CHARACTERIZING COLLEGE ALGEBRA STUDENTS'

MATHEMATICAL PROBLEM SOLVING

by

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Abstract

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This study examines the mathematical problem solving (MPS) practices of students enrolled in College Algebra at a large urban university in the southwestern United States. The primary research question explores how to characterize the MPS techniques, strategies or orientations used by College Algebra students. In addition, this study documents MPS approaches that appear to be most prevalent and examines how these approaches relate to student performance. A grounded theory approach is used to formulate a theory for characterizing the MPS of students in College Algebra. Data analysis shows that multiple student-held orientations identified in this theory correlate with improved performance in College Algebra with MPS techniques, strategies, or orientations being influenced by affective factors and problem types.

This qualitative study examines 19 MPS student interviews during which students answered questions about their typical approaches in MPS and discussed their solutions to mathematics problems completed both before and during the interview. Open-coding (Corbin & Strauss, 2008) was used to analyze recordings, transcriptions, and student work from the interviews. The MPS techniques, strategies, or orientations naturally arising from the data formed the basis for the theory formulated in this study. Of the eight orientations observed, three common primary orientations emerge from the data with the other five being used much less-commonly across student interviews. For each of the

three common primary orientations—formula application, reexamination, and big-picture focus—a particular set of strategies and techniques typically accompany the observed orientation. The formula application and reexamination orientations align with successful grade outcomes in College Algebra. Additional analysis reveals that strategies and techniques vary according to the type of problem presented, and the higher number of strategies used by a student correlates with higher College Algebra course grade outcomes. The characterization of the MPS techniques, strategies or orientations used by College Algebra students and how these relate to course grade outcomes raise important questions regarding the behaviors rewarded with success in College Algebra and the actual MPS capacity needed to succeed in a science, technology, engineering, or mathematics career path.

Table of Contents

List of Illustrations

List of Tables

Chapter 1

Introduction

With nationwide averages indicating that only 50% of the students enrolled in College Algebra earn a grade of C or better in the course, growing concern arises from this alarming failure rate (Saxe & Braddy, 2015). In particular, with current shortfalls in students completing science, technology, engineering, and mathematics (STEM) degrees, the fate of STEM students beginning in College Algebra underscores the need to understand the learners in College Algebra as well as strategies for increasing student success in this course (President's Council of Advisors on Science and Technology, 2012). With a 50% success rate, this allows only half of the 1 million students who enroll in College Algebra each year into the STEM pipeline at colleges and universities in the United States (Gordon, 2008). For those who are successful in College Algebra, Jarret (2000) found that only 6% to 8% later enrolled in Calculus I with only 35% of these earning a B or better in Calculus. Dunbar (2005) found that only 10% of the College Algebra students eventually enroll in Calculus I. Thus, for some time now, researchers have been examining possible changes to College Algebra courses as well as pathways to improving performance (Edwards, 2011; Wills, 2011; Ganter & Haver, 2011).

In determining avenues for increasing success rates in College Algebra and potentially increasing the numbers of students moving through the pipeline to Calculus, a look beyond the skills-based focus of many College Algebra courses to the mathematical thinking and problem-solving capacity being developed as a foundational pathway to Calculus provides an important lens for capturing the learners in College Algebra (Álvarez, Rhoads, & Campbell, 2017). Current mathematical problem solving (MPS) research consists of studies on secondary school students (e.g. Montague & Applegate, 2013), university students (e.g. Bookman & Friedman, 1994; Dawkins & Epperson, 2014), and graduate students and faculty (e.g. Carlson & Bloom, 2005). Although recent research (c.f. Rhoads, Epperson, Campbell, 2017; Epperson, Rhoads, & Campbell, 2016) aims to capture College Algebra students' MPS capacity as a way to identify non-procedural and non-conceptual content-based explanations for success in College Algebra as a pathway to Calculus, no studies attempt to develop a theory that characterizes College Algebra students' MPS. This study explores College Algebra students' engagement in MPS as way to characterize their use of MPS. The primary research question examined is:

> How can we characterize the mathematical problem-solving (MPS) techniques, strategies or orientations used by College Algebra students?

The sub-questions examined are:

1. What approaches appear to be most prevalent and how do these approaches relate to student performance in College Algebra?
2. How do affective factors impact College Algebra students' MPS?

Ultimately, understanding the learners in critical gateway courses to STEM may lead to revising curriculum and refining instruction in a manner that addresses key challenges that must be overcome and important student capacities that can be further developed.

In this study, a grounded theory approach is used to formulate a theory for characterizing the MPS of students in College Algebra. Data analysis shows that multiple student-held orientations identified in this theory correlate with improved performance in College Algebra with MPS techniques, strategies, or orientations being influenced by affective factors and problem types. The data consists of 19 MPS student interviews during which students answered questions about their typical approaches in MPS and discussed their solutions to mathematics problems completed both before and during the interview. The MPS techniques, strategies, or orientations naturally arising from the data formed the basis for the theory formulated in this study. Of the eight orientations observed, three common primary orientations emerge from the data with the other five being used much less-commonly across student interviews. For each of the three common primary orientations—formula application, reexamination, and big-picture focus—a particular set of strategies and techniques typically accompany the observed orientation. The formula application and reexamination orientations align with successful grade outcomes in College Algebra. Additional analysis reveals that strategies and techniques vary according to the type of problem presented, and the higher number of strategies used by a student correlates with higher College Algebra course grade outcomes.

The orientations held by students appears to change based on affective factors observed within the student interviews. These students show more instances of a positive attitude or positive beliefs about doing mathematics when not exhibiting a formula application orientation. Also, these students use different strategies and techniques from those exhibiting no or very few positive affect codes when working contextual problems compared with working on non-contextual problems that are driven by mathematical notation.

The characterization of the MPS techniques, strategies or orientations used by College Algebra students and how these relate to course grade outcomes raise important questions regarding the behaviors rewarded with success in College Algebra and the actual MPS capacity needed to succeed in a science, technology, engineering, or mathematics career path.

Chapter 2

Literature Review

2.1 Introduction

The research literature describes mathematical problem solving (MPS) in various ways; the question “What is mathematical problem solving?” may elicit distinct responses from different people. Campbell (2014) classified explicit or implicit definition of MPS used in the research literature which led to a characterization of these definitions into broad categories. These categories or domains were further refined by Epperson, Rhoads, and Campbell (2016). Four of these categories are used to frame the discussion of MPS. Each of sense-making or orienting, representing/connecting, reviewing or checking, and justification, describe a component of MPS arising from the research literature. An additional component of MPS defined in Epperson, et al. (2016) associates the difficulty of a problem with problem solving. That is, a problem that is too difficult or too easy may impede MPS or make it difficult to observe. The first four categories relate to students’ MPS behaviors whereas the difficulty category does not describe a behavior although it affects behavior. The MPS categories discussed in Sections 2.3-2.7 are not necessarily mutually exclusive but provide broad domains from which to consider various definitions and ways of thinking about MPS.

The existing knowledge of students alters the MPS behaviors and practice employed by the students (Wilson, Fernandez, & Hadaway, 1993). Since this study examines MPS by students in College Algebra, understanding curricular and learning issues in algebra must naturally accompany attempts to characterize students’ MPS in College Algebra. Significant work exists in the area of algebra instruction on both the grade school and collegiate levels (e.g. Kieran, 1989; 2007). In 2000, the National Council of Teachers of Mathematics’ (NCTM) recommendations initiated significant curricular change in this area. Researchers studied various changes to instructional methods since then (e.g. Ellington, 2005; Edwards, 2011; Wills, 2011). Lambert and Stylianou (2013) discussed a method to improve middle school students' algebraic problem solving. However, they also indicate it may be impossible to teach all the preferred methods of problem solving for the entire class. Lester (2013) discussed the weaknesses of problem-solving instruction 13 years after the NCTM (2000) recommendations. The role of variable in school algebra receives particular attention in the research literature (e.g. Chazan & Yerushalmy, 2003; Kieran, 2007). Additional confusion arises from the emphasis on symbolic manipulation (Daniel & Embretson, 2010; Radford, 2004). The relationship between the symbolic representations and the graphical representations has posed further challenges for students. Further, care must be taken to ensure that by comparing oral responses from students with their written work that the interview protocols do not alter the MPS processes observed in the students.

2.2 Foundational Work in Mathematical Problem Solving:

Schoenfeld's (cf. 1988, 1992, 2007, 2014) research in MPS provides much of the MPS perspectives embraced in this work. In 2007, his overview of MPS incorporates politics as a lens to frame the discussion while also noting the separation, in the United States, between the research in MPS, related curricular topics, and the actual development of the curriculum. He also asserts that this decoupling differs from the process in many countries. He explains that "high stakes testing" implemented in many states reduced the amount of problem-solving instruction because of a renewed focus on "skills-oriented assessments" (p. 538). Schoenfeld (2007) continues with a timeline overview of the research and theories in mathematics education. He notes the work of several authors, from before 1970, that studied MPS with a statistical or clinical framework. Later works in the 1980s studied the relations between problem-solving strategies and their success. Case studies and analysis of student problem-solving methods through an interview protocol or personal interaction served as the primary methods of research in this period. The 1980s research focused less on the success of students' problem-solving and more on finding the methods used by the students. Work in the field shifted again to focus on the direct influence of the problem-solving process on the success of the problem solver. The research in the 1990s examined heuristics and noted the important role of metacognition. Beyond the scope of Schoenfeld's timeline, Lesh and Zawojewski (2007) noted the need for heuristics to go beyond a prescriptive list and help students to "function better and within their ways of current thinking" (p. 770). The ability to self-monitor one's problem-solving processes showed particular importance among successful student MPS methods.

Additionally, Schoenfeld discusses many facets of mathematical thinking and problem solving in the *Handbook for Research on Mathematics Teaching and Learning* (1992). He explains the essence of mathematical thinking and later (2014) develops a theory for decision making in in any content-rich domain. Álvarez, et al., (2017) link Schoenfeld's (2014) theoretical framework to Campbell's (2014) domains.

2.3 Sense-Making

Sense-making and orienting emerges as a common characterization of MPS in the literature (Schoenfeld, 1988, 2010; Santos-Trigo, 1998). Garcia and Davis (2013) define problem solving as "Making sense of problems, reasoning abstractly and quantitatively, constructing viable arguments, and using mathematical modeling" (p. 350). They also write about the pitfalls of textbook instruction and discuss a method for expanding mathematical exercises into richer problem-solving activities calling this process "problem analysis" (p. 348). They focus mainly on classroom instruction and understanding key factors in the success and failure of a lesson. They go on to explain, the importance of orienting students or students knowing the value of the lesson. Students being left in mystery to figure out how the assessments they must complete and the need for learning the course content can cause some of the biggest hang-ups among students. A student knowing the assessment procedures for the course remains important, but in addition students should understand the importance of the lesson or course within the outside world. By accomplishing this, teachers and instructors can create an environment where the student can use sense-making to orient themselves within the problem and be aware of their goals and intentions. David Jonassen’s (2000) “design theory” for problem solving shows another method for increasing MPS opportunities in the classroom. He seeks to establish some ideas for the design of problem solving and more specifically, problems. He believes that students rarely receive adequate preparation for the problem solving that they will face in everyday life. He points to the need for problem-solving education, noting that such instruction lacks quality design. The textbooks, in particular, show a shortage of problem-solving designs. A culture of support forms another key aspect of the successful classroom (Duncan & Dick, 2000). Jonassen (2000) accentuates this importance, explaining the use of guiding students through problems as an important part of a supporting culture. However, care must be taken to not undo your problem analysis or problem designs by leading them through a problem too far.

Beyond the design theory orientation for MPS, Jonassen (1997) also examines sense-making as problem solving. After establishing the importance of problem solving, he proposes that problems then be central to discussion. He describes problems as having a difference between the goal state and the current state. The nature of a problem can range from a place to apply a classroom algorithm to social issues. He notes that problems should have some value to their solutions. The problem should be perceived, in fact, as a problem. The challenge of teaching MPS in the mathematics classroom stems from the difficulty in achieving this threshold of challenge for all students in the class while not going beyond some students' abilities. Some problem-solving models have been proposed in the past. However, these models lump most problems into the same classifications, leaving an incomplete understanding for many cases. Similarly, Sweller (1998) notes, experts have acquired knowledge of problems and solutions that they can apply in ways novices cannot. In a previous work, Jonassen (1997) discusses three types of problems: puzzle problems, well-structured, and ill-structured. The type of problem offers just one factor in problem design and problem solving. He notes other key factors influence problem variations, including complexity and domain-specificity. He states that more complex problems require more cognitive operations that simpler ones. Not to say that simple problems are easy. Access to additional resources further alters a problem's complexity. Availability of the internet has made many challenging scenarios much easier to solve. For domain-specificity, the nature of one's past experiences changes the nature of a problem. In particular, a problem requiring in-depth knowledge of a domain creates very different challenges from one that requires only general techniques to solve. These three factors are not solely independent or dependent on one another; they can be either or. Next, he gives rise to a typology (and to a lesser extent a taxonomy) for problem solving. The types of problems he lists include logical, algorithmic, story, rule listing, decision-making, troubleshooting, strategic performance, case analysis, design, and dilemmas. Teachers of MPS should examine each type of problem within the intended setting rather than assuming certain problems as superior or inferior. He notes that further examination of problems could give the notion of other types or lead to the merging of a couple of categories. By classifying the problems, the author hopes to foster a better understanding of how to teach MPS methods by improving the understanding of problem structure. The designs altered designs might give the instructors a better chance to sort their activities and assessments in creative ways to better maximize the problem-solving context of the classroom.

2.4 Representing/Connecting:

The capacity to represent and connect mathematical ideas also emerges from the research literature as an important feature of MPS (citations). Simon, et al. (2000) assert that a problem solver must draw on past experiences to access (or possibly develop) tools to solve problems. The student will use additional knowledge in solving the problem; this may take the form of a diagram that was not given in the problem or a related technique or strategy. Boaler (2002) discusses the problem solver's need to go beyond procedures to develop new solutions or novel paths to solving relevant problems. Wilson, Fernandez, and Hadaway (1993) discuss the use and conversion of domain and content knowledge into procedures for solving difficult problems.

Simon, et al. (2000) examined how changing mathematics instruction might affect the perceptions of teachers that are changing or have already changed their practices as a result of mathematics reform. They explain the need for reform and provide examples of successful interventions. These conceptions are as follows: mathematics is created through human activity; it is constrained by what is currently known, and then is a process of transforming one's ways of knowing and acting. Another claim offers that the teachers see the mathematical connections as applying to the objects themselves and not related to knowledge of the person studying. The teachers' perceptions connect to their previous knowledge and practices. The problem-solving perception of the teachers supports the notion of connecting to past understandings. As such, the teachers and students preceptions may not always align productively and matching the two points of view may be complicated. However, awareness of one's perceptions and the existence of others should inform one's teaching.

Boaler (2002) describes the relationship between existing knowledge and its expected uses. She explains a framework through which we study mathematics teaching, including MPS. Theory provides the framework through which we see current practices and understanding. Research in part challenges the accuracy of previous ideas. Ensuring the research implements the right framework becomes critical. Too narrow a framework may prevent the researcher from questioning the right accepted practices and may be limited by familiarity. Constructivism proposes one such framework. Its limitations come from cultural familiarity. Some students relate too well to the “social context of formal schooling.” These students MPS practices draw on past experiences and previous teachings as a resource. These pieces of knowledge that only some students possess but teachers expect students to use alter student problem solving. A theory must accommodate these past understandings in such a way that its does not limit our viewpoint by assuming too much as baseline knowledge. The theoretical framework must also not be so broad, and assume too little, as to have the research fail to lack cohesion and appear scattered. The educational researcher particularly must be willing to challenge the status quo and search out new ways of studying the field, including finding the hidden pieces of MPS the students and teachers may not perceive as critical to their use of MPS. Understanding what it means to “know” provides a key to mathematics education research. Rather studying mathematical knowledge, research knowledge, or test knowledge can make a large difference in the research direction. Someone may know mathematics and score poorly in testing. The opposite can happen as well. researcher must also acknowledge the different ways in which someone may know something; The researcher must consider the past knowledge, hidden understandings, and previously achieved proficiencies the student may possess prior to engaging in MPS.

Wilson, Fernandez, and Hadaway (1993) explore the meaning of MPS and how that influences or should alter instruction in the secondary mathematics classroom. Importantly, MPS understandings consider unique understandings for each person. A "problem" for one person may not be for another, and the difference between the two perspectives may not be entirely attributable to the difficulty of the problem. They write about three key factors in the process of MPS. First, domain or content knowledge plays an important in how the person will solve problems and what problems they will be able to solve. A set of algorithms exists within this knowledge. Application of these procedures alone does not show MPS. However, the discovery of an algorithm or the decisions regarding the implementation of such a scheme might be. Heuristics provide the third piece of problem-solving information students hold. Perhaps the most important facet of problem solving though involves how managing or controlling these factors. This metacognition, or monitoring, appears at the heart of the cyclic nature of problem solving. The evaluating must take place during a problem-solving period so the student knows when they should reexamine a line of thinking or determine their plan appears too complicated to implement. Looking back in the problem-solving process among students appears as a central piece of student learning in Carlson and Bloom’s (2005) framework. Content knowledge solidifies here, and algorithms or heuristics develop or consolidate. However, students and teachers also tend to avoid this phase. The willingness to self-evaluate one's correctness appears frequently absent of minimal.

Sometimes this added component might not be known before solving. Non-routine thinking or synthesis provides this element. Frank Lester's (1994; 2013) writings are the guiding influence for non-routine thinking in MPS. In both his discussion of the research (1994) and his more recent (2013) overview he discusses the ideas of information gathering, as well as knowledge connection and construction. In the 1994 article, Lester looks at the history and future of problem-solving research and poses some theories for how the research has been molded and what areas the research neglected. He states that the curriculums and literature emphasized MPS, it failed to become a central part of instruction. He notes the importance of problem solving to the math classroom. Using the NCTM (2000) recommendations to form this observation. However, he also mentions the lack of a coherent curriculum in MPS. The definition for problem solving presented here surrounds the dynamic nature connecting knowledge to solve a meaningful situation.

Lester (2013) separates the understanding of a problem from problem solving. Problem solving goes beyond merely the act of solving problems, but rather the coordinating and gathering of information and using it to create new information to solve problems or address situations that arise. Lester says that problem solvers require a certain level of experience to reach proficiency. Similarly, Lester (2013) says, “It is reasonable to expect that experience affects the teacher’s planning, thinking, affects, and actions in future situations” (p. 270). This experience plays a vital role in the problem solvers use of the representing/connecting domain.

2.5 Reviewing

Lester (2013) also notes little research exists about students' metacognition. It follows that we know little about how to teach it, and current thinking indicates that metacognition plays a major role in MPS. Artzt and Armour-Thomas (1992) also discuss metacognition in the context of Schoenfeld’s (1988) work on metacognition with roots in Polya’s (1971) work. They present four main components of the problem-solving process, orientation, organization, execution, and verification. Polya (1971) also discusses the cyclic nature of the process for expert problem solvers as well. From this, *reviewing* arises as an important feature of MPS.

Artzt and Armour-Thomas (1992) aimed to differentiate cognitive and metacognitive processes. They separated their observations into episodes, presented as periods of time “where an individual or a problem-solving group are engaged in one large task.” They categorized each episode under *read, analyze, explore, plan/implement,* or *verify*. After some initial observations, the authors made some modifications. Artzt and Armour-Thomas separated plan and implement since not all subjects put their plans into use. Also, the *read* and *analyze* phases separated themselves to need two other episodes added, understanding the problem and “watch and listen” (p.141). All of these components link together in the reviewing process for MPS.

Reviewing within MPS also arises in the work of Garofalo and Lester (1985). They note that interests of researchers have begun to focus on cognitive monitoring. This falls under metacognition. Their work studies the role of metacognition in mathematical performance. They also examine how the processes from other disciplines might fit into the metacognitive framework for MPS. They note that there have been few studies about the relationship between metacognition and MPS, due to the difficulty involved in observing this relationship. However, Blanco, Barona, and Carrasco (2013) did examine the MPS and metacognition of prospective teachers. They note the importance of separating the beliefs about MPS from the teachers’ personal beliefs about themselves. Emotions also play a key role. Research has indicated that higher anxiety levels tend to cause lower grades in students. Their program participants did declare some change in attitudes for the better. Also, the teachers were now aware of their own beliefs and emotions and how those affected the students. Teachers positively noted the development of a go-to approach. Considering their own thinking within an MPS process, the teachers identified reflective approaches unintentionally incorporated into their MPS methods. An approach should not be confused with a procedure. In conclusion, they indicated this change in teachers' attitudes reflects the importance of the reviewing approach. Additionally, beliefs and attitudes also impact the MPS approaches of prospective teachers.

Garofalo and Lester (1985) note the issues created by students self-reporting their thinking and their problem-solving processes. Some researchers see this invalid data collection. Others say that the process of reporting the data likely alters the thinking process and as such while offering valid data it fails to accurately reflect a typical process. Reviewing particularly accentuates this issue. Students may report checking their work or reexamining past actions when asked to explain their thinking, but may not perform these actions when unprompted by the research questions. Understanding mathematical tasks and knowledge of mathematical expectation in problem solving shows another area of metacognition relating specifically to mathematics. The effect that perceived problem difficulty has on the metacognitive processes shows one example. Strategy interactions play a role here too. Knowing that speed leads to mistakes makes students slow down. The slowing down comes about in part as a result of reexamining their own actions or thinking. Understanding a key word strategy leads to students not reading for meaning in the initial problem stage. The authors suggest framework should be created to study metacognition through. Some previous frameworks focused mostly on descriptive problem solving. In the development of their framework, they note three levels of knowledge they think should be considered. They are resources, control, and belief systems. Next, they present their framework that they position as relevant to a wide variety of tasks, not strictly "problems." They organized into four levels, orientation, organization, execution, and verification. The authors intend the levels of the framework to provide a tool not as a prescriptive list intended to be observed in all students. Their definition of problem solving related to any situations where a solver needed to apply cognitive or metacognitive processes.

Carlson and Bloom’s (2005) framework of cyclic problem solving shows another example of reviewing in the problem solving process. They looked into the characteristics of both problem solving and of problem solvers. Most notably the literature has noted differences in the techniques and methods of expert and novice problem solvers. The authors examine in detail the characteristics of expert problems solvers to better understand what makes them experts, and to see what traits can be identified as essential for success in MPS. Carlson and Bloom (2005) studied 12 experienced problem solvers, eight research mathematicians and four Ph.D. candidates at major university in the country. For this study, they developed a taxonomy for problem solving around which they based their theoretical framework. This taxonomy consists of five main areas: resources, control, method, heuristics, and affect. Control shows as the area most discussed. This closely relates to Schoenfeld's (1992) discussion of cognition and metacognition or monitoring. The researchers presented the subjects with five challenging problems. The researchers interviewed each subject while they were solving the problems with the discussions audio taped and later transcribed. They were asked to say why they were doing the steps they were and to explain their reasoning and justifications. The researchers coded these responses, according to the taxonomy. Using all the acquired information the authors develop an explanation for the problem-solving process. First, subjects use the techniques of sense-making in the orienting phase. Followed in the planning phase by conjecture or formulation of an approach that might serve to solve the problem. Further, while applying the conjecture or approach to the problem the problem solver evaluates the viability of their creation. In the executing stage the student carries out of the plan or testing of the conjecture using logic and mathematical constructions. Then the checking phase includes the students' verification of a solution or resolution. The four phases, *orienting*, *planning*, *executing*, and *checking* also closely relate to the four levels of Garofalo and Lester's (1985) framework. The authors note that this process rarely appears linear and straightforward and phases frequently intertwine with one another. Also, frequent cycles of the latter three phases can appear for a problem-solver engaged in challenging problem solving.

2.6 Justifying

The ability of a student to evaluate a plan in context and then support that plan with theory or underlying understandings an important factor in MPS (Szelta & Nicol, 2002). The Oregon Department of Education (2000) in its problem-solving rubric required a "justifying the solution outcome completely" to receive full credit.

Lampert (1990) argued the necessity for the mathematics classroom to consider conjecture and hypothesis. The classroom should go beyond the instructor handing out mathematical truths. Part of student difficulty in this area surrounds the teachers laying in a set of rules for students to follow. She argues that establishing rules precludes the possibility of "doing mathematics" (p.32). This justification discourse provides a critical component of MPS. Mathematics solutions and solution procedures rarely appear in a prescribed fashion, but frequently in the mathematics classroom students expect and rely on such prescriptions. ignoring the justification process and other MPS discourse (Chazan, 2000).

2.7 Challenging Problems (Difficulty)

Additionally, a necessary component of MPS requires the problems to be challenging enough that the student must engage with the problem on a deep level (Lester, 2013; Jonassen, 1997; Sweller, 1988). Bookman and Friedman (1994) wrote about the problem-solving performances of students in a lab based Calculus course when compared to student performances in a regular course. In their research, they look into quantifying the success of a new method for undergraduate Calculus. Their depth-focused Project CALC intervention contrasted with the traditional course which placed much greater emphasis on computational skills. Their findings suggest that emphasizing MPS in instruction improves performance on a similarly-focused pencil and paper test.

Student difficulty with mathematics and school mathematics problems results in various tracks for mathematics students secondary school. Montague and Applegate (2013) look at the problem-solving abilities of middle school students with learning disabilities. The desire to better the processes at work in MPS for students in math, with a particular focus on those might differ in students with learning disabilities when compared to those without any noted disability. The study gives particular attention to the cognitive and metacognitive strategies employed (or not employed) by the students. A statistically significant difference was observed between groups on a full-scale IQ test. The mean for the learning-disabled students was within the average range for this test, but was significantly lower that of their counterparts in the other groups (gifted and average achieving). They gave a series of tests to the students in the course during two separate hours on different days. They noted that the most evident difference for the learning-disabled students was that they were less able to use of representation strategies to help control their access to the problem. From this, it appears present needs include additional teaching of how students should approach a problem and help in devising plans, which can be executed, in solving problems. Montague and Applegate noted a reasonable level of computational efficacy present among the learning-disabled students but characterized each as poor problem solvers. The level of difficulty needed exists for all problem solvers. The authors consider that the reduced difficulty frequently found in mathematics teaching for the learning disabled may harm the MPS abilities of these students by not reaching the appropriate level for the students to develop the skills needed. The inability to apply techniques such as rewording or implementing a diagram, gave the reason for this separation between learning disabled and gifted students. The types of instruction and level of challenge presented to the learning-disabled students may explain these differences. They attribute this possibility to the curricula used for learning disabled students that heavily skews to “computational drill and practice.”

The type of problem asked of students impacts student problem-solving methods. Koedinger and Nathan (2004) studied the work of high school students on algebraically simple “story problems.” They showed the allowance for greater “verbal facilitation” allowed students to perform better on the story problems. They identified two main differences created by story problems not present in non-contextual problems. First. A context or “external representation” may be difficult for the student to understand. Second, one student may use a different representation than another. These issues minimize for non-contextual problems. Further, Lambert and Stylianou (2013) discuss the development of lessons to challenge all students, across multiple ability levels, and including learning disabilities. They emphasize the importance of cognitively demanding tasks, along with the fact that students engaged in challenging instruction show the highest gains (Silver & Stein, 1996). These studies point out that MPS has a base component of difficulty. A problem not challenging will not allow the student a window into MPS. The computational practice remains important but alone fails to promote MPS in students. An easy problem, even if presented as a word problem or behind an additional hurdle, will still serve a similar purpose as the computational repetition. This is not to say that computational practice is without value, but it does not, on its own, enhance the mathematical problem-solving opportunities for the student.

2.8 Understanding Algebra Achievement and Learning

Several factors have been shown to influence the achievement of students in algebra courses. The use of topics in other disciplines increased the number of students completing college algebra courses while maintaining achievement on a standardized math exam (Ellington, 2005). This suggests that the students in the course using cross-curricular examples possessed equal mathematical abilities after the course as those who took the traditional course. Further, Treisman-style programs including a problem-solving component in the course produced an increase in algebra achievement (Duncan & Dick, 2000). These interventions attempt to counter the difficulty universities have in retaining College Algebra students through calculus. As mentioned in Chapter 1, the failure rates for College Algebra students and persistence to Calculus is less than 50% and 10%, respectively (Saxe & Braddy, 2015; Dunbar, 2005; Jarrett, 2000). Changing the instruction within these courses with high failure rates should consider the suggested changes put forth in the literature (e.g. Bookman & Friedman, 1994; Cifarelli, Goodson-Espy, & Chae, 2010; Duncan & Dick, 2000). Those deciding to implement programs and methods to teach students problem solving must consider the meaning of problem solving. When deciding to change to algebra curriculum, administrators and instructors should understand the intention of the changes. “A primary goal of reform-based mathematics instruction is for students to develop into problem solvers who can self-initiate, monitor, and sustain their actions while solving problems” (Cifarelli, et al., 2010, p. 207). Further, these areas of initiating, monitoring, and sustaining, may not need to be emphasized equally in problem-solving instruction. Other facets of problem solving may prove more critical than previously identified aspects. If to advance in STEM fields certain areas prove more critical then we should give those parts of MPS greater scrutiny. This is the aim of this research.

The problems used in this study derive from secondary school algebra. High school algebra consists of many symbols and complex concepts. These ideas are not easy to classify or understand for students (Chazan & Yerushalmy, 2003). The challenge of developing students' mathematical understanding, and not simply the ability to perform rote procedures and apply prescribed techniques, has been emphasized by the NCTM (2000). For example, the standard, "the meaning of equivalent forms of expressions, equations, inequalities, and relations" (p. 296) differs from needing only be able to solve an equation for an unknown or identify expressions, inequalities, and relations, in a list. It implies a focus on student understandings of equivalence in these areas. A conceptual understanding of algebraic manipulations creates a challenging maze for students to navigate but also appears critical for success in later mathematics courses (Chazan & Yerushalmy, 2003).

Arguably from inception, algebra can be thought of as a tool for "manipulating symbols and solving problems" (Kieran, 2007, p. 707). The history of Algebra instruction follows this path and only beginning in the 1970s did research begin to examine the meaning students formed from the algebraic notation. Carry, Lewis, and Bernard (as cited in Kieran, 2007) demonstrated that exclusively-skills-based instruction did not yield highly skilled algebra students. Multiple competing views of how to improve the content of school algebra exist. Traditional algebra courses emphasize the use of symbols and the solving of equations by formal methods. These courses would include word or contextual problems spread throughout the sections. The emphasis on the student recognizing the form of the problems and identifying structures persists through these story problems. This traditional view of algebra places understanding of these forms at the highest level of importance (Saul, 1998). A reformist algebra curriculum places the importance on functions and relations alongside the "solution of 'real-world' problems by methods other than symbolic manipulation" (Kieran, 2007, p.709). Manipulative skills remain highly valued in the algebra curricula, but other models achieve a higher importance. Saul (1998) explains the traditional view that the study of functions and relations falls more into the domain of analysis than algebra.

After considering the curricular baselines and the role of functions and relations in algebraic instruction, the research considers where the students learn or assign meaning in algebraic processes and notations. Radford and Puig (2007) suggest the nature of the “unknown” differentiates algebra from arithmetic. Arithmetic requires no unknown while algebra inherently requires it. They point to natural language as one source of algebraic meaning. The student's understanding of the algebra relies on how the student fits the problem(s) into the appropriate context. The algebraic structure itself provides another source of meaning (Kieran, 2007). Bills (2001) showed that university students exhibited an inability to use both, contextual meaning and algebraic structure as sources of meaning together. Students could easily explain “shifting” on a coordinate plane when presented with a problem asking them to follow a known procedure, finding the equation of a line. Then when the asked to perform a similar task involving points along one of the axes the students could not shift between *x* as an unknown to *x* as given. Meaning can also follow from things external to the problem or the mathematics (Kieran, 2007). Students can place a high level of importance on factors they observe in the given contexts. For example, students identifying external factors, such as air resistance or fatigue, limits their effectiveness in using a linear representation in many situations. Radford (2000) points out these students bring in these sources from other domains before understanding the “complex algebraic meanings of contemporary school mathematics” (p.240). These meanings may prove useful in certain contexts, but also tend to confuse grade school algebra students.

Students, across the world, struggle with algebra. Kieran (2007) points to the results of Trends in International Mathematics and Science Study (TIMSS). The TIMSS showed accuracy measure of less than 50% among students on a standardized algebra test. The ability to manipulate “letter-symbolic” content landed under 25% across all countries in the study among lower secondary students. Reaching the upper secondary and college level students studies increasingly focus on the role of technology, specifically a graphing calculator, in algebraic achievement and reasoning. This focus leaves less understood about these students’ sources of algebraic meaning. The research identifies multiple drawbacks to instructional approaches overly reliant on technology. Asp et al. (as cited by Kieran, 2007) point to a need for students to physically construct tables for function values. Graphing calculators leave out this “essential” component of algebraic understanding. Warren and Pierce (as cited by Kieran, 2007) showed the need for “by-hand” procedures such as simple equations and standard techniques among students using a CAS. However, the place of these “by-hand” procedures within the curriculum remained an open question. Sutton (2015) showed that the use visualizing software in instruction must include appropriate “focusing features” even for students beyond College Algebra in the course sequence. Algebraic meaning provides a point of struggle for students through all levels of mathematics (Kieran, 2007).

Chapter 3

Methodology

3.1 Introduction

In creating a framework for studying student mathematical problem solving (MPS) we must first understand what MPS means. As noted by Wilson, Fernandez, and Hadaway (1993), “When two people talk about mathematics problem solving, they may not be talking about the same thing…Creative speakers and writers can put a twist on whatever topic or activity they have in mind to call it problem solving!” (p.57). The data analyzed for this study is part of a larger study funded by the National Science Foundation (DUE #1544545) led by principal investigators Álvarez and Rhoads which is developing MPS items that may be used to assess the MPS capacity of students in College Algebra (Epperson, Rhoads, & Campbell, 2016). This study examines College Algebra student MPS behaviors during one-hour think-aloud interviews. The 19 MPS-based interviews, student background information, and course grades provide the material examined in this study.

3.2 Setting

In fall 2015, students in both College Algebra and Calculus I completed a five-problem assessment (MPST) with 25 to 35 follow-up MPS Likert items gathering information on students’ MPS processes aligned with the MPS domains, sense-making, representing/connecting, justifying, reviewing, and challenge. More information on the MPST and its development is described in Álvarez, et al (2017), Epperson, et al. (2016), and Rhoads, et al. (2017). In spring 2016, students in College Algebra also completed an MPST. These students came from multiple sections of these courses at a major university in the southwestern United States with an enrollment of over 37,000 undergraduates. The MPST uses problems grounded in high school algebra to attempt to separate domain knowledge from mathematical problem-solving components. Students earned a score in each of four categories, sense-making, representing/connecting, reviewing, and justification. Additionally, students rated the difficulty of each problem completed in the MPSI. The data collected included the objective responses to the MPST items, the student's written work on the MPST problems, and responses to a demographic survey.

College Algebra sections included sixty students each. The sections meet once a week for an eighty-minute lecture. Each section also meets twice a week in an eighty-minute computer laboratory. During laboratory time, the students work in a computer-based program on course material with the assistance of the instructor and several undergraduate and graduate student lab assistants. The labs consist of 120 students since they combine two sections taught by the same lecture instructor. In fall of 2015, five instructors taught a total of nine sections while in spring 2016 three instructors taught a total of four sections. Since two of the instructors taught both semesters, there were six instructors across the two terms. Students needed to only attend lab for 36 total hours and were not required to complete the lab time only during the scheduled lab meetings. The students could attend any time the lab was open, though the instructor would only be in the lab during the scheduled lab meetings. Calculus sections met for either two eighty-minute lectures or three fifty-minute lectures. Each lecture was limited to 60 students. All sections regardless of lecture format met for two fifty-minute lab times per week. Graduate teaching assistants (GTA) facilitated the labs with group assignments and recitation time where the students could ask questions of the GTA. Calculus classes also included online homework, but it was not a requirement that it be done at school. In total, the researcher conducted 26 interviews with students across both semesters. This included 11 interviews with College Algebra students and seven interviews with calculus students during fall of 2015, and eight interviews with College Algebra students in spring of 2016. The calculus student interviews are not included in this study.

3.3 Student Testing

As described in Section 2.2, students completed a five-problem assessment (MPST) with 25 to 35 follow-up MPS Likert items gathering information on students' MPS processes aligned with the MPS domains, sense-making, representing/connecting, justifying, reviewing, and challenge. Each student also completes a short demographic survey when they complete an MPST. Completing an MPST takes one hour or less for the student. Though each student completes only five problems and follow-up items, the an MPST in fall 2015 consisted of five problems from a pool of 10 problems along with their 5-7 corresponding Likert items. During spring 2016, the MPSTs consisted of five problems drawn from a pool of 15 problems.

Each student was informed about the project at the beginning of the semester and provided a consent document explaining the project. There was both a pretest (MPST at the beginning of the semester) and a posttest (MPST near the end of the semester). Students completed the instrument outside of class time by attending their choice of several available meetings times over the course of the first two weeks of the semester. Students completed the posttest in a similar fashion by choosing a meeting time from a number of available meetings scheduled over the second to last week of the semester. For College Algebra, students completing the survey received a 100 quiz grade which could be used to replace a low score during the semester for completing either the pretest or posttest. Students completing both received a 100 homework to replace a low score in addition to the 100 quiz grade. In fall 2015, there were 70 pretests and 50 posttests collected including 12 students completing both. In spring 2016, there were 132 pretests and 29 posttests collected including 24 students completing both. The increase in pretests can be attributed to creating an option for the students to complete the pretest during the first lab meeting. Students could also earn the same incentive by completing an alternate assignment of problems similar to those in the instrument.

3.4 Interview Invitations

After recording the student responses, the answers given by the students were scored according to the alignment described in Epperson, et al. 2016. That is, the Likert items ranged from a desired response (scored a 6) which indicated that a participant was using a high-level of the linked domain to a response (scored a 1) which indicated that a participant was using a low-level of the linked domain. Average scores for each category were computed along with the highest single category score and the range of the category scores. Using this information, students were invited to complete an interview of one hour or less with the researcher. Participants were paid $20 for completing the interview.

The process for inviting students to interview included a letter of invitation explaining the offer to interview or an email with the same information. A sample invitation appears in Appendix A. All interviews occurred during the same semester as the MPSI. Invitations for an interview placed priority on students with particularly high or low category scores. An invited student received a letter from the researcher delivered via their instructor. The letter explained the interview process and informed the student of the confidentiality of their responses should they wish to participate (see Appendix A). As the letters were being distributed by instructors the selected students, the researcher sent an email to the selected students which contained the same details as in the physical letter. In fall of 2015, the researcher initially sent invitations to 25 students in College

Algebra. After giving the students adequate time to reply the process was repeated by distributing another 25 invitations among each course's participants. College Algebra required a third iteration of 25 invitations. In total, 75 College Algebra students were invited during fall of 2015. The researcher conducted 11 interviews with College Algebra students and seven interviews with calculus students in that semester. During spring of 2016, the researcher invited three groups of 25 students with the same procedures as the fall semester. Eight College Algebra students completed interviews in spring of 2016.

3.5 Student Background

The 19 interview participants were all enrolled in College Algebra as a STEM-intended course. The students either placed into College Algebra based on the results of a placement test taken immediately before enrolling or the student placed into a lower course and advanced to College Algebra in the sequence. All of the students in the College Algebra course intended to advance to at least Calculus I as a part of their degree track. Of the 19 students, 12 are female (63%), and seven are male (37%). Females accounted for 55% of College Algebra students completing an MPST. All the interview participants were between the ages 18 and 30 at the time of the interview (only 3.5% of students completing an MPST were over 30 years of age). Sixteen of the 19 students (84%) fell between ages 18 and 23, compared with 93% of those completing an MPST. Four of the 19 students (21%) identified as Hispanic whereas 28% of the College Algebra students completing an MPST identified as Hispanic. Of the 19 interview participants, four identified as Asian, two as African-American, and eight (42%) as White and non-Hispanic. One did not report her race. In the entire MPST sample, 27% reported as White and non-Hispanic. All of the students expected to do more work in college to earn the same grades they achieved in high school, and 15 of 19 reported taking at least one advanced placement, baccalaureate, or dual credit course in high school.

3.6 Student Interviews

After determining a time the student could meet with the researcher, the student reported to the researcher to complete the information needed to pay the student $20. After paying the student, the interview took place in either an available conference room or in an empty classroom. The researcher explained to the student that their responses would be kept confidential and that they should not feel pressured to give any particular answer. The researcher emphasized there were no expectations in the protocol that they perform any particular actions or steps, or offer a specific explanation. The researcher told the student that the interview would not last more than one hour. Each interview ultimately lasted between 23 and 58 minutes.

To guarantee anonymity, participants received a participant number as part of the MPST procedures. The researcher referred to the participant by this number during the interview and all notes and materials used only this number in evaluation and coding. College Algebra students in the fall of 2015 were assigned three-digit numbers with each set of 100 (e.g. 100-199) corresponding to a different section of the course. Students in the spring of 2016 were assigned four-digit numbers beginning with six. Then, each set of 100 corresponded to a different section of the course (e.g. 6100-6199 correspond to one sections students). Lastly, for ease of communicating about the 19 interview participants their participant numbers received a leading M for male students or a leading F for female students. The interviews examined within this study include 11 students from the fall with three-digit identifiers (four males and seven females) and eight students from the spring with four-digit identifiers (three males and five females).

3.5.1 Interview Protocol

The first question of the interview asked the students to, “Describe the usual process or steps you go through when you solve a challenging mathematics problem.”

The student responses to this question receive particular attention, as the problems and questions discussed later in the interview could possibly influence student responses. After the student's explanation, the student would be asked what they do first in solving difficult problems. The researcher asked for following steps until it appeared apparent that the student exhausted all preferred methods. Then, the researcher asked questions about each of the domains identified by the study of research articles. The questions asked for each domain appear in Table 3-1.

Table 3-1 Domain-Specific Questions

Do you break the problem down into smaller pieces or tend to think more globally?	Sense-Making
Do you like to use a diagram or picture or other representation when you are solving a tough problem?	Representing /Connecting
When solving a problem do you look back at what you've previously done and check or do you typically work all the way to the end of the problem?	Reviewing
How well do you feel like you can justify your solution? How often would you say you do that?	Justification

For the next part of the interview, the researcher identified at least three problems from the MPST the student completed and asked the student to explain their solution to a problem. The interviewer asked the student to explain what they believe was the process or steps they went through in solving the problem. The student was told to take as much time as needed to reexamine the problem since at least six weeks passed since they completed the problem. This repeated for at least three problems. The researcher identified certain writing or notations that he wanted to know more about and asked the student about those specifics if the student did not address them on their own. Some students chose on their own to look at each problem and depending on the time spent on the first three problems the researcher might ask the student about additional problems. If the student chose, they could write on the previously done problem or perform additional problem-solving methods to resolve the problem or check their previous work. If the student added anything to their past work, the researcher noted it on the page after the interview.

After looking at their past work, the student was given a different problem from the MPSI project's pool that had not been included on the participants MPST. The researcher asked the student to explain his or her thinking as they worked the problem. If unclear about the processes used the researcher asked the student to explain his or her thinking further. If the student was confused about the problem the researcher would attempt to offer advice that might start the student on the problem but would not offer proposed solution methods. Once the student decided they finished the problem, the researcher gave the student the Likert items associated with the problem. The student was asked to explain their choice and if the researcher believed the answer might not match with what he observed he would ask for further clarification.

Each interview was recorded and transcribed. The recordings capture the participant workspace but do not include images of the participants themselves in order to maintain anonymity and confidentiality. The researcher gathered all of the information collected for each interview participant, MPSI problems, items responses, and work during the interview in a single file along with notes taken by the interviewer during the interview. The researcher included notes taken during the interview about the problem-solving domains for each student. A copy of the interview protocol the researcher used during each interview appears in Appendix B.

3.6 Analysis of Student Interviews

Grounded Theory, as described by Corbin and Strauss (2008), allows the researcher to create constructs arising from the data collected. Both open coding and axial coding were used together in this study. The researcher examined all of the interview recordings and transcripts and coded each for orientations, strategies, techniques, and affective factors. This open coding procedure did not assume any particular constructs would exist within the codes. Each construct or set of codes arose from the student responses themselves. However, the open coding procedure combines with axial coding as the researcher related the codes into three distinct categories, orientations, strategies, and techniques. Axial coding relates the sets of codes to each other (Corbin & Strauss, 2008, p. 198). Additionally, the researcher coded for instances of affective factors in the interviews, such as beliefs or attitudes toward mathematics or MPS. After an initial coding of the interviews, the researcher reexamined each interview to ensure the accuracy and completeness of the coding. The codes attempt to cover all of the student orientations, strategies, and techniques of MPS. For a student to be considered to hold an orientation they needed at least two codes for that orientation. An orientation shows the overall structure and approach a student uses to attempt to reach a solution to an unfamiliar problem. The "body of resources and preferences" the student brings into the problem-solving situation with them (Schoenfeld, 2010). A strategy is defined as, a plan or method used for the specific goal of solving the problem. A specific action implemented or intended to be implemented during MPS defines a technique. Further, a student technique includes the application of a previously known skill, procedure or heuristic while a strategy may help create or identify the skill or procedure. While the researcher knows of the student's quantitative MPS scores, the theory is developed from the student responses independent of connections to the MPSI scores.

In general, the coding of orientations does not depend on the expected student behaviors based on the quantitative MPSI scores. Only justification appears as both an orientation and a problem-solving domain identified in the literature. Further, these characterizations are compared with the student's course grades, and demographic information to search for any clear connections.

3.6.1 Grounded Theory Framework

For the grounded theory perspective, the research holds no expectation that the coding will confirm or refute any existing theories. The data builds the possible theories arising from the data. The researcher finds the "theoretical constructs derived from qualitative analysis of data" (Corbin & Strauss, 2008). Though as in Charmaz (2006) it may be that the coding verifies a theory or idea proposed previously. Thus, to characterize the students' problem-solving techniques, strategies, and orientations the open-coding and axial-coding derived from the data provided the information used to construct a theory.

