A SNAPSHOT OF THE ALIGNMENT OF UNIVERSITY STUDENTS’ MATHEMATICAL PROBLEM SOLVING PRACTICES TO A LIKERT SCALE ASSESSMENT OF MATHEMATICAL PROBLEM SOLVING

by

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Abstract

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This study investigates the alignment of students’ actual problem-solving practices and compares them to the outcomes on a Likert scale mathematical problem solving (MPS) assessment. The Likert scale survey items were developed by the Mathematical Problem Solving Item Development Project (MPSI) to gather information on undergraduate’s MPS in five domains: sense-making, representing/connecting, reviewing, justifying, and challenge (Epperson, Rhoads, and Campbell, 2016).

This snapshot investigation analyzes two individual undergraduate student interviews to characterize the students’ MPS practices. Data analysis suggests a promising alignment between the MPSI survey results and students’ actual practices. The analysis also establishes preliminary reliability of the MPSI survey items and their links to the MPS domains from observations and responses during the student interviews.

In addition, the relationship between the undergraduate students’ subject-matter domain knowledge in algebra, their MPS behaviors, and their responses on the MPSI survey items is explored to determine what effects subject-matter domain knowledge may have on the MPSI survey results.
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Chapter 1

Introduction

Gagne (as cited in Jonassen, 2000) asserts that the focus of education is to teach students to think critically and improve their problem-solving ability. Although this is widely accepted among educators, there is no clear consensus on how problem solving should be taught in the classroom to achieve this goal (Lester & Cai, 2014). To study mathematical problem solving (MPS) capacity of undergraduate students, the Mathematical Problem Solving Item (MPSI) Development Project led by Álvarez and Rhoads is creating and refining MPS survey items to capture students' use of mathematical problem solving (Epperson, Rhoads, & Campbell, 2016). This study focuses on the MPS of two College Algebra students and compares data obtained from interviews with data obtained from the MPSI survey, a Likert scale assessment of MPS which will be described in Chapter 2.

Preliminary results from MPSI surveys have shown promising links between the MPS items and students' MPS. For example, the results reveal relatively consistent alignment between student behaviors linked to the "sense-making" domain to student responses on the MPSI survey (Epperson et al., 2016). Other related studies have examined how undergraduate students use representing/connecting in their solution paths and its alignment to the MPS items (Rhoads, Epperson, & Campbell, 2017). To further examine findings, analysis of students' problem solving during interviews are needed to validate the MPS items results.

This case study used data from Álvarez and Rhoads' MPSI Development Project which is partially supported by funding from the National Science Foundation, DUE grant #1544545. Data were collected and analyzed in an effort to establish the reliability of the MPS survey items by comparing outcomes on an MPSI survey and observed problem-
solving practices during interviews. This case study investigated the following research questions:

1. How do MPSI survey items align with observed problem-solving practices of university students?
2. What is captured well? What is not captured well?
3. How does students’ subject-matter knowledge impact their performance on the MPS items?

This investigation finds strong alignment between MPSI assessment results and the students’ actual problem-solving practices. In student interviews, the researchers observed that students’ choices on the Likert scale items reflected their actual solution paths in most instances. Findings from this work further inform the development of the MPSI assessment by providing supporting evidence for reliability of the items and indicators for further refinement.
Chapter 2
Literature Review

According to Jonassen (1997), “a problem is an unknown that results from any situation in which a person seeks to fulfill a need or accomplish a goal” (p.66), Jonassen also suggests that problems must have some cultural, social, or intellectual value to justify the need to solve it. Schoenfeld (as cited in Carlson & Bloom, 2005) reserves the word problem in mathematics as something the individual does not immediately know how to solve and that a problem without some elements of surprise should be classified as an exercise.

Jonassen (1997) also classified problems according to their features as being well-structured or ill-structured. However, his description of the problem involves problems that are encountered in everyday life or an academic environment. Greeno (as cited in Jonassen, 1997) stated that the problems seen in schools and universities are mostly well-structured problems. These problems have clear given information, goals, and well-defined constraints – the kinds of problems that can commonly be found in school textbooks. Chi & Glaser (1985) stated that ill-structured problems are the kinds that people confront every day, these problems usually not well defined and their solutions are often unpredictable. The issue of how to deal with pollution is an example of an ill-defined problem, the solutions to such problems are not clear and require the integration of multiple disciplines (Chi & Glaser, 1985).

Well-structured problems tend to be less complex than ill-structured problems due to less cognitive requirements (Jonassen, 1997). Simon described the different types of problems using a progression of well-structured to ill-structured continuum (as
cited in Jonassen, 2000). In regards to Simon’s continuum model, Hayes & Simon placed the commonly seen algorithmic problems towards the well-structured end of the problem continuum and common word problems toward the ill-structured end of the problem continuum (as cited in Jonassen, 2000). On the far ill-structured end of the problem continuum are dilemmas problems, which were considered very complex and unpredictable. When solving both well- and ill-structured problems, Jonassen (2000) suggested that problem-solving experiences based on familiarity do not transfer well to other problems, rather the transfer of effective problem-solving abilities required domain knowledge that is well-structured.

2.1 Mathematical problem solving (MPS)

2.1.1 What is a Mathematical Problem?

Lester and Cai (2014) describe a mathematics problem as: “a task presented to students in an instructional setting that poses a question to be answered but for which students do not have a readily available procedure or strategy for answering it.” (p. 122) Schoenfeld viewed mathematical problems as problems in any mathematically related scenario in which a person is responding to something he or she cannot solve comfortably (as cited in Carlson & Bloom, 2005).

Another aspect of mathematics problems involves the idea that sufficiently problematic problems can promote growth in students’ mathematical development by extending and stimulating students’ thinking (Lester & Cai, 2014). Lappan and Phillips (1998) developed criteria worthwhile mathematics problems when developing their Connected Mathematics middle school curriculum. Characterization of these problems includes criteria that the problems contain useful mathematics, can be approached in a variety of ways, allowing solvers to take different positions, engaging to students, provides higher-level thinking, bridging more than one mathematical ideas, and create
opportunities for the teacher to assess and assist student learning (Lappan & Phillips, 1998).

2.1.2. Characteristics of MPS

To characterize mathematical problem solving as defined in the research literature, Campbell (2014), analyzed over 20 research articles to characterize definitions of MPS either implied or stated explicitly. He identified five domains to characterize these definitions: sense making/orienting, representing/connecting, challenge/difficulty, reviewing/checking, and justifying/defending. Epperson, Rhoads, and Campbell (2016) refined the categories and their definitions and used this as a framework for developing survey items designed to provide information on students’ MPS. Sense-making during MPS has to do with “breaking apart the problem and identifying the key ideas and concepts.” Representing/connecting refers to the “bridging the problem to another idea, related mathematical approaches, or representations.” Reviewing behaviors include “self-monitoring or assessing progress as problem solving occurs, or assessing the problem solution once the problem-solving process has concluded.” Justifying means “communicating reasons for the methods and techniques used to arrive at a solution.” (Epperson et al., 2016 p. 2).

Evidence from multiple studies show similarities between different problem-solving models and serve as one way to characterize mathematical problem solving by the processes it entails (Carlson & Bloom 2005; Jonassen, 1997, 2000). As a synthesis of other problem models, Gick (as cited in Jonassen, 1997) proposed a problem-solving process that includes constructing a problem representation, searching for solutions, and implementing and monitor the solution. To clarify Gick’s model, Jonassen (1997) explained that the problem solver first needs to generate a representation that breaks apart the problem and connects the solver to prior knowledge. Next, the solver work
towards possible solutions, which are then implemented and assessed. This process of representing, searching for solutions, and testing the results continue until a correct solution is found. Similarly, Polya (as cited in Carlson & Bloom, 2005) suggested a problem-solving process that includes “understanding the problem, developing a plan, carrying out the plan, and looking back”. In their study investigating the problem-solving cycle of expert mathematicians, Carson and Bloom (2005) noticed that mathematicians showed behaviors orienting, planning, executing, and checking. They also noticed a cycle of planning, executing, and checking until a solution is reached.

In relation to MPS, Lithner (2000) cited Ross to suggest that reasoning is a crucial component of mathematics. He defined mathematical reasoning as “the line of though, the way of thinking, adopted to produce assertions and reach conclusions” (Lithner, 2000 p.166). He described solving mathematical problems using a four-step reasoning structure: problematic situation, strategy choice, strategy implementation, and conclusion.

2.2 Components of Mathematical Problem Solving

2.2.1 Resources

Polya, Schoenfeld, and Galbraith defined resources as “formal and informal knowledge about the content domain that includes facts, definitions, algorithmic procedures, routine procedures, and relevant competencies about rules of discourse” (as cited in Carlson & Bloom, 2005 p. 48). In an earlier study, Schoenfeld (as cited in Carlson & Bloom, 2005) defined resources as the mathematical knowledge possessed by the individual to be used on a problem. Having sufficient resources such as domain-knowledge is a strong predictor of problem-solving skills; how much someone knows about a domain is an indication of how likely they are to succeed in solving the task (Jonassen, 2000).
In studying mathematical reasoning, Lithner (2000) observed that the lack of resources negatively affected students’ reasoning and problem-solving outcomes. A participant in this study lacked domain knowledge led him to rely on routine procedures to determine the maximum of a function. He struggled to make sense of the outcome when the derivative method yields a minimum instead of a maximum like he had expected. In the same study, another participant attempted to sketch the graph of a derivative using the derivative algorithm and the superficial understanding of derivatives as slopes. He was able to use what he remembered from similar problems to sketch the graph of f’(x) given f(x). However, the student could not make sense of why the graph of f’(x) was discontinuous and ended up doubting his sketch. Lithner (2000) suggested that both students relied on familiar procedures while lacking the domain knowledge or conceptual understanding to interpret the results. In other words, their domain knowledge was not structured or connected which caused confusions during the problem-solving process (Jonassen, 2000). In 1998, Niss (as cited in Lithner, 2000) explained that although students are presented with theoretical definitions and concepts in the classroom, they will only acquire the concepts when it is embedded in exercises and tasks that they have worked.

Studies have shown that the understanding how topics within the same domain are related is a strong predictor of problem-solving success (Jonassen, 1999, 2000; Kieran, 1992; Carlson & Bloom, 2005). Carlson and Bloom (2005) found that formal knowledge such as well-connected conceptual knowledge plays a central role in the problem-solving process. While observing MPS of mathematics doctoral students, they saw the solvers’ well-connected conceptual knowledge influenced every step of their problem-solving process. In solving the given problems, participants relied on their connected understanding in multiple areas of mathematics to reason about their problem-
solving approaches. The study’s results suggested that the ability to access and apply learned information at the appropriate time is “highly dependent on the richness and connected of the individual’s conceptual knowledge” (Carlson & Bloom, 2015 p. 70).

