# WOODS TIRE MODEL

by

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Presented to the Faculty of the Graduate School of

The University of Texas at Arlington in Partial Fulfillment

of the Requirements for the Degree of

MASTER OF SCIENCE IN MECHANICAL ENGINEERING.

THE UNIVERSITY OF TEXAS AT ARLINGTON

August 2018

**Acknowledgements**

This thesis was made possible by the help of several individuals to whom I owe a great deal. I would like to acknowledge Professor Dr. Robert L. Woods for supervising my work and giving me an opportunity to work on a project of these scale. You encouraged me to do my best, while giving me guidance along the way. At the same time, you gave me the freedom necessary for this work to truly be my own.

I would like to Acknowledge Milliken Research Associates Inc. for providing all the resources in terms of numerous papers and insights on TTC forums. I would like to acknowledge all the volunteers and contributors of the FSAE TTC program, including Calspan Tire Research Facility. Your work has created many opportunities and helped hundreds of university students like me in expanding our knowledge and allowing us to grow as vehicle dynamic engineers. I would like to recognize all the volunteers of the many FSAE and FS competitions around the world.

Second, I would like to give a special thanks to University of Texas Arlington MAE department for your continued support of the FSAE program. Thanks also to all the current and past members of the UTA Racing team. Our current level of success would not have been possible without all the work you put in. Finally, I would like to thank my family for putting up with 5 years of my involvement in the FSAE program. My parents, you have always encouraged me to follow my dreams and have given me never ending support throughout my many years of school. You pushed me to do my best while giving me the space I needed.

August 8, 2018.

**Abstract**

SEMI-EMPERICAL (WOODS) TIRE MODEL

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The University of Texas at Arlington, 2018

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The performance of a racecar in a maneuver is almost totally determined by the characteristics of the tires and the suspension setup. If the suspension is properly tuned for the maneuver, then the limiting factor is the tire. Therefore, any racecar design and performance analysis must start with a full description of the performance of the tire [1].

Mathematical models for tire performance such as the Pacejka model have been in use for a long time and have become the standard for expressing how a tire will perform dynamically. It comprises of curve fit to experimental data and requires about 17 coefficients to describe the sensitivity of tire adhesion as a function of several variables. These coefficients are not easy to interpret or to estimate. Presented in this paper is the Woods model for tire performance that will provide a physical interpretation to each coefficient and allows an estimate of the coefficient of a

new tire based on knowledge of tested tires. Using Woods tire model, based on Pacejka model with different mathematic curve fits we were able represent the data with same accuracy.

The main objective of this project is to verify the assumptions and mathematical curve fits used in Woods tire model for tires with different compound, sizes and manufacturers and also to express degradation in the coefficient of friction and the value of slip that results in peak force as a function of normal load and camber.

## Contents

**List of Illustrations**

**Abbreviations**

SAE: Society of Automotive Engineers

FSAE: Formula SAE

TTC: Tire test consortium

$F_z$ = normal load on tire

$F_y$ = lateral force generated by the tire in a turn

$F_x$ = longitudinal force generated by the tire in acceleration or braking

$\alpha$ = slip angle in a turn

$\sigma$ = slip ratio in acceleration or braking

$\gamma$ = camber angle of tire

$D_y$ , $D_x$ = peak factor

$C_y$ , $C_x$ = shape factor

$B_y$ , $B_x$ = stiffness factor

$E_y$ , $E_x$ = curvature factor

$S_{yh}$ , $S_{xh}$ = horizontal shift

$S_{yv}$ , $S_{xv}$ = vertical shift

$a_i$ , $b_i$ = curve fit coefficients

## Chapter 1: Introduction

The objective in motor racing is to win races, whether one views racing as a sport, promotional entertainment, or corporate R& D activity. It is the dynamic behavior of the combination of high tech machines and infinitely complex human beings that makes the sport so intriguing for participants and spectators alike.

Vehicle performance is the function of how well the vehicle interacts with the road surface. Tires are the primary source of forces and torques which provide control and stability (handling) to the vehicle. The forces and torques developed by the pneumatic tire affect the vehicle in a variety of ways. Obviously, the tires support the vehicle weight and any other vertical loads but also take lateral forces and torques. The interactions between the tire and road surfaces generates tractive, braking and cornering forces for maneuvering the vehicle [1].

### Motivation

As mentioned previously, it is critical to understand the interactions between the tire and road surfaces, the forces and moments generated by the tire and how to take advantage of these effects on vehicle stability, control and performance. This creates two specific needs. First, there is a need for data on the force and moment characteristics of tire. Second, a direct expression on the maximum adhesion and maximizing slip which allows an engineer to determine car setup parameters and quantify adjustments[2].

## Chapter 2: Tire forces and moments

The tire is the principal means for creating tire forces and moments to produce vehicle motion. In this section, definitions of the various forces and moments, along with associated operating variables, are introduced. The fundamental tire force and moment axis system is shown in Figure 1. This appears in two Society of Automotive (SAE) standards documents, Surface Vehicle Recommended Practice [SAE J670E, 1976] and Tire Performance Technology [SAE J2047, 1998]. The definitions that follow are also based on these two sources.

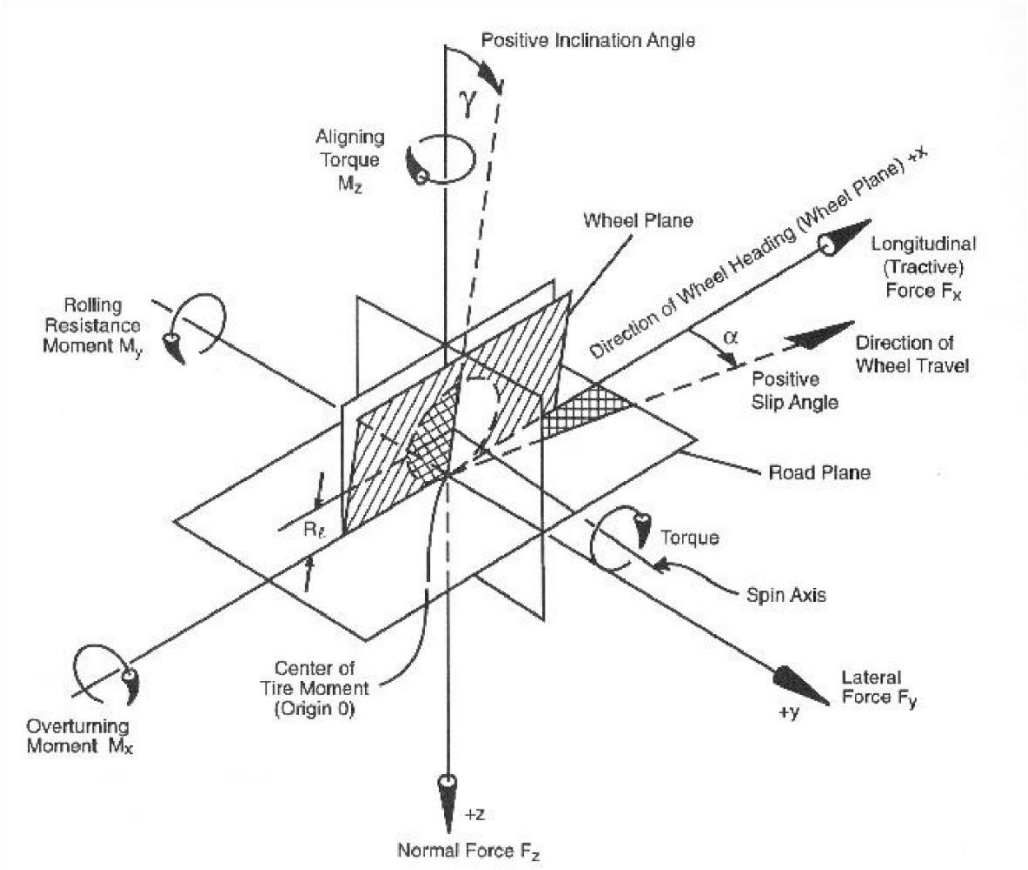



*Figure 1: SAE Standard tire axis system.*

### 2.1 Definition

There are four tire forces and moments of interest. They are described in a tire-relative axis system with origin located in the wheel plane below the wheel center on the ground. The x-axis points in the direction of the wheel plane in the ground plane. The y-axis points perpendicular to the wheel plane in the ground plane. The z-axis is normal to the ground plane and is positive downward [1]. The forces and moment produced by the tire are:

- Longitudinal force, Fx: Tire force produced in the X-axis direction principally as a function of slip ratio. Informally, this is the tractive/braking force.
- Lateral force, Fy: Tire force produced in the Y-axis direction principally as a function of slip angle, but also a weak but notable function of inclination angle. Informally, this is the cornering force.
- Aligning Torque, Mz: Tire moment about the Z-axis which usually acts as a restoring torque. Tires generally resist the introduction of slip angle through this mechanism. This is one source of the self-centering exhibited by automobile steering wheels.
- Overturning Moment, Mx: Tire moment about the X-axis resulting from the fact that the resultant normal load vector can experience a lateral offset from the origin of the SAE tire axis system. It is important for load transfer and suspension compliance calculations.

These four forces and moments are considered tire outputs. There is a fifth component, the rolling resistance moment, but this is often treated separately by vehicle dynamicists. There are five principal operating variables the tire experiences:

- Normal Load, Fz: Tire force in the Z-axis direction indicating the amount of weight being carried by a tire at a given instant in time. In the SAE system, tire loads are described as being

applied by the road to the vehicle. This is an upward direction so, strictly speaking, tire loads have a negative sign. It is commonplace however, to omit the negative sign whenever it doesn't affect the mathematics, such as during discussions or in written material.

- Slip angle, α: The difference between the tire's velocity and heading vectors as projected onto the ground plane. This angle is produced not only by steering the front wheels, but also by vehicle motions including yaw rate, sideslip and suspension kinematics/compliances. Positive slip angles produce negative lateral forces for turning left.
- Inclination angle, γ: The angle at which the top of the tire is tilted left or right from the vertical. Inclination angle results from suspension design and body roll during cornering. It has important, but not primary, effects on all four force and moment outputs. Inclination angle is sometimes incorrectly referred to as "camber angle". Both describe the tilt of the tire, but camber angle is relative to the vehicle such that when the top of the tire tilts toward the vehicle centerline it is referred to as "negative camber". Inclination angle is negative when the top of the tire tilts to the left of its velocity vector.
- Slip ratio, σ: While not shown in Figure, slip ratio is a measure of how fast the tire is rotating relative to how fast the roadway is passing by. There are a variety of expressions in use as summarized by [Milliken, 1995]. For the SAE definition used a slip ratio of zero means the tire is "free rolling" and not trying to develop any drive or brake forces. Positive slip ratios represent a faster rotation than needed for the free rolling condition, leading to tractive forces. Negative ratios indicate a slower rotation than needed for free rolling, which produces braking forces.
- Roadway Friction Coefficient, μ: Also, not shown the roadway friction coefficient is a metric to indicate the interaction between the roadway and the tire. Common asphalt or concrete has a value near 1.0, while packed snow is around 0.3-0.4 and wet ice can be as low as 0.1 or less [Wallingford, 1990].

It should be noted that the system presented above is the SAE standard and is in use across North America and, to a lesser extent, across the rest of the world.

### 2.2 Fundamental Tire force and moment characteristics.

One of the earliest publications on tire behavior was by Broulhiet [Broulheit, 1925] in which he established the concept of the slip angle. Until that time, the tire was largely seen as a suspension component (vertical response was studied) and as a source of power loss (rolling resistance). These were the major benefit and concern associated with the introduction of the pneumatic tire to bicycles in the 1880s. While tires had become more reliable, more complex and more effective in the decades since then, the overriding perception of the tire's role had changed little. The tire force & moment characteristics of significant interest to modern vehicle dynamicists were only beginning to be explored in the 1920s [3].

In 1931 Becker, Fromm and Maruhn [Becker, 1931] produced lateral force data on a rotating drum, a side-effect of their investigation into the problem of tire shimmy. Drum testing of tire forces and moments grew throughout the 1930s, with significant contributions being made by Olley and Evans [Evans, 1935]. By the end of the decade Bull was able to envision a full six component drum-type test machine [Bull, 1939]. The information obtained from these early testing machines, while crude by today's standards, was sufficient to establish fundamental relationships between operating variables and tire outputs. Figure 2.3 [Milliken, 1995] shows the generic shape of a lateral force versus slip angle curve. The same shape is also representative of longitudinal force vs. slip ratio curves. For small slip angle values the tire behaves linearly, producing a lateral force proportional slip angle in an amount defined as the "cornering stiffness". Nearly all normal passenger car driving occurs in this "linear range" of the tire performance curve [4].

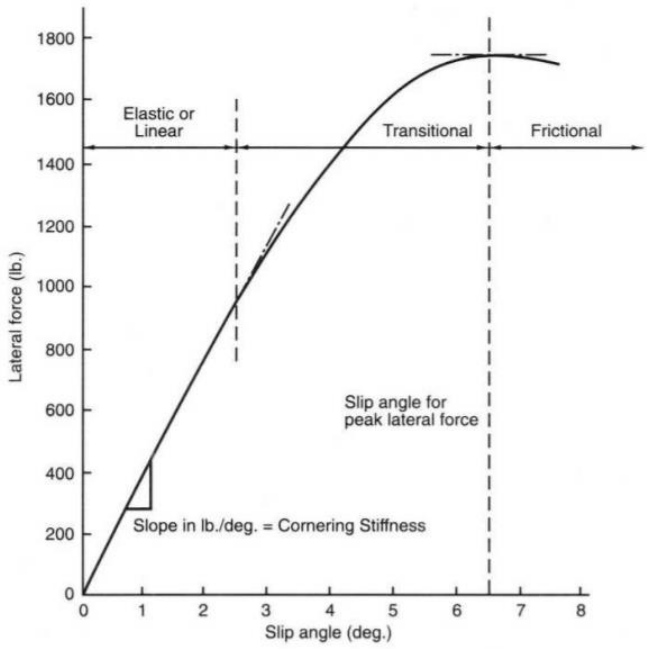



*Figure 2: Fundamental lateral force versus slip angle curve[1].*

There exists a certain slip angle value at which the lateral force is maximized. This is referred to as the peak of the curve. It is the area near the peak of the curve that race cars attempt to operate. Both the initial slope and the peak of the curve play a critical role throughout this dissertation. The curve in Figure 2.3 is drawn for a single normal load. As normal load is increased the height of the peak, location of the peak and initial slope of the curve all change, as shown in Figure 2.4 [Milliken, 1995]. Awareness of these trends is also essential to the work presented in later chapters [1].

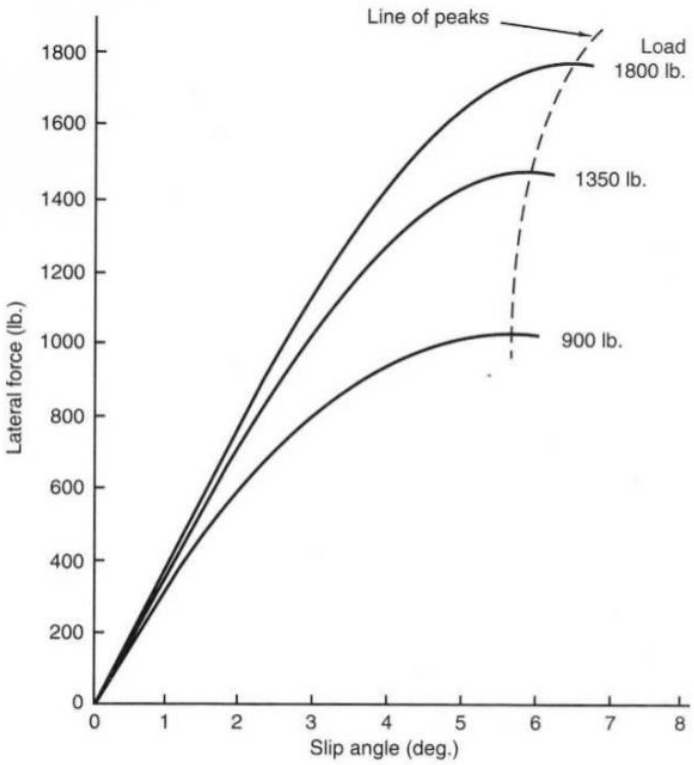



*Figure 3: Typical lateral force behavior across three different normal loads. [1]*

## Chapter 3: Tire modeling

A tire model is a way to describe real world phenomena as a series of mathematical equations. Engineers use numerous models to describe the world in a minimalist form, draw insights into how various properties interact and predict the outcome of future events. To make a model simple and easy to understand, we have to make certain assumptions. The key is to develop a model that supplies the desired accuracy and detail for a specific problem, while remaining solvable given the time and resources available. Hence, there is no surprise that there are several ways to develop a tire model.

The focus of this thesis is the WOODS tire model based on the normalization of Pajecka model. This model has been developed to accurately predict and corelate physical(tunable) parameters as a function of peak slip angle and adhesion.

Tire model can be divided into several categories based upon the purpose the model serves. For example, Ride model focuses on the tire vertical behavior like spring rate and damping treating the tire as a suspension component while the handling model focuses on the forces and moments that produce the vehicle motion. WOODS tire model comes under the handling model.

### 3.1 The Pacejka Tire Model

Of the many different tire models that are available today, the magic formula tire model proposed in 1, 2 and 3 is one of the most advanced and has proven to be very accurate when compared to experimental data. The approach of the model is semi empirical, meaning that the formulas aren't derived from a physical background that models the tire structure but rather are Mathematical approximations of curves that were recorded and experiments. For this purpose, scaling factors must be obtained from measurements.

The general form of the Magic Formula is [3]:

$$y = D\,\sin[C\,\arctan\{Bx - (Bx - \arctan(bx))\}]$$

Where y represents a tire force or torque and x is the slip quantity this force or torque depends on (i.e. longitudinal or lateral slip). B, C, D, E are factors to define the curve's shape in order to get an appearance similar to the recorded. Specifically,

| | |
|---|---|
| B | is a stiffness factor |
| C | is a shape factor |
| D | is the peak value |
| E | is a curvature factor |
| BCD | is the slope of curve at origin. |

Each of these factors must be approximated from measured data from experiments for the respective tire and environment. It is also possible to apply an offset in x and y direction with respect to origin to this general formula. An offset can arise due to ply sheer and conicity effects as well as wheel camber [3]. The shift in x and y can be performed by using the modified coordinates

$Y(X) = y(x) + S_V$ with $S_V$ being the vertical shift and

$x = X + S_H$ with $S_H$ being the horizontal shift.

As input variable X we can use tan α (lateral slip angle) or σ (Longitudinal slip) – which depend on vertical load $F_Z$ and camber angle γ. The output variable Y described by the formula might be $F_X$ (longitudinal force), $F_Y$ (lateral force) or $M_Z$ (self-aligning torque), depending on the problem.

As an example, the lateral force $F_{Y0}$ can be described as [3]:

$$F_{Y0} = D_Y \sin\left[C_Y \arctan\left\{B_Y\alpha_Y - E_y\left(B_y\alpha_y - \arctan\left(B_y\alpha_y\right)\right)\right\}\right] + S_{Vy}$$

Where $\alpha_y$ is the slip angle.

To display the agreement of the Magic Formula approach and experimental data figure 7 shows curves obtained from measurements compared to magic formula computed results.

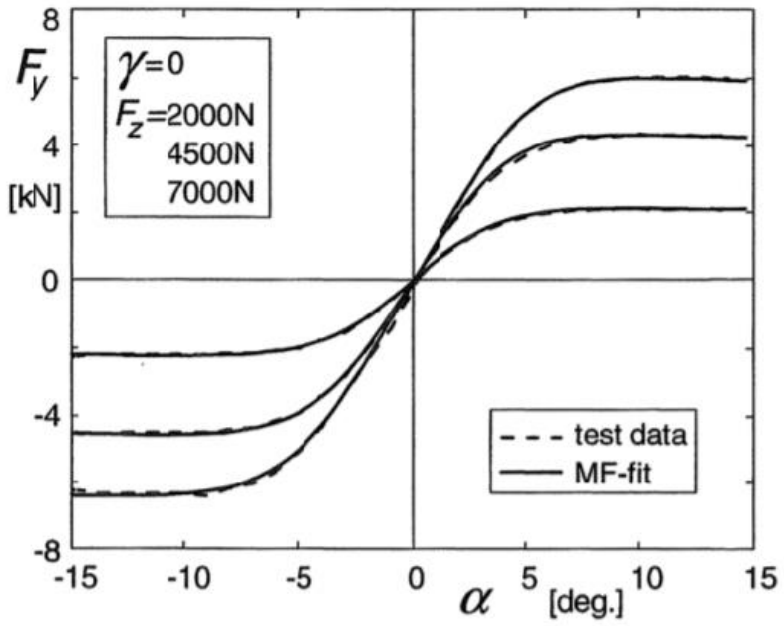



*Figure 4: Comparison of Magic Formula computed results with measured data [3].*

# Chapter 4: Tire test data

## 4.1 Tire testing

Formula SAE (FSAE) is a highly competitive series organized annually at 5 different continents. University teams from around the world contest in this unique competition where students virtually have complete freedom to design, develop and manufacture a racecar. UTA racing has a great legacy starting back in 1982, where the team has won various championships and accolades. UTA racing has been one of the top most competitive teams in the world.

The need for greater access to tire data and engineers familiar with tire data is addressed through the establishment of the Formula SAE Tire Test Consortium. This organization collects a modest fee from registered members, all of whom are colleges and universities participating in the international Formula SAE competitions. The consortium organizes tire force and moment tests and then distributes the raw data to all registered members. It is the first time that low-cost, high quality tire force and moment data has been available to academia. Cornering tests begin with a “cold to hot” series of twelve slip angle sweeps at one load [5].


*Figure 5: Main testing machine Calspan, TIRF.*

The cornering test is run with a free-rolling (no slip ratio) tire, while the drive/brake tests hold constant slip angle while varying slip ratio. Five loads, five inclination angles and four inflation pressures are tested. The first slip angle sweep block is longer than the rest because it contains a few “conditioning sweeps”. A lot of care is taken to exercise the tire and stabilize its performance before taking the main block of force and moment data.

## Chapter 5: Woods Tire Model

### 5.1 Pajecka Tire models

The Pacejka model has evolved slightly over the years and uses different signs for the coefficients in different coordinate systems used by various organizations, but the basic form is the sine of an inverse tangent function of the slip with various degradations and offsets as presented below [2].

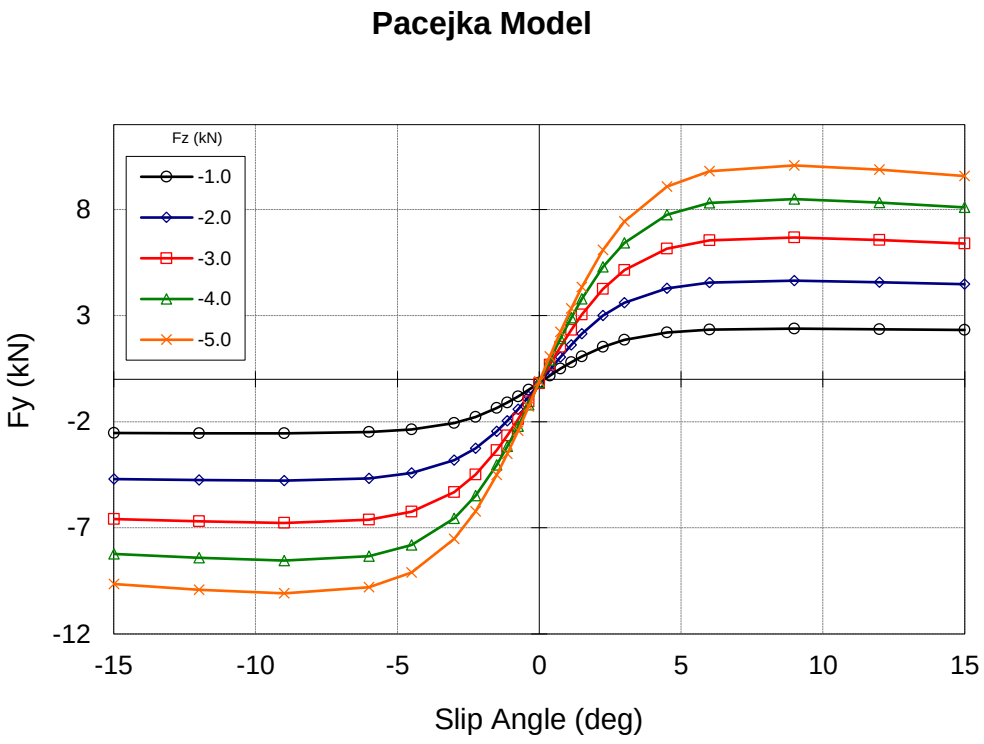



*Figure 6:Force and slip angle characteristics of a typical race tire.*

Figure 9 illustrates the force versus slip angle of a typical race tire as a function of normal load. In this graph, we are interested in two parameters, the slip angle that results in the maximum lateral force, and that lateral force relative to the normal load [2].

### 5.1.1 Pacejka Lateral Model

In lateral model 'slip' is the slip angle. The Pacejka formulation is given by the following equations. See the nomenclature in appendix A-1 for explanations of the variables and coefficients.

$$F_y = D_y \sin\left(C_y \, \theta_y\right) + S_{yv} \tag{1}$$

$$\theta_y = \tan^{-1}\left\{B_y\left(\alpha + S_{yh}\right) - E_y\left[B_y\left(\alpha + S_{yh}\right) - \tan^{-1}\left(B_y\left(\alpha + S_{yh}\right)\right)\right]\right\}$$

$$= \tan^{-1}\left\{B_y\left(\alpha + S_{yh}\right)\left(1 - E_y\left[1 - \frac{\tan^{-1}\left(B_y\left(\alpha + S_{yh}\right)\right)}{B_y\left(\alpha + S_{yh}\right)}\right]\right)\right\} \tag{2}$$

The coefficients $D_y$, $C_y$, $B_y$, $E_y$, $S_{yh}$, and $S_{yv}$ are curve fit parameters and are stated in terms of 18 constants, $a_i$, that are determined for each tire.

$$C_y = a_0 \tag{3}$$

$$D_y = \left[\left(a_1 \, F_z + a_2\right)\left(1 - a_{15} \, \gamma^2\right)\right] F_z \tag{4}$$

$$B_y = \frac{a_3 \sin\left(2 \tan^{-1}\left(\frac{F_z}{a_4}\right)\right)\left(1 - a_5 |\gamma|\right)}{C_y D_y} \quad (5)$$

$$E_y = \left(a_6 F_z + a_7\right)\left[1 - \left(a_{16}\gamma + a_{17}\right)\text{sign}\left(\alpha + S_{yh}\right)\right] \quad (6)$$

$$S_{yh} = a_8 F_z + a_9 + a_{10}\gamma \quad (7)$$

$$S_{yv} = a_{11} F_z + a_{12} + \left(a_{13} F_z + a_{14}\right) F_z \gamma \quad (8)$$

The *a* coefficients and the constants are expained at the end in appendix A-1.

### 5.1.2 Pacejka Longitudinal Model

In the longitudinal model, the "slip" is the slip ratio. The slip ratio is related to the normalized difference in the tangential speed of the tire, $R_t \omega$ and the longitudinal speed of the vehicle, *v*, (measured at the wheel axle). Various authors use differing conventions, but in this paper, the radius of the tire, $R_t$, is the loaded radius (considering speed) and $\omega$ is the rotational speed of the tire [2].

For acceleration, the slip ratio is the speed difference relative to the rotational speed.

$$\sigma = \frac{R_t \omega - v}{R_t \omega} = 1 - \frac{v}{R_t \omega} \quad (9a)$$

In acceleration, the tangential speed is greater than the vehicle speed, so the slip ratio is positive.

For deceleration, the slip ratio is the speed difference relative to the vehicle speed.

$$\sigma \;=\; \frac{R_t\;\omega - v}{v} \;=\; \frac{R_t\;\omega}{v} - 1 \tag{9b}$$

In deceleration, the vehicle speed is greater than the tangential speed, so the slip ratio is negative.

The longitudinal model is very similar to the lateral model, except camber effects are not considered.

$$F_x \;=\; D_x\;\sin\left(C_x\;\theta_x\right) + S_{xv} \tag{10}$$

$$\theta_x \;=\; \tan^{-1}\left\{B_x\left(\sigma + S_{xh}\right) - E_x\left[B_x\left(\sigma + S_{sh}\right) - \tan^{-1}\left(B_x\left(\sigma + S_{xh}\right)\right)\right]\right\}$$

$$=\; \tan^{-1}\left\{B_x\left(\sigma + S_{xh}\right)\left(1 - E_x\left[1 - \frac{\tan^{-1}\left(B_x\left(\sigma + S_{xh}\right)\right)}{B_x\left(\sigma + S_{xh}\right)}\right]\right)\right\} \tag{11}$$

For calculation of the longitudinal force, the $B_x$, $C_x$, $D_x$, $E_x$, $S_{xh}$, and $S_{xv}$ coefficients are given in terms of 14 constants, $b_i$, derived from the measured data.

$$C_x \;=\; b_0 \tag{12}$$

$$D_x \;=\; \left(b_1\;F_z + b_2\right)F_z \tag{13}$$

$$B_x \;=\; \frac{\left(b_3\;F_z + b_4\right)F_z\;e^{-b_5 F_z}}{C_x\;D_x} \tag{14}$$

$$E_x = \left(b_6 F_z^2 + b_7 F_z + b_8\right)\left[1 - b_{13} \operatorname{sign}\left(\sigma + S_{yh}\right)\right] \quad (15)$$

$$S_{xh} = b_9 F_z + b_{10} \quad (16)$$

$$S_{xv} = b_{11} F_z + b_{12} \quad (17)$$

The *b* coefficients and the constants are expained at the end in appendix.

### 5.2 Woods Tire Models

The Pacejka models do an acceptable job of representing tire data, they fall short on three aspects that are of interest to analysis and simulation. First, the equations are not presented in a normalized form so it is not clear at what value of slip the tire force reaches a peak. For racing applications, one would want to know the slip angle at peak adhesion, so a peak slip angle should be presented explicitly. Second, one should be able to isolate this peak slip function as an independent equation so it can be shown how the slip varies with normal load and camber. Third, the *a* and *b* coefficients do not have any physical interpretation to which one can relate. In most cases, the coefficients are even presented without any units. When units are presented, measurements of force are often not consistent (vertical load measured in *kN* and adhesion forces measured in *N*).[2]

The normalization is both in the adhesion force relative to the normal load, and the slip relative to the peak slip (slip at which the adhesion force reaches a peak). In this normalized form, the equation for the peak slip function is clear and can be written independently.[2]

First the Pacejka model is normalized with respect to the normal load, $F_z$, and second, the slip terms are normalized and scaled so that the maximizing slip can be observed directly. This normalizing approach makes use of some recognizble physical parameters such as the coefficient of friction. When the parameters are not obvious, they are expressed as the value that would cause a 10% change in performance. This will give a physical feel for the coefficients and for their units.[2]

### 5.2.1 Woods tire Model

My normalization of the Pacejka model for lateral acceleration can be stated in the following form (similar to equation 1).

$$\frac{F_y}{F_z} = \mu_y \sin\left(C_y \, \theta_y\right) + \frac{F_{yoff}}{F_z} \tag{18}$$

where $C_y$ = droop factor that adjusts the slope after peak adhesion.

The lateral coefficient of friction is $\mu_y$ and can be expressed as follows (similar to equation 4).

$$\mu_y = \frac{D_y}{F_z} = \mu_{y0}\left(1 - 0.1\frac{F_z}{F_{y\mu}}\right)\left[1 - 0.1\left(\frac{\gamma}{\gamma_{y\mu}}\right)^2\right] \tag{19}$$

where $\mu_{y0}$ = the coefficient of friction that would occur with zero load and zero camber

$F_{y\mu}$ = the load that will result in a 10% degradation in $\mu$

$\gamma_{y\mu}$ = the camber that will result in a 10% degradation in $\mu$

The vertical offset of the forces is expressed as a function of normal load and camber (similar to equation 8).

$$F_{yoff} = F_{yoff0} + S_{yoff}\left[1 + \left(1 + 0.1\frac{F_z}{F_{yoffz}}\right)\left(0.1\frac{\gamma}{\gamma_{yoff}}\right)\right]F_z \qquad (20)$$

where $F_{yoff0}$ = the offset (or force intercept) that would occur with zero load

$F_{yoffz}$ = the load that will result in a 10% change in offset related to camber

$S_{yoff}$ = the sensitivity of lateral force to normal force

$\gamma_{yoff}$ = a 10% normalizing factor for $\gamma$

The variable $\theta_y$ is a modified slip variable that is limited by an inverse tangent function (similar to equation 2).

$$\theta_y = \tan^{-1}\left\{\frac{G_y\left(\alpha+\alpha_{off}\right)}{\alpha_{peak}}\left(1-E_y\left[1-\frac{\tan^{-1}\left(\frac{G_y\left(\alpha+\alpha_{off}\right)}{\alpha_{peak}}\right)}{\frac{G_y\left(\alpha+\alpha_{off}\right)}{\alpha_{peak}}}\right]\right)\right\} \quad (21)$$

The term $G_y$ is introduced to make the sine function reach its maximum value when the slip is at its approximate peak value.

$$G_y = \tan\left(\frac{\pi/2}{C_y}\right) \quad (22)$$

Notice that if $\alpha$ is at its peak value, then $\theta_y \approx \tan^{-1}\left(G_y\right)$, and therefore $C_y\ \theta_y = \pi/2$.

Using the above equation, we can clearly observe the slip angle at which maximum lateral force occurs as an independent function [2] .

The horizontal offset is an offset in slip angle, $\alpha_{off}$ (similar to equation 7).

$$\alpha_{off} = S_{yh} = \alpha_{off0}\left[1-0.1\frac{F_z}{F_{\alpha off}}+0.1\frac{\gamma}{\gamma_{\alpha off}}\right] \quad (23)$$

where $\alpha_{off0}$ = the offset that would occur with zero load and zero camber

$F_{\alpha off}$ = the load that will result in a 10% decrease in offset

$\gamma_{\alpha off}$ = the camber that will result in a 10% increase in offset

The $E_y$ is a variable that modifies the slip function (similar to equation 6).

$$E_y = E_{y0}\left(1+0.1\frac{F_z}{F_{ye}}\right)\left[1-E_{y\gamma}\left(1+0.1\frac{\gamma}{\gamma_{ye}}\right)\text{sign}\left(\alpha+\alpha_{off}\right)\right] \quad (24)$$

where$E_{y0}$ = the slip modifier that would occur with zero load and zero camber

$F_{ye}$ = the load that will result in a 10% change in $E$

$E_{y\gamma}$ = another slip modifier constant

$\gamma_{ye}$ = the camber that will result in a 10% change of the modifier constant.

The original $B_y$ coefficient in the Pacejka model can be used (with some algebra) to find the slip angle at which the maximum adhesion occurs, $\alpha_{peak}$ (derived from equation 5 and others).[1]

$$\alpha_{peak} = \frac{G_y}{B_y} = \frac{\alpha_{peak0}\dfrac{\mu_y}{\mu_{y0}}}{\left(1-0.1\dfrac{|\gamma|}{\gamma_{\alpha peak}}\right)}\frac{2\dfrac{F_z}{\alpha_4}}{\sin\left(2\tan^{-1}\left(\dfrac{F_z}{\alpha_4}\right)\right)} \quad (25)$$

where $\alpha_{peak0}$ = the slip angle that would occur with zero load and zero camber

$\alpha_4$ = Pacejka coefficient

$\gamma_{\alpha peak}$ = the camber that will result in a 10% change in $\alpha_{peak}$

$$\alpha_{peak0} = \frac{C_y\, G_y\, \mu_{y0}\, F_{z\alpha}}{2\, S_{\alpha peak}} \quad (26)$$

where $S_{\alpha peak}$ = a load sensitivity = $a_3$

In the form expressed above, it is clear that the peaking slip angle is directly proportional to the coefficient of friction. In other words, as the $\mu$ increases, the slip angle will increase. [2]

The last term in the $\alpha_{peak}$ equation is mathematically equal to a parabolic function (which is much more simple than the trig functions). The Pacejka models do not make this simplification, but it will be used in all of the following equations even though it is not strictly a Pacejka model, it produces identical results and is easier to visualize than the original.[2]

$$\frac{2\dfrac{F_z}{\alpha_4}}{\sin\left(2\tan^{-1}\left(\dfrac{F_z}{\alpha_4}\right)\right)} = \left[1+\left(\frac{F_z}{\alpha_4}\right)^2\right] \tag{27}$$

Therefore, the $\alpha_{peak}$ equation can be simplified as follows.

$$\alpha_{peak} = \frac{\alpha_{peak0}\left(1+0.1\left(\dfrac{F_z}{F_{z\alpha}}\right)^2\right)\dfrac{\mu_y}{\mu_{y0}}}{\left(1-0.1\dfrac{|\gamma|}{\gamma_{\alpha peak}}\right)} \tag{28}$$

or

$$\alpha_{peak} = \frac{\alpha_{peak0}\left(1+0.1\left(\dfrac{F_z}{F_{z\alpha}}\right)^2\right)\left(1-0.1\dfrac{F_z}{F_{y\mu}}\right)\left[1-0.1\left(\dfrac{\gamma}{\gamma_{y\mu}}\right)^2\right]}{\left(1-0.1\dfrac{|\gamma|}{\gamma_{\alpha peak}}\right)} \tag{29}$$

where

$F_{z\alpha}$ = the load that will result in a 10% increase in $\alpha_{peak}$

For the conversions between the normalizing factors and the original Pacejka coefficients refer to appendix.

### 5.2.2 Normalized Longitudinal Pacejka Model

Normalization of the Pacejka model for acceleration and deceleration can be stated in the following form.

$$\frac{F_x}{F_z} = \mu_x \sin\left(C_x \theta_x\right) + \frac{F_{xoff}}{F_z} \tag{30}$$

$$\theta_x = \tan^{-1}\left\{\frac{G_x\left(\sigma+\sigma_{off}\right)}{\sigma_{peak}}\left(1-E_x\left[1-\frac{\tan^{-1}\left(\frac{G_x\left(\sigma+\sigma_{off}\right)}{\sigma_{peak}}\right)}{\frac{G_x\left(\sigma+\sigma_{off}\right)}{\sigma_{peak}}}\right]\right)\right\} \tag{31}$$

Again, the parameter $G_x$ is introduced to make the longitudinal force curve saturate at $\sigma_{peak}$.

$$G_x = \tan\left(\frac{\pi/2}{C_x}\right) \tag{32}$$

The longitudinal coefficient of friction is $\mu_x$ and can be expressed as follows.

$$\mu_x = \frac{D_x}{F_z} = \mu_{x0}\left(1-0.1\frac{F_z}{F_{x\mu}}\right) \tag{33}$$

where $\mu_{x0}$ = the coefficient of friction that would occur with zero load

$F_{x\mu}$ = the value of load that will result in a 10% degradation in $\mu$

The vertical offset is $F_{xoff}$ .

$$F_{xoff} = F_{xoff0}\left(1 + 0.1\frac{F_z}{F_{xoffz}}\right) \tag{34}$$

where $F_{xoff0}$ = the offset that would occur with zero load

$F_{xoffz}$ = the load that will result in a 10% change in offset

The horizontal shift is given as follows.

$$\sigma_{off} = S_{xh} = \sigma_{off0}\left[1 - 0.1\frac{F_z}{F_{\sigma off}}\right] \tag{35}$$

where $\sigma_{off0}$ = the offset that would occur with zero load

$F_{\sigma off}$ = the value of load that will result in a 10% decrease in offset

The $E_x$ is a variable that modifies the slip function.

$$E_x = E_{x0}\left[1 + 0.1\frac{F_z}{F_{xe1}} + 0.1\left(\frac{F_z}{F_{xe2}}\right)^2\right]\left[1 - E_{x\sigma}\ \mathrm{sign}\left(\sigma + \sigma_{off}\right)\right] \tag{36}$$

where $E_{x0}$ = the slip modifier that would occur with zero load and zero camber

$F_{xe1}$ = the value of load that will result in a 10% change in $E_x$

$F_{xe2}$ = the value of load that will result in a 10% change in $E_x$

$E_{x\sigma}$ = another slip modifier constant

By noting that $B_x = G_x / \sigma_{peak}$ , we can solve for the slip ratio at which the maximum adhesion occurs, $\sigma_{peak}$.

$$\sigma_{peak} = \frac{G_x}{B_x} = \frac{\sigma_{peak0} \frac{\mu_x}{\mu_{x0}}}{\left(1 - 0.1 \frac{F_z}{F_{\sigma z}}\right) e^{-\frac{F_z}{F_{\sigma e}}}} \quad (37)$$

where $\sigma_{peak0}$ = the slip ratio that would occur with zero load

$F_{\sigma z}$ = the value of load that will result in about a 10% change in $\sigma_{peak}$

$F_{\sigma e}$ = a normalizing load factor

$$\sigma_{peak0} = \frac{C_x \, G_x \, \mu_{x0}}{\mu_{\sigma peak0}} \quad (38)$$

where $\mu_{\sigma peak0}$ = a normalizing friction factor

we can also solve for μpeak at which maximum adhesion occurs

$$\mu peak = \mu_{0peak} * \left[1 - 0.1 \frac{F_z}{F_{\mu peak}}\right] * \left[1 - 0.1 \frac{\gamma}{\gamma_{\mu peak}}\right]^2 \quad (39)$$

Where, $\mu_{0peak}$ = the μ that would occur with zero load

$F_{\mu peak}$ = the value of load that will result in about a 10% change in $\mu_{0peak}$

$\gamma_{\mu peak}$ = normalising factor.

For the conversions between the normalizing factors and the original Pacejka coefficients refer to appendix.

### 5.3 Performance Interpretation of Woods tire model

Analytic tire models have several useful applications. One is the interpretation of the tire performance as the normal load and camber change. In driving applications of a vehicle of fixed weight, the normal load on a tire can change due to weight transfer in a driving maneuver. In particular, we are interested in how the coefficient of friction changes, and how the slip angle that results in peak adhesion changes with load and camber.[2]

This can be observed from the expressions for $\mu_y$ and $\alpha_{peak}$ derived in a previous section and presented below.[2]

$$\frac{\mu_y}{\mu_{y0}} = \left[1 - 0.1\frac{F_z}{F_{y\mu}}\right]\left[1 - 0.1\left(\frac{\gamma}{\gamma_{y\mu}}\right)^2\right] \tag{40}$$

$$\frac{\alpha_{peak}}{\alpha_{peak0}} = \frac{\left[1 + 0.1\left(\frac{F_z}{F_{z\alpha}}\right)^2\right]}{\left[1 - 0.1\frac{|\gamma|}{\gamma_{\alpha peak}}\right]}\frac{\mu_y}{\mu_{y0}} \tag{41}$$

or by substitution of $\mu_y$:

$$\frac{\alpha_{peak}}{\alpha_{peak0}} = \frac{\left[1-0.1\frac{F_z}{F_{y\mu}}\right]\left[1-0.1\left(\frac{\gamma}{\gamma_{y\mu}}\right)^2\right]}{\left[1-0.1\frac{|\gamma|}{\gamma_{\alpha peak}}\right]}\left[1+0.1\left(\frac{F_z}{F_{z\alpha}}\right)^2\right] \qquad (42)$$

By using the above equations we can determine how μ and maximizing slip vary with load and camber.

## Chapter 6: Verification

Using the equations in chapter 5, we can determine how μ and maximizing slip vary with load and camber. To verify our theory 4 different tire compounds from different manufacturers were selected:

- Hoosier R13 R25B.
- Hoosier R10 LCO.
- Goodyear R13 D2509.
- Avon R10 7*16.

These compounds were chosen because of the availability of testing data through Tire test consortium (TTC). Every tire is different in construction, material and geometry; hence a wide variety of manufacturers and compounds were selected to validate our tire model.

### 6.1 Data Parsing

The TTC records data over 16 channels at the frequency of 10-100Hz. All this raw data is dumped into a data file and is distributed to all the registered members. Due to the large number of input variables it's very important to understand the test procedure. The test accounts for tire break in and warm up to bring the tires up to temperature before every run. Hence it is important to parse the data and use the data that best simulates your purpose [5].

| Channel | Units | Description |
|---|---|---|
| AMBTMP | degC or degF | Ambient room temperature |
| ET | sec | Elapsed time for the test |
| FX | N or lb | Longitudinal Force |
| FY | N or lb | Lateral Force |
| FZ | N or lb | Normal Load |
| IA | deg | Inclination Angle |
| MX | N-m or lb-ft | Overturning Moment |
| MZ | N-m or lb-ft | Aligning Torque |
| N | rpm | Wheel rotational speed |
| NFX | unitless | Normalized longitudinal force (FX/FZ) |
| NFY | unitless | Normalized lateral force (FY/FZ) |
| P | kPa or psi | Tire pressure |
| RE | cm or in | Effective Radius |
| RL | cm or in | Loaded Radius |
| RST | degC or degF | Road surface temperature |
| SA | deg | Slip Angle |
| SL | unitless | Slip Ratio based on RE (such that SL=0 gives FX=0) |
| SR | unitless | Slip Ratio based on RL (used for Calspan machine control, SR=0 does not give FX=0) |
| TSTC | degC or degF | Tire Surface Temperature--Center |
| TSTI | degC or degF | Tire Surface Temperature--Inboard |
| TSTO | degC or degF | Tire Surface Temperature--Outboard |
| V | kph or mph | Road Speed |

*Figure 7: Data channels at TTC.*

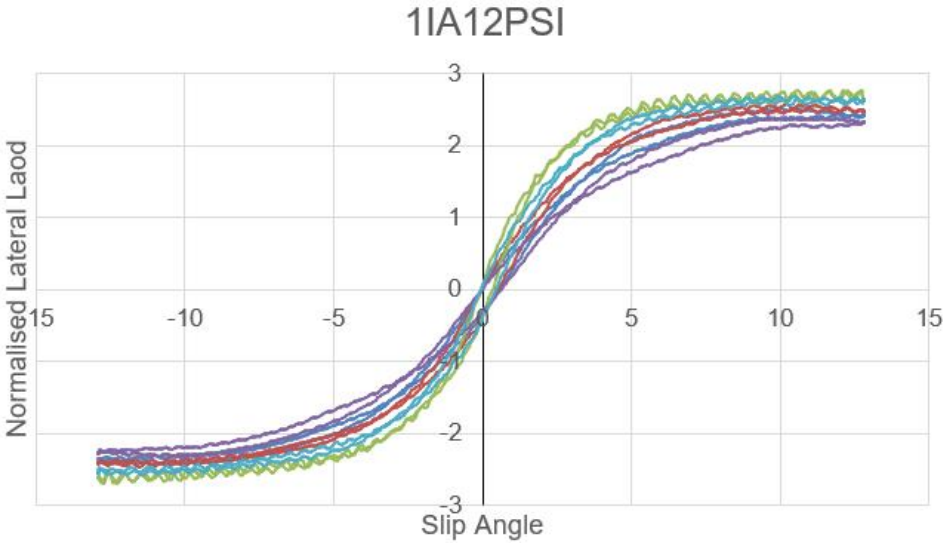


*Figure 8: Normalized Lateral Load vs Slip angle from TTC raw data.*

Once all the Data for a Tire has been parsed. We import the data into an excel spreadsheet where we procced to start with manually curve fitting our mathematical model over the TTC raw data for each variable. The nomenclature followed here was:

X IA XXX FZ XX PSI

Where numbers preceding IA denote inclination angle,

FZ denote Normal Load,

And PSI denote inflation pressure.

### 6.2 Curve fitting to TTC raw data.

Figure 9 shows a curve fit template. On the left side of the template we paste the TTC raw data. The numbers in blue on the right-hand side of the template are scaling factors for our mathematical model. Changing various parameters modifies the cure fit seen in the figure 9. The following equations are used to generate the curve fit model.

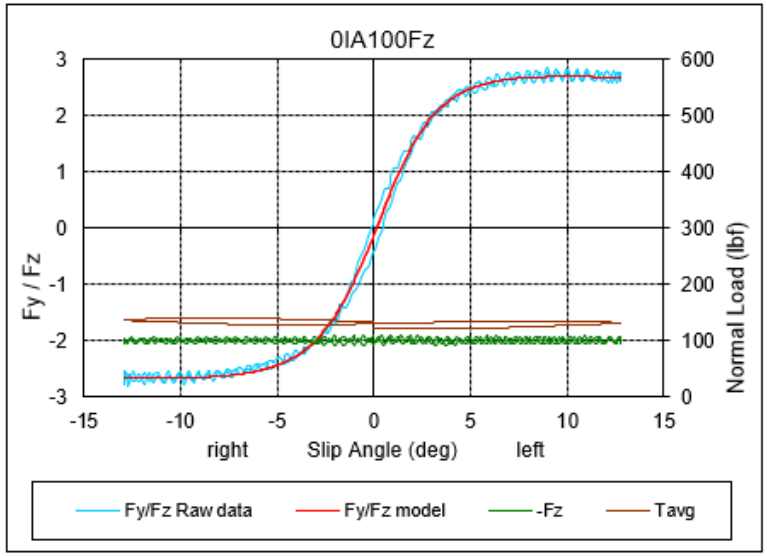
$$\frac{F_y}{F_z} = \mu_y \sin\theta_y$$

$$\theta_y = \frac{\pi}{2} \frac{tan^{-1}\left(\frac{\alpha - \alpha_{off}}{g_y}\right)}{tan^{-1}\left(\frac{\alpha_{peak} - \alpha_{off}}{g_y}\right)}$$

## Curve Fitting

| | |
|---|---|
| μy0 = | 2.7 |
| Fμy = | 160 |
| γμy = | 40.0 |
| Tpeak = | 190 |
| ΔTμy = | 80 |
| αpeak0 = | 9.5 |
| Fαpeak = | 150 |
| γαpeak = | 12 |
| gy0 = | 2.1 |
| Fgy = | 11 |
| γgy = | 2 |
| αoff0 = | -0.2 |
| Sbias = | -0.05 |
| Sγ = | 0.18 |
| Fαoff = | 55 |
| Fαmax = | 350 |
| Tmax = | 148 |
| Fzoff = | 210 |
| Ft = | 145 |
| μpeak0= | 2.6 |
| Fμpeak= | 160 |
| γμpeak= | 3.6 |

| modified numbers | | | |
|---|---|---|---|
| | | IA = | 0 |
| Fz@peak = | 100 | avg Fz = | 100 |
| α+ = | 10 | | |
| μ+ = | 2.7 | | |
| T+ = | 135 | | |
| α- = | -11.5 | | |
| μ- = | -2.67 | | |
| T- = | 140 | | |
| g = | 4 | | |
| μoff = | 0 | Nfy off = | -0.179 |
| αoff = | 0.15 | Fy0ff = | 19.123 |




$$\frac{F_y}{F_z} = \mu_y \sin\theta_y$$

$$\theta_y = \frac{\pi}{2} \frac{tan^{-1}\left(\frac{\alpha - \alpha_{off}}{g_y}\right)}{tan^{-1}\left(\frac{\alpha_{peak} - \alpha_{off}}{g_y}\right)}$$

*Figure 9: Curve Fit Template.*

The above listed coefficients were varied by hand to get a good curve fit to match raw data. Below is an example of a good curve fit over TTC raw data.