3.6.2 Additional Factors Impacting Student Coding

It is possible that the relationship between the student characterizations and the MPSI problems may alter student MPS. The researcher compared the codes arising from a participant interview to the problems completed and discussed in the interview. Using the coded data and the list of problems completed by each student allowed the researcher to investigate any trends. Some MPSI problems appear more abstract and less connected to everyday contexts, particularly when compared to the conventional contextual problems included in the problem pool. The former rely more on algebraic symbols and not on their contexts. The researcher determined if particular types of problems elicited certain types of thinking from the participants, which, in turn, would give rise to certain codes. Using not only the codes produced from the interview data, but also the artifacts of the participants' work on their MPST enabled the researcher to triangulate the emerging constructs related to the participants' orientations, techniques, and strategies. Additionally, the researcher coded for possible affective factors the student mentioned in the interview. Possible affective factors include beliefs about or attitudes toward math, a problem, or MPS. These factors could appear to be positive or negative. The researcher searched the affect codes and their frequency for correlations to certain orientations, strategies, and techniques, and also for any correlation to the correctness of the student's MPS.

3.6.3 Coding Methods

The researcher created codes and coded each interview using NVivo qualitative analysis software which allowed the researcher to identify and mark instances of each emergent code within a transcript of an interview. The codes for a particular orientation, strategy, or technique, could be displayed at once for all interview participants. The software also displays the codes appearing in a particular interview. After coding each of the interviews, the researcher placed the code frequencies in an Excel workbook along with the course grade. The researcher could examine any possible relationships between orientations, strategies, and techniques, as well as affective factors in this format. The NVivo software also allowed the researcher to look for words or phrases across all of the interviews to look for possible missed instances of a method not coded. After identifying an orientation, strategy, or technique by through open coding the researcher studied the written work to which the student referred that corresponded with a given code in the transcript. As mentioned, this additional evidence of supporting an emerging code helped triangulate the nature of the orientation, strategy, or technique. After the coding process was completed, code names were revised to succinctly reflect the intended interpretation of the code.

Once the coding concluded the researcher created “node diagrams” for each orientation expressed by each student. These diagrams connect orientations displayed to strategies and techniques that followed the orientation. Initially, the diagrams included all orientations for a student. These diagrams proved difficult to interpret due to the density of the coding. Thus, the diagrams were created per orientation which allowed the researcher to compare the diagrams for a particular orientation in order to identify any patterns. Further, the diagrams could be grouped according to other factors, such as course grade, to attempt to locate similarities. All of the node diagrams appear in Appendix C.

Chapter 4

Results

The College Algebra students in the problem-solving interviews answered questions about their mathematical problem solving (MPS) process and described those processes MPS both directly and indirectly. In the direct case, the interviewer asked the students about how they approached a challenging math problem. Additionally, students answered questions regarding the specific domains of MPS, sense-making, representing/connecting, reviewing, and justification. The students answered several questions related to specific aspects of their MPS process. Indirectly, the participants displayed their MPS techniques, preferences, and practices, while examining previously worked problems and working a new problem. The students completed an additional problem from the set of MPSI problems. The researcher asked the students to explain aloud what they were doing. The researcher also asked the students to clarify what they did when appropriate. The researcher reviewed the interviews coding for instances of orientations, strategies, and techniques, presented by the students.

A review of interviews identified eight orientations displayed by the students. Each student interview was coded for at least one orientation. The coding process did not suppose a maximum number of orientations for a student, but I identified no more than three orientations for any individual student. Each student displayed multiple instances of any orientations included in their profile. 11 strategies and 10 techniques appeared within the student interviews. Each strategy or technique received codes in multiple student interviews, but a single student may show only one instance of a particular strategy or technique.

4.1 Orientations

Each of eight orientations was observed in the student interviews including formula application, reexamination, big-picture focus, linear progression (step-by-step focus), replication, tinkering, streamlining, and justified reasoning. The names of the orientations stem from the observations of the students in the interview. The first three orientations appeared most commonly while the remaining orientations appeared less frequently. The number of students coded for each orientation appears in Table 4-1.

Table 4-1 Number of Interview Participants Coded for Each Orientation

Process	Number of Students (Total =19)
Formula Application	11
Reexamination	7
Big-Picture Focus	6
Linear Progression (step-by-step focus)	3
Streamlining	3
Replication	2
Tinkering	2
Justified Reasoning	1

4.1.1 Formula Application

Formula application appears among the most students of all observed orientations. Formula application is identifying and applying previously known formula(s) as primary anchors to discover solution approaches. Students engaging in formula application used a previously known formula or equation format (e.g. slope-intercept form for linear equations) to drive their MPS forward. The number of formula application instances appears in Table 4-2.

Table 4-2 Frequency of Formula Application Orientation

Student	Number of Instances (Instances in Initial Question)	Total Orientation Instances	Portion of Orientations classified as Formula Application
M105	3 (3)	9	33.3%
M106	4 (2)	10	40%
F209	8 (2)	14	57.1%
F303	8 (1)	11	72.7%
F304	4 (0)	12	33.3%
F311	11 (2)	11	100%
M506	7 (1)	7	100%
F6106	7 (2)	15	46.7%
F6145	9 (2)	13	69.2%
F6221	5 (0)	5	100%
F6259	9 (5)	9	100%
Total	78	195	40%

Students explain their MPS as involving equations, formulas, or applications of said equations and formulas. The students would work their problems with an apparent intention to move the situation back to a formula or equation that could be solved or applied. The presence of these analytical representations provided the structure the student used to attempt to solve the problem.

4.1.1.1 M105

The first student observed using formula application to structure their MPS practices was M105. He solved the Fun Golf problem during the interview. While completing the problem he initially checked the revenue for a $1 increase in price before stating, "I'm trying to think of a way that, to figure it out without just constantly guessing… I know what I'm trying to do is set up some kind of equation." This indicates his emphasis on using a formula or previously learned calculation method for determining the solution. He then uses the quadratic formula to solve the equation that he creates again preferring the analytic approach to a solution over more informal factoring methods. It can be seen in his work, shown in Figure 4-1, that he does check his solution fit with the problem by verifying that the values found do give the maximum revenue. However this technique was a secondary operation within his process of formula application.

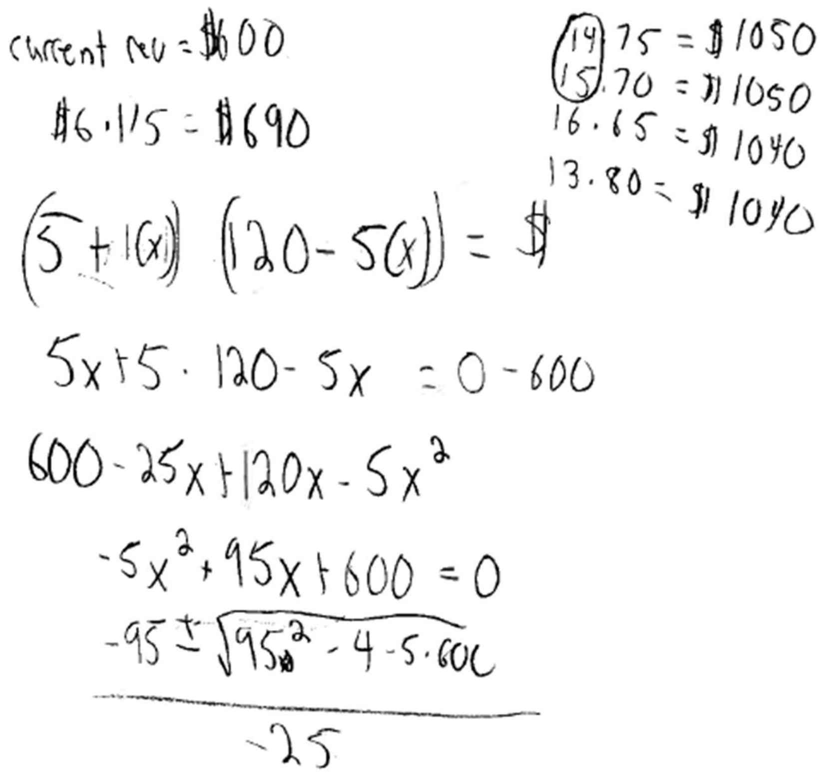


Figure 4-1 M105 Fun Golf Problem

Additionally, during the interview the student also discussed his work completed in the MPSI pretest done before the interview. In those problems, his discussions also mentioned a preference for a formula application process. For example, in the Cross-Country Race problem, shown in Figure 4-2, he included only a division algorithm to determine the solution. Further, when speaking about another problem, Book Stacks, he explained his preferred solution method would include, “have two equations on each side of each other and make them equal to each other.”

Figure 4-2 M105 Cross-Country Race Problem

Cross-Country Race

Brett is leading Charlie in a cross-country race. Charlie has 100 meters left in the race and Brett is 20 meters ahead of Charlie. Brett covers the final 100 meters in 16 seconds. If Charlie covers the final 100 meters in 14 seconds, who finishes the race first, Brett or Charlie? For each of the runners, write a position function that gives the distance in meters from the finish line after *t* seconds. Brett wins by 1.2s

c=100/14 = 7.14 m/s

~~b=80-6.25t~~ =

b=100/16 = 6.25 m/s

b 80/6.25 = 12.8 s

b=100- 6.25t

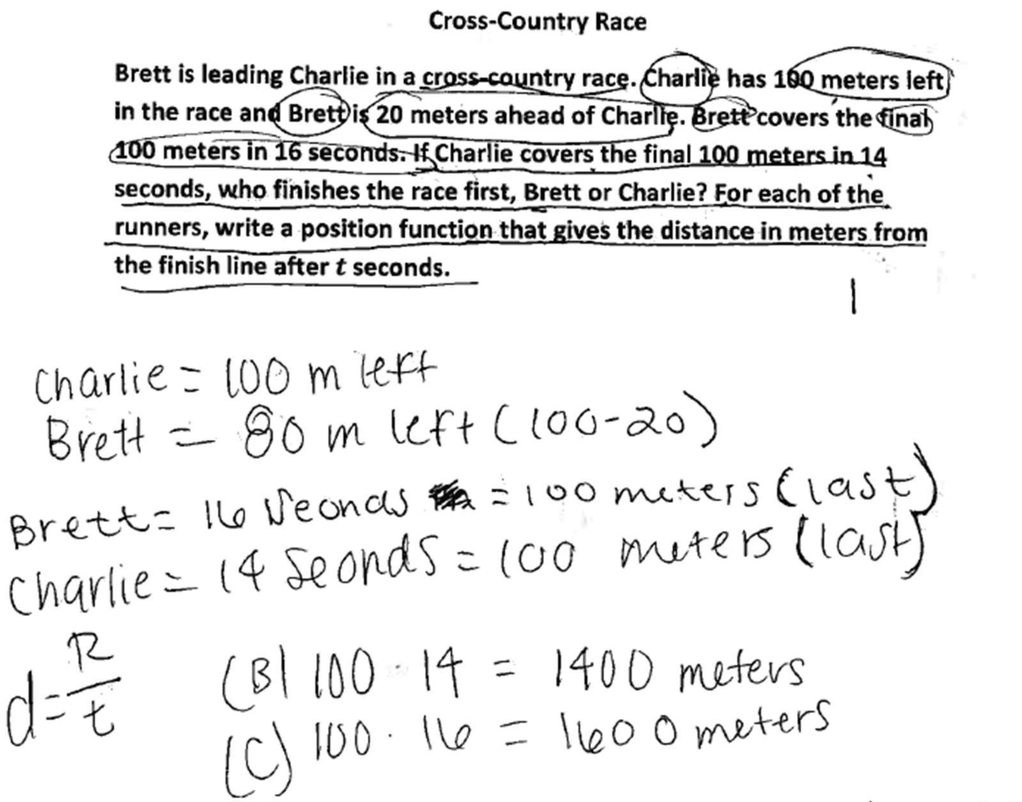
c=100- 7.14t

4.1.1.2 M106

M106 described at the beginning of the interview his MPS as being “I just tend to put it into kind of an equation, or a series of equations, in my head as it reads out.” He goes on to explain that he creates a “mock-equation in my head.” Two additional observed orientations also influence his MPS behaviors and he states behaviors or methods he would not do leading to fewer codes corresponding to instances of the formula application orientation.

4.1.1.3 F209

F209 relied on formula application as one of her MPS approaches as well. F209 was one of 11 students showing two different processes in her work. She completed the Cross-Country Race problem during the interview. She explained at the beginning of the problem, “First, I'm going to underline the question, what it’s asking to do …Then I'm going to circle the information…Then I'm going to label everything out…And then, going to write the formula.” The initial techniques provide the set-up for the formula driven process. Additionally, she wrote the formula $d = r/t$ in her solution, but for each of the problems revisited from the MPSI pretest that included a rate she invoked the formula “distance equals rate times time.” Her solution (Figure 4-3) shows only multiplication. The explanation of her work revolved around the computations within the formulaic representation.



Cross-Country Race

Brett is leading Charlie in a cross-country race. Charlie has 100 meters left in the race and Brett is 20 meters ahead of Charlie. Brett covers the final 100 meters in 16 seconds. If Charlie covers the final 100 meters in 14 seconds, who finishes the race first, Brett or Charlie? For each of the runners, write a position function that gives the distance in meters from the finish line after *t* seconds.

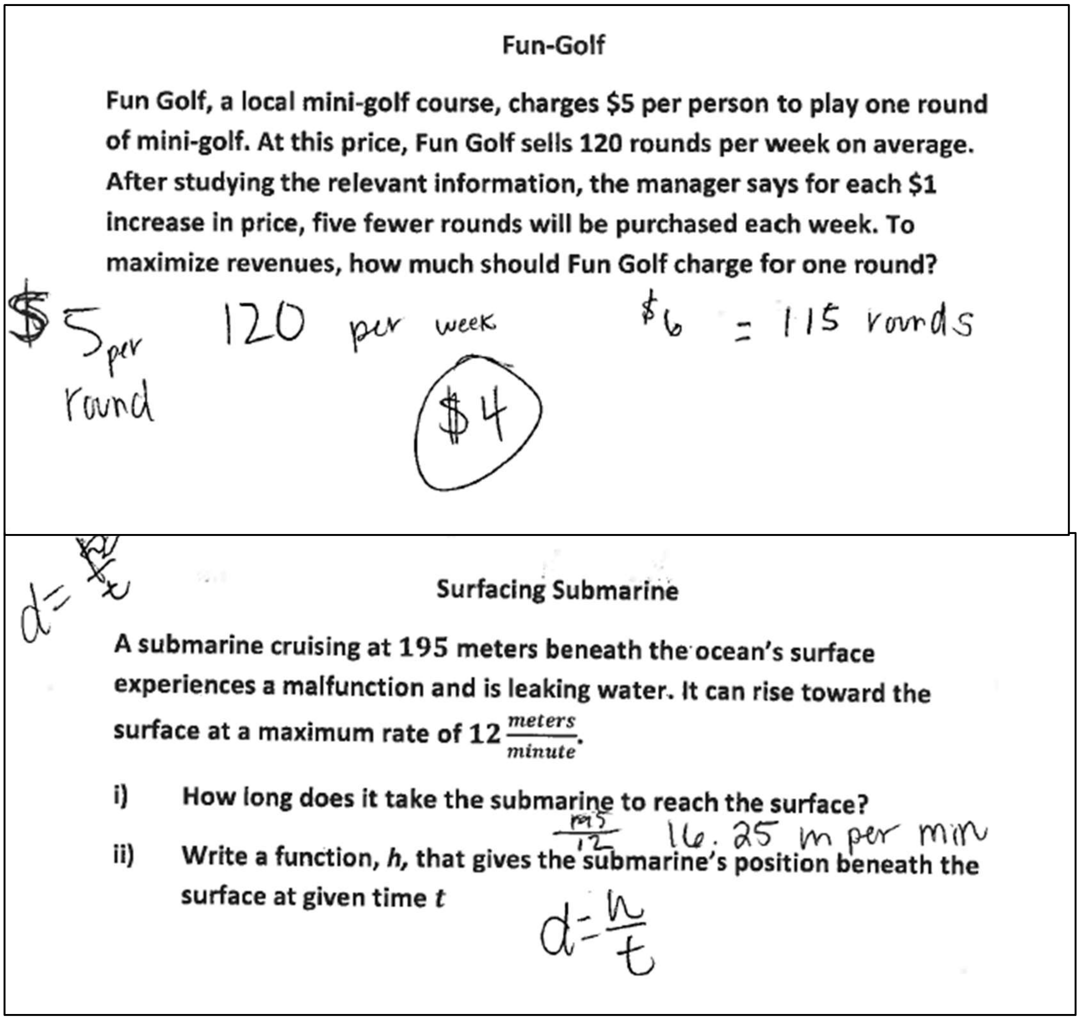
Charlie = 100 m left
Brett = 80 m left (100-20)
Brett = 16 seonds = 100 meters (last)
Charlie = 14 seonds = 100 meters (last)

$d = \frac{R}{t}$

(B) $100 \cdot 14 = 1400$ meters
(C) $100 \cdot 16 = 1600$ meters

Figure 4-3 F209 Cross-Country Race Problem

Also in looking at her previous MPSI problems, she explained on the Sonar problem, “I should have taken, like, this, and I don't know if I should have multiplied it or divided it by, like, 60 seconds. I don't know if I divided by one, or multiplied by one.” Her work shows the computational emphasis and application of known relationships and formulas. These formula application and uses are shown in Figure 4-4. F209 additionally displayed the reexamination process as discussed in that section.



Fun-Golf

Fun Golf, a local mini-golf course, charges $5 per person to play one round of mini-golf. At this price, Fun Golf sells 120 rounds per week on average. After studying the relevant information, the manager says for each $1 increase in price, five fewer rounds will be purchased each week. To maximize revenues, how much should Fun Golf charge for one round?

$5 per round 120 per week $6 = 115 rounds

$4

$d = \frac{r}{t}$

Surfacing Submarine

A submarine cruising at 195 meters beneath the ocean's surface experiences a malfunction and is leaking water. It can rise toward the surface at a maximum rate of 12 $\frac{meters}{minute}$.

i) How long does it take the submarine to reach the surface?

$\frac{195}{12}$ 16.25 m per min

ii) Write a function, *h*, that gives the submarine's position beneath the surface at given time *t*

$d = \frac{h}{t}$

Figure 4-4 F209 Formula examples from MPSI

4.1.1.4 F303

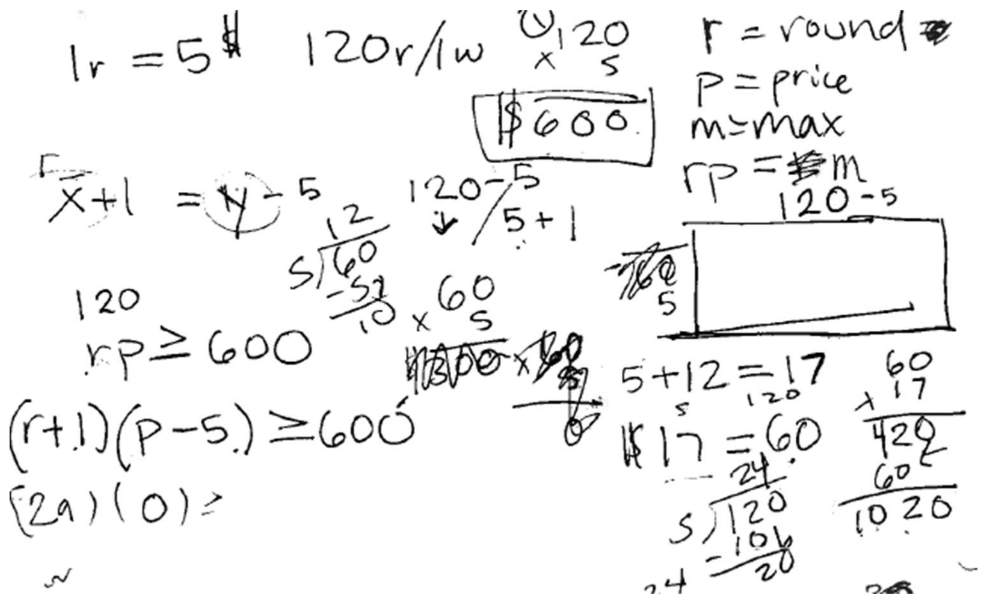
Student F303 worked the Sonar problem during the interview. After reading the problem, she immediately explained, “Okay. You have distance equals rate times time.” She would go on to reason about the relationship between the quantities to determine the following steps and calculations. F303 goes on to say, “Wouldn’t you have to convert into seconds because it says per second? So, wouldn't you multiply?" The application of the established formula provided her MPS method. In other problems, she explained the limitations of her formula base. On the Surfacing Submarine problem, she chose to multiply the quantities, explaining, "I think what I did was multiply those. I'm not sure." When asked about the $\frac{meters}{minute}$ notation she states, "No, I think it threw me off." F303 also used the reexamination orientation discussed later. The desire for a formulaic representation in spite of her self-identified limitation shows her overall structure forming her orientation.

4.1.1.5 F304

F304 when working the Fun Golf problem stated, "So, I have to – I think – I have to create a function." She goes on to explain that a trial and error approach will be too long and asks, "But how can I make this so I can see the maximum revenue?" She wants to create a formula or functional representation that will allow her to determine the solution without resorting to brute force methods. Though the formulas she applies are not previously known the methods used to develop them are.

She explains her use of algebraic notation, "I guess I can do, if it's *r* times *p* is going to equal the revenue, I can do that. So, *r* is – *r* times *p* is going to be greater than or equal to 600. I have something, but I still have to plug that in." The student begins to have difficulty deciding on her final answer and explains, "Hmm, the answer, honestly, right now, I'd probably guess some kind of range, since I can't find a function to put it in, I guess." In addition to formula application she also uses a big-picture focus orientation as explained in that section.

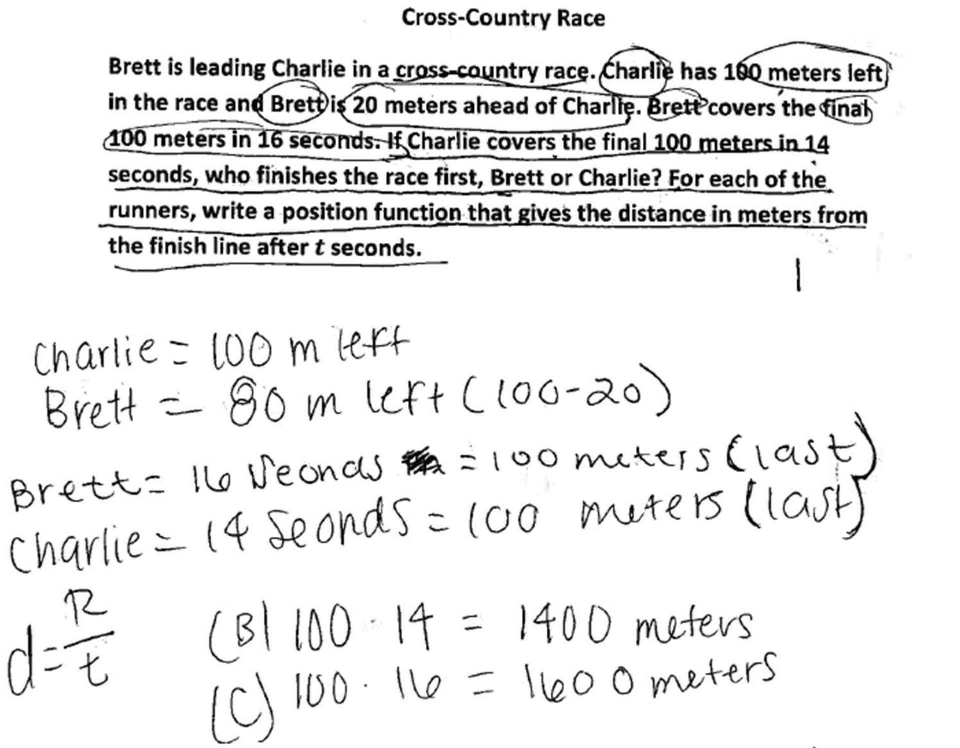
Figure 4-5 F304 Fun Golf Problem



4.1.1.6 F311

F311’s difficulty with the English language influenced her MPS behaviors and likely altered interview coding. However, the most formula application codes appeared in her interview compared to all interviews. She explains her solutions to the previously worked problems as, “I can imagine something – like an equation” and “What I did – I had a formula, and I just put down the formula, and then I solved it.” The primary focus of each explanation was the solving of her formula or equation.

F311 goes on in her interview to solve the Book Stacks problem. She quickly indicates the portions of the problem statement she wants to use and creates an algebraic representation of the problem. She goes on to directly solve her equation. Her preference for the precise formula appears in her work in Figure 4-6.


Cross-Country Race

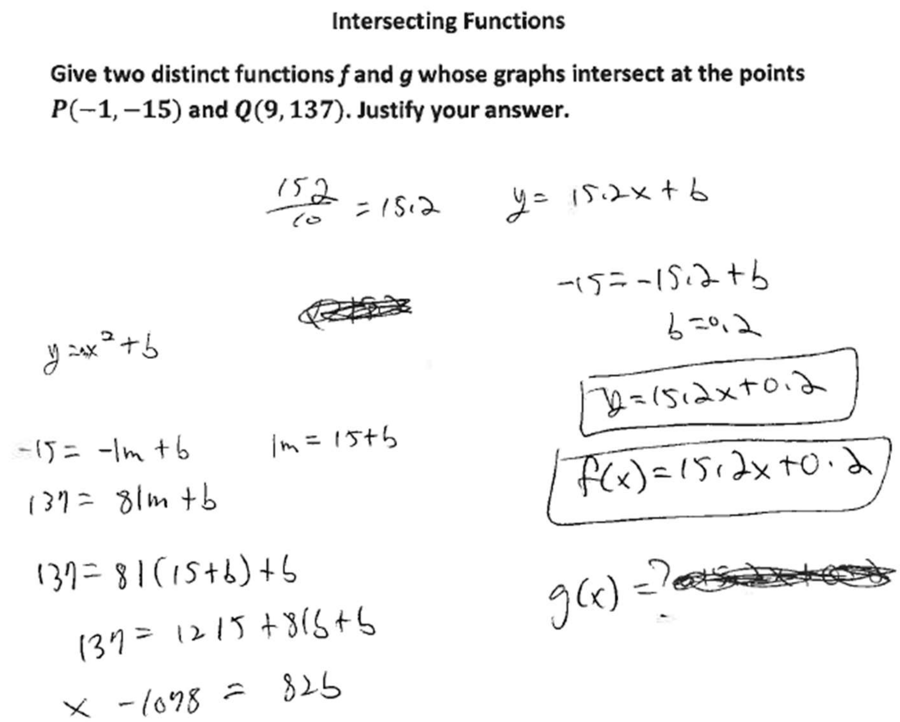
Brett is leading Charlie in a cross-country race. Charlie has 100 meters left in the race and Brett is 20 meters ahead of Charlie. Brett covers the final 100 meters in 16 seconds. If Charlie covers the final 100 meters in 14 seconds, who finishes the race first, Brett or Charlie? For each of the runners, write a position function that gives the distance in meters from the finish line after *t* seconds.

Charlie = 100 m left
Brett = 80 m left (100-20)
Brett = 16 seconds = 100 meters (last)
Charlie = 14 Seonds = 100 meters (last)
$d = \frac{R}{t}$
(B) 100 · 14 = 1400 meters
(C) 100 · 16 = 1600 meters

Figure 4-6 F311 Book Stacks Problem

4.1.1.7 M506

M506 noted in his interview that he “rarely draws things, but likes to write down the equations.” He noted in his solution of the Ken’s Garden problem that he “drew it out,” but clarified that it was only to help him find the model equations. The student stated, “for the word problem I wrote it in an equation and tried to solve it from [there].” Further, looking at his work on the Intersecting Graphs problem (Figure 4-7) no evidence of using a graphical representation appears despite the question specifically asking for functions “whose graphs intersect.”



Intersecting Functions

Give two distinct functions f and g whose graphs intersect at the points $P(-1,-15)$ and $Q(9,137)$. Justify your answer.

$\frac{152}{10} = 15.2$ $\quad y = 15.2x + b$

$-15 = -15.2 + b$

$b = 0.2$

$\boxed{y = 15.2x + 0.2}$

$\boxed{f(x) = 15.2x + 0.2}$

$g(x) = ?$

$y = ax^2 + b$

$-15 = -1m + b \qquad 1m = 15 + b$

$137 = 81m + b$

$137 = 81(15+b) + b$

$137 = 1215 + 81b + b$

$\times \; -1078 = 82b$

Figure 4-7 M506 Intersecting Graphs Problem

The interview with M506 coded for seven instances of the formula application orientation and no instances of other orientations, making the inherent structures used by the student more easily recognizable in his MPS. In the new problem worked during the interview, Book Stacks, the student was asked about his preference for developing a system of equations as opposed to using a trial and error approach. He explains his approach,

> Um...since the numbers are small I guess you can do trial and error too, but... I knew there would be two different...if I wanted to do it this way there would be two different equations. One for the height of the book and then one for the number of the books, and I just used substitution.

The identification of the appropriate system of equations as the model provides an appropriate strategy and substitution appears as an appropriate technique for solving the problem.

4.1.1.8 Other Students using Formula Application

Four other students coded for multiple instances of formula application orientations. Each of these students explained a portion of their MPS as focusing on the formulas they previously learned. An example of each student's explanation appears in Table 4-3. Student F106 provides an example of overlap between multiple orientations. She is one of three students to have instances of three separate orientations. Her responses are discussed in greater detail following the Reexamination section.

Table 4-3 Examples of Formula Application from other students

"With my online homework, uh...I feel like sometimes if I don't understand the question very well I do tend to like write it down a little more, like more steps, and more like ok, like x – 5=0, x= –5 kind of a thing."	F6106
"I know piece by piece, I mean you obviously need the pieces to eventually work the problem. But if you see how the pieces fit together in like an equation, then it can make sense of oh "How do I need to manipulate these number to get like an answer."	F6145
"Is just going to be that $5 times your 120 per week. And now your new equation will be your initial $5 plus a variable that we decide. We'll call it x, where x is equal to the increase in ticket prices."	M6221
"Um...distance equals time by velocity. It would make sense to multiply. I think I was assuming that we have the velocity...that it would make sense to multiply it with the .05 seconds."	F6259
"Ok, so. I'm trying to think, there has to be a sort of equation or something to use for this"	F6269

4.1.1.9 Formula Application and Course Grades

Nine of 11 students with multiple instances of the formula application orientation earned a grade of A, B, or C, in the College Algebra course allowing them to advance in the Calculus sequence. The study includes no direct examination of the role an orientation plays in the success of students within course grades. However, in contrast three of the eight students without codes of formula application received a D or F in the course.

Table 4-4 Summary of Course Grades and Formula Application Orientation

Student Orientation	Number of Interview participants	Number of Interview participants with A, B, or C grades	Number of Interview participants with D, F, or W grades
With Formula Application	11	9	2
Without Formula Application	8	5	3
Total	19	14	5

4.1.2 Reexamination

The next most frequent orientation coded amongst the students interviewed is reexamination. Reexamination students look back at previous steps ensuring their accuracy and/or appropriateness. Also, checking that the solution fits with the problem statement or question. Students holding this orientation will on multiple occasions use language referring to a look back or referencing a check of their previous steps. The student may identify previous locations in their attempt to solve a problem important to move forward. These examples do not provide an exhaustive list of possible coded instances, but should invoke the idea that a reexamination orientation involves the student looking back at past steps within their problem to ensure they are progressing correctly. The number of reexamination instances for the seven students coding for the orientation appear in Table 4-5.

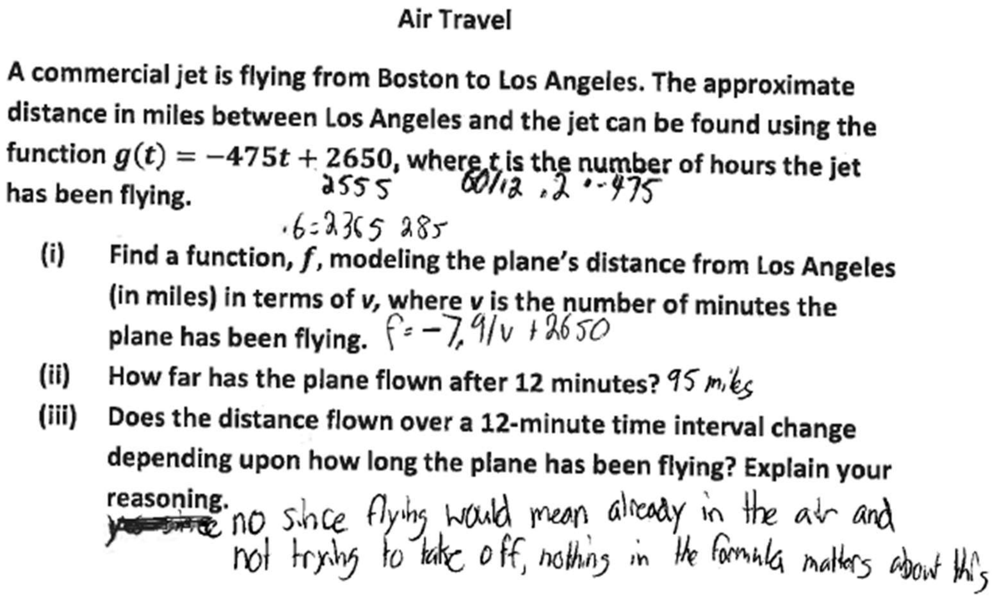
Table 4-5 Frequency of Reexamination Orientation

Student	Number of Instances (Instances in Initial Question)	Total Orientation Instances	Portion of Orientations classified as Reexamination
M105	6 (0)	9	66.7%
F209	6 (2)	14	42.9%
F303	8 (4)	11	72.7%
F304	2 (0)	12	8.3%
F315	4 (0)	10	69.2%
F318	12 (6)	12	100%
F6106	5 (0)	15	33.3%
Total	38	195	19.9%

4.1.2.1 M105

The interview with student M105 showed coded for more instances of reexamination than the previously discussed formula application. Immediately at the beginning of the interview he stated, “I like to reread the question. Quite a few times, to make sure I try to completely understand it before I start going off a wrong direction.” He goes on to explain, “I'm still kinda thinking about previous things. Just constantly thinking about all of it at the same time.” This frequent looking back idea comes with him into the MPS situation.

In discussing his work in the MPSI problems previously completed, he looked at the Air Travel problem. He explained, “I know I started to do it and went back and reread it a couple of times. I don't think that was the right way I did, but...” Then specifically referring to part (iii) he pointed out that his reasoning was distracting him from the problem statement, saying, “I was throwing in my own reasoning compared to what's actually in the problem.” This shows a reexamination of the problem statement after using alternate methods to decide on a solution. However, no explanation for this factor appears in his written work as seen in Figure 4-8.



Air Travel

A commercial jet is flying from Boston to Los Angeles. The approximate distance in miles between Los Angeles and the jet can be found using the function $g(t) = -475t + 2650$, where t is the number of hours the jet has been flying.

2555 60/12 .2 ·-475

.6=2365 285

(i) Find a function, f, modeling the plane's distance from Los Angeles (in miles) in terms of v, where v is the number of minutes the plane has been flying. f= -7.91/v +2650

(ii) How far has the plane flown after 12 minutes? 95 miles

(iii) Does the distance flown over a 12-minute time interval change depending upon how long the plane has been flying? Explain your reasoning.

no since flying would mean already in the air and not trying to take off, nothing in the formula matters about this

Figure 4-8 M105 Air Travel Problem

Also in his written work, it can be seen he wrote an incorrect calculation in the Surfacing Submarine problem, as seen in Figure 4-9. It is not clear that the reexamination orientation impacted his final solution, but it is possible that he adjusted his thinking after writing “12/195” to arrive at the correct solution, 16.25 seconds. This is not coded as an instance of reexamination since he did not discuss it during the interview. Though in the context of his statements in the interview it is possible that his orientation to reexamine his previous steps helped him to correctly solve the problem.

Surfacing Submarine

A submarine cruising at 195 meters beneath the ocean's surface experiences a malfunction and is leaking water. It can rise toward the surface at a maximum rate of 12 $\frac{meters}{minute}$.

i) How long does it take the submarine to reach the surface? 12 / 195 16.25 mins

ii) Write a function, *h*, that gives the submarine's position beneath the surface at given time *t*

h = 195 − 12t

Figure 4-9 M105 Surfacing Submarine Problem

4.1.2.2 F303

F303 coded for reexamination in addition to her codes for formula application. When asked if there is anything she tries each time she solves a challenging problem, she explained, “I would try to – if I got stuck, I would try to go back and see what I did wrong because if I did something wrong, then I'll be able to figure that out.” Her explanations mentioned that she would check to make sure her formulas were correct. In the new problem worked during the interview (Sonar, Figure 4-10) she explained, “Because per second, which is 60 seconds, and it says, ‘in five-hundredths of a second,’ so – I still want to go with the 73.15.” After using her equation that she developed in the earlier in the solving attempt, it was important for her to match the proposed solution with the problem statement.

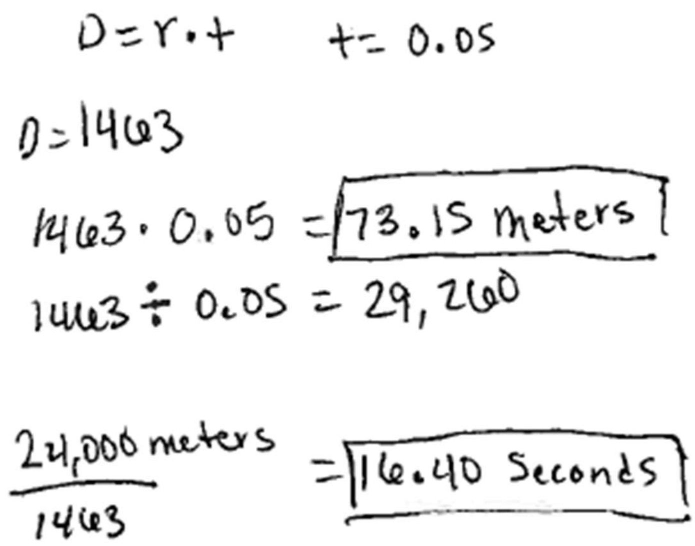
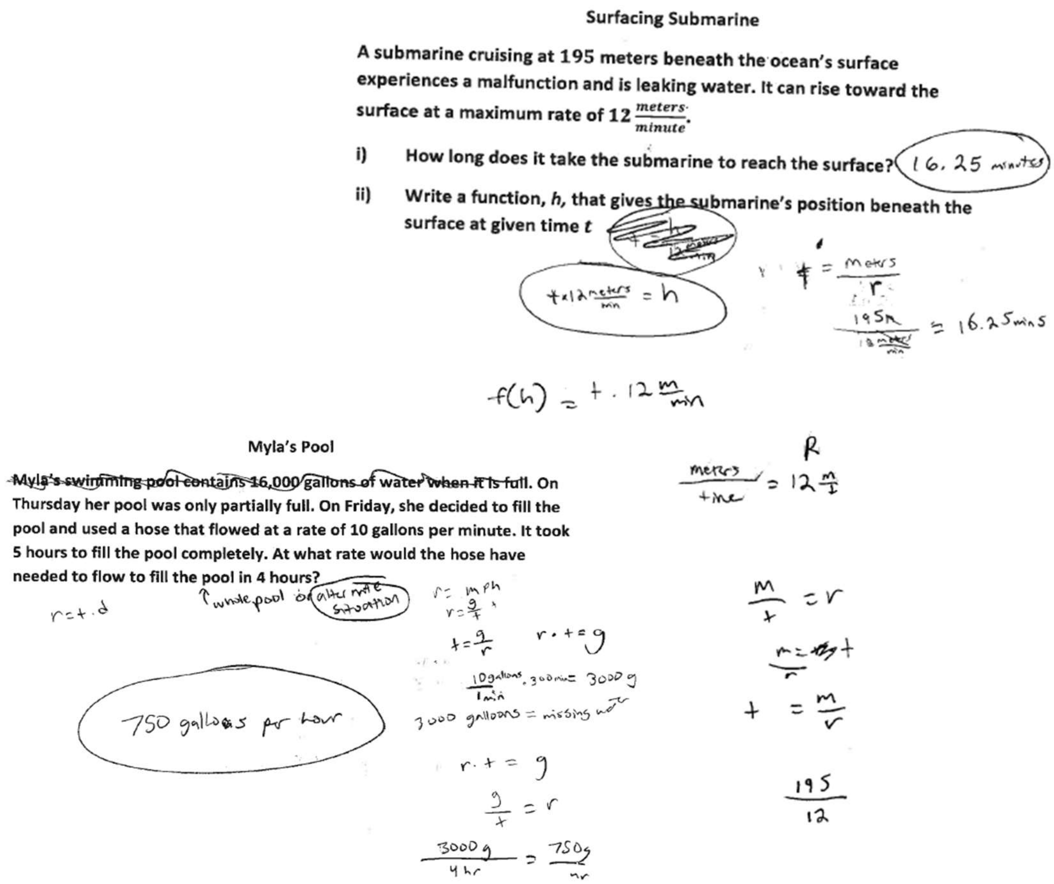


Figure 4-10 F303 Surfacing Submarine Problem

4.1.2.3 F318

Student F318's interview included the most instances of reexamination with 12. Throughout her work in both the new problem completed during the interview (Fun Golf) and her previously completed work earlier in the MSPI problems there are multiple scratch outs and write-overs indicating a reexamination of the previously done work. This work appears in Figure 4-11.



Surfacing Submarine

A submarine cruising at 195 meters beneath the ocean's surface experiences a malfunction and is leaking water. It can rise toward the surface at a maximum rate of 12 $\frac{meters}{minute}$.

i) How long does it take the submarine to reach the surface? 16.25 minutes

ii) Write a function, *h*, that gives the submarine's position beneath the surface at given time *t*

$t \times 12 \frac{meters}{min} = h$

$t = \frac{meters}{r}$

$\frac{195m}{12\frac{meters}{min}} = 16.25 mins$

$f(h) = t \cdot 12 \frac{m}{min}$

R

$\frac{meters}{time} = 12 \frac{m}{t}$

$\frac{m}{t} = r$

$t = \frac{m}{r}$

$\frac{195}{12}$

Myla's Pool

~~Myla's swimming pool contains 16,000 gallons of water when it is full.~~ On Thursday her pool was only partially full. On Friday, she decided to fill the pool and used a hose that flowed at a rate of 10 gallons per minute. It took 5 hours to fill the pool completely. At what rate would the hose have needed to flow to fill the pool in 4 hours?

$r = t \cdot d$

whole pool or alter rate situation

$r = mph$

$r = \frac{g}{t}$

$t = \frac{g}{r}$ $r \cdot t = g$

$\frac{10 gallons}{1 min} \cdot 300 min = 3000 g$

3000 gallons = missing water

$r \cdot t = g$

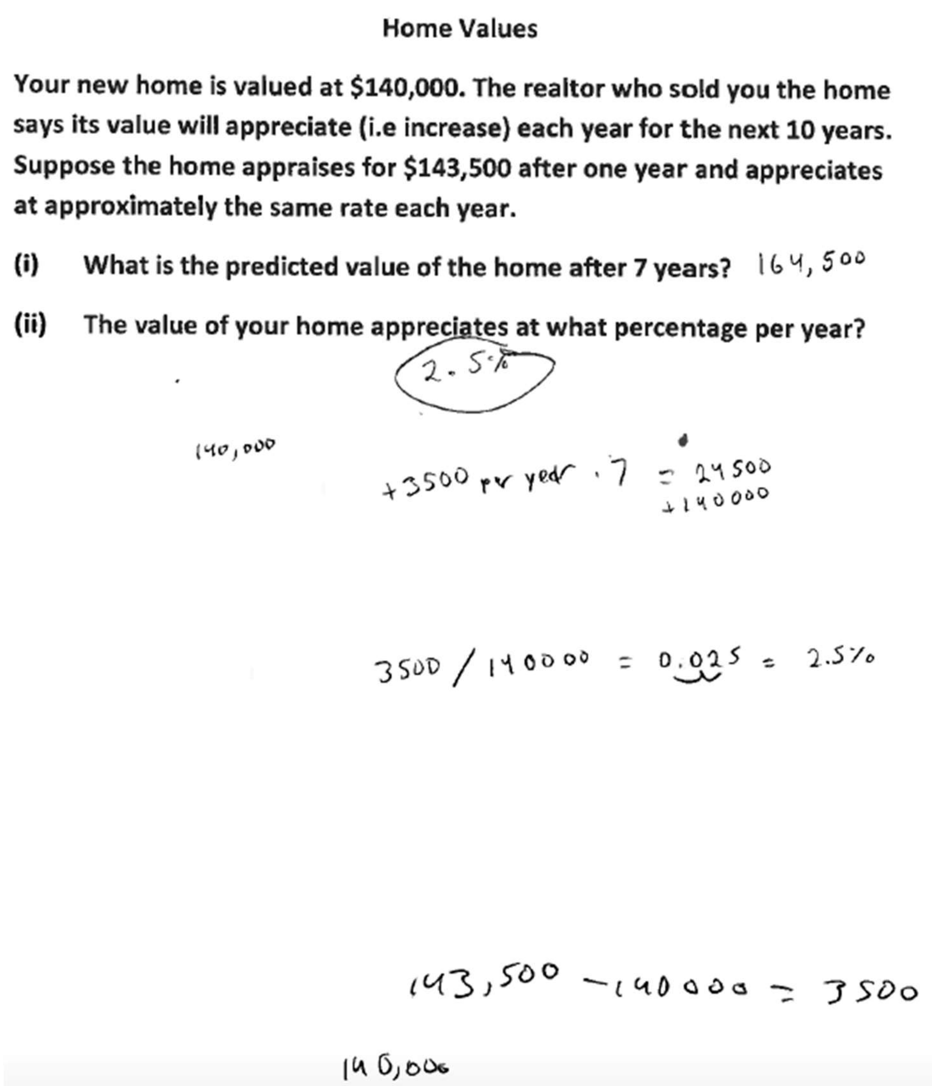
$\frac{g}{t} = r$

$\frac{3000 g}{4 hr} = \frac{750 g}{hr}$

750 gallons per hour

Figure 4-11 F318 Mark-outs and Write-overs

Further, the amount of space left between lines of work in the Home Values problem (Figure 4-12) was explained as being due to her sequencing of the steps. She indicated that the subtraction located near the bottom was done to ensure the correctness of the earlier calculation to answer part (i) (24,500 + 140,000). Each part of her MPS related back to an overall structure where each step was checked and verified that it fit into the wider problem-solving goal.



Home Values

Your new home is valued at $140,000. The realtor who sold you the home says its value will appreciate (i.e increase) each year for the next 10 years. Suppose the home appraises for $143,500 after one year and appreciates at approximately the same rate each year.

(i) What is the predicted value of the home after 7 years? 164,500

(ii) The value of your home appreciates at what percentage per year?

