{i,j,k}^{p}$$
(3.20)

In the second step, the following relation is used:

$$\delta Q_{i,j,k}^{*} = \delta Q_{i,j,k}^{0} + \left[\frac{3}{2}I + \Delta t J \left(r_{A} + r_{B} + r_{C}\right)I\right]^{-1} \times \left[\Delta t J \left(A^{+} \delta Q_{i-1,j,k}^{*} + B^{+} \delta Q_{i,j-1,k}^{*} + C^{+} \delta Q_{i,j,k-1}^{*}\right)\right]$$
(3.21)

For the last step, is obtained by

$$\delta Q_{i,j,k}^{p} = \delta Q_{i,j,k}^{*} + \left[\frac{3}{2}I + \Delta t J \left(r_{A} + r_{B} + r_{C}\right)I\right]^{-1} \times \left[\Delta t J \left(A^{-} \delta Q_{i+1,j,k}^{p} + B^{-} \delta Q_{i,j+1,k}^{p} + C^{-} \delta Q_{i,j,k+1}^{p}\right)\right]$$
(3.22)

The sweeping of the computational domain is performed along the planes of i + j + k = const, i.e. in the second step, sweeping is from the low-left corner of the grid to the high-right corner, and then vice versa in the third step.

### 3.3 Parallel Computation

The parallel version of the numerical simulation code based on the Message Passing Interface (MPI) has been developed to improve the performance. The parallel computing is combined with domain decomposition method. The computational domain is divided into *n* equal-sized subdomains along the  $\xi$  direction as shown in Figure (3.1), where *n* is the number of processors. This is a simple partition with a balanced load for each processor. During computation, a processor communicates with it neighbors through exchanging the data at left and right boundary of each subdomain. But this type of communication is not suitable for calculating derivative in the  $\xi$  direction while using

the compact finite difference scheme. If each grid node along a  $\xi$  grid line locates in the same processor, it will be straightforward to use the compact scheme. In Figure (3.2), a data structure with four processors is used as an example to illustrate a special type of data exchange which has been utilized to accomplish the data structure transformation. Figure (3.1) shows the original partition where the computational domain is divided along the  $\xi$  direction. This data structure can be transformed to a new structure shown in Figure (3.2) where the domain is divided along the  $\zeta$  direction. The transformation is accomplished by first defining two new data types and then calling a MPI routine "MPI\_ALLTOALL" from the MPI library. In the new data structure, all the grid nodes along a  $\xi$  grid line are stored in one processor. After the calculation of derivative is completed, an inverse transformation is used to the transfer the data structure back to original partition.



Figure 3.1 The domain decomposition along  $\xi$  direction in the computational space



Figure 3.2 The change of data structure for calculating derivative in  $\xi$  direction by way of the compact scheme

This parallel version of the code is based on domain decomposition. The computational domains are evenly divided along the streamwise direction, so that each processor has the same load. To run sequential simulations, we can use one processor. But local refinement hasn't been attempted.

A test case has been used to evaluate the performance of the MPI code to calculate the derivatives in the  $\xi$ ,  $\eta$ , and  $\zeta$  directions on a 480×160 ×80 grid. The performance of a parallel computing is measured by the speedup *S* (*n*, *p*) which is defined as the ratio of the runtime of a serial program to the runtime of the parallel program.

$$S(n,p) = \frac{T_{\delta}(n)}{T_{\pi}(n,p)}$$
(3.23)

where  $T_{\delta}(n)$  denotes the runtime of the serial program running with one process.  $T_{\pi}(n, p)$  denotes the runtime of the parallel code running with *p* processes. The meaning of *S* (*n*, *p*) was shown in the table 3.2.

Table 3.2 Meaning of S(n, p)

S(n,p) > p	the parallel program is exhibiting super-linear speedup
S(n,p) = p	the parallel program is exhibiting linear speedup
1 < S(n, p) < p	the parallel program is exhibiting speedup
S(n,p) < 1	the parallel program is exhibiting slowdown

An alternative to speedup is efficiency, which is a measure of process utilitization in a parallel program, relative to the serial program. It is defined as

$$E(n,p) = \frac{S(n,p)}{p} = \frac{T_{\delta}(n)}{pT_{\pi}(n,p)}$$
(3.24)

The meaning of E(n, p) was shown in the table 3.3.

T 11	<b>^</b>	<b>^</b>		•	<u> </u>	-	/	\	
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E(n,p) > 1	the parallel program is exhibiting super-linear speedup
E(n,p) = 1	the parallel program is exhibiting linear speedup
1/p < E(n, p) < 1	the parallel program is exhibiting speedup
E(n,p) < 1/p	the parallel program is exhibiting slowdown



Figure 3.3 Speedup *S*(*n*, *p*) of derivatives calculation by compact scheme



Figure 3.4 Efficiency E(n, p) of derivatives calculation by compact scheme

Figure (3.3)-(3.4) show the performance of MPI code to compute derivatives using the compact scheme. The speedups and efficiency on a SGI Origin 2000 computer are displayed as functions of the number of processors. Super-linear performance has been achieved to calculate derivatives in  $\eta$  and  $\zeta$  directions where no data exchange is required. In the  $\xi$  direction, the speedup is much lower because of the massive data exchange in the data structure transformation introduced above. Figure (3.5)-(3.6) compare the overall performance of the MPI Code with serial code complied with automatic parallelization option.



Figure 3.5 Speedup S(n, p) of the MPI code compared with the serial code complied with automatic parallelization option



Figure 3.6 Efficiency E(n, p) of the MPI code compared with the serial code complied with automatic parallelization option

#### 3.4 Boundary Conditions

The following boundary conditions are used during the preliminary validation

exercises described in the dissertation.

