AN INVESTIGATION OF TOPOLOGY OPTIMIZATION FOR COUPLED THERMO-STRUCTURAL LOADING FOR ADDITIVE MANUFACTURING

by

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Abstract

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This work investigates the ability of topology optimization methods to design multifunctional components with coupled thermo-structural loading for fabrication with additive manufacturing technologies. Topology Optimization is a mathematical approach developed to perform design optimization with the purpose of reducing material usage, while maximizing the structural and thermal performance, in accordance to specific design constraints. Additive manufacturing provides the capability to create internal structures and complex surfaces developed by topology optimization that would not be possible to produce by conventional manufacturing. The more structurally efficient configuration generated by topology optimization and fabricated with additive manufacturing can result in components with improved structural capability and reduced mass. Design optimization most often follows a single discipline approach in which analysis is carried out either by solving the problem from a structural or a thermal point view, but not both simultaneously. The present work involves the study of combined thermal and structural systems using a multidisciplinary optimization technique to design co-optimal systems.

The investigation for this work consists of two case studies: the design of a structural heat conductor and the design of an exhaust-washed structure. For a simple structural heat conductor with various thermal and structural boundary conditions, minimizing compliance is a structural objective whereas minimizing the maximum temperature is a thermal objective, hence carrying out structural analysis on a thermally optimized beam and vice versa. The presence of a thermal load may influence the structural performance by affecting component behavior and vice-versa for structural loading. To overcome this problem, combining both thermal and structural analysis is important. Thus, a systematic
study of obtaining a thermal objective using structural constraint and structural objective using a thermal constraint is carried out for the combined system to be co-optimal. Also, the design of aircraft exhaust washed structure has been carried out using a minimum compliance objective.
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Chapter 1
Introduction

An Additive Manufacturing is the process of creating a three-dimensional object from digital file. It is called additive because it generally involves building up thin layers of material, one by one. Technology can produce complex shapes that are not possible with traditional manufacturing. The ability which provides designer with high design freedom to optimize the part for optimum behavior rather than being limited by manufacturing constraint. Design part with optimal performance is a one of the important factors in the additive manufacturing. There are various design optimization methods which are used to achieve maximum geometric resolution to allow the fine features easily to manufacture by AM to be represented in the optimization model.

Topology optimization methods solve a material distribution over the domain to generate an optimal topology. It is a finite element method in which design domain to be defined as a design variable, allowing variation in the density or void-solid. Also, there are other methods that exist such as level set method and genetic algorithms, but they are not discussed here.

1.1 Motivation

The motivation behind this thesis is: 1) structural and thermal optimization of System, 2) Combined the thermal and structural optimization to get co-optimal system and 3) Optimization with thermal and structural loading. Here is the simple 2-D example of the thermal and structural optimization shown in figure 1-1 and combined Optimization shown in Figure 1-2
Figure 1-1 structural and thermal topology optimization of simple 2D problem [22]

Figure 1-2 Combined topology distribution [22]

Topology optimization is one of the three main types of structural optimization that are used to create a conceptual design. However, here we are introducing a thermal optimization with sequential and combined optimization of the
system. So, there always has been a demand to develop a system which is lighter in weight and continues withstand in both thermal and structural aspect.

1.2 Objective of Thesis

Topology optimization in AM has primarily been limited to isotropic material. Also, they need for both optimal material distribution and optimal orientation. Microstructures from AM process in topology optimization has largely been ignore. The focus of this thesis is on design and optimization of structural heat conductor and aircraft exhaust washed structure. After conducting a simultaneous study of structural and thermal behavior, then we got know that thermally optimized system is structurally unstable and vice-versa. So, it is an important task to design the system in such a way that they can withstand together in thermal and structural boundary condition and loading. For that, we are using a multi-objective technique. For structural optimization, minimum compliance is the objective and for thermal optimization, the objective is minimizing maximum temperature. After that combining loading and boundary condition to optimize system at time using thermal objective and structural constraint and conversely the same. Finally, in design of exhaust washed structure, optimizing system using structural objective with multidisciplinary loading. The objective is to produce the maximum stiffness in the structures.
Chapter 2
Background and Literature review

A lot of advancement is observed in the field of additive manufacturing, optimization, and modeling in recent years. Understanding minute details about the topic, the way it works and its’s application at the specific point is very important and is a major task. Topology optimization for additive manufacturing is used for designing various complex parts in different sectors of aerospace, automotive and civil engineering industries since many decades. All approaches in the past have focused on a single disciplinary approach like either structural or thermal environment. In this paper, we will be focusing on multidisciplinary problem with more than one objective at time using a unique design optimization tool known as topology optimization.

2.1 Additive Manufacturing

Additive Manufacturing is basically a layer based manufacturing approach. In Additive Manufacturing, a complete three-dimensional part is fabricated by adding materials layer by layer. Due to layer based approach, the fabrication of model with higher geometrical complexity has become easy, with no effect on the cost. It can also be defined by a set of technologies which are used sequentially to translate solid model data into physical model. Solid model data is firstly transformed into a series of 2D cross sections, which are further sent to a 3D printer, also called as AM machine. Layer by layer combination of all the cross sections in AM machine results in formation of a physical part. [21]

Different Methods for Additive Manufacturing [21]

1) Photopolymerization process

2) Extrusion based systems
3) Powder bed fusion process
4) Printing processes
5) Beam deposition processes
6) Sheet lamination processes
7) Direct write technologies

2.1.2 Design for Additive Manufacturing

Objective of DFEM is maximization of product performance through the synthesis of shapes, sizes, hierarchical structures, and material compositions, subject to the capabilities of Am technologies. AM enables the usage of complex geometry in achieving design goals without incurring time or cost penalties compared with simple geometry. Due to AM, it is possible to consolidate parts, integrating features into more complex parts and avoiding assembly issues. [21]

• With AM, the usage of customized geometry and parts by direct production from 3D data has become easier.
• AM allows designers to ignore all the constraints imposed by conventional manufacturing processes (although AM-specific constraints might be imposed).

The layer-by-layer fabrication approach means that the shapes of part cross sections can be arbitrarily complex, up to the resolution of the process. In many cases, several parts can be replaced with a single, more complex part. Even in case of relative motion between two or more components, AM can build fully assembled component, for eg. ball-and-socket joint. Since, number of parts are reduced, there is drastic reduction in the cost, while improving production efficiency.

2.2 Topology Optimization

Topology optimization is one of the three different types of structure optimization to create a conceptual design. Optimization process is discipline of adjusting process so as
to optimize some specific set of parameters without violating constraint. [17] The most common goal of topology optimization is minimization of cost, while maximizing efficiency. Another definition of optimization can be selecting the best design within available means. In this objective, function is the criterion for best design and constraints are the available means, also called as design requirements. Finally, design variables are used to describe different designs. Objective function is a value which is minimized or maximized. If the objective function does not converge to a solution, design variable must change. [2]

\[
\text{Minimize } f(x) = 0 \quad (2.1)
\]

\[
\text{Subject to } \begin{cases} g(x) \leq 0 \\ h(x) = 0 \end{cases} \quad (2.2)
\]

In above equation f(x) is the objective function, x is the design variable and g(x) and h(x) are constraints.

For computational design optimization, it is required to express objective functions and constraints as a function of design variable (or design vector X). The following flow chart illustrates steps in topology optimization process from conceptual design to achieving objective function goal.
2.1.1 Size optimization

Size optimization is defining ideal component parameters such as cross section dimensions, thickness and material values. It is used for determination of ideal thickness of material based on the performance goal and the forces expected to be placed on the component during its life. In an optimization process, it is generally used after freeform
optimization once the initial geometry of the component has been defined and interpreted. It generally comes after shape and topology optimization because it needs initial geometry before running for optimization. It is widely used to manufacture composite parts for giving better thickness using different proper layout of composites ply used to reduce the to produce that part. [18] Here we are considering simple Beam example for size optimization shown in the following figure 2-3

![Beam](image)

Figure 2-2  Size optimized beam [20]

(1) Design variable (x)  \( f(x) : \) compliance

X: thickness of each beam  \( g(x) : \) mass

(2) Number of design variables (ndv)

ndv: 5
Now we can see another simple size optimization example where cross section is reduced shown in the following figure.

