MEMS WIDEBAND ENERGY HARVESTING USING NONLINEAR SPRINGS AND MECHANICAL STOPPERS

by

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Abstract

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Vibrational energy harvesters convert kinetic energy from environment to useful electrical energy to power wireless sensor networks. These devices are finding applications in robotics, defense, medical, structural health monitoring, aerospace and wearables. The goal of this dissertation is to improve the design of vibrational electrostatic energy harvesters.

Wideband, out-of-plane gap-closing, electret-based electrostatic energy harvesters using nonlinear springs and mechanical stoppers are designed. A unique mechanism is developed to introduce nonlinearity in the force-displacement relationship by varying the spring anchor height relative to the level of the proof-mass in the vertical direction. Both spring softening and hardening responses are achieved. A mathematical model of the electrostatic energy harvester is derived and a linear energy harvester (with linear spring) is optimized. The system is then built using CoventorWare/MEMS+, where the linear springs are replaced by the novel nonlinear springs. Force-displacement curves are obtained by finite element analysis and fitted to obtain the nonlinear stiffness coefficients.

Mechanical stoppers are also designed and integrated with the hardening spring system to further enhance the operational bandwidth of the device. For comparison purposes, two different softening spring and one hardening spring systems, each with the same linear stiffness as the linear energy harvester are designed. The linear energy harvester obtained an acceleration-
normalized power density of 7.5 $\mu$Ws/cm$^3$-m$^2$ with full width at half maximum bandwidth of 11 Hz, whereas, a softening spring energy harvester produced a normalized power density of 6.6 $\mu$Ws/cm$^3$-m$^2$ and bandwidth of 82 Hz. Hardening system with mechanical stoppers produced the highest bandwidth of 231 Hz with a normalized power density of 8.9 $\mu$Ws/cm$^3$-m$^2$. Wideband energy harvesters produced significant improvement in bandwidth over the linear counterpart.

Moreover, arrays of four out-of-plane, gap-closing, electret-based electrostatic energy harvesters capable to harvest over a wide frequency range are designed. The novel design consists of energy harvesters with linear springs, softening springs, hardening springs and mechanical stoppers. This unique method of designing array with combination of linear and nonlinear generators allows the system to harvest significant amount of power even with shift in vibration frequency. A dynamic model of the system is derived and optimized using MATLAB. A maximum power of 110 nW is obtained over a frequency range of 600-1350 Hz. The design also takes 89 % less volume compared to an array of linear energy harvesters to generate over the same frequency range.
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<td>( V_{max} )</td>
</tr>
<tr>
<td>Pull-in voltage</td>
<td>( V_{PI} )</td>
</tr>
<tr>
<td>Voltage on capacitor at the start of cycle</td>
<td>( V_{start} )</td>
</tr>
<tr>
<td>Width of beam1</td>
<td>( W_1 )</td>
</tr>
<tr>
<td>Width of beam2</td>
<td>( W_2 )</td>
</tr>
<tr>
<td>Width of proof-mass</td>
<td>( W_{pm} )</td>
</tr>
<tr>
<td>Width of the spring</td>
<td>( W_s )</td>
</tr>
<tr>
<td>Distance at which center of mass is located</td>
<td>( x )</td>
</tr>
<tr>
<td>Input vibration signal</td>
<td>( y(t) )</td>
</tr>
<tr>
<td>Displacement of proof-mass relative to the reference frame</td>
<td>( z(t) )</td>
</tr>
<tr>
<td>Vibration amplitude at resonance</td>
<td>( Z_0 )</td>
</tr>
<tr>
<td>Magnitude of input displacement signal</td>
<td>(</td>
</tr>
<tr>
<td>Magnitude of displacement</td>
<td>(</td>
</tr>
<tr>
<td>Angle between two beams</td>
<td>( \alpha )</td>
</tr>
<tr>
<td>Electrostatic damping coefficient</td>
<td>( \zeta_e )</td>
</tr>
<tr>
<td>Mechanical damping coefficient</td>
<td>( \zeta_m )</td>
</tr>
<tr>
<td>Total damping coefficient</td>
<td>( \zeta_T )</td>
</tr>
<tr>
<td>Dimensionless load parameter of 1(^{st}) mode</td>
<td>( \lambda_1 )</td>
</tr>
<tr>
<td>Dimensionless load parameter of i-th mode</td>
<td>( \lambda_i )</td>
</tr>
<tr>
<td>Correction factor</td>
<td>( \beta(W_{pm} / L_{pm}) )</td>
</tr>
<tr>
<td>Function of nonlinear stiffness</td>
<td>( \kappa )</td>
</tr>
<tr>
<td>Permittivity of free space</td>
<td>( \varepsilon_0 )</td>
</tr>
<tr>
<td>Parameter</td>
<td>Symbol</td>
</tr>
<tr>
<td>------------------------------------------------</td>
<td>---------</td>
</tr>
<tr>
<td>Coefficient of viscosity</td>
<td>( \mu )</td>
</tr>
<tr>
<td>Angular frequency</td>
<td>( \omega )</td>
</tr>
<tr>
<td>Angular natural frequency</td>
<td>( \omega_n )</td>
</tr>
<tr>
<td>Amplitude dependent resonant frequency</td>
<td>( \omega_n' )</td>
</tr>
</tbody>
</table>
1 Introduction

1.1 Energy harvesting for self-powered wireless sensor networks (WSNs)

Energy harvesting or scavenging is the conversion of available environmental energy to useful electrical energy [1]. Energy scavenging usually refers to small scale (micro-to milli-watts) generation of energy contrary to big scale (megawatts) generation of renewable energy from sources such as solar, wind, pressure fluctuations, temperature gradient, tidal waves [1], [2], [3]. Vibration-to-energy converters produce electrical energy from kinetic energy available in the environment [4]. Vibrations are available in aircraft, ships, trains, automotive, bridges, power cables, buildings, machines and industrial environments among many other places [2], [5], [6].

Vibration energy harvesters are used in powering wireless sensor networks (WSNs). These work as an alternative to batteries as replacing and maintaining batteries is hard to impossible under such conditions [4], [5]. Therefore, the necessity to design and develop new vibration energy harvesters (VEHs) has never been of this high importance until now. The ability of VEHs to make low power WSNs self-sustaining brought these devices to limelight. Therefore, it is essential to improve the design of such energy harvesters which can generate energy from vibrations present in environment [2].

WSNs are mainly composed of sensing, computing and communication units driven by a power source [7]. The sensing unit detects a physical quantity (temperature, light, heat, motion) and produces an equivalent electrical quantity that is processed for communication needs forming a meshed network (Fig. 1-1). In WSNs, the bulk amount of power is consumed by the communication unit (60%); nearly 6%–20% is needed for the sensing unit. The rest is used by the computing system [7].

![Fig. 1-1 Sensor network of weather station [1]. (Reprinted with permission. Copyright © 2011, Springer-Verlag)]
Table 1-1 Power requirements for some of the sensors [8].

<table>
<thead>
<tr>
<th>Sensor</th>
<th>Current (mA)</th>
<th>Voltage (V)</th>
<th>Power (mW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Barometric pressure</td>
<td>7</td>
<td>5</td>
<td>35</td>
</tr>
<tr>
<td>Vibration</td>
<td>0.6</td>
<td>3.3</td>
<td>1.98</td>
</tr>
<tr>
<td>Humidity</td>
<td>0.3</td>
<td>3.3</td>
<td>0.99</td>
</tr>
<tr>
<td>Light</td>
<td>0.03</td>
<td>3.3</td>
<td>0.099</td>
</tr>
<tr>
<td>Temperature</td>
<td>0.008</td>
<td>3.3</td>
<td>0.026</td>
</tr>
</tbody>
</table>

Recent developments in the design of ultra-low power electronics has made it possible for EHs to work as the power source in WSNs. Current advances in microelectromechanical systems (MEMs) fabrication technology are allowing miniaturization of MEMs vibrational EHs. As a result, such devices can achieve high power densities to support the power requirements of WSNs [7]. Power requirements of some sensor networks are given in Table 1-1 [8].

1.2 Principle of vibration-to-electricity conversion model

Vibration-to-electricity energy harvesters are inertial devices which can be modelled as a second order mass-spring-damper system (Fig. 1-2) [2]. The mobile mass suspended by springs is free to move relative to the frame upon experiencing an external vibration.

![Fig. 1-2 Model of vibration to electricity converter [2].](image_url)
The dynamic equation of the electromechanical system is given by:

\[ m \ddot{z}(t) + (b_e + b_m) \dot{z}(t) + k_1 z(t) = -m \ddot{y}(t) \]  

(1-1)

where \( m \) is the mass of the proof mass, \( k_1 \) is the linear spring constant of the springs, \( b_e \) and \( b_m \) are the electrical damping and mechanical damping, respectively. The external vibration is \( y(t) \), with frequency, \( \omega \) and \( z(t) \) is the motion of the proof-mass relative to the frame. The electrical and mechanical forces are proportional to the velocity, \( v \). The power generated by the EH is the work done by the electrical damping and is given by:

\[ P = \int_0^\infty b_e v \, dv = \frac{1}{2} b_e v^2 = \frac{1}{2} b_e \dot{z}^2 \]  

(1-2)

Eq. (1-1) is solved using Laplace transformation to obtain the magnitude of the relative displacement, \( z \) as:

\[ |Z| = \left| \frac{-\omega^2}{-\omega^2 + 2(\zeta_e + \zeta_m) j \omega \omega_n + \omega_n^2} \right| |Y| \]  

(1-3)

where, unitless damping ratios are substituted into \( b_e \) and \( b_m \) following the relationship \( b = 2m \xi_\omega \), \( k_1 \) is replaced according to relationship \( \omega_n^2 = k_1/m \).

Therefore, the magnitude of the power generated is given by:

\[ |P| = \frac{m \xi_e \omega_n \omega^2 \left( \frac{\omega}{\omega_n} \right)^4 Y^2}{(2 \xi_T \frac{\omega}{\omega_n})^2 + (1 - (\frac{\omega}{\omega_n})^2)^2} \]  

(1-4)

where, \( \xi_T = \xi_e + \xi_m \).

When the external vibration frequency matches the resonant frequency of the mass-spring system, the energy harvester oscillates with the highest amplitude producing maximum power given by:
\[ \left| p \right|_{\text{max}} = \frac{m \xi_e a_v^2}{4 \omega \xi_T^2} \]  

where the magnitude of the input vibration \( Y \) is expressed as \( Y = \frac{a_v}{\omega^2} \).

From Eqs. (1-4) and (1-5), important design considerations can be deduced [2].

- Power is directly proportional to the mass of the proof-mass. Therefore, proof-mass should be made as large as possible within the given space.
- Power is directly proportional to the square of the input vibration magnitude.
- Power is inversely proportional to the frequency of vibration.
- The system should be designed so that its natural frequency matches the frequency of the external vibration.
- The system should be optimized to minimize \( \xi_m \) and \( \xi_e \) should be equal to \( \xi_m \).
  
\( \xi_e \) depends on the electrical parameters in the system which can be optimized to match \( \xi_m \). Moreover, very low mechanical damping can cause instability.

1.3 Common MEMs energy harvesters

MEMS energy harvesters can be broadly classified into three types: piezoelectric, electromagnetic and electrostatic based on the transduction mechanism [2]. Piezoelectric generators use an active piezoelectric material which produces charge when mechanically stressed [9], [10], [11]. Electromagnetic converters rely on the principal of electromagnetic induction between magnetic flux and a conductor [12], [13], [14]. Electrostatic energy scavengers operate based on parallel plate capacitor theory [15], [16], [17]. The vibration causes the distance between the plates to change, causing a change in the capacitance which can be conditioned for useful power.

This work focuses on electrostatic energy harvesters. Such converters are easily implemented using existing MEMS technology and can be incorporated with silicon-based electronics [18]. The proceeding sections discuss the fundamental theory of vibration-to-electricity conversion model and electrostatic converters.
1.3.1 Electrostatic energy harvesters

Electrostatic EHs use a MEMs variable capacitor for energy exchange. For a parallel plate capacitor, the capacitance and charge across the plates are given by:

\[ C = \frac{\varepsilon_0 A_0}{d} \]  \hspace{1cm} (1-6)

\[ Q = CV \]  \hspace{1cm} (1-7)

where, \( A_0 \) is the overlapping area of the plates, \( d \) is the separation between the plates, \( V \) is the voltage across the capacitor and \( \varepsilon_0 \) is the permittivity of free space.

When a charge is placed on the plates and mechanical vibration forces the plates to move, it causes a change in the capacitance. This change in capacitance is converted to electrical energy and stored. During the process either the voltage (voltage-constrained cycle) or the charge (charge-constrained cycle) is held constant (Fig. 1-3) [19].

![Conversion Cycles](image)

_Fig. 1-3 Conversion Cycles [19]._
1.3.1.1 Voltage-constrained energy cycle
The cycles are named on the property that is held constant during the conversion. For instance, pathway A-C-D-A shows a voltage-controlled conversion while path A-B-D-A depicts a charge-constrained conversion. Both cycles have a maximum allowable voltage due to system requirements.

The voltage-constrained cycle starts when the voltage across the capacitor is charged from a reservoir to \( V_{\text{max}} \) while the capacitance of the MEMS device is at \( C_{\text{max}} \). Through this time, the portion A-C is a straight line and \( C_{\text{MEMS}} \) is constant. The voltage during this period is held constant making it voltage-constrained cycle. When the distance between the plates changes and the capacitance gradually decreases to minimum (section C-D). During this time, the mechanical force does work while causing the charge to move from the capacitor bank into the reservoir. The remaining charge is then transferred following the path D-A where \( C_{\text{MEMS}} \) equals \( C_{\text{min}} \). The resultant energy gained, \( E_{\text{voltcons}} \), is depicted by the area ACD in Fig. 1-3 and is given by:

\[
E_{\text{voltcons}} = \frac{1}{2} \left( C_{\text{max}} - C_{\text{min}} \right) V_{\text{max}}^2
\]  

(1-8)

The problem with this type of conversion is that there must be a mechanism to hold the voltage constant across the MEMS device [19]. This adds another voltage source to the system apart from the one required to supply charge from the reservoir. It is highly preferred to use a single voltage source.

1.3.1.2 Charge-constrained energy cycle
To overcome the problem of using two voltage sources as in the voltage-constrained cycle, a charge-constrained conversion is proposed. In Fig. 1-3, the path A-B shows the condition of \( C_{\text{MEMS}} \) being charged to a voltage while at maximum capacitance. At D, the capacitor plates are at maximum displacement \( (C_{\text{MEMS}} = C_{\text{min}}) \), which causes the capacitance to decrease, increasing the voltage across the capacitor. While the voltage across the \( C_{\text{MEMS}} \) is \( V_{\text{max}} \), the initial charge to be placed at the capacitor is calculated. The MEMS device is open-circuited from the entire system and no current flows (B-D). Charge and voltage are held across the device. Therefore, \( V \) increases to fulfill \( Q = CV \) as \( C \) decreases. The charge is collected following the path D-A. The total energy during this cycle is certainly less than the case of voltage-constrained and is shown by the area ABD and is given by:
For the charge-constrained process, only a single voltage source is required, which is desirable [19].