2.2.2 Heuristics

Carson and Bloom (2005) defined heuristics to be the specific procedures and approaches used when solving a problem. Similarly, Lithner (2000) defined Heuristics as the “strategies and techniques for making progress on unfamiliar and non-standard problems” (p. 170). He suggested techniques like drawing a figure, relating to similar problems, and working backward as strategies when solving mathematical problems. Webb (1979) suggested that there’s a link between domain knowledge and heuristics; in order for concepts and information that required understanding to be include in the problem solver’s domain knowledge, some sort of heuristic problem-solving abilities must exist.

While compiling a list of heuristics suggestions for successful problem solving, Polya suggested draw-a-figure as a “good general advise” (as cited in Stylianou and Silver 2004, p. 354). Subsequently, in a study that explored the relationship between conceptual knowledge and processes in problem solving, Webb (1979) found that pictorial representation stood out as the most frequently used process. Stylianou and Silver (2004) observed how visual representation (VR) were used in problem solving of expert and novice problem solvers in five problems that encourages the use of VR. Most the participants drew a VR after reading the problem. Ninety percent of experts used their VR as an efficient tool to guide them through the problem-solving process and used their representation to justify their solutions. Fifty percent of all novices produced similar VR to the experts but lacked the conceptual understanding to make good use of their VR. The few students that could successfully apply their drawings produced correct solutions.
Jonassen (2000) suggests that people more successful at solving problems when they used domain-specific strategies. Although this is generally true, in a study of trade-offs between grounded and abstract representation, Koedinger, Alibali, and Nathan (2008) found that some students were more successful using informal strategies (less domain-specific strategies) to solve grounded story problems than their equation counterparts. They observed that students prefer to use informal strategies such as unwinding and guess-and-check to solve less complex single-reference word problem (where the unknown is referenced once in the problem statement). Koedinger suggested that this result is due to verbal advantage in simpler story problems; the real-world connection and relatability in the problem statements helped students make inferences and perform accurate arithmetic when using informal strategies.

2.2.3 Control

Lithner (2000) defined control as the conscious metacognitive action that influences the selection and implementation of resources and heuristics. He suggested that having sufficient control enable the problem solver to know when and how to implement what they know. Carlson and Bloom (2005) reported that “even when individuals appear to possess the resources to solve a particular problem, they often do not access those resources in the context of producing a problem solution” (p. 48). Likewise, Schoenfeld (1992) stated that “It’s not just what you know; it’s how, when and whether you use it” (as cited in Carlson & Bloom, 2005, p.48).

Lester (as cited in Carlson & Bloom, 2005) described that good mathematicians possess a high level of control in terms of awareness when solving problems; they regulate and monitor their own problem-solving effort. Carlson and Bloom (2005) observed that participants in their study closely monitored their work, accessed appropriate domain knowledge, managed their emotions, and verified their results. They
found that effective use of these behaviors and domain knowledge are a strong predictors of problem-solving success.

A lack of control was observed in Schoenfeld’s (1989) study on mathematical beliefs and behavior. He noticed that college and high school students could come up with correct proof for a mathematical problem but are unable to transfer their knowledge to a similar problem that required the application of the knowledge. Students came up with conclusions that violated what they’ve shown to be true in their proofs. Their behaviors indicated that knowing certain mathematical concepts do not directly transfer to applying it – students lacked control that comes from a well-connected domain knowledge.

2.2.4 Affect

Lithner (2000) defined affect or belief systems as one’s “mathematical world view” and the set of determinants of an individual’s behavior regarding mathematics. Similarly, Bandura (as cited in Pajares & Miller, 1995) stated that people’s judgments of their ability to execute a course of actions to obtain specific results strongly influence the choice they make, the effort they exert, their perseverance in adversity, and the amount of anxiety they experience. A similar view is shared by Carlson and Bloom (2005), they suggested that the role of variables relating to affect such as beliefs, attitudes, and emotions have a powerful impact on the problem-solving process. Transcription of a participant’s MPS consisted of affective behaviors that demonstrated how frustration, pride, ego, embarrassment and how it influenced his problem solving. The transcript showed that each frustrating incidence was followed by the “cycling back” phase where he had to re-strategize and plan before executing another approach. Even after he had found the satisfactory solution, the subject was embarrassed that he had made "sloppy calculations" (Carson and Bloom, 2005, p. 59).
In a study on self-efficacy and mathematics performance, Pajares and Miller (1995) used the MSES-Revised assessment to investigate three types of mathematical self-efficacy; “confidence to solve mathematics problems, confidence to succeed in mathematics-related courses, and confidence to perform mathematics-related tasks” (p. 1). Data from the study showed that student’s confidence to solve mathematics was a strong predictor of their actual ability to solve those problems. Also, their confidence to succeed in mathematics courses is highly correlated with their choice of major that required those courses. In exploring students’ mathematical beliefs and behavior, Schoenfeld (1989) found strong correlations between students’ academic performance, their expected mathematical performance, and their sense of personal ability. Those who were good at mathematics, with high grades in mathematics classes, were less likely to believe that the subject is mostly memorizing. Students who thought of mathematics as an interesting subject perceive themselves as harder working than their peers. On the other hand, students that were not confident in their mathematical ability believed success is based on luck and failures due to the lack of ability. These students tend to believe that memorization is crucial for success in mathematics.

2.3 Algebra

2.3.1 School Algebra

Kieran (1992) defined algebra as the branch of mathematics that symbolizes numerical relationships and mathematical structures with operations on those structures. She asserts that the study of Algebra has evolved from a procedural view with focus on numerical operations to a more to structural view, focusing more on generalized algebraic expressions. Kieran categorized school algebra activities into three types: generational, transformational, and global/meta-level. Radford (as cited in Kieran, 2007) stated that generational activities that involve forming expressions and equations are where algebra
expresses its meaning. Transformational activities are the rule-based, such as combining like terms, factoring, expanding, substitution, addition polynomials, etc. Global/meta-level activities are more rooted in algebraic structure, requiring a more generalized understanding of algebra needed for making conjectures, proving, and justifying.

According to Davis (as cited in Kieran, 1992), traditional algebra textbooks used in schools included topics such as (a) properties of real and complex numbers; (b) forming and solving of first- and second-degree equation; (c) simplification of polynomial and rational expressions; (d) symbolic and graphical representation of linear, quadratic, exponential, logarithmic, and trigonometric functions; and (e) sequences and series. Due to the widening gap between mathematics that is learned in the classroom and what is required for post-college jobs, especially in the sciences, modifications to the algebra curriculum was made to accommodate such changes (Kieran, 1992). New topics relating to a functional view of algebra was taught with the emphasis on structural understanding.

In 1991, Sfard suggested that most students will first acquire the operational (or process) conception before the transition to structural understanding (viewing mathematics concept as and manipulating mathematics as concrete objects) (as cited in Kieran, 1992). Kieran (1992) suggested that algebra students must eventually let go of the interpretation of expressions and equations as operations upon numbers and adopt the understanding that they are objects that can be operated upon. These operations are simplifying, factoring, rationalizing solving, differentiating, rather than just addition, subtraction, multiplication, and division. Topics such as literal terms relating to expressions and equations, simplifying expressions and solving equations, word or real world application problems, and functions with multiple representations are considered as the core of “school algebra.”
2.3.2 Challenges in the Learning of Algebra

A primary challenge in the learning of school algebra is a lack of structural understanding of literal terms and expressions (Kieran, 1992). This lack of understanding is revealed in studies that show that students unable to determine the equivalence of arithmetic expressions without computing the numerical value of each expression or unable to recognize letters or variables as generalized numbers (Kieran 1992). Furthermore, Kieran asserts that this lack of understanding begins at an early age and cites Booth’s study in which students were unable to recognize that the total of two sets, say a and b, can be written as a+b (Kieran, 1992).

Another difficulty is the lack of fundamental understanding of algebraic structure, which can often lead to inconsistencies when simplifying algebraic expressions (Kieran, 1992). Kieran (1992) suggested that when students have mastered how to simplify a particular structure of expressions, this knowledge is usually not transferred to other expressions. That is, students might simplify expressions with a similar structure differently due to differences in the variables used (Kieran, 1992). According to Matz (as cited in Kieran, 1992), students sometimes wrongly view variables as labels that are directly added on after arithmetic computations without regard to the like-terms.

Students’ lack of structural understanding often results in the conceptual misunderstanding regarding the use of equal signs in algebraic equations (Kieran, 1992). She suggested that some students might view the equal sign as a separator in an equation. Also, she raised the issue that students process the equal sign as a signal to “do something” rather than a symbol for equivalence between the expression to the left and right. According to Herscovics and Linchevski (as cited in Kieran, 2007), when given an equation such as $4 + n - 2 + 5 = 11 + 3 + 5$, some students might interpret the expression to the left as being equal to only 11 and not $11 + 3 + 5$. 
Some studies have suggested that students do not have a complete conceptual understanding of a variable when solving an equation (Kieran, 2007; Kieran, 1992). In a 2005 study by Vayavutjamai, Ellerton, and Clements, some college students thought that the first \( x \) in the equation \((x - 3)(x - 5) = 0\) is always 3 and the other \( x \) is always 5 (as cited in Kieran, 2007). Similarly, some middle school students also believed that the value of \( w \) and \( n \) in the equation \(7w + 22 = 109\) and \(7n + 22 = 109\) have different values and that \( w \) is somehow larger due to its position in the alphabet (Kieran, 1992).

Although Kieran (1992) reported that the most commonly used approach in teaching how to solve word problem is to formulate an equation using the relationship between the variables, representing a word problem using an equation is a well-known difficulty for algebra learner. Kieran (1992) suggested that this is due to the students’ weak understanding of the relationship between variables and algebraic structures. As a result, students often prefer to use arithmetic reasoning over the use of equations to solve word problems (Kieran 2007). Similar findings in Nathan and Koedinger (2000) have shown that students prefer to use and are successful at using informal strategies such as working backward to solve word problems. Furthermore, Kieran (2007) suggested that even when students can write the equation, they still prefer to use informal methods to reach the solution.

2.3.3 Issues in the Teaching of Algebra

One of the roles of a mathematics teacher is to select and implement essential mathematical tasks that promote growth and development of student's problem-solving ability (Lester & Cai, 2014). However, Lester and Cai (2014) suggested that teachers do not always implement these tasks in a meaningful way, resulting in a superficial understanding of mathematical ideas. Some teachers are also guilty of accepting student solutions that are vague and not grounded in algebraic structure (Lester and Cai, 2014).
They suggested that as long teachers set clear standards for their solutions, students will most likely strive to meet these standards.

