LOAD CARRYING CAPACITY MANAGEMENT OF COMPOUND ANISOTROPIC COMPOSITE RINGS UNDER EXTERNAL PRESSURE

by

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Abstract

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Composites have much higher strength than conventional materials such as steel. This makes it attractive to replace heavy loaded structural elements, made traditionally from steel, by elements made from composites. However, it is not a trivial task for statically indeterminate structures due to anisotropy of strength and elastic properties of composites. Present investigation is done for a ring subjected to external load. Such rings are used as bulkheads in many applications. Dependence of ultimate external pressure on thickness of the ring is nonlinear one. It is related to non-uniformity of distribution of circumferential stresses, with anisotropy of material, with set of other reasons. To improve performance of such structures, three ways were analyzed: increase of transversal compression strength of composite by redistribution of part of the fibers to axial direction, by redistribution of stresses in body using profiling and by making the ring from set of coaxial rings produced from different materials (step-wise non-homogeneity).

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Chapter 1

Introduction and Background

Composite materials have much better properties than conventional materials as they can be tailored to meet desired specifications. These properties make them desirable as replacements for conventional materials used in heavy structural elements. Improvements are made by attaining decrease in weight and increase in strength. However, replacing conventional materials with composites is not such a simple task as composites are highly anisotropic and heterogenous. Cylindrical structures are very commonly used as pressure vessels, tanks, etc. Composites are being considered in production of such cylinders due to their excellent properties. This investigation mainly focuses on thin-walled cylinders with low width, i.e., rings under external pressure. Thin walled cylinders or rings under external pressure are used as supporting bulkheads of cylindrical structures (ring frames of fuselages, rocket systems, deep water or underground cable protection systems, etc. and as stiffeners for pressure vessels. Predicting behavior of rings made from anisotropic materials is considerably problematic. To improve performance of such structures under external pressure, numerous formulations have been made in this study.

1.1 Thin and Thick-Walled Cylinders

Thin and thick-walled cylinders are widely used as pressure vessels in commercial as well as in military applications. The use of fiber reinforced and polymer-based composites have been increasing. Various numbers of applications have also been flourishing with this development. Fuel tanks, rocket motor cases, pipes are some examples of pressure vessels made of composite materials. Ever increasing use of this new class of materials in conventional applications is coupled with problems that are intrinsic to the material itself. Determination of material properties, mechanical analysis and design, failure of the structure are some examples which all require a non-conventional approach. Numerous applications concurrently are accompanied by various researches in the related field. Majority of the studies in the analysis of composite pressure vessels finds their origins in Lekhnitskii's approach [1]. The emergence of composite materials with high specific strength and stiffness has drawn both sectors (commercial and military) to consider using composites in the manufacturing of such cylinders.

(a)

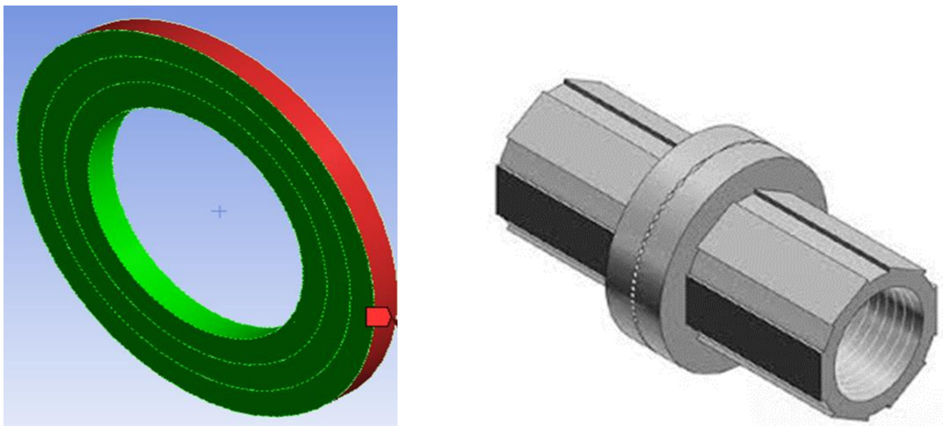
(b)

Figure 1-1 Examples of (a)Thin-walled Cylinders and (b) Thick-walled Cylinders

Thin-walled cylinders under internal pressure are used as main parts of pressure vessels, chemical reactors acting under pressure, pipelines transporting gas, oil, different liquids. Thin walled cylinders or rings under external pressure are used as supporting bulkheads of cylindrical structures (ring frames of fuselages, rocket systems, deep water or underground cable protection systems, etc.). Thick-walled cylinders and rings are used in the cases of extremely high levels of internal or external pressure or when they fulfill additional functions besides pressure resistance. Such functions can be thermal or electrical insulation, blast protection, radiation protection, etc.

In isotropic cylinders the difference between thin-walled and thick-walled cylinders is observed when wall thickness is bigger than 10% of the outer radius in bending problems and in thicknesses comparable to the few tens of percent for pressure problems.

The focus is mainly on analysis of composite cylinders, with low-width (rings), under external pressure in this study. These kinds of rings are used in applications like bulkheads, in different industries, and as stiffeners for pressure vessels, other cylindrical structures, etc.



(a) (b)

Figure 1-2 (a) Diagram of composite rings (b) Example of rings as stiffeners

1.2 Composite Materials

A composite is defined as a material composed of two or more constituents whose mechanical properties are distinctly different from each other, and phase separated such that at least one of the constituents forms a continuous interconnected region and one of the constituents acts as the reinforcement (and is typically discontinuous) [2]. The resulting composite has physical properties that differ from the original constituents. By nature, experimental stress analysis on composites can be considerably difficult, since composites can be highly anisotropic and heterogeneous. Additionally, for anisotropic materials the principal strain direction does not necessarily coincide with the principal stress direction. Failure mechanisms in composites can be also be significantly more complicated than for traditional isotropic materials [10].

The study of composite materials involves many topics, such as for example, manufacturing processes, anisotropic elasticity, strength of anisotropic materials, and micromechanics [11]. Composite materials have record high mechanical properties and they are pushing out many traditional materials. Modern spacecraft, defense industry, and many other industries are significantly relying on composite materials. Almost half of the body of a modern military aircraft is made from composites. Properties of composite materials can vary in wide range depending on selected constituents but mostly depending on material architecture (usually fiber architecture). Material architecture and properties can be designed for each point of the body, adjusted to acting stresses. Optimization of composite structure is a big branch of modern engineering.

1.3 Motivation and Objective

The equation shown below depicts the variance of ultimate pressure, that can be resisted, with thickness of the ring. The ultimate pressure displays linearity (proportional) with increase in thickness of ring. This is the governing equation for pressure variance with thickness of rings (theoretical). However, experimental data significantly deviate from this prediction.

$$q_{max} = \frac{\Pi^{-}.(t)}{R}$$

R = Outer Radius
t = Thickness of the Ring
Π⁻ = Compressive Strength of Material

(a)

$$\bar{\sigma}_\theta = \frac{p(R-t)-qR}{t}$$

(b)

Figure 1-3 (a)Governing equation for ultimate pressure variance with thickness of ring and (b) Integral averaged circumferential stresses equation

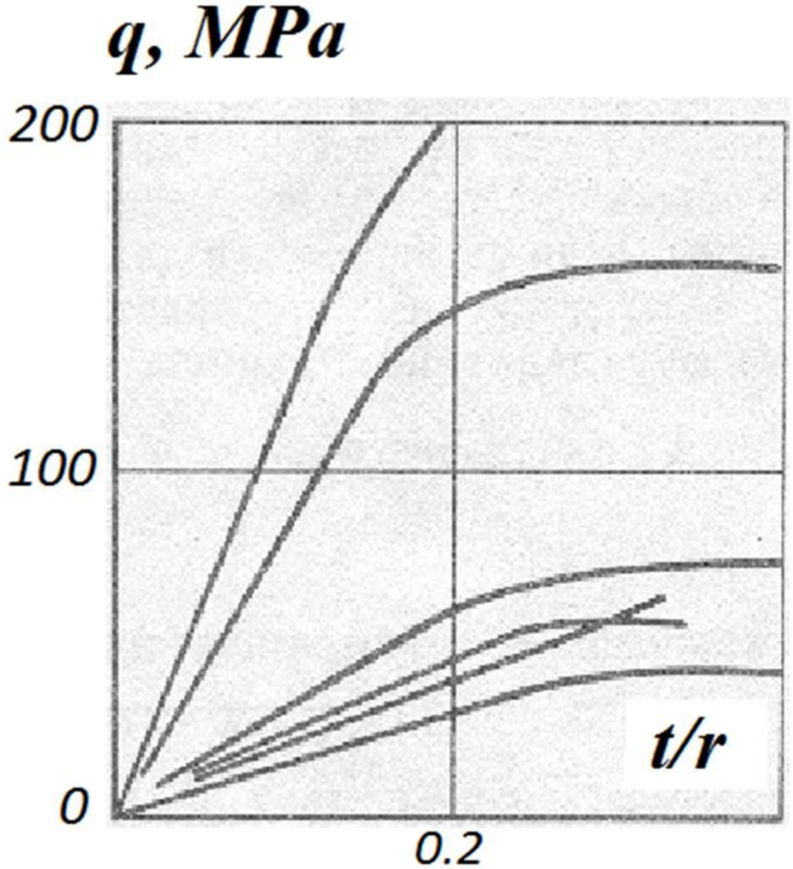


Figure 1-4 Graph showing experimental values of **q** vs. **t/r**

The graph illustrated depicts actual experimental values of variation of ultimate external pressure with the thickness of rings made of different composite materials. The limitations of this equation for thick-rings and anisotropy is clearly observed. The curves in the graph show non-uniform variance of thickness with bearable ultimate pressure for anisotropic materials [3].

One of the objective is to increase the ultimate pressure (leading to fracture) that can be resisted by anisotropic materials with increase of thickness.

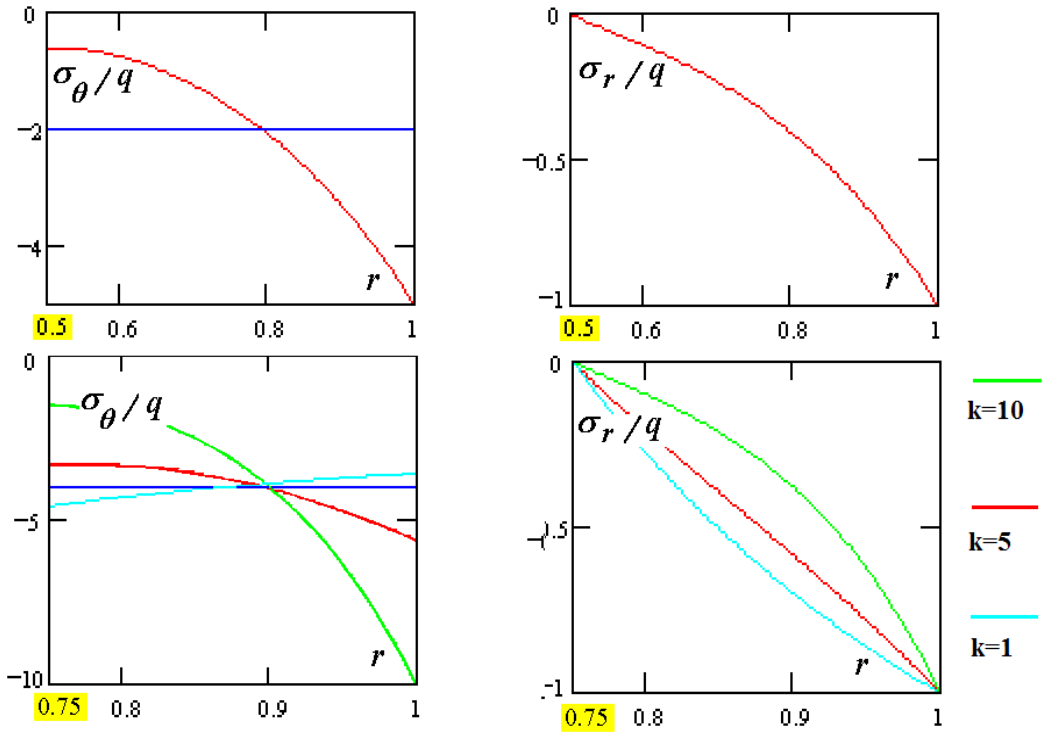


Figure 1-5 Graph depicting non-uniform distribution of stresses upon increase in thickness and anisotropy of ring material [3]

Here,

σ_θ = Tangential Stress

σ_r = Radial Stress

q = Pressure

k $=\sqrt{\frac{E_\theta}{E_r}}$ (Degree of Anisotropy)

r = Relative Thickness

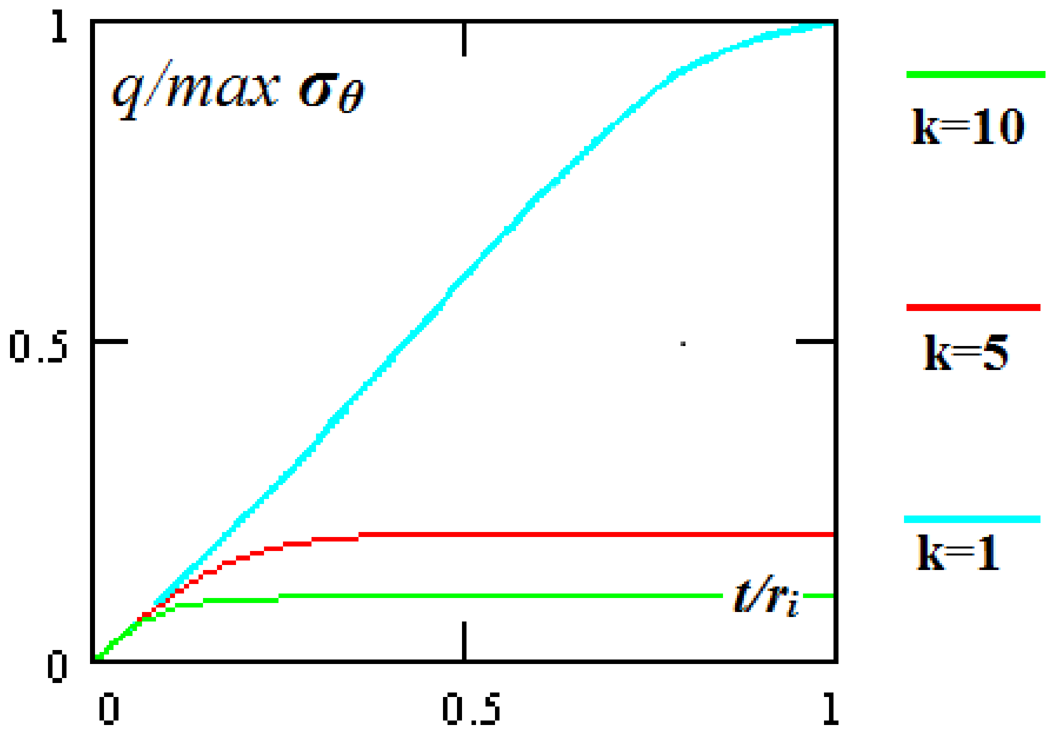


Figure 1-6 Graph showing limitation of radial compressive strength with increase in degree of anisotropy [3]

$$q \leq \left| \Pi_r^- \right|$$

The second limitation observed for the governing equation is that as the degree of anisotropy of the material increases, the more the restriction imposed by radial compression strength on the ultimate tolerable pressure of the ring.

The goal of the study is to overcome or make improvements on these limitations.

1.4 Materials Chosen and Validation of Limitations

The study involves anisotropic materials and their ultimate pressure capabilities. Composites with different aptitudes like high-stiffness (P120 Carbon Fiber), high-strength (T1000-G Carbon Fiber) and high tensile and compressive strengths (S2 Glass Fiber) were chosen to increase performance range. Both carbon fibers have excellent properties, P120 with a very high Young's modulus and T1000-G with high tensile strength [4]. The glass fiber (S2) was selected for its high compressive strength and to balance out the selection of the high stiffness and high strength composites.

The chosen composite materials were coupled with an epoxy matrix using a volume fraction of 0.7 and properties along principle directions were calculated by following method [11]:

I. Obtain the compliance matrix for the fiber and matrix C_1 and C_2.
II. Calculate for the two elements of compliances b_{ij} for the plane strain state for fiber and matrix.
III. Calculate bulk moduli K_{23} for the fiber and matrix
IV. Calculate the effective elastic properties of the Unidirectional composite
V. Substitute values of the effective elastic properties for the composite material in the compliance matrix
VI. Calculate the inverse of this compliance matrix

$$C1 := \begin{bmatrix} \frac{1}{E_1} & \frac{-v_{21}}{E_2} & \frac{\nu_{31}}{E_3} & 0 & 0 & 0 \\ \frac{-\nu_{12}}{E_1} & \frac{1}{E_2} & \frac{v_{32}}{E_3} & 0 & 0 & 0 \\ \frac{v_{13}}{E_1} & \frac{-\nu_{23}}{E_2} & \frac{1}{E_3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{23}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{31}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{12}} \end{bmatrix}$$

$$\frac{\nu 12}{E1} = \frac{\nu 21}{E2}$$

$$\frac{\nu 13}{E1} = \frac{\nu 31}{E3}$$

$$\frac{\nu 23}{E2} = \frac{\nu 32}{E3}$$

Table 1-1 Anisotropic material properties calculated for chosen fibers and matrix (calculated for 70% fiber)

Composite Material	E_θ (GPa)	E_r (GPa)	υ_{12}	υ_{23}	Π_θ(GPa)	Π_r(GPa)	Degree of Anisotropy (k)
P120 Carbon Fiber/Epoxy	582	8.1	0.31	0.34	2.1	0.09	8.477
T1000-G Carbon Fiber/Epoxy	194	8.1	0.321	0.34	4.4	0.1	4.894
S2 Glass Fiber/Epoxy	61.8	13.5	0.253	0.435	3.2	0.13	2.14
Epoxy Matrix	3	3	0.3	0.3	1.11	1.154	1

The ultimate pressure vs. relative thickness graphs (like Figure 1-4 and 1-6) are plotted for the chosen composite materials to validate the observed limitations in the previous chapter. These graphs were obtained using PTC Mathcad Prime 3.1.

Plots for chosen materials:

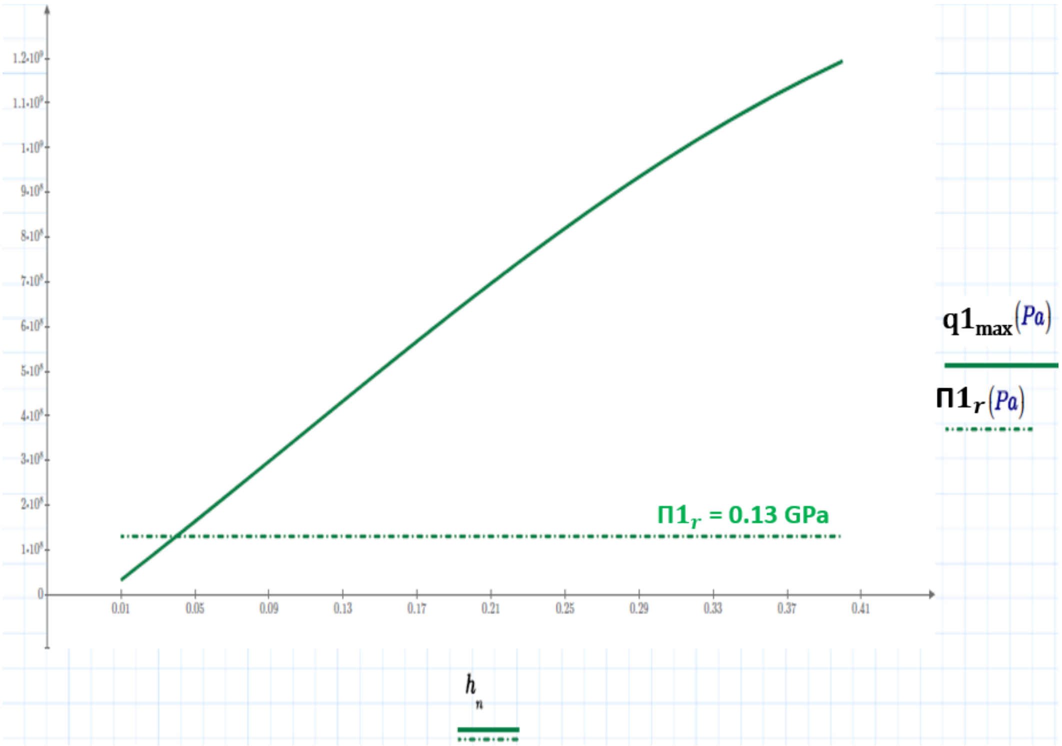


Figure 1-7 (a)Ultimate pressure vs. Relative thickness graph of Material: **S2 Glass/epoxy**

Here,

$q1_{max}$ = Ultimate pressure

h = Relative Thickness

$\Pi1_{r}$ = Compression Strength of **S2 Glass/epoxy**

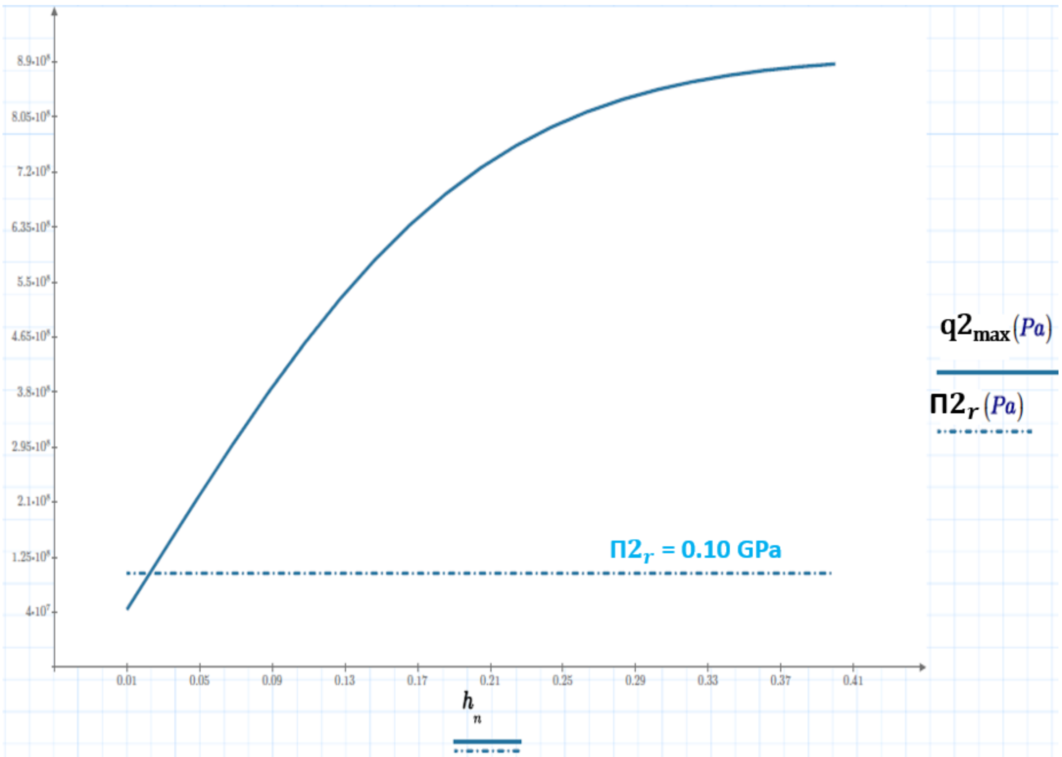


Figure 1-7 (b)Ultimate pressure vs. Relative thickness graph of Material: **T1000-G Carbon/epoxy**

Here,

$q2_{max}$ = Ultimate pressure

h = Relative Thickness

$\Pi 2_r$ = Compression Strength of **T1000-G Carbon/epoxy**

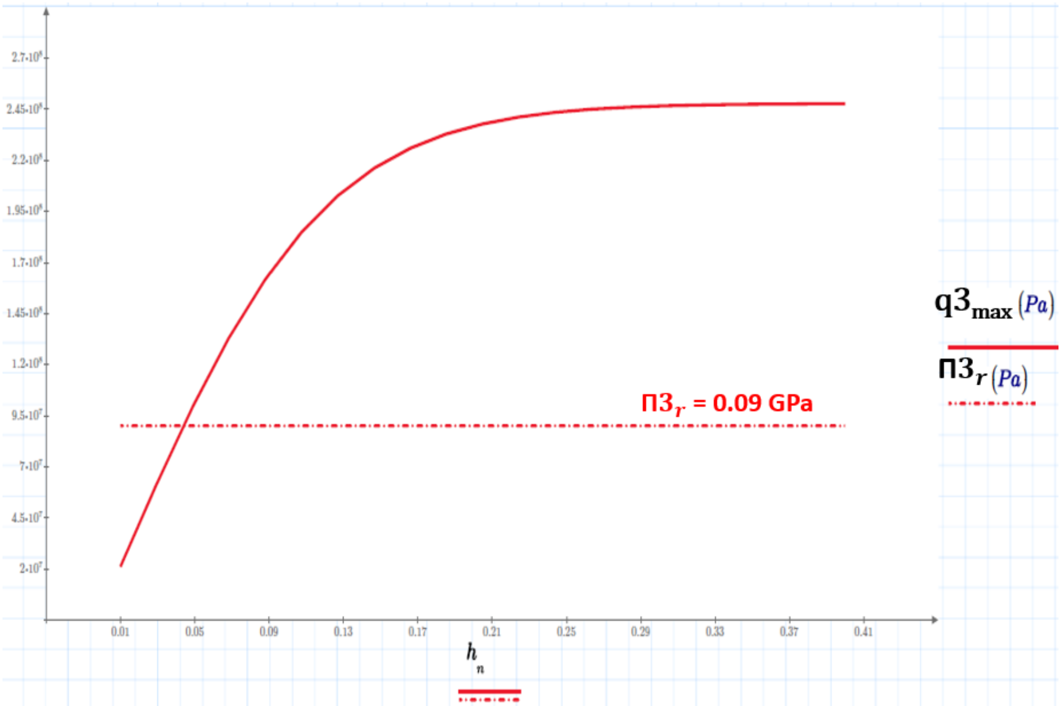


Figure 1-7 (c)Ultimate pressure vs. Relative thickness graph of Material: **P120 Carbon/epoxy**

Here,

$q3_{max}$ = Ultimate pressure

h = Relative Thickness

$\Pi 3_r$= Compression Strength of **P120 Carbon/epoxy**

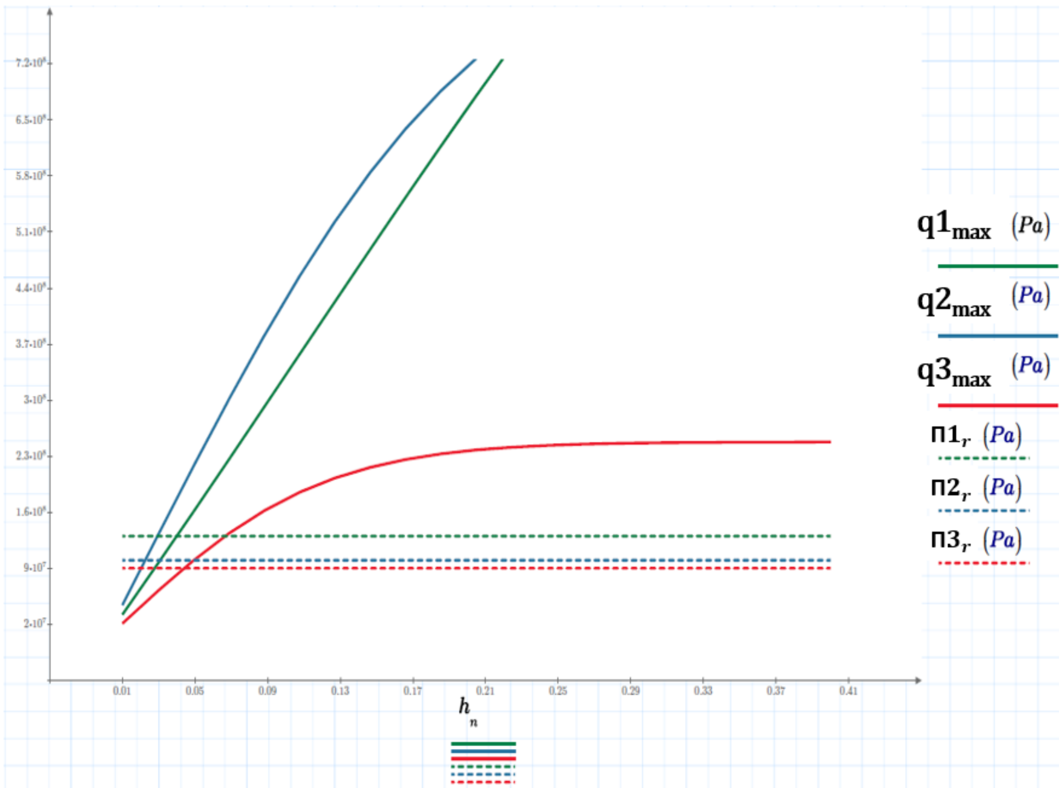


Figure 1-7 (d) Superimposition of graphs of chosen materials

The curves obtained for chosen materials validate the conclusion obtained earlier about variance becoming increasingly non-uniform and non-linear with increase in thickness of anisotropic materials.

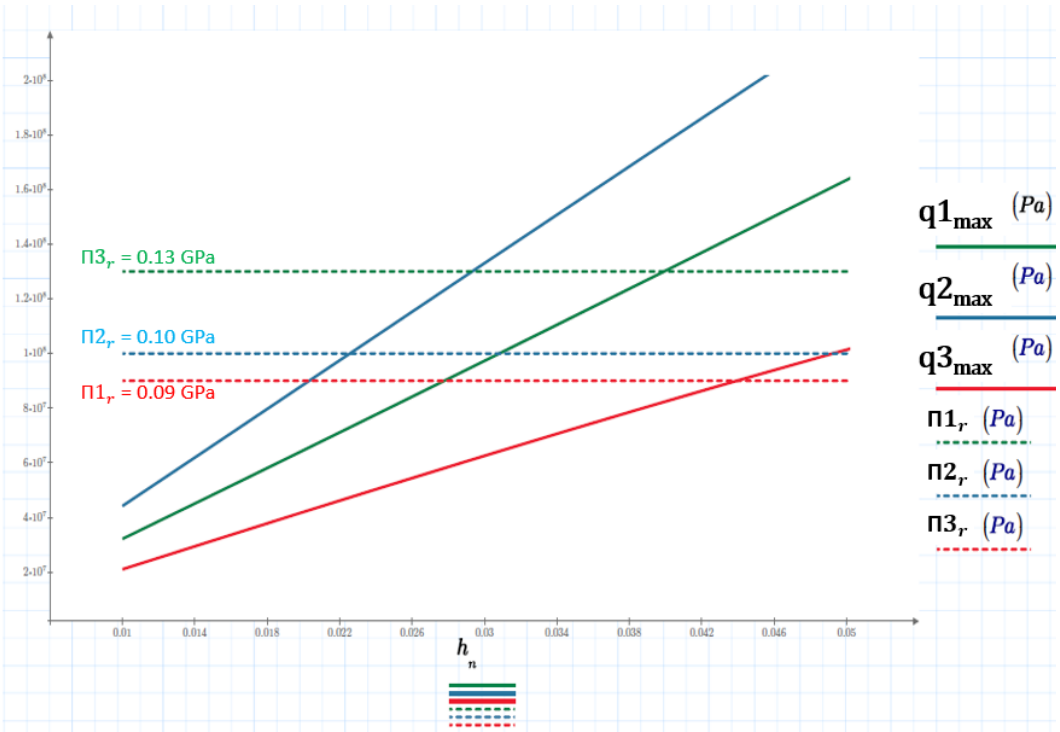


Figure 1-7 (e) Zoomed version of (d) depicting limitation imposed by radial compression strength

Limitation imposed by radial compression strength is observed here. Failure due to radial stresses increases as degree of anisotropy increases. The lowest envelope of these two lines determines the limitation of ultimate pressure. Both limitations have been validated by the graphs plotted for chosen materials.

$$\frac{\Pi_\theta^-}{\Pi_r^-} = \sqrt{\frac{E_\theta}{E_r}}$$

“Ideal” relationship of composite characteristics

Chapter 2

Methods Theorized to Increase Ultimate Tolerable Pressure

From the observations made from previous chapter, it is inferred that the equation becomes increasingly non-linear and non-uniform as the thickness of the ring and the degree of the anisotropy of the ring increases. The limitations of the governing equation for anisotropic rings are clearly observed, hence possible improvements are formulated to overcome these limitations.

Three ways have been theorized in this investigation to increase the ultimate tolerable pressure of the ring:

1) Artificial Heterogeneity
2) Ring Profiling
3) Redistribution of Fibers

2.1 Artificial Heterogeneity

The numerous different parameters such as dimensions, number of layers, thickness of layers and order of layers are altered around to achieve better performance and efficiency values.