The operation of charge-constrained electrostatic EH can also be explained with the help of a simple electrical circuit shown in Fig. 1-4.

\[ E_{\text{charcons}} = \frac{1}{2} (C_{\text{max}} - C_{\text{min}}) V_{\text{max}} V_{\text{start}} \]  

(1-9)

In Fig. 1-4, \( V_{\text{in}} \) is the battery needed for charged-constrained operation, variable capacitor \( C_v \) is the MEMS structure and \( C_{\text{par}} \) is the parasitic capacitance which is formed from the MEMS configuration and the interconnects. When the variable capacitance \( C_v \) is at maximum, \( C_{\text{max}} \), switch 1 (SW1) closes and the charge moves from the input to the variable capacitor. When both switches are open, the capacitance on the MEMs device goes from maximum to minimum. The gain in energy is given by Eq. (1-9).

When the capacitance is at minimum, switch 2 closes and the charge moves to \( C_{\text{stor}} \) from \( C_v \). Thus, mechanical vibrations increase the total energy on the system by doing work on the MEMS device.

1.3.1.3 Design topologies of electrostatic EHs

The variable capacitor in the electrostatic VEH mainly have three different design topologies as displayed in Fig. 1-5. In, in-plane overlap type converter, the proof-mass moves in the direction of the arrow, changing the overlap area of the interdigitated fingers that forms the capacitance (Fig. 1-5a). Gap-closing capacitor is formed as the distance between the adjacent fingers changes
when the proof mass oscillates in-plane in the direction demonstrated in Fig. 1-5b. Fig. 1-5c, is the out-of-plane gap closing EH, which forms a capacitance as the proof-mass oscillates out of plane [2].

![Fig. 1-5 a) In-plane overlap type b) In-plane gap-closing type c) Out-of-plane gap-closing type](image)

The capacitance, electrostatic and mechanical forces of each design topology are given in Table 1-2.

**Table 1-2 Capacitance, electrostatic and mechanical forces.**

<table>
<thead>
<tr>
<th>Topology</th>
<th>Capacitance</th>
<th>Electrostatic force</th>
<th>Mechanical force</th>
</tr>
</thead>
<tbody>
<tr>
<td>In-plane overlap</td>
<td>( C_v = \frac{N_g \varepsilon_0 L_f (z + z_0)}{d} )</td>
<td>( F_e = \frac{Q^2 d}{2N_g \varepsilon_0 h(z + z_0)^2} )</td>
<td>( F_m = \frac{N_g \mu L_f h}{d} \hat{z} )</td>
</tr>
<tr>
<td>In-plane gap- closing</td>
<td>( C_v = N_g d \varepsilon_0 L_f h \left( \frac{2d}{d^2 - z^2} \right) )</td>
<td>( F_e = \frac{Q^2 z}{2N_g d \varepsilon_0 L_f h} )</td>
<td>( F_m = \left( \frac{\mu A}{d_0} + 16\mu N_g L_f h^3 \left( \frac{1}{(d - z)^3} + \frac{1}{(d + z)^3} \right) \right) \hat{z} )</td>
</tr>
<tr>
<td>Out-of-plane gap-closing</td>
<td>( C_v = \frac{\varepsilon_0 W_{pm} L_{pm}}{z} )</td>
<td>( F_e = \frac{-Q^2}{2\varepsilon_0 W_{pm} L_{pm}} )</td>
<td>( F_m = \frac{16\mu W_{pm}^3 L_{pm}}{z^3} \hat{z} )</td>
</tr>
</tbody>
</table>

In the above expressions, \( N_g \) is the number of gaps per side formed by the interdigitated fingers, \( W_{pm} \) the width of the proof-mass, \( L_{pm} \) is the length of the proof-mass, \( L_f \) is the length of fingers,
$\varepsilon_0$ is the permittivity of free space, $\mu$ is the viscosity of air, $h$ is the initial overlap of the fingers, $d$ is the gap between fingers, and $z$ is the displacement of the proof mass.

1.3.1.4 Comparison of the design topologies

All three electrostatic EH topologies can harvest similar amount of power [2]. According to [2], in-plane gap-closing type design is the preferable topology as it can generate reasonable power with small deflection. With larger displacement, the capacitance decreases for this topology. Both in-plane overlap and out-of-plane gap-closing type EHs produces more power at large deflections. However, at large enough deflections these structurers are vulnerable to instability issues. For example, a small rotation for the in-plane overlap structure can cause the fingers to touch and short-circuit. In the out-of-plane gap-closing structure, large oscillations can cause electrostatic pull-in. Designing mechanical stoppers is difficult from fabrication aspect [2]. Out-of-plane gap-closing converter can harvest much more energy per cycle compared to other types when the displacement becomes comparable to the gap between the electrodes [20].

1.4 Wideband energy harvesters (EHs)

EHs that convert vibration into electrical energy are mostly mass-spring systems that generate maximum power when the frequency of the vibration matches the natural frequency of the device. Any mismatch in the two frequencies can cause the generated power to drop significantly [2]. This is also the major limitation of VEHs which reduces their use in real world applications [21]. Widening the bandwidth of the device by multiple generator arrays [3], [22] adding mechanical stoppers [20], [23], [24], and non-linear springs [25], [26], [27] can be a remedy to the problem [4]. Mechanical and electrical tuning of the device resonant frequency to match the external frequency can also be performed [4].

The next few sections discuss the well-practiced state-of-the-art strategies to enhance the operating frequency range of VEHs.

1.4.1 Mechanical tuning

Mechanical tuning of the resonator frequency to match the external vibration frequency can be obtained by various processes such as varying dimensions, changing center of gravity of the proof-mass and altering spring stiffness [4].
1.4.1.1 Variation in dimensions

In [28], a cantilever structure is used to tune the resonant frequency by changing the effective length, $l$ of the cantilever beam as shown in Fig. 1-6. The resonant frequency of such a cantilever with a mass is given by:

$$f_r = \frac{1}{2\pi} \sqrt{\frac{Ewt_b^3}{4l^3(m + 0.24m_c)}}$$  \hspace{1cm} (1-10)

where, $E$ is the Young’s modulus of the cantilever beam, $m_c$ is the mass of the cantilever, $w$, $l$ and $t_b$ are the width, length and thickness of the beam, respectively. A primary disadvantage of this method is the requirement of control system to adjust the length to achieve the desired frequency.

1.4.1.2 Variation of the center of gravity of the proof-mass

Wu et.al in [29] designed a piezoelectric cantilever-based energy harvester in which the resonant frequency of the device can be changed by changing the center of gravity of the proof-mass (Fig. 1-7).

![Fig. 1-7 a) Cantilever structure to adjust center of gravity [29] b) Piezoelectric cantilever-based EH with mobile mass [29]. (Reproduced with permission. Copyright © IOP Publishing. All rights reserved)](image-url)
The resonant frequency of the structure is given by:

\[ f_r = \frac{1}{2\pi} \sqrt{\frac{Em_t^3}{13ml^3} \cdot \frac{r^2 + 6r + 2}{8r^4 + 14r^2 + 21r^2 + 4r + \frac{2}{3}}} \]  

(1-11)

where, \( r \) is the ratio of \( x/l \) and all other symbols have the same meaning as the preceding section and by varying the \( x/l \) ratio, the natural frequency of the device can be tuned to that of the external frequency.

1.4.1.3 Variation in spring stiffness

The method of variable spring stiffness depends on addition of a negative spring in parallel to the mechanical spring (Fig. 1-8) [4]. Thus, the effective spring constant changes causing the resonant frequency of the device to vary according to Eqs. (1-12) and (1-13):

\[ k_{\text{eff}} = k_m + k_a \]  

(1-12)

\[ f_r = \frac{1}{2\pi} \sqrt{\frac{k_{\text{eff}}}{m}} \]  

(1-13)

where, \( k_m \) is the mechanical spring constant, \( k_a \) is the additional negative spring constant. The negative spring constant can be achieved by electrostatic [30], [31], magnetic [32], thermal [33] or piezoelectric process [34].

Fig. 1-8 Softened spring [4]. (Reproduced with permission. Copyright © IOP Publishing. All rights reserved)
1.4.1.3.1 Electrostatic method

In [30] and [31], an array of eight comb drive resonators, each with different resonant frequency was used. Each resonator was tuned by varying its voltage $V_{\text{run}}$, (Fig. 1-9). By changing the voltages, the electrostatic forces were altered, which had a similar effect to spring softening system.

![Comb-drive actuator for electrostatic tuning](image)

*Fig. 1-9 Comb-drive actuator for electrostatic tuning [30]. (Reproduced with permission. Copyright © IOP Publishing. All rights reserved)*

1.4.1.3.2 Piezoelectric method

Piezoelectric actuators require low power to produce large forces and can be used to mechanically stiffen the structure to tune the resonant frequency [34]. One of the two actuators shown in Fig. 1-10 is free to move. Upon application of an appropriate voltage, the actuator gets deformed increasing the stiffness of the total structure. Consequently, the resonant frequency is changed.

![Electrical tuning using piezoelectric actuators](image)

*Fig. 1-10 Electrical tuning using piezoelectric actuators [34]. (Reproduced with permission. Copyright © IOP Publishing. All rights reserved)*
1.4.1.3.3 Magnetic method

Fig. 1-11 Magnetically tunable piezoelectric harvester [32]. (Reproduced with permission. Copyright © IOP Publishing. All rights reserved)

In tuning the resonant frequency using magnetic method, attractive and repulsive magnetic forces are applied using permanent magnets perpendicular to the cantilever as depicted in Fig. 1-11 [32]. The forces are manually applied by controlling the distance between the two sets of permanent magnets, \( d_a \) and \( d_r \). A tuning range of 22-32 Hz was obtained around the original resonant frequency of 26 Hz in [32].

1.4.1.3.4 Thermal method and straining the structure

Resistive heating with a maximum temperature of 255° C was used in [33] to create thermal stress on the beams, which resulted in a 6.5% change from an initial resonant frequency of 31 kHz (Fig. 1-12). This method requires a constant power input to the system to maintain the thermal stress. Therefore, is not a suitable process for vibration energy harvesting.

Fig. 1-12 Thermally controlled comb drive actuator [33]. (Reproduced with permission. Copyright © IOP Publishing. All rights reserved)
Another method of tuning the resonant frequency of energy harvester is by applying stress to the structure and therefore staining it. An axial load is used to change the resonant frequency. The resonant frequency can be increased by applying axial tensile load and decreased by applying axial compressive load on a cantilever structure (Fig. 1-13) [35]. The resonant frequency of the structure under axial load, $f_{ri}'$, is given by:

$$f_{ri}' = f_{ri} \sqrt{1 + \frac{F}{F_b} \cdot \frac{\lambda^2}{\lambda_i^2}}$$  \hspace{1cm} (1-14)

where, $f_{ri}$ is the resonant frequency of the $i_{th}$ mode under no load. $F$ is the axial load, $F_b$ is the buckling axial load and $\lambda$ is dimensionless load parameter. A positive $F$ denotes a tensile axial load and a negative $F$ denotes a compressive axial load.

![Fig. 1-13 Axial tensile (a) and compressive load (b) [35]. (Reproduced with permission. Copyright © IOP Publishing. All rights reserved)](image)

1.4.2 Electrical tuning

Electrical tuning of vibration based EHs can be achieved by changing the electrical damping of system. A shift in the power spectrum was obtained by varying the capacitive load of EHs [4], [36]. All reported EHs that used electrical tuning are piezoelectric (Fig. 1-14). In [36], a piezoelectric EH was designed and some important design considerations were made regarding tunability of EHs.

![Fig. 1-14 EH with piezoelectric bimorph [36]. (Reproduced with permission. Copyright © IOP Publishing. All rights reserved)](image)
For example, a larger tuning range can be obtained using a piezoelectric material with high Young’s modulus, high strain coefficient and lower permittivity. Two piezoelectric layers (Fig. 1-15b) instead of single layer (Fig. 1-15a) can also increase the tuning range of the device.

![Diagram](image1.png)

*Fig. 1-15 a) Single layer b) Different layers [36]. (Reproduced with permission. Copyright © IOP Publishing. All rights reserved)*

1.4.3 Array of generators

This technique uses an array of smaller generators each with different dimensions, resulting in dissimilar resonant frequencies to design a wideband EH. Using this method, wider operating range can be achieved without sacrificing the quality factor. For example, a set of cantilever beams [3], or piezoelectric bimorphs [22] was used to obtain a band-pass filter like response from the device as shown in Fig. 1-16. One advantage of this method is that at a given frequency within the wider frequency range, one of the generators in the array will resonate. One major disadvantage is the design and fabrication complexity that arises in working with an array of smaller generators.

![Diagram](image2.png)

*Fig. 1-16 a) Array of cantilever beams of different dimensions b) Power spectrum [3]. (Reproduced with permission. Copyright © IOP Publishing. All rights reserved)*
1.4.4 Mechanical stopper

Mechanical stoppers can be incorporated into the design of the EHs to limit the vibration amplitude. This is a well-practiced method, which increases the bandwidth of the device during up-sweep of frequency [23], [24].

![Electrostatic EH with mechanical stoppers](image1)

**Fig. 1-17a** Electrostatic EH with mechanical stoppers  
**b)** End-stops are placed at a certain distance [24].

The impact of the proof-mass with the stopper introduces additional stiffness and damping into the system. Therefore, the system behaves like hardening springs system [20], [24]. The effect of the stopper can be visualized in the frequency response of a system displayed in Fig. 1-18 [37]. When the maximum displacement of the proof-mass is limited by a stopper, the bandwidth is widened at the cost of maximum power harnessed by the device.

![Frequency response of EH with stopper](image2)

**Fig. 1-18** Frequency response of EH with stopper [37].
1.4.5 Nonlinear springs

The idea of using nonlinear springs in vibration-based energy harvesters was first explored in [38]. The spring force in EHs that use nonlinear springs is a collection of linear and nonlinear forces [27]. The presence of the nonlinear force induces some interesting phenomena in the system behavior in the frequency domain [25]. Primarily, the system provides amplitude-dependent resonant frequency, which also depends on the frequency sweep direction [26]. The nonlinearity causes the frequency response to stay in the linear regime or shift either to lower (softening spring) or higher (hardening spring) resonant frequency. The direction in the shift in the resonant frequency is determined by the nonlinear stiffness and the oscillation magnitude [26].

This phenomenon is of great interest to researchers in the field of energy harvesting, since the system bandwidth is greatly enhanced during spring softening and hardening. Softening springs can also produce more average power compared to linear and hardening system [27].