#### 3.4.1 Wall boundary conditions

The Non-slip condition for velocity can be written as:

$$u = v = w = 0$$

The Adiabatic condition for temperature can be written as:

$$\frac{\partial T}{\partial n} = 0$$

The second order scheme is used to approximate the temperature derivative.

$$T_1 = (4T_2 - T_3)/3$$

The boundary condition for pressure can be written as:

$$\frac{\partial p}{\partial n} = 0$$

The second order scheme is used to approximate the pressure derivative.

$$p_1 = (4p_2 - p_3)/3$$

## 3.4.2 Non-reflecting boundary condition:

Non-reflecting boundary condition proposed in Dr. Li Jiang (Li1999) are used at the boundaries of far field and outflow. Non-reflecting boundary condition for inflow is also included in the code. Based on the characteristic analysis, Equation (2. 7) can be rewritten as

$$\begin{split} \frac{\partial p}{\partial t} + d_1 + V \frac{\partial \rho}{\partial \eta} + \rho \left(\eta_x \frac{\partial u}{\partial \eta} + \eta_y \frac{\partial v}{\partial \eta} + \eta_z \frac{\partial w}{\partial \eta}\right) + W \frac{\partial \rho}{\partial \xi} \\ &+ \rho \left(\zeta_x \frac{\partial u}{\partial \zeta} + \zeta_y \frac{\partial v}{\partial \zeta} + \zeta_z \frac{\partial w}{\partial \zeta}\right) + vis_1 = 0 \\ \frac{\partial u}{\partial t} + d_2 + V \frac{\partial u}{\partial \eta} + \frac{1}{\rho} \eta_x \frac{\partial p}{\partial \eta} + W \frac{\partial u}{\partial \zeta} + \frac{1}{\rho} \zeta_x \frac{\partial p}{\partial \zeta} + vis_2 = 0 \\ \frac{\partial v}{\partial t} + d_3 + V \frac{\partial v}{\partial \eta} + \frac{1}{\rho} \eta_y \frac{\partial p}{\partial \eta} + W \frac{\partial v}{\partial \zeta} + \frac{1}{\rho} \zeta_y \frac{\partial p}{\partial \zeta} + vis_3 = 0 \\ \frac{\partial w}{\partial t} + d_4 + V \frac{\partial w}{\partial \eta} + \frac{1}{\rho} \eta_z \frac{\partial p}{\partial \eta} + W \frac{\partial v}{\partial \zeta} + \frac{1}{\rho} \zeta_z \frac{\partial p}{\partial \zeta} + vis_4 = 0 \\ \frac{\partial p}{\partial t} + d_5 + V \frac{\partial p}{\partial \eta} + \gamma p \left(\eta_x \frac{\partial u}{\partial \eta} + \eta_y \frac{\partial v}{\partial \eta} + \eta_z \frac{\partial w}{\partial \eta} + \eta_z \frac{\partial w}{\partial \eta}\right) + W \frac{\partial p}{\partial \zeta} \\ &+ \gamma p \left(\zeta_x \frac{\partial u}{\partial \eta} + \zeta_y \frac{\partial v}{\partial \eta} + \zeta_z \frac{\partial w}{\partial \eta}\right) + vis_5 = 0 \end{split}$$

where vector  $\mathbf{d}$  is obtained from the characteristic analysis,
$$\begin{pmatrix} d_{1} \\ d_{2} \\ d_{3} \\ d_{4} \\ d_{5} \end{pmatrix} = \begin{pmatrix} \frac{1}{c^{2}} \left[ \frac{1}{2} (L_{1} + L_{5}) + L_{2} \right] \\ \frac{\xi_{2}}{2\beta\rho c} (L_{5} - L_{1}) - \frac{1}{\beta^{2}} (\xi_{y}L_{3} + \xi_{z}L_{4}) \\ \frac{\xi_{y}}{2\beta\rho c} (L_{5} - L_{1}) + \frac{1}{\beta^{2}\xi_{x}} \left[ (\xi_{x}^{2} + \xi_{z}^{2})L_{3} - \xi_{z}\xi_{y}L_{4} \right] \\ \frac{\xi_{z}}{2\beta\rho c} (L_{5} - L_{1}) - \frac{1}{\beta^{2}\xi_{x}} \left[ \xi_{y}\xi_{z}L_{3} - (\xi_{x}^{2} + \xi_{y}^{2})L_{4} \right] \\ \frac{1}{2} (L_{1} + L_{5}) \end{pmatrix}$$
(3.26)

where  $\mathbf{c}$  is the sound speed, and

$$\beta = \sqrt{\xi_x^2 + \xi_y^2 + \xi_z^2}$$
(3.27)

 $L_i$ 's represent the amplitude variations of the characteristic waves corresponding to the characteristic velocities  $\lambda_i$ 's, which are given by

$$\lambda_{1} = U - C_{\xi}$$

$$\lambda_{2} = \lambda_{3} = \lambda_{4} = U - C_{\xi}$$

$$\lambda_{5} = U + C_{\xi}$$
(3. 28)

Where  $C_{\xi} = c\beta$ , and  $L_i$ 's can be expressed as

$$L_{1} = \left(U - C_{\xi}\right) \left[-\frac{\rho c}{\beta} \left(\xi_{x} \frac{\partial u}{\partial \xi} + \xi_{y} \frac{\partial v}{\partial \xi} + \xi_{z} \frac{\partial w}{\partial \xi}\right) + \frac{\partial p}{\partial \xi}\right]$$

$$L_{2} = U\left(c^{2} \frac{\partial \rho}{\partial \xi} - \frac{\partial p}{\partial \xi}\right)$$

$$L_{3} = U\left(-\xi_{y} \frac{\partial u}{\partial \xi} + \xi_{x} \frac{\partial v}{\partial \xi}\right)$$

$$L_{4} = U\left(-\xi_{z} \frac{\partial u}{\partial \xi} + \xi_{x} \frac{\partial w}{\partial \xi}\right)$$

$$L_{5} = \left(U + C_{\xi}\right) \left[\frac{\rho c}{\beta} \left(\xi_{x} \frac{\partial u}{\partial \xi} + \xi_{y} \frac{\partial v}{\partial \xi} + \xi_{z} \frac{\partial w}{\partial \xi}\right) + \frac{\partial p}{\partial \xi}\right]$$
(3.29)

The terms  $vis_i$ 's in Equation (3. 25) represent the viscous terms. The analogous definitions can be made for the other two directions.

For subsonic outflow at  $\xi = N_x$ , four characteristic waves  $L_2$ ,  $L_3$ ,  $L_4$ ,  $L_5$  are going out of the computational domain, while  $L_1$  is entering the field. Therefore,  $L_2$ ,  $L_3$ ,  $L_4$ ,  $L_5$  can be calculated from the interior points using Equation (3. 29) with the compact finite difference scheme, while the  $L_1$  is set to zero.