A) Variation of ring material:

In our study, we chose 3 different composite materials for analysis and subsequently made rings out of the three materials.

Ring made from P120 composite/epoxy (Figure 2-1)

Ring made from T1000-G composite/epoxy (Figure 2-1)

Ring made from S2 glass composite/epoxy (Figure 2-1)

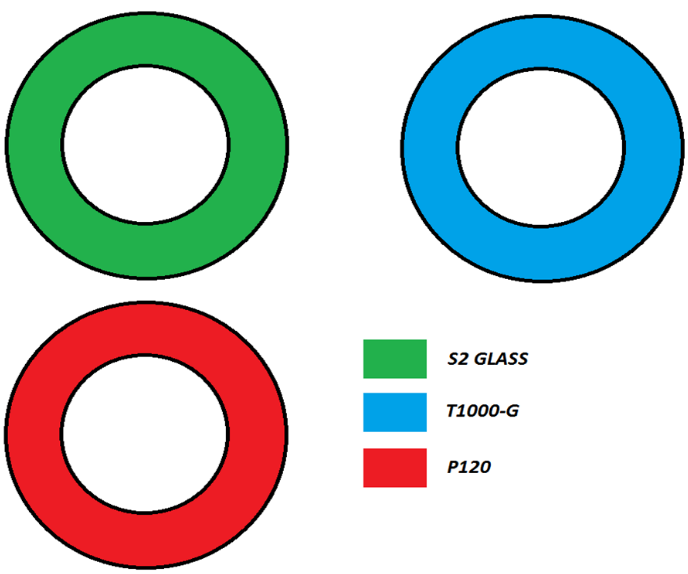


Figure 2-1 Rings made from chosen composite materials

B) Variation of order of compound layers:

Three layers of equal thickness (20cm in this case) made of materials chosen in various dissimilar combinations. Different orders of the material layers can be made.

The 1st number in the order represents the outermost layer , 2nd number representing the middle layer and 3rd number represents the innermost layer.

Different combinations:

I. 1 2 3

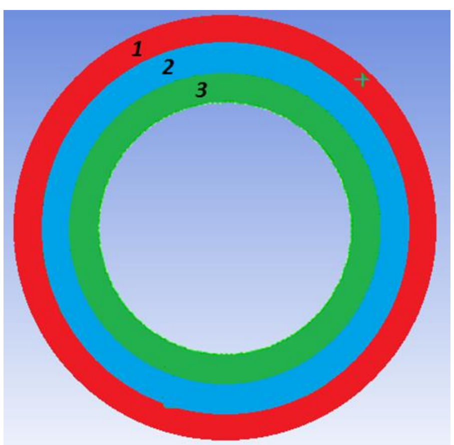
II. 1 3 2

III. 2 1 3

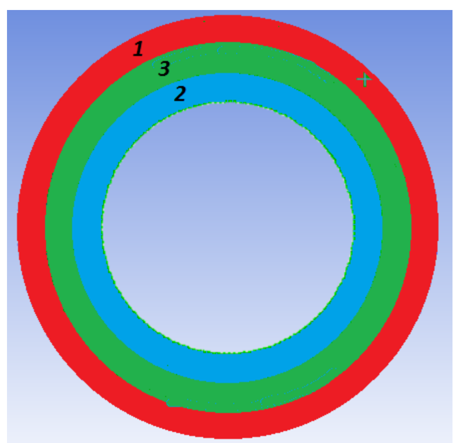
IV. 2 3 1

V. 3 1 2

VI. 3 2 1



(I)



(II)

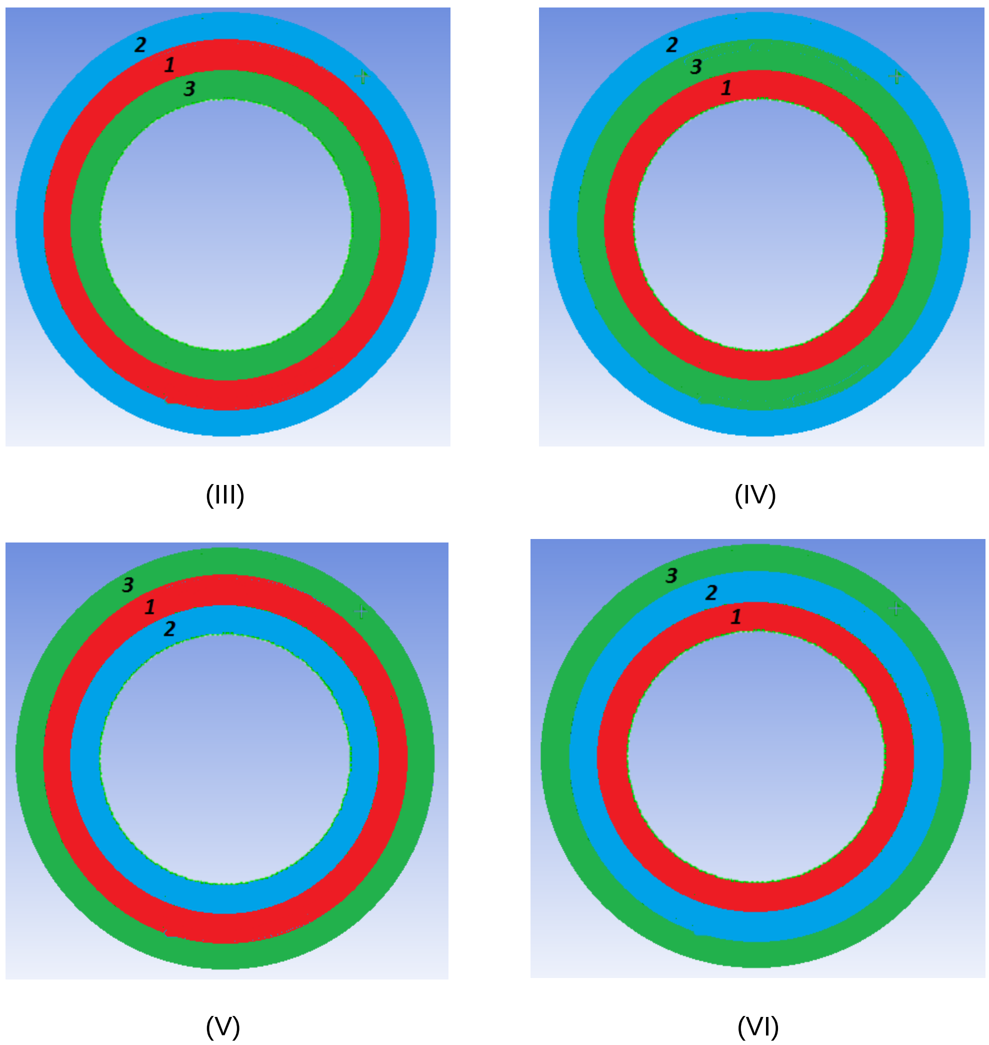


Figure 2-2: (I-VI) Compound rings made from different materials in various combinations

C) Variation of thicknesses of different layers:

The different compound material layers are made of different thickness:

In our study, we chose the best order of material layers (based on stress values and factors of safety) and then varied the thickness of the layers in this selected order.

Order of material layers carefully chosen: 3 2 1 (Figure 2-VI)

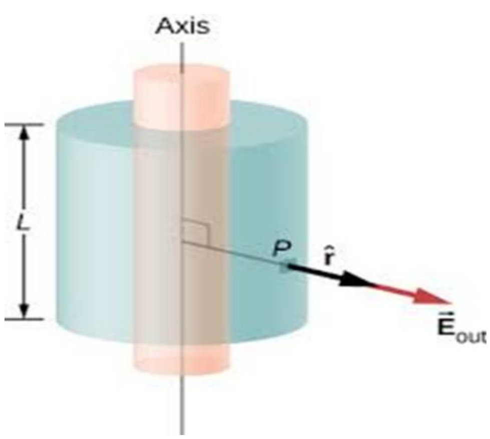
Table 2-1 Compound rings material layers in chosen order made of different thickness

No.	Layer '3' Thickness	Layer '2' Thickness	Layer '1' Thickness	Total Thickness of Ring
1	20 cm	20 cm	20 cm	60 cm
2	10 cm	20 cm	30 cm	60 cm
3	10 cm	15 cm	35 cm	60 cm
4	5 cm	7.5 cm	17.5 cm	30 cm
5	10 cm	10 cm	10 cm	30 cm
6	5 cm	5 cm	5 cm	15 cm
7	4 cm	3 cm	8 cm	15 cm
8	3.5 cm	3.5 cm	8 cm	15 cm
9	4 cm	2 cm	6 cm	12 cm
10	3.5 cm	0.5 cm	3.5 cm	7.5 cm

Colors represents the materials in their chosen order

2.2 Profiling

Profiling is the process of altering body dimensions and geometry to improve performance of structure. Performance increase is achieved through redistribution of stresses to thicker layers or profiles of the body or structure in question. This produces equal distribution of stresses through structure improving tolerance. This formulation was inspired by Stodola's design optimization of flywheels [7].



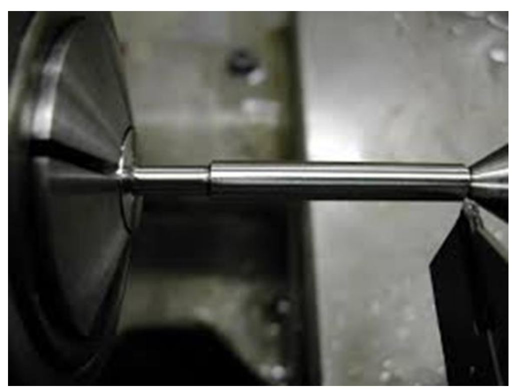


Figure 2-3 Examples of Profiling incorporated in cylindrical structures

2.3 Redistribution of Fibers

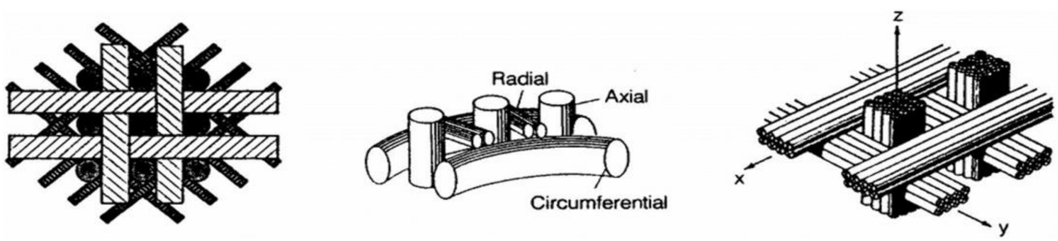
This is final formulation made to improve performance of ring. The material is made stronger by increasing tensile and compressive strengths by reinforcing structure with fibers in directions that need strengthening (z-direction for instance).

Through fiber distribution, following effects can be achieved:

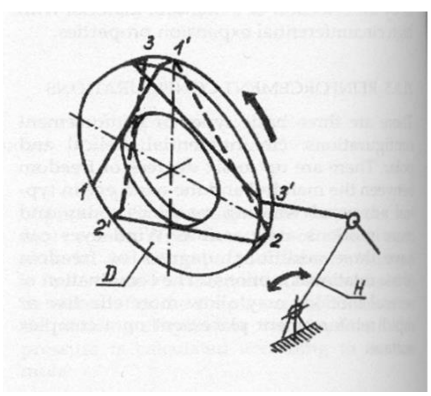
i. Radial reinforcement

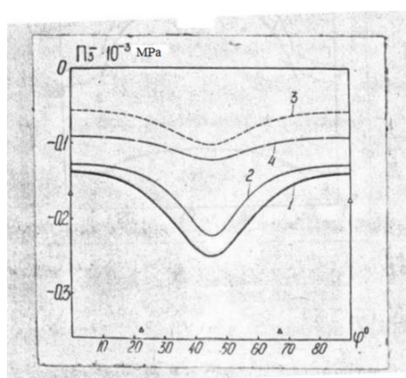
ii. Axial reinforcement

iii. Chord winding reinforcement



(a)





(b) (c)

Figure 2-4 (a) Schematic showing redistribution of fibers to strengthen structure in different directions (b) Chord Winding reinforcement (c) Graphs showing performance increase

Chapter 3

Analytical Model

The analytical solutions were obtained using Lekhnitskii's approach for stress distribution in a composite curvilinear anisotropic ring.

The diagram depicts n layers of composite rings of similar thickness under external pressure **q**. **a** is the internal radius(IR), **b** is the external radius(OR). $\mathbf{a_{m-1}}$, $\mathbf{a_m}$ are IR and OR of layer **m**. σ_θ^m, σ_r^m, u_r^m are stress components and displacements (in principle directions r and θ) of layer m. E_r^m, E_θ^m and υ_θ^m are the elastic constants of the material of layer m (in principle directions) [9].

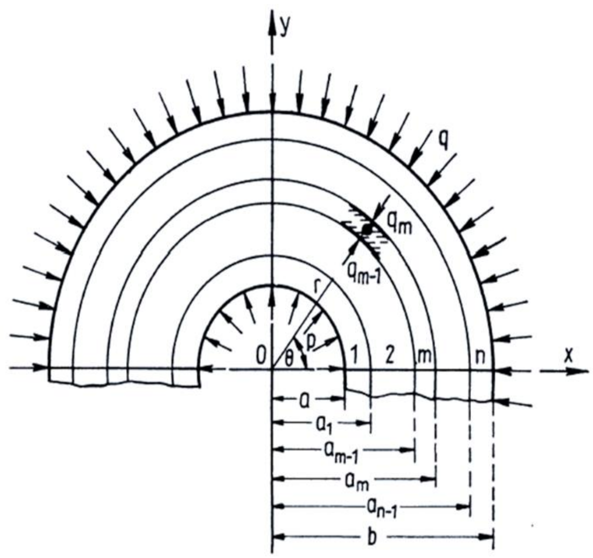


Figure 3-1 Lekhnitskii's Analytical Model for Composite Anisotropic Ring

The analytical model involves following assumptions:

i. Each layer is orthotropic

ii. The anisotropy poles of all layers are located at the center (origin of coordinates)

iii. All layers are rigidly connected, i.e., bonded along the contact surfaces

$$\sigma_{r_m} := q_{m-1} \cdot \frac{c_m^{k_m+1}}{1-c_m^{2k_m}} \cdot \left(\left(\frac{r}{a_m} \right)^{k_m-1} - \left(\frac{a_m}{r} \right)^{k_m+1} \right) + \left(\frac{q_m}{1-\left(c_m^{2k_m}\right)} \right) \cdot \left(-\left(\frac{r}{a_m} \right)^{k_1-1} + c_m^{2k_m} \cdot \left(\frac{a_m}{r} \right)^{k_m+1} \right)$$

$$\sigma_{\theta_m} := \left(q_{m-1}\right) \cdot \frac{c_m^{k_m+1} \cdot k_m}{1-c_m^{2k_m}} \cdot \left(\left(\frac{r}{a_m} \right)^{k_m-1} + \left(\frac{a_m}{r} \right)^{k_m+1} \right) - \left(\frac{q_m \cdot k_m}{1-c_m^{2k_m}} \right) \cdot \left(\left(\frac{r}{a_m} \right)^{k_m-1} + c_m^{2k_m} \cdot \left(\frac{a_m}{r} \right)^{k_m+1} \right)$$

Figure 3-2 (a) Formulae obtained to calculate stresses in principal directions

The stresses in the principal directions (r and θ) are calculated for each layer **m** using the above formulae.

$$q_{m+1} a_{m+1} \alpha_{m+1} + q_m a_m \beta_m + q_{m-1} a_{m-1} \alpha_{m-1} = 0$$

$$(m = 1, 2, \ldots, n-1).$$

$$\alpha_m = \frac{2k_m}{E_\theta^m} \frac{c_m^{k_m}}{1 - c_m^{2k_m}},$$

$$\beta_m = \frac{1}{E_\theta^{(m)}} \left(\nu_\theta^{(m)} - k_m \frac{1 + c_m^{2k_m}}{1 - c_m^{2k_m}} \right) - \frac{1}{E_\theta^{(m+1)}} \left(\nu_\theta^{(m+1)} + k_{m+1} \frac{1 + c_{m+1}^{2k_{m+1}}}{1 - c_{m+1}^{2k_{m+1}}} \right)$$

Figure 3-2 (b) System of equations formulated to obtain pressure values $\mathbf{q_m}$ acting on each layer **m**

Factors of safety of different layers are calculated with respect to radial stresses and circumferential stresses:

$$F1_{s1} = \left|\frac{\Pi 1_{\theta}}{\sigma 1_{\theta a}}\right| \qquad F2_{s1} = \left|\frac{\Pi 2_{\theta}}{\sigma_{\theta a}}\right| \qquad F3_{s1} = \left|\frac{\Pi 3_{\theta}}{\sigma_{\theta a}}\right|$$

$$F1_{s2} = \left|\frac{\Pi 1_{\theta}}{\sigma 1_{\theta b}}\right| \qquad F2_{s2} = \left|\frac{\Pi 2_{\theta}}{\sigma_{\theta b}}\right| \qquad F3_{s2} = \left|\frac{\Pi 3_{\theta}}{\sigma_{\theta b}}\right|$$

$$F2_{s3} = \left|\frac{\Pi 2_{r}}{q_1}\right| \qquad F3_{s3} = \left|\frac{\Pi 3_{r}}{q_2}\right|$$

$$F1_{s3} = \left|\frac{\Pi 1_{r}}{q_1}\right| \qquad F2_{s4} = \left|\frac{\Pi 2_{r}}{q_2}\right| \qquad F3_{s4} = \left|\frac{\Pi 3_{r}}{q_3}\right|$$

F1-Factors of safety for 1st layer, **F2**-Factors of safety for 2nd layer, **F3**-Factors of safety for 3rd layer. These notations are used throughout the report for uncomplicatedness.

s1 - Due to tangential stresses on inner boundary

s2 - Due to tangential stresses on outer boundary

s3 - Due to radial stresses on inner boundary

s4 - Due to radial stresses on outer boundary

Using these Analytical Relations:

(i) Stresses in principle directions for rings of different material with equal thickness are calculated and their factors of safety are determined.

(ii) Stresses in principle directions for compound layers of equal thicknesses (in various combinations) are calculated and subsequent factors of safety are obtained.

(iii) Stresses in principle directions for compound layers of different thicknesses are calculated and their factors of safety determined.

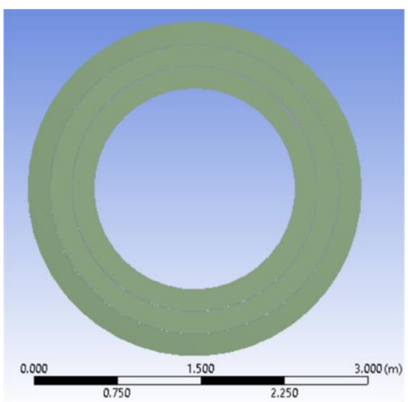
Chapter 4

Results

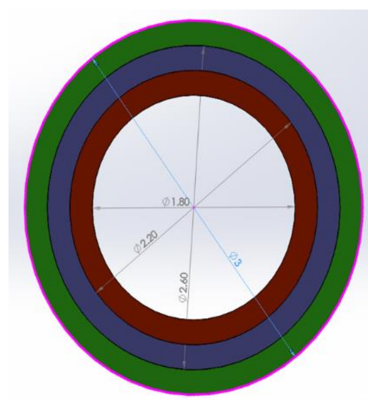
4.1 Base Geometry for Analytical Results

The base geometry was used for this investigation to obtain analytical results for the formulated improvements. Dimensions of geometry used for:

1) Single layer geometry: -
   Inner Radius = 0.9 m
   Outer Radius = 1.5 m
   Width of Ring = 20 cm
2) Compound layer geometry: –
   Inner Radius = 0.9 m
   Outer Radius = 1.5 m
   Width of Ring = 20 cm
   Thickness of each layer = 20 cm



(a)



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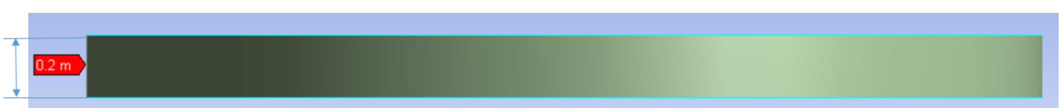


(c)

Figure 4-1 (a) Single layer base geometry (b) Base geometry for compound layers and (c) Width of ring

4.2 Analytical calculations for artificial heterogeneity

Analytical solutions are obtained for the formulations mentioned in previous chapter (2.1 Artificial Heterogeneity) using Lekhnitskii's method. [8] The solutions were calculated using PTC Mathcad Prime 3.1.

Mathcad calculations are shown in Appendix A.

A) Variation of ring material:

Stresses for single material rings with equal thickness are calculated and their factors of safety are determined.

Single material ring made from chosen composite materials:

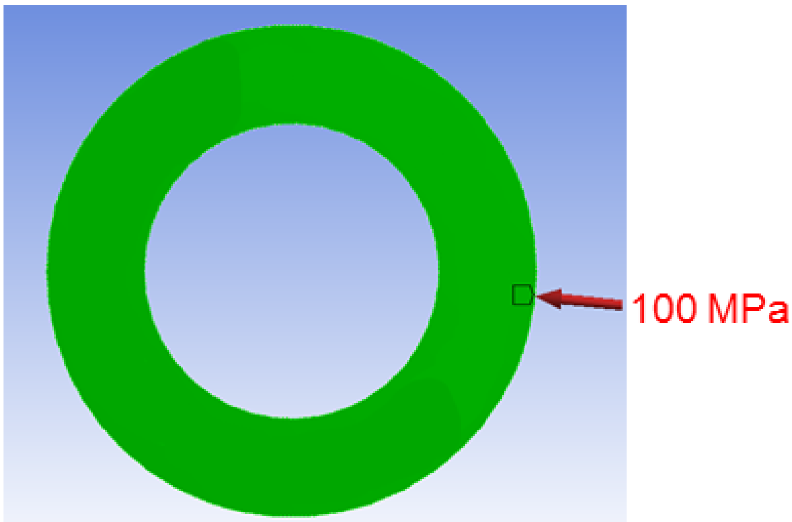


Figure 4-2 (a) Ring made of S2 Glass/epoxy with external radius 1.5m and internal radius 0.9m under an external pressure of 100 MPa

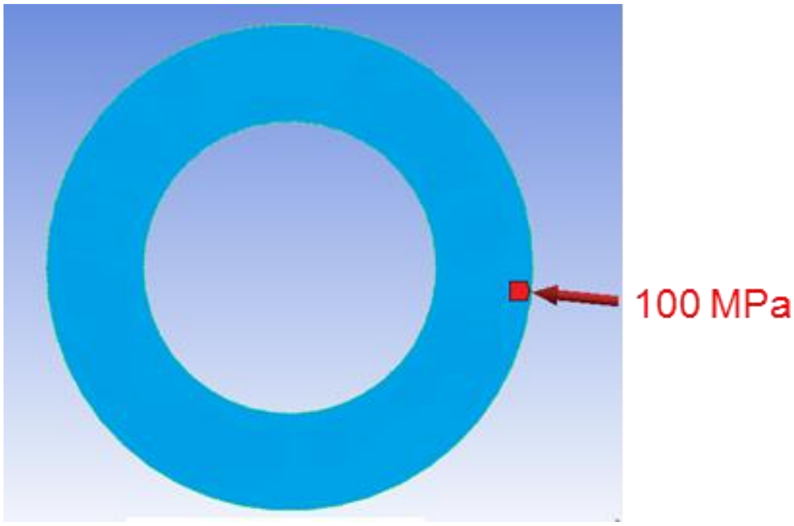


Figure 4.2 (b) Ring made of T1000-G carbon/epoxy with external radius 1.5m and internal radius 0.9m under an external pressure of 100 MPa

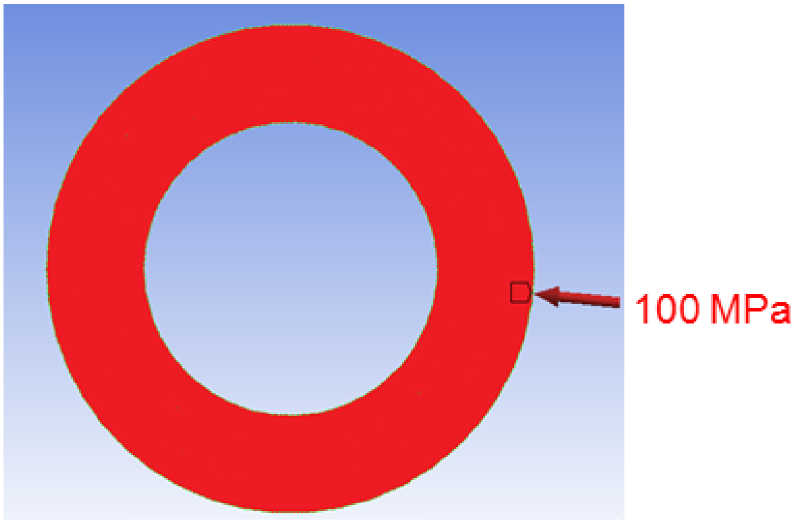


Figure 4-2 (c) Ring made of P120 composite with external radius 1.5m and internal radius 0.9m under an external pressure of 100 MPa