In developing instruction for a well-structured problem, Schoenfeld (1989) found that students’ problem solving often contradicts what they claimed to believe about mathematics. This is most likely a result of their experiences with numerous exercises assigned in the classroom that takes no longer than 45 seconds to 2 minutes to solve (Schoenfeld, 1989). Also, students believe that homework or test questions that take more than 12 minutes of work would be considered impossible (Schoenfeld, 1989). As a result of their classroom experiences, although students believe that mathematics allows them to be creative and think logically, they ironically claimed that mathematics is best learned by memorizing procedures (Schoenfeld, 1989).

As Schoenfeld (1989) claims that classroom experiences result in contradictory views about mathematics, this underscores the importance of classroom experiences and decisions teachers make about curriculum. Moreover, Kieran (1992) reported that there is little research about algebra teachers’ cognitions, but this affects the decisions they make in the classroom when dealing with new curriculum, in particular. Also, in a study conducted by Koedinger and Nathan (2000), teachers, researchers, and students were asked to arrange 12 mathematics problems from easiest to most difficult. Teachers and researchers predicted story- and word problems to be more difficult than symbol-equation problems, but students found symbol-equation problems to be the most difficult. The case of expert’s expert blind spot can sometimes hinder teachers from being aware of their student’s struggles, resulting in a misinterpretation of students’ needs (Kieran, 2007).

Research has also shown that, regardless of the nature of algebra teaching, sometimes students are unlikely to develop full-fledged structural understanding (Kieran, 1992). Kieran (1992) suggested that there is no direct approach to teaching that would
result directly in students’ structural understanding of algebra. Unfortunately, since many teachers only teach what is readily available in the textbook, they will not always meet the needs of their students (Kieran 1992). Even when a lesson is provided by the textbook, due to insufficient research, there will likely be a lack of support (or a gap) for how algebra teachers are to interpret and deliver that content (Kieran 1992). Kieran (2007) suggested that more research is required to determine the pedagogical and teaching content that work and those that do not work.
Chapter 3
Methodology

3.1 Participants and Setting

The Mathematical Problem Solving Item (MPSI) Development Project was conducted (and is ongoing) at a large urban university (37,000 students) in the Southwestern United States. This study draws from two of the student interviews conducted in Fall 2016. The data collected in fall 2016 included students enrolled in College Algebra and first-semester Calculus. Most of the participants were 18 to 19 years old with 238 males and 176 females in the College Algebra group and 248 males and 90 females in the Calculus group. Nearly three-fourths of the participants reported having completed a course at the level at or above second-year school algebra (Álvarez, Rhoads, & Campbell, 2017)

Participants completed a two-part survey or test. Álvarez et al. (2017) refer to the collection of five problem statements and their associated items as an MPS test (MPST). The MPST takes no more than one hour to complete. Participants solve five problems in part 1 if the MPST and record their work. After completing Part 1, they completed part 2. Part 2 begins with their rating the difficulty of each problem they worked and then they answer five to seven associated Likert items on each problem. College Algebra students took an MPST at the beginning of the semester (pre-MPST) and at the end of the semester (post-MPST).

Ten College Algebra students were selected, based upon the outcomes of the MPST, for follow-up interviews. As mentioned, this case study analyzes two of these interviews.
3.2 Data Analysis

In this study, the researcher analyzes the interviews of participants A6-309 and A6-022 to whom the monikers Kim and Bob are assigned, respectively. This section will first explain details regarding the MPST and the significance of the survey scores. Next, the author will discuss the methods and rubric involved in the transcribing, coding, and scoring of the individual interviews. Then the author will describe how students’ responses to the survey items were examined for reliability. Lastly, details regarding the comparison of the subject-matter (algebra) to survey item responses are discussed.

3.2.1 Mathematical problem solving Test (MPST)

The Likert items on the MPST are designed to give information about a students’ MPS in each of the five domains refined by Epperson et al. (2016) which are:

**Sense-Making (SM):** Breaking apart the problem and identifying the key ideas and concepts. Attending to the meaning of the problem posed

**Representing/connecting (R/C):** Bridging the problem to another idea, related mathematical approaches, or representations. Reformulating the problem by using a different representation or connecting the problem to seemingly disjoint prior knowledge.

**Reviewing (RV):** Self-monitoring or assessing progress as problem solving occur or assessing the problem solution (e.g. checking for reasonableness) once the problem solving as concluded.

**Justifying (JU):** Communicating reasons for the methods and techniques used to arrive at a solution. Justifying solution method(s) or approach(es).

**Challenge:** The problem must be challenging enough from the perspective of the problem solver to engage them in deep thinking or processes toward a goal, "without an immediate means of reaching the goal" (Wilson, Fernandez, and Hadaway, 1993, p. 57). (Epperson et al., 2016, p. 2)

As mentioned, participants took the pre-MPST at the beginning of the Fall 2016 semester. Kim and Bob were in the group of College Algebra students. Figure 3-1 shows the Ken’s Garden (KG), an example of the type of problem in an MPST. To address the
challenge domain, after solving the problems students are asked to rate the difficulty of each problem on a scale from “very easy” to “very difficult” (figure 3-2).

![Figure 3-1 One of the pretest question; the Ken's Garden problem](image1)

![Figure 3-2 Likert-item measure the “challenge” domain](image2)

An example of the subsequent associated Likert items is included in Figure 3-2. Except for the challenge domain, each Likert item is linked to one of the remaining MPS domains (sense-making, representing/connecting, justifying, and reviewing). For example, Figure 3-3 shows Item 1 in the KG problem as shown in is a Likert item associated with the sense-making domain. For this item, the choice “Only (A)” is predetermined to be associated with lowest use of sense-making (score of 1) and “Only (B)” with highest use of sense-making (score of 6). Each item is scored on a scale of 1 (low) to 6 (high) depending on student’s choices. Items where the desirable choice appeared in the “Only (A)” position were reverse coded for data analysis. In addition to computing an average the score for each domain from the 25 to 35 items on the MPST, researchers used the maximum average score (MAX score) received across all domains.
in data comparisons. Thus, each student receives a score for each of the MPS domains based on their responses in an MPST as well as a MAX score. Kim and Bob’s pre-MPST results will be referred to as “survey scores.” Figure 3-4 shows an example of a student’s survey scores who, on average, scored relatively high on all four domains.

![Figure 3-3 KG Item 1]

**Figure 3-3 KG Item 1**

![Figure 3-4 An example of a student’s survey scores]

**Figure 3-4 An example of a student’s survey scores**

### 3.2.2 Individual Interviews

To validate the results, for each pilot administration of the MPS items, interviews were carried out between pretest and posttest to observed student’s behaviors on the same mathematical problem. During the approximately one-hour think-aloud interview, students are asked to explain their reasoning while solving two mathematical problems, one problem previously seen on their pretest and one new problem. After solving each problem, like the pretest, they fill out an MPS survey for each problem. If time allows, the interviewer also asks students to explain their work on an additional pretest problem. Students were also asked general questions regarding their views on mathematics and general approach to solving mathematical problems.
Kim’s interview included the Fun Golf (FG) (see Figure 3-5) and Ken’s Garden (KG) problem. Participant Bob was assigned the KG and Cross-Country Problem (CCR) (see Figure 3-6). KG and CCR were previously worked problems on the pre-MPST for Kim and Bob, respectively. CCR has two parts, which will be referred to as CCR1 and CCR2.

Figure 3-5 The Fun Golf Problem (FG)

Fun Golf, a local mini-golf course, charges $5 to play one round of mini-golf. At this price, Fun Golf sells 120 rounds per week on average. After studying the relevant information, the manager says for each $1 increase in price, five fewer rounds will be purchased each week. To maximize revenues, how much should Fun Golf charge for one round?

Figure 3-6 The Cross-country Problem (CCR1 and CCR2)

3.2.3 Coding Scheme

Nvivo qualitative analysis software was used to transcribe and code each of the individual interviews. Nodes were created based upon the MPS domains (SM, R/C, RV, JU) to keep track of the number of instances one of these practices was observed (see Figure 3-7). In addition to the four MPS domains, nodes also included “affect” and “heuristics.” Affect (or belief system) is defined as the “mathematical world view” that includes variables such as “beliefs, attitudes, and emotions” during the problem-solving process (Carlson & Bloom, 2005, p. 1; Lithner, 2000, p. 7). Heuristics is defined as the specific procedures, approaches, and strategies used when solving a problem (Carlson & Bloom, 2005).
To better filter the coded content, the researcher re-coded each instance as either productive or nonproductive. A productive coded domain is generally aligned with the problem statement and serves to further the problem-solving progress and a nonproductive code indicates that the effort does not support meaningful progress in MPS. A coded instance does not have to directly lead to the correct solution to be productive, however it needs to provide evidences that could potentially lead to an accurate or reasonable solution. Figure 3-8 shows a coded instance of “justifying” that was re-coded as “productive.” Here student’s effort to justify his method was logical and led to a reasonable solution. Figure 3-9 shows an instance of JU that was nonproductive. The student made a “nonproductive” attempt to justify his method.
3.3 Assessing Students’ Problem-solving Practices

The researcher’s Initial approach was to simply use the coded interview transcript and student work to determine if each observed domain correlates to a low or high-level use. Although this may suffice for aligning survey scores with students’ observed problem-solving practices, it is limited to a problem by problem comparison and may be limited in determining a student’s usual problem-solving practices. To better compare student’s survey scores and actual problem-solving practices, an assessment rubric was created by the researcher using a scale of one to six to score observed problem-solving practices in relation to the MPS domains (see Table 3-1). The interview scoring rubric will be referred to as the “interview rubric” and students’ problem-solving domain scores based on the rubric as “interview scores.” As an effort to align the interview scores with the survey scores (from pretest), the scoring criteria for each domain was written based on the definitions of MPS domains with scores ranging from 1 to 6. Each student received two sets of scores, each set contained a score for each MPS domain.