![Figure 2-3 size optimized cross section](image)

2.1.2 Shape Optimization

Shape optimization involves defining ideal component parameters, such as material values, cross sectional dimensions, and thickness. Shape optimization is different from the topology optimization in the aspect that it is used once the component topology has already been defined. Topology optimization is used to generate material layout concept, whereas shape optimization refines and improves the topology within the concept. These sections are usually defined as members, walls, or shapes. [17] It is the second step in the structural optimization processes. In shape optimization, the outer boundary of the structure is modified to solve the optimization problem. Using finite element models, grid point location defines the shape. Hence, shape modification changes those locations.
Here we are considering simple B-spline in 2-5

Figure 2-4 Shape optimization B-spline [20]

Design Variables (x)

\[ x : \text{control points of the B-spline} \quad f(x) : \text{compliance} \]

(position of each control point)

\[ g(x) : \text{mass} \]

Number of design variables (ndv)

\[ ndv = 8 \]

2.1.3 Topology optimization approaches

Topology optimization is a mathematical method that optimizes material layout within a given design space and gives a set of loads, boundary conditions, and constraints with the goal of maximizing the performance of the system. The proportion of material to be used in the
The overall domain is limited to less than 100% of the volume of the domain. To solve this, it is discretized by using the finite element method and dividing the design domain into discrete elements (mesh). The resulting problem is then solved using optimization methods to find which elements that are material, and which are not. [18] Typical topology optimization problem as shown in the figure 2-6.

![Cells](image)

**Figure 2-5 Topology optimization. [20]**

Design variables (x)
- \(x\): Density of each cell
- \(f(x)\): Compliance

Number of design variables (ndv)
- \(g(x)\): Mass
- ndv: 27

There are various approaches for topology optimization, the two main solution strategies are as follows. [1]

1) Density method
2) Homogenization method

Here, in this research, we are mainly focusing on the density based approach. In the density based approach, one way to get a problem that can be solved is to relax the problem by letting the material density take any value between zero and one, i.e., 0% to 100% density. By making this relaxation, it is possible to use gradient based optimization method to find a minimum of the objective function. The design variable of the optimization problem is the density which is a function varying over the design domain. When considering elements in 2D the density could be represented as varying thickness of a plate. In 3D, there is no similar counterpart; a solid with 50% material is neither physically reasonable nor very intuitive. Topology optimization using this formulation is called the density method.

The density based approach further classified in to three different methods: [1]

1) SIMP Method
2) RAMP Method
3) SINH Method

In this research work, the main aim is topology optimization using SIMP (solid isotropic material with penalization) method. In SIMP method, density variable is penalized with basic power law (whose value is finite) and multiplied on to physical quantities such as material stiffness, cost, or conductivity. This is demonstrated in equation where the SIMP method is applied to the elastic modulus of an element. [2]

\[ E(\rho_e) = \rho_e^p E_0 \]  

(2.3)

Here, \( E(\rho_e) \) is scaled modulus, \( E_0 \) is the modulus of the solid material, and \( p \) is finite penalty parameter. We noted that for values of \( 0 \leq \rho_e^p \leq 1 \) and density \( \rho \) (commonly
taken as 3), E is bounded between zero at zero density and its solid value \( E_0 \) when \( \rho_e = 1 \). In the following figure which explains the relative stiffness as a function of density with different penalization factors.

Figure 2-6 Relative stiffness as function of density with different penalization factor [2]

When the densities are assumed constant over each element the density stiffness relation can be implemented simply by scaling the element stiffness matrices before assembling then into the global stiffness matrix: [2]

\[
K_e = \rho_e^p \ K_e^0
\]  

(2.4)

When \( p \) is a penalization factor greater than zero. The resulting cost-stiffness can be seen in Figure 2-7. The condition for this problem is use of isotropic material, which mean
the physical properties are same in all directions. Minimizing compliance is goal for doing
the optimization. [2]

Now we can see the SIMP topology optimization problem statement

\[
\text{Minimize } c(\rho) = U^T F = U^T KU = \sum_{e=1}^{N} \rho_e^p u_e^T k_0 u_e
\]  \hspace{1cm} (2.5)

Subjected to:
\[
\begin{aligned}
\frac{v(\rho)}{V_o} & \leq f \\
KU &= F \\
\rho_{\text{min}} & \leq \rho \leq 1
\end{aligned}
\]  \hspace{1cm} (2.6)

Where \( c \) is the compliance which is we are minimizing, \( U \) is displacement vector and \( F \) is
force vectors. \( K \) is global stiffness matrix \( u_e \) is the element displacement vector, \( k_0 \) is
stiffness matrix for element with unit young’s modulus, \( x_e \) is the vector of design variable,
\( N \) is the number of elements used \( \rho \) is penalization power, \( V(x) \) and \( V_o \) are the material
volume and given volume domain, Respectively, and \( f \) is the volume fraction, also
called as design constraint. [2]

Optistruct performs topology optimization based on the density method
which is also called known as SIMP method. It provides the user to control over the
element discretization using discrete parameters.

2.2 Structural topology optimization

Three parameters mainly describe structural optimization design problem, viz. objective
of the problem, design variables and constraints involved. A general optimization problem
is formulated as:
minimize or maximize an objective function subjected to behavioral and geometric constraint.

The objective function or behavioral constraints are described as follows: [8]

- Manufacturing or cost of material
- Structural weight or volume, storage capacity
- Local structural response such as stress, strain, or displacement at prescribed points; maximum stress, strain or displacement in whole structure, stress intensity factor.
- Global measure of structural performance such as stiffness, buckling load, natural frequency vibration, dynamic responses, plastic collapse load, etc.

Geometrical constraints are usually described as follows: [8]

- Physical limitation
- Manufacturing limitation
- Fabrication
- Availability of member size

Design variables are to be determined in the optimization process.

Use of numerical approach has been growing fast in topology optimization, with the advancement in technology & development of high speed computer. A numerical approach to topological design starts with a domain of material to which the external loads are boundary conditions are applied. The optimization algorithm then proceeds with removing out ineffectual material to generate the best structural solution. [8] Consider typical structural optimization in the figure 2.8.
Sizing and shape optimization problem are the complex features of topological structural optimization, amongst structural optimization problems. In the optimization process, trying to change topology and the shape during optimization processes tend to increase complexity of the problem. This class of problem is the most challenging one in the structural mechanics.

2.3 Thermal topology optimization

Thermal analysis is used for determination of temperature field and heat fluxes in the structure. Conduction and convection thermal analysis is done, by finite element method. Design sensitivity analysis and optimization have been enhanced to support thermal compliances in addition to previously implemented temperature responses. Thermal responses such as thermal compliance and temperature responses are supported either as objective function or constraint. They can be used simultaneously with responses from static analysis, normal mode analysis, buckling analysis, dynamic analysis, and user defined response. Conductive and convective problem can be solved using optistuct. The governing equation for linear steady state thermal analysis can be discretized by finite element method and expressed in the finite element form as. [13]
\[
\left( [K_c] + [H] \right) \{T\} = \{P_B\} + \{P_H\} + \{P_Q\}
\]  
(2.7)

Where,
\[
[K_c] = \int_V [B]^T [k] [B] dv
\]
\[
[H] = \int_S [N]^T [N] h \ ds
\]
\[
\{P_B\} = \int_S [N]^T f_B \ ds
\]
\[
\{P_H\} = \int_S [N]^T h T_f ds
\]
\[
\{P_Q\} = \int_V [N]^T Q dv
\]

\([Kc]\) is the conductivity matrix, \([H]\) is the convection matrix, \(\{T\}\) is the unknown temperature, \([N]\) is the shape function, \([B]\) is the derivative of \([N]\) with respect to coordinates, \([k]\) is the material thermal conductivity coefficient matrix, \(h\) is the convective heat transfer coefficient, \(f_B\) is the boundary heat flux, \(T_f\) is the ambient fluid temperature, \(Q\) is the volumetric heat generation, \(S\) is the boundary surface that has heat exchange, and \(V\) is the volume. The system of linear equations is solved to find the nodal temperature vector \(\{T\}\). [13]

