INVESTIGATION ON EFFECTIVE MODULUS AND STRESS CONCENTRATION FACTOR OF 3D REINFORCED COMPOSITE BY FINITE ELEMENT ANALYSIS METHOD

by

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April 30, 2018
Abstract

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The University of Texas at Arlington, 2018

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Micromechanics of Composites aims to analyze the localized stresses inside any given constituents of the composite structure. This Thesis is focused on Calculating the effective modulus and the Stress concentration factor of the 3D straight Fiber Composite using Finite Element(FE). The compressive and a Shear response of a 3D Straight fiber composite comprising three directional E Glass Fibers reinforced in an epoxy matrix is analysed. This result is the 10% of overall Composite Ductility and a 10% of overall Shear displacement load in the orthogonal directions. FE calculations are reported to analyse the response of 3d reinforcement as of with the reduced fiber volume fractions in the respective orthogonal directions. The FE calculations demonstrate that the three dimensionality of the microstructure constrains the kinks, and this results in the stable load response. Integrated Stresses of Constituents are formulated by FE to calculate the Effective Modulus and then Stress concentration factor(SCF) to understand the behavior along the orthogonally varied volume fraction models.

SCF Gives us the vital information on how concentrated the stress are in loaded constituents when there is change in volume fraction orthogonally and would help in predicting the failure methods and fatigue life of the material.
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Chapter

INTRODUCTION

Composite materials consist of a combination of materials that are layered together to achieve specific structural properties. The individual materials do not dissolve or merge completely in the composite, but they act together as one. Normally components can be physically identified as they interface with one another. The properties of the composite material are superior to the properties of the individual materials from which it is constructed. The composites provide various advantages like they are dimensionally stable in space during temperature changes. They constitute an outstanding feature of high strength to weight ratio & also possess high corrosion resistance property. This has provided the main motivation for the research and development of composite materials [1].

Many researchers (Hashin, 1983) have extensively addressed the effective properties for linear elastic composites. Based on the Eshelby–Mori–Tanaka theory, Zhao and Weng (1990) derived nine effective elastic constants of an orthotropic composite reinforced with monotonically aligned elliptic cylinders, and the five elastic moduli of a transversely isotropic composite reinforced with two-dimensional randomly-oriented elliptic cylinders. These moduli are given in terms of the cross-sectional aspect ratio and the volume fraction of the elliptic cylinders. In the approach using homogenized theory developed by Lene and Leguillon (1982), Lene (1986) and Jansson (1992), a unit cell problem governing effective mechanical properties for composites with periodic microstructure has been derived by employing an asymptotic expansion of the field variables in two length scales. Numerical computations using a finite element method are performed on a fiber reinforced material. Since the displacement field is interpolated with iso-parametric elements, selective reduced integration must be used to avoid locking (Jansson, 1992) [2].
An advanced composite material is made up of a fibrous material embedded in a resin matrix, generally laminated with fibers oriented in alternating directions to give the material strength and stiffness. Wood is the most common fibrous structural material.

Applications of composites on aircraft include:

- Fairings
- Flight control surfaces
- Landing gear doors
- Leading and trailing edge panels on the wing and stabilizer
- Interior components
- Floor beams and floor boards
- Vertical and horizontal stabilizer primary structure on large aircraft
- Primary wing and fuselage structure on new generation large aircraft
- Turbine engine fan blades
- Propellers.
1.1 Classification of Composites

Organic and inorganic fibers are used to reinforce composite materials. Almost all organic fibers have low density, flexibility, and elasticity. Inorganic fibers are of high modulus, high thermal stability and possess greater rigidity than organic fibers and notwithstanding the diverse advantages of organic fibers which render the composites in which they are used. The different types of fibers are glass fibers, silicon carbide fibers, high silica and quartz fibers, alumina fibers, metal fibers and wires, graphite fibers, boron fibers, Kevlar fibers and multi-phase fibers are used. Among the glass fibers, it is again classified into E-glass, S-glass, A-glass, R-glass etc.
Fibers are the principle constituents in the reinforcement. The various kinds of reinforcements are fiber reinforced composites, laminar composites, and particulate composites.

*Fiber Reinforced Composites* are further divided into continuous and discontinuous fibers. Continuous fibers are the one where the properties of reinforcement are same throughout the length of the fiber. Here the elastic modulus of the composite does not change with the increase/decrease in the length of the fiber. On the other hand, discontinuous fiber or short fiber composites are the one where the properties of reinforcement vary with the fiber length. Fibers are short in diameter and bend easily when pushed axially, although they have very good tensile properties. Therefore, fibers must be supported to avoid them from bending and buckling.

Fibers are usually circular or near to circular in cross-section. In composite, fibers occupy more volume than matrix and these are the major load carrying component. They are used as reinforcement in composites to provide strength and stiffness to the materials. They also provide heat resistance and protect the composite from corrosion. Performance of fiber depends upon its length, shape, orientation, and composition of the fibers. Anisotropic fibers have more strength in longitudinal direction when compared to transversal direction.

*Laminar Composites* are formed by stacking number of layers of materials together. Each layer has its own strength, stiffness, and modulus in metallic, ceramic, or polymeric matrix material. Coupling may occur between the layers of laminates depending upon the sequence of stacking. The most commonly used materials in laminar composites are carbon, glass, boron, and silicon carbide for fiber, and epoxies, polyether ether ketone,
aluminum, and titanium for matrix. Layers having different materials can be bounded, resulting in hybrid laminate.

*Particulate Composites* refers to the material where reinforcement is embedded in the matrix. One best example of particulate composite is concrete, here rock or gravel acts as the reinforcement and is integrated with the cement which is a matrix. In particulate composites, fibers are of various shapes like triangle, square, or round but the dimensions of their sides are found to be nearly equal. These composites can be very small, less than 0.25 microns, chopped glass fibers, carbon Nano-tubes, or hollow spheres. Each material provides different properties and is embedded in the matrix.

There is a greater market and higher degree of commercial movement of organic fibers. The potential of fibers of graphite, silicon carbide and boron are also exercising the scientific mind due to their applications in advanced composites.

In practice, most composites consist of a bulk material ('matrix'), and reinforcement added primarily to increase the strength and stiffness of the matrix. The role of a matrix is to transfer the stresses between the fibers, protect the surface from abrasion, provide a barrier against adverse environments, and to provide lateral support. The reinforcement used usually in the form of fiber, composites can be divided into three main groups:

*Polymer Matrix Composites* (PMC's) - Polymer composites materials are widely used in aerospace, aircraft, marine, sports and military industries. The materials provide unique mechanical and tribological properties combined with a low specific weight and a high resistance to degradation in order to ensure safety and economic efficiency. Also
known as FRP - Fiber Reinforced Polymers (or Plastics) - these materials use a polymer-based resin as the matrix, and a variety of fibers such as glass, carbon and Kevlar/aramid as the reinforcement.

*Metal Matrix Composites* (MMC's) - Increasingly found in the automotive industry, these materials use a metal such as aluminum as the matrix and reinforce it with fibers such as silicon carbide.

*Ceramic Matrix Composites* (CMC's) - Used in very high temperature environments, these materials use a ceramic as the matrix and reinforced with short fibers, or whiskers such as those made from silicon carbide and boron nitride.

1.2 Need of 3D Composites

Tailoring the alignment of fibers in a composite material or structure is crucial for maximizing properties like strength, stiffness, fracture toughness, damage resilience and in order to achieve good multiaxial performance in three dimensions. Three dimensional (3D) composites are characterized by yarns oriented not only in-plane but also out of plane resulting in higher through thickness strength and stiffness of the composite material. Freedom to manipulate cross-sectional size, shape and number of yarns means ‘3D Architecturing’ makes the design space much bigger. Examples of possible application of 3D composite in this regard are stiffeners and stringers of an aircraft where loads can be in-plane as well as out of plane. Limited growth of delamination in through thickness direction results in higher penetration resistance of 3D composites. Also, it has been shown that velocity corresponding to full penetration is higher for 3D orthogonal woven composite compared to 2D plane weave composite. Most composite beams used today in aerospace
industry are made by cutting, stacking and draping of 2D prepreg laminas. Key problems with this approach are difficulty to produce the desired shape and lower through thickness mechanical properties. In contrast, flexibility offered by 3D architecture in terms of control over various parameters like size, shape and types of yarns make it more like a structure rather than homogeneous material. Mechanical properties can be varied in different parts of the 3D profile opening up new design opportunities [3].

1.3 Objective

1.3.1 Motivation

The use of composites has started growing because of several stiffness and strength parameters. Composites can be significantly changed by the selection of fiber architecture, and composite materials can be designed individually for each particular application, thus expanding the scope of using 3D Composites. Designing and predicting the Mechanical Properties which influence the role of composite might be crucial in determining its fatigue life and predicting the hopeful outcomes of failure. This requires and intense study and understanding of 3D composites.

1.3.2 Goal and Objective

The primary objective of this work is to find the stress analysis of 3 Dimensional composites by finite element method. As compared to the other FE method modeling of unidirectional and bidirectional composites by reducing the fiber volume fraction orthogonally, here, the load on the cell is not prescribed but is calculated by taking into account the influence of the closest neighbors of the cell. This analysis is computed in ANSYS 17.0, and the results are compared with the analytical calculations from Mathcad.
Since the strength along the thickness of the composites is not high, precise prediction of it is important. It can be done by the detailed study of stress distribution on individual components of composite material. Independently of boundary conditions at infinity (applied stresses), the distribution of stresses in the main array is double periodic one.

Finite Element Modeling of a composite material is considered as of to compare the Effective modulus of 3dimensional cell model as of with the reduced fiber volume fraction cells in respective directions and also to determine the major difference in behavior of the respective components of composites materials.

Objective of this thesis can be summarized into

- To Estimate effectiveness of reinforcement in 3rd direction.
- To compare with approximate theoretical models.
- To estimate stress concentration.
Carbon fibre reinforced polymer (CFRP) composites are widely utilized in aerospace and automotive structures due to their high strength and stiffness to weight ratios. These long fibre composites are designed to possess high axial stiffness and tensile strength but the compressive strength of unidirectional composites rarely exceeds 60% of their tensile strength. The main competing mechanisms governing the compressive strength of long fibre composites are: (i) elastic micro-buckling (an elastic instability involving matrix shear); (ii) plastic microbuckling in which the matrix deforms plastically; (iii) fibre crushing (a compressive fibre failure mode); (iv) splitting by matrix cracking parallel to the main fibre direction; (v) buckle delamination and (vi) shear band formation at 45o to the main axis of loading due to matrix yielding [3].