For far field boundary at  $\xi = N_z$ , the directions of characteristic waves are determined automatically by local field values, and  $L_i$  of the outgoing waves are calculated from the interior points using Equation (3. 29) in  $\zeta$  direction. Those inward going waves are set to zero, i.e.

$$L_{i} = \begin{cases} L_{i}, & \text{for } \lambda_{i} > 0\\ 0, & \text{for } \lambda_{i} < 0 \end{cases}$$
(3.30)

For subsonic inflow at  $\xi = 1$ , four quantities should be specified, i.e. u, v, w, T, while the density  $\rho$  is obtained by solving Equation (3. 29). This arrangement is made based on the fact that the four characteristic waves  $L_2$ ,  $L_3$ ,  $L_4$ ,  $L_5$  are entering the computational domain, while  $L_1$  is going outward. Therefore,  $L_1$  is calculated from interior points using Equation (3. 29), where the spatial derivatives are calculated using the compact finite difference scheme, and  $L_2$ ,  $L_3$ ,  $L_4$ ,  $L_5$  are given in the equation (3. 31).

$$L_{2} = \frac{\rho}{M_{r}^{2}} \frac{\partial T}{\partial t} + \frac{1}{2} (L_{1} + L_{5})$$

$$L_{3} = \xi_{y} \frac{\partial u}{\partial t} - \xi_{x} \frac{\partial v}{\partial t}$$

$$L_{4} = \xi_{z} \frac{\partial u}{\partial t} - \xi_{x} \frac{\partial w}{\partial t}$$

$$L_{5} = L_{1} + \frac{2\beta\rho c}{\xi_{x}} \left[ \frac{1}{\beta^{2}} (\xi_{y}L_{3} + \xi_{z}L_{4}) - \frac{\partial u}{\partial t} \right]$$
(3.31)

### 3.5 Grid Generation

The physical configuration shown in Figure (3.7) will be replaced by a simplified computational domain, which includes a rectangular wing with a NACA0012 airfoil section and a rounded tip, as shown in Figure (3.7). The span-chord ratio of the wing is 0.75. In Figure (3.8), the bottom plane is assumed to be symmetric, corresponding to infinite long in the spanwise direction. The spanwise direction of the wing is perpendicular to the flat plate.



Figure 3.7 SUBOFF with Propulsors



Figure 3.8 Physical domain

In this paper, we use a one-block mesh for the computation. Generally a multiblock mesh is more flexible, but it may involve some additional special treatments at the boundary of each block that may introduce more numerical errors and reduce the accuracy of the computation. Therefore, the one-block mesh approach is used in the present work. For a configuration of the juncture of the wing and a flat plate, a single C-H topology is adopted for the grid generation. A C-type grid surrounding the wing on the flat plate is generated, then an elliptic grid generation method followed that of Spekreijse (1995) is used to redistribute and smoothen the grid inside the domain. After that a 3-D grid is generated algebraically.





An elliptic grid generation method first proposed by Spekreuse (1995) is used to generate 2D grids. The elliptic grid generation method is based on a composite

mapping, which is consisted of a nonlinear transfinite algebraic transformation and an elliptic transformation. The algebraic transformation maps the computational space C onto a parameter space P, and the elliptic transformation maps the parameter space on to the physical domain D. The computational space, parameter space, and the physical domain are illustrated in Figure (3.9).

The computational space *C* is defined as the unit square in a two-dimensional space with Cartesian coordinates  $(\xi, \eta)$ , and  $\xi \in [0,1]$ ,  $\eta \in [0,1]$  (see Figure (3.9)). The grids are uniformly distributed on the boundaries and in the interior area of the computational space. The mesh sizes are  $\frac{1}{N_{\xi}-1}$  in the  $\xi$  direction and  $\frac{1}{N_{\eta}-1}$  in the  $\eta$  direction, where  $N_{\xi}$  and  $N_{\eta}$  are the grid numbers in the corresponding direction. The parameter space *P* is defined as a unit space in a two-dimensional space with Cartesian coordinate (s, t), and  $s \in [0,1]$ ,  $t \in [0,1]$ . The boundary values of *s* and *t* are determined by the grid point distribution in the physical domain.

s and t satisfies the following condition:

#### Table 3.4 Boundary condition for *s* and *t*

$s = 0$ at edge $E_1$ and $s = 1$ at edge $E_2$ .
s is the normalized arc-length along edges $E_3$ and $E_4$
$t = 0$ at edge $E_3$ and $t = 1$ at edge $E_4$
t is the normalized arc-length along edges $E_1$ and $E_2$

An algebraic transformation  $s: C \rightarrow P$  is defined to map the computational space *C* onto the parameter space *P*. The grid distribution is specified by this algebraic transformation, which depends on the prescribed boundary grid point distribution. The interior grid point distribution inside the domain, generated by the algebraic transformation, is a good reflection of the prescribed boundary grid point distribution.

Let  $s_{E_3}(\xi) = s(\xi, 0)$  and  $s_{E_4}(\xi) = s(\xi, 1)$  denote the normalized arc-length along edges  $E_3$  and  $E_4$ ,  $t_{E_1}(\xi) = t(0, \eta)$  and  $t_{E_2}(\xi) = t(1, \eta)$  denote the normalized arc-length along edges  $E_1$  and  $E_2$ . The algebraic transformation  $s: C \to P$  is defined as

$$s = s_{E_3}(\xi)(1-t) + s_{E_4}(\xi)t$$
  

$$t = t_{E_1}(\eta)(1-s) + t_{E_2}(\eta)s$$
(3.32)

Equation (3. 32) is called the algebraic straight line transformation. It defines a differentiable one-to-one mapping because of the positiveness of the Jacobian:

$$s_{\xi}t_{\eta} - s_{\eta}t_{\xi} > 0$$

The elliptic transformation  $x: P \rightarrow D$ , which is independent of the prescribed boundary grid point distribution, is defined to map the parameter space P onto the physical domain D. The elliptic transformation is equivalent to a set of Laplace equations

$$s_{xx} + s_{yy} = 0$$
  
 $t_{xx} + t_{yy} = 0$ 
(3.33)

The elliptic transformation defined by the above equations is also differentiable and one-to-one.

Till now we have defined two transformations, i.e., the algebraic transformation  $s: C \rightarrow P$ , and the elliptic transformation  $x: P \rightarrow D$ . Because both the algebraic transformation and the elliptic transformation are differentiable and one-to-one, the composition the two transformation is also differentiable and one-to-one, so as to the inverse transformation.