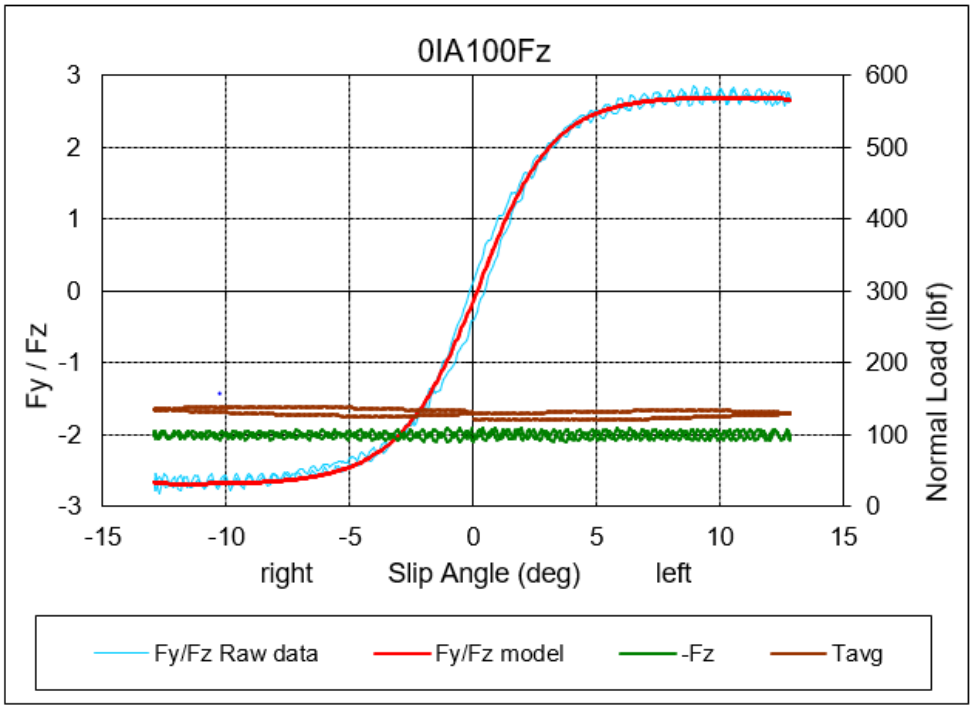



*Figure 10: Curve fitting over TTC raw data.*

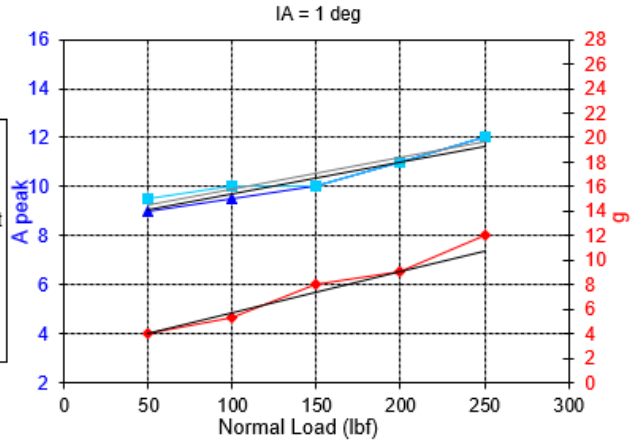
### 6.3 Curve fitting of scaling factors.

Trial and error method were used to achieve the best possible curve fit. For every inflation pressure 25 such curve fits were done per tire. Once you have achieved a satisfactory curve fit for all the parameters. The next step is to generate scaling coefficients for our mathematical model. Paying attention to every detail in the first step is key to getting accurate and quick results. Figure 11 shows a template to curve fit the scaling factors. After fine tuning every curve individually you can generate the scaling factors for our mathematical model.

## Curve Fitting

| | |
|---|---|
| μv0= | 3.3 |
| Fμv= | 160 |
| γμy= | 40.0 |
| Tpeak= | 205 |
| ΔTμv= | 55 |
| αpeak0= | 8.5 |
| Fαpeak= | 65 |
| γαpeak= | 12 |
| αv0= | 2.25 |
| Fαv= | 7 |
| γαv= | 2 |
| αoff0= | 0.15 |
| Sbias= | -0.05 |
| Sγ= | 0.15 |
| Fαoff= | 60 |
| Fαmax= | 300 |
| Tmax= | 140 |
| Fzoff= | 250 |
| Ft= | 250 |

| modified numbers | | | |
|---|---|---|---|
| | | IA = | 0 |
| Fz@peak = | 100 | avg Fz = | 100 |
| α+ = | 10 | | |
| μ+ = | 2.7 | | |
| T+ = | 135 | | |
| α- = | -11.5 | | |
| μ- = | -2.67 | | |
| T- = | 140 | | |
| g = | 4 | | |
| μoff = | 0 | Nfy off = | -0.179 |
| αoff = | 0.15 | Fy0ff = | 19.123 |




$$\mu_y = \mu_{y0} f_{\mu r} \left[1 - 0.1 \frac{F_z}{F_{\mu y}}\right]\left[1 + 0.1 \frac{\gamma}{\gamma_{\mu r}}\right]\left[1 - 0.1 \left(\frac{T - T_{peak}}{\Delta T_{\mu y}}\right)^2\right]$$

$$\alpha_{peak} = f_{\mu r}\, \alpha_{peak0}\left[1 + 0.1 \frac{F_z}{F_{\alpha peak}}\right]\left[1 - 0.1 \left(\frac{\gamma}{\gamma_{\alpha peak}}\right)\right]$$

$$g_y = g_{y0}\left[1 + 0.1 \frac{F_z}{F_{g y}}\right]\left[1 + 0.1 \frac{IA}{IA_{g y}}\right]$$

$$\alpha_{off} = f_{\mu r}\left[\alpha_{off\,0} + S_{bias}\left(\frac{F_z - F_{\alpha off}}{F_{\alpha max}}\right) + S_y\, IA \sin\left(\frac{\pi}{2}\frac{F_z - F_{\alpha off}}{F_{\alpha max}}\right)\right]$$

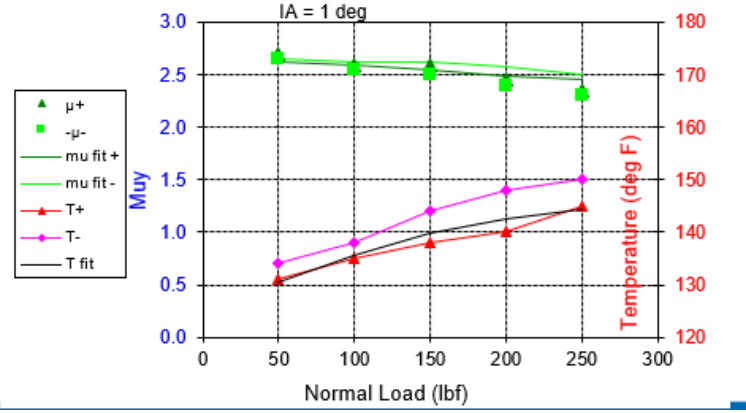



*Figure 11: Curve fit for obtaining scaling factors.*

| | Hoosier R10*18 LCO | Avon R10*16 | Hoosier R13*20 R25B | Goodyear R13*20 D2509 |
|---|---|---|---|---|
| μy0 = | 3.3 | 2.7 | 3.25 | 3.25 |
| Fμy = | 160 | 160 | 170 | 160 |
| γμy = | 40.0 | 40.0 | 60 | 40.0 |
| Tpeak = | 205 | 190 | 195 | 190 |
| ΔTμy = | 55 | 80 | 55 | 80 |
| αpeak0 = | 8.5 | 9.5 | 7.8 | 8.7 |
| Fαpeak = | 65 | 150 | 100 | 75 |
| γαpeak = | 12 | 12 | 9 | 12 |
| gy0 = | 2.5 | 2.1 | 1.7 | 0.27 |
| Fgy = | 9 | 11 | 15.0 | 0.6 |
| γgy = | 2 | 2 | 0.6 | 1.5 |
| αoff0 = | 0.15 | -0.2 | -0.020 | -0.2 |
| Sbias = | -0.05 | -0.05 | 0.180 | -0.05 |
| Sγ = | 0.15 | 0.18 | 0.155 | 0.18 |
| Fαoff = | 60 | 55 | 45 | 55 |
| Fαmax = | 300 | 350 | 330 | 350 |
| Tmax = | 135 | 148 | 118 | 132 |
| Fzoff = | 300 | 210 | 260 | 320 |
| Ft = | 250 | 145 | 260 | 300 |
| μpeak0= | 2.8 | 2.6 | 3 | 3.1 |
| Fμpeak= | 160 | 160 | 160 | 160 |
| γμpeak= | 4.5 | 3.6 | 5 | 6 |

*Figure 12: Woods coefficients for tested tires.*

### 6.4 Woods model vs TTC raw data.

Once the scaling factors are obtained, we can verify our mathematical model by superimposing it over TTC raw data. The mathematical model accurately represents tire behavior as seen in figure 15.

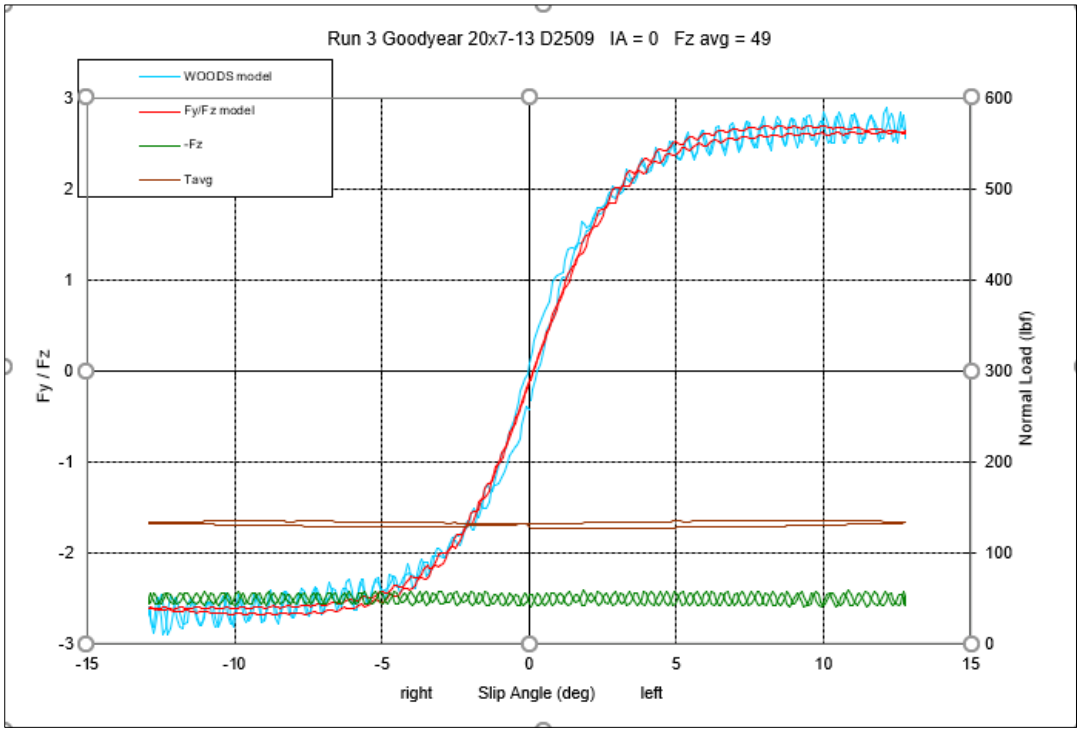



*Figure 13: Woods model vs TTC raw data.*

As seen in figure 13 all our assumptions and mathematical fits for obtaining scaling factors were accurate.

## Chapter 7: Results and conclusion

The WOODS tire model for tire performance was verified in such a way that the value of slip that results in a maximum force can be clearly identified. Having these equations for maximizing slip, we could also determine how $\mu$ and the maximizing slip vary with load and camber. These characteristics help the racecar engineer to set optimum toe settings and handling properties.

$$\alpha_{peak} = \frac{\alpha_{peak0}\left(1+0.1\left(\frac{F_z}{F_{z\alpha}}\right)^2\right)\left(1-0.1\frac{F_z}{F_{y\mu}}\right)\left[1-0.1\left(\frac{\gamma}{\gamma_{y\mu}}\right)^2\right]}{\left(1-0.1\frac{|\gamma|}{\gamma_{\alpha peak}}\right)}$$

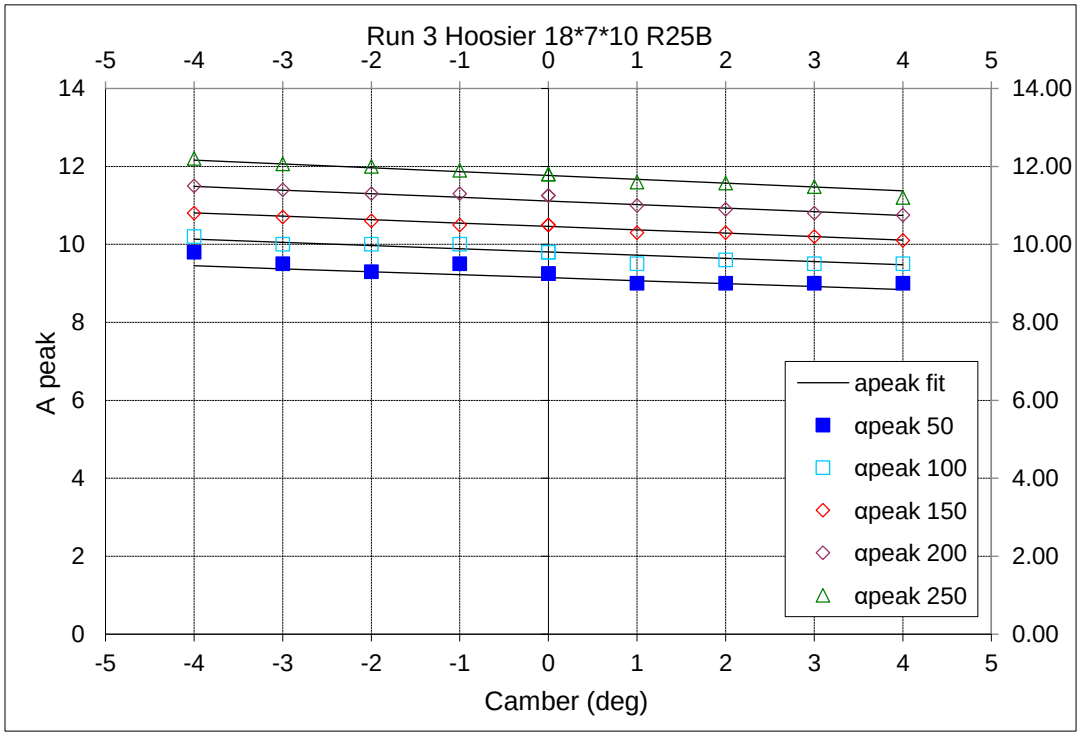



*Figure 14: Maximizing slip for various loads vs camber.*

Figure 14 shows us that we could successfully express maximizing slip as a function of normal loads and camber. As expected maximizing slip degrades with increase in normal load and inclination angle.

$$\mu peak = \mu_{0peak} * \left[1 - 0.1\frac{F_z}{F_{\mu peak}}\right] * \left[1 - 0.1\frac{\gamma}{\gamma_{\mu peak}}\right]^2$$

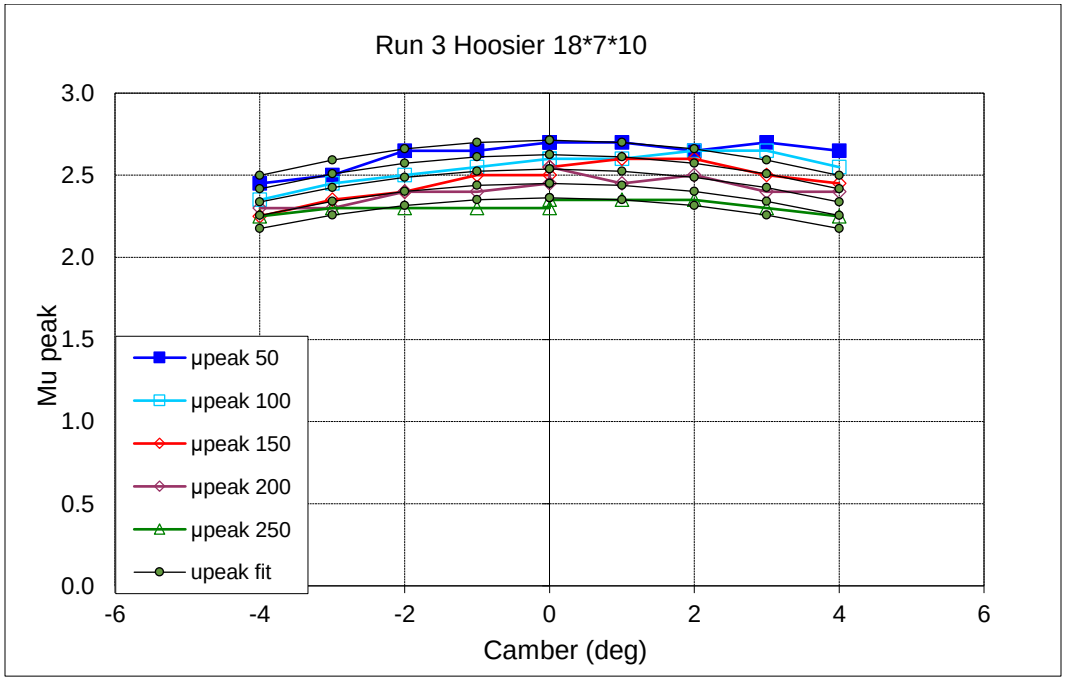



*Figure 15: µ for various loads vs Camber.*

Figure 15 shows us that we could successfully express µ as a function of normal loads and camber. As expected maximum adhesion degrades with increase in normal load.

For data on more tires please refer to appendix A-2.

## Appendix

### Appendix A-1

Conversions: The conversions between the original Pacejka coefficients and the physically relateable coefficients are given below. Note that in the SAE convention, some of the coefficients need to be divided by 1000 (to account for the conversion from kN to N) when they are used in the following conversion to normalized variables.

$C_y = a_0$ $\quad$ $a_0 = C_y$

$G_y = \tan\left(\frac{\pi/2}{a_0}\right)$ $\quad$ $a_1 = \frac{-0.1\,\mu_{y0} * 1000}{F_{\mu y}}$

$\mu_{y0} = a_2 / 1000$ $\quad$ $a_2 = \mu_{y0} * 1000$

$F_{y\mu} = \frac{-0.1\,a_2}{a_1}$ $\quad$ $a_3 = S_{\alpha peak}$

$\gamma_{y\mu} = \sqrt{\frac{0.1}{a_{15}}}$ $\quad$ $a_4 = F_{z\alpha}\sqrt{10}$

$F_{yoff0} = a_{12}$ $\quad$ $a_5 = \frac{0.1}{\gamma_{\alpha peak}}$

$F_{yoffz} = \frac{0.1 a_{14}}{a_{13}}$ $\quad$ $a_6 = \frac{0.1\,E_{y0}}{F_{ey1}}$

$S_{yoff} = \frac{a_{11}}{1000}$ $\quad$ $a_7 = E_{y0}$

$\gamma_{yoff} = \dfrac{0.1 a_{11}}{a_{14}}$

$\alpha_{off0} = a_9$

$F_{\alpha off} = \dfrac{-0.1 a_9}{a_8}$

$\gamma_{\alpha off} = \dfrac{0.1 a_9}{a_{10}}$

$E_{y0} = a_7$

$E_{y\gamma} = a_{17}$

$F_{ye} = \dfrac{0.1 a_7}{a_6}$

$\gamma_{ye} = \dfrac{0.1 a_{17}}{a_{16}}$

$S_{\alpha peak} = a_3$

$F_{z\alpha} = \dfrac{a_4}{\sqrt{10}}$

$\gamma_{\alpha peak} = \dfrac{0.1}{a_5}$

$a_8 = \dfrac{-0.1 \alpha_{off0}}{F_{\alpha off}}$

$a_9 = \alpha_{off0}$

$a_{10} = \dfrac{0.1 \alpha_{off0}}{\gamma_{\alpha off}}$

$a_{11} = S_{yoff} * 1000$

$a_{12} = F_{yoff0}$

$a_{13} = \dfrac{(0.1)^2 \, S_{yoff} * 1000}{\gamma_{yoff} \, F_{yoffz}}$

$a_{14} = \dfrac{0.1 S_{yoff} * 1000}{\gamma_{yoff}}$

$a_{15} = \dfrac{0.1}{\gamma_{\mu y}^2}$

$a_{16} = \dfrac{0.1 E_{y\gamma}}{\gamma_{ey}}$

$a_{17} = E_{y\gamma}$

and

$$\alpha_{peak0} = \frac{a_0 \tan\left(\frac{\pi/2}{a_0}\right) \frac{a_2}{1000} a_4}{2\, a_3}$$

Notice that I have defined two new coefficients, $G_y$ and $\alpha_{peak0}$, that are dependent on the original coefficients and does not increase the required information.

The normalizing factors can be related to the original Pacejka coefficients, *b*, as follows.

$C_x = b_0$ $\qquad$ $b_0 = C_x$

$G_x = \tan\left(\frac{\pi/2}{b_0}\right)$ $\qquad$ $b_1 = \frac{-0.1 * 1000\, \mu_{x0}}{F_{\mu x}}$

$\mu_{x0} = b_2 / 1000$ $\qquad$ $b_2 = 1000\, \mu_{x0}$

$F_{\mu x} = \frac{-0.1\, b_2}{b_1}$ $\qquad$ $b_3 = \frac{0.1\, \mu_{\sigma peak0}}{F_{\sigma z}}$

$F_{xoff0} = b_{12}$ $\qquad$ $b_4 = \mu_{\sigma peak0}$

$F_{xoffz} = 0.1 \frac{b_{12}}{b_{11}} * 1000$ $\qquad$ $b_5 = \frac{1}{F_{\sigma e}}$

$\sigma_{off0} = b_{10}$ $\qquad$ $b_6 = \frac{0.1\, E_{x0}}{F_{xe2}^2}$

$F_{\sigma off} = \frac{-0.1\, b_{10}}{b_9}$ $b_7 = \frac{0.1\, E_{x0}}{F_{xe1}}$

$E_{x0} = b_8$ $b_8 = E_{x0}$

$F_{xe1} = \frac{0.1\, b_8}{b_7}$ $b_9 = \frac{-0.1\, \sigma_{off0}}{F_{\sigma off}}$

$F_{xe2} = \sqrt{\frac{0.1\, b_8}{b_6}}$ $b_{10} = \sigma_{off0}$

$E_{x\sigma} = b_{13}$ $b_{11} = 0.1 \frac{F_{xoff0}}{F_{xoffz}} * 1000$

$F_{\sigma z} = \frac{-0.1\, b_4}{b_3}$ $b_{12} = F_{xoff0}$

$F_{\sigma e} = \frac{1}{b_5}$ $b_{13} = E_{x\sigma}$

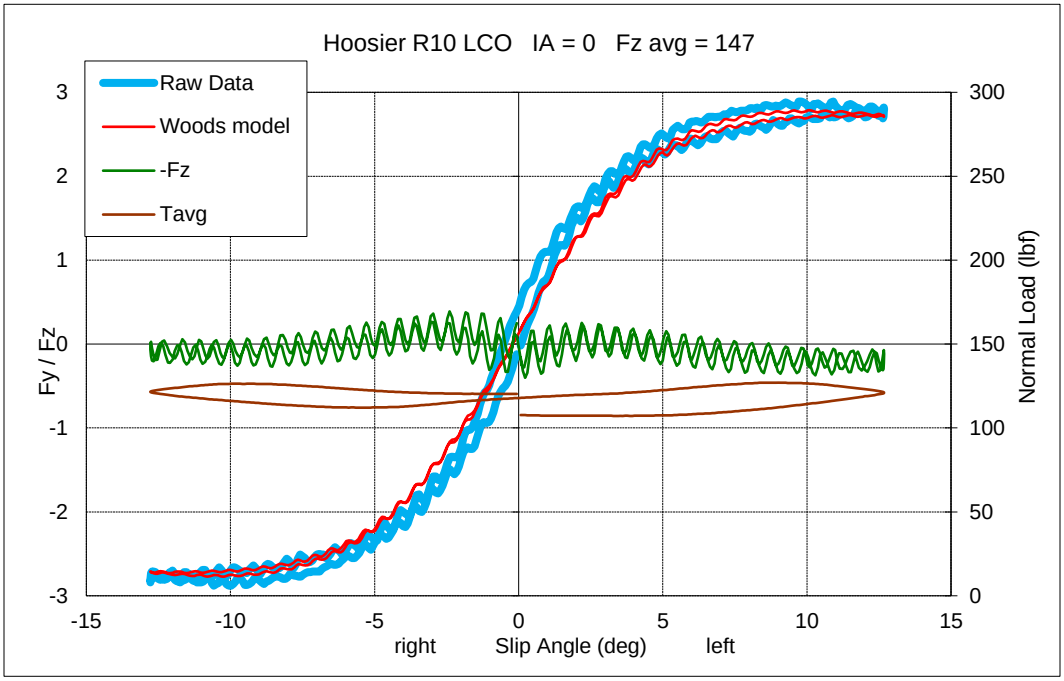
$\mu_{\sigma peak0} = b_4$

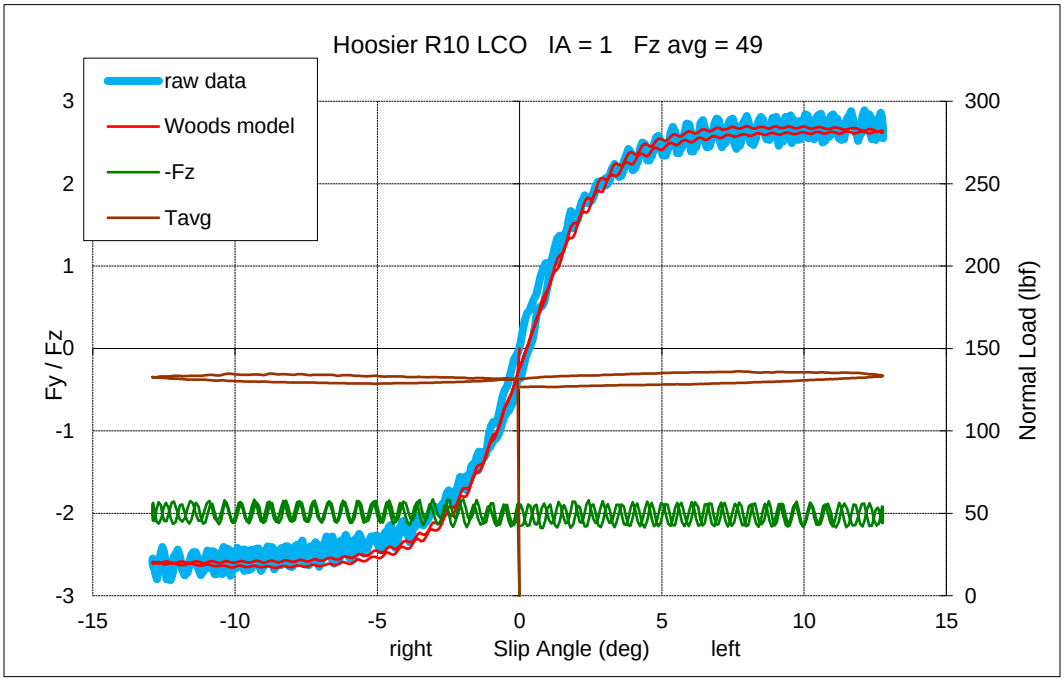
$$\sigma_{peak0} = \frac{b_0 \tan\left(\frac{\pi/2}{b_0}\right) \frac{b_2}{1000}}{b_4}$$

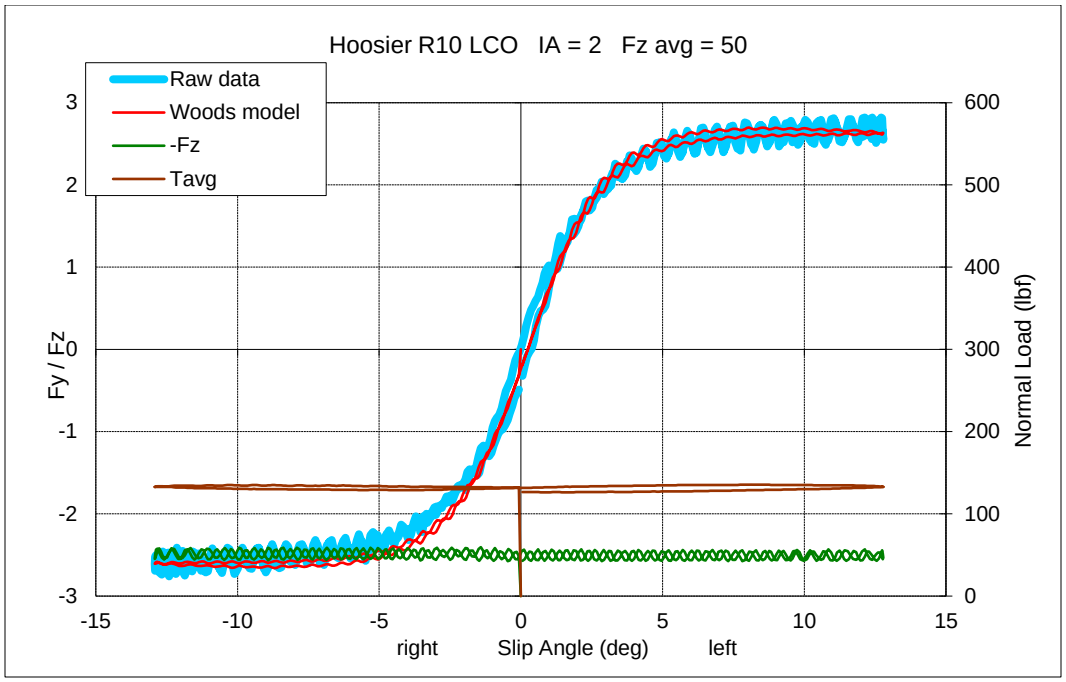
two more dependent variables are introduved $G_x$ and $\sigma_{peak0}$.

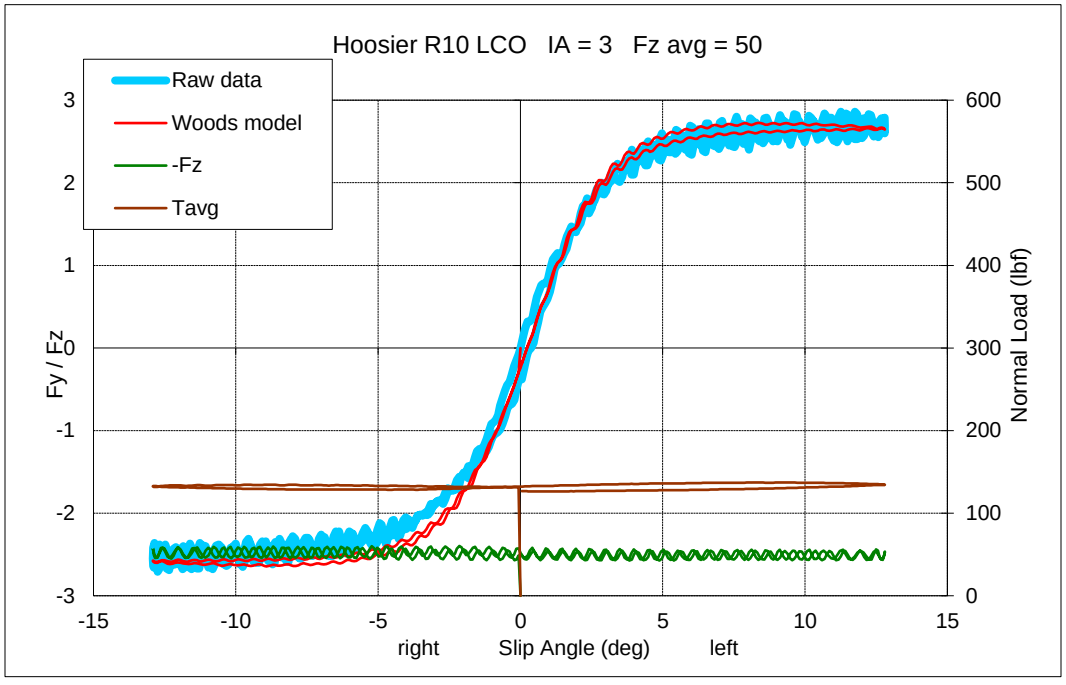
**Appendix A-2- graphs**

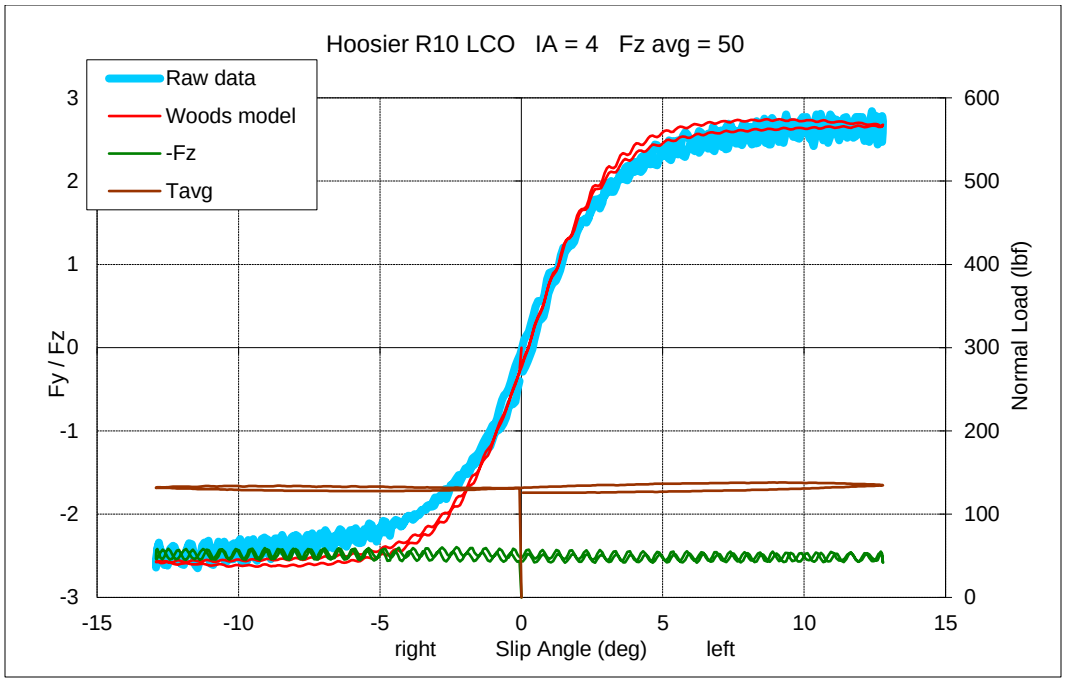
**Hoosier R10 LCO**

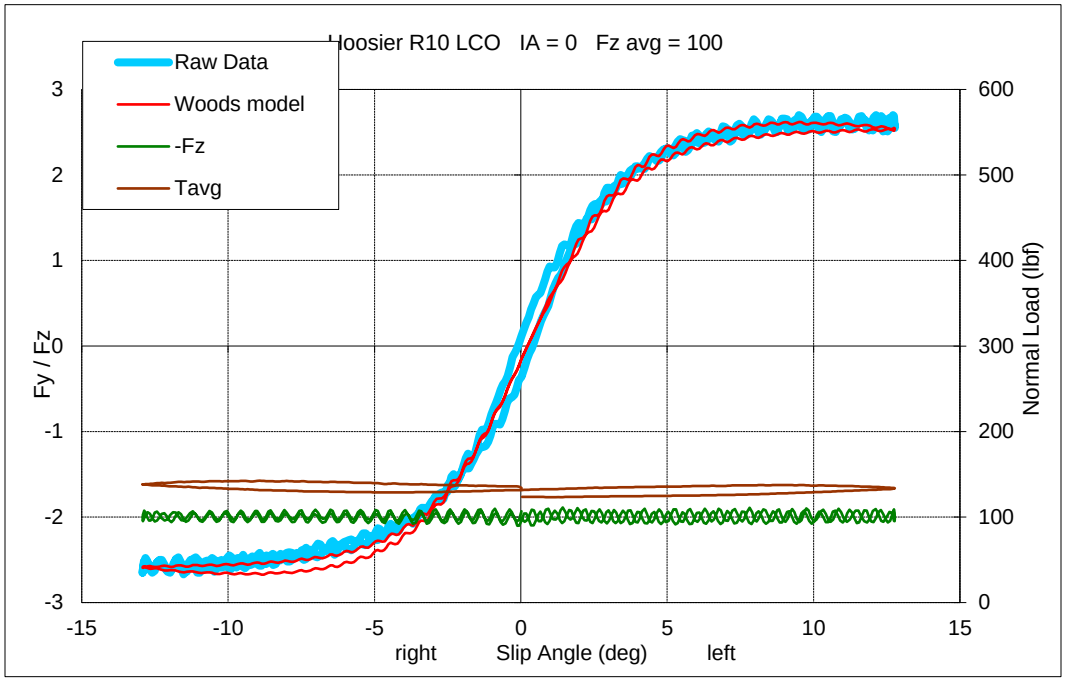


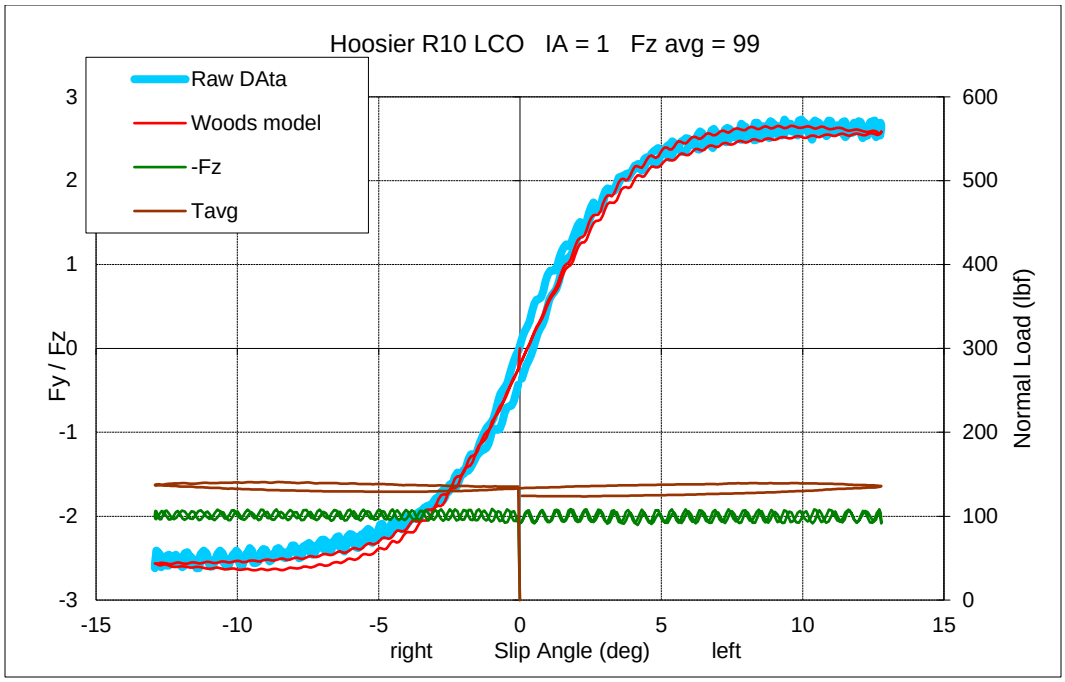



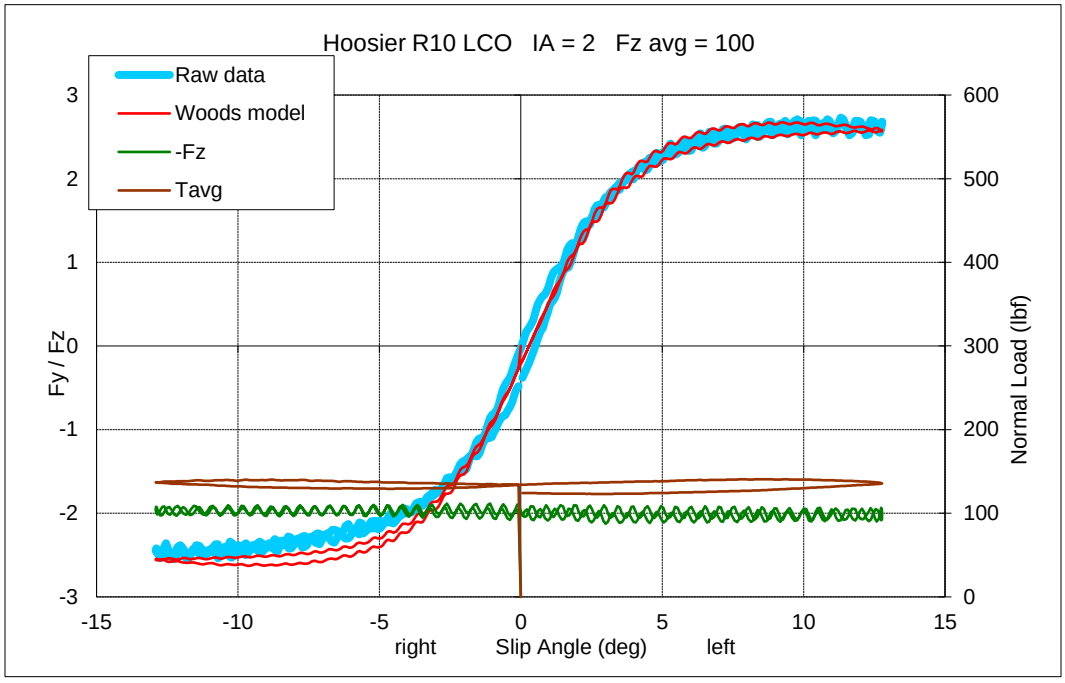

Hoosier R10 LCO IA = 2 Fz avg = 50
Raw data
Woods model
-Fz
Tavg
Fy / Fz
Normal Load (lbf)
Slip Angle (deg)
right
left
3
2
1
0
-1
-2
-3
600
500
400
300
200
100
0
-15
-10
-5
0
5
10
15

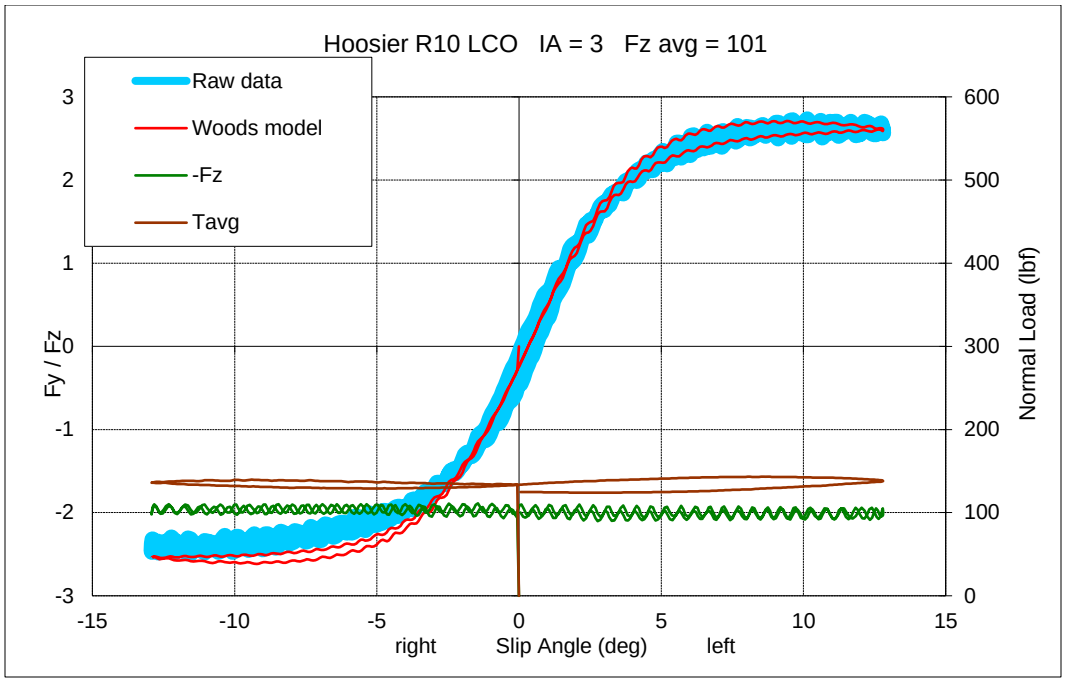

Hoosier R10 LCO IA = 3 Fz avg = 50
Raw data
Woods model
-Fz
Tavg
Fy / Fz
Normal Load (lbf)
Slip Angle (deg)
right
left
3
2
1
0
-1
-2
-3
600
500
400
300
200
100
0
-15
-10
-5
0
5
10
15

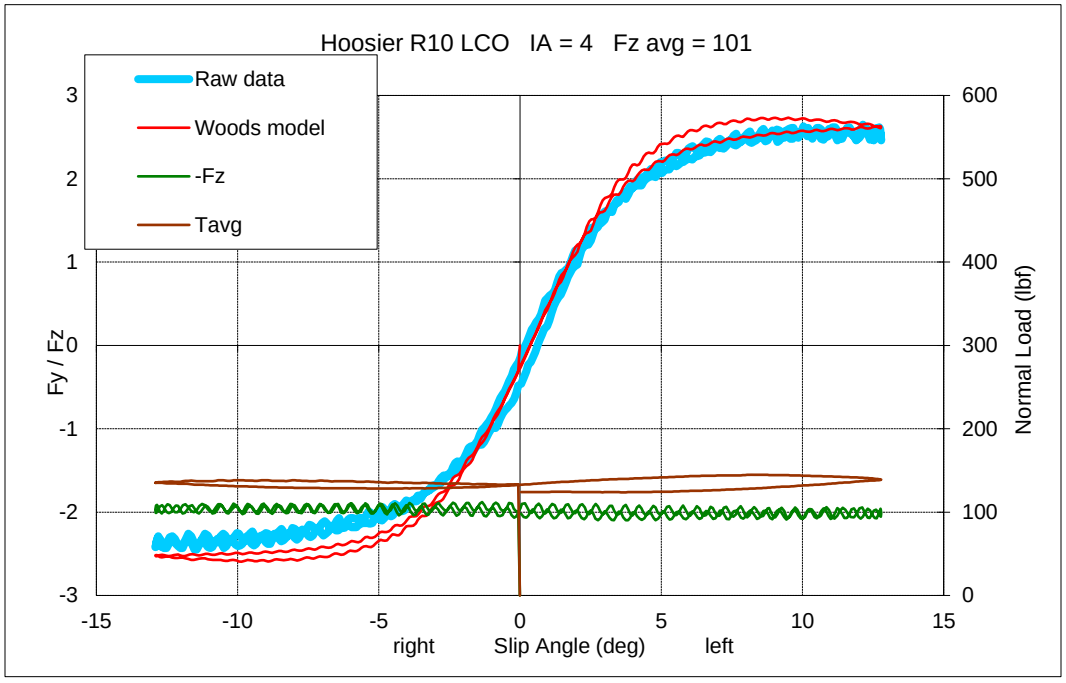

Hoosier R10 LCO IA = 4 Fz avg = 50
Raw data
Woods model
-Fz
Tavg
Fy / Fz
Normal Load (lbf)
Slip Angle (deg)
right
left

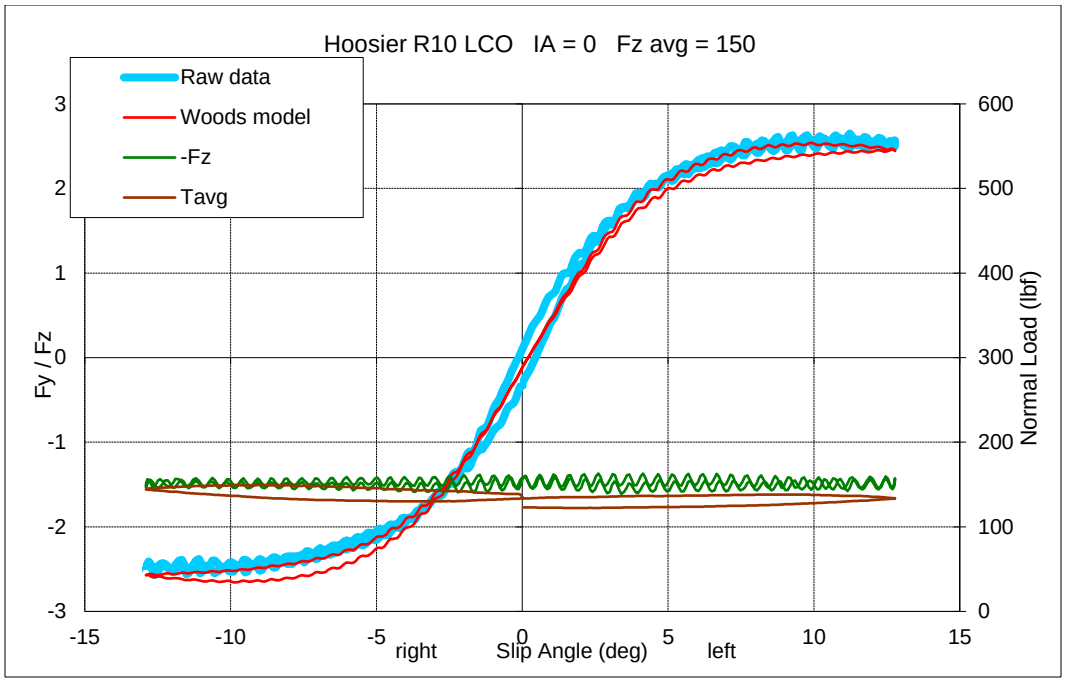

Raw Data
Woods model
-Fz
Tavg
Fy / Fz
Normal Load (lbf)
Slip Angle (deg)
right
left

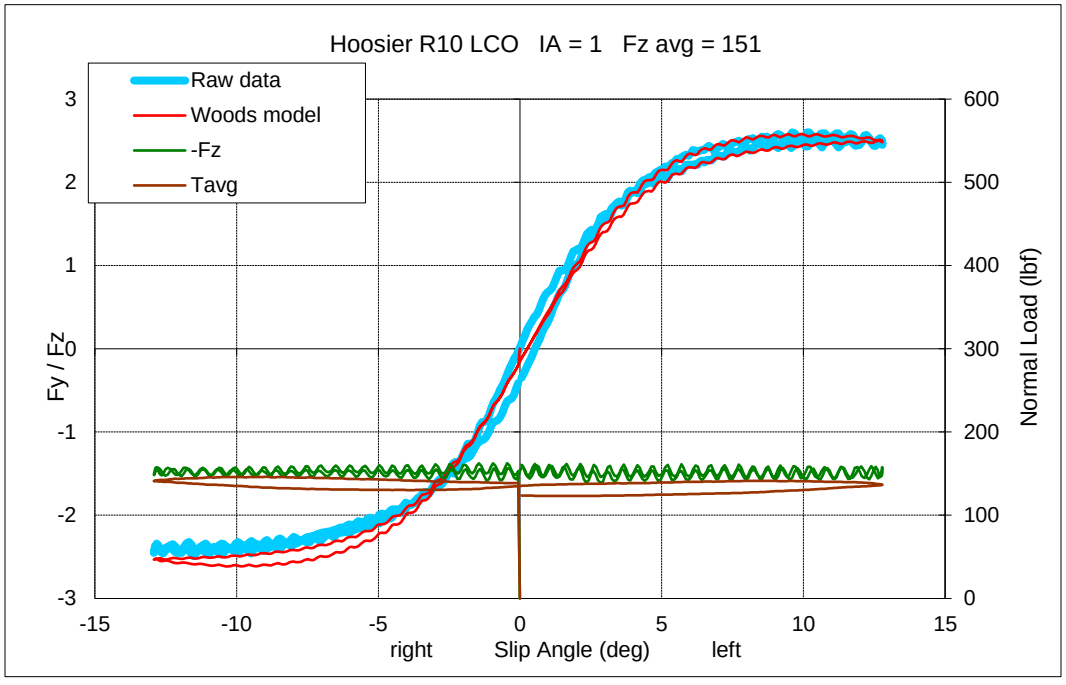

Hoosier R10 LCO IA = 1 Fz avg = 99
Raw DAta
Woods model
-Fz
Tavg
Fy / Fz
Normal Load (lbf)
Slip Angle (deg)
right
left