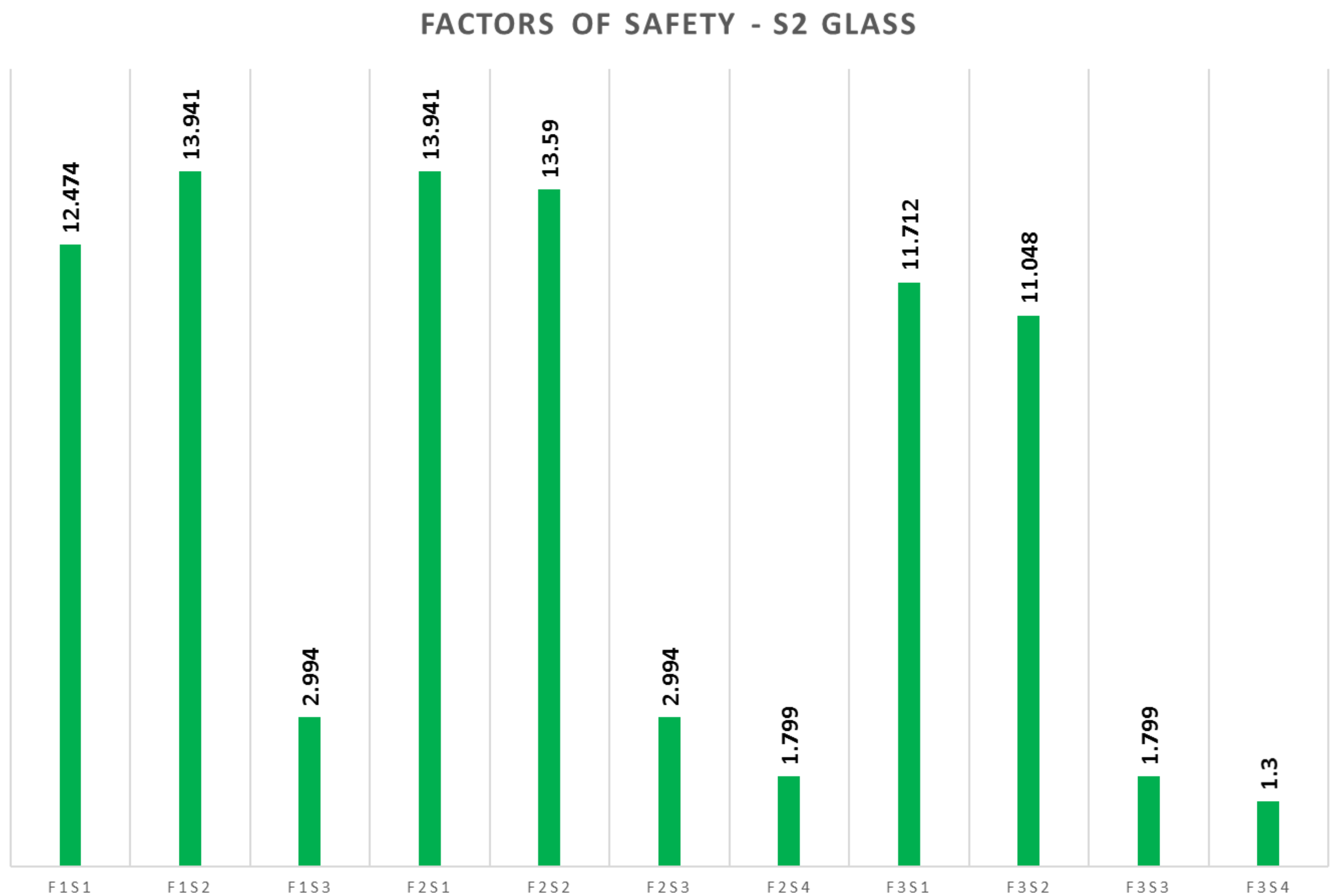


Figure 4-3 (a) Calculated factors of safety for S2 Glass/epoxy

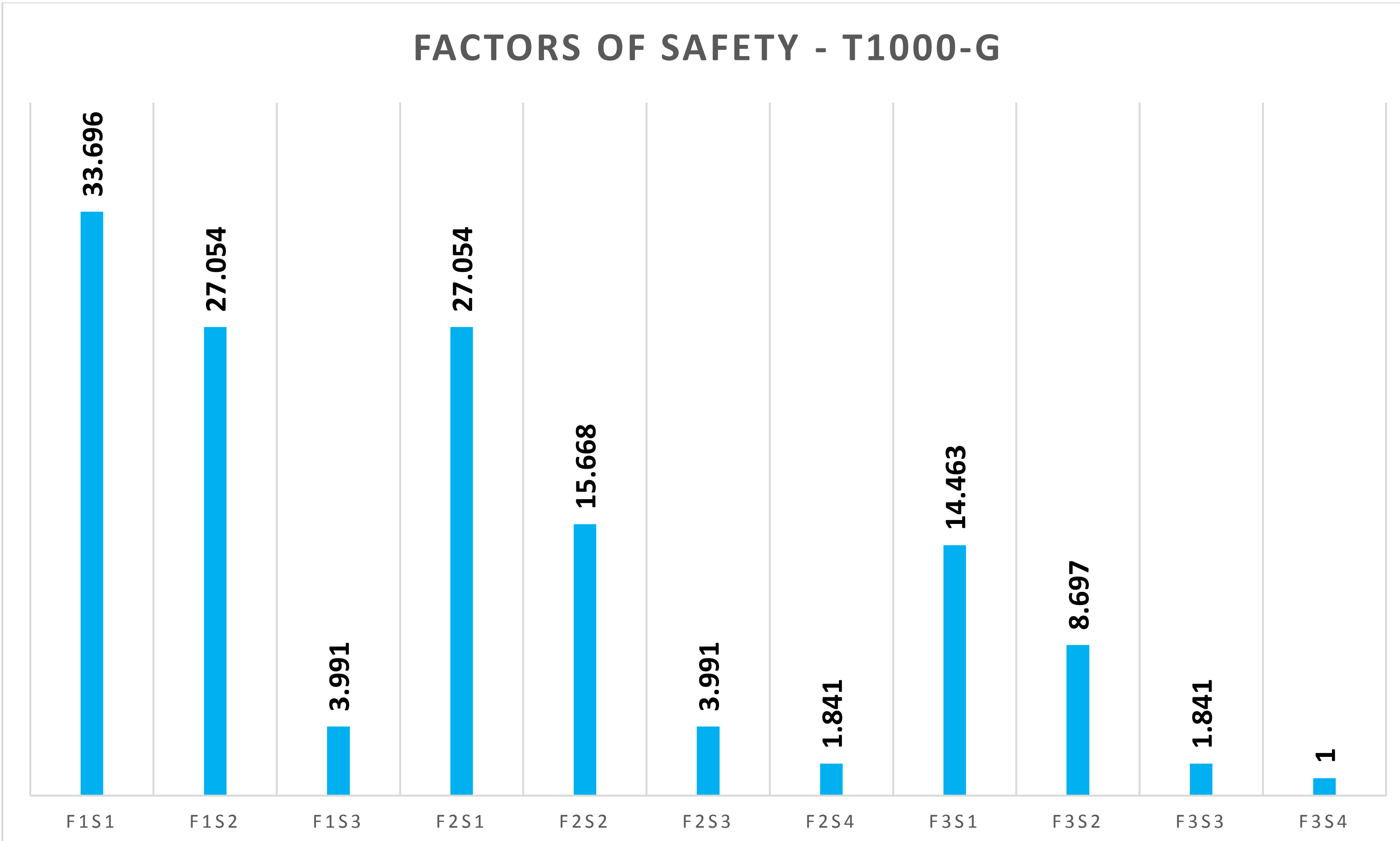


Figure 4-3 (b) Calculated factors of safety for T1000-G Carbon/epoxy

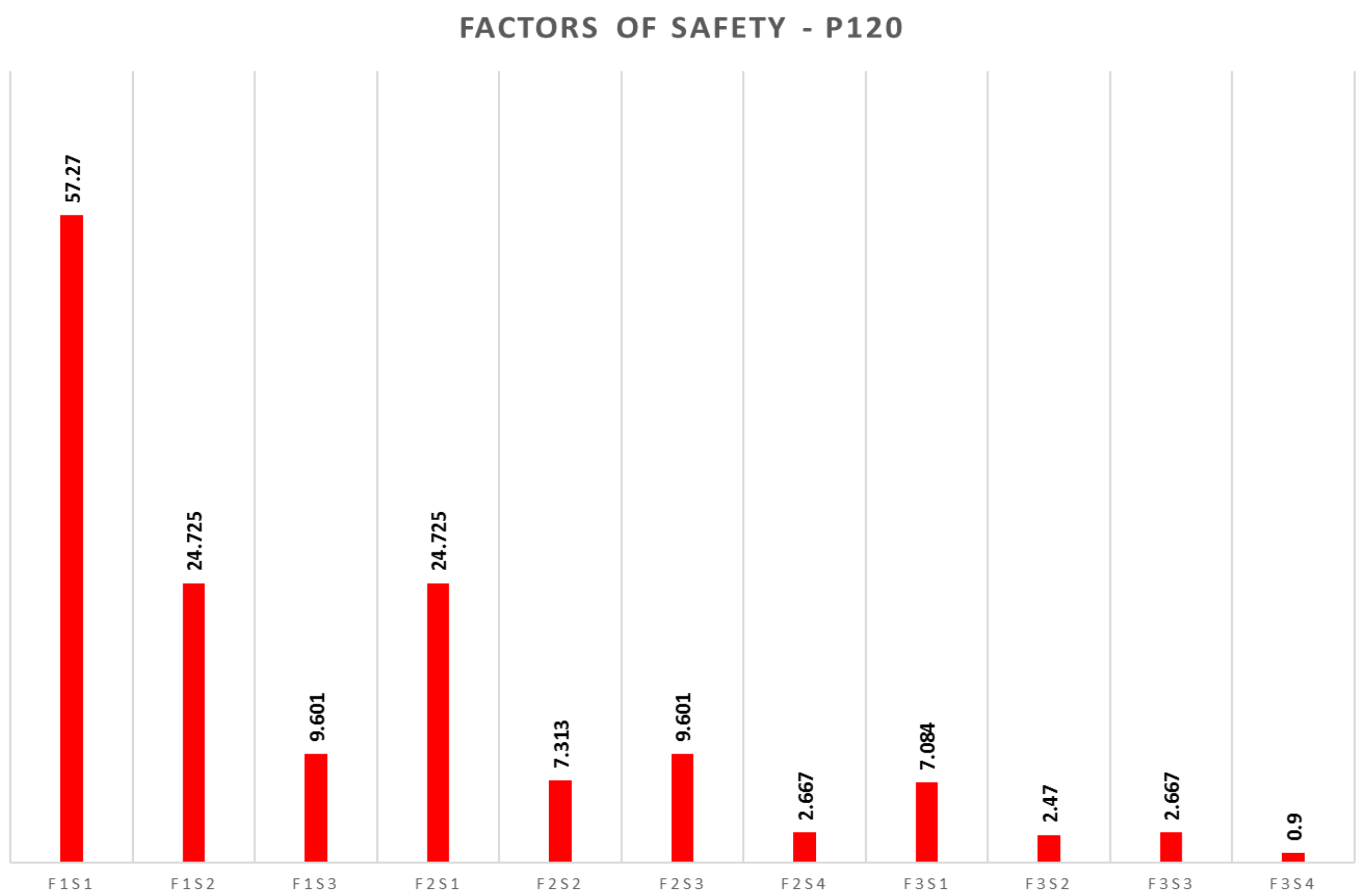


Figure 4-3 (c) Calculated factors of safety for P120 Carbon/epoxy

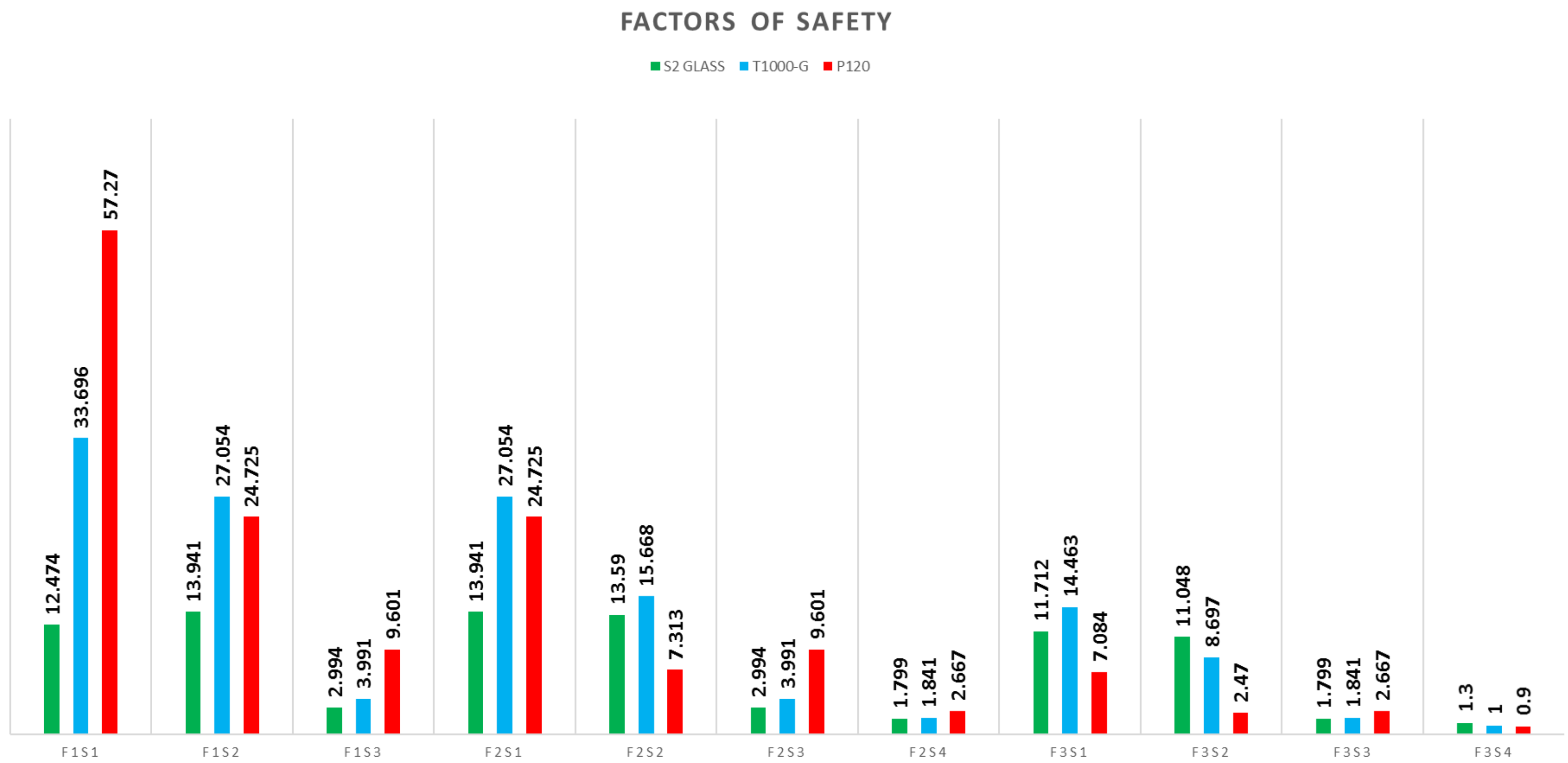


Figure 4-3 (d) Superimposition of the three calculated materials' factors of safety

Table 4-1 Factors of safety for ring made of different materials

Material	$F1_{s1}$	$F1_{s2}$	$F1_{s3}$	$F2_{s1}$	$F2_{s2}$	$F2_{s3}$	$F2_{s4}$	$F3_{s1}$	$F3_{s2}$	$F3_{s3}$	$F3_{s4}$
S2 Glass/epoxy	12.474	13.941	2.994	13.941	13.59	2.994	1.799	11.712	11.048	1.799	1.3
T1000-G Carbon/epoxy	33.696	27.054	3.991	27.054	15.668	3.991	1.841	14.463	8.697	1.841	1
P120 Carbon/epoxy	57.27	24.725	9.601	24.725	7.313	9.601	2.667	7.084	2.47	2.667	0.9

Inference from above results:

- The factors of safety of S2 Glass/epoxy are well within limits for principle directions in all three layers.
- The factors of safety of T1000-G Carbon/epoxy are mostly within boundaries for principle directions in all three layers except for the outer boundary where radial stress limits are reached ($F3_{s4}$ = 1).
- The factors of safety of P120 Carbon/epoxy are considerably above required restrictions for principle directions in all three layers except for failure due to radial stresses at outer boundary ($F3_{s4}$ = 0.9).

Factors of safety for S2 Glass are within limits in all layers but performances of other two composites are better in the other layers in the principle directions.

Thus, we move on to our next formulation about making rings with compound layers made from different materials and arranging in various combinations to increase the range of factor performance acquired.

(B) Variation of order of compound layers:

Stresses for compound layers of equal thicknesses in various order combinations are calculated and subsequent factors of safety are obtained.

The 1st number in the order represents the outermost layer, 2nd number representing the middle layer and 3rd number represents the innermost layer.

1: P120 Carbon/epoxy, **2**: T1000-G Carbon/epoxy and **3**: S2 Glass/epoxy

a) Rings made from chosen materials in order **1 2 3** of thickness 20 cm each under external pressure 100 MPa.
b) Rings made from chosen materials in order **1 3 2** of thickness 20 cm each under external pressure 100 MPa.
c) Rings made from chosen materials in order **2 1 3** of thickness 20 cm each under external pressure 100 MPa.
d) Rings made from chosen materials in order **2 3 1** of thickness 20 cm each under external pressure 100 MPa.
e) Rings made from chosen materials in order **3 1 2** of thickness 20 cm each under external pressure 100 MPa.
f) Rings made from chosen materials in order **3 2 1** of thickness 20 cm each under external pressure 100 MPa.

Illustration of above rings shown in Figure 2-2 (I-VI)

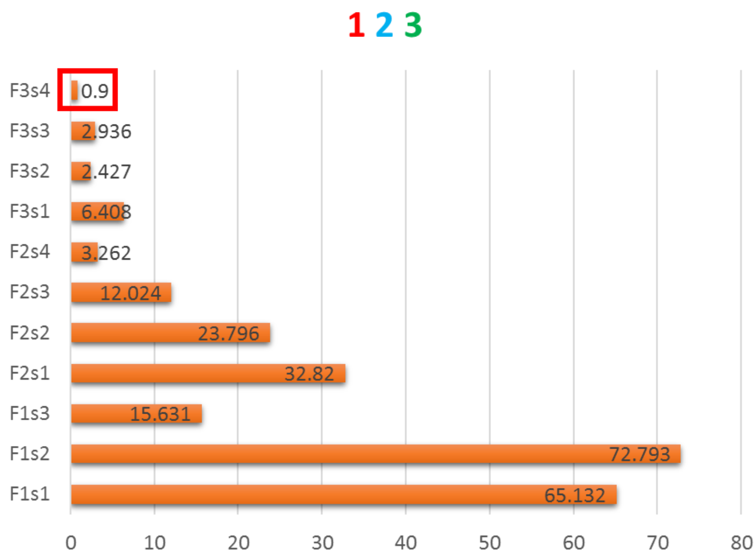


Figure 4-4 (a) Calculated factors of safety of layers' order **1 2 3**

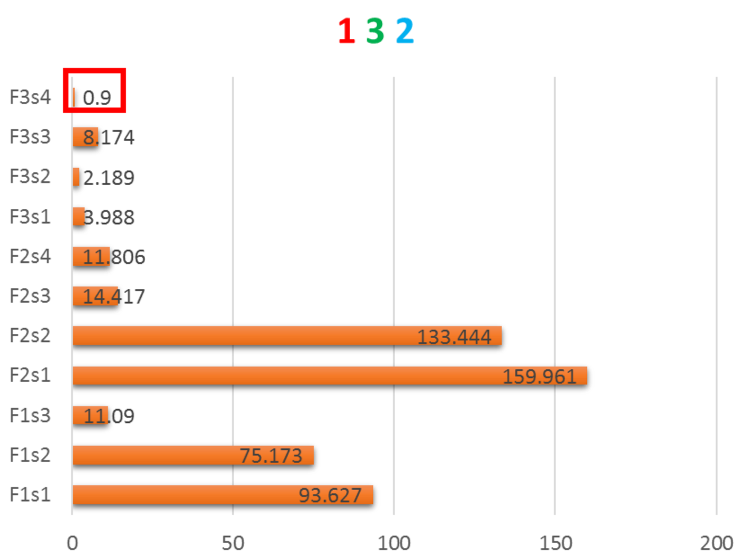


Figure 4-4 (b) Calculated factors of safety of layers' order **1 3 2**

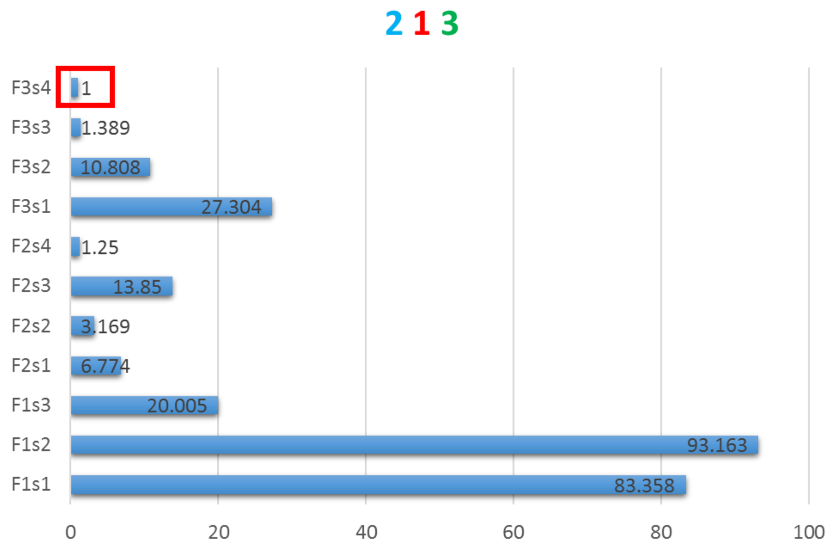


Figure 4-4 (c) Calculated factors of safety of layers' order **2 1 3**

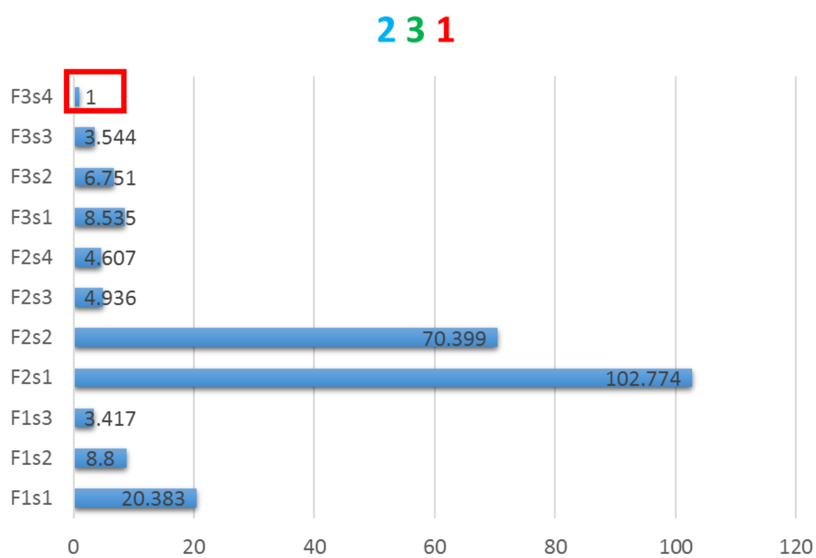


Figure 4-4 (d) Calculated factors of safety of layers' order **2 3 1**

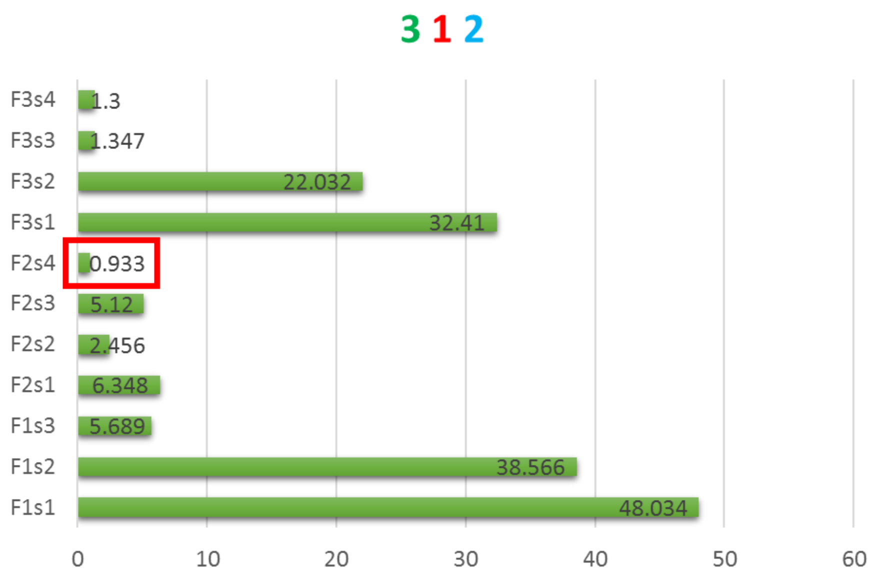


Figure 4-4 (e) Calculated factors of safety of layers' order **3 1 2**

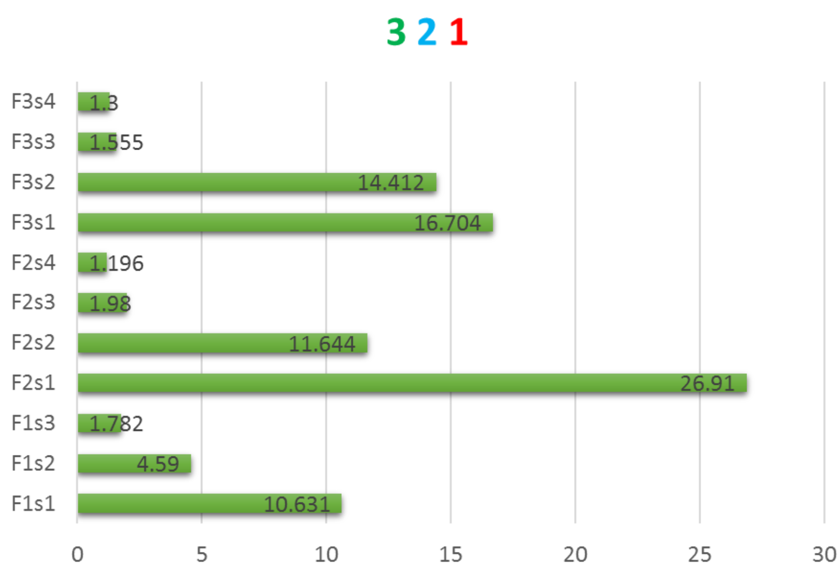


Figure 4-4 (f) Calculated factors of safety of layers' order **3 2 1**

Table 4-2 Factors of safety for different orders of compound layers

Order of layers	$F1_{s1}$	$F1_{s2}$	$F1_{s3}$	$F2_{s1}$	$F2_{s2}$	$F2_{s3}$	$F2_{s4}$	$F3_{s1}$	$F3_{s2}$	$F3_{s3}$	$F3_{s4}$
1 2 3	65.132	72.793	15.631	32.82	23.796	12.024	3.262	6.408	2.427	2.936	0.9
1 3 2	93.627	75.173	11.09	159.961	133.444	14.417	11.806	3.988	2.189	8.174	0.9
2 1 3	83.358	93.163	20.005	6.774	3.169	13.85	1.25	27.304	10.808	1.389	1
2 3 1	20.383	8.8	3.417	102.774	70.399	4.936	4.607	8.535	6.751	3.544	1
3 1 2	48.034	38.566	5.689	6.348	2.456	5.12	0.933	32.41	22.032	1.347	1.3
3 2 1	10.631	4.59	1.782	26.91	11.644	1.98	1.196	16.704	14.412	1.555	1.3

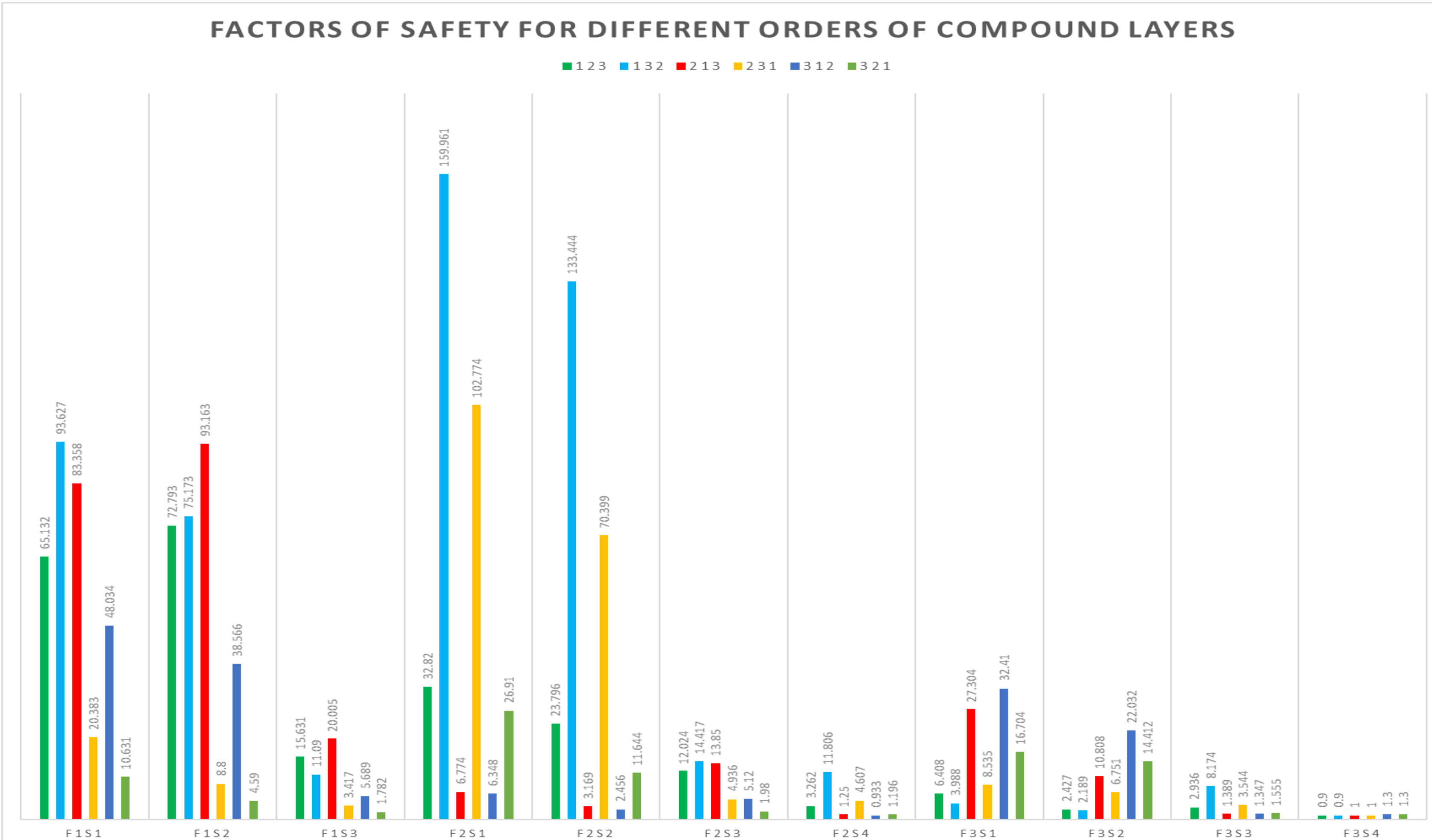


Figure 4-5 Chart depicting factors of safety of all different orders of layers

C) Variation of thicknesses of different layers:

The best order of material layers is selected based on the results obtained and the thickness of each layer is varied to achieve more suitable factors of safety.
Stresses for best selected order of compound layers are calculated with thickness variation and their factors of safety determined. From results of different orders of compound layers, it is observed that the order **3 2 1** has the best overall factors of safety relative to the other orders of layers. Order **3 2 1** signifies outermost layer made of S2 glass/epoxy, middle layer made of T1000-G carbon/epoxy and innermost layer made of P120 carbon/epoxy (Figure 2-2 VI).

Rings made of compound layers of different materials in selected order (**3 2 1**) of equal thickness:

Table 4-3 Rings made of compound layers of different materials in selected order (**3 2 1**) of equal thickness

No.	**S2 glass layer thickness (**Outermost: **3)**	**T1000-G layer thickness (**Middle: **2)**	**P120 layer thickness (**Innermost: **1)**	**Total Thickness**
1	20 cm	20 cm	20 cm	60 cm
2	10 cm	10 cm	10 cm	30 cm
3	5 cm	5 cm	5 cm	15 cm

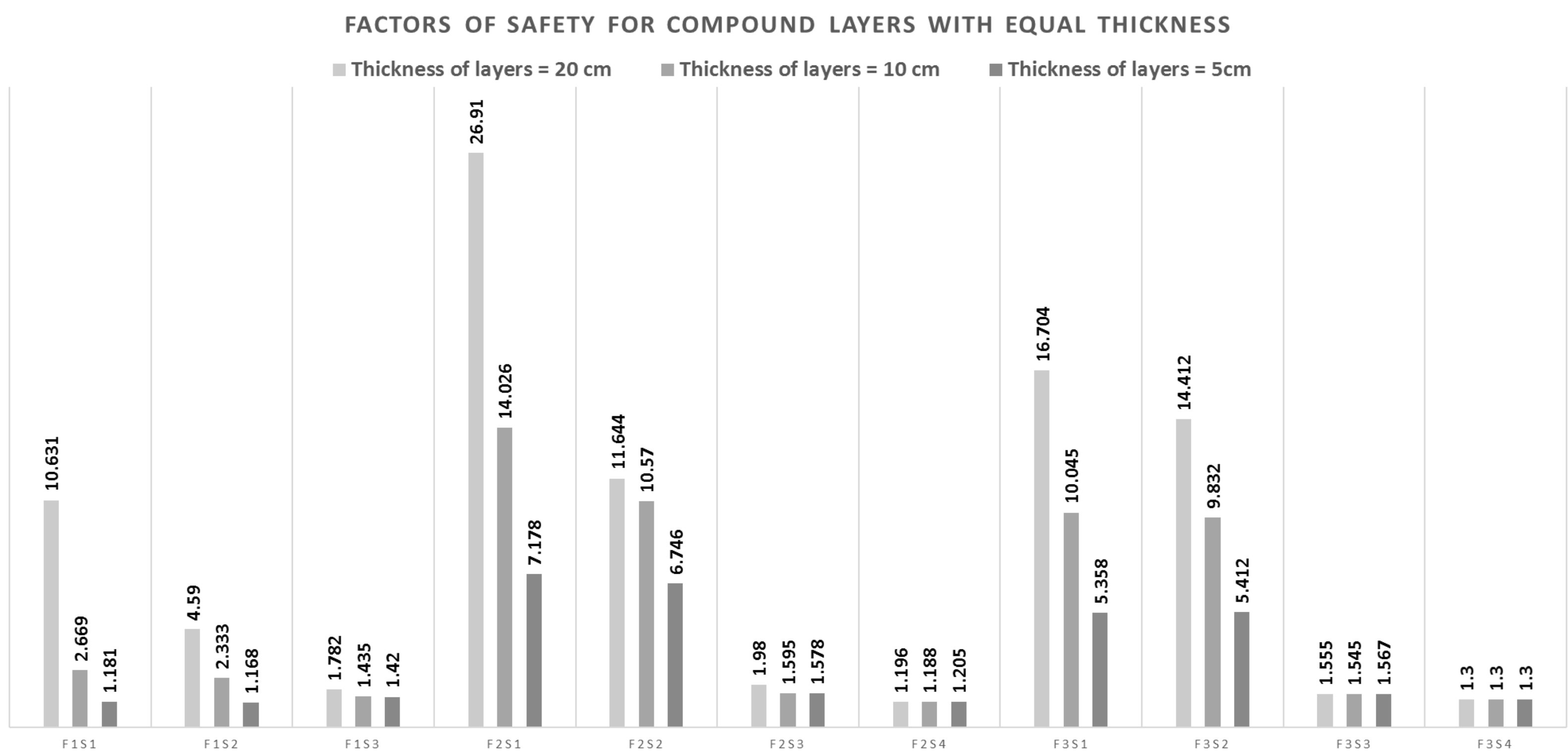


Figure 4-6 (a)Chart showing factors of safety for compound layers with equal thickness

Table 4-4 Rings made of compound layers in selected order (**3 2 1**) with different thicknesses:

No.	**S2 glass layer thickness** (Outermost**: 3)**	**T1000-G layer thickness** (Middle**: 2)**	**P120 layer thickness** (Innermost**: 1)**	**Total Thickness**
1	10 cm	20 cm	30 cm	60 cm
2	10 cm	15 cm	35 cm	60 cm
3	5 cm	7.5 cm	17.5 cm	30 cm
4	4 cm	4 cm	7 cm	15 cm
5	4 cm	3 cm	8 cm	15 cm
6	3 cm	3.5 cm	8.5 cm	15 cm
7	3.5 cm	3.5 cm	8 cm	15 cm
8	4 cm	2 cm	6 cm	12 cm
9	4 cm	2.5 cm	5.5 cm	12 cm
10	3 cm	3 cm	4 cm	10 cm
11	3.5 cm	0.5 cm	3.5 cm	7.5 cm
12	3 cm	1 cm	3.5 cm	7.5 cm
13	2.5 cm	1 cm	4 cm	7.5 cm