Thermal compliance is implemented as response in optistruct, it can be used as objective function or constraint. The following equation defines it [13]

\[
TCOMP = \frac{1}{2} \times T^T P = \frac{1}{2} \times T^T [K_c + H] T
\]

Where \(K_c\) is the conductivity matrix, \([H]\) is the convection matrix, \([T]\) is unknown temperature. When thermal compliance is minimized, temperature at the grids where power is applied is minimized, which typically highest in the structure.
Thermal compliance optimization formulation

To get maximum conduction, thermal compliance should be minimized for that power is applied to structures. The following equation results in lowest temperature.

\[ T_{COMP} = \frac{1}{2} \times T^T P = \frac{1}{2} \times P^T [K_c + H]^{-1} \times P \]

When enforced temperature is applied to the structure, Temperature field is fixed thermal compliance should be maximized to obtained maximum conduction given by following equation.

\[ T_{COMP} = \frac{1}{2} \times T^T P = \frac{1}{2} \times T^T [K_c + H]T \]

Convection is very common phenomenon in heat transfer problem. When convection is existing, the temperature is enforced, and power is applied. Thermal Compliance response is supported in additional to existing temperature Response. Thermal compliance optimization response is significantly faster than the temperature response to minimizing the maximum temperature of the entire surface. Minimizing the thermal compliance also minimizes temperature at the grid where power is applied. Those temperatures are typically highest in the structures. Computational cost of thermal compliance is lower because firstly, sensitivity of thermal compliance is single response, which does not require matrix inversion and forward-backward substitution unlike the sensitivity calculations of temperature response. Secondly, thermal compliance is single global response, the minmax temperature formulation needs to retain 500 critical temperature responses to achieve convergence. It is smooth convex function; the optimization converges much quickly than minimizing the maximum temperature of the entire surface.

In topology optimization, applying these conditions is challenging because the structural geometry, including the convection surfaces, constantly varies. Minimizing this functional
by varying the conductivity under a material usage constraint physically corresponds to finding the optimal conductivity distribution that produces less heat when the amount of high conductivity material is limited. Consider a simple 2D heat conductor in which design domain has adiabatic boundaries all around and is subjected to uniform volumetric heat generation and the temperature condition shown in the figure 2-9. [1]

In coupled thermal structural analysis, thermal analysis is performed first to determine temperature field of the structure. The temperature field is used as temperature load for subsequent structural analysis. [1] A single finite element mesh is usually used for both thermal and structural analysis. Thermal analysis affects subsequent structural analysis but structural analysis usually has no influence on thermal analysis. However, in coupled thermal optimization, the optimizer modifies structural design parameters to satisfy constraint with improved objective, which in turn affects thermal analysis. [1] In Optistruct, coupled thermal structure analysis is performed in one single analysis run. Thermal
analysis subcase and coupled structural analysis subcase are put together in the same
deck, where structural used TEMP case control card points to thermal Analysis. Thermal
analysis subcase is solved first to obtained temperature results, which is used as
temperature load in subsequent coupled thermal analysis. The concept of combining heat
transfer and structural topology optimization as is done in these works lends itself well to
the design of exhaust-washed structures. With such capability, topologies may be
obtained with efficient conduction pathways and convective cooling to reduce their overall
temperature levels. This would lead to reduce thermal expansion and thermal stresses,
perhaps with more effect than any pure structurally motivated modifications.

A more common application of coupled thermal-structural topology optimization is
the design of thermally compliant mechanism or micro actuator. Here, the design problem
is to find the optimal topology in the designable region that when subjected to an elevated
temperature maximizes the work done in the spring. We see that for different stiffness
springs, alternative topologies are developed that provide different magnitudes of force
and displacement.

Figure 2-9(a) Thermal actuator design domain [1]
Figure 2-9(b) Optimal topologies for actuator [1]
Chapter 3
Methodology

In this chapter, the focus is on how the model design of a structural heat conductor is done. The structural heat conductor is mainly dependent on both structural and thermal behavior. The design of structural heat conductor varies according to application, in this thesis we are considering three-dimensional rectangular shape structural heat conductor. Along with this it contains 3D solid fine meshing, also various boundary conditions and constraints are applied for the analysis.

3.1 Structural Heat conductor

Structural heat conductor having excellent thermal and structural property which will an advantage for doing density based topology optimization.

3.1.1 Meshing and component

Meshing is one of the important part in the finite element analysis. Mesh is a partition of an arbitrary domain into simple geometrical element. Those elements are compound of edges, faces, nodes and relation between them. Finite element method reduces degree of freedom from infinite to finite with the help of discretization. There are various types of meshing are used based on the type of element. Type of element can be categorized as 1D, 2D, 3D and other. In this chapter our center of attention is on 3D meshing. 3D elements types are further divided into following types.

1) Tetra
2) Penta or wedge
3) Hex or brick
4) Pyramid

Tetra meshing is used in the structural heat conductor because it is very important save
lot of time to run the analysis. Corner nodes are used in the element. This can do using auto mesh feature in the optistruct. Also, it is handy while doing structural analysis.

Following figure shows the structural heat conductor with 3D mesh.

![Figure 3-1 Structural heat conductor with fine meshing](image)

Converting single degree of freedom topology optimization problem into a multi degree of freedom problem is done by using discretizing design domain into finite elements. The amount of volume occupied by geometry is known as design space. It is finely meshed into finite element and iterated as per the optimization algorithm is working. The design space is displayed as reddish-brown color and non-design space is on both end of the structural heat conductor with green color in figure 3-1 respectively. This non-design region remains unchanged even after optimization. Design space can have any shape or topology if it is a single solid volume. The part that is used as design space should be as simple as possible otherwise it takes too long to run the optimization problem. If we increase the number of elements in the design space or if make it more
fine mesh, then that will also take long time for running optimization problem. To save
time for 3D meshing tetras are preferred over hexas in the structural heat conductor
loads, and boundary conditions are not directly applied on the design space it’s because
it often leads to give an incorrect result. Therefore, it is important to have a non-design
space so that we can apply different boundary conditions and loads on that.

3.1.2 Material and property selection

Material and property selection is arbitrary part of any design problem without that it is
impossible to reach out the proper results. In case of structural heat, conductor steel is
the best suitable material and has good linear isotropic properties which we are
considering here. The values for elastic modulus and poisons ratio are $2 \times 10^5$ N/mm$^2$ &
0.30 respectively. These values are entered as input. For both design and non-design
region assigning same material as steel. Finally, material property is assigned to mass.
The P-solid property was assigned. Material property details shown in the following table.

<table>
<thead>
<tr>
<th>Material</th>
<th>Elastic modulus (N/mm$^2$)</th>
<th>Poisson's Ratio</th>
<th>Density (tone/mm$^3$)</th>
<th>Yield Strength (N/mm$^2$)</th>
<th>Ultimate Strength (N/mm$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel</td>
<td>$2.10 \times 10^5$</td>
<td>0.3</td>
<td>$7.89 \times 10^{-9}$</td>
<td>250</td>
<td>420</td>
</tr>
</tbody>
</table>

Table 3-2 material Property

3.1.2 Dimensions

The rectangular structural heat conductor is ready for optimizations once all meshing,
material and property selection are done. The maximum dimensions are 100mm x 50mm
x15mm, the length, width, and height respectively, with each element having mesh size
3-3 Dimensions

3.1.3 Boundary conditions and constraint

After completing all the steps our geometry is ready for applying various loads and boundary conditions. Here, the major aim is doing structural and thermal topology optimization to get a structurally and thermally optimized rectangular structural heat conductor. Figure 3-4 shows structural boundary conditions and figure 3-5 shows thermal boundary conditions.