According to Spatially Reinforced Composites Book by I. G. Zhigun, Ù. M. Tarnopol'skii, V. A. Polyakov [10]. In material with a regular picture, it is easy to isolate the repeating elements in the form of a plane laminae. Disregarding the inhomogeneity of structure in each lamina and finding out the significant characteristics as those of a quasi-homogeneous material, the deformation model of material can be represented in the form of an inhomogeneous block composed of different laminae. The elastic characteristics of each lamina are determined by the properties of the components and their volume fraction; construction of the calculation model of material is completed by imposition of laminae on each other. For this reason, it is necessary to write out the stiffness components for each lamina in the system of coordinates 1,2,3, turned relative to the initial, in the general case, non-orthogonal vectors \( a_i = 1, 2, 3 \), and employ, with the second assumption taken into
account, the general formulas that correspond to the joint deformation of a package of laminae. In modelling of a laminated medium, the macro stresses relate to an individual lamina, having its own deformation characteristics. The integral averaging of these stresses throughout the volume of material comprising all the laminae leads to mean stresses.

![Figure 2-1: Composite Structure from Spatially reinforced composite book.](image)

The procedure of calculation of the elastic constants is reduced to the following stages. Let the reinforcement be represented by a system of three straight threads, whose direction coincides with three non-coplanar vectors $a_1$, $a_2$, $a_3$ (Figure 2-2). Let us isolate the plane of a lamina, which passes through vectors $a_1$, $a_2$. Two laminae are modelled in the material parallel to this plane, containing conditionally only the fibers of $a_3$ directions and, respectively, $a_2$, $a_3$ [5].
The modelling of the laminae for a three-dimensional fibrous material is based on two assumptions, corresponding to constant density of fiber packing. With bulk reinforcement coefficient $\mu_1$, $\mu_2$, $\mu_3$ respectively for $a_1$, $a_2$, $a_3$ directions, two laminae, being parallel to the plane, and passing through $a_1$, $a_2$ vectors, have

1. Identical reinforcement coefficient equal to $\mu_3$ (in $a_3$ direction) and $\mu_1 + \mu_2$ (in $a_3$, direction for the first lamina and $a_2$ for the second lamina)
2. Different relative thicknesses, equal to $\mu_1/ (\mu_2 + \mu_3)$ and $\mu_2/ (\mu_1 + \mu_3)$ respectively.

The elastic characteristics of each lamina are determined by the properties of the components and their volume fraction; construction of the calculation model of material is completed by imposition of laminae on each other. For this reason, it is necessary to write out the stiffness components for each lamina in the system of coordinates 1,2,3, turned relative to the initial, in the general case, non-orthogonal! vectors $a_i$, $i = 1, 2, 3$, and employ, with the second assumption taken into account, the general formulas that correspond to

Figure 2-2: 3-Dimensional Thread alignments
the joint deformation of a package of laminae. In modelling of a laminated medium, the macro stresses relate to an individual lamina, having its own deformation characteristics. The integral averaging of these stresses throughout the volume of material comprising all the laminae leads to mean stresses [5].

For models of material, which will be examined further, the orthogonality of $a_3$ vector to $a_1$ and $a_2$ vectors ((Figure 2-3) is characteristic. This special feature simplifies the calculation of the characteristics of material and efficient constants of material through imposition of laminae.

According to Das [3], The tailoring of fiber/tow architectures in CFRPs has also been widely used to improve properties including the compressive response. The most common approaches include modifying 2D composites by adding out-of-plane reinforcements. This is typically achieved by Z-pinning, stitching and knitting. More recently, a range of techniques has been developed to manufacture three-dimensional (3D) fabrics wherein tows are present in at-least three orthogonal directions; see [15] for a detailed review of these techniques. In brief, 3D fabrics fall into three categories: (i) 2D woven 3D fabrics produced by usual 2D weaving methods with monodirectional shedding; (ii) 3D woven fabrics produced by a dual-direction shedding system and (iii) non-woven 3D fabrics without interlacing or interweaving produced by a technique known as “noobing” that is described in Section 4.2. The ability to manipulate the volume fractions of fiber in three directions not only allows tailoring of the multi-axial properties of composites; it also reduces the susceptibility to delamination, which results in an improvement in the impact performance of CFRPs. Moreover, 3D composites can add more functionality to any eventual component as discussed by Stig.
(Whitney and Riley, 1965-66; Hermans, 1967) did the study on effective elastic moduli of fiber-reinforced composite. The model they used was concentric cylinder matrix with cylindrical fiber embedded in it, by considering Reuss and Voigt'type estimation for elastic moduli to find out the results. This work was followed by Hashin and Rosen. They used the model that consisted of fiber embedded in an unbounded solid possessing the effective moduli of the composite. They were able to formulate the result, but they never explicitly solved the problem.
One of the basic requirements to accurately run a model in ANSYS is the materials and its properties. It is necessary to have all the required material properties to find the stresses and deformation in a composite material. The properties required to run a model in ANSYS are:

- Young 's modulus
- Poison 's ratio
- Density
- Shear modulus
- Coefficient of thermal expansion

Since, composite is made up of two constituents, i.e., matrix and fiber, each of the constituent can be described with different materials and their respective properties [4].

3.1 MATRIX

A matrix in a composite is a surrounding medium in which fiber is cast or shaped. Its function is to transfer stresses between the fibers and provide a barrier against adverse environments. It also protects the surface from abrasion. The matrix materials used in this work are:

3.1.1 Epoxy

Epoxies are polymerizable thermosetting resins and are available in a variety of viscosities from liquid to solid. Epoxies are used widely in resins for prepreg materials and structural adhesives. The processing or curing of epoxies is slower than polyester resins. Processing techniques include autoclave molding, filament winding, press molding, vacuum bag molding, resin transfer molding, and pultrusion [4]. Curing temperatures vary
from room temperature to approximately 350 °F (180 °C) [16]. The most common cure temperatures range between 250° and 350°F (120–180 °C). According to L. S. Penn and T. T. Chiao., when epoxy resin reacts with a curing agent, it does not release any volatiles or water. Therefore, epoxy does not easily shrink as compared to polyesters or phenolic resins. Moreover, epoxy resins which are cured provide excellent electrical insulation and are resistant to chemicals (1982).

The advantages of epoxies are high strength and modulus, low levels of volatiles, excellent adhesion, low shrinkage, good chemical resistance, and ease of processing. Their major disadvantages are brittleness and the reduction of properties in the presence of moisture.
<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Density</strong></td>
<td>1.17 gm/cm$^3$</td>
</tr>
<tr>
<td><strong>Tensile Strength</strong></td>
<td>80 MPa</td>
</tr>
<tr>
<td><strong>Youngs Modulus</strong></td>
<td>3.4 GPa</td>
</tr>
<tr>
<td><strong>Poisson’s Ratio</strong></td>
<td>0.36</td>
</tr>
</tbody>
</table>

Table 1: Material Properties of Epoxy Matrix

3.2 FIBER

3.2.1 E Glass

Fibers are used to strengthen the composites; they are the principal constituents in fiber reinforced composite materials. Fibers carry majority of the load in composites. They occupy the largest volume fraction of the composite. As the amount of fiber increases the specific strength of the composite increases, since fibers have low weight density. Based upon the material characterization, various kinds of fibers are used.

![Figure 3-2: Fiber - E Glass](image)
<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density</td>
<td>2.54 gm/cm³</td>
</tr>
<tr>
<td>Tensile Strength</td>
<td>3.45 GPa</td>
</tr>
<tr>
<td>Youngs Modulus</td>
<td>72.4 GPa</td>
</tr>
<tr>
<td>Poisson’s Ratio</td>
<td>0.21</td>
</tr>
</tbody>
</table>

Table 2: Material Properties of E Glass Fiber
4. Chapter  
MODELLING AND SIMULATION  

4.1 Modeling of 3D reinforced composites  

This chapter deals with the FEA modeling and simulation techniques used for unidirectional reinforced composites. The procedure used for the analysis is discussed in the following steps:  

1. Modeling: Create the geometric model according to the required dimensions in Catia V5.  
2. Preprocessing: Import the model from Catia V5 to ANSYS v14.5, define material properties.  
3. Meshing: Creating a refined mesh to provide a better approximation of the solution.  
4. Solution process: Applying boundary conditions such as loads and supports to the body, selecting output control, and obtaining the results.  
5. Post processing: Review the obtained results and taking the values, plot the graphs.  

Few assumptions were made during processing of finite element analysis.  

- The fibers are cylinders with circular in cross-section.  
- The displacements are continuous across the fiber and matrix interface.  
- The temperature is uniform, and the material properties do not vary with temperature.  

Normal to the surface and shear stresses are continuous.
4.2 GEOMETRY

The Standard Model Discussed in Book Spatially Reinforced Composites Book by by Ivan Grigor'evich Zhigun, IU. M. Tarnopol'skii, V. A. and also Considering the work Described by Satyajit Das [3], The Specimen Model was considered is 3D reinforced composite comprising E-Glass Fiber (non-twisted E-Glass Fibers) in a epoxy matrix with a glass transition temperature of 180$^\circ$ C. The fibers are approximately $d = 2$ mm in diameter and the 3D composite was anisotropic with 7.8 % of the total number of tows in $Z$-direction and 46.07% each in the $X$ and $Y$-directions. Blocks of the 3D reinforced composite of size 9 ($X$)mm $\times$ 9 ($Y$)mm $\times$9.2 ($Z$)mm were Designed in CAD Software (CATIA V5 R20).