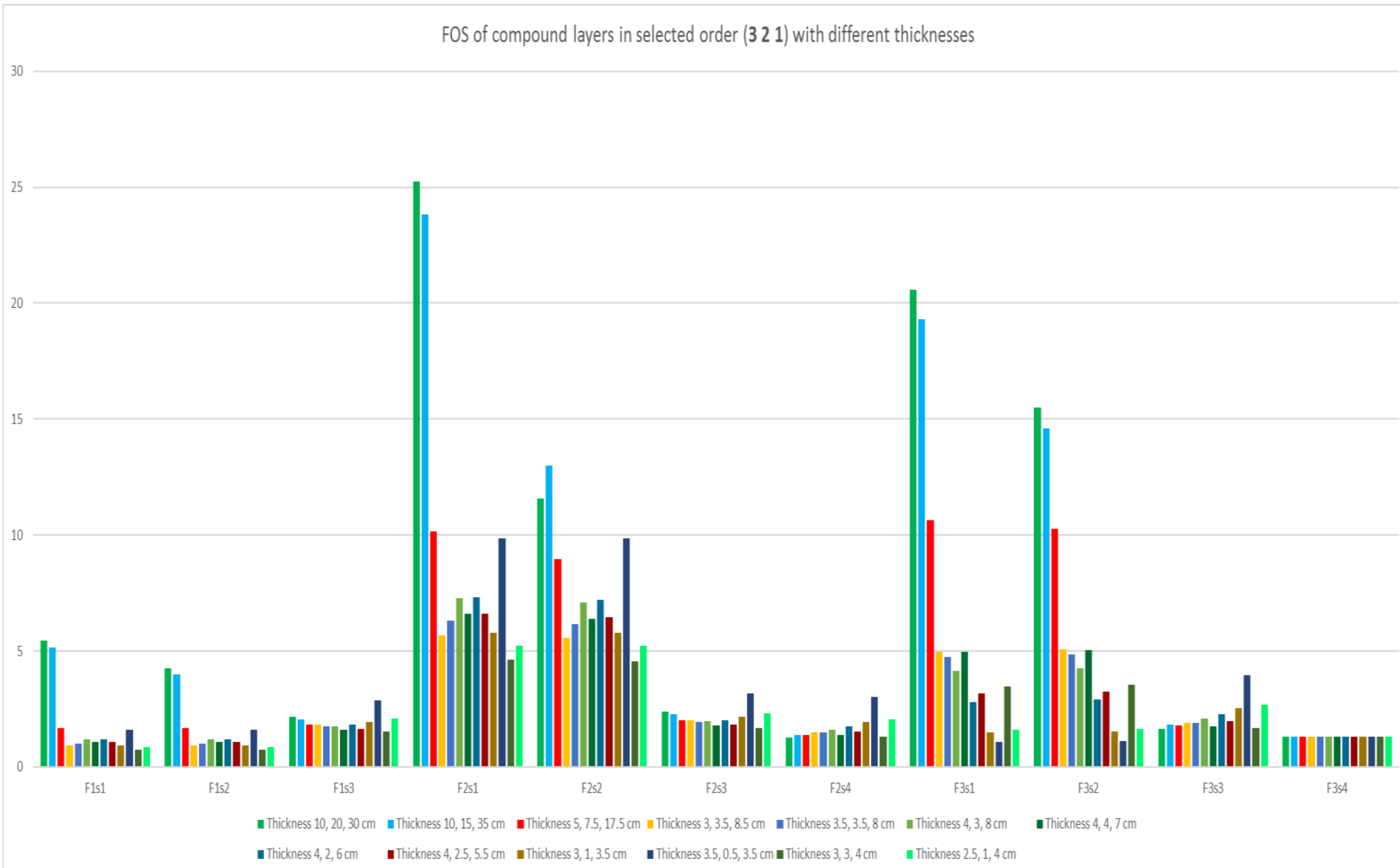
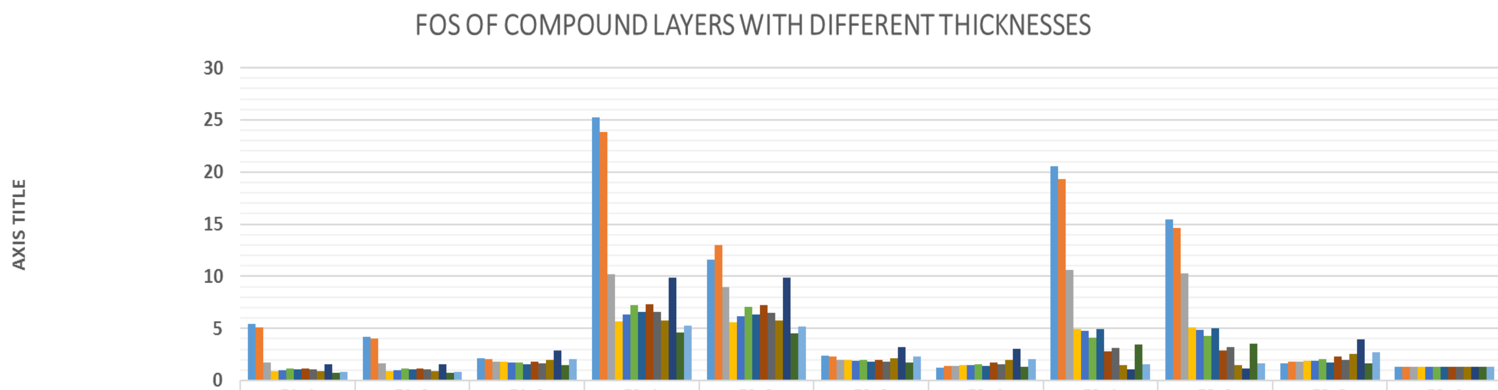


Figure 4-6 (b)Chart showing factors of safety of compound layers in selected order (**3 2 1**) with different thicknesses



	F1s1	F1s2	F1s3	F2s1	F2s2	F2s3	F2s4	F3s1	F3s2	F3s3	F3s4
Thickness 10, 20, 30 cm	5.435	4.235	2.158	25.232	11.584	2.398	1.255	20.565	15.492	1.631	1.3
Thickness 10, 15, 35 cm	5.13	3.997	2.037	23.816	12.994	2.263	1.389	19.299	14.608	1.805	1.3
Thickness 5, 7.5, 17.5 cm	1.69	1.66	1.812	10.173	8.96	2.013	1.372	10.642	10.256	1.784	1.3
Thickness 3, 3.5, 8.5 cm	0.913	0.918	1.81	5.687	5.574	2.011	1.475	4.969	5.084	1.917	1.3
Thickness 3.5, 3.5, 8 cm	1.017	1.019	1.733	6.301	6.14	1.925	1.469	4.74	4.851	1.909	1.3
Thickness 4, 3, 8 cm	1.18	1.179	1.765	7.273	7.085	1.961	1.591	4.151	4.268	2.068	1.3
Thickness 4, 4, 7 cm	1.071	1.07	1.602	6.6	6.363	1.78	1.359	4.953	5.044	1.767	1.3
Thickness 4, 2, 6 cm	1.185	1.184	1.811	7.311	7.198	2.012	1.741	2.811	2.89	2.264	1.3
Thickness 4, 2.5, 5.5 cm	1.07	1.07	1.636	6.603	6.472	1.818	1.522	3.158	3.232	1.978	1.3
Thickness 3, 1, 3.5 cm	0.928	0.933	1.939	5.787	5.766	2.155	1.95	1.481	1.511	2.534	1.3
Thickness 3.5, 0.5, 3.5 cm	1.589	1.594	2.855	9.864	9.84	3.172	3.035	1.089	1.112	3.946	1.3
Thickness 3, 3, 4 cm	0.74	0.744	1.52	4.612	4.545	1.689	1.286	3.48	3.532	1.672	1.3
Thickness 2.5, 1, 4 cm	0.836	0.841	2.088	5.228	5.217	2.32	2.058	1.6	1.637	2.676	1.3

Figure 4-6 (c) Chart depicting values of factors of safety of compound layers in selected order (**3 2 1**) with different thicknesses and occurrence of failure

Table 4-5 Factors of safety of compound layers in selected order (**3 2 1**) with all thickness variations with locations of failure marked:

Thickness of layers 3 2 1	$F1_{s1}$	$F1_{s2}$	$F1_{s3}$	$F2_{s1}$	$F2_{s2}$	$F2_{s3}$	$F2_{s4}$	$F3_{s1}$	$F3_{s2}$	$F3_{s3}$	$F3_{s4}$
Thickness of layers = **20 cm**	10.631	4.59	1.782	26.91	11.644	1.98	1.196	16.704	14.412	1.555	1.3
Thickness of layers = **10 cm**	2.669	2.333	1.435	14.026	10.57	1.595	1.188	10.045	9.832	1.545	1.3
Thickness of layers = **5 cm**	1.181	1.168	1.42	7.178	6.746	1.578	1.205	5.358	5.412	1.567	1.3
Thickness **10, 20, 30 cm**	5.435	4.235	2.158	25.232	11.584	2.398	1.255	20.565	15.492	1.631	1.3
Thickness **10, 15, 35 cm**	5.13	3.997	2.037	23.816	12.994	2.263	1.389	19.299	14.608	1.805	1.3
Thickness **5, 7.5, 17.5 cm**	1.69	1.66	1.812	10.173	8.96	2.013	1.372	10.642	10.256	1.784	1.3
Thickness **2.5, 3.75, 8.75 cm**	0.791	0.796	1.873	4.942	4.864	2.081	1.417	5.504	5.611	1.842	1.3
Thickness **3, 3.25, 8.75 cm**	0.935	0.94	1.853	5.823	5.718	2.059	1.539	4.741	4.865	2.001	1.3
Thickness **3, 3.5, 8.5 cm**	0.913	0.918	1.81	5.687	5.574	2.011	1.475	4.969	5.084	1.917	1.3
Thickness **3.5, 3.5, 8 cm**	1.017	1.019	1.733	6.301	6.14	1.925	1.469	4.74	4.851	1.909	1.3
Thickness **4, 3, 8 cm**	1.18	1.179	1.765	7.273	7.085	1.961	1.591	4.151	4.268	2.068	1.3
Thickness **4, 4, 7 cm**	1.071	1.07	1.602	6.6	6.363	1.78	1.359	4.953	5.044	1.767	1.3
Thickness **4, 4, 4 cm**	0.934	0.933	1.427	5.761	5.566	1.586	1.211	4.326	4.377	1.574	1.3
Thickness **4, 3, 3 cm**	0.882	0.882	1.367	5.446	5.318	1.519	1.232	3.108	3.146	1.602	1.3
Thickness **4, 3, 5 cm**	1.002	1.001	1.531	6.18	6.029	1.701	1.38	3.527	3.595	1.794	1.3
Thickness **4, 2.5, 5.5 cm**	1.07	1.07	1.636	6.603	6.472	1.818	1.522	3.158	3.232	1.978	1.3
Thickness **4, 2, 6 cm**	1.185	1.184	1.811	7.311	7.198	2.012	1.741	2.811	2.89	2.264	1.3
Thickness **3, 3, 4 cm**	0.74	0.744	1.52	4.612	4.545	1.689	1.286	3.48	3.532	1.672	1.3
Thickness **2.5, 1, 4 cm**	0.836	0.841	2.088	5.228	5.217	2.32	2.058	1.6	1.637	2.676	1.3
Thickness **3, 1, 3.5 cm**	0.928	0.933	1.939	5.787	5.766	2.155	1.95	1.481	1.511	2.534	1.3
Thickness **3.5, 0.5, 3.5 cm**	1.589	1.594	2.855	9.864	9.84	3.172	3.035	1.089	1.112	3.946	1.3

From the results, best performing geometries were found for different orders of compound layers and for different total thickness of compound layers of selected order.

Table 4.6 (a) Best geometry for **different orders** of compound layers with the same dimensions

Order of layers	$F1_{s1}$	$F1_{s2}$	$F1_{s3}$	$F2_{s1}$	$F2_{s2}$	$F2_{s3}$	$F2_{s4}$	$F3_{s1}$	$F3_{s2}$	$F3_{s3}$	$F3_{s4}$
3 2 1	10.631	4.59	1.782	26.91	11.644	1.98	1.196	16.704	14.412	1.555	1.3

Table 4.6 (b) Best geometry for compound layers of selected order **3 2 1** with total thickness of layers = **7.5 cm**

Order of Layers 3 2 1	$F1_{s1}$	$F1_{s2}$	$F1_{s3}$	$F2_{s1}$	$F2_{s2}$	$F2_{s3}$	$F2_{s4}$	$F3_{s1}$	$F3_{s2}$	$F3_{s3}$	$F3_{s4}$
Thickness 3.5, 0.5, 3.5 cm	1.589	1.594	2.855	9.864	9.84	3.172	3.035	1.089	1.112	3.946	1.3

Table 4.6 (c) Best geometry for compound layers of selected order **3 2 1** with total thickness of layers = **15 cm**

Order of Layers 3 2 1	$F1_{s1}$	$F1_{s2}$	$F1_{s3}$	$F2_{s1}$	$F2_{s2}$	$F2_{s3}$	$F2_{s4}$	$F3_{s1}$	$F3_{s2}$	$F3_{s3}$	$F3_{s4}$
Thickness 4, 3, 8 cm	1.18	1.179	1.765	7.273	7.085	1.961	1.591	4.151	4.268	2.068	1.3

Table 4.6 (d) Best geometry for compound layers of selected order **3 2 1** with total thickness of layers = **30 cm**

Order of Layers 3 2 1	**F1$_{s1}$**	**F1$_{s2}$**	**F1$_{s3}$**	**F2$_{s1}$**	**F2$_{s2}$**	**F2$_{s3}$**	**F2$_{s4}$**	**F3$_{s1}$**	**F3$_{s2}$**	**F3$_{s3}$**	**F3$_{s4}$**
Thickness 5, 7.5, 17.5 cm	1.69	1.66	1.812	10.173	8.96	2.013	1.372	10.642	10.256	1.784	1.3

Table 4.6 (e) Best geometry for compound layers of selected order **3 2 1** with total thickness of layers = **60 cm**

Order of Layers 3 2 1	**F1$_{s1}$**	**F1$_{s2}$**	**F1$_{s3}$**	**F2$_{s1}$**	**F2$_{s2}$**	**F2$_{s3}$**	**F2$_{s4}$**	**F3$_{s1}$**	**F3$_{s2}$**	**F3$_{s3}$**	**F3$_{s4}$**
Thickness 10, 15, 35 cm	5.13	3.997	2.037	23.816	12.994	2.263	1.389	19.299	14.608	1.805	1.3

4.3 Profiling to increase ring performance

From formulation made earlier for increasing performance we implement a profile for the geometry used in this study. The best order of compound layers is selected (**3 2 1)** and changes are made to increase the stress tolerance of the ring. The width of the innermost ring layer is increased by 4 cm (2 cm each side). This increase is being carried out as the order **3 2 1** has innermost layer (P120 Carbon) with strongest resistance to tangential stresses (high-stiffness).

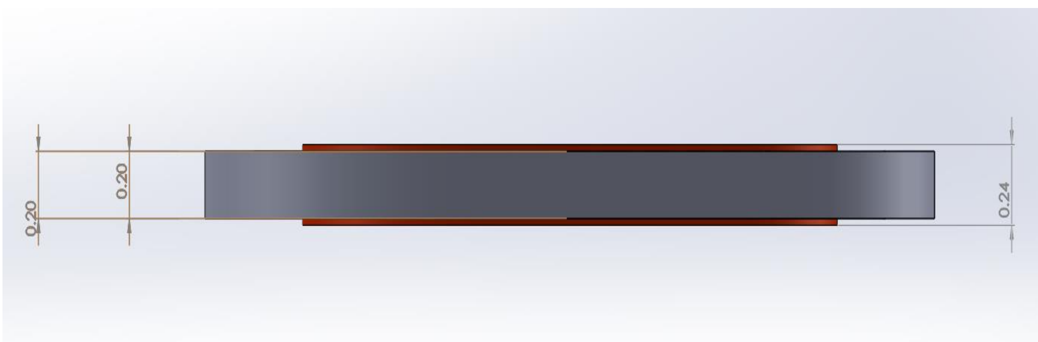
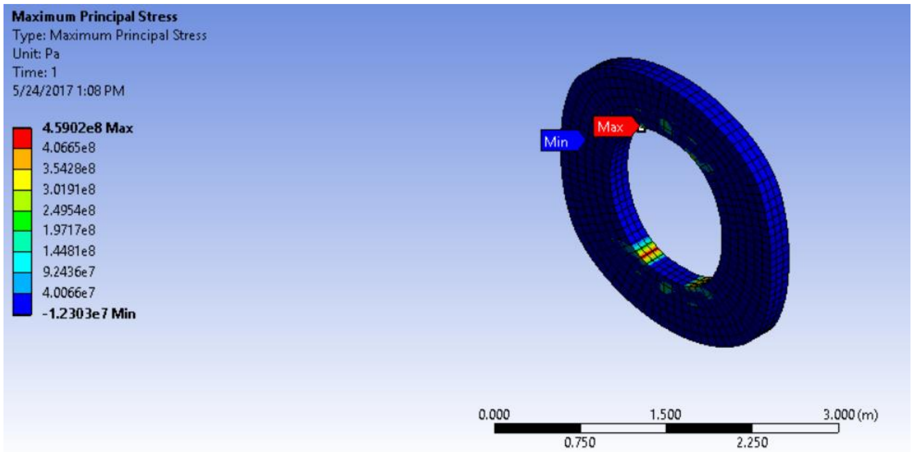
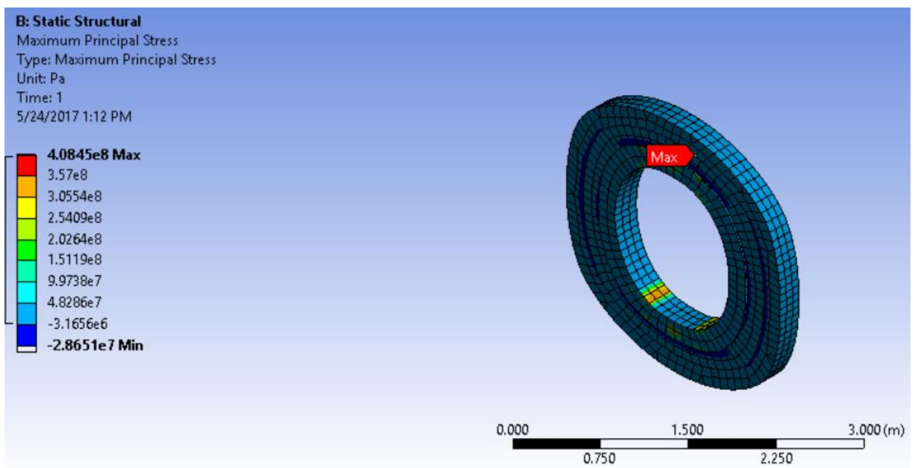


Figure 4-7 Figure showing profiling done for base geometry of compound layers



(a)



(b)

Figure 4-8 Stress distribution for ring (a) without profiling and (b) with profiling

FEA analysis is done using ANSYS Workbench 17 Academic Edition, the stress distribution of the two geometries (one with profiling and one without) is obtained. The material properties of chosen materials are input in the engineering data.

It is observed that the maximum stress experienced by structure is being reduced by about 12.38%, which is a noteworthy decrease.

Chapter 5

Conclusion and Future Work

Conclusion

In the investigation, methods were formulated to increase ultimate pressure performance on thin-walled concentric anisotropic rings by changing different parameters and observe the increase in performance with each iteration.

The best performing geometries were also obtained for different dimensions of compound layers of selected order (**3 2 1)** and also for similar proportions of different materials. Improvements made by profiling were validated by Finite Element Analysis (ANSYS Workbench 17 Academic Edition).

Future Work

Future work can be done in this area by devising a superimposition of the three approaches that were conceptualized.

The ring heterogeneity, different orders of compound layers and thicknesses can be varied along with incorporation of profiling and redistribution of fibers along the z-direction.

The results can then be validated with a thorough Finite Element Analysis (FEA) with deviation of different constraints to obtain optimum properties.

Appendix A

Calculation of Analytical Results using PTC MathCAD Prime 3.1

All analytical solutions were calculated using PTC Mathcad Prime 3.1 using Lekhnitskii's solution for stress distribution in composite anisotropic rings. The Mathcad calculations are attached as images in this section.

Solution for rings of single material: (a) **S2 glass**

$$a_0 \equiv 0.9\ m \qquad a_1 \equiv 1.1\ m \qquad a_2 \equiv 1.3\ m \qquad a_3 \equiv 1.5\ m \qquad q_0 \equiv 0\ GPa \qquad q_3 \equiv 100\ MPa$$

$$c_1 \equiv \frac{a_0}{a_1} = 0.818 \qquad c_2 \equiv \frac{a_1}{a_2} = 0.846 \qquad c_3 \equiv \frac{a_2}{a_3} = 0.867$$

$$E1_\theta \equiv 61.8\ GPa \quad E2_\theta \equiv 61.8\ GPa \quad E3_\theta \equiv 61.8\ GPa \quad E4_\theta \equiv 0\ GPa$$

$$E1_r \equiv 13.5\ GPa \quad E2_r \equiv 13.5\ GPa \quad E3_r \equiv 13.5\ GPa$$

$$\nu1_\theta \equiv 0.253 \qquad \nu2_\theta \equiv 0.253 \qquad \nu3_\theta \equiv 0.253 \qquad \nu4_\theta \equiv 0 \qquad \tau1_\theta \equiv 3.2\ GPa \quad \tau2_\theta \equiv 3.2\ GPa \quad \tau3_\theta \equiv 3.2\ GPa$$

$$k_1 \equiv \sqrt{\frac{E1_\theta}{E1_r}} \qquad k_2 \equiv \sqrt{\frac{E2_\theta}{E2_r}} \qquad k_3 \equiv \sqrt{\frac{E3_\theta}{E3_r}} = 2.14 \qquad \tau1_r \equiv 0.13\ GPa \quad \tau2_r \equiv 0.13\ GPa \quad \tau3_r \equiv 0.13\ GPa$$

$$\alpha_1 \equiv 2.\left(\frac{k_1}{E1_\theta}\right)\left(\frac{{c_1}^{k_1}}{1-{c_1}^{2k_1}}\right) = \left(7.821\cdot10^{-11}\right)\frac{1}{Pa} \qquad \beta_1 \equiv \frac{1}{E1_\theta}\left(\nu1_\theta-(k_1)\frac{\left(1+{c_1}^{2k_1}\right)}{\left(1-{c_1}^{2k_1}\right)}\right)-\frac{1}{E2_\theta}\left(\nu2_\theta+(k_2)\frac{\left(1+{c_2}^{2k_2}\right)}{\left(1-{c_2}^{2k_2}\right)}\right) = -1.865\cdot10^{-10}\frac{1}{Pa}$$

$$\alpha_2 \equiv 2.\left(\frac{k_2}{E2_\theta}\right)\left(\frac{{c_2}^{k_2}}{1-{c_2}^{2k_2}}\right) = \left(9.483\cdot10^{-11}\right)\frac{1}{Pa} \qquad \beta_2 \equiv \frac{1}{E2_\theta}\left(\nu2_\theta-(k_2)\frac{\left(1+{c_2}^{2k_2}\right)}{\left(1-{c_2}^{2k_2}\right)}\right)-\frac{1}{E3_\theta}\left(\nu3_\theta+(k_3)\frac{\left(1+{c_3}^{2k_3}\right)}{\left(1-{c_3}^{2k_3}\right)}\right) = -2.175\cdot10^{-10}\frac{1}{Pa}$$

$$\alpha_3 \equiv 2.\left(\frac{k_3}{E3_\theta}\right)\left(\frac{{c_3}^{k_3}}{1-{c_3}^{2k_3}}\right) = \left(1.113\cdot10^{-10}\right)\frac{1}{Pa} \qquad \beta_3 \equiv \frac{1}{E3_\theta}\left(\nu3_\theta-(k_3)\frac{\left(1+{c_3}^{2k_3}\right)}{\left(1-{c_3}^{2k_3}\right)}\right) = -1.125\cdot10^{-10}\frac{1}{Pa}$$

$$q_2 \equiv \frac{\left((q_3)\ (\alpha_3)\right)}{\left(\frac{(\alpha_1)\ (\alpha_2)}{\beta_1}-\beta_2\right)}\left(\frac{a_3}{a_2}\right) = \left(7.226\cdot10^7\right) Pa \qquad q_1 \equiv \frac{\left((-q_2)\ (\alpha_2)\right)}{\beta_1}\left(\frac{a_2}{a_1}\right) = \left(4.343\cdot10^7\right) Pa$$

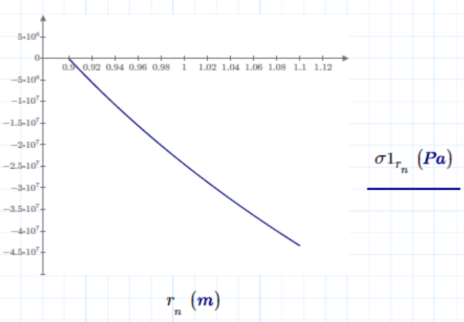
$$n := 0..20$$

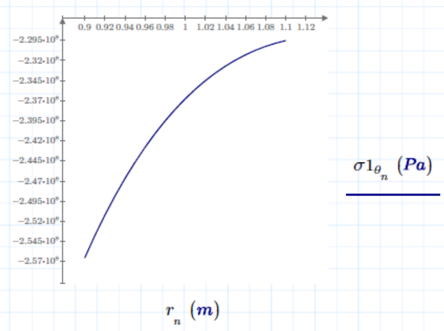
$$r_n := (0.9+0.01\ (n))\ m$$

$$\sigma1_{r_n} := (q_0)\cdot\frac{{c_1}^{k_1+1}}{1-{c_1}^{2k_1}}\cdot\left(\left(\frac{r_n}{a_1}\right)^{k_1-1}-\left(\frac{a_1}{r_n}\right)^{k_1+1}\right)+\left(\frac{q_1}{1-\left({c_1}^{2k_1}\right)}\right)\cdot\left(-\left(\frac{r_n}{a_1}\right)^{k_1-1}+{c_1}^{2k_1}\cdot\left(\frac{a_1}{r_n}\right)^{k_1+1}\right)$$

$$\sigma1_{\theta_n} := (q_0)\cdot\frac{{c_1}^{k_1+1}\cdot k_1}{1-{c_1}^{2k_1}}\cdot\left(\left(\frac{r_n}{a_1}\right)^{k_1-1}+\left(\frac{a_1}{r_n}\right)^{k_1+1}\right)-\left(\frac{q_1\cdot k_1}{1-{c_1}^{2k_1}}\right)\cdot\left(\left(\frac{r_n}{a_1}\right)^{k_1-1}+{c_1}^{2k_1}\cdot\left(\frac{a_1}{r_n}\right)^{k_1+1}\right)$$

$$u1_{r_n} := \left(\frac{q_0}{E1_\theta}\right)\cdot\frac{{c_1}^{k_1+1}\cdot a_1}{1-{c_1}^{2k_1}}\cdot\left((k_1-\nu1_\theta)\left(\frac{r_n}{a_1}\right)^{k_1}+(k_1-\nu1_\theta)\left(\frac{a_1}{r_n}\right)^{k_1}\right)-\left(\frac{q_1\cdot a_1}{E1_\theta\cdot\left(1-{c_1}^{2k_1}\right)}\right)\cdot\left((k_1-\nu1_\theta)\left(\frac{r_n}{a_1}\right)^{k_1}+(k_1+\nu1_\theta)\cdot{c_1}^{2k_1}\cdot\left(\frac{a_1}{r_n}\right)^{k_1}\right)$$





$$\sigma1_{\theta_0} := (q_0)\cdot\frac{c_1^{k_1+1}\cdot k_1}{1-c_1^{2k_1}}\cdot\left(\left(\frac{r_0}{a_1}\right)^{k_1-1}+\left(\frac{a_1}{r_0}\right)^{k_1+1}\right)-\left(\frac{q_1\cdot k_1}{1-c_1^{2k_1}}\right)\cdot\left(\left(\frac{r_0}{a_1}\right)^{k_1-1}+c_1^{2k_1}\cdot\left(\frac{a_1}{r_0}\right)^{k_1+1}\right)=-2.565\cdot10^8\ Pa \qquad F1_{s1}:=\left|\frac{\tau1_\theta}{\sigma1_{\theta_0}}\right|=12.474$$

$$\sigma1_{\theta_{20}} := (q_0)\cdot\frac{c_1^{k_1+1}\cdot k_1}{1-c_1^{2k_1}}\cdot\left(\left(\frac{r_{20}}{a_1}\right)^{k_1-1}+\left(\frac{a_1}{r_{20}}\right)^{k_1+1}\right)-\left(\frac{q_1\cdot k_1}{1-c_1^{2k_1}}\right)\cdot\left(\left(\frac{r_{20}}{a_1}\right)^{k_1-1}+c_1^{2k_1}\cdot\left(\frac{a_1}{r_{20}}\right)^{k_1+1}\right)=-2.295\cdot10^8\ Pa \qquad F1_{s2}:=\left|\frac{\tau1_\theta}{\sigma1_{\theta_{20}}}\right|=13.941$$

$$n:=0..20$$

$$F1_{s3}:=\left|\frac{\tau1_r}{q_1}\right|=2.994$$

$$r_n:=(1.1+0.01\ (n))\ m$$

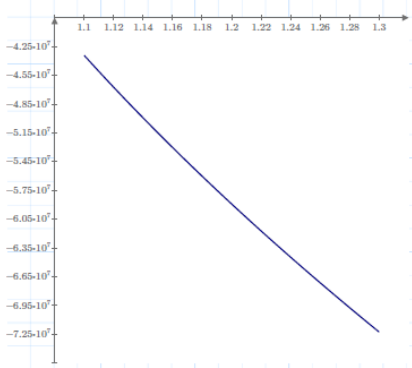
$$\sigma2_{r_n} := (q_1)\cdot\frac{c_2^{k_2+1}}{1-c_2^{2k_2}}\cdot\left(\left(\frac{r_n}{a_2}\right)^{k_2-1}-\left(\frac{a_2}{r_n}\right)^{k_2+1}\right)+\left(\frac{q_2}{1-\left(c_2^{2k_2}\right)}\right)\cdot\left(-\left(\frac{r_n}{a_2}\right)^{k_2-1}+c_2^{2k_2}\cdot\left(\frac{a_2}{r_n}\right)^{k_2+1}\right)$$

$$\sigma2_{\theta_n} := (q_1)\cdot\frac{c_2^{k_2+1}\cdot k_2}{1-c_2^{2k_2}}\cdot\left(\left(\frac{r_n}{a_2}\right)^{k_2-1}+\left(\frac{a_2}{r_n}\right)^{k_2+1}\right)-\left(\frac{q_2\cdot k_2}{1-c_2^{2k_2}}\right)\cdot\left(\left(\frac{r_n}{a_2}\right)^{k_2-1}+c_2^{2k_2}\cdot\left(\frac{a_2}{r_n}\right)^{k_2+1}\right)$$

$$u2_{r_n} := \left(\frac{q_1}{E2_\theta}\right)\cdot\frac{c_2^{k_2+1}\cdot a_2}{1-c_2^{2k_2}}\cdot\left((k_2-\nu2_\theta)\left(\frac{r_n}{a_2}\right)^{k_2}+(k_2-\nu2_\theta)\left(\frac{a_2}{r_n}\right)^{k_2}\right)-\left(\frac{q_2\cdot a_2}{E2_\theta\cdot\left(1-c_2^{2k_2}\right)}\right)\cdot\left((k_2-\nu2_\theta)\left(\frac{r_n}{a_2}\right)^{k_2}+(k_2+\nu2_\theta)\cdot c_2^{2k_2}\cdot\left(\frac{a_2}{r_n}\right)^{k_2}\right)$$

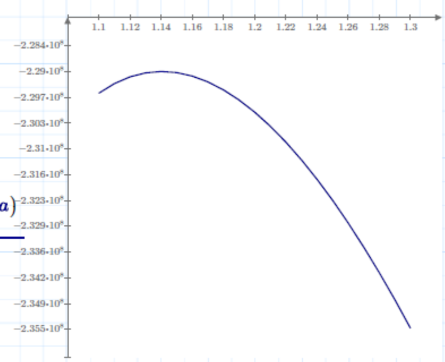
$$\sigma2_{\theta_0} := (q_1)\cdot\frac{c_2^{k_2+1}\cdot k_2}{1-c_2^{2k_2}}\cdot\left(\left(\frac{r_0}{a_2}\right)^{k_2-1}+\left(\frac{a_2}{r_0}\right)^{k_2+1}\right)-\left(\frac{q_2\cdot k_2}{1-c_2^{2k_2}}\right)\cdot\left(\left(\frac{r_0}{a_2}\right)^{k_2-1}+c_2^{2k_2}\cdot\left(\frac{a_2}{r_0}\right)^{k_2+1}\right)=-2.295\cdot10^8\ Pa \qquad F2_{s1}:=\left|\frac{\tau2_\theta}{\sigma2_{\theta_0}}\right|=13.941$$

$$\sigma2_{\theta_{20}} := (q_1)\cdot\frac{c_2^{k_2+1}\cdot k_2}{1-c_2^{2k_2}}\cdot\left(\left(\frac{r_{20}}{a_2}\right)^{k_2-1}+\left(\frac{a_2}{r_{20}}\right)^{k_2+1}\right)-\left(\frac{q_2\cdot k_2}{1-c_2^{2k_2}}\right)\cdot\left(\left(\frac{r_{20}}{a_2}\right)^{k_2-1}+c_2^{2k_2}\cdot\left(\frac{a_2}{r_{20}}\right)^{k_2+1}\right)=-2.355\cdot10^8\ Pa \qquad F2_{s2}:=\left|\frac{\tau2_\theta}{\sigma2_{\theta_{20}}}\right|=13.59$$