1.5 Comparison of the frequency tuning and bandwidth widening strategies of EHs

Most resonant frequency tuning methods require additional control logic to be built into the energy harvesting system. Therefore, the design complexity and power requirements are increased. Such as, control system is required while tuning the resonant frequency either by changing the dimensions of the device or controlling the temperature to create local stress in the structure. Similarly, tuning by electrical means of piezoelectric generators requires a mechanism of adjusting the capacitive load to improve the system performance.

Contrary to the frequency tuning methods by mechanical or electrical means, widening the bandwidth methods work by increasing the operating range of the energy harvester. Each method of bandwidth enhancement has some merits and demerits. For example, an array of EHs can greatly increase the overall size of the device. Moreover, mechanical stoppers are hard to incorporate into the design. In addition, if the mechanical stoppers are not properly designed, the impact of the proof-mass and the stopper can create instability and fatigue reducing the device lifetime.
Unlike the other methods, nonlinear spring EHs can be designed within a space constraint and require no additional power supply to enlarge the operating bandwidth and the output power. For this reason, this method has received widespread attention recently.

1.6 Objectives of this work

Real life environmental vibrations are random in nature or display shifts in frequency. Mismatch between the external vibration and the resonant frequency of the device is bound to occur at some point of time, which can largely decrease the harvester efficiency.

In this work, out-of-plane, gap-closing, electret-based wideband electrostatic energy harvesters are designed. Novel nonlinear springs are developed that produces nonlinear force-displacement relationship in the vertical direction. A unique method is developed here to obtain softening or hardening responses by varying the gap between the anchor height and the level of the proof-mass. The designed nonlinear spring force system exhibited strong nonlinear behavior within the desired displacement range and widened the operating bandwidth of the device harnessing more average power compared to the linear spring force device. In addition, mechanical stoppers are integrated with the hardening system to further improve the EH bandwidth.

Moreover, a completely new design of an electrostatic VEH array is proposed where linear, softening springs, hardening springs and hardening springs with mechanical stoppers are utilized to enhance the frequency range of the system.

1.7 Structure of this thesis

This dissertation contains six chapters.

The potential of vibrational energy harvesters (VEHs) to power WSNs is discussed in Chapter 1. Various types of VEHs are also explained with their working principles. Different topologies of electrostatic EHs are compared. Important and well-practiced strategies to develop wideband EHs are discussed in detail before establishing the goals of this work.

Chapter 2 is focused on the design, modeling and optimization of the linear electrostatic VEH. The optimization process of the device is explained and the performance of the designed linear VEH is compared with the state-of-the-art designs.
**Chapter 3** explains the principle of nonlinear vibration and the influence of nonlinear spring force on the system frequency response. The geometry of the novel nonlinear springs designed in this work and the methodology to introduce nonlinearity in the spring force are described. The design of the mechanical stoppers is also justified. The overall dynamics of the VEH with nonlinear springs and mechanical stoppers are developed.

The design and modeling of the array of electrostatic EHs containing linear, softening springs, hardening springs and hardening springs with mechanical stoppers are explained in **Chapter 4**. The connection of the individual EHs in the array is described. The dynamics of the system is also derived.

**Chapter 5** primarily contains the simulation results. VEH frequency response is explained with different spring types and mechanical stoppers. VEHs array response is also provided. Stress analysis results on the springs and stoppers are shown. Performance of the wideband EHs designed in this work is compared with state-of-the-art designs in this chapter.

**Chapter 6** rounds up the dissertation focusing on major challenges of VEHs, emphasizing on the contributions of this work and discusses scopes of the future work.
2 Linear electrostatic VEH

2.1 Introduction

Traditional linear spring electrostatic EHs are designed such that their natural frequency matches the external vibration frequency for maximum power generation [2]. The geometry of the device is designed to minimize the mechanical damping. The electrostatic force is optimized by varying the electrical parameters to match the mechanical damping. Significant amount of work has been done in developing electrostatic energy harvesters over the years. Hybrid structures are also developed that incorporates multiple topologies of variable capacitors to increase power density [39], [40].

In [15], ventricular wall motion of human, vibrating at 1~2 Hz was used as the source of mechanical energy. A honeycomb structure was used as the variable capacitor as shown in Fig. 2-1a. An average power of 36 mW was generated. Three different topologies of EHs were analyzed and compared in [2]. An optimized in-plane, gap-closing type converter achieved the maximum power density of 116 μW/cm³ from an input vibration source with 2.25 m/s² at 120 Hz. G-J. Sheu et al. in [41] designed an in-plane overlap type converter which worked in charge-constrained mode (Fig. 2-1b). It was claimed to be the smallest (3000 µm × 3000 µm × 500 µm) electrostatic EH to date operating at 105 Hz. An average power of 0.0924 µW was generated. An interesting concept of using dielectrics with varying equivalent permittivity to improve the power generation in electrostatic energy harvesters was proposed in [42]. Their proposed method produced 54 times more power compared to conventional approaches where a single dielectric is used in electrostatic EHs. A hybrid structure shown in Fig. 2-1c, incorporating in-plane overlap and gap-closing type capacitors was fabricated in [40]. Another hybrid EH consisting of out-of-plane, gap-closing and overlap capacitors on a silicon-on-insulator wafer was developed in [39]. The device depicted in Fig. 2-1d required a small bias voltage of 1 volt and produced maximum power at 120 Hz.

In this work, an out-of-plane, gap-closing, electret-based converter is designed. The subsequent sections provide design detail, device dynamic model, optimization procedure, and the mechanical simulation results for the linear electrostatic EH.
Fig. 2-1a) Honeycomb capacitor structure [15] b) In-plane overlap type EH[41] (Reproduced with permission. Copyright © Elsevier. All rights reserved) c) SEM of hybrid EH [40] (Reproduced with permission. Copyright © IOP Publishing. All rights reserved) d) Solid model of out-of-plane EH [39]. (Reproduced with permission. Copyright © IEEE Publishing. All rights reserved)
2.2 Design and modelling of the linear spring electrostatic EH

The electrostatic EH is designed using CoventorWare/MEMS+® suite as four springs suspending a proof-mass (Fig. 2-2a). Nickel is selected for both components because of the ease with which it can be deposited by electroplating and its high density [40]. The proof-mass together with the aluminium ground electrode form an out-of-plane, gap-closing capacitor as shown in Fig. 2-2c. The electromechanical system can be represented by an equivalent electrical circuit, (Fig. 2-3), where a voltage source, \( V_{bias} \) of 20 V, and a bias capacitance, \( C_{bias} \) of \( 1 \times 10^{-12} \) F are used to depict the behavior of charged electret [43]. The MEMS device forms the variable capacitance, \( C_v \). The equivalent circuit is used to analyse the power delivered by the EHs to the load resistor, \( R \).

The dynamic equation of the system is derived in terms of the design parameters listed in Table 2-1.

![Fig. 2-2 Design of the electrostatic EH](image-url)

*a) Side view of the electrostatic EH b) top view shows two design parameters c) close-up view of the selected portion showing gap between the electrodes and gap between proof-mass and anchor level.*
Table 2-1 List of design parameters (Depicted in Fig. 2-2)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Design</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proof-mass length</td>
<td>$L_{pm}$</td>
<td>Optimized</td>
</tr>
<tr>
<td>Proof-mass width</td>
<td>$W_{pm}$</td>
<td>Optimized</td>
</tr>
<tr>
<td>Proof-mass thickness</td>
<td>$T_{pm}$</td>
<td>Optimized</td>
</tr>
<tr>
<td>Spring length</td>
<td>$L_s$</td>
<td>796 µm</td>
</tr>
<tr>
<td>Spring width</td>
<td>$W_s$</td>
<td>25 µm</td>
</tr>
<tr>
<td>Spring thickness</td>
<td>$T_s$</td>
<td>5 µm, 6 µm, 10 µm</td>
</tr>
<tr>
<td>Density of nickel</td>
<td></td>
<td>8910 kgm$^{-3}$</td>
</tr>
<tr>
<td>Initial gap between proof-mass and bottom electrode</td>
<td>$l_0$</td>
<td>Optimized</td>
</tr>
<tr>
<td>Load resistance</td>
<td>$R$</td>
<td>Optimized</td>
</tr>
<tr>
<td>Electret voltage</td>
<td>$V_{bias}$</td>
<td>20 V</td>
</tr>
<tr>
<td>Electret capacitance</td>
<td>$C_{bias}$</td>
<td>1×10$^{-12}$ F</td>
</tr>
<tr>
<td>Anchor position from the center of the device</td>
<td>$A$</td>
<td>55 µm</td>
</tr>
<tr>
<td>Gap between level of anchor and proof-mass</td>
<td>$H$</td>
<td>0-50 µm</td>
</tr>
</tbody>
</table>

The variable capacitance, $C_v$, is written in terms of the proof-mass displacement in the vertical direction, $z$ as:

$$C_v(z) = \frac{\varepsilon_0 L_{pm} W_{pm}}{(l_0 - z)}$$  \hspace{1cm} (2-1)

From which, the voltage across the variable capacitor, $V$, electrostatic energy, $U$, and the electrostatic force, $F_e$, are found as:

$$V(q, C_v) = q \frac{C_v(z)}{C_v(z)} = q \frac{(l_0 - z)}{\varepsilon_0 L_{pm} W_{pm}}$$  \hspace{1cm} (2-2)

$$U(z) = \frac{q^2}{2C_v(z)}$$  \hspace{1cm} (2-3)

$$F_e(q, z) = \frac{dU}{dz} = \frac{1}{2} \frac{q^2}{\varepsilon_0 L_{pm} W_{pm}}$$  \hspace{1cm} (2-4)

where, $q$ is the charge on the capacitor, $\varepsilon_0$ the permittivity of free space, $l_0$ is the initial gap between the electrodes, $L_{pm}$ and $W_{pm}$ are the length and width of the proof-mass, respectively.

The mechanical damping, $b_m$, is dominated by squeeze film damping between the proof-mass and the bottom electrode and is given by:
\[ b_m = \frac{\mu W_{pm}^3 L_{pm}}{(l_o - z)^3} \beta \left( \frac{W_{pm}}{L_{pm}} \right) \]  

(2-5)

Here, \( \beta(W_{pm} / L_{pm}) \) is the correction term that depends on the lateral aspect ratio \( W_{pm} / L_{pm} \) [44], and \( \mu \) is the viscosity of air [41].

The electromechanical system, then, can be described with the mechanical equation for the balance of forces and the electrical circuit equation for Fig. 2-3:

\[ m\ddot{z} + F_s + b_m \dot{z} - F_e(q, z) - mg = -ma(t) \]  

(2-6)

\[ \frac{dq}{dt} = \frac{1}{R} \left[ V_{bias} - \frac{q}{C_{bias}} - V(q, z) \right] \]  

(2-7)

\[ Fig. 2-3 \text{ Equivalent electrical circuit of the EH.} \]

Here, the restoring force can be expressed as \( F_s = k_z \) for linear springs. The external acceleration \( a(t) = a_v \sin(2\pi ft) \) is expressed in terms of amplitude of the vibration \( a_v \), frequency \( f \), and time \( t \).

2.3 Design optimization

A linear spring EH with a resonant frequency \( f_r \) of 803 Hz and spring constant of 15.3 Nm\(^{-1}\) is optimized for \( a_v = 9.8 \) ms\(^{-2}\) (1g). The frequency 803 Hz is the strongest component in the vibration spectrum measured on a typical aircraft fuselage skin [6]. In order to maximize the average generated power, \( P_{avg} \) during the time duration \( t_1 \) to \( t_2 \) in Eq. 2-8, design parameters in the electromechanical system are optimized such as length, width, thickness of the proof-mass, load resistance and the ambient pressure (Table 2-1).

\[ P_{avg} = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} \left( \frac{dq}{dt} \right)^2 R \, dt \]  

(2-8)
Table 2-2 shows $P_{avg}$ for different proof-mass dimensions at 1 and 0.1 atmospheric pressures (atm.) for an initial gap of 30 µm. The dimensions are varied such that the total mass of the proof-mass remains constant to have resonance at $f_r = 803$ Hz. It can be seen from Table 2-2 that higher power is generated when $W_{pm} < L_{pm}$. This is mainly due to reduced squeeze film damping, which depends on the overlapping area of the electrodes. In addition, much higher power is generated at 0.1 atm. than 1 atm. due to the higher displacement of the proof-mass at reduced pressure, which causes greater capacitance change. An average power of 60 nW is delivered from the optimized system with $L_{pm} \times W_{pm} \times T_{pm} = 1500 \, \mu m \times 300 \, \mu m \times 150 \, \mu m$, $l_0 = 30 \, \mu m$, $R = 150$ MΩ and at the ambient pressure of 0.1 atm.

Table 2-2 Average power generated at 1 atm. and 0.1 atm. for different combination of proof-mass dimensions. $l_o = 30 \mu m$, $a_v = 9.8 \, ms^{-2}$, $f_r = 803$ Hz, $R = 150$ MΩ.

<table>
<thead>
<tr>
<th>$L_{pm}$ (µm)</th>
<th>$W_{pm}$ (µm)</th>
<th>$T_{pm}$ (µm)</th>
<th>$P_{avg}$ (W) 1 atm.</th>
<th>$P_{avg}$ (W) 0.1 atm.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1500</td>
<td>300</td>
<td>150</td>
<td>8.98×10^{-9}</td>
<td>5.99×10^{-8}</td>
</tr>
<tr>
<td>1500</td>
<td>600</td>
<td>75</td>
<td>9.40×10^{-10}</td>
<td>2.33×10^{-8}</td>
</tr>
<tr>
<td>1500</td>
<td>900</td>
<td>50</td>
<td>1.81×10^{-10}</td>
<td>9.65×10^{-9}</td>
</tr>
<tr>
<td>1500</td>
<td>1200</td>
<td>37.5</td>
<td>5.87×10^{-11}</td>
<td>4.34×10^{-9}</td>
</tr>
<tr>
<td>1500</td>
<td>1500</td>
<td>30</td>
<td>2.79×10^{-11}</td>
<td>2.34×10^{-9}</td>
</tr>
</tbody>
</table>

Fig. 2-4 illustrates the average power generated for different values of initial gap between the electrodes. A higher gap allows the proof-mass to oscillate with larger amplitude resulting in greater output power. An initial gap of 30 µm is chosen for the design with a goal to develop nonlinear springs to obtain softening and hardening responses from the EH within the displacement range of ± 30 µm. The contour plot, Fig. 2-5 shows the variation of $P_{avg}$ with external vibration frequency and load resistance. Resonant frequency $f_r$, of 803 Hz for all load values indicates the absence of any electromechanical nonlinearity. A load resistance value of 150 MΩ is chosen since the increase in power for higher $R$ is incremental.
Fig. 2-4 Average output power of the linear EH increases with initial gap between the electrodes. The curve is obtained for $L_{pm} = 1500 \, \mu m$, $W_{pm} = 300 \, \mu m$, $T_{pm} = 150 \, \mu m$, $a_v = 9.8 \, ms^{-2}$, $f_r = 803 \, Hz$ and $R = 150 \, M\Omega$ at 0.1 atm.