2.5%

140,000

+3500 per year · 7 = 24500
+140000

3500 / 140000 = 0.025 = 2.5%

143,500 − 140000 = 3500

140,000

Figure 4-12 F318 Home Values Problem

To distinguish F318's reexamination orientation from the more common formula application it is necessary to examine her explanations as she worked the Fun Golf problem during the interview. Figure 4-11 shows her work on the problem during the interview. There are multiple formulas and algebraic equations present in her work. However, her statements indicate that each operation was reexamined as it was completed. She explained her work in the middle of the page,

> Okay, so for five plus A equals X minus 5A. And then X is number of rounds – yeah. That looks about right. So I want as much money as I can get. I think I'm going to start out with a 10 for my values of money and then I think later I will do $1 just to kind of see the different things. So I'm going to add – make this 15 dollars per person, so five plus – so I'm going to say A equals 10. So five plus 10 equals X minus five times 10. And then I'm going to see how that compares to the original 120 rounds per week. Here we go – I'm on my way to actually solving this problem. So I guess 120 equals the number of rounds, so X would equal 120 minus 50 – what? Oh, okay – so I actually have to do 15 times – I guess I need another number.

The student verifies each step with the past steps. She questions each operation or matches it to another operation to ensure a correct pathway. Then she continues, "So let me think. I guess B is new rounds. What is this? I guess this would be new rounds that are being sold." She explains that she found a misstep in her work by her checking process.

> So we have 25 dollars, and that's for 100 rounds, so they're making 25 dollars with – okay, that's – I did that wrong. I messed up, because I didn't multiply this out by five. Okay, 20 times five equals 100. See, that's a good thing – I caught that thing because I was double-checking my work.

She goes on to explain her solution method,

> I could probably use an inequality for this, but I feel like if I do that I'm going to get confused, so I'm going to go back in the middle, check out 15. So five plus 15, and then I'm going to invent a new weird symbol, it's going to be this triangle with three dots. I guess I could do a "therefore" symbol, I don't know."

Her checking protocol appears all the way through her solution even once she switches ideas from a more algebraic approach (seen in the middle) to a trial and error style (around the edges) using the triangular symbol.

4.1.2.4 F6106

F6106 discussed her problem-solving methods both in terms of formula application and reexamination. Initially when asked to explain her approach she points to a desire to make sure she has used the correct pathway stating, "basically I'll just plug it back in and see if works out and see if there is something that makes sense." In the earlier example of her formula application in Table 4-3 she stated, "I like to write it down a little more." She explains her thinking by referencing formulas and the creation of an equation, along with an underlying second look at the problem. She explains, "I want to make sure I'm actually doing the steps right."

While working on the Fun Golf problem in the interview F6016 tells the interviewer that "I want to say like 120 minus five but that doesn't seem right." In solving the problem she displays her dual approach using both a formula reliant plan while also reexamining her steps and solution path for appropriateness and correctness. She explains while pointing at her work (shown in Figure 4-13),

> Um so I just went through what I was thinking. I went 120 divided by six which came out to be 20 and just took the five out which was 15 times six which is what… and that's roughly around (pointing to table) how many rounds they're selling just multiply six times 90 which was 540 which shows that they're losing money which is what the manager doesn't want. So I'm trying to do...50 times 120 to see if there will be a drastic change.

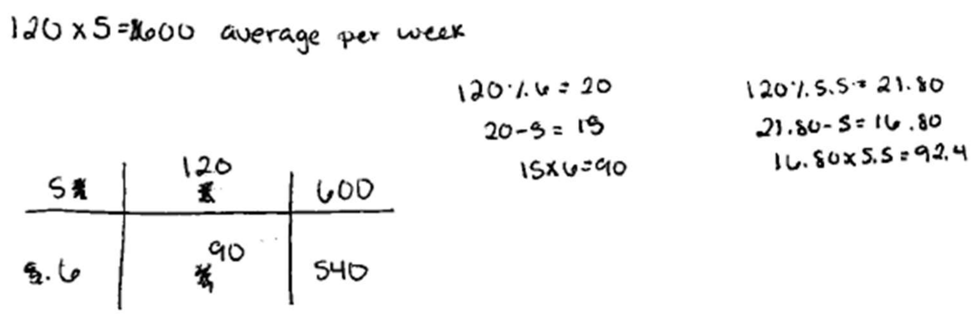


Figure 4-13 F6106 Fun Golf Problem

At each step she appears to be adherent to both the need for an equation based representation but also that her work fits into a neat solution.

4.1.2.5 Other Students using Reexamination

Three other students displayed instances of Reexamination. The need to look back at the previous steps of the problem shows in their new work during the interview and also when reviewing their past work. Figure 4-14 shows F304's work on the Cross Country Race problem showing the multiple reworks and rechecks in the student's problem solving for that problem.

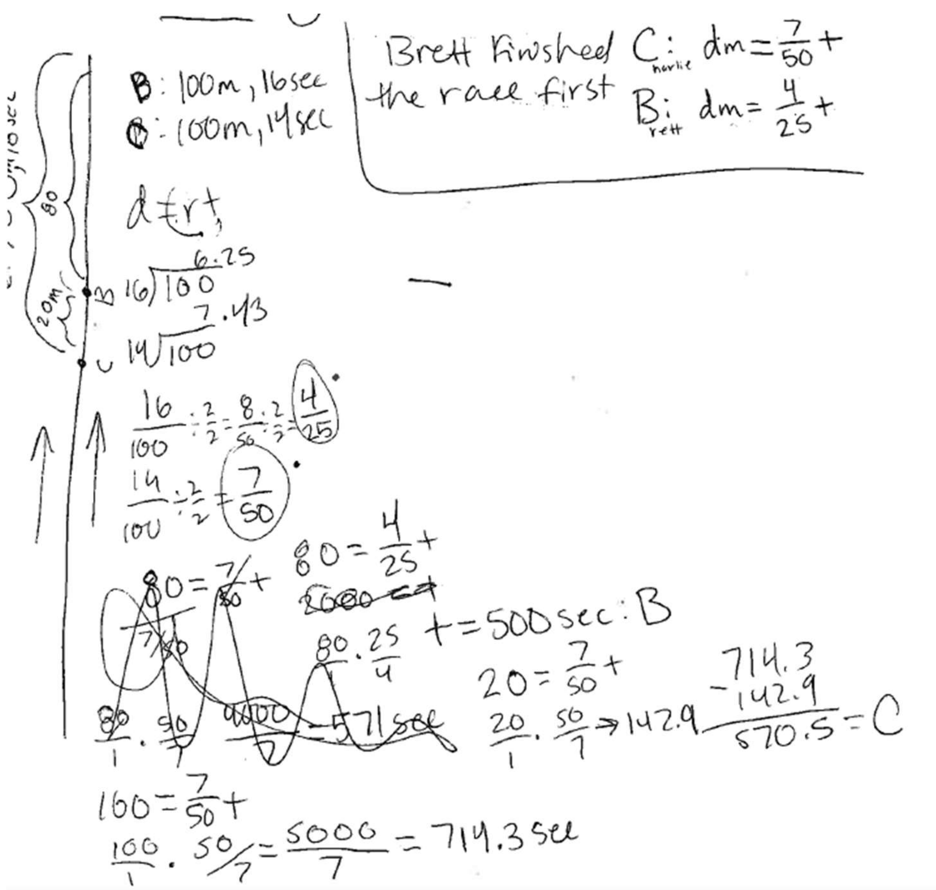


Figure 4-14 F304 Cross Country Race Problem

Additionally, the remaining students are quoted in Table 4-6 with an example of their use of reexamination in the interview. Student F209 exhibited the orientation while examining previous work. However, she followed those instances with multiple reexamination instances while solving the new problem. Another student, F315 explained her reexamination orientation as part of her attitude toward MPS. She pointed to being uncomfortable with certain types of problems and the need to review those problems in addition to a built-in frustration with her own actions in solving math problems.

Table 4-6 Examples of Reexamination from other students

"Oh! In the first one, seconds pops out at me that I probably should have done something with that I didn't. Oh, because I probably should have done, I don't know, should I have done something with the seconds?"	F209
"I'm thinking I went through some simple multiplication and addition in my head, and like, this one seems okay. I'm going to write it and check it."	F304
"No, I just really like algebra, and then when I don't get questions right, I get mad about it, but most of the time – I mean, so word problems, they're really difficult for me."	F315

4.1.3 Big-Picture Focus

Big-picture focus appeared third most among orientations displayed by the interview participants. A characterization of big-picture focus includes examining the problem from a wide overview. Big-picture users told the researcher about their desire to see the entire problem or question. Students coded for big-picture might indicate a desire to work from a broader viewpoint of the problem. These students might use a graph or diagram to obtain this wider view, but no strategy or technique serves as a requirement of a big-picture focus orientation. Big-picture students might also hold a formula application orientation but would determine the correct formula or equation based on a holistic view of the problem. Table 4-7 shows the number of codes for big-picture focus tabulated for each of the interview participants.

Table 4-7 Frequency of Big-Picture Focus Orientation

Student	Number of Instances (Instances in Initial Question)	Total Orientation Instances	Portion of Orientations classified as Big-Picture Focus
M106	4 (1)	10	40%
F304	2 (2)	12	16.7%
F6141	5 (1)	5	100%
F6145	4 (2)	13	30.7%
F6238	3 (1)	12	25%
F6269	5 (0)	8	62.5%
Total	23 (7)	195	11.8%

4.1.3.1 F6145

Student F6145 tells the researcher that when doing MPS she wants "To make sure you read the whole thing. I used to be kind of hasty and just like read whatever just the question part was and then look back first, but I feel reading from top to bottom completely helps me." Then she goes on to state that she takes more of a global view. The latter statement is a response to a question that asked the student if she preferred to work "piece by piece" or with a "global view." However, in each problem worked before the interview the student wrote a conclusion for each problem (Figure 4-15).

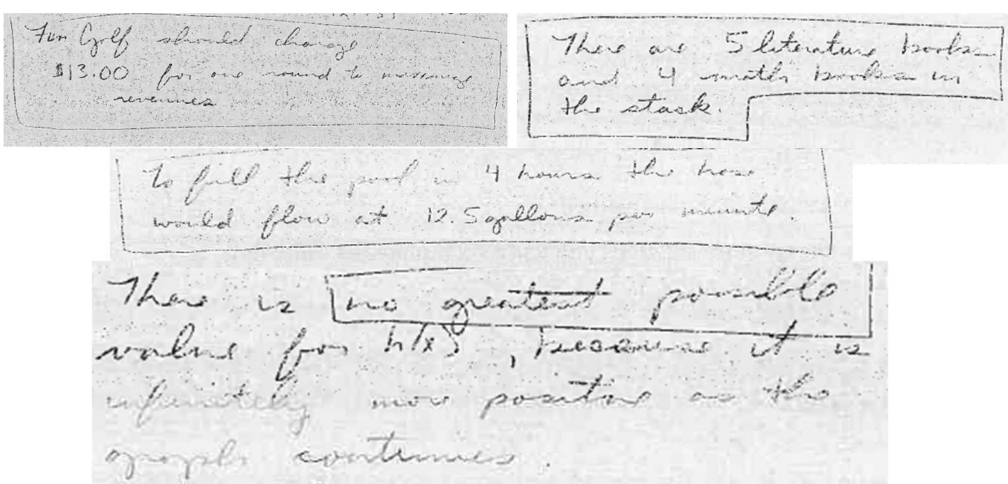


Figure 4-15 F6145 Written Explanation in Previous Problems

In contrast, her solution to the Ken’s Garden problem in the interview did not show a written explanation. Her verbal responses while solving the problem reflected a wider look at the problem statement. She begins, “To start with, um...the question reads... So, there's a new garden and it wants to be about half the size of the current garden. So, half the size probably means half the area of the current garden.” She begins with a broad look at the problem’s statement before closing in on the wording and operations needed for solving the problem. She ultimately worked the problem back to the formula application orientation that she also coded for.

4.1.3.2 F6269

Student F6269 worked the Cross Country Race problem with a broad beginning outlook. She began by writing down each of the problem constraints and attempting to determine how each fits with the problem statement. She explains,

> I found, I just found the, what they're asking for, so wait. Who finishes first? And write a position function. Right now, I'm not knowing where I should go from this, so I need to write down what they're asking for. So, it says that Charlie has 100 meters left, and he does 100 meters in 14 seconds, but Brett does 100 meters in 16 seconds, but he's 20 meters ahead. So I need to see if Brett catches up or if Charlie just wins. I feel like it's painfully obvious. Ok, I found the rate, but I don't really know what to do with it anymore. Or I never knew if this was what you had to do at the beginning.

Additionally, in Figure 4-16 her work shows the broad to narrow direction of her focus.

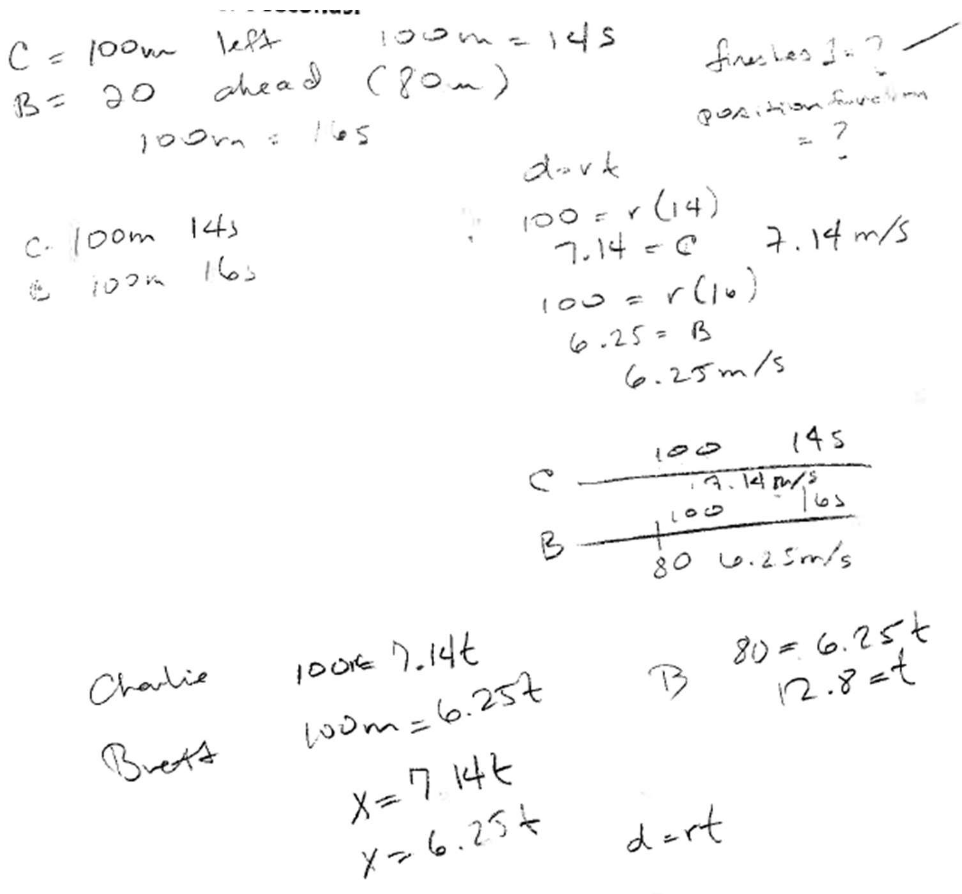
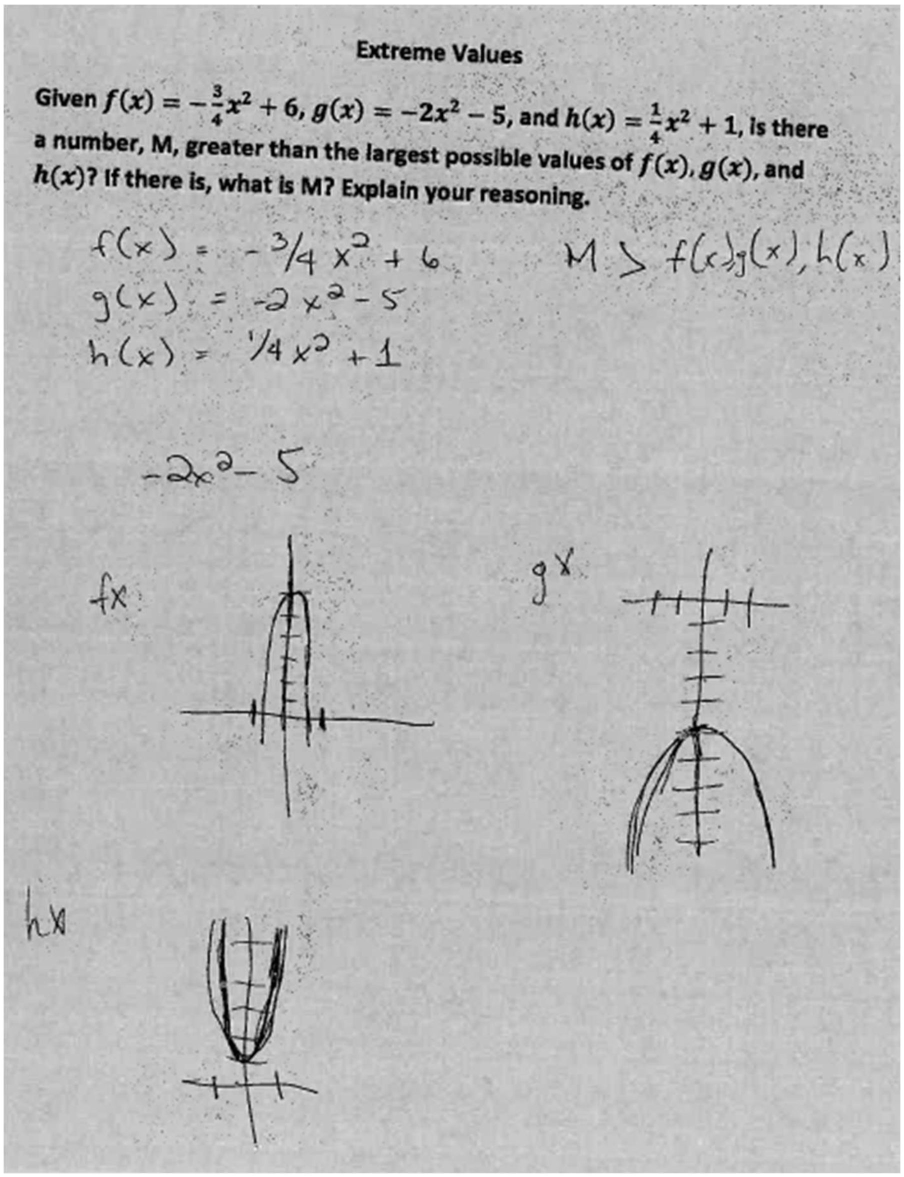


Figure 4-16 F6269 Cross Country Race Problem

Similar to F6145, she would work each part of the problem back to a formula representation. However, her discussions of the previously worked problems did not code for any instances of the formula application orientation. Given the graphical representations seen in her previous problems possibly a formula application orientation hides within her work. The use of these graphs suggests a further emphasis in her big-picture focus. For example, in the Extreme Values problem (Figure 4-17) she explains, “Yeah, writing them down. And then I didn't really know what to do, because, it says it wants to know if any values, any positive values [reading] wait, they want to know if there is a larger number than anything that those formulas can give, which is M.” This referring to the problem statement provides evidence of a broad look at the problem as her orientation. The nature and type of problems worked by a student may also impact which orientations the student displays in the interview.


Extreme Values

Given $f(x) = -\frac{3}{4}x^2 + 6$, $g(x) = -2x^2 - 5$, and $h(x) = \frac{1}{4}x^2 + 1$, is there a number, M, greater than the largest possible values of $f(x)$, $g(x)$, and $h(x)$? If there is, what is M? Explain your reasoning.

$f(x) = -3/4\, x^2 + 6$ $\quad$ $M > f(x), g(x), h(x)$

$g(x) = -2x^2 - 5$

$h(x) = 1/4\, x^2 + 1$

$-2x^2 - 5$

Figure 4-17 F6269 Extreme Values Problem

Student F304 coded for all three of the most frequent orientations. In her interview, she explained to the researcher,

> I think I look at it more globally, in general. I feel like when I try to solve it in pieces, I tend to get caught up in certain things, and then I miss other parts, if I don't look at the whole picture. But I know that I have a bad habit of not checking for minor things, so I'm still putting it in my mind you always have to go back and check, because I'm always looking at it more globally, so I miss one thing.

However, while solving the new problem (Fun Golf, shown in Figure 4-5) she coded for no instances of big-picture focus. The relationship between what she believes she does in her MPS and what she actually does provides a challenge in analyzing her interview. Similar to F6145, the presence of graphical representations suggests the use of a big-picture focus orientation for the student to progress in their problem solving.

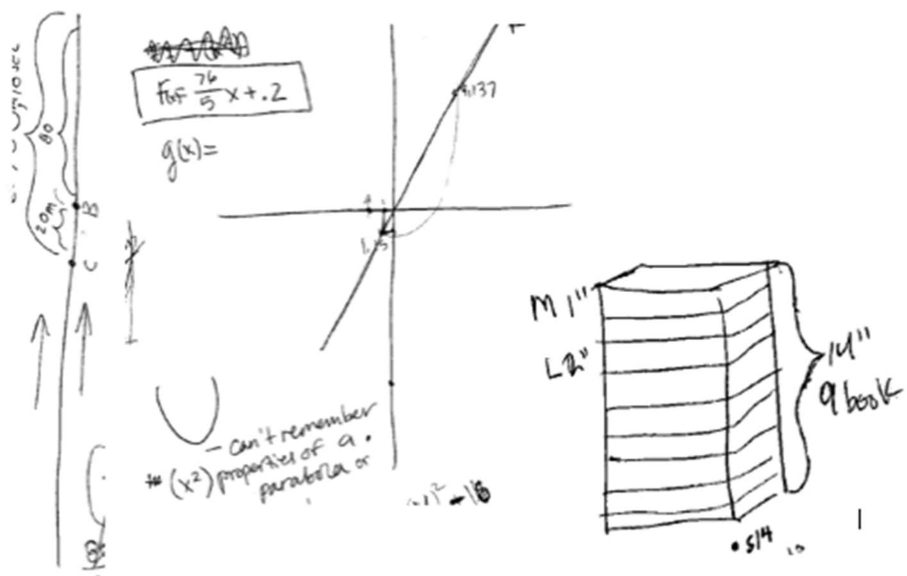


Figure 4-18 F6269 Book Stacks Problem

4.1.4 Linear Progression – Step by Step

Next, three students displayed a linear progression orientation. These students pick an initial MPS step and use the information gained to inform the next choice of strategy or technique. In contrast to Big-picture, these students do not plan to begin the problem. These students make an informed decision in their direction of MPS but only an initial decision and not an overall plan or intention. Students holding this orientation scaffold their way to a solution path. The students may hold other orientations as well, even within the same problem but there is an initial attempt of a strategy or technique that refers them to another of their held orientations. The number of linear progression instances appears in Table 4-8.

Table 4-8 Frequency of Linear Progression Orientation

Student	Number of Instances (Instances in Initial Question)	Total Orientation Instances	Portion of Orientations classified as Linear Progression
M106	2 (0)	10	20%
F315	6 (0)	10	60%
F6106	3 (0)	14	21.4%
Total	11 (0)	195	5.6%

4.1.4.1M106

Student M106 looked back at the Home Values problem he worked previously. The problem includes a part (i) and (ii) in the problem statement. He explained, "I basically found – at least, I think I found part two in part one just from doing, how I solved it so I just drug it over there. That happens in a lot of problems, you'll find the answer to some other part in like an earlier part if you do it a certain way, at least." This type of scaffolded thinking arises in his solution to the Fun Golf problem done in the interview. M106 states, "So, it says $5 per person to play one round and on average, they do 120 rounds. So, that'd be about… $600 in revenue for… $5 a person. Normally, I would just make a graph mentally, but it's my thought process, and that's per each dollar increase… they use five rounds." His calculations then gave rise to a pattern he identified. "So what I'm starting to notice is that every dollar that it's starting to go up, it's losing $10. It's going up $10 less each time. So, it keeps going by that." The influence of the problem statement being given in parts is unclear, but only one other student explicitly stated they used another part of a problem to find a solution to a different part. M310 indicated to the researcher that he used a part (ii) equation to find a solution to part (i), but only after the interviewer asked why he wrote down an equation first. Thus, no assumption can be made that a problem separated into parts will encourage this orientation for a student.

4.1.4.2 F315

Student F315 built her MPS method upon her previous steps in solving the Fun Golf problem in the interview. She explains her initial approach,

> So it charges $5 per person – so we can do 5X to play one round of mini golf. At this price, they sell 120 rounds per week, on average. So I'll just write that down because it sounds like an important piece of information. After studying the relevant information, the manager says for each $1 increase in price – so plus one – five fewer rounds will be purchased each week. Okay. I guess with that, this would be 115, because he said five fewer rounds each other. To maximize revenues, how much should Fun Golf charge for one round?

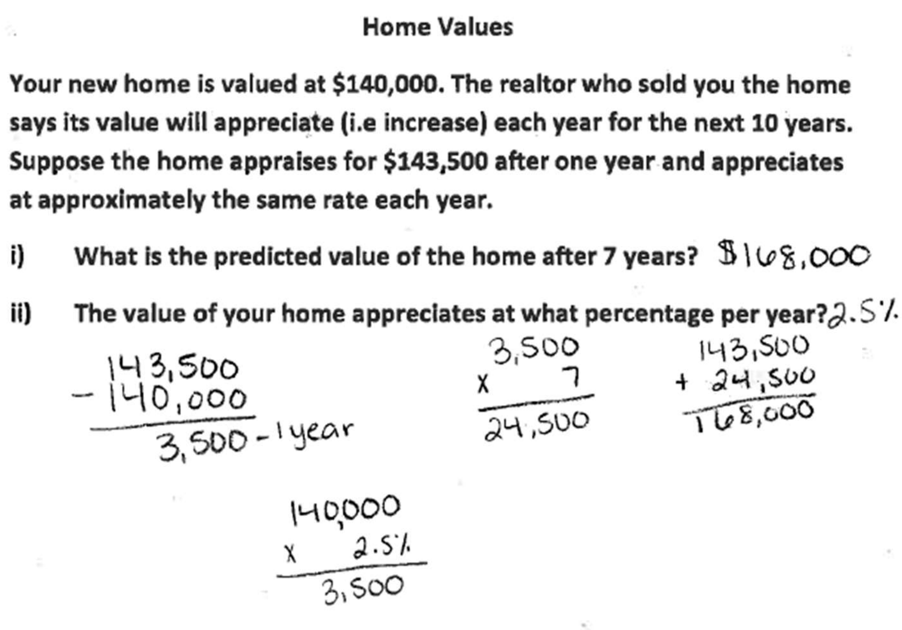
So, she attaches an initial representation for 5X to represent the revenue multiplication. She explains that she next found the important piece that 115 would be the number of rounds sold for a higher price. Then sees the need to maximize revenues. She goes on to say, "The five was the dollar, too. I don't know what I did. Could I have done 6*X* equals 115?" She uses the previously written 5X to determine what her next step should be. She continues, "So I can't multiply the dollars because they're adding a dollar. And then so 120 rounds per week – can we do 120 times seven?" Then she sees that the relationship between 120 and seven is not as intended. She states, "So he makes 720 per week. Actually, no, can you multiply by five? Okay, so 600 per week. And then if you increase the price, so which would be six dollars, and then it says five fewer rounds will be purchased each week, so times 115." Each operation or strategy helped F315 form her next course of action.

4.1.4.3 F6106

F6106 showed her step-by-step focus in the previously worked problem. For the Home Values problem she explains, “I think Um I did everything by steps. First this then this then this.” She goes on to indicate her steps as she moves from left to right in her work shown in Figure 4-19,

> It says that it was first $143,500. I just got the difference from the value, you know and that was one year right? At seven years I multiplied it and then I have no idea where I got the 24...From original price to 24. That's where I got that.

The progression through the problem appears in her work and her explanation. F6106 displayed no initial plan, but rather each operation gave rise to the next choice.



Home Values

Your new home is valued at $140,000. The realtor who sold you the home says its value will appreciate (i.e increase) each year for the next 10 years. Suppose the home appraises for $143,500 after one year and appreciates at approximately the same rate each year.

i) What is the predicted value of the home after 7 years? $168,000

ii) The value of your home appreciates at what percentage per year? 2.5%

143,500
− 140,000
3,500 - 1 year

3,500
x 7
24,500

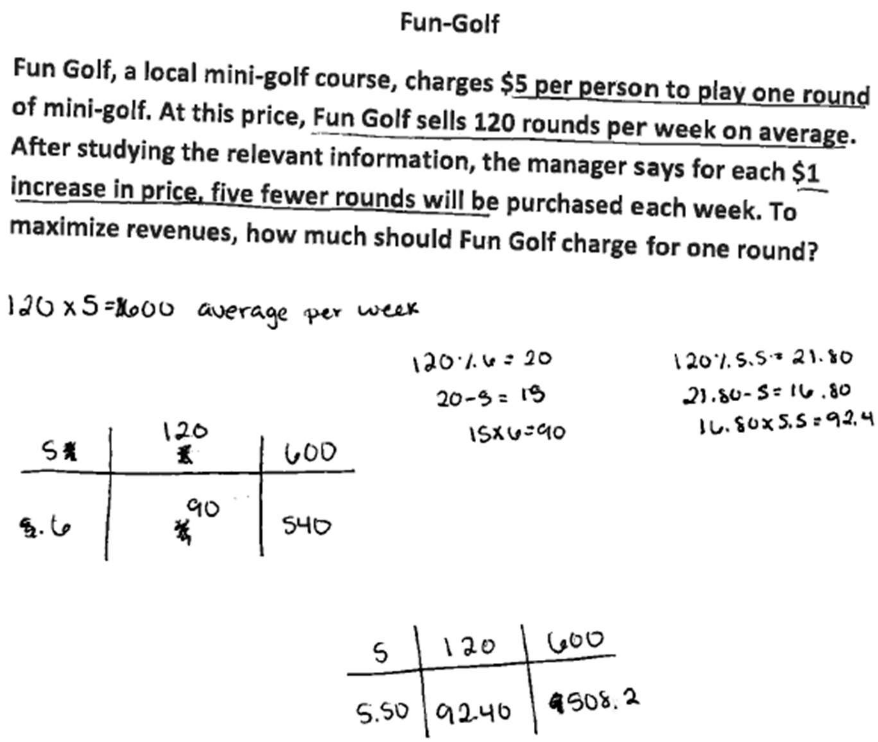
143,500
+ 24,500
168,000

140,000
x 2.5%
3,500

Figure 4-19 F6106 Work on Home Values Problem

In the Fun Golf problem worked in the interview. F6016 showed unproductive instances of the step-by-step focus. Since the student only used the previous steps she did move from the unproductive pathway. As she explains, I went 120 divided by six which came out to be 20 and just took the five out which was 15 times six which is what and that's roughly around (pointing to table) how many rounds they're selling just multiply six times 90 which was 540 which shows that they're losing money which is what the manager doesn't want.

Her work appears shown in Figure 4-20. Each step gave rise to her next choice and no rechecking of the procedure presented.



Fun-Golf

Fun Golf, a local mini-golf course, charges $5 per person to play one round of mini-golf. At this price, Fun Golf sells 120 rounds per week on average. After studying the relevant information, the manager says for each $1 increase in price, five fewer rounds will be purchased each week. To maximize revenues, how much should Fun Golf charge for one round?

120 x 5 = 600 average per week

120 ÷ 6 = 20
20 - 5 = 15
15 x 6 = 90

120 ÷ 5.5 = 21.80
21.80 - 5 = 16.80
16.80 x 5.5 = 92.4

5	120	600
6	90	540

5	120	600
5.50	92.40	508.2

Figure 4-20 F6106 Work on Fun Golf Problem

4.1.4.4 Additional Student Comments

Two other students mentioned a “step” process in their response to the initial interview question. However, neither of these students coded for any instances of a linear progression orientation in their interview. F209 said, “I decide what formula, or problem solving steps I need to do.” The reference to formulas matched with eight coded instances of the formula application orientation. The steps did not pair to any orientation and, during the interview, did not appear to be a part of the student's overall structure. Similarly, F318 answered the researcher's follow-up question about a go-to technique or strategy with, "Not particularly. Just kind of follow the steps." Again, the researcher coded no instances of a linear progression orientation, and this is not considered part of her overall structure. However, the interview protocol might "hide" additional orientations from the coding process.

4.1.5 Replication

Two student coded for instances of a *replication* orientation. Students holding a replication orientation identify previously worked or seen examples similar to the problem at hand and match their work to the style seen or used previously by the student as the focus of MPS. Students holding this orientation use their previous knowledge to provide the structure used in their problem-solving methods. The instances of *replication* are shown in Table 4-9.

Table 4-9 Frequency of Replication Orientation

Student	Number of Instances (Instances in Initial Question)	Total Orientation Instances	Portion of Orientations classified as Replication
M310	2 (1)	4	50%
F505	6 (2)	20	30%
Total	8 (3)	195	4.1%

Observing students with this orientation could be challenging in the interview if the student does not mention it as part of their initial approach in answering the researchers first question. A student may rely on replicating work he used or observed previously, but does not verbalize this portion of their approach. In this situation, it may appear to show the researcher a different orientation than replication. This limits the interview protocol throughout the study.

4.1.5.1 M310

Student M310 stated directly to the initial question from the interview, “I just use – based on what I’ve learned in class.” Later in the interview M310 worked on the Air Travel problem. He struggled with how to rewrite the function in terms of a number of seconds, *V*, rather than the number minutes, *T*. He explains, “I think you just replace *T* with *V*. It would be easier if it already told me the number of hours.” Thus, M310 needs a way out of his current pathway. He then says, “I could try to put in an example of…” He goes on to explain that when he has trouble he will refer back to what he has seen previously. His solution continues mentioning that previously teachers have told him to check multiple values. Showcasing his orientation of replicating previously done examples.

4.1.5.2 F505

P505 tells the researcher she would first focus on finding measurements in the problem. This followed an initial response asking if she would get to see the question being asked about. After the interviewer explained that the question targets a general explanation she offered, “I would really just try and follow what I learned in class.” Explaining to the interviewer, “So pretty much just follow the same exact steps, but with different numbers.”

Later in the interview F505 answers questions about previously worked problem in the MPSI packet. For the Intersecting Graphs problem, she attempted to determine two graphs that intersected at two given points using only linear equations. She explained that one of the two would require a negative slope and that she could only use one formula “because I didn’t really know how to find two formulas with one point on each one.” Then she claimed that “I know I learned it back in high school.” Holding to the orientation that replicating previously learned procedures or techniques could complete the problem. Additionally, F505 coded for instances of justified reasoning. She was the only student noted with this orientation. However, this orientation relates to statements about matching to things she learned or used previously. She explained, “I went to school up in New York, and in the classes, when we did stuff like this, we had to write out an explanation at the end.” This justified reasoning appears to correspond to matching to previous styles employed in her MPS background.

4.1.6 Tinkering

Two students coded for instances of a Tinkering orientation. Students holding a tinkering orientation appear to choose at random from previously known strategies or techniques. Little observed regard for the appropriateness or usefulness of chosen strategy, but can help the student determine productive pathway. This orientation may be useful for the student’s MPS despite its haphazard nature. Though the researcher observed minimal planning or intended progression for the problem solver, the student may have not verbalized or noted in some way the actions or thoughts used in their MPS method. As a result, the student may be considered to have used deficient explanation leading to the researcher being unable to uncover the student’s true orientation. The instances of tinkering are summarized in Table 4-10.

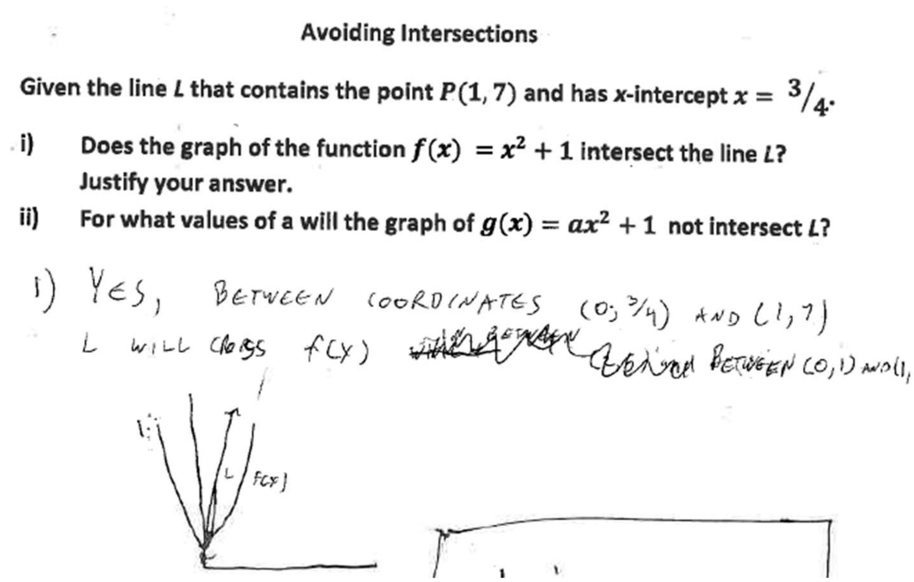
Table 4-10 Frequency of Tinkering Orientation

Student	Number of Instances (Instances in Initial Question)	Total Orientation Instances	Portion of Orientations classified as Tinkering
M6115	10 (5)	15	66.7%
M6221	2 (0)	7	28.6%
Total	12 (5)	195	6.2%

4.1.6.1 M6115

Student M6115 displayed a quick moving MPS approach immediately in answering the researcher’s initial question. As in every interview, the first question to the student asked, “Can you describe kind of the usual process or steps you go through when you're solving a challenging math problem?” Part of the challenge in categorizing the student’s methods includes the verbose nature of his responses. For this question alone his answer consumed one minute and 20 seconds. His answered jumped from possibility to possibility. Initially he stated, “Probably the first thing is that if I see a problem that I’m going to maybe write it out.” He continues, “my first impulse it to just try doing things in my head.” Next, he draws attention on his own to the frequent direction changes in MPS approach. He tells the interviewer, “If you were looking at grading my paper what you'll probably see is the point in time where I sort of half wrote down something until I got to a train of thought and then the point where I probably missed something.” Then later still in the answer to the initial inquiry, “Step one I'm going to write in full you know all of this next step that happens the PV the DV the AVS or something like that.” He did not explain the meaning of his abbreviations. Finally, he finishes his answer with, “I can learn from my own mistakes which seems to be most of the problem I have in math.” In this one answer alone, his approach ranged from drawing out solutions, to a formula based idea with the unknown abbreviations, to a checking or reexamination possibility. Since none of these appear to explain his “overall structure and approach” they did not code in the reexamination or formula application orientations.

Later in the interview the researcher asks M6115 if he would “make a representation or diagram of any sort usually?” Even with this targeted question the student indicates the movement in his approach, “Rarely, I'd say that something occasionally if it's some kind of a function or something and it still doesn't feel right...I'll try to draw a graph or something, but usually that's...uh...forestalling the inevitable of settling on an answer I don't like anyway.” Next, M6115 reviews the previously completed Avoiding Intersections problem (Figure 4-21). He explains, “I know that x^2+1 starts on the y-intercept and then increases exponentially and so looking at ... and I probably did this wrong...but my idea was at the time...yeah I think I did [do it wrong] (pointing at graph) because I think I did run over rise."



Avoiding Intersections

Given the line L that contains the point $P(1, 7)$ and has x-intercept $x = {}^{3}/_{4}$.

i) **Does the graph of the function $f(x) = x^2 + 1$ intersect the line L? Justify your answer.**

ii) **For what values of a will the graph of $g(x) = ax^2 + 1$ not intersect L?**

1) YES, BETWEEN COORDINATES (0, 3/4) AND (1, 7)
L WILL CROSS f(x) BETWEEN (0, 1) AND (1,

Figure 4-21 M6615 Work on Avoiding Intersections Problem

His full answer lasts two minutes and 58 seconds. During his explanation, he tries multiple approaches. Initially, he states, "sketch out the line and kind of go the, see if it would intersect with that intersection." Then he explains a more formula based understanding of the graph, "I guess what threw me off it's like if it said negative *x* then I would know it would reverse the parabola and it would go down instead of up...uh...but is I guess because it seemed like its a different parabola." In each step, he does not appear to plan a method or strategy in advance but perform one technique and then another attempting to relate the resulting information, but not knowing if there will be additional information gained. He continues until he reaches what he feels is a reasonable result.

In discussing the Ken's Garden problem, M6115 again displays a varied and quick moving approach in which he "tinkers" with the problem using one possible technique at a time. He says, "Ok, this is a garden is 17 feet long and 12 feet wide [reading problem to himself] ...I think this one I got flustered because I just sort of started plugging and playing." He then continues to work on the problem through a series of calculations.

> So we know that uh, the final area of the garden is 102, so let's see... and we know that the square it's going to be square it's going to be even...so let's see…I guess...we what we need to know is essentially I guess what we're looking at is for the square root of 102, because if we want these things to be the same widths and lengths and you know its in a square and since we know it's that's supposed to be the final area, so...

In each case, his approach involves choosing a couple of ideas with no observed evidence of his reasoning for those choices.

In the Fun Golf problem completed during the interview, he immediately jumps in after reading the problem with, "So, we know the base price $5.00 times 120 rounds (writing this down) so that is $600.00 and then.... let's see I'm not sure this describes a function." Again, he chooses his methods and runs into the problem. In this example, his chosen idea proves to be a productive path. After computing several values for the businesses' revenue, he decides, "Ok, so we get to a point of diminishing returns at 14 times 75 or $14 per ticket and 75 rounds...15 spot we did the same number so hypothetically it would be either of these, but I'll go with $14.00 per ticket." The "jump in" nature of his MPS provides a productive pathway in some cases in the interview and may be even more useful in his regular problem solving in the course.

4.1.6.2 M6221

M6221 displayed only two instances of the tinkering orientation alongside the more frequently displayed formula application. In the Fun Golf problem worked in the interview, he jumps quickly from his more usual formula based approach saying,

> That 120 that you're selling on average per week less the--what did we say--five fewer per dollar. So now it's going to be that five fewer times x again, being the increase in ticket price, and that will tell you your final revenue. Ok, now to maximize revenue, we just need to find the vertex of our parabola.

He quickly jumped from one pathway, direct calculations using algebraic notation, to another using the graphical representation. Then he redirects again to a brute force approach explaining, “Ok, there we go. So now we're left with plugging in.” In his other work, he articulated a more important structure for his formula application while in the interview problem he jumped quickly from one strategy to the next. Each “jump” incorporated the information gained in the previous technique, but no coordinated plan appeared evident.

4.1.7 Streamlining

Three students coded for instances of a streamlining orientation. Students using a streamlining orientation desire to focus on relevant information by eliminating unproductive pathways. These students will also eliminate or disregard parts of the problem’s wording they believe make the problem more difficult. A streamlining student may eliminate possible techniques or strategies that appear will not be helpful. These students usually simplify where possible and write down as few calculations or steps as possible. This orientation may work in concert with other orientations as the student intends to remove possible errant pathways within a big-picture focus or replication orientation. The instances of tinkering are summarized in Table 4-11.

Table 4-11 Frequency of Streamlining Orientation

Student	Number of Instances (Instances in Initial Question)	Total Orientation Instances	Portion of Orientations classified as Streamlining
M310	2 (1)	4	50%
M6115	5 (1)	14	35.7%
M6238	9 (0)	12	75%
Total	15 (2)	195	7.7%

4.1.7.1M310

In the initial question asked in the interview, he followed his explanation of matching what is done in class with, “I tend to block out unnecessary information.” Later in the interview M310 works the Air Travel problem and says, “It would be easier if it already told me the number of hours.” He goes on to explain that the 12 minutes quantity makes his calculation difficult. He chooses a method removing the minutes unit from the situation. He explains, “I could change T into one-fifth and answer it like that.” M310 did not offer the same amount of explanation as some of the other students interviewed leading to fewer orientation codes than other students.

4.1.7.2 M6238

Streamlining can take multiple forms. For M6238, he displayed both written evidence of his orientation and spoken tendencies. Streamlining includes not solely the elimination of information, but also the intention to make the written portions of the solution as clear and concise as possible. M6238 explains his method in response to the initial interview question, “I try to recognize core elements or functions of the problem that I already do know.” He goes on to elaborate,

> I typically will write them separately and give them an alpha value. That way, I can sort of go, ok, this was used to recognize this type of thing. Such as, if a problem would give me say the rate, whether I know what the rate is for or not, I would r, capital R, rate equals whatever value it gave me. And just sort of separate the information I do know from the information I don't.

His orientation to keeping the problem solution easy to see and work with is further explained by his interpretation of the researcher’s questions about if he uses a diagram or representation. He explains, “I line things very specifically so I can see the elements that are lining up, and it's very visually organized.”

Discussing the Myla’s Pool problem with the interviewer, M6238 stated, “I pulled out all the numbers and tried to give them some sort of representation. I said, ok this was the sixteen thousand gallons is full.” He did not mention his elimination of 16,000 gallons as an important piece as evidenced in his work (Figure 4-22) when asked to explain his solution method. Later in the interview when asked about the mark through, he explained, “extraneous, yeah because at no point in my actual work towards the solution did I need or use that number, and so while it was information I was given, and I'm sure it's useful, I didn't use it. It was extraneous towards my solution path.”

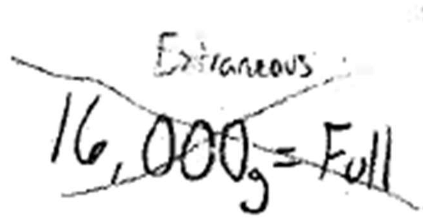


Figure 4-22 M6238 Myla’s Pool Information Eliminated

Though other students discussed the same problem only M6238 marked through the information in his written work. Additionally, he streamlined his thought processes for the building functions problem by marking over the lettered answer choices (Figure 4-23).