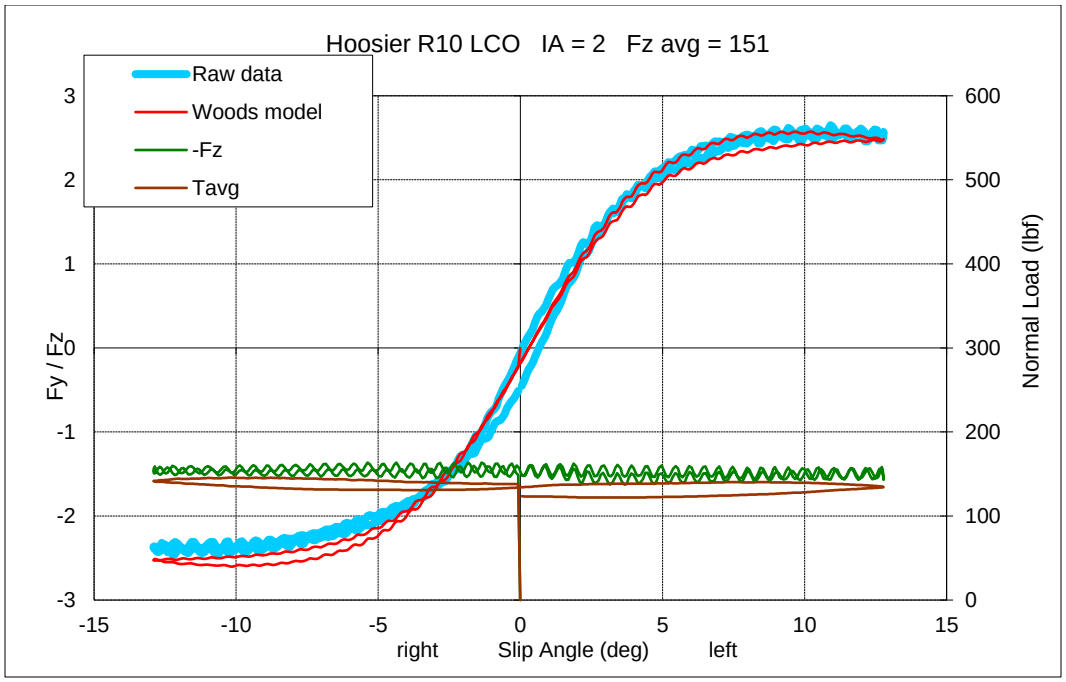

Hoosier R10 LCO IA = 2 Fz avg = 100
Raw data
Woods model
-Fz
Tavg
Fy / Fz
Normal Load (lbf)
Slip Angle (deg)
right
left

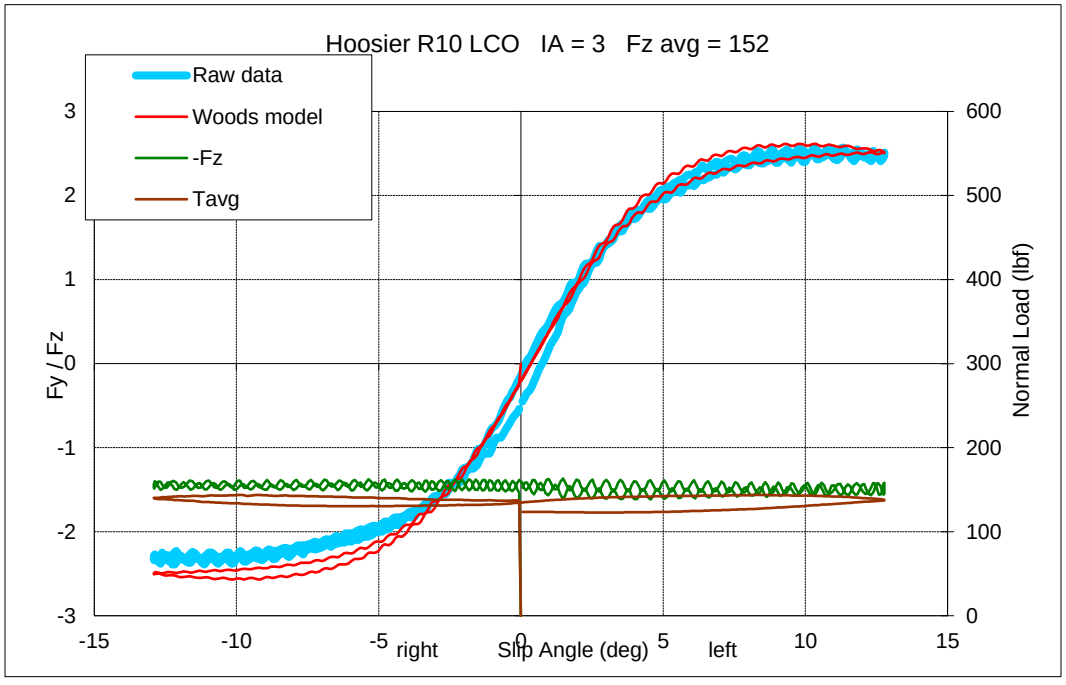

Hoosier R10 LCO IA = 3 Fz avg = 101
Raw data
Woods model
-Fz
Tavg
Fy / Fz
Normal Load (lbf)
Slip Angle (deg)
right
left

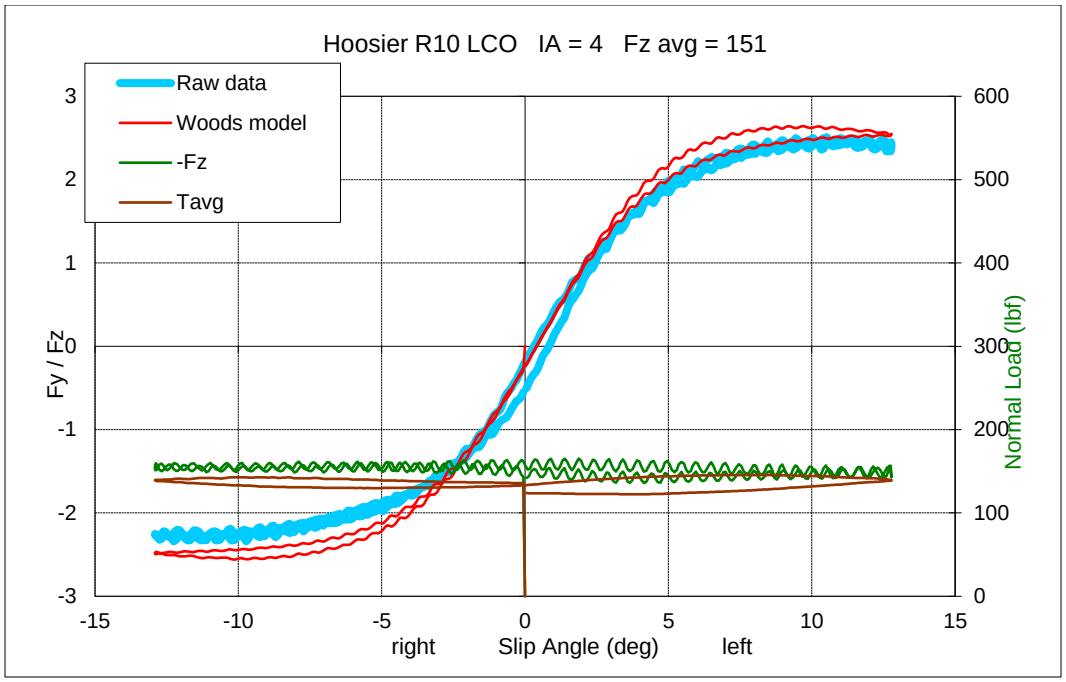

Hoosier R10 LCO IA = 4 Fz avg = 101
Raw data
Woods model
-Fz
Tavg
Fy / Fz
Normal Load (lbf)
Slip Angle (deg)
right
left

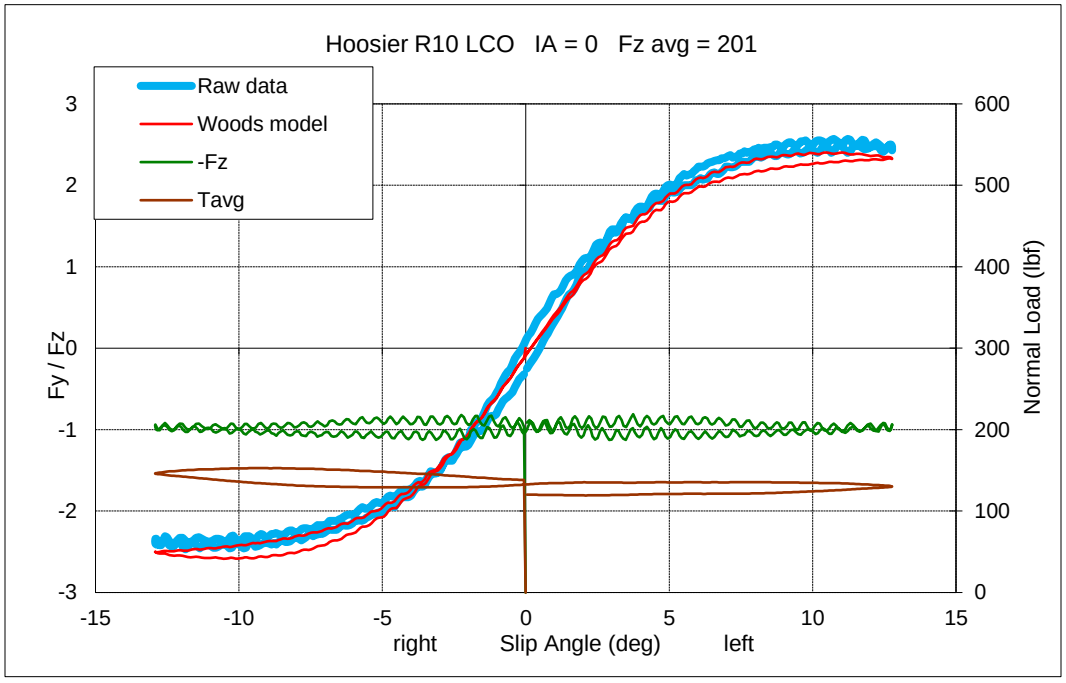

Hoosier R10 LCO IA = 0 Fz avg = 150
Raw data
Woods model
-Fz
Tavg
Fy / Fz
Normal Load (lbf)
Slip Angle (deg)
right
left
3
2
1
0
-1
-2
-3
600
500
400
300
200
100
0
-15
-10
-5
0
5
10
15

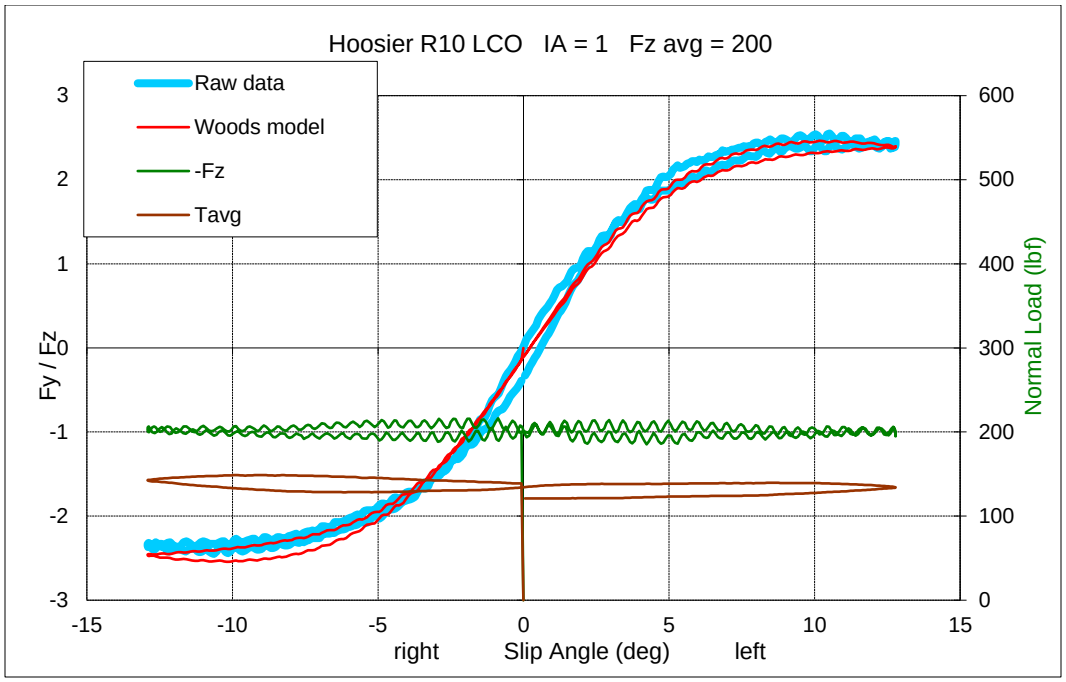

Hoosier R10 LCO IA = 1 Fz avg = 151
Raw data
Woods model
-Fz
Tavg
Fy / Fz
Normal Load (lbf)
Slip Angle (deg)
right
left
3
2
1
0
-1
-2
-3
600
500
400
300
200
100
0
-15
-10
-5
0
5
10
15

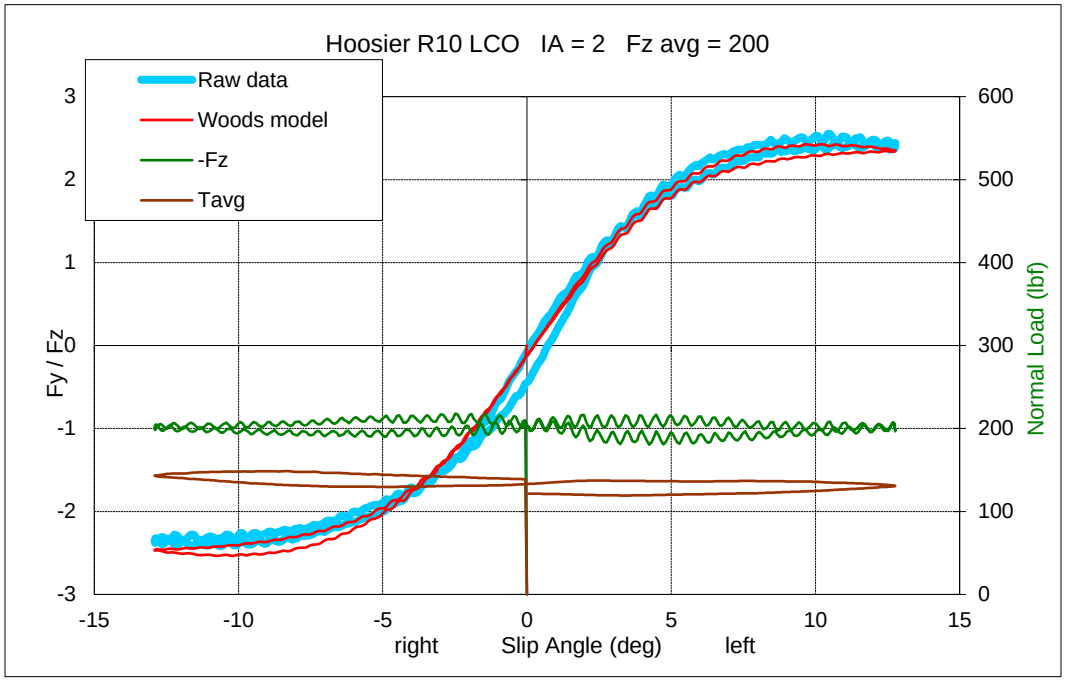

Hoosier R10 LCO IA = 2 Fz avg = 151
Raw data
Woods model
-Fz
Tavg
Fy / Fz
Normal Load (lbf)
Slip Angle (deg)
right
left
3
2
1
0
-1
-2
-3
600
500
400
300
200
100
0
-15
-10
-5
0
5
10
15

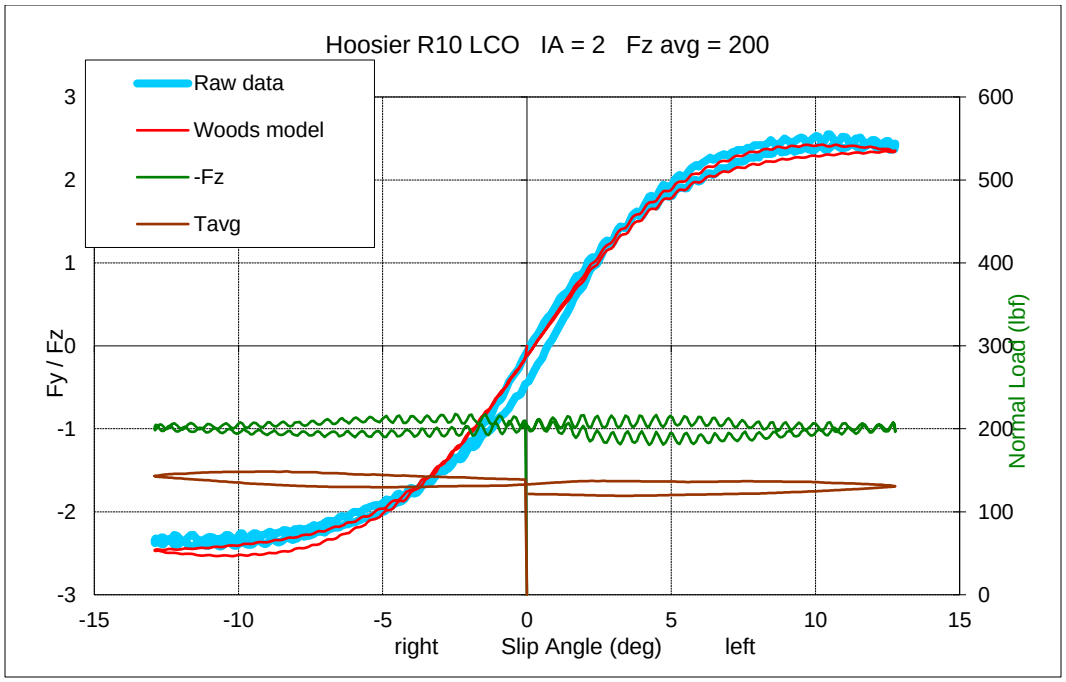

Hoosier R10 LCO IA = 3 Fz avg = 152
Raw data
Woods model
-Fz
Tavg
Fy / Fz
Normal Load (lbf)
Slip Angle (deg)
right
left
3
2
1
0
-1
-2
-3
600
500
400
300
200
100
0
-15
-10
-5
0
5
10
15

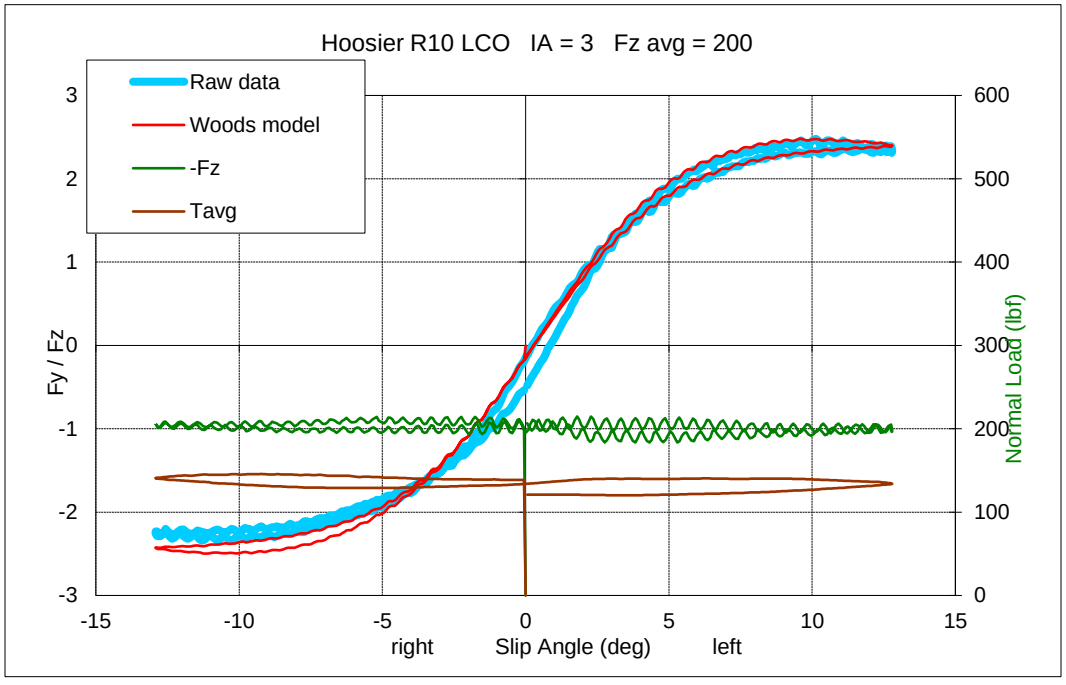

Hoosier R10 LCO IA = 4 Fz avg = 151
Raw data
Woods model
-Fz
Tavg
Fy / Fz
Normal Load (lbf)
Slip Angle (deg)
right
left

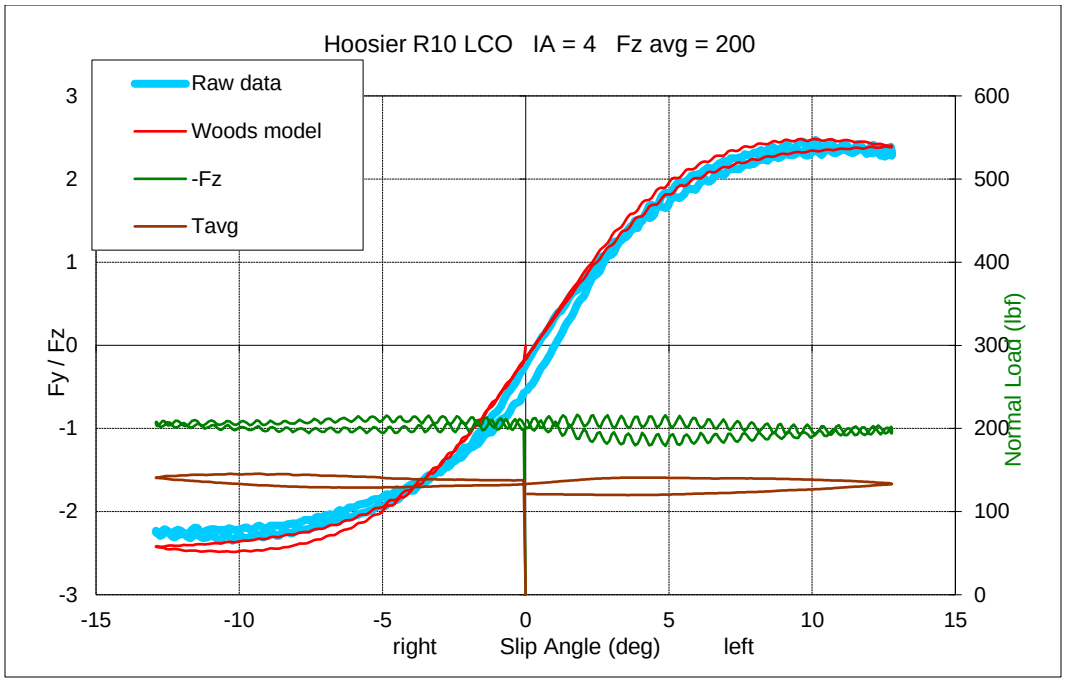

Hoosier R10 LCO IA = 0 Fz avg = 201
Raw data
Woods model
-Fz
Tavg
Fy / Fz
Normal Load (lbf)
Slip Angle (deg)
right
left

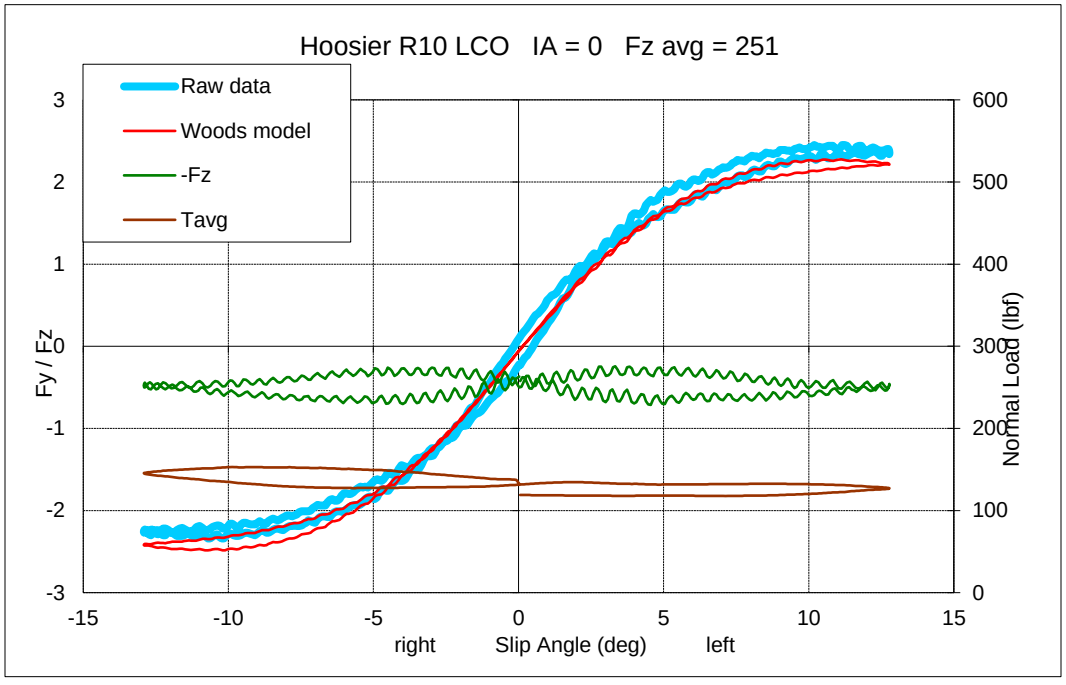

Hoosier R10 LCO IA = 1 Fz avg = 200
Raw data
Woods model
-Fz
Tavg
Fy / Fz
Normal Load (lbf)
Slip Angle (deg)
right
left
3
2
1
0
-1
-2
-3
600
500
400
300
200
100
0
-15
-10
-5
0
5
10
15

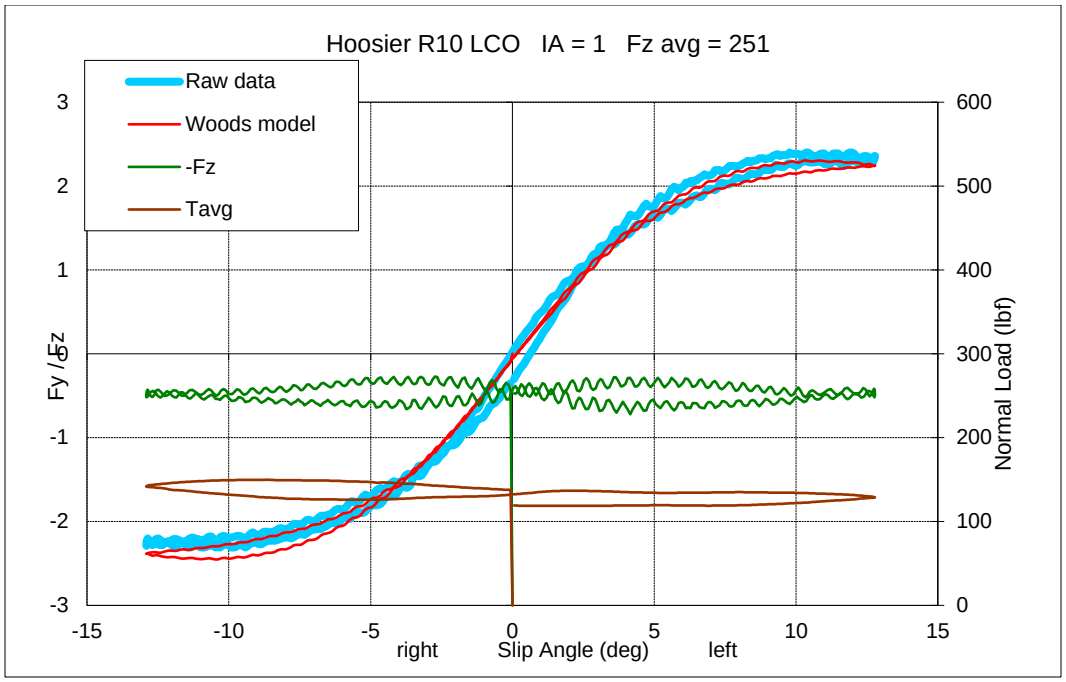

Hoosier R10 LCO IA = 2 Fz avg = 200
Raw data
Woods model
-Fz
Tavg
Fy / Fz
Normal Load (lbf)
Slip Angle (deg)
right
left
3
2
1
0
-1
-2
-3
600
500
400
300
200
100
0
-15
-10
-5
0
5
10
15

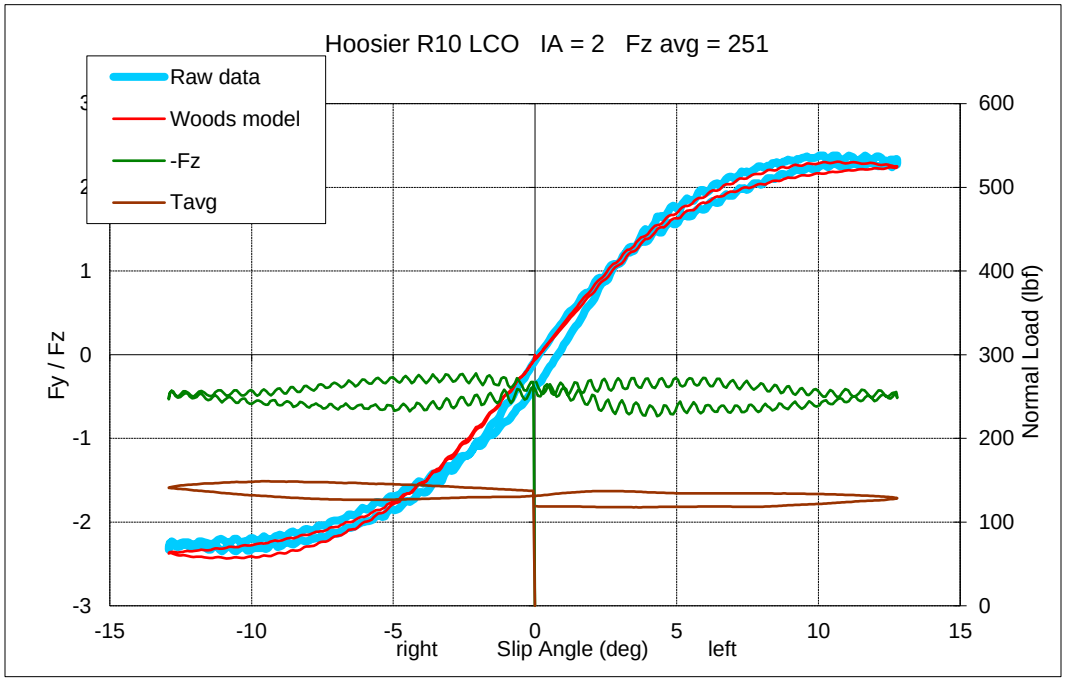

Hoosier R10 LCO IA = 2 Fz avg = 200
Raw data
Woods model
-Fz
Tavg
Fy / Fz
Normal Load (lbf)
Slip Angle (deg)
right
left
3
2
1
0
-1
-2
-3
600
500
400
300
200
100
0
-15
-10
-5
0
5
10
15

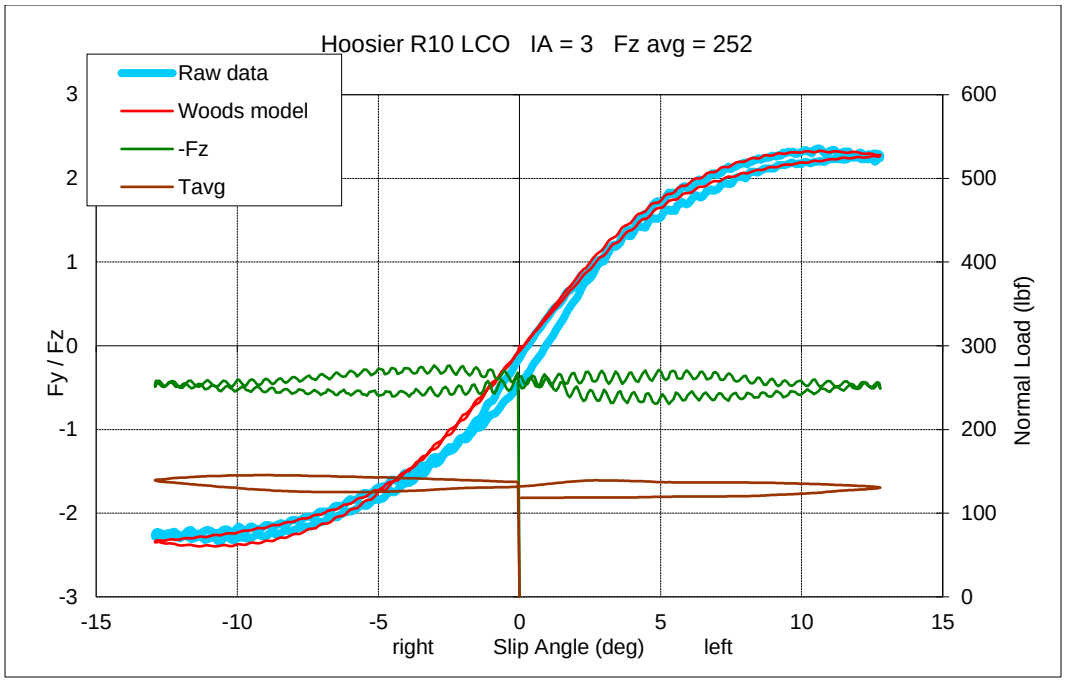

Hoosier R10 LCO IA = 3 Fz avg = 200
Raw data
Woods model
-Fz
Tavg
Fy / Fz
Normal Load (lbf)
Slip Angle (deg)
right
left
3
2
1
0
-1
-2
-3
600
500
400
300
200
100
0
-15
-10
-5
0
5
10
15

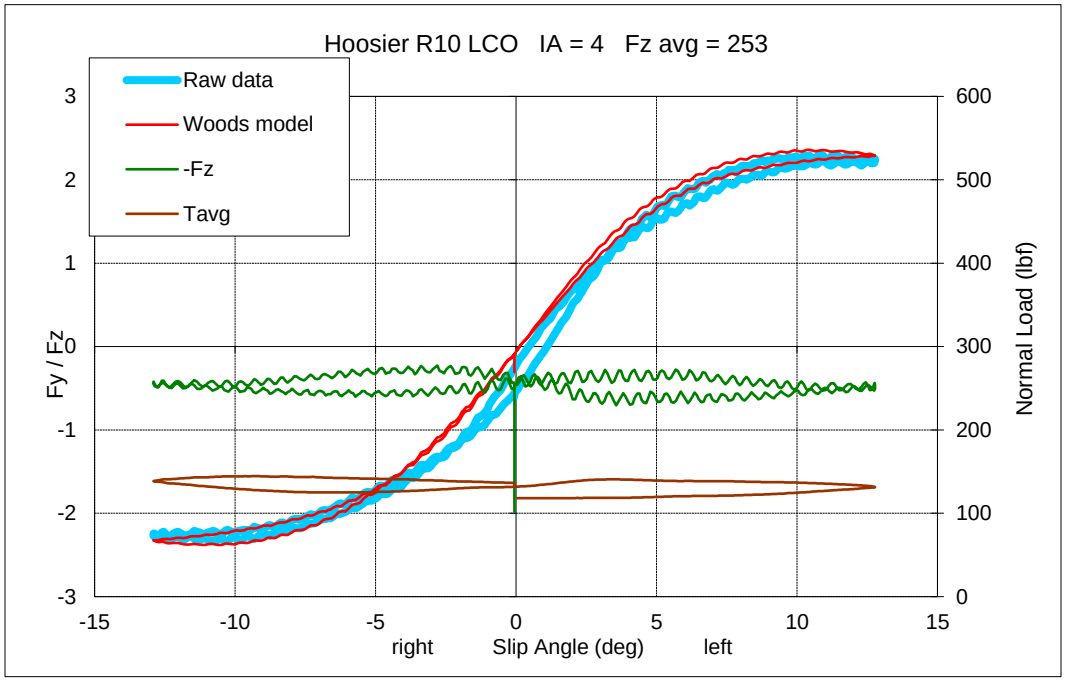

Hoosier R10 LCO IA = 4 Fz avg = 200
Raw data
Woods model
-Fz
Tavg
Fy / Fz
Normal Load (lbf)
Slip Angle (deg)
right
left
3
2
1
0
-1
-2
-3
600
500
400
300
200
100
0
-15
-10
-5
0
5
10
15

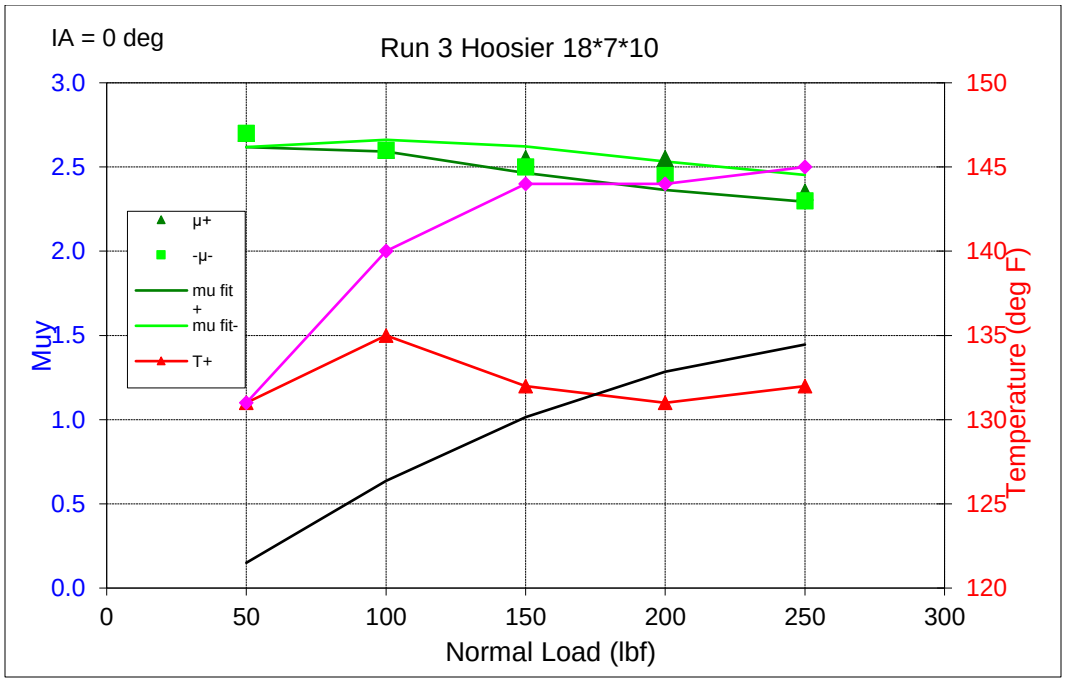

Hoosier R10 LCO IA = 0 Fz avg = 251
Raw data
Woods model
-Fz
Tavg
Fy / Fz
Normal Load (lbf)
Slip Angle (deg)
right
left
3
2
1
0
-1
-2
-3
600
500
400
300
200
100
0
-15
-10
-5
0
5
10
15

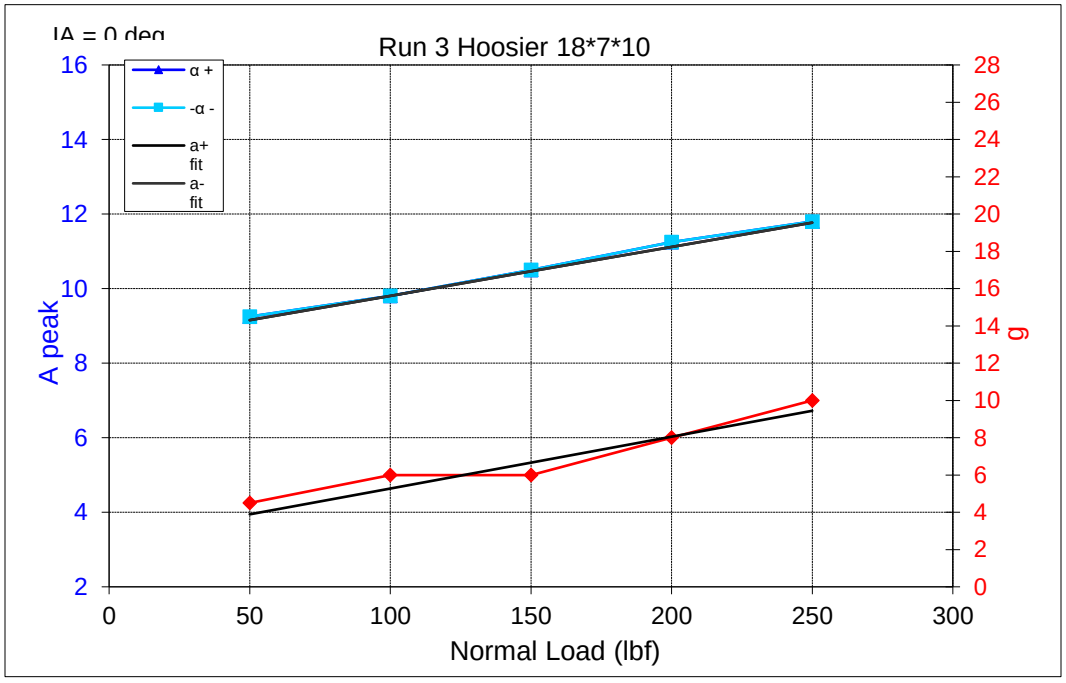

Hoosier R10 LCO IA = 1 Fz avg = 251
Raw data
Woods model
-Fz
Tavg
Fy / Fz
Normal Load (lbf)
Slip Angle (deg)
right
left

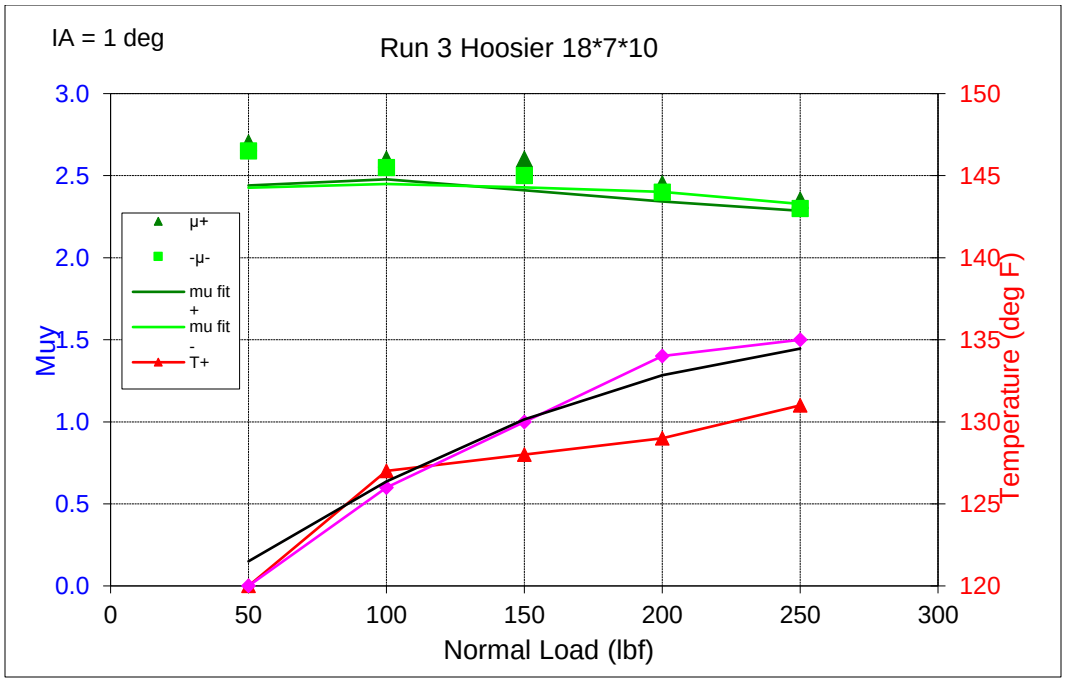

Hoosier R10 LCO IA = 2 Fz avg = 251
Raw data
Woods model
-Fz
Tavg
Fy / Fz
Normal Load (lbf)
Slip Angle (deg)
right
left

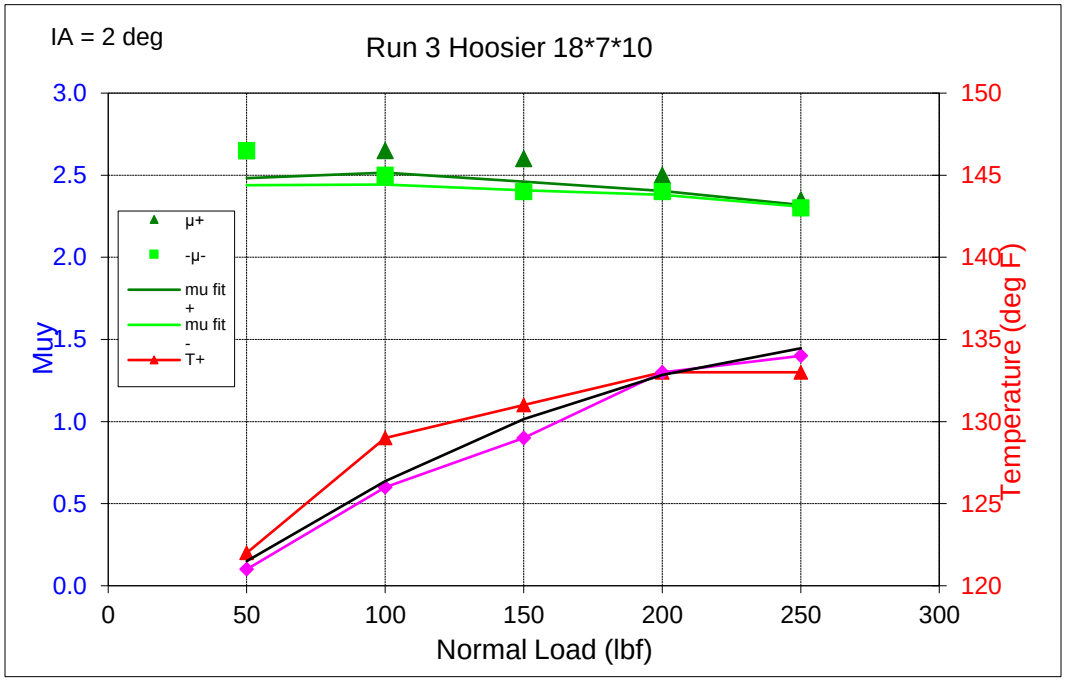

Hoosier R10 LCO IA = 3 Fz avg = 252
Raw data
Woods model
-Fz
Tavg
Fy / Fz
Normal Load (lbf)
Slip Angle (deg)
right
left

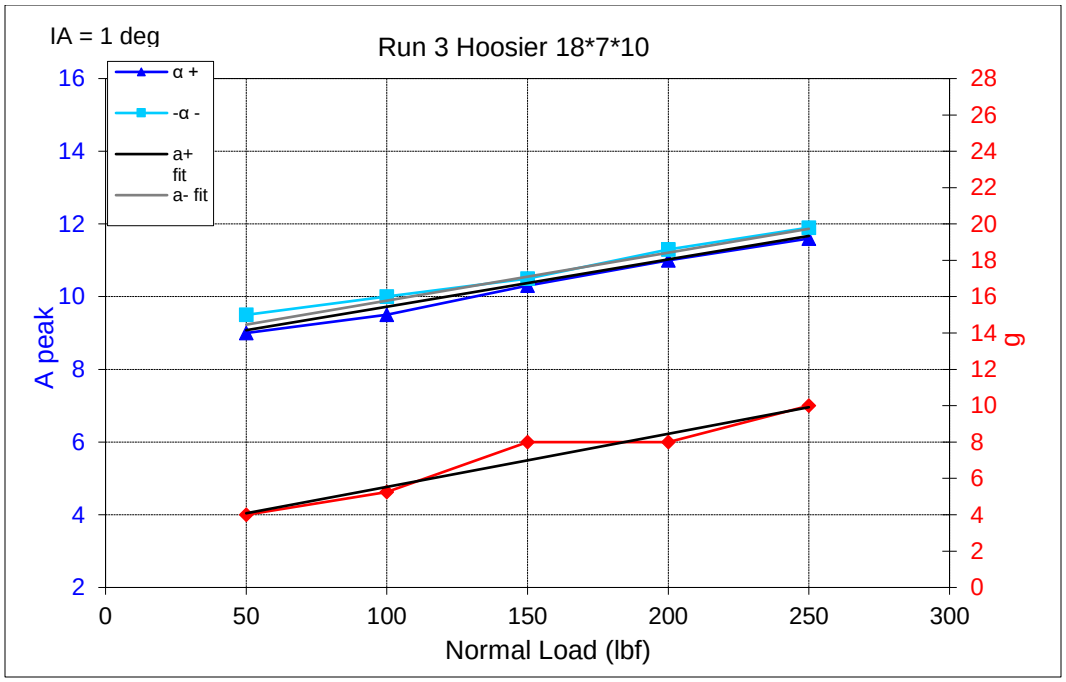

Hoosier R10 LCO IA = 4 Fz avg = 253
Raw data
Woods model
-Fz
Tavg
Fy / Fz
Normal Load (lbf)
Slip Angle (deg)
right
left

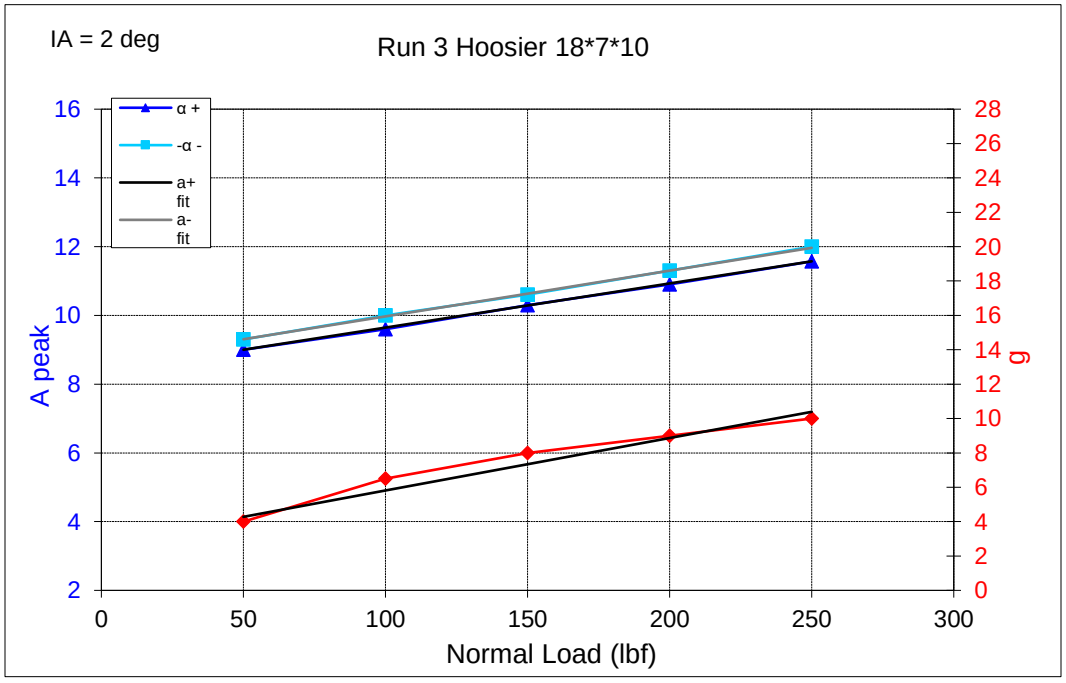

IA = 0 deg
Run 3 Hoosier 18*7*10
3.0
2.5
2.0
1.5
1.0
0.5
0.0
Muy
150
145
140
135
130
125
120
Temperature (deg F)
0
50
100
150
200
250
300
Normal Load (lbf)
μ+
-μ-
mu fit +
mu fit-
T+

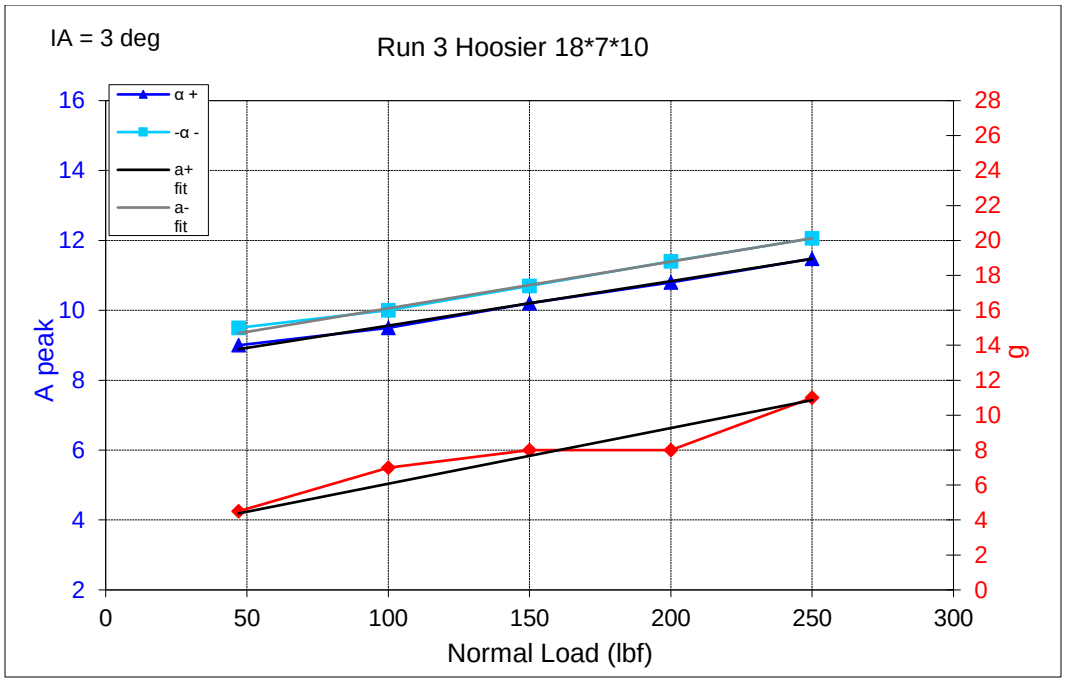

IA = 0 deg
Run 3 Hoosier 18*7*10
16
14
12
10
8
6
4
2
A peak
28
26
24
22
20
18
16
14
12
10
8
6
4
2
0
g
0
50
100
150
200
250
300
Normal Load (lbf)
α +
-α -
a+ fit
a- fit