Figure 3-4 structural Boundary conditions
In structural boundary conditions as we can see from the figure 3-4, Y-Z section is fixed having 6 degrees of freedom which can be applied on the one side of the non-design space. Also, 1000N load is applied on the other side of non-design space at the center in the positive Y-direction. This condition of constraint and boundary condition will continue for all the Volume fractions. In the thermal boundary condition heat flux is applied on some elements in Y-Z section and other side of non-design space is adiabatic (T=0). After applying these constraint and boundary conditions on the structural heat conductor, they are ready for running analysis, before that we must define load cases and load steps in optistruct for each thermal and structural condition. In both thermal and structural analysis we are using same material properties and mesh size will be easier for comparing both results.

Now we are applying both thermal and structural boundary conditions together on structural heat conductor which is shown in the following figure 3.6
3.2 Exhaust-washed structure

For designing exhaust washed structure using topology optimization, goal is to develop stiffening concept for pre-existing thermal structures. It involves coupled physics of heat transfer and structural mechanics. There are variety of exhaust washed structure configurations are possible. To demonstrate this considering an exhaust-washed structure configuration belonging to an Efficient Supersonic Air Vehicle. Following 3-6 figure shows concept of ESAV.

![Efficient supersonic Air Vehicle (ESAV) Concept](image)

Figure 3.6 Combined boundary conditions

![Combined boundary conditions](image)

Figure 3-7 Efficient supersonic Air Vehicle (ESAV) Concept [1]
Exhaust washed structure included aft-deck (exhaust-washed regions on the exterior of the vehicle), adjoining structures and duct nozzle (internal to aircraft). These components are subjected to multidisciplinary environment where both thermal and structural loading produce combine response. Figure 3-7 explains these in detail.

Figure 3-8 Exhaust washes structure [1]

For designing this using topology optimization, considering two dimensional schematics of EWS which shows in the following figure 3-8 with hot gases are flowing on that.

Figure 3-9 2D schematic of exhaust-washed structure [1]
Now we can see how the design of exhaust washed structure via topology optimization is done using optistruct. We can consider from meshing, component, material, properties etc. in detail in next section.

3.2.1 Meshing and Component

As we have seen in the structural heat conductor meshing used is 3D solid meshing, but here we are dealing with two-dimensional strip model. Therefore, meshing is done by using 2D meshing using optistruct. This was done using 2D quad element with minimum size 2.5mm. Also, we differentiate the design domain and non-design domain for applying loads and boundary condition. for 2D meshing there 2 basic element shapes Quad and Tri. Quad is preferred over tri because Tri is CST element. Also, we need an additional data from user is thickness, here we are assuming thickness is 1mm. There are various types of element but out of that we are using thin shell element. Figure 3-9 shows EWS with 2D mesh.

![Figure 3-10 2D strip meshed exhaust washed structure](image)

Purple region consists of design domain and blue region consists of non-design region. Any open region between outer aircraft skins and exhaust-washed nozzle surface could be used as a topology design in which to develop a stiffening structure. Topology
optimization have potential to incorporate all design structure, including sub-structures and supporting.

3.2.2 Material and Property selection

Two-dimensional strip EWS consists of standard steel material having coefficient of thermal expansion is $1.25 \times 10^{-5} \text{C}^{-1}$. In this analysis, we are considering the coefficient of thermal expansion for steel because hot gas passes through it which causes a change in the structure. Therefore, this is one of the important input property while doing optimization. The values for elastic modulus and poisons ratio are $2 \times 10^5 \text{N/mm}^2$ & 0.30 respectively. These values are entered as input. For both design and non-design domain assigning same material as steel. Here we are selecting PSHELL as card image while selecting the property. Also selecting 1mm thickness in the properties.

<table>
<thead>
<tr>
<th>Material</th>
<th>Elastic modulus (N/mm$^2$)</th>
<th>Poisson's Ratio</th>
<th>Density (tone/mm$^3$)</th>
<th>Yield Strength (N/mm$^2$)</th>
<th>Ultimate Strength (N/mm$^2$)</th>
<th>Coefficient of thermal Expansion(1/$^\circ$C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel</td>
<td>$2.10 \times 10^5$</td>
<td>0.3</td>
<td>$7.89 \times 10^{-9}$</td>
<td>250</td>
<td>420</td>
<td>$1.25 \times 10^5$</td>
</tr>
</tbody>
</table>

Table 3.11 Material Property EWS

3.2.3 Dimensions

2D strip EWS consists of 305 mm length and 102mm height respectively. Along with that each element having mesh size 2.5 mm. The thickness of strip in 1mm constant throughout analysis. The dimensions of EWS shown in the figure 3-12
3.2.4 Boundary condition and constraints

As we have seen in the figure 3.8, hot exhaust gas is passed through exhaust washed nozzle and has a temperature of 900°F, therefore we are considering thermal loading on the top face of EWS strip on the entire non-design region. After that constraining both end of non-design domain with six degrees of freedom also called as clamped boundaries. Applying uniformly distributed mechanical load having magnitude of 10N in the positive Y-direction. The primary aim behind applying uniformly distributed load is to design stiffness for restraining expansion. In formulating the topology optimization problem, the minimum compliance objective function is retained with the compliance determined in the absence of thermal loading and subjected to artificial loading. Following 3-10 figure represents the boundary conditions and constraint of EWS.

![Figure 3-13 boundary conditions and constraint of EWS](image)

In the above figure, the blue region is non-design region on which all the boundary conditions are applied to get suitable results in the design domain. If the thermal load is significantly higher than mechanical load or if there is no mechanical load present, then it is difficult to produce suitable designs using minimum compliance as the objective. That is the reason behind assuming 50N is a uniformly distributed load.
3.3 Optimization Problem setup

In this section, methodology obtained while setting up a topology optimization problem. There are various computational approaches. In this work, all the computational work is done using optistruct. While setting up a topology optimization we are using DRDO approach in optistruct which states that

D: Design variable
   Area on which an element should remove a material or change of the space.

R: Responses
   In this we defined the responses for constraint and objective

D: Constraint
   Maximum or minimum value for any variable for example stress, displacement, temperature etc.

O: Objective
   It can be anything that we must achieve either minimize, maximize and minmax or maximin of any variable

In this optimization, process we cannot define more than one objective at time. We are using single objective at time. This thesis focusing on the two different types of problem which are explained in detail in further sections.

3.3.1 Multiobjective problem

In multiobjective problem we mainly consider structural heat conductor. Which will be used to optimized using structural and thermal objective. After that structural analysis is done on structurally optimized model and vice versa. Finally, using thermal constraint and structural objective and vice versa. This will have done for getting combined results of structural heat conductor. So, we can design the system having consideration of both
thermal and structural environment. Topology optimization allows designer to start with design that already has advantage of optimal material distribution and is ready of design fine tuning with shape or size optimization. The optimization problem for 4 different cases of structural heat conductor shown in the following table 3-14, 3-15, 3-16, 3-16

<table>
<thead>
<tr>
<th>Design Variables</th>
<th>• Density</th>
</tr>
</thead>
</table>
| Topology Optimisation Responses | • Volume fraction  
|                   | • Weighted compliance                        |
| Design Constraints | • Volume Fraction  
|                   | • 0.3 ≤ VF ≥ 0.5                            |
| Objective         | • Minimize Weighted compliance                |

Figure 3.14 Structural Optimization Problem

<table>
<thead>
<tr>
<th>Design Variables</th>
<th>• Density</th>
</tr>
</thead>
</table>
| Topology Optimisation Responses | • Volume fraction  
|                   | • Thermal compliance                        |
| Design Constraints | • Volume Fraction  
|                   | • 0.3 ≤ VF ≥ 0.5                            |
| Objective         | • Minimize Thermal compliance                |

Figure 3.15 Thermal Optimization Problem
### Figure 3.16 Structural objective with Thermal Constraint

<table>
<thead>
<tr>
<th>Design Variables</th>
<th>• Density</th>
</tr>
</thead>
</table>
| Topology Optimisation Responses | • Volume fraction  
• Thermal compliance  
• Weighted compliance |
| Design Constraints | • Volume Fraction ($0.3 \leq VF \geq 0.5$)  
• Thermal compliance (26000) |
| Objective | • Minimize weighted compliance |

### Figure 3.17 Thermal Objective with structural constraint

<table>
<thead>
<tr>
<th>Design Variables</th>
<th>• Density</th>
</tr>
</thead>
</table>
| Topology Optimisation Responses | • Volume fraction  
• Thermal compliance  
• Weighted compliance |
| Design Constraints | • Volume Fraction ($0.3 \leq VF \geq 0.5$)  
• Displacement (0.07mm) |
| Objective | • Minimize Thermal compliance |
Setting up a design variable is the same for all types of optimization. For example, thermal, structural etc. because here design variable is density which is constant throughout in the topology optimization. The setup of design variable in Altair Optistruct 14.0 shown in the following figure.