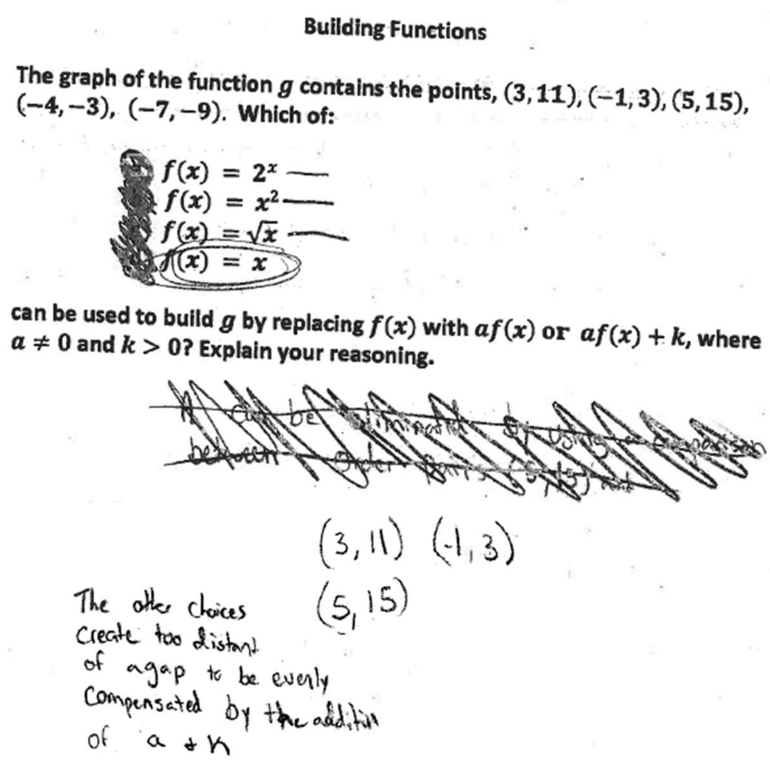


Figure 4-23 M6238 Building Functions Information Eliminated

M6238 explained his solution to the Home Values problem in another streamlined manner. “Yeah, but it looks like what I did was I tried to save myself the trouble of calculating too much, and I just tried to determine what the growth rate was between the first gap, the second gap, and then just use that number over the extension of each year.” He shows a clear disposition to eliminating unneeded information and presenting his work in clear, streamlined way.

4.1.8 Justified Reasoning – F505

One student, F505, used justified reasoning as her overall structure and approach in the problem-solving interview. Each action taken by the student was based on the justification or reasoning for that action. The selection of strategies and techniques occurs through understanding the reasoning for doing so. Each step is planned in advance and thought through the intentions of taking such actions. This student’s MPS orientation was unique among the interview participants and partially explained by the other orientation she coded for, Replication. As she mentioned, “I went to school up in New York, and in the classes, when we did stuff like this, we had to write out an explanation at the end.”

This relationship with proof does not always provide a successful platform for her. For example, in the Book Stacks problem she explains,

> I just really tried going to the ratios. If there is nine books, and there’s nine books, it has to equal the two to one ratio of two math books to one literature book. So I just went into that and did the math of that to see if there is two literature books and there’s one math book, how many does it take to get to nine books?

At the end of the problem she wrote out an explanation for her solution (Figure 4-24).

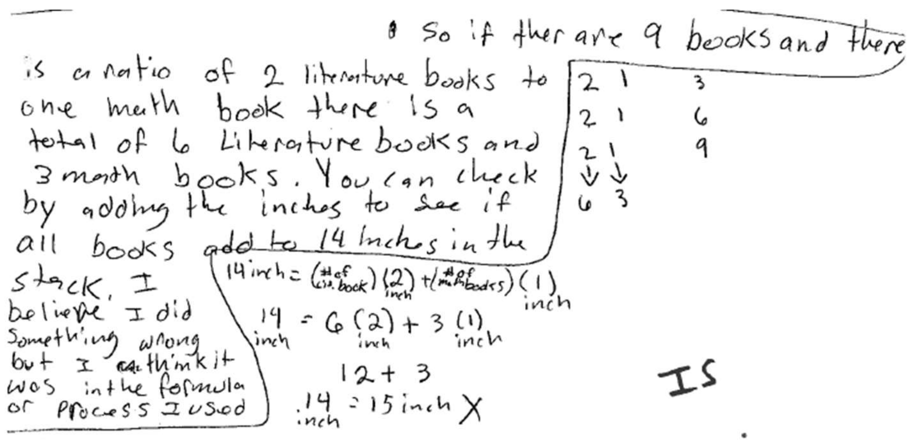


Figure 4-24 F505 Justification of Solution

F505 wrote similar explanations for each problem. In the Cross-Country Race problem she wrote, “Brett finishes first because he runs at a faster rate and a shorter distance.” Further in her work on the problem it can be seen the number of checks that she performed and continued proving and verifying her calculations. Her conclusion is ultimately correct, but her reasoning was confused in her desire to prove the solution.

Uniquely among the students that completing the Fun Golf problem during the interview F505 checked the situation where the price was lowered. “I’m going to say that it’ll – oh, so it’ll increase by five rounds for every dollar that it goes down. I’m assuming. So, if it’s four, it’ll go up to 125, and if it’s three, it’ll go up to 130.” Using this information, she created two tables, one each for the case where the price is lowered and where the price is raised (Figure 4-25).

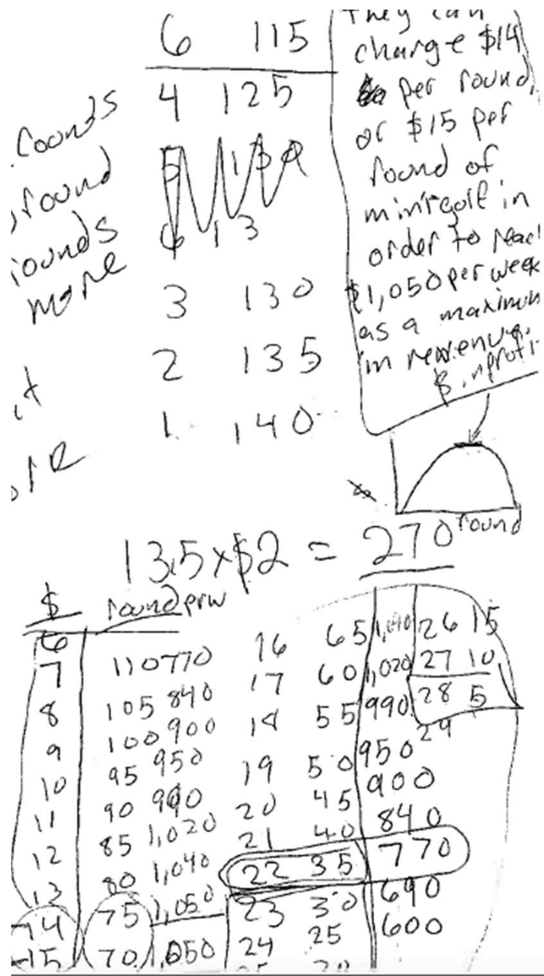


Figure 4-25 F505 Checking Tables

Her tables showed many more possible prices than other interview participants that worked the same problem. F505 checked all 25 whole dollar prices on between $1 and $25, but checked no prices involving parts of a dollar. The most prices checked by other interview participants, used all the integer values between $5 and $20. Multiple students checked those 16 prices. Some of the students checked other partial values between, but no one checked as many prices beyond the answer threshold of $15 as F505. This systematic approach limited the completeness of her solution. When answering the MPSI items she explained that, “if you put it in the calculator, it would probably give the exact answer, which is probably $14.50, which it is halfway between them, or something like that.” Even after finishing the problem F505 was examining the validity of the full solution. Each step of her process is thought through before completion. The correctness of her just justifications proves not as reliable.

4.2 Strategies

The 19 interview participants engaged in multiple strategies within their MPS actions and behaviors. A strategy is defined as a plan or method used for the specific goal of solving the problem. Further, a strategy can include establishing a goal or goals and identifying or creating a plan or method to use toward that goal. A strategy may include a technique or series of techniques, but also may not rely on a particular technique or group of techniques. A student might consider the utility of certain techniques as part of strategy even if the student does not use the techniques themselves. For example, a student using a graphing strategy might consider multiple models or methods for creating the graph while still falling under the same graphing strategy. A technique shows a specific action while a strategy appears as a broader group of actions or considered actions. 11 different groups of strategies were coded in the 19 interviews. The actions and verbalizations of the students landed in these 11 groups and each group of codes appears similar in many ways, but the precise strategy employed by each student is not expected to be identical within a group. Within a strategy group students did not necessarily perform the same actions or discuss the problem in the exact same way, but students coding for the same strategy group share similar themes and used similar plans or methods. A student did not have to be successful in their use of a strategy to be coded within that strategy group. The most common groups coded were approximation, algebraic representation, identification of formula, identification of model, pattern thinking, and information reuse. Less common strategy groups were elimination, functional relationships, graphing, new representations, and separation of parts.

4.2.1 Approximation

Multiple students used an approximation strategy. The approximation strategy group splits into two primary categories. One type of approximation involves students choosing and testing a value relating to the problem statement and then intending to use the information to further their next choice of strategy or technique. The second type of approximation includes students estimating the answer to either the entire problem or a part of the problem. Particularly in the second type, approximation's efficacy ties directly to the problem. For example, the Ken's Garden problem includes the word "approximately" in the problem statement.

4.2.1.1 M105

Student M105 coded for two instances of approximation, both while working the Fun Golf problem during the interview. M105 engaged in the first type of approximation. He used the information already obtained about the possible maximum price to decide that he needed to examine a price point in between two similar values obtained before. He explains, "I mean I know you could actually go in and figure out for a 50-cent increment. But, then you'd have a 2.5 reduced rounds." He used the information gained previously in the problem and the context of problem to approximate where the next test value should be. Further, his "but" statement shows the consideration of his strategy and its implications for the broader problem situation. This separates it from a technique of merely calculating a value between two known or previously determined values.

4.2.1.2 M106

M106 used the first type of approximation in explaining his solution to the Ken's Garden problem. The Ken's Garden problem specifically says, "If he wants the new garden to be approximately half the size, what dimensions are appropriate for Ken's new garden." Thus, the problem privileges an approximation strategy in some. Notably, M106 referenced both types of approximation in his explanation. He explains his solution, "I just narrowed it down to – you know – I would do… I think I erased most of the things I tried, but – so I started with six times maybe 15 or something like that, then I'll be like, 'Okay, it's not quite there,' so then I go a little above six, a little below 15, keep doing that until it narrows all the way down to 102 feet squared. And yeah, that's how I solved that one." This is the first type of approximation where M106 has chosen values chosen according to the information in the problem. Then using the calculations relating to those values he determines additional test cases to triangulate a solution. Only M106 performed this type of approximation for the Ken's Garden problem. Six students coded for instances of an approximation strategy while working or discussing the Ken's Garden problem, and five used the second type approximating the final solution rather than a deductive process.

4.2.1.3 F505

Student F505 coded for the most instances of approximation among the interview participants. She included both types of approximation in her work and explanations. While solving the Fun Golf problem she explains,

> If they charge a dollar, let's say it'll increase by five times the amount of 120 – that's about 500 dollars that they'll be – 120 rounds. 120 rounds times a dollar would mean it's 120 dollars, but if they increase it to a dollar per person, then it'll be 120 dollars per person times 5. Yeah, so it'll be about 500 dollars they'll be making a week. And that's just keeping it at a normal price.

Her strategy appears to use the information in the problem to make judgments about the direction her solution should take. She appears to estimate the situation and makes a decision based on her estimation of the price points, associating with the second type of approximation. In this case, she decides that she believes the business needs to lower the price. After calculating several prices and determining the price actually needs to increase to raise revenues, she explains, “So, if they’re doing 140 rounds times $1, it means they’re only making 140 dollars. That would mean that even though the price is increasing, unless people are coming, they may still be making more money.” From this she moves into the first type of approximation. Now she has acquired a number of points and begins to expand them into a new path. She continues her solution, “So seven is 110, then we go 105, then 100, then 95, then 90, then 85, then 80, 75, 70.” Continuing, “I’m thinking that’s probably going to be a low number. It’s probably not going to be right – there’s probably some number in the middle.” Now she begins to triangulate a solution, eventually locating her solution.

> So 60 times 17 – that’s 1,020 dollars. I’m going to start writing in green, just so it doesn’t get too mixed up. 16 times 65 – 1,040 dollars. I’m going to come down here – 15 times 70 – that’s 1,050 dollars. Then we have 14 times 75 – 1,050. Now, these two are the same, so what I can assume that this is the real median, not over here, especially with the prices continuing to increase this way. But what it shows me is these are the same, and these are where the price goes down, all the way to the end.

As discussed in her orientations, the verbose nature of her responses led to additional opportunities for coding within her interview. This could cause an overrepresentation of certain strategies.

4.2.1.4 Relationship with Course Grades

Students M106 and F505 coded for the most instances of the approximation strategy among interview participants. Notably, M106 earned an A in the College Algebra course while F505 received an F. The possible overrepresentation of F505’s strategies could relate to this opposition in the possible efficacy of the approximation strategy. The number of approximation instances coded and the students’ course grades are shown in

Table 4-12. Interestingly, only F505 failed the course among students coding for instances of approximation. Further, five of the seven students earned an A or B in the course compared with a total of 10 of the 19 students interviewed in the study.

Table 4-12 Frequency of Approximation Strategy

Student	Number of Instances	Course Grade
M105	2	B
M106	5	A
F304	1	A
F318	2	B
F505	6	F
M6115	1	A
M6259	2	C

4.2.2 Algebraic Relationship

Additional students deployed an algebraic relationship strategy. Students engaging in this strategy attempt to create an algebraic rule or symbolic depiction to represent a problem statement. These students use procedures such as translation, where they interpret the problem text into an algebraic representation. Students could create a representation that then later gives rise to a graph or other visual representation. Students using this strategy indicate an importance of these representations and relate the problem statement into a new symbolic depiction or algebraic structure. When a student codes for this strategy, he or she falls short of the instance providing an overall structure for the student's MPS.

4.2.2.1 F318

Student F318 coded for the algebraic relationship strategy more than any other interview participant. She explains her intentions on the Fun Golf problem she worked during the interview,

> So he says that – okay, so what I'm trying to get is – 120 equals X for the original. And so they were selling 120 times – I guess this is an X and a

> Y equation. So I'm going to say X is person – no, X equals number of rounds, and then Y equals the person.

She continues to explain her actions by discussing the representations she writes for each of the problem's needed quantities. Her work (shown in Figure 4-26) demonstrates this tendency. She matches each of 120 rounds, \$5, and a change quantity, to variables *x*, *y*, and *a*, respectively. Together she attempts to manipulate these variables into a single algebraic relationship she can use to solve the problem. Each definition step fit beneath the broader strategy of algebraic relationship, fitting within her reexamination orientation. She would look back at her previous steps and place importance on the correctness of her previous step.

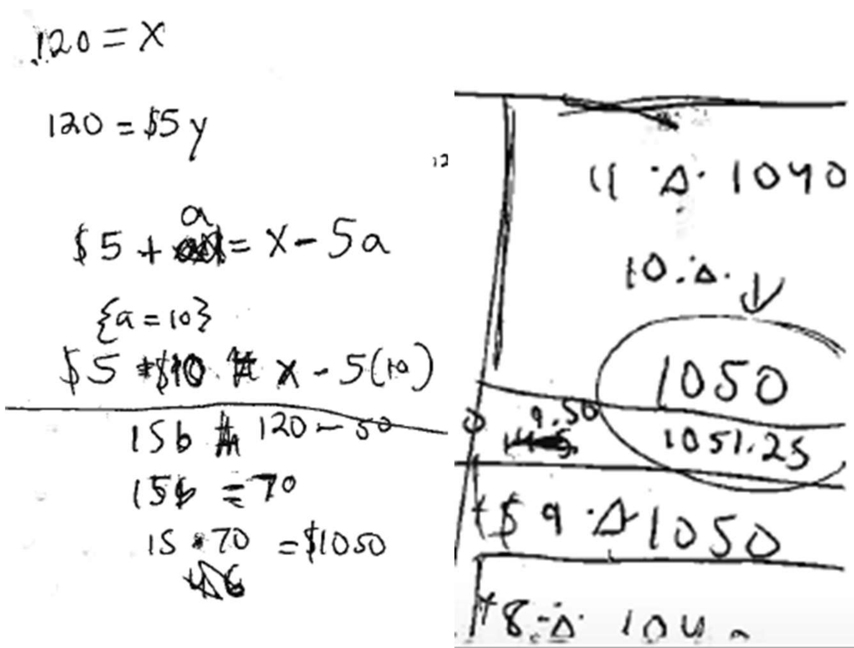


Figure 4-26 F318 Fun Golf Algebraic Relationships

Later in her solution, she develops a symbol (seen on the right of Figure 4-26), "a triangle with three dots," that she uses to connect two quantities. Here, she failed to solve the problem directly with the algebraic relationship and representations, but continued to preserve the notion of a symbolic relationship between the quantities to determine the answer. She explains her method and symbol, “so I’m going to go back in the middle, check out 15. So five plus 15, and then I’m going to invent a new weird symbol, it’s going to be this triangle with three dots. I guess I could do a therefore symbol, I don’t know.”

4.2.2.2 F6145

Notably, F6145 explains her strategy before reaching the portion of the interview discussing specific problems. After the researcher asked if she would break down problems into pieces she explained, “But if you see how the pieces fit together in like an equation, then it can make sense of oh ‘How do I need to manipulate these number to get like an answer.” This corroborated with her explanations about previously completed problems. In the Fun Golf problem, she states, “There's a direct correlation between the two variables.” However, she did not solve the problem using this strategy though it remained her desire even seeing the problem several weeks after having completed it. As she explains and her work (shown in Figure 4-27) demonstrates,

> Uh, so the idea was that they would start with the $5 per person and they sell 120 rounds, you multiply that together and that's $600 that they would make, but for each dollar increase that they do then five less rounds would be purchased, and so...I don't know how to make it easier though. There's probably, I know there's like an easier way.

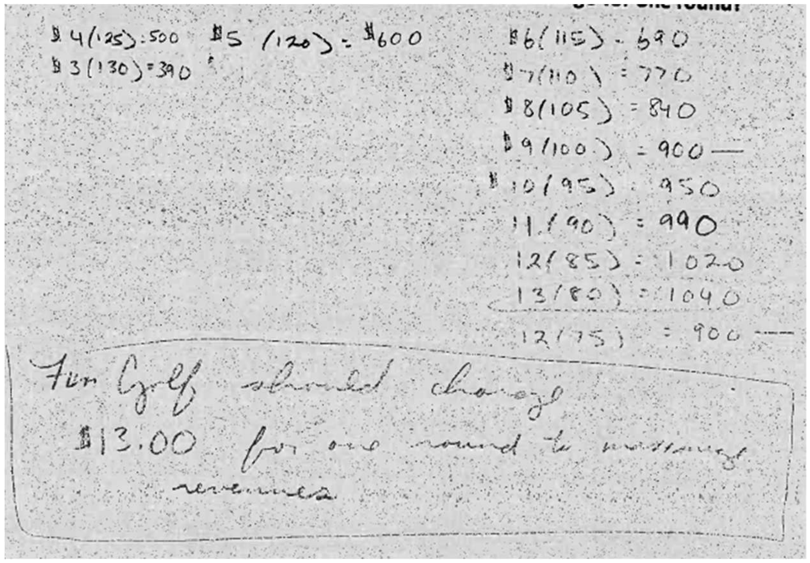


Figure 4-27 F6145 Fun Golf Problem

She believes that there must be an easier way because a correlation exists between the two variables. She assigns the idea of variables to the two quantities without any prodding from the interviewer or problem statement. In asking about her answers to the MPSI items the researcher pointed out an answer where she preferred an “algebraic representation or graph.” F6145 said, “I would prefer the algebraic representation or the graph.” When asked why she preferred such representations, she followed with, “because you can solve for *x*,” before confirming to the interviewer “that it feels more reliable.”

Later in the interview F6145 completes the Ken’s Garden problem. After reading the problem, she immediately begins to write down an algebraic relationship. She explains, “To reduce the length and increase the width. Wants to reduce the length and increase the width by the same amount. Ok. Wants…ok so *x* is going to be...the distance or the amount of...ok...however much is taken off the length but...its like the same.” She directly begins to assign variables to the problem constraints and establish a representation using algebraic symbols. She continues, “The area equals length times width so it would be like the 17 - *x* times 12 + *x*, what would equal your 102 feet squared." Her work in Figure 4-28 shows the sole strategy employed was an algebraic representation. This problem elicited visual representations for most interview participants and also included "approximately" in the problem statement, making her an important instance of this strategy.

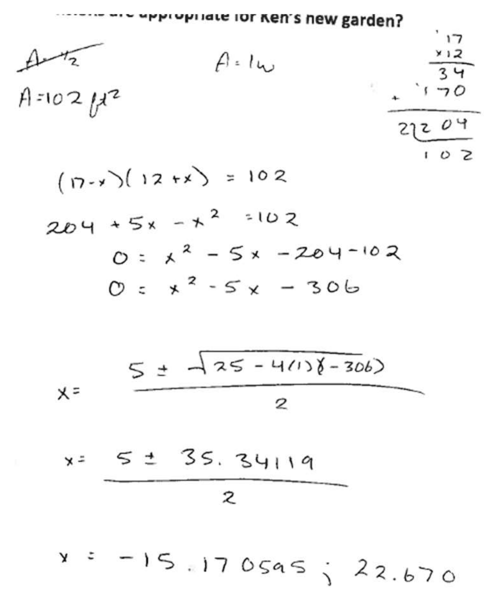


Figure 4-28 F6145 Fun Golf Problem

4.2.2.3 Functional Relationship

Within searching for and using algebraic relationships certain students placed an importance on a functional relationship specifically. The only codes for a functional relationship came while discussing the Surfacing Submarine problem. This problem specifically asks the student to, "write a function, h, that gives the submarine's position beneath the surface at a given time t." As F209 says, "I would think that would be rate, distance, and time is what all should go together there. But see, I think I, because if says H here, I think that's why I put the H there." The problem has necessitated that she include these algebraic representations in her MPS method. Of particular interest, students did not code for this strategy when working or discussing the Cross-Country Race problem. That problem asks the student, “For each of the runners, write a position function that gives the distance in meters from the finish line after t seconds.” There are two notable differences in the presentation of the question to the students that could account for the difference in deployed strategies. First, in the Surfacing Submarine problem the part of the question asking for a function appears as a part (b) while in the Cross-Country Race problem the instruction appears at the end of a paragraph rather than showing as a separate part of the problem. Additionally, the Surfacing Submarine problem names the requested function, “h,” while Cross-Country Race merely asks for functions and does not name them (e.g. “b” for Brett or “c” for Charlie).

4.2.3 Elimination

A portion of the students interviewed used elimination as an MPS strategy. In elimination students delete possible outcomes to facilitate their method. Further, students using elimination intend to simplify the problem in some way. Examples of the elimination strategy include, blocking out parts of the problem, deciding against certain techniques or procedures, or by avoiding certain information as it may create a conflict with previous information.

4.2.3.1 F304

Student F304 used elimination in discussing multiple problems in the interview. First, she explains her solution to the Intersecting Graphs problem. Initially, she portrays her idea as using a simple formula or procedure. She claims, “Man, if I remember these simple, I guess… not formulas, but rules, I guess, it would be a lot easier.” However, she dismisses this explaining, “Okay, so since I can’t really remember those, I’ll just go with the coordinates on the graph.” She goes on to point out the places in her work where she eliminates various representations, both algebraic and graphical. These eliminations are shown in Figure 4-29. Similarly, in the Book Stacks problem she claims, “I went through the options in my head.” The simplicity of the calculations needed in the problem may have prevented further use of the strategy in the problem. Then in discussing the Cross-Country Race problem she says, “There was less variables to deal with, the way I see it,” to explain her use of the diagram at the left of the page (included first in Figure 4-29). She appears to use the modeling technique to help her eliminate the algebraic solution pathway.

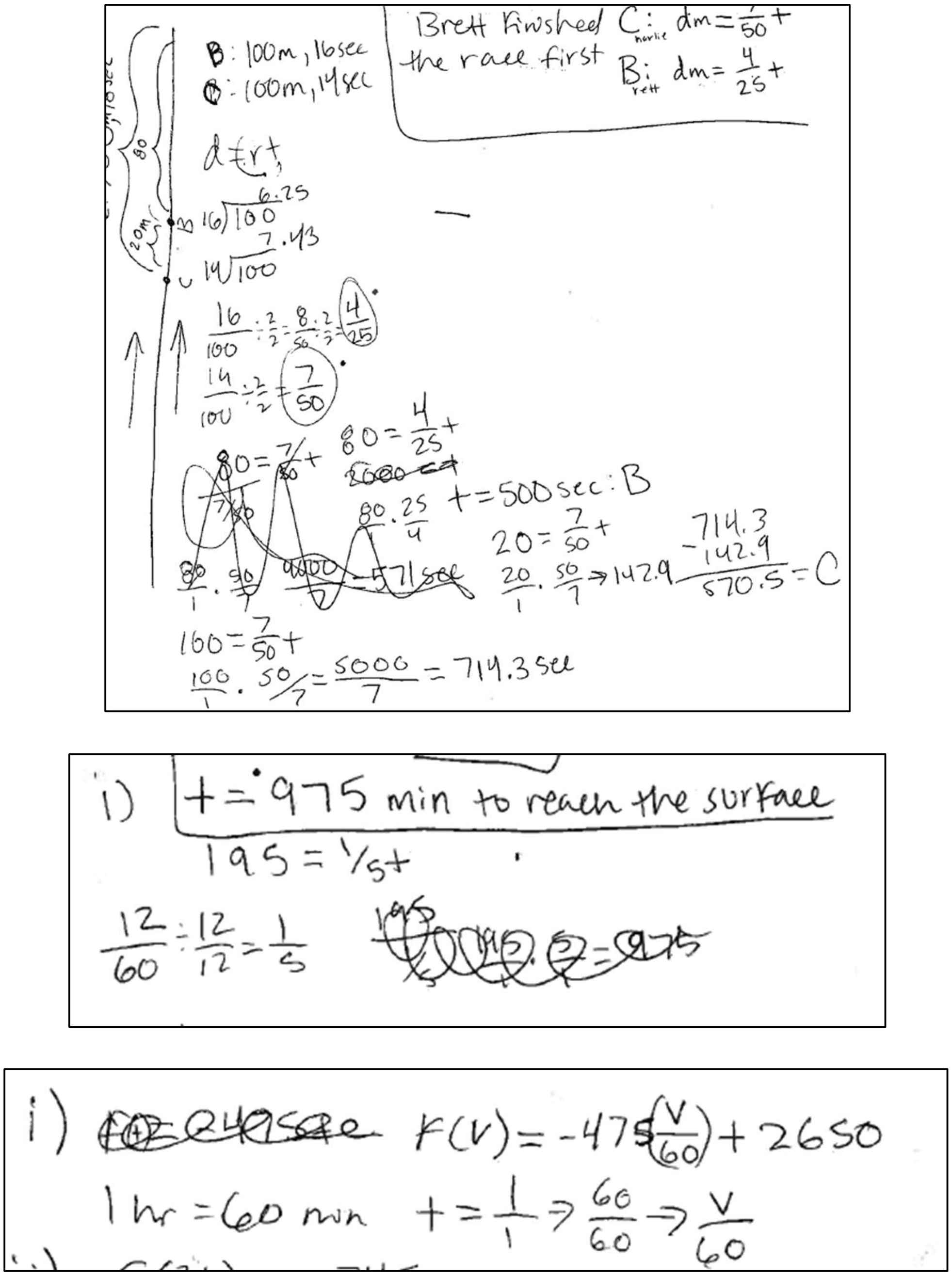


Figure 4-29 F304 Elimination of Possible Pathways

4.2.3.2 F318

Student F318 eliminated possible pathways through her MPS method. In explaining her work on the Ken's Garden problem, she first wrote a quadratic equation that she needed to solve. She claimed to have abandoned that choice, "well, I had a square root of something, and I didn't want to mess with that, I guess. But I think it was just – it was supposed to be approximately half the size, which means that that measurement wasn't going to be a precise number." The evaluation of this possible solution pathway took place and then it was blocked from further pursuit. Continuing in her examination of the previously worked problems, she explains the Intersecting Graphs problem. The researcher asked, "So you feel like you need another line? You needed the second curve to be a line?" She explained, "And then I guess I could have done a square root, or something. But I'm not sure. It would be kind of complicated." She again evaluates the efficacy of a method and decides not to pursue it. Ultimately this was detrimental to her success on the problem. Additionally, evidence of her elimination strategy can be seen in her work in Figure 4-11 as this relates closely to her reexamination orientation.

4.2.3.3 F505

F505 discussed eliminating alternatives in the Fun Golf problem she completed in the interview. After reading the problem, she begins, "if they already charge $5 a person to play, and when they increase the price, less people play, all I can assume is, the less they charge, the more people come." Based on her interpretation she eliminates raising the price as a productive pathway. Notably, the use of this strategy does not require that a possible solution path be eliminated permanently as she ultimately does check the values for price increases. In a similar manner to F318, F505 determines an algebraic pathway to be too difficult for her to navigate and discards it. She explains, "In my head, I know that there's probably a way to figure out – you know, 515 divided by five will get you to the answer, but I feel more comfortable writing it out, just to make sure that I get to the right answer." Her language can be confusing but "515 divided by 5" corresponded to her (ultimately incorrect) findings when checking lower price points. Further, she continues to elaborate on her method, "In my head, one of the things I think about is double-digits times double-digits. And then this is double-digits times a single digit. And up here, it's a single digit times triple-digits. If I go down here, this is four places all together, this is three places, and this is four places." She uses multiplication principles to determine what values can be eliminated before continuing to search for her solution.

4.2.4 Graphing

Graphing or the use of a graph served as a notable strategy for some students. Students using a graphing strategy would determine before creating a formula or function that a graph would likely be helpful for solving the problem. Then these students may use techniques such as, plotting and connecting points, substitution, or curve sketching. The strategy involves the student privileging graphing as a beneficial method. For example, students using the graphing strategy might ask, "how can I make a graph?" Then, other students using a different strategy might decide to use a graph after using an algebraic strategy, concluding for example that they should plot some points.

4.2.4.1 F6145

F6145 talked about her use of a graph in the Building Functions problem. The problem asks to the student to determine what base function can be used to build a function that passes through a set of five points. She explains her strategy, "I made a graph. Um...its not like completely reliable because it was like hand drawn and the spaces aren't equal, but I had like a rough visual on what the general shape of the graph would be. Even if you drew these out they would be very different shapes." The graph helped her decide what to do next and then explained that she would "do all the slopes." Her work shows (Figure 4-30) the checking of only one slope, but she chose direct calculation technique for the slope based on the graphical relationship she observed in her graphing strategy.

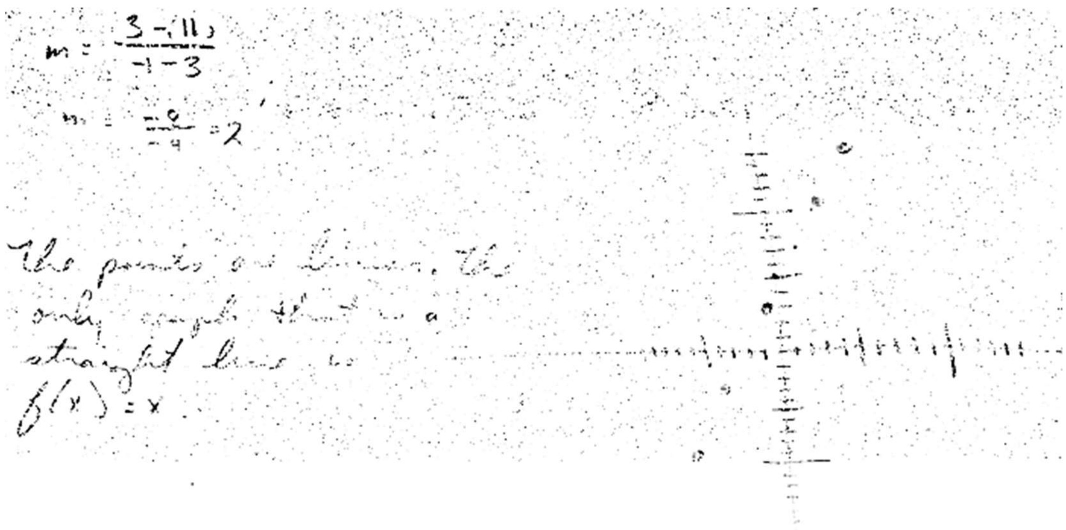


Figure 4-30 F6145 Building Functions Problem

In the Extreme Values problem, F6145 explains after thinking about the graphs of the three given functions. She moves to a computational strategy explaining, "I tried using the derivative, but then I realized that's obviously not going to work" (F6145 previously completed a calculus course in high school). Then she claims, "Just visualizing it would be simpler." The graphing strategy provided the influence to move toward a computational technique and then later provided the push toward a final solution.

4.2.4.2 F6269

F6269 used a graphing strategy in the Extreme Values problem as well. She explains, "I just tried to do a graph…to visualize." She answers the problem by saying there can be no largest value. “I don't think so, because we can plug any number into x, right?” The student also set a graph as her initial strategy in the Avoiding Intersections problem. She shows no other calculations, representations, or tools for determining her answer beyond the graph shown at the left of her work. F6269’s graphs are shown in Figure 4-31.

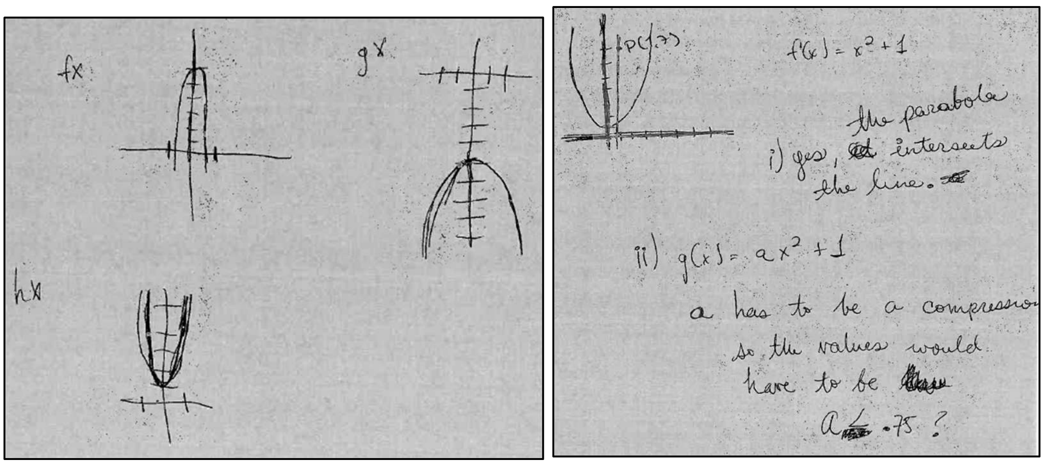


Figure 4-31 F6269 Extreme Values and Avoiding Intersections Graphs

F6269 establishes a clear difference in her MPS strategy when the problem invokes graphing. The researcher asked, “How often would you say you draw a diagram or a picture or something like that?” She replies, “Not that often. Unless we're graphing and I visually need to do the answer.” The importance of the problem statement in the chosen strategy creates a challenge in understanding what a student’s “typical” strategies might be.

4.2.4.3 F304

Student F304 portrayed the use of a graphing strategy only on the Intersecting Graphs problem. As she explained, “Okay, so since I can’t really remember those [formulas], I’ll just go with the coordinates on the graph.” So, using the points and trying to remember how to make the line there, and then plugging stuff in to see what the resulting graph would be. So that's what I remember doing." The graphing strategy provided a secondary approach that sits within her multiple orientations (formula application, reexamination, and big-picture focus). She did not know where the graph would lead her solution. She explains, "I was trying to remember the graphs. Because you know how there's – usually a certain formula will create a certain graph, so I was trying to think, 'is this a parabola? Is this formula a parabola?'" The graphing strategy does not provide her a direct solution, but she appears to use the graph to hopefully locate additional techniques or calculations that she can perform to find the answer. Her work shows (Figure 4-32) the many calculations she performed after some graphing attempts and also indicates her understanding of a function as a rule requiring "f(x)" style notation. Outside of this problem F304 showed no function graphing in any of her work. She did included diagrams in both the Cross-Country Race and Book Stacks problems.

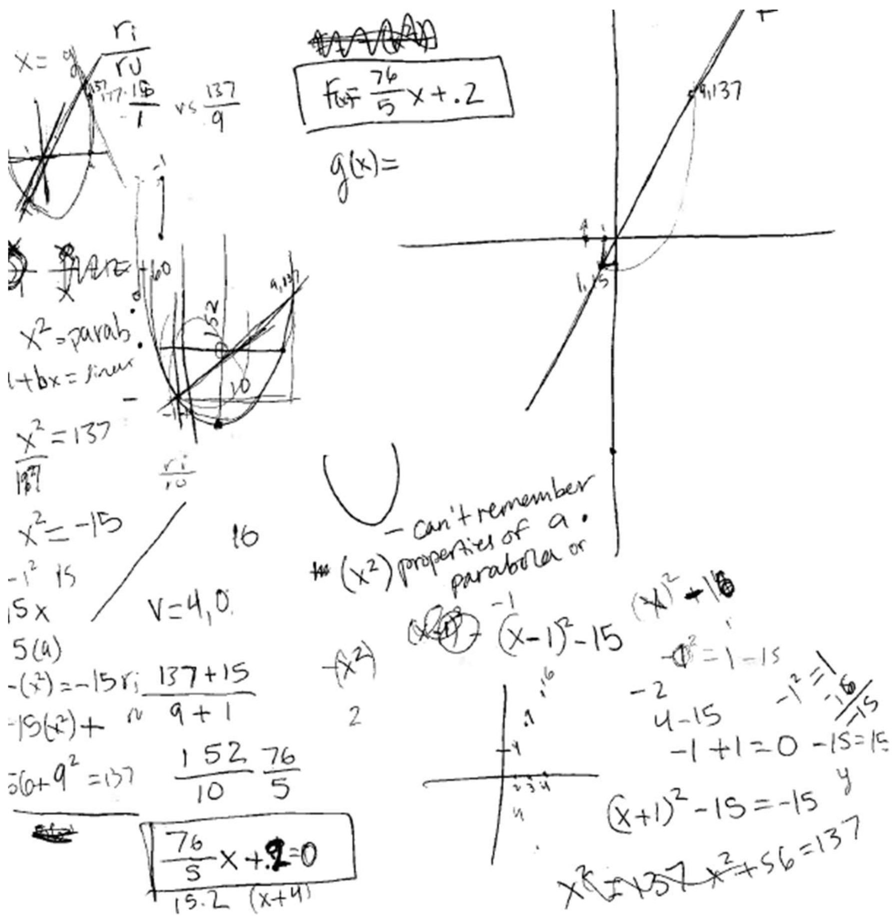


Figure 4-32 F304 Intersecting Graphs Problem

4.2.4.4 Problem Dependency

The usage of the graphing strategy appears to rely on the problem contexts. Only two usages of the strategy were coded on problems other than Extreme Values, Intersecting Graphs, Avoiding Intersections, and Intersecting Quadratics. The two instances included one each on the Fun Golf and Ken's Garden problems. These four problems included either functions in the problem statement or asked the student to graph two functions. The graphing strategy appears to only be used by students solving problems that either specifically request graphs or give the student function notation (e.g. *f(x)*) in the question. Insight into the efficacy of the strategy proved more difficult to obtain.

No decision could be made about whether the student struggled with the type of problem or if using a graphing strategy caused additional confusion.

4.2.5 Identification of Formula

Another group of students used the identification of a formula for their strategy. Usage of this strategy involves the search for a correct formula to use in a problem. This serves a different function than simply applying a formula or calculating a value or values based on the application of the formula. This also contrasts from a formula application orientation in that the search for a formula does not require the student to use the formula application for their MPS structure. As a strategy, the student searches through their previously known information in an attempt to locate a formula or rule they feel will help in solving the problem. The student might use clues in the problem or perform other techniques or strategies to decide if the formula will be helpful. All students coding for a formula application orientation coded for at least two instances of this strategy. To separate the orientation from the strategy, the researcher considers the existence of a cyclic process (Carlson & Bloom, 2005). The student attempting to go back to a formula from a strategy or technique provides evidence of a formula application orientation and the usage of the formulas as the overall structure of the student's MPS. Usage of the strategy may only move the student forward in their calculations and not be a part of the student's overall structure.

4.2.5.1 M106 and F311

M106 explained his MPS method as,

> You know, so I'll start, and then I'll figure out what I need – you know, what are the other variables that I could possibly pull from, some things and some variables that I need to figure out from other numbers that are given… After I've read everything through, and I have the little, you know, mock-equation in my head.

Similarly, F311 claimed her MPS method, “Just something I didn’t know that then, I don’t know how to do that. Things that I already know, I can imagine something – like an equation, I can imagine a relation about it.” Both of these students attempt to search for and hope to create or locate a formula, rule, or equation, they can use to solve the problem. The intent appears to be advancing the solution pathway to another technique or strategy.

4.2.5.2 M6221

M6221 worked the Fun Golf problem during the interview. After calculating the initial revenue as $600 he describes his method, “And now your new equation will be your initial $5 plus a variable that we decide. We'll call it *x*, where *x* is equal to the increase in ticket prices.” At this point, he discovers the resulting equation is a quadratic, and that he needs to find the vertex. His identification of a formula led him to the equation for the vertex of a parabola. Giving him the opportunity to employ a substitution strategy. Earlier in the interview he explained his desire to use a formula, telling the researcher, “I would not want to have to go through a guess and check method.” Thus, the importance of identifying a formula or equation is clear for the student. This follows his orientation using the formula application as the structure of his MPS.

4.2.6 Identification of Model

Students using a similar search strategy may not search for a formula but rather a more flexible model. Students valuing a model strategy look for applications they can apply to the problem context. A model could range from a diagram of the problem, to a visual representation, to a pneumonic device that can aid the student’s MPS behaviors. Students using an identification of model strategy could use a graph. For a model, the student would refer to properties of the graph. For example, one might suggest a linear model or a quadratic graph. The formula identification would include references to a variable or an equation. Model identification would focus on "straight" or "parabola." Students could use their model to help them identify a formula. The usage of a tree diagram or consecutive calculations could be a part of a model identification process.

4.2.6.1 F6269

The identification of model strategy related with the type of problem presented to the students. The problems using a non-abstract problem statement showed more instances of this strategy. F6269 explained her approach, "Realistic, real-world problems, I would try to draw." The usage of the model strategy carries into the problems that do not rely on the constructs of mathematical notation or language. For each of the problems F6269 discussed or worked in the interview she codes for instances of the identification of model strategy. Table 4-13 presents an example for each problem.

Table 4-13 F6269 Examples of Identification of Model Strategy

"So, because it said it's 1463 meters, that's like the total, and we're trying to see how far the ship is from the fish, if it reaches in like half the time, well not really half the time, point zero five seconds."	Sonar
"Because, I mean it's going up, up, up, and finally, these numbers aren't changing, so if it does end up being 1050 for both, then that means the curve got to the peak, and it leveled out a little, and it's, it makes sense that it's going to go back down, because the numbers keep changing."	Fun Golf
"Ok, so obviously I had to visualize it."	Ken's Garden
"I know that I have to find out a way to figure out, I need to find a way to figure out if Brett, if he passes Charlie…I can do like the time thing and do, see where Brett was when he was. I'm trying to do 100 and can do that in 16 seconds, and Brett can do 100. I'll do 100 in 16 seconds, and then Brett does 100."	Cross-Country Race

4.2.6.2 F6259

Similar to F6259, F6259 indicated for the Sonar problem, "Like I drew the ship and a school of fish, and drawing an arrow back to return five hundredths of a second." F6259 explains her strategy on the Ken's Garden problem she worked before the interview. She explains, "I would solve it right no by draw...17 feet and then 12 feet (draws rectangle). And he wants to decrease the width the increase the width and so drop off here and add a little bit here." This modeling gave way to her techniques of direct computation and ultimately solving equations. The modeling approach did provide her initial strategy beginning. Further, F6259 worked the Cross-Country Race problem in the interview. She explains,

> Well for starters, I'm drawing it obviously. Finish and then Charlie is 100 meters from the finish line and then Brett is 20 meters ahead of Charlie, so Brett is 80 meters from the finish line. Brett covers the final 100 meters in 16 seconds. So he goes 16 seconds. Brett covers the final 100 meters in 16 seconds. So actually it's more than this (points at 80) it's all of this (points at whole line). Then Charlie covers the final 100 meters in 14 seconds. "Who finishes the race first Brett or Charlie?"

The model identification provides a starting point around which the student could further develop their problem-solving methods for a particular problem. Student's overall structure also contributes to the direction taken after identifying a model.

4.2.6.3 F304

F304 looked at her previous work on Fun Golf problem during the interview. She explained her thinking as, "So the first thing – yeah, the first thing I did was draw this picture. Brett covers the final 100 meters in 16 seconds. Charlie covers the final 100 meters in 13 seconds. Who finishes the race first? Brett finished first. Which one is Brett? Brett started here." The diagram or model offered her window into the problem set-up. The self-questioning of the utility of the model reveals a portion of her strategy includes identifying the appropriate model. The model strategy appears to lead students into additional strategies or techniques, rather than serving as a terminal strategy.

4.2.7 New Directions

Instead of attempting to identify a model that represents the existing viewpoint of the problem, other students prefer to use a new direction as their strategic approach to the problem. New directions should be a departure in some way from the conventional approach or the approach previously taken by the student. The new direction strategy provides a metaphorical restart for the student. Previously attainted information may inform this new direction and the choice of direction likely fits within the students held orientations.

4.2.7.1 F304

F304's new direction strategy fits within the reexamination orientation. She states that he takes a different approach as her "go-to" strategy or technique. The researcher asked what she typically does in her MPS method. M105 asked in response, "Oh, if I get stuck or something?" She then goes on to explain, "Probably, what I think of is kind of just blanking my mind, if that makes sense, like just saying, Okay, this is not working, so I have to just think of something else.” As with her reexamination orientation, the need to look back and refocus on the problem plays an important factor in this strategy. Within the interview, F304 discussed the Intersecting Graphs problem she worked previously. Her approach shifts from formulas, to graphs, and back to formulas. Further, the deviations in her approach and the “reset” of the new directions appears in her work in Figure 4-33.