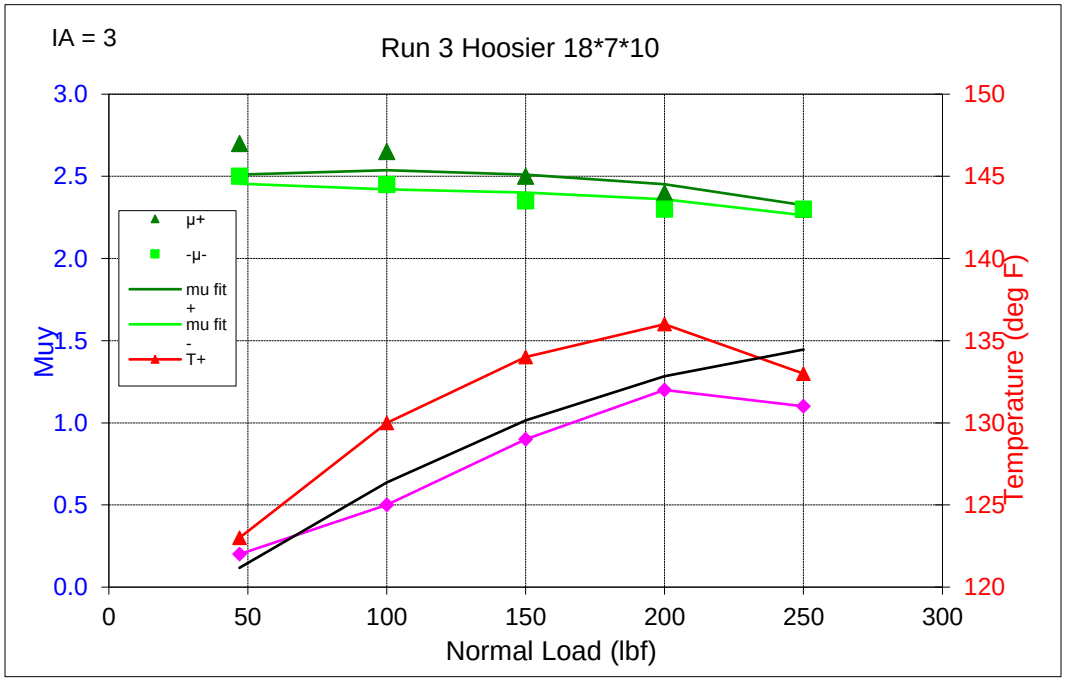

IA = 1 deg
Run 3 Hoosier 18*7*10
3.0
2.5
2.0
1.5
1.0
0.5
0.0
Muy
150
145
140
135
130
125
120
Temperature (deg F)
0
50
100
150
200
250
300
Normal Load (lbf)
μ+
-μ-
mu fit +
mu fit -
T+

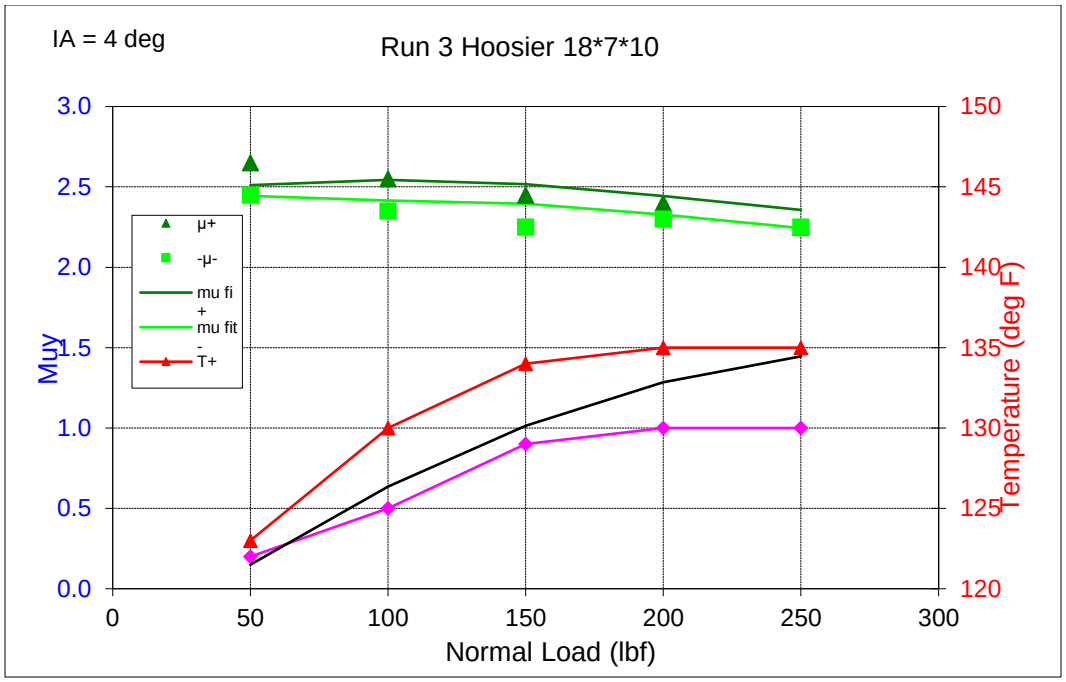

IA = 2 deg
Run 3 Hoosier 18*7*10
3.0
2.5
2.0
1.5
1.0
0.5
0.0
Muy
150
145
140
135
130
125
120
Temperature (deg F)
0
50
100
150
200
250
300
Normal Load (lbf)
μ+
-μ-
mu fit +
mu fit -
T+

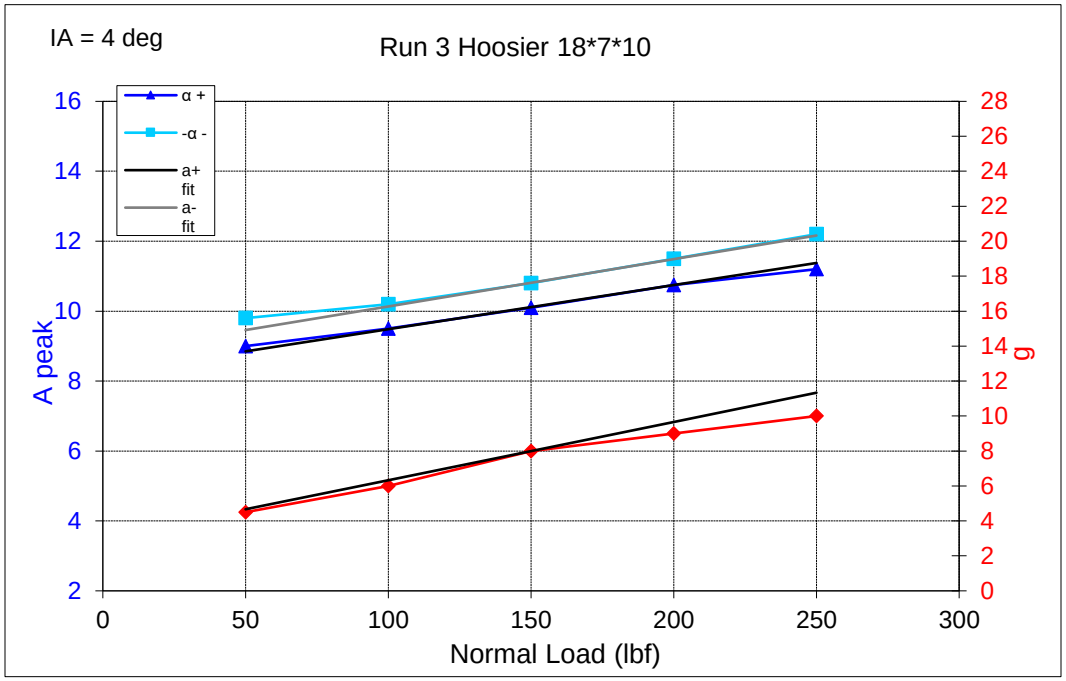

IA = 1 deg
Run 3 Hoosier 18*7*10
α +
-α -
a+ fit
a- fit
A peak
g
Normal Load (lbf)
0
50
100
150
200
250
300
2
4
6
8
10
12
14
16
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28


IA = 2 deg
Run 3 Hoosier 18*7*10
α +
-α -
a+ fit
a- fit
A peak
g
Normal Load (lbf)
0
50
100
150
200
250
300
2
4
6
8
10
12
14
16
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28

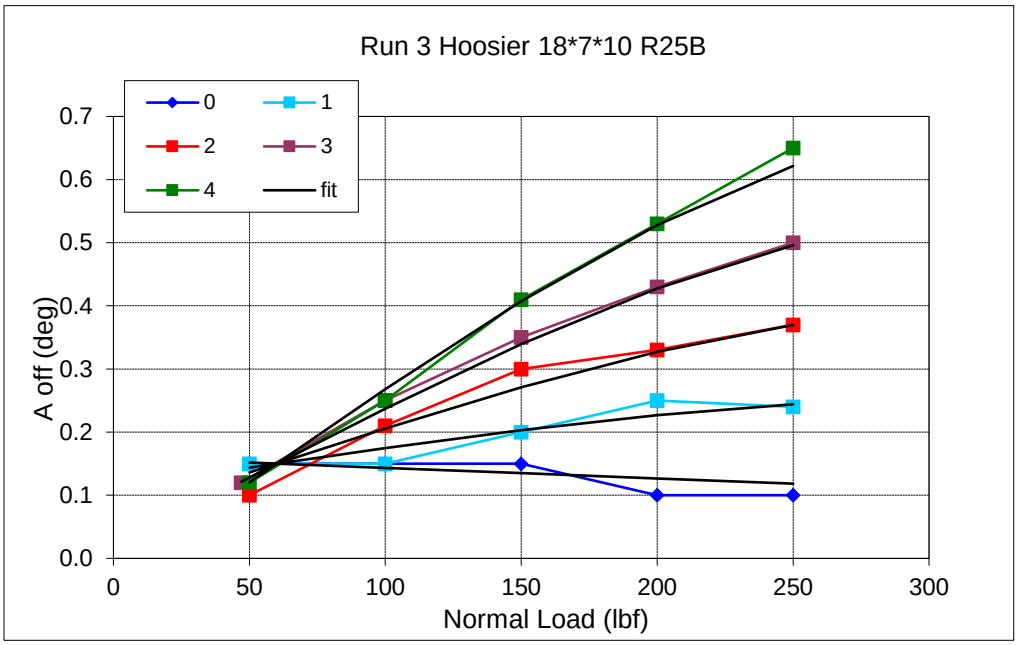

IA = 3 deg
Run 3 Hoosier 18*7*10
α +
-α -
a+ fit
a- fit
A peak
g
Normal Load (lbf)
16
14
12
10
8
6
4
2
28
26
24
22
20
18
16
14
12
10
8
6
4
2
0
0
50
100
150
200
250
300

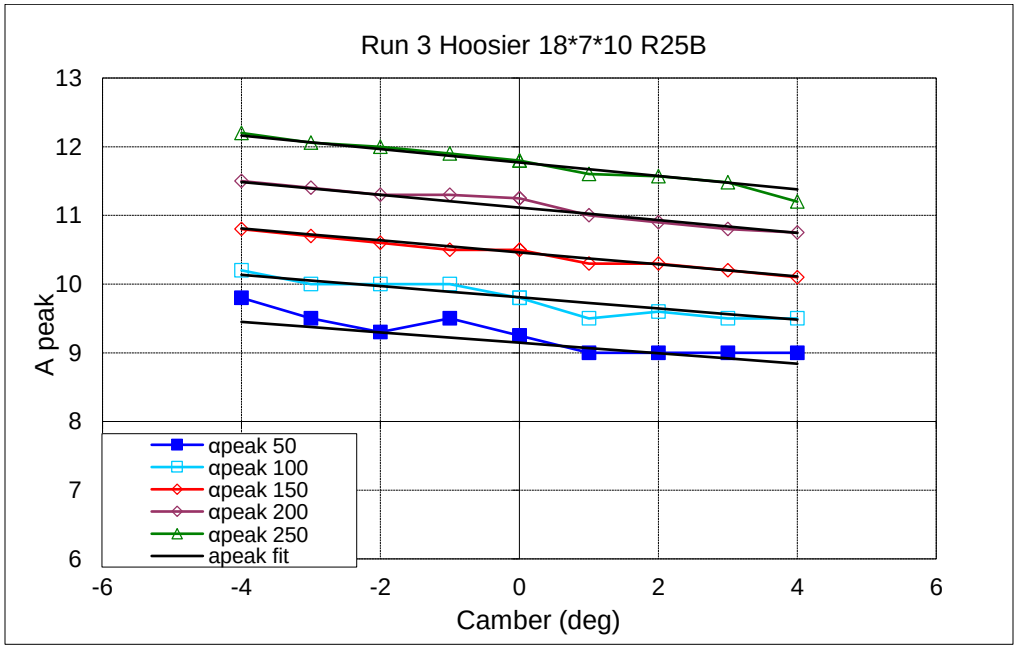

IA = 3
Run 3 Hoosier 18*7*10
μ+
-μ-
mu fit +
mu fit -
T+
Muy
Temperature (deg F)
Normal Load (lbf)
3.0
2.5
2.0
1.5
1.0
0.5
0.0
150
145
140
135
130
125
120
0
50
100
150
200
250
300

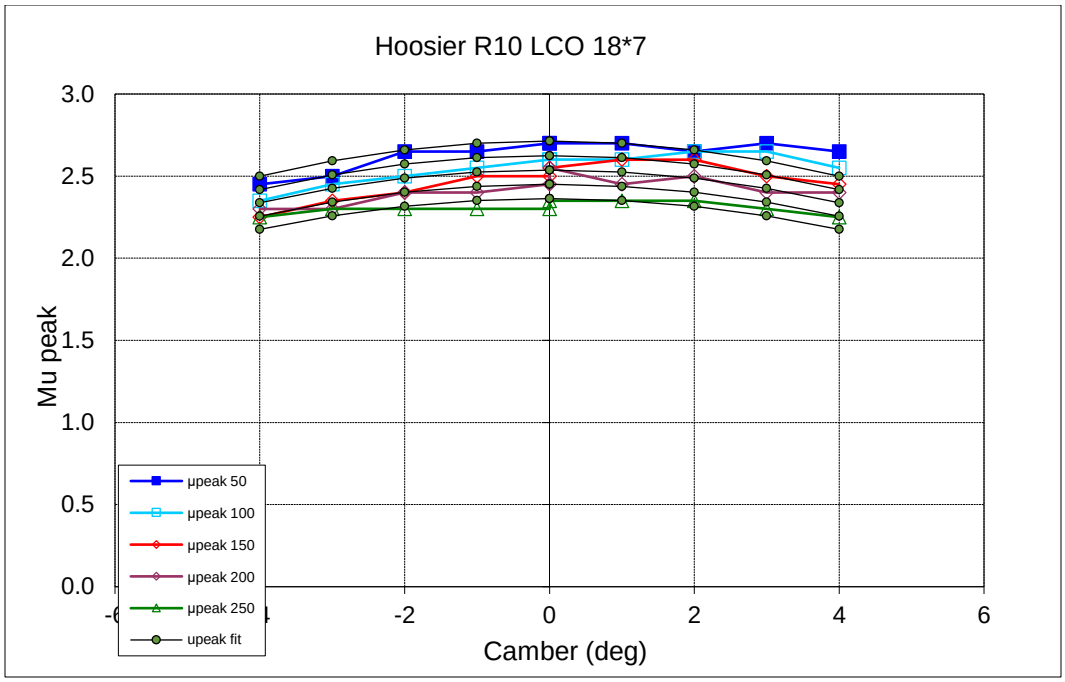

IA = 4 deg
Run 3 Hoosier 18*7*10
Muy
Temperature (deg F)
Normal Load (lbf)
μ+
-μ-
mu fi +
mu fit -
T+
3.0
2.5
2.0
1.5
1.0
0.5
0.0
150
145
140
135
130
125
120
0
50
100
150
200
250
300


IA = 4 deg
Run 3 Hoosier 18*7*10
A peak
g
Normal Load (lbf)
α +
-α -
a+ fit
a- fit
16
14
12
10
8
6
4
2
28
26
24
22
20
18
16
14
12
10
8
6
4
2
0
0
50
100
150
200
250
300

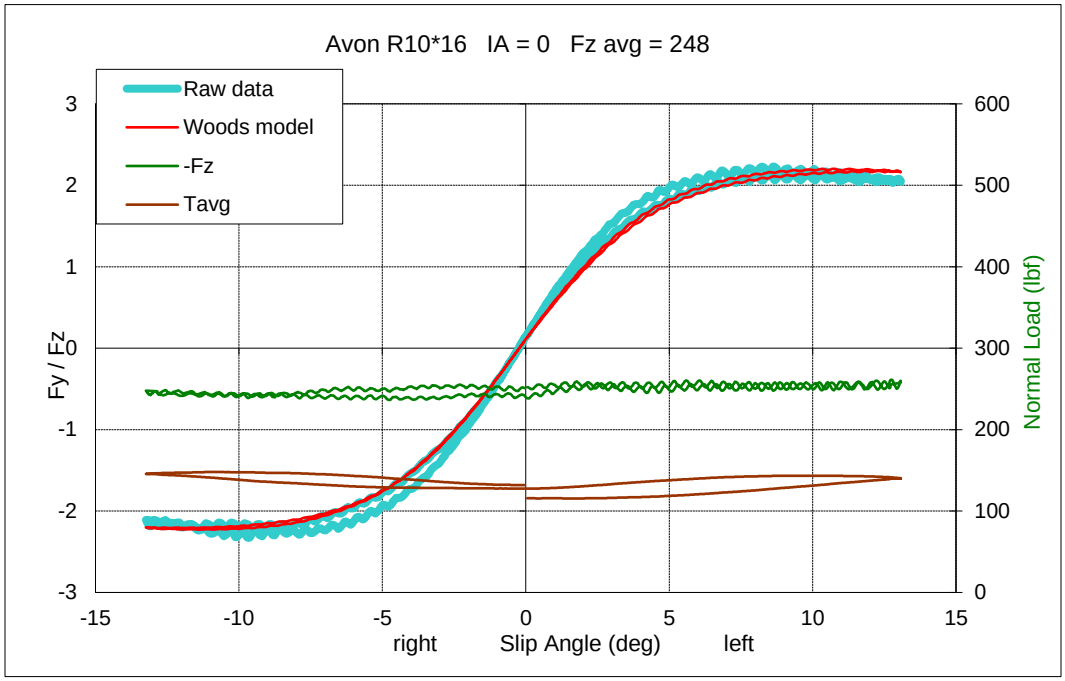
Parametric Graphs

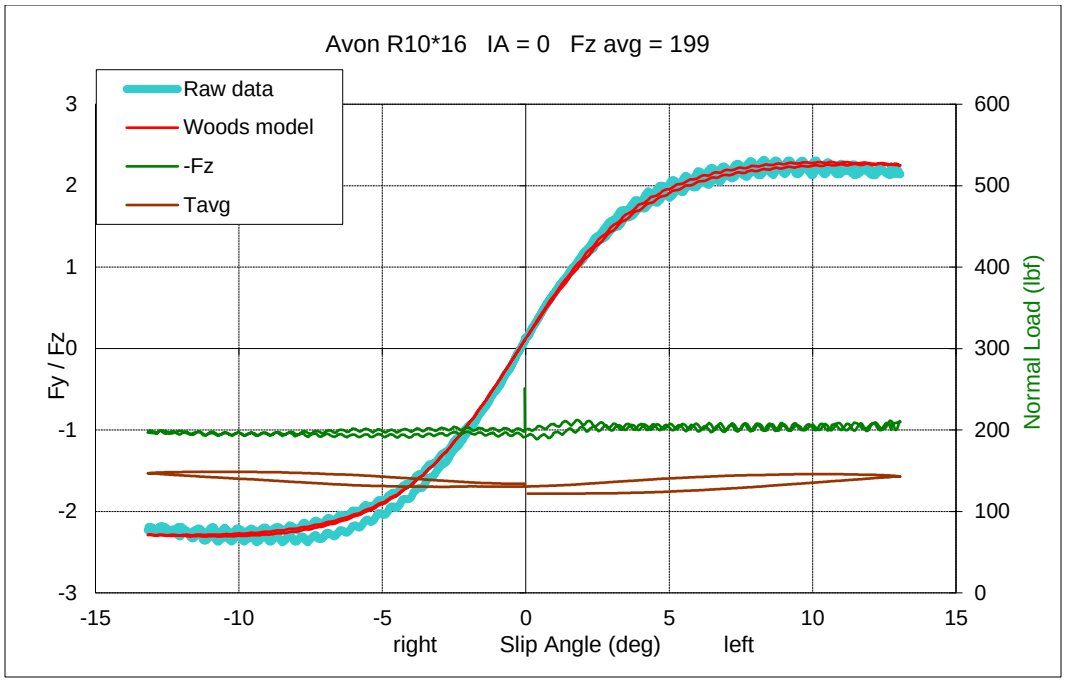


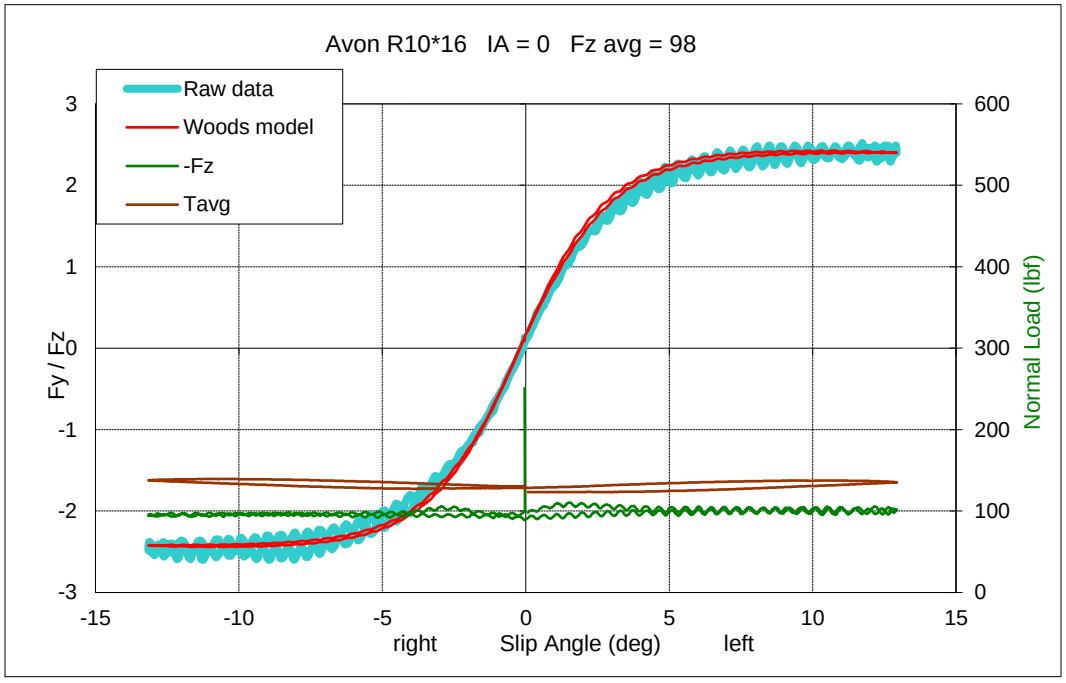



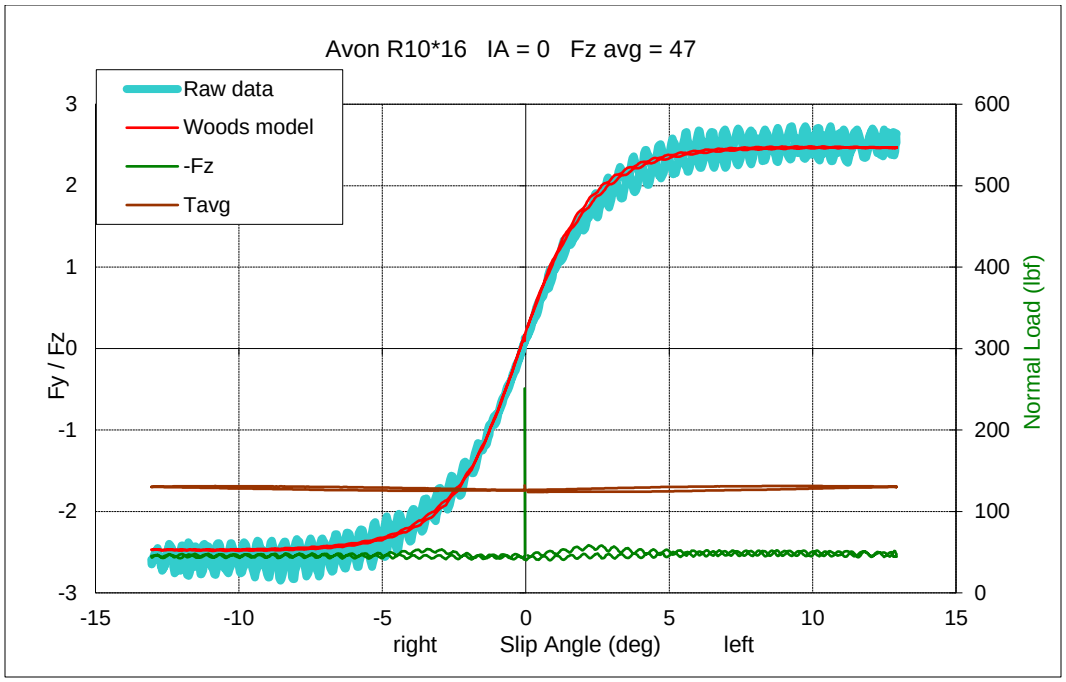

Hoosier R10 LCO 18*7
Mu peak
Camber (deg)
μpeak 50
μpeak 100
μpeak 150
μpeak 200
μpeak 250
μpeak fit
3.0
2.5
2.0
1.5
1.0
0.5
0.0
-2
0
2
4
6

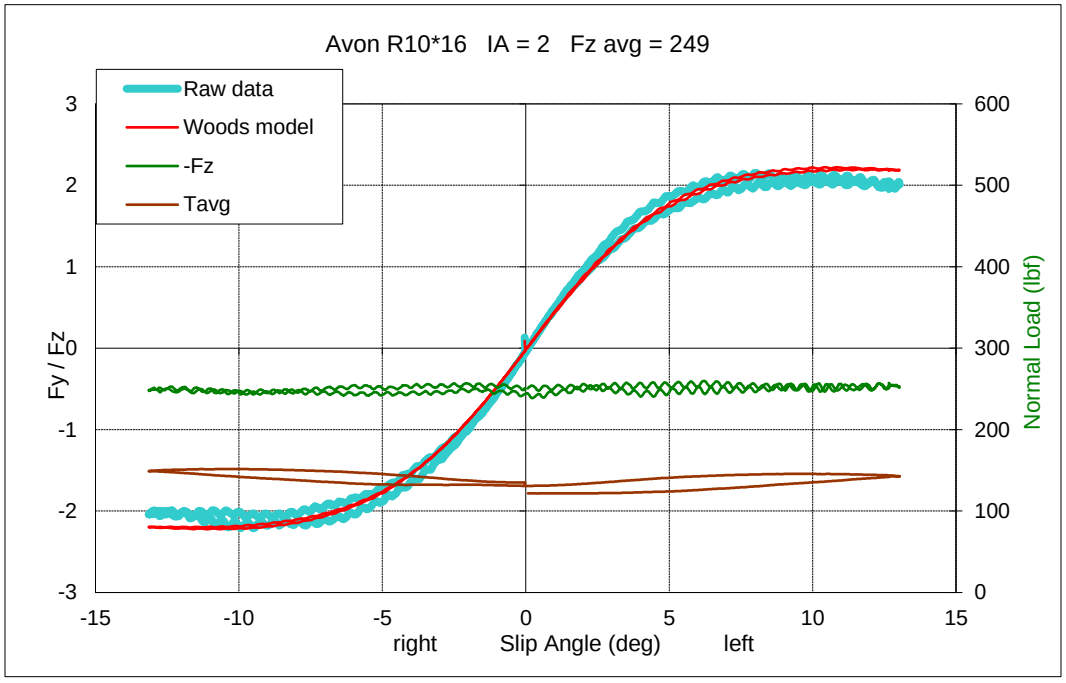
**Avon R10*16**

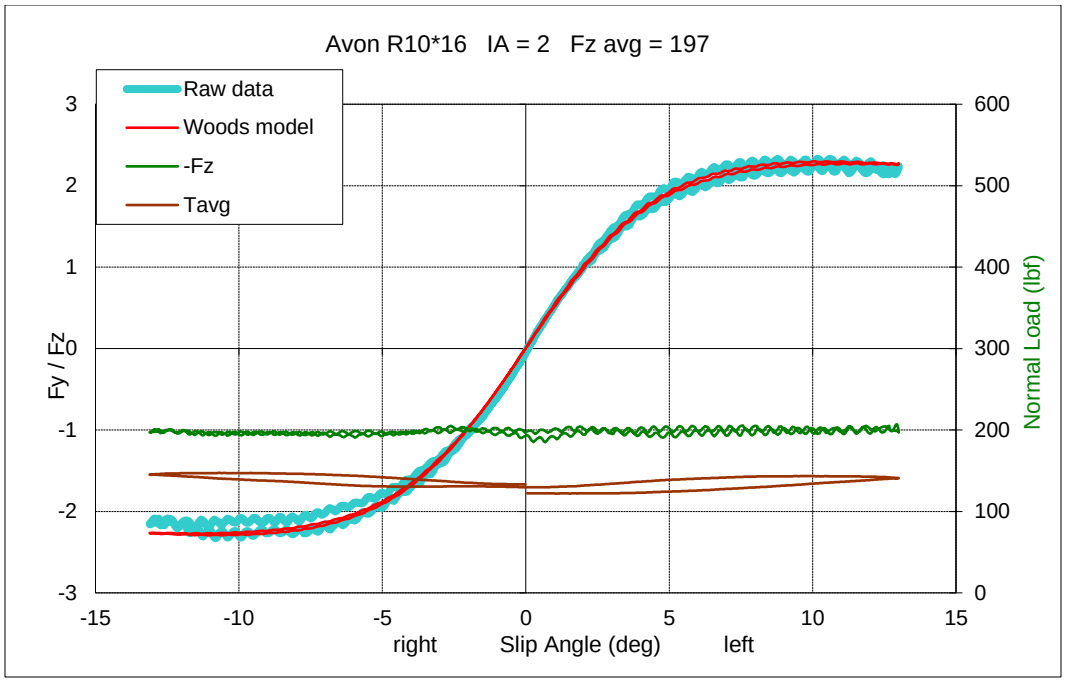

Avon R10*16 IA = 0 Fz avg = 248
Raw data
Woods model
-Fz
Tavg
Fy / Fz
Normal Load (lbf)
Slip Angle (deg)
right
left
3
2
1
0
-1
-2
-3
600
500
400
300
200
100
0
-15
-10
-5
0
5
10
15

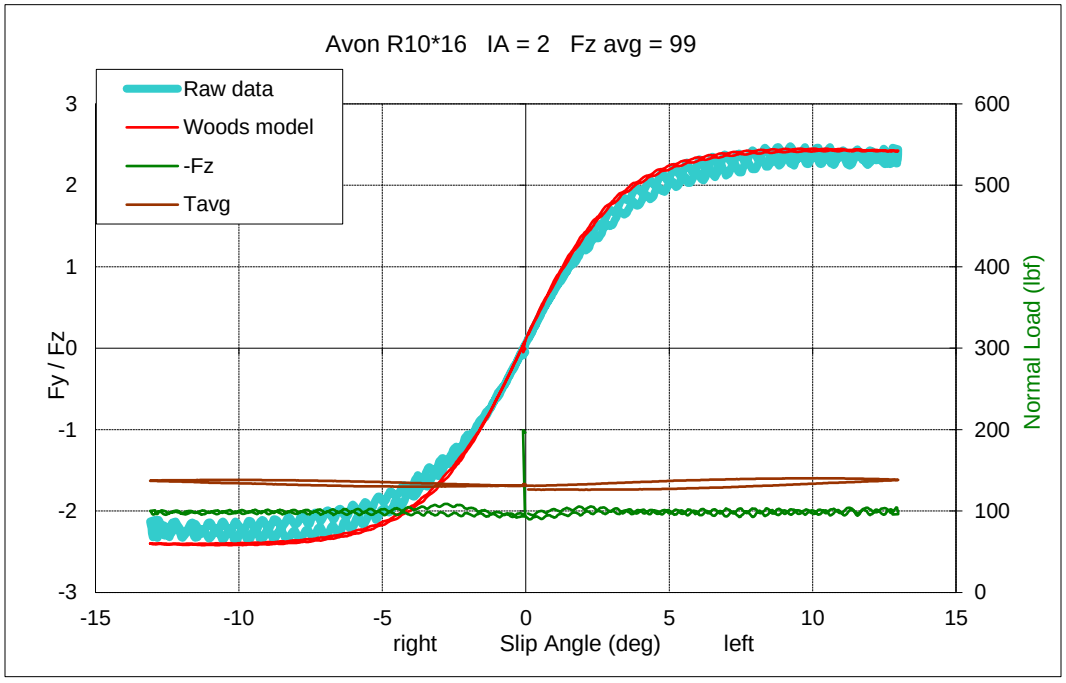

Avon R10*16 IA = 0 Fz avg = 199
Raw data
Woods model
-Fz
Tavg
Fy / Fz
Normal Load (lbf)
3
2
1
0
-1
-2
-3
600
500
400
300
200
100
0
-15
-10
-5
0
5
10
15
right
Slip Angle (deg)
left

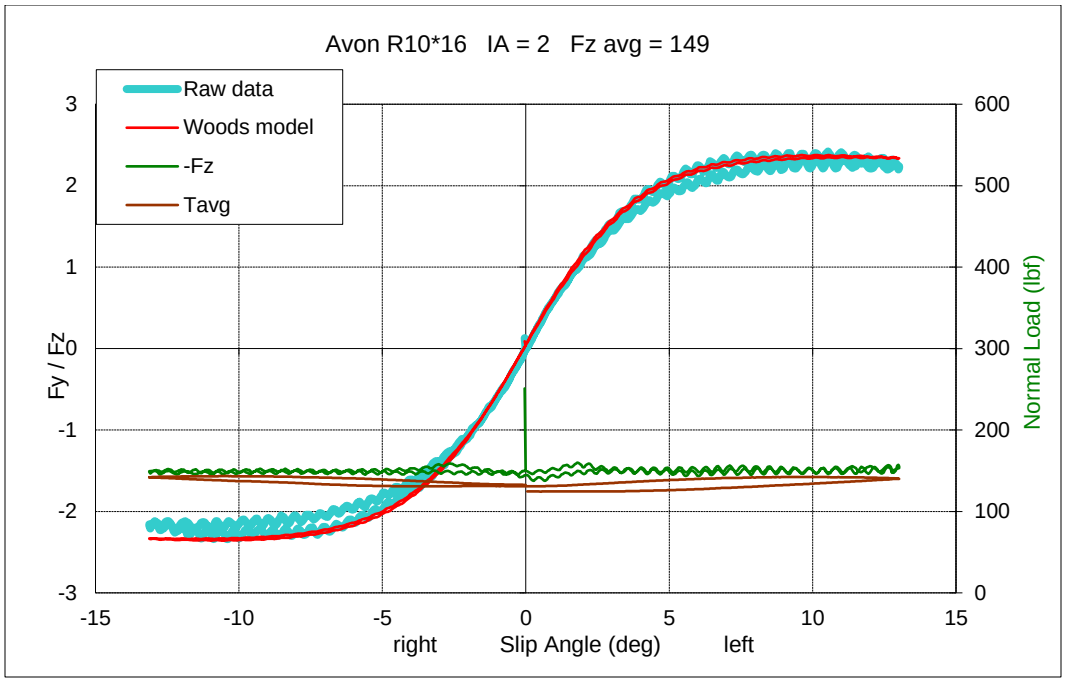

Avon R10*16 IA = 0 Fz avg = 98
Raw data
Woods model
-Fz
Tavg
Fy / Fz
Normal Load (lbf)
3
2
1
0
-1
-2
-3
600
500
400
300
200
100
0
-15
-10
-5
0
5
10
15
right
Slip Angle (deg)
left

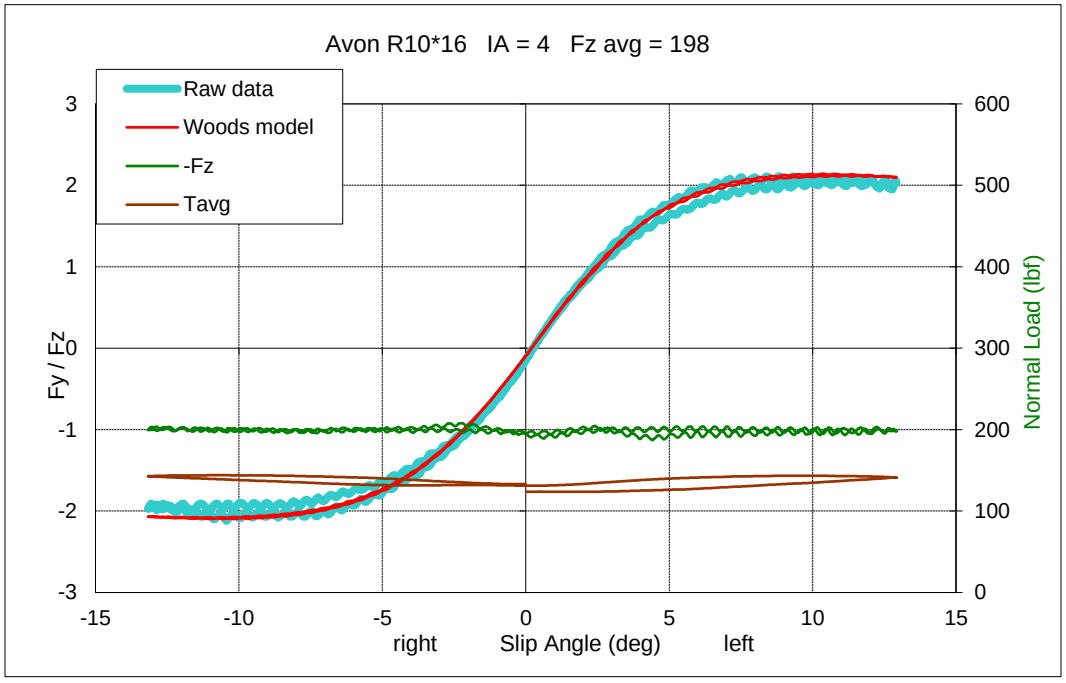

Avon R10*16 IA = 0 Fz avg = 47
Raw data
Woods model
-Fz
Tavg
Fy / Fz
Normal Load (lbf)
Slip Angle (deg)
right
left
3
2
1
0
-1
-2
-3
600
500
400
300
200
100
0
-15
-10
-5
0
5
10
15

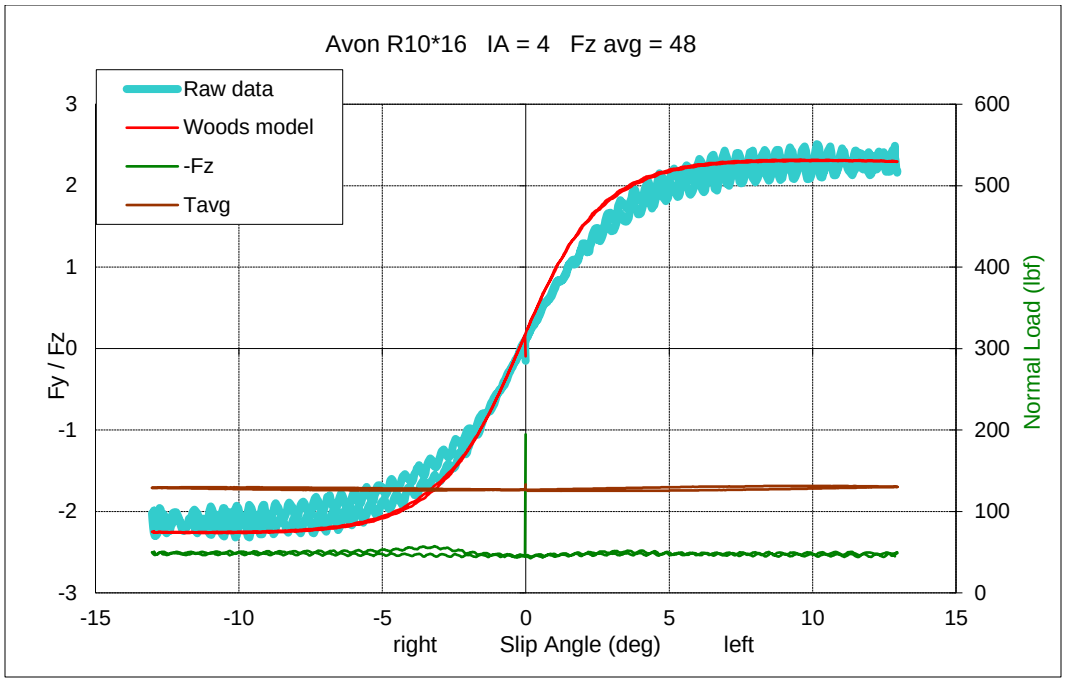

Avon R10*16 IA = 2 Fz avg = 249
Raw data
Woods model
-Fz
Tavg
Fy / Fz
Normal Load (lbf)
Slip Angle (deg)
right
left
3
2
1
0
-1
-2
-3
600
500
400
300
200
100
0
-15
-10
-5
0
5
10
15

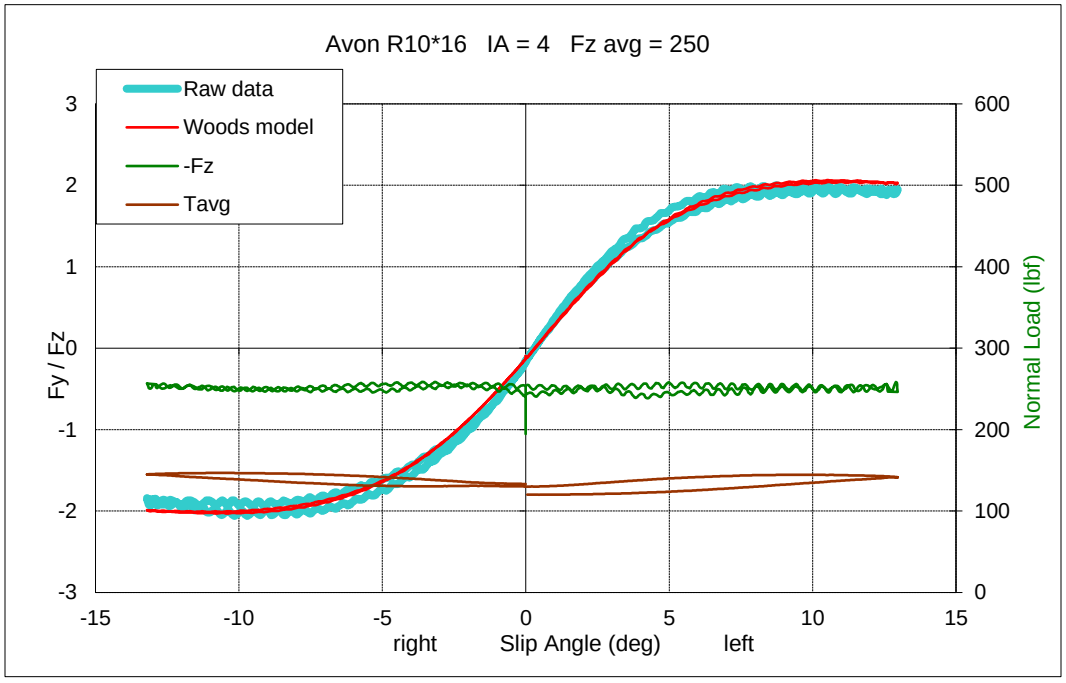

Avon R10*16 IA = 2 Fz avg = 197
Raw data
Woods model
-Fz
Tavg
Fy / Fz
Normal Load (lbf)
3
2
1
0
-1
-2
-3
600
500
400
300
200
100
0
-15
-10
-5
0
5
10
15
right
Slip Angle (deg)
left

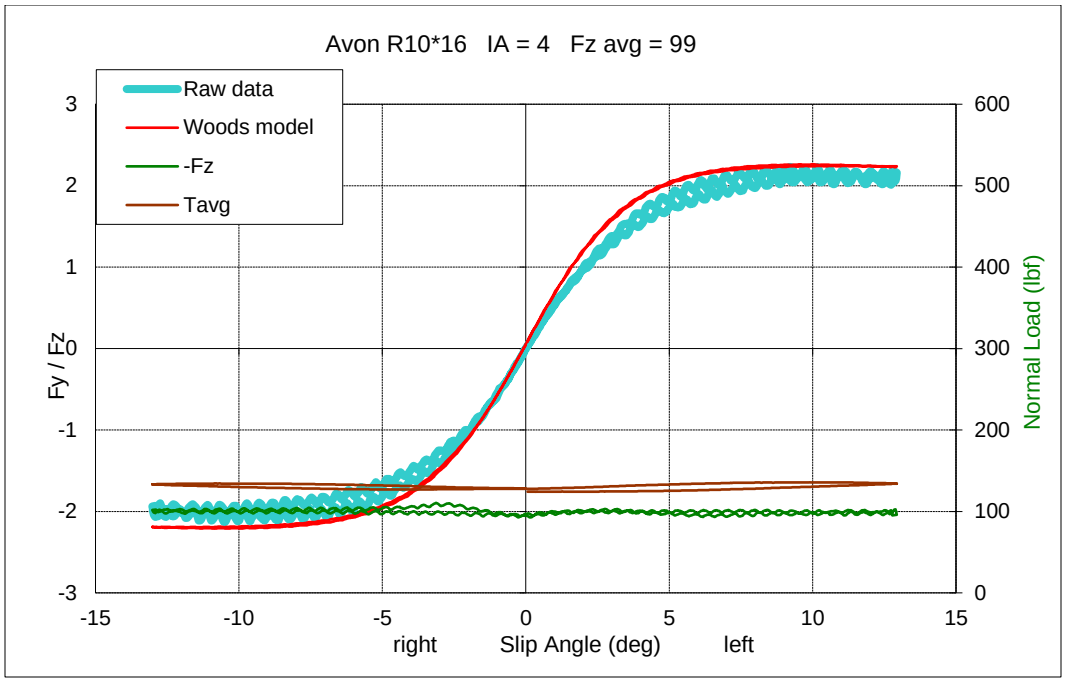

Avon R10*16 IA = 2 Fz avg = 99
Raw data
Woods model
-Fz
Tavg
Fy / Fz
Normal Load (lbf)
3
2
1
0
-1
-2
-3
600
500
400
300
200
100
0
-15
-10
-5
0
5
10
15
right
Slip Angle (deg)
left

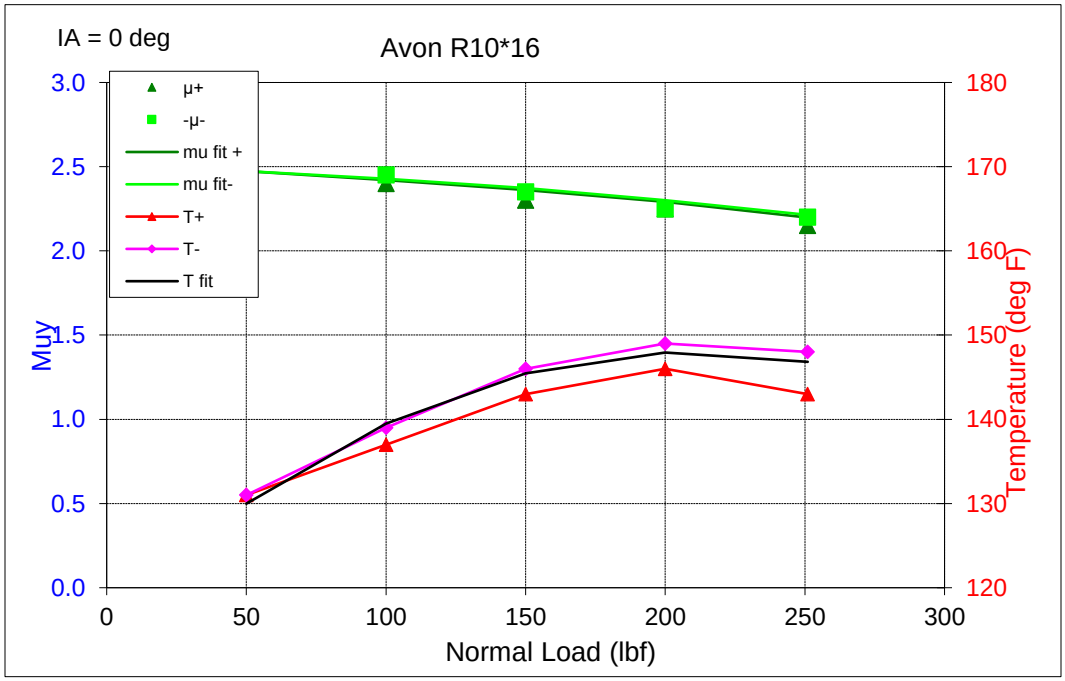

Avon R10*16 IA = 2 Fz avg = 149
Raw data
Woods model
-Fz
Tavg
Fy / Fz
Normal Load (lbf)
3
2
1
0
-1
-2
-3
600
500
400
300
200
100
0
-15
-10
-5
0
5
10
15
right
Slip Angle (deg)
left

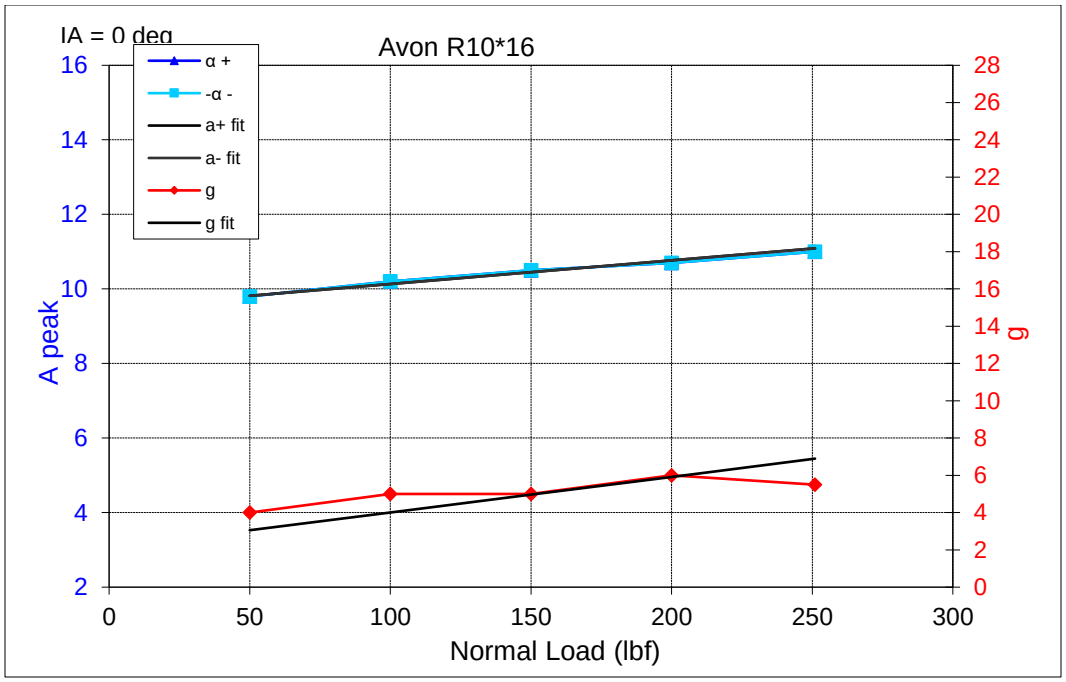

Avon R10*16 IA = 4 Fz avg = 198
Raw data
Woods model
-Fz
Tavg
Fy / Fz
Normal Load (lbf)
3
2
1
0
-1
-2
-3
600
500
400
300
200
100
0
-15
-10
-5
0
5
10
15
right
Slip Angle (deg)
left