The scoring criteria for this rubric was based on the MPS domain definitions (Epperson et al., 2016), the researcher’s interpretation of low and high-level use of each
domain, and reference to the study by Carlson and Bloom (2005). Carlson and Bloom studied MPS of four expert mathematicians and provided open code interview transcripts for each participant in their paper (Carson & Bloom, 2005). The experts’ problem-solving practices in Carlson and Bloom’s study were aligned with the researcher’s interpretation of high-level use of the MPS domains. The experts in Carlson and Bloom’s paper all identified key information and other instances of SM were scattered throughout their problem-solving process. Based on James’ transcription in Carlson and Bloom (2005), the researcher decided that highest-level use of SM should include the identifying of key information and at least five” instances of SM (pp. 58-59). After identifying the given information, James showed five other instances of SM throughout the problem-solving process. In the other three interview transcripts, the participants also identified given information and showed no more than 6 instances of SM. Scoring criteria for R/C, RV, and JU did not require a certain number of coded instances due to the participants' localized display of each domain. In other words, a single coded instance of any of the three domains could suggest a high-level use (whereas it would be low-level for SM). For example, receiving a 5 on R/C could mean that the student’s diagram clearly and accurately represents the problem statement but usage of the diagram was not extensively (although there were evidence of usage).
Table 3-1 Interview Scoring Rubric

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2-3</th>
<th>4-5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>SM</td>
<td>No Evidence of SM</td>
<td>• May or may not identified key ideas or given conditions &lt;br&gt; • Less than three coded instances of SM</td>
<td>• Key ideas or given condition(s) were identified &lt;br&gt; • At least 3 additional coded instances of SM</td>
<td>• Key ideas or given condition(s) were identified &lt;br&gt; • At least five additional coded instances of SM</td>
</tr>
<tr>
<td>R/C</td>
<td>No Evidence of R/C</td>
<td>• Picture, diagrams, equations, formulas or other representation were present but not used in problem-solving process</td>
<td>• One of the following &lt;br&gt; (1) Adequate use of pictures, diagrams, equations, formulas, or other ideas to bridge the problem statement. &lt;br&gt; (2) Sufficient reformulation of the problem using different representation or connection of the problem prior knowledge &lt;br&gt; • Evidence of using (1) or (2) as part of the problem-solving process</td>
<td>• One of the following &lt;br&gt; (1) Clear and accurate use of pictures, diagrams, equations, formulas or other ideas to bridge the problem statement. &lt;br&gt; (2) Clearly and accurately Reformulate the problem using different representation or connected the problem to prior knowledge &lt;br&gt; • Extensive use (1) or (2) as part of the problem-solving process</td>
</tr>
<tr>
<td>RV</td>
<td>No evidence of RV</td>
<td>• limited monitoring or assessing progress as problem-solving occurs &lt;br&gt; • Solution was vaguely checked or not checked</td>
<td>• Moderate self-monitoring or assessing progress for reasonableness as problem-solving occurs &lt;br&gt; • Moderate assessment of the solution once problem-solving has concluded</td>
<td>• Extensive self-monitoring or assessing progress for reasonableness as problem-solving occurs &lt;br&gt; • Complete assessment of the solution once problem-solving has concluded</td>
</tr>
<tr>
<td>JU</td>
<td>No evidence of JU</td>
<td>• Reasons for the methods and techniques used were vaguely or inadequately explained</td>
<td>• Adequately explained reasons for the methods and techniques used</td>
<td>• Clearly communicated reasons for the methods and techniques used</td>
</tr>
</tbody>
</table>
The resulting rubric was used to score each student’s MPS practices and assigned each student a set of four scores (four domains) for each problem they solved in the interview. The researcher will refer to each set of four scores as “interview scores” (not to be confused with “survey scores”). Like the survey scores, interview scores of 1-3 suggest a low-level use and 4-6 high-level use of a domain. The author compared each student’s interview scores to their survey scores and discuss implications regarding the alignment of student’s actual practices to MPS items. It is worth noting that the “challenge” domain was not scored, although it will be discussed in the Discussion chapter.

3.3.1 Assessing Likert Item Responses

Solution pathways provided in the Likert items were designed to capture student’s MPS levels based on their responses. Students’ choices that do not reflect their actual problem-solving practices could potentially result in domain scores that do not accurately present their MPS levels. To assess the alignment students’ responses to their actual problem-solving practices, responses for each item were compared to evidence during the problem-solving process. More specifically, 11 responses from Kim’s interview that came from the FG and KG problem (one for each Likert item) and 11 responses from Bob’s interview that came from the KG and CCR problem were analyzed and examined for the alignment of each choice to the student’s practices while solving the problem.

3.3.2 Connecting Domain knowledge and MPS Items

To further investigate how subject-matter domain knowledge may be affecting students’ problem-solving approaches, the researcher coded instances of heuristic use. Students’ problem-solving strategies, their written work, and any verbalization of algebraic concepts was used as evidence of subject-matter (algebra) knowledge. Based upon this evidence, the researcher compared the evidence of subject-matter (algebra) knowledge to the problem approaches observed and documented in the survey to
determine what role this played in the problem-solving approaches observed. Thus, students’ problem approaches were documented on the details of their problem-solving process. In addition, the researcher searched for student responses in the survey that may have been directly influenced by the documented subject-matter (algebra) knowledge (domain knowledge). The responses that were influenced by specific subject-matter knowledge will be discussed later in chapter 5.
Chapter 4
Results

Individual interviews and pretest data for participants Kim and Bob were the principal sources of data for this investigation. As mentioned in the previous chapter, their survey scores from a Fall 2016 administration of an MPST were provided along with interview video recordings and interview assessments. Each Interview was transcribed and coded using sense-making (SM), representing/connecting (R/C), justifying (JU), reviewing (RV), heuristics, and affect as the primary coding framework. Recall that each student solved two problems in their think-aloud interview; one new problem and one previously seen on the pretest. Kim was given Fun Golf (FG) as the new problem and Ken’s Garden (KG) was from the pretest. KG was the new problem for Bob and Cross-Country Race (CCR) was from the pretest. CCR has two parts so will refer to them as CCR1 and CCR2.

After both interviews had been transcribed and coded, each problem was scored using the interview rubric (see Table 3-1) and each of the Likert items was compared to students’ problem solving observed during the interview. In this chapter, the author reports on the results of this process as well as implications from coded instances related to affect and heuristics. First, instances of affect throughout the interview are explained regarding what they suggest about student’s beliefs and disposition towards MPS. Then observed instances of SM, R/C, JU, and RV in each problem for Kim and Bob are elaborated to explain the interview scores that each domain was assigned. Then students’ choices for each Likert items are compared to their MPS practices in the
interview. Finally, the author discusses student's domain knowledge based on coded instances of heuristics.

4.1 Participants' Commentary Related to Affect

Instances of affect were code to provide a better understand student behaviors in MPS that include personal beliefs, attitudes, and emotions expressed during the interview. Participants' responses were guided by the interviewer’s questions. In both interviews, the interviewer asked both participants questions regarding their attitude towards mathematics and whether there are any processes/strategies that they often use.

When asked how she feels about mathematics, Kim declared that mathematics is her favorite subject.

Interviewer: Okay Would you say you like math or afraid of math?
Kim: Yea I actually love math, my favorite subject
Interviewer: Okay, um, is there any reason why that you know off the top of your head?
Kim: Um, I think it's just there's always gonna be a right or a wrong answer

Kim also suggested that she preferred problems with formulas because it is "easier to apply." Other than mathematics, Kim mentioned that she preferred science over subjects like English and history. She explained that she doesn't like history "you kina [sic] have to memorize information, it's like uh, you have to know the exact dates and that doesn't work well with me." When asked if there’s any problem-solving strategies that she normally uses, Kim did not have a specific answer, she said that she would try to apply what she had “learned over the years.” Towards the end of working the KG problem, Kim suggested that she might have approached the problems differently if she was not in an interview. For instance, Kim said, "…usually when I'm just doing it by myself I kind of do
trial and error or do whatever is best because I have it in my mind but here I have to show some kind of work…”

When asked if there are any usual process that he would use when solving a challenging mathematical problem, Bob explained that he would engage a problem by first making sense of the given information. Regarding his attitude towards mathematics classes, Bob said, “I've always kind of enjoy mathematics classes and it has always come semi-naturally to me, like I understood it better than other subjects.” Bob's responses suggested that he, like Kim, also prefers mathematics over other classes. When the interviewer asked if there are any particular areas of mathematics that he was good at, Bob answered, “yeah, I feel like I'm very good at I guess equations…like algebra, I'm pretty good at algebra.” Bob suggested that he is not comfortable with math topics like calculus because, to him, it is abstract and “not like real world.” Regarding abstract mathematical topics such as those in calculus, Bob said, “in those situations you would need to know the specific equations.” Bob expressed that he preferred situation where he can visualize and perform trial and error to get “closer to the answer.” At one point during the problem-solving process, Bob expressed his fatigue: “Man…I've been in Algebra all day…at this point in the day…”

Even though the interviewer suggested that he takes a quick break, Bob decided to continue to reason about the problem. Bob then mentioned that if he was not in an interview, he could have performed better on certain problems; he felt like the interview pressure might have thrown him off. Even so, Bob suggested, “as far as the things I would normally do, I normally yeah…I wouldn't work it any differently. Towards the end of the interview, the interviewer asked Bob why his work on the CCR problem in the pretest was slightly different from his work in the interview. Bob stated that there was no particular reason why he did it differently and that he just happened to process the
information differently on a different day. The interviewer also asked about Bob’s informal strategies and if his background had anything to do with the approach. Bob suggested that he prefers to look at smaller parts of a problem instead of the big picture.

4.2 Kim’s Commentary Related to the MPS Domains

In the interview, Kim was able to solve both the KG and FG problems. For her, KG was from the pretest and FG was a new problem. Table 4-1 shows Kim’s interview scores for both problems and their average. As mentioned, interview scores were results of a student’s actual problem-solving practices that were scored using the interview scoring rubric, not to be confused with survey scores which came from the pretest.

Table 4-1 Kim’s interview scores for the FG and KG problem

<table>
<thead>
<tr>
<th></th>
<th>SM</th>
<th>R/C</th>
<th>RV</th>
<th>JU</th>
</tr>
</thead>
<tbody>
<tr>
<td>FG</td>
<td>4</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>KG</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>Mean</td>
<td>3.5</td>
<td>4</td>
<td>5</td>
<td>4</td>
</tr>
</tbody>
</table>

4.2.1 Sense-Making (SM)

In solving the FG problem, instances of SM were observed as Kim identified given information, reread the question, and referenced conditions from the problem statement as problem solving occurs.

Figure 4-1 shows an instance where Kim wrote down given information after reading the problem – she used the problem’s constraints to calculate the number of rounds and weekly revenue for $5 and $6 per round, demonstrating her understanding of the given conditions. From the problem statement, Kim interpreted the keyword “maximize” to mean finding the maximum revenue. There were some instances where Kim showed nonverbal behavior that could be related to SM (pointing back to the
problem statement). In one instance, when the interviewer asked Kim what she was thinking:

Interviewer:  Okay, what have you done so far?
Kim:  So it’s saying 5 dollars for each round and they sell about 120 on average per week. So every time they increase in dollars so instead of charging 5 dollars they charge 6 dollars, they’ll only sell 115 rounds...

There were instances that suggested SM behavior but was unclear due to the lack of verbalization (possibly rereading the problem or making sense of given conditions). Kim was assigned an SM score of 4 since she identified key information and three additional instances of SM were observed.