In physical domain, the curvilinear coordinate system satisfies a system of Laplace equations:

$$\Delta r = 0 \tag{3.34}$$

where  $r = (x, y)^T$ . The inherent smoothness of the Laplace operator makes the grids smoothly distributed in the physical domain. Being transformed to the computational space, this Laplace system becomes a set of Poisson equations. The control functions are determined by the composed transformation according to the following procedures. First, Equation (3. 33) is transformed into the computational space *C*:

$$\Delta s = g^{11} s_{\xi\xi} + 2g^{12} s_{\xi\eta} + g^{22} s_{\eta\eta} + \Delta \xi s_{\xi}$$
  

$$\Delta t = g^{11} t_{\xi\xi} + 2g^{12} t_{\xi\eta} + g^{22} t_{\eta\eta} + \Delta \eta t_{\eta}$$
(3.35)

where  $g^{11}, g^{12}, g^{22}$  are the components of the contravariant metric tensor, which can be calculated from the covariant metric tensor

$$g^{11} = \frac{1}{J^2} g_{22} = (r_{\eta}, r_{\eta}) / J^2$$
  

$$g^{12} = -\frac{1}{J^2} g_{12} = -(r_{\xi}, r_{\eta}) / J^2$$
  

$$g^{22} = \frac{1}{J^2} g_{11} = (r_{\xi}, r_{\xi}) / J^2$$
(3.36)

*J* is defined as  $\sqrt{\det |g_{ij}|}$ . From Equation (3. 33) and (3. 35), we have

$$\binom{\Delta\xi}{\Delta\eta} = g^{11}P_{11} + 2g^{12}P_{12} + g^{22}P_{22}$$
(3.37)

Where

$$P_{11} = \begin{pmatrix} P_{11}^{(1)} \\ P_{11}^{(2)} \end{pmatrix} = -T^{-1} \begin{pmatrix} s_{\xi\xi} \\ t_{\xi\xi} \end{pmatrix}$$

$$P_{12} = \begin{pmatrix} P_{12}^{(1)} \\ P_{12}^{(2)} \end{pmatrix} = -T^{-1} \begin{pmatrix} s_{\xi\eta} \\ t_{\xi\eta} \end{pmatrix}$$

$$P_{22} = \begin{pmatrix} P_{22}^{(1)} \\ P_{22}^{(2)} \end{pmatrix} = -T^{-1} \begin{pmatrix} s_{\eta\eta} \\ t_{\eta\eta} \end{pmatrix}$$
(3. 38)

and the matrix T is defined as

$$T = \begin{pmatrix} s_{\xi} & s_{\eta} \\ t_{\xi} & t_{\eta} \end{pmatrix}$$
(3.39)

Then the Laplace system Equation (3.34) is transformed to the computational space C:

$$g^{11}r_{\xi\xi} + 2g^{12}r_{\xi\eta} + g^{22}r_{\eta\eta} + \Delta\xi r_{\xi} + \Delta\eta r_{\eta} = 0$$
(3.40)

Substitute Equation (3. 37) into Equation (3. 40),  $\Delta \xi$  and  $\Delta \eta$  are replaced by the control functions on the right-hand-side of Equation (3. 40), and we obtain the Poisson equations for the grid generation as follows:

$$g^{11}r_{\xi\xi} + 2g^{12}r_{\xi\eta} + g^{22}r_{\eta\eta} + (g^{11}P_{11}^{(1)} + 2g^{12}P_{12}^{(1)} + g^{22}P_{22}^{(1)})r_{\xi} + (g^{11}P_{11}^{(2)} + 2g^{12}P_{12}^{(2)} + g^{22}P_{22}^{(2)})r_{\eta} = 0$$
(3.41)

where the control functions  $P_{11}^{(1)}$ ,  $P_{12}^{(1)}$ ,  $P_{22}^{(1)}$ ,  $P_{11}^{(2)}$ ,  $P_{12}^{(2)}$ ,  $P_{22}^{(2)}$  are determined by the algebraic transformation, as defined previously in Equation (3. 38).

The elliptic transformation is carried by solving a set of Poisson equations. The control functions are specified by the algebraic transformation only and it is, therefore, not needed to compute the control functions at the boundary and to interpolate them into the interior of the domain, as required in the case for all well-known elliptic grid generation systems based on Poisson systems.

The computed grids are in general not orthogonal at the boundary. The algebraic transformation can be redefined to obtain a grid which is orthogonal at the boundary. First, redefine the elliptic transformation by imposing the following boundary conditions for s and t:

Table 3.5 Boundary condition for s and t

s = 0 at edge  $E_1$  and s = 1 at edge  $E_2$ 

 $\frac{\partial s}{\partial n} = 0$  along edges  $E_3$  and  $E_4$ , where *n* is the outward normal direction

$$t = 0$$
 at edge  $E_3$  and  $t = 1$  at edge  $E_4$   
 $\frac{\partial t}{\partial n} = 0$  along edges  $E_1$  and  $E_2$ , where *n* is the outward normal direction

Second, redefine the algebraic transformation  $s: C \rightarrow P$  according to two algebraic equations,

$$s = s_{E_3}(\xi)H_0(t) + s_{E_4}(\xi)H_1(t)$$
  

$$t = t_{E_1}(\eta)H_0(s) + t_{E_2}(\eta)H_1(s)$$
(3.42)

where  $H_0$  and  $H_1$  are cubic Hermite interpolation functions defined as

$$H_0(s) = (1+2s)(1-s)^2$$
  

$$H_1(s) = (3-2s)s^2$$
(3.43)

Grid orthogonality at boundaries is obtained in three steps.

1. Compute an initial grid based on the Poisson grid generation system with control functions specified according to the algebraic straight line transformation defined by Equation (3. 32);

2. Solve the two Laplace equations given by Equation (3. 33) with the above specified boundary conditions;

3. Re-compute the grid based on the Poisson system but with control functions specified according to the algebraic transformation defined by Equation (3. 42).

The two-dimensional grid used in this work is shown in Figure (3.10). The grid near the leading and trailing edges of the airfoil is shown in Figure (3.10) and (3.11).



Figure 3.10 Illustration of C-type grid on the flat plate

Figure 3.11 Illustration of grid near the trailing edge on the flat plate

In order to avoid any singularity in the computation of the Jacobian of the coordinate transformation, a narrow slot with around  $10^{-5}$  of the chord width was assigned at the trailing edge of the airfoil, as shown in Figure (3.11). Here we only gave out the illustration. The grid spacing size in actual computation is much smaller than the above illustration.