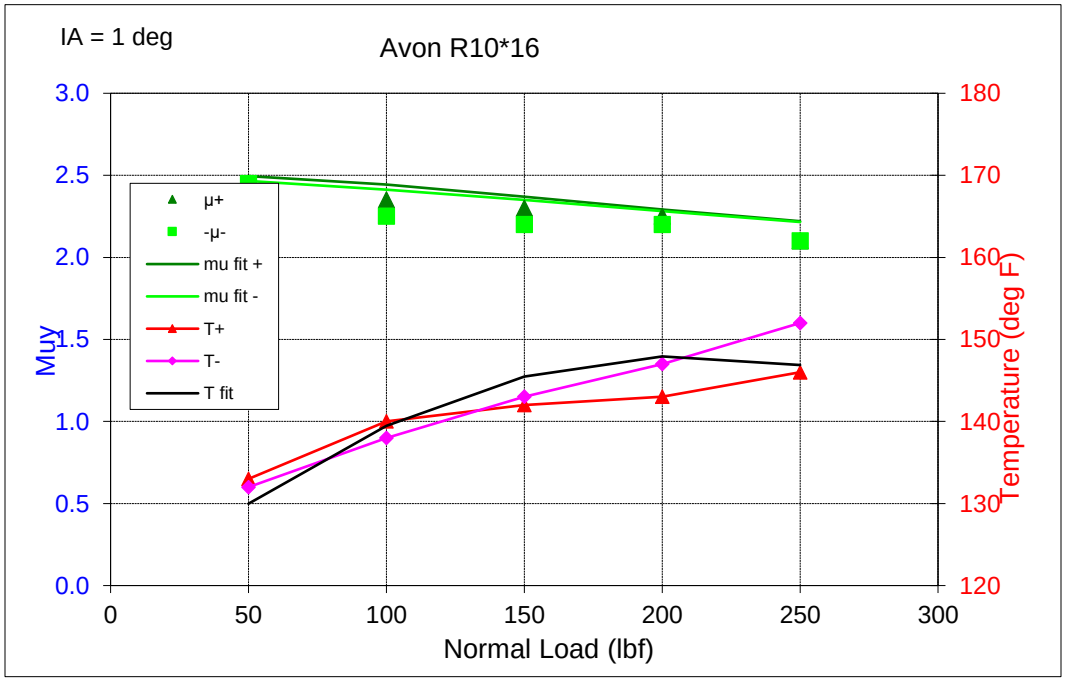

Avon R10*16 IA = 4 Fz avg = 48
Raw data
Woods model
-Fz
Tavg
Fy / Fz
Normal Load (lbf)
3
2
1
0
-1
-2
-3
600
500
400
300
200
100
0
-15
-10
-5
0
5
10
15
right
Slip Angle (deg)
left

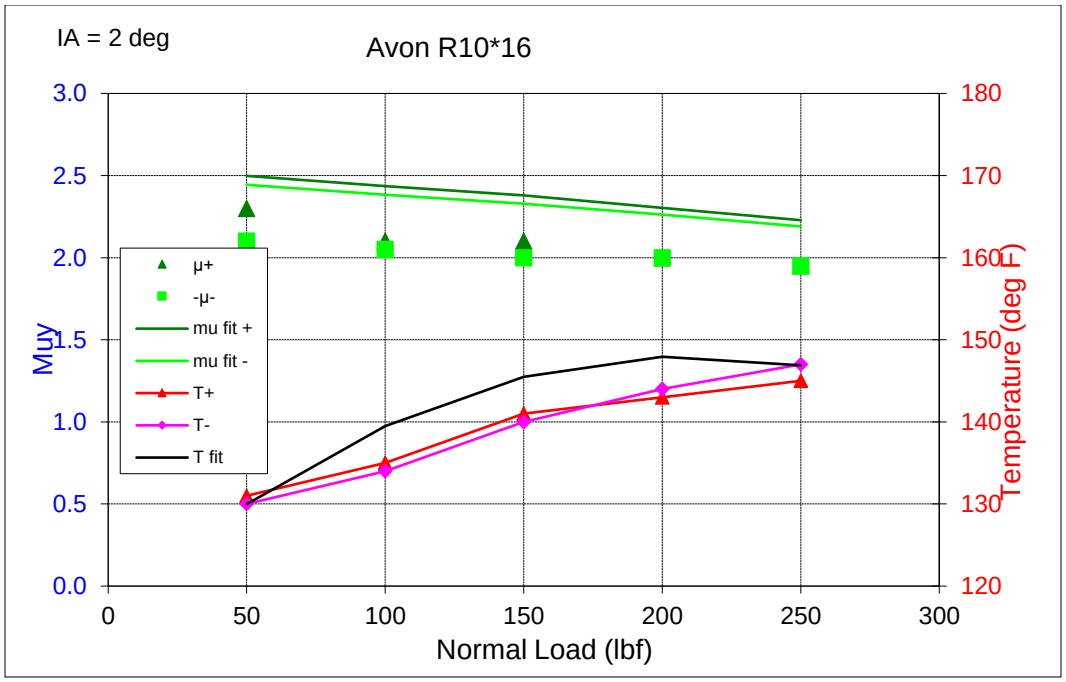

Avon R10*16 IA = 4 Fz avg = 250
Raw data
Woods model
-Fz
Tavg
Fy / Fz
Normal Load (lbf)
3
2
1
0
-1
-2
-3
600
500
400
300
200
100
0
-15
-10
-5
0
5
10
15
right
Slip Angle (deg)
left

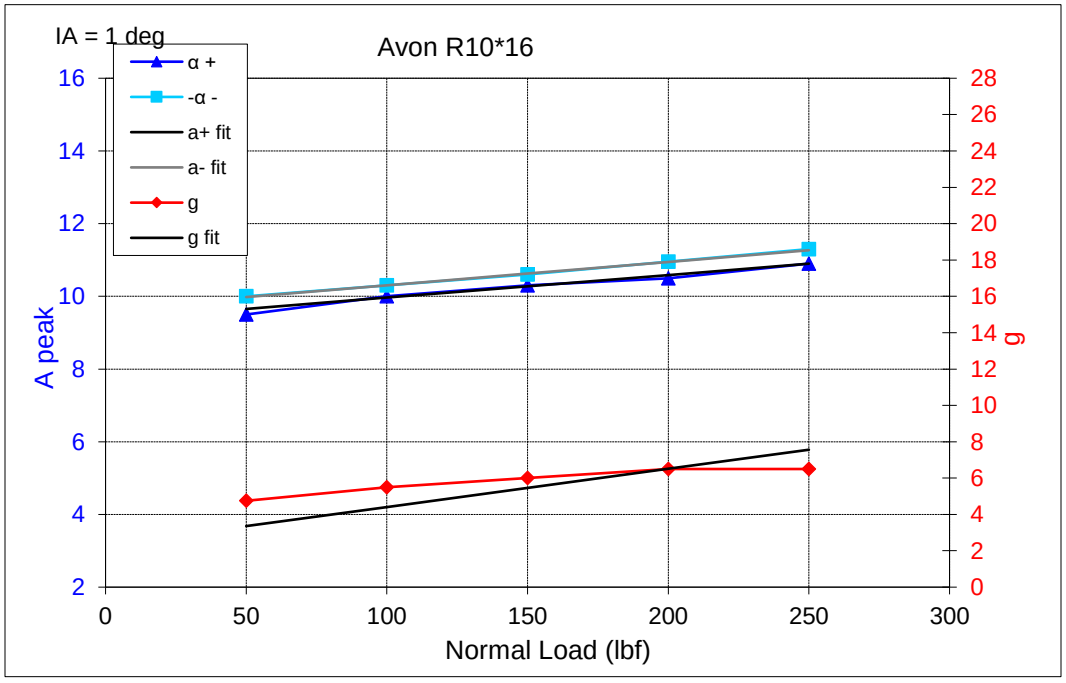

Avon R10*16 IA = 4 Fz avg = 99
Raw data
Woods model
-Fz
Tavg
Fy / Fz
Normal Load (lbf)
3
2
1
0
-1
-2
-3
600
500
400
300
200
100
0
-15
-10
-5
0
5
10
15
right
Slip Angle (deg)
left

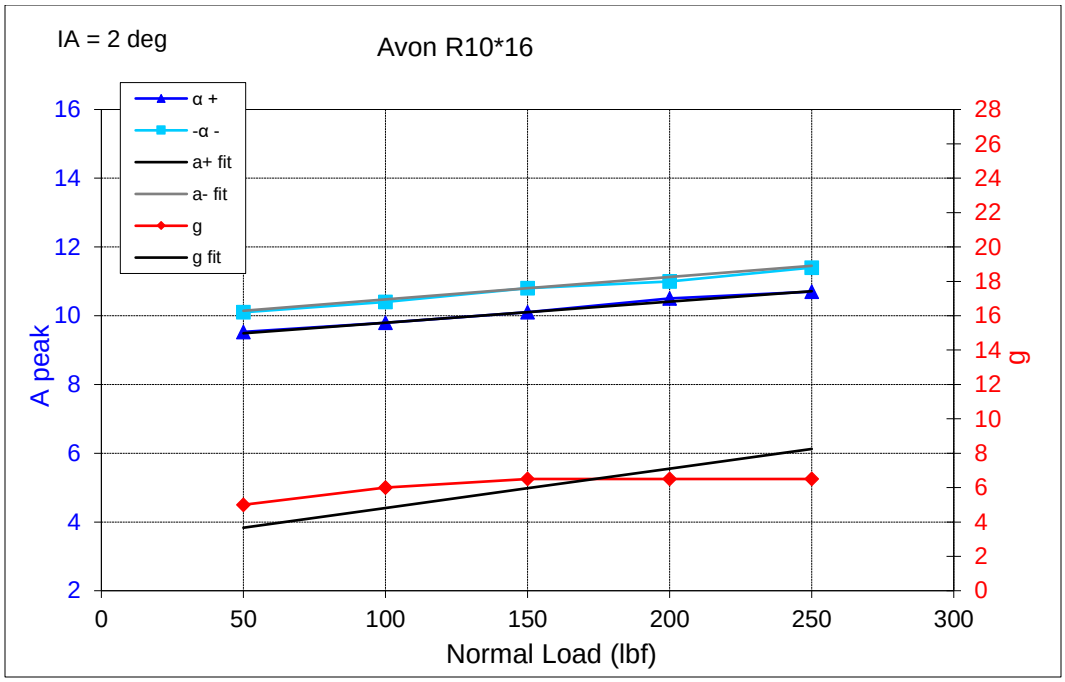

IA = 0 deg
Avon R10*16
μ+
-μ-
mu fit +
mu fit-
T+
T-
T fit
Muy
Temperature (deg F)
3.0
2.5
2.0
1.5
1.0
0.5
0.0
180
170
160
150
140
130
120
0
50
100
150
200
250
300
Normal Load (lbf)

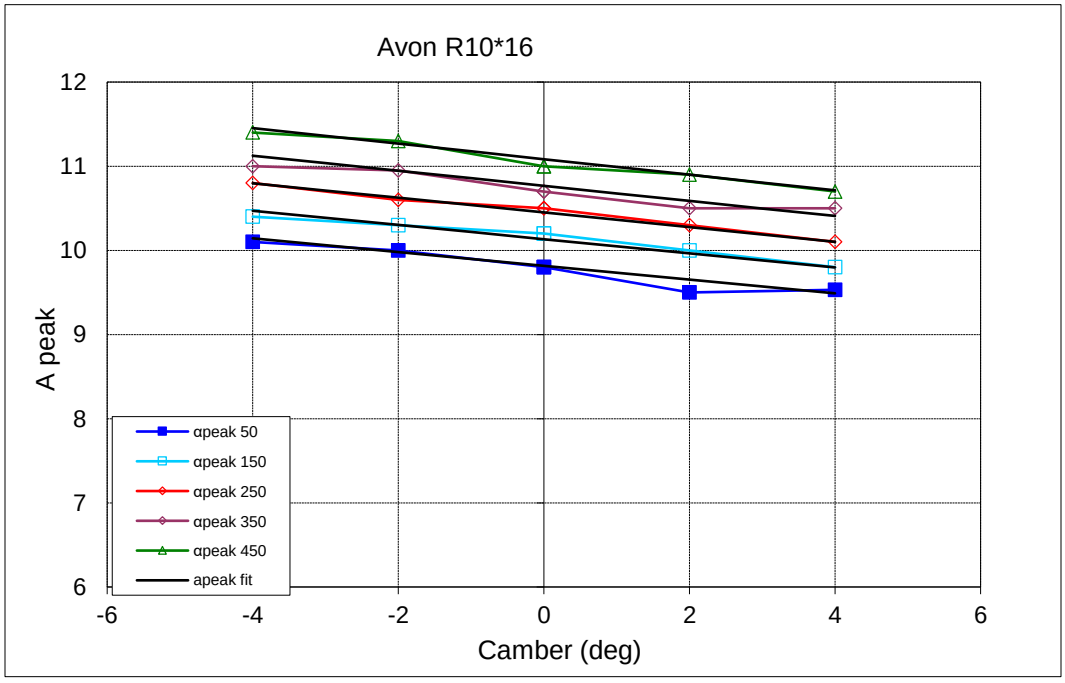

IA = 0 deg
Avon R10*16
α +
-α -
a+ fit
a- fit
g
g fit
A peak
g
Normal Load (lbf)
0
50
100
150
200
250
300
2
4
6
8
10
12
14
16
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28

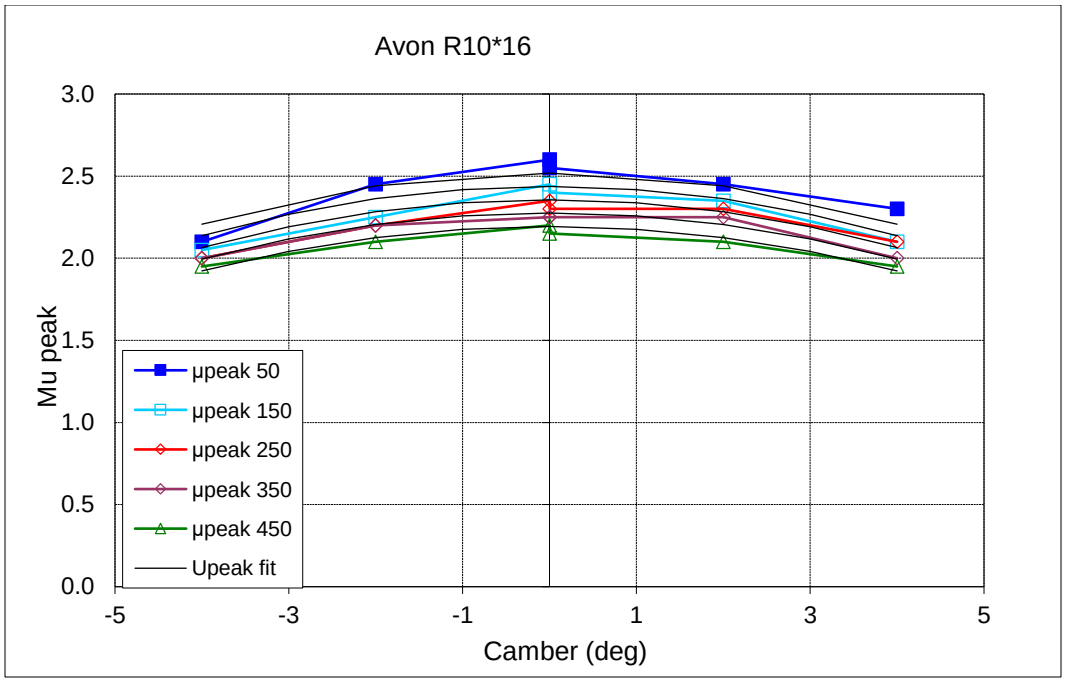

IA = 1 deg
Avon R10*16
μ+
-μ-
mu fit +
mu fit -
T+
T-
T fit
Muy
Temperature (deg F)
Normal Load (lbf)
0
50
100
150
200
250
300
0.0
0.5
1.0
1.5
2.0
2.5
3.0
120
130
140
150
160
170
180

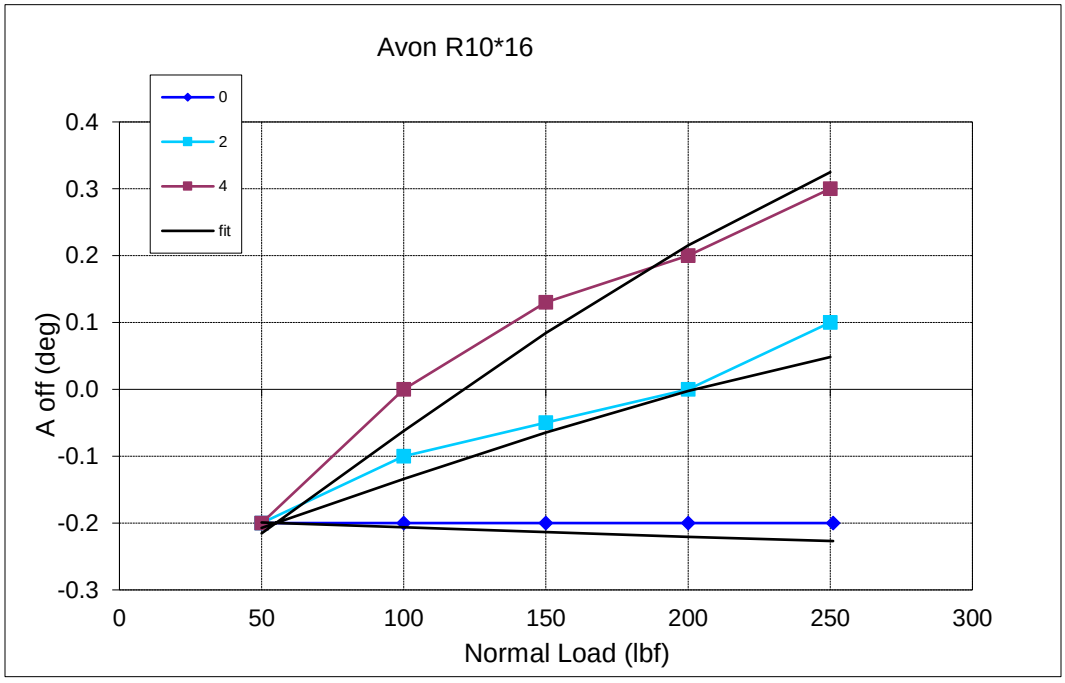

IA = 2 deg
Avon R10*16
3.0
2.5
2.0
1.5
1.0
0.5
0.0
Muy
180
170
160
150
140
130
120
Temperature (deg F)
μ+
-μ-
mu fit +
mu fit -
T+
T-
T fit
0
50
100
150
200
250
300
Normal Load (lbf)


IA = 1 deg
Avon R10*16
α +
-α -
a+ fit
a- fit
g
g fit
16
14
12
10
8
6
4
2
A peak
28
26
24
22
20
18
16
14
12
10
8
6
4
2
0
g
0
50
100
150
200
250
300
Normal Load (lbf)

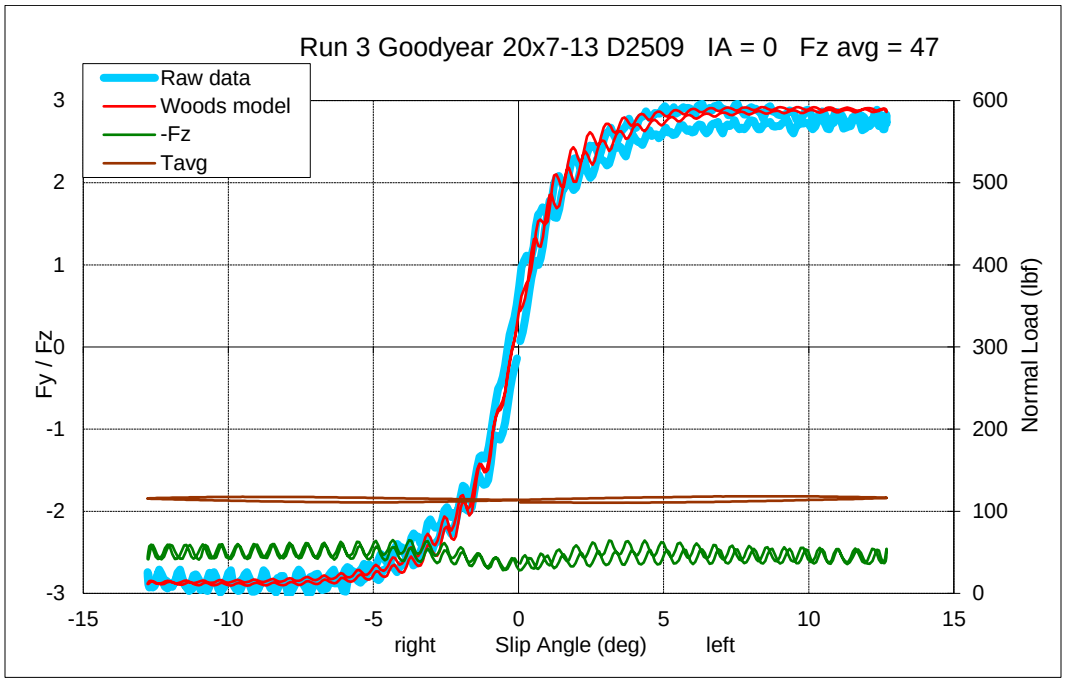

IA = 2 deg
Avon R10*16
α +
-α -
a+ fit
a- fit
g
g fit
A peak
g
Normal Load (lbf)
16
14
12
10
8
6
4
2
28
26
24
22
20
18
16
14
12
10
8
6
4
2
0
0
50
100
150
200
250
300

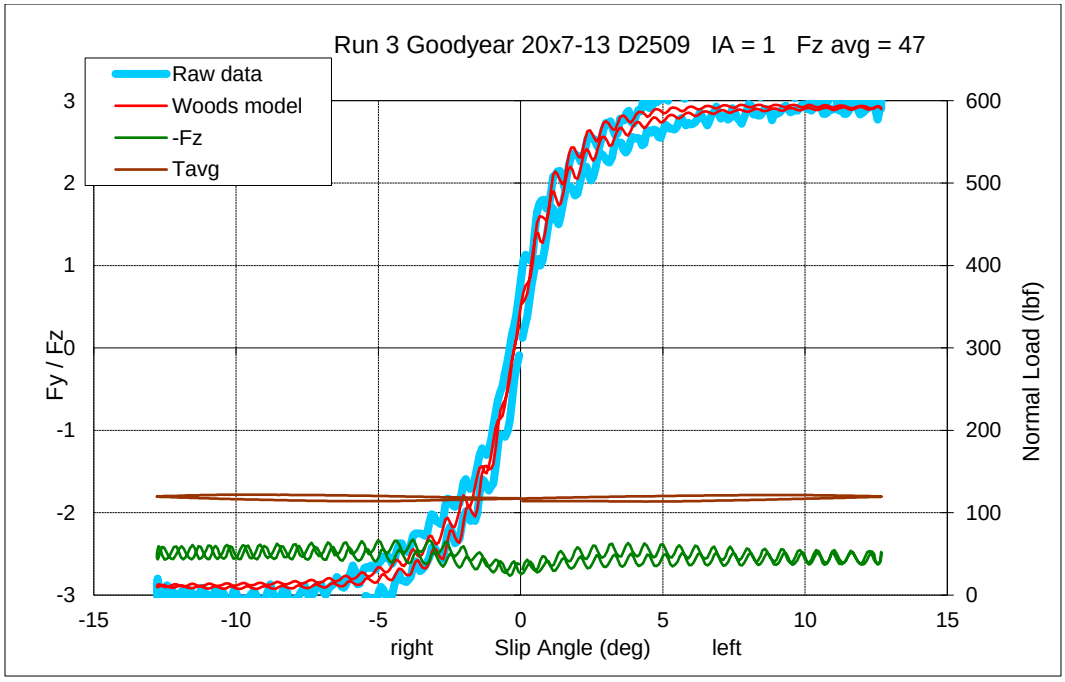

Avon R10*16
A peak
Camber (deg)
αpeak 50
αpeak 150
αpeak 250
αpeak 350
αpeak 450
apeak fit
12
11
10
9
8
7
6
-6
-4
-2
0
2
4
6

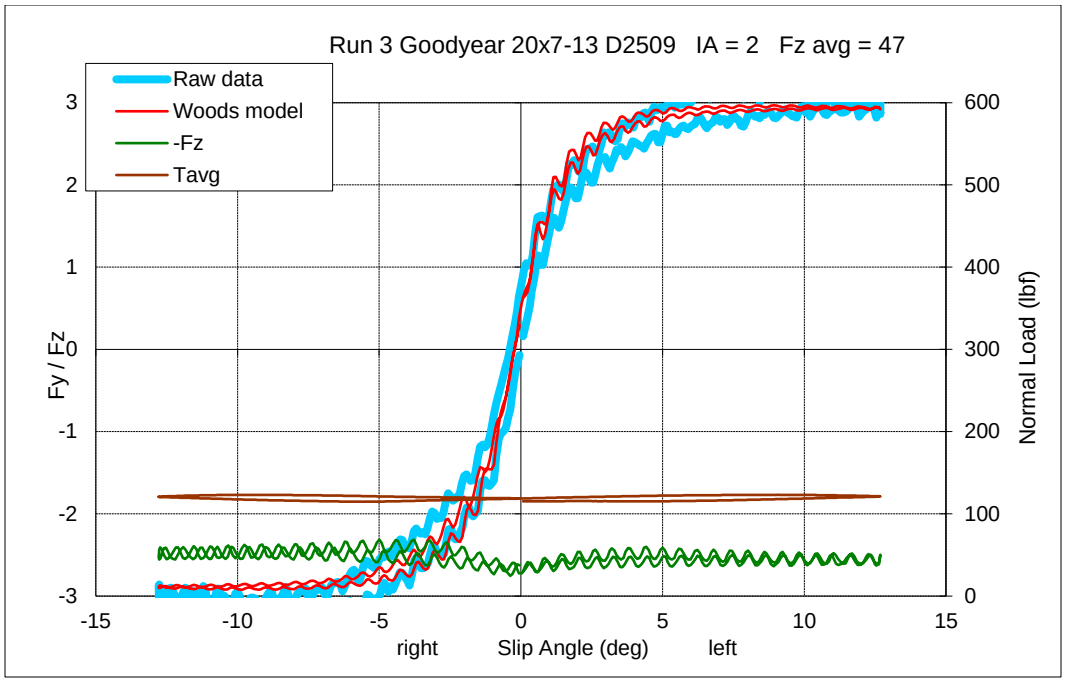

Avon R10*16
3.0
2.5
2.0
1.5
1.0
0.5
0.0
Mu peak
-5
-3
-1
1
3
5
Camber (deg)
µpeak 50
µpeak 150
µpeak 250
µpeak 350
µpeak 450
Upeak fit

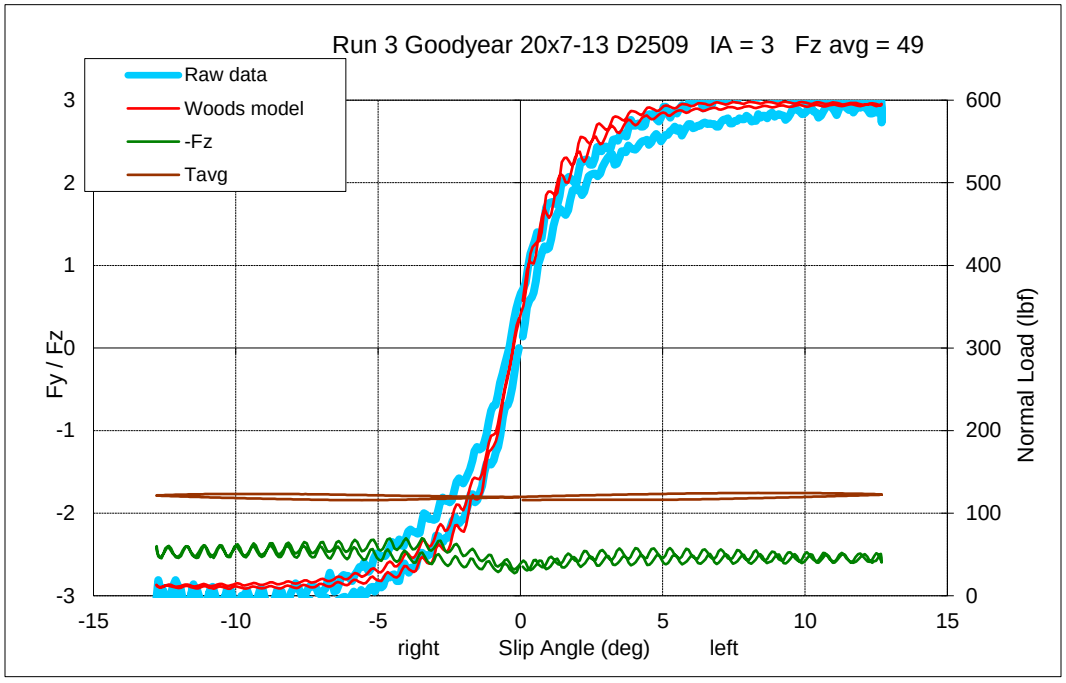

Avon R10*16
0
2
4
fit
0.4
0.3
0.2
0.1
0.0
-0.1
-0.2
-0.3
A off (deg)
0
50
100
150
200
250
300
Normal Load (lbf)

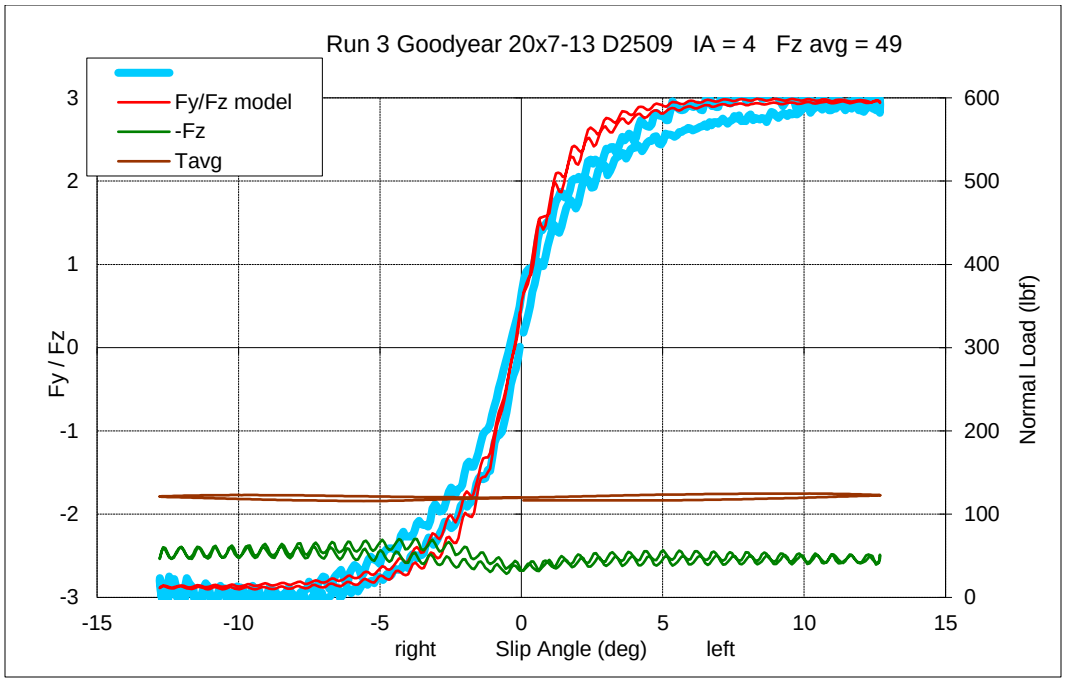
**Goodyear 20x7-13 D2509**

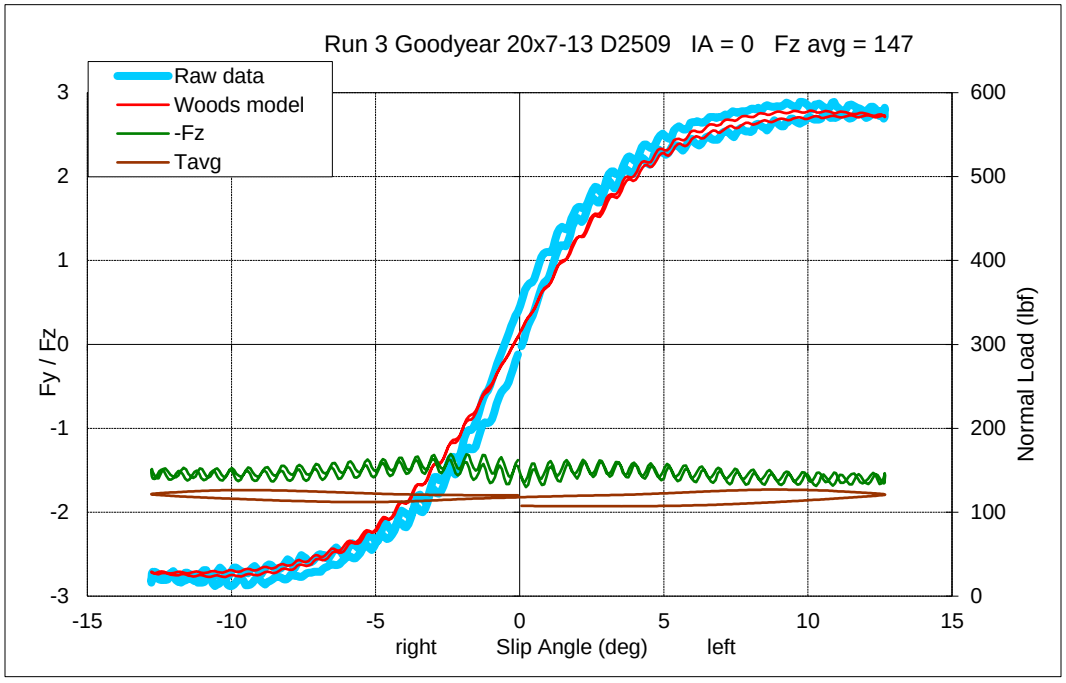


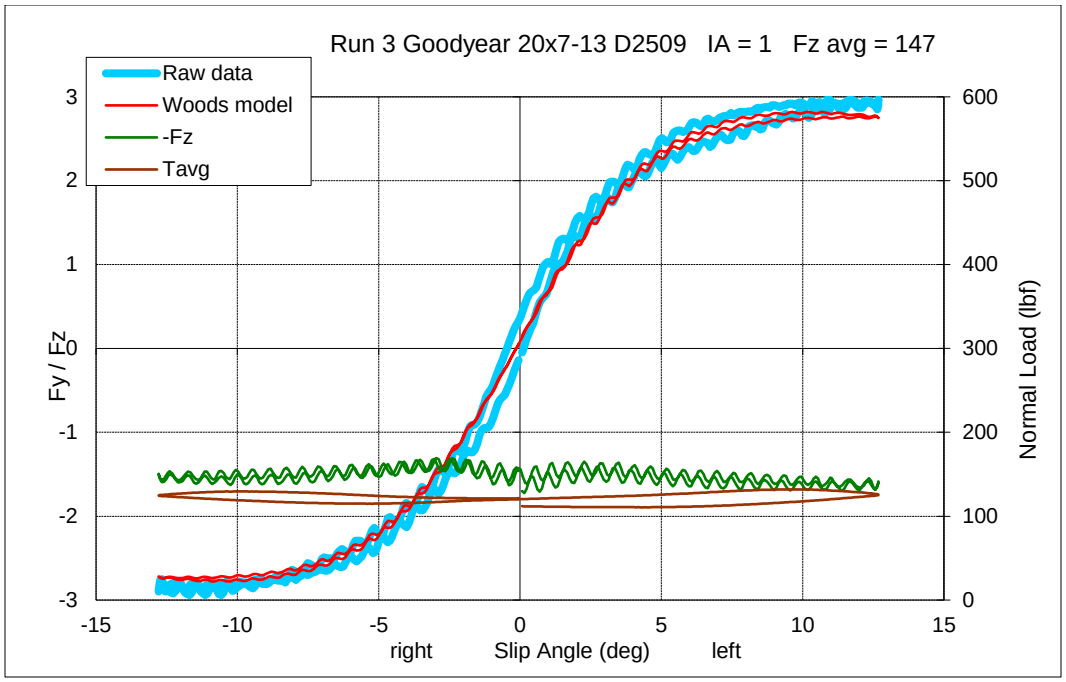



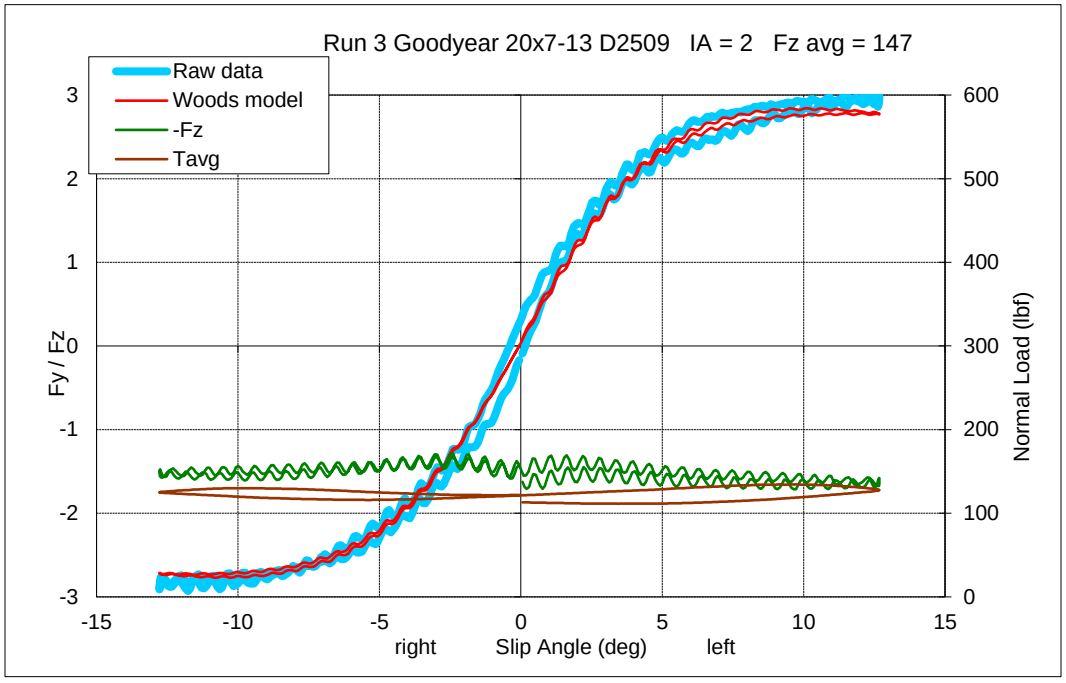

Run 3 Goodyear 20x7-13 D2509 IA = 2 Fz avg = 47
Raw data
Woods model
-Fz
Tavg
Fy / Fz
Normal Load (lbf)
Slip Angle (deg)
right
left

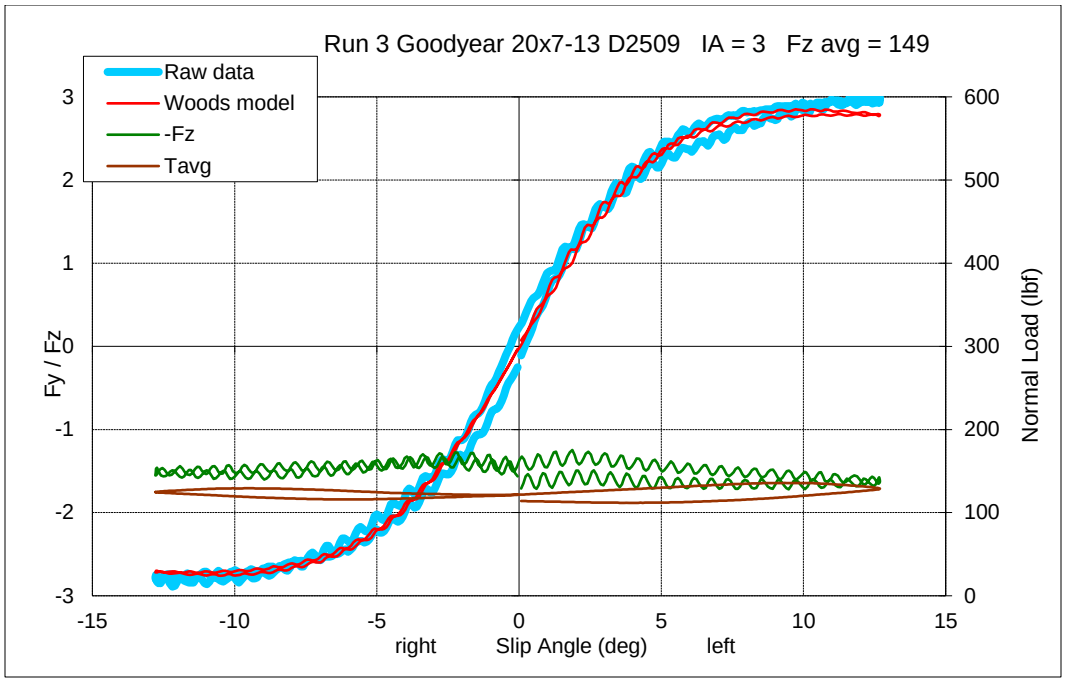

Run 3 Goodyear 20x7-13 D2509 IA = 3 Fz avg = 49
Raw data
Woods model
-Fz
Tavg
Fy / Fz
Normal Load (lbf)
Slip Angle (deg)
right
left

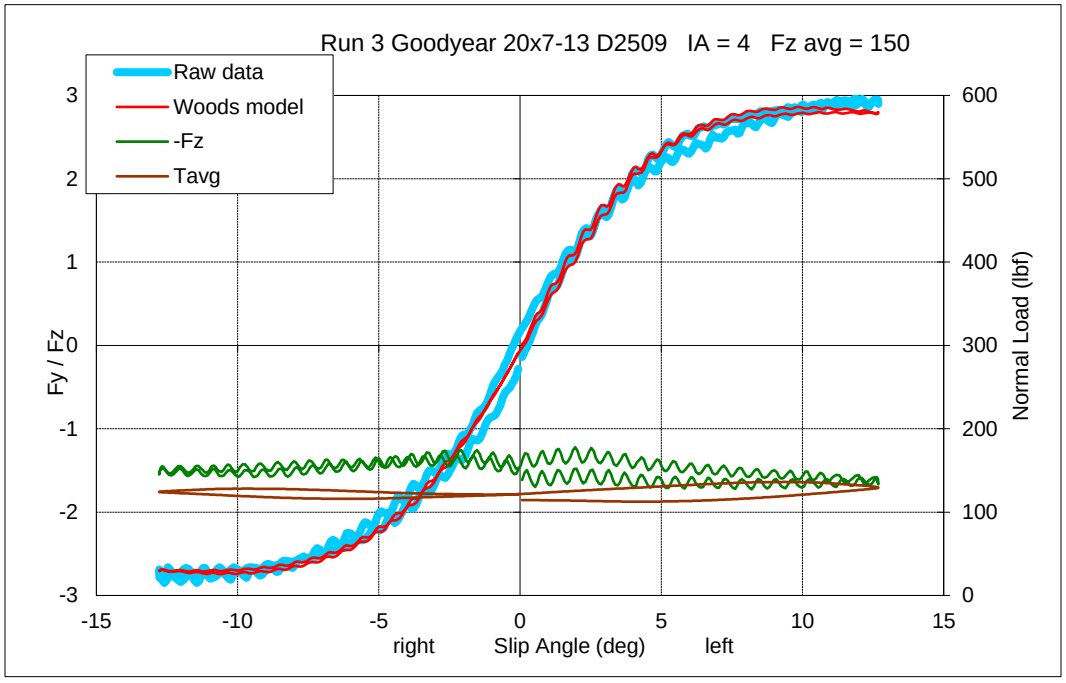

Run 3 Goodyear 20x7-13 D2509 IA = 4 Fz avg = 49
Fy/Fz model
-Fz
Tavg
Fy / Fz
Normal Load (lbf)
Slip Angle (deg)
right
left

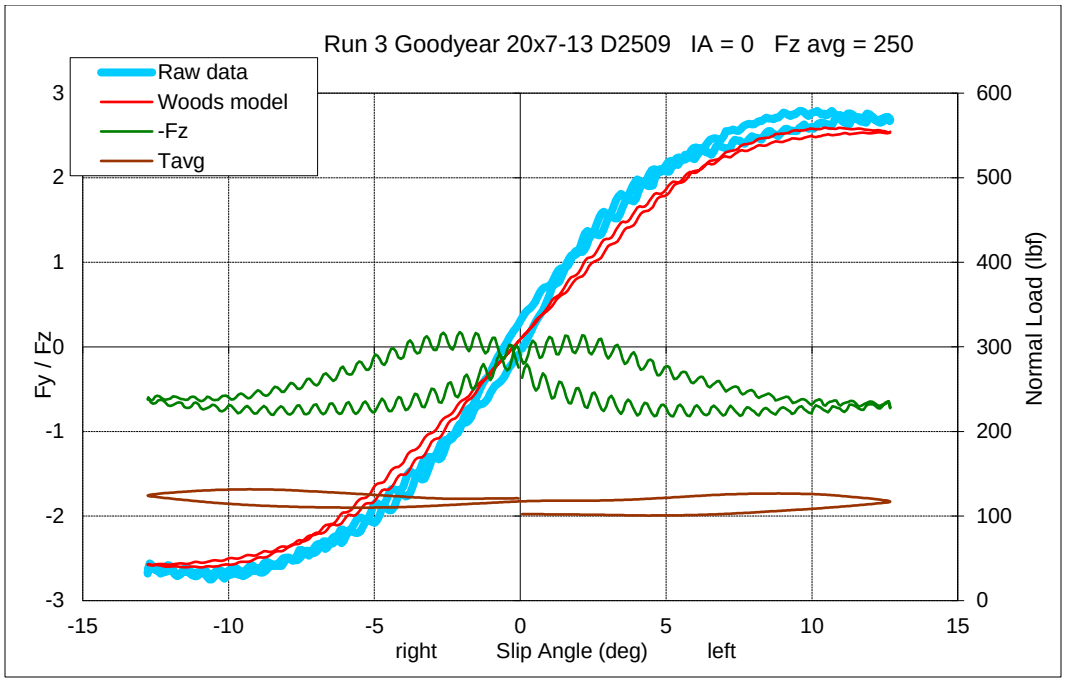

Run 3 Goodyear 20x7-13 D2509 IA = 0 Fz avg = 147
Raw data
Woods model
-Fz
Tavg
Fy / Fz
Normal Load (lbf)
Slip Angle (deg)
right
left

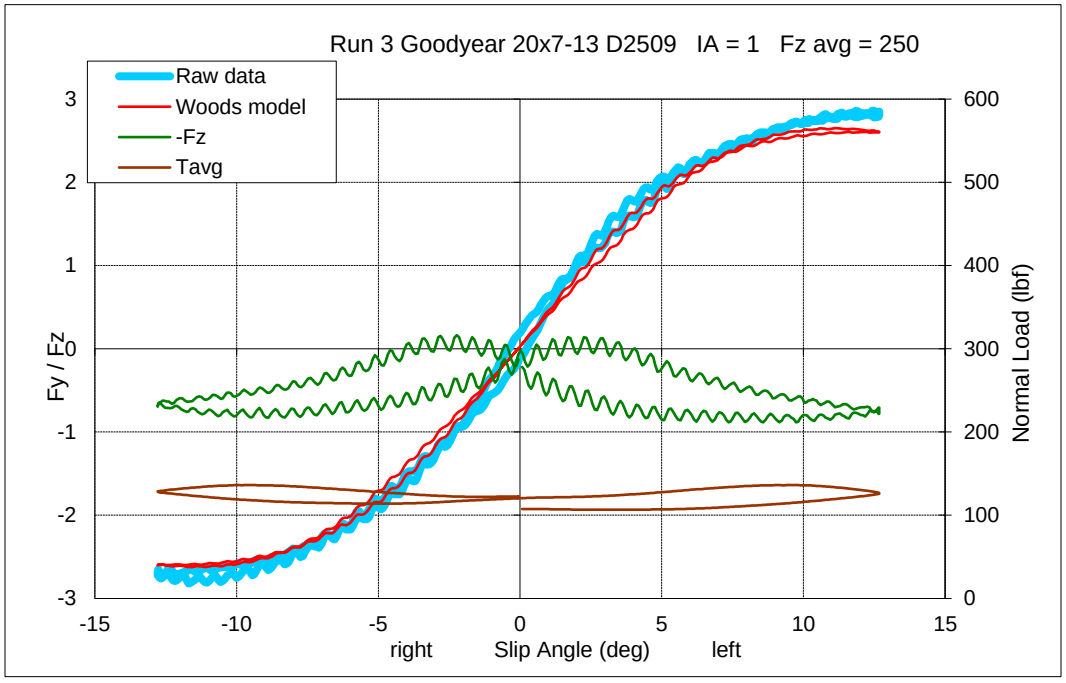

Run 3 Goodyear 20x7-13 D2509 IA = 1 Fz avg = 147
Raw data
Woods model
-Fz
Tavg
Fy / Fz
Normal Load (lbf)
Slip Angle (deg)
right
left
3
2
1
0
-1
-2
-3
600
500
400
300
200
100
0
-15
-10
-5
0
5
10
15

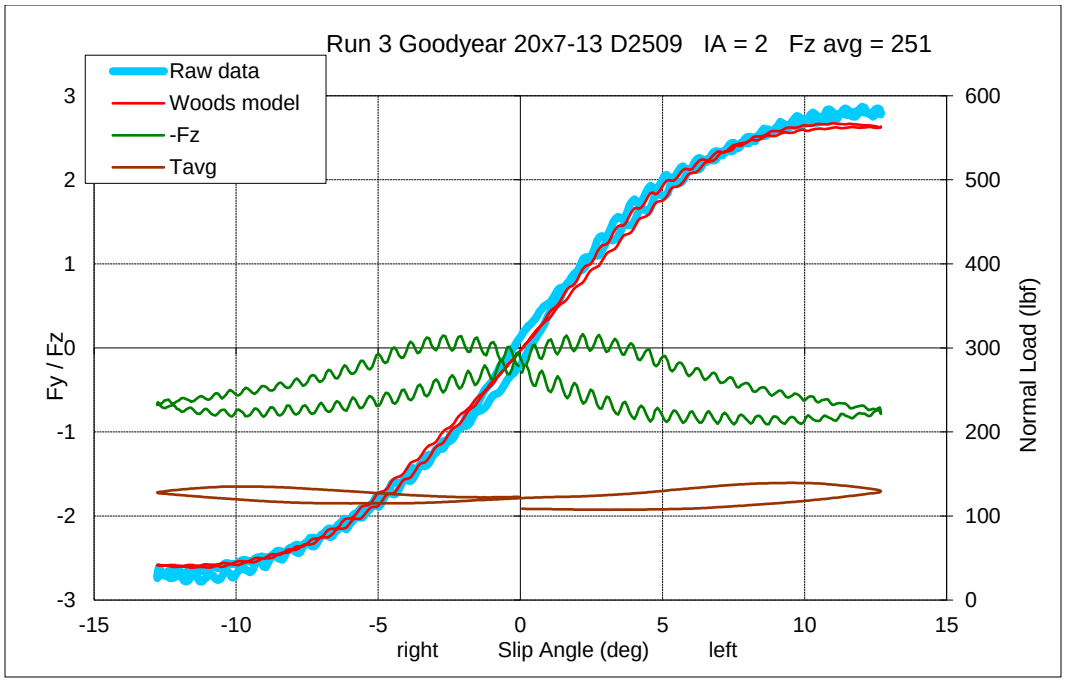

Run 3 Goodyear 20x7-13 D2509 IA = 2 Fz avg = 147
Raw data
Woods model
-Fz
Tavg
Fy / Fz
Normal Load (lbf)
Slip Angle (deg)
right
left
3
2
1
0
-1
-2
-3
600
500
400
300
200
100
0
-15
-10
-5
0
5
10
15