![Image of handwritten notes]

Figure 4-1 Kim identified key information in the FG problem

In the KG problem, Instances of SM were few and not always clear. Kim did not identify given information. The first instance of SM was her implied interpretation of “size” to mean area. Kim did not verbalize her interpretation but implied it by writing down “204 ft²” and “102” (for half the original area) in her diagrams. For the second instance of SM, Kim reread the problem to clarify that she had misread the problem. She corrected herself by saying, “reduce the length and increase the width.” She was assigned SM score of 3 since there were few (only two) instances of SM observed.
4.2.2 Representing and Connecting (R/C)

In the FG problem, Kim used two representations during the problem-solving process. She first created a table to organize the price per round, the number of rounds, and weekly revenue. In her search for the solution, Kim connected the keyword "maximize" in the problem statement to the image of a vertex of a parabola. Kim reflected on how she used her mental picture as a guide in her problem solving, "I thought it would just keeping going up, but I realized maximize and minimum mean quadratic." She used this connection to a quadratic function to determine the appropriate values that should be input in her table and which quantity would represent the solution. There was sufficient connection to prior knowledge and Kim used her connection as a part of the problem-solving process, so she was assigned R/C score of 4.

In the KG problem, Kim represented the gardens in three ways. Kim started out by drawing two diagrams; a large rectangle for the original and a smaller rectangle for the new garden (see Figure 4-2). Although Kim did not explain or describe her diagrams, evidence from her work suggested that she referred to her drawings as a reasoning tool in her solution paths. Second, she implemented the content of her diagram to represent the area of the new garden algebraically using an equation, \((17 – x)(12 + x) = 102\). Kim used the equation extensively in her problem solving by applying algebraic manipulation and completing the square to solve for \(x\), but she was unable to arrive at a solution due to an arithmetic error. Although the equation did not lead directly to the solution, her work suggested that she relied on the algebraic structure of the equation to create a table that eventually led to a solution. Since Kim drew accurate diagrams and used it to formulate her equation which was adequately implemented during the problem-solving process, she was assigned R/C score of 4.
4.2.3 Reviewing (RV)

Throughout the problem, Kim was actively monitoring her work to make sure it correlated with the given conditions. She also stopped to make sure that her quantities were behaving in a pattern that she expected (quadratic). There was one instance when Kim made progress after pausing for two minutes to (quietly) assess her work. After reaching the solution, Kim assessed her answer: “so it hit $15…is started doing $16 a round it went back down, so the maximum was $1050…I should charge $15.” Kim was assigned an RV score of 5 since there was adequate monitoring and assessing of the solution.

In the KG problem, Kim showed RV related behaviors by monitoring her work and recognizing errors. In solving this problem, Kim had made several errors that halted progress. Through careful monitoring, she was able to locate and correct two of her errors. Kim found her first mistake through monitoring her results: “Well, I misread the question, cuz it says it reduce the length and increase the width…” After finding the solution, Kim briefly assessed her answer: “… have to be 4 feet by 25 feet since it is approximately 102, not exactly…” Kim was assigned an RV score of 5 since she showed efficient use of monitoring and assessing the solution paths as well as the solution.
4.2.4 Justifying (JU)

There were instances of JU towards the end of the FG problem. Kim justified her method by relating it to what she understood about parabolas. Kim explained that if she continues to increase the price per round, the revenue would eventually reach a maximum:

Interviewer: Okay, and can you just explain how you got to that answer?
Kim: …so each time they increase in dollars…would subtract 5 from the total per week to see how much the total was and it was a quadratic function. So, once it hit to $15 per round it got to the max of 150 and once and I start doing $16 a round it went back down, so the maximum was 1050 for the total so I should charge $15

Interviewer: …I notice you kina [sic] got stuck there after 7
Kim: Yea, cuz I thought it would just going keep going up but I realized maximizing and minimum would mean quadratic.

Kim explained how she knew to continue increasing the input values and when to stop inputting values. Kim was assigned JU score of 5 since she adequately explained the reasoning behind her problem-solving method.

Kim may have used her written work as a way to justify her methods, but there were no coded incidents of JU in the KG problem. Kim did not communicate the reasons behind any of her solution paths. Her justification of her problem solving was vague which resulted in a JU score of 2.

4.3 Bob’s Commentary Related to the MPS Domains

In the interview, Bob was assigned the KG and Cross-Country problem. Table 4-2 shows Bob’s interview scores to each problem and their average. He was able to arrive at a solution for the KG and part one of the CCR problem (CCR1) but was unable to solve CCR2. CCR2 was not scored since the majority of progress made on CCR2 were assisted by the interviewer.
### Table 4-2 Bob’s interview scores for the KG and CCR problem

<table>
<thead>
<tr>
<th></th>
<th>SM</th>
<th>R/C</th>
<th>RV</th>
<th>JU</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>KG</strong></td>
<td>5</td>
<td>5</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td><strong>CCR1</strong></td>
<td>5</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td><strong>CCR2</strong></td>
<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
</tr>
<tr>
<td><strong>Mean</strong></td>
<td>5</td>
<td>3.5</td>
<td>3</td>
<td>5</td>
</tr>
</tbody>
</table>

#### 4.3.1 Sense-Making (SM)

Bob showed extensive use of SM in the KG problem. Instances of SM was scattered throughout the KG problem. Bob quickly identified key ideas after reading the problem: “I know that his existing garden is 12 feet wide and 17 feet long...reduce the length and increase the width...new garden to be approximately half...” At first, Bob was a bit hesitant about the meaning of the word “size,” but quickly decided that it means area. During the solving process, Bob continued to refer to the given conditions, verbalizing key ideas to form strategies. For example, he referenced the problem before writing an expression, “he wants to reduce the length and increase the width...I don’t know what that amount is so say that it is x.” Bob was assigned an SM score of 5 since he made sense of the given information, and his transcript showed four additional coded instances of SM.

Bob showed similar use of SM in the CCR1 problem. He summarized the problem statement and identified key ideas. After reading the problem, Bob said:

…so the given information says that it takes him...it takes Brett 100…um, 6 seconds to complete the last 100 meters. However, he only has at the point that we’re talking about, he only has 80 meters left of the race to complete.
He set a goal to determine Brett’s time for 80 meters. Bob made repeated reference to the problem statement and given information throughout his work.

Bob had a hard time with CCR2. He decided that he wouldn’t be able to do the problem after only two minutes: “Can I say I don’t know?” He was urged on by the interview: “…but what do you know?” Bob struggled to form a relationship between the time “t” and the given quantities. Bob claimed that he could “see it” but didn’t know how to incorporate it, “putting it into a function.” For CCR1, Bob was assigned SM score of 5 since he identified the given information, and his transcript showed additional five instances of SM.

4.3.2 Representing/Connecting (R/C)

Figure 4-3 shows Bob’s diagrams for the original garden and new garden. On the new garden, Bob depicted the change in the original length and width by using two different expressions to represent the changes. During problem solving, he indirectly used his diagrams and expression to reason about his strategies to determine the new length and width that satisfy the condition of new area. After arriving at the solution, Bob drew a third rectangle to represent the final dimensions. Bob was assigned R/C score of 5 since he drew accurate diagrams and used it to reason about Ken’s new garden.

In CCR1, Bob implied that he divided up the total distance into increments but did not clearly describe or use any visual representations. The interviewer suggested that
perhaps Bob processed the relationship between the numbers in terms of common
increments or "chunks." Bob was assigned R/C score of 2 since he did not display any
further connections to mathematical concepts or visual images.

4.3.3 Reviewing (RV)

Only two instances of RV were coded in Bob’s interview episode of the KG
problem. In one instance, Bob reminded himself that area of the new garden "should be
roughly 100." Bob appeared to be rushing and did stop to monitor his work. After he had
found the answer using the calculator, Bob wrote “17 – 13” and “12 + 13” to verify that his
new length and width is 4 by 25; satisfying the approximate dimensions that yield the
approximated area of 100 square feet. Bob was assigned RV score of 3, although he
checked his solution, Bob showed limited instances of monitoring and assessing his work
his work during the problem-solving process.

In solving the CCR1, instances of RV were observed but the purpose of the
behavior was somewhat unclear. Bob’s dialogue during problem solving hinted that he
was monitoring his work:

So it takes him 3.2 seconds to run each 20 meters, so you can apply
that, so you can say [writing] 3.2 times 4, uh, since he's doing it four,
un…since those are in 20 seconds interval and it's 80 which is 12.8
seconds…

He was speaking while writing to monitor his steps. Bob made sure that his steps were
aligned with his overall goal of breaking the problem into smaller “chunks”. Towards the
end, he suggested that Brett finished the race first due to his shorter time but did not
clearly check his solution. Bob was assigned RV score of 3 since instances of RV were
unclear and Bob’s solution was not complete.

4.3.4 Justifying (JU)

Instances of JU of the KG problem during Bob’s interview were both stated
verbally and written by him. During the interview, Bob explained that he used trial and
error to find the solution by determining the value of x: “I guess I did a bunch of trial and errors to see what it could be... just trying to get closer and closer to 100... so x is 13.” “What it could be” was referring to the change in dimensions or “x” value in the expressions “12 + x” and “17 - x.” Figure 4.4 shows Bob’s written work that supported his explanation of why x equals to 13 would yield the appropriate result; he substituted 13 for x into the two expressions (above) to yield dimensions of 4 feet and 5 feet. Bob was assigned JU score of 6 since he clearly explained how the result of his guess-and-check method yields the appropriate solution.

![Figure 4-4 Bob’s work showing his justification for x =13](image)

In the CCR1 problem, Bob explained his strategy of breaking the problem down into “chunks” to as a mean to calculate Brett’s time:

I guess I can break that down into five different um... I guess 20 goes into a 100 five times... so then you can break down 16 seconds into 5 times [use calculator to calculate 16/5 = 3.2] to see how long it takes for him to run that 80

After thinking about the given information, Bob broke 100 and 80 into “chunks” of 20. In doing so, Bob explained that he needed to find how long Brett takes to run 20 meters (and multiply it by four) to determine Brett’s time to the finish line. After he calculated the
time of 3.2 seconds for distance of 20 meters, Bob said, “3.2 times 4…which is 12.8.” Although his explanation of the method used could be clearer, Bob was assigned a JU score of 4 because it was adequately aligned with how he progressed through his work.

4.4 Comparing Survey Item Responses to Problem-Solving Practices

This section compares students’ responses in the MPS surveys taken during the interview to actual problem-solving practices scored by the researcher. Most of the students’ choices were supported by evidence in their problem solving and verbalization documented during the individual interviews. The survey items associated with the Fun Golf problem are represented using the scheme “FG item X” which would refer to “Fun Golf Item X.” This scheme is also used for survey items associated with the KG and CCR problem.