Fig. 2-5 Average output power of the linear EH for the variation of frequency and load resistance. The curve is obtained for $L_{pm} = 1500 \, \mu m$, $W_{pm} = 300 \, \mu m$, $T_{pm} = 150 \, \mu m$, $l_o = 30 \, \mu m$, $a_v = 9.8 \, ms^{-2}$, at 0.1 atm..
The study of electrostatic pull-in voltage is important for MEMs devices that uses parallel plate capacitors. The electrostatic force of attraction between the electrodes becomes significantly high at pull-in voltage and the electrodes stick together causing short circuit. Therefore, the voltage across the MEMs electrostatic EH should be less than the pull-in voltage during the period of operation. Electrostatic pull-in voltage, \( V_{\text{pl}} \) for an out-of-plane gap closing EH is calculated using [45]:

\[
V_{\text{pl}} = \sqrt{\frac{8k_l l_0^3}{27\varepsilon_0 L_{\text{pm}} W_{\text{pm}}}}
\]

For an initial gap of 30 \( \mu \text{m} \), spring constant of 15.3 Nm\(^{-1}\) and an overlapping area of 1500 \( \mu \text{m} \times 300 \mu \text{m} \), the pull-in voltage is 175.2 V. A much smaller \( V_{\text{bias}} \) compared to the pull-in voltage ensures the capacitor plates do not collapse together.

The dynamic response of the optimized system is shown in Fig. 2-6. A sinusoidal signal \( 9.8\sin(\omega t) \) is used as the external vibration signal driving the electrostatic EH (Fig. 2-6a). The electromechanical system responds to the external vibration and the proof-mass oscillates between the maximum and the minimum position (Fig. 2-6b). The proof-mass is closest to the bottom electrode at the maximum positive z value (minimum position) while it is farthest at the minimum negative z value (maximum position) of the displacement curve (Fig. 2-6b). When the proof-mass is at the maximum position, the gap between the electrodes is highest and the charge is low. This low charge occurs when the proof-mass moves beyond a certain distance away from the bottom electrode, resulting in nearly a flat response near the minimum point of the charge curve (Fig. 2-6c). During this period, there is no significant current flowing in the electromechanical system. The lack of current is observed by the flat region on both sides of the zero-current level (Fig. 2-6d). Due to the low current in the system, the instantaneous power drops nearly to zero and remains at low values while the current is insignificant (Fig. 2-6e). When the proof-mass moves closer to the bottom electrode, the capacitance, the charge and current increase sharply to the maximum value. Thus, the instantaneous power curve contains spikes corresponding to the maximum and the minimum currents followed by flat regions.
corresponding to the low level current. Because of this nature, the average power is used instead of maximum power when comparing performance of the EHs.

2.4 Linear spring design

Four U-shaped springs with length, width and thickness of \( L_s = 796 \mu m \), \( W_s = 25 \mu m \) and \( T_s = 10 \mu m \), respectively, were placed such that the anchor position relative to the center of the device was \( A = 55 \mu m \), with the proof-mass and anchor at the same level (\( H = 0 \mu m \)). Fig. 2-7 depicts that the spring constants in the lateral directions, \( k_y = 655.2 \text{ Nm}^{-1} \), \( k_x = 5624.3 \text{ Nm}^{-1} \), are much higher than the vertical direction. Therefore, as such, vibrations in the lateral directions are to be suppressed. Moreover, it can be seen that the force-displacement relationship is linear within the desired displacement range.

Fig. 2-6 Average output power of the linear EH for the variation of frequency and load resistance. The curve is obtained for \( L_{pm} = 1500 \mu m \), \( W_{pm} = 300 \mu m \), \( T_{pm} = 150 \mu m \), \( l_o = 30 \mu m \), \( a_v = 9.8 \text{ ms}^{-2} \), at 0.1 atm..
2.4.1 Stress analysis
Stress analysis ensured that the springs can hold the proof-mass without collapsing. The device is portioned into four equal parts using the symmetry property in Coventorware (Fig. 2-8a). A converged mesh is used for finite element analysis on the quarter device. The proof-mass is displaced by 30 µm in the negative z-direction; maximum distance the proof-mass can move before hitting the bottom electrode and the stresses on the beams are found out (Fig. 2-8b). Maximum stress of 300 MPa found on the beams is less than the maximum yield strength of nickel (500 MPa) [46]. The beams are specially designed to have curved edges near the corners as shown in Fig. 2-8c. Moreover, fillets are added at the junction of the beams and the anchors (Fig. 2-8d). These distribute the stress over a larger area and minimize the maximum stress at the beam corners.

![Force-displacement curves of the linear EH for different directions.](image)

*Fig. 2-7 Average output power of the linear EH for the variation of frequency and load resistance. The curve is obtained for \( L_{pm} = 1500 \, \mu m \), \( W_{pm} = 300 \, \mu m \), \( T_{pm} = 150 \, \mu m \), \( l_o = 30 \, \mu m \), \( a_v = 9.8 \, ms^2 \), at 0.1 atm.*
Fig. 2-8 Average output power of the linear EH for the variation of frequency and load resistance. The curve is obtained for $L_{pm} = 1500 \, \mu m$, $W_{pm} = 300 \, \mu m$, $T_{pm} = 150 \, \mu m$, $l_o = 30 \, \mu m$, $a_v = 9.8 \, ms^{-2}$, at 0.1 atm.
2.5 Electrostatic energy harvester comparison

The performance of the designed EH is compared with the state-of-the-art and is provided in Table 2-3. The in-plane gap-closing converter in [18] produced the highest acceleration-normalized power density among all EH designs. The proof-mass was large compared to MEMS standard (10 mm × 10 mm). Moreover, high density alloy of tungsten and nickel was used to obtain heavy proof-mass. The device worked with a small input vibration of magnitude 2.25 ms\(^{-2}\). The hybrid EH in [40] achieved a power density of 3.37 \(\mu\)Ws\(^4/cm^3\)-m\(^2\). Novelty of the design was the incorporation of the in-plane, gap-closing and overlap type capacitors within a small volume of 1.4 cm\(^3\). In addition, the device worked with a large input vibration signal of magnitude 78.4 ms\(^{-2}\).

In-plane overlap type converter built using a SOI wafer produced a power of 92.4 nW at 105 Hz [41]. The 650 \(\mu\)m long fingers produced significant capacitance when the 4.9 mg proof-mass oscillated with a \(\pm\) 300 \(\mu\)m amplitude. A noteworthy achievement of this work was the design of the four very low stiffness springs (0.544 Nm\(^{-1}\)) that held the proof-mass and provided such large vibration. Sterken et al. developed an electret-based EH that generated 3.3 \(\mu\)Ws\(^4/cm^3\)-m\(^2\) acceleration-normalized power density [47]. However, the device worked with a large input vibration of magnitude 164 ms\(^{-2}\).

The linear EH developed in this work, improves the out-of-plane, gap-closing design in multiple ways. The springs are tucked around the proof-mass, which reduces the overall device size. Squeeze-film damping is reduced by careful selection of proof-mass dimensions. Lower mechanical damping allows larger displacement of the proof-mass producing more capacitance change. The linear EH finds itself in the 4\(^{th}\) position in terms of power density and in the 2\(^{nd}\) place in terms of power density normalized to magnitude of acceleration squared among some notable works in energy harvesting.

The linear EH developed here is used as the basis for the novel, nonlinear design described in the next sections.
### Table 2-3 Comparison of linear VEH with state-of-the-art.

<table>
<thead>
<tr>
<th>Source</th>
<th>Model</th>
<th>Input vibration- $a_v$(m/s²)</th>
<th>Volume (cm³)</th>
<th>Power density- PD (µW/cm³)</th>
<th>PD/$a_v^2$ (µWs⁴/cm³-m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>[41]</td>
<td>In-plane overlap</td>
<td>4.35</td>
<td>4.40×10⁻³</td>
<td>21</td>
<td>1.1</td>
</tr>
<tr>
<td>[47]</td>
<td>In-plane overlap</td>
<td>164</td>
<td>5.63×10⁻⁴</td>
<td>8.88×10⁴</td>
<td>3.30</td>
</tr>
<tr>
<td>[48]</td>
<td>In-plane overlap</td>
<td>164</td>
<td>5.20×10⁻⁴</td>
<td>9.62×10⁴</td>
<td>3.58</td>
</tr>
<tr>
<td>[40]</td>
<td>Hybrid</td>
<td>78.4</td>
<td>1.40×10⁻³</td>
<td>2.07×10⁴</td>
<td>3.37</td>
</tr>
<tr>
<td>[18]</td>
<td>In-plane gap-closing</td>
<td>2.25</td>
<td>1.16×10²</td>
<td>22.91</td>
<td></td>
</tr>
<tr>
<td>[50]</td>
<td>Out-of-plane overlap</td>
<td>2.45</td>
<td>1.98×10⁻²</td>
<td>3.08×10⁰</td>
<td>0.51</td>
</tr>
<tr>
<td><strong>This work</strong></td>
<td>Out-of-plane gap closing</td>
<td>9.8</td>
<td>8.33×10⁻⁵</td>
<td>7.20×10²</td>
<td>7.50</td>
</tr>
</tbody>
</table>

### 2.6 Summary

The linear out-of-plane, gap-closing, electrostatic EH designed is capable of generating 60 nW of power for a sinusoidal input acceleration of 1g at 803 Hz. The device has a linear stiffness of 15.3 N/m. Mechanical simulations indicated the maximum stress on the springs to be well under the yield strength of nickel. Therefore, oscillations in the z-direction is achieved without the beams being at a risk of fracture.
3 Design of wideband electrostatic energy harvester using nonlinear springs and mechanical stoppers

3.1 Introduction

VEHs with nonlinear springs are gaining vast attention recently because of their ability to change the resonant frequency of the device with vibration amplitude and thus achieving wide frequency range. Moreover, system bandwidth can be enhanced by using nonlinear springs. Real life vibrations often display shifts in frequency which makes linear VEHs less efficient [21]. Wideband VEHs with nonlinear springs proved to be much more effective when the vibration displays variation in frequency [21]. In [27], numerical analysis showed considerable improvements in power and bandwidth over linear VEHs by the use of softening and hardening springs. A crab-leg spring shown in Fig. 3-1a was modified, and nonlinear force-displacement relationship was achieved to improve power generation of nonlinear EH by 2.6 times due to its larger bandwidth [51]. Nonlinear responses were obtained by a L-shaped beam depicted in Fig. 3-1b with a joint angle of 131.2° giving larger bandwidth [52]. H-shaped hardening springs were designed in [53] to improve the power generation (Fig. 3-1c). A maximum power of 160 nW and a bandwidth of 52 Hz were generated by a device with nonlinear polyimide springs for 1g sinusoidal input vibration [54]. 13 times bandwidth and 68% power improvements over linear harvester using angled beams for broadband random vibration of \(7 \times 10^{-4} \text{g}^2 \text{Hz}^{-1}\) were reported in [55]. In [5], a curved beam depicted in Fig. 3-1d was designed to introduce nonlinear behavior in the system to improve the bandwidth of the system.

The use of mechanical stoppers to create wideband VEHs has been studied in literature as well. In [56], the effect of mechanical stoppers in electrostatic, comb-drive, in-plane and overlap-type EHs was investigated. Soliman et.al [23] developed an electromagnetic VEH with mechanical stoppers, which maintained the same frequency response for both up- and down-sweeps (Fig. 3-2a). A cantilever-based piezoelectric VEH with end stops broadened the bandwidth by 18 Hz and power varied from 34 nW to 100 nW for a base excitation of 0.6g (Fig. 3-2b) [57]. A novel two-mass piezoelectric cantilever system with end stops was developed in [58] that improved the bandwidth by 289% over the one without end stops.
Fig. 3-2 a) Electromagnetic EH with mechanical stoppers [23] (Reproduced with permission. Copyright © IOP Publishing. All rights reserved) b) Cantilever based EH with mechanical stoppers on both ends [57] (Reproduced with permission. Copyright © IOP Publishing. All rights reserved)
The main objective of this work is to create wideband VEHs. Nonlinear springs and mechanical stoppers are developed to achieve the purpose. This chapter explains the principle of nonlinear vibration. The spring geometry and procedure to obtain softening and hardening springs are explained. The mechanical stopper design is described. The dynamic models of the VEHs are derived with nonlinear springs and mechanical stoppers.

3.2 Principle of nonlinear spring force vibration

A mechanical system with a mass, damper and nonlinear spring is shown in Fig. 3-3.

![Fig. 3-3 Schematic of a mechanical system with nonlinear spring.](image)

The nonlinear spring force can be represented by:

\[
F_x = -k_1 z - k_2 z^2 - k_3 z^3 + \ldots
\]  

(3-1)

where, \(k_2\) and \(k_3\) are quadratic and cubic spring constants respectively. It can also contain higher order stiffness parameters.

Mass-spring-damper system with nonlinear spring force is represented by:

\[
m \dddot{z} + b_m \ddot{z} + k z + k_1 \dot{z}^2 + k_2 z^3 = F \cos \omega t
\]  

(3-2)

where, \(F\) and \(\omega\) are the magnitude and frequency of the forcing term, respectively. The nonlinear differential of motion (Eq. 3-2) can be analytically solved using Lindstedt-Pointcare method [59]. The response of the system displays an amplitude dependent resonant frequency given by:
\[ \omega_n = \omega_n + \kappa Z_0^2 \]  

(3-3)

where, \( \omega_n \) is the natural frequency of the system, given by \( \omega_n = \sqrt{\frac{k_1}{m}} \), \( Z_0 \) is the amplitude of vibration at resonance and \( \kappa \) is a function of nonlinear stiffnesses [59]:

\[ \kappa = \left[ \frac{3k_3}{8k_1} \omega_n - \frac{5k_2^2}{12k_1^2} \omega_n \right] \]  

(3-4)

The vibration amplitude of the system is expressed by:

\[ |Z_0| = \frac{F/m}{\sqrt{(\omega^2 - \omega_n^2)^2 + (\omega \omega_n/Q_f)^2}} \]  

(3-5)

where, \( Q_f = \omega_n m/b_m \)

Eq. 3-3 indicates that a positive \( \kappa \) will increase the resonant frequency and a negative \( \kappa \) will shift the resonant frequency to a lower value than the linear resonant frequency of \( \omega_n \). \( \kappa = 0 \) indicates a linear spring with a resonant frequency of \( \omega_n \). A negative \( \kappa \) represents spring softening while a positive \( \kappa \) spring hardening as shown in Fig. 3-4a. For both springs (softening and hardening), the resonant frequency of the system changes.