![Figure 4-1: Microstructure unit cell Selection from the standard 3D composite structure](image)
4.3 VOLUME FRACTION

The composite comprises four principal phases: (i) the $X$, $Y$ and $Z$-direction Fibers and (ii) matrix pockets. Based on the unit cell with dimensions sketched in Figure 4-1, the $X$ and $Y$-direction fibers comprise a volume fraction $v_X = v_Y \approx 22.75\%$ each of the composite while the $Z$-direction fiber occupy a volume fraction $v_Z \approx 3.9\%$ of the composite. The remainder $v_M = 50.61\%$ of the volume is occupied by the matrix pockets. It now remains to specify the overall fiber volume fraction within the composite.

![Figure 4-2: Designed 3D Composite Structure](image)

To Compare the 3D Reinforced Model with the reduced Volume Fractions orthogonally, we have considered the other models which have reduced volume fractions in respective orthogonal directions as shown in the Figures Below.
Figure 4-3: Reduced Z Fibers (Left) and Reduced Y Fibers (Right).

The Reduced Z Fiber Model (Figure 4-3) consists of the $X$ and $Y$-direction fibers comprise a volume fraction $v_X = v_Y = 22.75\%$ each of the composite while the $Z$-direction Fiber occupy a volume fraction $v_Z = 0\%$ of the composite. The remainder $v_M = 54.5\%$ of the volume is occupied by the matrix pockets. Hence increase in Matrix Volume as of with 3D Reinforced Model.

The Reduced Y Fiber Model (Figure 4-3) consists of the $X$ and $Y$-direction fibers comprise a volume fraction $v_X = 22.75\%$ and $v_Y = 0\%$ respectively of the composite while the $Z$-direction Fiber occupy a volume fraction $v_Z = 3.8\%$ of the composite. The remainder $v_M = 73.3\%$ of the volume is occupied by the matrix pockets. Hence increase in Matrix Volume as of with 3D Reinforced Model.
The Reduced Y and Z Fibers Model (Figure 4-6) consists of the $X$ and $Y$-direction fibers comprise a volume fraction $v_X = 22.75\%$ and $v_Y = 0\%$ respectively of the composite while the $Z$-direction Fiber occupy a volume fraction $v_Z = 0\%$ of the composite. The remainder $v_M = 77.25\%$ of the volume is occupied by the matrix pockets. Hence increase in Matrix Volume as of with 3D Reinforced Model.

The Reduced Z Fiber Model (Figure 4-6) consists of the $X$ and $Y$-direction fibers comprise a volume fraction $v_X = v_Y = 0\%$ each of the composite while the $Z$-direction Fiber occupy a volume fraction $v_Z = 3.8\%$ of the composite. The remainder $v_M = 96.2\%$ of the volume is occupied by the matrix pockets. Hence increase in Matrix Volume as of with 3D Reinforced Model.
4.4  FINITE ELEMENT/ MESH GENERATION

The goal of meshing in ANSYS Workbench is to provide robust, easy to use meshing tools that will simplify the mesh generation process. These tools have the benefit of being highly automated along with having a moderate to high degree of user control.

The required Model is meshed to Refinement till the values converge. The convergence of the Refinement is judged by noting the results at different locations and parameters in order to make it consistent and to overcome the singularities which might affect the study of values which may be crucial in analyzing composite.

Mesh Convergence Report is generated by applying some test loads on the models. The Table shows the stress report for convergence.

![Mesh Convergence Plot](image)

**Figure 4-5 : Mesh Convergence plot**
The model was fully meshed in ANSYS with a body sizing of $15\mu m$ and face meshing of size $10\mu m$ was done at the interface between matrix and fiber. Since the stresses calculated are crucial in differentiating the difference between Reduced models, the meshing has to be fine to obtain more accurate results.

No of Nodes: 5014494
Min Edge Length: $2.2927e-003$ mm
Max Edge Length: $2.9027e-003$ mm
Error Ratio: 2.67 %

Figure 4-6: Locations considered for mesh convergence test
4.5 ELASTIC PROPERTIES

As discussed in The Stress Analysis Method for Three-Dimensional Composite Materials by Kanehiro Nagai, Atsushi Yokoyama, Zen'ichiro Maekawa and Hiroyuki Hamada. We will treat a unit cell volume as 3-D anisotropic space shown in Figure 4-10, in which material properties would continuously vary in three orthogonal directions and derive the fundamental formulas for equivalent mechanical properties to this space.
These formulas are deduced for Young's modulus, shear modulus, and Poisson's ratio. The size of unit cell volume is represented by \( L_x, L_y, \) and \( L_z \) as shown in Figure 4-10, and the material properties of an infinitesimal representative volume with edges \( dx, dy, \) and \( dz \) at an arbitrary point \( (x, y, z) \) in this space are defined. Where the following Maxwell's Reciprocal Theorem between Young's moduli and Poisson's ratio is assumed.

\[
\frac{E_i(x, y, z)}{E_j(x, y, z)} = \frac{\nu_{ij}(x, y, z)}{\nu_{ji}(x, y, z)} \quad (i, j = x, y, z).
\]

In the case of 3-D composite material, each point in this unit cell volume would corresponds to fiber, resin, void, crack and so on.

4.5.1 Youngs Modulus

To understand the present analytical methodology for Young's modulus, firstly we will derive the formulas about 2-D area as shown in Figure 4-9. In this region, Young's moduli in \( \zeta \) and \( \eta \) direction are defined as \( E_\zeta(\zeta) \) and \( E_\eta(\zeta) \) respectively, which are assumed here to be functions of \( \zeta \). For the \( \zeta \)-direction, the relation between the total strain \( \epsilon_\zeta \) to this
region and the local strain $\varepsilon(\zeta)$ to an infinitesimal rectangular area with sides $d\zeta$ and $L_\eta$ at distance $\zeta$ is formed as follows:

$$
\varepsilon_\zeta = \int_0^{L_\zeta} \bar{\varepsilon}_\zeta(\zeta)d\zeta / L_\zeta.
$$

As the $\zeta$-directional stress $\sigma_\zeta$ is independent of $\zeta$, the strain $E_\zeta(\zeta)$ is

$$
\bar{\varepsilon}_\zeta(\zeta) = \sigma_\zeta / \bar{E}_\zeta(\zeta).
$$

Accordingly the $\zeta$-directional total Young's Modulus $E_\zeta$ (can be obtained from Equations (5) and (6) as follows:

For the $\eta$-direction, the relation between the total strain $\varepsilon_\eta$ to this region and the local stress $\bar{\sigma}_\eta(\zeta)$ to an infinitesimal area with sides $d\zeta$ and $L_\eta$ at an arbitrary if, would be formed as follows:

$$
\bar{\sigma}_\eta(\zeta) = \bar{E}_\eta(\zeta) \cdot \varepsilon_\eta.
$$

The $\eta$-directional total stress $\sigma_\eta$ is obtained as the following integrated form:

$$
\sigma_\eta = \int_0^{L_\zeta} \bar{\sigma}_\eta(\zeta)d\zeta / L_\zeta.
$$
\[ E_\eta = \sigma_\eta / \varepsilon_\eta \]
\[ = \int_0^{L_\zeta} E_\eta(\zeta) d\zeta / L_\zeta. \]  

(5, from 3 and 4)

Figure 4-10: Explanatory illustration of analytical approach for Young's modulus (3-D).

Hence the Resulting equation is.

\[ \bar{E}_\alpha(x) = \int_0^{L_x} \int_0^{L_y} \hat{E}_\alpha(x, y, z) dy dz / (L_y \cdot L_z). \]  

(6)
To understand the present analytical methodology for the shear modulus, a formula on 2-D area as shown in Figure 4-11 will be derived. In this region, the shear modulus \( G_{\zeta \eta}(\zeta) \) is assumed to be a function of \( \zeta \). The relation between the total shear strain \( \Upsilon \), and the local shear strain \( \Upsilon(\zeta) \) for an infinitesimal rectangular area with sides \( d\zeta \) and \( L_\eta \) at \( \zeta \), is given by

\[
\gamma = \int_0^{L_\zeta} \bar{\gamma}(\zeta) d\zeta / L_\zeta. 
\]  
(7)

As the shear stress \( \tau_{\zeta \eta} \) is independent of \( \zeta \), the following relation is formed.

\[
\tau_{\zeta \eta} = \bar{\gamma}(\zeta) G_{\zeta \eta}(\zeta). 
\]  
(8)

Thus, using Equations (7) and (8), the total shear modulus \( G_{\zeta \eta} \) could be expressed as follows:

\[
G_{\zeta \eta} = \frac{\tau_{\zeta \eta}}{\gamma} = \frac{L_\zeta}{\int_0^{L_\zeta} (1 / G_{\zeta \eta}(\zeta)) d\zeta}. 
\]  
(9)
Figure 4-12: Explanatory illustration of analytical approach for Shear modulus (3-D).

Hence Shear Modulus $G_{\eta\eta}$ is

$$G_{xy} = \int_0^{L_z} \overline{G}_{xy}(z) \, dz / L_z,$$
4.6 LOAD SETUP

Knowing that the composite needs to be analyzed with both compressive and shear loads. It requires a proper load set up. The following Figures shows the Load set for the desired Composite unit cell.

A displacement of total 10% of the edge length is applied on the either sides of the composite. Each side is applied a 0.45mm of displacement to form a compressive load. Then the results are evaluated accordingly to note the changes and post process the output.

Figure 4-13 : Compressive Load setup 2D Representation
A displacement of total 10% of the edge length is applied on one side(A) of the composite making other end fixed(B). Load acting side is applied a 0.45mm of
displacement to form a shear load in the plane. Then the results are evaluated accordingly
to note the changes and post process the output.

Figure 4-16: Shear Load Setup 3d isometric representation
For finding the force exerted on individual components which can be further used to find the Effective Modulus, we need a Volume integral Stress value. We can obtain Integrated Avg Stress value directly from Ansys 14.5 User defined Values feature.