$\sigma2_{r_n}\ (Pa)$

$r_n\ (m)$



$\sigma2_{\theta_n}\ (Pa)$

$r_n\ (m)$

$$F2_{s3}:=\left|\frac{\tau2_r}{q_1}\right|=2.994$$

$$F2_{s4}:=\left|\frac{\tau2_r}{q_2}\right|=1.799$$

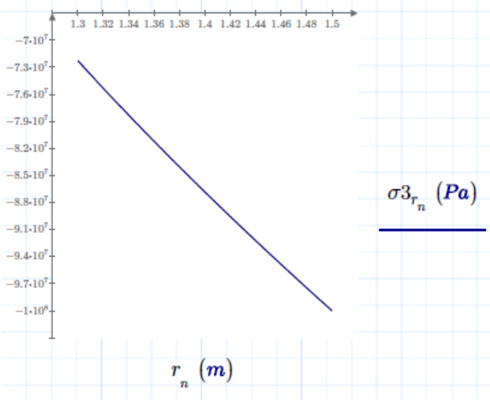
$$n:=0..20$$

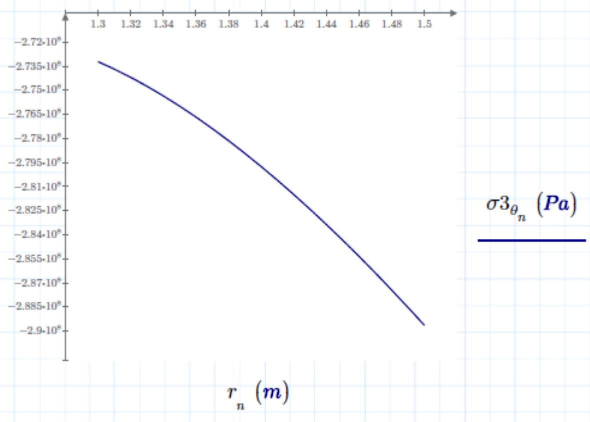
$$r_n:=(1.3+0.01\ (n))\ m$$

$$\sigma3_{r_n} := (q_2)\cdot\frac{c_3^{k_3+1}}{1-c_3^{2k_3}}\cdot\left(\left(\frac{r_n}{a_3}\right)^{k_3-1}-\left(\frac{a_3}{r_n}\right)^{k_3+1}\right)+\left(\frac{q_3}{1-\left(c_3^{2k_3}\right)}\right)\cdot\left(-\left(\frac{r_n}{a_3}\right)^{k_3-1}+c_3^{2k_3}\cdot\left(\frac{a_3}{r_n}\right)^{k_3+1}\right)$$

$$\sigma3_{\theta_n} := (q_2)\cdot\frac{c_3^{k_3+1}\cdot k_3}{1-c_3^{2k_3}}\cdot\left(\left(\frac{r_n}{a_3}\right)^{k_3-1}+\left(\frac{a_3}{r_n}\right)^{k_3+1}\right)-\left(\frac{q_3\cdot k_3}{1-c_3^{2k_3}}\right)\cdot\left(\left(\frac{r_n}{a_3}\right)^{k_3-1}+c_3^{2k_3}\cdot\left(\frac{a_3}{r_n}\right)^{k_3+1}\right)$$

$$u3_{r_n} := \left(\frac{q_2}{E3_\theta}\right)\cdot\frac{c_3^{k_3+1}\cdot a_3}{1-c_3^{2k_3}}\cdot\left((k_3-\nu3_\theta)\left(\frac{r_n}{a_3}\right)^{k_3}+(k_3-\nu3_\theta)\left(\frac{a_3}{r_n}\right)^{k_3}\right)-\left(\frac{q_3\cdot a_3}{E3_\theta\cdot\left(1-c_3^{2k_3}\right)}\right)\cdot\left((k_3-\nu3_\theta)\left(\frac{r_n}{a_3}\right)^{k_3}+(k_3+\nu3_\theta)\cdot c_3^{2k_3}\cdot\left(\frac{a_3}{r_n}\right)^{k_3}\right)$$





$$\sigma3_{\theta_0} := (q_2)\cdot\frac{c_3^{k_3+1}\cdot k_3}{1-c_3^{2k_3}}\cdot\left(\left(\frac{r_0}{a_3}\right)^{k_3-1}+\left(\frac{a_3}{r_0}\right)^{k_3+1}\right)-\left(\frac{q_3\cdot k_3}{1-c_3^{2k_3}}\right)\cdot\left(\left(\frac{r_0}{a_3}\right)^{k_3-1}+c_3^{2k_3}\cdot\left(\frac{a_3}{r_0}\right)^{k_3+1}\right)$$

$$\sigma3_{\theta_{20}} := (q_2)\cdot\frac{c_3^{k_3+1}\cdot k_3}{1-c_3^{2k_3}}\cdot\left(\left(\frac{r_{20}}{a_3}\right)^{k_3-1}+\left(\frac{a_3}{r_{20}}\right)^{k_3+1}\right)-\left(\frac{q_3\cdot k_3}{1-c_3^{2k_3}}\right)\cdot\left(\left(\frac{r_{20}}{a_3}\right)^{k_3-1}+c_3^{2k_3}\cdot\left(\frac{a_3}{r_{20}}\right)^{k_3+1}\right)$$

$$F3_{s1} := \left|\frac{\tau3_\theta}{\sigma3_{\theta_0}}\right| = 11.712$$

$$F3_{s2} := \left|\frac{\tau3_\theta}{\sigma3_{\theta_{20}}}\right| = 11.048$$

$$F3_{s3} := \left|\frac{\tau3_r}{q_2}\right| = 1.799$$

$$F3_{s4} := \left|\frac{\tau3_r}{q_3}\right| = 1.3$$

(b) **T1000-G**

$a_0 \equiv 0.9\ m \quad a_1 \equiv 1.1\ m \quad a_2 \equiv 1.3\ m \quad a_3 \equiv 1.5\ m \quad q_0 \equiv 0\ GPa \quad q_3 \equiv 100\ MPa$

$c_1 \equiv \frac{a_0}{a_1} = 0.818 \quad c_2 \equiv \frac{a_1}{a_2} = 0.846 \quad c_3 \equiv \frac{a_2}{a_3} = 0.867$

$E1_\theta \equiv 194\ GPa \quad E2_\theta \equiv 194\ GPa \quad E3_\theta \equiv 194\ GPa \quad E4_\theta \equiv 0\ GPa$

$E1_r \equiv 8.1\ GPa \quad E2_r \equiv 8.1\ GPa \quad E3_r \equiv 8.1\ GPa$

$\nu1_\theta \equiv 0.321 \quad \nu2_\theta \equiv 0.321 \quad \nu3_\theta \equiv 0.321 \quad \nu4_\theta \equiv 0 \quad \tau1_\theta \equiv 4.4\ GPa \quad \tau2_\theta \equiv 4.4\ GPa \quad \tau3_\theta \equiv 4.4\ GPa$

$k_1 \equiv \sqrt{\frac{E1_\theta}{E1_r}} \quad k_2 \equiv \sqrt{\frac{E2_\theta}{E2_r}} \quad k_3 \equiv \sqrt{\frac{E3_\theta}{E3_r}} = 4.894 \quad \tau1_r \equiv 0.10\ GPa \quad \tau2_r \equiv 0.10\ GPa \quad \tau3_r \equiv 0.10\ GPa$

$\alpha_1 \equiv 2. \left(\frac{k_1}{E1_\theta}\right)\left(\frac{c_1^{k_1}}{1-c_1^{2k_1}}\right) = \left(2.198 \cdot 10^{-11}\right)\frac{1}{Pa} \quad \beta_1 \equiv \frac{1}{E1_\theta}\left(\nu1_\theta - (k_1)\frac{\left(1+c_1^{2k_1}\right)}{\left(1-c_1^{2k_1}\right)}\right) - \frac{1}{E2_\theta}\left(\nu2_\theta + (k_2)\frac{\left(1+c_2^{2k_2}\right)}{\left(1-c_2^{2k_2}\right)}\right) = -7.09 \cdot 10^{-11}\ \frac{1}{Pa}$

$\alpha_2 \equiv 2. \left(\frac{k_2}{E2_\theta}\right)\left(\frac{c_2^{k_2}}{1-c_2^{2k_2}}\right) = \left(2.767 \cdot 10^{-11}\right)\frac{1}{Pa} \quad \beta_2 \equiv \frac{1}{E2_\theta}\left(\nu2_\theta - (k_2)\frac{\left(1+c_2^{2k_2}\right)}{\left(1-c_2^{2k_2}\right)}\right) - \frac{1}{E3_\theta}\left(\nu3_\theta + (k_3)\frac{\left(1+c_3^{2k_3}\right)}{\left(1-c_3^{2k_3}\right)}\right) = -7.917 \cdot 10^{-11}\ \frac{1}{Pa}$

$\alpha_3 \equiv 2. \left(\frac{k_3}{E3_\theta}\right)\left(\frac{c_3^{k_3}}{1-c_3^{2k_3}}\right) = \left(3.324 \cdot 10^{-11}\right)\frac{1}{Pa} \quad \beta_3 \equiv \frac{1}{E3_\theta}\left(\nu3_\theta - (k_3)\frac{\left(1+c_3^{2k_3}\right)}{\left(1-c_3^{2k_3}\right)}\right) = -4.007 \cdot 10^{-11}\ \frac{1}{Pa}$

$q_2 \equiv \frac{((q_3)(\alpha_3))}{\left(\frac{(\alpha_1)(\alpha_2)}{\beta_1} - \beta_2\right)}\left(\frac{a_3}{a_2}\right) = \left(5.433 \cdot 10^{7}\right) Pa \quad q_1 \equiv \frac{((-q_2)(\alpha_2))}{\beta_1}\left(\frac{a_2}{a_1}\right) = \left(2.506 \cdot 10^{7}\right) Pa$

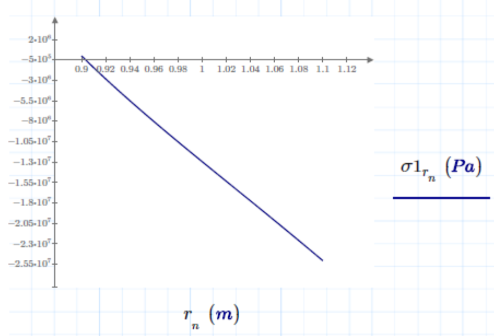
$n := 0..20$

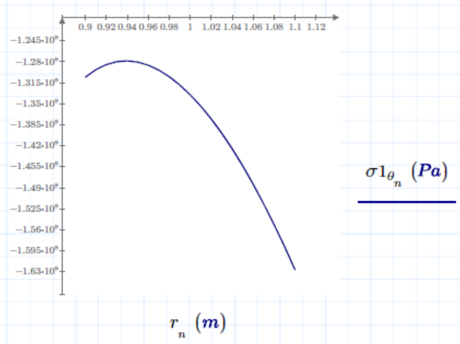
$r_n := (0.9 + 0.01\,(n))\ m$

$$\sigma1_{r_n} := (q_0)\cdot\frac{c_1^{k_1+1}}{1-c_1^{2k_1}}\cdot\left(\left(\frac{r_n}{a_1}\right)^{k_1-1} - \left(\frac{a_1}{r_n}\right)^{k_1+1}\right) + \left(\frac{q_1}{1-\left(c_1^{2k_1}\right)}\right)\cdot\left(-\left(\frac{r_n}{a_1}\right)^{k_1-1} + c_1^{2k_1}\cdot\left(\frac{a_1}{r_n}\right)^{k_1+1}\right)$$

$$\sigma1_{\theta_n} := (q_0)\cdot\frac{c_1^{k_1+1}\cdot k_1}{1-c_1^{2k_1}}\cdot\left(\left(\frac{r_n}{a_1}\right)^{k_1-1} + \left(\frac{a_1}{r_n}\right)^{k_1+1}\right) - \left(\frac{q_1\cdot k_1}{1-c_1^{2k_1}}\right)\cdot\left(\left(\frac{r_n}{a_1}\right)^{k_1-1} + c_1^{2k_1}\cdot\left(\frac{a_1}{r_n}\right)^{k_1+1}\right)$$

$$u1_{r_n} := \left(\frac{q_0}{E1_\theta}\right)\cdot\frac{c_1^{k_1+1}\cdot a_1}{1-c_1^{2k_1}}\cdot\left((k_1-\nu1_\theta)\left(\frac{r_n}{a_1}\right)^{k_1} + (k_1-\nu1_\theta)\left(\frac{a_1}{r_n}\right)^{k_1}\right) - \left(\frac{q_1\cdot a_1}{E1_\theta\cdot\left(1-c_1^{2k_1}\right)}\right)\cdot\left((k_1-\nu1_\theta)\left(\frac{r_n}{a_1}\right)^{k_1} + (k_1+\nu1_\theta)\cdot c_1^{2k_1}\cdot\left(\frac{a_1}{r_n}\right)^{k_1}\right)$$





$$\sigma1_{\theta_0} := (q_0)\cdot\frac{c_1^{k_1+1}\cdot k_1}{1-c_1^{2k_1}}\cdot\left(\left(\frac{r_0}{a_1}\right)^{k_1-1}+\left(\frac{a_1}{r_0}\right)^{k_1+1}\right)-\left(\frac{q_1\cdot k_1}{1-c_1^{2k_1}}\right)\cdot\left(\left(\frac{r_0}{a_1}\right)^{k_1-1}+c_1^{2k_1}\cdot\left(\frac{a_1}{r_0}\right)^{k_1+1}\right)=-1.306\cdot10^{8}\ Pa$$

$$F1_{s1} := \left|\frac{\tau1_\theta}{\sigma1_{\theta_0}}\right| = 33.696$$

$$\sigma1_{\theta_{20}} := (q_0)\cdot\frac{c_1^{k_1+1}\cdot k_1}{1-c_1^{2k_1}}\cdot\left(\left(\frac{r_{20}}{a_1}\right)^{k_1-1}+\left(\frac{a_1}{r_{20}}\right)^{k_1+1}\right)-\left(\frac{q_1\cdot k_1}{1-c_1^{2k_1}}\right)\cdot\left(\left(\frac{r_{20}}{a_1}\right)^{k_1-1}+c_1^{2k_1}\cdot\left(\frac{a_1}{r_{20}}\right)^{k_1+1}\right)=-1.626\cdot10^{8}\ Pa$$

$$F1_{s2} := \left|\frac{\tau1_\theta}{\sigma1_{\theta_{20}}}\right| = 27.054$$

$n := 0..20$

$$F1_{s3} := \left|\frac{\tau1_r}{q_1}\right| = 3.991$$

$r_n := (1.1+0.01\ (n))\ m$

$$\sigma2_{r_n} := (q_1)\cdot\frac{c_2^{k_2+1}}{1-c_2^{2k_2}}\cdot\left(\left(\frac{r_n}{a_2}\right)^{k_2-1}-\left(\frac{a_2}{r_n}\right)^{k_2+1}\right)+\left(\frac{q_2}{1-\left(c_2^{2k_2}\right)}\right)\cdot\left(-\left(\frac{r_n}{a_2}\right)^{k_2-1}+c_2^{2k_2}\cdot\left(\frac{a_2}{r_n}\right)^{k_2+1}\right)$$

$$\sigma2_{\theta_n} := (q_1)\cdot\frac{c_2^{k_2+1}\cdot k_2}{1-c_2^{2k_2}}\cdot\left(\left(\frac{r_n}{a_2}\right)^{k_2-1}+\left(\frac{a_2}{r_n}\right)^{k_2+1}\right)-\left(\frac{q_2\cdot k_2}{1-c_2^{2k_2}}\right)\cdot\left(\left(\frac{r_n}{a_2}\right)^{k_2-1}+c_2^{2k_2}\cdot\left(\frac{a_2}{r_n}\right)^{k_2+1}\right)$$

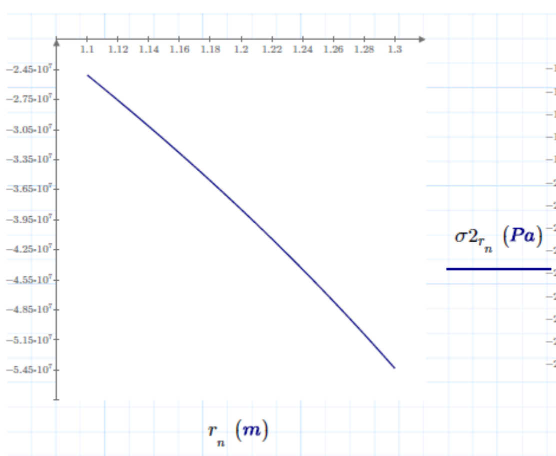
$$u2_{r_n} := \left(\frac{q_1}{E2_\theta}\right)\cdot\frac{c_2^{k_2+1}\cdot a_2}{1-c_2^{2k_2}}\cdot\left(\left(k_2-\nu2_\theta\right)\left(\frac{r_n}{a_2}\right)^{k_2}+\left(k_2-\nu2_\theta\right)\left(\frac{a_2}{r_n}\right)^{k_2}\right)-\left(\frac{q_2\cdot a_2}{E2_\theta\cdot\left(1-c_2^{2k_2}\right)}\right)\cdot\left(\left(k_2-\nu2_\theta\right)\left(\frac{r_n}{a_2}\right)^{k_2}+\left(k_2+\nu2_\theta\right)\cdot c_2^{2k_2}\cdot\left(\frac{a_2}{r_n}\right)^{k_2}\right)$$

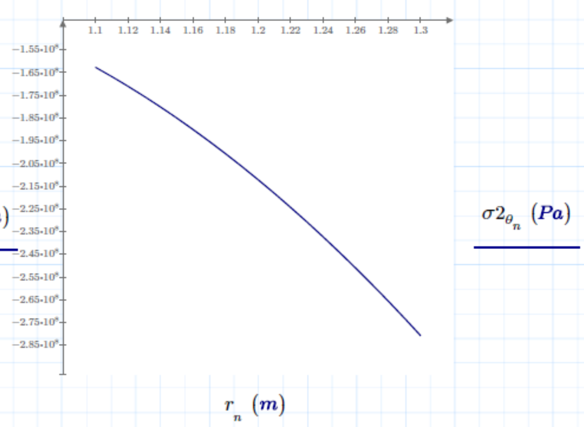
$$\sigma2_{\theta_0} := (q_1)\cdot\frac{c_2^{k_2+1}\cdot k_2}{1-c_2^{2k_2}}\cdot\left(\left(\frac{r_0}{a_2}\right)^{k_2-1}+\left(\frac{a_2}{r_0}\right)^{k_2+1}\right)-\left(\frac{q_2\cdot k_2}{1-c_2^{2k_2}}\right)\cdot\left(\left(\frac{r_0}{a_2}\right)^{k_2-1}+c_2^{2k_2}\cdot\left(\frac{a_2}{r_0}\right)^{k_2+1}\right)=-1.626\cdot10^{8}\ Pa$$

$$F2_{s1} := \left|\frac{\tau2_\theta}{\sigma2_{\theta_0}}\right| = 27.054$$

$$\sigma2_{\theta_{20}} := (q_1)\cdot\frac{c_2^{k_2+1}\cdot k_2}{1-c_2^{2k_2}}\cdot\left(\left(\frac{r_{20}}{a_2}\right)^{k_2-1}+\left(\frac{a_2}{r_{20}}\right)^{k_2+1}\right)-\left(\frac{q_2\cdot k_2}{1-c_2^{2k_2}}\right)\cdot\left(\left(\frac{r_{20}}{a_2}\right)^{k_2-1}+c_2^{2k_2}\cdot\left(\frac{a_2}{r_{20}}\right)^{k_2+1}\right)=-2.808\cdot10^{8}\ Pa$$

$$F2_{s2} := \left|\frac{\tau2_\theta}{\sigma2_{\theta_{20}}}\right| = 15.668$$





$$F2_{s3} := \left|\frac{\tau2_r}{q_1}\right| = 3.991$$

$$F2_{s4} := \left|\frac{\tau2_r}{q_2}\right| = 1.841$$

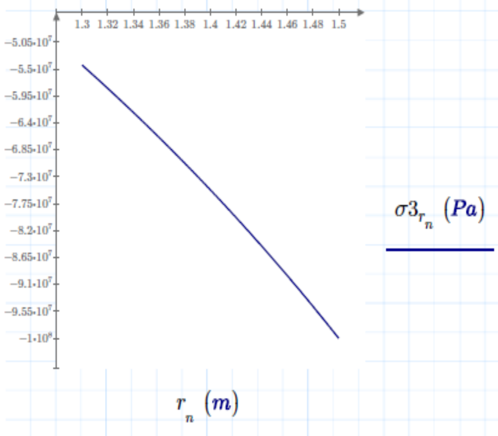
$n := 0..20$

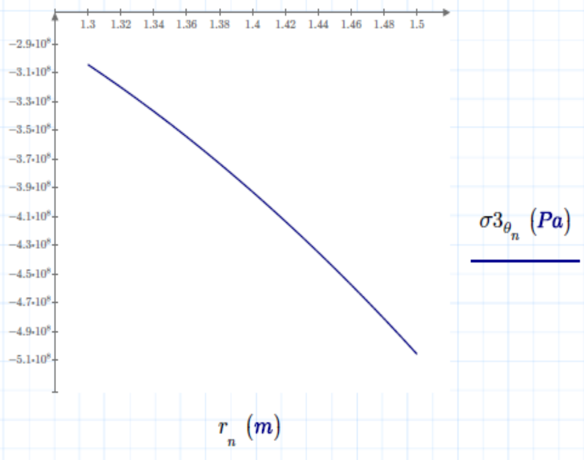
$r_n := (1.3+0.01\ (n))\ m$

$$\sigma3_{r_n} := (q_2)\cdot\frac{c_3^{k_3+1}}{1-c_3^{2k_3}}\cdot\left(\left(\frac{r_n}{a_3}\right)^{k_3-1}-\left(\frac{a_3}{r_n}\right)^{k_3+1}\right)+\left(\frac{q_3}{1-\left(c_3^{2k_3}\right)}\right)\cdot\left(-\left(\frac{r_n}{a_3}\right)^{k_3-1}+c_3^{2k_3}\cdot\left(\frac{a_3}{r_n}\right)^{k_3+1}\right)$$

$$\sigma3_{\theta_n} := (q_2)\cdot\frac{c_3^{k_3+1}\cdot k_3}{1-c_3^{2k_3}}\cdot\left(\left(\frac{r_n}{a_3}\right)^{k_3-1}+\left(\frac{a_3}{r_n}\right)^{k_3+1}\right)-\left(\frac{q_3\cdot k_3}{1-c_3^{2k_3}}\right)\cdot\left(\left(\frac{r_n}{a_3}\right)^{k_3-1}+c_3^{2k_3}\cdot\left(\frac{a_3}{r_n}\right)^{k_3+1}\right)$$

$$u3_{r_n} := \left(\frac{q_2}{E3_\theta}\right)\cdot\frac{c_3^{k_3+1}\cdot a_3}{1-c_3^{2k_3}}\cdot\left(\left(k_3-\nu3_\theta\right)\left(\frac{r_n}{a_3}\right)^{k_3}+\left(k_3-\nu3_\theta\right)\left(\frac{a_3}{r_n}\right)^{k_3}\right)-\left(\frac{q_3\cdot a_3}{E3_\theta\cdot\left(1-c_3^{2k_3}\right)}\right)\cdot\left(\left(k_3-\nu3_\theta\right)\left(\frac{r_n}{a_3}\right)^{k_3}+\left(k_3+\nu3_\theta\right)\cdot c_3^{2k_3}\cdot\left(\frac{a_3}{r_n}\right)^{k_3}\right)$$





$$\sigma3_{\theta_0} := (q_2)\cdot\frac{c_3^{k_3+1}\cdot k_3}{1-c_3^{2k_3}}\cdot\left(\left(\frac{r_0}{a_3}\right)^{k_3-1}+\left(\frac{a_3}{r_0}\right)^{k_3+1}\right)-\left(\frac{q_3\cdot k_3}{1-c_3^{2k_3}}\right)\cdot\left(\left(\frac{r_0}{a_3}\right)^{k_3-1}+c_3^{2k_3}\cdot\left(\frac{a_3}{r_0}\right)^{k_3+1}\right)$$

$$\sigma3_{\theta_{20}} := (q_2)\cdot\frac{c_3^{k_3+1}\cdot k_3}{1-c_3^{2k_3}}\cdot\left(\left(\frac{r_{20}}{a_3}\right)^{k_3-1}+\left(\frac{a_3}{r_{20}}\right)^{k_3+1}\right)-\left(\frac{q_3\cdot k_3}{1-c_3^{2k_3}}\right)\cdot\left(\left(\frac{r_{20}}{a_3}\right)^{k_3-1}+c_3^{2k_3}\cdot\left(\frac{a_3}{r_{20}}\right)^{k_3+1}\right)$$

$$F3_{s1} := \left|\frac{\tau3_\theta}{\sigma3_{\theta_0}}\right| = 14.463$$

$$F3_{s2} := \left|\frac{\tau3_\theta}{\sigma3_{\theta_{20}}}\right| = 8.697$$

$$F3_{s3} := \left|\frac{\tau3_r}{q_2}\right| = 1.841$$

$$F3_{s4} := \left|\frac{\tau3_r}{q_3}\right| = 1$$

(c) **P120**

$a_0 \equiv 0.9\ m$ $\quad a_1 \equiv 1.1\ m$ $\quad a_2 \equiv 1.3\ m$ $\quad a_3 \equiv 1.5\ m$ $\quad q_0 \equiv 0\ GPa$ $\quad q_3 \equiv 100\ MPa$

$c_1 \equiv \frac{a_0}{a_1} = 0.818 \quad c_2 \equiv \frac{a_1}{a_2} = 0.846 \quad c_3 \equiv \frac{a_2}{a_3} = 0.867$

$E1_\theta \equiv 582\ GPa \quad E2_\theta \equiv 582\ GPa \quad E3_\theta \equiv 582\ GPa \quad E4_\theta \equiv 0\ GPa$

$E1_r \equiv 8.1\ GPa \quad E2_r \equiv 8.1\ GPa \quad E3_r \equiv 8.1\ GPa$

$\nu1_\theta \equiv 0.31 \quad \nu2_\theta \equiv 0.31 \quad \nu3_\theta \equiv 0.31 \quad \nu4_\theta \equiv 0$

$\tau1_\theta \equiv 2.1\ GPa \quad \tau2_\theta \equiv 2.1\ GPa \quad \tau3_\theta \equiv 2.1\ GPa$

$k_1 \equiv \sqrt{\frac{E1_\theta}{E1_r}} \quad k_2 \equiv \sqrt{\frac{E2_\theta}{E2_r}} \quad k_3 \equiv \sqrt{\frac{E3_\theta}{E3_r}} = 8.477$

$\tau1_r \equiv 0.09\ GPa \quad \tau2_r \equiv 0.09\ GPa \quad \tau3_r \equiv 0.09\ GPa$

$$\alpha_1 \equiv 2.\left(\frac{k_1}{E1_\theta}\right)\left(\frac{c_1^{\,k_1}}{1-c_1^{\,2k_1}}\right) = \left(5.499 \cdot 10^{-12}\right)\frac{1}{Pa} \quad \beta_1 \equiv \frac{1}{E1_\theta}\left(\nu1_\theta - (k_1)\frac{\left(1+c_1^{\,2k_1}\right)}{\left(1-c_1^{\,2k_1}\right)}\right) - \frac{1}{E2_\theta}\left(\nu2_\theta + (k_2)\frac{\left(1+c_2^{\,2k_2}\right)}{\left(1-c_2^{\,2k_2}\right)}\right) = -3.196 \cdot 10^{-11}\frac{1}{Pa}$$

$$\alpha_2 \equiv 2.\left(\frac{k_2}{E2_\theta}\right)\left(\frac{c_2^{\,k_2}}{1-c_2^{\,2k_2}}\right) = \left(7.511 \cdot 10^{-12}\right)\frac{1}{Pa} \quad \beta_2 \equiv \frac{1}{E2_\theta}\left(\nu2_\theta - (k_2)\frac{\left(1+c_2^{\,2k_2}\right)}{\left(1-c_2^{\,2k_2}\right)}\right) - \frac{1}{E3_\theta}\left(\nu3_\theta + (k_3)\frac{\left(1+c_3^{\,2k_3}\right)}{\left(1-c_3^{\,2k_3}\right)}\right) = -3.378 \cdot 10^{-11}\frac{1}{Pa}$$

$$\alpha_3 \equiv 2.\left(\frac{k_3}{E3_\theta}\right)\left(\frac{c_3^{\,k_3}}{1-c_3^{\,2k_3}}\right) = \left(9.5 \cdot 10^{-12}\right)\frac{1}{Pa} \quad \beta_3 \equiv \frac{1}{E3_\theta}\left(\nu3_\theta - (k_3)\frac{\left(1+c_3^{\,2k_3}\right)}{\left(1-c_3^{\,2k_3}\right)}\right) = -1.686 \cdot 10^{-11}\frac{1}{Pa}$$

$$q_2 \equiv \frac{\left((q_3)\ (\alpha_3)\right)}{\left(\frac{(\alpha_1)\ (\alpha_2)}{\beta_1} - \beta_2\right)}\left(\frac{a_3}{a_2}\right) = \left(3.374 \cdot 10^{7}\right) Pa \quad q_1 \equiv \frac{\left((-q_2)\ (\alpha_2)\right)}{\beta_1}\left(\frac{a_2}{a_1}\right) = \left(9.374 \cdot 10^{6}\right) Pa$$

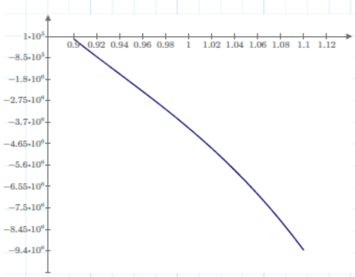
$n := 0..20$

$r_n := (0.9 + 0.01\ (n))\ m$

$$\sigma1_{r_n} := (q_0) \cdot \frac{c_1^{\,k_1+1}}{1-c_1^{\,2k_1}} \cdot \left(\left(\frac{r_n}{a_1}\right)^{k_1-1} - \left(\frac{a_1}{r_n}\right)^{k_1+1}\right) + \left(\frac{q_1}{1-\left(c_1^{\,2k_1}\right)}\right) \cdot \left(-\left(\frac{r_n}{a_1}\right)^{k_1-1} + c_1^{\,2k_1} \cdot \left(\frac{a_1}{r_n}\right)^{k_1+1}\right)$$

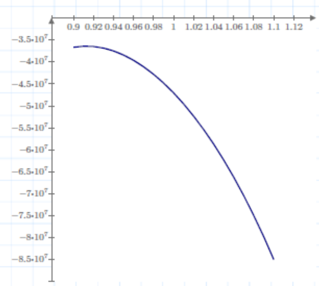
$$\sigma1_{\theta_n} := (q_0) \cdot \frac{c_1^{\,k_1+1} \cdot k_1}{1-c_1^{\,2k_1}} \cdot \left(\left(\frac{r_n}{a_1}\right)^{k_1-1} + \left(\frac{a_1}{r_n}\right)^{k_1+1}\right) - \left(\frac{q_1 \cdot k_1}{1-c_1^{\,2k_1}}\right) \cdot \left(\left(\frac{r_n}{a_1}\right)^{k_1-1} + c_1^{\,2k_1} \cdot \left(\frac{a_1}{r_n}\right)^{k_1+1}\right)$$

$$u1_{r_n} := \left(\frac{q_0}{E1_\theta}\right) \cdot \frac{c_1^{\,k_1+1} \cdot a_1}{1-c_1^{\,2k_1}} \cdot \left((k_1 - \nu1_\theta)\left(\frac{r_n}{a_1}\right)^{k_1} + (k_1 - \nu1_\theta)\left(\frac{a_1}{r_n}\right)^{k_1}\right) - \left(\frac{q_1 \cdot a_1}{E1_\theta \cdot \left(1-c_1^{\,2k_1}\right)}\right) \cdot \left((k_1 - \nu1_\theta)\left(\frac{r_n}{a_1}\right)^{k_1} + (k_1 + \nu1_\theta) \cdot c_1^{\,2k_1} \cdot \left(\frac{a_1}{r_n}\right)^{k_1}\right)$$



$\sigma1_{r_n}\ (Pa)$

$r_n\ (m)$



$\sigma1_{\theta_n}\ (Pa)$

$r_n\ (m)$

$$\sigma 1_{\theta_0} := (q_0) \cdot \frac{c_1^{k_1+1} \cdot k_1}{1-c_1^{2k_1}} \cdot \left(\left(\frac{r_0}{a_1} \right)^{k_1-1} + \left(\frac{a_1}{r_0} \right)^{k_1+1} \right) - \left(\frac{q_1 \cdot k_1}{1-c_1^{2k_1}} \right) \cdot \left(\left(\frac{r_0}{a_1} \right)^{k_1-1} + c_1^{2k_1} \cdot \left(\frac{a_1}{r_0} \right)^{k_1+1} \right) = -3.667 \cdot 10^7 \ Pa$$

$$F1_{s1} := \left| \frac{\tau 1_\theta}{\sigma 1_{\theta_0}} \right| = 57.27$$

$$\sigma 1_{\theta_{20}} := (q_0) \cdot \frac{c_1^{k_1+1} \cdot k_1}{1-c_1^{2k_1}} \cdot \left(\left(\frac{r_{20}}{a_1} \right)^{k_1-1} + \left(\frac{a_1}{r_{20}} \right)^{k_1+1} \right) - \left(\frac{q_1 \cdot k_1}{1-c_1^{2k_1}} \right) \cdot \left(\left(\frac{r_{20}}{a_1} \right)^{k_1-1} + c_1^{2k_1} \cdot \left(\frac{a_1}{r_{20}} \right)^{k_1+1} \right) = -8.493 \cdot 10^7 \ Pa$$

$$F1_{s2} := \left| \frac{\tau 1_\theta}{\sigma 1_{\theta_{20}}} \right| = 24.725$$