The behavior of the nonlinear system can be further explained with the help of hardening frequency response depicted in Fig. 3-4b. For a hardening spring system, when the excitation frequency increases, the response takes the path ABC and then there is a jump to E, followed by F. During down-sweep, the response takes the path FEDBA and there is a discontinuity at D. However, between BCDE, the system can have 3 different solutions for a single frequency. Between D and C, the solutions are unstable, and the system cannot vibrate at those amplitudes. During frequency up-sweep, while the response takes the path ABCEF, the system bandwidth is greatly increased. In addition, the response stays at high vibration level between B and C. This behavior of the nonlinear system is very promising for energy harvesting.
Fig. 3-4 a) Effect of nonlinearity on resonant frequency b) Hardening amplitude response [25] (Reproduced with permission. Copyright © IEEE Publishing. All rights reserved).

Typical response of linear, softening and hardening spring systems can be explained with the help of Fig. 3-5a [27]. For both up- and down-sweeps, the response of the linear spring system follows the same path. Moreover, the resonant frequency is the same for all magnitudes of input vibration (Fig. 3-5a). Unlike the linear spring system response, the response of hardening spring system is different for up- and down-sweeps of frequency. High-level vibration is obtained during up-sweep and resonant frequency varies with the external vibration magnitude (Fig. 3-5b). The softening spring system behaves similarly to the hardening spring system except that the high amplitude vibration is observed during down-sweep.

In addition, as the excitation is increased, the nonlinearity in the system becomes pronounced and the resonant frequency shifts depending on the vibration amplitude. The response shifts to a lower resonant frequency during down-sweep for softening springs and to a higher frequency for hardening springs during up-sweep. Moreover, softening and hardening system responses do not follow the same path for frequency up- and down-sweeps. The bandwidth increases during frequency down-sweep for softening systems and during up-sweep for hardening spring systems.
The responses taking different paths during frequency sweeps is also a test for the presence of nonlinearity in the system.

![Output voltage of the EH for frequency up- and down-sweeps. A sinusoidal input vibration with varying magnitudes of 0.05g, 0.1g, 0.15g, 0.2g, 0.25g, 0.3g, 0.35g, 0.4g was used for a) linear spring system b) hardening spring system c) softening spring system [27].]
3.3 Spring geometry design

Our novel nonlinear spring design is illustrated in Fig. 3-6. Nonlinearity in the force-displacement relationship is introduced by changing the gap between the proof-mass level and the anchor, $H$, while keeping other design parameters constant. Fig. 3-6b-d shows how the gap, $H$ changes the spring design. Nonlinear springs are shown for the gap, $H$ of 10 $\mu$m, 20 $\mu$m and 30 $\mu$m in Fig. 3-6b-d.

3.3.1 Nonlinear spring designs

For nonlinear springs, three different designs were done, each with $k_1 = 15.3$ Nm$^{-1}$. The optimized proof-mass dimensions of 1500 $\mu$m × 300 $\mu$m × 150 $\mu$m and the initial gap of 30 $\mu$m were used. For each case $R$ was kept at 150 M$\Omega$ and the pressure was 0.1 atm.

Nonlinear force-displacement relation in the z- direction was obtained by varying the design parameters $L_z$, $T_z$ and $H$ while keeping $W_z$ constant at 25 $\mu$m. The length of the beam was altered by designing the anchor at different positions with respect to the center of the device, thus

*Fig. 3-6 a) EH with nonlinear springs showing the gap between the proof-mass level and anchor b) Nonlinear spring with $H = 10$ $\mu$m c) Nonlinear spring with $H = 20$ $\mu$m d) Nonlinear spring with $H = 30$ $\mu$m.*
varying $A$. Three different spring designs referred to here as S1, S2 and S3 ($A(300)-L_s(551)-T_s(5)$), ($A(100)-L_s(701)-T_s(5)$) and ($A(350)-L_s(501)-T_s(6)$) were investigated. These values were chosen such that $k_1 = 15.3 \text{ Nm}^{-1}$ is achievable for $H < 50 \mu$m and manufacturable dimensions for the springs. For each, the gap $H$ between the proof-mass and the anchor was varied between 0-50 $\mu$m and the force-displacement curves were obtained using finite element analysis (FEA) in CoventorWare/MEMS+. The curves were then fitted using MATLAB to the nonlinear spring equation of the form $F_s = k_1 z + k_2 z^2 + k_3 z^3$ to obtain the linear, quadratic and cubic stiffness coefficients, $k_1$, $k_2$, and $k_3$, respectively. The stiffness coefficients of the three designs for the variation in $H$ are provided in Table 3-1.

<table>
<thead>
<tr>
<th>No.</th>
<th>$T_s(\mu$m)</th>
<th>$A(\mu$m)</th>
<th>$H(\mu$m)</th>
<th>$L (\mu$m)</th>
<th>$k_1$(N/m)</th>
<th>$k_2$(N/m$^2$)</th>
<th>$k_3$(N/m$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>350</td>
<td>0</td>
<td>$501$</td>
<td>$1.46 \times 10^1$</td>
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<td>5</td>
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<td>$1.53 \times 10^1$</td>
<td>$1.49 \times 10^5$</td>
<td>$2.26 \times 10^{10}$</td>
</tr>
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<td>350</td>
<td>10</td>
<td>$501$</td>
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<td>$2.14 \times 10^5$</td>
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</tr>
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<td>15</td>
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<td>$2.41 \times 10^{10}$</td>
</tr>
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<td>30</td>
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<td>$3.84 \times 10^5$</td>
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<td>50</td>
<td>$501$</td>
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<td>$4.15 \times 10^5$</td>
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<td>0</td>
<td>$551$</td>
<td>$8.04 \times 10^0$</td>
<td>$-4.25 \times 10^2$</td>
<td>$3.31 \times 10^9$</td>
</tr>
<tr>
<td>8</td>
<td>5</td>
<td>300</td>
<td>10</td>
<td>$551$</td>
<td>$8.71 \times 10^0$</td>
<td>$9.29 \times 10^4$</td>
<td>$2.97 \times 10^9$</td>
</tr>
<tr>
<td>9</td>
<td>5</td>
<td>300</td>
<td>20</td>
<td>$551$</td>
<td>$1.05 \times 10^1$</td>
<td>$1.79 \times 10^5$</td>
<td>$2.46 \times 10^9$</td>
</tr>
<tr>
<td>10</td>
<td>5</td>
<td>300</td>
<td>30</td>
<td>$551$</td>
<td>$1.33 \times 10^1$</td>
<td>$2.52 \times 10^5$</td>
<td>$1.65 \times 10^9$</td>
</tr>
<tr>
<td>11</td>
<td>5</td>
<td>300</td>
<td>40</td>
<td>$551$</td>
<td>$1.72 \times 10^1$</td>
<td>$3.10 \times 10^5$</td>
<td>$-9.27 \times 10^7$</td>
</tr>
<tr>
<td>12</td>
<td>5</td>
<td>300</td>
<td>50</td>
<td>$551$</td>
<td>$2.15 \times 10^1$</td>
<td>$3.27 \times 10^5$</td>
<td>$6.04 \times 10^7$</td>
</tr>
<tr>
<td>19</td>
<td>5</td>
<td>100</td>
<td>0</td>
<td>$701$</td>
<td>$4.57 \times 10^0$</td>
<td>$-1.33 \times 10^3$</td>
<td>$2.46 \times 10^9$</td>
</tr>
<tr>
<td>20</td>
<td>5</td>
<td>100</td>
<td>10</td>
<td>$701$</td>
<td>$5.07 \times 10^0$</td>
<td>$7.29 \times 10^4$</td>
<td>$2.27 \times 10^9$</td>
</tr>
<tr>
<td>21</td>
<td>5</td>
<td>100</td>
<td>20</td>
<td>$701$</td>
<td>$6.47 \times 10^0$</td>
<td>$1.42 \times 10^5$</td>
<td>$2.03 \times 10^9$</td>
</tr>
<tr>
<td>22</td>
<td>5</td>
<td>100</td>
<td>30</td>
<td>$701$</td>
<td>$8.79 \times 10^0$</td>
<td>$2.03 \times 10^5$</td>
<td>$1.77 \times 10^9$</td>
</tr>
<tr>
<td>23</td>
<td>5</td>
<td>100</td>
<td>40</td>
<td>$701$</td>
<td>$1.19 \times 10^1$</td>
<td>$2.56 \times 10^5$</td>
<td>$1.39 \times 10^9$</td>
</tr>
<tr>
<td>24</td>
<td>5</td>
<td>100</td>
<td>50</td>
<td>$701$</td>
<td>$1.57 \times 10^1$</td>
<td>$3.01 \times 10^5$</td>
<td>$8.46 \times 10^8$</td>
</tr>
</tbody>
</table>
This unique method of varying $H$ and keeping other spring design parameters constant allows us to obtain nonlinear force-displacement relation in the $z$-direction. Fig. 3-6 depicts the different force-displacement curves obtained through FEA for different values of $H$ for $A = 100 \, \mu m$, $T_s = 5 \, \mu m$ and $L_s = 701 \, \mu m$. $H$ introduces asymmetry in the force-displacement curves. The spring becomes less stiff as the proof-mass moves down and stiffer while moving up. The nature of the spring design inherently develops an axial compressive force as the proof-mass moves down and produces spring softening [60]. This effect becomes prominent with the increase in $H$.

Fig. 3-7 Force-displacement curves of S1. Spring design parameter $H$ changes the force-displacement curves of the spring. $A = 100 \, \mu m$, $T_s = 5 \, \mu m$ and $L_s = 701 \, \mu m$. Nonlinearity is enhanced with increasing height difference between the anchor and the proof-mass.
Since \( H \) also changes \( k_1 \), and our aim is to keep the linear spring constant the same between different nonlinear spring designs, the variation in \( k_1 \) with respect to \( H \) was investigated. Fig. 3-8 displays the plot of linear stiffness against \( H \) for the designs. The intersection of the curves with the 15.3 Nm\(^{-1}\) line provides the gap between the proof-mass and the anchor, \( H \), that is required for each design to attain a linear stiffness of 15.3 Nm\(^{-1}\). For example, S1 (red curve in Fig. 3-8) design requires the height difference between the anchor and proof-mass to be 5 µm.

The quadratic and cubic stiffness coefficients for each spring design are listed in Table 3-2.

**Table 3-2 Stiffness coefficients for the nonlinear springs.**

<table>
<thead>
<tr>
<th>Spring</th>
<th>( k_2 ) (Nm(^{-2}))</th>
<th>( k_3 ) (Nm(^{-3}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>2.85×10(^5)</td>
<td>7.55×10(^8)</td>
</tr>
<tr>
<td>S2</td>
<td>3.00×10(^5)</td>
<td>8.64×10(^8)</td>
</tr>
<tr>
<td>S3</td>
<td>1.50×10(^5)</td>
<td>2.29×10(^10)</td>
</tr>
</tbody>
</table>

Linear spring constant, \( k_1 \) for all the springs is 15.3 Nm\(^{-1}\).
Fig. 3-9 shows the three designs S1, S2 and S3. S1 has a small gap of 5 µm between the level of proof-mass and the anchor (Fig. 3-9a) S2 and S3 has \( H \) of 35 µm and 49 µm respectively. In addition, anchor position and the beam length are different (Fig. 3-9b-c).

3.4 Design of mechanical stoppers
Four mechanical stoppers are placed at the corners of the bottom electrode as depicted in Fig. 3-10, which limits the maximum displacement of the proof-mass. The zoomed-in view shows the rectangular mechanical stoppers and their position on the bottom electrode (Fig. 3-10b). The stoppers are made of Polydimethylsiloxane (PDMS). The low Young’s modulus, \( Y = 870 \text{ kPa} \) [61] makes PDMS an ideal material for stoppers. The mechanical stoppers and the maximum allowable displacement both have influence on the system response. To investigate that, stoppers with a stiffness of 600 Nm\(^{-1}\) and 900 Nm\(^{-1}\) for two stopping distances of 22 µm and 24 µm were designed. The maximum allowable displacement determines...
the stopper thickness, $t_{st} = l_o - z_{max}$. A specific $k_{st}$ value was achieved using Eq. (3-6) by changing the length ($l_{st}$) and width ($w_{st}$) once the thickness ($t_{st}$) was fixed.

$$k_{st} = \frac{4Y_{st}w_{st}l_{st}}{t_{st}} \quad (3-6)$$

*Fig. 3-10 Design of the mechanical stoppers (a) EH with mechanical stoppers. (b) zoomed-in view. (c) stopper design parameters.*
Table 3-3 lists the required stopper dimensions thus calculated to obtain the desired stiffness for stopping distances of 24 µm and 22 µm.

Table 3-3 Mechanical stopper dimensions.

<table>
<thead>
<tr>
<th>Stopper stiffness (Nm⁻¹)</th>
<th>( l_{st} \times w_{st} \times t_{st} ) (µm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( z_{max} = 24 , \mu m ) ( z_{max} = 22 , \mu m )</td>
</tr>
<tr>
<td>600</td>
<td>34.5 × 30 × 6 40 × 34.5 × 8</td>
</tr>
<tr>
<td>900</td>
<td>40 × 39 × 6 50 × 41.5 × 8</td>
</tr>
</tbody>
</table>

The stoppers with stiffness 600 Nm⁻¹ for stopping distances of 24 µm and 22 µm are shown in Fig. 3-11a-b while Fig. 3-10c-d depicts the mechanical stoppers with 900 Nm⁻¹ for the two stopping distances.

Fig. 3-11 Mechanical stoppers a) 600 Nm⁻¹, \( t_{st} = 6 \, \mu m \) b) 600 Nm⁻¹, \( t_{st} = 8 \, \mu m \) c) 900 Nm⁻¹, \( t_{st} = 6 \, \mu m \) d) 900 Nm⁻¹, \( t_{st} = 8 \, \mu m \)
3.5 Modelling of the electrostatic energy harvester with nonlinear springs and mechanical stoppers

The nonlinear springs are modelled using a third order polynomial containing the linear, quadratic and cubic spring constants. A higher order function was not necessary because the third order polynomial perfectly fitted the force-displacement curves obtained from FEA in Coventorware/MEMS+ and captured the behavior of the springs.

Mechanical stoppers limit the maximum displacement of the proof-mass and can prevent short circuit between the two electrodes in the case of high vibrations [2]. The impact between the proof-mass and mechanical stoppers increases the effective stiffness of the system. Therefore, the resonant frequency increases similarly like a hardening spring system [57]. As a result, frequency response is widened.

The impact of the proof-mass with the stopper was modelled as an elastic collision [56]. A stopper force, $F_{st}$ was added to the dynamical model which opposes the proof-mass motion when engaged. The stopper force is proportional to its stiffness and changes the overall system frequency response during collision which happens only when the proof-mass moves more than the maximum allowable displacement.