The 3-D integration of Rectangular blocks

\[
\int_{-1}^{1} \int_{-1}^{1} \int_{-1}^{1} f(x, y, z) \, dx \, dy \, dz = \sum_{k=1}^{n} \sum_{j=1}^{m} \sum_{i=1}^{l} H_k H_j H_i f(x_i, y_j, z_k)
\]

Where, \( f(x, y, z) = \sigma(x, y, z) \)

The numerical integration of general axisymmetric elements gives:

\[
\int_{0}^{\pi} \int_{-1}^{1} f(r, z, \theta) \, dr \, dz \, 2\pi r \, d\theta = \sum_{k=1}^{n} \sum_{j=1}^{m} \sum_{k=1}^{l} H_k H_j H_i f(r_j, z_j, \theta_k)
\]

Where, \( f(r, z, \theta) = \sigma(r, z, \theta) \)
5.1 Tensile Compressive Loads

5.1.1 X Compressive Loads

From following figures, Compressive stress(-ve) integral at points concentrate
Maximum at outer bound and minimum occurs at interaction point with other fiber. This
stress is computed by integrating the stress at volumes on X Fibers, and then finding out
the required average value of Stress that can be computed to find total force on the
elements.

To find the Sum of Forces on System

\[
\int_V \sigma(f) \, dV(x) + \int_V \sigma(m) \, dV(y + z + m) = \sum F
\]

Where, \( \int_V \sigma(f) \, dV(x) \) is normal stress(x) integral along X fibers Volume
And \( \int_V \sigma(m) \, dV(y + z + m) \) is normal stress(x) integral along Y Fibers, Z Fibers and Matrix
Volumes

Here, the contribution of Y and Z fibers Stresses are quite less, almost equal to Matrix
stress. Hence, Neglecting the individual contribution of Y and Z fibers and taking whole Y,
Z fibers and Matrix together with Matrix Stresses.
To find the Total Stress Induced on System

\[ \sum \frac{F}{A} = \langle \sigma \rangle, \text{ Where } \sum A \text{ is the total Cross-sectional area of the Unit Cell.} \]

Therefore, Required Effective Modulus can be found by

\[ \langle E \rangle = \frac{\langle \sigma \rangle}{\langle \varepsilon \rangle} \]

Where, Strain \( \langle \varepsilon \rangle = \frac{d}{L} \) d- Displacement, L- Edge Length.

And the computed Modulus can be compared to find the accuracy by \( \langle E_{\text{Roughly}} \rangle \)

\[ \langle E_{\text{Roughly}} \rangle := E_f \cdot C_{fx} \]

Where \( E_f \) is the Youngs modulus of Fiber and \( C_{fx} \) is the Volume fraction of Fibers in X Direction.

Figure 5-3: Normal Stress(X) Integral in X Fibers and Matrix (Vf = 49.38%, 3D Composite)
Figure 5-4: Normal Stress(X) Integral on X Fibers and Matrix (Vf=45.5% Reduced Z Fibers)

Figure 5-5: Normal Stress(X) Integral on X Fibers and Matrix (Vf=26.63% Reduced Y Fibers)

Figure 5-6: Normal Stress(X) Integral on X Fibers and Matrix (Vf=22.75% Reduced Y and Z Fibers)
<table>
<thead>
<tr>
<th>Fiber (Stress)</th>
<th>Matrix (Stress)</th>
<th>Strain€</th>
<th>Sigma $F$</th>
<th>$&lt;\text{Stress}&gt;$</th>
<th>$&lt;\text{E}&gt;$</th>
<th>$&lt;\text{E}_{\text{Roughly}}&gt;\text{ Difference}$</th>
<th>Diff (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_f=49.38%$ All Fibers</td>
<td>-6929.1</td>
<td>-285.22</td>
<td>0.1</td>
<td>-148786.92</td>
<td>-1796.94</td>
<td>17969.43</td>
<td>16473.6</td>
</tr>
<tr>
<td>$V_f=45.5%$ Reduced $Z$ Fibers</td>
<td>-7224.4</td>
<td>-395.54</td>
<td>0.1</td>
<td>-161406.43</td>
<td>-1949.35</td>
<td>19493.53</td>
<td>16473.6</td>
</tr>
<tr>
<td>$V_f=26.63%$ Reduced $Y$ Fibers</td>
<td>-6899.1</td>
<td>-278.54</td>
<td>0.1</td>
<td>-147794.46</td>
<td>-1784.9</td>
<td>17849.57</td>
<td>16473.6</td>
</tr>
<tr>
<td>$V_f=22.75%$ Reduced $Y$ and $Z$ Fibers</td>
<td>-7227.1</td>
<td>-346.49</td>
<td>0.1</td>
<td>-158320.06</td>
<td>-1912.08</td>
<td>19120.78</td>
<td>16473.6</td>
</tr>
</tbody>
</table>

*All Stress and Modulus values are in MPa*

Table 3: Effective Modulus results for X Compressive Load setup
From the above table we can clearly state that the computed results of Effective Modulus of the System are quite close to the $<E_{\text{Roughly}}>$ Calculated from the Classic modulus Equation for Uni directional Composite. The highest deviation is 12.49 % at Reduced Z Fibers model.

Hence the Stress Concentration Factor for the required model can be found by

$$K_{lx} = \frac{\sigma_{\text{Max}}}{\sigma_{\text{Avg}}}$$

Where, $\sigma_{\text{Max}}$ is the Max Stress Induced on the component when X Compressive load is applied.

and $\sigma_{\text{Avg}}$ is the Avg Stress / Nominal Stress Computed when X Compressive load is applied.

Figure 5-7: Used Defined Plots for Stress Max on X Fibers and Matrix (Vf=49.38%)
Figure 5-8: User Defined Plots for Stress Max on X Fibers and Matrix (Vf=45.5% Reduced Z Fibers)

Figure 5-9: User Defined Plot for Max Stress on X Fibers and Matrix (Vf=26.63% Reduced Y Fibers)

Figure 5-10: User Defined Plot for Max Stress on X Fibers and Matrix (Vf=22.75% Reduced Y and Z Fibers)
### Table 4: Computed Stress Concentration Factors of X Fibers and Matrix when X Compressive Load is applied

<table>
<thead>
<tr>
<th>Vf (%)</th>
<th>Fibers (X Fibers)</th>
<th>Matrix</th>
<th>$K_{fx}$</th>
<th>$K_{mx}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>49.38% All Fibers</td>
<td>$\sigma_{\text{Max}(\text{Fibers})}$, $\sigma_{\text{Avg}(\text{Fibers})}$, $\sigma_{\text{Max}(\text{Matrix})}$, $\sigma_{\text{Avg}(\text{Matrix})}$, $K_{fx}$, $K_{mx}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vf = 45.5% Reduced Z Fibers</td>
<td>8954.8, 7224.4, 2989.9, 395.54, 1.24, 7.56</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vf = 26.63% Reduced Y Fibers</td>
<td>7506.2, 6899.1, 1015.2, 278.54, 1.09, 3.64</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vf = 22.75% Reduced Y and Z Fibers</td>
<td>7239.6, 7227.1, 365.59, 346.49, 1.00, 1.06</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* All Stress values are in MPa
5.1.2 Y Compressive Load

Similarly, Y Compressive Computations on Unit cell are performed as X Compressive Computations, and then the results are Post processed to obtain effective modulus(<E>) and stress concentration factor(K).

Figure 5-11: Y Compressive Load Setup (0.45 mm of displacement on each face)

Figure 5-12: Normal Stress(Y) Integral in Y Fibers and Matrix (Vf = 49.38%, 3D Composite)
Figure 5-13: Normal Stress(Y) Integral on Y Fibers and Matrix (Vf=45.5% Reduced Z Fibers)
<table>
<thead>
<tr>
<th>Y Compressive Fiber (Stress)</th>
<th>Matrix (Stress)</th>
<th>Strain€</th>
<th>Sigma F</th>
<th>&lt;Stress&gt;</th>
<th>&lt;E&gt;</th>
<th>E Rough</th>
<th>Difference</th>
<th>Diff (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vf = 49.38 % All Fibers</td>
<td>-6936.1</td>
<td>-309.7</td>
<td>0.1</td>
<td>-148786.92</td>
<td>-1817.45</td>
<td>18174.46</td>
<td>16473.62</td>
<td>1700.838</td>
</tr>
<tr>
<td>Vf = 45.5 % Reduced Z Fibers</td>
<td>-6893.5</td>
<td>-257.94</td>
<td>0.1</td>
<td>-146371.38</td>
<td>-1767.77</td>
<td>17677.7</td>
<td>16473.62</td>
<td>1204.08</td>
</tr>
</tbody>
</table>

Table 5: Effective Modulus results for Y Compressive Load setup

*All Stress and Modulus values are in MPa*
From the above table we can clearly state that the computed results of Effective Modulus of the System are quite close to the $<E_{\text{Roughly}}> \text{ Calculated from the Classic modulus Equation for Uni directional Composite.}$

Hence the Stress Concentration Factor for the required model can be found by

$$K_{iy} = \frac{\sigma_{\text{Max}}}{\sigma_{\text{Avg}}}$$

Where, $\sigma_{\text{Max}}$ is the Max Stress Induced on the component when Y Compressive load is applied.

and $\sigma_{\text{Avg}}$ is the Avg Stress / Nominal Stress Computed when Y Compressive load is applied.

Figure 5-14: Used Defined Plots for Stress Max on Y Fibers and Matrix (Vf=49.38%)

Figure 5-15: User Defined Plots for Stress Max on Y Fibers and Matrix (Vf=45.50% Reduced Z Fibers)
### Table 6: Computed Stress Concentration Factors of Y Fibers and Matrix when Y Compressive Load is applied

<table>
<thead>
<tr>
<th></th>
<th>$\sigma_{\text{Max}}$(Fibers)</th>
<th>$\sigma_{\text{Avg}}$(Fibers)</th>
<th>$\sigma_{\text{Max}}$(Matrix)</th>
<th>$\sigma_{\text{Avg}}$(Matrix)</th>
<th>$K_{fy}$</th>
<th>$K_{my}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Vf = 49.38%$ All Fibers</td>
<td>-8873.3</td>
<td>-6936.1</td>
<td>-4027.4</td>
<td>-309.7</td>
<td>1.28</td>
<td>13.00</td>
</tr>
<tr>
<td>$Vf = 45.5%$ Reduced Z Fibers</td>
<td>-8933.9</td>
<td>-6893.5</td>
<td>-2949.4</td>
<td>-257.94</td>
<td>1.29</td>
<td>11.43</td>
</tr>
</tbody>
</table>

*All Stress and Modulus values are in MPa*
5.1.3 Z Compressive Load

Similarly, Z Compressive Computations on Unit cell are performed as X and Y Compressive Computations, and then the results are Post processed to obtain effective modulus(\(\langle E \rangle\)) and stress concentration factor(K).