$n := 0 .. 20$

$$F1_{s3} := \left| \frac{\tau 1_r}{q_1} \right| = 9.601$$

$r_n := (1.1 + 0.01\ (n))\ m$

$$\sigma 2_{r_n} := (q_1) \cdot \frac{c_2^{k_2+1}}{1-c_2^{2k_2}} \cdot \left(\left(\frac{r_n}{a_2} \right)^{k_2-1} - \left(\frac{a_2}{r_n} \right)^{k_2+1} \right) + \left(\frac{q_2}{1-\left(c_2^{2k_2}\right)} \right) \cdot \left(-\left(\frac{r_n}{a_2} \right)^{k_2-1} + c_2^{2k_2} \cdot \left(\frac{a_2}{r_n} \right)^{k_2+1} \right)$$

$$\sigma 2_{\theta_n} := (q_1) \cdot \frac{c_2^{k_2+1} \cdot k_2}{1-c_2^{2k_2}} \cdot \left(\left(\frac{r_n}{a_2} \right)^{k_2-1} + \left(\frac{a_2}{r_n} \right)^{k_2+1} \right) - \left(\frac{q_2 \cdot k_2}{1-c_2^{2k_2}} \right) \cdot \left(\left(\frac{r_n}{a_2} \right)^{k_2-1} + c_2^{2k_2} \cdot \left(\frac{a_2}{r_n} \right)^{k_2+1} \right)$$

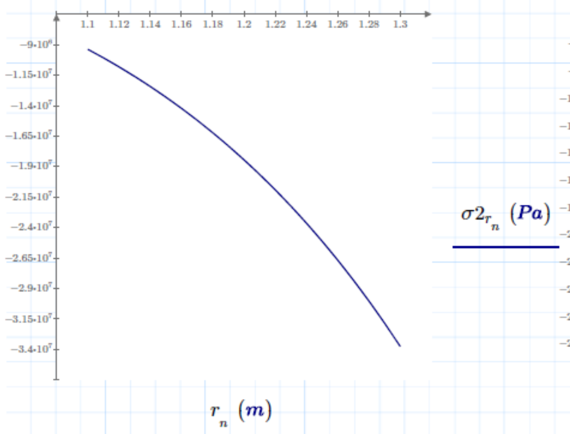
$$u2_{r_n} := \left(\frac{q_1}{E2_\theta} \right) \cdot \frac{c_2^{k_2+1} \cdot a_2}{1-c_2^{2k_2}} \cdot \left((k_2 - \nu 2_\theta) \left(\frac{r_n}{a_2} \right)^{k_2} + (k_2 - \nu 2_\theta) \left(\frac{a_2}{r_n} \right)^{k_2} \right) - \left(\frac{q_2 \cdot a_2}{E2_\theta \cdot \left(1-c_2^{2k_2}\right)} \right) \cdot \left((k_2 - \nu 2_\theta) \left(\frac{r_n}{a_2} \right)^{k_2} + (k_2 + \nu 2_\theta) \cdot c_2^{2k_2} \cdot \left(\frac{a_2}{r_n} \right)^{k_2} \right)$$

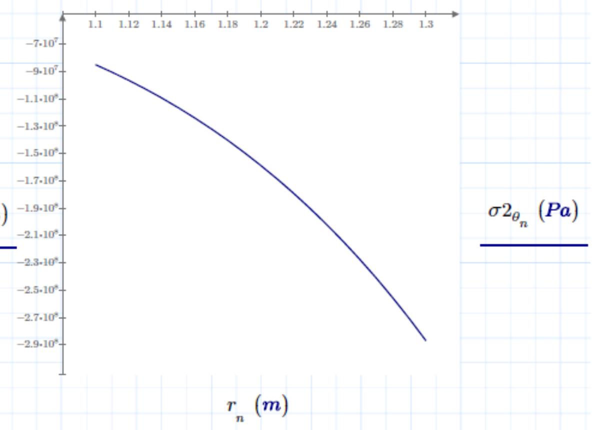
$$\sigma 2_{\theta_0} := (q_1) \cdot \frac{c_2^{k_2+1} \cdot k_2}{1-c_2^{2k_2}} \cdot \left(\left(\frac{r_0}{a_2} \right)^{k_2-1} + \left(\frac{a_2}{r_0} \right)^{k_2+1} \right) - \left(\frac{q_2 \cdot k_2}{1-c_2^{2k_2}} \right) \cdot \left(\left(\frac{r_0}{a_2} \right)^{k_2-1} + c_2^{2k_2} \cdot \left(\frac{a_2}{r_0} \right)^{k_2+1} \right) = -8.493 \cdot 10^7 \ Pa$$

$$F2_{s1} := \left| \frac{\tau 2_\theta}{\sigma 2_{\theta_0}} \right| = 24.725$$

$$\sigma 2_{\theta_{20}} := (q_1) \cdot \frac{c_2^{k_2+1} \cdot k_2}{1-c_2^{2k_2}} \cdot \left(\left(\frac{r_{20}}{a_2} \right)^{k_2-1} + \left(\frac{a_2}{r_{20}} \right)^{k_2+1} \right) - \left(\frac{q_2 \cdot k_2}{1-c_2^{2k_2}} \right) \cdot \left(\left(\frac{r_{20}}{a_2} \right)^{k_2-1} + c_2^{2k_2} \cdot \left(\frac{a_2}{r_{20}} \right)^{k_2+1} \right) = -2.872 \cdot 10^8 \ Pa$$

$$F2_{s2} := \left| \frac{\tau 2_\theta}{\sigma 2_{\theta_{20}}} \right| = 7.313$$





$$F2_{s3} := \left| \frac{\tau 2_r}{q_1} \right| = 9.601$$

$$F2_{s4} := \left| \frac{\tau 2_r}{q_2} \right| = 2.667$$

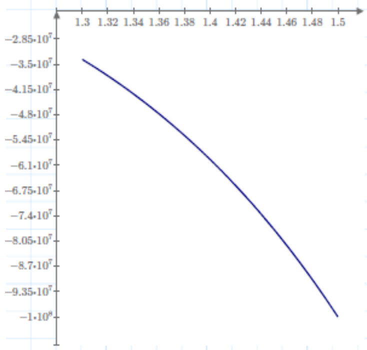
$n := 0 .. 20$

$r_n := (1.3 + 0.01\ (n))\ m$

$$\sigma3_{r_n} := (q_2)\cdot\frac{c_3^{k_3+1}}{1-c_3^{2k_3}}\cdot\left(\left(\frac{r_n}{a_3}\right)^{k_3-1}-\left(\frac{a_3}{r_n}\right)^{k_3+1}\right)+\left(\frac{q_3}{1-\left(c_3^{2k_3}\right)}\right)\cdot\left(-\left(\frac{r_n}{a_3}\right)^{k_3-1}+c_3^{2k_3}\cdot\left(\frac{a_3}{r_n}\right)^{k_3+1}\right)$$

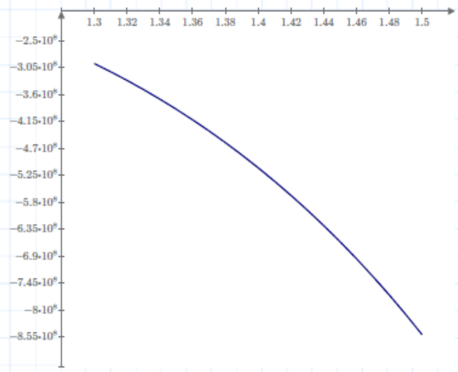
$$\sigma3_{\theta_n} := (q_2)\cdot\frac{c_3^{k_3+1}\cdot k_3}{1-c_3^{2k_3}}\cdot\left(\left(\frac{r_n}{a_3}\right)^{k_3-1}+\left(\frac{a_3}{r_n}\right)^{k_3+1}\right)-\left(\frac{q_3\cdot k_3}{1-c_3^{2k_3}}\right)\cdot\left(\left(\frac{r_n}{a_3}\right)^{k_3-1}+c_3^{2k_3}\cdot\left(\frac{a_3}{r_n}\right)^{k_3+1}\right)$$

$$u3_{r_n} := \left(\frac{q_2}{E3_\theta}\right)\cdot\frac{c_3^{k_3+1}\cdot a_3}{1-c_3^{2k_3}}\cdot\left(\left(k_3-\nu3_\theta\right)\left(\frac{r_n}{a_3}\right)^{k_3}+\left(k_3-\nu3_\theta\right)\left(\frac{a_3}{r_n}\right)^{k_3}\right)-\left(\frac{q_3\cdot a_3}{E3_\theta\cdot\left(1-c_3^{2k_3}\right)}\right)\cdot\left(\left(k_3-\nu3_\theta\right)\left(\frac{r_n}{a_3}\right)^{k_3}+\left(k_3+\nu3_\theta\right)\cdot c_3^{2k_3}\cdot\left(\frac{a_3}{r_n}\right)^{k_3}\right)$$



$\sigma3_{r_n}$ (Pa)

r_n (m)



$\sigma3_{\theta_n}$ (Pa)

r_n (m)

$$\sigma3_{\theta_0} := (q_2)\cdot\frac{c_3^{k_3+1}\cdot k_3}{1-c_3^{2k_3}}\cdot\left(\left(\frac{r_0}{a_3}\right)^{k_3-1}+\left(\frac{a_3}{r_0}\right)^{k_3+1}\right)-\left(\frac{q_3\cdot k_3}{1-c_3^{2k_3}}\right)\cdot\left(\left(\frac{r_0}{a_3}\right)^{k_3-1}+c_3^{2k_3}\cdot\left(\frac{a_3}{r_0}\right)^{k_3+1}\right)$$

$$\sigma3_{\theta_{20}} := (q_2)\cdot\frac{c_3^{k_3+1}\cdot k_3}{1-c_3^{2k_3}}\cdot\left(\left(\frac{r_{20}}{a_3}\right)^{k_3-1}+\left(\frac{a_3}{r_{20}}\right)^{k_3+1}\right)-\left(\frac{q_3\cdot k_3}{1-c_3^{2k_3}}\right)\cdot\left(\left(\frac{r_{20}}{a_3}\right)^{k_3-1}+c_3^{2k_3}\cdot\left(\frac{a_3}{r_{20}}\right)^{k_3+1}\right)$$

$$F3_{s1} := \left|\frac{\tau3_\theta}{\sigma3_{\theta_0}}\right| = 7.084$$

$$F3_{s2} := \left|\frac{\tau3_\theta}{\sigma3_{\theta_{20}}}\right| = 2.47$$

$$F3_{s3} := \left|\frac{\tau3_r}{q_2}\right| = 2.667$$

$$F3_{s4} := \left|\frac{\tau3_r}{q_3}\right| = 0.9$$

Solution for rings of compound layers in different orders: (i) **1 2 3**

$a_0 \equiv 0.9\ m \quad a_1 \equiv 1.1\ m \quad a_2 \equiv 1.3\ m \quad a_3 \equiv 1.5\ m \quad q_0 \equiv 0\ GPa \quad q_3 \equiv 100\ MPa$

$c_1 \equiv \frac{a_0}{a_1} = 0.818 \quad c_2 \equiv \frac{a_1}{a_2} = 0.846 \quad c_3 \equiv \frac{a_2}{a_3} = 0.867$

$E1_\theta \equiv 61.8\ GPa \quad E2_\theta \equiv 194\ GPa \quad E3_\theta \equiv 582\ GPa \quad E4_\theta \equiv 0\ GPa$

$E1_r \equiv 13.5\ GPa \quad E2_r \equiv 8.1\ GPa \quad E3_r \equiv 8.1\ GPa$

$\nu1_\theta \equiv 0.253 \quad \nu2_\theta \equiv 0.321 \quad \nu3_\theta \equiv 0.31 \quad \nu4_\theta \equiv 0 \quad \tau1_\theta \equiv 3.2\ GPa \quad \tau2_\theta \equiv 4.4\ GPa \quad \tau3_\theta \equiv 2.1\ GPa$

$k_1 \equiv \sqrt{\frac{E1_\theta}{E1_r}} \quad k_2 \equiv \sqrt{\frac{E2_\theta}{E2_r}} \quad k_3 \equiv \sqrt{\frac{E3_\theta}{E3_r}} = 8.477$

$\tau1_r \equiv 0.13\ GPa \quad \tau2_r \equiv 0.10\ GPa \quad \tau3_r \equiv 0.09\ GPa$

$$\alpha_1 \equiv 2.\left(\frac{k_1}{E1_\theta}\right)\left(\frac{c_1^{k_1}}{1-c_1^{2k_1}}\right) = (7.821\cdot10^{-11})\ \frac{1}{Pa} \qquad \beta_1 \equiv \frac{1}{E1_\theta}\left(\nu1_\theta-(k_1)\ \frac{(1+c_1^{2k_1})}{(1-c_1^{2k_1})}\right) - \frac{1}{E2_\theta}\left(\nu2_\theta+(k_2)\ \frac{(1+c_2^{2k_2})}{(1-c_2^{2k_2})}\right) = -1.205\cdot10^{-10}\ \frac{1}{Pa}$$

$$\alpha_2 \equiv 2.\left(\frac{k_2}{E2_\theta}\right)\left(\frac{c_2^{k_2}}{1-c_2^{2k_2}}\right) = (2.767\cdot10^{-11})\ \frac{1}{Pa} \qquad \beta_2 \equiv \frac{1}{E2_\theta}\left(\nu2_\theta-(k_2)\ \frac{(1+c_2^{2k_2})}{(1-c_2^{2k_2})}\right) - \frac{1}{E3_\theta}\left(\nu3_\theta+(k_3)\ \frac{(1+c_3^{2k_3})}{(1-c_3^{2k_3})}\right) = -5.371\cdot10^{-11}\ \frac{1}{Pa}$$

$$\alpha_3 \equiv 2.\left(\frac{k_3}{E3_\theta}\right)\left(\frac{c_3^{k_3}}{1-c_3^{2k_3}}\right) = (9.5\cdot10^{-12})\ \frac{1}{Pa} \qquad \beta_3 \equiv \frac{1}{E3_\theta}\left(\nu3_\theta-(k_3)\ \frac{(1+c_3^{2k_3})}{(1-c_3^{2k_3})}\right) = -1.686\cdot10^{-11}\ \frac{1}{Pa}$$

$$q_2 \equiv \frac{((q_3)\ (\alpha_3))}{\left(\frac{(\alpha_1)\ (\alpha_2)}{\beta_1}-\beta_2\right)}\left(\frac{a_3}{a_2}\right) = (3.066\cdot10^{7})\ Pa \qquad q_1 \equiv \frac{((-q_2)\ (\alpha_2))}{\beta_1}\left(\frac{a_2}{a_1}\right) = (8.317\cdot10^{6})\ Pa$$

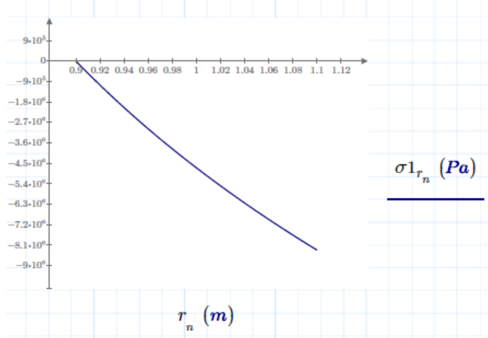
$n := 0\,..\,20$

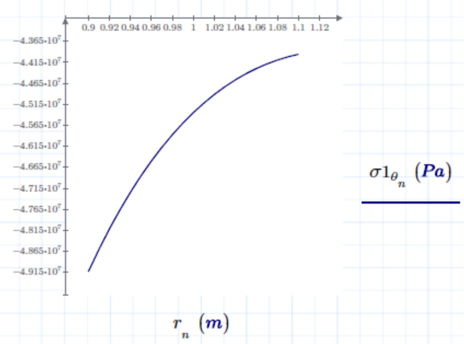
$r_n := (0.9+0.01\ (n))\ m$

$$\sigma1_{r_n} := (q_0)\cdot\frac{c_1^{k_1+1}}{1-c_1^{2k_1}}\cdot\left(\left(\frac{r_n}{a_1}\right)^{k_1-1}-\left(\frac{a_1}{r_n}\right)^{k_1+1}\right)+\left(\frac{q_1}{1-(c_1^{2k_1})}\right)\cdot\left(-\left(\frac{r_n}{a_1}\right)^{k_1-1}+c_1^{2k_1}\cdot\left(\frac{a_1}{r_n}\right)^{k_1+1}\right)$$

$$\sigma1_{\theta_n} := (q_0)\cdot\frac{c_1^{k_1+1}\cdot k_1}{1-c_1^{2k_1}}\cdot\left(\left(\frac{r_n}{a_1}\right)^{k_1-1}+\left(\frac{a_1}{r_n}\right)^{k_1+1}\right)-\left(\frac{q_1\cdot k_1}{1-c_1^{2k_1}}\right)\cdot\left(\left(\frac{r_n}{a_1}\right)^{k_1-1}+c_1^{2k_1}\cdot\left(\frac{a_1}{r_n}\right)^{k_1+1}\right)$$

$$u1_{r_n} := \left(\frac{q_0}{E1_\theta}\right)\cdot\frac{c_1^{k_1+1}\cdot a_1}{1-c_1^{2k_1}}\cdot\left((k_1-\nu1_\theta)\left(\frac{r_n}{a_1}\right)^{k_1}+(k_1-\nu1_\theta)\left(\frac{a_1}{r_n}\right)^{k_1}\right)-\left(\frac{q_1\cdot a_1}{E1_\theta\cdot(1-c_1^{2k_1})}\right)\cdot\left((k_1-\nu1_\theta)\left(\frac{r_n}{a_1}\right)^{k_1}+(k_1+\nu1_\theta)\cdot c_1^{2k_1}\cdot\left(\frac{a_1}{r_n}\right)^{k_1}\right)$$





$$\sigma 1_{\theta_0} := (q_0)\cdot\frac{c_1^{k_1+1}\cdot k_1}{1-c_1^{2k_1}}\cdot\left(\left(\frac{r_0}{a_1}\right)^{k_1-1}+\left(\frac{a_1}{r_0}\right)^{k_1+1}\right)-\left(\frac{q_1\cdot k_1}{1-c_1^{2k_1}}\right)\cdot\left(\left(\frac{r_0}{a_1}\right)^{k_1-1}+c_1^{2k_1}\cdot\left(\frac{a_1}{r_0}\right)^{k_1+1}\right)=-4.913\cdot 10^{7}\ \mathbf{Pa}$$

$$F1_{s1} := \left|\frac{\tau 1_\theta}{\sigma 1_{\theta_0}}\right| = 65.132$$

$$\sigma 1_{\theta_{20}} := (q_0)\cdot\frac{c_1^{k_1+1}\cdot k_1}{1-c_1^{2k_1}}\cdot\left(\left(\frac{r_{20}}{a_1}\right)^{k_1-1}+\left(\frac{a_1}{r_{20}}\right)^{k_1+1}\right)-\left(\frac{q_1\cdot k_1}{1-c_1^{2k_1}}\right)\cdot\left(\left(\frac{r_{20}}{a_1}\right)^{k_1-1}+c_1^{2k_1}\cdot\left(\frac{a_1}{r_{20}}\right)^{k_1+1}\right)=-4.396\cdot 10^{7}\ \mathbf{Pa}$$

$$F1_{s2} := \left|\frac{\tau 1_\theta}{\sigma 1_{\theta_{20}}}\right| = 72.793$$

$n := 0..20$

$$F1_{s3} := \left|\frac{\tau 1_r}{q_1}\right| = 15.631$$

$r_n := (1.1 + 0.01\ (n))\ \mathbf{m}$

$$\sigma 2_{r_n} := (q_1)\cdot\frac{c_2^{k_2+1}}{1-c_2^{2k_2}}\cdot\left(\left(\frac{r_n}{a_2}\right)^{k_2-1}-\left(\frac{a_2}{r_n}\right)^{k_2+1}\right)+\left(\frac{q_2}{1-\left(c_2^{2k_2}\right)}\right)\cdot\left(-\left(\frac{r_n}{a_2}\right)^{k_2-1}+c_2^{2k_2}\cdot\left(\frac{a_2}{r_n}\right)^{k_2+1}\right)$$

$$\sigma 2_{\theta_n} := (q_1)\cdot\frac{c_2^{k_2+1}\cdot k_2}{1-c_2^{2k_2}}\cdot\left(\left(\frac{r_n}{a_2}\right)^{k_2-1}+\left(\frac{a_2}{r_n}\right)^{k_2+1}\right)-\left(\frac{q_2\cdot k_2}{1-c_2^{2k_2}}\right)\cdot\left(\left(\frac{r_n}{a_2}\right)^{k_2-1}+c_2^{2k_2}\cdot\left(\frac{a_2}{r_n}\right)^{k_2+1}\right)$$

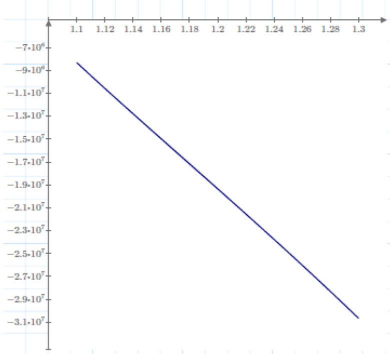
$$u2_{r_n} := \left(\frac{q_1}{E2_\theta}\right)\cdot\frac{c_2^{k_2+1}\cdot a_2}{1-c_2^{2k_2}}\cdot\left((k_2-\nu 2_\theta)\left(\frac{r_n}{a_2}\right)^{k_2}+(k_2-\nu 2_\theta)\left(\frac{a_2}{r_n}\right)^{k_2}\right)-\left(\frac{q_2\cdot a_2}{E2_\theta\cdot\left(1-c_2^{2k_2}\right)}\right)\cdot\left((k_2-\nu 2_\theta)\left(\frac{r_n}{a_2}\right)^{k_2}+(k_2+\nu 2_\theta)\cdot c_2^{2k_2}\cdot\left(\frac{a_2}{r_n}\right)^{k_2}\right)$$

$$\sigma 2_{\theta_0} := (q_1)\cdot\frac{c_2^{k_2+1}\cdot k_2}{1-c_2^{2k_2}}\cdot\left(\left(\frac{r_0}{a_2}\right)^{k_2-1}+\left(\frac{a_2}{r_0}\right)^{k_2+1}\right)-\left(\frac{q_2\cdot k_2}{1-c_2^{2k_2}}\right)\cdot\left(\left(\frac{r_0}{a_2}\right)^{k_2-1}+c_2^{2k_2}\cdot\left(\frac{a_2}{r_0}\right)^{k_2+1}\right)=-1.341\cdot 10^{8}\ \mathbf{Pa}$$

$$F2_{s1} := \left|\frac{\tau 2_\theta}{\sigma 2_{\theta_0}}\right| = 32.82$$

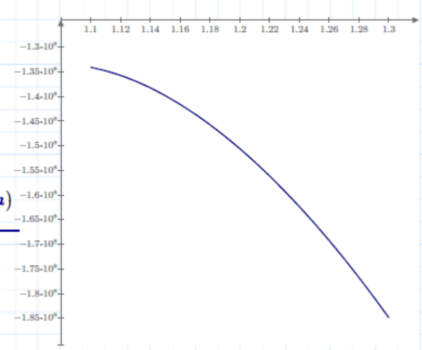
$$\sigma 2_{\theta_{20}} := (q_1)\cdot\frac{c_2^{k_2+1}\cdot k_2}{1-c_2^{2k_2}}\cdot\left(\left(\frac{r_{20}}{a_2}\right)^{k_2-1}+\left(\frac{a_2}{r_{20}}\right)^{k_2+1}\right)-\left(\frac{q_2\cdot k_2}{1-c_2^{2k_2}}\right)\cdot\left(\left(\frac{r_{20}}{a_2}\right)^{k_2-1}+c_2^{2k_2}\cdot\left(\frac{a_2}{r_{20}}\right)^{k_2+1}\right)=-1.849\cdot 10^{8}\ \mathbf{Pa}$$

$$F2_{s2} := \left|\frac{\tau 2_\theta}{\sigma 2_{\theta_{20}}}\right| = 23.796$$



$\sigma 2_{r_n}\ (\mathbf{Pa})$

$r_n\ (\mathbf{m})$



$\sigma 2_{\theta_n}\ (\mathbf{Pa})$

$r_n\ (\mathbf{m})$

$$F2_{s3} := \left|\frac{\tau 2_r}{q_1}\right| = 12.024$$

$$F2_{s4} := \left|\frac{\tau 2_r}{q_2}\right| = 3.262$$

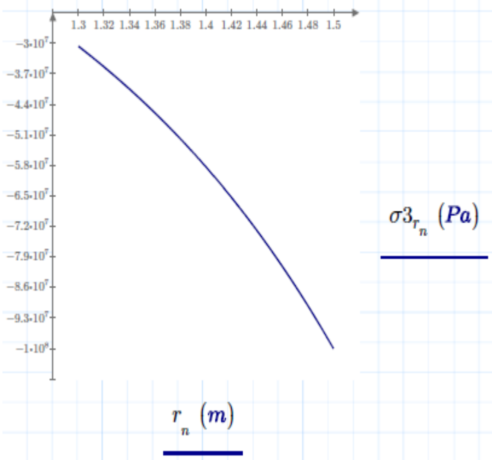
$n := 0..20$

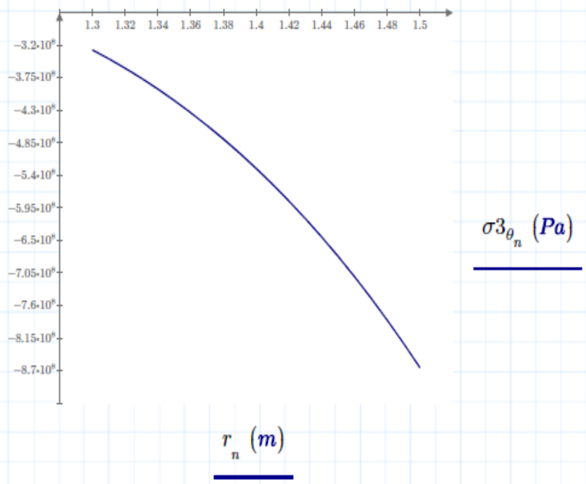
$r_n := (1.3 + 0.01\ (n))\ \mathbf{m}$

$$\sigma3_{r_n} := (q_2)\cdot\frac{c_3^{k_3+1}}{1-c_3^{2k_3}}\cdot\left(\left(\frac{r_n}{a_3}\right)^{k_3-1}-\left(\frac{a_3}{r_n}\right)^{k_3+1}\right)+\left(\frac{q_3}{1-\left(c_3^{2k_3}\right)}\right)\cdot\left(-\left(\frac{r_n}{a_3}\right)^{k_3-1}+c_3^{2k_3}\cdot\left(\frac{a_3}{r_n}\right)^{k_3+1}\right)$$

$$\sigma3_{\theta_n} := (q_2)\cdot\frac{c_3^{k_3+1}\cdot k_3}{1-c_3^{2k_3}}\cdot\left(\left(\frac{r_n}{a_3}\right)^{k_3-1}+\left(\frac{a_3}{r_n}\right)^{k_3+1}\right)-\left(\frac{q_3\cdot k_3}{1-c_3^{2k_3}}\right)\cdot\left(\left(\frac{r_n}{a_3}\right)^{k_3-1}+c_3^{2k_3}\cdot\left(\frac{a_3}{r_n}\right)^{k_3+1}\right)$$

$$u3_{r_n} := \left(\frac{q_2}{E3_\theta}\right)\cdot\frac{c_3^{k_3+1}\cdot a_3}{1-c_3^{2k_3}}\cdot\left(\left(k_3-\nu3_\theta\right)\left(\frac{r_n}{a_3}\right)^{k_3}+\left(k_3-\nu3_\theta\right)\left(\frac{a_3}{r_n}\right)^{k_3}\right)-\left(\frac{q_3\cdot a_3}{E3_\theta\cdot\left(1-c_3^{2k_3}\right)}\right)\cdot\left(\left(k_3-\nu3_\theta\right)\left(\frac{r_n}{a_3}\right)^{k_3}+\left(k_3+\nu3_\theta\right)\cdot c_3^{2k_3}\cdot\left(\frac{a_3}{r_n}\right)^{k_3}\right)$$





$$\sigma3_{\theta_0} := (q_2)\cdot\frac{c_3^{k_3+1}\cdot k_3}{1-c_3^{2k_3}}\cdot\left(\left(\frac{r_0}{a_3}\right)^{k_3-1}+\left(\frac{a_3}{r_0}\right)^{k_3+1}\right)-\left(\frac{q_3\cdot k_3}{1-c_3^{2k_3}}\right)\cdot\left(\left(\frac{r_0}{a_3}\right)^{k_3-1}+c_3^{2k_3}\cdot\left(\frac{a_3}{r_0}\right)^{k_3+1}\right)$$

$$\sigma3_{\theta_{20}} := (q_2)\cdot\frac{c_3^{k_3+1}\cdot k_3}{1-c_3^{2k_3}}\cdot\left(\left(\frac{r_{20}}{a_3}\right)^{k_3-1}+\left(\frac{a_3}{r_{20}}\right)^{k_3+1}\right)-\left(\frac{q_3\cdot k_3}{1-c_3^{2k_3}}\right)\cdot\left(\left(\frac{r_{20}}{a_3}\right)^{k_3-1}+c_3^{2k_3}\cdot\left(\frac{a_3}{r_{20}}\right)^{k_3+1}\right)$$

$$F3_{s1} := \left|\frac{\tau3_\theta}{\sigma3_{\theta_0}}\right| = 6.408$$

$$F3_{s2} := \left|\frac{\tau3_\theta}{\sigma3_{\theta_{20}}}\right| = 2.427$$

$$F3_{s3} := \left|\frac{\tau3_r}{q_2}\right| = 2.936$$

$$F3_{s4} := \left|\frac{\tau3_r}{q_3}\right| = 0.9$$

(ii) **1 3 2**

$a_0 \equiv 0.9\ m$ $a_1 \equiv 1.1\ m$ $a_2 \equiv 1.3\ m$ $a_3 \equiv 1.5\ m$ $q_0 \equiv 0\ GPa$ $q_3 \equiv 100\ MPa$

$c_1 \equiv \frac{a_0}{a_1} = 0.818$ $c_2 \equiv \frac{a_1}{a_2} = 0.846$ $c_3 \equiv \frac{a_2}{a_3} = 0.867$

$E2_\theta \equiv 61.8\ GPa$ $E1_\theta \equiv 194\ GPa$ $E3_\theta \equiv 582\ GPa$ $E4_\theta \equiv 0\ GPa$

$E2_r \equiv 13.5\ GPa$ $E1_r \equiv 8.1\ GPa$ $E3_r \equiv 8.1\ GPa$

$\nu 2_\theta \equiv 0.253$ $\nu 1_\theta \equiv 0.321$ $\nu 3_\theta \equiv 0.31$ $\nu 4_\theta \equiv 0$ $\tau 2_\theta \equiv 3.2\ GPa$ $\tau 1_\theta \equiv 4.4\ GPa$ $\tau 3_\theta \equiv 2.1\ GPa$

$k_1 \equiv \sqrt{\frac{E1_\theta}{E1_r}}$ $k_2 \equiv \sqrt{\frac{E2_\theta}{E2_r}}$ $k_3 \equiv \sqrt{\frac{E3_\theta}{E3_r}} = 8.477$

$\tau 2_r \equiv 0.13\ GPa$ $\tau 1_r \equiv 0.10\ GPa$ $\tau 3_r \equiv 0.09\ GPa$

$$\alpha_1 \equiv 2.\left(\frac{k_1}{E1_\theta}\right)\left(\frac{c_1^{k_1}}{1-c_1^{2k_1}}\right) = \left(2.198 \cdot 10^{-11}\right)\frac{1}{Pa} \quad \beta_1 \equiv \frac{1}{E1_\theta}\left(\nu 1_\theta - (k_1)\frac{\left(1+c_1^{2k_1}\right)}{\left(1-c_1^{2k_1}\right)}\right) - \frac{1}{E2_\theta}\left(\nu 2_\theta + (k_2)\frac{\left(1+c_2^{2k_2}\right)}{\left(1-c_2^{2k_2}\right)}\right) = -1.369 \cdot 10^{-10}\ \frac{1}{Pa}$$

$$\alpha_2 \equiv 2.\left(\frac{k_2}{E2_\theta}\right)\left(\frac{c_2^{k_2}}{1-c_2^{2k_2}}\right) = \left(9.483 \cdot 10^{-11}\right)\frac{1}{Pa} \quad \beta_2 \equiv \frac{1}{E2_\theta}\left(\nu 2_\theta - (k_2)\frac{\left(1+c_2^{2k_2}\right)}{\left(1-c_2^{2k_2}\right)}\right) - \frac{1}{E3_\theta}\left(\nu 3_\theta + (k_3)\frac{\left(1+c_3^{2k_3}\right)}{\left(1-c_3^{2k_3}\right)}\right) = -1.148 \cdot 10^{-10}\ \frac{1}{Pa}$$

$$\alpha_3 \equiv 2.\left(\frac{k_3}{E3_\theta}\right)\left(\frac{c_3^{k_3}}{1-c_3^{2k_3}}\right) = \left(9.5 \cdot 10^{-12}\right)\frac{1}{Pa} \quad \beta_3 \equiv \frac{1}{E3_\theta}\left(\nu 3_\theta - (k_3)\frac{\left(1+c_3^{2k_3}\right)}{\left(1-c_3^{2k_3}\right)}\right) = -1.686 \cdot 10^{-11}\ \frac{1}{Pa}$$

$$q_2 \equiv \frac{\left((q_3)\ (\alpha_3)\right)}{\left(\frac{(\alpha_1)\ (\alpha_2)}{\beta_1} - \beta_2\right)}\left(\frac{a_3}{a_2}\right) = \left(1.101 \cdot 10^{7}\right) Pa \quad q_1 \equiv \frac{\left((-q_2)\ (\alpha_2)\right)}{\beta_1}\left(\frac{a_2}{a_1}\right) = \left(9.017 \cdot 10^{6}\right) Pa$$