4.4.1 Kim’s Likert-Item Responses

For FG item 1 (see Figure 4-5), Kim indicated “Mostly A” which suggested that her initial step in solving the problem mostly involved “first thinking about each quantity in the problem and how it could change with a price increase.” Kim’s written work supports this choice selection – she suggested the thinking of price increase by writing down the quantities (revenues) that were associated with the price per round of $5 and $6.

![Image](image.png)

Figure 4-5 A snapshot of Kim’s response to FG Item 1
For FG item 2, Kim indicated that to reach the solution, her approach mostly involved “Only (A)” or “finding a pattern between price, and revenue using a table, function, or graph.” This is supported by her work – she made a table that included the price per round, the number of rounds, and revenue per week (see Figure 4-6) and relied on this table during the problem-solving process to arrive at the solution.

![Figure 4-6 Table created by Kim in the FG problem](image)

For FG item 3, Kim indicated “mostly A” which means that she verified her solution by mostly “showing a table of values with the greatest revenue value circled.” This is supported by her work. Although Kim did not include the “greatest revenue” in her table, she used the table to establish a pattern then use the calculator to determine the desired value and wrote down “$15 for 1 round.”

For FG item 4, Kim chose “Mostly A”, suggesting that to point out why her answer yields the maximum value, her approach was to mostly “point out that increasing or decreasing the round would give less revenue.” This choice is supported by evidence from the interview. Kim implied decreasing the round would give less revenue by saying, “once it hit to $15 per round it got to the max of 1050 and once and I start doing $16 a round it went back down, so the maximum was 1050 for the total so I should charge $15.”
For FG item 5 (see Figure 4-7), Kim chose “Mostly (B).” This choice suggested that if a student was to indicate that higher price per round will raise revenue, Kim thinks it would be mostly helpful for this student to compute and compare the prices of $5 and $10. This choice did not reflect Kim’s work in the FG problem. In problem solving, Kim showed that the revenue continues to increase from $5 to $10 per round, giving no indication that she supported the idea that “any price per round will raise more revenue.”

![Figure 4-7 A snapshot of Kim’s response to FG item 5](image)

For KG item 1, Kim indicated that that her initial approach to solving the problem involved “Only (A)” or “first calculating the area of Ken's garden.” Kim’s work is aligned with this choice. Right after reading the problem, Kim drew a rectangle and labeled its length, width, and area as 204 ft².

For KG Item 2, Kim indicated “mostly (B)” which suggested that in the process of solving this problem, her problem-solving approach mostly "relied extensively on a diagram (e.g. a rectangle with labeled edges)." This choice is aligned with Kim’s solution path. In solving this problem, Kim drew two rectangles to represent the two gardens and formulate an equation that aligns with her drawing.

For KG item 3 (see Figure 4-8), Kim indicated “only (B)” to suggest that if she was provided a rectangular diagram, she would “mark the sides with the original lengths of the garden.” This is supported by her work. Figure 4-9 shows that Kim labeled the
diagram of Ken’s original garden with the given length and width. To further support this choice, Kim did not label the dimensions for her diagram of Ken’s new garden.

![Diagram of Ken's original garden with the given length and width.](image)

**Figure 4-8 KG item 3**

For KG Item 4, Kim indicated “mostly (B)” which suggest that to explain why Kim’s solution is correct, she would mostly compare Ken’s new garden to Ken’s original garden. This choice is supported by evidence in the interview. After determining the appropriate dimension for length and width of Ken’s new garden, Kim indicated that the dimensions yield the desired value of half the original garden’s area, “So I'll say this new garden will have to be 4ft by 25ft since its approximately 102, not exactly.”

For KG item 5, Kim chose “Only (A)” indicating that if a classmate divides the length of the garden in half to find its solution, it would be more helpful for him to “only check that his approach agrees with the conditions given in the problem.” Evidence in
interview supports Kim’s response. Kim made a mistake of reducing both the length and width while solving the KG problem, which she later clarified, “I misread the question…it says reduce the length and increase the width…” This statement suggests that Kim was well aware of the problem’s given condition that the length of Ken’s garden cannot be reduced without an equivalent increase in the width.

In KG Item 6, Kim chose “only (B)” suggesting that if a classmate divided both the length and width of the garden by 2, it would be more helpful for her to only “verify that her approach to the problem agrees with the conditions.” Similar to the explanation for KG item 5, since Kim caught herself making the mistake of reducing both the length and width, it is appropriate to assume that she would suggest checking the given condition to someone making the same mistake.

4.4.2 Bob’s Likert-Item Responses

For KG item 1, Bob indicated “Mostly (A)” suggesting that his initial approach to solving the KG problem is mostly similar to “first calculating the area of Ken’s Garden.” Bob’s problem-solving approach supports this choice; his first course of action was calculating the area of Ken’s original garden.

For KG item 2, Bob indicated “mostly (B)” to assert that in the process of solving the KG problem, his approach mostly “relied extensively on a diagram (e.g. a rectangle with labeled edges.” This choice is supported by his work. Bob drew two rectangles in which he labeled the second one with dimensions of \((17 - x)\) and \((12 + x)\) to represent changes in length and width. Bob used his diagram and written expression to reason about the dimensions of the new garden.

For KG item 3, Bob chose “Lean Toward (B)” which suggests that when given a rectangular diagram, he would most likely lean toward “marking the sides with the original lengths.” Bob’s work supports this choice. Although he labeled the first rectangle with
original dimensions, Bob also labeled the new rectangle with the modified dimensions (see Figure 4-10).

![Figure 4-10 Bob's diagrams representing Ken's original and new garden](image)

For KG item 4, Bob chose “Mostly (B)” which indicates to best explain why his solution is correct, he would mostly “compare the new area of Ken’s new garden with the area of his original garden.” Bob’s verbalization of his approach during the interview supports this choice. He explained his approach in saying, “plugging in different answers…how close I could get to half the area.” Bob’s informal approach to this problem would also suggest that he would compare the areas rather than use words, graphs, or equations.

For KG item 5, Bob indicated “Mostly (A)” to suggest that if a classmate divides the length of the garden in half to find his solution, it would be mostly helpful for him to “check that his approach agrees with the conditions given in the problem.” Bob’s problem-solving approach supports this choice. In problem-solving, Bob showed that he had a clear understanding of the conditions for the new length and width when he chose different values for his trial-and-error approach.

For KG item 6, Bob chose “mostly (B)” to indicates that if a classmate divides both length and width by 2, it would be mostly helpful for her to “verify that her approach agrees with the conditions given in the problem.” This choice is aligned with Bob’s choice
on KG item 5; Bob’s clear understanding of the problem’s constraints would support his recommendation in checking the given conditions.

For CCR item 1, Bob chose "only (B)" which suggests that his initial idea for solving the problem is most similar to only “first drawing a diagram or consider the position of the two runners at different times.” This is somewhat supported by evidence in the interview. In the interview, Bob considered the position of each runner at the same time rather than different time; Brett at 80 meters and Charlie at 100 meters.

For CCR item 2, Bob chose “only (A)” to suggest that to reach a conclusion his approach “did not use graphs of the function at all.” This choice is supported by his work since there were no usage or mention of graphs or functions in the problem-solving process.

For CCR item 3 (see Figure 4-11), Bob chose “None” since he was unable to complete CCR2. Bob explained that he would check his conclusion about who finished first by comparing finish times.

![Figure 4-11 A snapshot of Bob's response to CCR item 3](image)

For CCR item 4, Bob indicated “Only (B)” suggesting that to explain why Brett, who runs slower than Charlie for the last 100 meters of the race, wins, he would only “show how both the position of each runner when t = 0 and the rate of each run
influences who wins." Evidence from the interview somewhat supported this. Bob did consider the position of each runner at t = 0 but he did not calculate their rate.

For CCR item 5 (see Figure 4-12), Bob indicates “Mostly (A)” which means that if a classmate concludes that Charlie finished first since 14 seconds < 16 seconds, it would be more helpful for him to mostly "consider how far each runner must run to reach the finish line." Bob’s work supports this choice. He recognized that the two runners are not at the same place in the race, so he calculated the time it takes for Brett to reach the finish line from 80 meters away and not 100 meters away (where Charlie is).

Figure 4-12 A snapshot of Bob's response to CCR item 5

4.5 Subject-Matter Knowledge

Students’ written work and evidence from the interview suggested that both Kim and Bob preferred to use informal strategies when solving mathematics problems. In this section, Kim’s and Bob’s subject-matter knowledge or algebra knowledge demonstrated during the interviews is interpreted based upon the coded instances of heuristics in the interviews.

4.5.1 Kim’s Algebra Knowledge

Kim’s work on the FG problem (see Figure 4-13) depicted a combination of pattern-seeking and trial-and-error approaches. In solving this problem, Kim mentioned, “…I thought it would just keep going up but I realized maximizing and minimum would mean quadratic.” This would suggest that Kim has some understanding of quadratic
functions and their visual characteristics. In this case, Kim suggested that the quadratic relationship would increase and eventually decrease at some point after reaching the maximum. Relying on this knowledge, Kim created a table with a set of quantities to establish a quadratic pattern.

Kim’s work in the KG problem consisted of trial-and-error, pattern-seeking, and an algebraic approach. Kim started solving using trial-and-error then switched to writing an equation to represent the problem’s constraint (see Figure 4-14). In solving the KG problem, Kim showed that she could translate the given word problems to a quadratic equation. As shown in Figure 4-14, Kim was able to perform algebraic manipulations on her equation – Kim used the distributive property to expand her equation in to solve for the variable x. While manipulating her equation, she also displayed the ability to perform “complete the square” procedure. Although she correctly executed the procedure, this method did not lead to a solution due to the arithmetic error Kim resorted to using trial-
and-error and appeared to have relied on the structure of her equation to guide the informal problem-solving process.

![Figure 4-14 Kim’s work for the KG problem](image)

### 4.5.2 Bob’s Algebra Knowledge

In solving the KG problem, instances of heuristics showed that Bob relied mostly on the use of informal trial-and-error strategy. Although he did not use any formal algebraic strategies in his problem solving, Bob was able to write expressions using the variable “x” to represent the change in dimensions. Figure 4-15 shows that expressions “12 + x” and “17 – x” were dimensions for the diagram of Ken’s new garden. After sketching the diagrams, Bob immediately uses the calculator to input possible values for the variable “x” that would yield the desirable area.
In solving the CCR problem, Bob also did not use any formal algebra related strategies, instead, he relied mostly on basic arithmetic operations to manipulate the given information (see Figure 4-16). Bob was able to arrive at the correct solution for CCR1 but was unable to develop an equation to meet the requirement of CCR2.
Chapter 5
Discussion

Analysis of the interviews of Kim and Bob and comparisons to their MPST scores follows several interpretations which will be discussed in this chapter. The consideration of these results in relation to existing studies, implications and remaining questions for further investigation comprise the central focus of this chapter.