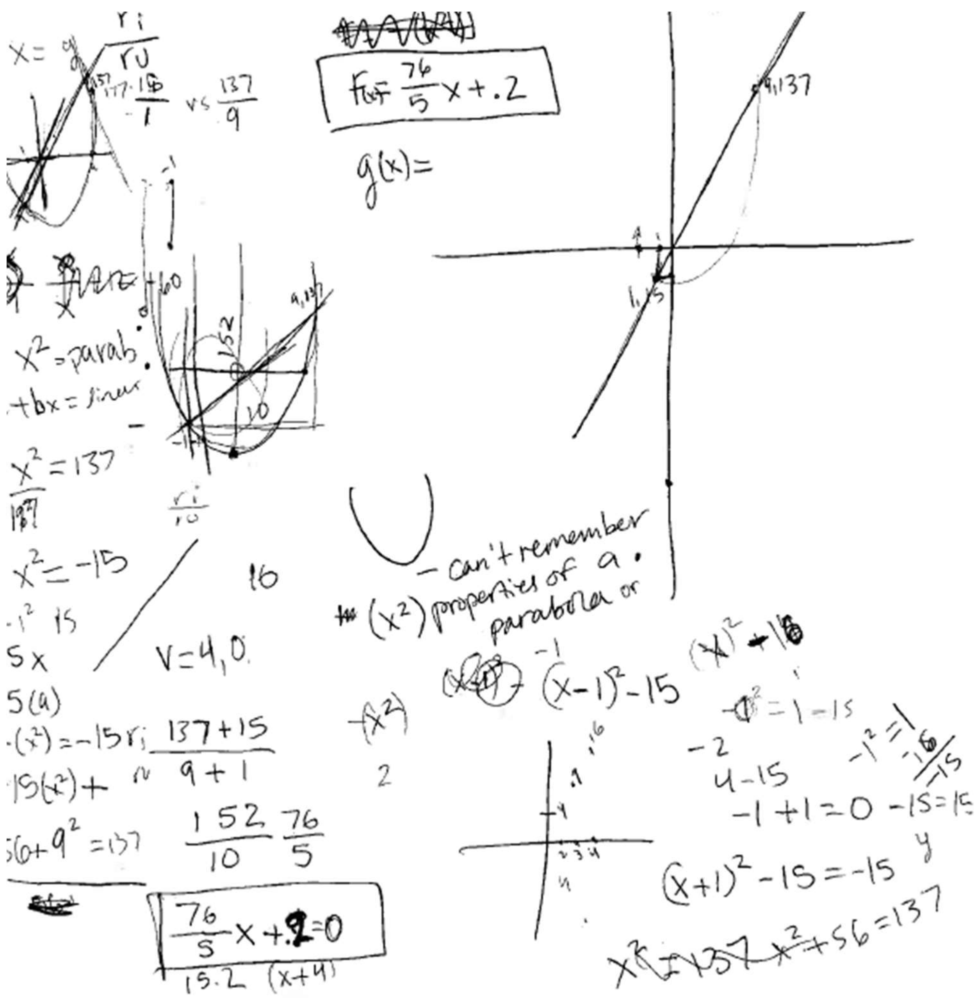


Figure 4-33 F304 Intersecting Graphs Problem

Additionally, in the interview, she completed the Fun Golf problem. In her language as she worked the problem, multiple shifts in direction can be observed. After reading the problem, she first states, “So, I have to – I think – I have to create a function. I’m leaning towards some kind of inequality to be able to find the maximum revenue they would be able to make would be. Let me find another way to create this.” Immediately her approach shifts from a function to “another way.” Next, she considers a trial and error approach, stating, “But how do I make this so I can see the maximum revenue? What I’m thinking of doing but I don’t really want to because I feel like it will take a long time is.” Then in the next passage she shifts again, moving from a trial and error or systematic pattern checking idea to a formula or algebraic notation approach.

> Should I use 120 or replace it…? Well, I guess I’ll start seeing how much they’re making right now. So 120 times five is going to be that much. They make 600 dollars in revenue each week from when they have this system. So, each dollar increase… Okay. I’ll just mark these just in case. So they’re trying to maximize it, so it’ll probably be greater than 600. I guess I can do, if it’s *r* times *p* is going to equal the revenue, I can do that. So *r* is – *r* times *p* is going to be greater than or equal to 600. I have something, but I still have to plug that in.

She falls short of a tinkering orientation because the shifts in thinking occur apart from a planned structure. Her structure involved the application of a formula or function rather than indicating a desire to use multiple options in an apparently unstructured approach. Later she invokes an approximation idea, explaining, “Hmm, the answer, honestly, right now, I’d probably guess some kind of range.” She later shifts to an area model, appearing in her work shown in Figure 4-34.

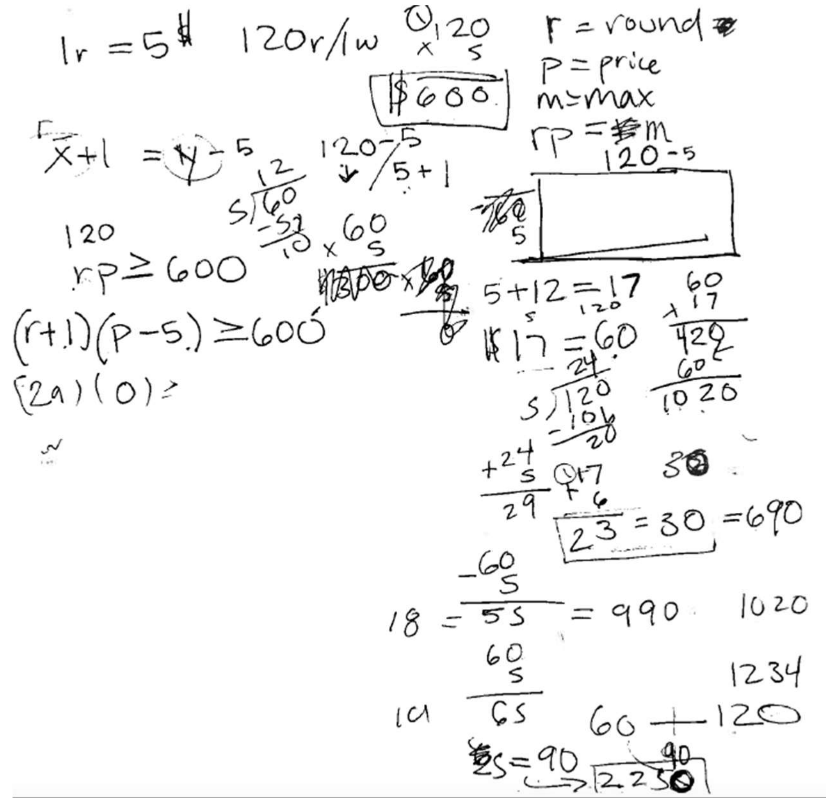


Figure 4-34 F304 Fun Golf Problem

4.2.7.2 M105 and M6221

Other instances of new directions showed the students redirecting around a difficult concept of calculation. First, student M6221 indicated that he needed to find the vertex of the parabola based on his equation for the Fun Golf problem. He stated, “And so now we're left with determining our vertex, which escaped me for the moment, so I might just try to find a way around it.” Additionally, M105 redirected his thinking on the Air Travel problem. He explained, “Ok. So, I know for #2 [part ii], I had just plugged in. Well I converted 12 minutes to a decimal form from an hour and just plugged that in for *t* in the equation. Got the 95 miles.” However, he continues to consider the implications of his previous calculations and says, “You know, from like real world experiences that it would take longer to get up in the air and start flying. But, in the equation that doesn't actually account for that." The context of the problem appears to push him to redirect his plan in a new direction. For both M105 and M6221, these instances were the only coded for the new directions strategy. Further emphasizing the importance of the problem type and context influence on student strategy usage.

4.2.8 Pattern Thinking

Some interview participants used patterns to help deduce a solution within their MPS method. These students attempt to reach a solution by relying on a repeatable pattern that may help them to determine a technique to use in later operations. Further, the student would not necessarily perform operations or technique with the intention of creating or locating a pattern. The pattern thinking strategy can apply after a series of computations or after the collection of information in the problem.

4.2.8.1 M106

Student M106 worked the Fun Golf problem in the interview. He explained his plan for solving the problem as "plug and chug" and "noticing a pattern." He goes on to explain, "So what I'm starting to notice is that every dollar that it's starting to go up, it's losing $10. It's going up $10 less each time." 11 of the 19 interview participants completed the Fun Golf problem in the interview, and this observation was made by only one other student in that group. Though his initial approach involved the direct calculation and systematic pattern checking techniques, and no specific mention of looking for a pattern in the setup. After just four values checked, he explained, "I'm starting to notice a pattern. I just want to do a couple more just to make sure it's right." Without the verbal explanations, no evidence of the pattern usage appears in the student's work (Figure 4-35).

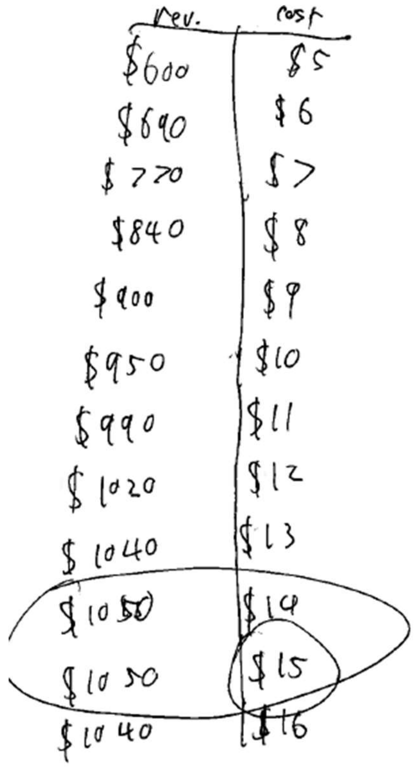


Figure 4-35 M106 Fun Golf Problem

4.2.8.2 M6238

M6238 explained his work on the Home Values problem during the interview. He explains his plan,

> I just tried to determine what the growth rate was between the first gap, the second gap, and then just use that number over the extension of each year and say, ok, so what's the value at three? What's the value at four? What's the value at five? What's the value at six? What's the value at seven?

This systematic pattern checking technique fits beneath his big-picture focus orientation and his intention of establishing a pattern to solve the problem. He also explains his preference over other techniques or strategies that could be employed on the problem. M6238 told the researcher that he could have used a graph or possibly found the increase in a different way, but he indicated that it was not necessary to go beyond his initial systematic checking technique and pattern strategy.

Additionally, M6238 worked the Fun Golf problem during the interview. He explained his initial approach, “I'm going to go for an extending set of dollar amounts and I'm going to take an extreme to the left and an extreme to the right and try to find a central set of dollar amounts for the extreme that I'm looking at.” The quote does not indicate the pattern thinking intention at face value. He continues, “And I'll take 10, 15, so that's my left value.” He continues to explain the various points he checks in his solution. He uses the multiplicative properties for raising the prices by increments greater than $1. He uses the pattern thinking within an approximation strategy, explaining, “Fifteen times five is seventy-five. 120 minus seventy-five is forty-five. So equal 900. So, it's definitely somewhere between fifteen and twenty.” The pattern formed by the various values helps him to approximate the location of the solution and then use the pattern to determine a precise solution. His work further demonstrates this pattern thinking in Figure 4-36.

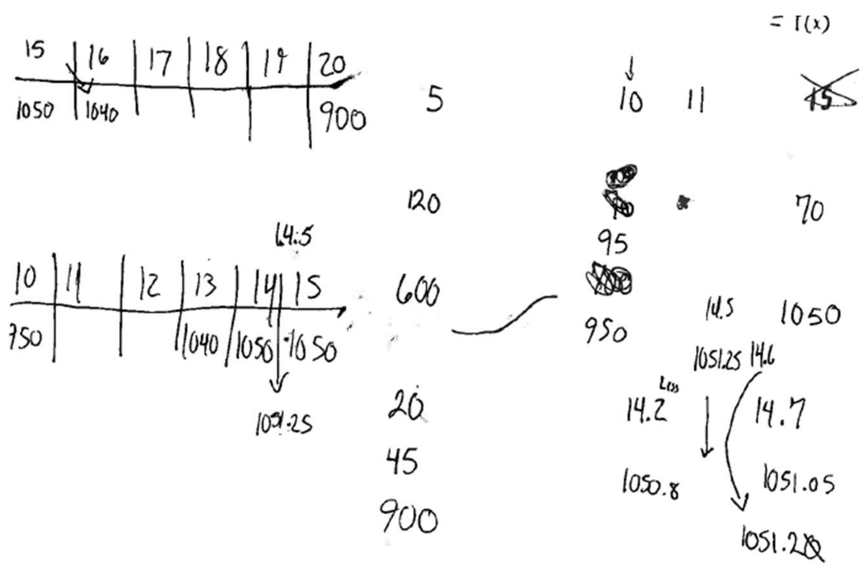


Figure 4-36 M6238 Fun Golf Problem

4.2.9 Information Reuse

Students use the previous information in a problem to decide on the future direction of their MPS. Another type of information reuse involves previous examples, possibly from class time or other problems done before. Students using this strategy attempt insert the previous information into the new environment. This can fit within a linear progression orientation, but also can be a matter of convenience.

4.2.9.1 Multi-step Problems

This strategy most often appeared in multi-step problems. For example, each of the Home Values, Air Travel, Robert's Crew, and Surfacing Submarine problems have parts (i) and (ii) visually separated on the participant's copy. In 15 coded instances of the information reuse strategy, 11 of them occurred when working or discussing one of these four problems. In contrast, Cross-Country Race asks two questions of the student, but keeps both inline in the paragraph of the problem statement. The researcher observed no instances of this strategy for the problem.

Multiple students explained their strategy for the problems with listed parts. First, M106 explained for the Home Values problem, "I basically found – at least, I think I found part two in part one just from doing, how I solved it so I just drug it over them." M310 mentioned an inverse use of this strategy on the Surfacing Submarine claiming that he solved part (ii) first before using that function to determine part (i). F315 explained for the same problem that she could verify her first part using the function found in part (ii). The appearance of the problem seems to correlate with the usage of this strategy by students.

4.2.9.2 Identifying Known Information

The other instances of the reuse information strategy revolved around using preferred information the student previously learned. This results from identifying information, facts, or formulas, the student has previously worked with. For example, F6145 pointed to past information as an important part of her MPS. She explained that she finds “a possible equation” or something “did before.” When explaining her typical steps F318 explained that she, “usually kind of take what I know and then I work from there.” Further, F311 explained that “the example[s] guide me.” Then the final code of this type F303 claimed, “If there was the same problem like that, and I get stuck because I forget what to do next, I do look back at the problem.” The students in this group showed no instances of this strategy outside of their answer to the initial interview question. The intentions of a student’s strategy and their execution appear to be at odds.

4.2.10 Separation of Parts

Another set of students preferred to separate the problem into individual parts. This separation contrasts to a streamlining orientation in that the separation does not provide a structure for the problem solver. Rather, it separates the problem into smaller pieces that the student then can apply their typical methods or other orientations.

4.2.10.1 F505

F505 indicated a separation between pieces of the problem in multiple examples in the interview. First, in the Intersecting Graphs problem, she explained, “what I was trying to do was just put these two points on the same line. So, for P, that’s X and Y, and for Q, that’s X and Y, and so if I find the line for this one, and it intersects with that one.” This strategy proves unsuccessful for this problem. The student proves unable to position both functions in such a way that they meet the problem constraints. Here the strategy blocked further successful MPS since the problem required manipulating both functions together. Later when asked if the separating of the problem was a “typical thing” she would do, she explained her approach involving reading the problem and “every time it says, this side is this, I write down that side.”

F505 completed the Fun Golf problem in the interview. She first checked several values before settling on a "starting point" of 17. She then explained, "Let's continue that. So, 60 times 17 – that's $1,020. I'm going to start writing in green, just so it doesn't get too mixed up. 16 times 65 – $1,040. I'm going to come down here – 15 times 70 – that's $1,050. Then we have 14 times 75 – 1,050." She separated the parts of the solution that she felt were needed to determine the solution, using multiple colors to further delineate her separation. Her explanation of her table of values in her solution further displays her separation strategy. She describes her table, "What it looks like, to me, in my head, I'm looking at it, is a chart that goes like that. If this is the money and profit – so this is money, and this is amount of rounds."

4.2.10.2 M6238

M6238 explained the strategy of separation in his response to the initial question asked in the interview. He stated his usual process for MPS uses "sort of separate the information I do know from the information I don't." Then he explained that he would identify the end goal of the problem and determine the appropriate separation of parts. He also explained that he became more systematic in his approach during the College Algebra course. He claimed that before, "It was sort of, I know how to do this, or I don't know how to do this. Make something up, plug stuff into the calculator, maybe see if I graph this it makes more sense to me." He went on to say that he preferred to break the problem into smaller pieces. When asked about the Intersecting Quadratics problem he explained, "I think I just tried to break down each graph as a separate thing and then overlay and see, did they intersect."

4.3 Techniques

Techniques are a smaller unit of a student's problem-solving methods. The codes within the interviews defined techniques as specific actions implemented or intended to be implemented during MPS. A technique could include the application of a previously known skill, procedure or heuristic. These are individual actions that do not serve the broad purpose that a student's orientation fills. They may be put together as part of a strategy or they can be performed independently. A technique is relatively inflexible. The specific actions fill an intended role. The student may not carry out the technique but only intend to apply it and then decide against its use.

The techniques discussed appeared in at least three participant interviews. The participants may have used other techniques, but unless the researcher could code an instance in the minimum number of students these techniques were excluded. This threshold attempts to establish what typical techniques College Algebra students use. The researcher did not presume that a technique would appear multiple times within one student interview, and there are multiple examples of a student coding for a technique only once during an interview. The inclusion of these techniques that may have a limited number of occurrences accounts for the limited number of problems the student could discuss during the interview and the influences of problem type on the techniques observed. The researcher identified 10 techniques; three techniques have sub categories that fall within the same technique. One additional technique, direct computation, overwhelmingly appears subordinate to other techniques, though a limited number of instances were not tied to a previous technique. For each student, the researcher computed the ratio of codes for a technique over the number of total technique instances coded for that student. Showing how often a student used a particular technique. The ranges of these ratios for a particular technique and the median are shown along with the frequencies of each technique are listed in Table 4-14.

Table 4-14 Summary of Techniques Observed

Technique	Students (n=19)	Instances	Range	Median
Rereading	17	70	.05-.70	.18
Modeling	13	25	.07-.40	.09
Distance	*7*	*12*	*.02-.33*	*.10*
Area	*6*	*13*	*.02-.40*	*.08*
Marking the Problem	11	35	.03-.63	.09
Formula Writing	11	25	.02-.20	.10
with Substitution	*9*	*21*	*.03-.20*	*.10*
without Substitution	*2*	*4*	*.02-.16*	*.09*
Reasonability Check	12	38	.04-.50	.21
Solving Equations	6	11	.05-.16	.10
Systematic Pattern Checking	10	20	.02-.18	.09
Unit Attention	8	16	.02-.28	.11
Dimensional Analysis	*4*	*8*	*.02-.28*	*.10*
Creating Definitions	8	14	.04-.33	.10
Guessing	12	50	.05-.46	.22
(Subordinate) Direct Computation	18	68	.07-.38	.26

4.3.1 Rereading

A *rereading* technique involves the student consciously choosing to reread the problem or revisit previous statements in their solution attempt. Students may do this on account of confusion or because an initial reading was unclear. As student F6141 explained her actions, “Just re-reading it. I feel like I kind of got stuck.” A student with a reexamination orientation may perform this action as a part of their overall MPS structure, but any orientation can include this technique. F310 explained that for her usual MPS method she “read[s] the problem over again.” Another student, F505, encountered a difficulty with the existence of two possible solutions and explained, “I’m just going to read over this to see if there’s a way that they’re telling me that there’s only one answer, or there can be multiple answers.” This technique often clears up confusions the student may encounter. The codes of the rereading technique appear more often than any other technique. The nature of the action may make it more apparent than other techniques as the students often completed the action verbally during the interview. Other techniques may be more easily hidden within the student's MPS.

4.3.2 Modeling

The modeling technique includes students using multiple types of models to solve the problem. These models fell into two categories, distance and area. The two categories appeared approximately equally within instances of modeling. Modeling appeared among the second most students behind only rereading.

4.3.2.1 Area

The area model technique appears most often in the students discussing the Ken's Garden problem. This technique appears to be limited to specific problem types. For the Ken's Garden problem M106 explained, "so whenever I think of like size and that I just know it's area so I just know whenever you know you have something that's a dimension – well, yeah [pause] yeah it's a square, and then so the things that actually make up the area." Notably, student F304 engaged an area model in the Fun Golf problem and notably any problem involving multiplication could use an area model as tool. When creating her area model (Figure 4-37), she explained, "I'm thinking if there is a way I can compare these two to each other instead of comparing it to this to see if I can find some kind of relation here." The area model technique can be extended to non-geometric problems but only F304 displayed that technique in that way during the interview.

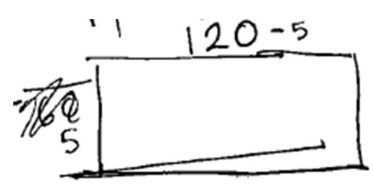


Figure 4-37 F304 Fun Golf Area Model

4.3.2.2 Distance

As with the area model the use of a distance model correlates with the problem being worked. The problems lending to instances of a distance model are Cross-Country Race, Sonar, and Surfacing Submarine. Each of the three students working the Cross-Country Race problem used a distance model technique. F6259 explained her method, “Well for starters, I'm drawing it obviously. Finish and then Charlie is 100 meters from the finish line and then Brett is 20 meters ahead of Charlie, so Brett is 80 meters from the finish line.” Her area model appears in Figure 4-38.

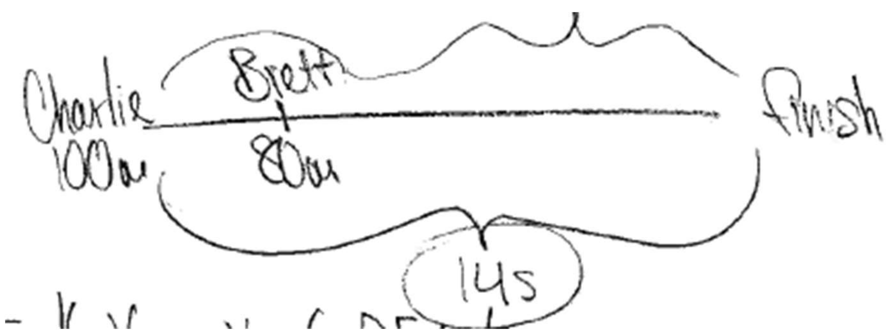


Figure 4-38 F6259 Cross-Country Race Distance Model

A student coding for a distance model technique does not mean the model was an integral part of their solution method. It may provide an initial point the student uses to determine later methods and techniques or it may provide a more important referral point or anchor for a reexamination orientation. For example, F209 used a distance model as a reference point within her reexamination orientation. However, she showed no visual distance model on her work. She stated, “I would do the same distance for Charlie.” Then later, she asks “wouldn’t they run the same distance?” The importance of the distance in her calculation and verbalizations along with the multiple references to it within her method shows the existence of an underlying model.

4.3.3 Creating Definitions

Students create definitions for parts of the problem. They may do this verbally (e.g. “I’m going to call this…”) or with notation, rather mathematical or not. One common example involves using an equals sign as a definition tool. Students may use their definitions to inform their choice of strategy or other technique. Some students defined certain parts of a problem and then did not refer to those definitions again. Possibly, some of these students created or used definitions but did not verbalize or indicated the action in their work. These students would receive no coding for the technique.

Many definition technique instances involve the use of an equals sign. This usage many times involves the misuse of that equals sign. Students will write definitions “equal” to functions or equations. For example, student F505 explained her work in the Book Stacks problem, “Because up here it says that the literature books are two inches thick, and the math books are one inch thick.” The corresponding work (Figure 4-39) displays the definition and the use of the equal sign as a definition.

Figure 4-39 F505 Book Stacks Problem with Equals Sign as Definition

However, some students use the equals sign carefully and their definition notation in a mathematically correct fashion. For example, student F304 used an equal sign and colons to define functions for both runners in the Cross-Country Race problem (Figure 4-40). Though the functions themselves do not meet the problem constraints. F304 made no mention of definitions or defining variables in the interview.

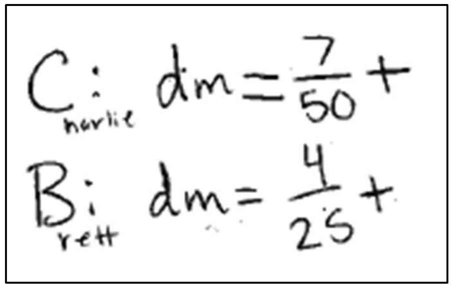


Figure 4-40 F304 Cross-Country Race Definition

Other students only explain their definition verbally and leave no visual indication of the technique. Student F506 explained her choice of M for math books and L for literature books in the book stacks problem, but her work showed no indication of these definitions. The definition technique may frequently be underlying within student work and not easily observed.

4.3.4 Guessing

Students using a guessing technique, choose values or information to examine or work with seemingly at random. As in the Tinkering orientation, this can be a productive technique. Also, students using this technique will use another technique or strategy to act on their guess. The students pick various values within the problem and then jump into another technique with that value. The guessing technique does not fall under the heading of a strategy as it lacks plan or intended method put forth ahead of time. The student only guesses a possible input or starting value and then determines future techniques or strategies. This technique closely relates to the tinkering orientation, but students may guess at a next step within a problem even if they do not hold that orientation.

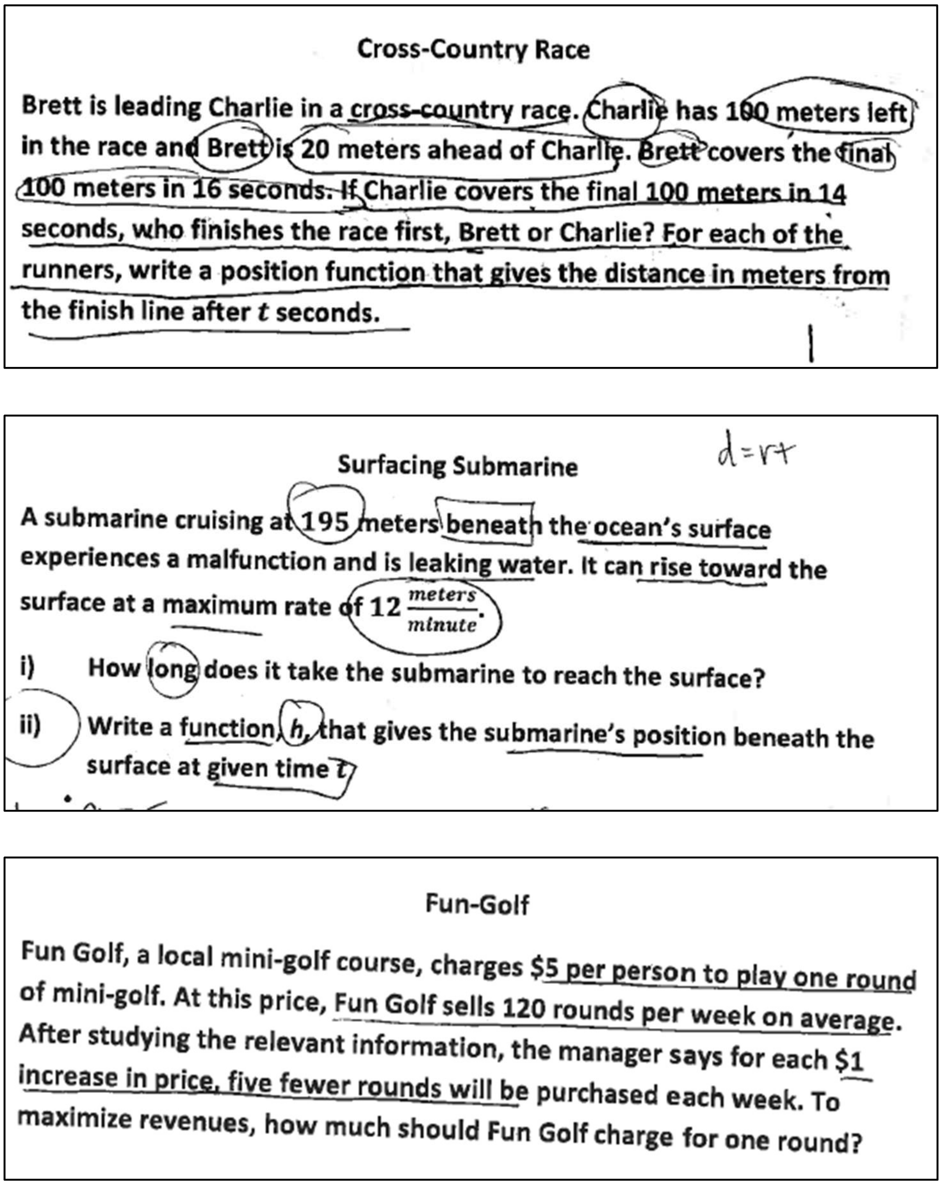
Student F209 discussed the Ken's Garden problem during the interview. She explained that "I honestly just like guesstimated, because he wanted to reduce it so I made – I don't know, I'm thinking that I did like, since this is like 1.5, like seven, 7.5 is like, it's one from here and 1.5 from here, so I think I just subtracted and added 1.5." The student performed some calculations before hand, dividing both length and width by two, but then guessed at the next steps. Another student, F310, discussed the Air Travel problem during the interview. She explained, "You want the same, because it's the same amount of time. I would have tried trial and error right here. I would try the 60 times 475, but that wouldn't have worked. Put it with one hour, that wouldn't have worked." Her technique accentuates the repeated nature usually seen in this technique. The student tests a guess or idea and then uses the information to determine the next action. Trial and error plays a role through many orientations, strategies, and techniques, but the methods tie back to one of the codes already included. Trial and error ties onto many of the orientations, strategies, and techniques observed in the interviews.

4.3.5 Marking the Problem

The marking the problem technique displays in a number of ways. Most commonly the student underlines or emphasizes words in the problem statement. The student may also reword or replace language in the problem they feel distracts or to make it clearer. This technique most commonly occurs at the beginning of the problem attempt, but can occur at any time. Students using a reexamination orientation included more instances outside of the initial reading of the problem than other students.

Multiple students mentioned an importance on locating "important" information. nine of the 11 students with codes for this technique received codes relating to their use of the word "important" to describe the marking technique in their MPS. For example, F209 described her usual MPS method, "I look for important words, like what it's asking." Another student F6141 explained that her first step "would be to write down important things from the problem." Further, multiple students shows examples of underlining or other marking actions to identify notable parts of the problem for later use or emphasis.

Some examples of these marking techniques appear in Figure 4-41. F209 appears at the top, F304 in the middle, and F6106 at the bottom.



Cross-Country Race

Brett is leading Charlie in a cross-country race. Charlie has 100 meters left in the race and Brett is 20 meters ahead of Charlie. Brett covers the final 100 meters in 16 seconds. If Charlie covers the final 100 meters in 14 seconds, who finishes the race first, Brett or Charlie? For each of the runners, write a position function that gives the distance in meters from the finish line after t seconds.

Surfacing Submarine

d=rt

A submarine cruising at 195 meters $\boxed{\text{beneath}}$ the ocean's surface experiences a malfunction and is leaking water. It can rise toward the surface at a maximum rate of 12 $\frac{meters}{minute}$.

i) How long does it take the submarine to reach the surface?

ii) Write a function, h, that gives the submarine's position beneath the surface at given time t.

Fun-Golf

Fun Golf, a local mini-golf course, charges \$5 per person to play one round of mini-golf. At this price, Fun Golf sells 120 rounds per week on average. After studying the relevant information, the manager says for each \$1 increase in price, five fewer rounds will be purchased each week. To maximize revenues, how much should Fun Golf charge for one round?

Figure 4-41 Marking the Problem Examples

4.3.6 Reasonability Check

The reasonability check technique has the student look at the likelihood their previous work includes a miscalculation or mistake. Reasonability checks appeared within 12 student interviews, one of six techniques coded in at least 10 interviews. The reasonability check technique falls short of a verification or true checking of previous work or computations. A complete verification will show as an instance of reexamination or any of several strategies. The techniques used in such verifications will be direct computations, solving equations, or substitution. The less complete reasonability check provides the student a less formal consideration of the correctness of their current work and occurs in a shorter period of time than a complete verification.

Reasonability checks received the fourth most codes among the techniques. The problem type and interview setting may lead to student performing more of these checks than they would in the course of their regular MPS. Various students pointed out or completed reasonability checks throughout their MPS. M106 explained his method within the Ken's Garden problem,

> I think I erased most of the things I tried, but – so I started with six times maybe 15 or something like that, then I'll be like, Okay, it's not quite there, so then I go a little above six, a little below 15, keep doing that until it narrows all the way down to 102 feet squared. And yeah, that's how I solved that one."

Another student F209 made an initial claim and then reevaluated that claim after. While working the Cross-Country Race problem in the interview, she first explained, "But it's asking me who finishes the race first if Charlie goes the final 100 meters in 14 seconds and then…I mean Brett finished first because he was 20 meters ahead of Charlie, and like even if he would've ran quicker he was still behind." Then she reevaluates her thinking with a reasonability check, "Now I'm rethinking it, because if he ran 14 even though he was 20 meters behind, that last two seconds he could've passed him in that time period.” Other students used the magnitude of their answers to check for reasonability. F303 explained for the Sonar problem, “I just thought this was too big of a number, and especially because it’s only 1,463 meters. It feels a little bit bigger number than I would understand.” F505 pointed out an issue in her method for the Fun Golf problem. After finding the initial revenue at $600, she calculated lower price points than $5 and realized, “135 times $2 [pause] it means they would be making 270 – something’s not right.”

4.3.7 Solving Equations

The solving equations technique involves the student engaging in the algebraic procedures to solve an equation that correctly relates two equal quantities. Using a formula does not by itself indicate the student solves an equation. A substitution technique may later lead to an equation solving technique. Calculations may occur within, before or after an equation solving technique. Students solving equations show an operation or series of steps to attempt to determine an unknown value. Also, a solving equations technique may include a system of simultaneous equations. M105 (left) and F506 (right) show examples of solving equations in their work in Figure 4-42.

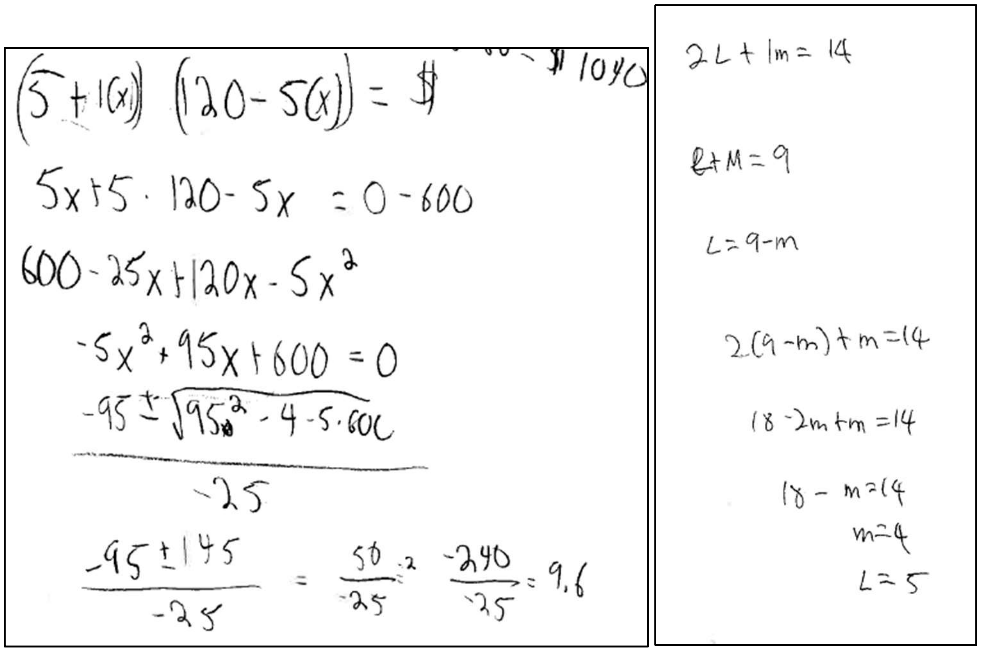


Figure 4-42 Solving Equations Examples

4.3.8 Substitution

Substitution can occur following the writing of a formula, the solving of an equation, or within a calculation technique. For a substitution technique, the student replaces something within the problem. Substitution does not require that a number replace a symbol, but indicates the most common usage for the technique. Other substitutions include, replacing a number with a symbol or variable or replacing terms with other terms the student prefers or replacing a term with a formula or alternate representation. Most commonly students using substitution mention “plugging in.” The student may have determined information earlier in the problem that needs to be used in an existing formula or procedure. Student F304 pointed out in the Cross-Country Race problem, “Yeah, I think those are the rates for each of the runners. Yeah, these are the rates because I plugged them in here.” Other examples involve replacing variables with ones clearer or more appropriate for the question being asked. F310 explained the Air

Travel problem, “I think you just replace T with V,” and telling the researcher, “It would be easier if it already told me the number of hours.” The substitution technique can also be used as a checking procedure. Student F6238 described her actions saying, “My method of checking is I like to plug in and see if the things that I'm doing are actually working. And it's mostly towards the end when I actually am capable of plugging things back in and seeing.” The usage of substitution can be done alongside multiple other techniques.

4.3.9 Formula Writing without Substitution

Students using a formula approach used multiple techniques. One such technique involved writing down a formula the student believed would be important to the problem. Contrasting from solving equations using the formula writing technique the student writes down a formula in abstract notation. The student may then apply substitution technique, but no assumption is made that the student will. Generally, this technique was not observed as students usually moved past this possible intermediate step to solving or direct computations. Student F6269 explained her solution to the Extreme Values problem. “I don't think I really understood the problem. I still don't understand it that much, but obviously here's me writing the…wrote the formulas.” The student explained that she could not solve the problem due to her confusion, but by writing the formulas, or functions, in this case it offered her a mechanism “to visualize.” Another student, F505, pointed out her work on the Intersecting Graphs problem explaining, “I really only found one formula because I didn’t really know how to find two formulas with one point on each one.” Her slope-intercept formula writing appears in Figure 4-43.

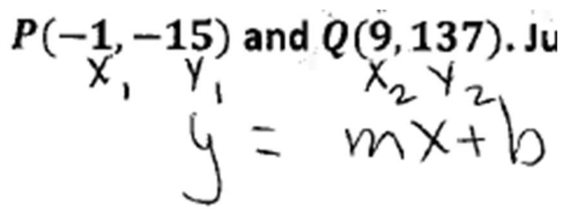


Figure 4-43 F505 Intersecting Graphs Formula Writing

The writing of the formula before using it she hoped would provide some indication of her next steps. This technique could be considered similar to rereading. The repeating of the formula or, as in F6269's case, problem constraint may offer a similar clarifying effect. This similarity may explain the low of number of instances for this technique.

4.3.10 Systematic Pattern Checking

The systematic pattern checking technique involves the student checking each value with an increase of a unit or uniform group of units. This systematic technique includes uniformity in the increase or decrease in the inputs for the test values. Other valid techniques are similar where the student uses test values or non-uniform intervals to move to or zoom in on the solution. Direct computation closely relates, and students using systematic pattern checking likely perform direct computations, but the systematic pattern checking technique does not require it for an instance to be coded. The student could abandon the technique prior to computations being performed. This uniform increase occurred with students working the Fun Golf problem most often. Eight of the 10 students coded for this technique included it in the Fun Golf problem. Examples of students using the systematic pattern checking technique appear in Figure 4-44. Students F105 and F6115's work appear at the left and right respectively.

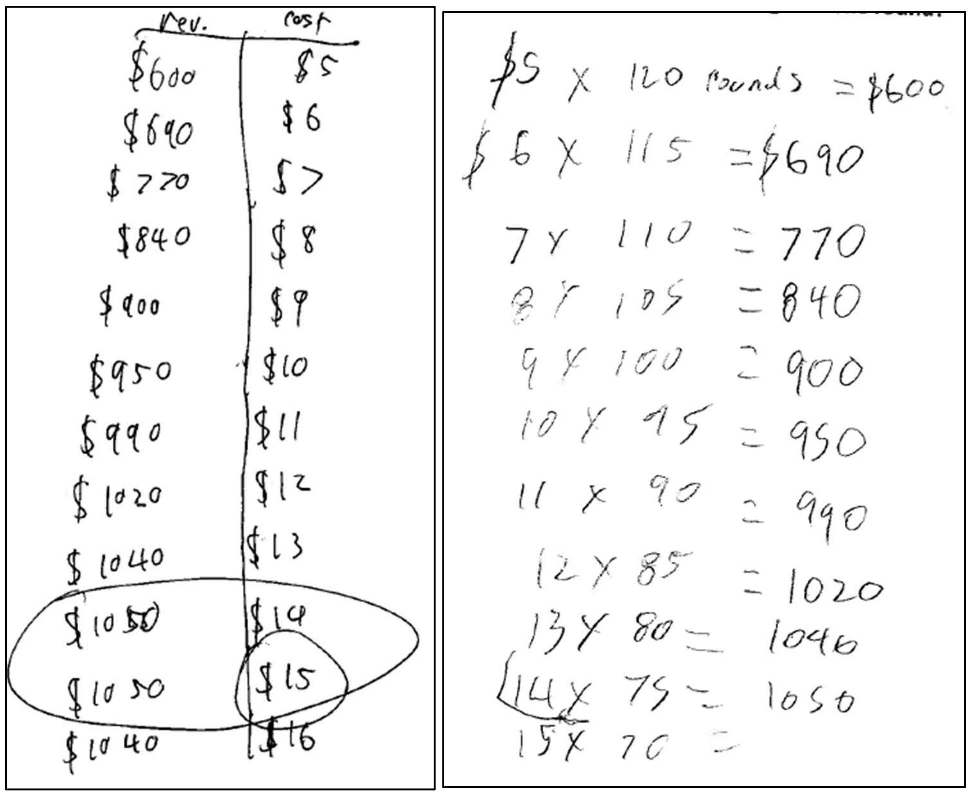


Figure 4-44 Systematic Pattern Checking Examples

Some students calculated more values than necessary as part of this technique. The technique requires another strategy or technique to identify an appropriate value to stop calculations, as in F505's work shown before in Figure 4-25.

4.3.11 Unit Attention

Unit attention requires the student to use the type of unit to match the computation method or solution. A dimensional analysis action shows a unit attention technique. Unit attention can also be performed as a part of a reasonability check. To do this, the student checks that the units on the proposed solution match with those of the problem. A unit attention technique examines the type of units assigned to the quantities in the problem. The student may use the units to determine the type of calculations needed. Dimensional analysis provides one example of this technique. F318 explained her calculations in the Surfacing Submarine problem, “I did time equals meters over rate. I guess I must have taken all the units, because there’s 12 meters per minute, so you have to have meters over time, which means that you’re going to have – the units are going to match up with that, kind of?” She used the meters unit with the meters per minute unit noted for the rate of ascent to match her calculations. F6145 determined the necessary calculations by matching the units within the Myla’s Pool problem. She explained, “The first thing I would notice would be the rate, this is per minute and they're asking per hours. So with dimensional analysis, um you're like multiplying through, you have to cancel out like units.” The unit attention technique allows the student to understand the needed actions or to ensure the previous actions were correct.

4.3.12 Direct Computation

Students frequently attempt to calculate the needed information directly. Though this technique appears as the second most common technique code, likely that some instances fail to receive coding as a result of the computations being done silently or without other evidence. Further, this technique most often appears subordinate to another technique. The speak aloud nature of the interview leads to some missed instances of strategies or techniques. Some students explained in detail their calculation steps during the interview while others preferred to work quietly or to discuss the reason they chose a method rather than the details of their computations. The identification, pattern thinking, and information reuse strategies corresponded to more coded instances of direct computation. Three techniques served as the primary technique with only five instances where the primary technique could not be identified. Formula writing with substitution, solving equations, and systematic pattern checking included this subordinate technique. The number of subordinate instances appears in Table 4-15.

Table 4-15 Summary of Direct Computations Techniques

Subordinate to Technique	**Students**	**Instances**
Formula Writing w/ Substitution	15	31
Solving Equations	5	20
Systematic Pattern Checking	4	12
None	4	5
Total	18	68

Students offered verbal indications of their computations in multiple parts of the interview. For example, when discussing previously completed problems F209 explained for the Sonar problem, “I did, five divided by 100 to get 1/500th, to get 0.05, then divide that by the 1463 to get the 29,260.” Others did not point out the numerical components but rather noted which parts of the problem they performed the computations for. Saying “this one” or “I added these together.” Students using the calculator also coded for the direct computations technique. The calculator can supplement other techniques and strategies beyond direct computation, but the physical action of entering numbers into the calculator falls within a direct computation. Notably, an instance of direct computation may include multiple calculations or operations but occurs as part of the same choice of technique by the student.

4.4 Connections between Orientations, Strategies, and Techniques

The coding of orientations, strategies, and techniques assumed no connection between the three areas. However, the appearance of codes group in certain ways. Particular orientations connected to certain strategies. Other strategies connected with particular techniques. Additionally, better course grades show a correlation to some orientation to strategy connections. The number of instances coded also shows a correlation to higher grades. Within each section, the node diagrams show connections between the various orientations, strategies, and techniques. Thicker connecting lines and bold text indicate the appearance of multiple instances of the connection in the student interview, while thinner connecting lines indicate single instances of the connection. Each diagram shows the connections for one student orientation. A single dot next to a strategy or technique indicates the researcher could not determine a connection leading to that strategy or technique. The student used the strategy or technique, but it was not clear that it originated from an orientation. Each student has as many diagrams as they have coded orientations (e.g. A student with two coded orientations will have two node diagrams). All of the diagrams are included in Appendix C.

4.4.1 Relationship Between Formula Application Orientation and Strategies

The formula application orientation codes correlate with instances of algebraic representations and identification models and formulas. Table 4-16 shows the number of instances coded for each of the strategy groups among the students coding for formula application compared with those not coding for formula application. The students displaying a formula application orientation also showed no instances of separation of parts while three of the eight students coding for no instances of formula application displayed that strategy.