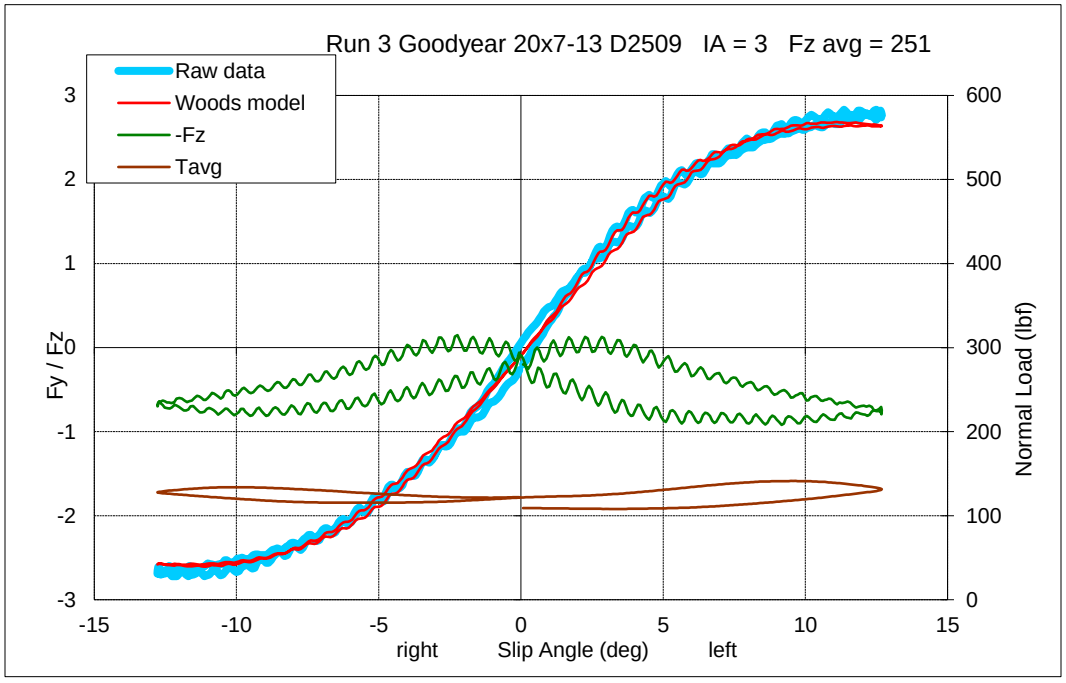

Run 3 Goodyear 20x7-13 D2509 IA = 3 Fz avg = 149
Raw data
Woods model
-Fz
Tavg
Fy / Fz
Normal Load (lbf)
Slip Angle (deg)
right
left

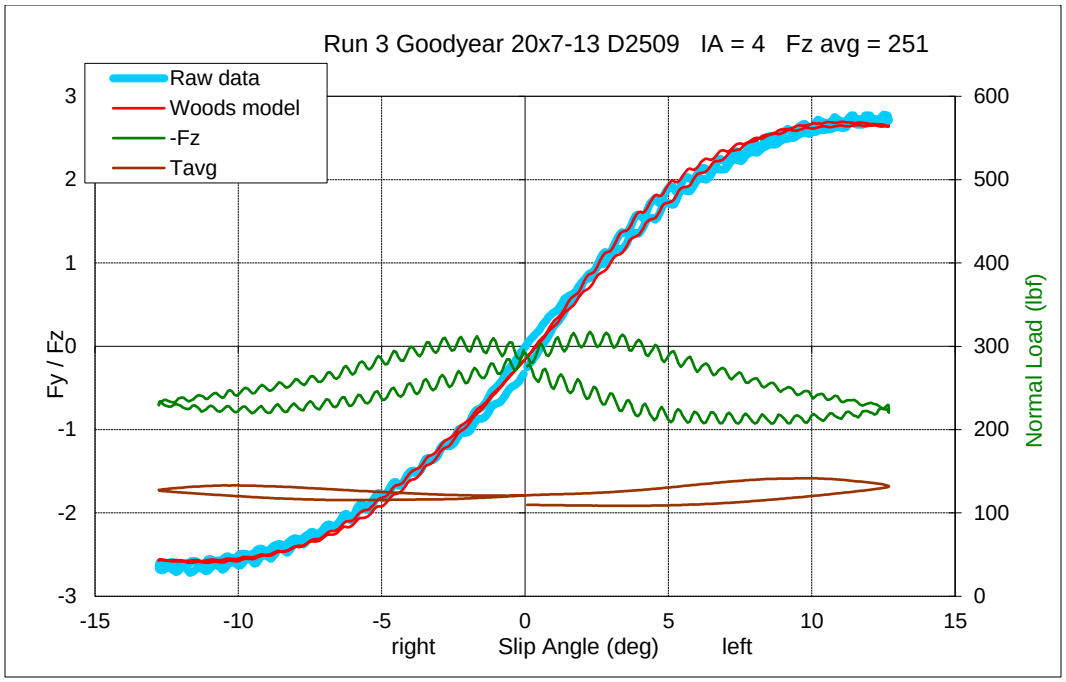

Run 3 Goodyear 20x7-13 D2509 IA = 4 Fz avg = 150
Raw data
Woods model
-Fz
Tavg
Fy / Fz
Normal Load (lbf)
Slip Angle (deg)
right
left

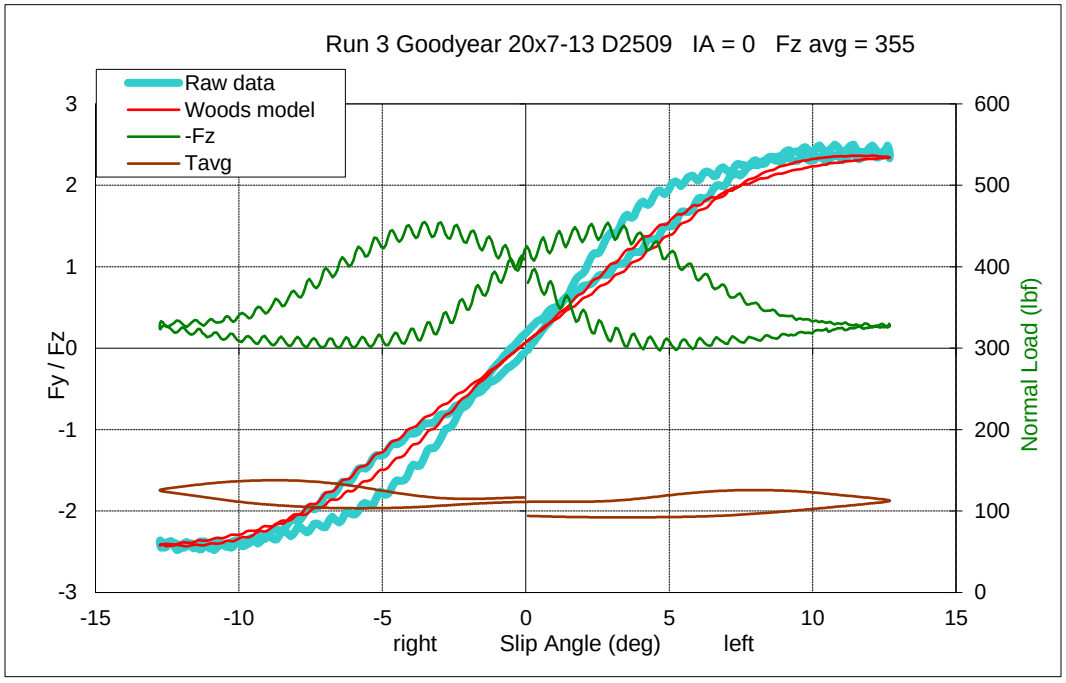

Run 3 Goodyear 20x7-13 D2509 IA = 0 Fz avg = 250
Raw data
Woods model
-Fz
Tavg
Fy / Fz
Normal Load (lbf)
Slip Angle (deg)
right
left

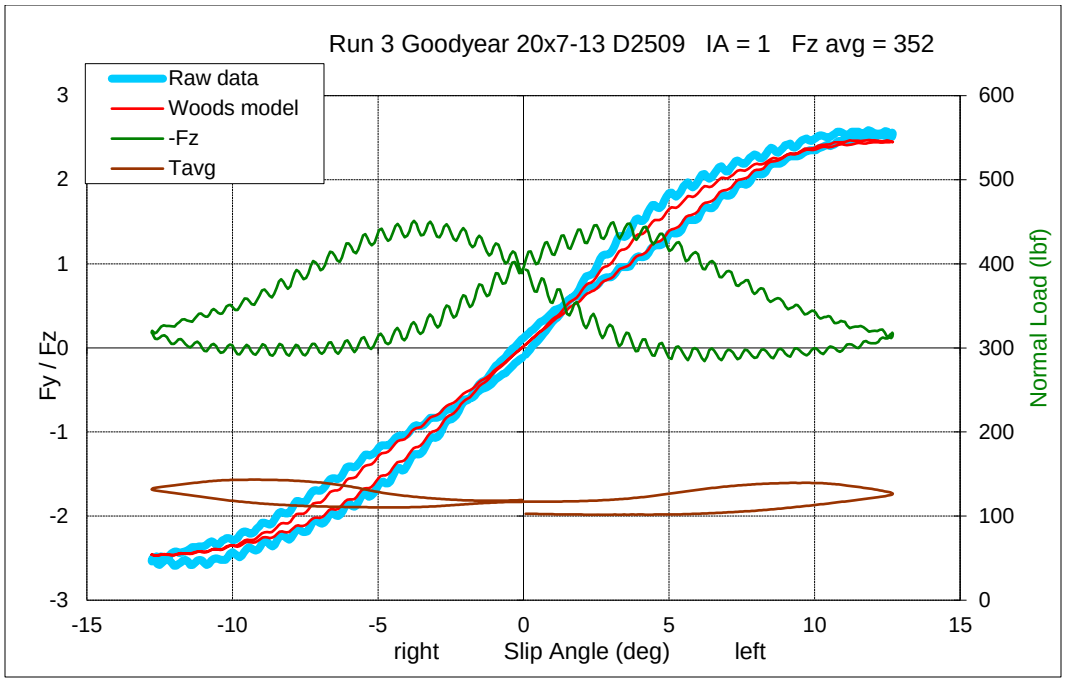

Run 3 Goodyear 20x7-13 D2509 IA = 1 Fz avg = 250
Raw data
Woods model
-Fz
Tavg
Fy / Fz
Normal Load (lbf)
Slip Angle (deg)
right
left

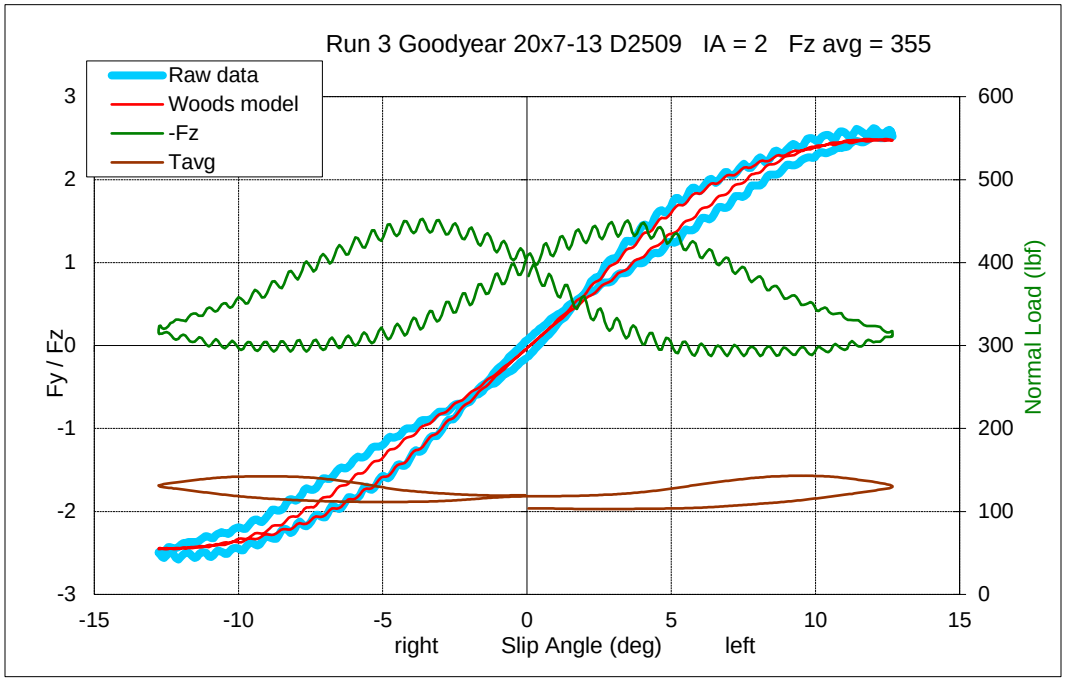

Run 3 Goodyear 20x7-13 D2509 IA = 2 Fz avg = 251
Raw data
Woods model
-Fz
Tavg
Fy / Fz
Normal Load (lbf)
Slip Angle (deg)
right
left
3
2
1
0
-1
-2
-3
600
500
400
300
200
100
0
-15
-10
-5
0
5
10
15

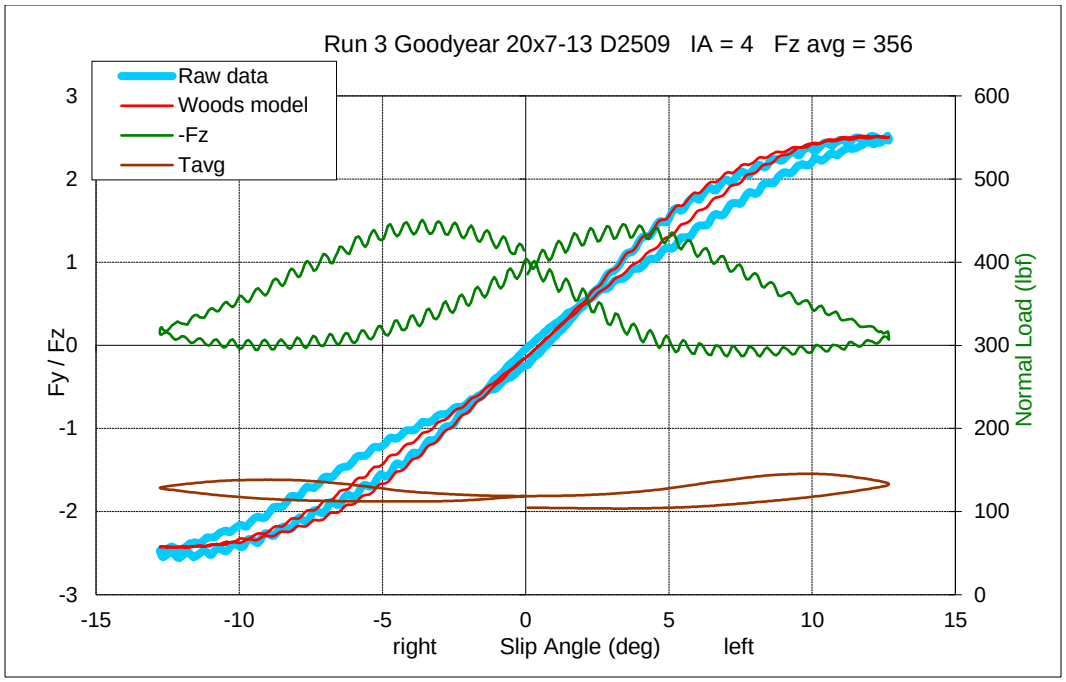

Run 3 Goodyear 20x7-13 D2509 IA = 3 Fz avg = 251
Raw data
Woods model
-Fz
Tavg
Fy / Fz
Normal Load (lbf)
Slip Angle (deg)
right
left
3
2
1
0
-1
-2
-3
600
500
400
300
200
100
0
-15
-10
-5
0
5
10
15

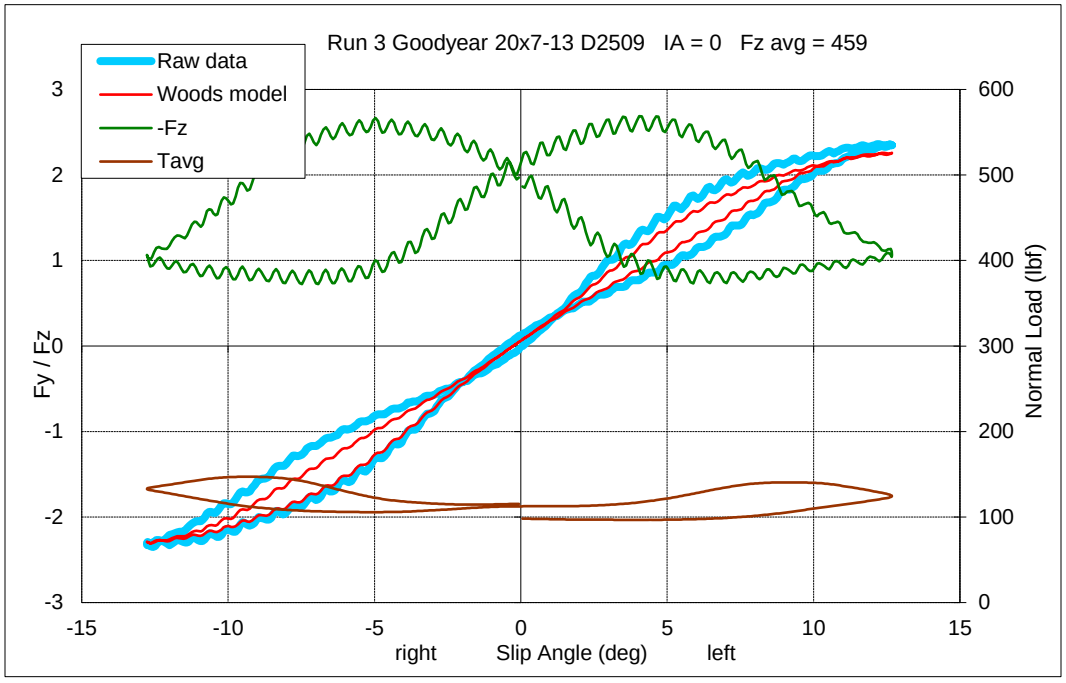

Run 3 Goodyear 20x7-13 D2509 IA = 4 Fz avg = 251
Raw data
Woods model
-Fz
Tavg
Fy / Fz
Normal Load (lbf)
Slip Angle (deg)
right
left

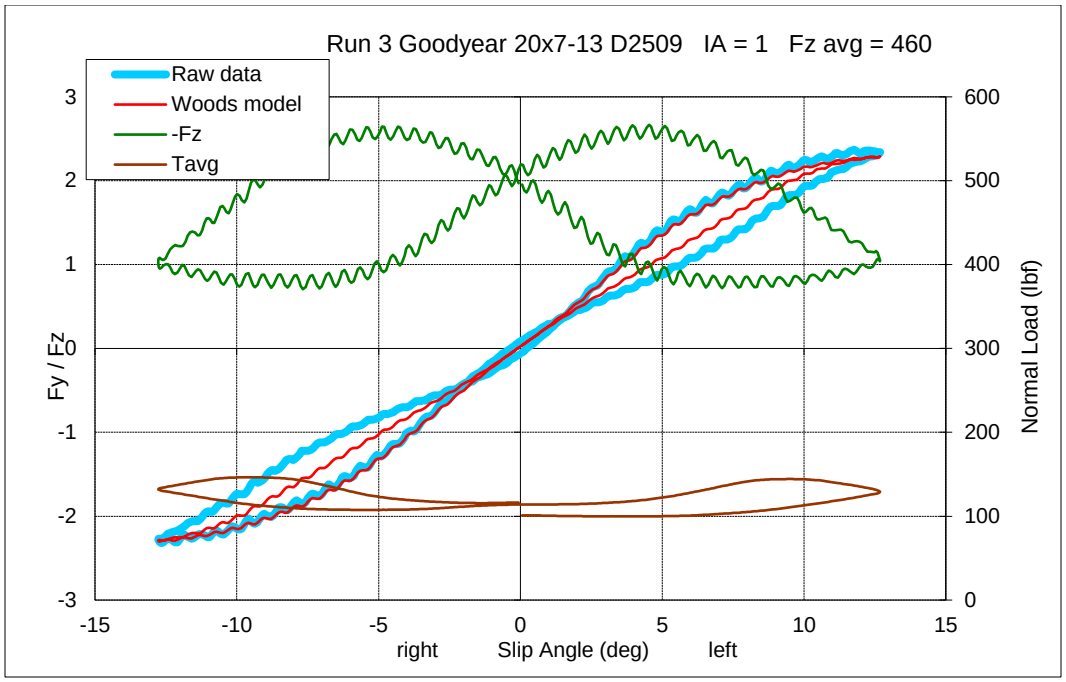

Run 3 Goodyear 20x7-13 D2509 IA = 0 Fz avg = 355
Raw data
Woods model
-Fz
Tavg
Fy / Fz
Normal Load (lbf)
Slip Angle (deg)
right
left

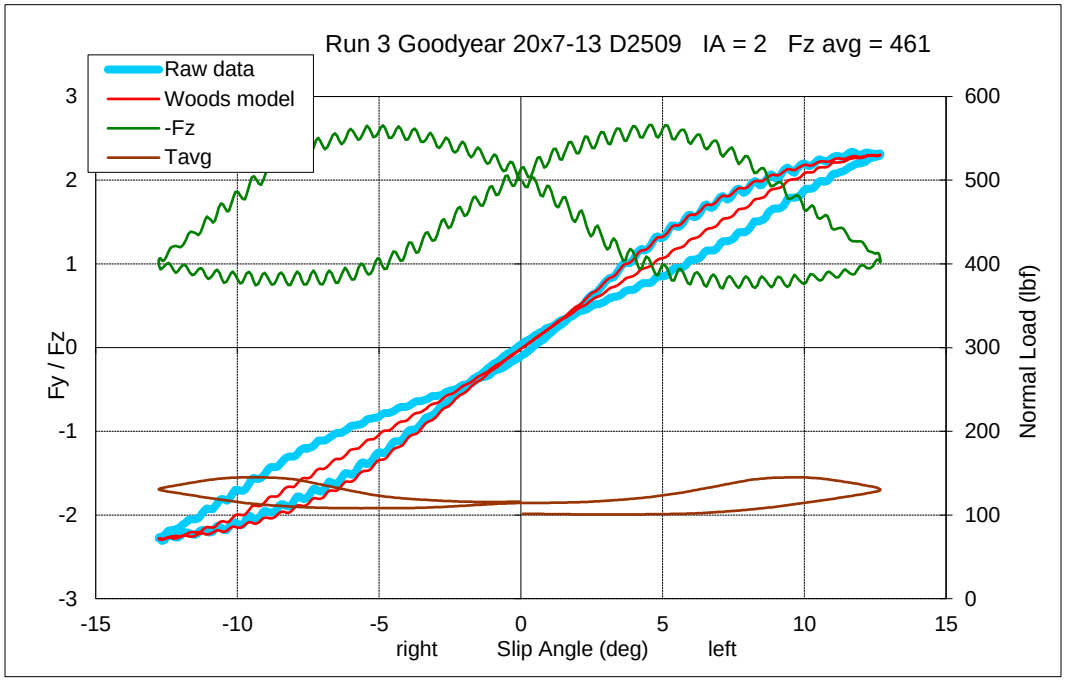

Run 3 Goodyear 20x7-13 D2509 IA = 1 Fz avg = 352
Raw data
Woods model
-Fz
Tavg
Fy / Fz
Normal Load (lbf)
Slip Angle (deg)
right
left

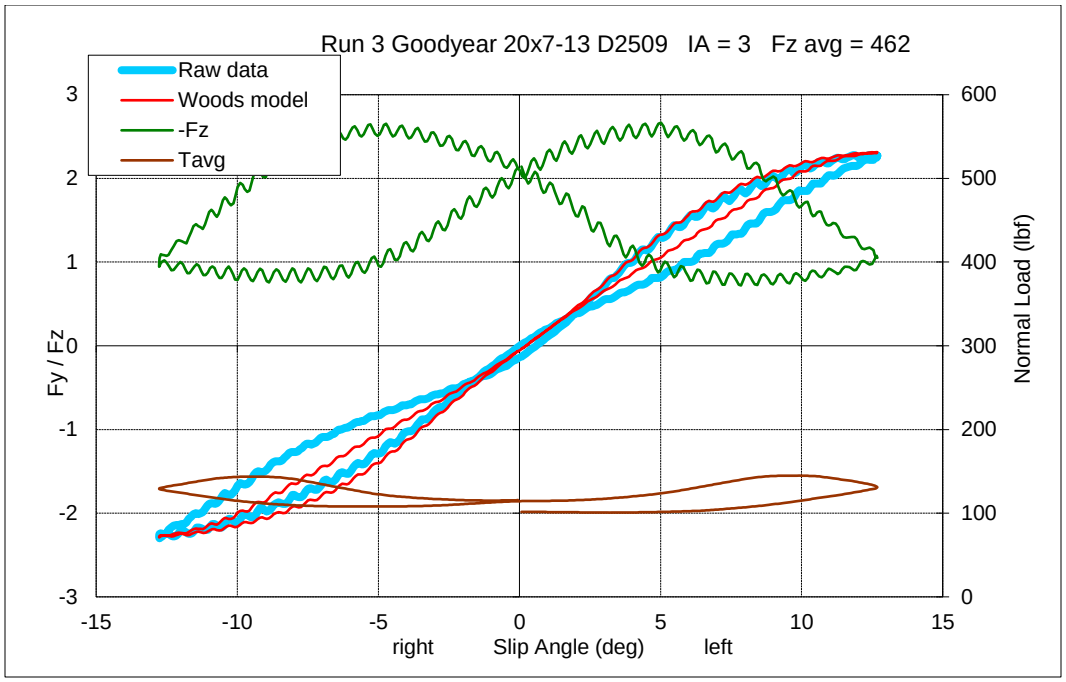

Run 3 Goodyear 20x7-13 D2509 IA = 2 Fz avg = 355
Raw data
Woods model
-Fz
Tavg
Fy / Fz
Normal Load (lbf)
Slip Angle (deg)
right
left

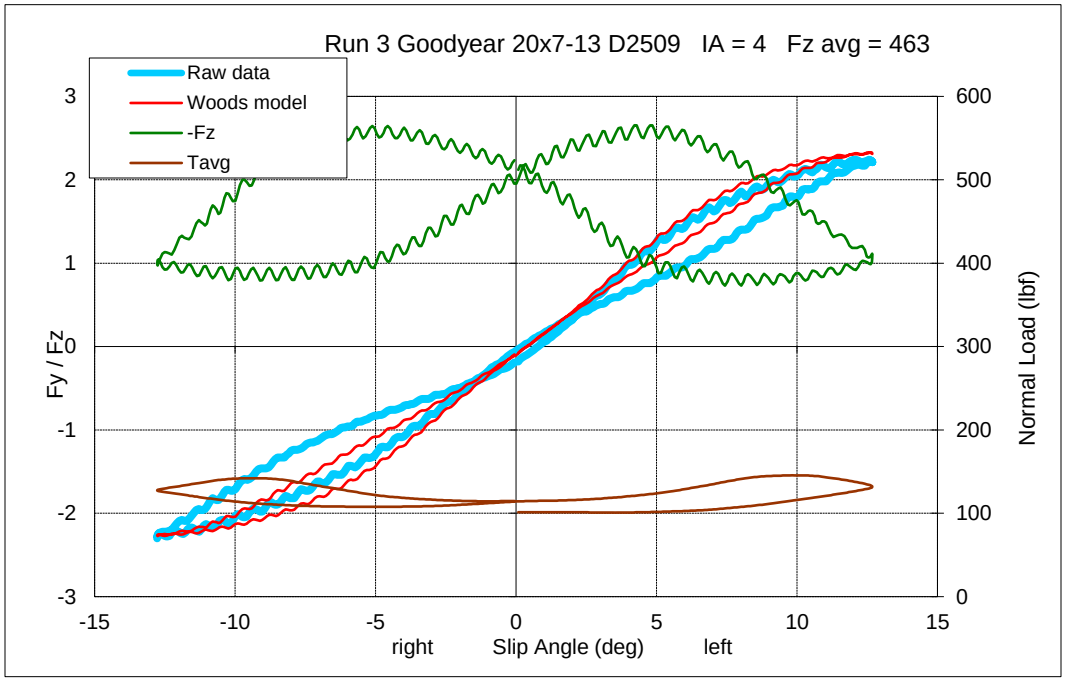

Run 3 Goodyear 20x7-13 D2509 IA = 4 Fz avg = 356
Raw data
Woods model
-Fz
Tavg
Fy / Fz
Normal Load (lbf)
Slip Angle (deg)
right
left
3
2
1
0
-1
-2
-3
600
500
400
300
200
100
0
-15
-10
-5
0
5
10
15

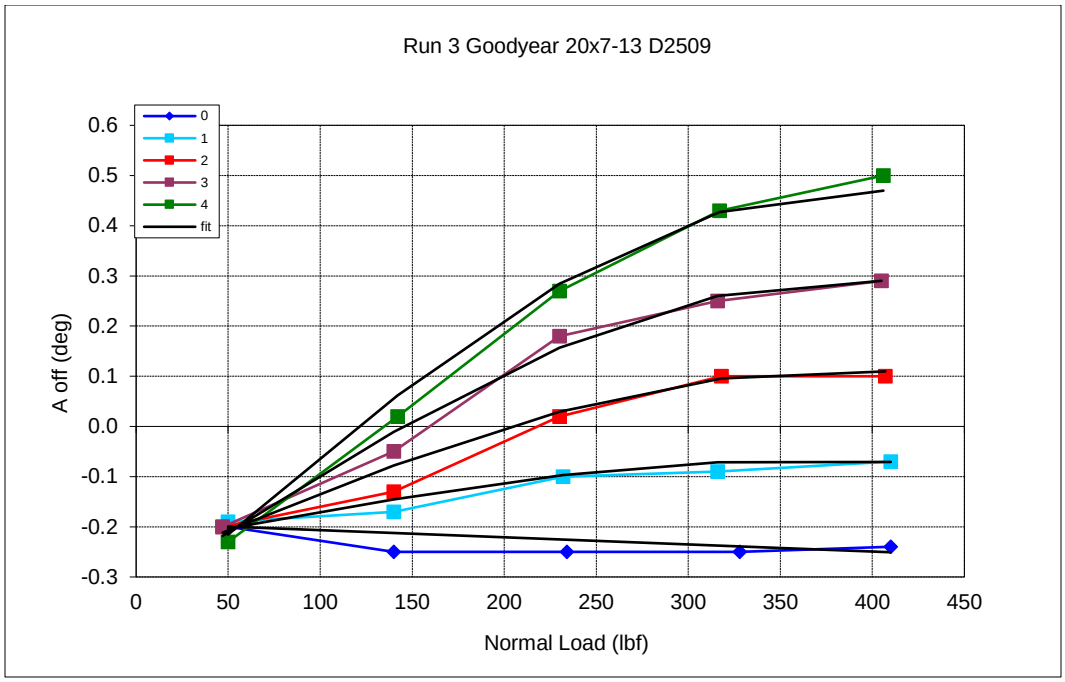

Run 3 Goodyear 20x7-13 D2509 IA = 0 Fz avg = 459
Raw data
Woods model
-Fz
Tavg
Fy / Fz
Normal Load (lbf)
Slip Angle (deg)
right
left
3
2
1
0
-1
-2
-3
600
500
400
300
200
100
0
-15
-10
-5
0
5
10
15

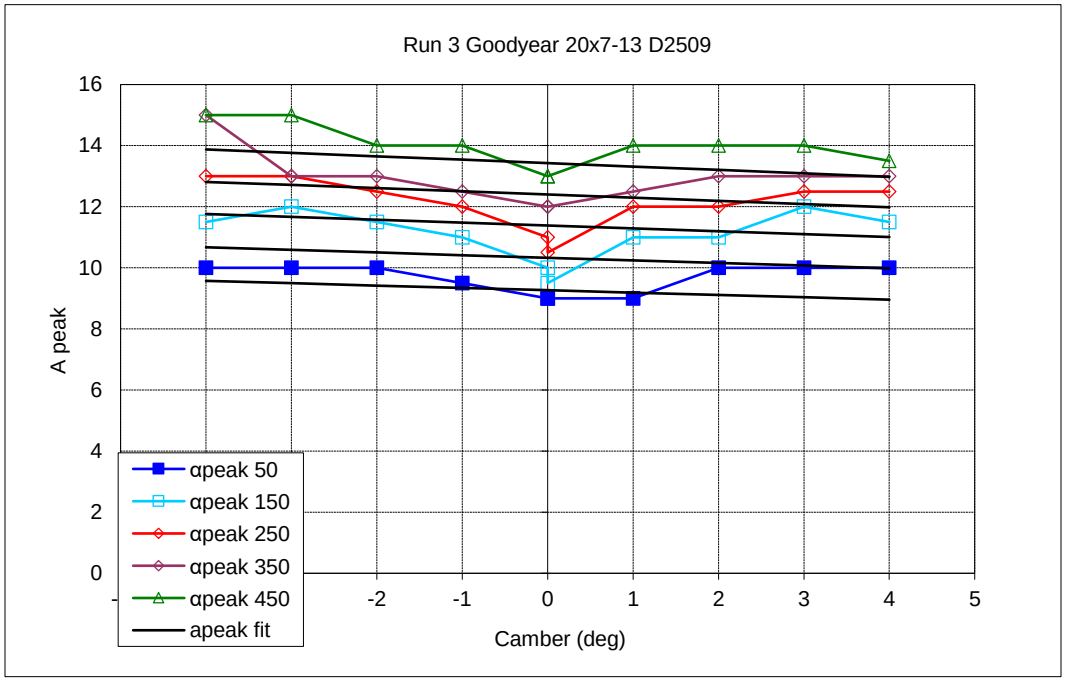

Run 3 Goodyear 20x7-13 D2509 IA = 1 Fz avg = 460
Raw data
Woods model
-Fz
Tavg
Fy / Fz
Normal Load (lbf)
Slip Angle (deg)
right
left
3
2
1
0
-1
-2
-3
600
500
400
300
200
100
0
-15
-10
-5
0
5
10
15

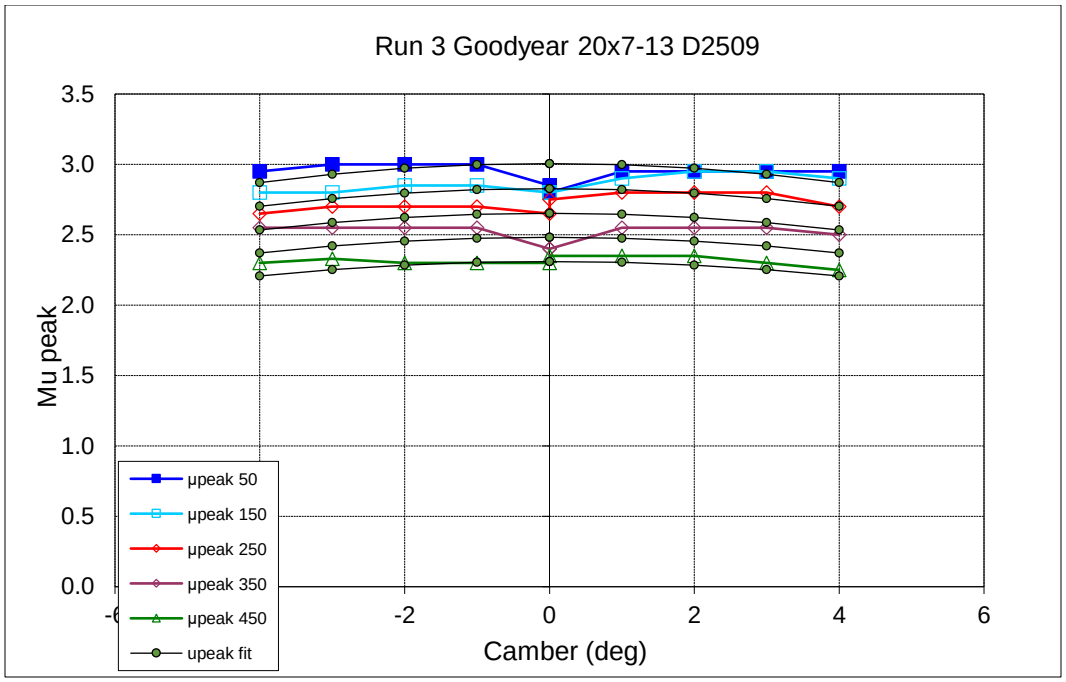

Run 3 Goodyear 20x7-13 D2509 IA = 2 Fz avg = 461
Raw data
Woods model
-Fz
Tavg
Fy / Fz
Normal Load (lbf)
Slip Angle (deg)
right
left
3
2
1
0
-1
-2
-3
600
500
400
300
200
100
0
-15
-10
-5
0
5
10
15

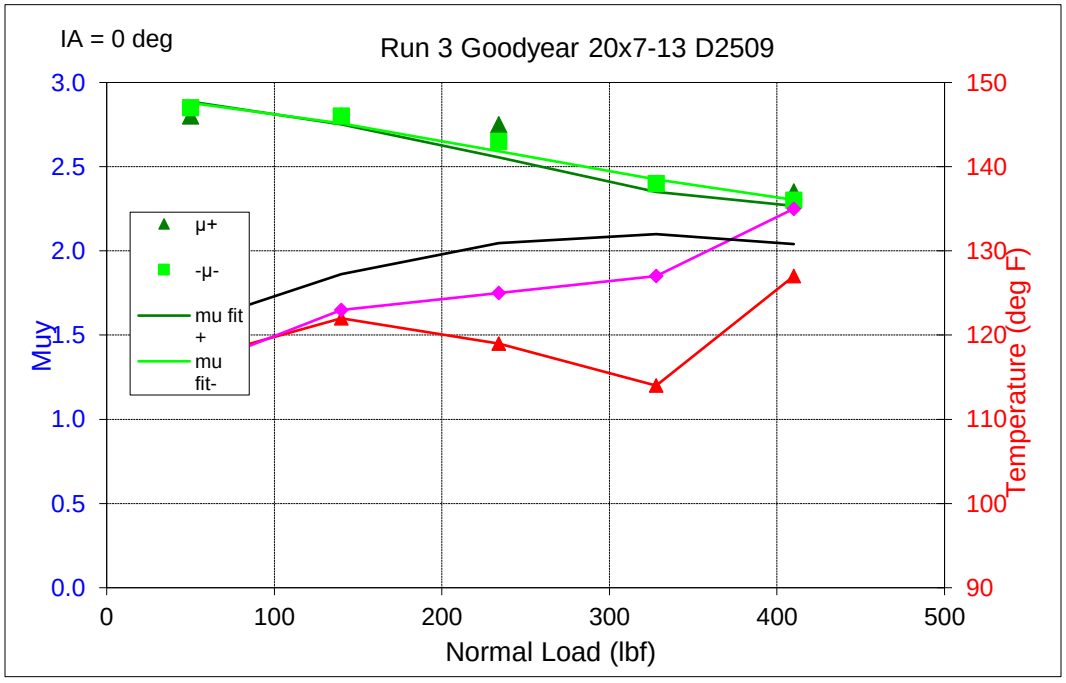

Run 3 Goodyear 20x7-13 D2509 IA = 3 Fz avg = 462
Raw data
Woods model
-Fz
Tavg
Fy / Fz
Normal Load (lbf)
Slip Angle (deg)
right
left

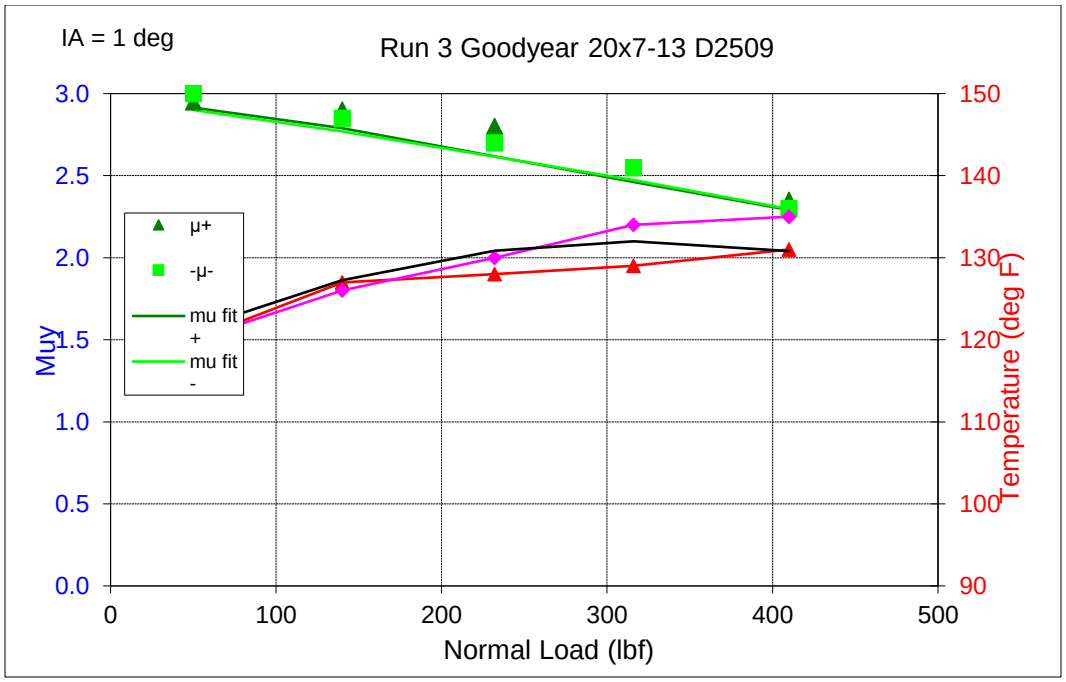

Run 3 Goodyear 20x7-13 D2509 IA = 4 Fz avg = 463
Raw data
Woods model
-Fz
Tavg
Fy / Fz
Normal Load (lbf)
Slip Angle (deg)
right
left


Run 3 Goodyear 20x7-13 D2509
0
1
2
3
4
fit
0.6
0.5
0.4
0.3
0.2
0.1
0.0
-0.1
-0.2
-0.3
A off (deg)
0
50
100
150
200
250
300
350
400
450
Normal Load (lbf)

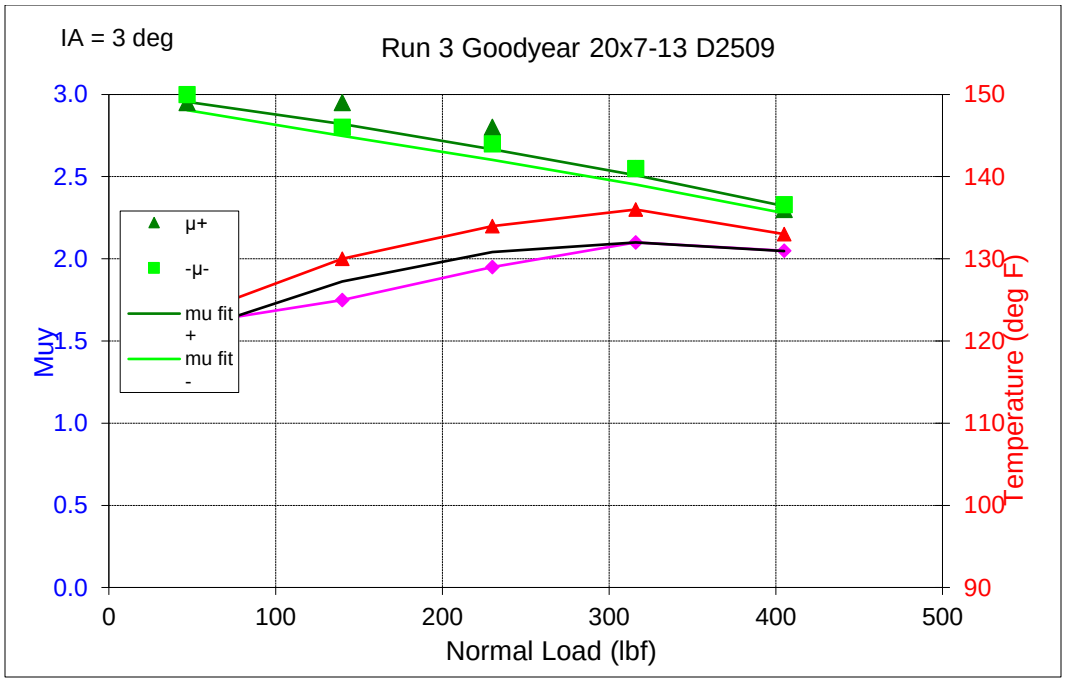

Run 3 Goodyear 20x7-13 D2509
16
14
12
10
8
6
4
2
0
A peak
αpeak 50
αpeak 150
αpeak 250
αpeak 350
αpeak 450
apeak fit
-2
-1
0
1
2
3
4
5
Camber (deg)

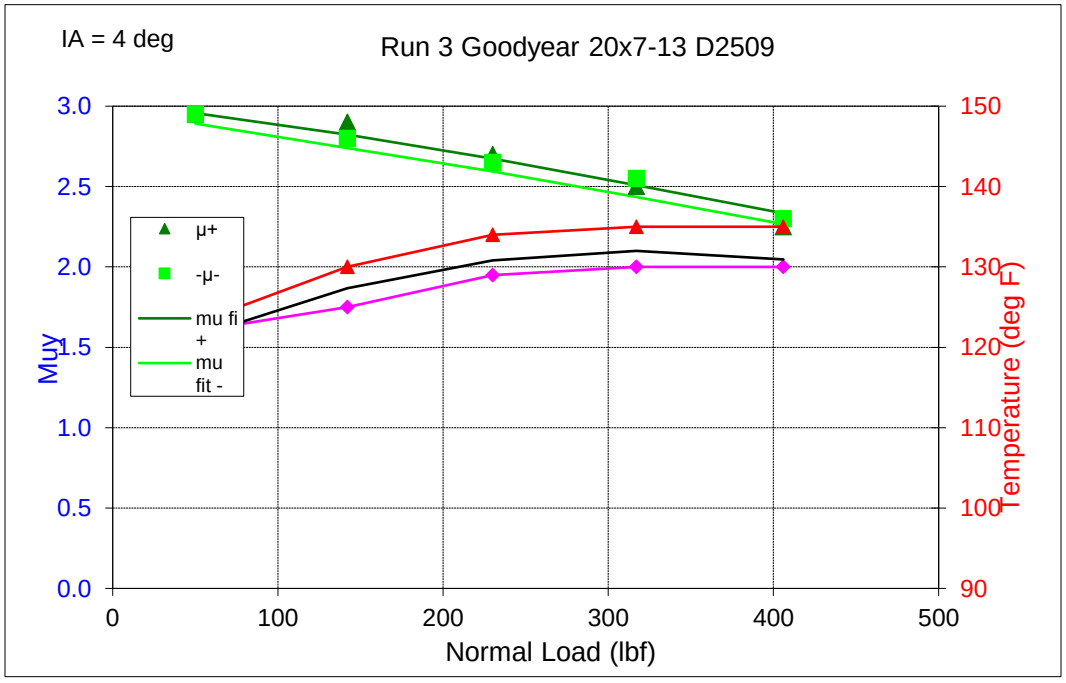

Run 3 Goodyear 20x7-13 D2509
Mu peak
Camber (deg)
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
-2
0
2
4
6
µpeak 50
µpeak 150
µpeak 250
µpeak 350
µpeak 450
upeak fit


IA = 0 deg
Run 3 Goodyear 20x7-13 D2509
Muy
Temperature (deg F)
Normal Load (lbf)
3.0
2.5
2.0
1.5
1.0
0.5
0.0
150
140
130
120
110
100
90
0
100
200
300
400
500
µ+
-µ-
mu fit +
mu fit-

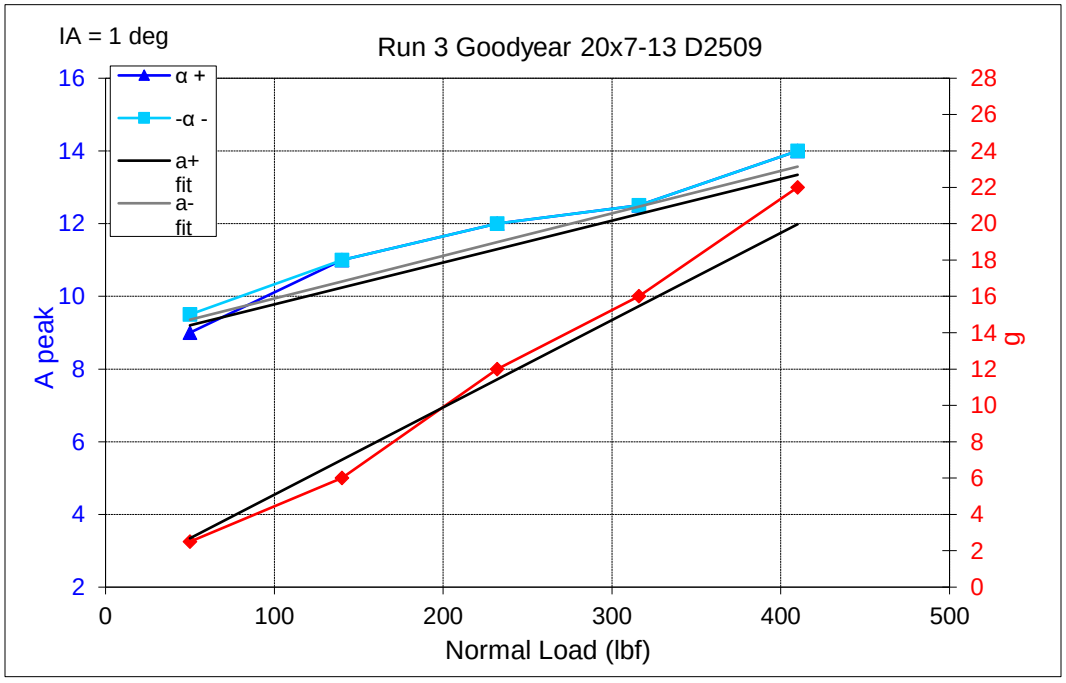

IA = 1 deg
Run 3 Goodyear 20x7-13 D2509
3.0
2.5
2.0
1.5
1.0
0.5
0.0
Muy
150
140
130
120
110
100
90
Temperature (deg F)
0
100
200
300
400
500
Normal Load (lbf)
μ+
-μ-
mu fit +
mu fit -

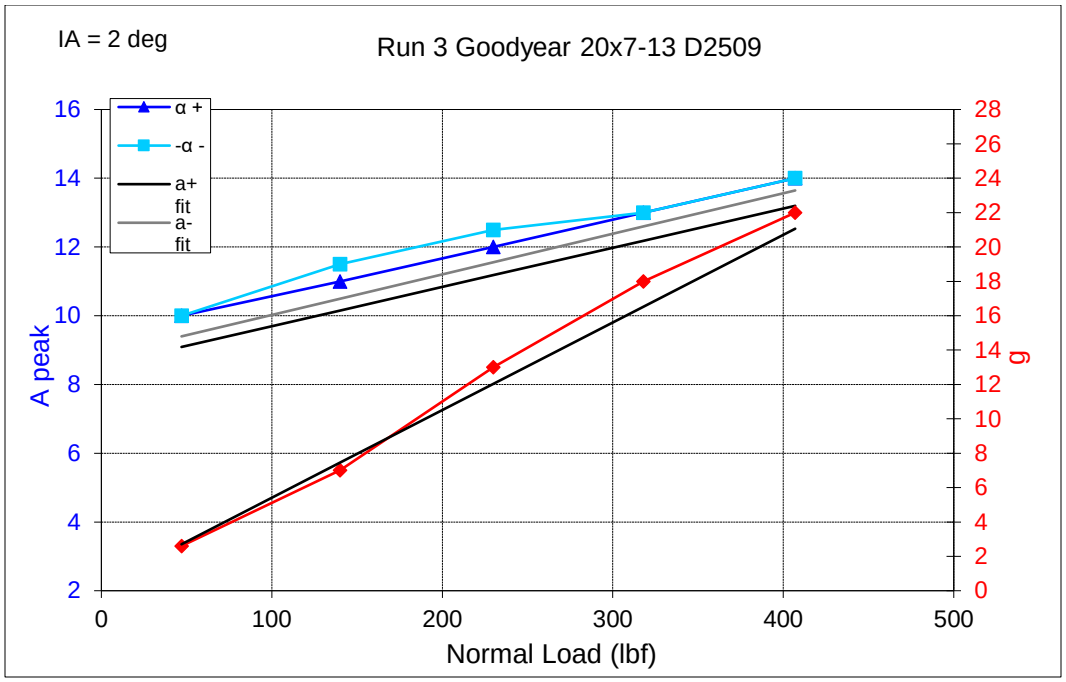

IA = 2 deg
Run 3 Goodyear 20x7-13 D2509
3.0
2.5
2.0
1.5
1.0
0.5
0.0
Muy
150
140
130
120
110
100
90
Temperature (deg F)
0
100
200
300
400
500
Normal Load (lbf)
μ+
-μ-
mu fit +
mu fit -