![Figure 3.18(a) Setting up design variable](image)

Once we have finished defining the design variable, the next step is to define design responses. For structural optimization we are defining two responses, first is volume fraction and second is weighted compliance. For thermal optimization, we are defining two responses first are same as in structural optimization, but second is thermal compliance. After that, responses define in the combined optimization, for structural objective and thermal constraint having responses volume fraction, thermal compliance and weighed compliance. For thermal objective and structural constraint having responses same as previous analysis (structural objective and thermal constraint) except thermal compliance we are using temperature response. Volume fraction values taken for this analysis is 0.3, 0.4 and 0.5. setting up design response in Altair optistruct 14.0 shown in the following figure.
### Voltec

<table>
<thead>
<tr>
<th>response</th>
<th>Voltec</th>
<th>6</th>
<th>no regional</th>
</tr>
</thead>
<tbody>
<tr>
<td>response type</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>weighted</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Tempcom

<table>
<thead>
<tr>
<th>response</th>
<th>Tempcom</th>
<th>6</th>
<th>no regional</th>
</tr>
</thead>
<tbody>
<tr>
<td>response type</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>thermal</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Temper

<table>
<thead>
<tr>
<th>response</th>
<th>Temper</th>
<th>6</th>
<th>no regional</th>
</tr>
</thead>
<tbody>
<tr>
<td>response type</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>temperature</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Trip Disp

<table>
<thead>
<tr>
<th>response</th>
<th>Trip Disp</th>
<th>6</th>
<th>no regional</th>
</tr>
</thead>
<tbody>
<tr>
<td>response type</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>client displacement</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Weighted

<table>
<thead>
<tr>
<th>response</th>
<th>Weighted</th>
<th>6</th>
<th>no regional</th>
</tr>
</thead>
<tbody>
<tr>
<td>volume</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>total</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Constraining the one or more responses is one of the important factor in the optimization process without that it is impossible to reach out suitable results. In topology optimization, our main aim is to constrain volume fraction because we must remove unwanted material. Therefore, for structural and thermal optimization volume fraction is constraint.

For combined optimization we are constraining one more variable along with volume fraction. Firstly, for structural objective constraining the thermal compliance of value 26000 and for thermal objective constraining the displacement of value 0.07. which will give a more correct result of combined optimization. Following figure shows setup of optimization constraint in Altair Optistruct 14.0
Setting up optimization objective

Setting up an optimization objective is final step in the formulation of optimization process. In structural heat conductor, we are dealing with mainly two objectives which are structural and thermal objective. For structural objective, we are defining minimizing weighted compliance and thermal objective we are defining minimizing thermal compliance. Following figure 3.15(d) shows setup of optimization objective in Altair optistruct 14.0.

![Figure 3.18(d) setting up optimization problem](image)

3.3.2 Multidisciplinary loading

The introduction of finite element analysis to thermal structures also enables application of optimization method and automated design practiced by the structural and multidisciplinary design optimization. Design of thermal structures for elevated temperature application yields two basic design rules: 1) accommodate thermal expansion 2) minimize temperature and gradient. Here we are considering the example of exhaust washed structure in which hot exhaust gases creates an extreme thermal-structure design environment as it is ducted to rear of the aircraft. In this environment, damaging effects of elevated temperature including excessive deformation, thermal buckling, creep and thermal stresses. The optimization problem for exhaust washed
structure shown in the following figure 3.18

![Figure 3.19 EWS optimization problem](image)

The formulation investigated is the basic minimum compliance objective with a volume fraction constraint. In analysis of EWS, we are using thermal loading, constraint and uniformly distributed mechanical load. Following steps shows setting up an optimization problem.

(a) Setting a design variable

For EWS analysis, density is design variable for optimization problem. The setting up design variable in Altair optistruct 14.0 shown in following figure.

![3.19(a) setting up design variable](image)
(b) Setting up a response

Optimization of EWS contains two responses which are volume fraction and weighted Compliance. Setting up an optimization response shown in the following figure 3.16(b)

![Response setup](image)

Figure 3.19(b) setting up a design response

(b) Setting up a design constraint

The setup of optimization constraint in as in Altair optistruct 14.0 shown below in following figure

![Constraint setup](image)

Figure 3.19(c) setting up a design constraint

(c) setting up an optimization

The setup of optimization objective as in Altair optistruct 14.0 shown in the following figure
figure 3.18(d) setting up optimization objective
Chapter 4
Case studies

For getting detailed insights about coupled thermal structures topology optimization, here considering two case studies which are as follows: 1) simple structural heat conductor 2) Exhaust washed structure.

4.1 Structural heat conductor

In structural heat conductor we mainly perform two operations structural optimization and thermal optimization. As we have seen, methodology and setting up an optimization problem for structural heat conductor in chapter 3. We mainly perform the following analysis sequentially.

1) Structural optimization
2) Thermal optimization
3) Structural objective and Thermal constraint
4) Thermal objective and structural constraint

Now we can see step by step detailed analysis of structural heat conductor optimization problem.

4.1.1 Structural optimization

As we have seen in the chapter 3 figure 3.2 all the loads and boundary condition are Applied on heat conductor. As per the optimization problem define in the figure 3.11 structural optimization is done using in the Altair optistruct 14.0 which will give an structurally optimize structural heat conductor (VF=0.3) which shown in the following figure 4.1
Figure 4.1 Structurally optimized structural heat conductor

As seen in the above figure 4.1, unwanted material is removed from the heat conductor with the 0.3 volume fraction value. As we know the one side of the SHC is fixed with six degrees of freedom and other end corner force is applied center of the non-design region, that point having maximum stress and displacement which causes less material is removed from that point. On the other hand, SHC is subjected to bending therefore stress is more along x-z plane which shows more amount of material is present in the optimized part. Also, the right hand upper corner material is completely removed because there is no bending stress is present. Apart from this due bending more amount of stress is acting on the both end of the constraint root having more material, and which will starts reducing after point. The main motivation behind this research to study the domain with combined loading, to reach out validation of combined optimization we must prove that a structurally optimized system is thermally weak and thermally optimize system is structurally weak. Therefore, we must design the system like they can withstand in the both thermal and structural environment. To prove the above statement, we have to the
further analysis of the structurally optimized model. We mainly doing following two analyses on this optimized beam.

1) Structural analysis on structurally optimize heat conductor

2) Thermal analysis on structurally optimize heat conductor

For doing this further analysis we have to apply the both loads and boundary condition on the structurally optimized model. Following figure 4.2 (a) & (b) shows the applied loads and BC.

Figure 4.2(a) Structural Boundary conditions and constraint on structurally optimized heat conductor
Figure 4.2 (b) Thermal boundary condition and constraint on structurally optimized heat Conductor

Once we have completed applying both types of loads and boundary conditions on structurally optimized heat conductor, it is ready for running analysis in the Altair optistruct 14.0. As we know from previous chapter structural analysis having minimized weighted compliance objective and thermal analysis having minimized thermal compliance as objective. Here, we are dealing with parameters which are measured as objective function. For weighted compliance we are dealing with displacement of the model and for thermal compliance we are dealing with temperature. So, our focus on these values while doing analysis. Also, we find out von mises stress in the structural heat conductor while doing structural analysis. The reason behind considering the displacement as output of weighed compliance is shows in the following equations.