Table 4-16 Relationship Between Formula Application and Strategies

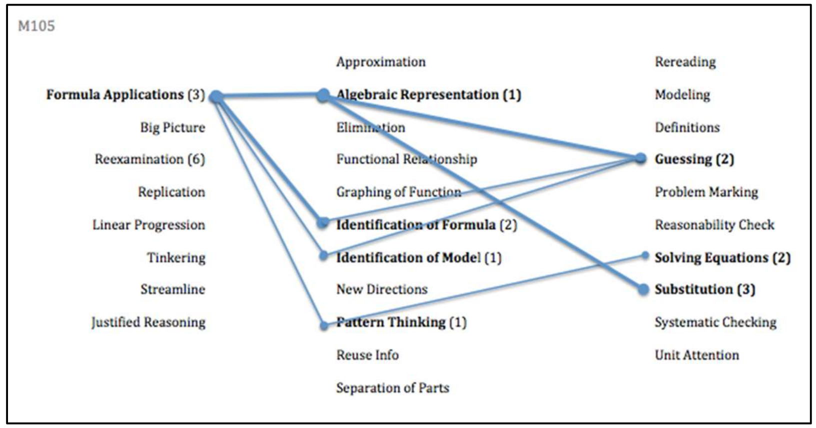
Formula Application Codes	**Students**	**Algebraic Relationship Students**	**Identification of Model Students**	**Identification of Formula Students**	**Separation of Parts Students**
Yes	11	6	6	8	0
No	8	1	3	3	4
Total	19	7	9	11	4

The connection between formula application and identification of models and formulas appears straightforward. Students display a desire to use a formula and route their MPS onto formulaic approaches and structure their MPS around these formula interactions. Then within the method the student attempts to identify appropriate models and formulas that aid in their completion of the problem. For example, student F6145 displayed her orientation saying, “If there were like two or three in a systems of equation you could still solve it if you had the different variables there.” She follows this statement by discussing her solution to the building functions problem with an identification of model strategy. She explains, “This is obviously a very linear graph so it would have to be that, but I also solved for the slope.”

The connection between formula application and an algebraic representation strategy also shows clearly. F318 explains her method on the Fun Golf problem including “I guess this is an X and a Y equation.” She goes on to attempt to locate an algebraic expression or relationship that will lead to her solution. This fits beneath her orientation of formula application. She believes she can fit the problem into a formula that follows from her algebraic expression. She explains that she tried “to make into this weird equation.” The need within the formula application orientation she held to organize the problem into a formula or known procedure held her back from using the algebraic representation strategy to solve the problem though the algebraic representation followed from her desire to understand the problem through the formula application.

The node diagrams in Figure 4-45 show the connection between the formula application orientation and the strategies used by the students. These student’s codes display the connection between the identification strategies and the formula application orientation. Comparing these diagrams with those of the students not coding for formula application we can perceive the apparent difference in the two profiles. The reader should see the connection from formula application in the orientation column, to the identification strategies in middle of the center strategies column.

Figure 4-45 Formula Application Orientation Node Diagrams



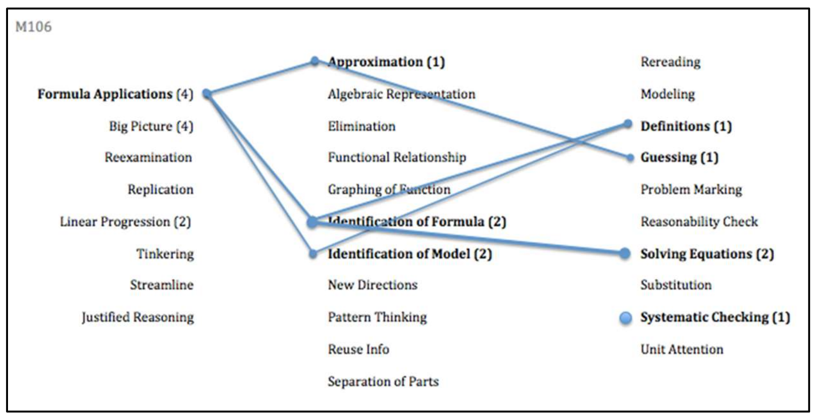


Figure 4-45 Formula Application Orientation Node Diagrams

4.4.2 Correlation Between Big-Picture Focus Orientation, Approximation Strategy, and Unit Attention

Students coding for instances of the big-picture focus orientation coded for an increased number of approximation strategy instances and unit attention techniques. Six of the 19 interview participants displayed instances of the big-picture focus orientation. Within these six students five of them incorporated the approximation strategy within their problem-solving methods. Further, four of these five students showed a unit attention technique and five of the six big-picture focus students used the unit attention technique. Figure 4-46 shows the node diagrams for the six big-picture focus orientation students. It is not the case that the “pathway” for each of these students passes from the big-picture orientation, to the approximation strategy, to the unit attention technique, rather that the student coded for each of those in the interview. Only three students incorporated the unit attention technique that did not code for big-picture focus orientation, and only three students using the approximation strategy did not code for big-picture. In each diagram, the approximation strategy appears at the top of the middle column and the unit attention technique appears at the bottom of the right column. The broader view of the problem situation undertaken by holders of the big-picture focus orientation appears to yield this strategy and technique.

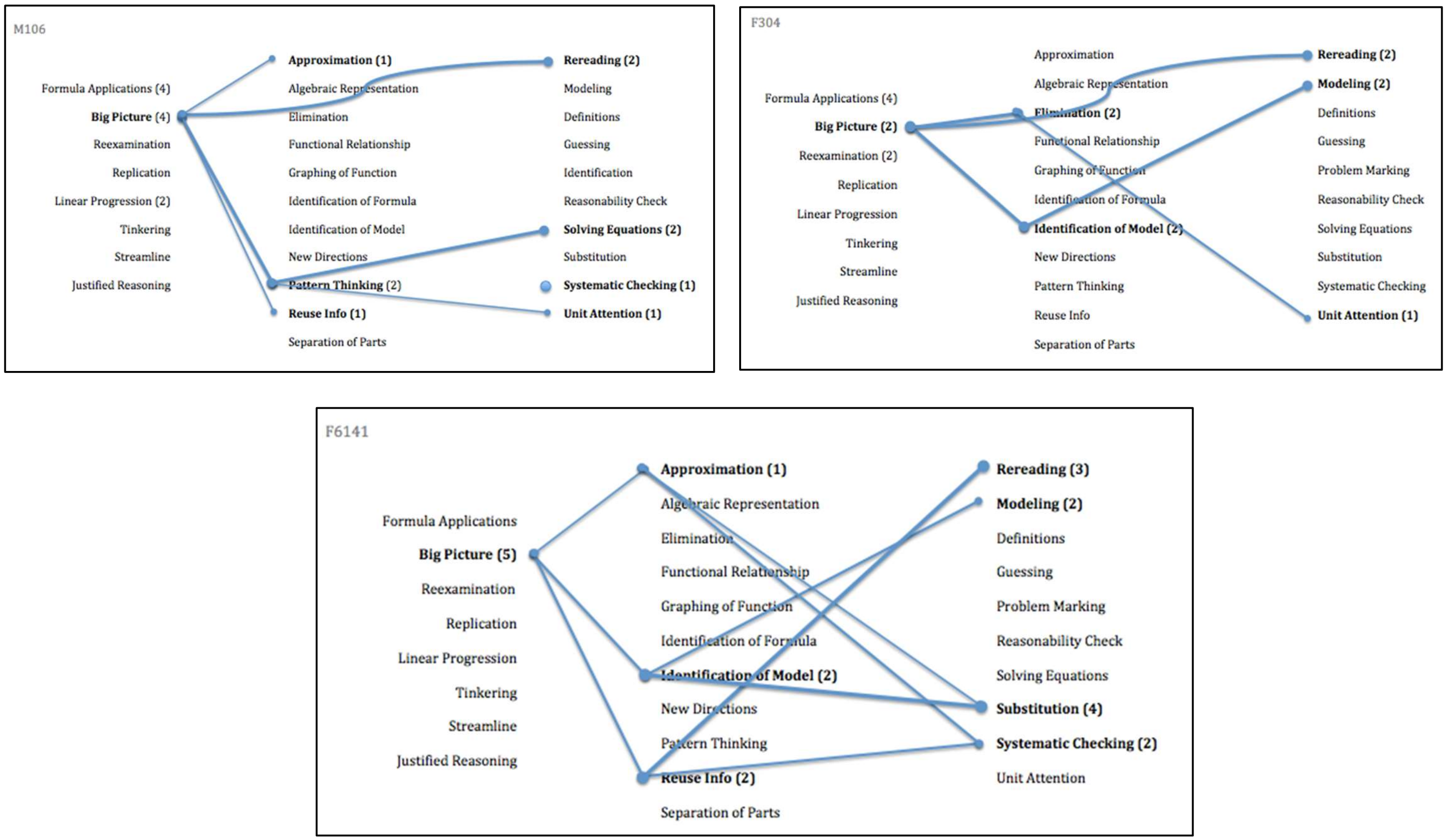


Figure 4-46 Big-Picture Focus Orientation Node Diagrams

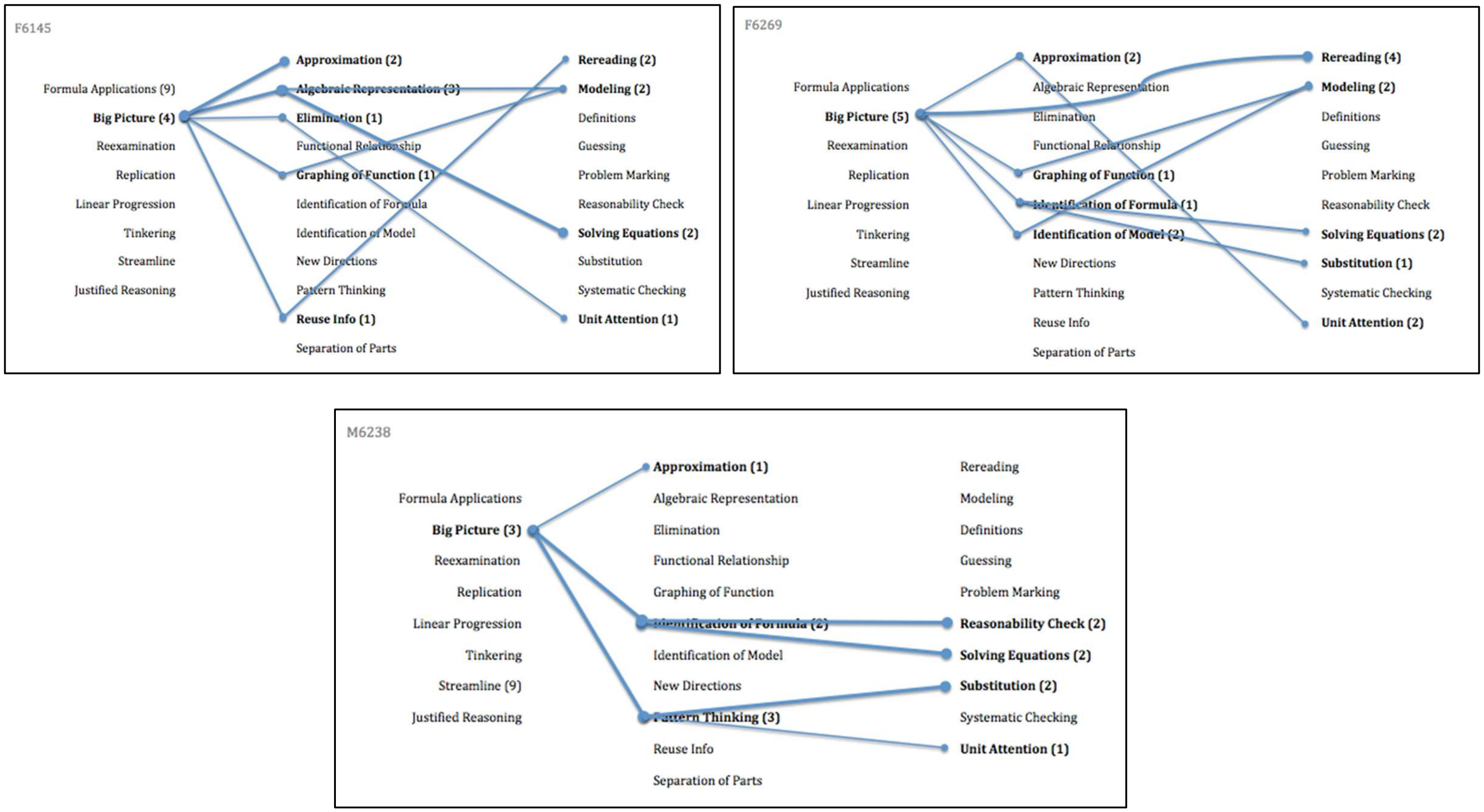


Figure 4-46 Big-Picture Focus Orientation Node Diagrams

4.4.3 Correlation Between Reexamination Orientation and Reasonability Check and Guessing Techniques

Reexamination appeared within the second most students across orientations. Within this subgroup of students six of the seven students coded for instances of reasonability check technique. Similarly, six of the seven students coded for the guessing technique. The students displaying the reexamination orientation also coded for similar strategies in some cases. The “pathways” from orientation to strategy to technique show no apparent trends among those with the reexamination orientation, but the “destination” of the technique appears frequently among those students showing this orientation. The ability or desire to reexamine the problem may lead to the usage of these techniques since each of reasonability check and guessing provide an opportunity to look back at the previous work for the student, possibly fitting into the overall structure of reexamination. The node diagrams in Figure 4-47 show the reexamination orientation students and their techniques.

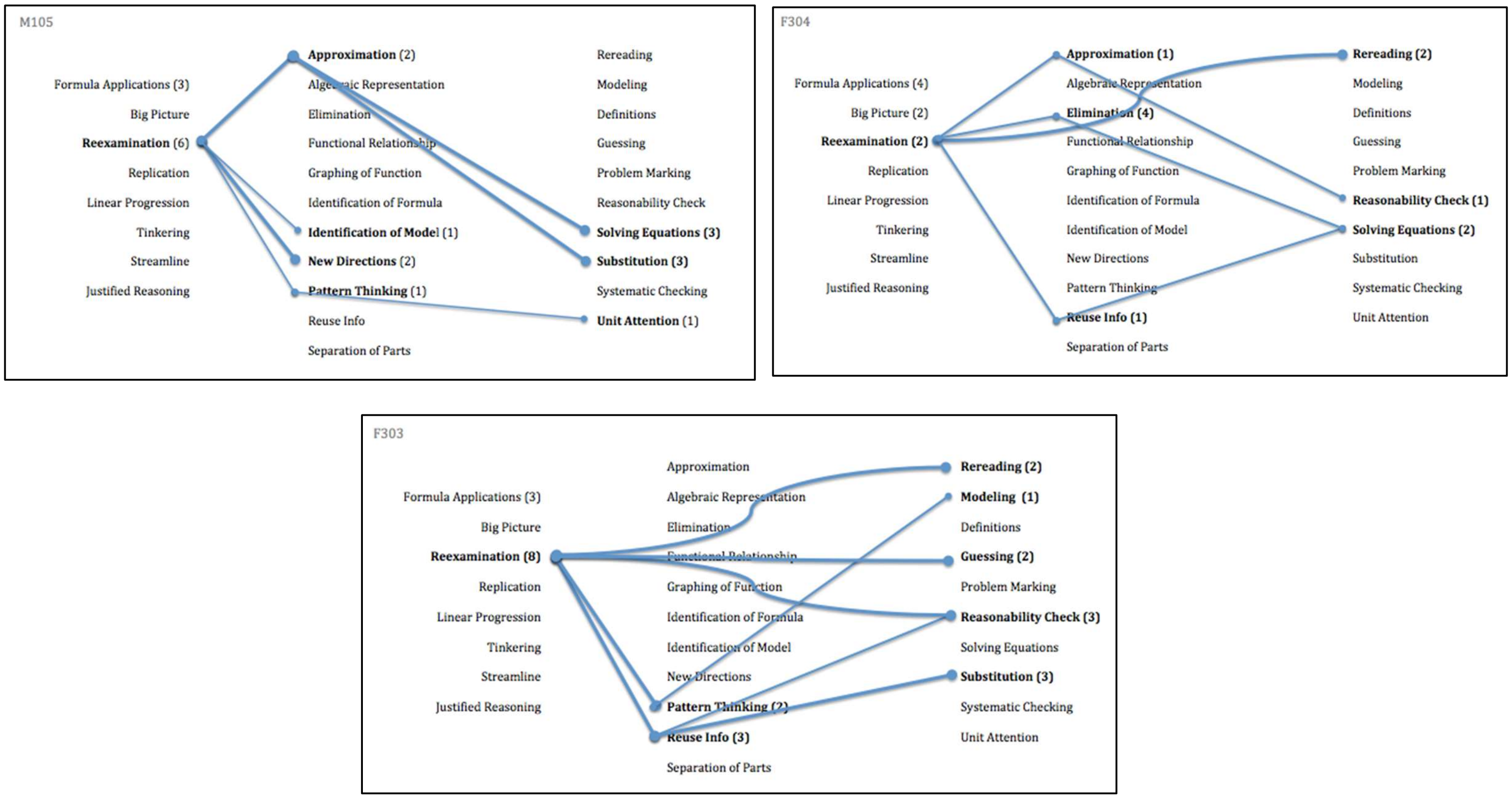


Figure 4-47 Reexamination Orientation Node Diagrams

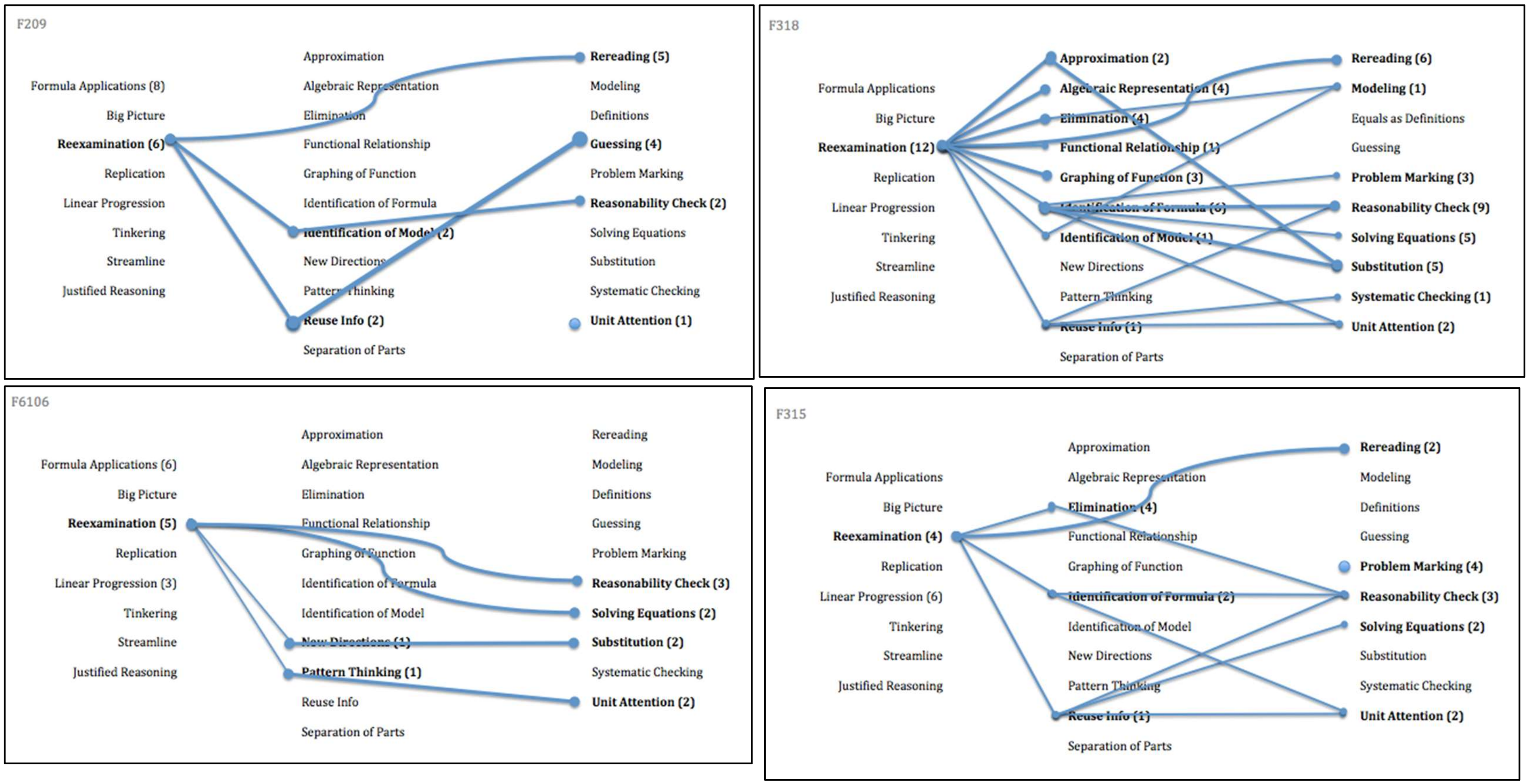


Figure 4-47 Reexamination Orientation Node Diagrams

4.4.4 High Performing Students' Orientations and Strategies

In addition to the MPSI responses and written work on the problems, the researcher also collected the students' course grades. The students earning an A or B grade in the course display a distinct profile of orientations, strategies, and techniques. Other students display a similar profile, but also many of the students earning below a B in the course display a different profile than their higher scoring counterparts. For example, student F318, earning a B in the course, shows an increased number of strategies in her profile, using eight different strategies. This is typical among the high achievers in the course. The 10 students earning A or B coded for an average of 4.7 strategies each while the remaining eight students coded for an average of 3.75 strategies. Reexamination appears primarily among high performing students. Six of the seven students coding for the reexamination orientation earned an A or B in the course and five of those six students coded for multiple orientations. Also, all three students coding for the linear progression orientation earned an A or B in the course. In Figure 4-48, the node diagrams of student F304 shows the density of coding across multiple orientations, strategies, and techniques for an A student using each of the three primary common orientations.

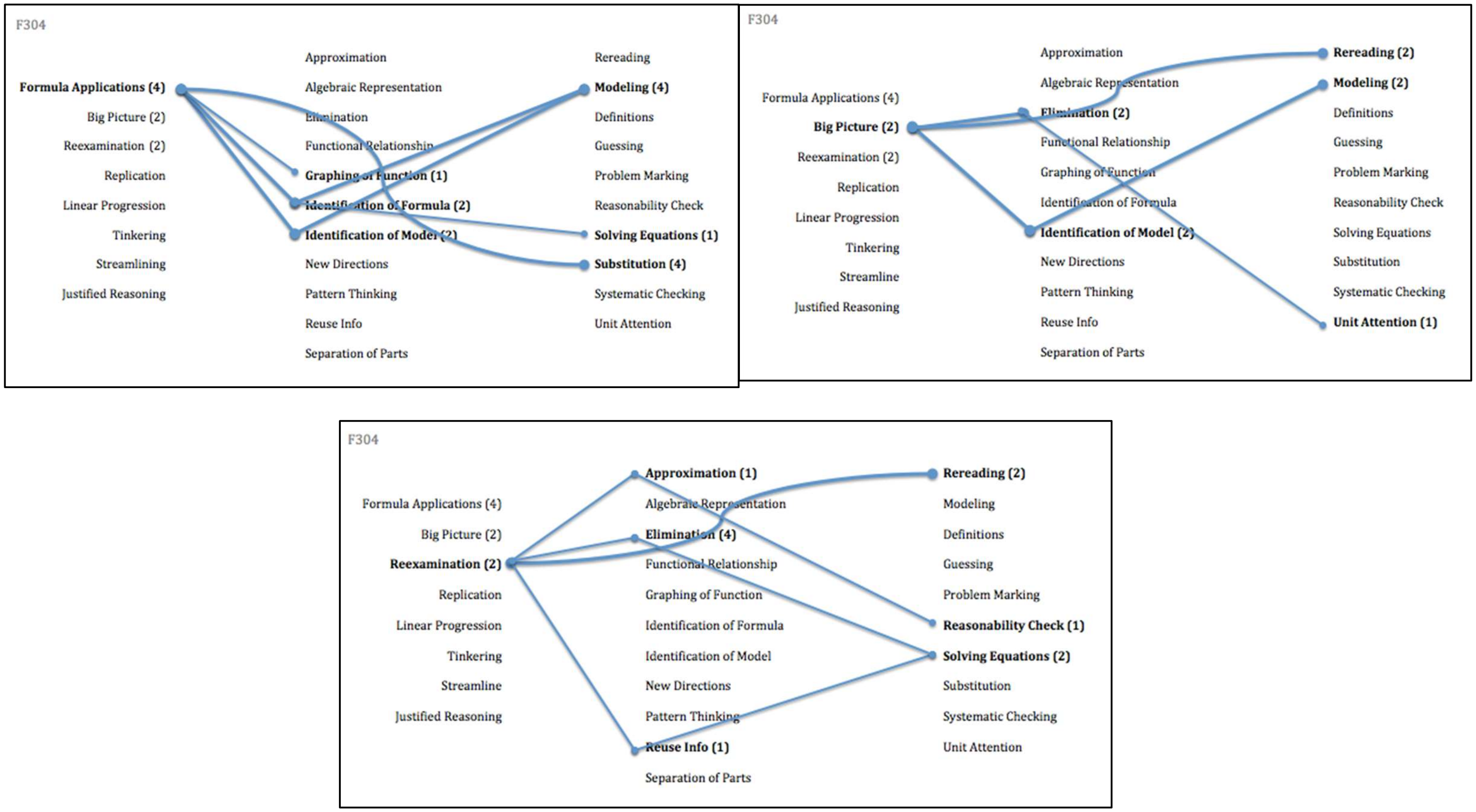


Figure 4-48 F304 Multiple Problem Solving Pathways

Additionally, the students earning an A or B in the course show an increased number of connections directly from orientation to technique, as denoted by the curved lines in the node diagrams. These direct connections may offer evidence of an increase in implicit strategies or methods. This automaticity may help students increase their efficacy and efficiency in MPS.

4.4.4.1 Number of Instances Correlated with Course Grade

In the interview, students with higher course grades coded for more instances of orientations, strategies, and techniques than lower scoring students. The number of coded instances of any orientation, strategy, or technique appears in Table 4-17.

Table 4-17 Course Grades Compared with Coding

Student Course Grades	**Students**	**Orientation Instances (Per Student)**	**Strategy Instances (Per Student)**	**Technique Instances (Per Student)**	**Total Instances (Per Student)**
A or B	10	107 (10.7)	121 (12.1)	170 (17.0)	398 (39.8)
C, D, F, W, or I	9	88 (9.8)	68 (7.6)	144 (16.0)	300 (33.3)

Notably, in each type of code the number of instances increased by at least one instance. In total, the number of codes for high performing students increased by six on average over lower performing students. The median number of codes displayed the same trend. A or B course grades coded for a median of 10.5 instances of orientations while C, D, F, or W, students coded for a median of nine instances. Similarly, high performing students included a median of 10 instances of strategies and 17 instances of techniques. Low performing students coded for seven instances of strategies and 16 instances of techniques. In each case these differences are not statistically significant except for the total instances of all three types.

4.5 Connections Between Affect and Orientations, Strategies, and Techniques

Students hold beliefs and attitudes about mathematics and MPS before the interview. These affective factors manifest in the student's speech during the interview. The number of instances coded within a student interview appears to correlate with certain coded aspects of student MPS. In particular, the appearance of affective factors correlates with certain orientations. Further, the frequency of codes and the length of the interview also correlate with number of affect codes observed by the researcher.

4.5.1 Positive Affect Codes Correlate to Non-Formula Application Orientations

11 of the 19 interview participants coded for instances of the formula application orientation. Only three of these students coded for only formula application instances in their orientation. This arranges the students into three groups, the first group, "No Formula Applications," includes the eight students that did not code for any instances of formula application. The second group, "Displayed Formula Applications," includes the students that coded for formula application and at least one additional orientation. The third group, "Displayed Only Formula Applications holders," includes only the students that coded for formula application only. The number of positive affect codes observed in the interview correlated with the group the student codes into. Table 4-18 shows the number of positive affect codes for each group.

Table 4-18 Positive Affect Codes Sorted by Formula Application

	Number of Students	**Positive Affect Instances**	**Instances Per Student**
No Formula Application	8	31	3.875
Displayed Formula Application and Other Orientation(s)	8	22	2.75
Displayed Only Formula Application	3	5	1.67

4.5.1 Negative Affect Codes Correspond to Shorter Interviews and Fewer Codes

The number of negative affect codes correlated to both shorter interviews and fewer coded instances of orientations, strategies, and techniques. Students coded for negative affect when they indicated a negative or apprehensive belief or attitude toward mathematics or problem solving. This would not necessarily mean they did poorly on the problem, rather they felt poor about math or problem solving going into the problem. The number of these codes weakly correlated negatively with the length of the interview and the number of codes the interviewer made in the interview. The graph in Figure 4-49 shows the weak correlation, with correlation coefficient, $r = -.24$. The correlation between negative affect instances and the length of the interview lacks a strong correlation, but the limited number of data points at 19 leaves the possibility that a trend where students with negative attitudes toward mathematics discuss their work in a different and less thorough way.

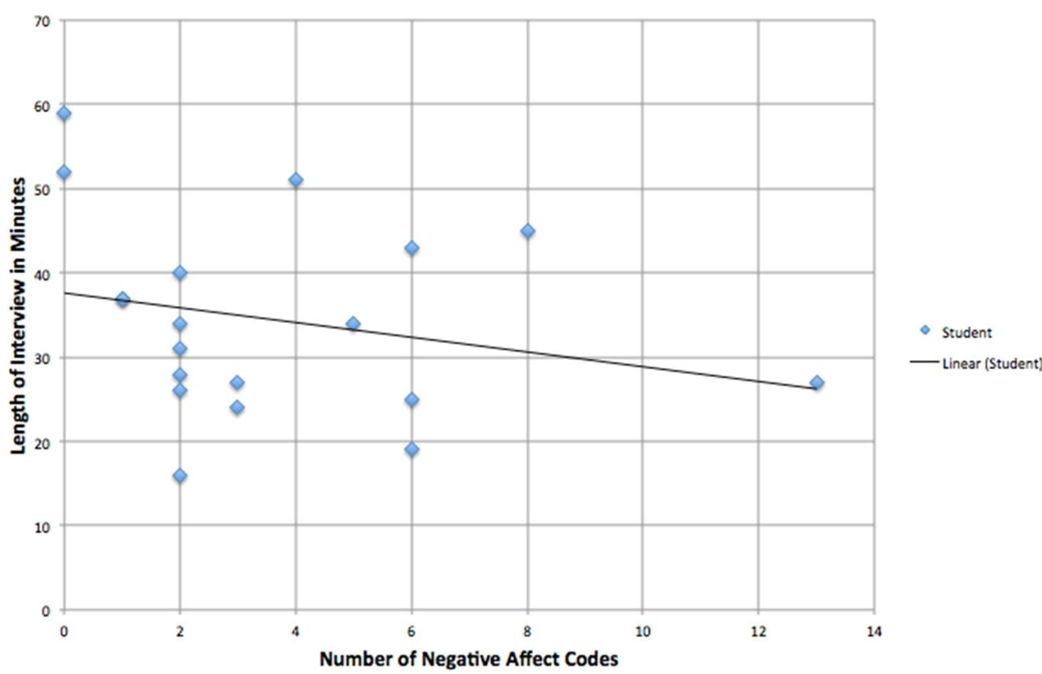


Figure 4-49 Correlation Between Interview Length and Frequency of Negative Affect

Chapter 5

Discussion

Using a Grounded Theory approach as described in Corbin and Strauss (2008), I examine the methods in mathematical problem solving (MPS) behaviors and artifacts of the 19 College Algebra interview participants. To search for “theoretical constructs derived from qualitative analysis of the data.” I followed a combination of open and axial coding as described in Corbin and Strauss (pp.61-74). The open coding created the constructs without presupposition of any particular code or set of codes. Axial coding helped to decide the grouping of the sets of codes into the three categories. Along with the axial coding, the grounded theory approach was used to examine the interrelationship between these constructs as described in Creswell (2014).

For this investigation, I modified the grounded theory structure in one key way. I predetermined the existence of three different grain-sized constructs of MPS. No assumptions existed about the number of constructs within each category and the names of the three categories evolved from the data and coding collected. I presumed no connections between constructs prior to their emergence from the data. This modification is akin to the grounded theory framework described by Charmaz (2006). The incorporation of existing understandings of MPS allows the researcher to attend to expected constructs while remaining sensitive to the emerging constructs and attempt to frame the existing information with the emerging constructs. The theory I constructed proposes three primary orientations held by College Algebra students, *formula application, big-picture focus,* and *reexamination*. Each orientation links to a subset of strategies or techniques used in conjunction with the orientation. Usage of formula application connects to using an increased number of strategies in the students’ MPS. Further, *formula application* holders displayed greater emphasis on quantitative strategies and techniques being less likely to use methods involving qualitative or "common sense" methods. *Big-picture focus* students performed the worst among the primary orientations. These students showed more instances of qualitative techniques, such as approximation or separation of parts. *Reexamination* students displayed more instances of techniques than other students. Codes of *reasonability checks* and redirection strategies, *new directions* or *information reuse*, appeared within this orientation. Among other orientations, Students using a *linear progression* orientation, or step-by-step focus, scored highly in the course. Additionally, an overemphasis on formula application may create affective challenges for students. Beliefs and attitudes toward mathematics appear to alter the orientations held by students. Positive attitudes and belief correlate with using less of the *formula application* orientation. Negative attitudes and beliefs correlate with being less willing to explain their MPS. Lastly, word problems appear to change the types of problem-solving strategies implemented by the students when compared with more abstract problems.

5.1 Orientations, Strategies, and Techniques Used by College Algebra Students

Within the student MPS, three particular orientations appear more frequently than others. These primary orientations form a structure of MPS ideas that College Algebra students appear to value. By understanding this structure, instruction can address components of MPS that appear to be missing. Also, certain strategies and techniques preferred by a typical College Algebra student may or may not be productive for learning algebra. Understanding those orientations, strategies or techniques privileged by students, instruction can cultivate productive strategies that are underdeveloped or less-used by students.

5.1.1 Primary Orientations of College Algebra Problem Solving

The codes for *formula application*, *big-picture focus*, and *reexamination* arose more than other orientations. Each of these orientations appeared in at least six participant interviews while the most prevalent of the other five orientations appeared in only three interviews. The most prevalent orientations may relate to the way high school teachers and college instructors convey or model information about MPS. By understanding the primary orientations held by students in College Algebra, we may discover the additional critical components of MPS that typical students lack as well as discover other possible course-related factors linked to STEM attrition.

5.1.1.1 Formula Application

Formula application students adhere to an understanding that a formula, equation, or other mathematically precise notation will hold the key to completing the problem. For example, student F304 adheres so firmly to the formula application orientation that she cannot abandon an equation solving technique to complete the problem. This strict adherence created a barrier for some students. The students may privilege the type of thinking used in this orientation even while not believing it to be the best MPS practice. Possibly, he or she may feel they should be using ideas and structures close to this orientation even if it may not be the best orientation for the student. Student M106 expressed his preferences brought into the MPS process in the Fun Golf problem in the interview. He says, “I know there’s an easier way to do it, but at this point it’s kind of plug and chug for me.” Though no direct evidence appears in this statement of a formula based approach reasonably the “easier” way includes the use of a formula or other mathematical representation. Mathematics courses frequently reference the importance for knowing formulas and procedures. The number of students holding this orientation suggests an external factor may push students to this orientation.

However, the formula application orientation appears to serve as a productive orientation for many students in this course. Nine of 11 students showing this orientation in the interview earned an A, B, or C in the course. Comparatively, more students received a D, F, or W, in the course among those not coded for formula application. Finding the correct balance between the formula application orientation and other orientations, strategies, and techniques presents a challenge for both instructors and students. The relationship between a formula application structure and student beliefs and attitudes is further explored in section 5.3.

Formula application may be a heuristic (or set of heuristics) that can be taught to improve students problem-solving abilities. Santos-Trigo (1998) discussed the success of Schoenfeld's courses in this regard. For this to be successful, as described by Santos-Trigo, the students need to see a wide variety of problems and this may not be the case in the College Algebra curriculum. The variety observed in the curriculum for this course appears to be limited (Green, 2016). Thus, formula application may both match up with successful orientations in the literature and in these cases be beneficial only because of course format.

5.1.1.2 Reexamination

The reexamination orientation appears among the second most student interviews. The looking back protocols exhibited by these students were easily observed within the interviews. Notably, students exhibiting this orientation expressed it in their answer to the initial interview question. In this question about their usual problem-solving methods or steps each of the seven students voiced a desire to recheck or look back within their MPS method. Further, five of the seven students earned an A or B in the course, and a sixth passed the course with a C grade. Again, this may point to a following of teacher instructions in a student's usual MPS. Students are frequently encouraged to check their work and specifically in the College Algebra course the students enrolled in instructors told the students directly to use all available time on exams and to maximize their use of scratch paper and other tools. The efficacy of this orientation follows the cyclic nature of problem solving described by Carlson and Bloom (2005). The cycling back done in reexamination relates to the problem solving of the experts studied by Carlson and Bloom. The apparent success the students achieved in the course suggests a potential relationship between this orientation and course success. The effects of the course on student learning are not explored by this study though it interestingly three of the seven students did not solve the interview problem correctly. One possible explanation for the coding of the students into this orientation considers the problem the students worked in the interview. Five of these seven students completed the Fun Golf problem in the interview. However, since each of the seven pointed to reexamination as a part of their usual MPS structure this seems unlikely.

5.1.1.3 Big-Picture Focus

Big-picture focus emerged as an MPS orientation for a significant number of interview participants. Six students coded for instances of big-picture focus in their interviews. Three of these students received a D, F, or W in the course. Four of these students noted a broad view or a “start[ing] out as the whole problem” in their answer to the initial questions about their MPS. The researcher has the least influence on the student responses at this stage of the interview. The poorer course grades among this group suggest this orientation offers the least help to students in College Algebra. Further, only three of the students coded for this orientation then coded for another of the primary orientations that appear to correlate to better course performance. The big-picture focus orientation appears to offer fewer benefits to College Algebra students. Often students are asked to “take a step back” when struggling with a problem to look back over the entire problem situation. However, participants doing this as their orientation only solved the interview problem correctly in three of six cases. The utility of the orientation for the participants is not as clear as it is for the two primary orientations.

5.1.1.4 Other Orientations

Linear Progression, or step-by-step focus, surfaced for only three students but all of them performed well in the course. That is, they earned an A or B in the course. These students also account for two of the three students coded for big-picture focus that passed the course with an A, B, or C, grade. Further, each of these three linear progression orientation students coded for one of the primary orientations. One possible explanation involves the computer program used for course assessments in the lab section of College Algebra. In this program, several of the problems require the student to answer the question in parts. On a homework problem, the student will be asked to answer one question, followed by another using the first question's answer. In some cases, the latter many repeat for five or more parts in some cases. Exam questions are not presented in the same way, but often contain multiple questions that must be answered for a single problem. The possible emphasis on linear thinking or step-by-step procedural format of the course may explain the success of linear progression orientation students in course. Only one of the students expressed this orientation in the initial interview question further suggesting the students may be lured into this thinking by the type of problems they work from the curriculum.

Each of the other four orientations not included above appeared for at least one low performing student. The limited number of student coding for these orientations makes trends difficult to identify. One notable case involves student F505. Her interview coded for 14 instances of a justification orientation, the only student to code for that orientation. Her willingness to provide detailed explanations in the interview appears to correspond with her work. This type of thinking and willingness to challenge one's own solution would be typically viewed as an approach that leads to success in MPS, but only led to two correct answers in the six problems collected from her and she failed the course. The added time used for this approach may inhibit performance in a formal course setting with timed exams.

The two students coding for streamlining passing this course aligns with the work of Lavie (2010). That is, the elimination of distractions and the possible lessening of cognitive load may contribute to the course success of those students. M310 earned a D in the course while coding for streamlining may relate to his additional replication orientation where the other student coding for this orientation earned an F.

5.1.2 Preferred Strategies and Techniques and the Relationship to Course Success

Certain techniques and strategies appear with greater frequency among student interviews. Since the majority of interview participants hold a formula application orientation, the modeling and formula based strategies appear among the greatest number of students. Less expectedly, the number of students using approximation as a strategy exceeded the number of students using strategies tied to College Algebra course content. For example, 10 students coded for instances of approximation as a strategy, while only two students displayed a functions-based strategy relating to the functions material in the course. Among techniques the expected methods typically taught or modeled by instructors appear most often. Students reread the problem to identify any information they view as important. Many students mark the problem, holding to the technique of circling numbers or identifying units or other important information using underlining or highlighting. Secondary teachers frequently expect their students to perform these steps or emphasize such procedures in preparing for standardized tests.

The modeling and formula based strategies form a base of methods the students use in their MPS. Each student coded for at least one strategy where they seek a construct to place into the problem. These strategies include identifying a model, identifying a formula, graphing, and functional relationships. No strategies alone showed a significant difference in passing and failing students.

Among techniques the simplest techniques appeared most often. These include rereading, guessing, computations, and reasonability checks. The level of mathematical understanding needed to complete the first three stays relatively minimal. Any student can reread the problem regardless or understanding. Guessing and computations stay at a low level since the student can use a calculator. These steps require minimal need understanding of the mathematical concepts underlying the skills. Reasonability checks require additional understanding

5.2 Profiles of College Algebra Student Problem Solving

The profiles of College Algebra students’ MPS may provide information regarding factors or attributes that support or detract from a College Algebra student’s success in the course. The student-held orientations that convey a structure within which a student works problems may reveal other areas for development that will lead to success. Certain strategies and techniques also link to particular orientations. By identifying these profiles, we can begin to examine the efficacy in applications beyond these problem-solving interviews. Further, by identifying a student’s primary orientations the instructor can be aware of the strategies and techniques commonly coupled with them. Alternatively, it may be possible to identify techniques to emphasize with students that can help a student to gain a beneficial orientation or structure to aid in their MPS.

5.2.1 Formula Application with Many Strategies

Among students coded for instances of the formula application orientation the number different strategies coded in an interview increased compared with students that did not show any instances of the formula application orientation. From the 11 students displaying formula application, all coded strategy groups appeared in at least two interviews, and the 11 interviews averaged 4.1 different strategies per student. Meanwhile, the students not coding for the orientation coded for only 3.2 different strategies on average. Table 5-1 shows the strategy information comparing the two groups.

Table 5-1 Summary of Course Grades and Formula Application Orientation

Student Orientation	**Number of Interview Participants**	**Number of Different Strategies Coded**		**Number of Interview Participants with Grades of**	
		Among Entire Group	Average per Student	A, B, or C	D, F, W, or **I**
With Formula Application	11	11	4.1	9	2
Without Formula Application	8	10	3.4	5	3
Total	19	11 Possible Strategies		14	5

Comparing the two groups can be done using the Wilcoxon Rank-Sum test. This test is analogous to the Student's T-test, but non-parametric to consider the small sample size and discrete nature of the data. The test produces a p-value of .16, not a significant correlation. However, student F318 coded for eight different strategies and did not code for formula application. None of the other students without formula application coded for more than four different strategies. If we exclude her data from the calculations the average number of strategies for the group not using formula application falls to 2.9 and the p-value for the statistical test drops to .05, a possibly significant correlation. No claim is being made that a formula application orientation causes these students to use more strategies or if students using more strategies are more likely to hold a formula application orientation. However, the possible correlation may provide further insight.

Additionally, the students coded showing no evidence of the formula application orientation did not code for any instances of the new directions strategy and only one of the eight students coded for instances of a functional relationship strategy. The clear connection between formula application and the strategies of algebraic representation and functional relationships may explain a piece of this separation. However, students using the new directions strategy to redirect their thinking would not appear to have a clear connection with formula application. Combining the willingness to use multiple strategies with the utility of formulas and procedures appears to give the students hold this orientation an advantage over those students that did not.

Placing the increase in different strategy usage next to the observations about the success of students using the formula application orientation, it suggests this orientation may be among most important for students in College Algebra. Only two of the 11 students holding a formula application orientation did not advance in the STEM sequence by earning a D or F in the course. The course format and assessment methods may be a driver in this connection, and a differently formatted College Algebra course may produce different results with respect to course grades. The instructor will need to balance the course set-up with the number of students leaving STEM sequences due to “uninspiring” course work being a leading cause of students leaving STEM education (The President’s Council of Advisors on Science and Technology, 2012). Though for these students not enrolled in a traditional course set-up, meeting in lecture only 80 minutes per week, the more conventional orientation of formula application, valuing equations, formulas, and procedures, as the primary problem-solving structure, appears to offer the greatest utility in increasing course success.

5.2.2 Big-Picture Focus with Approximation and Separation of Parts

Big-picture focus students exhibited two specific pathways among the six students. Five of the six students coding for the orientation also coded for the approximation strategy. Of those five four connected it to a unit attention technique. Further only four of the 19 interview participants coded for instances of the separation of parts strategy and three of them held a big-picture focus orientation.

These students perform a particular path of steps. First, students explain they look at the whole problem, or as F6238 explained, “I accumulate all of the things I know,” and then she identifies “What it’s asking me for.” The approximation strategy then follows for all but one of these students. The approximation strategy shows the student attempting to “zoom in” on the answer. Starting from a broad viewpoint with many possible answers the student finds information and begins to try and locate a solution pathway. The challenge students face involves finding a correct pathway. The broad view leaves them with incomplete information and too many possible pathways to examine. Though the approximation strategy helps some of the students to find the way through the problem, only two of those five students were able to use the strategy to find a productive solution path.

In contrast, each of the three students coding for the separation of parts strategy and big-picture focus solved their interview problem correctly. This strategy has the student separate the problem in a way that will be more easily accessible for the students. Student F6141 explains her MPS process,

> “Well, I try to read the question all the way through, maybe a few times before, and then I take the information and I take the information out. And then I try to read through it again and see, decide the process and put it in whatever order I need to put it in to get an answer.”

She goes on to say that she works the problem, “piece-by-piece.” That she needs to see only smaller parts. So, her structure examines the entire problem, but then as a strategy she breaks the problem into smaller pieces. She explains for the Book Stacks problem, “Ok, so I tried to look at the numbers of, the numbers to make it equal 14 and be nine books. So, I did the literature books first.” This breaking apart process helped her after examining the broader problem constraints.

Big-picture focus can be a productive orientation when paired with the certain strategies. Interview participants became overly concerned with the broader picture of the problem if they did not pare down the problem with a strategy in some way. This direction can be important for instructors to consider when encouraging students to “step back” in a problem, as it appears the student will later need to “zoom back in” in their approach. Notably, no connection between the profiles of the big-picture focus students appeared in the demographic information. Each of the three non-passing students finished high school with a different math class. The same occurred for the passing students. Two of the six students holding the orientation identified as Hispanic and one of each passed the course. Two of students graduated from high school at least five years before enrolling in the course and one of the two passed College Algebra. Though the research did not include the students IQ the lack of connection between orientation and specific profiles suggests the correctness of Mandler (1989). Discovering clear connections between student MPS and conventional metrics proves difficult.