The dynamic equation with nonlinear springs and mechanical stoppers for the electromechanical system is expressed by:

$$mz'' + k_1z + k_2z^2 + k_3z^3 + F_{st} + b_nz' - F_e(q, z) - mg = -ma(t)$$

$$\frac{dq}{dt} = \frac{1}{R} \left[ V_{bias} - \frac{q}{C_{bias}} - V(q, z) \right]$$

$$F_{st} = \begin{cases} 
0, & z < z_{max} \\
 k_{st}(z - z_{max}), & z > z_{max} 
\end{cases}$$

(3-7)

(3-8)

where, $z_{max}$ is the proof-mass maximum allowable displacement and $k_{st}$ is the stopper stiffness.

EH with only nonlinear springs can be obtained from Eq. 3-7 by omitting the stopper force, $F_{st}$. 


3.6 Summary
In this chapter, the nonlinear spring and mechanical stopper designs are explained. Three nonlinear springs with the same linear stiffness of 15.3 Nm$^{-1}$ are developed. Mechanical stoppers with two different stiffness are designed. Overall dynamic equations with nonlinear springs and mechanical stoppers are derived.
4. Design of array of electrostatic EHs using linear and nonlinear generators

4.1 Introduction
Continuous effort is being made to improve the energy harvester designs to have a broad frequency response. For a single device it is only possible to enhance the frequency operating range by a small margin. Moreover, for a linear spring energy harvester, higher bandwidth often comes with the trade-off between maximum power. In order to obtain high output over a wide frequency range, energy harvester array design has been proposed. Array design consists of numerous generators, each with different dimensions resulting in different resonant frequency. The resonant frequencies of the devices are selected such that the individual energy harvester frequency responses overlap each other. As a result, wideband frequency response is achieved from the system.

An electromagnetic VEHs array connected in series delivered 0.5 µW power with a bandwidth of 300 Hz in [62]. Series and parallel connections of piezoelectric bimorphs with varying aspect ratios were studied in [63]. It was also shown that operating frequency range can be maneuvered using different configurations. Four circular, diaphragm-based piezoelectric generators, each with different tip mass were arranged in an array to obtain a wideband response from 120 Hz to 225 Hz shown in Fig. 4-1a [64]. Peak powers varied between 5.14 mW to 10 mW for an input vibration of 1g. A novel piezoelectric cantilever structure was developed in [65] which delivered power in milli-to-micro Watt range within the frequency range of 300-800 Hz. Upadrashta et al. in [66] developed a piezomagnetoelastic energy harvester array shown in Fig. 4-1b-c. The array consisting of nonlinear piezomagnetoelastic energy harvesters (NPEH) that used micro fiber composite (MFC) showed improvement in bandwidth over linear piezoelectric EH (LPEH) array. The array produced 100 µW power with a bandwidth of 3.3 Hz. Magnetic force of attraction and repulsion were used to introduce the nonlinear response in the energy harvesters. Attractive and repulsive forces were realized by changing the polarity of the fixed magnet. The nonlinear EH design achieved a 100% bandwidth increase compared to the linear one. An array of 8 piezoelectric EHS with unequal cross-sectional area was designed to have a broadband response in [67]. Five natural frequencies were observed within the frequency range of 10 Hz to 240 Hz. The cantilevers were connected such that a polygon like structure was formed as shown in Fig. 4-2. Meruane et al. in [68] developed an interesting method of creating broadband response from energy harvesters. Cantilever beam array was designed with beams connected by springs. The method improved both power and bandwidth of the system.
Fig. 4-1  a) 4 disc-shaped piezoelectric energy harvester array [64])  b) Single piezomagnetoelastic EH with magnet positions to create force of attraction and repulsion. The influence of the magnets shown in the frequency response. c) The array design with attractive and repulsive force. The overall system bandwidth is enhanced [66].
All these above-mentioned works achieved wideband response from the energy harvesting system. However, there are two major shortcomings. First, the power densities of the systems are low due to the large volume required to accommodate enough VEHs in the array to harvest over a large frequency range. Second, each generator in the array uses linear springs which will cause the output power to drop significantly whenever there is a difference between the resonant and the external vibration frequency. Hence, the power density of the entire system will be lower.

Our work overcomes these problems while maintaining a broadband response. An array design of electrostatic VEHs with linear, softening springs, hardening springs and hardening springs with mechanical stoppers is proposed for the first time to enhance the frequency range of the system. The overall volume of the energy harvesting system is greatly reduced using a combination of linear and nonlinear generators in an array. As such, higher power density is achieved compared to array of linear spring VEHs. The use of nonlinear springs ensures the energy harvesting system responds to any shifts in vibration frequency, producing higher power.

4.2 Design of the electrostatic energy harvester arrays
The design consists of 4 electrostatic energy harvesters in parallel configuration. Energy harvesters with a linear spring, softening spring, hardening spring and hardening spring with mechanical stoppers are used in the array. The concept of the array design is depicted in Fig.4-3.

Fig. 4-2 Eight EHs connected in a polygon shaped structure. Each EH has different size, therefore various natural frequency [67].
The individual EH bottom electrodes are all connected to form the array bottom electrode. Similarly, top individual electrodes are connected to create the system top electrode.

Two array designs are proposed in this work. In one of the arrays (array 1), all the EHs have a linear spring constant of 15.3 N/m. That indicates a resonant frequency of 803 Hz at small vibration level. The EHs used in the array are: linear EH described in Chapter 2, two nonlinear EHs using the spring designs S2 and S3 explained in Chapter 3 and one EH with S3 and mechanical stoppers of 900 N/m with 24 µm maximum allowable displacement. This array is developed to create an energy harvesting system with a broadband response around mid-level frequency of 803 Hz.

The second array (array 2) is designed to create an energy harvesting system with a broadband response around a smaller center frequency of 503 Hz. The EHs used in the array are: the linear EH, two nonlinear EHs using the spring designs S4 and S5 and one EH with S5 and mechanical stoppers of 1500 N/m with 24 µm maximum allowable displacement. The details of the EHs and the springs to be used in this array are described in this chapter.
4.3 Modeling of the electrostatic energy harvester array

The 4 EHs are connected in parallel. A parallel configuration allows the gap-closing capacitance formed in the individual EHs to add up. Therefore, the power delivered by the array is the sum of power generated by the individual EHs in the system.

$C_{vn}$ is the variable capacitance that forms across the gap-closing capacitor of the $n$-th VEH in the array. The capacitance $C_{vn}$, formed as the proof-mass is displaced in the $z$-direction, $z_n$ can be written as

$$C_{vn}(z_n) = \frac{\varepsilon_0 L_{pm} W_{pm}}{(l_0 - z_n)} \quad (4-1)$$

The voltage $V_n$, electrostatic energy $U_n$ and force $F_{en}$ are

$$V_n(q_n, C_{vn}) = \frac{q_n}{C_{vn}(z_n)} = q_e \frac{(l_0 - z_n)}{\varepsilon_0 L_{pm} W_{pm}} \quad (4-2)$$

$$U_n(z_n) = \frac{q_n^2}{2C_{vn}(z_n)} \quad (4-3)$$

$$F_{en}(q_n, z_n) = -\frac{dU_n}{dz_n} = \frac{1}{2} \frac{q_n^2}{\varepsilon_0 L_{pm} W_{pm}} \quad (4-4)$$

Squeeze film damping, $b_{mn}$ is given by [44]

$$b_{mn} = \frac{\mu W_{pm}^3 L_{pm}}{(l_0 - z_n)^3} \beta \left( \frac{W_{pm}}{L_{pm}} \right) \quad (4-5)$$

Using the above expressions, the dynamic equations of electrostatic VEH are derived.

$$m_n \ddot{z}_n + k_1 z_n + k_2 z_n^2 + k_3 z_n^3 + F_{st} + b_{mn} \dot{z}_n - F_{en}(q_n, z_n) - m_n g = -m_n a(t) \quad (4-6)$$

$$\frac{dq_n}{dt} = \frac{1}{R} \left[ V_{bias} - \frac{q_n}{C_{bias}} - V_n(q_n, z_n) \right] \quad (4-7)$$

$$F_{st} = \left\{ \begin{array}{ll} 0, & z_n < z_{max} \\ k_{st} (z_n - z_{max}) & z_n > z_{max} \end{array} \right\} \quad (4-8)$$
It is worth mentioning that the stopper force is only present with one EH in the array. Three of the four EHs have nonlinear springs.

Total average power $P_{avg}$, delivered to the load by the array between the time interval $(t_2 - t_1)$ is expressed by:

$$P_{avg} = \frac{1}{t_2 - t_1} \sum_{i=1}^{4} \left( \frac{dq_{ix}}{dt} \right)^2 R \, dt (4.9)$$

4.4 Design and optimization of a linear energy harvester with 503 Hz resonant frequency

A linear spring EH with resonant frequency of 503 Hz is designed and optimized in a similar way as explained in Chapter 2. The VEH is optimized for an input vibration source of $9.8 \sin(2\pi ft) \text{ m/s}^2$ at first. Mid-level frequency, $f$ of 503 Hz is selected for the vibration source.

Table 4-1 illustrates the output power of the VEH for different proof-mass dimensions at 1 atm. and 0.1 atm. As before, a thicker proof-mass produced more power than a wider one. Fig. 4-4 indicates the output power improves with increase in the initial gap between the electrodes. Resonant frequency of 503 Hz is observed for all load values as shown in Fig. 4-5. A 250 MΩ load resistance is selected as the rate of increase of power decreases at higher load values.

Table 4-1 Average power generated at 1 atm. and 0.1 atm. for different combination of proof-mass dimensions. $l_o = 30\mu m$, $a_v = 9.8 \text{ m/s}^2$, $f_r = 503 \text{ Hz}$, $R = 250 \text{ M\Omega}$.

<table>
<thead>
<tr>
<th>$L_{pm} , (\mu m)$</th>
<th>$W_{pm} , (\mu m)$</th>
<th>$T_{pm} , (\mu m)$</th>
<th>$P_{avg} , (W)$ 1 atm.</th>
<th>$P_{avg} , (W)$ 0.1 atm.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1500</td>
<td>300</td>
<td>150</td>
<td>9.6×10^{-9}</td>
<td>4.2×10^{-8}</td>
</tr>
<tr>
<td>1500</td>
<td>600</td>
<td>75</td>
<td>1.3×10^{-9}</td>
<td>2.1×10^{-8}</td>
</tr>
<tr>
<td>1500</td>
<td>900</td>
<td>50</td>
<td>2.7×10^{-10}</td>
<td>1.0×10^{-8}</td>
</tr>
<tr>
<td>1500</td>
<td>1200</td>
<td>37.5</td>
<td>8.8×10^{-11}</td>
<td>5.2×10^{-9}</td>
</tr>
<tr>
<td>1500</td>
<td>1500</td>
<td>30</td>
<td>4.2×10^{-11}</td>
<td>3.0×10^{-9}</td>
</tr>
</tbody>
</table>
Fig. 4-4 Average output power of the linear EH increases with initial gap between the electrodes. The curve is obtained for $L_{pm} = 1500 \, \mu m$, $W_{pm} = 300 \, \mu m$, $T_{pm} = 150 \, \mu m$, $a_v = 9.8 \, ms^2$, $f_r = 503 \, Hz$ and $R = 250 \, M\Omega$ at 0.1 atm.

Fig. 4-5 Average output power of the linear EH as the frequency and the load resistance are varied. The curve is obtained for $L_{pm} = 1500 \, \mu m$, $W_{pm} = 300 \, \mu m$, $T_{pm} = 150 \, \mu m$, $l_o = 30 \, \mu m$, $a_v = 9.8 \, ms^2$, at 0.1 atm.
Therefore, proof-mass dimensions of 1500 µm × 300 µm × 150 µm, 250 MΩ load resistance, 0.1 atm ambient pressure, 30 µm initial gap, 20 V \( V_{bias} \) and 1×10-12 F \( C_{bias} \) are chosen for the design. Pull-in voltage of 110 V for the design was much higher than the \( V_{bias} \) used. A linear spring \((A(0)-L_s(751)-T_s(7)-H(0))\) with spring constant of 6 N/m produced the desired resonant frequency. The optimized system harnessed an average output power of 42 nW.

4.4.1 Design of nonlinear springs with 6 N/m linear stiffness

Two nonlinear springs with 6 N/m spring constant are developed following the design procedure explained in Chapter 3. Two different spring designs referred to here S4 with design specifications \((A(0)-L_s(851)-T_s(5))\) and S5 with \((A(0)-L_s(801)-T_s(6))\) are investigated.

![Fig. 4-6 Linear spring constants of S4, S5 changes with H. For each, the H value yielding \( k_1 = 6 \text{ Nm}^{-1} \) is noted.](image)

The variation of the linear spring constant with \( H \) is shown in Fig. 4-6. The two curves intersected the 6 N/m line at 12 and 36. Therefore, \( H \) values of 12 µm and 36 µm are used to design the nonlinear springs.

Table 4-2 shows the spring stiffness coefficients.
4.4.2 Design of mechanical stoppers with 1500 N/m stiffness
Four PDMS mechanical stoppers with total 1500 N/m stiffness are designed. Desired stopper stiffness was achieved for \( t_\text{st} = 6 \, \mu\text{m} \), \( w_\text{st} = 50 \, \mu\text{m} \) and \( l_\text{st} = 52 \, \mu\text{m} \) for a stopping distance of 24 \( \mu\text{m} \). This stopper is used in the second array design.

4.5 The two arrays of linear and nonlinear energy harvesters
The array of generators with a center frequency of 503 Hz is depicted in Fig. 4-7. It consists of softening spring EH (S-EH), linear spring EH (L-EH), hardening spring EH (H-EH) and

<table>
<thead>
<tr>
<th>Spring</th>
<th>( k_1 ) (N/m)</th>
<th>( k_2 ) (N/m(^2))</th>
<th>( k_3 ) (N/m(^3))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td>6</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>S4</td>
<td>6</td>
<td>1.26\times10^5</td>
<td>1.00\times10^9</td>
</tr>
<tr>
<td>S5</td>
<td>5.93</td>
<td>7.82\times10^4</td>
<td>2.40\times10^9</td>
</tr>
</tbody>
</table>

Fig. 4-7 Design of electrostatic energy harvester array (array 2) with linear and nonlinear springs with linear spring constant of 6 N/m.
hardening spring with mechanical stopper EH (H-EH+M$_{st}$).

Fig. 4-8 shows the array of EHs for a center frequency of 803 Hz.