In optistruct compliance is strain energy. In general, when force is applied, displacement is results. Compliance, force, displacement can be represented in the following equation.

\[ C = FT \times U \]
Where

U is displacement

FT is transpose of force

C is compliance (strain energy)

The weighed compliance is the method used to consider multiple subcases, such as load step and load cases in the topology optimization. For checking effect of many subcases we use weighted compliance. The response is weighed sum of each individual subcase, this would typically input a weighting factor for compliances for individual subcases, this factor is multiplied with individual compliance of each subcase and then this product is added together.

As we have seen in the figure 4.2 (a) & (b) both analysis is performed simultaneously for Volume fraction 0.3. Result of structural and thermal analysis on structurally optimized heat conductor shown in the following figures.4.3 (a), (b) & (c)

![Figure 4.3 (a) displacement plot (top view)](image-url)
Values of both results shown in the following table

| Structural and Thermal analysis on structurally optimized beam (VF:0.3) |
|---------------------------------|-----------------|-----------------|
| Temperature (°C)                | Displacement (mm) | Stress at constraint root (N/mm²) |
| 19.88                           | 3.563 x 10⁻²      | 4.191 x 10¹      |

Table 4.4 Analysis results for structurally optimized heat conductor
As we seen in above table value of temperature is higher than the expected, but the values of stress and displacement are within the expected range. Our focus is on displacement to get stiffer design, we are designing structures for minimum structural compliance which is reciprocal of stiffness. Therefore, our aim is to reduce the displacement is the structure to get stiffer structure. To get a better understanding of this concept we must do further analysis. The next step is to do thermal optimization. Which will be explained in detail in next section.

4.1.2 Thermal Optimization

As discussed in the chapter 3, once the thermal loads and boundary conditions are applied on the structural heat conductor as shown in figure 3.5. after that we define optimization problem as shown in figure 3.12. Using that problem thermal optimization is done in Altair optistruct 14.0 on structural heat conductor which will give a following result shown in the figure 4.5

![Thermally optimize structural heat conductor](image)

Figure 4.5 Thermally optimize structural heat conductor [23]
As shown in the above figure, the unwanted material is removed in the optimization process which will result in a thermally optimized structural heat conductor. Due heat flux is applied at center of the one end temperature is distributed throughout the SHC from the point where heat is applied whereas other end having adiabatic condition because of this less material is removed at that end and more material is removed at the end where heat flux is applied. Material is not removed where point of application of heat flux and temperature value varies along the z direction which also plays an important role in the removing material. Same as in the previous Section, here we are doing structural and thermal analysis on thermally optimized beam to prove that structurally optimized system is thermally weak and vice versa. Once done with optimized model now apply both types of constraint and boundary conditions above model. Following figure 4.5 (a) & (b) shows applied loads and boundary conditions.

![Figure 4.6 (a) Thermal boundary condition and constraint on thermally optimized heat Conductor [23]](image-url)
Figure 4.6 (b) Structural Boundary conditions and constraint on thermally optimized heat conductor.

After both thermal and structural loads are applied, it is ready to run the for analysis in Altair optistruct 14.0 using minimum thermal compliance as thermal objective which will be measure in terms of temperature values. Now following figure 4.6 (a), (b) & (c) shows the both analysis results of thermally optimized structural heat conductor.

Figure 4.7 (a) displacement plot (top view)
Figure 4.7 (b) Stress plot (top view)

Figure 4.7 (C) Temperature Plot (top view) [23]

The resulting values of above result shown in the following table 4.7

<table>
<thead>
<tr>
<th>Temperature (°C)</th>
<th>Displacement (mm)</th>
<th>Stress at constraint root (N/mm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.5</td>
<td>2.258 x 10⁻¹</td>
<td>9.493 x 10¹</td>
</tr>
</tbody>
</table>

Table 4.8 Analysis results for thermally optimized heat conductor
As we have seen in table 4.4 and table 4.7 which shows that temperature value on the structurally optimize structural heat conductor is much higher as compared to the temperature on thermally optimized model. On the other hand, structurally optimized heat conductor having higher displacement and stress value as compared to thermally optimized heat conductor. Due to higher displacement in thermally optimized SHC then it will be less stiff compared to previous structurally optimized SHC Hence, it is proving that Thermally optimized heat conductor structurally weak and vice-versa. Therefore, we must go for combined analysis to get an optimized system which are structurally and thermally strong.

4.1.3 Combined optimization

In combined optimization, we are dealing with following two analyses on the structural heat conductor which are as follows:

1) Structural objective with thermal constraint

2) Thermal objective with structural constraint

In chapter 3, we have seen the detail study about structural heat conductor dimension, component, material selection and properties also all the boundary conditions and constraints are applied on that. To design the system which are stable in both thermal and structural environment. As seen in the figure 3… both thermal and structural boundary conditions are applied together, once we have done with this as shown in the figure 3.13 & 3.14 optimization problem is defined. Displacement is constrained in the thermal optimization and thermal compliance is constrained in the structural optimization to get the system which are suitable in both types of conditions. Firstly, consider the case of structural objective with thermal constraint, after optimization done on structural heat conductor, optimized model shown in the following figure 4.8
Figure 4.9 Structurally Optimized (combined) structural heat conductor

As seen in above figure 4.8 material is removed from the locations model which was less as compared to the individual optimized model, less material is removed in combined optimization. Due to heat flux is applied on the positive x direction on the non-design region which will not remove material along x direction. Also, it is giving more uniform distribution of material as compared to individual optimization due to this SHC giving more strength and less displacement. After that, all the loads and boundary conditions are applied on the optimized structural heat conductor which shows in the following figure

Figure 4.10 combined boundary condition and constraint on structurally optimized
Furthermore, we are doing analysis on structurally optimized beam using thermal compliance as constraint (26000), which will produce the following results of displacement, stress, and temperature shown in the following figure 4.10 (a), (b) & (c).

4.11 (a) displacement contour (Top View)

4.11 (b) stress Contour (Top View)
Results shown in the above figure 4.10 are all for 0.3 volume fraction, from above results it is clear structural heat conductor gives suitable results in combined optimization and values of displacement, stress and temperature are within expected values which shows the combined optimization is better compared to individual optimization. Following table 4.11 shows results values for combined optimization (structural objective).

| Structural analysis using thermal constraint on optimized heat conductor (VF:0.3) |
|---------------------------------|-----------------|-----------------|
| Temperature (°C)                | Displacement (mm) | Stress at constraint root (N/mm²) |
| 13.22                          | 3.450 x 10⁻²     | 4.476 x 10¹     |

Table 4.12 Structural analysis using thermal constraint on optimized heat conductor

From above table 4.12 temperature value is reduce as compared to single discipline structural analysis, which shows that system behaves significantly under both loading. Displacement and stress values are nearly same as single discipline structural analysis. Displacement values are same even after adding thermal loads and boundary condition. Structural heat conductor having less displacement in the combined loading condition
and more uniform material distribution which will give the stiffer structure under multidisciplinary loading.

Now we are focusing on second analysis, which is using thermal objective and structural Constraint. As studied in chapter 3 we initially define combined optimization problem (Thermal Objective) shown in the figure 3.14. After that, we perform optimization on structural heat conductor using thermal objective and structural constraint (displacement) which will give a following result shown in figure 4.13

![Figure 4.13 Thermally optimized (combined) structural heat conductor](image)

Again, optimized model is ready to perform Thermal analysis using structural constraint.