5.2.3 Reexamination Orientation with Identification Strategies and Look Back Techniques

Students using the reexamination orientation frequently look back at previous steps in their MPS. The importance of a correct method appears in their MPS structure. These students then work through a problem using ideas and materials obtained previously in the problem. The success of these students in the course, with six of seven passing, indicates some value to this orientation. By looking back at previous work in the problem the student engages in a metacognitive process. In thinking about their methods a second time the student appears to better understand their previous steps. These students establish a structure where each piece completed must be reexamined based on new information. Student M105 explains, “I'm still kind of thinking about previous things. Just constantly thinking about all of it at the same time.” By doing so later in the interview he identifies a mistake in the Fun Golf problem, telling the researcher, “So, somewhere I have in my problem something wrong. Since my *X*s cancel out.” Another opportunity for the student to work with a part of a problem or to understand the operations completed provides that additional learning opportunity to use later in the problem or in another problem.

Certain strategies and techniques appeared most often among these seven students. By identifying models and formulas the students recheck their understanding of the problem. The techniques looking back at previous parts of the problem, such as reasonability checks and unit attention appear among all seven students holding a reexamination orientation. The causality of reexamination leading to the use of the techniques is unclear. It may be that using the techniques caused the student to appear to hold the orientation. Since three of the students coding for reexamination did not do so in response to the initial interviewer questions, it seems possible that the techniques may cause the orientation as much as the orientation causes the techniques. This suggests teaching students the techniques of paying attention to the units or checking for reasonability may be equally as effective as teaching the students the importance of verifying steps or fitting a solution to the problem statement.

5.2.4 Profiles of Better Performing College Algebra Students

Interview participants coded with the two primary common orientations, *formula application* and *reexamination,* performed better in the course. These two orientations may indicate a profile for higher performing students in this College Algebra course or courses with a similar emphasis and focus. Further, there seems to be a strong relationship between grades and the use of multiple strategies.

Eleven of the 13 students earning an A, B, or C, in the course (passing students) coded for at least one of formula application or reexamination orientations. Among the five students receiving a D, F, or W in the course (failing students) only two coded for either of those orientations. Among strategies the number of different strategies correlated between the two groups as well. Passing students used all 11 strategies among them and an average of 4.2 different strategies per student. Failing students included an average of only 2.4 different strategies per student. Using the Wilcoxon Rank-Sum test to compare the two groups shows a statistically significant difference at the $p<.05$ level. Table 5-2 shows the information for number of different strategies. The importance of multiple strategies in a student's MPS approach appears evident. The utility of being flexible within one's approach regardless of orientation appears to clearly connect to success in the course. Santos-Trigo (1998) showed the importance for a problem-solving course to "discuss the importance of using diverse types of strategies" (p.645). The better performance among students using more strategies confirms this earlier research finding.

Table 5-2 Summary of Course Grades and Number of Strategies

Student Course Grades	**Number of Interview Participants**	**Number of Different Strategies Coded**	
		Among Entire Group	Average per Student
A, B, or C	13	11	4.2
D, F, or W	5	10	2.4
Total	18*	11 Possible Strategies	

*One student received an incomplete

5.3 Affective Factors Influence on College Algebra Student Problem Solving

Affective factors, such as beliefs and attitudes toward mathematics, appear to be related to the orientations held by students. Further, the ability of a student to express his or her problem-solving methods relates to their apprehension about mathematics. Also, the nature of the interview may prevent a full understanding of each student's MPS.

5.3.1 Positive Affect and Formula Application

The relationship between positive affect codes and the formula application orientation suggests the direction of the course instruction must be careful to not influence students away from STEM fields. Given the success of students holding the formula application orientation in the course the lack of positive affect codes among those students suggests fewer positive feelings toward mathematics and mathematics courses. Those students coding for only formula application among orientations exhibited the fewest positive affect instances while the students coding for the no instances of formula application showed the most instances of positive beliefs and attitudes about mathematics. Though the formula driven structure of many students' MPS appears to relate well to course success it does not appear to coincide with a positive outlook toward mathematics.

The procedural nature of the formula application orientation and mathematical notation driving the problems may not elicit expressions of positive feelings about mathematics. For example, student F304 described herself as being good problem solver, explaining, "I have to be able to [justify]." She earned an A in the course, but when presented with the Book Stacks problem she tells the researcher,

> I've actually always had – not a problem I guess, but an issue with these questions, because I never – like I remember in one instance, in particular – or multiple, but the same situation, where we would have a problem like this, and I would have a friend who would be able to make the math – to do this without having to do all of this [points to paper] but it

> never really clicked, I suppose. And so, I always use this longer method and more pictures again.

She indicates her belief that the problem should have been solved with a system of equations and displays a negative belief about her ability to do so. However, she solved the problem using an alternate approach while still holding her formula application orientation. For a student less able to use additional orientations or strategies this may have been a greater obstacle. This further points to a need for instructors to emphasize varied approaches in MPS.

Positive interview participant attitudes and beliefs appeared highest among those not coded for formula application. Student F318 explained that she “likes to check [her work] as often as I can.” She told the researcher, “I'd like to think I'm really good at defending my solutions to problems.” Her confidence also appeared through her solution to the new problem in the interview. She spoke the most of any student interviewed in Fall of 2015 during her solution, and included no comments about confusion or being unsure of her solution. F318 coded for no instances of formula application. In contrast, student F303 coded for formula application and reexamination. She explained to the interviewer, “But if it’s really hard, then I can’t [justify]” and throughout the solving of the Sonar problem she seemed unsure of her next steps. F303 told the researcher, “Well, okay, so [pause] I’m trying to think” and asked the interviewer to explain multiple points of the problem. For example, F303 asked, “Wouldn’t you have to convert into seconds because it says, “per second”? So, wouldn’t you multiply?” and “So, wouldn’t that be the answer for how far the ship is from the fish?” These shaky indications and the lack of positive affect instances identify her struggles despite coding for the two orientations shared among most of the successful students.

5.3.2 Negative Affect and Lesser Student Explanations

Students displaying negative affect instances took significantly less time in explaining solutions—leading to shorter interviews. These students also coded for fewer instances of all orientations, strategies, and techniques throughout the interview. It appears more negative attitudes and beliefs among a student leads to the student being less able or willing to explain their MPS practices. This may make the negative affect students harder to identify. This may also explain the limited number of codes for students F6141 and F6269 who received an F and D in the course respectively. These lowered expectations appear to shorten the student explanations to the researcher and thus may indicate a reduced willingness to consider their own thinking. By lessening the metacognitive process for a student, that student may have a more difficult time recalling previously learned information or using higher order thinking skills. The fewer number of codes may only be a result shortened time in the interview, but also could indicate incomplete understandings for these students. The inability to discuss their solution matches to Santos-Trigo's finding that students should be able to consistently "communicate their ideas in written and oral forms" (1998, p.645). Interestingly, merely the belief the student cannot express their MPS orally or on paper, and the negative affective components along with it, appear to be enough to inhibit student MPS performance regardless of ability as the participants who were unable to solve the new problem in the interview coded for more instances of negative affective components.

5.3.3 Think-Aloud Protocols and Incomplete Picture of Student Problem Solving

Students in the interview are asked to "think aloud" about their MPS process and thinking. Think aloud protocols help the researcher to identify the understanding and ideas the student uses in their MPS practices. Students rarely engage in this protocol and are typically asked to work in silence (Lucas & Ball, 2005). The information obtained by similar protocols often results in a researcher's incomplete understanding. Likely, the interview participants did not disclose all components of their MPS practices. For example, the replication orientation was coded in only two student interviews, yet students in this course were often encouraged to match a new problem to a previously worked example or problem. It seems likely the students may hold this orientation, but did not verbalize this orientation as they enacted the primary orientations that emerged from the data. Other aspects of student MPS practices and orientations, strategies, and techniques, such as replication, tinkering, or separation of parts strategies, may be under-observed by the think-aloud nature of the interview.

5.4 Problem Type Influence on Strategies and Techniques

The type of problem the student completes appears to influence their use of strategies and techniques. In 1997, Jonassen noted this in his research on well-structured problems. Though the difference in problems explored here differs from those discussed by Jonassen, the results may be applicable for many problem structures. Koedinger and Nathan (2004) explored the differences in cognitive process and student performance on "story problems" compared with other problem types. The orientations held by students provide a broader structure that appears unchanged for most students, regardless of problem type. Students completing a contextual problem use a differing set of strategies than those completing a non-contextual problem. A contextual problem uses a real world setting to provide the context for the problem. The problem may ask the student to create certain mathematical representations or to use typical mathematical notations, but the question is based on the setting rather than the specific mathematics. For example, a problem may ask for a function, but will not use *"f(x)"* notation in the problem statement or require the student to understand mathematical notation to read the problem. An abstract problem relies on the mathematical notation to define the question.

Students are asked to answer questions about given equations, points, functions, or mathematical constructs. Mathematical notation is inherently required to understand the problem.

5.4.1 Strategies and Techniques used in Contextual Problems

In contextual problems, the interview participants preferred a specific group of strategies and techniques in the problem-solving methods, particularly if solving the Fun Golf problem. Other strategies appeared nearly absent among contextual problems. First, students only used the systematic pattern checking technique in contextual problems. Identification of model strategies displayed in 12 of the student interviews while discussing a contextual problem. Only one student exhibited this strategy during the interview, but showed only while discussing a non-contextual problem. Though relatively few students used pattern thinking, only five, the uses of the strategy occurred only while discussing a contextual problem.

Each of the 19 interviews used a contextual problem as the problem the student worked without having seen it before. The strategies used during this period included graphing for only one student. Further, only two students used function notation among just six students using algebraic relationships. Students would indicate a desire to use more algebraic strategies but when presented a contextual problem shied away from those strategies. For example, one of the six students using algebraic representations, M105, stated, “I figured that would be better for the interview than just going in and guessing.” This indicates that he might not use algebraic representations in his regular problem solving. When teaching MPS and mathematics, the writers of the problem need to consider the nature of the problem and the responses it will generate to reach the targeted goal for the instruction. Rather a contextual problem or not, the specifics of a problem may yield certain responses more than others. Further, the contextual problems or applications questions are typically the ones the instructor builds to by showing a series of worked examples. These algebraic- and notation-based strategies then fail to appear when students solve contextual problems out of the context of class. The format of a question appears to have a substantial impact on the types of problem-solving strategies and techniques employed by the students. These repeated strategies may reflect the different types of knowledge needed for solving contextual problems. Similar to contextual problems, research shows story problems require unique verbal comprehension skills not needed for other types of problems (Koedinger & Nathan, 2004).

5.4.1.1 Methods Unique to the Fun Golf Problem

Ten of the 19 interview participants completed the Fun Golf problem during the interview. Within the nine working a different problem in the interview, five discussed the Fun Golf problem based upon their work on the MPST. Among these 15 students that either worked on the Fun Golf problem or discussed it in the interview, 11 included systematic pattern checking in their methods. Among the four students not using that technique, two used an equation based approach and strategy, one used only direct calculations, and the last student used an area model and equations. The clear preference for a certain strategy shows clearly in the student work. Comparatively, only the students working Fun Golf problem indicated this clear preference among contextual problems. The Cross-Country Race problem that would appear to privilege a modeling strategy and distance model technique included just three of seven students using the expected privileged strategy. The only other observed instances of the systematic pattern checking arose while students worked the Ken’s Garden problem. Despite the need to check 10 or more values in these two problems students used this technique in the Fun Golf and Ken’s Garden problem, but completely omitted it from the Book Stacks problem only that required checking nine values at most. Also of note, students showed no instances of systematic pattern checking on the Cross-Country Race or Myla's Pool Problems despite their relatively low quantities. The presence of a rate (speed or flow rate in these problems) seems to eliminate the technique for many students. Problem writers must consider the difficulty created by the quantities in the problem. If the writer wants students to use more algebraic approaches the quantities either need to be small enough the student can complete the calculations easily or so large that the systematic pattern checking technique will take too long. If the writer wants the student to use a checking or logical progression approach they need to fit the problem into a small window where algebraic procedures provide no obvious short path to a solution nor are the numbers so large that checking the values looks impossible.

5.4.2 Graphing Strategies used in Non-Contextual Problems

The graphing strategy occurred with high frequency in non-contextual problems. For these problems, the students answered questions pertaining to functions, graphs and their intersections, points, among other mathematical constructs. Only one student engaged in a graphing strategy in a contextual problem compared with six on abstract problems. The *intersecting graphs, avoiding intersections* and *extreme values* problems accounted for all apparent instances of a graphing strategy. These six students account for all but one use of the word "graph" while discussing a problem. Student M106 indicated he would make "rough graph" as part of his MPS, but he showed no evidence in the interview, and his work on the Intersecting Graphs problem shows no graphing indications. There were relatively fewer non-contextual problems and this may have contributed to a limited use of graphing strategies. Only five of the 15 problems would be considered non-contextual, and the 11 interview participants during fall of 2015 saw only nine total non-contextual problems in an MPST. The eight students interviewed in spring of 2016 saw 16 non-contextual problems, but instances of the graphing strategy still appeared less often than modeling or equation based strategies. In this course and for these students, it may be that the graphing strategy will only be used as strategy if the problem leads toward the strategy in a more deliberate manner. Dawkins and Epperson (2014) found that calculus students do not employ a graphing strategy in their MPS even when it may be the most insightful method for arriving at a resolution to a problem situation.

5.5 Limitations of the Study

One issue with the grounded theory approach relates to its reliance on the oral communication and written work of the interview participants. Though portions of the interview intend to guide the student into a situation where they will display their MPS methods, the coding relies on what the student actually says. Charmaz points to this as a limitation of grounded theory in general because of its reliance “on participants' prior writing skills and practices” (2006, p.36). The researcher may also have inadvertently guided an interview participant to certain responses or methods. To elicit verbal interactions from the student, the interviewer needed to ask the student to further discuss certain aspects. In doing so, the student may use methods different than those used unprompted. To attempt to counter for these possible inadvertent guides, the initial question asked in the interview received particular attention for the coding of orientations.

Some students struggle with certain aspects of the MPS done in the interview. Student F311 struggled with English; thus, her interview likely yields an incomplete understanding of her MPS. Another student, M310, struggled to understand the Air Travel problem he worked during the interview. The interplay between that problem and his MPS possibly made observations of his methods and approach problematic. Similarly, two students completed the Book Stacks problem during the interview. Both of these students completed the problem in less than five minutes leading to a shorter overall interview and creating fewer opportunities for the researcher to observe the student's methods for coding. Further, the nature of analyzing the transcript and recording of the workspace only creates a challenge to understanding the underlying student thinking when the student works quietly. This challenge becomes even greater when the student only thinks quietly. Though the interviewer noted things during the interview while interacting with the student, incorporating these observations into the coding process provides minimal additional insight.

The apparent positive effects of certain techniques may stem in part from the course format. Notably, Green (2016) studied the course materials for this College Algebra course and found only 15% of the homework problems used in the course would elicit behaviors associated with sense-making, representing/connecting, justifying, reviewing, or challenge. Moreover, the majority of the problems in the course relied on rote memorization or "straightforward instructions." The abundance of these problems in the course may lead to a privileging of the formula application orientation. Thus, the students coding for the orientation more frequently would likely perform well in the course. Additionally, Green's research showed the 671 problems included 1167 parts (p. 35). The number of problems formatted into parts may help explain the success of the three students coding for a linear progression orientation. This course format may also help to explain the lack of success seen among big-picture focus students.

The distinct nature of the interview to other student experiences in the course poses another challenge for this research. The design of the MPSI problems, and thus those discussed in the interview, did not necessarily mimic the course format or course materials. The wording or format of the problems in the course follows different patterns or usage of words and questions than the problems presented on an MPST. Also, the course relies almost exclusively on a computer-based program rather than the paper and pencil explanations asked for in the MPSI problems. An evaluation of the homework problems used in the course at this particular university found 74% of problems assigned used a fill-in-the-blank format and none of the MPSI problems use a similar format (Green, 2016). The relative lack of participation in the interview process among the students made targeting the interviews to any particular group impossible. Less than 10% of students invited to interview responded to invitations. Similar response rates existed in both semesters.

5.6 Future Directions

Further investigation of the secondary orientations may be warranted. In addition, a deep look at the effects of course structure on the orientations arising as well as those aligning with student success in the course. All students in this study attended College Algebra sections with the same course format of once weekly lectures and twice weekly computer lab meetings. The instructors also used the same course materials and the students took the same assessments. Differences in course format and curricular emphasis might yield different orientations or provide a meaningful observation that student orientations persist across course differences.

5.7 Summary of Conclusions

Overall relatively few orientations emerged from the interview participant data during the coding process. These orientations include formula application, reexamination, and big-picture focus, and they align with behaviors students in College Algebra and high-school mathematics courses are typically told to do (Bransford, 1984). Research proposed similar methods for Algebra instruction dating back to 1984 (Glaser). Thus, the theory offers a recommendation for implementation of similar methods to those used previously. Emphasizing reexamination, or a checking procedure, may alter student orientations and push them toward more productive methods. The underlying strategies and techniques offer little evidence of a trend in the success of the students. These pieces of student problem appear to rely on the problem specific components for success or failure. Certain problems may require certain techniques or strategies for better performance, but in general no evidence for particular strategies and techniques improving MPS performance in general emerged from the 19 interviews.

Within the three primary orientations, strategies and techniques appeared more frequently than others. The challenge for teachers using the information will be to not overly bias students into particular approaches. The students in the study using only one orientation showed a more negative outlook toward mathematics particularly if that orientation was formula application. This creates a trap for teachers, privileging this orientation appears to improve student performance in the course but hurt student beliefs about mathematics. One orientation, big-picture focus, appears to hurt student performance in the course relative to other orientations. The implications of this prove difficult to pin down. The course format may unintentionally create a bias against these students, but also students may “get lost” in the process of MPS. The approach mentioned by Glaser (1984) “decomposing a problem and recombining elements” bears similarity to this orientation, and the reasons for why students using this orientation struggle are not obvious.

The five less common orientations coded in the interviews provide a view of only a few students MPS. The apparent success of students using the linear progression orientation provides a point of interest. Again, this may relate to the format of the course. The large number of homework problems in parts (Green, 2016) possibly provides a course structure where the students are more successful due to this orientation. Other orientations may seem problematic to separate from strategies, but the student used the method as their “overall structure.” This further complicates the implications of holding these orientations.

The MPS techniques, strategies, and orientations used by College Algebra students varied across the 19 interview participants. Identification strategies and computational techniques appeared most often among the interview participants, but the strategies and techniques spread across many different methods. In characterizing College Algebra students’ MPS, this work suggests three primary common orientations, *formula application, big-picture focus,* and *reexamination.* These most prevalent approaches relate to student performance in this type of College Algebra course. Formula application and reexamination appears to connect with improved course performance while big-picture focus seems unhelpful for student performance in this course. Further investigations should follow that profile affective factors coupled with these primary orientations, strategies, and techniques, and relate them to course outcomes for various settings of a College Algebra course as well as work on linking these profiles to persistence in STEM.

Appendix A

Sample Interview Invitation

Dear Student,

Thank you for participating in my research study in your class at the beginning of the semester. As described in the consent document you signed, the goal of the project is, “to explore the problem solving practices and understanding of problem solving among university math students.” As such, the researchers plan to use feedback from students that took the test to evaluate the accuracy of the instrument.

To that end, you are being invited to a brief interview, as described in the consent. For your participation and as compensation for your time **you will be paid $20**, if you participate in the interview. You will discuss with me your responses and examine them together to better understand the nature of your responses. The students chosen to interview were chosen with the intent of composing a representative sample of all students in studied courses.

The interview should last between 30 minutes and 1 hour, and does not require any preparation on your part. The interview will be recorded as described in the consent to allow the researchers to review the interactions. Any publication or sharing of the video will protect your identity through methods including, but not limited to, transcription or reenactment.

To set up a time to complete your interview, please contact me by email at robertcc@uta.edu or visiting me at my office. If you have any questions I will be happy to discuss those with you at the same contact information. Please email me or visit me in my office to set up a time so I can be sure to not schedule multiple people at the same time. Please let me know a time that would work for you. If you need to meet later in the evening I can likely work that out as well.

Thank you again for participating in my research,

Appendix B

Script Framework for Student Interview

Interview Script

Describe the usual process or steps you go through when you solve a (challenging) mathematics problem.

Let's take a look at your work from the Problem Solving Instrument…

- Researcher will have identified interesting parts of student work and ask: Explain what you were thinking when you (e.g. drew this diagram and labeled the quantities)…?
- How well do you think your problem solving method fit within the choices given? Explain.
- When solving this problem (researcher shows a problem from the student's work on the Problem Solving Instrument) were you:
    - Trying to make sense of the problem and how things fit together
    - Representing/Connecting
    - Challenging/Difficult
    - Reviewing/Evaluating
    - Justifying

For each "yes" ask them to explain.

Let's take a look at another problem…

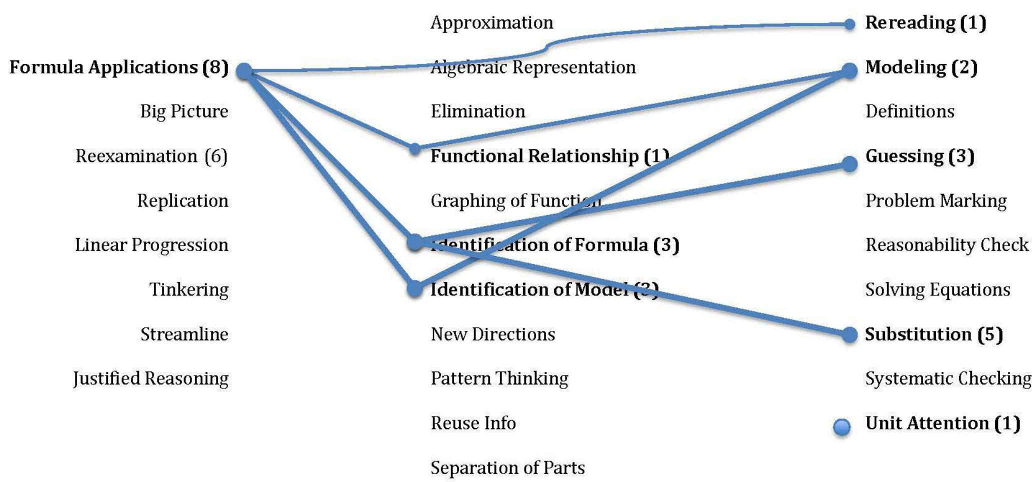
Researcher says: Take a few minutes to read over the problem and then begin working it.

- As you work this problem please explain your thinking aloud.
- I may ask you to clarify at some points (what do you mean by… what are you thinking when you say…

Appendix C

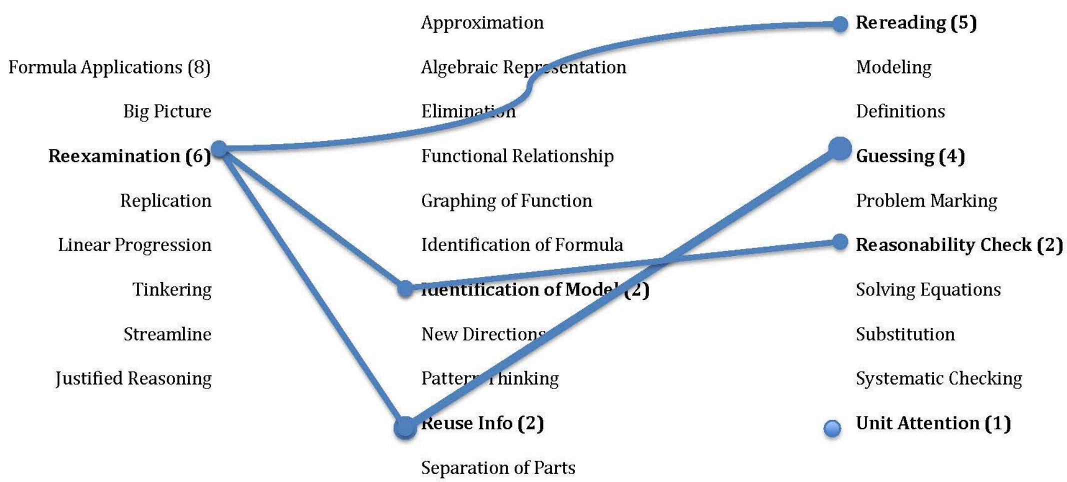
All Node Diagrams

F209



Formula Applications (8)
Big Picture
Reexamination (6)
Replication
Linear Progression
Tinkering
Streamline
Justified Reasoning
Approximation
Algebraic Representation
Elimination
Functional Relationship (1)
Graphing of Function
Identification of Formula (3)
Identification of Model (2)
New Directions
Pattern Thinking
Reuse Info
Separation of Parts
Rereading (1)
Modeling (2)
Definitions
Guessing (3)
Problem Marking
Reasonability Check
Solving Equations
Substitution (5)
Systematic Checking
Unit Attention (1)

F209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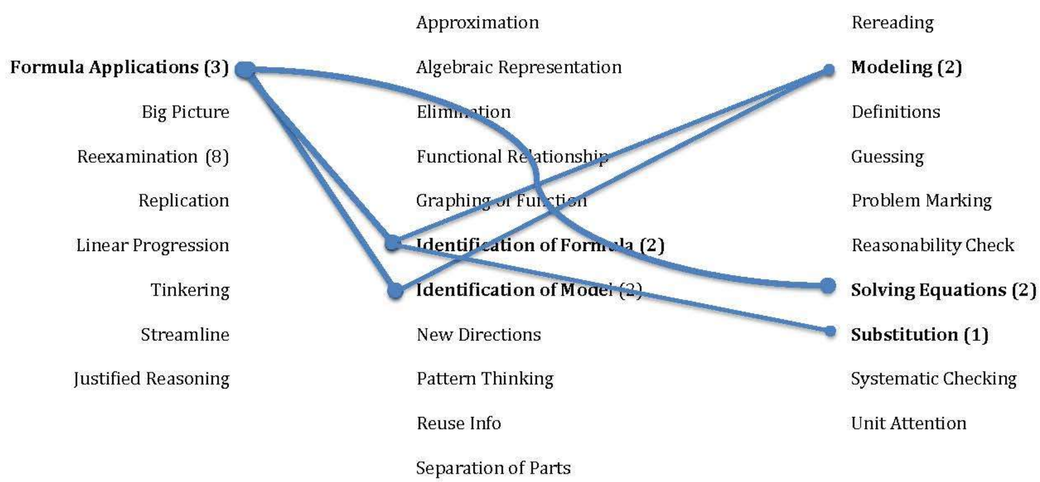
Algebraic Representation
Modeling
Big Picture
Elimination
Definitions
Reexamination (6)
Functional Relationship
Guessing (4)
Replication
Graphing of Function
Problem Marking
Linear Progression
Identification of Formula
Reasonability Check (2)
Tinkering
Identification of Model (2)
Solving Equations
Streamline
New Directions
Substitution
Justified Reasoning
Pattern Thinking
Systematic Checking
Reuse Info (2)
Unit Attention (1)
Separation of Parts

F303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Reexamination (8)
Replication
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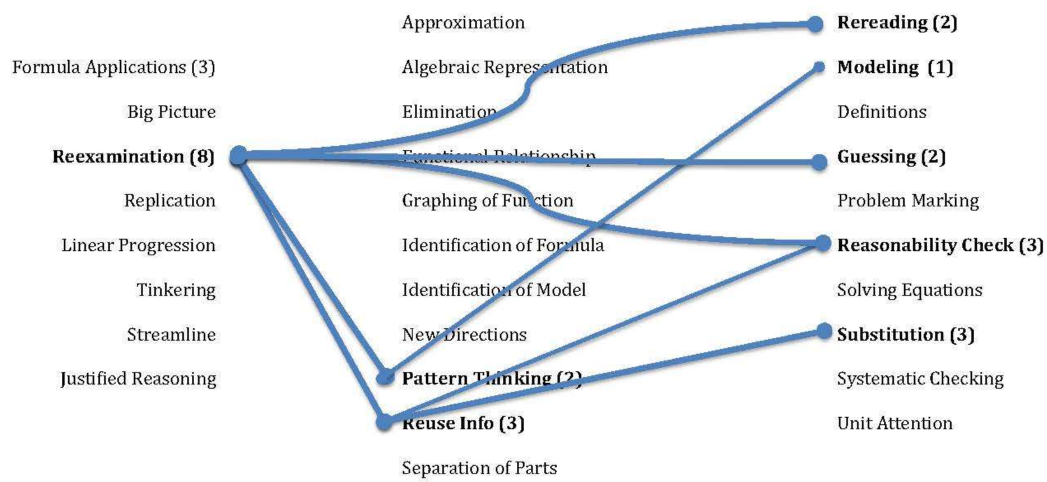
Tinkering
Streamline
Justified Reasoning
Approximation
Algebraic Representation
Elimination
Functional Relationship
Graphing of Function
Identification of Formula (2)
Identification of Model (2)
New Directions
Pattern Thinking
Reuse Info
Separation of Parts
Rereading
Modeling (2)
Definitions
Guessing
Problem Marking
Reasonability Check
Solving Equations (2)
Substitution (1)
Systematic Checking
Unit Attention

F303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Reexamination (8)
Replication
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Tinkering
Streamline
Justified Reasoning
Approximation
Algebraic Representation
Elimination
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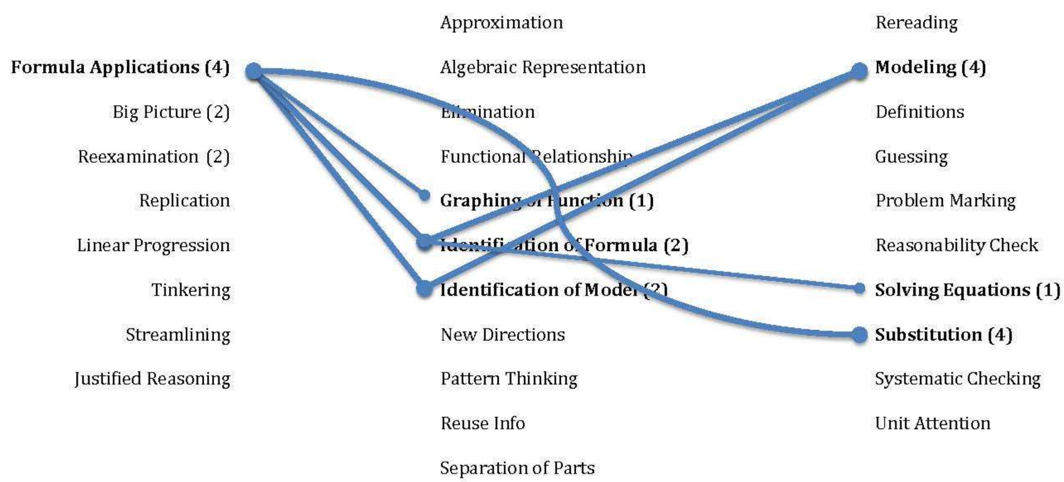
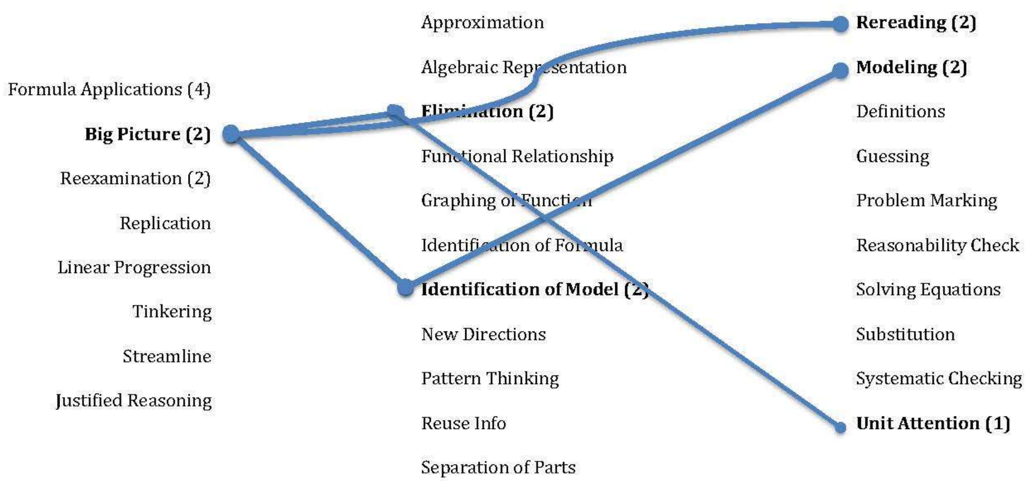
Graphing of Function
Identification of Formula
Identification of Model
New Directions
Pattern Thinking (2)
Reuse Info (3)
Separation of Parts
Rereading (2)
Modeling (1)
Definitions
Guessing (2)
Problem Marking
Reasonability Check (3)
Solving Equations
Substitution (3)
Systematic Checking
Unit Attention

F304



Formula Applications (4)
Big Picture (2)
Reexamination (2)
Replication
Linear Progression
Tinkering
Streamlining
Justified Reasoning
Approximation
Algebraic Representation
Elimination
Functional Relationship
Graphing of Function (1)
Identification of Formula (2)
Identification of Model (2)
New Directions
Pattern Thinking
Reuse Info
Separation of Parts
Rereading
Modeling (4)
Definitions
Guessing
Problem Marking
Reasonability Check
Solving Equations (1)
Substitution (4)
Systematic Checking
Unit Attention

F304


Formula Applications (4)
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Reexamination (2)
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Algebraic Representation
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Functional Relationship
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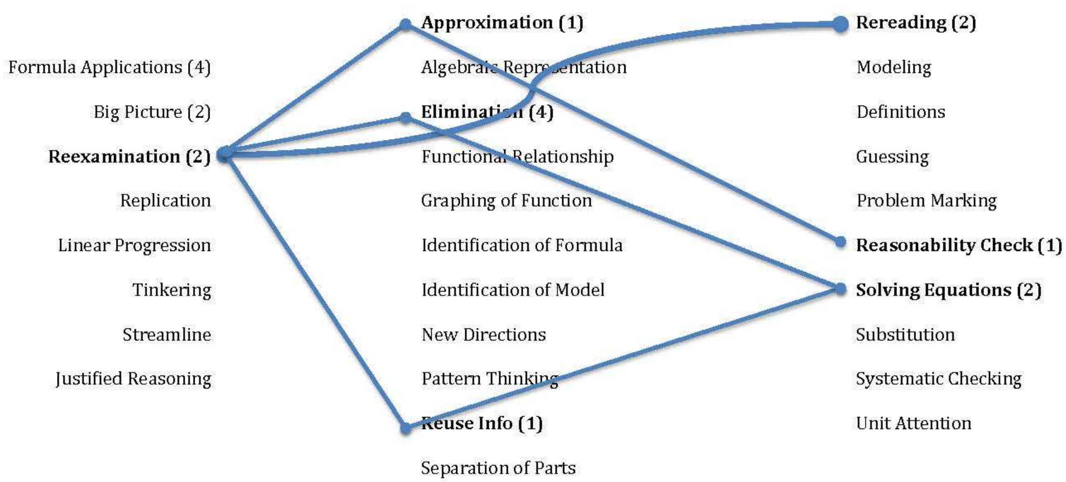
Solving Equations
Substitution
Systematic Checking
Unit Attention (1)

F304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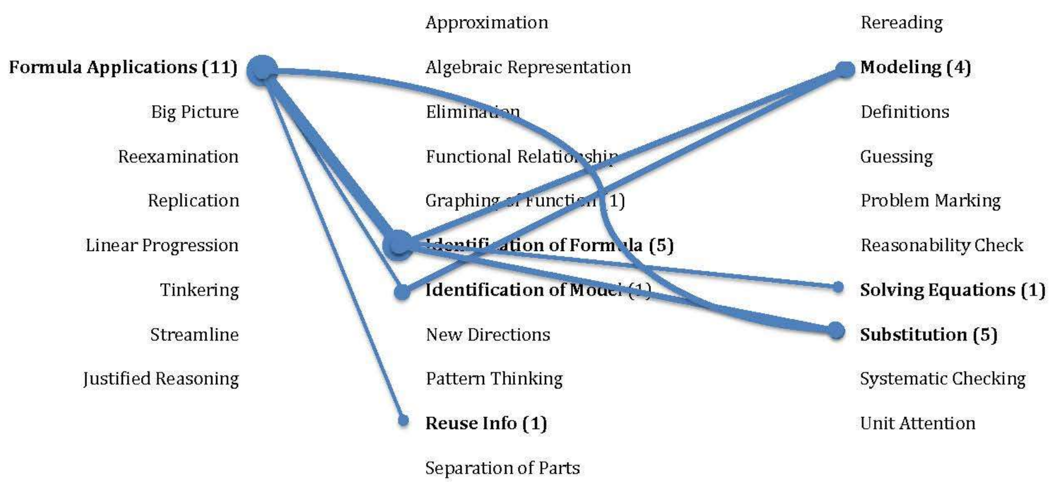
Replication
Linear Progression
Tinkering
Streamline
Justified Reasoning
Approximation (1)
Algebraic Representation
Elimination (4)
Functional Relationship
Graphing of Function
Identification of Formula
Identification of Model
New Directions
Pattern Thinking
Reuse Info (1)
Separation of Parts
Rereading (2)
Modeling
Definitions
Guessing
Problem Marking
Reasonability Check (1)
Solving Equations (2)
Substitution
Systematic Checking
Unit Attention

F311



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Reexamination
Replication
Linear Progression
Tinkering
Streamline
Justified Reasoning
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Algebraic Representation
Elimination
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Graphing of Function (1)
Identification of Formula (5)
Identification of Model (1)
New Directions
Pattern Thinking
Reuse Info (1)
Separation of Parts
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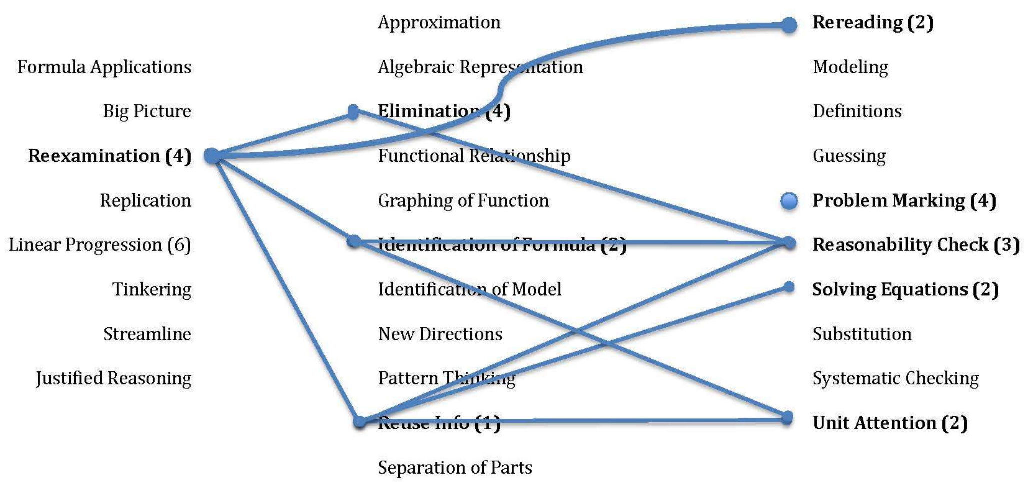
Guessing
Problem Marking
Reasonability Check
Solving Equations (1)
Substitution (5)
Systematic Checking
Unit Attention

F315



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Functional Relationship
Graphing of Function
Identification of Formula (2)
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Pattern Thinking
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Separation of Parts
Formula Applications
Big Picture
Reexamination (4)
Replication
Linear Progression (6)
Tinkering
Streamline
Justified Reasoning
Rereading (2)
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Definitions
Guessing
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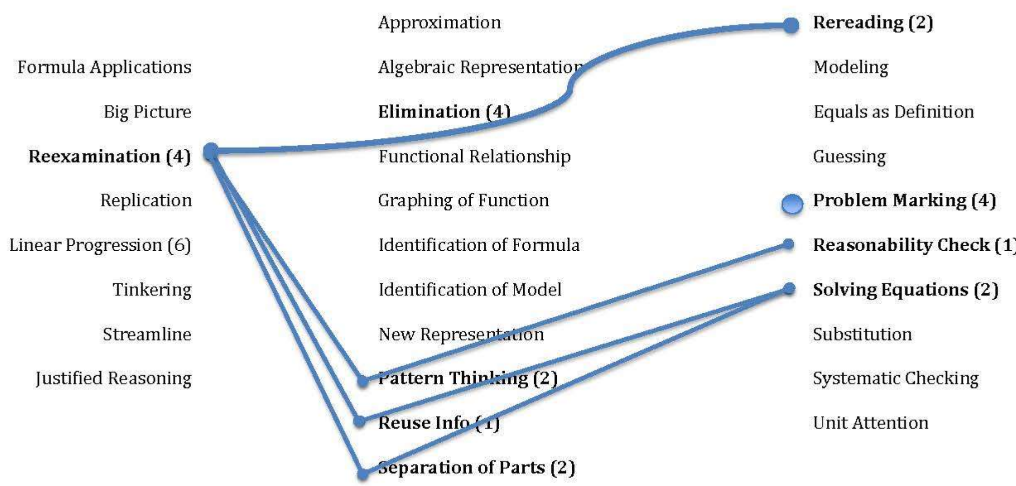
Solving Equations (2)
Substitution
Systematic Checking
Unit Attention (2)

F315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Reexamination (4)
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Streamline
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Functional Relationship
Graphing of Function
Identification of Formula
Identification of Model
New Representation
Pattern Thinking (2)
Reuse Info (1)
Separation of Parts (2)
Rereading (2)
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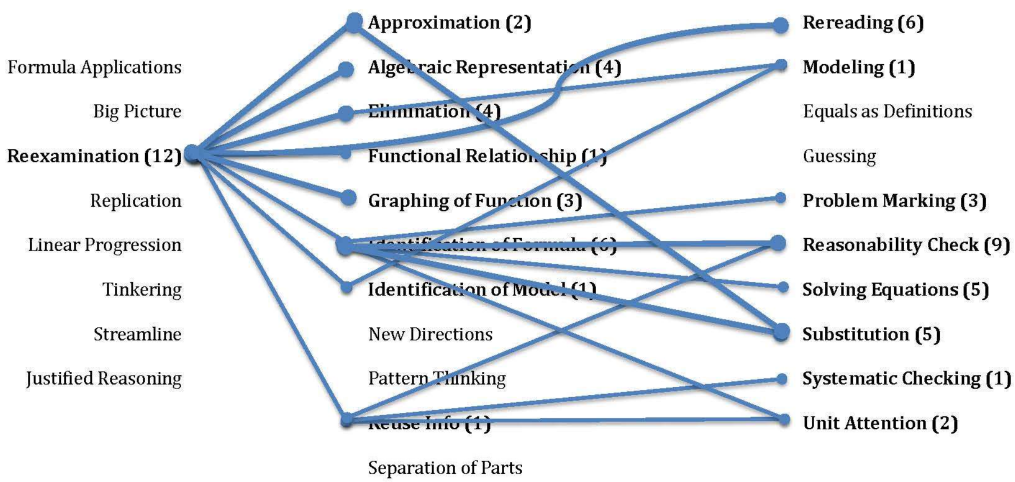
Substitution
Systematic Checking
Unit Attention

F318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Functional Relationship (1)
Graphing of Function (3)
Identification of Model (1)
New Directions
Pattern Thinking
Separation of Parts
Rereading (6)
Modeling (1)
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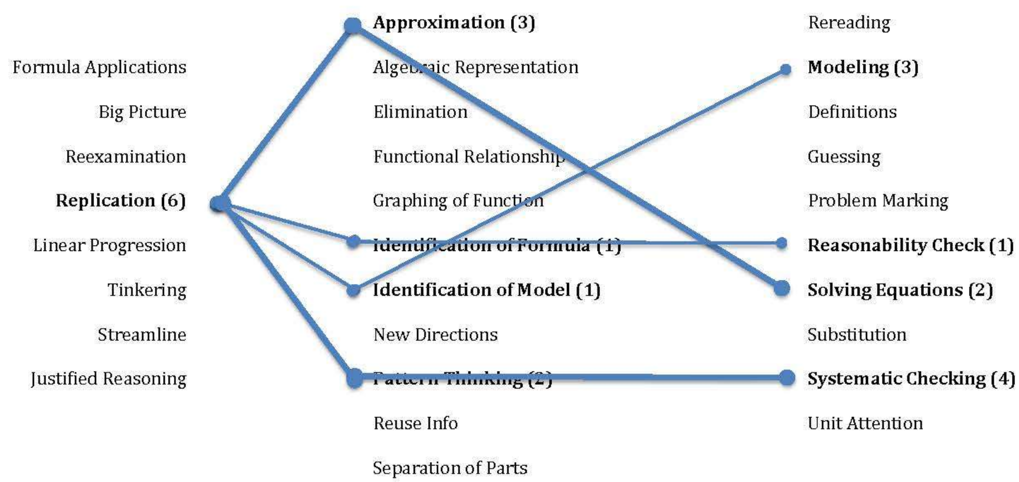
F505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Separation of Parts
Formula Applications
Big Picture
Reexamination
Replication (6)
Linear Progression
Tinkering
Streamline
Justified Reasoning
Rereading
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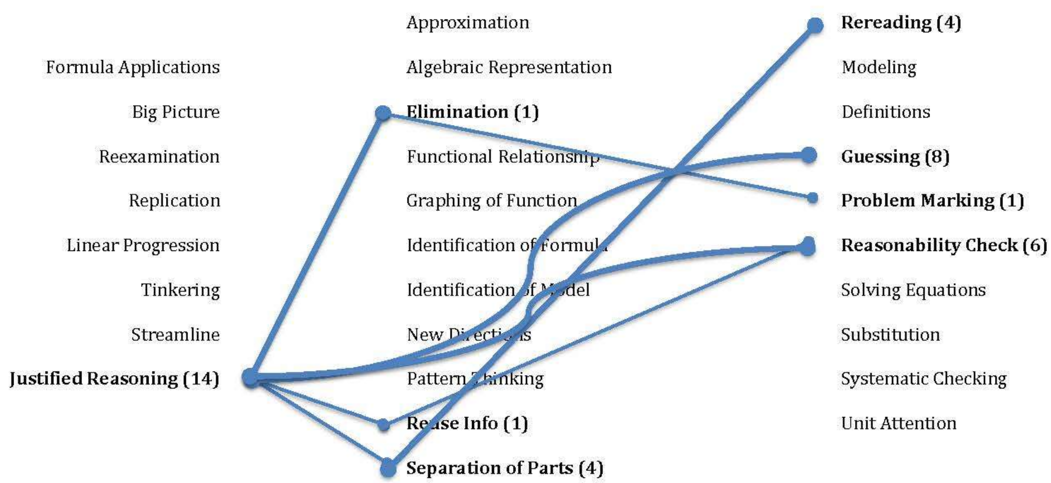
Unit Attention