IA = 3 deg
Run 3 Goodyear 20x7-13 D2509
3.0
2.5
2.0
1.5
1.0
0.5
0.0
Muy
150
140
130
120
110
100
90
Temperature (deg F)
0
100
200
300
400
500
Normal Load (lbf)
μ+
-μ-
mu fit +
mu fit -

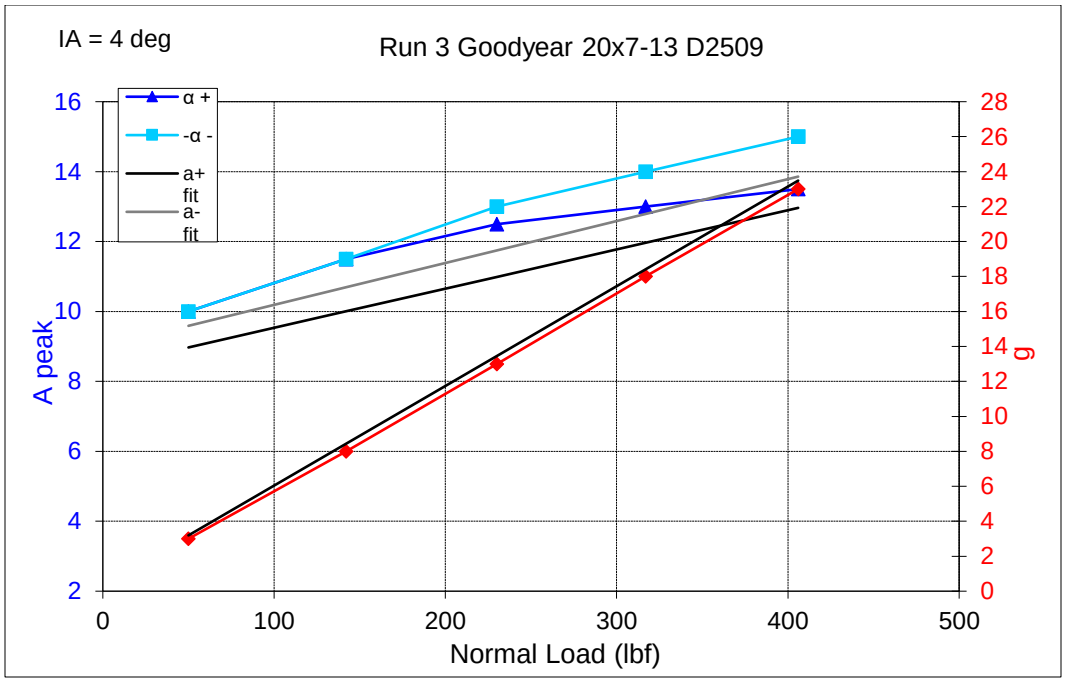

IA = 4 deg
Run 3 Goodyear 20x7-13 D2509
3.0
2.5
2.0
1.5
1.0
0.5
0.0
Muy
150
140
130
120
110
100
90
Temperature (deg F)
0
100
200
300
400
500
Normal Load (lbf)
μ+
-μ-
mu fi +
mu fit -


IA = 0 deg
Run 3 Goodyear 20x7-13 D2509
α +
-α -
a+ fit
a- fit
A peak
g
Normal Load (lbf)
0
100
200
300
400
500
2
4
6
8
10
12
14
16
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28

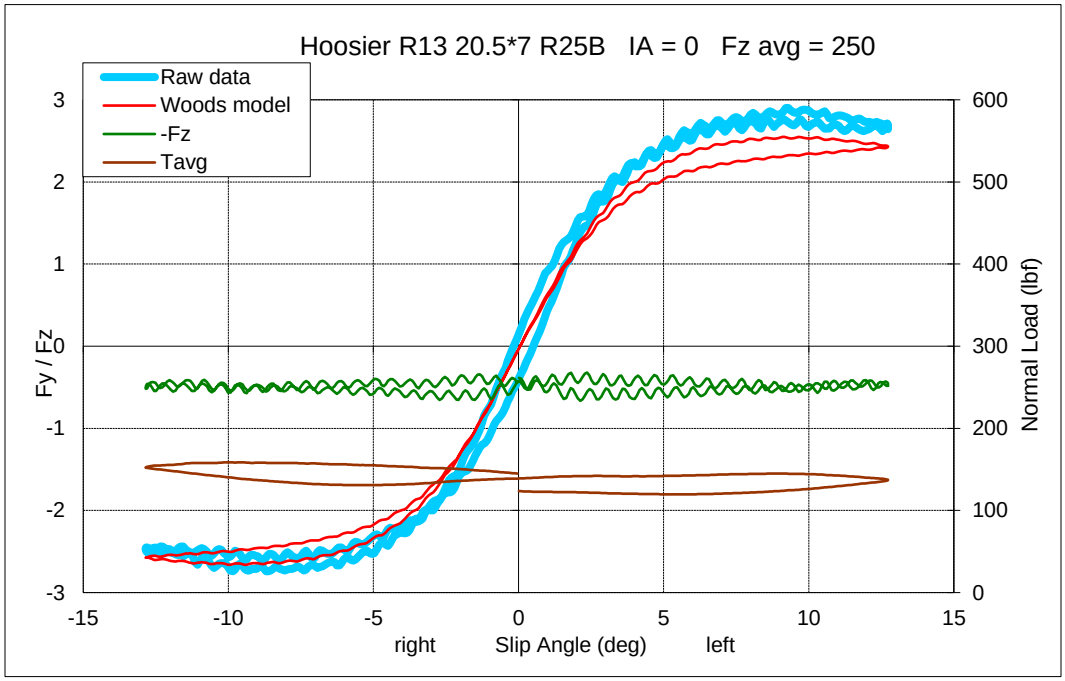

IA = 1 deg
Run 3 Goodyear 20x7-13 D2509
α +
-α -
a+ fit
a- fit
A peak
g
Normal Load (lbf)
0
100
200
300
400
500
2
4
6
8
10
12
14
16
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28

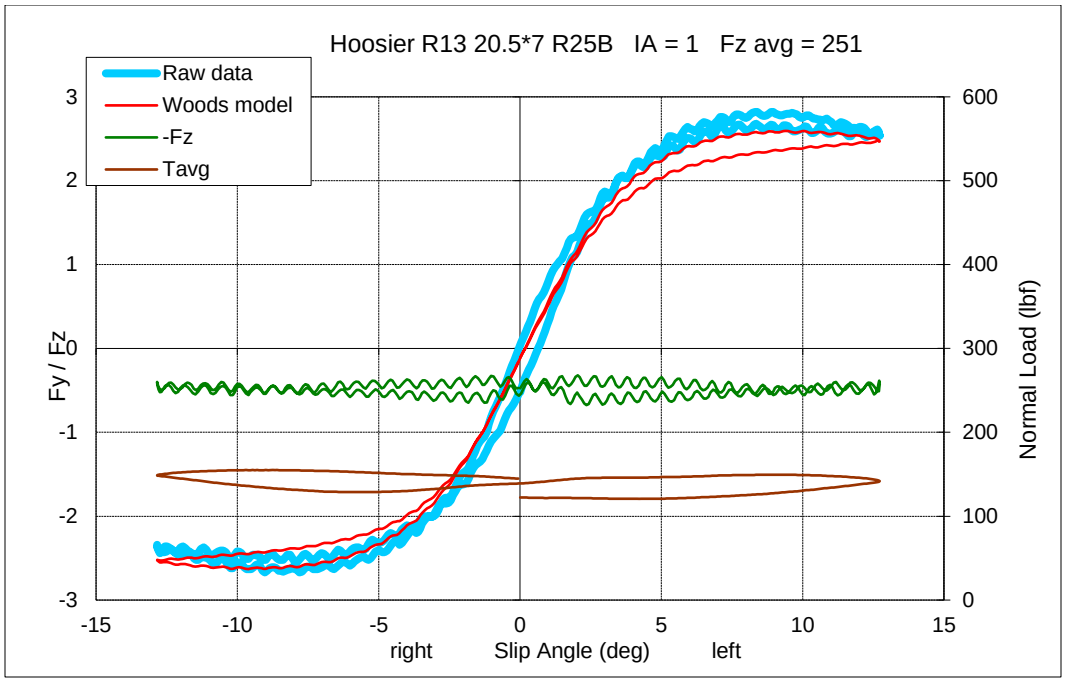

IA = 2 deg
Run 3 Goodyear 20x7-13 D2509
α +
-α -
a+ fit
a- fit
A peak
g
Normal Load (lbf)
0
100
200
300
400
500
2
4
6
8
10
12
14
16
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28

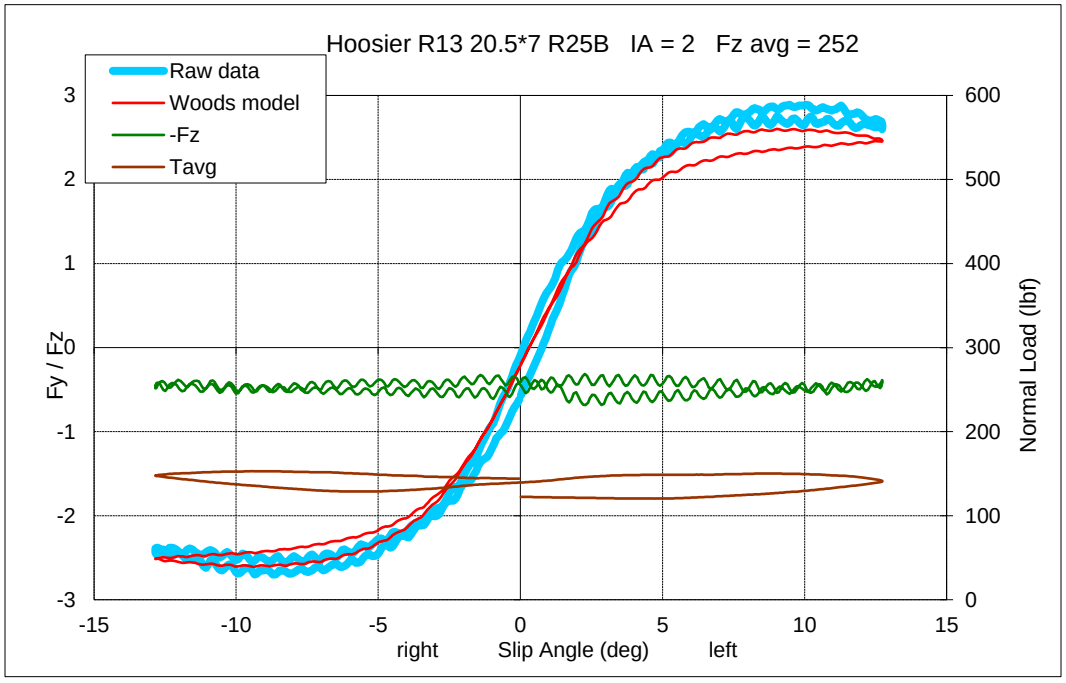

IA = 3 deg
Run 3 Goodyear 20x7-13 D2509
α +
-α -
a+ fit
a- fit
A peak
g
Normal Load (lbf)
0
100
200
300
400
500
2
4
6
8
10
12
14
16
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28

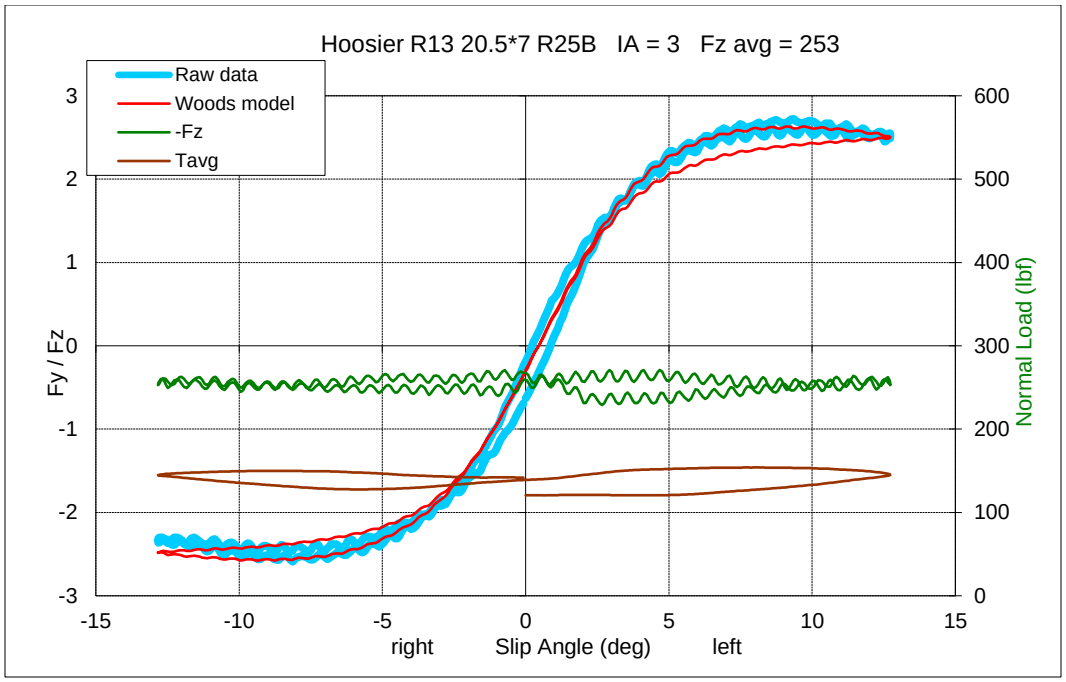


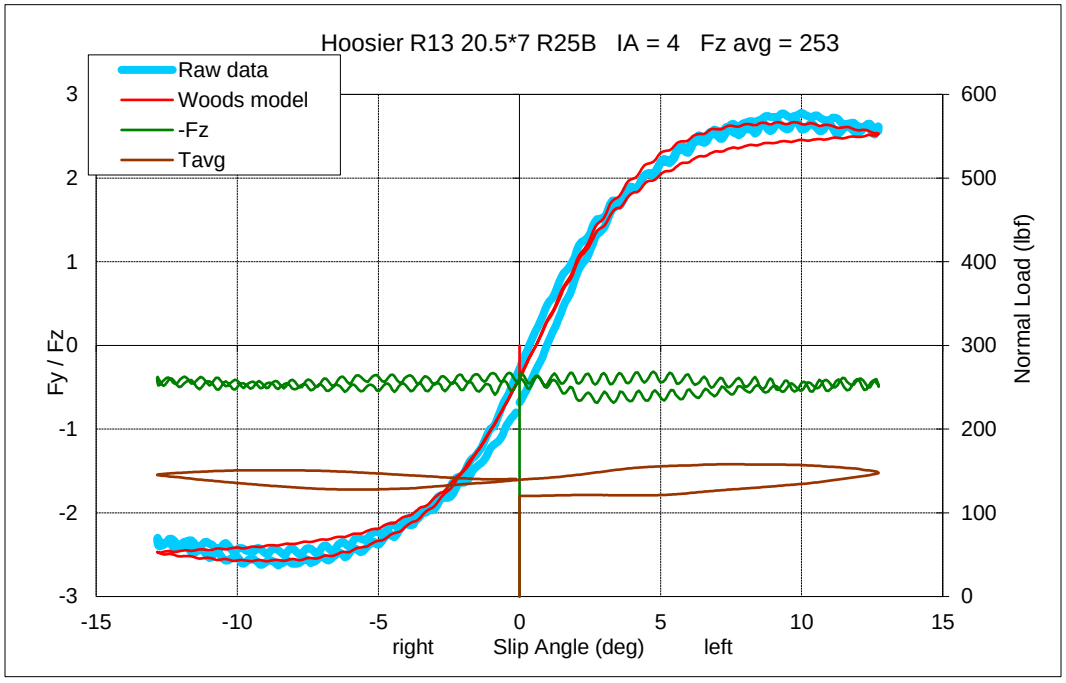

**Hoosier R13 20.5*7 R25B**

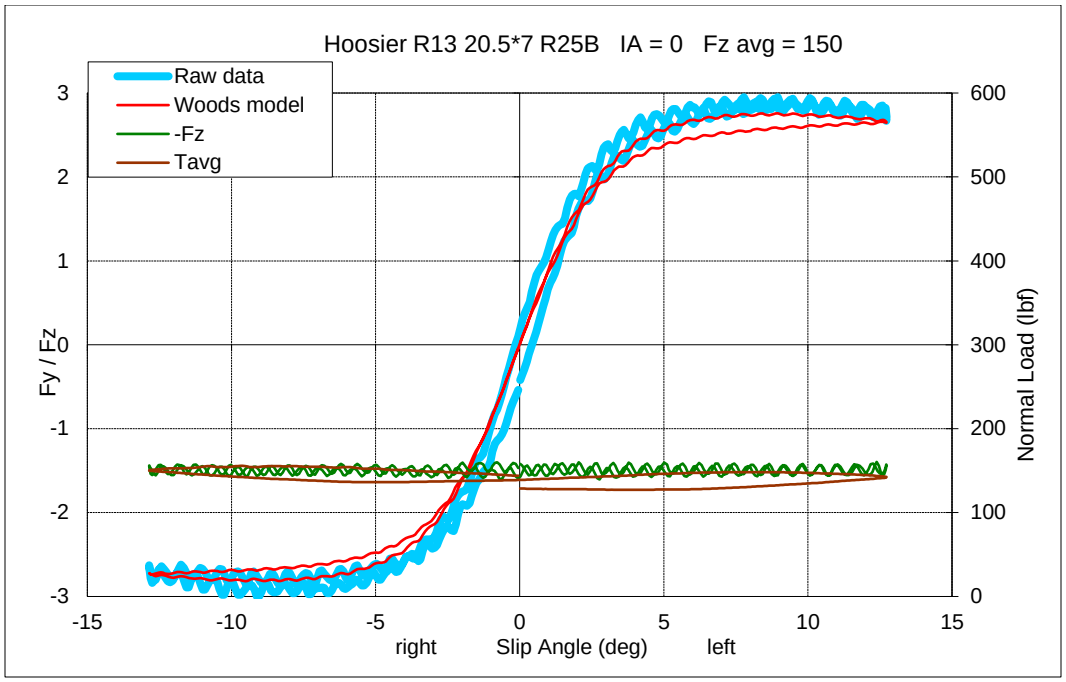


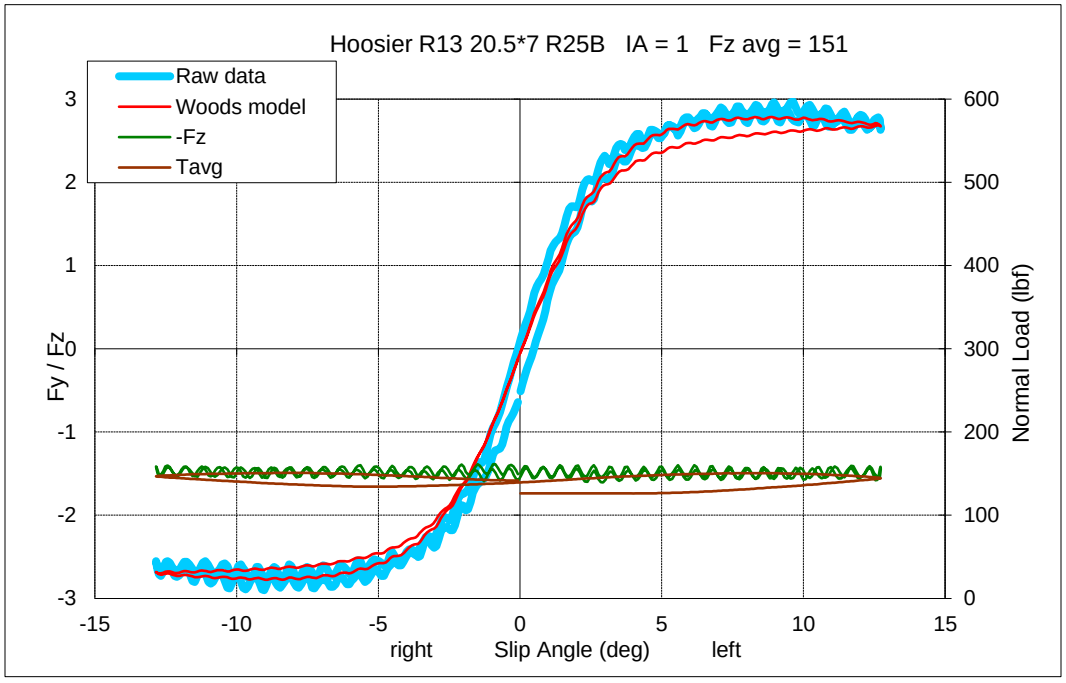

Hoosier R13 20.5*7 R25B IA = 1 Fz avg = 251
Raw data
Woods model
-Fz
Tavg
Fy / Fz
Normal Load (lbf)
Slip Angle (deg)
right
left

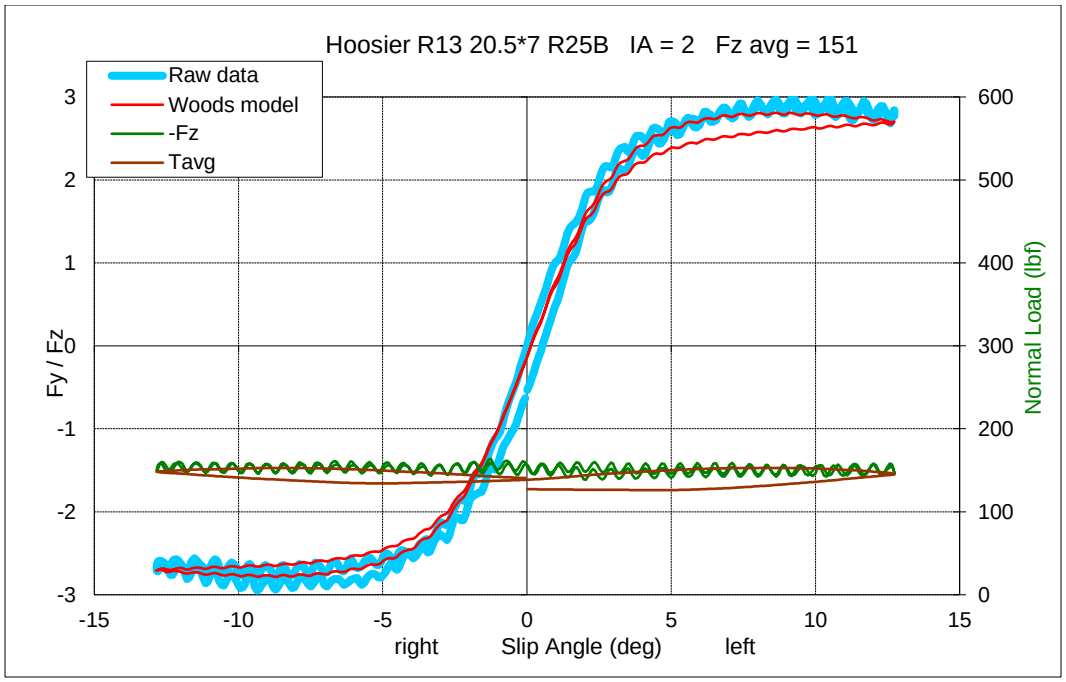

Hoosier R13 20.5*7 R25B IA = 2 Fz avg = 252
Raw data
Woods model
-Fz
Tavg
Fy / Fz
Normal Load (lbf)
Slip Angle (deg)
right
left

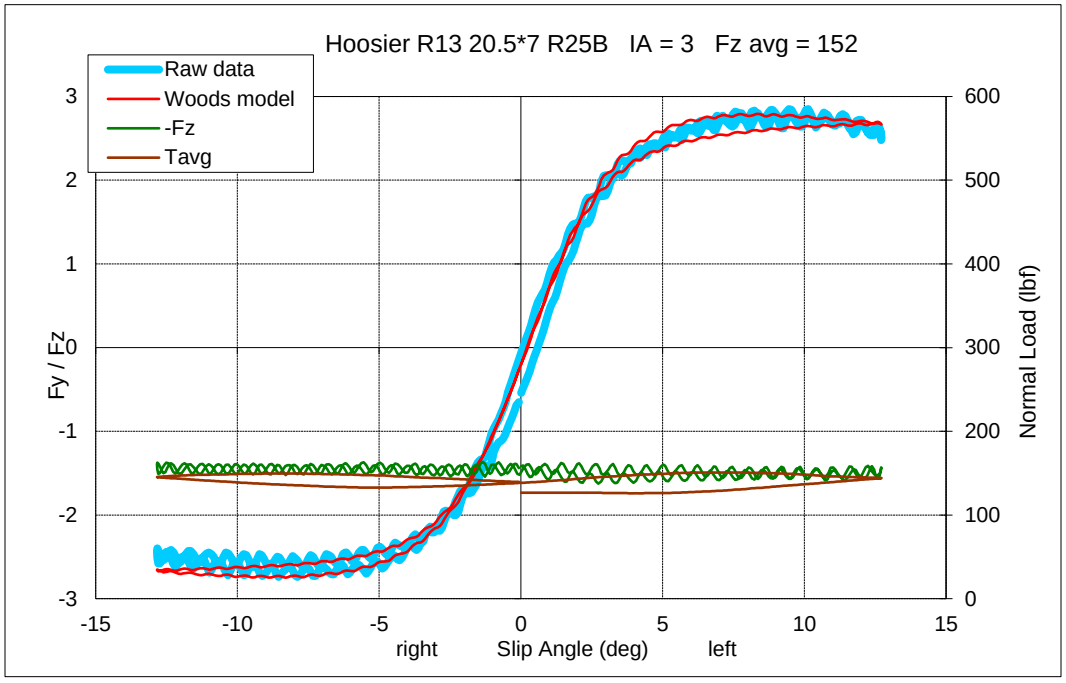

Hoosier R13 20.5*7 R25B IA = 3 Fz avg = 253
Raw data
Woods model
-Fz
Tavg
Fy / Fz
Normal Load (lbf)
Slip Angle (deg)
right
left

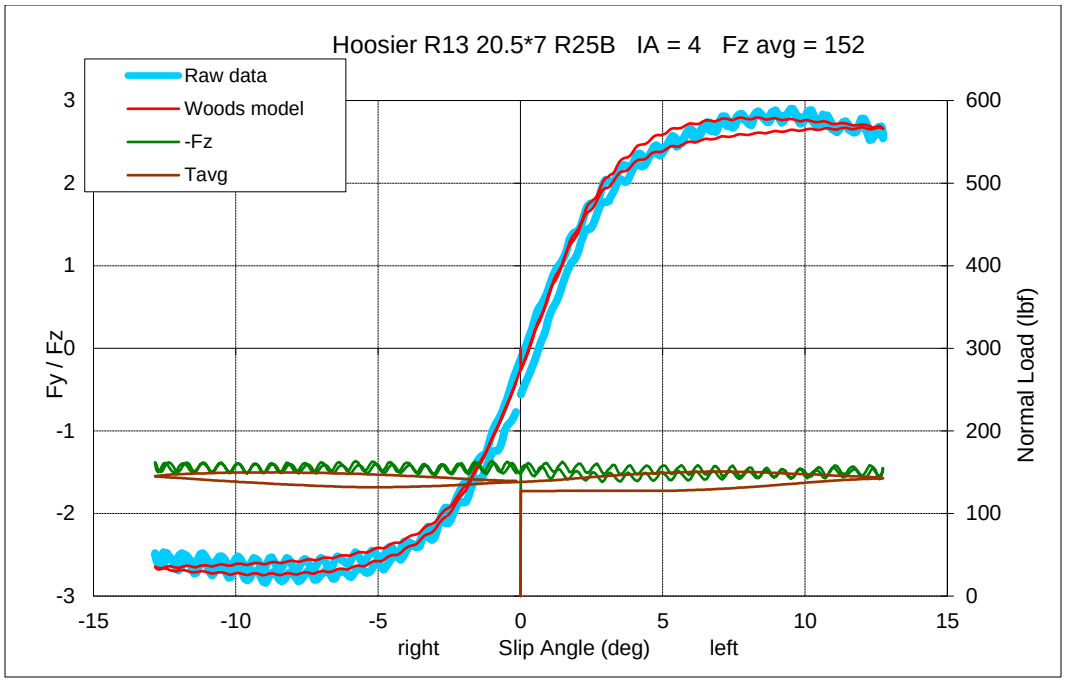

Hoosier R13 20.5*7 R25B IA = 4 Fz avg = 253
Raw data
Woods model
-Fz
Tavg
Fy / Fz
Normal Load (lbf)
Slip Angle (deg)
right
left

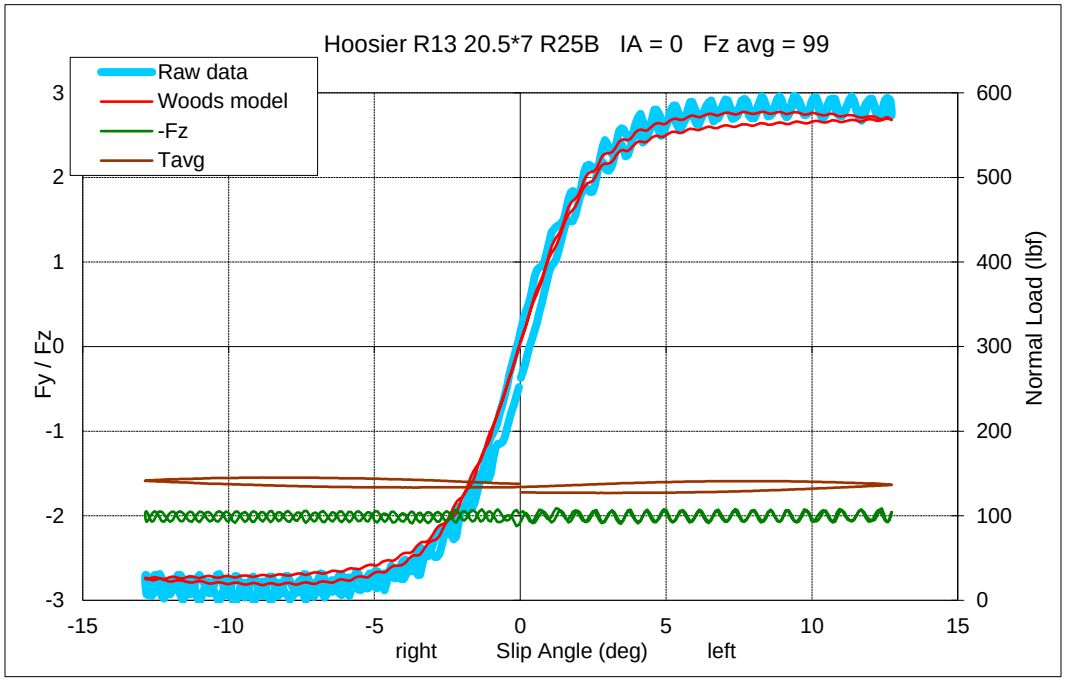

Hoosier R13 20.5*7 R25B IA = 0 Fz avg = 150
Raw data
Woods model
-Fz
Tavg
Fy / Fz
Normal Load (lbf)
Slip Angle (deg)
right
left

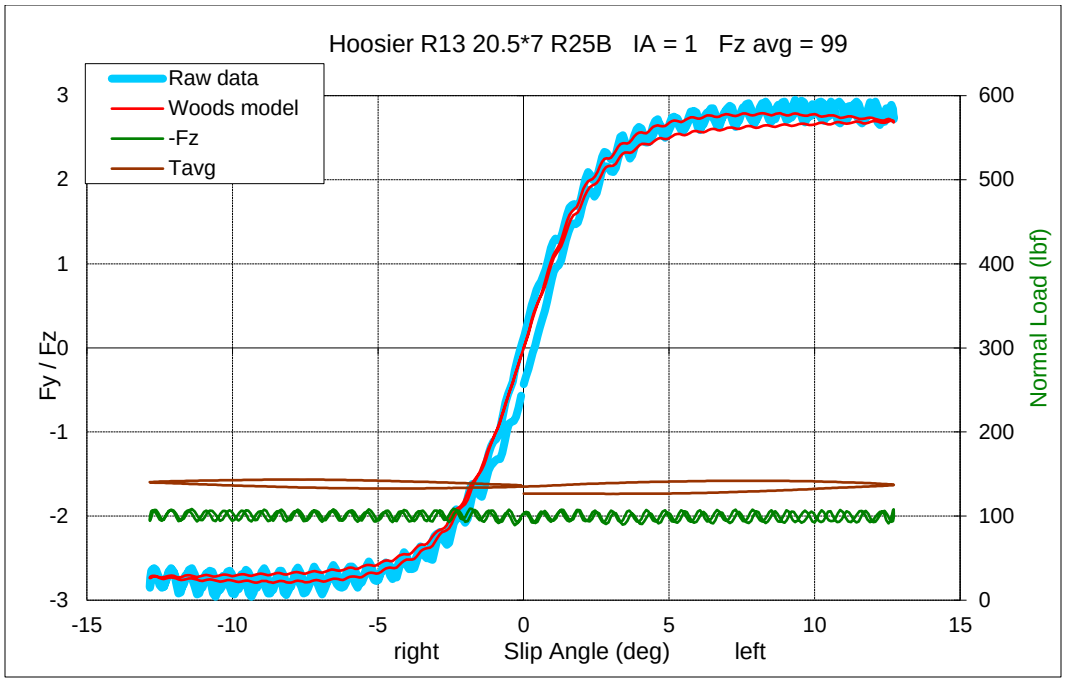

Hoosier R13 20.5*7 R25B IA = 1 Fz avg = 151
Raw data
Woods model
-Fz
Tavg
Fy / Fz
Normal Load (lbf)
Slip Angle (deg)
right
left

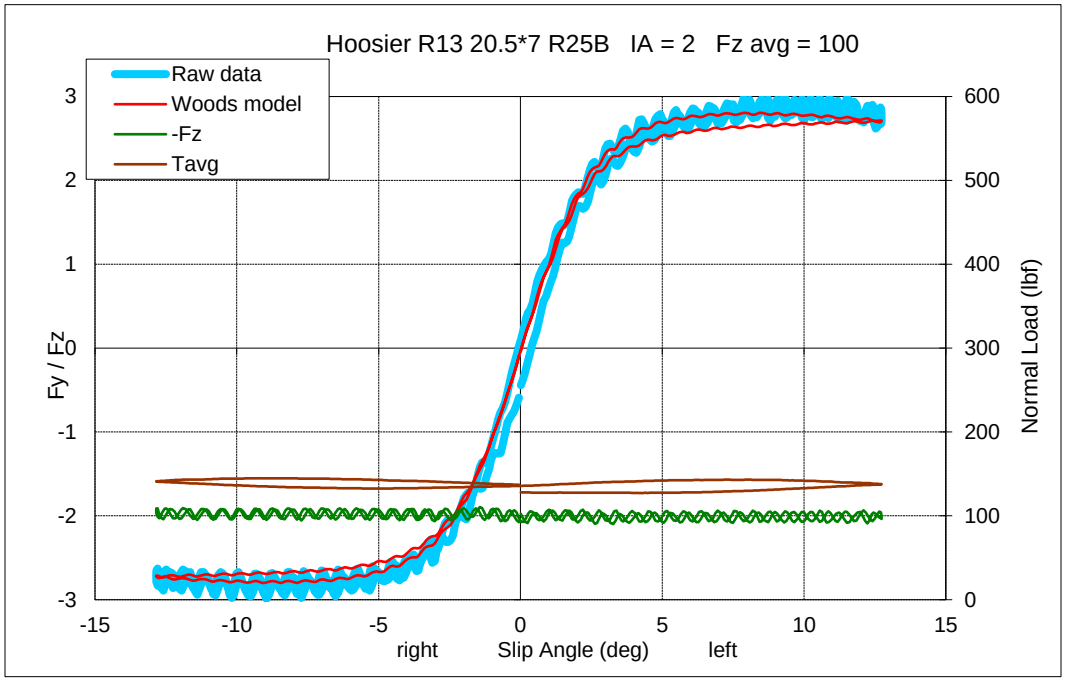

Hoosier R13 20.5*7 R25B IA = 2 Fz avg = 151
Raw data
Woods model
-Fz
Tavg
Fy / Fz
Normal Load (lbf)
Slip Angle (deg)
right
left

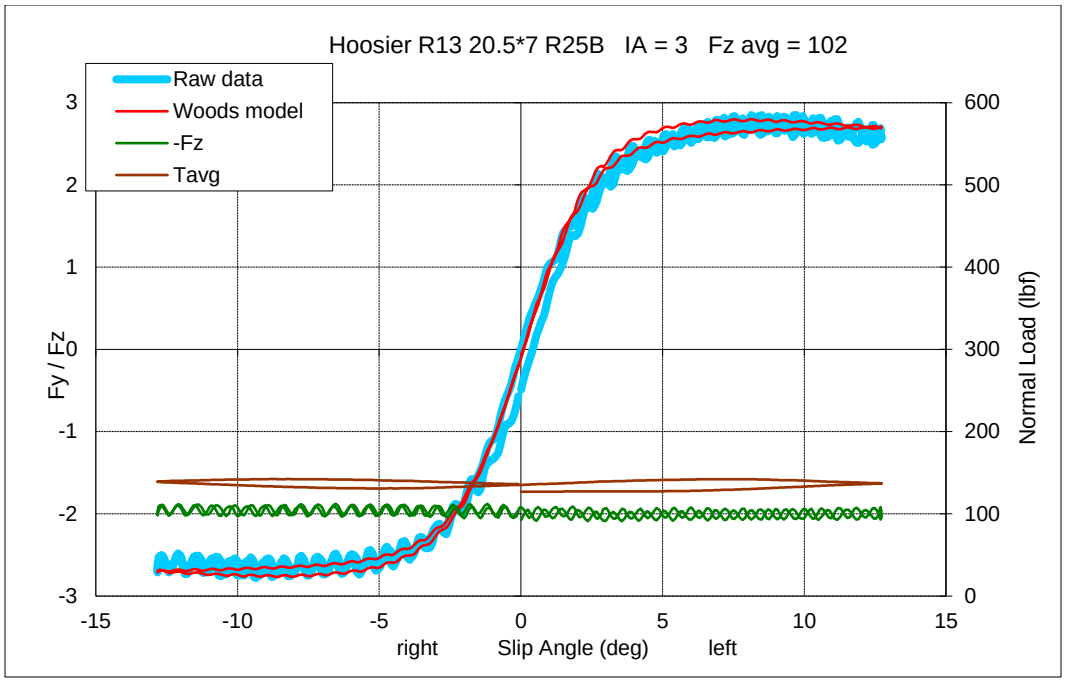

Hoosier R13 20.5*7 R25B IA = 3 Fz avg = 152
Raw data
Woods model
-Fz
Tavg
Fy / Fz
Normal Load (lbf)
Slip Angle (deg)
right
left

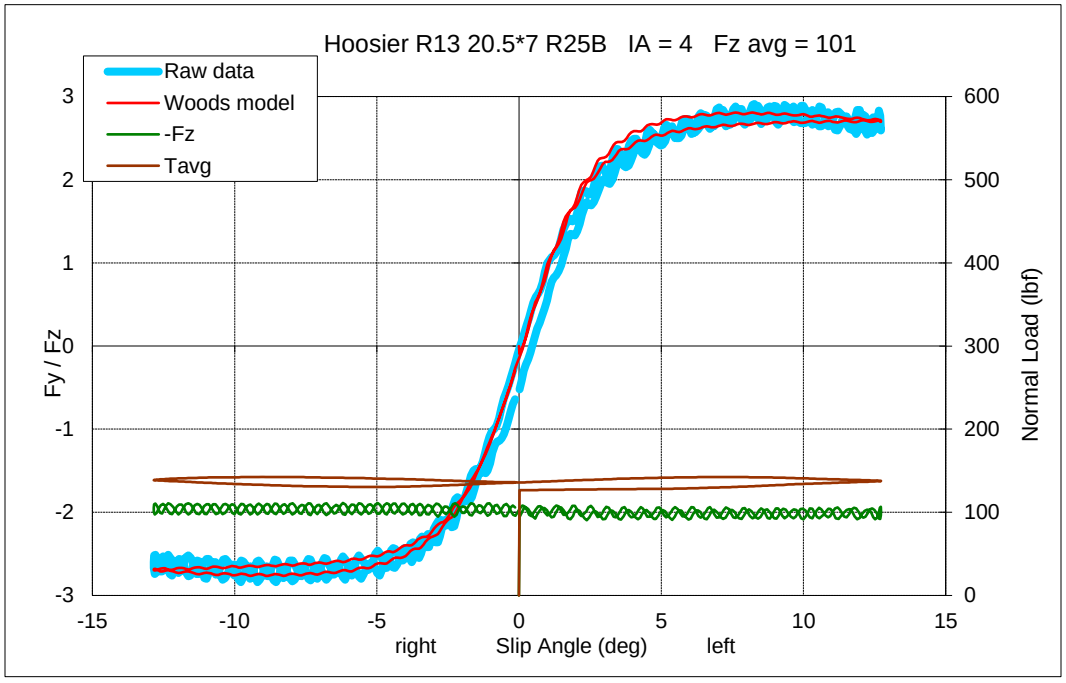

Hoosier R13 20.5*7 R25B IA = 4 Fz avg = 152
Raw data
Woods model
-Fz
Tavg
Fy / Fz
Normal Load (lbf)
Slip Angle (deg)
right
left

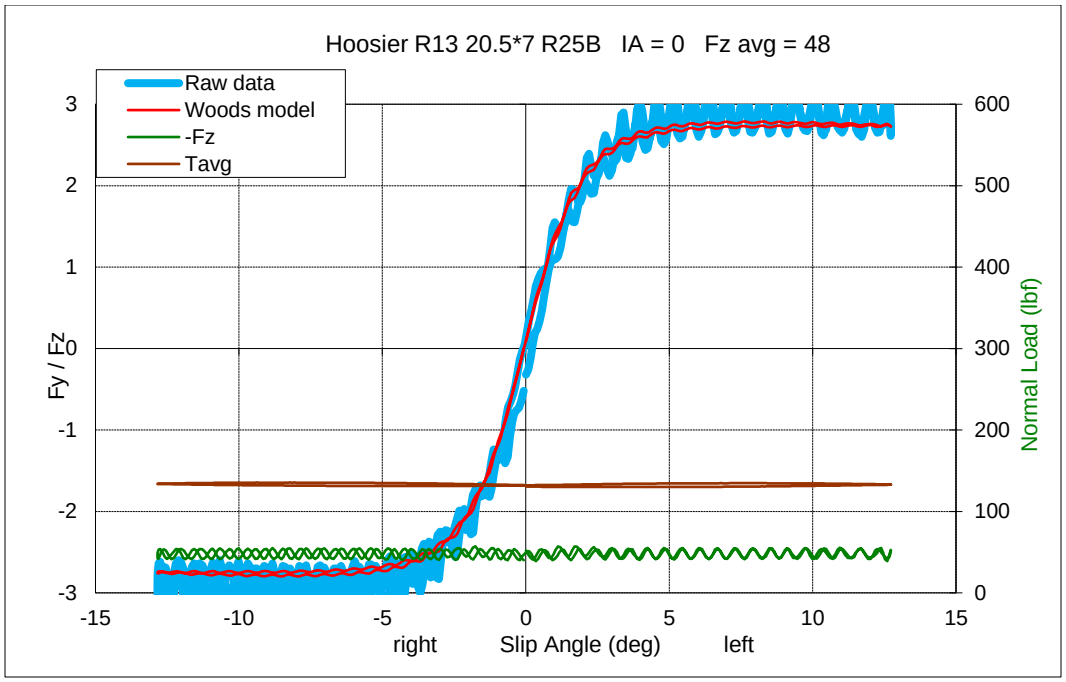

Hoosier R13 20.5*7 R25B IA = 0 Fz avg = 99
Raw data
Woods model
-Fz
Tavg
Fy / Fz
Normal Load (lbf)
Slip Angle (deg)
right
left

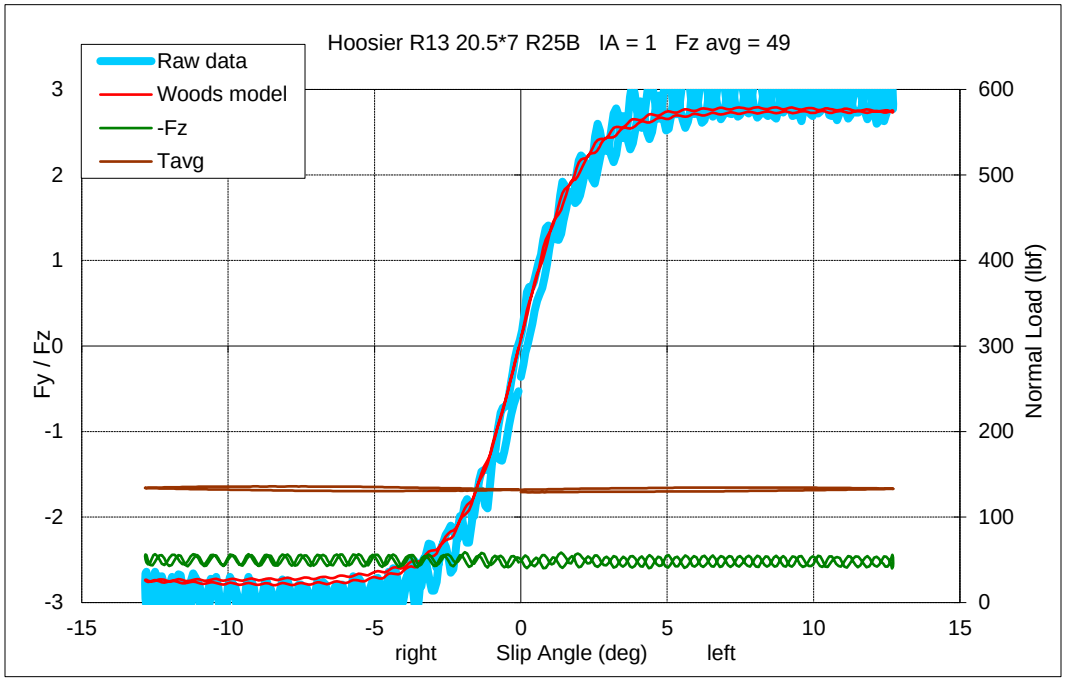

Hoosier R13 20.5*7 R25B IA = 1 Fz avg = 99
Raw data
Woods model
-Fz
Tavg
Fy / Fz
Normal Load (lbf)
Slip Angle (deg)
right
left

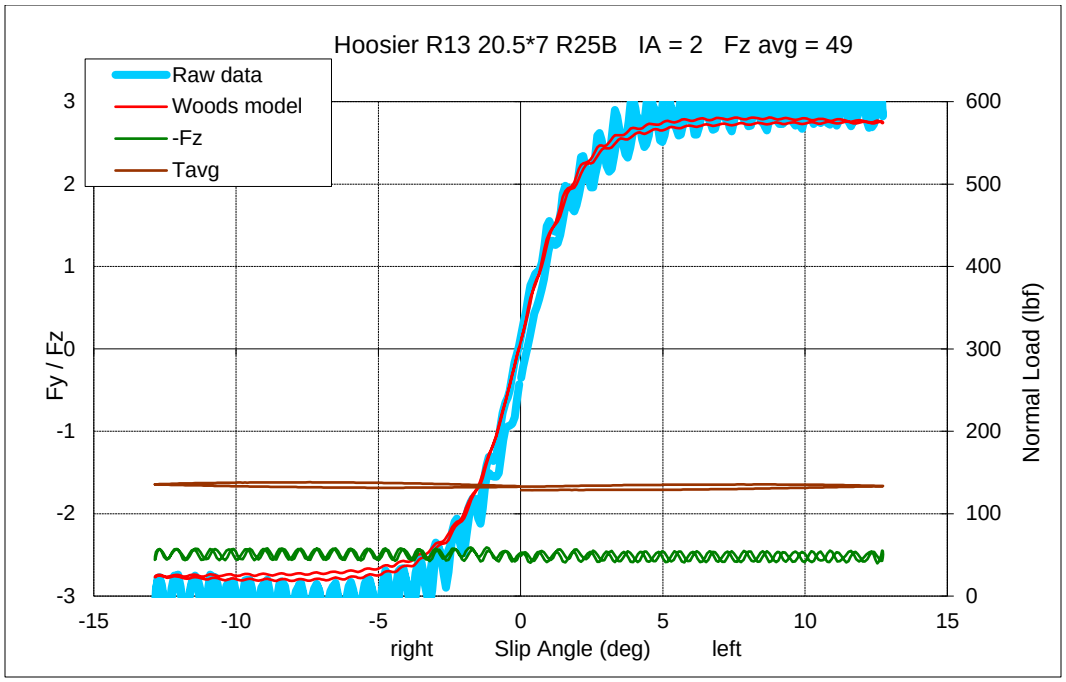

Hoosier R13 20.5*7 R25B IA = 2 Fz avg = 100
Raw data
Woods model
-Fz
Tavg
Fy / Fz
Normal Load (lbf)
Slip Angle (deg)
right
left

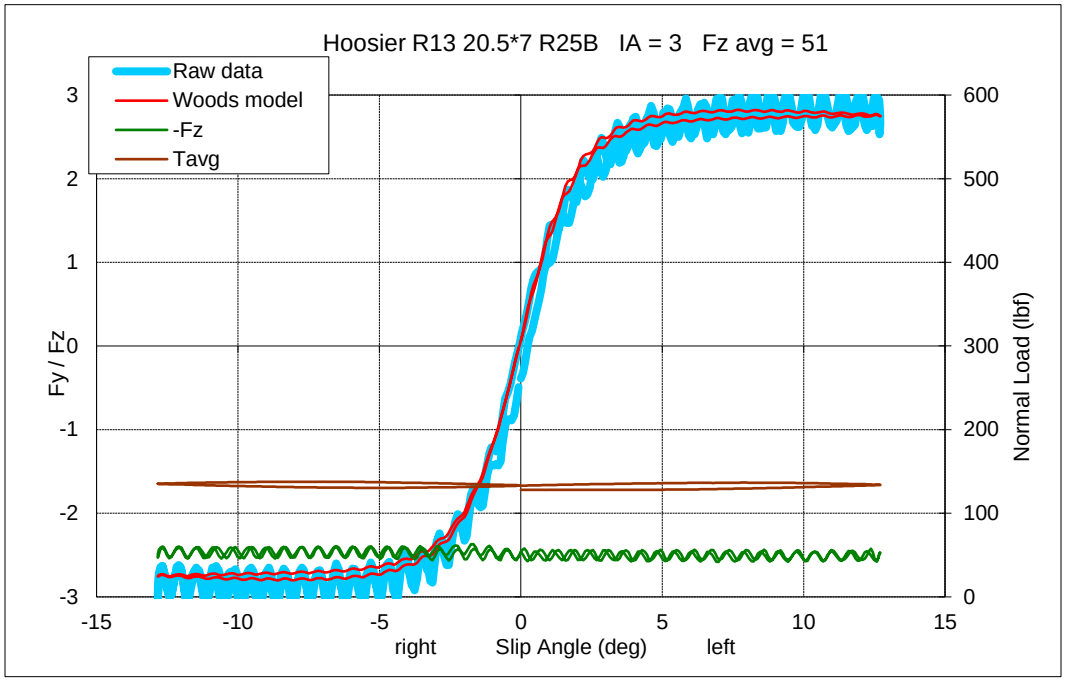

Hoosier R13 20.5*7 R25B IA = 3 Fz avg = 102
Raw data
Woods model
-Fz
Tavg
Fy / Fz
Normal Load (lbf)
Slip Angle (deg)
right
left
3
2
1
0
-1
-2
-3
600
500
400
300
200
100
0
-15
-10
-5
0
5
10
15