## 3.5.2 3D grid generation

The overall computational domain is show in the Figure (3.12). Then the overall grid geometry is composed of the surface grids shown in Figure (3.13).



Figure 3.12 Physical domain



Figure 3.13 Grid topology

The procedure of grid generation for the three dimensional grid is as following. Step1: Plate grid generation by way of Spekreuse's method. That was introduced in the section 3.5.1. Step 2: Surface grid generations for the airfoil surface, outer boundary, and exit boundary. In this step, the algebra grid generation method was used. Step 3: the volume grid generation. The volume grid was also generated by way of the algebraic method. The whole volume grid was composed of a group of surface grids, as shown in Figure (3.13).



Figure 3.14 Vertical Surface Grid



Now we turned to surface grid generation in the step 2. In the directions that are corresponding to the  $\eta$  and  $\zeta$  directions in the transformed computational domain, the grid is of H-type, such as the vertical plane shown in Figure (3.14), where the grid at the downstream boundary of the domain is displayed. To avoid any singularity in the computation of the Jacobian of the coordinate transformation, a narrow slot with around  $10^{-5}$  of the chord width was assigned at the trailing edge of the airfoil, as shown in Figure (3.15).

The outer boundary surface grid and details are shown in Figure (3.16) and (3.17)





Figure 3.16 Outer Boundary Surface Grid

Figure 3.17 Grid Detail

The sharp edge of the wingtip is replaced by a rounded corner with a radius of no more than  $10^{-5}$  of the chord length in order to avoid any singularity in the computation of the Jacobian of the coordinate transformation. The detail of the rounded corner is shown in Figure (3.18). If the whole wing tip is rounded, as Chow's experiment did, the grid is showed in Figure (3.19). In the LES computation running now, the rounded wingtip is used for the sake of the Chow's experiment. The sharp edged wingtip can be used in the future to compare with other experiments.





Figure 3.18 Grids for the Sharp Edge

Figure 3.19 Grids for the Rounded Edge

Our experience shows that grid spacing plays a critical role in the computation. The wingtip vortex will not be captured accurately if the grid spacing is not fine enough near the wingtip vortex core region. Therefore the better way is to locate the vortex core region by a coarse grid simulation. Then refine the grid near the core region.

The grid points are clustered near the vortex core region and boundary layer so that the grid sizes in wall units satisfied that  $\Delta x_{+\min}$  is around 17.5 in the streamwise direction,  $\Delta y_{+\min}$  around 35 (in the spanwise direction) and  $\Delta z_{+\min}$  around 2.6 (in the wall-normal direction). The numbers of grid points in  $\xi$ ,  $\eta$  and  $\zeta$  directions are 1024, 160 and 160, respectively.

#### 3.6 Code Validation

In order to validate the code, DNS of the flow transition on a flat plate is shown as the first case here. In this case, the LES model is switched off. The computational domain is displayed in Figure (3. 20).



Figure 3.20 Computational domain of flow over a flat plate



In the test case, the Mach number  $Ma_{\infty} = 0.5$ , the Reynolds number based on the free-stream velocity and the displacement thickness at the inflow boundary  $Re_{\delta_m} = 1000$ . Here  $\delta$  in is the inlet displacement thickness. The Prandtl number Pr = 0.7. The x-coordinate of the inflow boundary measured from the leading edge of the fat plate is  $x_{in} = 300.8 \delta_{in}$ . The linear spatial evolution of the small disturbance imposed at the inlet is simulated in the test case. At the inlet boundary, the most amplified eigenmode of the 2-D Tollmien-Schlichting (T-S) waves is enforced. The eigenfunction has a frequency  $\omega = 0.0957$ , and a space wave number  $\alpha = 0.25792$ - i 6.72054  $\times 10^{-3}$ . The disturbance imposed at inflow boundary is  $A_{2d}q'_{2d}$ . The amplitude of T-S wave is specified as  $A2d = 5 \times 10^{-4}$ , and

 $q'_{2d} = \phi_r \cos(\omega t) + \phi_i \sin(\omega t)$ 

where  $\phi_r$  and  $\phi_i$  are the respective real and imaginary parts of the eigen-function obtained from linear stability theory (LST). The length of computational domain along the streamwise direction includes 8 T-S wavelengths, and height of the domain at the inflow boundary is  $40 \delta_{in}$ . For a 2-D simulation, the grid size is of 128×61 representing the number of grids in the streamwise (x), and wall normal (z) directions.

After eight time periods of calculation, the T-S wave has been fully established over the entire domain, the Fourier transform in time is conducted on physical variables. The amplitude of disturbance is defined as Fourier amplitude. At a certain streamwise location, the Fourier amplitude is a function of z – the wall direction coordinate.

For a 3-D DNS of flow transition over the flat plate, the flow parameters are similar to the previous case. The length of computational domain along the streamwise direction is  $800 \delta_{in}$ , the width along the spanwise direction is  $22 \delta_{in}$ , and height at the inflow boundary is  $40 \delta_{in}$ . The grid size is  $640 \times 64 \times 60$  representing the number of grids in the streamwise (x), spanwise (y), and wall normal (z) directions. The inflow consists of a 2-D T-S wave eigenmode and a 3-D random disturbance. The frequency of the T-S eigenmode is  $\omega = 0.114027$ . The space wavenumber of the mode is given by  $\alpha = 0.29919$ - i  $5.00586 \times 10^{-3}$ . The amplitude of the 2-D mode is specified as  $A_{2d} = 0.02$ , which is much larger than that used in the 2-D simulation. The 3-D random disturbance at inflow is given by white noise with amplitude of 0.01.

In Figure (3.21), the time- and spanwise-averaged velocity profiles at different streamwise locations are plotted in logarithmic wall unit. The curves of the linear law near the wall and the log law are also plotted for comparison.

The profiles of the disturbance amplitude of the streamwise velocity u, and the wall normal velocity w, are shown in Figure (3.22) by solid lines, while the LST results are plotted by square symbols in the same figure for comparison.



Figure 3.22 Comparison of the numerical and LST velocity profiles at Rex=394300. (a) disturbance amplitude of u; (b) disturbance amplitude of w

FROM THIS CLASSICAL FLOW TRANSITION PROBLEM, WE CAN FIND OUR HIGH ORDER DNS CODE IS RIGHT AND CAN PROVIDE A HIGH ORDER SOLUTION FOR TIME DEPENDENT PROBLEMS.