Figure 5-16: Z Compressive Load Setup (0.45 mm of displacement on each face)

Figure 5-17: Normal Stress(Z) Integral in Z Fibers and Matrix (Vf = 49.38%, 3D Composite)
Figure 5-18: Normal Stress(Z) Integral on Y Fibers and Matrix (Vf=26.63% Reduced Y Fibers)

Figure 5-19: Normal Stress(Z) Integral on Y Fibers and Matrix (Vf=3.87% Reduced X and Y Fibers)
<table>
<thead>
<tr>
<th>Vf</th>
<th>Fiber Stress</th>
<th>Matrix Stress</th>
<th>Strain€</th>
<th>Sigma F (Stress)</th>
<th>&lt;Stress&gt;</th>
<th>&lt;E&gt;</th>
<th>E Rough</th>
<th>Difference</th>
<th>Diff (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>All Fibers</td>
<td>-7084.6</td>
<td>-871.09</td>
<td>0.0978261</td>
<td>-90068.71</td>
<td>-1111.96</td>
<td>11366.7</td>
<td>2806.617</td>
<td>8560.079</td>
<td>75.31</td>
</tr>
<tr>
<td>Reduced Y Fibers</td>
<td>-7331.9</td>
<td>-253.92</td>
<td>0.0978261</td>
<td>-42792.38</td>
<td>-528.301</td>
<td>5400.41</td>
<td>2806.617</td>
<td>2593.792</td>
<td>48.03</td>
</tr>
<tr>
<td>Reduced X and Y Fibers</td>
<td>-7077.3</td>
<td>-333.04</td>
<td>0.0978261</td>
<td>-48153.22</td>
<td>-594.484</td>
<td>6076.949</td>
<td>2806.617</td>
<td>3270.332</td>
<td>53.82</td>
</tr>
</tbody>
</table>

Table 7: Effective Modulus results for Z Compressive Load setup

*All Stress and Modulus values are in MPa
From the above table we can clearly state that the computed results of Effective Modulus of the System possess quite huge difference, this is due to low volume fraction in Z Direction and the concentration of stress is directly impended on Matrix.

Hence the Stress Concentration Factor for the required model can be found by

\[ K_{iz} = \frac{\sigma_{Max}}{\sigma_{Avg}} \]

Where, \( \sigma_{Max} \) is the Max Stress Induced on the component when Z Compressive load is applied.

and \( \sigma_{Avg} \) is the Avg Stress / Nominal Stress Computed when Z Compressive load is applied.

Figure 5-20: Used Defined Plots for Stress Max on Z Fibers and Matrix (Vf=49.38%)

![Stress Distribution](image1)

Figure 5-21: Used Defined Plots for Stress Max on Z Fibers and Matrix (Vf=22.75% Reduced Y Fibers)

![Stress Distribution](image2)
Figure 5-22 : Used Defined Plots for Stress Max on Z Fibers and Matrix (Vf=3.87% Reduced X and Y Fibers)

Looking at Stress Concentration factor of Matrix in this case, The Matrix is experiencing more stress than any other directions. The X and Y Fibers Reduced Model seems to clearly shows the Stress concentration on the fiber and matrix is Completely concentrated throughout volume attaining its maximum capability.
<table>
<thead>
<tr>
<th>Vf</th>
<th>Fibers (Fibers)</th>
<th>Matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\sigma_{\text{Max}}$ (Fibers)</td>
<td>$\sigma_{\text{Avg}}$ (Fibers)</td>
</tr>
<tr>
<td>All Fibers</td>
<td>-8045.5</td>
<td>-7084.6</td>
</tr>
<tr>
<td>Reduced Y Fibers</td>
<td>-8728.8</td>
<td>-7331.9</td>
</tr>
<tr>
<td>Reduced X and Y Fibers</td>
<td>-7089.1</td>
<td>-7077.3</td>
</tr>
</tbody>
</table>

Table 8: Computed Stress Concentration Factors of Z Fibers and Matrix when Z Compressive Load is applied

*All Stress and Modulus values are in MPa*
5.2 Shear Loads

5.2.1 X Axis Y Shear (XY Shear)

From following figures, Shear stress integral at points concentrate Maximum at outer bound and minimum occurs at interaction point with other fiber. This Shear stress is computed by integrating the stress at volumes on individual components, and then finding out the required average value of Shear Stress that can be computed to find total force on the elements.

To find the Sum of Forces on System

\[ \int_V \tau(x) \, dV(x) + \int_V \tau(y) \, dV(y) + \int_V \tau(z) \, dV(z) + \int_V \tau(m) \, dV(m) = \sum F \]

Where, \( \int_V \tau(x) \, dV(x) \) is Shear stress integral along X fibers Volume in acting plane

\( \int_V \tau(y) \, dV(y) \) is Shear stress integral along Y fibers Volume in acting plane.
\[ \int_V \tau(fz) \, dV(Z) \] is Shear stress integral along Z fibers Volume in acting plane

And \( \int_V \tau(m) \, dV(m) \) is Shear stress integral along Matrix Volume in acting plane

To find the Total Stress Induced on System

\[ \frac{\Sigma F}{\Sigma V} = < \tau > , \] Where \( \Sigma V \) is the total Volume of the Unit Cell.

Formulae used for estimation of the effective properties of 3D composites

1. Recalculation of the properties of fiber (isotropic) and matrix

\[
K = \frac{E}{3(1-\nu)} \text{; bulk modulus} \\
G = \frac{E}{2(1+\nu)} \text{; shear modulus}
\]

2. Shear modulus of modified matrix

\[
G_\perp = G^M + \frac{4c_\perp \left( G^F_{23} - G^M_{23} \right) (1-V^M)}{K^M} \frac{1}{1 + c_\perp + (1-c_\perp) \frac{G^F_{23}}{G^M}}; \\
c_\perp \text{ is the volume concentration of all fibers perpendicular to the shear plane}
\]

3. Effective shear modulus in the plane

\[
G_\parallel = G_\perp + \frac{2c_\parallel \left( G^F - G^M \right)}{1 + c_\parallel + (1-c_\parallel) \frac{G^F}{G^M}} \\
c_\parallel \text{ is the total volume concentration of all fibers in the plane of shear or parallel to it}
\]

After applying load on the unit cell, the following plots have been noted for Avg Shear Stress in XY Plane.
Figure 5-24: Shear Stress(XY) Integral in X and Y Fibers Resp. (Vf = 49.38%, 3D Composite)

Figure 5-25: Shear Stress(XY) Integral in Z Fiber and Matrix Resp. (Vf = 49.38%, 3D Composite)

Figure 5-26: Shear Stress(XY) Integral in X and Y Fibers Resp. (Vf = 45.5%, Reduced Z Fibers)
Figure 5-27: Shear Stress (XY) Integral in Matrix (Vf=45.5%, Reduced Z Fibers)

Figure 5-28: Shear Stress (XY) Integral in X and Z Fibers Resp. (Vf=26.63%, Reduced Y Fibers)

Figure 5-29: Shear Stress (XY) Integral in Matrix. (Vf=26.63%, Reduced Y Fibers)
Figure 5-30: Shear Stress (XY) Integral in X Fibers and Matrix Resp. (Vf=22.75%, Reduced Y and Z Fibers)

Figure 5-31: Shear Stress (XY) Integral in Z Fibers and Matrix Resp. (Vf=3.87%, Reduced X and Y Fibers)
Table 9: Effective Shear Modulus results for X Y Shear Load setup

<table>
<thead>
<tr>
<th>X axis Y Shear</th>
<th>$\tau_x$ Avg</th>
<th>$\tau_y$ Avg</th>
<th>$\tau_z$ Avg</th>
<th>$\tau_f$</th>
<th>$\varepsilon_f$</th>
<th>$\tau$</th>
<th>$&lt;G&gt;$</th>
<th>$G_\perp$</th>
<th>$G_\parallel$</th>
<th>Difference</th>
<th>Difference %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vf =49.38% All Fibers</td>
<td>792.975</td>
<td>433.11</td>
<td>226.48</td>
<td>62.902</td>
<td>238163.7</td>
<td>0.1</td>
<td>319.597</td>
<td>3195.97</td>
<td>1250.07</td>
<td>3049.881</td>
<td>146.0891</td>
</tr>
<tr>
<td>Vf =45.5% Reduced Z Fibers</td>
<td>505.94</td>
<td>233.29</td>
<td>0</td>
<td>52.939</td>
<td>146841.3</td>
<td>0.1</td>
<td>197.0495</td>
<td>1970.495</td>
<td>1250</td>
<td>2060.992</td>
<td>-90.4967</td>
</tr>
<tr>
<td>Vf =26.63% Reduced Y Fibers</td>
<td>515.27</td>
<td>0</td>
<td>155.21</td>
<td>119.96</td>
<td>157441.3</td>
<td>0.1</td>
<td>211.2738</td>
<td>2112.738</td>
<td>1250.07</td>
<td>1911.811</td>
<td>200.9269</td>
</tr>
<tr>
<td>Vf =22.75% Reduced Y and Z Fibers</td>
<td>511.07</td>
<td>0</td>
<td>0</td>
<td>93.492</td>
<td>140474.8</td>
<td>0.1</td>
<td>188.5061</td>
<td>1885.061</td>
<td>1250</td>
<td>1911.709</td>
<td>-26.6475</td>
</tr>
<tr>
<td>Vf =3.87% Reduced X and Y Fibers</td>
<td>0</td>
<td>0</td>
<td>219.7</td>
<td>113.566</td>
<td>87695.38</td>
<td>0.1</td>
<td>117.6803</td>
<td>1176.803</td>
<td>1250.07</td>
<td>1250.07</td>
<td>-73.2664</td>
</tr>
</tbody>
</table>

*All Stress and Modulus values are in MPa*
Hence the Stress Concentration Factor for the required model can be found by

$$K_i = \frac{\tau_{\text{Max}}}{\tau_{\text{Avg}}}$$

Where, $\tau_{\text{Max}}$ is the Max Stress Induced on the component when XY Shear load is applied.

