ANALYSIS AND OPTIMAL DESIGN OF BEAMS USING RADIAL BASIS FUNCTION

by

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Dedicated to my parents, Mr. Heng Sun and Mrs. Li-Yun Hung

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ABSTRACT

ANALYSIS AND OPTIMAL DESIGN OF BEAMS USING RADIAL BASIS FUNCTION

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The application of Multiquadric Radial Basis Function (MQ RBF) in the analysis and optimal design of beams are addressed in this dissertation. For static analysis, a new Least Square Collocation (LSC) method is introduced. For buckling and vibration analysis, Rayleigh-Ritz analysis procedures are presented using MQ RBF basis function. Numerical results show that LSC can provide better results than the classical collocation method when the same number of collocation points are used. In vibration analysis, MQ RBF-based Rayleigh-Ritz can be used to calculate natural frequency accurately for several hundred modes. The important question of choosing shift parameters is discussed and a guide line for choosing this parameter is developed. Finally, the MQ RBF is also used for beam cross-section shape parameterization.

Minimum weight design of beam under buckling and natural frequency constraints are also presented in this dissertation.

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CHAPTER 1

INTRODUCTION

1.1 Review of Beam Equation

The beam is an important structural member designed to support loadings applied perpendicular to its longitudinal axes. In structural terminology, a beam can resist axial, lateral and twisting loads[2]. In the application on the civil engineering, beams are used to support roof and floor loadings[1]. Thanks to the powerful computers developed over 30 years, engineers can simulate stresses and strains on different loading conditions. It not only reduces the budget to make prototypes for testing, but also prevents buckling and fatigue before building structures. Before developing computer codes in the application of the beam, the equation should be derived from the beam's theory.

The equation is derived from the beam's theory based on the elastic curve. From figure 1.1, the equation is written as

$$\frac{1}{\rho} = \frac{\varepsilon}{y} \quad \text{.............................. (1)}$$

where ρ is the radius of curvature at a specific point on the elastic curve, and ε is the strain. Because $\varepsilon = \frac{\sigma}{E}$ and $\sigma = -\frac{My}{I}$, equation (1) becomes

$$\frac{1}{\rho} = \frac{M}{EI} \quad \text{.......................} \quad (2)$$

where M is the internal moment in the beam at the point when ρ is to be determined, E is the material's modulus of elasticity, and I is the beam's moment of inertia computed about the neutral axis.

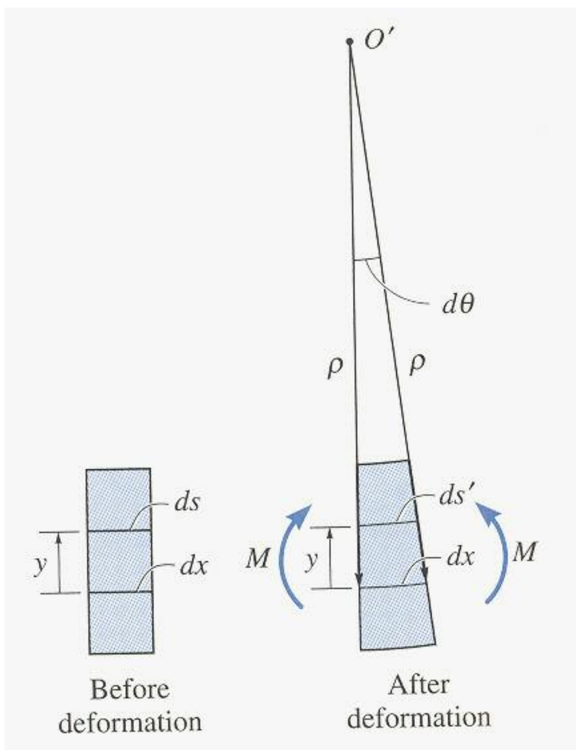

Figure 1.1 The elastic curve[1]

The elastic curve for a beam can be expressed as υ=f(x). The relationship between υ and $\frac{1}{\rho}$ is represented as

$$\frac{1}{\rho} = \frac{\frac{d^2 v}{dx^2}}{\left[1 + (\frac{dv}{dx})^2\right]^{\frac{3}{2}}} \text{ .... } (3)$$

Now substitute $\frac{1}{\rho}$ from equation (3) to equation (2), the new equation represents as

$$\frac{\frac{d^2 v}{dx^2}}{\left[1 + (\frac{dv}{dx})^2\right]^{\frac{3}{2}}} = \frac{M}{EI} \text{ .... } (4)$$

Because $\frac{1}{\rho} = \frac{d^2 v}{dx^2}$ based on the approximation, equation (4) is rewritten as

$$\frac{d^2 v}{dx^2} = \frac{M}{EI} \text{ .... } (5)$$

From equation (5), it is possible to write the equation in two alternative forms. Assume $v = \frac{dM}{dx}$, the new equation is shown as

$$\frac{d}{dx}(EI \frac{d^2 v}{dx^2}) = V(x) \text{ .... } (6)$$

where V is the shear force. Now using the equation $-p(x) = \frac{dV}{dx}$ to substitute equation (6), the beam equation derived from the beam's theory is shown as

$$\frac{d^2}{dx^2}(EI \frac{d^2 v}{dx^2}) = -p(x) \text{ .... } (7)$$

where p(x) is the distributing load on the beam[1].

In order to get the deformation equation, the boundary conditions are applied to equation (7). The following is the boundary conditions with the mathematical expression on the beam:

Fixed-Fixed: $v(0) = 0, v'(0) = 0, v(L) = 0, v'(L) = 0$

Cantilever Beam (Fixed-Free): $v(0) = 0, v'(0) = 0, v''(L) = 0, v'''(L) = 0$

Fixed-Pin: $v(0) = 0, v'(0) = 0, v(L) = 0, v''(L) = 0$

Pin-Pin: $v(0) = 0, v''(0) = 0, v(L) = 0, v''(L) = 0$

Also, Rayleigh-Ritz method is used for finding the deformation. Rayleigh-Ritz method, also called the energy method, is the powerful method in the beam equation. Figure 1.2 shows the general forces on the beam with changes of sectional areas.

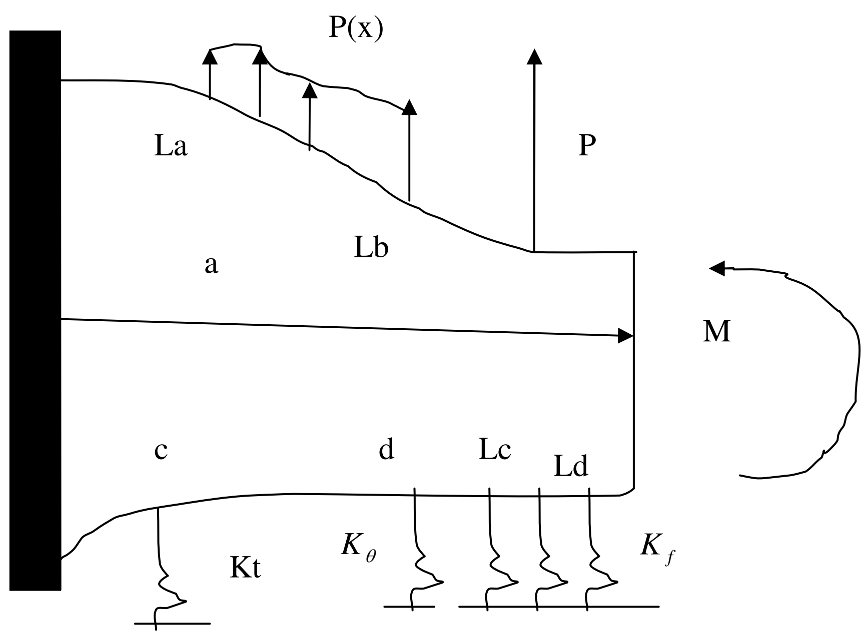

Figure 1.2 General forces on the beam with changes of sectional areas

Noted that in Figure 1.2,

P(x) =applied distributed load

L_a, L_b =bounds of P(x)

P =applied concentrated load

K_t =translational spring constant

c =location of K_t

K_θ =rotational spring

d =location of K_θ

K_f =foundational constant

L_c, L_d =lower and upper bounds of elastic foundations

M =applied bending moment at x=L

The problem can be solved by Rayleigh-Ritz method. The total potential energy of the system can be written as:

$$\pi = \frac{1}{2}\int_0^L EI(\nu'')^2\,dx - \int_{La}^{Lb} p(x)\nu dx + \frac{1}{2}\int_{Lc}^{Ld} K_f \nu^2 dx - p\,\nu(a) - M\nu'(L) + \frac{1}{2}K_t\nu^2(c) + \frac{1}{2}K_\theta \nu'^2(d)$$

………………………………………………………… (8)

The first step in Rayleigh-Ritz method is to select a set of basis functions [F]. By assume $[\nu] = [F]\{c\}$, where $\{c\}$ is a set of unknown coefficients, we have $\nu' = [F'][c], \nu'' = [F''][c]$. Substitute ν, ν', ν'' into equation (8); the new equation is represented as

$$\pi = \frac{1}{2}\int_{0}^{L} EI([c]^T[F'']^T[F''][c])dx - \int_{La}^{Lb} p(x)[F][c]dx + \frac{1}{2}\int_{Lc}^{Ld} K_f([c]^T[F]^T[F][c])dx$$

$$- p[F(a)][[c] - M[F'(L)][c] + \frac{1}{2}K_t([c]^T[F(c)]^T[F(c)][c]) + \frac{1}{2}K_{\vartheta}([c]^T[F'(d)][F'(d)][c])$$

……………………………………………………………………………………….. (9)

Equation (9) can be written as

$$\pi = \frac{1}{2}c^T K_c c^T - c^T F_C \quad \text{(9A)}$$

where

$$\left[K_c\right] = \int_{0}^{L} EI[F'']^T[F'']dx + \int_{L_c}^{L_d} K_f[F]^T[F]dx + K_t[F(c)]^T[F(c)] + K_{\vartheta}[F'(d)]^T[F'(d)]$$

and ……………………………………………………………………..(10)

$$[F_c] = \int_{La}^{Lb} p(x)[F]dx + M[F'(L)]^T + p[F(a)]^T$$

Note that the $\{c\}$ coordinates are not linearly independent coordinates. To get a set of linearly independent coordinates, we apply boundary conditions to get the following set of constraint equation:

$$\left[A\right]\{c\} = \left[0\right] \quad \text{(11A)}$$

From Equation (11A), a set of independent coordinates can be defined:

$$c = Tq \quad \text{(11B)}$$

Where

T=null (A)…………. (11C)

Note that

[A]= [Function F(x) operated by the boundary conditions]

Substitute (11B) into Equation (9A), we get

$$\pi = \frac{1}{2} q^T K_q q^T - q^T F_q \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(11D)}$$

where $K_q = T^T K_e K$, $F_q = T^T F_C$.

Since the q coordinates are linearly independent, we may impose the necessary condition for minimum potential energy

$$\frac{\partial \pi}{\partial q} = 0$$

This leads to the following set of equation

$$K_q q = F_q \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(11E)}$$

Once q is computed from (11E), we may use Equation (11B) to compute c and finally compute $\nu(x) = [F]\{c\}$.

1.2 Method of Weighted Residual

Three weight residual methods, collocation method, Galerkin method and Rayleigh-Ritz method, are commonly used in mathematical approximation, and are applied in this research. The following sections introduce the basic concept-collocation, Galerkin and Rayleigh-Ritz methods.

1.2.1. Collocation Method[25]

Collocation method is very popular in the approximation approaches with ordinal and partial differential equations. With limited scope, we consider the onedimensional differential equation as the example. Traditionally, any ordinary or partial differential equations can be presented as

$$Lu + g = 0,\ x \in [a,b] \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \ (12)$$

where u(x) is the unknown function, g(x) is a known function, and L denotes a linear differential operator, which specifies the actual form of the differential equation (12). In the collocation method, we assume a solution of the form

$$u = \sum_{i=1}^{n} c_i f_i(x) \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \ (13)$$

where $f_i(x)$ is the admissible function that is satisfied with the boundary conditions. Equation (14) will be rewritten as

$$[u] = [f]\ [c] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \ (14)$$

where [f] is the function matrix, and [c] is the unknown coefficient matrix.

In collocation method, the unknown coefficients are computed by requiring u(x) satisfy the boundary conditions and the differential equation at the specified points. In this research, we present a Least Square Collocation method. In the Least Square Collocation (LSC) method, we select more collocation points than the unknown coefficients. Thus, in LSC, we require the assumed solution satisfy all the boundary conditions, while minimize the errors in the differential equation at the collocation points. The following is a derivation of the proposed method. Let Du=P and Bu=s, where D is the differential operator, and B is the boundary condition operator. Then

$$\begin{aligned} D(\textstyle\sum c_i f_i(x)) &\approx P \\ B(\textstyle\sum c_i f_i(x)) &= s \end{aligned} \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \ (15)$$

Now equation (15) can be rewritten the new form as

$$\begin{aligned} AEQ \cdot c &\approx bEQ \\ ABC \cdot c &= bBC \end{aligned} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(16)}$$

where AEQ is the equation matrix on the left-hand side of the linear equation, ABC is the boundary condition matrix on the left-hand side of the linear equation, bEQ is the equation matrix on the right-hand side, bBC is the boundary matrix on the right-hand side, and c is the unknown coefficient matrix. Here the unknown coefficients are solved by the following constrained minimization problem:

Find c to minimize $F = e^T e$

where $e = AEQ \cdot c - bEQ$ ………………………………. (17)

subject to $ABC \cdot c = bBC$

Using Lagrange multiplier method, the above problem can be solved from the solution of the following set of linear equations:

$$\begin{bmatrix} AEQ^T AEQ & ABC^T \\ ABC & 0 \end{bmatrix} \begin{bmatrix} c \\ \lambda \end{bmatrix} = \begin{bmatrix} bEQ \\ bBC \end{bmatrix} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(18)}$$

Or, in matrix form, we have

[AA][cc]= [bb]…………………………………………… (19)

where $[AA] = \begin{bmatrix} AEQ^T AEQ & ABC^T \\ ABC & 0 \end{bmatrix}$, $[cc] = \begin{bmatrix} c \\ \lambda \end{bmatrix}$, $[bb] = \begin{bmatrix} bEQ \\ bBC \end{bmatrix}$

The unknown coefficient matrix $[cc]$ can be obtained from Equation (19). When getting the unknown coefficient matrix, substitute back Equation (14) to get the approximate solution.

1.2.2. Galerkin Method

Galerkin method is one of weight residual methods in approximation applications. Like a variation statement of a problem, a Galerkin statement incorporates differential equations in the weak formulation, so that they are satisfied over a domain in an integral or average sense rather than at any point. Traditionally, Galerkin method uses in the finite element formulation, especially in structural mechanics. This section introduces basic concepts of Galerkin method.

The mathematical statement of a physical problem is as

In domain V: Du-f=0……………… (20)

where D is a differential operator, u=u(x) is dependent variables, and f is a function of x which may be constant or zero. Equation (21) stated in strong form and appropriate boundary conditions, which imply the differential equation, must be satisfied at every internal point and boundary conditions at every boundary point. In general, an approximating function $\tilde{u}$ does not satisfy equation (21) at every point. Thus a residual R=R(x) remains:

Residual in domain V: $R = D\tilde{u} - f$ ……… (21)

where $\tilde{u} = \tilde{u}(x)$ is approximate solution. Let $\tilde{u}$ be a linear combination of basic functions. Typically $\tilde{u}$ is a polynomial of n terms whose ith term is multiplied by a generalized degree of freedom a_i. The n values of the a_i are to be selected so that R is small. According to a weighted residual method, values of the a_i that are best satisfy the following expression of governing equation in the weak form as

$$\int w_i R dV = 0 \text{, for i=1, 2…n} \quad (22)$$

where each $w_i = w_i(x)$ is a weight function. In Galerkin weighted residual method, each w_i is the multiplier of the corresponding a_i in $\tilde{u}$.

1.2.3. Rayleigh-Ritz Method

The Rayleigh-Ritz method, also known as the energy method, has a classical form and a Finite Element form. In the 1870s, Lord Rayleigh originated for studies of vibration problems. He used an approximating field contained a single degree of freedom. In 1909, Ritz generalized the method by building an approximating field from several functions. Each function is satisfied essential boundary conditions, and associated with a separate degree of freedom. Ritz applied the method to equilibrium problems and to eigenvalue problems. In general, the Rayleigh-Ritz method is a procedure for determining parameter in an approximating field so as to achieve an extremum of a function F of the field. In the practical application of the Rayleigh-Ritz method, vibration analysis and buckling problems are always used. The detail of Rayleigh-Ritz method for static analysis is presented in section 1.

1.3 Review of Beam Buckling

Buckling is one of the main concerns in the structural design. Based on the definition from energy consideration, buckling means loss of the stability of an equilibrium configuration without fracture or separation of the material or at least prior to it. Usually the buckling occurs when the compression or tension is on the axial load[2]. Traditionally, columns are divided into three types: short column, intermediate column, and long columns. This section reviews the equation of beam buckling from Eular's equation and energy method with different boundary conditions.