**Table 4-3** shows the spring design used in the EHs and their expected behavior in the array design.

**Table 4-3 Energy harvesters used in the array designs.**

<table>
<thead>
<tr>
<th>Design</th>
<th>L-EH</th>
<th>S-EH</th>
<th>H-EH</th>
<th>H-EH+M$_{st}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Array 1</td>
<td>Linear (803Hz)-15.3 N/m</td>
<td>S2</td>
<td>S3</td>
<td>S3+M$_{st, 900}$</td>
</tr>
<tr>
<td>Array 2</td>
<td>Linear (503Hz)-6 N/m</td>
<td>S4</td>
<td>S5</td>
<td>S5+M$_{st, 1500}$</td>
</tr>
<tr>
<td>Behavior</td>
<td>Linear</td>
<td>Softening</td>
<td>Hardening</td>
<td>Hardening + stoppers</td>
</tr>
</tbody>
</table>
4.6 Summary
In this chapter, array design to achieve broadband response from energy harvesting system is explained. The dynamic equations are derived. Linear and nonlinear EHs with 6 N/m linear spring constant are developed. Two array designs with center frequencies of 503 Hz and 803 Hz are explained. The results are presented in Chapter 5.
5. Simulation results

5.1 Introduction
The primary objective of this work was to develop wideband EHs. To achieve the purpose, at first, linear electrostatic EHs are designed. The linear EH formed the basis of the novel nonlinear design. Then novel nonlinear springs and mechanical stoppers are developed to create broadband EHs. In addition, arrays are established using the linear and nonlinear EHs to harvest over a wide frequency range otherwise not possible with a single nonlinear device.

This chapter provides the simulation results. Frequency response of the EHs is obtained. The linear and nonlinear springs are tested with different signals. Sinusoidal and chirp signals are used as the external vibration source. All simulation results showed performance improvement in terms of full width at half maximum (FWHM) bandwidths and output power for nonlinear spring EHs compared to the linear EHs. Stress analysis on the beams and mechanical stoppers are performed. FEA analysis showed that the maximum stress on the beams and the mechanical stoppers are well within the maximum yield strength of the respective material. The performance of EHs developed in this work is also compared with state-of-art designs.

5.2 Simulation of EHs with 15.3 N/m linear spring constant
5.2.1 Frequency sweeps
Frequency sweeps were done on the EHs developed in this work with a sinusoidal input vibration of magnitude 1g. Frequency up-and down-sweeps on EHs help to see the bandwidth widening of EHs due to nonlinear springs. Fig. 5-1 depicts the frequency sweeps on EHs with S1 S2, S3 and linear spring with 15.3 N/m spring constant at 1atm. ambient pressure. The presence of nonlinear spring force brings upon interesting changes in the behavior of the EHs. Unlike linear EHs, the resonant frequency of the nonlinear EHs becomes a function of the vibration amplitude and spring constants [59]. For example, the resonant frequency shifts to lower values for softening springs and higher values for hardening springs [54]. Moreover, the increase or decrease of external vibration frequency from the resonant frequency of the device determines if the system will produce low-or-high-level oscillation for a spring type [26]. The dotted lines represent the frequency down-sweeps and bold lines frequency up-sweeps. During the down-sweep, the resonant frequency shifts to lower values indicating spring softening with S1 and S2. The system produces much lower power due to the low-level vibration which is an inherent...
property of the softening springs during frequency up-sweeps. On the other hand, the resonant frequency shifts to high values during frequency up-sweep for S3 while producing much lower power during frequency down-sweep indicates spring hardening. These variations in the responses during frequency sweeps indicate the presence of mechanical nonlinearity due to the geometry of the springs. However, the response of the linear EH is the same for both up-and down-sweeps as the resonant frequency of the linear system does not depend on the direction of frequency shifts in the external vibration source.

For all the nonlinear EHs, the bandwidth is higher than the linear EH as shown in Table 5-1. Average power of nonlinear spring EHs is also found to be more than the linear EHs as indicated by the ratio between average power of nonlinear EH to linear EH in Table 5-1.

![Graph showing frequency up-and down-sweeps on EHs at 1 atm.](image)

*Fig. 5-1 Frequency up-and down-sweeps on EHs at 1 atm.*

<table>
<thead>
<tr>
<th>System</th>
<th>FWHM bandwidth (Hz)</th>
<th>Average power/average power of linear system</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 atm.</td>
<td>0.1 atm.</td>
</tr>
<tr>
<td>Linear</td>
<td>15.7</td>
<td>11</td>
</tr>
<tr>
<td>S1</td>
<td>28.6</td>
<td>62</td>
</tr>
<tr>
<td>S2</td>
<td>31.8</td>
<td>82</td>
</tr>
<tr>
<td>S3</td>
<td>47.8</td>
<td>52</td>
</tr>
</tbody>
</table>

*Table 5-1 FWHM bandwidth and average harvested power.*
The performances of the EHs are evaluated at 0.1 atm ambient pressure as well. Fig. 5-2 shows the responses. The maximum power and bandwidth for each EHs increased significantly compared to 1atm. pressure condition. At low ambient pressure, the proof-mass oscillates with higher amplitude due to less mechanical damping in the system. Higher displacement increases the effect of nonlinearity and hence the bandwidth is increased. The bandwidth and average power are given in Table 5-1. The full width at half maximum (FWHM) bandwidth of the linear EH is 11 Hz. The use of the nonlinear springs increased the bandwidth to 62 Hz, 82 Hz and 52 Hz for S1, S2 and S3 respectively, with the corresponding maximum harvested power of 54.6 nW, 52.2 nW and 74 nW.

![Figure 5-2 Frequency up- and down-sweeps for S1, S2 and S3. Power generated by softening springs during up-sweep is much less than the down-sweep which makes it hard to see in the figure.](image)

Fig. 5-2 Frequency up- and down-sweeps for S1, S2 and S3. Power generated by softening springs during up-sweep is much less than the down-sweep which makes it hard to see in the figure. $\ell_{pm} = 1500 \mu m$, $w_{pm} = 300 \mu m$, $\tau_{pm} = 150 \mu m$, $a_v = 9.8 \text{ ms}^{-2}$, and $R = 150 \text{ M}\Omega$ at 0.1 atm.

5.2.2 Response to chirp signal
A chirp signal is a frequency-swept cosine signal given by Eq. 5-1.

\[
y(t) = a_v \cos(2\pi f(t)t) \\
f(t) = kt + f_o; k = \frac{f'_i - f_o}{T}
\]  

(5-1)
where, $a_v$ is the magnitude, $T$ is the time duration of the signal, $f_0$ and $f_1$ are the starting and ending frequency of the signal. Different systems are simulated with a chirp signal of magnitude 1g and the displacement responses are depicted in Fig. 5-3. Real life vibration sources may not behave like a chirp signal. However, nonlinear spring systems are often tested with this signal [69] because the change in frequency in the vibration signal instigates the mechanical nonlinearity. Hence, the effect of nonlinear springs can be monitored.

![Graphs showing displacement vs frequency for different springs](image)

*Fig. 5-3 Frequency up-and down-sweeps of linear and nonlinear EHs to chirp. a) linear b) EH with S1 c) EH with S2 d) EH with S3 spring.*

For both up-and down-sweeps of frequency, maximum amplitude is observed at 803 Hz for the linear spring EH as shown in Fig. 5-3a. EHs with softening spring designs S1 and S2 produce high vibration level over a wider frequency range during frequency down-sweep (Fig. 5-3b-c). The EHs oscillate with lower vibration amplitude during frequency up-sweep. Hardening spring design S3, maintains higher amplitude during frequency up-sweep while broadening the frequency operating range, (Fig. 5-3d). Frequency down-sweep is accompanied by low-level oscillation. Compared to linear EH, the nonlinear EHs utilizing spring designs S1, S2 and S3 produced higher vibration amplitude over a wider frequency range. Consequently, higher output power will be generated by the nonlinear EHs compared to the linear EH.
5.3 Simulation of EHs with 6 N/m linear spring constant
The responses of the EHs with 6 N/m linear spring, S4 and S5 are shown in Fig. 5-4. S4 behaves as a softening spring, producing maximum power at a lower resonant frequency than 503 Hz. S5 is a hardening spring, which shifts the resonant frequency to higher values compared to linear EH resonant frequency during frequency up-sweep. Linear EH maintains the same response during both frequency up-and down-sweeps. S4 and S5 produces softening and hardening behaviors respectively. However, not much is gained in terms of bandwidth and power compared to linear EH as shown in Table 5-2. These springs, in addition to mechanical stoppers are used in array 1 to create wideband energy harvesting system.

![Graph showing power vs. frequency for S4 and S5 with 6 N/m linear spring constant](image)

**Fig. 5-4** Frequency up-and down-sweeps on EHs with 6N/m linear spring constant, S4 and S5 at 0.1 atm.

<table>
<thead>
<tr>
<th>System</th>
<th>FWHM Bandwidth (Hz)</th>
<th>Average power/average power of linear system</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td>19</td>
<td>1.0</td>
</tr>
<tr>
<td>S4</td>
<td>22</td>
<td>1.4</td>
</tr>
<tr>
<td>S5</td>
<td>20</td>
<td>1.3</td>
</tr>
</tbody>
</table>

*Table 5-2 Bandwidth and average power at 0.1 atm.*
5.4 Simulation of EHs with S3 and mechanical stoppers
The combined effect of hardening spring and mechanical stoppers is investigated. The influence of mechanical stoppers with different stiffness and maximum allowable displacement on the bandwidth of the EHs is also studied. Design S3 is a hardening spring as higher power with increasing resonant frequency is observed during frequency up-sweep. For this reason, mechanical stoppers are designed and integrated only to the EH with S3. Fig. 5-5 shows the frequency response of the EH with S3, with and without mechanical stoppers of two different stiffnesses (600 Nm\(^{-1}\) and 900 Nm\(^{-1}\)). Each stopper is designed for two thicknesses \(t_{st} = 6 \mu m\) and \(t_{st} = 8 \mu m\) which correspond to stopping distances of 24 \(\mu m\) \((t_{st} = 6 \mu m)\) and 22 \(\mu m\) \((t_{st} = 8 \mu m)\), respectively. For both values of stopping distance, the maximum power is more for a less stiff stopper \(k_{st} = 600 \text{ Nm}^{-1}\). However, FWHM bandwidth is more for the stiffer stopper.

![Graph showing frequency response of EHs with S3 and mechanical stoppers](image)

**Fig. 5-5 Impact of mechanical stoppers and stopping distances on frequency response.** EHs with S3 and mechanical stoppers produce a considerable bandwidth enhancement. Power generated by the EHs during the down-sweeps are the same for all the EHs and are indistinguishable from zero in this scale. \(L_{pm} = 1500 \mu m, W_{pm} = 300 \mu m, T_{pm} = 150 \mu m, a_v = 9.8 \text{ m} \text{s}^{-2}, \text{ and } R = 150 M\Omega \text{ at } 0.1 \text{ atm.} \)
\( k_{st} = 900 \text{ Nm}^{-1} \). For a stopping distance of 22 \( \mu\text{m} \), the proof-mass hits the stopper with higher velocity and can compress the less stiff stopper more. This causes the gap between the capacitor plates to be less in case of the less stiff stopper. Hence the maximum power is more for \( \text{H-EH+M}_{st, 600} \) than \( \text{H-EH+M}_{st, 900} \). Moreover, the proof-mass gains energy from the impact and oscillates more increasing the bandwidth of the device. For all cases, the resonant frequency increases after impact, increasing the bandwidth.

\[ 0 \quad 10 \quad 20 \quad 30 \quad 40 \quad 50 \quad 60 \quad 70 \quad 80 \]
\[ 600 \quad 700 \quad 800 \quad 900 \quad 1000 \quad 1100 \quad 1200 \quad 1300 \quad 1400 \]

\( \text{Power (nW)} \)

\( \text{Frequency (Hz)} \)

\[ 0.2g \quad 0.4g \quad 0.6g \quad 0.8g \quad 1g \quad 1.2g \]

**Fig. 5-6** Impact of input vibration magnitude on bandwidth. Bandwidth of the EH with S3 design increases with increasing magnitude of vibration for frequency up-sweep. \( k_{st} = 900 \text{ Nm}^{-1} \), \( \zeta_{\text{max}} = 24 \mu\text{m} \), \( L_{\text{pm}} = 1500 \mu\text{m} \), \( W_{\text{pm}} = 300 \mu\text{m} \), \( T_{\text{pm}} = 150 \mu\text{m} \), and \( R = 150 \text{ M\Omega} \) at 0.1 atm..

5.4.1 Response of EH with S3 and mechanical stoppers to varying input vibration magnitude

The frequency up-sweep responses to varying input vibration magnitudes of \( \text{H-EH+M}_{st, 900} \) are depicted in Fig. 5-6 for a maximum allowable displacement of 24 \( \mu\text{m} \). At low levels of input vibration (0.2g, 0.4g) when the stopper is not engaged, the response follows that of the nonlinear EH without a stopper. Above the vibration magnitude of 0.6g, the proof-mass impacts the stopper and the resonant frequency starts to increase. For higher magnitudes of vibration, the resonant frequency increases further, and bandwidth is widened.
5.5 Simulation of EH arrays

5.5.1 Frequency sweeps on array 1

Array 1 was designed using linear and nonlinear springs with 15.3 N/m linear spring constant. Responses of the individual EHs in array 1 are shown in Fig. 5-7. Frequency up-and down-sweeps with sinusoidal signal of 1g magnitude for array 1 are depicted in Fig. 5-8. The peaks near the 600 Hz and 803 Hz are due to the softening spring and linear spring EHs (S-EH and L-EH), respectively. A large peak occurs between 1000 Hz to 1100 Hz due to hardening spring and hardening spring with mechanical stoppers (H-EH and H-EH+M_{st,900}). The peak around 1350 Hz is due to the EH with hardening spring and mechanical stoppers (H-EH+M_{st,900}). Linear EH response is superimposed on the entire frequency range (600 Hz-1350 Hz). 37 linear EHs would be required to harvest over the frequency range as opposed to only 4 EHs proposed by our design. Due to the nature of the softening and hardening springs (S2 and S3), the response of the EHs shifts away from the linear EH. Thus, the array does not produce significant power around 803 Hz. This array design reduces the overall volume of the system by 89%.

![Graph showing frequency sweeps on array 1](image)

*Fig. 5-7 Responses of individual EHs in array 1.*
5.5.2 Frequency sweeps on array 2
Array 2 was designed with all EHs having a linear spring constant of 6 N/m. Which corresponds to a 503 Hz resonant frequency at small vibration of the proof-mass. Nonlinear spring S4 and S5 are used. One of the 4 EHs was designed with S5 and mechanical stopper with stiffness 1500 N/m and a maximum allowable displacement of 24 µm. The individual responses of the EHs in the array are shown in Fig. 5-9. The overall response of the array 2 is depicted in Fig. 5-10. The maxima for the softening spring, linear spring and hardening spring EHs occur at around 450 Hz, 503 Hz and 550 Hz, respectively. The EH (H-EH+M_{st,1500}) with mechanical stoppers and hardening springs produces maximum power at 760 Hz. Response of linear EHs reproduced over the frequency range indicates 16 linear EHs would have been required to harvest over the same frequency range (400 Hz – 760 Hz). That is a volume reduction by 75 % of the entire system compared to a linear EHs array.
Fig. 5-9 Responses of individual EHs in array 2.