and $\tau_{\text{Avg}}$ is the Avg Stress / Nominal Stress Computed when XY Shear load is applied.
<table>
<thead>
<tr>
<th>( \tau_x \text{ Max} )</th>
<th>( \tau_y \text{ Max} )</th>
<th>( \tau_z \text{ Max} )</th>
<th>( \tau_M \text{ Max} )</th>
<th>( \tau_x \text{ Avg} )</th>
<th>( \tau_y \text{ Avg} )</th>
<th>( \tau_z \text{ Avg} )</th>
<th>( \tau_M \text{ Avg} )</th>
<th>( K_{fx} )</th>
<th>( K_{fy} )</th>
<th>( K_{fz} )</th>
<th>( K_m )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{Vf} = 49.38 % \text{ All Fibers} )</td>
<td>1169.1</td>
<td>703.43</td>
<td>687.17</td>
<td>1250.6</td>
<td>792.97</td>
<td>433.11</td>
<td>226.48</td>
<td>62.902</td>
<td>1.47</td>
<td>1.62</td>
<td>3.03</td>
</tr>
<tr>
<td>( \text{Vf} = 45.5 % \text{ Reduced Z Fibers} )</td>
<td>1139.3</td>
<td>615.17</td>
<td>0</td>
<td>1061.7</td>
<td>505.94</td>
<td>233.29</td>
<td>0</td>
<td>52.939</td>
<td>2.25</td>
<td>2.64</td>
<td>0</td>
</tr>
<tr>
<td>( \text{Vf} = 26.63 % \text{ Reduced Y Fibers} )</td>
<td>1031.5</td>
<td>0</td>
<td>392.77</td>
<td>1112.6</td>
<td>515.27</td>
<td>0</td>
<td>155.21</td>
<td>119.96</td>
<td>2.00</td>
<td>0</td>
<td>2.53</td>
</tr>
<tr>
<td>( \text{Vf} = 22.75 % \text{ Reduced Y and Z Fibers} )</td>
<td>985.75</td>
<td>0</td>
<td>0</td>
<td>1057.8</td>
<td>511.07</td>
<td>0</td>
<td>0</td>
<td>93.492</td>
<td>1.93</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( \text{Vf} = 3.87 % \text{ Reduced X and Y Fibers} )</td>
<td>0</td>
<td>0</td>
<td>304.68</td>
<td>138.52</td>
<td>0</td>
<td>0</td>
<td>219.7</td>
<td>113.566</td>
<td>0</td>
<td>0</td>
<td>1.39</td>
</tr>
</tbody>
</table>

Table 10: Computed Stress Concentration Factors of Fibers and Matrix when X Y Shear Load is applied

*All Stress and Modulus values are in MPa*
Similarly, the procedure is followed for all the shear loads in different planes. Hence the results follow.

### 5.2.2 X Axis Z Shear (XZ Shear)

<table>
<thead>
<tr>
<th>X axis Z Shear</th>
<th>$\tau_x$</th>
<th>$\tau_y$</th>
<th>$\tau_z$</th>
<th>$\tau_{H}$</th>
<th>$\sum F$</th>
<th>$\varepsilon_{Strain}$</th>
<th>$\tau$</th>
<th>$&lt;G&gt;$</th>
<th>$G_{\perp}$</th>
<th>$G_{\parallel}$</th>
<th>Difference</th>
<th>Difference %</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_f=49.38%$ All Fibers</td>
<td>370.33</td>
<td>306.75</td>
<td>201.76</td>
<td>127.40</td>
<td>168688.39</td>
<td>0.10</td>
<td>226.37</td>
<td>2263.67</td>
<td>1250.50</td>
<td>2061.79</td>
<td>201.88</td>
<td>8.92</td>
</tr>
<tr>
<td>$V_f=45.5%$ Reduced Z Fibers</td>
<td>378.65</td>
<td>319.39</td>
<td>0.00</td>
<td>127.70</td>
<td>170216.08</td>
<td>0.10</td>
<td>228.42</td>
<td>2284.17</td>
<td>1250.50</td>
<td>1912.45</td>
<td>371.72</td>
<td>16.27</td>
</tr>
<tr>
<td>$V_f=26.63%$ Reduced Y Fibers</td>
<td>448.36</td>
<td>0.00</td>
<td>260.52</td>
<td>100.31</td>
<td>138394.52</td>
<td>0.10</td>
<td>185.71</td>
<td>1857.15</td>
<td>1250.00</td>
<td>2060.99</td>
<td>-203.85</td>
<td>-10.98</td>
</tr>
<tr>
<td>$V_f=22.75%$ Reduced Y and Z Fibers</td>
<td>449.82</td>
<td>0.00</td>
<td>0.00</td>
<td>100.01</td>
<td>133841.24</td>
<td>0.10</td>
<td>179.60</td>
<td>1796.04</td>
<td>1250.00</td>
<td>1911.71</td>
<td>-115.66</td>
<td>-6.44</td>
</tr>
<tr>
<td>$V_f=3.87%$ Reduced X and Y Fibers</td>
<td>0.00</td>
<td>0.00</td>
<td>219.84</td>
<td>133.88</td>
<td>102252.02</td>
<td>0.10</td>
<td>137.21</td>
<td>1372.14</td>
<td>1250.00</td>
<td>1342.44</td>
<td>29.71</td>
<td>2.16</td>
</tr>
</tbody>
</table>

Table 11: Effective Shear Modulus results for X Z Shear Load setup

*All Stress and Modulus values are in MPa*
Table 12: Computed Stress Concentration Factors of Fibers and Matrix when X Z Shear Load is applied

<table>
<thead>
<tr>
<th>Vf</th>
<th>Reduced Z Fibers</th>
<th>Reduced Y Fibers</th>
<th>Reduced Y and Z Fibers</th>
<th>Reduced X and Y Fibers</th>
</tr>
</thead>
<tbody>
<tr>
<td>49.38%</td>
<td>1062.90</td>
<td>694.65</td>
<td>1014.70</td>
<td>370.33</td>
</tr>
<tr>
<td>45.5%</td>
<td>1110.40</td>
<td>980.97</td>
<td>981.23</td>
<td>378.65</td>
</tr>
<tr>
<td>26.63%</td>
<td>664.16</td>
<td>0.00</td>
<td>780.22</td>
<td>282.41</td>
</tr>
<tr>
<td>22.75%</td>
<td>665.61</td>
<td>0.00</td>
<td>0.00</td>
<td>279.07</td>
</tr>
<tr>
<td>3.87%</td>
<td>0.00</td>
<td>0.00</td>
<td>352.20</td>
<td>153.41</td>
</tr>
</tbody>
</table>

*All Stress and Modulus values are in MPa*
### 5.2.3 Y Axis X Shear (YX Shear)

<table>
<thead>
<tr>
<th>Y axis X Shear</th>
<th>$\tau_x$ Avg</th>
<th>$\tau_y$ Avg</th>
<th>$\tau_z$ Avg</th>
<th>$\tau_M$ Avg</th>
<th>$\sum F$</th>
<th>$\varepsilon$, Strain</th>
<th>$\tau$</th>
<th>$&lt;G&gt;$</th>
<th>$G_z$</th>
<th>$G_y$</th>
<th>Difference</th>
<th>Difference %</th>
</tr>
</thead>
<tbody>
<tr>
<td>VF = 49.38% All Fibers</td>
<td>433.11</td>
<td>792.98</td>
<td>230.59</td>
<td>62.90</td>
<td>238282.3</td>
<td>0.10</td>
<td>319.76</td>
<td>3197.56</td>
<td>1250.07</td>
<td>3049.88</td>
<td>147.68</td>
<td>4.62</td>
</tr>
<tr>
<td>VF = 45.5% Reduced Z Fibers</td>
<td>767.37</td>
<td>355.44</td>
<td>0.00</td>
<td>113.10</td>
<td>236311.3</td>
<td>0.10</td>
<td>317.11</td>
<td>3171.11</td>
<td>1250.00</td>
<td>3049.72</td>
<td>121.39</td>
<td>3.83</td>
</tr>
<tr>
<td>VF = 26.63% Reduced Y Fibers</td>
<td>267.09</td>
<td>0.00</td>
<td>285.91</td>
<td>159.86</td>
<td>140950.9</td>
<td>0.10</td>
<td>189.15</td>
<td>1891.45</td>
<td>1250.07</td>
<td>1911.81</td>
<td>-20.36</td>
<td>-1.08</td>
</tr>
<tr>
<td>VF = 22.75% Reduced Y and Z Fibers</td>
<td>300.08</td>
<td>0.00</td>
<td>0.00</td>
<td>126.23</td>
<td>123543.7</td>
<td>0.10</td>
<td>165.79</td>
<td>1657.86</td>
<td>1250.00</td>
<td>1911.71</td>
<td>-253.85</td>
<td>-15.31</td>
</tr>
<tr>
<td>VF = 3.87% Reduced X and Y Fibers</td>
<td>0.00</td>
<td>0.00</td>
<td>164.81</td>
<td>113.57</td>
<td>86113.87</td>
<td>0.10</td>
<td>115.56</td>
<td>1155.58</td>
<td>1250.07</td>
<td>1250.07</td>
<td>-94.49</td>
<td>-8.18</td>
</tr>
</tbody>
</table>