In 1757, Leonard Euler developed a relationship for the critical column load which would produce buckling. For the system below in Figure 1.3, the governing equation for the computing buckling load is presenting

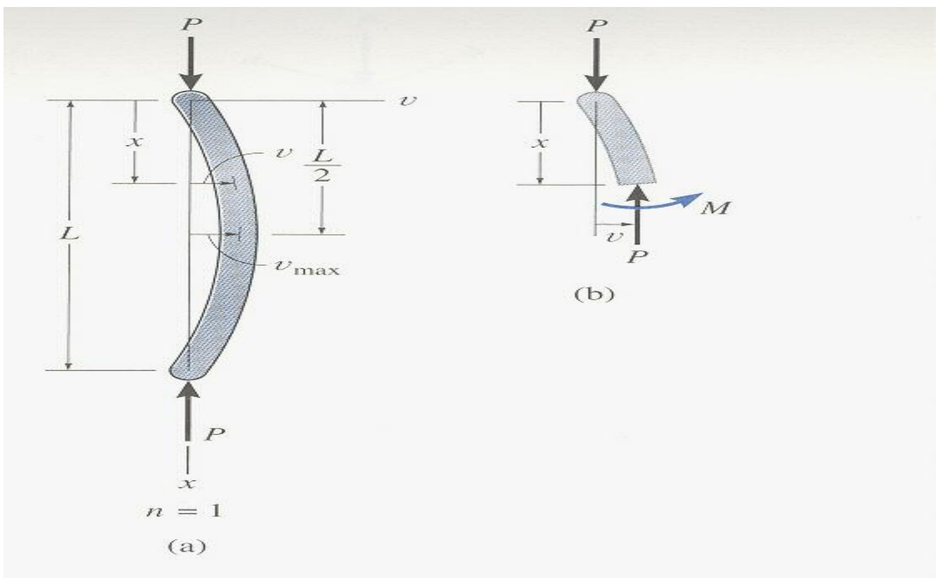

Figure 1.3 Loaded Pined-Pined Columns[1]

$$\frac{d^2}{dx^2}\left[EI\frac{d^2\upsilon}{dx^2}\right]+\frac{d^2}{dx^2}\left[P\frac{d\upsilon}{dx}\right]=w \qquad (23)$$

where E is the elasticity of the strength material, I is the moment of inertia, υ is the deflection distance, w is the distributed load, and P is the axial load.

From equation (23), the general solution form is

$$\nu = A\sin\sqrt{\frac{P}{EI}}x + B\cos\sqrt{\frac{P}{EI}}x \qquad (24)$$

Now we apply the boundary condition of the pinned-pinned case:

$$\nu(0) = 0, \upsilon''(0) = 0, \upsilon(L) = 0, \upsilon''(L) = 0 \text{ .... (25)}$$

Here we get the critical load as:

$$P_{cr} = \frac{\pi^2 EI}{L^2} \text{ .... (26)[26]}$$

The critical load is important in the engineering design of the beam. It avoids structural buckling on the axial axis with tension or compression loads. Figure 1.4 shows the requirement of length and K on the following boundary conditions.

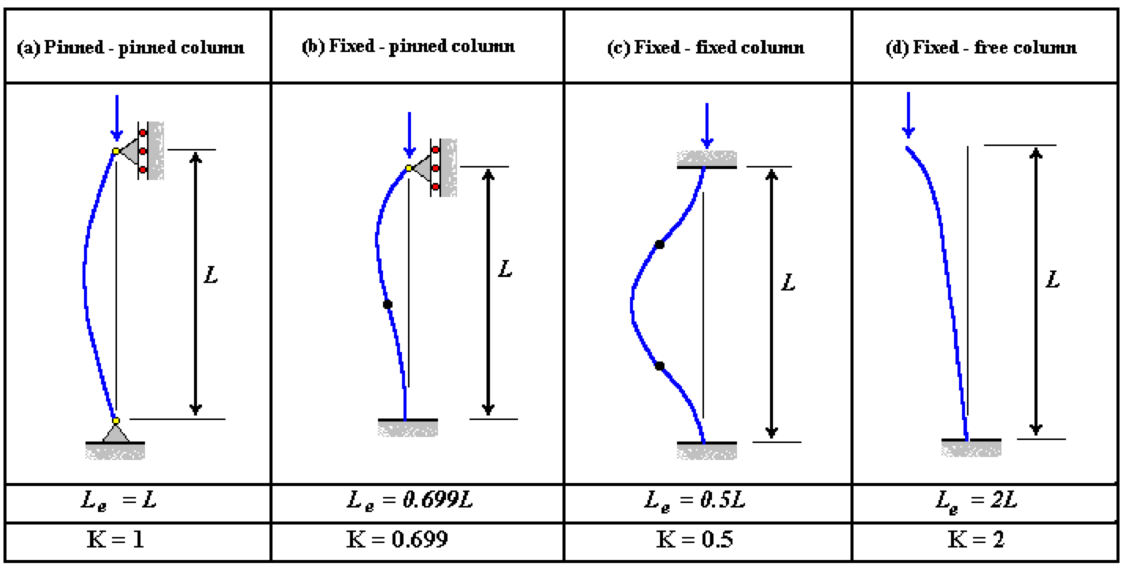

Figure 1.4 Requirement of length and K on the pin support column

In energy method, the beam buckling with the total energy is represented as:

$$\pi = \frac{1}{2}\int_0^L EI(\upsilon'')^2 dx + \frac{1}{2}\int_0^L P(\upsilon')^2 dx \text{ .... (27)}$$

Assume $\upsilon = [f][c], \upsilon' = [f'][c], \upsilon'' = [f''][c]$ and [f] is the 1-D basis function matrix.

Substitute $\upsilon, \upsilon', \upsilon''$ into equation (27), we get

$$\pi = \frac{1}{2}c^T K_c c + \frac{1}{2}c^T K_g c \text{ .... (28)}$$

where $K_c = \frac{1}{2}\int_0^L EI([c]^T[f'']^T[f''][c])dx,\ K_g = \frac{1}{2}\int_0^L P([c]^T[f']^T[f'][c])dx$ ……… (29)

Note that the $\{c\}$ coordinates are not linearly independent. To get a set of linearly independent coordinates, we apply boundary conditions to get the following set of constraint equation:

$$[A]\{c\} = \{0\} \text{…………………..} (30)$$

From Equation (30), a set of independent coordinates can be defined:

C=Tq……………… (31)

Where

T=null (A)………………. (32)

Note that [A] = [Function F(x) operated by the boundary condition]

Substitute (31) into Equation (28), we get

$$\pi = \frac{1}{2}q^T K_q q^T + \frac{1}{2}q^T K_{gq} q$$

Where $K_q = T^T K_c T,\ K_{gq} = T^T K_g T$

Since the q coordinates are linearly independent, we may impose the necessary condition for minimum of Rayleigh quotient

$$-P = \frac{q^T K_q q}{q^T K_{gq} q}$$

This leads to the following eigenvalue problem

$$K_q q = \lambda K_{qg} q \text{ …………………. } (33)$$

Where $\lambda = -P$ .Once q is computed from (33), we may use Equation (31) to compute c and finally q compute the buckling vector by using

$$u(x) = [F]\{c\}.$$

1.4 Review of Beam Vibration

Structural response is time-dependent if loading is time-dependent. If loading is of higher frequency or is applied suddenly, dynamic analysis is required. Dynamic analysis uses the same stiffness matrix as static analysis, but also requires mass and damping matrices. For a given magnitude of loading, dynamic response may be greater or less than static response. It will be much greater if loading is cycle with frequency close to a natural frequency of the structure. In the analysis of the structural dynamics, we need to get the natural frequencies and mode shapes. To calculate frequencies and mode shapes, the eigenvalue problem needs to be solved.

To calculate frequencies and mode shapes, we need to get the mass and stiffness matrices in the non-damping vibration system. Assume $u = [f]\{c\}$, the stiffness and mass matrices in c coordinates are shown as:

$$[K] = T^T (\int_0^L EI([f'']^T [f''])dx)T \text{ .... (34)}$$

$$[M] = T^T (\int_0^L [f]^T \rho A[f]dx)T \text{ .... (35)}$$

where E is the elasticity of strength material, I is the moment of inertia, ρ is the density of the material, A is the area of the beam, [K] is the stiffness matrix, [M] is the mass matrix, and [f] is the function used in the beam. Note that the $\{c\}$coordinates are not linearly independent. To get a set of linearly independent coordinates, we apply boundary conditions to get the following set of constraint equation:

$$[A]\{c\} = \{0\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{ (36)}$$

From Equation (36), a set of independent coordinates can be defined:

$$C=Tq \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{ (37)}$$

Where

$$T=\text{null (A)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{ (38)}$$

Note that [A] = [Function F(x) operated by the boundary condition].

Using the transformation of Equation (37), the stiffness and mass for the independent coordinates q are obtained as

$$K_q = T^T KT$$
$$M_q = T^T MT$$

and the natural frequencies can be solved from the following eigenvalue problem.

$$\lfloor K_q \rfloor q = \lambda \lfloor M_q \rfloor q \ldots\ldots\ldots\ldots\ldots\ldots \text{ (39)}$$

where q is the eigenvectors. Once q is known, the associated mode shape can be calculated from the following equation

$$u(x) = [f(x)][T][q]$$

1.5 Motivation of This Research

Traditionally, engineers use polynomial methods in the beam vibration and buckling cases. However, the higher polynomial terms reduce the calculation time and accuracy, and the error between the approximate value and exact solution is increased.

Although the polynomial method has been used in the numerical approximation for forty years, the error gap is still the main obstacle for engineering analysis.

The following is the motivation of this dissertation:

1. Develop an innovate technique using the Radial Basis Function (RBF to be defined in Chapter 2) on both strong and weak formulations.
2. Investigate approximate methods with the Radial Basis Function to reduce computational time and memory.
3. Evaluate advantages and technical obstacles of applying the Radial Basis Function in structural analysis.

At the end of this chapter, the author points out three main contributions in this research:

1. The Radial Basis Function cooperates with the Least Square Collocation method for structural static analysis. This allows the user to obtain more accurate solution using the same number of collocation or the traditional collocation method.
2. The Radial Basis Function is applied in the beam design optimization. This is the first time the Radial Basis Function is applied in engineering optimization problems. The examples of using the Radial Basis Function in the beam design will be introduced in chapter 5 of this dissertation.
3. A guideline in choosing RBF parameter is proposed in this dissertation.

CHAPTER 2

LITERATURE REVIEW

2.1 Radial Basis Function

Radial Basis Function (RBF) is popular for interpolating scattered data as the associated system of linear equations guaranteed to be invertible under very mild conditions on the locations of the data points. Theorically, the Radial Basis Function does not require the data lied on the regular grid.

Here is the definition of the Radial Basis Function. A radial basis function is any function that has a radial symmetry. RBF can be used to approximate a nonlinear function in the form:

$$s(x) = p(x) + \sum_{i=1}^{N} c_i \varphi_i (|x - x_i|),\ x \in R \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (40)$$

In the numerical application, three popular Radial Basis Function have been used: The Thin-Plate Spline, Gaussian and the Multiquadric (MQ) function. The following are the definition of each of the above Radial Basis Function type[20]:

- The Thin-Plate Spline: $\varphi(r) = r^2 \log(r)$ ……………………(41)
- The Gaussian: $\varphi(r) = e^{-hr}$ …………………………………(42)
- The Multiquadratic (MQ): $\varphi(r) = \sqrt{r^2 + h}$ …………………(43)

where r is the distance as $|x - x_i|$, and h is the shift or smooth parameter. Figure 2.1 shows the Radial Basis Function with the Thin-Plate Spline type; Figure 2.2 shows the Radial Basis Function with Gaussian type, and Figure 2.3 shows the Radial Basis Function with Multiquadratic (MQ) type.

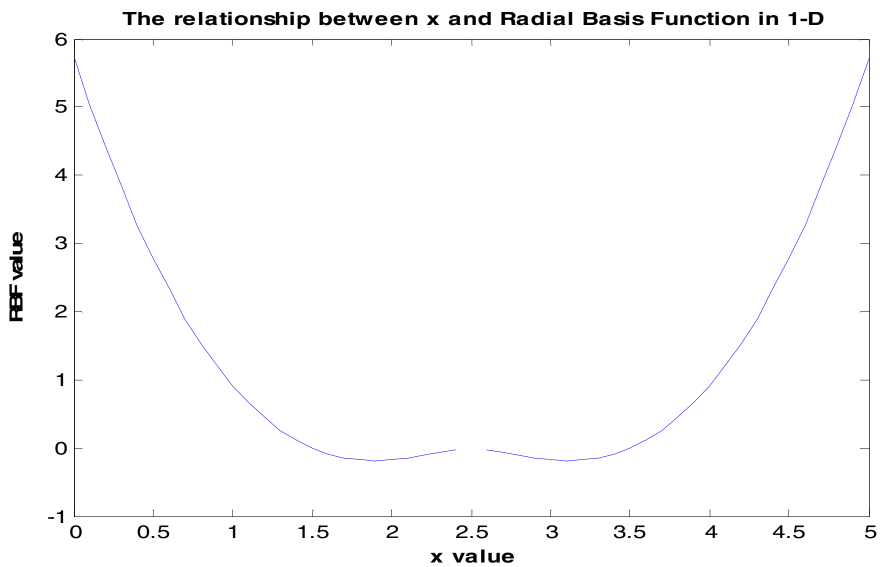

Figure 2.1 Thin-Plate Spline Type

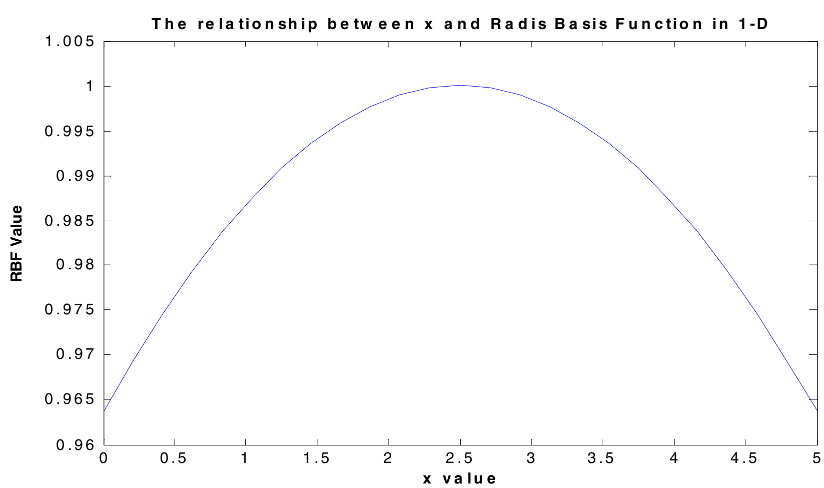

Figure 2.2 Gaussian Type

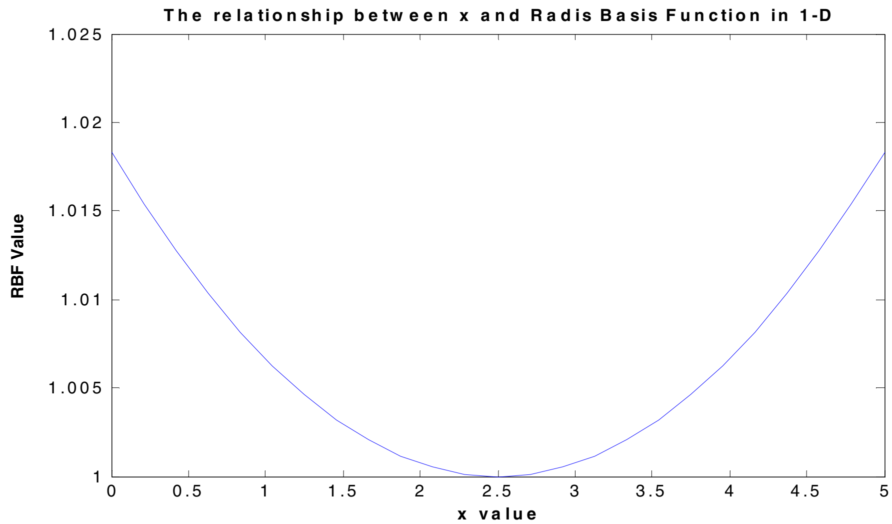

Figure 2.3 Multiquadric (MQ) Type

In this research we use Multiquadratic (MQ) function in the Least Square Collocation method for static analysis. MQ function is also used in Optimal Design and vibration and buckling analysis that use Rayleigh-Ritz method. More discussions of MQ function are given in Chapter 6.

2.2 Radial Basis Function for Engineering Applications

The Radial Basis Function was applied for different types of interpolation problems in the 1970s. Originally, this method was to solve problems in the networking and curves and surface fitting. In 2006, Dr. Subbarao introduced the idea of using the Radial Basis Function in the direction dependent learning approach networks[3]. In the 1980s, the Radial Basis Function was introduced in solving structural problems in the civil engineering field. Because the Radial Basis Function does not need any mesh to approach the numerical calculation, this method is also called the meshless method. From 1998 to 2005, many scholars presented their ideas of using the Radial Basis

Function in the engineering analysis. For example, Dr. Grindeanu introduced using the Radial Basis Function (Meshless method) in the application of the design sensitivity analysis and optimization in the field of hyperelastic structures[6, 7]. Dr. Wang presented his idea to analyze the parameter optimization and application of solving Boundary Value Problems with the Radial Basis Function[9, 10, 20]. Dr. Chen presented using coupling finite element and meshless local Petro-Galerkin methods for 2-D potential problems[11]. Other applications of using the Radial Basis Function are as follows[20]:

1. Curves and Surface Fitting
2. Photogrammetry
3. Surveying and Mapping
4. Geology and Mining
5. Hydrography
6. Solution of Partial Differential Equations
7. CFD
8. Optimization

CHAPTER 3

RADIAL BASIS FUNCTION FOR 1-D PROBLEM

3.1 Bar

This chapter discusses the application of using the Radial Basis Function with the strong formulation in the collocation method and the proposed Least Square Collocation (LSC) method for the 1-D bar, 1-D beam and higher order system problems. This section introduces the basic concept of applying the Radial Basis Function with the strong formulation in the Collocation method. The following sections will use the same concept in the 1-D beam and higher order system.