### CHAPTER 4

### NUMERICAL RESULTS

The flow configuration in the simulations is based on the wing tip experiment by Chow et al (Chow, et al. 1997). The computational domain includes a rectangular half-wing with a NACA 0012 airfoil section, a rounded wing tip and the surrounding boundaries. The wing has an aspect ratio of 0.75. The angle of attack is 10 degree. The upstream boundary is three chord lengths away from the leading edge of the wing. The upper and lower boundaries are four chord lengths from the airfoil surface. The outflow boundary is located at 5 chord lengths downstream. The free-stream velocity  $U_{\infty}$ , the free-stream pressure  $p_{\infty}$ , the free-stream temperature  $T_{\infty}$ , and the chord length of the airfoil C are selected as the reference parameters for nondimensionalization.

The Reynolds number based on free-stream velocity and the chord length is  $4.6 \times 10^6$ . In the simulations, the free-stream Mach number is set to 0.2.  $\xi$ ,  $\eta$  and  $\zeta$  are computational coordinates. The numbers of grid points in  $\xi$ ,  $\eta$  and  $\zeta$  directions are 1024, 160 and 160, respectively.

### 4.1 Instantaneous flow field

Instantaneous data are picked at t= 3.15c/U<sub>∞</sub> when the flow field is fully developed. Figure (4.1) shows the instantaneous field of the axial vorticity. Figure (4.1)(a) is the perspective view of the iso-surface of axial vorticity, which originates from the wing tip in the form of small vortical structures and evolves into a smooth vortex tube in the further downstream wake. On the suction side of the wing, the flow near the wing tip is highly three-dimensional turbulent. Small vortical structures are clearly seen inside and around the tip vortex. Near the trailing edge, spiral wake surrounding the tip vortex is formed as the wake is skewed and laterally stretched and curved by the rotating velocity field associated with the vortex. In the further downstream region (about one chord length from the trailing edge), the wing tip vortex is stabilized, where the small vortical structures are not visible around the primary tip vortex. Figure (4.1)(b) shows the contours of axial vorticity in a vertical plane at *y*=0.72 which approximately intercepts with the tip vortex core. The evolution of the vortical structures inside and around the tip vortex core be seen clearly on the plane.



(a) (b) Figure 4.1 Instantaneous field of axial vorticity. (a) Iso-surface of vorticity component  $\omega_x=3$ ; (b) Contours of vorticity component  $\omega_x$  in the x-z plane at y=0.72

The distribution of the y- and z- components of the vorticity are plotted in Figure (4.2) and Figure (4.3) respectively. In Figure (4.2)(a), the iso-surfaces of positive and negative y- component vorticity are parallel to each other in a vertical layout, and both extend along the axis of the wing tip vortex in the wake, representing a typical spiral motion of velocity field. Similar observation can be found in Figure (4.3)(a), where the iso-surfaces corresponding to the positive and the negative z- component of vorticity are parallel to each other in a horizontal layout. The evolution of the y- and z- components of the vorticity on the vertical plane at y=0.72 is shown in Figure (4.2)(b) and Figure (4.3)(b).



(a) (b) Figure 4.2 Instantaneous field of spanwise vorticity. (a) Iso-surface of vorticity component  $\omega_y = \pm 3$ ; (b) Contours of vorticity component  $\omega_y$  in the x-z plane at y=0.72.



Figure 4.3 Instantaneous field of vorticity in z direction. (a) Iso-surface of vorticity component  $\omega_z = \pm 3$ ; (b) Contours of vorticity component  $\omega_z$  in the x-z plane at y=0.72.

Figure (4.4), (4.5) and (4.6) show the pressure contours and velocity vectors on the cross-sections at different streamwise locations.



Figure 4.4 Pressure contours and velocity vectors in the cross plane at different streamwise locations. (a) x/c=0.9; (b) x/c=0.995.



(a) (b) Figure 4.5 Pressure contours and velocity vectors in the cross plane at different streamwise locations. (a) x/c=1.06; (b) x/c=1.125.



Figure 4.6 Pressure contours and velocity vectors in the cross plane at different streamwise locations. (a) x/c=1.25; (b) x/c=1.45.

At x/c=0.90, the vortex structures are highly unsteady. The shear layer separates from the rounded wing tip and rolls up generating the primary vortex. The core area of the primary vortex is very unstable and the outer edge of the primary vortex is not smooth. Near the junction of the wing and the rounded tip, the secondary vortex is induced by the primary vortex and the secondary vortex rotates in the opposite direction of the primary vortex, while the vorticity of small vortices shedding from wing tip shear layer has the same sign as the primary vortex. At x/c=0.995, on the cross-section that is very close to the trailing edge, the center of the primary vortex moves upward away from the surface of the wing and the size of vortex core grows.

The secondary vortex also grows as it rolls up and merges into the primary vortex and brings unsteadiness and instability into the core of the primary vortex. The interaction between the primary and the secondary vortices is quite intensive, thus generates small vortical structures in the core area. At x/c=1.06, on the cross-section

that locates at the immediate down-stream of the trailing edge, due to the absence of wall, there is not any newly generated secondary vortices. The visible secondary vortical structures on this cross-section are actually the secondary vortices shed from the wing surface at the upstream location and convected downstream. As a matter of fact, the secondary vortex becomes weaker as they are traveling downstream, and thus introduces less disturbance to the core of the primary vortex. On this cross-section, the primary vortex is able to maintain an unbroken core all the time. Further downstream from the trailing edge of the wing, on the cross-section at x/c=1.125, the secondary vortex is barely seen, because they are significantly dissipated as they are convected downstream. On the cross-sections located on the downstream of the trailing edge, the shear layer associated with the wing tip also disappears. Without the secondary vortex and the shear layer, no more disturbances are fed into the primary vortex. Therefore, the primary vortex is stabilized and is able to maintain a shape of a smooth regular circle. Sometimes, the trailing edge wake (from the lower left corner of the plot), being entrained by the primary vortex, rolls up and is entangled with the primary vortex. When this happens, the shape and the position of the vortex core are affected. As one goes to further downstream locations, the primary vortex becomes more stable.