5.1 Comparing Kim’s Problem-solving Practices to Survey Scores

Table 5-1 shows Kim’s survey scores (from the pre-MPSI assessment) and her interview domain scores for each problem. Her survey scores suggested high-level use all four MPS domains when solving mathematical problems.

<table>
<thead>
<tr>
<th>Survey scores</th>
<th>SM</th>
<th>R/C</th>
<th>RV</th>
<th>JU</th>
</tr>
</thead>
<tbody>
<tr>
<td>FG interview scores</td>
<td>4</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>KG interview scores</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>Mean interview scores</td>
<td>3.5</td>
<td>4</td>
<td>5</td>
<td>4</td>
</tr>
</tbody>
</table>

5.1.1 Interview Scores for FG Problem and Survey Scores

Based on Kim’s problem-solving practices in the FG problem, the author scored her use of SM, R/C, RV, and JU as 4, 4, 5, and 6 respectively (see Table 5-1). Kim’s interview scores indicated that her problem-solving practices in regards to the MPS domains ranged from relatively high to high. With the exception of SM, the other MPS domains were easily observed during the interview. Although Kim’s interview scores show a relatively high level of SM, it should be noted that some instances of SM during the FG
problem were initiated by the interviewer and others were based on body language. If the interviewer had not intervened and asked what Kim has done so far, she might not have explained her thoughts that included instances of SM. That is, there were instances where Kim remained quiet for an extended period without explaining what she was thinking. However, this does not mean that instances of SM originated only from the interviewer’s questions. The researcher coded Kim’s other instances of SM from Kim’s behaviors such as: reading quietly to herself, reviewing given information, rereading the question, and thinking. Although these instances were not verbalized, these behaviors display deep thinking suggested that Kim could be engaging in SM (although this is only speculation). This finding is similar to what was mentioned in Carlson and Bloom (2005), they explained that SM usually occur within the orienting phase that is often characterized by intense thinking about the solver is trying to construct a “personal representation” of the problem (p. 62). Kim’s behavior could be link to intense thinking about prior knowledge regarding quadratic relationship.

5.1.2 The KG problem and Survey scores

Based on evidence from the interview, the researcher scored Kim’s use of SM, R/C, RV, and JU as 3, 4, 5, and 2 respectively. Observed instances of SM and JU were relatively low-level and instances of R/C and RV were on the higher end. As mentioned, there was a lack of verbalization during Kim’s problem solving, which resulted in limited coded instances of SM. As mentioned in Chapter 4, Kim showed very little evidence of justifying her solution paths which resulted in JU interview score of 2. Based on her problem approaches in the KG problem, the lack of JU instances could be due to the time consumed by Kim’s use of two different methods in this problem; she cycled between the trial-and-error and algebraic approaches several times before coming to a conclusion using the algebraic method. Kim reached her solution using the informal trial-and-error
approach – she substituted different values for x in the equation \((17 - x)(12 + x) = y\).

Perhaps if Kim had reached the solution relying fully on the algebraic representation, it might have served as a better guideline for Kim to justify her work. This relationship between representation and justification resembled what was mentioned in Stylianou (2014) where she suggested that the student’s representation of the problem is a tool that could impact the way in which they justify their work and explain their results. Since Kim did not arrive at the solution from algebraic manipulation of her equation, she may not have considered her equation as a useful tool (or representation) that needed justification. That is, Kim was not able to justify her solution using the algebraic method due to her lack of success in using the equation. Kim reached the solution using trial-and-error, which may not evoke the same level of justification or basis for it.

5.1.3 Alignment of Actual Practices and MPS items

Comparison of Kim’s Interview scores and survey scores for the FG problem suggested that her problem-solving practices were mostly aligned with her responses in the MPSI assessment. The R/C and RV domains in the KG problem were aligned between the interview scores and survey scores (both relatively high), but the SM and JU were not aligned. Domain averages for the interview scores show a strong correlation in three of the domains – R/C, RV, and JU. Taking explanations for the low-level use of the JU domain during the KG problem and averages of the two problems into consideration, R/C, RV, and JU domains were well captured by the survey scores. Although the SM domain was scored relatively high (score of 4) in the FG problem according to interview rubric, the true score may be lower since some coded instances were initiated by the interviewer. Overall, Kim’s practices regarding the SM domain were largely inconclusive.
5-2 MPS Items and Bob’s MPS Practices

Table 5-2 shows Bob’s survey scores, indicating a high-level use of SM and JU and low-level use of R/C and RV. As mentioned, Bob was able to complete the KG problem and part (i) of the CCR problem (CCR1), but was unable to complete part (ii) of CCR (CCR2). CCR1 was scored as a separate problem, taking into account only instances of MPS associating with only CCR1 and not the CCR as a whole. Although some progress was made on CCR2, any progress made was a result of the interviewer’s prompting.

Table 5-2 Bob’s survey scores and interview scores

<table>
<thead>
<tr>
<th></th>
<th>SM</th>
<th>R/C</th>
<th>RV</th>
<th>JU</th>
</tr>
</thead>
<tbody>
<tr>
<td>Survey Scores</td>
<td>4.5</td>
<td>2.0</td>
<td>3.0</td>
<td>5.0</td>
</tr>
<tr>
<td>KG interview scores</td>
<td>5</td>
<td>5</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>CCR1 interview scores</td>
<td>5</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>Mean Interview scores</td>
<td>5</td>
<td>3.5</td>
<td>3</td>
<td>5</td>
</tr>
</tbody>
</table>

5.2.1 The KG Problem and Survey scores

In solving the KG problem, Bob’s interview scores for SM, RV, and JU of 5, 3, and 6 respectively were very similar to his survey scores. His extensive use of diagrams that resulted in an interview score of 5 for R/C was not aligned with survey score of 2. It is possible that his extensive use of the diagrams was linked to the type problem that KG represented. Bob mentioned during the interview that he preferred “real-world application” word problems, which were aligned with the characteristics of the KG problem. Bob’s problem-solving behaviors showed almost no struggle while working on the KG problem. The problem statement was a type that Bob suggested he would prefer – a situation based on a “real world application.” Once the diagram was drawn and
labeled, Bob relied on the information included (dimensions of “new” length and width) in his diagram (the expressions for length and width) as he applied the trial-and-error method to find the solution.

5.2.2 The CCR1 Problem and Survey scores

Bob’s interview scores for all four domains indicated that his problem-solving practices for the CCR1 problem are consistent with what the MPS items suggested regarding Bob’s problem-solving domains. Bob showed a relatively high-level use of SM and JU and low-level use of R/C and RV. Unlike the KG problem, his interview score for R/C reflected the survey scores. It should be noted that although the CCR problem statement also described a real-world situation, Bob did not use any visual representation to assist with his problem-solving approach. In regards to the lack of R/C in CCR1, Rhoads et al. (2017) found in their study on undergraduates’ use of R/C that students are more likely to use a representation when one (such as a graph or diagram) is specifically mentioned in the problem statement. Unlike the KG problem, the CCR problem statement did not mention the mention any form of representation which could explain Bob’s low score on R/C.

5.2.3 Alignment of Bob’s Actual Problem-solving Practices and MPS items

Generally, Bob’s behavior during the interview reflected the results indicated by the MPSI assessment. In the case of R/C, he showed extensive use in KG but low-level use in CCR1. As mentioned, it is possible that Bob’s use of R/C is more extensive in problems that suggest the use of graph/diagram in the problem statement as reported by Rhoads et al. (2017). Overall, Ken’s problem-solving results showed alignment between the SM, RV, and JU domains. The use of R/C was inconsistent between the two problems.
5.3 Regarding Visual Representation

Rhoads et al. (2017) reported that undergraduate students use R/C in three ways: (1) visualize/sketch graphs (2) draw diagrams, and (3) making mathematical connections. This is consistent with how R/C was used in this study, Kim and Bob both use R/C by drawing a diagram or connecting to other mathematical ideas. In studying the role of visual representation, Stylianou & Silver (2004) stated that novice problem solvers usually abandon their diagrams in favor of familiar computations. Kim and Bob’s use of their diagrams were unlike the majority of the novices in Stylianou and Silver’s study. Kim referenced her diagram to write an equation which she then used extensively and Bob used his diagram to reason through his guess-and-check method. This observation could also be due to the problem’s level of difficulty. A surprising observation is that Bob, who score very low on his R/C domain in the pretest, possibly showed a more extensive use of his diagram then Kim who score relatively high on her R/C domain. This may suggest that the R/C score for this problem may be dependent on behaviors that are not representative of R/C or that the R/C domain, as indicated by the items, is dependent on the type of problem posed.

5.4 MPS items and Student’s Actual Practices

For the MPS items to accurately assess student’s problem-solving capacity, students’ choices on the Likert-scale items must reflect their actual problem-solving practices. As mentioned, Kim has not seen the Likert-items for the FG problem (since it was a new problem) before the interview but she had seen the survey for the KG problem (which was a “redo”) in the pretest. Likewise, Bob had not seen the survey for KG problem prior to the interview but had seen the survey for CCR in the pretest. Thus, their survey scores did align with their interview scores perhaps because Kim and Bob’s
responses to the Likert items were an accurate reflection of their actual problem solving practice.

5.4.1 Kim’s Likert Items

The FG problem had 5 associated Likert items in which Kim’s choices on four out of five were supported by her work. Based on evidence from the interview, Kim’s choice on FG Item 5 did not reflect her work. Kim suggested that if a classmate were to indicate that higher price per round will raise more revenue, then it would be most helpful for this student to compute and compare the revenues for prices of $5 and $10. This decision is not consistent with the work shown in the FG problem. Kim had shown a clear understanding that the relationship was quadratic; the increase in price would eventually result in a decrease in revenue. Furthermore, Kim concluded that revenue started to decrease at the prices of $16 per round. A re-examining of the recording shows that Kim read the item twice before making her choice. It is possible that Kim might have misinterpreted the provided pathways. Perhaps by rewording to emphasize the faulty indicates that…"

All 6 assessed Likert items associated with the KG problem were supported by evidence in Kim’s problem-solving-practices. Item 2 was mentioned in an earlier study – some students claimed to have used the diagram extensively in their solution path but their actual problem-solving practices during the interview showed otherwise (Epperson et al., 2016). This was not the case with Kim, her choice of having relied on a diagram was supported by evidence in the interview.