The differential equation for a fixed-free bar under axial static loading can be represented as

$$AEu'' - Ku' = -F \text{ , B.C.: } u(x_0) = 0, u'(x_1) = 0 \text{ .... (44)}$$

In order to apply the collocation method with the Radial Basis Function, the function u is rewritten as

$$[u] = [f]\{c\} \text{ .... (45)}$$

and $$[u'] = [f']\{c\} \text{ .... (46)}$$

$$[u''] = [f'']\{c\} \text{ .... (47)}$$

where [f] = [Radial Basis Function Polynomial Term] and {c} = { $c_0, c_1, \cdots, c_N$ }. Now we substitute $[u], [u'], [u'']$ into equation (45). The new equation will be presented as

$$(AE[f''] - K[f])\{c\} = F \text{ ....} (48)$$

Next, we apply boundary conditions and equation (48) with collocation points into the Radial Basis Function. The classical collocation method will be reformed to the matrix form as

$$\begin{bmatrix} f(x_0) \\ f'(x_1) \\ AE[f''(x_c)] - K[f(x_c)] \end{bmatrix} [c] = \begin{bmatrix} a \\ b \\ [F] \end{bmatrix} \text{ ....} (49)$$

Here we get [c] through the matrix operation, and substitute [c] to equation (45) to get each value in the collocation point. The following shows operations of [c] and [u] matrices.

$$[c] = \begin{bmatrix} f(x_0) \\ f'(x_1) \\ AE[f''(x_c)] - K[f(x_c)] \end{bmatrix}^{-1} \begin{bmatrix} a \\ b \\ [F] \end{bmatrix} \text{ ....} (50)$$

Once c is known, we can calculate u(x) from

[u]= [f] [c]………………………………………….. (51)

Here we introduce the concept of the Radial Basis Function in the Least Square Collocation method for the 1-D bar problem. This is the contribution for the 1-D strong formulation in this dissertation. The same steps will be followed in the classical collocation method described in previous paragraphs. Now we introduce Du=p and Bu=s into the strong form equation, where D is the differential operator, and B is the boundary operator. Applying the differential operator and the boundary operator into the strong formulation, the equation is written as

$$D(\sum c_i f_i(x)) - p = e$$
$$B(\sum c_i f_i(x)) = s \quad \text{……………………………..} \quad (52)$$

Using Equation (45) to (47), Equation (52) becomes $e = [AEQ]\{c\} - bEQ$ and $[ABC]\{c\} = bEQ$. Where

$$AEQ = [AE[f''(x_c)] - K[f(x_c)]] \text{…………………} (53)$$

$$bEQ = [[F]] \text{………………………………………} (54)$$

$$ABC = \begin{bmatrix} f(x_0) \\ f(x_1) \end{bmatrix} \text{……………………………………} (55)$$

$$bBC = \begin{bmatrix} a \\ b \end{bmatrix} \text{……………………………………………} (56)$$

$$[c] = [c_0, c_1, \cdots c_N] \text{…………………………………} (57)$$

Next, we will find the [c] through the concept of the optimization. The formulation is shown as

Find [c] to minimize $F^* = e^T e$

where $e = AEQ \cdot c - bEQ$ ……………………………. (58)

subject to ABC·c=bBC

Using Lagrange multiplier method, the solution of the above equation can be found by solving the following system of linear equation:

$$\begin{bmatrix} AEQ^T AEQ & ABC^T \\ ABC & [0] \end{bmatrix} \begin{bmatrix} c \\ \lambda \end{bmatrix} = \begin{bmatrix} bEQ \\ bBC \end{bmatrix} \text{…………………………} (59)$$

That is

$$\begin{bmatrix} c \\ \lambda \end{bmatrix} = \begin{bmatrix} AEQ^T AEQ & ABC^T \\ ABC & [0] \end{bmatrix}^{-1} \begin{bmatrix} bEQ \\ bBC \end{bmatrix} \qquad (60)$$

Once c is known, we can calculate

$$[u] = [f][c] \qquad (61)$$

As a numerical example, consider the following problem:

$$u'' - 5u = -1,\ B.C.: u(0) = 0,\ u'(1) = 0 \qquad (62)$$

The problem is solved using Radial Basis Function with classical collocation method and the Least Square Collocation method.

The Matlab code including the classical collocation method, the least square collocation method and exact solution for solving this problem is in Appendix A. Figure 3.1 shows the comparison of using the classical collocation method and the least square collocation method with the exact solution. We found that the least square collocation method is close to the exact solution if inputting many geometrical points on the 1-D bar. This is the first advantage of applying the least square collocation method in the truss analysis.

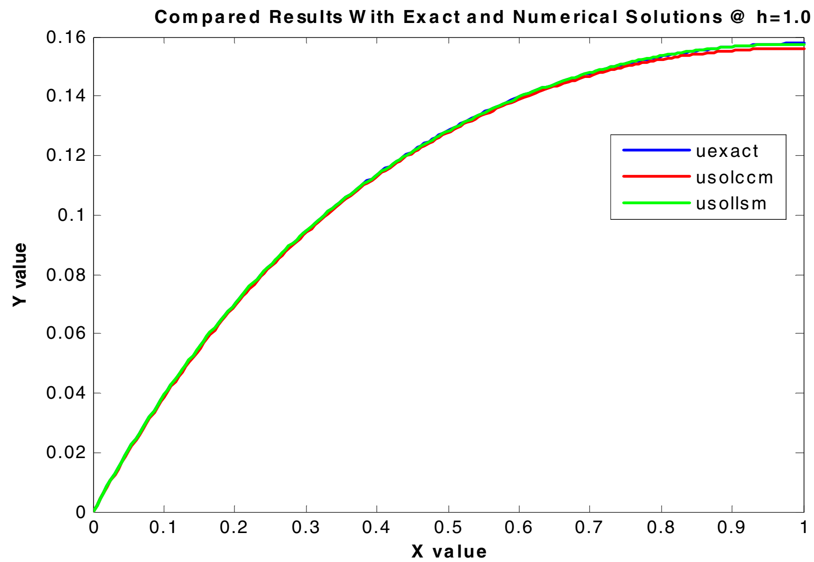

Figure 3.1 Compared Results with Three Method in 1-D Bar

3.2 Beam

The classical formulation for a uniform beam on elastic foundation is:

$$EIu'''' - ku = w \text{ With Boundary Conditions} \quad \text{(63)}$$

In order to apply the collocation method with the Radial Basis Function, the function u is assume to be

$$[u] = [f]\{c\} \quad \text{(64)}$$

then $$[u'] = [f']\{c\} \quad \text{(65)}$$

$$[u''] = [f'']\{c\} \quad \text{(66)}$$

$$[u'''] = [f''']\{c\} \quad \text{(67)}$$

$$[u''''] = [f'''']\{c\} \quad \text{(68)}$$

where [f] = [Radial Basis Function   Polynomial Term] and {c} = { $c_0, c_1, \cdots, c_N$ }. Now we substitute $[u],[u'],[u'']$ into equation (45). The new equation will be presented as

$$(EI[f''''] - K[f])\{c\} = w \ldots\ldots\ldots\ldots\ldots\ldots\ldots.. (69)$$

Next, we apply boundary conditions and equation (70) with collocation points into the Radial Basis Function. The classical collocation method will be reformed to the matrix form as

$$\begin{bmatrix} [f(boundary\ conditions)] \\ [EI[f''''(x_c)] - K[f(x_c)]] \end{bmatrix} [c] = \begin{bmatrix} [0] \\ [w] \end{bmatrix} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (70)$$

Here we get [c] through the matrix operation, and substitute [c] to equation (65) to get each value in the collocation point. The following shows operations of [c] and [u] matrices.

$$[c] = \begin{bmatrix} [f(boundary\ conditions)] \\ [EI[f''''(x_c)] - K[f(x_c)]] \end{bmatrix}^{-1} \begin{bmatrix} [0] \\ [w] \end{bmatrix} \ldots\ldots\ldots\ldots\ldots\ldots (71)$$

[u]= [f] [c]……………………………………………….. (72)

Here we introduce the concept of the Radial Basis Function in the Least Square Collocation method for the 1-D beam problem. This is the contribution for the 1-D strong formulation in this dissertation. The same steps will be followed in the classical collocation method described in previous paragraphs. Now we introduce Du=p and Bu=s into the strong form equation, where D is the differential operator, and B is the boundary operator. Applying the differential operator and the boundary operator into the strong formulation, the equation is written as

$$\begin{aligned} &D(\sum c_i f_i(x)) - p = e \\ &B(\sum c_i f_i(x)) = s \end{aligned} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. (73)$$

Using Equation (64) to (68), Equation (73) becomes

$$[AEQ]\{c\} - bEQ = e$$
$$[ABC]\{c\} = bBC$$

Where

$$AEQ = \left[EI\left[f''''(x_c)\right] - K\left[f(x_c)\right]\right] \ldots\ldots\ldots\ldots\ldots\ldots\ldots (74)$$

$$bEQ = [[w]] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (75)$$

$$ABC = \left[f(boundary\ conditions)\right] \ldots\ldots\ldots\ldots\ldots\ldots (76)$$

$$bBC = [[0]] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (77)$$

$$[c] = [c_0, c_1, \cdots c_N] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (78)$$

Next, we will find the [c] through the concept of the optimization. The formulation is shown as

Find [c] to minimize $F^* = e^T e$

where $e = AEQ \cdot c - bEQ$ ……………………………. (79)

subject to ABC·c=bBC

The above problem can be solved by Lagrange multiplier method, and the solution can be obtained from the following set of linear equation:

$$\begin{bmatrix} AEQ^T AEQ & ABC^T \\ ABC & [0] \end{bmatrix} \begin{bmatrix} c \\ \lambda \end{bmatrix} = \begin{bmatrix} bEQ \\ bBC \end{bmatrix} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (80)$$

Note that λ is the vector of Lagrange multipliers. Explicitly, we have

$$\begin{bmatrix} c \\ \lambda \end{bmatrix} = \begin{bmatrix} AEQ^T AEQ & ABC^T \\ ABC & [0] \end{bmatrix}^{-1} \begin{bmatrix} bEQ \\ bBC \end{bmatrix} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (81)$$

Once c is known, we an compute u(x) from

$$[u] = [f][c] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (82)$$

The following fixed-fixed beam example is used to demonstrate the classical collocation method and the Least Square Collocation method using Radial Basis Function. The governing equation and the boundary conditions in the following:

$$2.5u'''' + 2u = 1,\ B.C.: u(0) = 0,\ u'(0) = 0,\ u''(20) = 0,\ u'''(20) = 0 \quad (83)$$

The Matlab code including the classical collocation method, the least square collocation method and exact solution for solving this problem is in Appendix B. Figure 3.2 shows the comparison of using the classical collocation method and the least square collocation method with the exact solution. We found that the least square collocation method is close to the exact solution if inputting many geometrical points on the 1-D beam. This is another advantage of applying the least square collocation method in the truss analysis, and we conclude that the least square collocation method is the best approximating method in the bar and beam simulation.

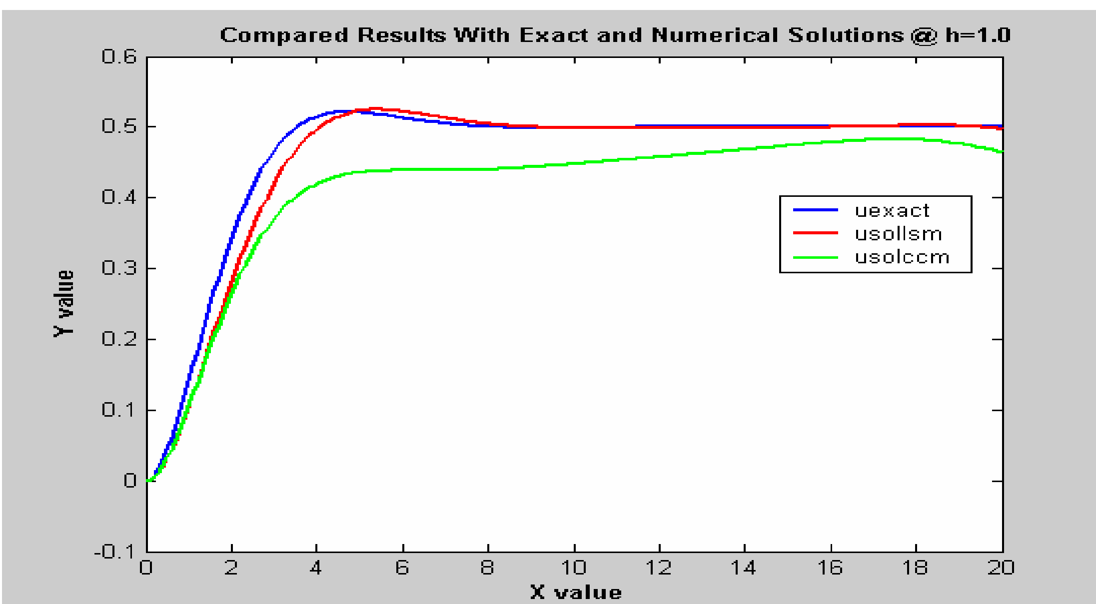

Figure 3.2 Compared Results with Three Method in 1-D Beam

3.3 Other Example

The governing equation 1-D bar and beam are the 2nd and 4th order Ordinary Differential Equations (ODEs). Because the value on each point calculated from the collocation method and the Least Square Collocation method with the Radial Basis Function are close to the exact solution in the 1-D bar and beam cases, the collocation method will thus be of value in the application of higher order systems. Here is the example of using the collocation method and the least square collocation method for a general 4th order system. The question is to solve the equation with the boundary conditions in the following:

$$x^4 y'''' - 4x^3 y''' + x^2(12 - x^2) y'' + 2x(x^2 - 12) y' + 2(12 - x^2) y = 2x^5$$

$$x(0) = 0, x'(0) = 0, x(11) = 0, x'(11) = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots.. (84)$$

The original Matlab code including the classical collocation method, the least square collocation method and exact solution for solving this problem is in Appendix C. Figure 3.3 shows the comparison of using the classical collocation method and the exact solution. We found that the classical collocation method is close to the exact solution if inputting many collocation points on this equation. This is the advantage of applying the collocation method in the higher order system, and we conclude that the collocation method also uses properly in the higher order system.

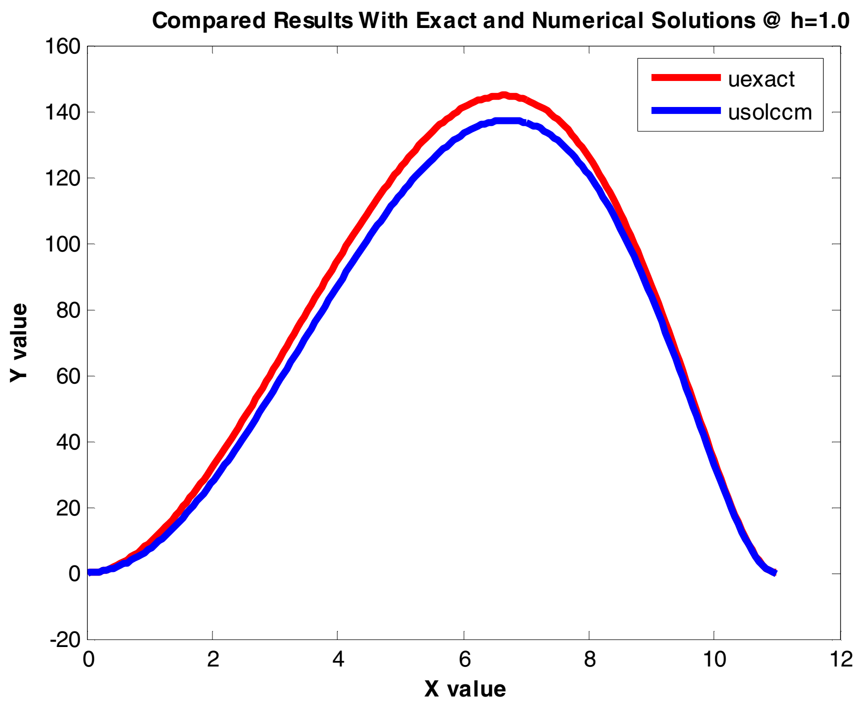

Figure 3.3 Compared results with Exact Solution and Collocation Method

CHAPTER 4

RADIAL BASIS FUNCTION FOR EIGENVALUE PROBLEM

4.1 Beam Buckling

Chapter 1 introduces the theories of beam buckling and vibration from the original concepts of physics. We also provide general formulation for Rayleigh-Ritz method to solve equations of beam buckling and vibration. This chapter discusses about using the Radial Basis Function with Rayleigh-Ritz method in the beam buckling and vibration problems.

Here we discuss how to apply the Radial Basis Function with Rayleigh-Ritz method in this section. Because the beam buckling is axial load-dependent, we need to calculate geometrical stiffness matrices for finding eigenvalues and eigenvectors. TO solve the buckling problem, assume

$$u = [f]\{c\} \qquad \text{.................... (85)}$$

Where [f] = [Radial Basis Function | Polynomial Term]

The following figures show the first mode shape and critical load computed by the current method with different boundary conditions. The original Matlab code for solving this problem is in Appendix D. Figure 4.1 shows the beam buckling with fixed-fixed condition; Figure 4.2 shows the beam buckling with fixed-free condition; Figure

4.3 shows the beam buckling with fixed-pin condition, and figure 4.4 shows the beam buckling with pin-pin condition.