#### Table 13: Effective Shear Modulus results for YX Shear Load setup

<table>
<thead>
<tr>
<th>Y axis X Shear</th>
<th>$\tau_x$ Max</th>
<th>$\tau_y$ Max</th>
<th>$\tau_z$ Max</th>
<th>$\tau_M$ Max</th>
<th>$\tau_x$ Avg</th>
<th>$\tau_y$ Avg</th>
<th>$\tau_z$ Avg</th>
<th>$\tau_M$ Avg</th>
<th>$K_{fx}$</th>
<th>$K_{fy}$</th>
<th>$K_{fz}$</th>
<th>$K_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>VF = 49.38% All Fibers</td>
<td>692.60</td>
<td>1096.50</td>
<td>632.99</td>
<td>1173.70</td>
<td>433.11</td>
<td>792.98</td>
<td>230.59</td>
<td>62.90</td>
<td>1.60</td>
<td>1.38</td>
<td>2.75</td>
<td>18.66</td>
</tr>
<tr>
<td>VF = 45.5% Reduced Z Fibers</td>
<td>767.37</td>
<td>1146.40</td>
<td>0.00</td>
<td>1113.00</td>
<td>626.36</td>
<td>355.44</td>
<td>0.00</td>
<td>113.10</td>
<td>1.23</td>
<td>3.23</td>
<td>0.00</td>
<td>9.84</td>
</tr>
<tr>
<td>VF = 26.63% Reduced Y Fibers</td>
<td>609.88</td>
<td>0.00</td>
<td>708.29</td>
<td>647.08</td>
<td>267.09</td>
<td>0.00</td>
<td>285.91</td>
<td>159.86</td>
<td>2.28</td>
<td>0.00</td>
<td>2.48</td>
<td>4.05</td>
</tr>
<tr>
<td>VF = 22.75% Reduced Y and Z Fibers</td>
<td>463.07</td>
<td>0.00</td>
<td>0.00</td>
<td>497.02</td>
<td>300.08</td>
<td>0.00</td>
<td>0.00</td>
<td>126.23</td>
<td>1.54</td>
<td>0.00</td>
<td>0.00</td>
<td>3.94</td>
</tr>
<tr>
<td>VF = 3.87% Reduced X and Y Fibers</td>
<td>0.00</td>
<td>0.00</td>
<td>304.64</td>
<td>138.28</td>
<td>0.00</td>
<td>0.00</td>
<td>164.81</td>
<td>113.57</td>
<td>0.00</td>
<td>0.00</td>
<td>1.85</td>
<td>1.22</td>
</tr>
</tbody>
</table>

#### Table 14: Computed Stress Concentration Factors of Fibers and Matrix when YX Shear Load is applied

*All Stress and Modulus values are in MPa*
5.2.4 Y Axis Z Shear (YZ Shear)

Table 15: Effective Shear Modulus results for YZ Shear Load setup

<table>
<thead>
<tr>
<th>Y axis Z Shear</th>
<th>$\tau_x$ Avg</th>
<th>$\tau_y$ Avg</th>
<th>$\tau_z$ Avg</th>
<th>$\tau_M$ Avg</th>
<th>$\sum F$</th>
<th>$\varepsilon$, Strain</th>
<th>$\tau$</th>
<th>$&lt;G&gt;$</th>
<th>$G_A$</th>
<th>$G_I$</th>
<th>Difference</th>
<th>Difference %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vf = 49.38 % All Fibers</td>
<td>322.29</td>
<td>380.95</td>
<td>196.57</td>
<td>127.92</td>
<td>173170.2</td>
<td>0.10</td>
<td>232.38</td>
<td>2323.81</td>
<td>1250.50</td>
<td>2061.79</td>
<td>262.02</td>
<td>11.28</td>
</tr>
<tr>
<td>Vf = 45.5 % Reduced Z Fibers</td>
<td>304.29</td>
<td>368.61</td>
<td>0.00</td>
<td>127.16</td>
<td>165734.0</td>
<td>0.10</td>
<td>222.40</td>
<td>2224.02</td>
<td>1250.50</td>
<td>1912.45</td>
<td>311.57</td>
<td>14.01</td>
</tr>
<tr>
<td>Vf = 26.63 % Reduced Y Fibers</td>
<td>243.44</td>
<td>0.00</td>
<td>350.26</td>
<td>133.10</td>
<td>124170.0</td>
<td>0.10</td>
<td>166.63</td>
<td>1666.26</td>
<td>1250.50</td>
<td>1250.50</td>
<td>215.13</td>
<td>14.68</td>
</tr>
<tr>
<td>Vf = 22.75 % Reduced Y and Z Fibers</td>
<td>191.87</td>
<td>0.00</td>
<td>0.00</td>
<td>133.22</td>
<td>109218.7</td>
<td>0.10</td>
<td>146.56</td>
<td>1465.63</td>
<td>1250.50</td>
<td>1912.45</td>
<td>311.57</td>
<td>14.77</td>
</tr>
<tr>
<td>Vf = 3.87 % Reduced X and Y Fibers</td>
<td>0.00</td>
<td>0.00</td>
<td>214.52</td>
<td>111.42</td>
<td>86007.32</td>
<td>0.10</td>
<td>115.42</td>
<td>1154.15</td>
<td>1250.50</td>
<td>1250.50</td>
<td>215.13</td>
<td>14.68</td>
</tr>
</tbody>
</table>

Table 16: Computed Stress Concentration Factors of Fibers and Matrix when YZ Shear Load is applied

<table>
<thead>
<tr>
<th>Y axis Z Shear</th>
<th>$\tau_x$ Max</th>
<th>$\tau_y$ Max</th>
<th>$\tau_z$ Max</th>
<th>$\tau_M$ Max</th>
<th>$\tau_x$ Avg</th>
<th>$\tau_y$ Avg</th>
<th>$\tau_z$ Avg</th>
<th>$\tau_M$ Avg</th>
<th>$K_{fx}$</th>
<th>$K_{fy}$</th>
<th>$K_{fz}$</th>
<th>$K_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vf = 49.38 % All Fibers</td>
<td>996.67</td>
<td>1082.90</td>
<td>724.60</td>
<td>1003.00</td>
<td>322.29</td>
<td>380.95</td>
<td>196.57</td>
<td>127.92</td>
<td>3.09</td>
<td>2.84</td>
<td>3.69</td>
<td>7.84</td>
</tr>
<tr>
<td>Vf = 45.5 % Reduced Z Fibers</td>
<td>999.77</td>
<td>1085.40</td>
<td>0.00</td>
<td>993.48</td>
<td>304.29</td>
<td>368.61</td>
<td>0.00</td>
<td>127.16</td>
<td>3.29</td>
<td>2.94</td>
<td>0.00</td>
<td>7.81</td>
</tr>
<tr>
<td>Vf = 26.63 % Reduced Y Fibers</td>
<td>294.31</td>
<td>0.00</td>
<td>766.94</td>
<td>327.03</td>
<td>243.44</td>
<td>0.00</td>
<td>350.26</td>
<td>133.10</td>
<td>1.21</td>
<td>0.00</td>
<td>2.19</td>
<td>2.46</td>
</tr>
<tr>
<td>Vf = 22.75 % Reduced Y and Z Fibers</td>
<td>228.30</td>
<td>0.00</td>
<td>0.00</td>
<td>227.62</td>
<td>191.87</td>
<td>0.00</td>
<td>0.00</td>
<td>133.22</td>
<td>1.19</td>
<td>0.00</td>
<td>0.00</td>
<td>1.71</td>
</tr>
<tr>
<td>Vf = 3.87 % Reduced X and Y Fibers</td>
<td>0.00</td>
<td>0.00</td>
<td>352.16</td>
<td>153.75</td>
<td>0.00</td>
<td>0.00</td>
<td>214.52</td>
<td>111.42</td>
<td>0.00</td>
<td>0.00</td>
<td>1.64</td>
<td>1.38</td>
</tr>
</tbody>
</table>

*All Stress and Modulus values are in MPa
### 5.2.5 Z Axis X Shear (ZX Shear)

<table>
<thead>
<tr>
<th>Z axis X Shear</th>
<th>$\tau_x$ Avg</th>
<th>$\tau_y$ Avg</th>
<th>$\tau_z$ Avg</th>
<th>$\tau_M$ Avg</th>
<th>$\sum F$</th>
<th>$\varepsilon_x$ Strain</th>
<th>$\tau$</th>
<th>$&lt;G&gt;$</th>
<th>$G_{\perp}$</th>
<th>$G_I$</th>
<th>Difference</th>
<th>Difference %</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Vf=49.38%$ All Fibers</td>
<td>280.43</td>
<td>223.25</td>
<td>132214.3</td>
<td>0.10</td>
<td>177.42</td>
<td>1813.64</td>
<td>1250.50</td>
<td>2061.79</td>
<td>-248.15</td>
<td>-13.68</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Vf=45.5%$ Reduced Z Fibers</td>
<td>214.38</td>
<td>244.46</td>
<td>120487.6</td>
<td>0.10</td>
<td>161.68</td>
<td>1652.78</td>
<td>1250.50</td>
<td>1912.45</td>
<td>-259.67</td>
<td>-15.71</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Vf=26.63%$ Reduced Y Fibers</td>
<td>239.88</td>
<td>223.25</td>
<td>161710.2</td>
<td>0.10</td>
<td>217.00</td>
<td>2218.25</td>
<td>1250.00</td>
<td>2060.99</td>
<td>157.26</td>
<td>7.09</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Vf=22.75%$ Reduced Y and Z Fibers</td>
<td>217.06</td>
<td>229.26</td>
<td>143159.9</td>
<td>0.10</td>
<td>192.11</td>
<td>1963.79</td>
<td>1250.00</td>
<td>1911.71</td>
<td>52.08</td>
<td>2.65</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Vf=3.87%$ Reduced X and Y Fibers</td>
<td>0.00</td>
<td>0.00</td>
<td>115621.6</td>
<td>0.10</td>
<td>155.16</td>
<td>1586.03</td>
<td>1250.00</td>
<td>1342.44</td>
<td>243.59</td>
<td>15.36</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 17: Effective Shear Modulus results for Z X Shear Load setup