F505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Replication
Linear Progression
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Graphing of Function
Identification of Formula
Identification of Model
New Directions
Pattern Thinking
Reuse Info (1)
Separation of Parts (4)
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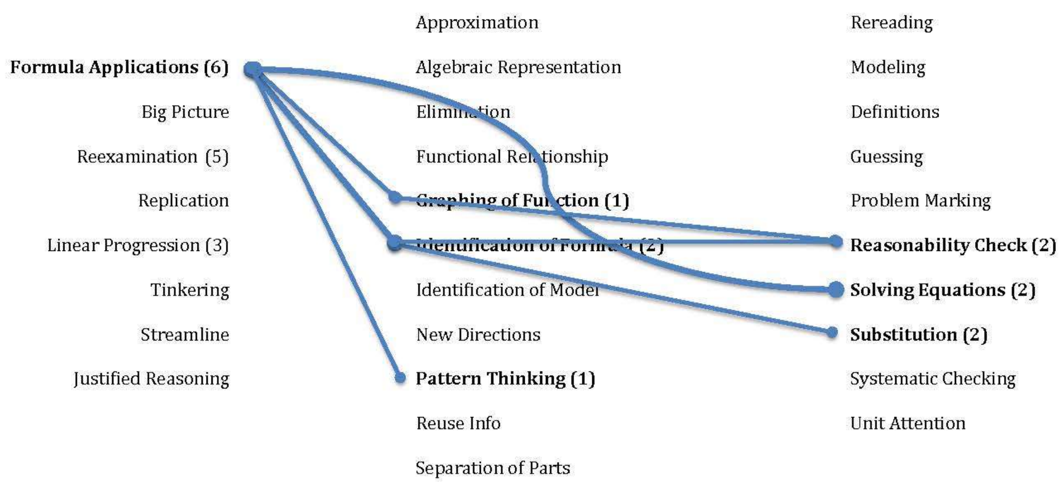
Solving Equations
Substitution
Systematic Checking
Unit Attention

F6106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Functional Relationship
Graphing of Function (1)
Identification of Formula (2)
Identification of Model
New Directions
Pattern Thinking (1)
Reuse Info
Separation of Parts
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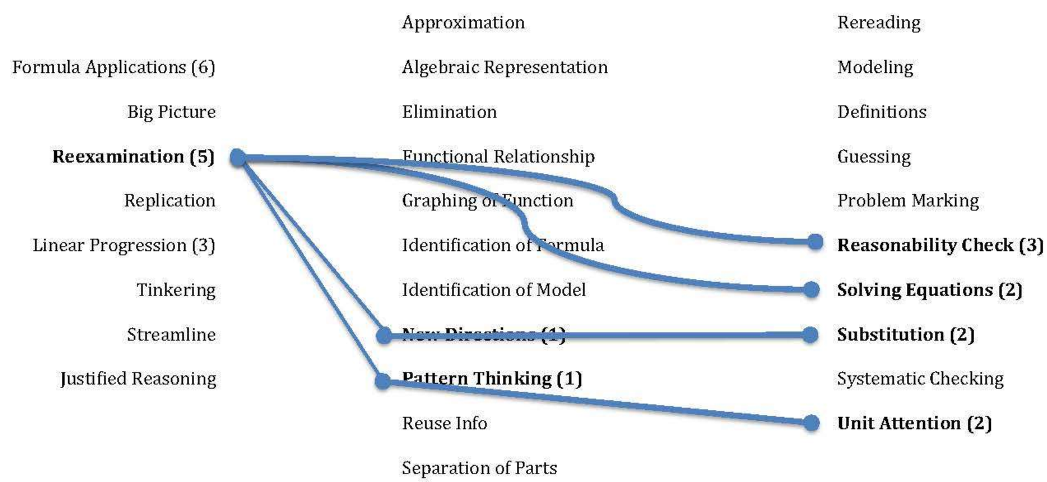
Unit Attention

F6106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Elimination
Functional Relationship
Graphing of Function
Identification of Formula
Identification of Model
New Directions (1)
Pattern Thinking (1)
Reuse Info
Separation of Parts
Rereading
Modeling
Definitions
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Systematic Checking
Unit Attention (2)

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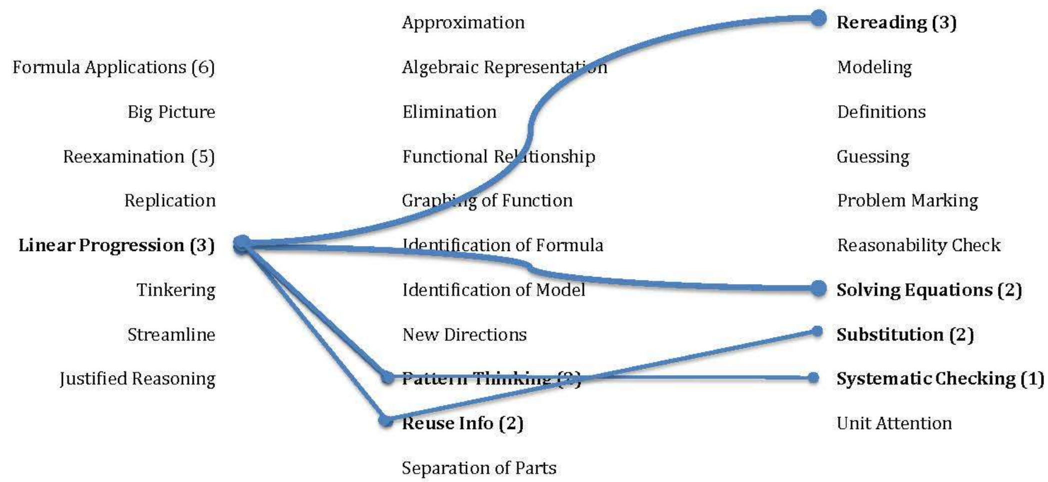

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Reexamination (5)
Replication
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Tinkering
Streamline
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Approximation
Algebraic Representation
Elimination
Functional Relationship
Graphing of Function
Identification of Formula
Identification of Model
New Directions
Pattern Thinking
Reuse Info (2)
Separation of Parts
Rereading (3)
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Definitions
Guessing
Problem Marking
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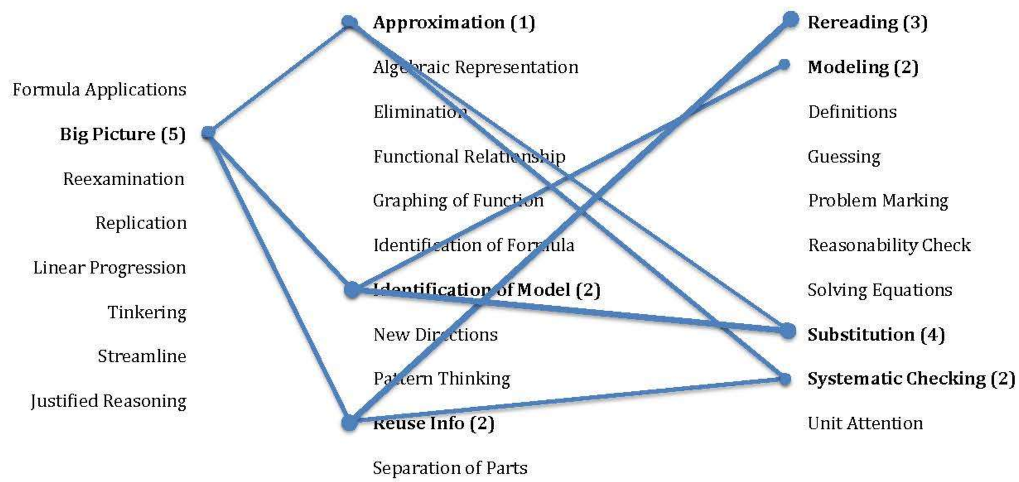
Unit Attention

F6141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Tinkering
Streamline
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Functional Relationship
Graphing of Function
Identification of Formula
Identification of Model (2)
New Directions
Pattern Thinking
Reuse Info (2)
Separation of Parts
Rereading (3)
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Definitions
Guessing
Problem Marking
Reasonability Check
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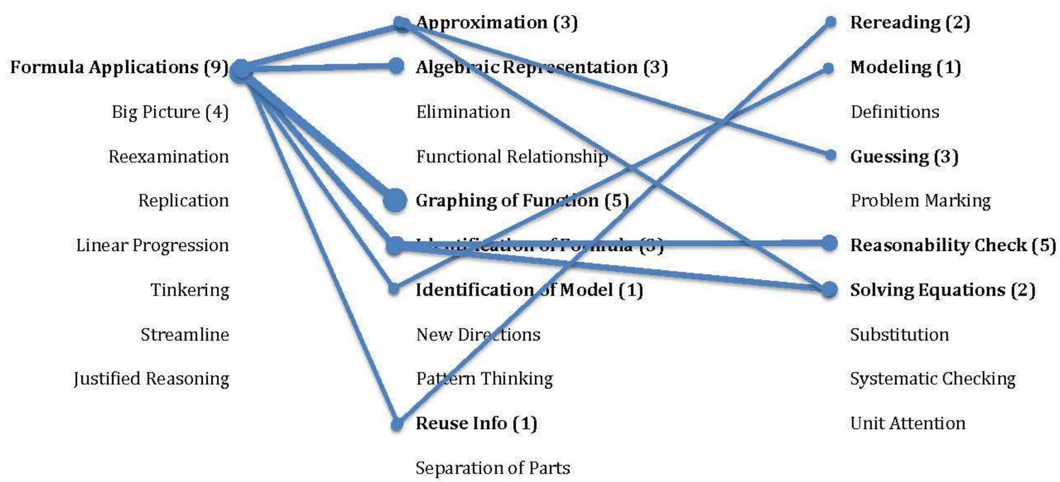
Systematic Checking (2)
Unit Attention

F6145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Replication
Linear Progression
Tinkering
Streamline
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Elimination
Functional Relationship
Graphing of Function (5)
Identification of Formula (3)
Identification of Model (1)
New Directions
Pattern Thinking
Reuse Info (1)
Separation of Parts
Rereading (2)
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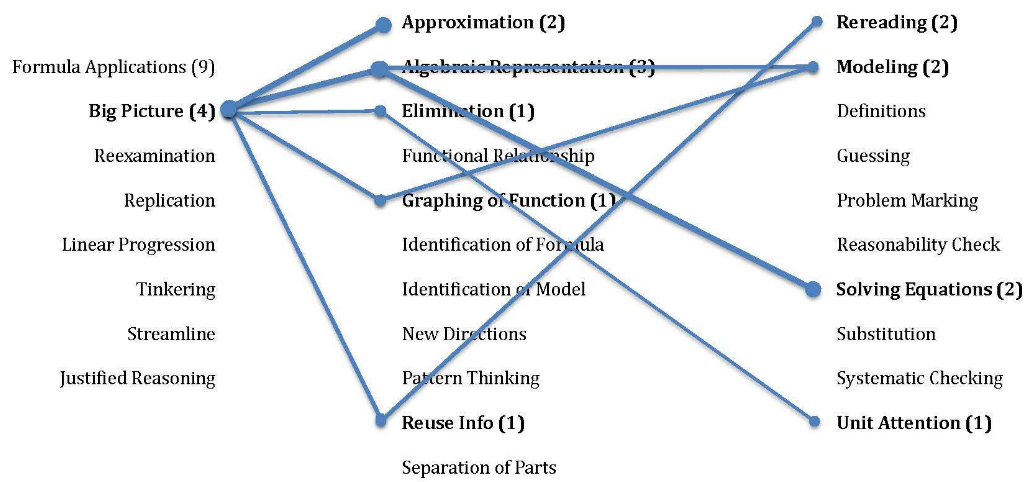
Substitution
Systematic Checking
Unit Attention

F6145



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Big Picture (4)
Reexamination
Replication
Linear Progression
Tinkering
Streamline
Justified Reasoning
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Algebraic Representation (3)
Elimination (1)
Functional Relationship
Graphing of Function (1)
Identification of Formula
Identification of Model
New Directions
Pattern Thinking
Reuse Info (1)
Separation of Parts
Rereading (2)
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Guessing
Problem Marking
Reasonability Check
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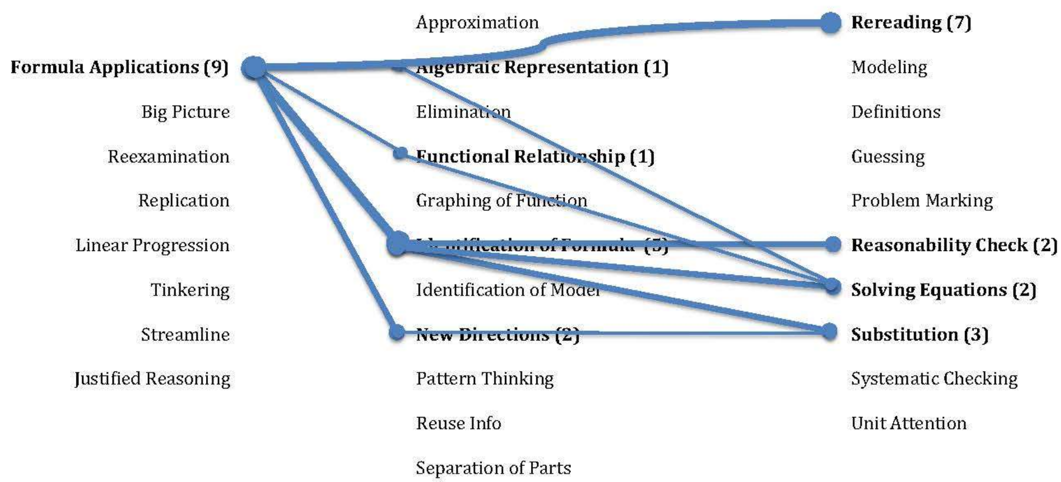
Substitution
Systematic Checking
Unit Attention (1)

F6259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Reexamination
Replication
Linear Progression
Tinkering
Streamline
Justified Reasoning
Approximation
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Functional Relationship (1)
Graphing of Function
Identification of Formula (5)
Identification of Model
New Directions (2)
Pattern Thinking
Reuse Info
Separation of Parts
Rereading (7)
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Definitions
Guessing
Problem Marking
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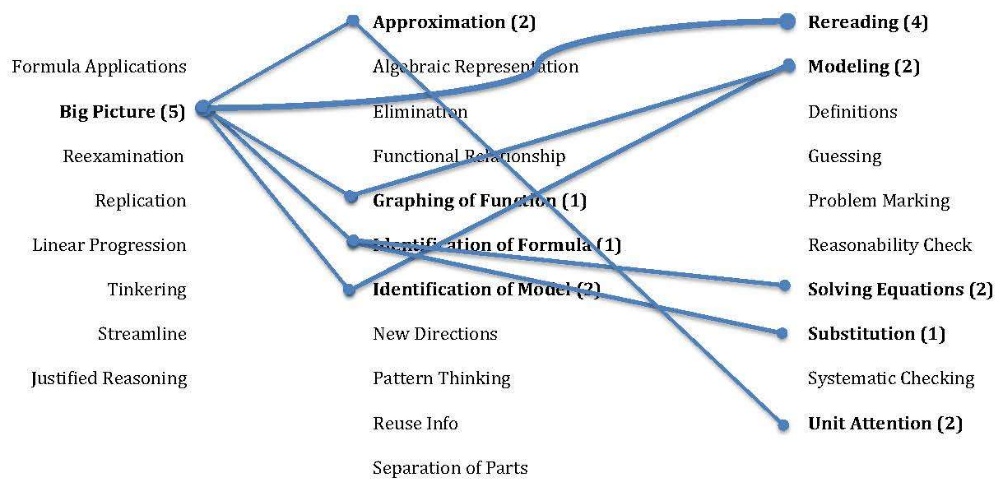
Substitution (3)
Systematic Checking
Unit Attention

F6269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Reexamination
Replication
Linear Progression
Tinkering
Streamline
Justified Reasoning
Approximation (2)
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Elimination
Functional Relationship
Graphing of Function (1)
Identification of Formula (1)
Identification of Model (2)
New Directions
Pattern Thinking
Reuse Info
Separation of Parts
Rereading (4)
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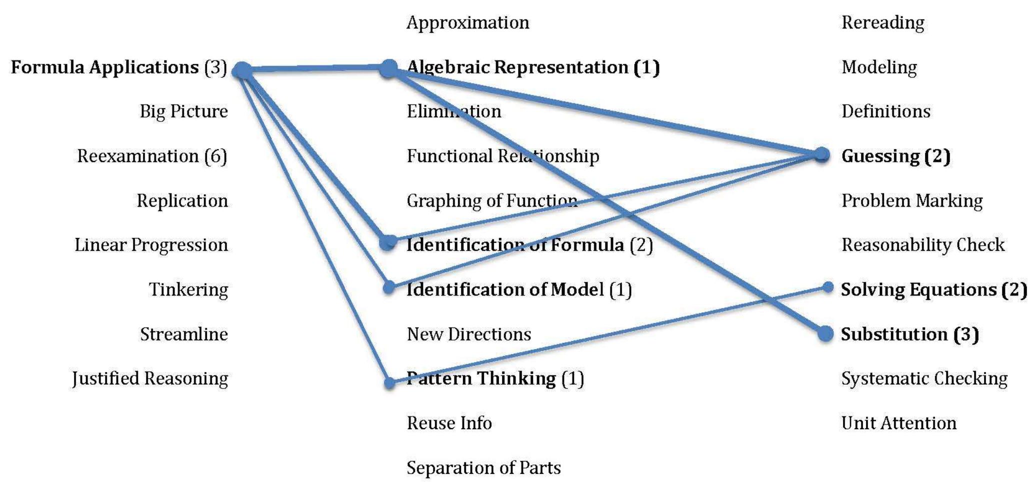
Definitions
Guessing
Problem Marking
Reasonability Check
Solving Equations (2)
Substitution (1)
Systematic Checking
Unit Attention (2)

M105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Reexamination (6)
Replication
Linear Progression
Tinkering
Streamline
Justified Reasoning
Approximation
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Elimination
Functional Relationship
Graphing of Function
Identification of Formula (2)
Identification of Model (1)
New Directions
Pattern Thinking (1)
Reuse Info
Separation of Parts
Rereading
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Definitions
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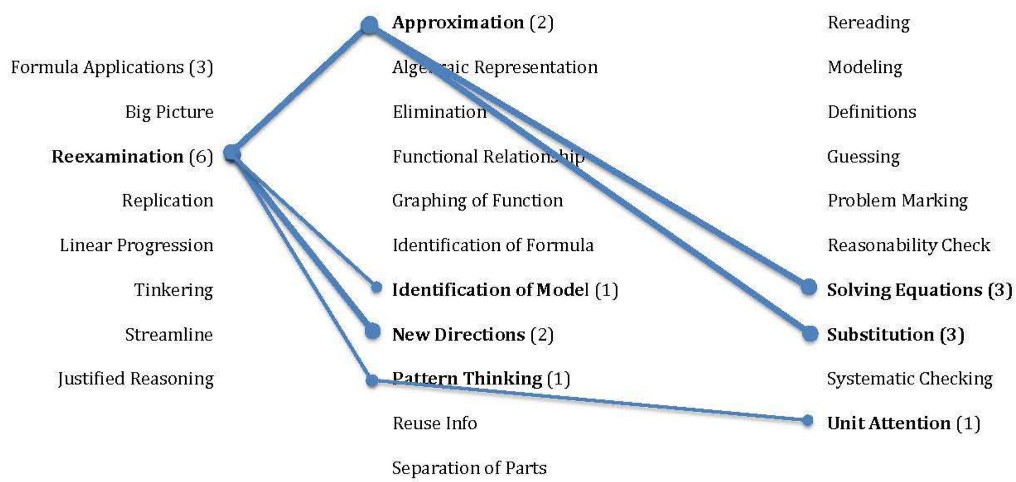
Reasonability Check
Solving Equations (2)
Substitution (3)
Systematic Checking
Unit Attention

M105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Reexamination (6)
Replication
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Tinkering
Streamline
Justified Reasoning
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Elimination
Functional Relationship
Graphing of Function
Identification of Formula
Identification of Model (1)
New Directions (2)
Pattern Thinking (1)
Reuse Info
Separation of Parts
Rereading
Modeling
Definitions
Guessing
Problem Marking
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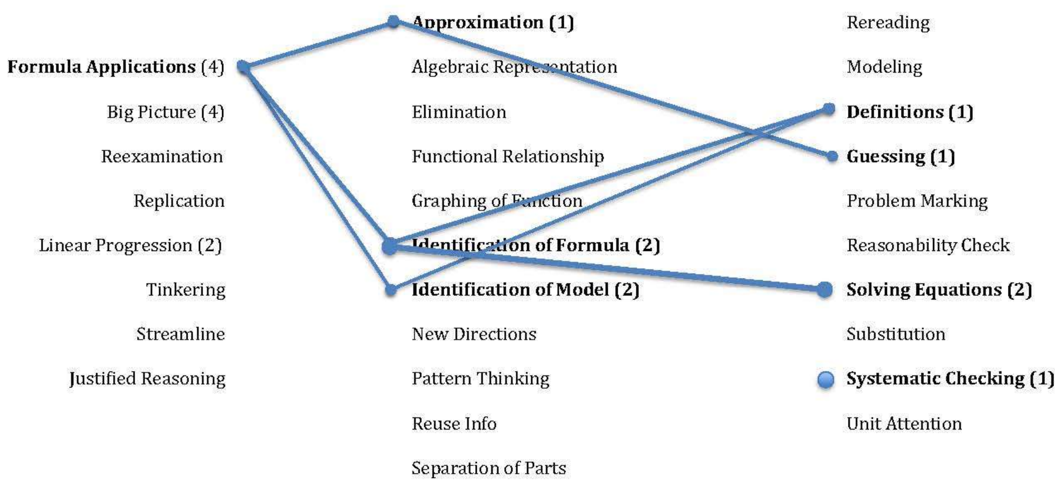
Substitution (3)
Systematic Checking
Unit Attention (1)

M106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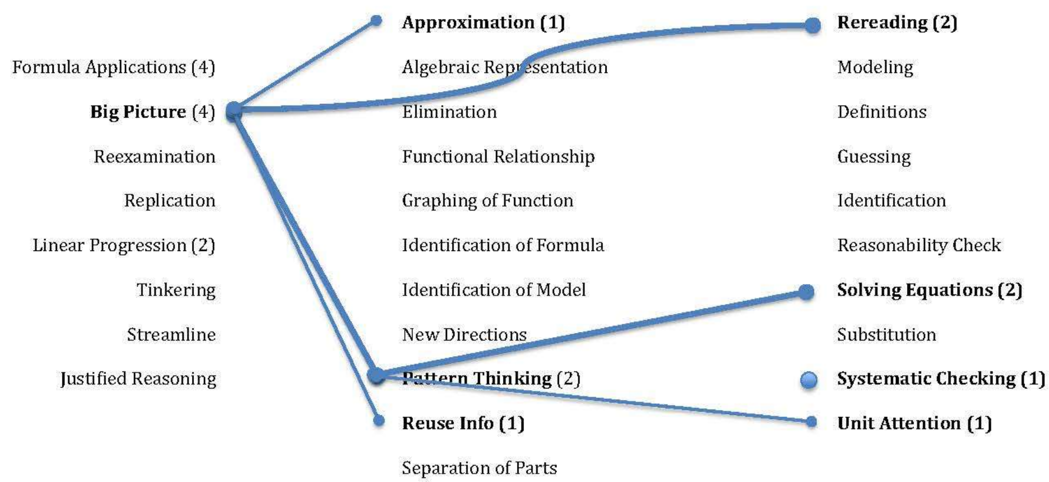
Big Picture (4)
Reexamination
Replication
Linear Progression (2)
Tinkering
Streamline
Justified Reasoning
Approximation (1)
Algebraic Representation
Elimination
Functional Relationship
Graphing of Function
Identification of Formula (2)
Identification of Model (2)
New Directions
Pattern Thinking
Reuse Info
Separation of Parts
Rereading
Modeling
Definitions (1)
Guessing (1)
Problem Marking
Reasonability Check
Solving Equations (2)
Substitution
Systematic Checking (1)
Unit Attention

M106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Big Picture (4)
Reexamination
Replication
Linear Progression (2)
Tinkering
Streamline
Justified Reasoning
Approximation (1)
Algebraic Representation
Elimination
Functional Relationship
Graphing of Function
Identification of Formula
Identification of Model
New Directions
Pattern Thinking (2)
Reuse Info (1)
Separation of Parts
Rereading (2)
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Definitions
Guessing
Identification
Reasonability Check
Solving Equations (2)
Substitution
Systematic Checking (1)
Unit Attention (1)

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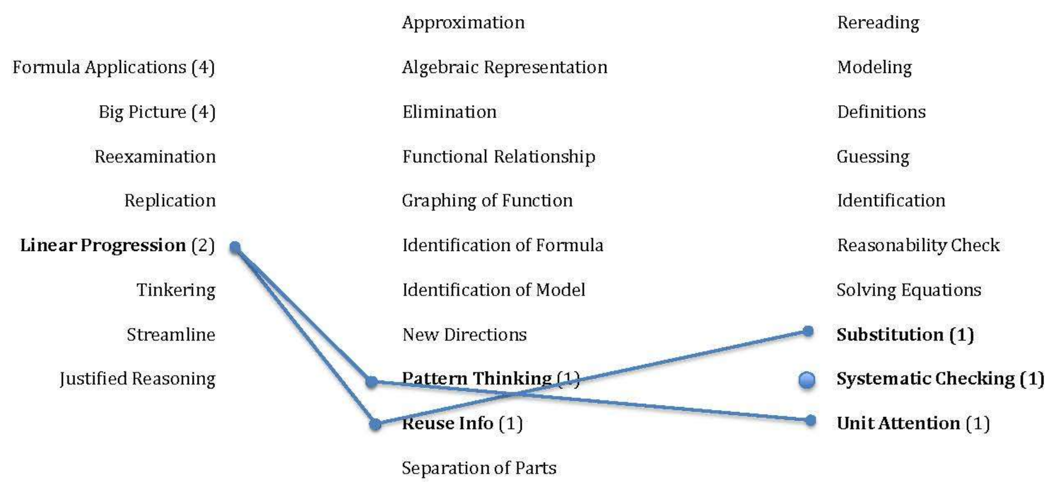

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Big Picture (4)
Reexamination
Replication
Linear Progression (2)
Tinkering
Streamline
Justified Reasoning
Approximation
Algebraic Representation
Elimination
Functional Relationship
Graphing of Function
Identification of Formula
Identification of Model
New Directions
Pattern Thinking (1)
Reuse Info (1)
Separation of Parts
Rereading
Modeling
Definitions
Guessing
Identification
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Substitution (1)
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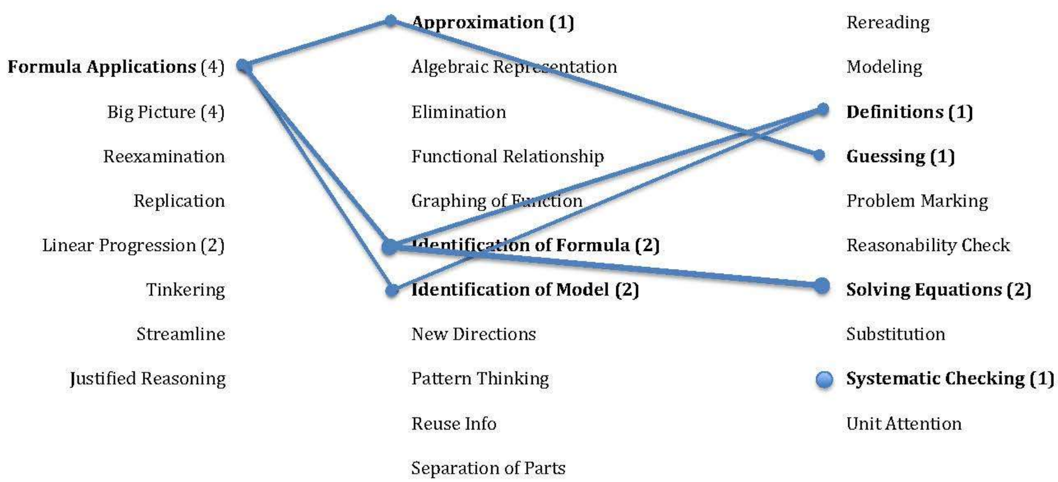
Unit Attention (1)

M310



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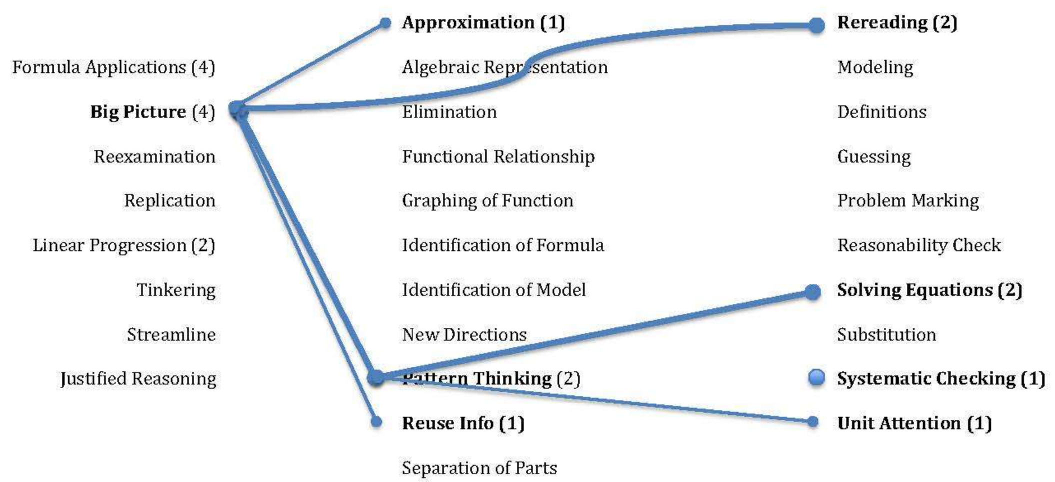
Big Picture (4)
Reexamination
Replication
Linear Progression (2)
Tinkering
Streamline
Justified Reasoning
Approximation (1)
Algebraic Representation
Elimination
Functional Relationship
Graphing of Function
Identification of Formula (2)
Identification of Model (2)
New Directions
Pattern Thinking
Reuse Info
Separation of Parts
Rereading
Modeling
Definitions (1)
Guessing (1)
Problem Marking
Reasonability Check
Solving Equations (2)
Substitution
Systematic Checking (1)
Unit Attention

M310



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Big Picture (4)
Reexamination
Replication
Linear Progression (2)
Tinkering
Streamline
Justified Reasoning
Approximation (1)
Algebraic Representation
Elimination
Functional Relationship
Graphing of Function
Identification of Formula
Identification of Model
New Directions
Pattern Thinking (2)
Reuse Info (1)
Separation of Parts
Rereading (2)
Modeling
Definitions
Guessing
Problem Marking
Reasonability Check
Solving Equations (2)
Substitution
Systematic Checking (1)
Unit Attention (1)

M310

	Approximation	Rereading
Formula Applications (4)	Algebraic Representation	Modeling
Big Picture (4)	Elimination	Definitions
Reexamination	Functional Relationship	Guessing
Replication	Graphing of Function	Problem Marking
Linear Progression (2)	Identification of Formula	Reasonability Check
Tinkering	Identification of Model	Solving Equations
Streamline	New Directions	**Substitution (1)**
Justified Reasoning	**Pattern Thinking** (1)	**Systematic Checking (1)**
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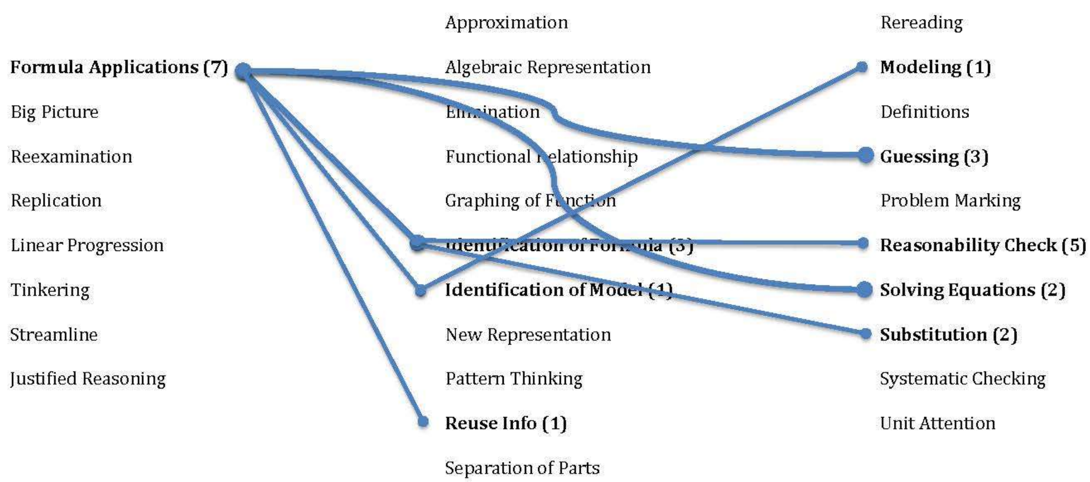
	Separation of Parts	

M506



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Big Picture
Reexamination
Replication
Linear Progression
Tinkering
Streamline
Justified Reasoning
Approximation
Algebraic Representation
Elimination
Functional Relationship
Graphing of Function
Identification of Formula (3)
Identification of Model (1)
New Representation
Pattern Thinking
Reuse Info (1)
Separation of Parts
Rereading
Modeling (1)
Definitions
Guessing (3)
Problem Marking
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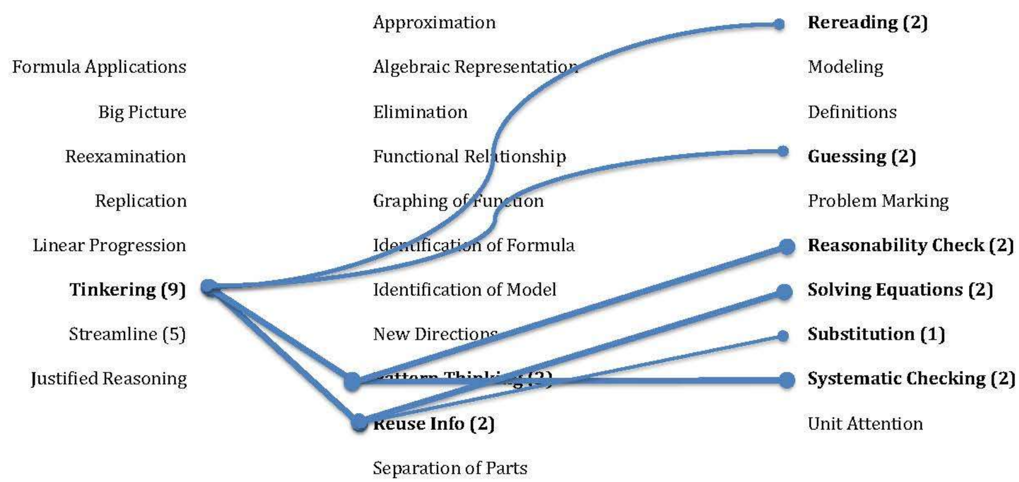
Solving Equations (2)
Substitution (2)
Systematic Checking
Unit Attention

M6115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Formula Applications
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Modeling
Big Picture
Elimination
Definitions
Reexamination
Functional Relationship
Guessing (2)
Replication
Graphing of Function
Problem Marking
Linear Progression
Identification of Formula
Reasonability Check (2)
Tinkering (9)
Identification of Model
Solving Equations (2)
Streamline (5)
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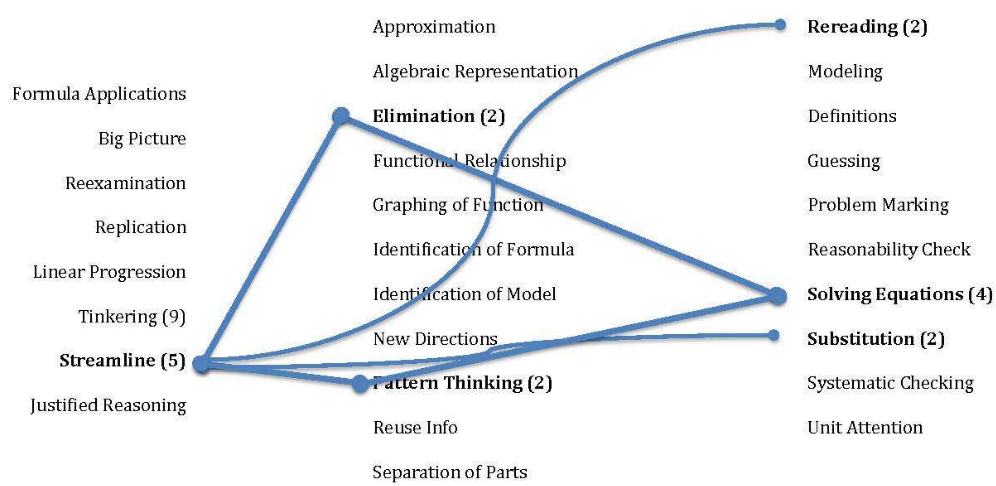
Substitution (1)
Justified Reasoning
Systematic Checking (2)
Reuse Info (2)
Unit Attention
Separation of Parts

M6115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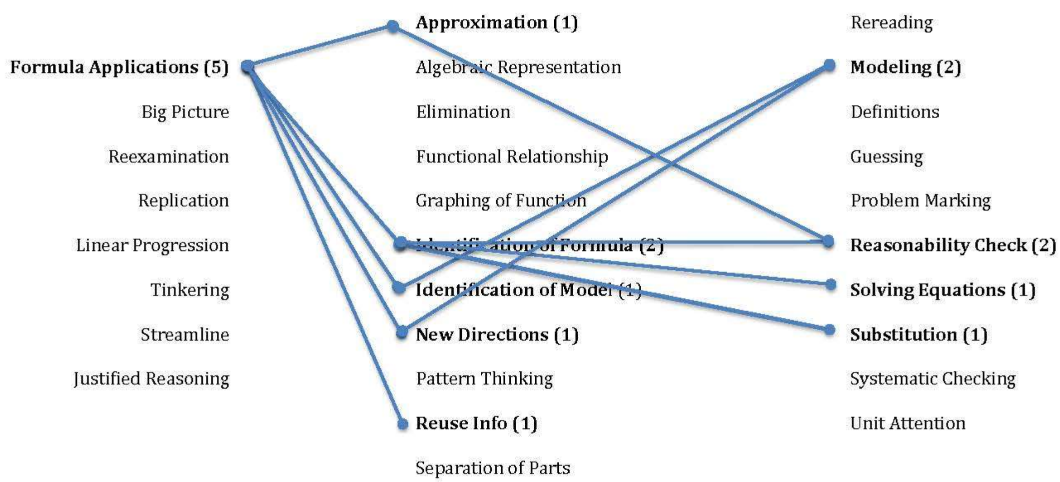
Reexamination
Replication
Linear Progression
Tinkering (9)
Streamline (5)
Justified Reasoning
Approximation
Algebraic Representation
Elimination (2)
Functional Relationship
Graphing of Function
Identification of Formula
Identification of Model
New Directions
Pattern Thinking (2)
Reuse Info
Separation of Parts
Rereading (2)
Modeling
Definitions
Guessing
Problem Marking
Reasonability Check
Solving Equations (4)
Substitution (2)
Systematic Checking
Unit Attention

M6221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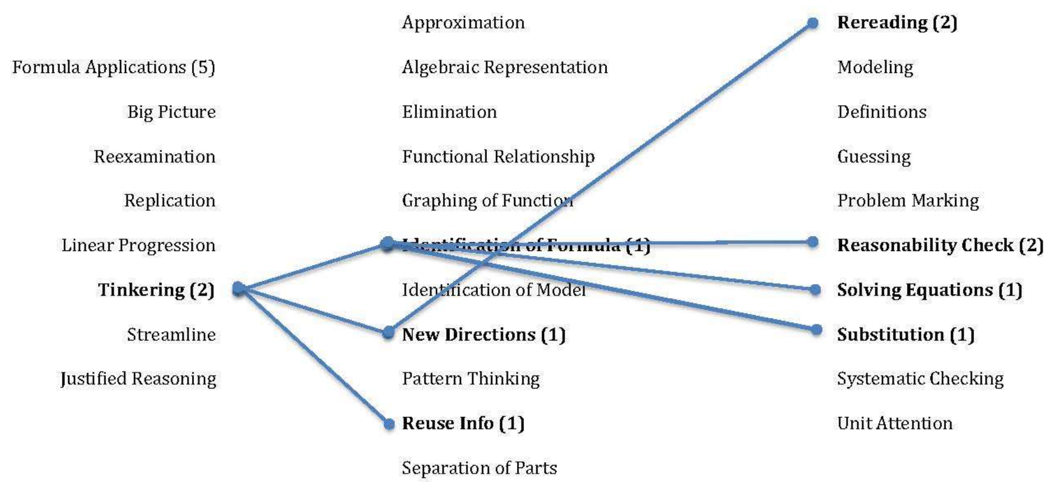
Reexamination
Replication
Linear Progression
Tinkering
Streamline
Justified Reasoning
Approximation (1)
Algebraic Representation
Elimination
Functional Relationship
Graphing of Function
Identification of Formula (2)
Identification of Model (1)
New Directions (1)
Pattern Thinking
Reuse Info (1)
Separation of Parts
Rereading
Modeling (2)
Definitions
Guessing
Problem Marking
Reasonability Check (2)
Solving Equations (1)
Substitution (1)
Systematic Checking
Unit Attention

M6221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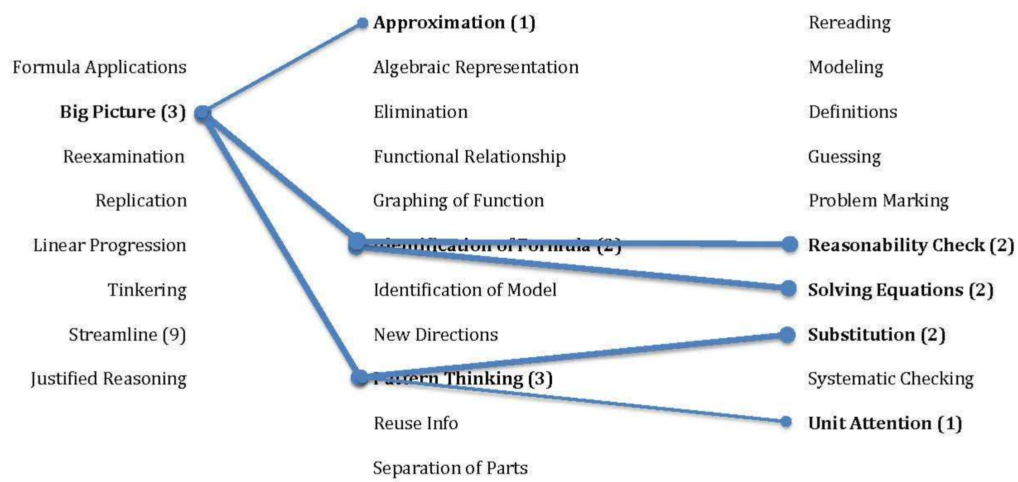
Big Picture
Reexamination
Replication
Linear Progression
Tinkering (2)
Streamline
Justified Reasoning
Approximation
Algebraic Representation
Elimination
Functional Relationship
Graphing of Function
Identification of Formula (1)
Identification of Model
New Directions (1)
Pattern Thinking
Reuse Info (1)
Separation of Parts
Rereading (2)
Modeling
Definitions
Guessing
Problem Marking
Reasonability Check (2)
Solving Equations (1)
Substitution (1)
Systematic Checking
Unit Attention

M6238



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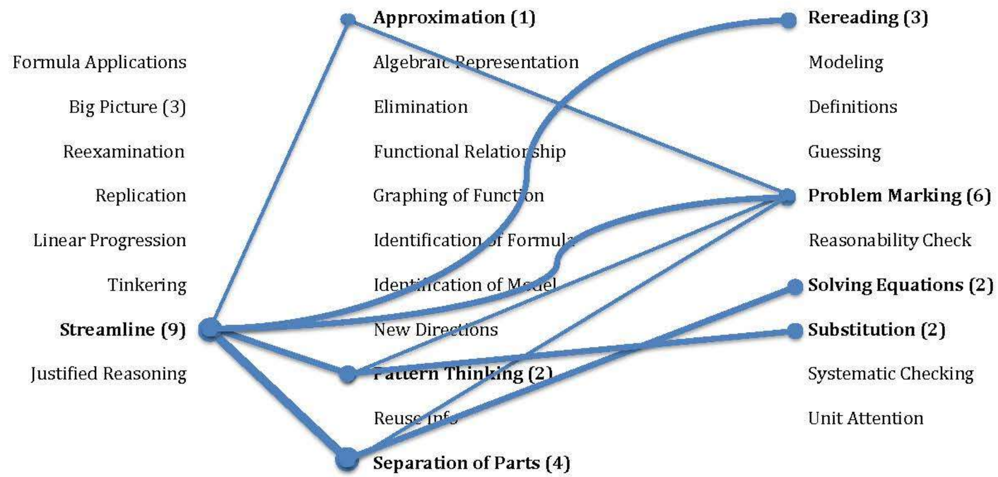
Rereading
Formula Applications
Algebraic Representation
Modeling
Big Picture (3)
Elimination
Definitions
Reexamination
Functional Relationship
Guessing
Replication
Graphing of Function
Problem Marking
Linear Progression
Reasonability Check (2)
Tinkering
Identification of Model
Solving Equations (2)
Streamline (9)
New Directions
Substitution (2)
Justified Reasoning
Thinking (3)
Systematic Checking
Reuse Info
Unit Attention (1)
Separation of Parts

M6238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Formula Applications
Algebraic Representation
Big Picture (3)
Elimination
Reexamination
Functional Relationship
Replication
Graphing of Function
Linear Progression
Identification of Formula
Tinkering
Identification of Model
Streamline (9)
New Directions
Justified Reasoning
Pattern Thinking (2)
Reuse Info
Separation of Parts (4)
Rereading (3)
Modeling
Definitions
Guessing
Problem Marking (6)
Reasonability Check
Solving Equations (2)
Substitution (2)
Systematic Checking
Unit Attention

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