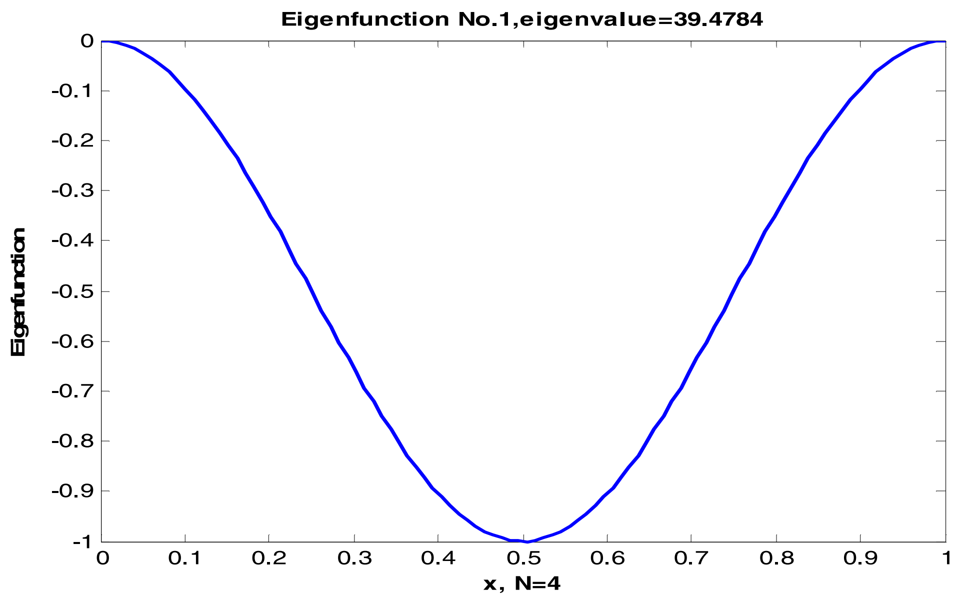

Figure 4.1 Beam Buckling in Fixed-Fixed Condition

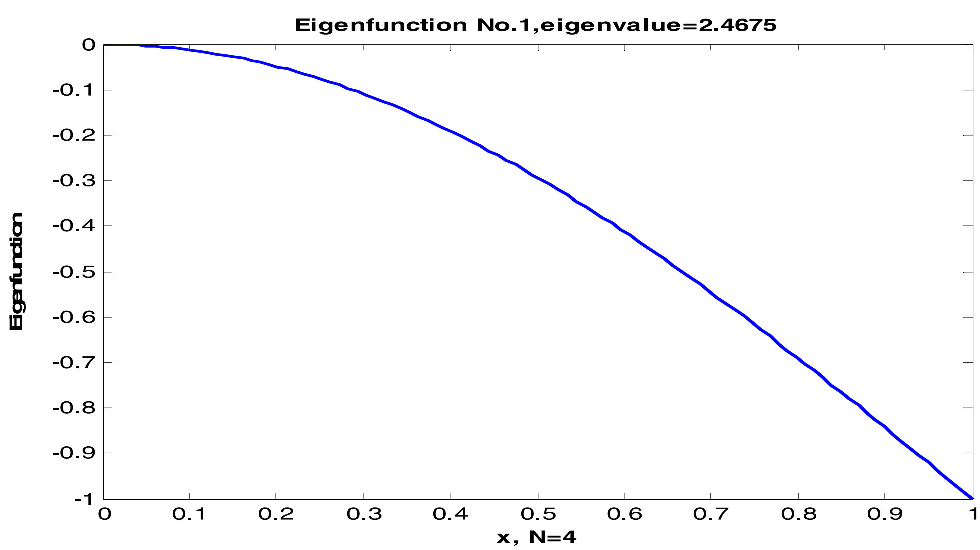

Figure 4.2 Beam Buckling in Fixed-Free Condition

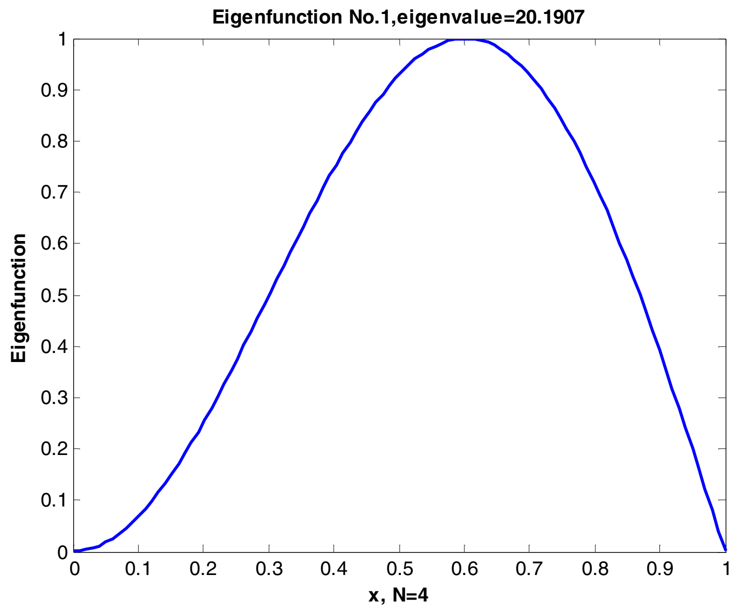

Figure 4.3 Beam Buckling in Fixed-Pin Condition

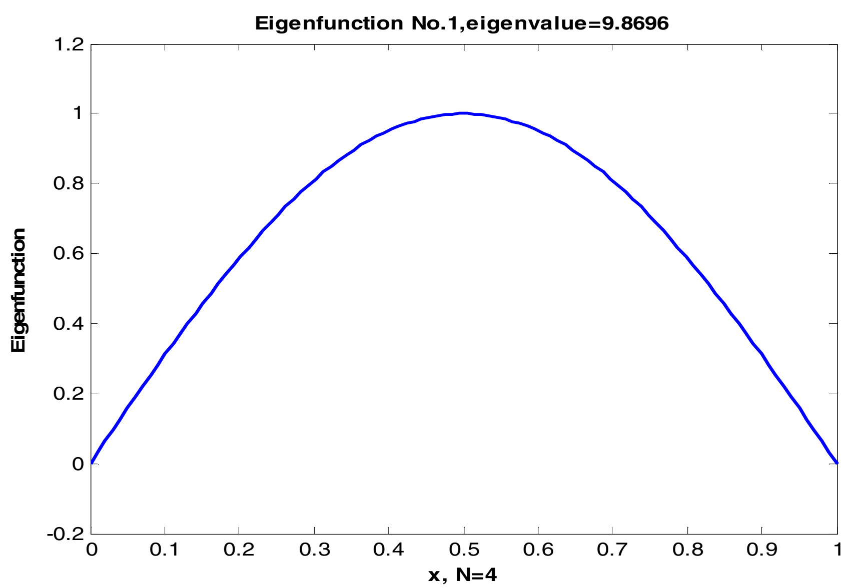

Figure 4.4 Beam Buckling in Pin-Pin Condition

The following tables present the comparison of error percentage of using the Radial Basis Function, polynomial term and exact solution. Table 4.1 shows errors between the exact solution and RBF method in the fixed-fixed condition; Table 4.2 shows errors between the exact solution and RBF method in the fixed-free condition; Table 4.3 shows errors between the exact solution and RBF method in the fixed-pin condition, and Table 4.4 shows errors between the exact solution and RBF method in the pin-pin condition We found the Radial Basis Function gets the closest solution compared to the exact solution. However, the Radial Basis Function has the barrier. If choosing more than twenty collocation points and more than 5 polynomial terms, the solution will become unstable. Other than that, the Radial Basis Function performs the great job in the 1-D beam buckling.

Table 4.1 Errors between the exact solution and RBF method in the fixed-free condition

Mode	Exact-RBF
1	0.01%

Table 4.2 Errors between the exact solution and RBF method in the fixed-fixed condition

Mode	Exact-RBF
1	0.018%

Table 4.3 Errors between the exact solution and RBF method in the fixed-pin condition

Mode	Exact-RBF
1	0%

Table 4.4 Errors between the exact solution and RBF method in the pin-pin condition

Mode	Exact-RBF
1	0%

4.2 Beam Vibration

This section discusses about using the Radial Basis Function in the numerical analysis of the beam vibration. Rayleigh method is used to calculate the natural frequencies and the associated mode shapes. The general formulation for beam vibration analysis by Rayleigh-Ritz method has been presented in Chapter1. Here we assume

$$u = [f]\{c\}$$

Where [f] = [Radial basis function polynomial].

The following figures show the first two natural frequencies and their mode shapes with different boundary conditions. The original Matlab code for solving this problem is in Appendix E. Figure 4.5 and Figure 4.6 show the beam vibration with fixed-fixed condition; Figure 4.7 and Figure 4.8 show the beam vibration with fixed-free condition;

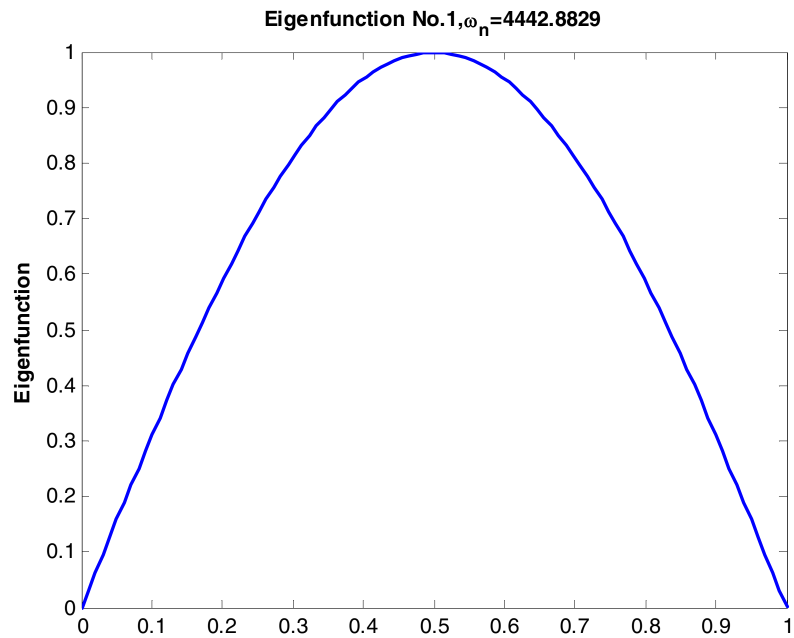

Figure 4.5 First Mode Shape in Fixed-Fixed Condition

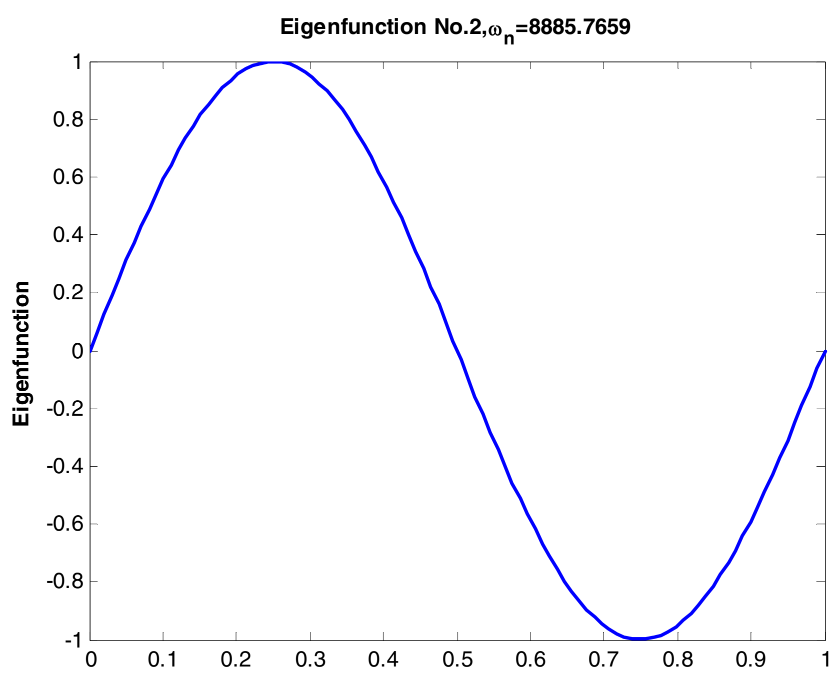

Figure 4.6 Second Mode Shape in Fixed-Fixed Condition

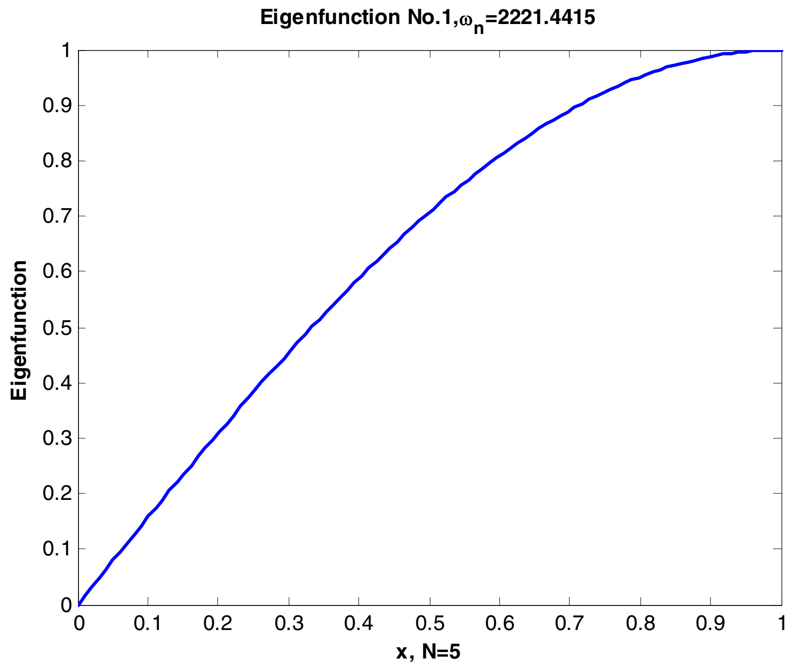

Figure 4.7 First Mode Shape in Fixed-Free Condition

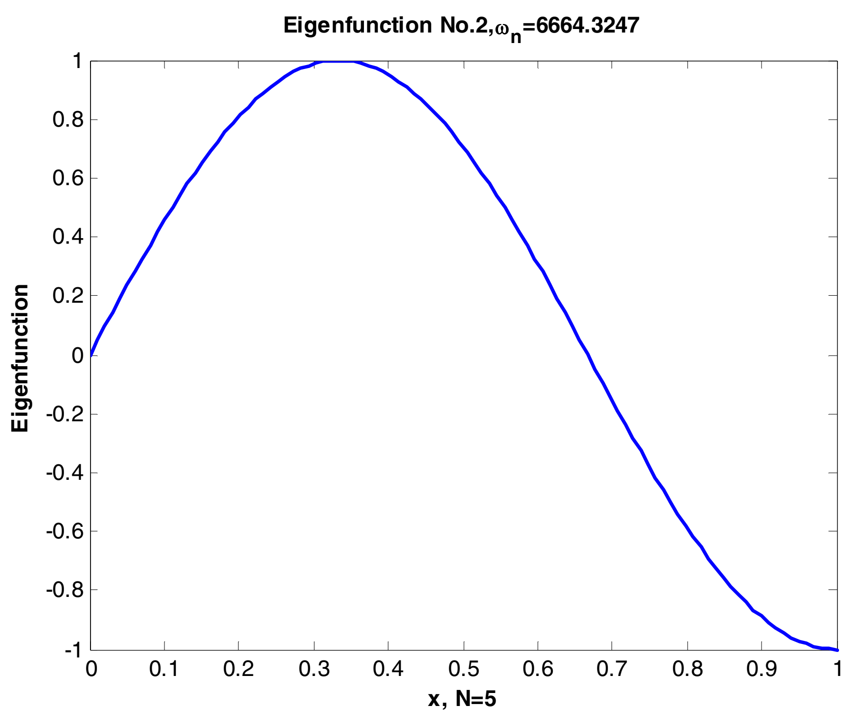

Figure 4.8 Second Mode Shape in Fixed-Free Condition

The following tables present the comparison of error percentage of using the Radial Basis Function, polynomial term and exact solution. Table 4.5 compares errors among the exact solution, polynomial method and Radial Basis Function method in the fixed-free condition, and Table 4.6 compares errors among the exact solution, polynomial method and Radial Basis Function method in the fixed-fixed condition. We found the Radial Basis Function gets the closest solution compared to the exact solution. However, the Radial Basis Function has a problem in the weak formulation like the beam buckling. If choosing more than twenty collocation points and more than 5 polynomial terms, the solution will become unstable. Also, the value of the parameter h in the Radial Basis Function could not be higher than 5. Other than that, using Rayleigh-Ritz method with the Radial Basis Function is the best choice in the beam vibration analysis.

Table 4.5 Errors among Three Method in the Fixed-Free Condition

Modes	Exact-Polynomial	Exact-RBF
1	0.01%	0.01%
2	0.56%	0.004%

Table 4.6 Errors among Three Method in the Fixed-Fixed Condition

Modes	Exact-Polynomial	Exact-RBF
1	0.34%	0.002%
2	2.04%	0.01%

CHAPTER 5

RADIAL BASIS FUNCTION FOR OPTIMIZATION DESIGN

5.1 Beam Buckling

Using the Radial Basis Function for the optimal design is another contribution in this research. Traditionally, the optimization helps engineers find the optimized value in the engineering design. This technology saves much time and development procedures. This chapter introduces how to apply the Radial Basis Function for the optimal design in the 1-D beam buckling and vibration applications.

In order to reduce the weight of the beam and avoid being occurred the buckling or vibration, the following optimization problem is solved in this research.

$$\text{Find } x_i \text{ to minimize weight} = \int_0^L A_i \rho dx \quad (86)$$

Such that $P_{cr} \geq \lambda_{\max}$, $x_U \leq x_i \leq x_{UP}$

where x_i is the design variable, A_i is the area of the cross-section, ρ is the density of the material, P_{cr} and $\lambda_{\max}$ are eigenvalues. In this research, x_i represents as a width of a beam with cross-section. In the proposed foundation, the cross-section dimension at N_x selected points are selected as design variables. Then the variation of the dimension is represented by the following RBF equation:

$$a(x) = \sum_{i=1}^{N_x} \phi_i(x) c_i \text{ .... (87)}$$

The coefficients of c_i are computed by imposing interpolation conditions. Once a(x) is known, the cross-sectional area and the sectional moment of inertia are computed from

$$A(x) = a^2(x)$$
$$I(x) = \frac{a^4(x)}{12}$$

As a design example, consider a beam with the following data:

- Density of the material: 0.1 lbf
- Length of the beam: 100 feet
- Young's Modulus: 10000000 psi
- Maximum requirement of the first eigenvalue: 10000
- Design limit of the width: $1 \le width_i \le 10,\ i = 1,2$
- Initial gauss of the design width: 3 feet and 2 feet on both ends.
- Optimal design width: 6.04 feet and 6.07 feet in the uniform type beam

Figure 5.1 and Figure 5.2 show the optimal beam design in the regular square and the first mode shape and eigenvalue in the fixed-fixed condition. Figure 5.3 and Figure 5.4 presents the optimal design in the tapered-line square beam and the first mode shape with the eigenvalue. The original program codes are listed in Appendix F.