<table>
<thead>
<tr>
<th>Z axis X Shear</th>
<th>$\tau_x$ Max</th>
<th>$\tau_y$ Max</th>
<th>$\tau_z$ Max</th>
<th>$\tau_M$ Avg</th>
<th>$\tau_x$ Avg</th>
<th>$\tau_y$ Avg</th>
<th>$\tau_z$ Avg</th>
<th>$\tau_M$ Avg</th>
<th>$K_{fx}$</th>
<th>$K_{fy}$</th>
<th>$K_{fz}$</th>
<th>$K_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Vf=49.38%$ All Fibers</td>
<td>510.59</td>
<td>625.43</td>
<td>922.23</td>
<td>658.59</td>
<td>280.43</td>
<td>223.25</td>
<td>154.00</td>
<td>112.31</td>
<td>1.82</td>
<td>2.80</td>
<td>5.99</td>
<td>5.86</td>
</tr>
<tr>
<td>$Vf=45.5%$ Reduced Z Fibers</td>
<td>471.76</td>
<td>575.56</td>
<td>0.00</td>
<td>599.61</td>
<td>214.38</td>
<td>224.46</td>
<td>0.00</td>
<td>113.47</td>
<td>2.20</td>
<td>2.56</td>
<td>0.00</td>
<td>5.28</td>
</tr>
<tr>
<td>$Vf=26.63%$ Reduced Y Fibers</td>
<td>383.30</td>
<td>0.00</td>
<td>571.21</td>
<td>320.70</td>
<td>239.88</td>
<td>0.00</td>
<td>229.64</td>
<td>209.24</td>
<td>1.60</td>
<td>0.00</td>
<td>2.49</td>
<td>1.53</td>
</tr>
<tr>
<td>$Vf=22.75%$ Reduced Y and Z Fibers</td>
<td>261.24</td>
<td>0.00</td>
<td>0.00</td>
<td>229.36</td>
<td>217.06</td>
<td>0.00</td>
<td>0.00</td>
<td>184.76</td>
<td>1.20</td>
<td>0.00</td>
<td>0.00</td>
<td>1.24</td>
</tr>
<tr>
<td>$Vf=3.87%$ Reduced X and Y Fibers</td>
<td>0.00</td>
<td>0.00</td>
<td>612.93</td>
<td>180.43</td>
<td>0.00</td>
<td>0.00</td>
<td>179.96</td>
<td>154.16</td>
<td>0.00</td>
<td>0.00</td>
<td>3.41</td>
<td>1.17</td>
</tr>
</tbody>
</table>

Table 18: Computed Stress Concentration Factors of Fibers and Matrix when ZX Shear Load is applied

*All Stress and Modulus values are in MPa*
### 5.2.6 Z Axis Y Shear (ZY Shear)

<table>
<thead>
<tr>
<th>Z axis Y Shear</th>
<th>τₓ Avg</th>
<th>τᵧ Avg</th>
<th>τ𝑧 Avg</th>
<th>τ𝑀 Avg</th>
<th>∑F</th>
<th>ε Strain</th>
<th>τ</th>
<th>&lt;G&gt;</th>
<th>G⊥</th>
<th>G∥</th>
<th>Differece</th>
<th>Difference %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vf = 49.38 % All Fibers</td>
<td>274.43</td>
<td>206.38</td>
<td>166.48</td>
<td>119.57</td>
<td>131435.4</td>
<td>0.10</td>
<td>176.38</td>
<td>1802.95</td>
<td>1250.50</td>
<td>2061.79</td>
<td>-258.83</td>
<td>-14.36</td>
</tr>
<tr>
<td>Vf = 45.5 % Reduced Z Fibers</td>
<td>316.92</td>
<td>357.89</td>
<td>0.00</td>
<td>94.61</td>
<td>152837.9</td>
<td>0.10</td>
<td>205.10</td>
<td>2096.54</td>
<td>1250.50</td>
<td>1912.45</td>
<td>184.09</td>
<td>8.78</td>
</tr>
<tr>
<td>Vf = 26.63 % Reduced Y Fibers</td>
<td>227.04</td>
<td>263.08</td>
<td>131.95</td>
<td>118238.5</td>
<td>99156.14</td>
<td>0.10</td>
<td>133.06</td>
<td>1360.17</td>
<td>1250.50</td>
<td>1463.26</td>
<td>158.67</td>
<td>9.78</td>
</tr>
<tr>
<td>Vf = 22.75 % Reduced Y and Z Fibers</td>
<td>202.22</td>
<td>0.00</td>
<td>0.00</td>
<td>112.69</td>
<td>152837.9</td>
<td>0.10</td>
<td>155.15</td>
<td>1585.98</td>
<td>1250.50</td>
<td>1342.44</td>
<td>243.54</td>
<td>15.36</td>
</tr>
<tr>
<td>Vf = 3.87 % Reduced X and Y Fibers</td>
<td>0.00</td>
<td>0.00</td>
<td>179.82</td>
<td>154.16</td>
<td>115617.7</td>
<td>0.10</td>
<td>133.06</td>
<td>1360.17</td>
<td>1250.50</td>
<td>1250.50</td>
<td>109.66</td>
<td>8.06</td>
</tr>
</tbody>
</table>

Table 19: Effective Shear Modulus results for Z Y Shear Load setup

<table>
<thead>
<tr>
<th>Z axis Y Shear</th>
<th>τₓ Max</th>
<th>τᵧ Max</th>
<th>τ𝑧 Max</th>
<th>τ𝑀 Avg</th>
<th>τᵧ Avg</th>
<th>τ𝑧 Avg</th>
<th>τ𝑀 Avg</th>
<th>Kₓ</th>
<th>Kᵧ</th>
<th>Kᵧ</th>
<th>Km</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vf = 49.38 % All Fibers</td>
<td>324.17</td>
<td>206.38</td>
<td>207.98</td>
<td>180.04</td>
<td>274.43</td>
<td>140.89</td>
<td>166.48</td>
<td>119.57</td>
<td>1.18</td>
<td>1.46</td>
<td>1.25</td>
</tr>
<tr>
<td>Vf = 45.5 % Reduced Z Fibers</td>
<td>521.29</td>
<td>456.55</td>
<td>0.00</td>
<td>560.32</td>
<td>316.92</td>
<td>357.89</td>
<td>0.00</td>
<td>94.61</td>
<td>1.64</td>
<td>1.28</td>
<td>0.00</td>
</tr>
<tr>
<td>Vf = 26.63 % Reduced Y Fibers</td>
<td>349.43</td>
<td>0.00</td>
<td>626.05</td>
<td>399.42</td>
<td>227.04</td>
<td>0.00</td>
<td>263.08</td>
<td>131.95</td>
<td>1.54</td>
<td>0.00</td>
<td>2.38</td>
</tr>
<tr>
<td>Vf = 22.75 % Reduced Y and Z Fibers</td>
<td>208.99</td>
<td>0.00</td>
<td>0.00</td>
<td>231.16</td>
<td>202.22</td>
<td>0.00</td>
<td>0.00</td>
<td>112.69</td>
<td>1.03</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Vf = 3.87 % Reduced X and Y Fibers</td>
<td>0.00</td>
<td>0.00</td>
<td>613.01</td>
<td>180.42</td>
<td>0.00</td>
<td>0.00</td>
<td>179.82</td>
<td>154.16</td>
<td>0.00</td>
<td>0.00</td>
<td>3.41</td>
</tr>
</tbody>
</table>

Table 20: Computed Stress Concentration Factors of Fibers and Matrix when Z Y Shear Load is applied

*All Stress and Modulus values are in MPa*
6.1 Discussion of the results

Prediction of the \(X\) and \(Y\)-direction compressive response is included in Section 5.1.1 and Section 5.1.2 with deformed configurations shown in Figures mentioned in Sections at applied load setups. The FE prediction of the Effective Modulus versus approximate theoretical modulus response is in reasonable agreement with the measurements except for the fact that the FE calculations over-predict the peak strength. This is because no initial imperfection is included in these FE calculations since the \(X\)-direction fibers are not explicitly modelled. Consistent with observations no significant localised deformation is observed prior to the peak stress though the FE calculations. The Stress Concentration Factor of Matrix is observed to be reduced more than ten times from 13.41 in 3D Model to 1.06 in reduced unidirectional model (Table 4 & 6). This reduction clearly states the response from low fiber volume fraction in the structure.

Prediction of the \(Z\)-direction compressive response is included in Section 5.1.3 with deformed configurations shown in Section 5.1.3 at applied load setup. The FE Prediction of the effective modulus versus approximate theoretical modulus has comparatively huge variation due non-considering the role of horizontal fibers contribution (\(X\&Y\) Fibers) which take up most of the cross section along \(Z\) Direction. But clearly the Stress concentration factor shows us the response of change in volume fractions by making the matrix concentrated with peak stresses throughout in Reduced models unlike to the 3d model.

In Case of Shear forces the Effective modulus computed by FE simulations where having reasonable variations as compared to the derived theoretical modulus Max difference being around 16\%. The Stress concentration factor of the whole models were
reasonable with Maximum stress concentration factor being for the 3d Model as of with the
reduced models. The reduced models bare a concentrated stress throughout volume due
to low fiber volume fraction.

6.2 Future Wok

- Overall Structure failure prediction can be performed in order to understand the
growth rate and the fatigue life of the composite model.
- The Ultimate Tensile Strength of the model can be computed to compare the
results with reduced model.
- Considering the popularity of braided materials, this study can be performed on
Considered 3D Braided Composite material.
- Effect of Matrix/Fiber debonding can be studied by applying tensile loading in
transversal direction.
- Stress analysis of composites under thermal loading can be performed.
References


[6] Lotfi Toubal, Moussa Karama, Bernard Lorrian “Stress Concentration in a Circular hole in composite plate”.


BIOGRAPHICAL STATEMENT

Chandra Bhaskar Reddy Pittu received his bachelor’s degree in Aeronautical Engineering from Jawaharlal Nehru Technological University, Hyderabad in the year 2015. He then decided to pursue his masters in mechanical engineering from the University of Texas at Arlington, USA in Fall 2016 and received his degree in Spring 2018.

Master’s Degree provided him a comprehensive background in Mechanical Technology and kept him abreast of the latest evolutions in this area. In four years of study of this science, subjects like Finite Element, Additive Manufacturing, Composites, and Fluid Dynamics dragged his interest. Of all the courses, Computer aided design (CAD) has evoked a particular stake in him. As a result, he got certified by Dassault systems as “CATIA V5R20 Part Design Specialist”. Not only Catia, He have also learned other CAD/CAE software’s like AutoCAD, ANSYS and Solidworks.

He did Internships at National Aerospace laboratories and Boeing when he was pursuing his Bachelors in Hyderabad. He also published his work on Micro Aerial vehicles in International Journals on Science and Technology(IJST).

Coming from a business background, Chandra was always ambitious to be an entrepreneur and start his industry in the field of mechanical engineering. Chandra would like to use his skills and experience to develop technology for the comfort of humankind by keeping environmental factors in mind.