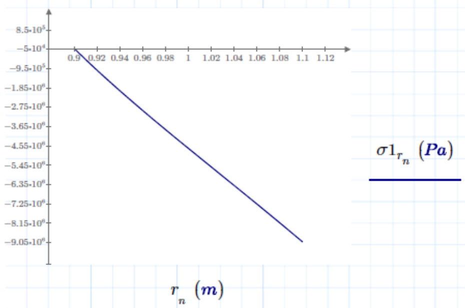
$n := 0..20$

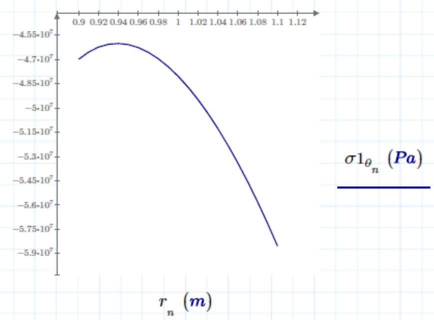
$r_n := (0.9 + 0.01\ (n))\ m$

$$\sigma 1_{r_n} := (q_0) \cdot \frac{c_1^{k_1+1}}{1-c_1^{2k_1}} \cdot \left(\left(\frac{r_n}{a_1}\right)^{k_1-1} - \left(\frac{a_1}{r_n}\right)^{k_1+1}\right) + \left(\frac{q_1}{1-\left(c_1^{2k_1}\right)}\right) \cdot \left(-\left(\frac{r_n}{a_1}\right)^{k_1-1} + c_1^{2k_1} \cdot \left(\frac{a_1}{r_n}\right)^{k_1+1}\right)$$

$$\sigma 1_{\theta_n} := (q_0) \cdot \frac{c_1^{k_1+1} \cdot k_1}{1-c_1^{2k_1}} \cdot \left(\left(\frac{r_n}{a_1}\right)^{k_1-1} + \left(\frac{a_1}{r_n}\right)^{k_1+1}\right) - \left(\frac{q_1 \cdot k_1}{1-c_1^{2k_1}}\right) \cdot \left(\left(\frac{r_n}{a_1}\right)^{k_1-1} + c_1^{2k_1} \cdot \left(\frac{a_1}{r_n}\right)^{k_1+1}\right)$$

$$u1_{r_n} := \left(\frac{q_0}{E1_\theta}\right) \cdot \frac{c_1^{k_1+1} \cdot a_1}{1-c_1^{2k_1}} \cdot \left((k_1 - \nu 1_\theta)\left(\frac{r_n}{a_1}\right)^{k_1} + (k_1 - \nu 1_\theta)\left(\frac{a_1}{r_n}\right)^{k_1}\right) - \left(\frac{q_1 \cdot a_1}{E1_\theta \cdot \left(1-c_1^{2k_1}\right)}\right) \cdot \left((k_1 - \nu 1_\theta)\left(\frac{r_n}{a_1}\right)^{k_1} + (k_1 + \nu 1_\theta) \cdot c_1^{2k_1} \cdot \left(\frac{a_1}{r_n}\right)^{k_1}\right)$$





$$\sigma 1_{\theta_0} := (q_0)\cdot\frac{c_1^{k_1+1}\cdot k_1}{1-c_1^{2k_1}}\cdot\left(\left(\frac{r_0}{a_1}\right)^{k_1-1}+\left(\frac{a_1}{r_0}\right)^{k_1+1}\right)-\left(\frac{q_1\cdot k_1}{1-c_1^{2k_1}}\right)\cdot\left(\left(\frac{r_0}{a_1}\right)^{k_1-1}+c_1^{2k_1}\cdot\left(\frac{a_1}{r_0}\right)^{k_1+1}\right)=-4.699\cdot 10^{7}\ Pa$$

$$F1_{s1} := \left|\frac{\tau 1_\theta}{\sigma 1_{\theta_0}}\right| = 93.627$$

$$\sigma 1_{\theta_{20}} := (q_0)\cdot\frac{c_1^{k_1+1}\cdot k_1}{1-c_1^{2k_1}}\cdot\left(\left(\frac{r_{20}}{a_1}\right)^{k_1-1}+\left(\frac{a_1}{r_{20}}\right)^{k_1+1}\right)-\left(\frac{q_1\cdot k_1}{1-c_1^{2k_1}}\right)\cdot\left(\left(\frac{r_{20}}{a_1}\right)^{k_1-1}+c_1^{2k_1}\cdot\left(\frac{a_1}{r_{20}}\right)^{k_1+1}\right)=-5.853\cdot 10^{7}\ Pa$$

$$F1_{s2} := \left|\frac{\tau 1_\theta}{\sigma 1_{\theta_{20}}}\right| = 75.173$$

$n := 0..20$

$$F1_{s3} := \left|\frac{\tau 1_r}{q_1}\right| = 11.09$$

$r_n := (1.1+0.01\ (n))\ m$

$$\sigma 2_{r_n} := (q_1)\cdot\frac{c_2^{k_2+1}}{1-c_2^{2k_2}}\cdot\left(\left(\frac{r_n}{a_2}\right)^{k_2-1}-\left(\frac{a_2}{r_n}\right)^{k_2+1}\right)+\left(\frac{q_2}{1-\left(c_2^{2k_2}\right)}\right)\cdot\left(-\left(\frac{r_n}{a_2}\right)^{k_2-1}+c_2^{2k_2}\cdot\left(\frac{a_2}{r_n}\right)^{k_2+1}\right)$$

$$\sigma 2_{\theta_n} := (q_1)\cdot\frac{c_2^{k_2+1}\cdot k_2}{1-c_2^{2k_2}}\cdot\left(\left(\frac{r_n}{a_2}\right)^{k_2-1}+\left(\frac{a_2}{r_n}\right)^{k_2+1}\right)-\left(\frac{q_2\cdot k_2}{1-c_2^{2k_2}}\right)\cdot\left(\left(\frac{r_n}{a_2}\right)^{k_2-1}+c_2^{2k_2}\cdot\left(\frac{a_2}{r_n}\right)^{k_2+1}\right)$$

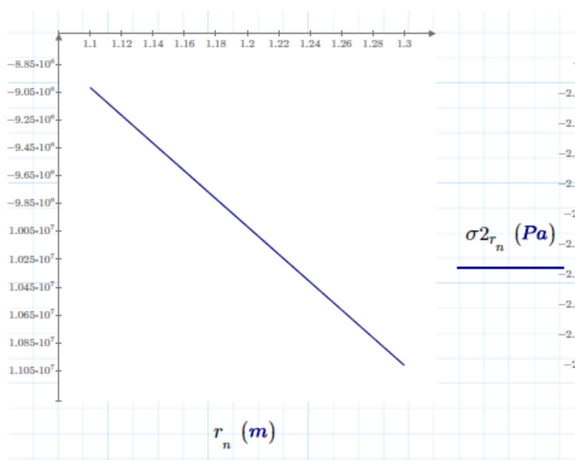
$$u2_{r_n} := \left(\frac{q_1}{E2_\theta}\right)\cdot\frac{c_2^{k_2+1}\cdot a_2}{1-c_2^{2k_2}}\cdot\left((k_2-\nu 2_\theta)\left(\frac{r_n}{a_2}\right)^{k_2}+(k_2-\nu 2_\theta)\left(\frac{a_2}{r_n}\right)^{k_2}\right)-\left(\frac{q_2\cdot a_2}{E2_\theta\cdot\left(1-c_2^{2k_2}\right)}\right)\cdot\left((k_2-\nu 2_\theta)\left(\frac{r_n}{a_2}\right)^{k_2}+(k_2+\nu 2_\theta)\cdot c_2^{2k_2}\cdot\left(\frac{a_2}{r_n}\right)^{k_2}\right)$$

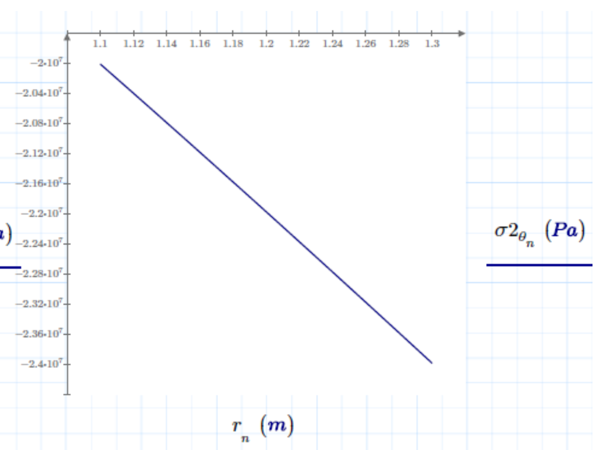
$$\sigma 2_{\theta_0} := (q_1)\cdot\frac{c_2^{k_2+1}\cdot k_2}{1-c_2^{2k_2}}\cdot\left(\left(\frac{r_0}{a_2}\right)^{k_2-1}+\left(\frac{a_2}{r_0}\right)^{k_2+1}\right)-\left(\frac{q_2\cdot k_2}{1-c_2^{2k_2}}\right)\cdot\left(\left(\frac{r_0}{a_2}\right)^{k_2-1}+c_2^{2k_2}\cdot\left(\frac{a_2}{r_0}\right)^{k_2+1}\right)=-2\cdot 10^{7}\ Pa$$

$$F2_{s1} := \left|\frac{\tau 2_\theta}{\sigma 2_{\theta_0}}\right| = 159.961$$

$$\sigma 2_{\theta_{20}} := (q_1)\cdot\frac{c_2^{k_2+1}\cdot k_2}{1-c_2^{2k_2}}\cdot\left(\left(\frac{r_{20}}{a_2}\right)^{k_2-1}+\left(\frac{a_2}{r_{20}}\right)^{k_2+1}\right)-\left(\frac{q_2\cdot k_2}{1-c_2^{2k_2}}\right)\cdot\left(\left(\frac{r_{20}}{a_2}\right)^{k_2-1}+c_2^{2k_2}\cdot\left(\frac{a_2}{r_{20}}\right)^{k_2+1}\right)=-2.398\cdot 10^{7}\ Pa$$

$$F2_{s2} := \left|\frac{\tau 2_\theta}{\sigma 2_{\theta_{20}}}\right| = 133.444$$





$$F2_{s3} := \left|\frac{\tau 2_r}{q_1}\right| = 14.417$$

$$F2_{s4} := \left|\frac{\tau 2_r}{q_2}\right| = 11.806$$

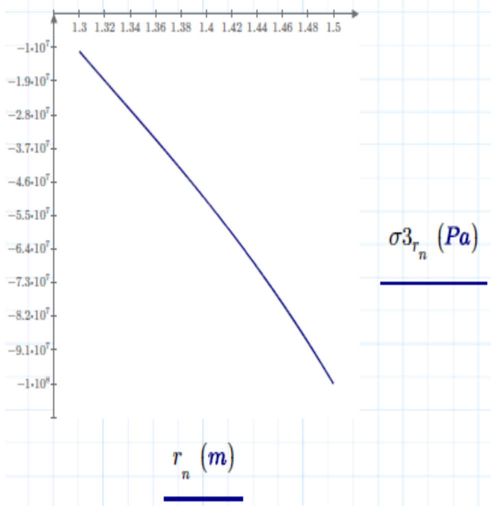
$n := 0..20$

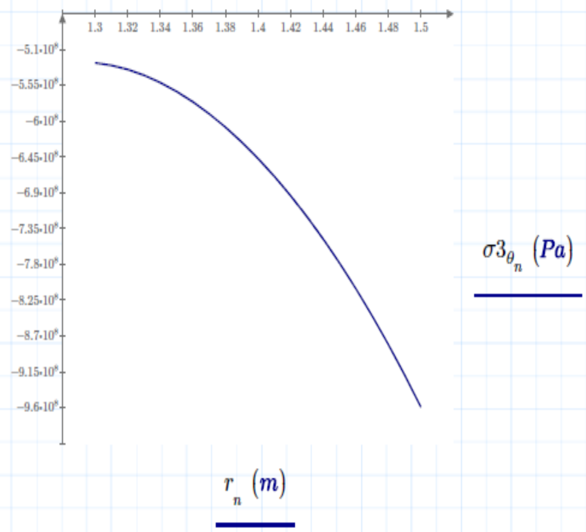
$r_n := (1.3+0.01\ (n))\ m$

$$\sigma 3_{r_n} := (q_2)\cdot\frac{c_3^{k_3+1}}{1-c_3^{2k_3}}\cdot\left(\left(\frac{r_n}{a_3}\right)^{k_3-1}-\left(\frac{a_3}{r_n}\right)^{k_3+1}\right)+\left(\frac{q_3}{1-\left(c_3^{2k_3}\right)}\right)\cdot\left(-\left(\frac{r_n}{a_3}\right)^{k_3-1}+c_3^{2k_3}\cdot\left(\frac{a_3}{r_n}\right)^{k_3+1}\right)$$

$$\sigma 3_{\theta_n} := (q_2)\cdot\frac{c_3^{k_3+1}\cdot k_3}{1-c_3^{2k_3}}\cdot\left(\left(\frac{r_n}{a_3}\right)^{k_3-1}+\left(\frac{a_3}{r_n}\right)^{k_3+1}\right)-\left(\frac{q_3\cdot k_3}{1-c_3^{2k_3}}\right)\cdot\left(\left(\frac{r_n}{a_3}\right)^{k_3-1}+c_3^{2k_3}\cdot\left(\frac{a_3}{r_n}\right)^{k_3+1}\right)$$

$$u3_{r_n} := \left(\frac{q_2}{E3_\theta}\right)\cdot\frac{c_3^{k_3+1}\cdot a_3}{1-c_3^{2k_3}}\cdot\left(\left(k_3-\nu 3_\theta\right)\left(\frac{r_n}{a_3}\right)^{k_3}+\left(k_3-\nu 3_\theta\right)\left(\frac{a_3}{r_n}\right)^{k_3}\right)-\left(\frac{q_3\cdot a_3}{E3_\theta\cdot\left(1-c_3^{2k_3}\right)}\right)\cdot\left(\left(k_3-\nu 3_\theta\right)\left(\frac{r_n}{a_3}\right)^{k_3}+\left(k_3+\nu 3_\theta\right)\cdot c_3^{2k_3}\cdot\left(\frac{a_3}{r_n}\right)^{k_3}\right)$$





$$\sigma 3_{\theta_0} := (q_2)\cdot\frac{c_3^{k_3+1}\cdot k_3}{1-c_3^{2k_3}}\cdot\left(\left(\frac{r_0}{a_3}\right)^{k_3-1}+\left(\frac{a_3}{r_0}\right)^{k_3+1}\right)-\left(\frac{q_3\cdot k_3}{1-c_3^{2k_3}}\right)\cdot\left(\left(\frac{r_0}{a_3}\right)^{k_3-1}+c_3^{2k_3}\cdot\left(\frac{a_3}{r_0}\right)^{k_3+1}\right)$$

$$\sigma 3_{\theta_{20}} := (q_2)\cdot\frac{c_3^{k_3+1}\cdot k_3}{1-c_3^{2k_3}}\cdot\left(\left(\frac{r_{20}}{a_3}\right)^{k_3-1}+\left(\frac{a_3}{r_{20}}\right)^{k_3+1}\right)-\left(\frac{q_3\cdot k_3}{1-c_3^{2k_3}}\right)\cdot\left(\left(\frac{r_{20}}{a_3}\right)^{k_3-1}+c_3^{2k_3}\cdot\left(\frac{a_3}{r_{20}}\right)^{k_3+1}\right)$$

$$F3_{s1} := \left|\frac{\tau 3_\theta}{\sigma 3_{\theta_0}}\right| = 3.988$$

$$F3_{s2} := \left|\frac{\tau 3_\theta}{\sigma 3_{\theta_{20}}}\right| = 2.189$$

$$F3_{s3} := \left|\frac{\tau 3_r}{q_2}\right| = 8.174$$

$$F3_{s4} := \left|\frac{\tau 3_r}{q_3}\right| = 0.9$$

(iii) **2 1 3**

$a_0 \equiv 0.9\ m$ $a_1 \equiv 1.1\ m$ $a_2 \equiv 1.3\ m$ $a_3 \equiv 1.5\ m$ $q_0 \equiv 0\ GPa$ $q_3 \equiv 100\ MPa$

$c_1 \equiv \frac{a_0}{a_1} = 0.818$ $c_2 \equiv \frac{a_1}{a_2} = 0.846$ $c_3 \equiv \frac{a_2}{a_3} = 0.867$

$E1_\theta \equiv 61.8\ GPa$ $E3_\theta \equiv 194\ GPa$ $E2_\theta \equiv 582\ GPa$ $E4_\theta \equiv 0\ GPa$

$E1_r \equiv 13.5\ GPa$ $E3_r \equiv 8.1\ GPa$ $E2_r \equiv 8.1\ GPa$

$\nu1_\theta \equiv 0.253$ $\nu3_\theta \equiv 0.321$ $\nu2_\theta \equiv 0.31$ $\nu4_\theta \equiv 0$ $\tau1_\theta \equiv 3.2\ GPa$ $\tau3_\theta \equiv 4.4\ GPa$ $\tau2_\theta \equiv 2.1\ GPa$

$k_1 \equiv \sqrt{\frac{E1_\theta}{E1_r}}$ $k_2 \equiv \sqrt{\frac{E2_\theta}{E2_r}}$ $k_3 \equiv \sqrt{\frac{E3_\theta}{E3_r}} = 4.894$ $\tau1_r \equiv 0.13\ GPa$ $\tau3_r \equiv 0.10\ GPa$ $\tau2_r \equiv 0.09\ GPa$

$$\alpha_1 \equiv 2.\left(\frac{k_1}{E1_\theta}\right)\left(\frac{c_1^{k_1}}{1-c_1^{2k_1}}\right) = \left(7.821 \cdot 10^{-11}\right)\frac{1}{Pa} \qquad \beta_1 \equiv \frac{1}{E1_\theta}\left(\nu1_\theta - (k_1)\frac{\left(1+c_1^{2k_1}\right)}{\left(1-c_1^{2k_1}\right)}\right) - \frac{1}{E2_\theta}\left(\nu2_\theta + (k_2)\frac{\left(1+c_2^{2k_2}\right)}{\left(1-c_2^{2k_2}\right)}\right) = -9.836 \cdot 10^{-11}\ \frac{1}{Pa}$$

$$\alpha_2 \equiv 2.\left(\frac{k_2}{E2_\theta}\right)\left(\frac{c_2^{k_2}}{1-c_2^{2k_2}}\right) = \left(7.511 \cdot 10^{-12}\right)\frac{1}{Pa} \qquad \beta_2 \equiv \frac{1}{E2_\theta}\left(\nu2_\theta - (k_2)\frac{\left(1+c_2^{2k_2}\right)}{\left(1-c_2^{2k_2}\right)}\right) - \frac{1}{E3_\theta}\left(\nu3_\theta + (k_3)\frac{\left(1+c_3^{2k_3}\right)}{\left(1-c_3^{2k_3}\right)}\right) = -5.924 \cdot 10^{-11}\ \frac{1}{Pa}$$

$$\alpha_3 \equiv 2.\left(\frac{k_3}{E3_\theta}\right)\left(\frac{c_3^{k_3}}{1-c_3^{2k_3}}\right) = \left(3.324 \cdot 10^{-11}\right)\frac{1}{Pa} \qquad \beta_3 \equiv \frac{1}{E3_\theta}\left(\nu3_\theta - (k_3)\frac{\left(1+c_3^{2k_3}\right)}{\left(1-c_3^{2k_3}\right)}\right) = -4.007 \cdot 10^{-11}\ \frac{1}{Pa}$$

$$q_2 \equiv \frac{\left((q_3)\ (\alpha_3)\right)}{\left(\frac{(\alpha_1)\ (\alpha_2)}{\beta_1} - \beta_2\right)}\left(\frac{a_3}{a_2}\right) = \left(7.2 \cdot 10^{7}\right) Pa \qquad q_1 \equiv \frac{\left((-q_2)\ (\alpha_2)\right)}{\beta_1}\left(\frac{a_2}{a_1}\right) = \left(6.498 \cdot 10^{6}\right) Pa$$

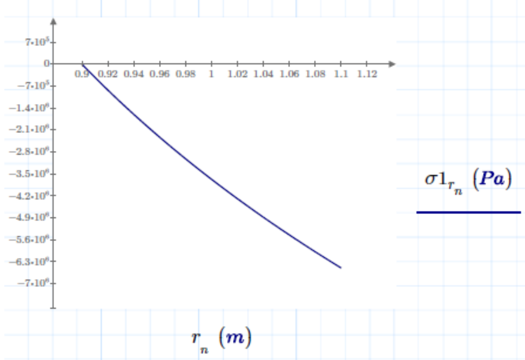
$n := 0\,..\,20$

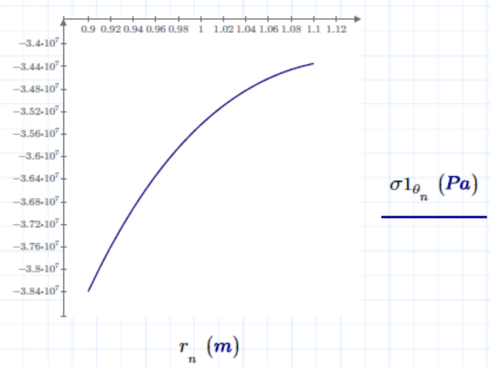
$r_n := (0.9 + 0.01\ (n))\ m$

$$\sigma1_{r_n} := (q_0) \cdot \frac{c_1^{k_1+1}}{1-c_1^{2k_1}} \cdot \left(\left(\frac{r_n}{a_1}\right)^{k_1-1} - \left(\frac{a_1}{r_n}\right)^{k_1+1}\right) + \left(\frac{q_1}{1-\left(c_1^{2k_1}\right)}\right) \cdot \left(-\left(\frac{r_n}{a_1}\right)^{k_1-1} + c_1^{2k_1} \cdot \left(\frac{a_1}{r_n}\right)^{k_1+1}\right)$$

$$\sigma1_{\theta_n} := (q_0) \cdot \frac{c_1^{k_1+1} \cdot k_1}{1-c_1^{2k_1}} \cdot \left(\left(\frac{r_n}{a_1}\right)^{k_1-1} + \left(\frac{a_1}{r_n}\right)^{k_1+1}\right) - \left(\frac{q_1 \cdot k_1}{1-c_1^{2k_1}}\right) \cdot \left(\left(\frac{r_n}{a_1}\right)^{k_1-1} + c_1^{2k_1} \cdot \left(\frac{a_1}{r_n}\right)^{k_1+1}\right)$$

$$u1_{r_n} := \left(\frac{q_0}{E1_\theta}\right) \cdot \frac{c_1^{k_1+1} \cdot a_1}{1-c_1^{2k_1}} \cdot \left((k_1 - \nu1_\theta)\left(\frac{r_n}{a_1}\right)^{k_1} + (k_1 - \nu1_\theta)\left(\frac{a_1}{r_n}\right)^{k_1}\right) - \left(\frac{q_1 \cdot a_1}{E1_\theta \cdot \left(1-c_1^{2k_1}\right)}\right) \cdot \left((k_1 - \nu1_\theta)\left(\frac{r_n}{a_1}\right)^{k_1} + (k_1 + \nu1_\theta) \cdot c_1^{2k_1} \cdot \left(\frac{a_1}{r_n}\right)^{k_1}\right)$$





$$\sigma 1_{\theta_0} := (q_0)\cdot\frac{c_1^{k_1+1}\cdot k_1}{1-c_1^{2k_1}}\cdot\left(\left(\frac{r_0}{a_1}\right)^{k_1-1}+\left(\frac{a_1}{r_0}\right)^{k_1+1}\right)-\left(\frac{q_1\cdot k_1}{1-c_1^{2k_1}}\right)\cdot\left(\left(\frac{r_0}{a_1}\right)^{k_1-1}+c_1^{2k_1}\cdot\left(\frac{a_1}{r_0}\right)^{k_1+1}\right)=-3.839\cdot 10^{7}\ \boldsymbol{Pa}$$

$$F1_{s1} := \left|\frac{\tau 1_\theta}{\sigma 1_{\theta_0}}\right| = 83.358$$

$$\sigma 1_{\theta_{20}} := (q_0)\cdot\frac{c_1^{k_1+1}\cdot k_1}{1-c_1^{2k_1}}\cdot\left(\left(\frac{r_{20}}{a_1}\right)^{k_1-1}+\left(\frac{a_1}{r_{20}}\right)^{k_1+1}\right)-\left(\frac{q_1\cdot k_1}{1-c_1^{2k_1}}\right)\cdot\left(\left(\frac{r_{20}}{a_1}\right)^{k_1-1}+c_1^{2k_1}\cdot\left(\frac{a_1}{r_{20}}\right)^{k_1+1}\right)=-3.435\cdot 10^{7}\ \boldsymbol{Pa}$$

$$F1_{s2} := \left|\frac{\tau 1_\theta}{\sigma 1_{\theta_{20}}}\right| = 93.163$$

$n := 0..20$

$$F1_{s3} := \left|\frac{\tau 1_r}{q_1}\right| = 20.005$$

$r_n := (1.1+0.01\ (n))\ \boldsymbol{m}$

$$\sigma 2_{r_n} := (q_1)\cdot\frac{c_2^{k_2+1}}{1-c_2^{2k_2}}\cdot\left(\left(\frac{r_n}{a_2}\right)^{k_2-1}-\left(\frac{a_2}{r_n}\right)^{k_2+1}\right)+\left(\frac{q_2}{1-\left(c_2^{2k_2}\right)}\right)\cdot\left(-\left(\frac{r_n}{a_2}\right)^{k_2-1}+c_2^{2k_2}\cdot\left(\frac{a_2}{r_n}\right)^{k_2+1}\right)$$

$$\sigma 2_{\theta_n} := (q_1)\cdot\frac{c_2^{k_2+1}\cdot k_2}{1-c_2^{2k_2}}\cdot\left(\left(\frac{r_n}{a_2}\right)^{k_2-1}+\left(\frac{a_2}{r_n}\right)^{k_2+1}\right)-\left(\frac{q_2\cdot k_2}{1-c_2^{2k_2}}\right)\cdot\left(\left(\frac{r_n}{a_2}\right)^{k_2-1}+c_2^{2k_2}\cdot\left(\frac{a_2}{r_n}\right)^{k_2+1}\right)$$

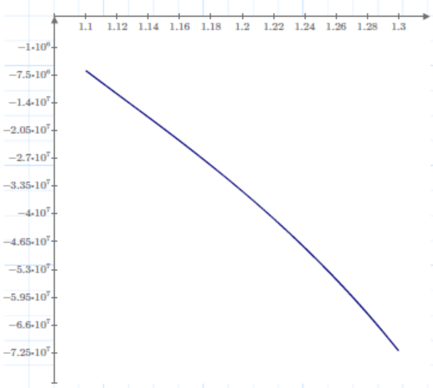
$$u2_{r_n} := \left(\frac{q_1}{E2_\theta}\right)\cdot\frac{c_2^{k_2+1}\cdot a_2}{1-c_2^{2k_2}}\cdot\left((k_2-\nu 2_\theta)\left(\frac{r_n}{a_2}\right)^{k_2}+(k_2-\nu 2_\theta)\left(\frac{a_2}{r_n}\right)^{k_2}\right)-\left(\frac{q_2\cdot a_2}{E2_\theta\cdot\left(1-c_2^{2k_2}\right)}\right)\cdot\left((k_2-\nu 2_\theta)\left(\frac{r_n}{a_2}\right)^{k_2}+(k_2+\nu 2_\theta)\cdot c_2^{2k_2}\cdot\left(\frac{a_2}{r_n}\right)^{k_2}\right)$$

$$\sigma 2_{\theta_0} := (q_1)\cdot\frac{c_2^{k_2+1}\cdot k_2}{1-c_2^{2k_2}}\cdot\left(\left(\frac{r_0}{a_2}\right)^{k_2-1}+\left(\frac{a_2}{r_0}\right)^{k_2+1}\right)-\left(\frac{q_2\cdot k_2}{1-c_2^{2k_2}}\right)\cdot\left(\left(\frac{r_0}{a_2}\right)^{k_2-1}+c_2^{2k_2}\cdot\left(\frac{a_2}{r_0}\right)^{k_2+1}\right)=-3.1\cdot 10^{8}\ \boldsymbol{Pa}$$

$$F2_{s1} := \left|\frac{\tau 2_\theta}{\sigma 2_{\theta_0}}\right| = 6.774$$

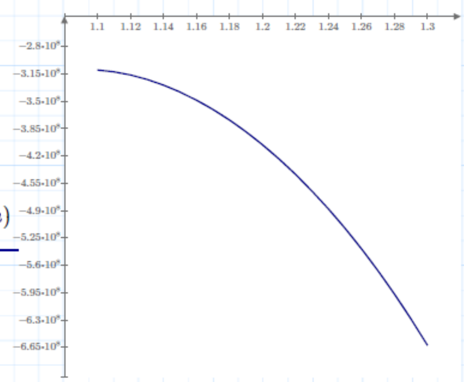
$$\sigma 2_{\theta_{20}} := (q_1)\cdot\frac{c_2^{k_2+1}\cdot k_2}{1-c_2^{2k_2}}\cdot\left(\left(\frac{r_{20}}{a_2}\right)^{k_2-1}+\left(\frac{a_2}{r_{20}}\right)^{k_2+1}\right)-\left(\frac{q_2\cdot k_2}{1-c_2^{2k_2}}\right)\cdot\left(\left(\frac{r_{20}}{a_2}\right)^{k_2-1}+c_2^{2k_2}\cdot\left(\frac{a_2}{r_{20}}\right)^{k_2+1}\right)=-6.627\cdot 10^{8}\ \boldsymbol{Pa}$$

$$F2_{s2} := \left|\frac{\tau 2_\theta}{\sigma 2_{\theta_{20}}}\right| = 3.169$$



$\sigma 2_{r_n}$ $(\boldsymbol{Pa})$

r_n $(\boldsymbol{m})$



$\sigma 2_{\theta_n}$ $(\boldsymbol{Pa})$

r_n $(\boldsymbol{m})$

$$F2_{s3} := \left|\frac{\tau 2_r}{q_1}\right| = 13.85$$

$$F2_{s4} := \left|\frac{\tau 2_r}{q_2}\right| = 1.25$$

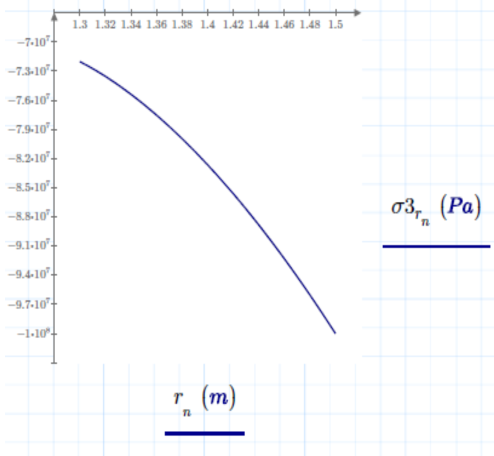
$n := 0..20$

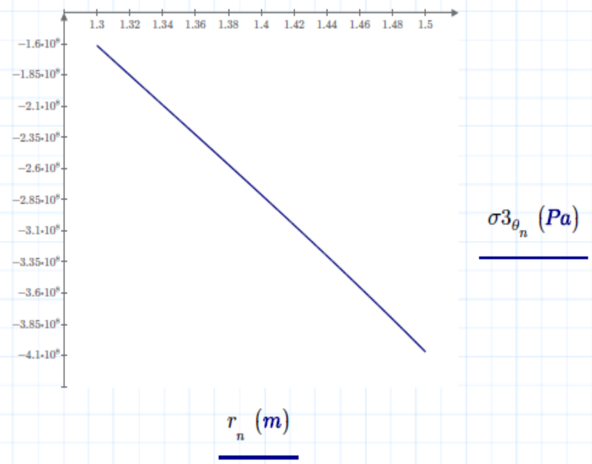
$r_n := (1.3+0.01\ (n))\ \boldsymbol{m}$

$$\sigma3_{r_n} := (q_2)\cdot\frac{c_3^{k_3+1}}{1-c_3^{2k_3}}\cdot\left(\left(\frac{r_n}{a_3}\right)^{k_3-1}-\left(\frac{a_3}{r_n}\right)^{k_3+1}\right)+\left(\frac{q_3}{1-\left(c_3^{2k_3}\right)}\right)\cdot\left(-\left(\frac{r_n}{a_3}\right)^{k_3-1}+c_3^{2k_3}\cdot\left(\frac{a_3}{r_n}\right)^{k_3+1}\right)$$

$$\sigma3_{\theta_n} := (q_2)\cdot\frac{c_3^{k_3+1}\cdot k_3}{1-c_3^{2k_3}}\cdot\left(\left(\frac{r_n}{a_3}\right)^{k_3-1}+\left(\frac{a_3}{r_n}\right)^{k_3+1}\right)-\left(\frac{q_3\cdot k_3}{1-c_3^{2k_3}}\right)\cdot\left(\left(\frac{r_n}{a_3}\right)^{k_3-1}+c_3^{2k_3}\cdot\left(\frac{a_3}{r_n}\right)^{k_3+1}\right)$$

$$u3_{r_n} := \left(\frac{q_2}{E3_\theta}\right)\cdot\frac{c_3^{k_3+1}\cdot a_3}{1-c_3^{2k_3}}\cdot\left(\left(k_3-\nu3_\theta\right)\left(\frac{r_n}{a_3}\right)^{k_3}+\left(k_3-\nu3_\theta\right)\left(\frac{a_3}{r_n}\right)^{k_3}\right)-\left(\frac{q_3\cdot a_3}{E3_\theta\cdot\left(1-c_3^{2k_3}\right)}\right)\cdot\left(\left(k_3-\nu3_\theta\right)\left(\frac{r_n}{a_3}\right)^{k_3}+\left(k_3+\nu3_\theta\right)\cdot c_3^{2k_3}\cdot\left(\frac{a_3}{r_n}\right)^{k_3}\right)$$





$$\sigma3_{\theta_0} := (q_2)\cdot\frac{c_3^{k_3+1}\cdot k_3}{1-c_3^{2k_3}}\cdot\left(\left(\frac{r_0}{a_3}\right)^{k_3-1}+\left(\frac{a_3}{r_0}\right)^{k_3+1}\right)-\left(\frac{q_3\cdot k_3}{1-c_3^{2k_3}}\right)\cdot\left(\left(\frac{r_0}{a_3}\right)^{k_3-1}+c_3^{2k_3}\cdot\left(\frac{a_3}{r_0}\right)^{k_3+1}\right)$$

$$\sigma3_{\theta_{20}} := (q_2)\cdot\frac{c_3^{k_3+1}\cdot k_3}{1-c_3^{2k_3}}\cdot\left(\left(\frac{r_{20}}{a_3}\right)^{k_3-1}+\left(\frac{a_3}{r_{20}}\right)^{k_3+1}\right)-\left(\frac{q_3\cdot k_3}{1-c_3^{2k_3}}\right)\cdot\left(\left(\frac{r_{20}}{a_3}\right)^{k_3-1}+c_3^{2k_3}\cdot\left(\frac{a_3}{r_{20}}\right)^{k_3+1}\right)$$

$$F3_{s1} := \left|\frac{\tau3_\theta}{\sigma3_{\theta_0}}\right| = 27.304$$

$$F3_{s2} := \left|\frac{\tau3_\theta}{\sigma3_{\theta_{20}}}\right| = 10.808$$

$$F3_{s3} := \left|\frac{\tau3_r}{q_2}\right| = 1.389$$

$$F3_{s4} := \left|\frac{\tau3_r}{q_3}\right| = 1$$

(iv) **2 3 1**

$a_0 \equiv 0.9\ m \quad a_1 \equiv 1.1\ m \quad a_2 \equiv 1.3\ m \quad a_3 \equiv 1.5\ m \quad q_0 \equiv 0\ GPa \quad q_3 \equiv 100\ MPa$