It is also worth noting that for the KG problem, although Kim’s general problem-solving strategies did not change, her choice on KG item 4 on the pretest survey was “Mostly (A)” but Kim chose “Mostly (B)” in the interview. When the interviewer noticed this change, he asked Kim if she would consider choosing a choice closer to (A). After
pointing out to Kim that although she did do the things in (B), she also did most of the items that described in (A), Kim said, “I mean, now that you have explained it I would go towards A.” In the result section, neither (A) or (B) were regarded as the better choice since Kim did display the use of both methods in her problem solving. The interview format could be responsible for this change.

Out of 11 Likert items, only one was not supported by Kim’s work. This one item could be ruled out due to interview format or carelessness. Overall, Kim’s choices in the MPS items do align with her actual problem-solving practices.

5.4.2 Bob’s Likert Items

For the KG problem, Bob’s choices on the Likert items were all supported by evidence from the interview. For the CCR problem, Bob’s choices were also mostly supported by evidence from the interview, although some “only” choices might have been more appropriate as “mostly.” These cases can be seen in the “Only (B)” choice in item 1 and 4. For Item 1, Bob claimed that he considered the positions of the two runners at different times, but his work showed that he considered them at the same time – a “Lean Toward (B)” or “Mostly (B)” would be more appropriate. For Item 4, Bob claimed that to explain why Brett finishes first (even though he runs slower), he would show how both the position of each runner when t = 0 and the rate of each runner influences who wins. Bob did consider t = 0 but did determine the rate of each runner – a “Lean Toward (B)” or “Mostly (B)” choice would be more appropriate.

Furthermore, Bob’s choices in the pretest and interview for the CCR problem were mostly consistent. Bob’s choices in the MPS items were aligned with his actual problem-solving practices.
5.4.3 Interview Context Affecting Student’s Practices and Choices on Likert items

Bob suggested that he “might have been able” to solve CCR2 outside the interview, he claimed that the pressure from the interview might have affected his problem solving. After the conclusion of the KG problem, Kim suggested that she only attempted different methods and showed additional work because she thought it was expected of her in a task-solving interview. Both participants suggested that they were at most negatively affected by the interview environment.

An interesting incident occurred in Bob’s interview could be responsible for his “only” choices on CCR item 1 and 4. The interviewer made a comment that none of Bob’s choices were “only (A)” or “only (B)” during in the KG survey. Bob suggested that choice was somewhat influenced by one of the provided paths: “that’s how I see it because I don’t say ultimatum like only…it has a little, just a little leeway…” The fact that the interviewer asked him could put some constraints on his choices or trigger some emotional reaction that resulted in his decision in choosing “Only” in the following survey.

These incidents were similar to what was suggested in Goldin’s (1997) study on observing MPS through task-based interviews. He suggested that contextual influences, like Kim’s belief that she had to show more work in an interview than she would normally, can hinder the observation of her actual competencies. Goldin stated that “task-based interviews do not take place outside of a social and psychological context” and that the context affects the interview’s interaction (Goldin, 1997 p. 58). Perhaps Bob would have been more successful with CCR2 without the interview pressure, and Kim would show even less work on paper.

5.5 Subject-Matter Knowledge Affecting MPSI Assessment

Both Kim and Bob’s Subject matter knowledge based on their problem solving strategies was documented in Chapter 4. Instances of content-specific heuristics in the
interview indicated that the only formal algebraic domain knowledge that Bob demonstrated was the ability to write simple algebraic expressions (in the KG problem). Bob’s problem solving consisted mostly of informal strategies. Kim, on the other hand, displayed some formal algebraic domain knowledge in both problems. Kim was able to sufficiently manipulate algebraic equation, perform “Complete the square” procedure, and displayed some understanding of the characteristics of quadratic equations. Like Bob, Kim also relied mostly on informal problem solving methods rather than algebraic approaches.

While responding to the MPSI assessment in the interview, Kim and Bob gave some indications that their subject matter knowledge influenced their responses on some items. After choosing “Mostly (A)” for FG item 4, Kim said, “I knew it was it was a quadratic so it would have to turn around.” Kim suggested that her knowledge regarding the shape (or behavior) of a quadratic function lead her to this response. For KG item 3, Bob indicated that he chose “Lean Toward (B)” because he would “Mark the original length of the problem and then change this...” Bob suggested that his response was influenced by the fact that he was able to write the algebraic expressions to represent the dimensions of Ken’s new garden.

Since only two MPS items were observed to have been influenced by the students’ subject-matter knowledge, the rest of the items might have been influenced mostly by students’ problem solving strategies. The results also indicated that although domain knowledge was not observed to have directly impacted the MPS items, the lack of influence of domain knowledge on problem-solving strategies could have impacted Kim and Bob’s the survey responses. In other words, domain knowledge could have been observed to impact the MPS items if the knowledge was applied during problem solving. Jonassen (2000) suggested that although knowledge about a particular domain
can greatly influence the problem solver’s success, that knowledge must be structured and integrated (perhaps through practice) to positively affect the outcome of one’s problem solving. The lack of reference to algebra knowledge observed in Bob’s problem solving may be the result of a lack of general algebra domain knowledge. Kim, on the other hand, indicated that she had the algebraic knowledge associated with each problem but was unable effectively use what she knows. As a result, both participants depended on informal strategies (e.g. guess-and-check) as the main approach in their problem solving. These strategies in turn affect their responses to the MPS items.

5.6 Affective Factors Influence Problem solving

At the beginning of the interview, both Kim and Bob expressed that they enjoy mathematics and prefer mathematics over the other classes. Kim expressed that mathematics was her favorite subject and suggested that she is good at problems that are application driven. Like Kim, Bob suggested that he preferred problems that based on “real world” setting and are not “abstract” (like Calculus)."

There appears to be a link between how the student feels about mathematics and their approach to problem solving. This observation can be seen throughout the interview as Kim and Bob worked on problems were concrete and rooted in real life situation. Kim appeared confident throughout the problem solving process and even with some difficulties while solving the KG problems, she was able to reach a satisfactory solution for all the problems. Similarly, Bob appeared comfortable with problems that are concrete and based on realistic situations. There was no sign of struggles when he worked on the KG problem, and he was found a creative way to solve CCR1. However, this trend came to a halt when Bob reached the CCR2, which asked the solver to write two equations. Bob spent no more than a minute after reading this problem before deciding that he could not do it – “can I say I don’t know?” it appears that Bob had quickly
made up his mind due to the problem statement not being concrete or based on a real-world situation. These instances suggested that Bob and Kim somewhat predicted what they can solve mathematically before even seeing the problems. Furthermore, these findings suggest that either the students were well aware of their mathematical abilities or they are only willing to perform up to their perception of personal mathematical ability. Although the results are too limited to support these claims, other studies have shown that students perform well on things that they believe to be good at (Schoenfeld, 1989).

5.7 The “Challenge” Domain

Comparisons between interview scores and survey scores possibly reveal that on the problems experienced as more challenging for Kim or Bob the alignment between the two sets of scores was less pronounced. For instance, the interview transcript shows that Kim had more difficulties on the KG problem than the FG problem and the results show that the KG interview scores were less aligned with her survey scores. As for Bob, he was able to solve CCR1 but not CCR2, making it the more difficult problem. Although his interview scores for CCR1 aligned with the survey scores, the outcomes were inconclusive. Although these findings are limited due to the small sample of interviews, it is possible that the difficulty level of a problem may be interacting with how well the MPST items align with actual practices. However, both students engaged in MPS at several levels, so the problems examined were “problems” for Kim and Bob in the sense of Lester and Cai (2014).
Chapter 6

Conclusion

Results suggest that the Kim and Bob’s survey scores obtained from the MPSI assessment followed a similar pattern to the interview scores determined by the researchers when analyzing their think-aloud interviews. This similarity suggests that the survey scores may be good indicators of students’ actual problem solving capacity as determined by the outcomes linked to each MPS domain. In Kim’s case, her interview scores aligned closely with her MPS item scores in R/C, JU, and RV. However, there was a discrepancy in her SM domain scores which may be explained by her tendency not to think-aloud or verbalize during the interview making it difficult to classify her use of SM in the interview. For Bob, his interview scores aligned closely with his MPS item scores in SM, RV, and JU domains. However, contrary to what his survey scores indicate, Bob demonstrated a high-level use of R/C in solving the KG problem which may indicate that on the MPST items that link to R/C may have been problem-dependent for Bob or in general and should be investigated further. With the exception of SM for Kim and R/C for Bob, the close alignment between their survey scores and interview scores linked to the MPS domains is encouraging.

Examining students’ choices on the Likert items during the interview and comparing them to their actions and behaviors shows that students’ choices mostly align with their actual practice. On all the Likert items associated with the problems given in the interview, only one item (FG item 5 in Kim’s interview) did not support the student’s actual practices. There were two responses (in two MPS items) that could have been more accurate (“lean toward” or “mostly” instead of “only”). With a broader lens, the alignment of students’ responses to their actual problem solving practices suggests that the solution
paths presented in the MPS items may be effectively providing information regarding as
student’s MPS capacity and linked to the appropriate domains which supports the
reliability of the survey scores.

With respect to domain-knowledge and its role in student responses to the MPST
items, both Kim and Bob showed that they preferred informal approaches over formal
algebraic methods. Both participants solved their problems using mostly using a trial-and-
error approach and there was some indication that this was used due to lack of domain
knowledge (i.e. solving the equations) instead of a preference for using a particular
heuristic over purely algebraic approaches. For example, there was one instance each
for Kim and Bob that suggested that their algebra domain knowledge was linked to their
chosen responses.

As Goldin (1997) asserts, it may be the case that the interview environment may
be affecting students’ MPS performance in a manner that students underperform or that
students may actually exhibit a higher capacity in the MPS domains in a non-interview
setting. However, the close alignment between survey scores and interview scores in the
MPS domains may indicate that this issue was not a major factor in this analysis.

Further work analyzing more student interviews and correlating them with MPST
scores is needed to corroborate the findings in this snapshot of Kim and Bob’s work. This
study also raises some interesting questions for future work related to the MPSI
Development project such as exploring the influence of the “challenge” domain on the
MPS Items in relation to the reliability of the outcomes per student and exploring
correlations between the accuracy of student work on the MPST to their domain scores
and answers on the MPST items.
References


Biographical Information

Duy Phan was born in Danang, Vietnam on February 2, 1988. He graduated from Southwest High School in the Fort Worth Independent School District (FWISD) in the top ten percent of his class in 2006. He earned his Bachelor of Science in mathematics from the University of Texas at Arlington in 2010. In 2011, he began teaching mathematics at Southwest High School and is currently teaching pre-calculus at Eastern Hills High School in FWISD. In May 2017, he received his Master of Science in mathematics from the University of Texas at Arlington in Arlington, Texas.