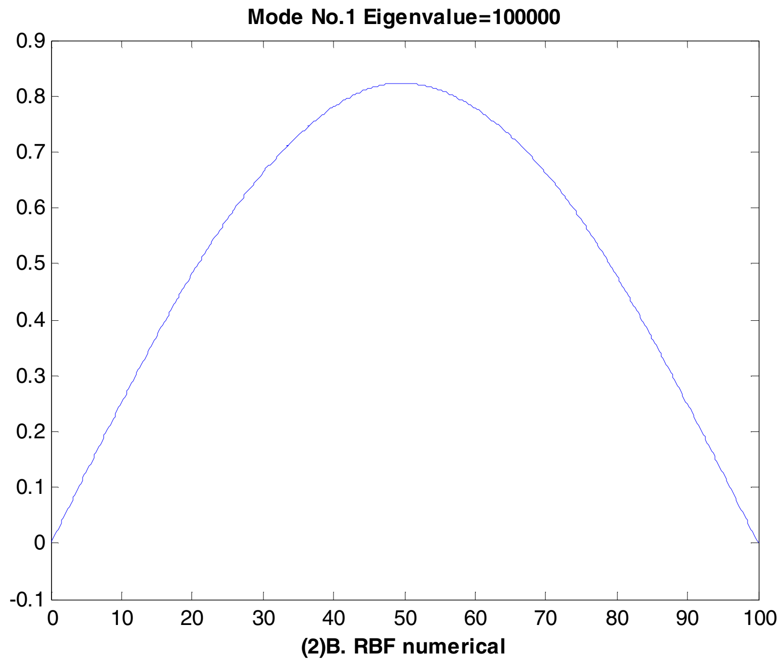

Figure 5.1 First mode shape and eigenvalue for buckling in fixed-fixed condition

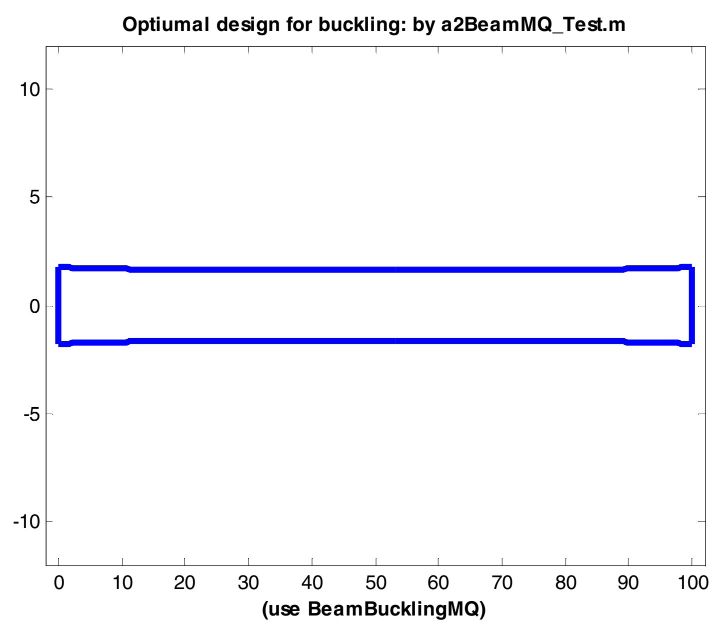

Figure 5.2 Optimal beam design for buckling in fixed-fixed condition

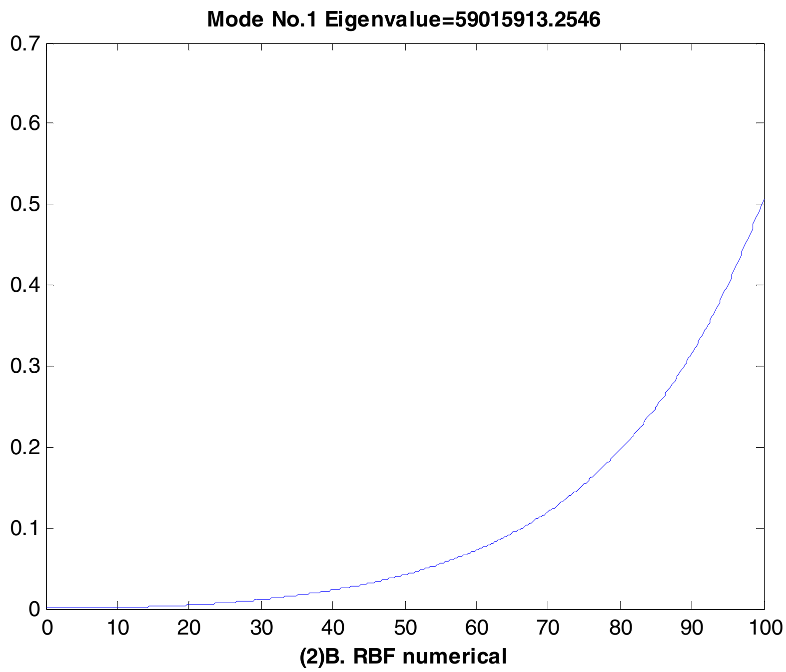

Figure 5.3 First mode shape and eigenvalue for buckling in fixed-free condition

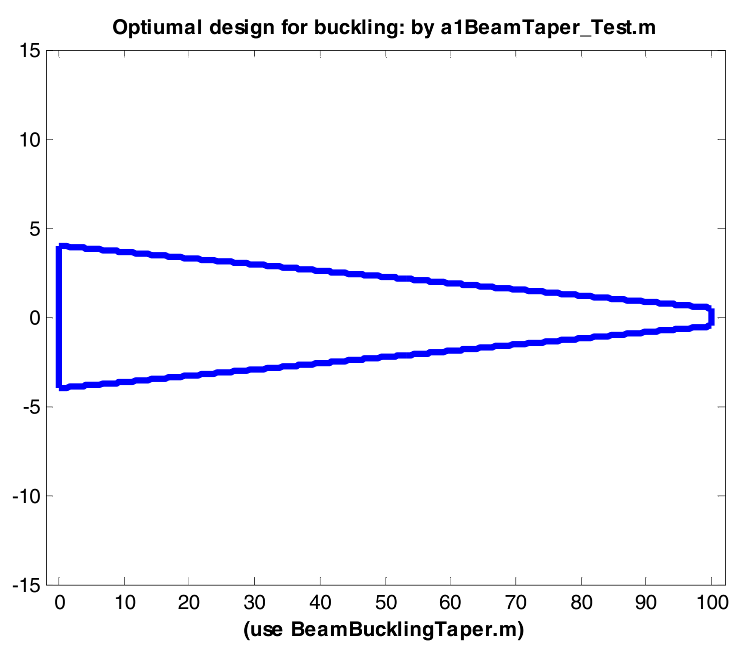

Figure 5.4 Optimal beam design for buckling in fixed-free condition

In order to avoid beam buckling, the critical load should be large, so that the beam will not have the buckling problem in the first critical load on the beam. Also, we set the boundary limit in the design variable because we will have infinite optimal results without limiting the range.

5.2 Beam Vibration

This section discusses the application of using the Radial Basis Function in the optimal design of the beam vibration problem. In the engineering design, the beam should be light-weighted and no vibration at the first natural frequency within the design range. The following is the minimum weight design in the beam vibration problem:

$$\text{Find } x_i \text{ to minimize weight} = \int_0^L A_i \rho dx \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (88)$$

$$\text{Such that } \begin{array}{l} \omega_1 \geq \sqrt{\lambda_{\max}} \\ x_{UB} \leq x_i \leq x_{UP} \end{array}$$

where ω_1 is the first natural frequency. The procedures used to solve this problem are the same as the buckling optimal design problem.. Figure 5.5 and figure 5.6 show the first mode shape and eigenvalue in the uniform square cross-section and the optimal beam design in the regular square. As a fixed-free example, consider a beam with the following data:

- Density of the material: 0.1 lbf
- Length of the beam: 100 feet
- Young's Modulus: 10000000 psi

- Maximum requirement of the first eigenvalue: 10000
- Design limit of the width: $1 \le width_i \le 10,\ i = 1,2$
- Initial gauss of the design width: 3 feet and 2 feet on both ends.
- Optimal design width: 6.04 feet and 6.07 feet in the uniform type beam

The original Matlab codes are listed in Appendix G. Figure 5.5 and Figure 5.6 show the first mode shape and eigenvalue in the uniform square cross-section and the optimal beam design in the regular square. Figure 5.7 and Figure 5.8 present the first mode shape and eigenvalue in the taper linear cross-section and the optimal beam design in the taper linear type beam. The original Matlab codes are listed in Appendix G.

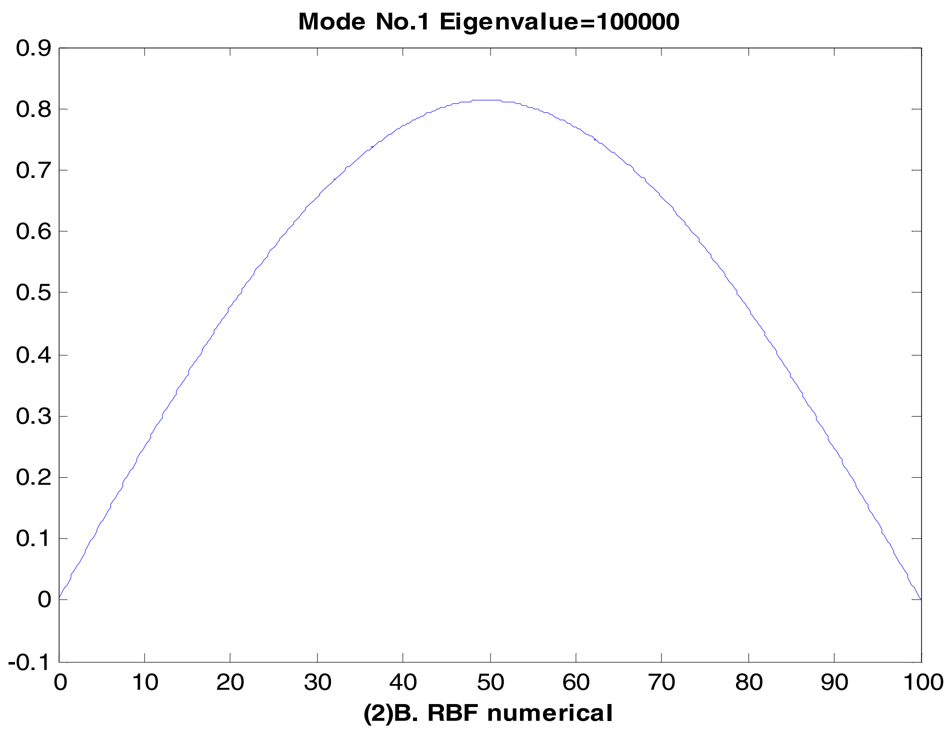

Figure 5.5 First mode shape and eigenvalue for vibration in fixed-fixed condition

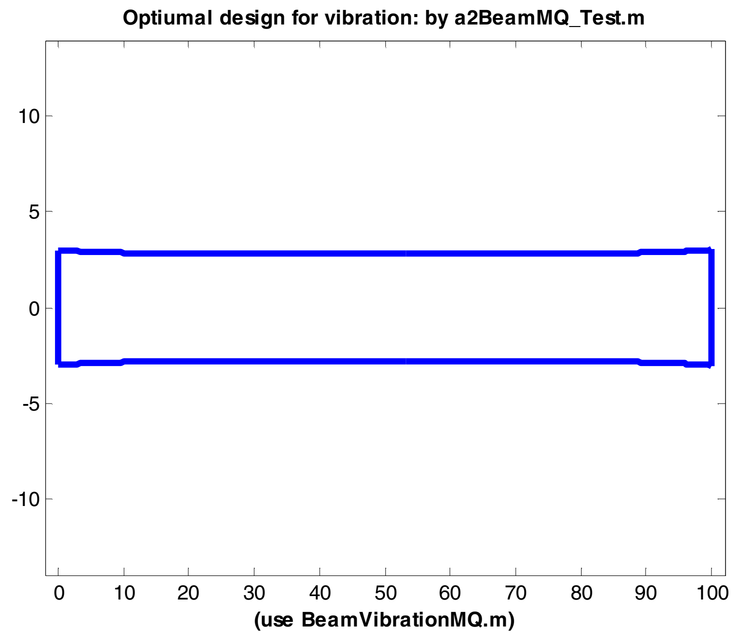

Figure 5.6 Optimal beam design for vibration in fixed-fixed condition

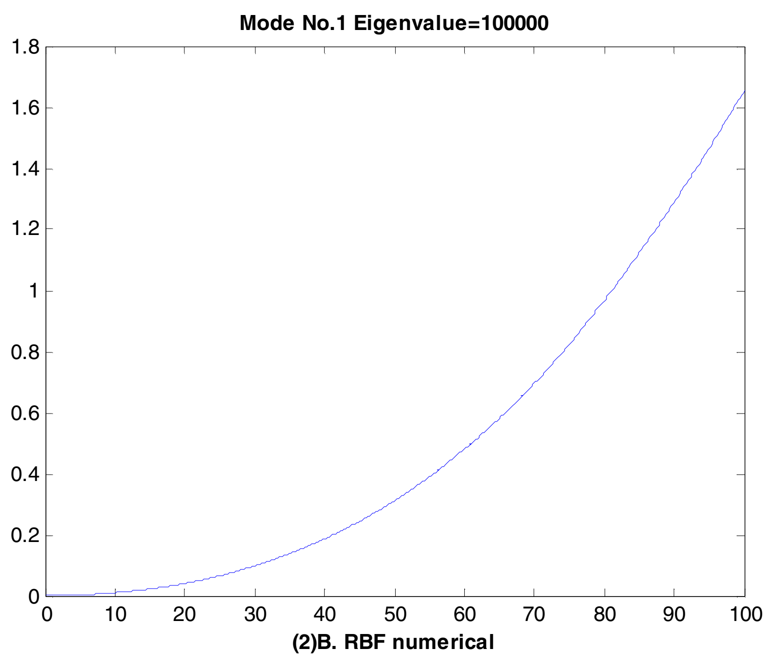

Figure 5.7 First mode shape and eigenvalue for vibration in fixed-free condition

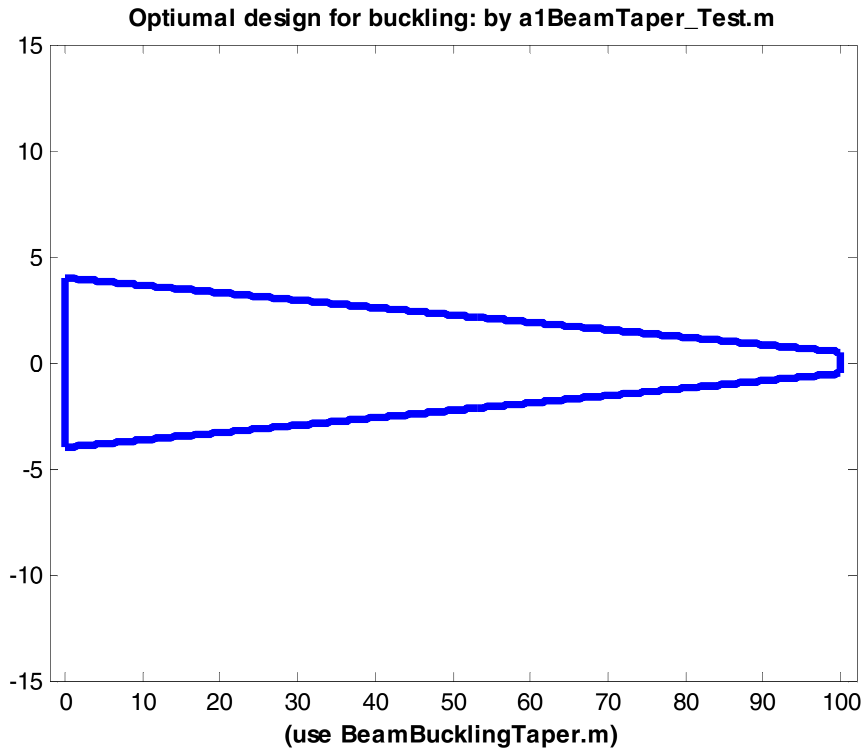

Figure 5.8 Optimal beam design for vibration in fixed-free condition

CHAPER 6

PARAMETRIC STUDY

6.1 Introduction

In previous chapters, it has been noted that the parameter h is the MQ basis function has a great effect on the solution. Recall the 1-D MQ function with center at x_i:

$$\phi(x) = \sqrt{(x - x_i)^2 + h} \text{ …………………….. (89)}$$

Note that at $x = x_i$

$$\phi(x_i) = \sqrt{h} \text{ ………………… (90)}$$

$$\phi'(x_i) = 0 \text{ ……………….... (91)}$$

$$\phi''(x_i) = \frac{1}{\sqrt{h}} \text{ ……………….. (92)}$$

Because of Equation (92), h is referred to as the shift parameter of the MQ function, since $\sqrt{h}$ is the value (shift) of the function from zero. On the other hand, Equation (92) says $\frac{1}{\sqrt{h}}$ is the rate of change of the slope at x_i, hence large h yields small change of slope. Thus, h is also called the smooth parameter of the MQ basis function in the literature. In this research, we will call h the shift parameter.