Figure 4.7 Contours of streamwise vorticity  $\omega_x$  in cross planes at different locations

Figure (4.7) shows the contours of the instantaneous stream-wise vorticity  $\omega_x$  on cross-sections at different stream-wise locations. At the mid-chord location x/c=0.606, the separated shear layer is very unstable, as some small vortical structures are continuously shedding from the shear layer and reattaching to the suction surface. At a further downstream location, the rolling up of the separated shear layer produces the primary vortex (red color) between the shear layer and the wing surface. The counterrotating vortical structures shown in blue color is the secondary vortex. The highly unstable shear layer and the interaction between the primary and the secondary vortices

serves as an external resource of disturbance that is fed into the primary vortex, which now has a very unstable center area – in the form of a broken core with many small vortical structures. At downstream of the trailing edge, in the absence of wall surface, without additional disturbance/energy input from the secondary vortex and shear layer, the primary vortex core becomes more stable, and small structures dissipate quickly.



Figure 4.8 Contours of axial velocity in cross planes at different locations

The large favorable axial pressure gradient in the core of the primary vortex accelerates the incoming fluid to produce high axial velocity. The instantaneous axial velocity can reach as high as  $2U_{\infty}$ , as shown by Figure (4.8), which displays the

contours of the instantaneous axial velocity on cross-sections at different streamwise locations. Strong axial flow occurs on cross-sections over the wing. In the further downstream of the wake, the axial velocity decays rapidly.



Figure 4.9 Time history of axial velocity at different streamwise locations inside the vortex core

The time history of the instantaneous velocity reveals more features of the flow field, as those in Figure (4.9), which shows the time history of axial velocity at the streamwise locations ranging from 0.9c to 1.452c inside the vortex core. At these two stations of x/c=0.9 and 0.99 located above the wing surface on the suction side, the signal of velocity fluctuation is highly random and has a broadband spectrum. From the

trailing edge to further downstream, fluctuation amplitudes become smaller and high frequency fluctuation gradually disappear



Figure 4.10 Time history of cross-flow velocity magnitude (v2+w2)1/2 at different streamwise locations inside the vortex core

Figure (4.10) shows the time history of cross-flow velocity at the location inside the vortex core. In comparison with the time history of axial velocity, the evolution of the cross-flow velocity shows the same trend. The flow inside the core is highly unsteady and random with a broadband spectrum. At further downstream in the wake region, the cross-flow velocity fluctuations have much lower frequency and smaller amplitude. This result indicates that the flow inside the vortex core is becoming more stable and laminar in the downstream.

### 4.2 Mean flow field

The mean flow field is obtained from a time-averaging process. Figure (4.11) shows contours of time averaged axial vorticity in cross planes at different streamwise locations. The location and the size of the primary vortex core can be seen clearly. Over the wing surface, the time averaged vortex core is deformed and stretched with an irregular edge. In the contrast, the shape of the vortex core becomes more regular in the wake region, and becomes even more circular in the further downstream.



Figure 4.11 Contours of axial vorticity in cross planes at different locations



Figure 4.12 Distribution of axial vorticity along a line with constant *z* through vortex core

In Figure (4.12), the time averaged profiles of the axial vorticity along a horizontal centerline at different streamwise locations clearly show the negative value of vorticity near wingtip on the first four profiles, which are either located over the wing surface or close to the trailing edge, indicating the existence of the secondary vortex with an opposite direction of rotation of the primary vortex. The vorticity strength is kept at the same level in the immediate wake region. Since the flow has not been fully developed yet, the vorticity strength is weaker at x/c=1.45. Figure (4.13) shows the contours of mean (time averaged) pressure on the cross-sections at different streamwise locations. On each cross-section, a region with a low mean pressure usually reflects the location of the vortex core. At x/c=0.9, the low-pressure zone is attached to the wing

surface. On the downstream cross-sections, e.g. at  $x/c=1.06\sim1.452$ , the locations of the mean vortex core corresponding to the low-pressure area move upward and toward the symmetric plane.



Figure 4.13 Contours of mean pressure in cross-sections at different streamwise locations



Figure 4.14 Contours of mean axial velocity on cross-sections at different streamwise locations.

The distribution of pressure in the core area of the primary vortex indicates the favorable axial pressure gradient, which accelerates the axial velocity and produce axial velocity surplus. Figure (4.14) shows the contours of the mean axial velocity on cross-sections at different streamwise locations. The time averaged axial velocity can reach as high as  $1.2U_{\infty}$ .



Figure 4.15 Contours of mean cross-flow velocity magnitude in cross-sections at different axial locations.

The contours of the magnitude of mean cross-flow velocity on cross-sections at different axial locations are shown in Figure (4.15). On cross-sections at x/c=0.803 and 0.9, high-speed cross-flow circumvents the wing tip. On the suction side of the wing, starting from the mid-chord, both the size of the high cross-flow region and the magnitude of the cross-flow velocity increase as x increases. After the primary tip vortex and secondary vortex have established, the size of the area with low cross-flow velocity becomes smaller, surrounded by cross-flow with relatively higher speed. In the

further downstream of the trailing edge, there is only one area with low cross-flow velocity corresponding to vortex core.

# 4.3 Turbulence Character

The contours of the Root-Mean-Squared (RMS) of axial velocity fluctuation u' on cross-sections at different streamwise locations are shown in Figure (4.16).



Figure 4.16 Contours of  $u'_{\rm rms}$  on cross-sections at different streamwise locations

On the cross-sections that intersect with the wing, the peak value of velocity fluctuation u' appears at locations where the shear layer separates from the surface of wing tip. The axial velocity fluctuation reaches its maximum value of  $0.32U_{\infty}$  near the trailing edge. The high level fluctuations are wrapped up into the vortex core and convected downstream. On the cross-sections located in the wake, peak value of velocity fluctuation u' appears in the center of the vortex core. In the streamwise direction, the fluctuation level decreases rapidly in the further downstream of the wake.



Figure 4.17 Distribution of  $u'_{rms}$  along a line with constant z through vortex core

The profiles of  $u'_{rms}$  along a line that intersects with the vortex core with z=constant are shown in Figure(4.17). In the wake, the level of the axial velocity fluctuation decreases as x increases.


Figure 4.18 Contours of  $v'_{rms}$  on cross-sections at different streamwise locations

Figure (4.18) shows the contours of velocity fluctuation  $v'_{rms}$  on the crosssections at different streamwise locations. On cross-sections that intersect with the wing, the velocity fluctuation  $v'_{rms}$  reaches its peak value of  $0.36U_{\infty}$  near the trailing edge of the wing. On the cross-sections located in the wake near the trailing edge (at x/c=1.06), strong interactions between wake and the primary vortex is obvious. The wake/vortex interaction becomes much weaker as the vortex core moves upward and away from the wake in the further downstream.