To perform this, we must apply all loads and boundary condition aging on optimized part which shows in the following figure 4.13
Figure 4.14 Combined loads and Boundary conditions on thermally optimized heat conductor

Boundary conditions and constraint applied is done then it is ready to perform analysis of this thermally optimized conductor which will produce the following contour shown in the figure 4.15 (a), (b) & (c)

Figure 4.15 (a) Displacement contour (top View)
From above all the optimized contour it clearly shows that it is having more material as compared to single discipline Thermal optimization. Following result table 4.15 shows the values of temperature, displacement, and stress contour.
 Thermal analysis using structural constraint on optimized heat conductor (VF:0.3)

<table>
<thead>
<tr>
<th>Temperature (°C)</th>
<th>Displacement (mm)</th>
<th>Stress at constraint root (N/mm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>12.69</td>
<td>2.361 x 10⁻¹</td>
<td>1.995 x 10²</td>
</tr>
</tbody>
</table>

Table 4.16 Thermal analysis using Structural constraint on optimized heat conductor

Above table represents temperature value which will be slightly increased as compared to single disciplinary thermal optimization and displacement and stress value is reduced slightly which proves combined optimized model having more strength compared to single optimized conductor, that is shown in table 4.11 and 4.7. Temperature is slightly increases due to structural boundary conditions are added which causes small rise in the temperature in the SHC. Also, reduction in displacement values causes increase in stiffness which is our structural objective.

4.2 Exhaust washed structure

In Exhaust washed structure we are dealing with design of thermal structure. The structures are subjected to elevated temperature has long been an important area of study in aerospace industry. Here, we are focusing on beam strip stiffening application, considering 2D beam strip structure over that hot gas is passes having temperature of 900 °F. Initially, applying thermal load and constraining both ends of the non-design region as shown in the figure 3.10. Once we have finished with boundary condition and constraint optimization, the problem is set in Altair optistruct 14.0 as shown in the figure 3.16, once we perform optimization it will produce the following results shown in the 4.12
Figure 4.17 ISO contour

As seen in above figure only non-design region remains after optimization which shows that it will not produce suitable design. This indicates that existence of the elements and accompanying increased thermal loads, serve to only increase compliance. From results in case of significant thermal loads and absence of mechanical effects, the minimum compliance topology optimization is unable to produce suitable design. Hence, for getting suitable design of exhaust washed structure design we are applying uniformly distributed mechanical load of 10N on the non-design region. Again, we are doing same optimization problem with minimum compliance for volume fraction 0.2 which will produce the following results shown in the figure 4.18 (a), (b), (c)

Figure 4.17(a) Density Contour
As we have seen in above figure 4.1, it is obvious that potentially useful stiffening structures are obtained. This is possible because well posted minimum, compliance with purely mechanical loading problem was utilized. As shown in above figure optimum structures span the entire depth of the designable region and contain lower inverted arch structure. This inverted arch is connected to upper non-design region at locations where the factious mechanical load is applied. By creating this stiffener, it will help to reduce thermal stress in the structure by adding adjoining component and fasteners.
Chapter 5
Results

5.1 Structural heat conductor results
For structural heat conductor we are producing results of displacement, displacement with displaced contour and temperature. The optimization analysis is done using Altair 14.0 optistruct. We have already seen the optimization and analysis results for volume fraction 0.3 in the previous chapter. Here we are looking results for various volume fraction and all types of optimization results.

5.1.1 Structural Optimization
The optimization for structural heat conductor is done as per defined in the chapter 3 and 4. In this part, we are observing various results of structural optimization for different volume fraction. Firstly, we are observing the structural analysis results on structurally optimized conductor shown in the following figures.

Structural Analysis on structurally optimized conductor
For Volume Fraction 0.3

(a) Displacement contour
(b) Displacement with displaced contour
Figure 5.1 structural Analysis Results on structurally optimized conductor for 0.3 VF

For volume Fraction 0.4

Figure 5.2 structural Analysis Results on structurally optimized conductor for 0.4 VF
For Volume fraction 0.5

![Displacement Contour](image1)

(a) Displacement Contour

![Displacement with displaced contour](image2)

(b) Displacement with displaced contour

![stress Contour](image3)

(c) stress Contour

![Cross section along z-direction](image4)

(d) Cross section along z-direction

Figure 5.3 structural Analysis Results on structurally optimized conductor for 0.5 VF

As we can see from above structural analysis on structurally optimized SHC for 3 different VF which shows that displacement reduces as we increase VF shown in the figure (b) of all VF. Also, the displacement is more at corner where force is applied and stress concentration area showing with red and yellow region in the figure. We perform the analysis on the various VF to check the structural stability of SHC in the various types of material distribution and the topologies.

Now we perform structural analysis on thermally optimized conductor in Altair optistruct 14.0. As discussed in the previous chapter, structural boundary conditions are applied on thermally optimized conductor. After that, analysis is performed using minimum weighted compliance as objective which will give a following result for various volume fraction.

For Volume Fraction 0.3
Figure 5.4 structural Analysis Results on Thermally optimized conductor for 0.3 VF

For Volume Fraction 0.4
Figure 5.5 structural Analysis Results on Thermally optimized conductor for 0.4 VF
For Volume Fraction 0.5

Figure 5.6 structural Analysis Results on Thermally optimized conductor for 0.5 VF
We have performed structural analysis on the thermally optimized SHC to check the
Thermally optimized SHC structurally robust or not. For that analysis is performed of
various VF for example 0.3, 0.4, 0.5 etc. as shown in the above figure. In all type of VF,
we get the solid cross section result. As we are designing for the minimum structural
compliance displacement is the major factor in it. Which is slightly higher in the thermally optimized SHC that is reason they are less robust as compare to structurally optimized SHC. Even with increase in VF displacement reduces as seen in above contour which gives more stiffer structure but this is less stiff as compared to structurally optimized SHC.

5.1.2 Thermal Optimization

Topology optimization using thermal objective is an important factor in the structural heat conductor. Thermal optimization is done based on the all the boundary conditions, constraint and optimization problem defined in the chapter 3 and 4. In thermal optimization, a measure parameter is to check the temperature of the structural heat conductor. Once optimization is done, thermal analysis is performed in Altair Optistruct 14.0 with objective of minimum compliance which produced following results shown in figure 5.4

For Volume Fraction 0.3

![Figure 5.7 Thermal Analysis Result on Thermally optimized conductor for 0.3 VF](image)

For Volume Fraction 0.4
Figure 5.8 Thermal Analysis Result on Thermally optimized conductor for 0.4 VF [23]

For Volume Fraction 0.5

Figure 5.9 Thermal Analysis Result on Thermally optimized conductor for 0.5VF [23]

As seen in above 3 figures for various VF temperature contour, shows that temperature is reduces as we increase the amount of material in the design space but all VF giving maximum temperature at the end where heat flux is applied, and it gradually decreases to zero at the other end having adiabatic boundary condition.

Now performing thermal analysis on structurally optimized structural heat conductor. As we have seen in the previous chapter thermal analysis is done as per the methodology and problem definition in the chapter 4 and 5 which will produce the following temperature plot shown in the figure below.
For Volume Fraction 0.3 [23]

Figure 5.10 Thermal Analysis Result on structurally optimized conductor for 0.3 VF

For Volume Fraction 0.4 [23]

Figure 5.11 Thermal Analysis Result on structurally optimized conductor for 0.4 VF

For Volume Fraction 0.5 [23]
Figure 5.12 Thermal Analysis Result on structurally optimized conductor for 0.5 VF

In structurally optimized SHC temperature is much higher in case of 0.3 VF as compared to 0.4 and 0.5 VF. As we can see form temperature contour red region indicates highest temperature among entire SHC. As we increase VF we get different topologies for same boundary conditions and constraint. Which helps to understands temperature behavior on both type of environment.

5.1.3 Combined optimization results

Once we are done with individual optimization and analysis on optimized structural heat conductor, this proves that thermally optimized system is structurally weak vice-versa. So, next step is to do the combined analysis which define in the chapter 3 and 4 in detail from we got the following results shown in figure

Structural objective using thermal constraint (Thermal Compliance: 26000)

For Volume Fraction 0.3
(c) stress Contour  
(d) Cross section along z- direction

(d) Temperature plot

Figure 5.13 structural objective using thermal constraint on conductor for 0.3 VF

For volume Fraction 0.4

(a) Displacement Contour  
(b) Displacement with displaced contour
Figure 5.14 structural objective using thermal constraint on conductor for 0.4 VF

For Volume Fraction 0.5
As we can see various contours for different types of VF shows the combined analysis results of SHC using structural objective and thermal constraint. Here we are constraining the thermal compliance to design the SHC in structural point of view also in the thermal environment which shows in the above results. As comparing these results with single disciplinary problem having same displacement even with multiple loading which prove that SHC stiffer when we design using combined problem. Also, the topology distribution is more uniform compare to previous result. Temperature values are reducing which was much higher in the single optimization. Temperature profile trend is same which is decreasing with increasing VF. All the VF having solid cross section. And stress values are reduced as VF increased.