The effect of the shift parameter and its selection will be discussed in this chapter. We will perform the study use the vibration analysis of a uniform simply supported beam whose natural frequencies have the following well known solutions.

$$\omega_n^2 = (n\pi)^2 \sqrt{\frac{EI}{\rho L^4}} \text{ ………. (93)}$$

Where n=mode number

E=Young's modulus

I=cross-section area moment of inertia

ρ =mode density

L=beam length

For simplicity, unless otherwise specified, we use E=1, I=1, ρ =1, L=1 in our study.

Before we study the effect of shift parameter, we will first review the problem associated with beam vibration using polynomial basis function.

6.2 Vibration Analysis by Polynomial Ritz Methods

6.2.1. Limitation: polynomial Order Problem

The beam vibration analysis using polynomial basis is a special care of our generation function in chapter 4, that is, in Equation (85) we use polynomial and beam only. In this case, it is well known that we can only get solutions for polynomial when up to order 12 or so. The following Figure 6.1 shows that using polynomial order 11, we can get about 6 accurate natural frequencies and the solution deteriates quickly. For polynomial of order 12 or above, the solution fails to yield valid solutions.

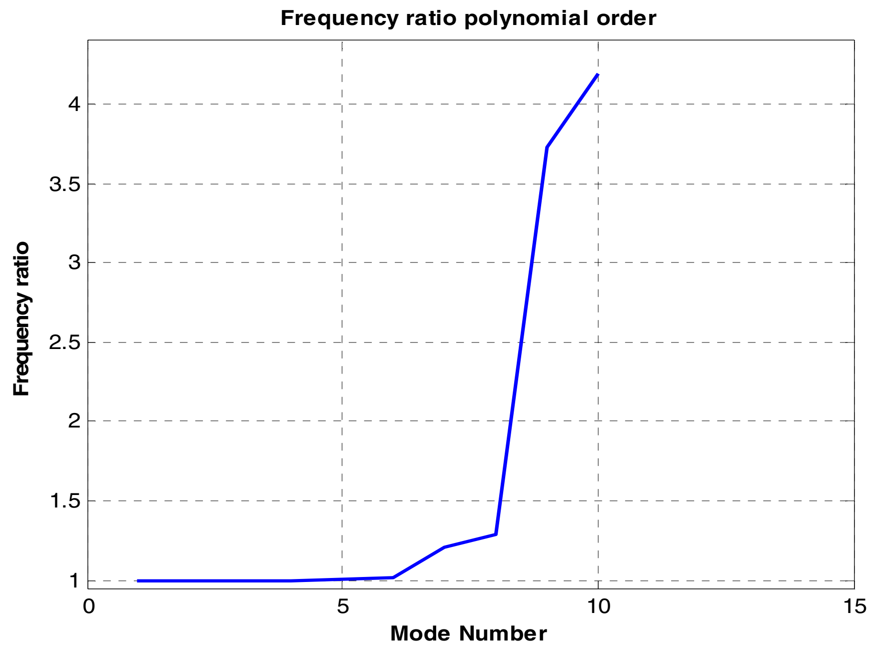

Figure 6.1 Frequency Ratio Polynomial Order

6.2.2. Reason: Condition Number and Basis Function Plots

The reason for numerical difficulty for polynomial basis function is that the higher terms becomes similar to each other and consequently they lose their linear independence, see Figure 6.2. Computationally, this loss of linear independence leads to high conditioned number for the mass and stiffness matrices. Consequently, the eigenvalues problem can not be solved accurately.

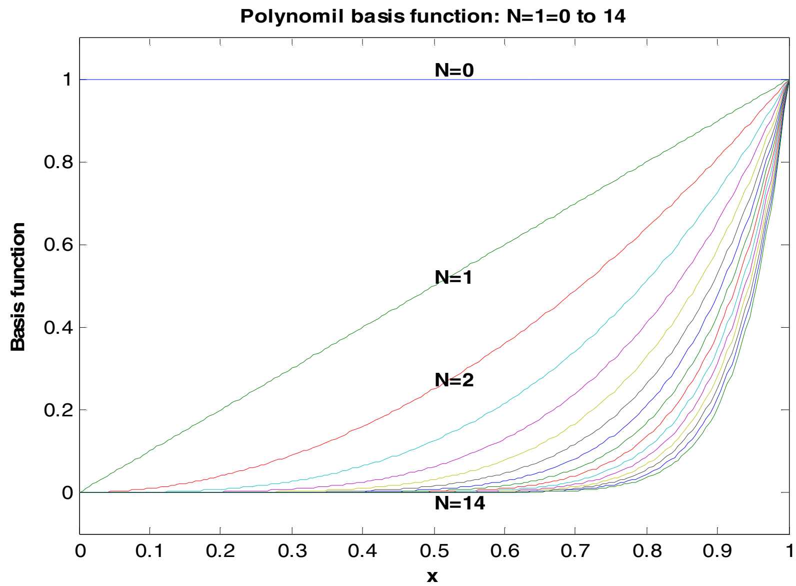

Figure 6.2 Polynomial Basis Function

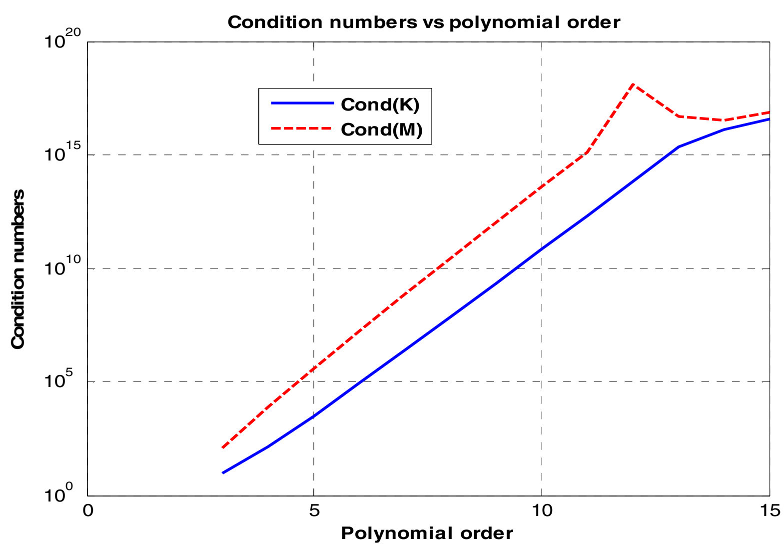

Figure 6.3 Condition Numbers vs. Polynomial Order

6.3 Vibration Analysis by RBF Methods

To study the effect of shift parameter on the accuracy of the natural frequency of a simple-support beam, the vibration problem is solved with basis consists of the constant term, x and 14 MQ basis with centers uniformly distributed between 0 and L for various value of h. Note that the frequency ratio is defined as

$$R = \frac{\overline{\omega}_n}{\omega_n} \quad \text{.................. (94)}$$

Where $\overline{\omega}_n$ is the calculated natural frequency and ω_n is the exact nth natural frequency given by Equation (94). The results are shown in Figure 6.4.

Note that R=1 indicated exact solution. Thus, it can be seen that for a wide range of choice of h, there are 8 computed mode with less than 0.5% error. If larger value of h is used, however, numerical difficulty will appear. This is evident by the complex solution for the eigenvalue problem, which theoretically have only real solutions.

In Figure 6.9, we plot one MQ basis for various values of h. As h increases, the basis function becomes flatter and the differences between various basis are getting smaller. The consequence is that the condition number for the stiffness and mass matrices becomes large see Figure 6.10. This leads to numerical difficulty in solving the eigenvalue problem.

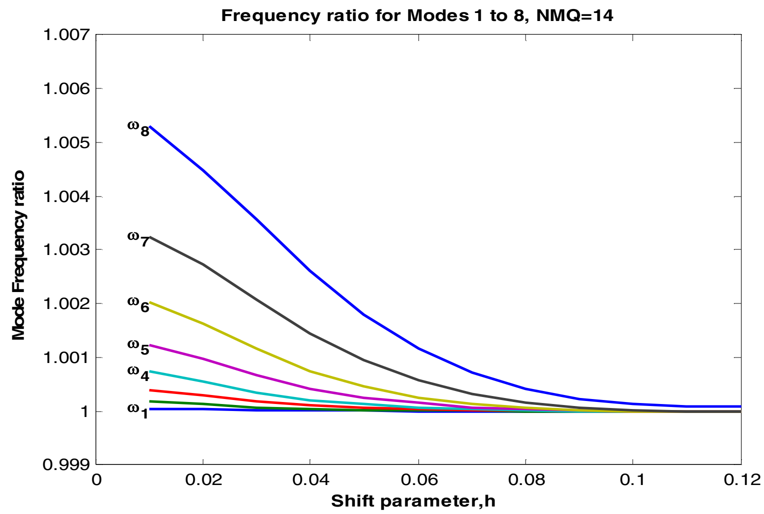

Figure 6.4 Frequency Ratios for Modes 1 to 8

6.3.1. MQ Basis Function: Effect of Shift Parameters

To study the effect of shift parameters, the 14 MQ basis functions for h=0.025, 0.05, 0.1, and 0.15 are plotted in Figures 6.5 to 6.8. Note that an h becomes larger, the functions become flatter. This leads to large condition number for the mass and stiffness matrices, see Figure 6.10.