Fig. 5-10 Frequency up-and down-sweeps of array 2.
5.6 Stress analysis on springs S1, S2, S3, S4 and S5
Stress analysis was also performed to confirm that the maximum stress is within the yield strength of nickel, the material for the proof-mass and the springs. The proof-mass is displaced by 30 µm which is the maximum displacement possible in the downward direction before hitting the bottom electrode. Subsequently, the maximum stresses on the springs are found, (Fig. 5-11 and Fig. 5-12). The maximum von Mises stress in S1 design is 390 MPa; in S2 design it is 360 MPa and in S3 design it is 350 MPa. Maximum von Mises stress in 6 N/m linear spring is 250 MPa; in S4 design it is 190 MPa and in S5 design it is 140 MPa. In all the cases, the maximum stress is below the yield strength of nickel [46]. Quarter of the device was used for simulation purposes using the symmetry property in CoventorWare® to reduce the simulation time.

Fig. 5-11 Maximum stress observed at the junction of the beam and the anchor. Maximum von mises stress of 390 MPa, 360 MPa and 350 MPa are observed for EH with a) S1 b) S2 and c) S3, respectively.
5.7 Stress analysis on mechanical stoppers

Stress analysis was also performed on the mechanical stoppers to ensure they can withstand the force imparted by the proof-mass during collision. Displacement versus frequency curves indicated maximum deformations of 2 µm and 2.7 µm of the stoppers after impact for the case of 24 µm and 22 µm stopping distances, respectively for S3. For this reason, the stoppers with stopping distances of 24 µm and 22 µm were compressed by 2 µm and 3 µm, respectively and the generated stress was determined using CoventorWare® (Fig. 5-13a-d). For H-EH+M_{st,600}, the maximum von Mises stresses are 1.2 MPa and 0.95 MPa for $z_{\text{max}}$ of 24 µm and 22 µm, respectively. Maximum stresses of 1.4 MPa and 1 MPa are observed for stopping distances of 24 µm and 22 µm, respectively in H-EH+M_{st,900}. The proof-mass after impact with the mechanical

Fig. 5-12 Maximum stress on EH with a) 6 N/m linear spring is 250 MPa b) S4 is 190 MPa and c) S5 is 140 MPa.
stoppers in H-EH+M\textsubscript{st,1500} (one of the EHs in array 2) was deformed by 2.6 µm. This stopper when compressed by 2.6 µm, incurred a maximum stress of 1.6 MPa. The maximum stress on the stoppers in all the cases is less than the compressive strength of PDMS [70].

**Fig. 5-13** Mechanical stopper with (a) $k_{st} = 600 \text{ Nm}^{-1}$, $l_g = 6 \text{ µm}$ (b) $k_{st} = 900 \text{ Nm}^{-1}$, $l_g = 6 \text{ µm}$ (c) $k_{st} = 600 \text{ Nm}^{-1}$, $l_g = 8 \text{ µm}$ (d) $k_{st} = 900 \text{ Nm}^{-1}$, $l_g = 8 \text{ µm}$ (e) $k_{st} = 1500 \text{ Nm}^{-1}$, $l_g = 6 \text{ µm}$. The maximum von Mises stress in each case is much less than the compressive yield strength of PDMS.
5.8 Comparison of nonlinear spring EHs with the state-of-art designs

The EH designs reported in this work are compared with the state-of-art designs in Table 5-3. The linear EH achieved a power density of $7.2 \times 10^2 \mu W/cm^3$ with FWHM of 11 Hz. EH with softening S2 spring design produced a maximum power density of $6.3 \times 10^2 \mu W/cm^3$ and FWHM bandwidth of 82 Hz. The combination of hardening S3 spring design and mechanical stopper (H-EH+Ms_{st=900}) for a stopping distance of 22 µm achieved the highest FWHM bandwidth of 231 Hz and maximum power of 74 nW over the frequency range of 1122-1353 Hz.

Nguyen in [71] developed bistable springs based on curved beams. The proof-mass of the electrostatic EH with comb-drive capacitors had a displacement range of -100 µm to 22 µm in the lateral direction. The use of 1700 µm curved beams produced a bandwidth of 587 Hz. However, such long beams increased the overall size of the device. Furthermore, high electret voltage of 150 V required for the system to work can be hard to achieve.

The large proof-mass (9 mm × 2 mm ×1.5 mm) of the electrostatic EH with stoppers worked with small vibration of 0.08g in [20]. It produced a very high acceleration-normalized power density of 45.2 µWs$^4$/cm$^3$-m$^2$. However, the device with the proof-mass and springs is large compared to MEMS standard. The broadband EH with end-stops working in the frequency range of 1135 Hz -1475 Hz produced a power density of 0.022 µWs$^4$/cm$^3$-m$^2$ [24]. The mechanical stoppers significantly increased the bandwidth of the device with 5.5g input vibration signal.

These notable results were of pioneering nature in wideband energy harvesting and formed the basis for the work reported here. Nevertheless, there were some drawbacks, such as, requirement of larger proof-mass than MEMS standard, high electret voltage, very long springs or large input vibration signal. The size of the EHs developed in this work is significantly smaller. The new spring designs introduced here achieve nonlinear force-displacement relationships within a small proof-mass displacement range of ± 30 µm. Moreover, the EH can operate with a nominal electret voltage of 20 V and input vibration signal of 1g.
Table 5-3 Performance comparison with the state-of-art designs.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Method</th>
<th>Transduction</th>
<th>Vibration magnitude $v_a$ (m/s²)</th>
<th>Volume (cm³)</th>
<th>Power (µW)</th>
<th>Power density-PD (µW/cm³)</th>
<th>PD/ $\alpha_v^2$ (µWs/cm³·m²)*</th>
<th>Bandwidth (Hz)*</th>
<th>Operating frequency range (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>71</td>
<td>Softening</td>
<td>Electrostatic</td>
<td>9.8</td>
<td>1.50×10⁻²</td>
<td>3.40×10⁰</td>
<td>2.26×10²</td>
<td>2.353</td>
<td>587</td>
<td>380-967</td>
</tr>
<tr>
<td>72</td>
<td>Stoppers</td>
<td>Piezoelectric</td>
<td>7.8</td>
<td>5.52×10⁻³</td>
<td>8.80×10⁻¹</td>
<td>1.59×10²</td>
<td>2.593</td>
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<td>12-26</td>
</tr>
<tr>
<td>73</td>
<td>Stoppers</td>
<td>Piezoelectric</td>
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<td>1.70×10¹</td>
<td></td>
<td></td>
<td></td>
<td>45</td>
<td>460-505</td>
</tr>
<tr>
<td>74</td>
<td>Stoppers</td>
<td>Electrostatic</td>
<td>9.8</td>
<td>4.20×10⁻²*</td>
<td>2.00×10⁰</td>
<td>4.76×10¹*</td>
<td>0.496</td>
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</tr>
<tr>
<td>75</td>
<td>Stoppers</td>
<td>Electrostatic</td>
<td>0.78</td>
<td>3.60×10¹*</td>
<td>1.00×10³</td>
<td>2.78×10¹*</td>
<td>45.29</td>
<td>9</td>
<td>86-95</td>
</tr>
<tr>
<td>76</td>
<td>Stoppers</td>
<td>Electrostatic</td>
<td>53.9</td>
<td>1.92×10⁻³*</td>
<td>1.25×10¹</td>
<td>6.51×10¹*</td>
<td>0.022</td>
<td>340</td>
<td>1135-1475</td>
</tr>
<tr>
<td>77</td>
<td>Stoppers</td>
<td>Piezoelectric</td>
<td>9.8</td>
<td>1.60×10⁰</td>
<td>1.60×10⁻¹</td>
<td>1.00×10⁻¹</td>
<td>0.001</td>
<td>32</td>
<td>228-260</td>
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<td>2.70×10⁻²</td>
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<td>6.00×10⁰</td>
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<td>8.33×10⁻⁵</td>
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<td>7.20×10⁻¹</td>
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</table>

* Calculated from the information provided in the paper.

Table 5-4 Performance comparison with the state-of-art array designs.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Transduction</th>
<th>Vibration magnitude $v_a$ (m/s²)</th>
<th>Volume (cm³)</th>
<th>Power (µW)</th>
<th>Power density-PD (µW/cm³)</th>
<th>PD/ $\alpha_v^2$ (µWs/cm³·m²)*</th>
<th>Operating frequency range (Hz)</th>
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<td>450-760</td>
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* Calculated from the information provided in the paper.
5.9 Performance comparison of array design with the state-of-art
Several works attempted array design of VEHs mostly with individual VEHs utilizing piezoelectric or electromagnetic transduction methods. This work develops for the first time an array of electrostatic VEHs. Furthermore, array design with nonlinear spring generators has never been tried before. Some of the key features of this design are compared with other existing array designs for wideband energy harvesting in Table 5-4.

5.10 Summary
Simulation results indicated that softening and hardening spring forces can be obtained utilizing the spring design technique developed in this work. The nonlinear springs produced wideband frequency response. The energy harvester with hardening spring design S3 utilizing a mechanical stopper (H-EH+M_{st,900}) produced the highest FWHM bandwidth of 231 Hz. In addition, array using linear and nonlinear EHs produced broadband frequency response from the energy harvesting system. Array design 1 can work over a broad 600 Hz to 1350 Hz frequency range and produce maximum power of 110 nW.
6. Conclusion and future work
Vibrational energy harvesters (VEHs) have the potential to replace batteries in wireless sensor networks (WSNs). These devices can work in environments where harvesting energy from sun, light, temperature, pressure, etc. is not possible. In a hybrid system, VEH can supply power in the absence of other sources. The energy scavenging nature of VEHs opens the possibility of making completely wireless and self-sustainable systems. Moreover, VEHs are environmentally friendly. All these features made VEHs an important research topic.

MEMs VEHs capture energy from the environment mainly following three transduction mechanisms; piezoelectric, electromagnetic and electrostatic. MEMs electrostatic EHs form vibration-induced capacitor that converts the vibration energy to electrical energy. This work develops the design of an electrostatic EH.

Two electret-based, out-of-plane, gap-closing electrostatic energy harvesters with natural frequency of 803 Hz and 503 Hz are developed in this work. The EH with resonant frequency of 803 Hz produced 7.5 \( \mu \text{Ws}^4/\text{cm}^3\cdot\text{m}^2 \) acceleration-normalized power density while the EH with 503 Hz produced 5.3 \( \mu \text{Ws}^4/\text{cm}^3\cdot\text{m}^2 \) for 1g sinusoidal signal at 0.1 atm. ambient pressure. The design utilized a small bias voltage and capacitance of 20 V and \( 1\times10^{-12} \) F, respectively. Optimal load resistance of 150 M\( \Omega \) and 250 M\( \Omega \) are used in the designs. High power densities are obtained from the designs by utilizing thicker proof-mass instead of wider ones. Squeeze-film damping is reduced significantly which allowed the proof-mass to oscillate with a larger amplitude, leading to greater capacitance change. Reduced volume was attained because of the spring design. The springs are tucked around the proof-mass which lowered the overall volume of the device.

VEHs are usually custom designed for a vibration source. Moreover, the randomness of the vibration source is often neglected. For a VEHs to be functional in real world applications of robotics, defense, medical, structural health monitoring, aerospace and wearables, it must be able to work over a wide frequency range to account for the randomness of the vibration source. Linear spring EHs lose efficiency as a shift in the vibration source frequency. To account for this problem, frequency operating range of a device is enhanced by designing novel nonlinear springs and mechanical stoppers.
Nonlinear springs can alter the frequency response of a mass-spring system and therefore increase the bandwidth. In this work, completely new springs are developed to introduce nonlinear force-displacement behavior in the vertical direction of proof-mass motion. A simple technique of introducing height gap between the proof-mass and the anchor causes asymmetric spring force. Force-displacement curves are obtained by FEA using Coventorware/MEM+® and different stiffness coefficients are obtained by fitting the curves to a 3\textsuperscript{rd} order polynomial using MATLAB. Several spring designs are studied. Following the method developed in this work, two softening spring designs (S1 and S2) produced 464\% and 646\% increase in FWHM with only 9\% and 13\% decrease in maximum power respectively compared to the linear EH.

Mechanical stoppers are also designed. Mechanical stoppers change the frequency response of the system when the proof-mass moves beyond a maximum allowable displacement. It has the same effect as hardening springs. Mechanical stoppers are implemented with hardening springs to further improve the system bandwidth. Both maximum allowable displacement and stopper stiffness influence the bandwidth of the EH. Mechanical stoppers with 900 N/m stiffness and maximum allowable displacement 22 µm implemented with the hardening spring design, S3 further enhanced the FWHM by 2000\% with a 23\% increase in maximum power compared to the 15.3 N/m linear spring EH.

Bandwidth of a EH can be enhanced only by a limited amount utilizing nonlinear springs and mechanical stoppers. To be able to harvest over a broad frequency range, EH arrays are developed. Unlike conventional array design formula, a new method is proposed. An array with linear EH, softening spring EH, hardening spring EH and hardening spring EH with mechanical stoppers is proposed. Parallel connection between the EHs ensures the total power is a collection of individual powers. All EHs in the array have the same linear spring constant indicating same resonant frequency at small vibration level. Softening and hardening spring energy harvesters respond to any shift in the excitation frequency. Linear energy harvester produces maximum power when there is no shift in the excitation frequency while mechanical stoppers farther widen the bandwidth of the hardening spring energy harvester. The ability of this energy harvesting system to respond to randomness and shift in the excitation frequency makes it ideal for use with real world vibration sources. Array 1 design generated maximum acceleration-normalized power density of 1.97 µWs\(^4\)/cm\(^3\)-m\(^2\) with a frequency operating range of 600 Hz to 1350 Hz. Array 2
operated within 450 Hz-760 Hz and produced 0.97 μWs/ cm²-m² power density. The array design method significantly reduced the system volume compared to the linear EHs array.

Nonlinear spring design technique developed in this work allows to design nonlinear springs with any linear spring constant meaning wideband EHs around any center frequency. By simply changing the beam length, thickness and $H$, the purpose can be achieved.

As part of the future work, the developed model can be tested with some real-world vibration sources that are random in nature. The device once fabricated, can be characterized with various types of signals to validate the theoretical work. The nonlinear springs can be designed for an electromagnetic or piezoelectric energy harvesting system as well.
References


Biography

Shaikh M. Tousif received his B.Sc. degree in Electrical and Electronic Engineering from American International University-Bangladesh in 2009 with highest academic distinction of Summa Cum Laude. He received his M.Sc. degree in Electrical Engineering from University of Texas at Arlington in 2014. His research focus is on MEMs vibrational energy harvesting and sensor design. He worked as a lecturer in the department of Electrical Engineering in American International University-Bangladesh between 2009 to 2012. At UTA, he was a teaching assistant at Electrical Engineering department. He also taught EE 4314 (Control System Engineering) course at UTA. He received recognition for his excellent contribution towards the department and was acknowledged with outstanding graduate teaching assistant award. He received his PhD from University of Texas at Arlington in 2019.