Figure 4.19 Distribution of v'rms along a line with constant z through vortex core

In Figure (4.19), the profiles of  $v'_{rms}$  along a constant-z line that cuts through vortex core are plotted as functions of y. Figure (4.19) shows that the velocity fluctuation  $v'_{rms}$  along the spanwise direction decreases monotonically as x increases, except at x/c=1.25, where the increase of the fluctuation level can be caused by the interaction between the primary vortex and the wake near the trailing edge.



Figure 4.20 Contours of w'<sub>rms</sub> on cross-sections at different stremwise locations

Figure (4.20) shows the contours of the velocity fluctuation  $w'_{rms}$  on crosssections at different streamwise locations. On cross-sections that intersect with the wing, the velocity fluctuation reaches its peak value of  $0.28U_{\infty}$  near the trailing edge of the wing. The contours of  $w'_{rms}$  on cross-sections located in the wake near the trailing edge show strong interactions between wake and the primary vortex. The wake/vortex interaction becomes weaker in the further downstream of the wake. The profiles of  $w'_{rms}$ along a constant-z line that cuts through vortex core are plotted as functions of y in Figure (4.21).



Figure 4.21 Distribution of  $w'_{rms}$  along a line with constant z through vortex core

The contours of the components of Reynolds stress u'v', u'w', and v'w' are plotted on cross-sections with different streamwise locations in Figure (4.22), Figure (4.24), and Figure (4.26), respectively.

Figure (4.23), Figure (4.25), and Figure (4.27) show the profiles of the three components of the Reynolds stress along a line with constant z through vortex core as functions of y.



Figure 4.22 Contours of Reynolds shear stress component u'v' on cross-sections at different streamwise locations

The contours on the cross-section shown in Figure (4.22) indicate that the separated shear layer has maximum Reynolds shear stress u'v'. The two-lobe structure identified by opposite sign of Reynolds shear stress u'v' in the vortex core can be seen at cross-sections over the wing surface and in the wake. Similar observation has also been made by the experiments (Chow, et al, 1997). In the wake of the wing, the Reynolds shear stress u'v' decreases rapidly along the axial direction.



Figure 4.23 Profiles of Reynolds shear stress u'v' along a line with constant z through vortex core

In Figure (4.24), the contour on the cross-section also indicates that the separated shear layer has the maximum Reynolds shear stress u'w'. Again, the two-lobe structure identified by the opposite sign of u'w' in the vortex core can be seen at cross-sections over the wing surface and in the wake. The u'w' component of the Reynolds stress decreases rapidly along the axial direction in the wake. In Figure (4.26), the contours of the v'w' show a four-leaf clove pattern which was also observed in the experiments (Chow, et al, 1997). This pattern becomes more clear in the wake where the distortion effect of the shear layer vanishes. The distribution of v'w' in the plot

indicates that strong turbulent activity appears in the center of the vortex core over the wing surface. The Reynolds stress v'w' also decays along the axial direction in the wake



Figure 4.24 Contours of Reynolds shear stress component u'w' on cross-sections at different streamwise locations



Figure 4.25 Profiles of Reynolds shear stress *u'w'* along a line with constant *z* through vortex core



Figure 4.26 Contours of Reynolds shear stress component *v'w'* on cross-sections at different streamwise locations



Figure 4.27 Profiles of Reynolds shear stress *v*'*w*' along a line with constant *z* through vortex core

Figure (4.28) shows the contours of turbulence kinetic energy on cross-sections at different stream-wise locations. The profiles of turbulence kinetic energy along a line with constant z cutting through vortex core are plotted as functions of y in Figure (4.29). The maximum turbulence kinetic energy occurs in the separated shear layer near the wing tip, as shown in Figure (4.28). After the primary vortex is formed, the peak turbulence kinetic energy appears at the center of the vortex core. In the wake, the turbulence kinetic energy decreases rapidly along the axial direction.



Figure 4.28 Contours of turbulence kinetic energy on cross-sections at different streamwise locations



Figure 4.29 Profiles of turbulence kinetic energy along a line with constant *z* through vortex core

## CHAPTER 5

## CONCLUSION

The compact scheme with high order accuracy and high resolution is critical to direct numerical simulation and large eddy simulation which require resolve small length scales (high frequencies) as much as possible. For same resolution, the high order compact scheme requires much fewer grid points than the low order scheme.

The numerical simulation was carried out to investigate the formation and the near field evolution of a wing tip vortex at high Reynolds number  $(4.6 \times 10^6)$ . The flow configuration in the simulations is based on the wing tip experiment by Chow et al (Chow, et al. 1997). The high-order, and high-resolution compact scheme used in the simulation has captured the major features on a mesh with around 24 million grid points.

On the suction side of the wing surface, the rolling up of the separated shear layer at the rounded wing tip creates the primary tip vortex. The rotational flow field of the primary tip vortex induces the counter-rotating secondary vortex near the wing surface. The separated shear layer contains high level of fluctuations and wraps up the secondary vortex into the primary vortex. Both the shear layer fluctuations and the secondary vortex contribute high disturbance energy to the primary vortex core to produce small vortical structures within the core region, which becomes highly turbulent. Therefore, the turbulence inside the primary tip vortex is not created by the tip vortex itself. Instead, the turbulent shear layer and the interaction between the primary and the secondary vortices are the major sources of turbulent activity in the core.

The primary tip vortex is generated near the wing tip onboard the suction surface and is convected downstream. The rotation of the tip vortex produces low pressure in the core region. The favorable axial pressure gradient accelerates the axial flow and produces axial velocity surplus. The instantaneous axial velocity can be as high as  $2.0U_{\infty}$ .

In the near wake region, there is no more energy input from either the shear layer or the secondary vortex due to the absence of the wall surface. In the wake, the small vortical structures convected from the upstream dissipate rapidly. The near field wake is screwed and laterally stretched and curved by the rotating primary vortex. The wake disturbances also contribute to the fluctuation of the primary vortex and dissipate quickly when traveling downstream. In further downstream, the tip vortex becomes more stable and flow in the core region is more axisymmetric.

On the cross-sections that are intersected with the wing, peak values of velocity fluctuations are found over the suction side of the wing where the shear layer separates from the rounded wing tip. The high level fluctuations are wrapped up into the vortex core and convected downstream. In the wake region, the peak of velocity fluctuation appears in the center of the vortex core and fluctuation level decreases rapidly downstream along the axial direction.

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