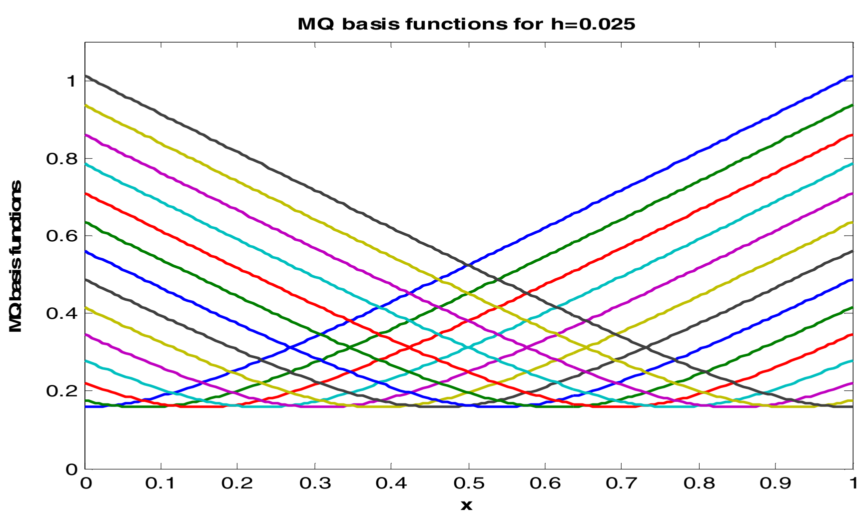

Figure 6.5 MQ Basis Function for h=0.025

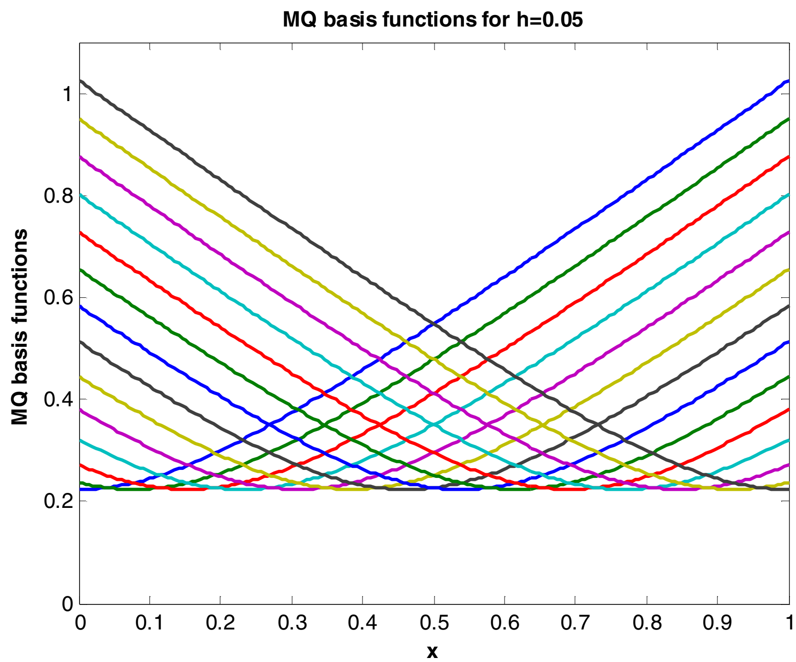

Figure 6.6 MQ Basis Function for h=0.05

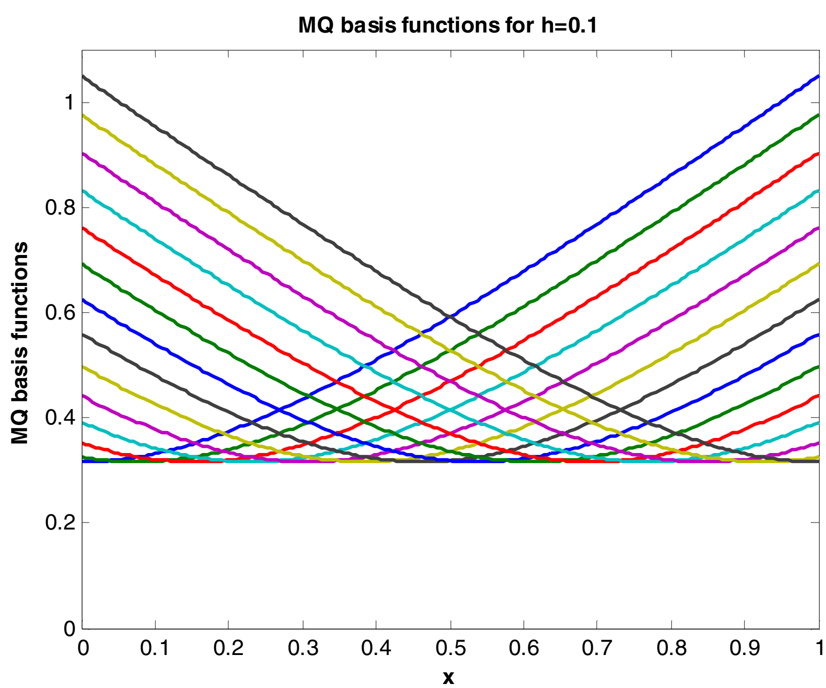

Figure 6.7 MQ Basis Function for h=0.1

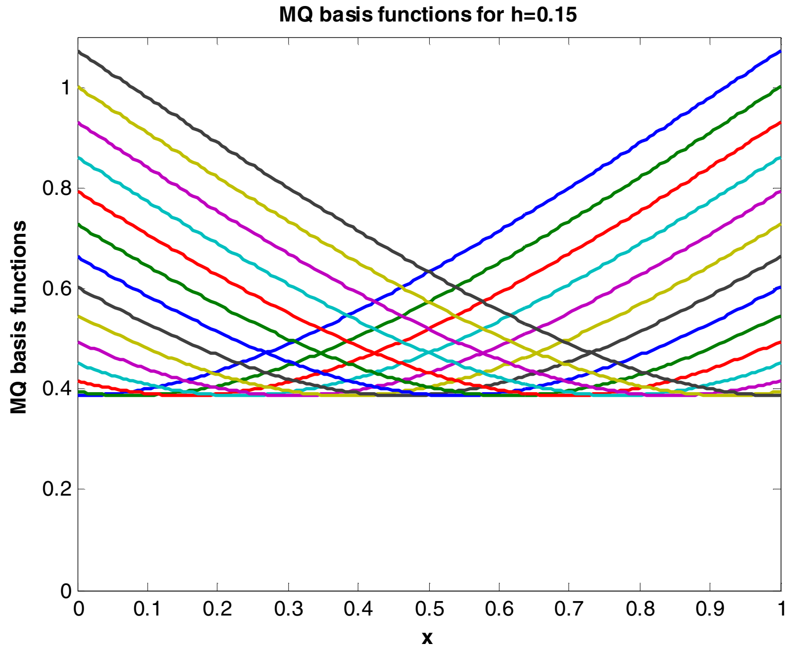

Figure 6.8 MQ Basis Function for h=0.15

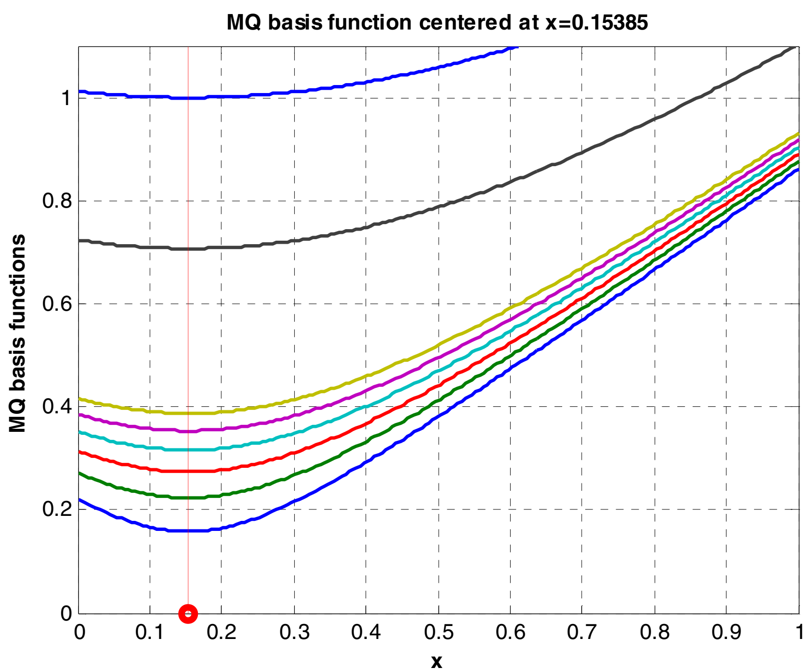

Figure 6.9 MQ Basis Function Centered at x=0.15385

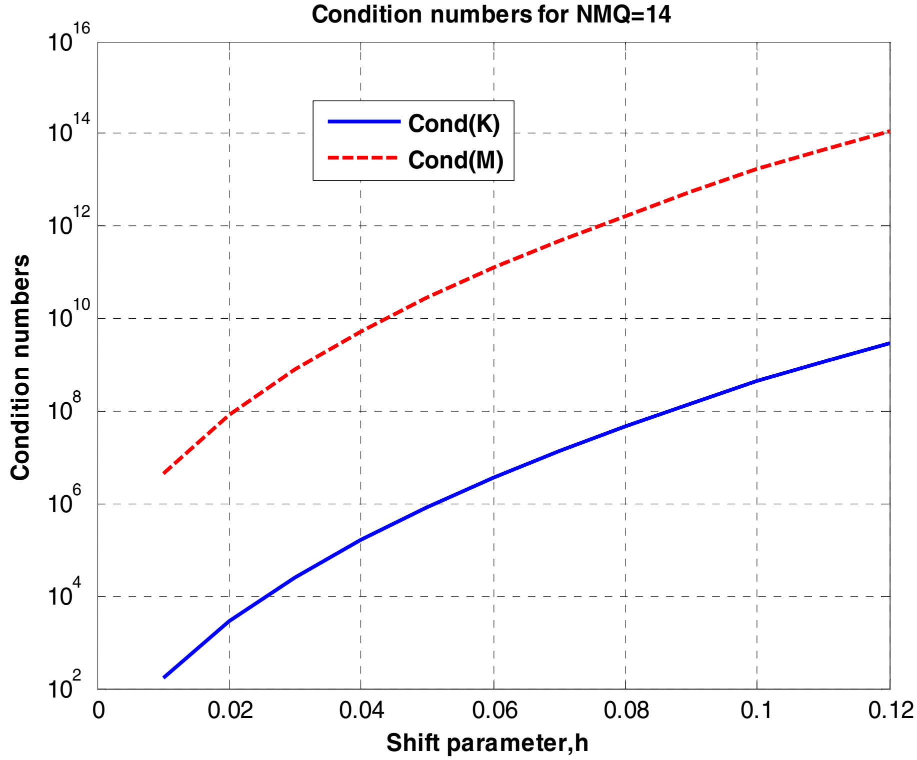

Figure 6.10 Condition Numbers for NMQ=14

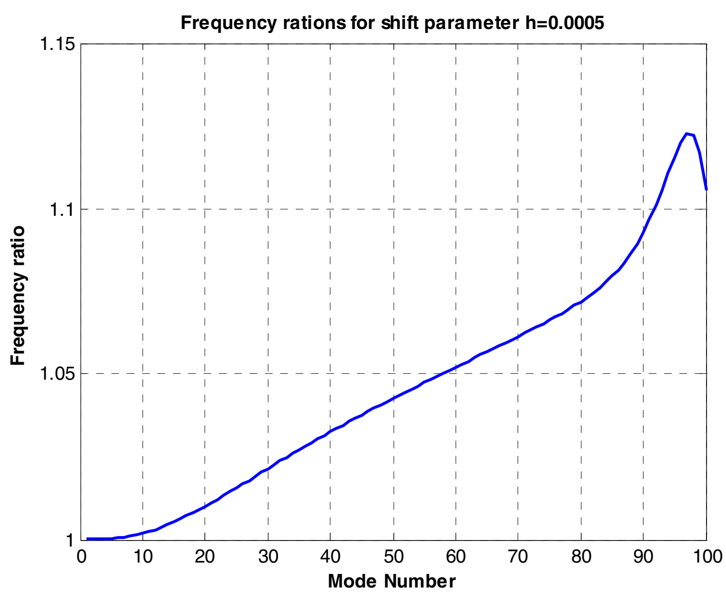

Figure 6.11 Frequency Rations for Shift Parameter h=0.0005

6.4 Recommendation of Choosing Shift Parameter for Vibration Analysis by RBF Method

Based on the numerical experiments performed during this research, the following value of shift parameters is recommended for beam vibration analysis using MQ basis function. The shift parameter should be chosen in the following range:

$$h_L \le h \le h_U \;\ldots\ldots\ldots\ldots\ldots\ldots \text{(95)}$$

$$\text{where}\quad \begin{aligned} h_L &= \frac{\bar{h}}{10} \\ h_U &= 10\bar{h} \\ \bar{h} &= \frac{L}{(NMQ)^2} \end{aligned}$$

Where NMQ is the number of MQ basis used in the analysis, L is the beam length.

To illustrate the choice of shift parameter, the beam is modeled using 100 MQ basis functions. For this model,

$$\begin{aligned} \bar{h} &= \frac{1}{(100)^2} = 0.0001 \\ h_L &= 0.00001 \\ h_U &= 0.001 \end{aligned}$$

The result for using shift parameter h=0.0005 is shown in Figure 6.12. It should be noted that all the 100 natural frequencies computed by this model has less than 13% errors. This is a great achievement when compare with polynomial model that can only provide about 8 orders with less than 13% even (see Figure 6.1).

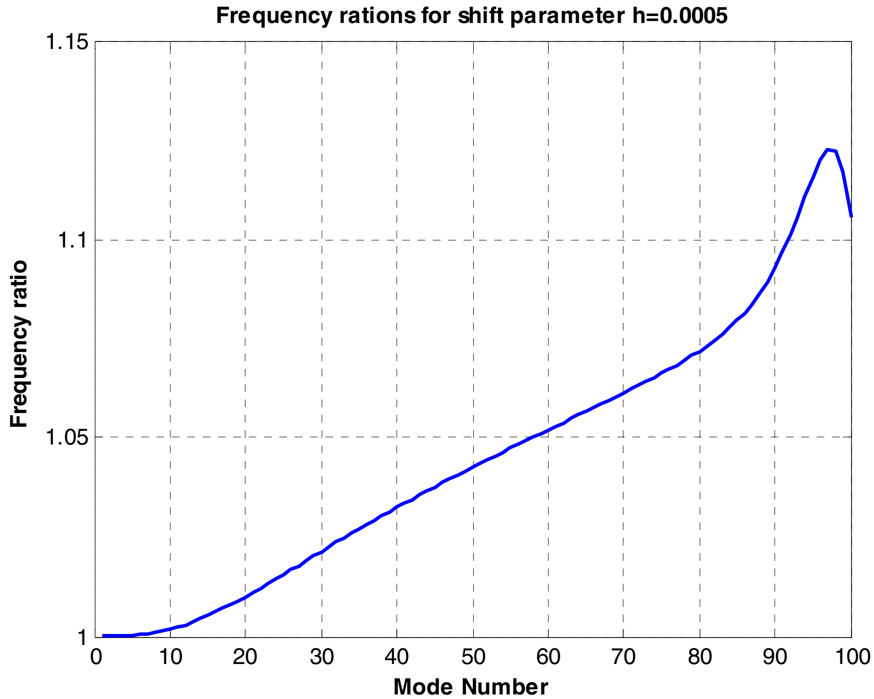

Figure 6.12 Frequency Rations for Shift Parameter h=0.0005

For static analysis using Least Square Collocation method, the shift parameter h should be written the following range:

$$h_{LS} \leq h \leq h_{US} \ldots\ldots\ldots\ldots\ldots\ldots \text{(96)}$$

$$h_{LS} = \frac{h_o}{10}$$

where $h_{Us} = 10h_o$

$$h_o = \frac{L}{NMQ}$$

The above criterion is formed by numerical experiences.

CHAPTER 7

CONCLUSION AND FUTURE WORK

7.1 Conclusion

In this dissertation, MQ RBF have been used to solve beam static response, buckling and vibration problems. The results show that accurate solution can be obtained for both strong (differential equation) and weak (energy method) formulations.

The main contribution of this dissertation are:

1. Least Square Collocation (LSC) for beam static analysis using strong formulation. The results show that LSC can provide better results than classical collocation method using same number of collocation points.
2. Beam buckling and vibration analysis using Rayleigh-Ritz method based on Radial Basis Function. A criterion for choosing the shift parameter for MQ function is also presented.
3. Application of RBF for beam cross-sectional shape parameterization and design optimization.

The advantages of using Radial Basis Function in structural analysis of beam is the direct results of the ability of RBF function to represent an arbitrary function accurately.

Other contributions of this dissertation include:

1. The demonstration that in vibration analysis, RBF based Rayleigh-Ritz method can be used to compute accurate natural frequencies for up to several hundred modes while polynomial based method can only capture the first few orders.
2. A discussion of the limitation of polynomial-based Rayleigh-Ritz method for beam vibration analysis.

7.2 Future Work

The following are suggested future work in using Radial Basis Function in the structural analysis and design optimization:

1. Extend the RBF least square collocation method to 2-D plate static analysis problem.
2. Extend the RBF Rayleigh-Ritz method for plate vibration analysis and develop a similar producer for selecting the shift parameter in the RBF basis function.
3. Extend the Radial Basis Function applying Least Square Collocation method and design optimization in the 2-D and 3-D structural components under static loading.
4. Extend the RBF Rayleigh-Ritz method for optimal design of plates under vibration and buckling constraints.
5. The future research should investigate the Radial Basis Function in the sensitivity in the bar, beam, shell and plate components. The sensitivity analysis is the new field that the Radial Basis Function can apply in the engineering application. This method will help engineers develop efficient numerical method for engineering analysis. Also, this can be applied with other numerical

techniques, such as the design optimization and static analysis in the structural components.

APPENDIX A

MATLAB PROGRAM CODE FOR 1-D BAR EXAMPLE

A1.OVREVIEW

This program presents the example of using the Radial Basis Function with the Collocation method and the Least Square Collocation method, and compares numerical results in the plot. This program solves the following problem:

$$u'' - 5u = -1,\ B.C.: u(0) = 0,\ u'(1) = 0$$

The red line represents the classical collocation method, the green line represents the least square collocation method, and the blue line shows the exact solution.

A2. MATLAB PROGRAM CODE

```
%Bar Problem with Collocation Method and Least Square Method
k=5; p=1;n=1;N=5; Le=1; Segment=200;
xp=linspace(0.00001,Le,Segment)';
%Set boundary conditions
%u=[f]{c}
%u(0)=[f(0)]{c}, u'(1)=0
x11=0;
xc=(1:N-1)*(Le/N);
[f,df]=RBF1D(x11,xc,n,1);
ABC(1,:)=f;
x2=Le;
[f1,df1]=RBF1D(x2,xc,n,1);
ABC(2,:)=df1;
bBC=[0;0];
%For EQ with Collocation Method
xcp1=xc;
np1=length(xcp1);
for i=1:np1
   x121=xcp1(i);
   %F=u''+ku
   for j=1:length(x121)
   [f2(j,:),df2(j,:),df22(j,:),df23(j,:),df24(j,:)]=RBF1D(x121,xc,n,1);
end
F1=df22+k*f2;
   AEQ1(i,:)=F1;
   bEQ1(i,1)=p;
end
```

```
A1=[ABC;AEQ1];
b1=[bBC;bEQ1];
c2=A1\b1;
%For EQ with Least Square Method
xcp=xp;
np=length(xcp);
for i=1:np
   x12=xcp(i);
   %F=u''+ku
   for j=1:length(x12)
   [f(j,:),df(j,:),df2(j,:),df3(j,:),df4(j,:)]=RBF1D(x12,xc,n,1);
end
F=df2+k*f;
   AEQ(i,:)=F;
   bEQ(i,1)=p;
end
ZZ=zeros(n+1,n+1);
A=[AEQ'*AEQ ABC';ABC ZZ];
b=[AEQ'*bEQ;bBC];
c=A\b;
c1=c(1:(N+n));
%Plotting
x13=xp;
for i=1:length(xp)
   xx=xp(i,:);
[f(i,:)]=RBF1D(xx,xc,n,1);
end
usolccm=f*c2;
usollsm=f*c1;
x=xp;
uexact=1/5-1/5.*tan(5.^(1/2)).*sin(5^(1/2).*x)-1/5.*cos(5^(1/2).*x);
plot(x,uexact, ’LineWidth’,3)
hold on
plot(x,usolccm,’LineWidth’,3,’r’)
hold on
plot(x13,usollsm, ’LineWidth’,3,’g’)
title('\bf          Compared Results With Exact and Numerical Solutions @ h=1.0')
xlabel('\bf X value')
ylabel('\bf Y value')
legend('uexact','usolccm','usollsm')
```

APPENDIX B

MATLAB PROGRAM CODE FOR 1-D BEAM EXAMPLE

B1.OVREVIEW

This program presents the example of using the Radial Basis Function with the Collocation method and the Least Square Collocation method, and compares numerical results in the plot. This program solves the following problem:

$$2.5u'''' + 2u = 1,\ B.C.: u(0) = 0,\ u'(0) = 0,\ u''(20) = 0,\ u'''(20) = 0$$

The red line represents the least square collocation method, the green line represents the classical collocation method, and the blue line shows the exact solution.

B2. MATLAB PROGRAM CODE

```
%Generalize Program
EI=2.5; w0=1; k=2; Le=20; N=25;
xp=linspace(0,20,200)';
%Set boundary conditions
%u=[f]{c}
%u(0)=[f(0)]{c}, u'(1)=0
x11=0;
xc=(1:N-1)*(Le/N);
[f1,df1,df12,df13,df14]=RBF1D(x11,xc,1);
ABC(1,:)=f1;
ABC(2,:)=df1;
x2=Le;
[f,df,df2,df3,df4]=RBF1D(x2,xc,1);
ABC(3,:)=df2;
ABC(4,:)=df3;
bBC=[0;0;0;0];
%For EQ
%Define collocation points (Collocation Method)
xcp=xc;
np=length(xcp);
for i=1:np
   x12=xcp(i);
   %F=EI*df2+k*f
   for j=1:length(x12)
   [f(j,:),df(j,:),df2(j,:),df3(j,:),df4(j,:)]=RBF1D(x12,xc,1);
end
   F=EI*df4+k*f;
```

```
    AEQ(i,:)=F;
    bEQ(i,1)=w0;
end
A=[ABC;AEQ];
b=[bBC;bEQ];
c=A\b;
%Least Square method
xcp1=xp;
np=length(xcp1);
for i=1:np
    x3=xcp1(i);
    %F=EI*df2+k*f
    for j=1:length(x3)
    [f2(j,:),df2(j,:),df22(j,:),df23(j,:),df24(j,:)]=RBF1D(x3,xc,1);
end
    F1=EI*df24+k*f2;
    AEQ1(i,:)=F1;
    bEQ1(i,1)=w0;
end
ZZ=zeros(4,4);
AA=[AEQ1'*AEQ1 ABC';ABC ZZ];
bb=[AEQ1'*bEQ1;bBC];
cc=AA\bb;
c1=cc(1:N+3);
%Plotting
x13=xp;
for i=1:length(xp)
    xx=xp(i,:);
[f(i,:)]=RBF1D(xx,xc,1);
end
usolccm=f*c;
usollsm=f*c1;
u=dsolve('2.5*D4u+2*u=1','u(0)=0','Du(0)=0','D2u(20)=0','D3u(20)=0','x');
t=linspace(0,20,200)';
un=subs(u,'x',t);
uexact=real(un);
x=linspace(0,20,200)';
plot(x,uexact,'LineWidth',3)
hold on
plot(x13,usollsm,'LineWidth',3,'Color','r')
hold on
plot(x,usolccm,'LineWidth',3,'Color','g')
title('\bf          Compared Results With Exact and Numerical Solutions @ h=1.0')
xlabel('\bf X value')
```

```
ylabel('\bf Y value')
legend('uexact','usollsm','usolccm')
```

APPENDIX C

MATLAB PROGRAM CODE FOR 1-D HIGHER ORDER SYSTEM

C1.OVREVIEW

This program presents the example of using the Radial Basis Function with the Collocation method, and compares numerical result with the exact solution in the plot.

This program solves the following problem:

$$x^4 y'''' - 4x^3 y''' + x^2(12 - x^2) y'' + 2x(x^2 - 12) y' + 2(12 - x^2) y = 2x^5, \quad x(0)=0, \quad x'(0)=0, \quad x(11)=0$$
$$x'(11)=0$$

The red line represents exact solution, and the blue line represents the classical collocation method.

C2. MATLAB PROGRAM CODE

```
%Example Problem 1 @ N. Mai-Duy's Paper
Le=11; N=55; Segment=200; h=1.0;
xp=linspace(0,Le,Segment)';
%Set boundary conditions
%u=[f]{c}
%u(0)=[f(0)]{c}, u'(1)=0
x11=0;
xc=(1:N-1)*(Le/N);
% h=Le/9;
[f1,df1,df12,df13,df14]=RBF1D(x11,xc,h);
ABC(1,:)=f1;
ABC(2,:)=df1;
x2=Le;
[f,df,df2,df3,df4]=RBF1D(x2,xc,h);
ABC(3,:)=f;
ABC(4,:)=df;
bBC=[0;0;0;0];
%For EQ
%Define collocation points (Collocation Method)
xcp=xc;
np=length(xcp);
for i=1:np
   x12=xcp(i);
```

```
   %F=x^4y''''-4x^3y'''+x^2(12-x^2)y''+2x(x^2-12)y'+2(12-x^2)y
   for j=1:length(x12)
   [f(j,:),df(j,:),df2(j,:),df3(j,:),df4(j,:)]=RBF1D(x12,xc,h);
  end
   F=x12.^4*df4-4.*x12.^3*df3+x12.^2.*(12-x12.^2)*df2+2.*x12.*(x12.^2-
12)*df+2.*(12-x12.^2)*f;
   AEQ(i,:)=F;
   bEQ(i,1)=2.*x12.^5;
end
A=[ABC;AEQ];
b=[bBC;bEQ];
c=A\b;
%Plotting
x13=xp;
for i=1:length(xp)
   xx=xp(i,:);
[f(i,:)]=RBF1D(xx,xc,h);
end
usolccm=f*c;
usollsm=f*c1;
x=linspace(0,Le,Segment)';
uexact=-x.^3-Le.^2./(1-exp(Le)+Le.*exp(Le)).*x+(2.*Le.*exp(Le)-
2.*Le.*exp(Le).^2+Le.^2.*exp(Le).^2)./(1-
exp(Le)+Le.*exp(Le))./exp(Le).*x.^2+Le.^2./(1-exp(Le)+Le.*exp(Le)).*x.*exp(x);
plot(x,uexact,'r','LineWidth',3)
hold on
plot(x,usolccm,'b','linewidth',3)
title('\bf          Compared Results With Exact and Numerical Solutions @ h=1.0')
xlabel('\bf X value')
ylabel('\bf Y value')
legend('uexact','usolccm')
```

APPENDIX D

MATLAB PROGRAM CODE FOR 1-D BEAM BUCKLING

D1.OVREVIEW

This program presents the example of using the Radial Basis Function to solve beam buckling problems with different conditions, and plot first two mode shapes and critical loads. The requirements of the beam are: A=10, E=1,000,000, ρ=0.5, L=1, h=0.1.