Thermal Objective Using structural constraint (Displacement: 0.07)

For Volume Fraction 0.3
5.16 Thermal Objective using structural constraint for VF 0.3

For Volume Fraction 0.4
5.17 Thermal Objective using structural constraint for VF 0.4

For Volume Fraction 0.5

(a) Displacement Contour                             (b) Displacement with displaced contour

(c) stress Contour                             (d) Cross section along z- direction

(e) Temperature plot

5.18 Thermal Objective using structural constraint for VF 0.5

In combined Thermal objective and structural constraint, we are constraining displacement at 0.07 to get the displacement which should be less than this to satisfy the design constraint. Displacement values are higher as compared to the structural objective
and similarly, it is showing increasing trend as we increase the VF without violating constraint. For the temperature it is slightly increases compared with single optimization but then in combined optimization we combined both BC and constraints which showing trend of decreasing value of temperate as VF increases. Stress concentration is more at the constraint of the root as we can see from the above images. Also, they are giving solid cross section in all VF.

Result Table

<table>
<thead>
<tr>
<th>VF</th>
<th>Thermally optimized</th>
<th>Structurally optimized</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( T_{\text{max}} ) (°C)</td>
<td>Displacement (mm)</td>
</tr>
<tr>
<td>0.3</td>
<td>9.5</td>
<td>( 2.25 \times 10^{-1} )</td>
</tr>
<tr>
<td>0.4</td>
<td>7.3</td>
<td>( 1.25 \times 10^{-1} )</td>
</tr>
<tr>
<td>0.5</td>
<td>6.4</td>
<td>( 7.45 \times 10^{-2} )</td>
</tr>
</tbody>
</table>

Table 5.18(a) Structural and Thermal optimization

<table>
<thead>
<tr>
<th>VF</th>
<th>Combined Thermal Optimization (Thermal Objective)</th>
<th>Combined Structural Optimization (Structural Objective)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( T_{\text{max}} ) (°C)</td>
<td>Displacement (mm)</td>
</tr>
<tr>
<td>0.3</td>
<td>12.69</td>
<td>( 2.36 \times 10^{-2} )</td>
</tr>
<tr>
<td>0.4</td>
<td>9.36</td>
<td>( 1.31 \times 10^{-3} )</td>
</tr>
<tr>
<td>0.5</td>
<td>7.549</td>
<td>( 5.19 \times 10^{-2} )</td>
</tr>
</tbody>
</table>

Table 5.18(b) Combined Optimization

5.2 Exhaust Washed structure

The aim for designing exhaust washed structure via topology optimization is to study the coupled thermal and structural environment to design a domain to withstand in both conditions. As we have seen in the chapter 3 and 4 optimization problems, this is defined
and solved in the Altair optistruct 14.0 with minimum weighted compliance as objective to give more stiffness to structures. Results of 2D beam strip exhaust washed structure shown in following figure for various volume fraction.

For volume Fraction 0.2

For Volume Fraction 0.25

(a) Displacement Contour (c) stress Contour

(d) Density plot (e) ISO plot
In EWS our goal is to design a structure to reduce thermal stresses which causes due to high temperature gas passing through it. After analysis, we got results for various VFs such as 0.2, 0.25, 0.30, as we can see there is very less variation in the VF similarly results having very less variations. In the density red region, material should be there, which is called stiffeners which help to reduce the thermal stresses. As we increase VF which will give more fine stiffeners which can clearly be viewed in the ISO plots.
contour. Displacement and stress values are decreases as we increase VF. Displacement is maximum at center shown in the red region and stresses are maximum where the stiffeners are created to withstand in that condition.

Result Table

<table>
<thead>
<tr>
<th>Volume Fraction</th>
<th>Displacement (mm)</th>
<th>Stress (N/mm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.20</td>
<td>8.402 x 10⁻¹</td>
<td>5.995 x 10⁴</td>
</tr>
<tr>
<td>0.25</td>
<td>6.798 x 10⁻¹</td>
<td>4.935 x 10⁴</td>
</tr>
<tr>
<td>0.30</td>
<td>5.865 x 10⁻¹</td>
<td>4.393 x 10⁴</td>
</tr>
</tbody>
</table>

Table 5.20 EWS result table

As we have seen in both result tables, the displacement and stress value decrease as we increase the volume fraction (more material is added). In result table 5.12, temperature values are less in thermal optimization, but it drastically increases in structural optimization. Similarly, displacement and stress values are less in structural optimization while it is larger in thermal optimization which proves structurally optimized domain is weak in thermal environment and vice versa. So, we perform the combined optimization to get better results which are shown in table 5.12.
Chapter 6
Conclusion and future work

6.1 Conclusion

The research work divided in two parts where our main objective is to reduce volume of design space using topology optimization for reducing cost. Additive manufacturing is technology that will enable engineers to easily create complex geometries allowing optimized design to directly translated into physical design. Firstly, design of structural heat conductor using topology optimization. In the study, we perform various analyses to reach the conclusion which are as follows

1) Structurally optimized design space thermally weak.
2) Thermally optimized design space structurally weak.

To overcome this, we have performed combined optimization using coupled thermal structural boundary conditions and constraint, using thermal objective with structural constraint as displacement and structural objective with thermal constraint as objective which produces the suitable results to withstand in all types of environment.

Secondly, we have design EWS using topology optimization as in application of coupled thermal structural problem. The fundamental design goal is to reduce thermal stress produced due to hot gas as it passes over it. It is impossible to reduce thermal stresses directly from the structure. A characteristic of beam strip structure the breakdown of typical formulation of topology optimization was demonstrated for thermo-elastic structure whose thermal loads are significant in comparison to mechanical loads. Therefore, we added mechanical loading which gives appropriate stiffening configuration for thermal structures which are in practical application adjoining structure and fasteners.
Coupled thermal structure having wide range of application, majorly used in aerospace industry where have maintain temperatures in the structures. Along with that future application in automobile, civil and machinery sector. The presented idea is helpful for designers to overcome problem.

Chapter 6.2 Future work

Topology optimization technique is rapidly developing in all aspects of the design. Here we have done the analysis using Altair optistruct 14.0. Topology optimization for thermal-structure loading is in its developing stage, with still a lot of research going on in this area. In this investigation, mainly focusing on simple rectangular heat conductor. We can implement this idea on complex model for example hollow cylinder. Also, we only consider static loading condition. In the future, studies can be done for dynamic loading to get a clear picture about real world problems. For thermal analysis, we can use this idea for transient system optimization. Future studies will include incorporation of AM design constraint directly into topology optimization algorithm. As additive manufacturing is widely used in the industry, topology optimization will grow to support constraints for AM process. While in design of exhaust washed structure, our aim is to reduce thermal stresses for that we must implement different techniques such as application of mechanical loads. Therefor there is lack of direct treatment of stresses even though our primary design goal is to reduce stresses. Also, we can use complex 3D model in future study instead of simple 2D beam strip model for analysis which will give more clear view of the structures as compared to part we are considering here.
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Biographic Information

Vaibhav Gaikwad received his Bachelor’s degree in Mechanical Engineering from University of Mumbai, India in July 2014. He has work experience as a production executive from July 2014 to May 2015 in Valvoline Cummins. He, then, decided to pursue his Master of Science in Mechanical Engineering from University of Texas Arlington in Fall 2015. He has been actively involved in the research and interest in FEA, Optimization and Design. He also gives an inspiration and motivation to new students in the same area. Vaibhav Gaikwad plans to pursue a career in the direction where his experience and expertise will be utilized.