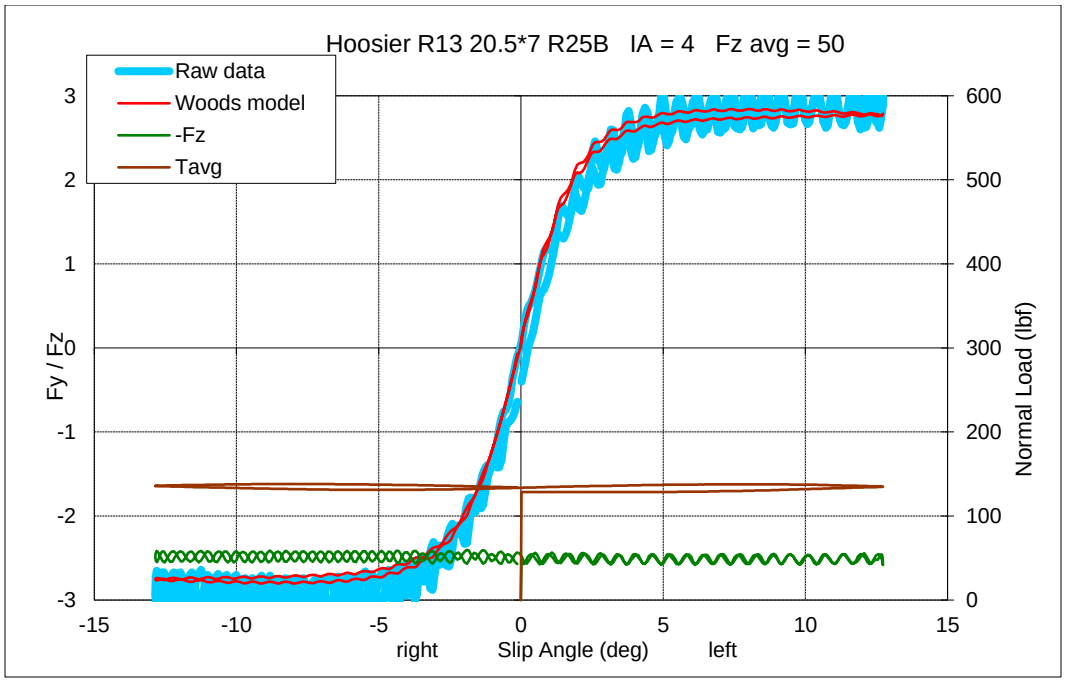

Hoosier R13 20.5*7 R25B IA = 4 Fz avg = 101
Raw data
Woods model
-Fz
Tavg
Fy / Fz
Normal Load (lbf)
Slip Angle (deg)
right
left
3
2
1
0
-1
-2
-3
600
500
400
300
200
100
0
-15
-10
-5
0
5
10
15

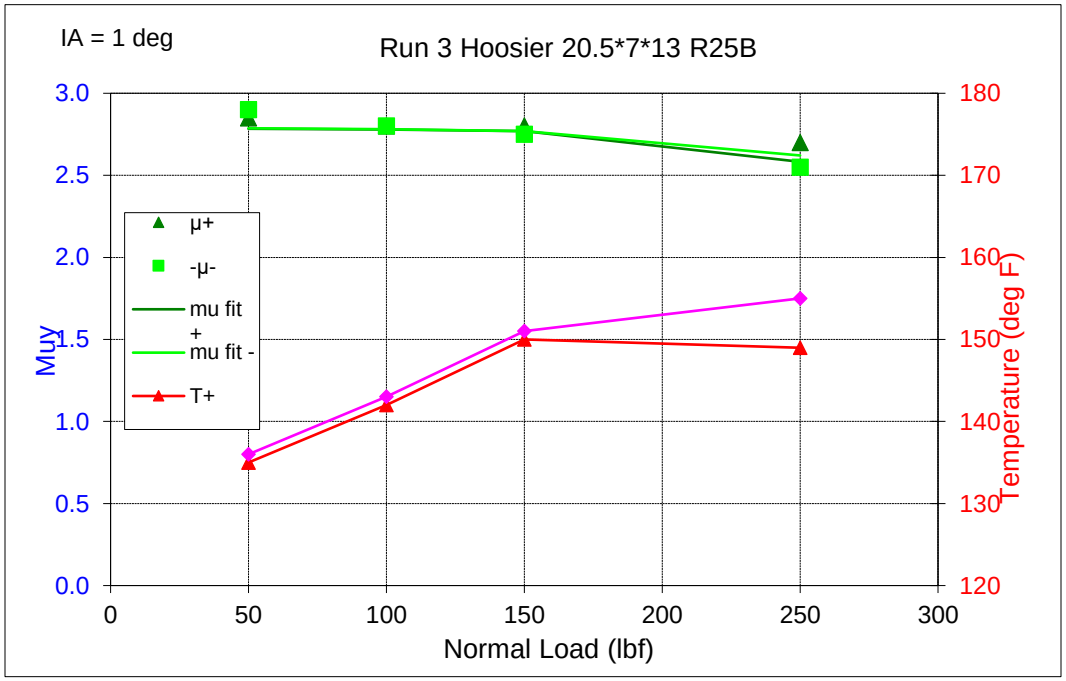

Hoosier R13 20.5*7 R25B IA = 0 Fz avg = 48
Raw data
Woods model
-Fz
Tavg
Fy / Fz
Normal Load (lbf)
3
2
1
0
-1
-2
-3
600
500
400
300
200
100
0
-15
-10
-5
0
5
10
15
right
Slip Angle (deg)
left

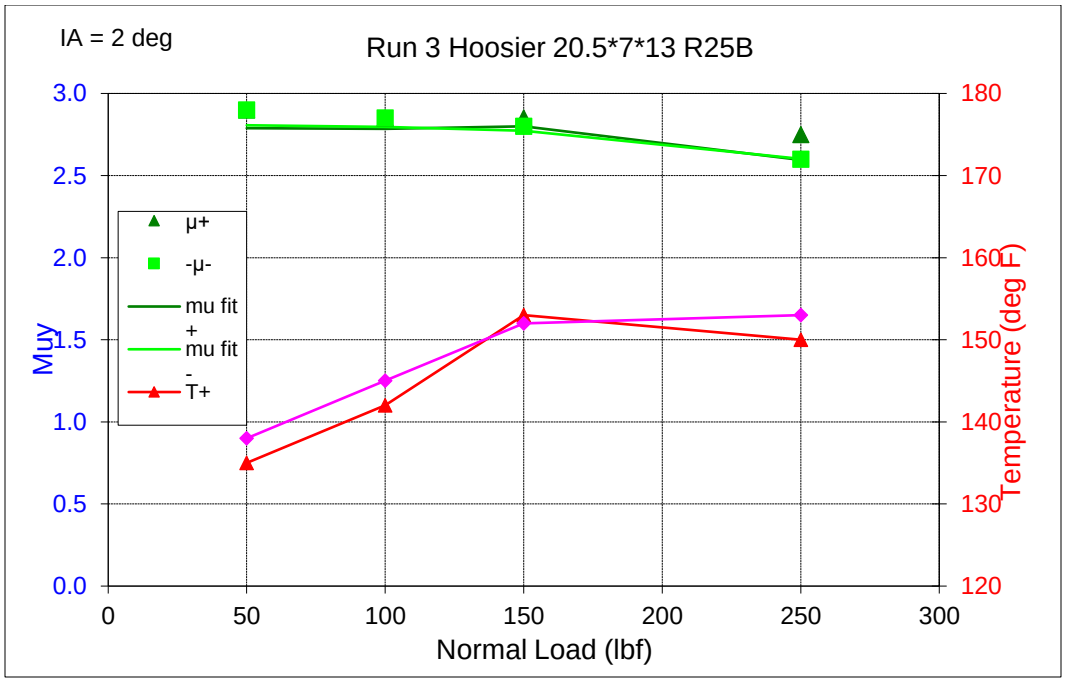

Hoosier R13 20.5*7 R25B IA = 1 Fz avg = 49
Raw data
Woods model
-Fz
Tavg
Fy / Fz
Normal Load (lbf)
3
2
1
0
-1
-2
-3
600
500
400
300
200
100
0
-15
-10
-5
0
5
10
15
right
Slip Angle (deg)
left

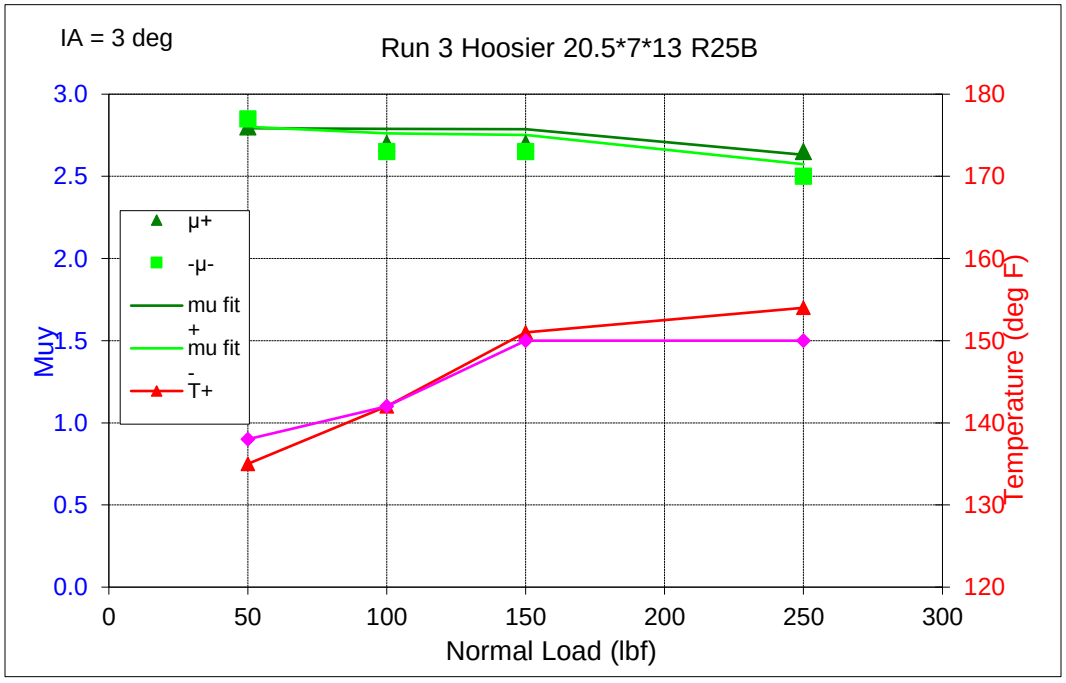

Hoosier R13 20.5*7 R25B IA = 2 Fz avg = 49
Raw data
Woods model
-Fz
Tavg
Fy / Fz
Normal Load (lbf)
Slip Angle (deg)
right
left
3
2
1
0
-1
-2
-3
600
500
400
300
200
100
0
-15
-10
-5
0
5
10
15

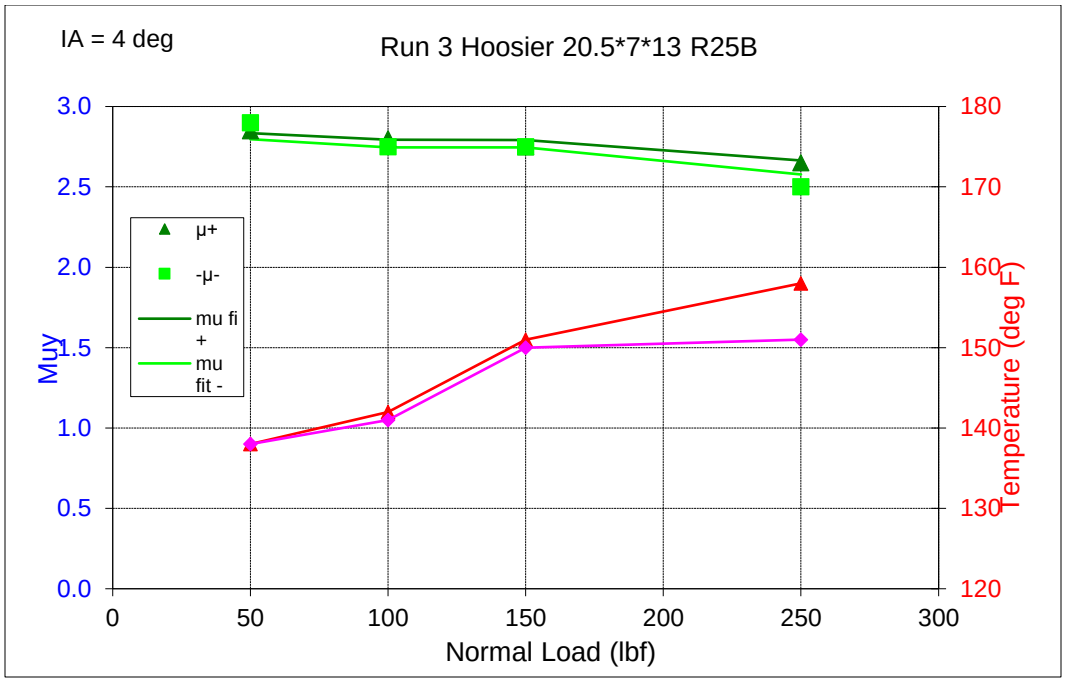

Hoosier R13 20.5*7 R25B IA = 3 Fz avg = 51
Raw data
Woods model
-Fz
Tavg
Fy / Fz
Normal Load (lbf)
Slip Angle (deg)
right
left
3
2
1
0
-1
-2
-3
600
500
400
300
200
100
0
-15
-10
-5
0
5
10
15

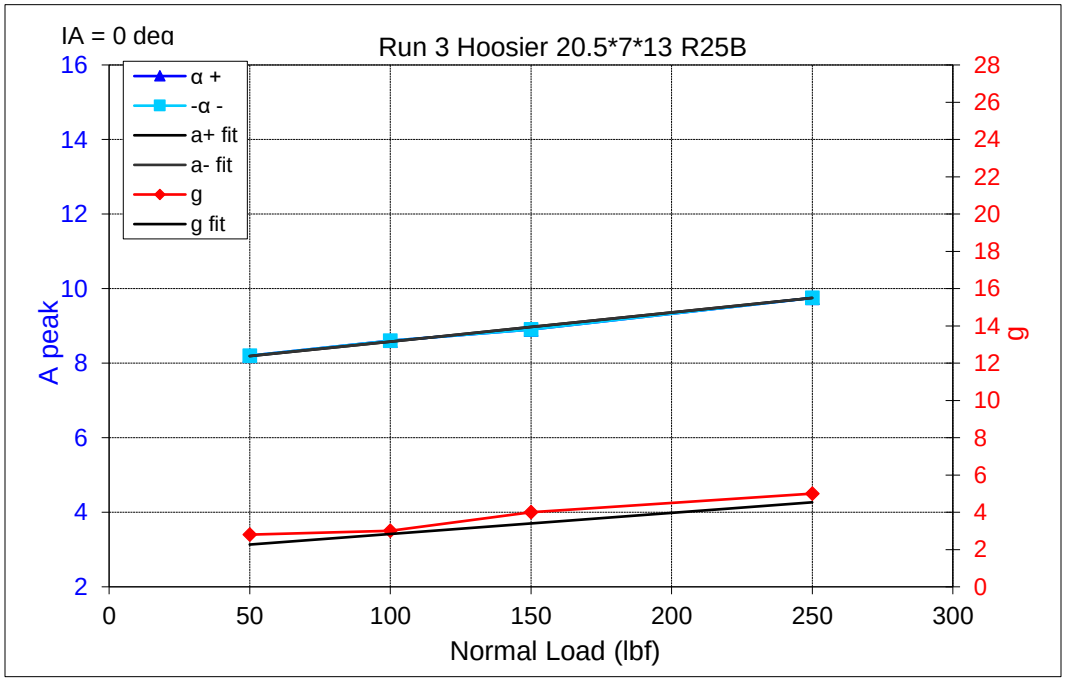

Hoosier R13 20.5*7 R25B IA = 4 Fz avg = 50
Raw data
Woods model
-Fz
Tavg
Fy / Fz
Normal Load (lbf)
Slip Angle (deg)
right
left
3
2
1
0
-1
-2
-3
600
500
400
300
200
100
0
-15
-10
-5
0
5
10
15

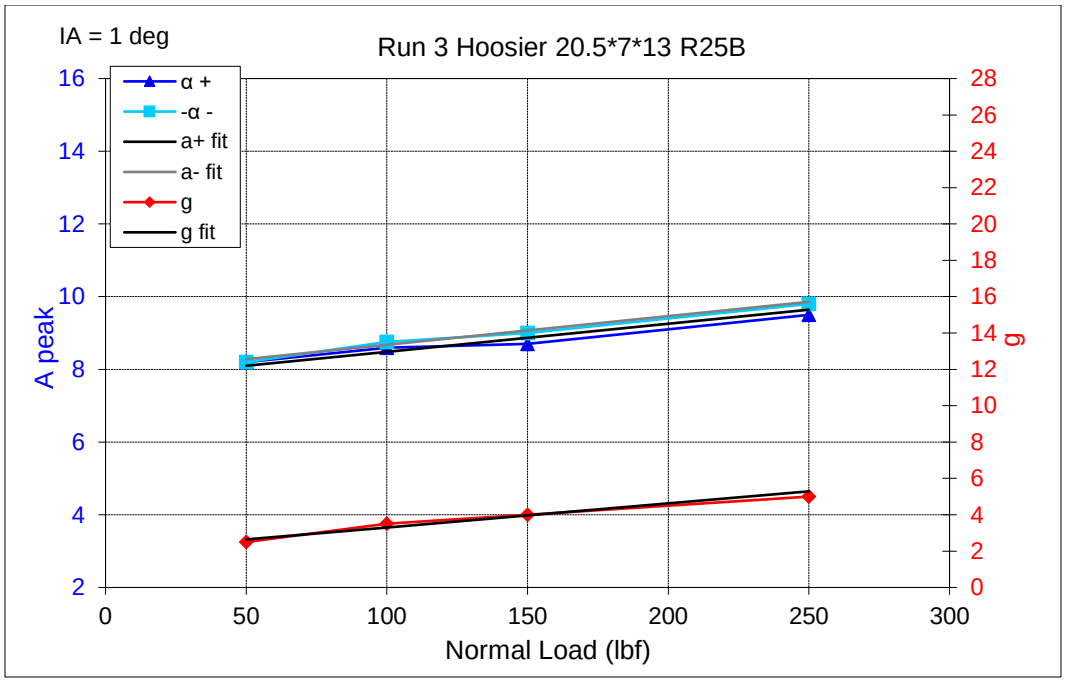

IA = 1 deg
Run 3 Hoosier 20.5*7*13 R25B
μ+
-μ-
mu fit +
mu fit -
T+
Muy
Temperature (deg F)
Normal Load (lbf)
3.0
2.5
2.0
1.5
1.0
0.5
0.0
180
170
160
150
140
130
120
0
50
100
150
200
250
300


IA = 2 deg
Run 3 Hoosier 20.5*7*13 R25B
3.0
2.5
2.0
1.5
1.0
0.5
0.0
Muy
180
170
160
150
140
130
120
Temperature (deg F)
0
50
100
150
200
250
300
Normal Load (lbf)
μ+
-μ-
mu fit +
mu fit -
T+

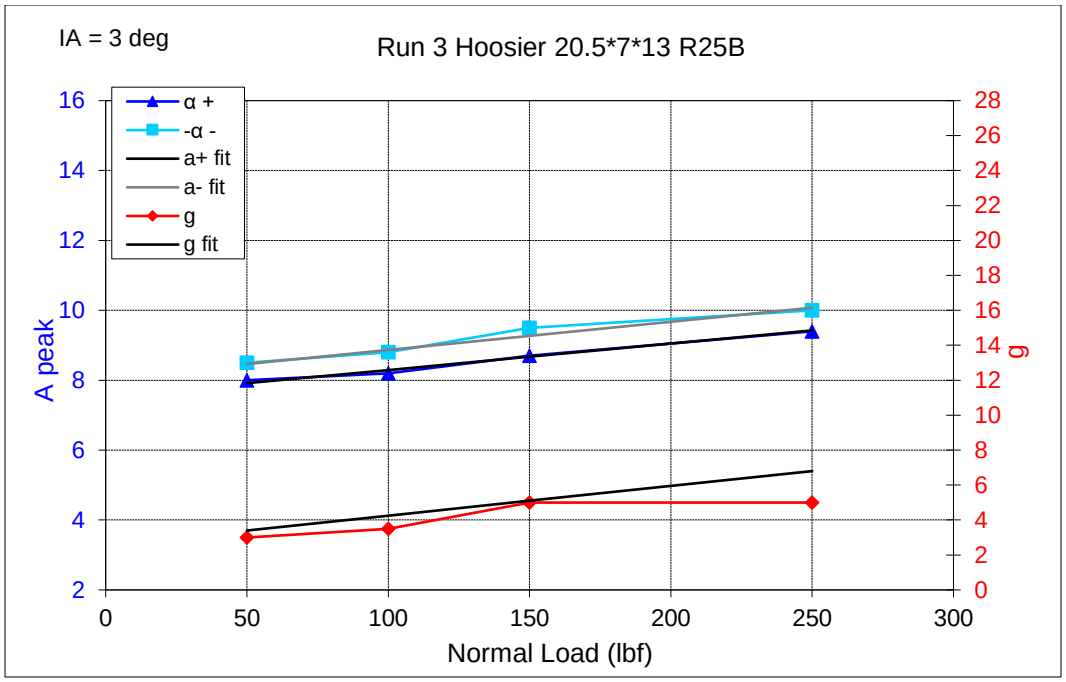

IA = 3 deg
Run 3 Hoosier 20.5*7*13 R25B
3.0
2.5
2.0
1.5
1.0
0.5
0.0
Muy
180
170
160
150
140
130
120
Temperature (deg F)
0
50
100
150
200
250
300
Normal Load (lbf)
μ+
-μ-
mu fit +
mu fit -
T+

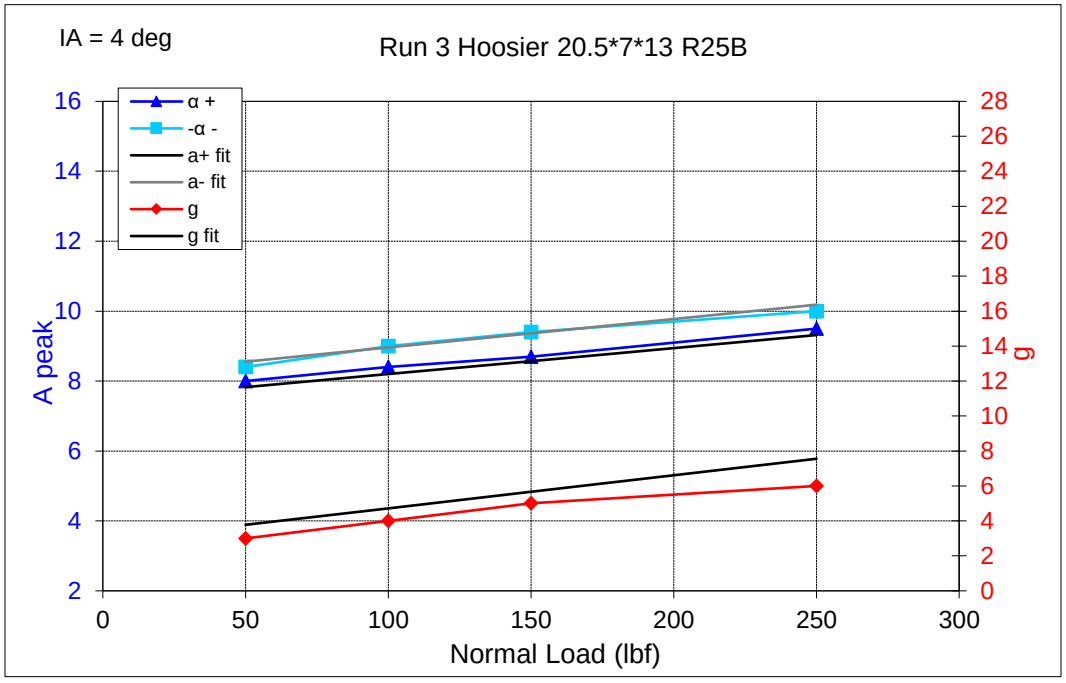

IA = 4 deg
Run 3 Hoosier 20.5*7*13 R25B
Muy
Temperature (deg F)
Normal Load (lbf)
μ+
-μ-
mu fi +
mu fit -
3.0
2.5
2.0
1.5
1.0
0.5
0.0
180
170
160
150
140
130
120
0
50
100
150
200
250
300

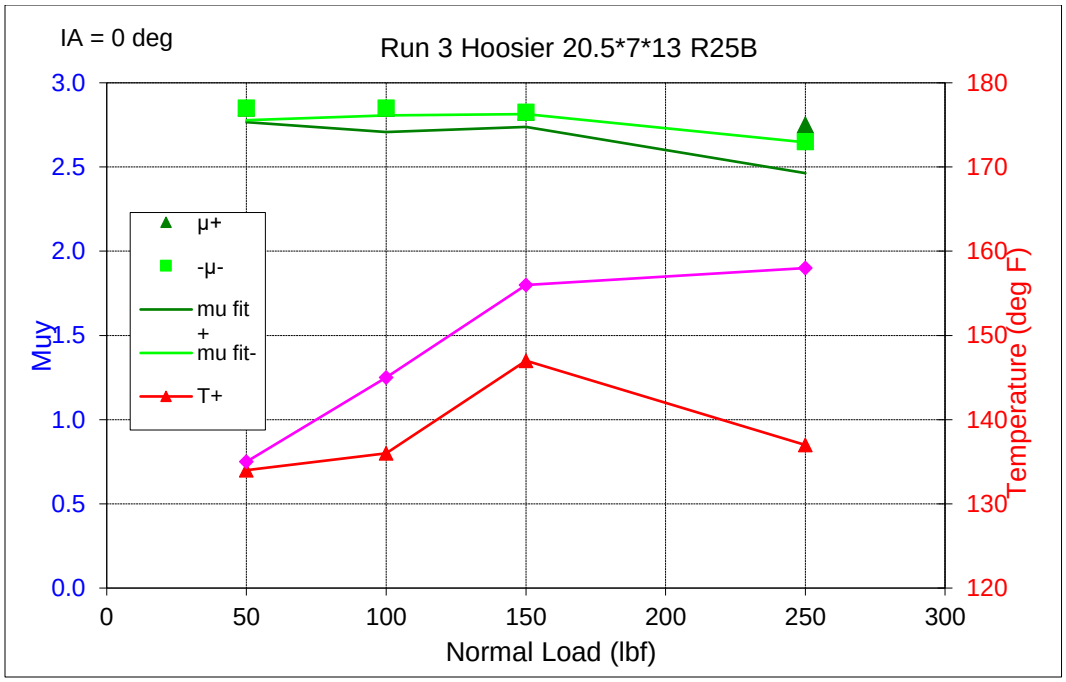

IA = 0 deg
Run 3 Hoosier 20.5*7*13 R25B
A peak
g
Normal Load (lbf)
α +
-α -
a+ fit
a- fit
g
g fit
16
14
12
10
8
6
4
2
28
26
24
22
20
18
16
14
12
10
8
6
4
2
0
0
50
100
150
200
250
300

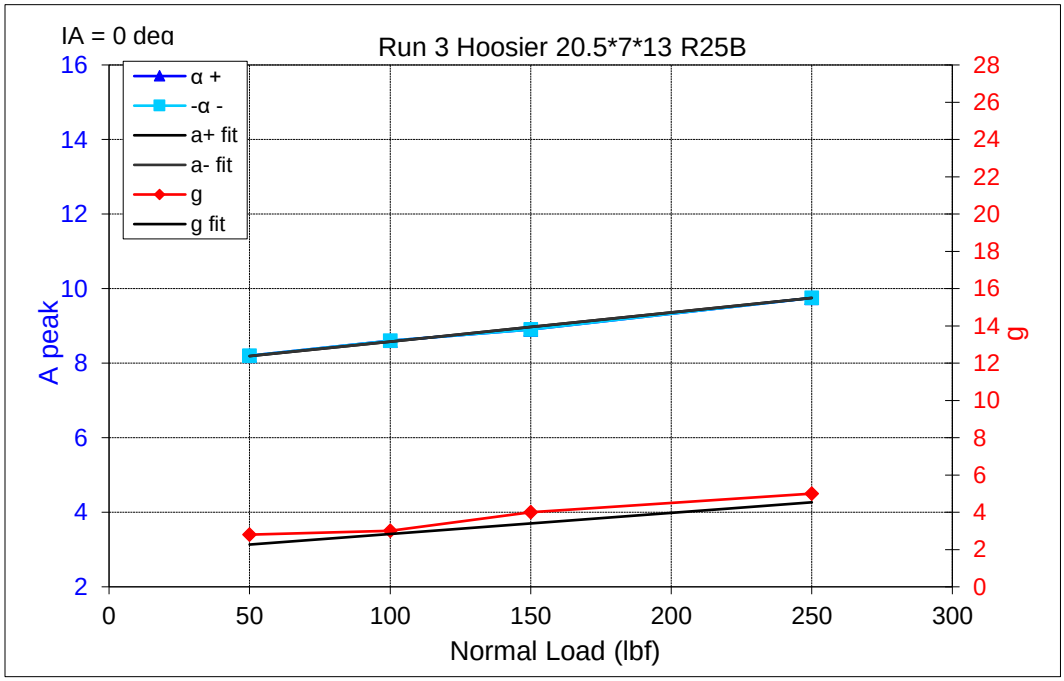

IA = 1 deg
Run 3 Hoosier 20.5*7*13 R25B
α +
-α -
a+ fit
a- fit
g
g fit
A peak
g
Normal Load (lbf)
0
50
100
150
200
250
300
2
4
6
8
10
12
14
16
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28

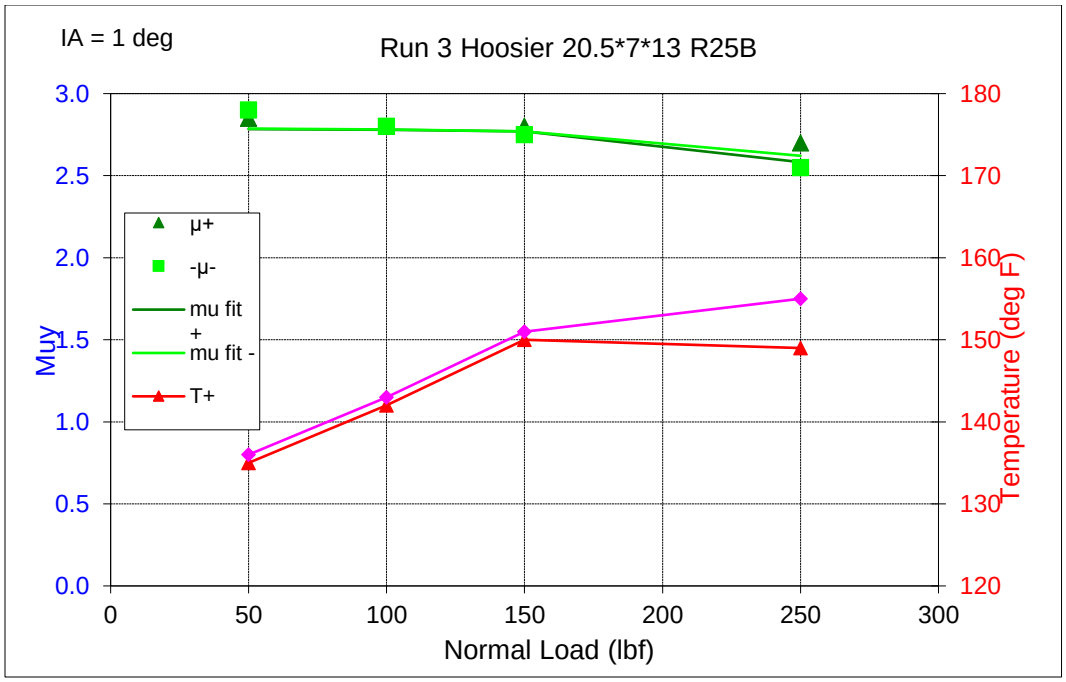

IA = 2 deg
Run 3 Hoosier 20.5*7*13 R25B
α +
-α -
a+ fit
a- fit
g
g fit
A peak
g
Normal Load (lbf)
0
50
100
150
200
250
300
2
4
6
8
10
12
14
16
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28


IA = 3 deg
Run 3 Hoosier 20.5*7*13 R25B
α +
-α -
a+ fit
a- fit
g
g fit
A peak
g
Normal Load (lbf)
0
50
100
150
200
250
300
2
4
6
8
10
12
14
16
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28

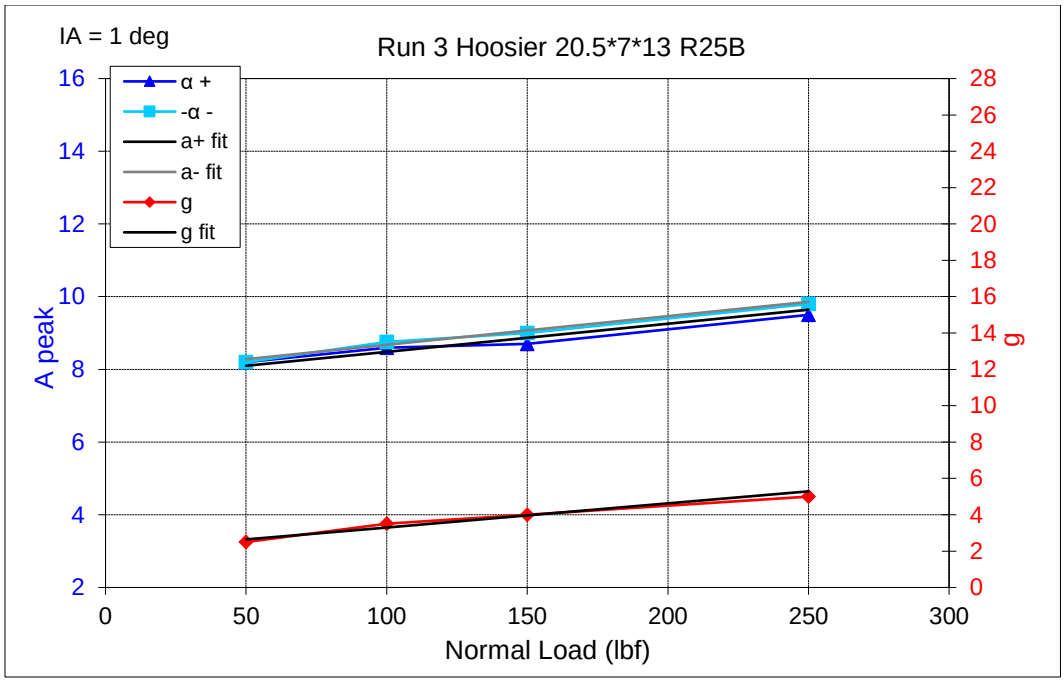

IA = 4 deg
Run 3 Hoosier 20.5*7*13 R25B
α +
-α -
a+ fit
a- fit
g
g fit
A peak
g
Normal Load (lbf)
0
50
100
150
200
250
300
2
4
6
8
10
12
14
16
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28


IA = 0 deg
Run 3 Hoosier 20.5*7*13 R25B
3.0
2.5
2.0
1.5
1.0
0.5
0.0
Muy
180
170
160
150
140
130
120
Temperature (deg F)
0
50
100
150
200
250
300
Normal Load (lbf)
μ+
-μ-
mu fit +
mu fit-
T+

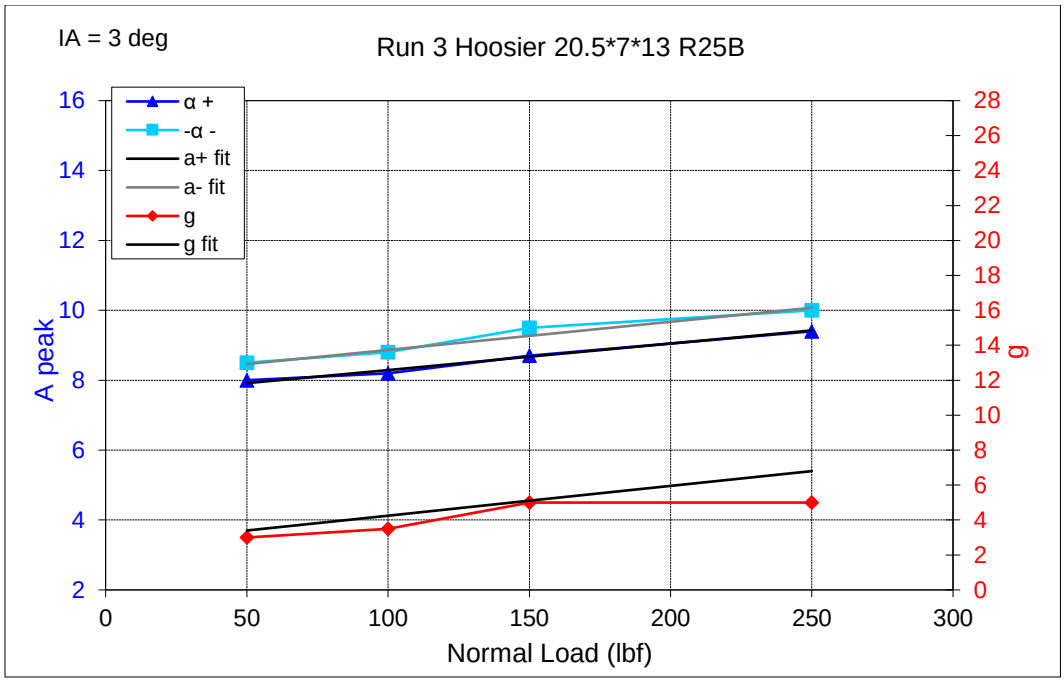

IA = 0 deg
Run 3 Hoosier 20.5*7*13 R25B
16
14
12
10
8
6
4
2
A peak
28
26
24
22
20
18
16
14
12
10
8
6
4
2
0
g
0
50
100
150
200
250
300
Normal Load (lbf)
α +
-α -
a+ fit
a- fit
g
g fit

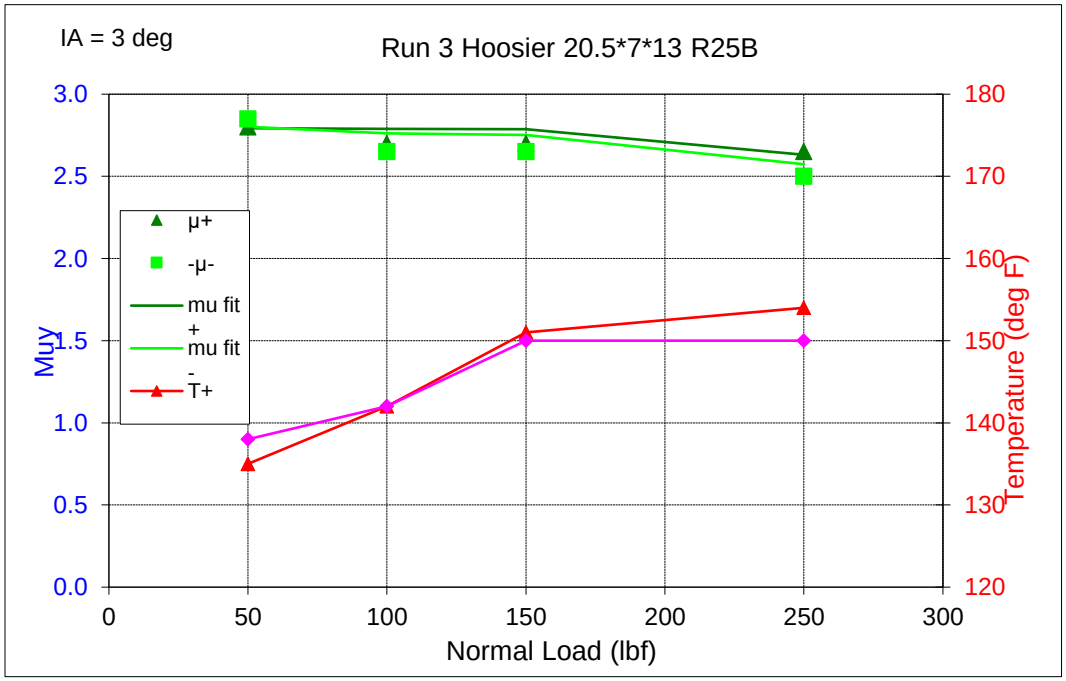

IA = 1 deg
Run 3 Hoosier 20.5*7*13 R25B
3.0
2.5
2.0
1.5
1.0
0.5
0.0
Muy
180
170
160
150
140
130
120
Temperature (deg F)
μ+
-μ-
mu fit +
mu fit -
T+
0
50
100
150
200
250
300
Normal Load (lbf)

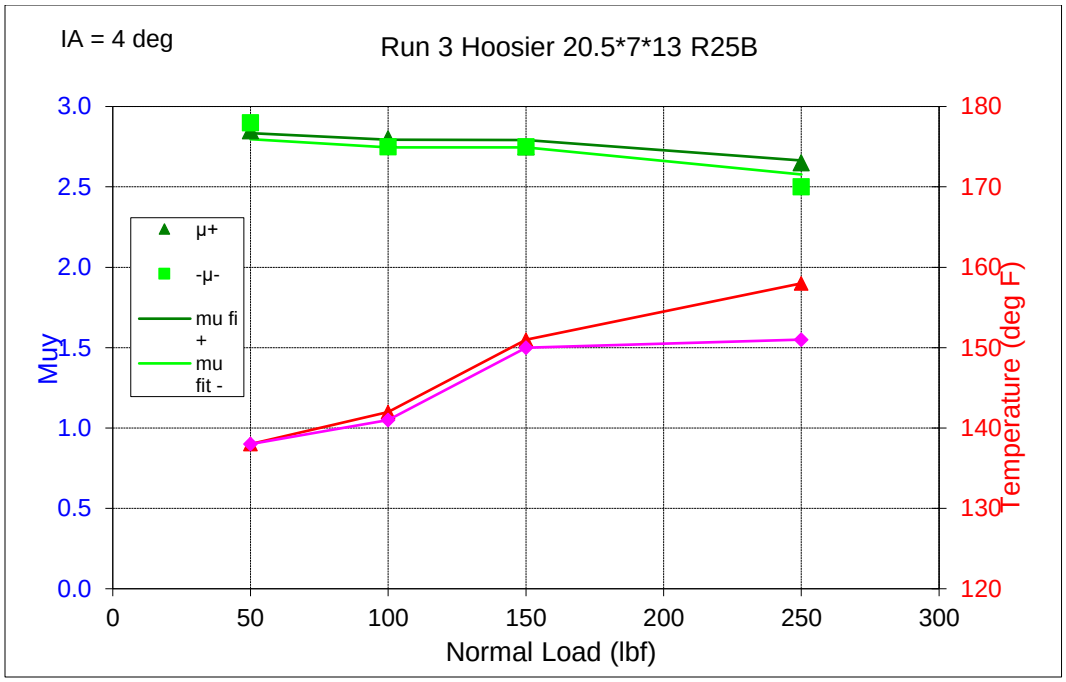

IA = 2 deg
Run 3 Hoosier 20.5*7*13 R25B
3.0
2.5
2.0
1.5
1.0
0.5
0.0
Muy
180
170
160
150
140
130
120
Temperature (deg F)
μ+
-μ-
mu fit +
mu fit -
T+
0
50
100
150
200
250
300
Normal Load (lbf)

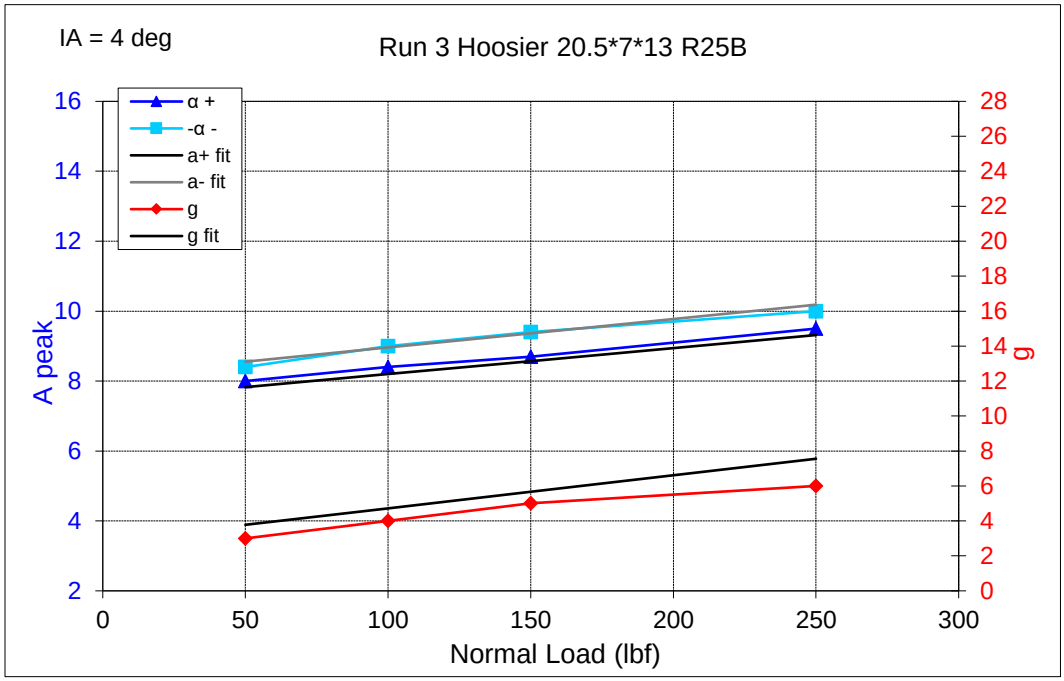

IA = 1 deg
Run 3 Hoosier 20.5*7*13 R25B
α +
-α -
a+ fit
a- fit
g
g fit
A peak
g
Normal Load (lbf)
0
50
100
150
200
250
300
2
4
6
8
10
12
14
16
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28

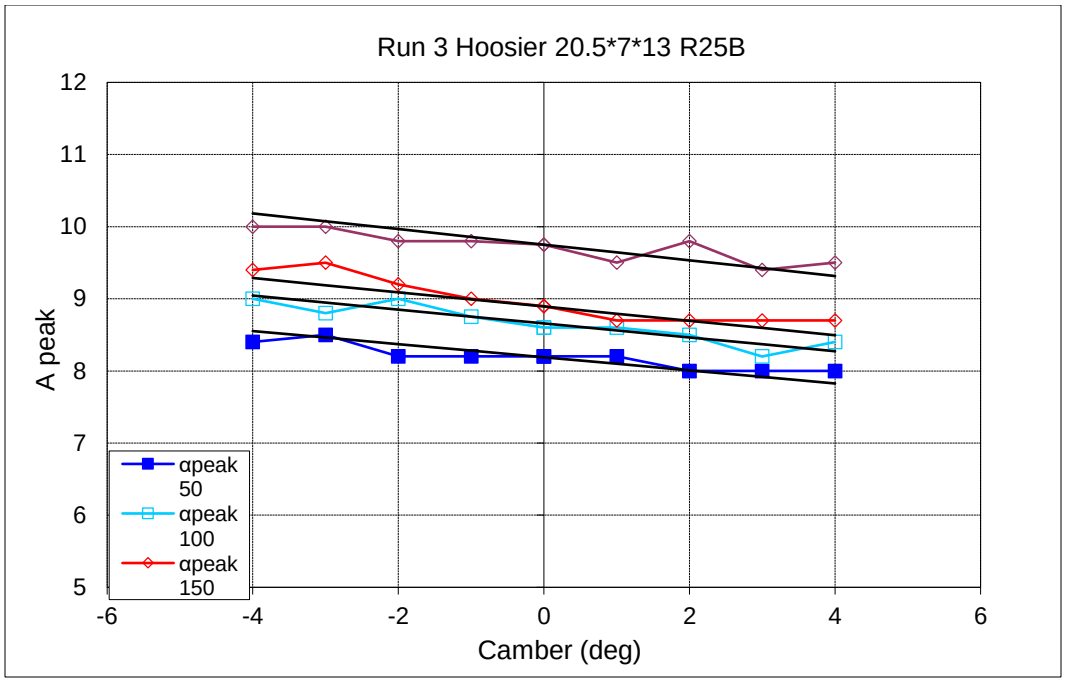

IA = 2 deg
Run 3 Hoosier 20.5*7*13 R25B
α +
-α -
a+ fit
a- fit
g
g fit
A peak
g
Normal Load (lbf)
0
50
100
150
200
250
300
2
4
6
8
10
12
14
16
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28


IA = 3 deg
Run 3 Hoosier 20.5*7*13 R25B
α +
-α -
a+ fit
a- fit
g
g fit
A peak
g
Normal Load (lbf)
0
50
100
150
200
250
300
2
4
6
8
10
12
14
16
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28

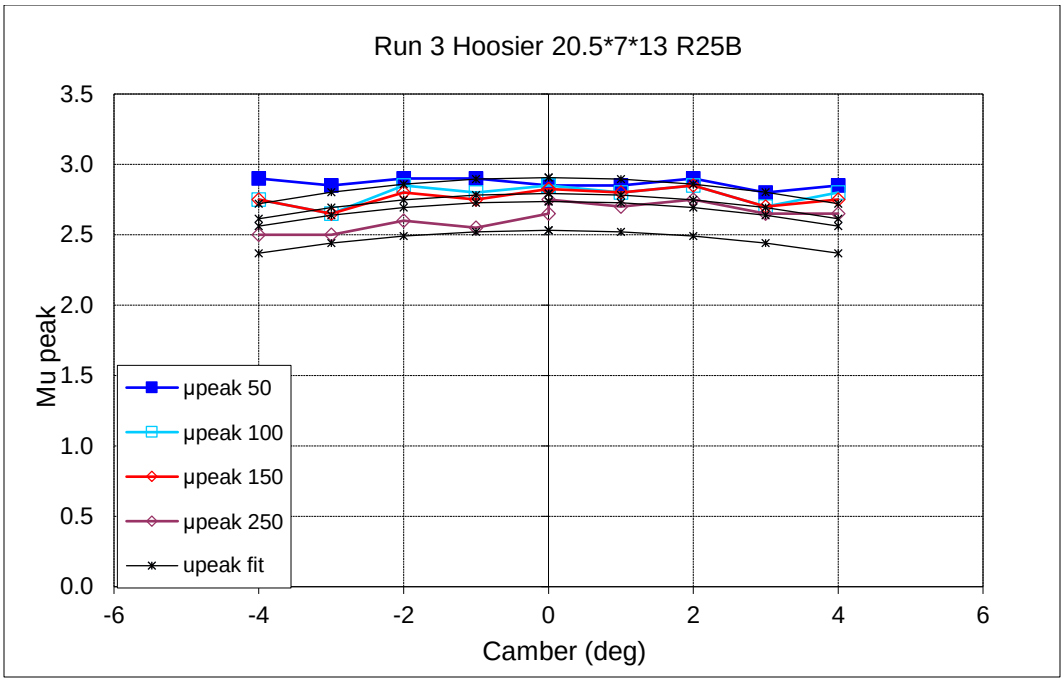

IA = 3 deg
Run 3 Hoosier 20.5*7*13 R25B
μ+
-μ-
mu fit +
mu fit -
T+
Muy
Temperature (deg F)
Normal Load (lbf)
0
50
100
150
200
250
300
0.0
0.5
1.0
1.5
2.0
2.5
3.0
120
130
140
150
160
170
180


IA = 4 deg
Run 3 Hoosier 20.5*7*13 R25B
3.0
2.5
2.0
1.5
1.0
0.5
0.0
Muy
180
170
160
150
140
130
120
Temperature (deg F)
μ+
-μ-
mu fi +
mu fit -
0
50
100
150
200
250
300
Normal Load (lbf)


IA = 4 deg
Run 3 Hoosier 20.5*7*13 R25B
16
14
12
10
8
6
4
2
A peak
28
26
24
22
20
18
16
14
12
10
8
6
4
2
0
g
α +
-α -
a+ fit
a- fit
g
g fit
0
50
100
150
200
250
300
Normal Load (lbf)


Run 3 Hoosier 20.5*7*13 R25B
A peak
Camber (deg)
αpeak 50
αpeak 100
αpeak 150


Run 3 Hoosier 20.5*7*13 R25B
A off (deg)
Normal Load (lbf)
0
1
2
3
4
fit


Run 3 Hoosier 20.5*7*13 R25B
Mu peak
Camber (deg)
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
-6
-4
-2
0
2
4
6
µpeak 50
µpeak 100
µpeak 150
µpeak 250
upeak fit

## References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