D2. MATLAB PROGRAM CODE

1. **Fixed-Fixed Condition**

```
syms x
E=1; I=1; P=1; nn=100; xc=[1:10]/11; N=4; L=1; h=0.2; L=1; NM=2;
F=[sqrt((x-xc).^2+h) x.^(0:N)];
dF=diff(F,x);
dF2=diff(dF,x);
dF3=diff(dF2,x);
K=real(double(int(E*I*dF2.'*dF2,x,0,L)));
Kg=real(double(int(P*dF.'*dF,x,0,L)));
%Apply BC: Fixed-fixed beam
BC=[subs(F,x,0);subs(dF,x,0);subs(F,x,L);subs(dF,x,L)];
T=double(null(BC));
Kgs=T.'*Kg*T;
Ks=T.'*K*T;
%Compute enginvalues
[PP,EE]=eig(Ks,Kgs);
[Eg,ii]=sort(diag(EE));
PP=PP(:,ii);
%Compute eigenfunctions
Psic=T*PP;
%Plot modes
xn=linspace(0,L,nn)';
for i=1:length(xn)
   xx=xn(i,:);
   [Fn(i,:)]=RBF1DTESTN(N,xx,xc,h);
end
for i=1:NM
   Psix=Fn*Psic(:,i);
   a1=max(abs(Psix));
   Psix=Psix/a1;
```

```
    figure
    plot(xn,Psix,'linewidth',2)
    xlabel(['\bfx, N=',int2str(N)])
    ylabel('\bfEigenfunction')
    title(['\bfEigenfunction No.',int2str(i),',\bfeigenvalue=',num2str(Eg(i))])
       end
```

2. Fixed-Free Condition

```
syms x
E=1e6; I=1e-8;
E=1;I=1;
P=1; nn=100;
xc=[1:8]/9;
N=3; L=1; h=.2; L=1; NM=2;
F=[sqrt((x-xc).^2+h) x.^(0:N)];
dF=diff(F,x);
dF2=diff(dF,x);
dF3=diff(dF2,x);
K=real(double(int(E*I*dF2.'*dF2,x,0,L)));
Kg=real(double(int(P*dF.'*dF,x,0,L)));
%Apply BC: Fixed-free beam
BC=[subs(F,x,0);subs(dF,x,0);subs(dF2,x,L);subs(dF3,x,L)];
T=double(null(BC));
Kgs=T.'*Kg*T;
Ks=T.'*K*T;
Kgs=(Kgs+Kgs')/2;  % enforced symmetry
Ks=(Ks+Ks')/2;

%Compute enginvalues
[PP,EE]=eig(Ks,Kgs);
[Eg,ii]=sort(diag(EE));
PP=PP(:,ii);
Eg

%Compute eigenfunctions
Psic=T*PP;
%Plot modes
xn=linspace(0,L,nn)';
for i=1:length(xn)
    xx=xn(i,:);
    [Fn(i,:)]=RBF1DTESTN(N,xx,xc,h);
end
for i=1:NM
```

```
   Psix=Fn*Psic(:,i);
   a1=max(abs(Psix));
   Psix=Psix/a1;
   figure
   plot(xn,Psix,'linewidth',2)
   xlabel(['\bfx, N=',int2str(N)])
   ylabel('\bfEigenfunction')
   title(['\bfEigenfunction No.',int2str(i),',\bfeigenvalue=',num2str(Eg(i))])
      end
```

3. Fixed-Pin Condition

```
syms x
E=1; I=1; P=1; nn=100; xc=[1:10]/11; N=4; L=1; h=0.2; L=1; NM=2;
F=[sqrt((x-xc).^2+h) x.^(0:N)];
dF=diff(F,x);
dF2=diff(dF,x);
dF3=diff(dF2,x);
K=real(double(int(E*I*dF2.'*dF2,x,0,L)));
Kg=real(double(int(P*dF.'*dF,x,0,L)));
%Apply BC: Fixed-Pin beam
BC=[subs(F,x,0);subs(dF,x,0);subs(F,x,L);subs(dF2,x,L)];
T=double(null(BC));
Kgs=T.'*Kg*T;
Ks=T.'*K*T;
%Compute enginvalues
[PP,EE]=eig(Ks,Kgs);
[Eg,ii]=sort(diag(EE));
PP=PP(:,ii);
%Compute eigenfunctions
Psic=T*PP;
%Plot modes
xn=linspace(0,L,nn)';
for i=1:length(xn)
   xx=xn(i,:);
   [Fn(i,:)]=RBF1DTESTN(N,xx,xc,h);
end
for i=1:NM
   Psix=Fn*Psic(:,i);
   a1=max(abs(Psix));
   Psix=Psix/a1;
   figure
   plot(xn,Psix,'linewidth',2)
   xlabel(['\bfx, N=',int2str(N)])
```

```
    ylabel('\bfEigenfunction')
    title(['\bfEigenfunction No.',int2str(i),',\bfeigenvalue=',num2str(Eg(i))])
       end
```

4. Pin-Pin Condition

```
syms x
E=1; I=1; P=1; nn=100; xc=[1:10]/11; N=4; L=1; h=0.2; L=1; NM=2;
F=[sqrt((x-xc).^2+h) x.^(0:N)];
dF=diff(F,x);
dF2=diff(dF,x);
dF3=diff(dF2,x);
K=real(double(int(E*I*dF2.'*dF2,x,0,L)));
Kg=real(double(int(P*dF.'*dF,x,0,L)));
%Apply BC: Fixed-Pin beam
BC=[subs(F,x,0);subs(dF2,x,0);subs(F,x,L);subs(dF2,x,L)];
T=double(null(BC));
Kgs=T.'*Kg*T;
Ks=T.'*K*T;
%Compute enginvalues
[PP,EE]=eig(Ks,Kgs);
[Eg,ii]=sort(diag(EE));
PP=PP(:,ii);
%Compute eigenfunctions
Psic=T*PP;
%Plot modes
xn=linspace(0,L,nn)';
for i=1:length(xn)
    xx=xn(i,:);
    [Fn(i,:)]=RBF1DTESTN(N,xx,xc,h);
end
for i=1:NM
    Psix=Fn*Psic(:,i);
    a1=max(abs(Psix));
    Psix=Psix/a1;
    figure
    plot(xn,Psix,'linewidth',2)
    xlabel(['\bfx, N=',int2str(N)])
    ylabel('\bfEigenfunction')
    title(['\bfEigenfunction No.',int2str(i),',\bfeigenvalue=',num2str(Eg(i))])
       end
```

APPENDIX E

MATLAB PROGRAM CODE FOR 1-D BEAM VIBRATION

E1.OVREVIEW

This program presents the example of using the Radial Basis Function to solve beam vibration problems with different conditions, and plot first two mode shapes and critical loads. The requirements of the beam are: A=10, E=1,000,000, ρ=0.5, L=1, h=0.1.

.

E2. MATLAB PROGRAM CODE

1. Fixed-Free Condition

```
syms x
A=10; E=1e6; Rho=0.5; h=0.1; L=1; nn=100; xc=[1/5 2/5 3/5 4/5]; N=5; NM=2; I=10;
F=[sqrt((x-xc).^2+h) x.^(0:N)];
dF=diff(F,x);
dF2=diff(dF,x);
K=real(double(int(E*I*dF2.'*dF2,x,0,L)));
M=real(double(int(Rho*A*F.'*F,x,0,L)));
%Apply BC: Fixed-fixed beam
BC=[subs(F,x,0);subs(dF,x,0);subs(F,x,L);subs(dF,x,L)];
T=double(null(BC));
Ks=T.'*K*T;
Ms=T.'*M*T;
%Compute enginvalues
[PP,EE]=eig(Ks,Ms);
[Eg,ii]=sort(diag(EE));
PP=PP(:,ii);
%Compute eigenfunctions
Psic=T*PP;
%Plot modes
xn=linspace(0,L,nn)';
for i=1:length(xn)
   xx=xn(i,:);
   [Fn(i,:)]=RBF1DTESTN(N,xx,xc,h);
end
for i=1:NM
   Psix=Fn*Psic(:,i);
   a1=max(abs(Psix));
```

```
    Psix=Psix/a1;
    figure
    plot(xn,Psix,'linewidth',2)
    xlabel(['\bfx, N=',int2str(N)])
    ylabel('\bfEigenfunction')
    title(['\bfEigenfunction No.',int2str(i),',\omega_n=',num2str(sqrt(Eg(i)))])
        end
```

2. Fixed-Fixed Condition

```
syms x
A=10; E=1e6; Rho=0.5; h=0.1; L=1; nn=100; xc=[1/5 2/5 3/5 4/5]; N=5; NM=2; I=10;
F=[sqrt((x-xc).^2+h) x.^(0:N)];
dF=diff(F,x);
dF2=diff(dF,x);
K=real(double(int(E*I*dF2.'*dF2,x,0,L)));
M=real(double(int(Rho*A*F.'*F,x,0,L)));
%Apply BC: Fixed-free beam
BC=[subs(F,x,0);subs(dF,x,0)];
T=double(null(BC));
Ks=T.'*K*T;
Ms=T.'*M*T;
%Compute enginvalues
[PP,EE]=eig(Ks,Ms);
[Eg,ii]=sort(diag(EE));
PP=PP(:,ii);
%Compute eigenfunctions
Psic=T*PP;
%Plot modes
xn=linspace(0,L,nn)';
for i=1:length(xn)
    xx=xn(i,:);
    [Fn(i,:)]=RBF1DTESTN(N,xx,xc,h);
end
for i=1:NM
    Psix=Fn*Psic(:,i);
    a1=max(abs(Psix));
    Psix=Psix/a1;
    figure
    plot(xn,Psix,'linewidth',2)
    xlabel(['\bfx, N=',int2str(N)])
    ylabel('\bfEigenfunction')
    title(['\bfEigenfunction No.',int2str(i),',\omega_n=',num2str(sqrt(Eg(i)))])
        end
```

APPENDIX F

MATLAB PROGRAM CODE FOR 1-D DESIDN OPTIMIZATION FOR UNIFORM CROSS-SECTION TYPE

F1.OVREVIEW

This program presents the example of using the Radial Basis Function for beam design optimization in the beam buckling and vibration problems with the fixed-fixed condition, and plot first mode shape, critical load and optimal design shape. The requirements of the beam design are:

- Density of the material: 0.1 lbf
- Length of the beam: 100 feet
- Young's Modulus: 10000000 psi
- Maximum requirement of the first eigenvalue: 10000
- Design limit of the width: $1 \le width_i \le 10, i = 1,2$
- Initial gauss of the design width: 3 feet and 2 feet on both ends.
- Optimal design width: 6.04 feet and 6.07 feet

F2. MATLAB PROGRAM CODE

```
%Set Up Initial Data
NPoly=1;
NMQ= 5; % number of of RBF;
h =200; % MQ shift parameter
E = 1e7; % Young's modulus
Rho=.1/386.4; % mass density
L = 100; % beam length
Xt= [0]; %location with translational dof fixed
Xr= [0]; % location with rotational dof fixed
Xt= [0 L], Xr= [];
NPlot= 501; % number of points for plotting;
%SecProp=@Sec1;
SecProp=@Sec1MQ;
LB= [1 1]';
UB=[10 10 ]';
EigD=1e5;
%  Part A. Analysis runs of beams
```

```
DES= [3 2];
% Fixed-Fixed beam
NInt=100;
NInt=60
XC=linspace (0, L, length (LB));
DES=UB;
%Buckling optimal design
SENOPT=1; X0=UB;
OUTb=BeamBucklingMQ('Optimal
design',NPoly,NMQ,h,E,Rho,L,Xt,Xr,SecProp,NPlot,EigD,LB,UB,SENOPT,NInt,X0)
DES=OUTb.X
OUTbP=BeamBucklingMQ('PlotModes',DES,NPoly,NMQ,h,E,Rho,L,Xt,Xr,SecProp,N
Plot,NInt,1,XC)
%Plot solution
OUTbP=BeamBucklingMQ('Section Plot',OUTb.X,L)
title('\bfOptiumal design for buckling: by a2BeamMQ\_Test.m')
xlabel('\bf(use BeamBucklingMQ)')
%Vibration optimal design
OUTv=BeamVibrationMQ('Optimal
design',NPoly,NMQ,h,E,Rho,L,Xt,Xr,SecProp,NPlot,EigD,LB,UB,SENOPT,NInt,X0)
DES=OUTv.X
OUTbV=BeamVibrationMQ('PlotModes',DES,NPoly,NMQ,h,E,Rho,L,Xt,Xr,SecProp,
NPlot,NInt,1,XC)
%Plot solution
OUTbP=BeamVibrationMQ('Section Plot',OUTv.X,L)
title('\bfOptiumal design for vibration: by a2BeamMQ\_Test.m')
xlabel('\bf(use BeamVibrationMQ.m)')
```

APPENDIX G

MATLAB PROGRAM CODE FOR 1-D DESIDN OPTIMIZATION FOR TAPER CROSS-SECTION TYPE

G1.OVREVIEW

This program presents the example of using the Radial Basis Function for beam design optimization in the beam buckling and vibration problems with the fixed-free condition, and plot first mode shape, critical load and optimal design shape. The requirements of the beam design are:

- Density of the material: 0.1 lbf
- Length of the beam: 100 feet
- Young's Modulus: 10000000 psi
- Maximum requirement of the first eigenvalue: 10000
- Design limit of the width: $1 \le width_i \le 10,\ i = 1,2$
- Initial gauss of the design width: 3 feet and 2 feet on both ends.
- Optimal design width: 6.04 feet and 6.07 feet

G2. MATLAB PROGRAM CODE

```
% Set up the data
NPoly=1;
NMQ= 5; % number of RBF;
h =200; % MQ shift parameter
E = 1e7; % Young's modulus
Rho=.1/386.4; % mass density
L = 100; % beam length
Xt=[ 0 ]; %location with translational dof fixed
Xr= [0]; % location with rotational dof fixed
NPlot=  501; % number of points for plotting;
SecProp=@Sec1;
LB=[1 1];
UB=[10 10];
EigD=1e5;
%  Part A. Analysis runs of uniform beams
DES=[3 2];
%    Case A-1-CF  : clamped-free beam
```

```
NInt=100;
NInt=60
X0=UB;
SENOPT=1;
MODES=1;
%Vibration optimal design
OUTv=BeamVibrationTaper('Optimal
design',NPoly,NMQ,h,E,Rho,L,Xt,Xr,SecProp,NPlot,EigD,LB,UB,SENOPT,NInt,X0)
DES=OUTv.X;
XC=L;
OUTvV=BeamVibrationTaper('PlotModes',DES,NPoly,NMQ,h,E,Rho,L,Xt,Xr,SecPro
p,NPlot,NInt,1,XC)
%Plot solution
OUTvP=BeamVibrationTaper('Section Plot',OUTv.X,L)
title('\bfOptiumal design for vibration: by BeamTaper\_Test.m')
title('\bf(Use BeamVibrationTaper)')
%Buckling analysis optimal design
OUTb=BeamBucklingTaper('Optimal
design',NPoly,NMQ,h,E,Rho,L,Xt,Xr,SecProp,NPlot,EigD,LB,UB,SENOPT,NInt,X0)
DES=OUTv.X;
XC=L;
OUTbV=BeamBucklingTaper('PlotModes',DES,NPoly,NMQ,h,E,Rho,L,Xt,Xr,SecProp
,NPlot,NInt,1,XC)
%Plot solution
OUTbP=BeamBucklingTaper('Section Plot',OUTv.X,L)
title('\bfOptiumal design for buckling: by a1BeamTaper\_Test.m')
xlabel('\bf(use BeamBucklingTaper.m)')
% %------------  end of BeamTaper_Test.m
```

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BIOGRAPHICAL INFORMATION

Yung-Kang Sun received his B.S. in Mechanical Engineering at Chinese Culture University, Taipei, Taiwan in 1996, M.S. in Engineering Technology at West Texas A&M University, Canyon, Texas in 2001, and Ph.D. in Mechanical Engineering at the University of Texas at Arlington, Arlington, Texas in 2006. Since 2003, he has presented several technical papers in the fields of renewable energy and advanced space propulsion technology at AIAA conferences. In 2004, He passed the foundational engineering exam and registered as an Engineer-In-Training in Texas. His research interests are finite element analysis, the meshless Galerkin method in structural analysis, the matrix calculation, structural dynamic analysis, image processing, future propulsion technology, renewable energy, and computer aided engineering. He is a member at American Institute of Aeronautics and Astronautics (AIAA), American Society of Mechanical Engineers (ASME) and National Society of Professional Engineers (NSPE)