$c_1 \equiv \frac{a_0}{a_1} = 0.818 \quad c_2 \equiv \frac{a_1}{a_2} = 0.846 \quad c_3 \equiv \frac{a_2}{a_3} = 0.867$

$E2_\theta \equiv 61.8\ GPa \quad E3_\theta \equiv 194\ GPa \quad E1_\theta \equiv 582\ GPa \quad E4_\theta \equiv 0\ GPa$

$E2_r \equiv 13.5\ GPa \quad E3_r \equiv 8.1\ GPa \quad E1_r \equiv 8.1\ GPa$

$\nu2_\theta \equiv 0.253 \quad \nu3_\theta \equiv 0.321 \quad \nu1_\theta \equiv 0.31 \quad \nu4_\theta \equiv 0 \quad \tau2_\theta \equiv 3.2\ GPa \quad \tau3_\theta \equiv 4.4\ GPa \quad \tau1_\theta \equiv 2.1\ GPa$

$k_1 \equiv \sqrt{\frac{E1_\theta}{E1_r}} \quad k_2 \equiv \sqrt{\frac{E2_\theta}{E2_r}} \quad k_3 \equiv \sqrt{\frac{E3_\theta}{E3_r}} = 4.894$

$\tau2_r \equiv 0.13\ GPa \quad \tau3_r \equiv 0.10\ GPa \quad \tau1_r \equiv 0.09\ GPa$

$$\alpha_1 \equiv 2.\left(\frac{k_1}{E1_\theta}\right)\left(\frac{{c_1}^{k_1}}{1-{c_1}^{2k_1}}\right) = \left(5.499 \cdot 10^{-12}\right)\frac{1}{Pa} \qquad \beta_1 \equiv \frac{1}{E1_\theta}\left(\nu1_\theta - (k_1)\,\frac{\left(1+{c_1}^{2k_1}\right)}{\left(1-{c_1}^{2k_1}\right)}\right) - \frac{1}{E2_\theta}\left(\nu2_\theta + (k_2)\,\frac{\left(1+{c_2}^{2k_2}\right)}{\left(1-{c_2}^{2k_2}\right)}\right) = -1.201 \cdot 10^{-10}\ \frac{1}{Pa}$$

$$\alpha_2 \equiv 2.\left(\frac{k_2}{E2_\theta}\right)\left(\frac{{c_2}^{k_2}}{1-{c_2}^{2k_2}}\right) = \left(9.483 \cdot 10^{-11}\right)\frac{1}{Pa} \qquad \beta_2 \equiv \frac{1}{E2_\theta}\left(\nu2_\theta - (k_2)\,\frac{\left(1+{c_2}^{2k_2}\right)}{\left(1-{c_2}^{2k_2}\right)}\right) - \frac{1}{E3_\theta}\left(\nu3_\theta + (k_3)\,\frac{\left(1+{c_3}^{2k_3}\right)}{\left(1-{c_3}^{2k_3}\right)}\right) = -1.402 \cdot 10^{-10}\ \frac{1}{Pa}$$

$$\alpha_3 \equiv 2.\left(\frac{k_3}{E3_\theta}\right)\left(\frac{{c_3}^{k_3}}{1-{c_3}^{2k_3}}\right) = \left(3.324 \cdot 10^{-11}\right)\frac{1}{Pa} \qquad \beta_3 \equiv \frac{1}{E3_\theta}\left(\nu3_\theta - (k_3)\,\frac{\left(1+{c_3}^{2k_3}\right)}{\left(1-{c_3}^{2k_3}\right)}\right) = -4.007 \cdot 10^{-11}\ \frac{1}{Pa}$$

$$q_2 \equiv \frac{\left((q_3)\ (\alpha_3)\right)}{\left(\frac{(\alpha_1)\ (\alpha_2)}{\beta_1} - \beta_2\right)}\left(\frac{a_3}{a_2}\right) = \left(2.822 \cdot 10^{7}\right) Pa \qquad q_1 \equiv \frac{\left((-q_2)\ (\alpha_2)\right)}{\beta_1}\left(\frac{a_2}{a_1}\right) = \left(2.634 \cdot 10^{7}\right) Pa$$

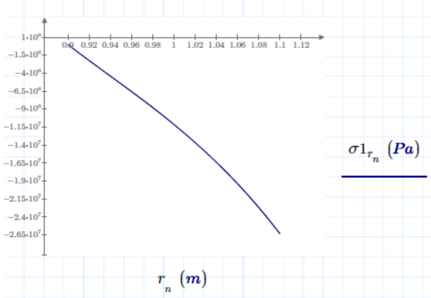
$n := 0..20$

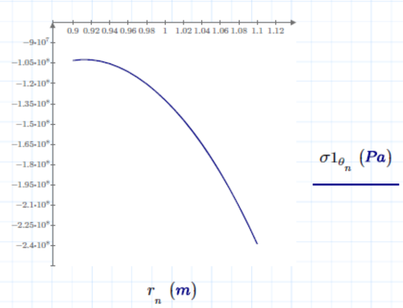
$r_n := (0.9 + 0.01\ (n))\ m$

$$\sigma1_{r_n} := (q_0) \cdot \frac{{c_1}^{k_1+1}}{1-{c_1}^{2k_1}} \cdot \left(\left(\frac{r_n}{a_1}\right)^{k_1-1} - \left(\frac{a_1}{r_n}\right)^{k_1+1}\right) + \left(\frac{q_1}{1-\left({c_1}^{2k_1}\right)}\right) \cdot \left(-\left(\frac{r_n}{a_1}\right)^{k_1-1} + {c_1}^{2k_1} \cdot \left(\frac{a_1}{r_n}\right)^{k_1+1}\right)$$

$$\sigma1_{\theta_n} := (q_0) \cdot \frac{{c_1}^{k_1+1} \cdot k_1}{1-{c_1}^{2k_1}} \cdot \left(\left(\frac{r_n}{a_1}\right)^{k_1-1} + \left(\frac{a_1}{r_n}\right)^{k_1+1}\right) - \left(\frac{q_1 \cdot k_1}{1-{c_1}^{2k_1}}\right) \cdot \left(\left(\frac{r_n}{a_1}\right)^{k_1-1} + {c_1}^{2k_1} \cdot \left(\frac{a_1}{r_n}\right)^{k_1+1}\right)$$

$$u1_{r_n} := \left(\frac{q_0}{E1_\theta}\right) \cdot \frac{{c_1}^{k_1+1} \cdot a_1}{1-{c_1}^{2k_1}} \cdot \left((k_1 - \nu1_\theta)\left(\frac{r_n}{a_1}\right)^{k_1} + (k_1 - \nu1_\theta)\left(\frac{a_1}{r_n}\right)^{k_1}\right) - \left(\frac{q_1 \cdot a_1}{E1_\theta \cdot \left(1-{c_1}^{2k_1}\right)}\right) \cdot \left((k_1 - \nu1_\theta)\left(\frac{r_n}{a_1}\right)^{k_1} + (k_1 + \nu1_\theta) \cdot {c_1}^{2k_1} \cdot \left(\frac{a_1}{r_n}\right)^{k_1}\right)$$





$$\sigma 1_{\theta_0} := (q_0)\cdot\frac{c_1^{k_1+1}\cdot k_1}{1-c_1^{2k_1}}\cdot\left(\left(\frac{r_0}{a_1}\right)^{k_1-1}+\left(\frac{a_1}{r_0}\right)^{k_1+1}\right)-\left(\frac{q_1\cdot k_1}{1-c_1^{2k_1}}\right)\cdot\left(\left(\frac{r_0}{a_1}\right)^{k_1-1}+c_1^{2k_1}\cdot\left(\frac{a_1}{r_0}\right)^{k_1+1}\right)=-1.03\cdot 10^{8}\ Pa \qquad F1_{s1} := \left|\frac{\tau 1_\theta}{\sigma 1_{\theta_0}}\right| = 20.383$$

$$\sigma 1_{\theta_{20}} := (q_0)\cdot\frac{c_1^{k_1+1}\cdot k_1}{1-c_1^{2k_1}}\cdot\left(\left(\frac{r_{20}}{a_1}\right)^{k_1-1}+\left(\frac{a_1}{r_{20}}\right)^{k_1+1}\right)-\left(\frac{q_1\cdot k_1}{1-c_1^{2k_1}}\right)\cdot\left(\left(\frac{r_{20}}{a_1}\right)^{k_1-1}+c_1^{2k_1}\cdot\left(\frac{a_1}{r_{20}}\right)^{k_1+1}\right)=-2.386\cdot 10^{8}\ Pa \qquad F1_{s2} := \left|\frac{\tau 1_\theta}{\sigma 1_{\theta_{20}}}\right| = 8.8$$

$n := 0..20$

$$F1_{s3} := \left|\frac{\tau 1_r}{q_1}\right| = 3.417$$

$r_n := (1.1+0.01\ (n))\ m$

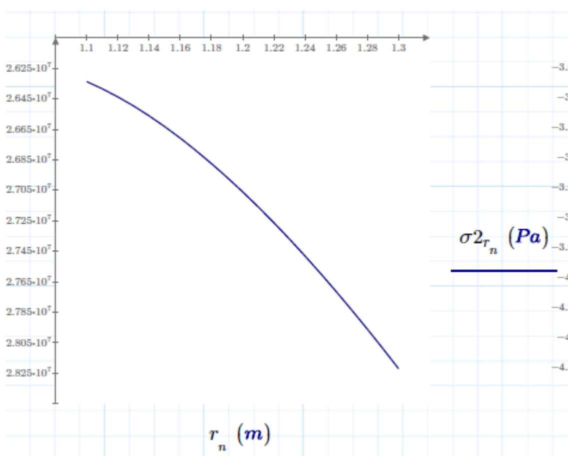
$$\sigma 2_{r_n} := (q_1)\cdot\frac{c_2^{k_2+1}}{1-c_2^{2k_2}}\cdot\left(\left(\frac{r_n}{a_2}\right)^{k_2-1}-\left(\frac{a_2}{r_n}\right)^{k_2+1}\right)+\left(\frac{q_2}{1-\left(c_2^{2k_2}\right)}\right)\cdot\left(-\left(\frac{r_n}{a_2}\right)^{k_2-1}+c_2^{2k_2}\cdot\left(\frac{a_2}{r_n}\right)^{k_2+1}\right)$$

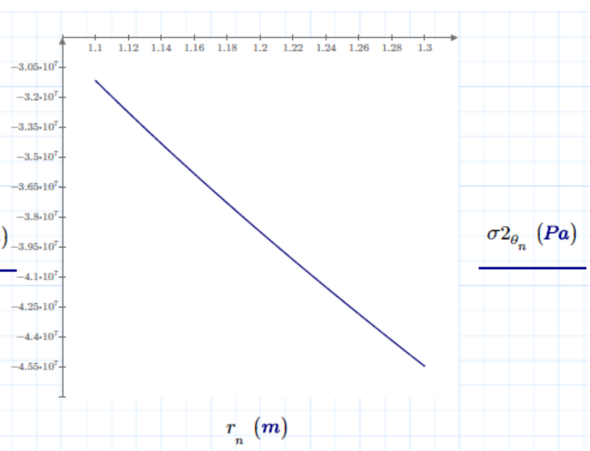
$$\sigma 2_{\theta_n} := (q_1)\cdot\frac{c_2^{k_2+1}\cdot k_2}{1-c_2^{2k_2}}\cdot\left(\left(\frac{r_n}{a_2}\right)^{k_2-1}+\left(\frac{a_2}{r_n}\right)^{k_2+1}\right)-\left(\frac{q_2\cdot k_2}{1-c_2^{2k_2}}\right)\cdot\left(\left(\frac{r_n}{a_2}\right)^{k_2-1}+c_2^{2k_2}\cdot\left(\frac{a_2}{r_n}\right)^{k_2+1}\right)$$

$$u2_{r_n} := \left(\frac{q_1}{E2_\theta}\right)\cdot\frac{c_2^{k_2+1}\cdot a_2}{1-c_2^{2k_2}}\cdot\left((k_2-\nu 2_\theta)\left(\frac{r_n}{a_2}\right)^{k_2}+(k_2-\nu 2_\theta)\left(\frac{a_2}{r_n}\right)^{k_2}\right)-\left(\frac{q_2\cdot a_2}{E2_\theta\cdot\left(1-c_2^{2k_2}\right)}\right)\cdot\left((k_2-\nu 2_\theta)\left(\frac{r_n}{a_2}\right)^{k_2}+(k_2+\nu 2_\theta)\cdot c_2^{2k_2}\cdot\left(\frac{a_2}{r_n}\right)^{k_2}\right)$$

$$\sigma 2_{\theta_0} := (q_1)\cdot\frac{c_2^{k_2+1}\cdot k_2}{1-c_2^{2k_2}}\cdot\left(\left(\frac{r_0}{a_2}\right)^{k_2-1}+\left(\frac{a_2}{r_0}\right)^{k_2+1}\right)-\left(\frac{q_2\cdot k_2}{1-c_2^{2k_2}}\right)\cdot\left(\left(\frac{r_0}{a_2}\right)^{k_2-1}+c_2^{2k_2}\cdot\left(\frac{a_2}{r_0}\right)^{k_2+1}\right)=-3.114\cdot 10^{7}\ Pa \qquad F2_{s1} := \left|\frac{\tau 2_\theta}{\sigma 2_{\theta_0}}\right| = 102.774$$

$$\sigma 2_{\theta_{20}} := (q_1)\cdot\frac{c_2^{k_2+1}\cdot k_2}{1-c_2^{2k_2}}\cdot\left(\left(\frac{r_{20}}{a_2}\right)^{k_2-1}+\left(\frac{a_2}{r_{20}}\right)^{k_2+1}\right)-\left(\frac{q_2\cdot k_2}{1-c_2^{2k_2}}\right)\cdot\left(\left(\frac{r_{20}}{a_2}\right)^{k_2-1}+c_2^{2k_2}\cdot\left(\frac{a_2}{r_{20}}\right)^{k_2+1}\right)=-4.546\cdot 10^{7}\ Pa \qquad F2_{s2} := \left|\frac{\tau 2_\theta}{\sigma 2_{\theta_{20}}}\right| = 70.399$$





$$F2_{s3} := \left|\frac{\tau 2_r}{q_1}\right| = 4.936$$

$$F2_{s4} := \left|\frac{\tau 2_r}{q_2}\right| = 4.607$$

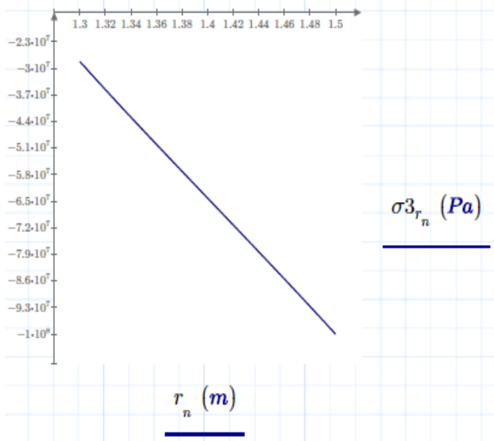
$n := 0..20$

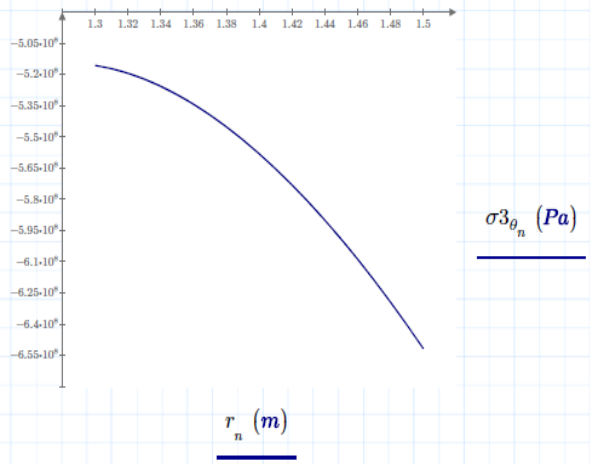
$r_n := (1.3+0.01\ (n))\ m$

$$\sigma3_{r_n} := (q_2)\cdot\frac{c_3^{k_3+1}}{1-c_3^{2k_3}}\cdot\left(\left(\frac{r_n}{a_3}\right)^{k_3-1}-\left(\frac{a_3}{r_n}\right)^{k_3+1}\right)+\left(\frac{q_3}{1-\left(c_3^{2k_3}\right)}\right)\cdot\left(-\left(\frac{r_n}{a_3}\right)^{k_3-1}+c_3^{2k_3}\cdot\left(\frac{a_3}{r_n}\right)^{k_3+1}\right)$$

$$\sigma3_{\theta_n} := (q_2)\cdot\frac{c_3^{k_3+1}\cdot k_3}{1-c_3^{2k_3}}\cdot\left(\left(\frac{r_n}{a_3}\right)^{k_3-1}+\left(\frac{a_3}{r_n}\right)^{k_3+1}\right)-\left(\frac{q_3\cdot k_3}{1-c_3^{2k_3}}\right)\cdot\left(\left(\frac{r_n}{a_3}\right)^{k_3-1}+c_3^{2k_3}\cdot\left(\frac{a_3}{r_n}\right)^{k_3+1}\right)$$

$$u3_{r_n} := \left(\frac{q_2}{E3_\theta}\right)\cdot\frac{c_3^{k_3+1}\cdot a_3}{1-c_3^{2k_3}}\cdot\left(\left(k_3-\nu3_\theta\right)\left(\frac{r_n}{a_3}\right)^{k_3}+\left(k_3-\nu3_\theta\right)\left(\frac{a_3}{r_n}\right)^{k_3}\right)-\left(\frac{q_3\cdot a_3}{E3_\theta\cdot\left(1-c_3^{2k_3}\right)}\right)\cdot\left(\left(k_3-\nu3_\theta\right)\left(\frac{r_n}{a_3}\right)^{k_3}+\left(k_3+\nu3_\theta\right)\cdot c_3^{2k_3}\cdot\left(\frac{a_3}{r_n}\right)^{k_3}\right)$$





$$\sigma3_{\theta_0} := (q_2)\cdot\frac{c_3^{k_3+1}\cdot k_3}{1-c_3^{2k_3}}\cdot\left(\left(\frac{r_0}{a_3}\right)^{k_3-1}+\left(\frac{a_3}{r_0}\right)^{k_3+1}\right)-\left(\frac{q_3\cdot k_3}{1-c_3^{2k_3}}\right)\cdot\left(\left(\frac{r_0}{a_3}\right)^{k_3-1}+c_3^{2k_3}\cdot\left(\frac{a_3}{r_0}\right)^{k_3+1}\right)$$

$$\sigma3_{\theta_{20}} := (q_2)\cdot\frac{c_3^{k_3+1}\cdot k_3}{1-c_3^{2k_3}}\cdot\left(\left(\frac{r_{20}}{a_3}\right)^{k_3-1}+\left(\frac{a_3}{r_{20}}\right)^{k_3+1}\right)-\left(\frac{q_3\cdot k_3}{1-c_3^{2k_3}}\right)\cdot\left(\left(\frac{r_{20}}{a_3}\right)^{k_3-1}+c_3^{2k_3}\cdot\left(\frac{a_3}{r_{20}}\right)^{k_3+1}\right)$$

$$F3_{s1} := \left|\frac{\tau3_\theta}{\sigma3_{\theta_0}}\right| = 8.535$$

$$F3_{s2} := \left|\frac{\tau3_\theta}{\sigma3_{\theta_{20}}}\right| = 6.751$$

$$F3_{s3} := \left|\frac{\tau3_r}{q_2}\right| = 3.544$$

$$F3_{s4} := \left|\frac{\tau3_r}{q_3}\right| = 1$$

(v) **3 1 2**

$a_0 \equiv 0.9\ m \quad a_1 \equiv 1.1\ m \quad a_2 \equiv 1.3\ m \quad a_3 \equiv 1.5\ m \quad q_0 \equiv 0\ GPa \quad q_3 \equiv 100\ MPa$

$c_1 \equiv \frac{a_0}{a_1} = 0.818 \quad c_2 \equiv \frac{a_1}{a_2} = 0.846 \quad c_3 \equiv \frac{a_2}{a_3} = 0.867$

$E3_\theta \equiv 61.8\ GPa \quad E1_\theta \equiv 194\ GPa \quad E2_\theta \equiv 582\ GPa \quad E4_\theta \equiv 0\ GPa$

$E3_r \equiv 13.5\ GPa \quad E1_r \equiv 8.1\ GPa \quad E2_r \equiv 8.1\ GPa$

$\nu3_\theta \equiv 0.253 \quad \nu1_\theta \equiv 0.321 \quad \nu2_\theta \equiv 0.31 \quad \nu4_\theta \equiv 0 \quad \tau3_\theta \equiv 3.2\ GPa \quad \tau1_\theta \equiv 4.4\ GPa \quad \tau2_\theta \equiv 2.1\ GPa$

$k_1 \equiv \sqrt{\frac{E1_\theta}{E1_r}} \quad k_2 \equiv \sqrt{\frac{E2_\theta}{E2_r}} \quad k_3 \equiv \sqrt{\frac{E3_\theta}{E3_r}} = 2.14 \quad \tau3_r \equiv 0.13\ GPa \quad \tau1_r \equiv 0.10\ GPa \quad \tau2_r \equiv 0.09\ GPa$

$$\alpha_1 \equiv 2.\left(\frac{k_1}{E1_\theta}\right)\left(\frac{c_1^{k_1}}{1-c_1^{2k_1}}\right) = \left(2.198\cdot 10^{-11}\right)\frac{1}{Pa} \quad \beta_1 \equiv \frac{1}{E1_\theta}\left(\nu1_\theta - (k_1)\frac{\left(1+c_1^{2k_1}\right)}{\left(1-c_1^{2k_1}\right)}\right) - \frac{1}{E2_\theta}\left(\nu2_\theta + (k_2)\frac{\left(1+c_2^{2k_2}\right)}{\left(1-c_2^{2k_2}\right)}\right) = -4.872\cdot 10^{-11}\ \frac{1}{Pa}$$

$$\alpha_2 \equiv 2.\left(\frac{k_2}{E2_\theta}\right)\left(\frac{c_2^{k_2}}{1-c_2^{2k_2}}\right) = \left(7.511\cdot 10^{-12}\right)\frac{1}{Pa} \quad \beta_2 \equiv \frac{1}{E2_\theta}\left(\nu2_\theta - (k_2)\frac{\left(1+c_2^{2k_2}\right)}{\left(1-c_2^{2k_2}\right)}\right) - \frac{1}{E3_\theta}\left(\nu3_\theta + (k_3)\frac{\left(1+c_3^{2k_3}\right)}{\left(1-c_3^{2k_3}\right)}\right) = -1.365\cdot 10^{-10}\ \frac{1}{Pa}$$

$$\alpha_3 \equiv 2.\left(\frac{k_3}{E3_\theta}\right)\left(\frac{c_3^{k_3}}{1-c_3^{2k_3}}\right) = \left(1.113\cdot 10^{-10}\right)\frac{1}{Pa} \quad \beta_3 \equiv \frac{1}{E3_\theta}\left(\nu3_\theta - (k_3)\frac{\left(1+c_3^{2k_3}\right)}{\left(1-c_3^{2k_3}\right)}\right) = -1.125\cdot 10^{-10}\ \frac{1}{Pa}$$

$$q_2 \equiv \frac{\left((q_3)\ (\alpha_3)\right)}{\left(\frac{(\alpha_1)\ (\alpha_2)}{\beta_1} - \beta_2\right)}\left(\frac{a_3}{a_2}\right) = \left(9.648\cdot 10^{7}\right)\ Pa \quad q_1 \equiv \frac{\left((-q_2)\ (\alpha_2)\right)}{\beta_1}\left(\frac{a_2}{a_1}\right) = \left(1.758\cdot 10^{7}\right)\ Pa$$

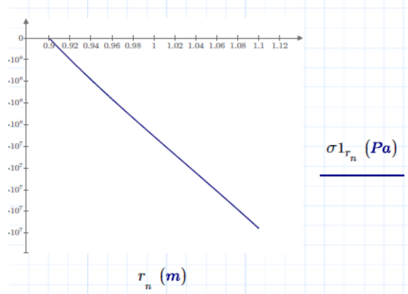
$n := 0..20$

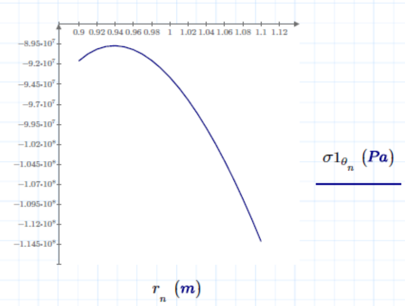
$r_n := (0.9 + 0.01\ (n))\ m$

$$\sigma1_{r_n} := (q_0)\cdot\frac{c_1^{k_1+1}}{1-c_1^{2k_1}}\cdot\left(\left(\frac{r_n}{a_1}\right)^{k_1-1} - \left(\frac{a_1}{r_n}\right)^{k_1+1}\right) + \left(\frac{q_1}{1-\left(c_1^{2k_1}\right)}\right)\cdot\left(-\left(\frac{r_n}{a_1}\right)^{k_1-1} + c_1^{2k_1}\cdot\left(\frac{a_1}{r_n}\right)^{k_1+1}\right)$$

$$\sigma1_{\theta_n} := (q_0)\cdot\frac{c_1^{k_1+1}\cdot k_1}{1-c_1^{2k_1}}\cdot\left(\left(\frac{r_n}{a_1}\right)^{k_1-1} + \left(\frac{a_1}{r_n}\right)^{k_1+1}\right) - \left(\frac{q_1\cdot k_1}{1-c_1^{2k_1}}\right)\cdot\left(\left(\frac{r_n}{a_1}\right)^{k_1-1} + c_1^{2k_1}\cdot\left(\frac{a_1}{r_n}\right)^{k_1+1}\right)$$

$$u1_{r_n} := \left(\frac{q_0}{E1_\theta}\right)\cdot\frac{c_1^{k_1+1}\cdot a_1}{1-c_1^{2k_1}}\cdot\left((k_1-\nu1_\theta)\left(\frac{r_n}{a_1}\right)^{k_1} + (k_1-\nu1_\theta)\left(\frac{a_1}{r_n}\right)^{k_1}\right) - \left(\frac{q_1\cdot a_1}{E1_\theta\cdot\left(1-c_1^{2k_1}\right)}\right)\cdot\left((k_1-\nu1_\theta)\left(\frac{r_n}{a_1}\right)^{k_1} + (k_1+\nu1_\theta)\cdot c_1^{2k_1}\cdot\left(\frac{a_1}{r_n}\right)^{k_1}\right)$$





$$\sigma1_{\theta_0} := (q_0) \cdot \frac{c_1^{\,k_1+1} \cdot k_1}{1-c_1^{\,2k_1}} \cdot \left(\left(\frac{r_0}{a_1}\right)^{k_1-1} + \left(\frac{a_1}{r_0}\right)^{k_1+1} \right) - \left(\frac{q_1 \cdot k_1}{1-c_1^{\,2k_1}}\right) \cdot \left(\left(\frac{r_0}{a_1}\right)^{k_1-1} + c_1^{\,2k_1} \cdot \left(\frac{a_1}{r_0}\right)^{k_1+1} \right) = -9.16 \cdot 10^7 \; Pa \qquad F1_{s1} := \left| \frac{\tau1_\theta}{\sigma1_{\theta_0}} \right| = 48.034$$

$$\sigma1_{\theta_{20}} := (q_0) \cdot \frac{c_1^{\,k_1+1} \cdot k_1}{1-c_1^{\,2k_1}} \cdot \left(\left(\frac{r_{20}}{a_1}\right)^{k_1-1} + \left(\frac{a_1}{r_{20}}\right)^{k_1+1} \right) - \left(\frac{q_1 \cdot k_1}{1-c_1^{\,2k_1}}\right) \cdot \left(\left(\frac{r_{20}}{a_1}\right)^{k_1-1} + c_1^{\,2k_1} \cdot \left(\frac{a_1}{r_{20}}\right)^{k_1+1} \right) = -1.141 \cdot 10^8 \; Pa \qquad F1_{s2} := \left| \frac{\tau1_\theta}{\sigma1_{\theta_{20}}} \right| = 38.566$$

$n := 0\,..\,20$

$$F1_{s3} := \left| \frac{\tau1_r}{q_1} \right| = 5.689$$

$r_n := (1.1 + 0.01\,(n))\; m$

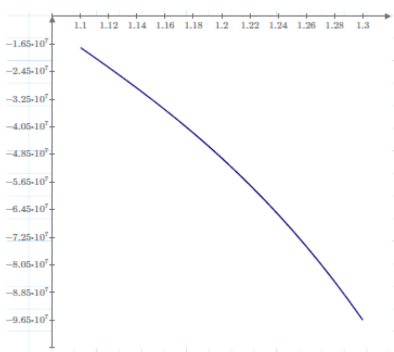
$$\sigma2_{r_n} := (q_1) \cdot \frac{c_2^{\,k_2+1}}{1-c_2^{\,2k_2}} \cdot \left(\left(\frac{r_n}{a_2}\right)^{k_2-1} - \left(\frac{a_2}{r_n}\right)^{k_2+1} \right) + \left(\frac{q_2}{1-\left(c_2^{\,2k_2}\right)}\right) \cdot \left(-\left(\frac{r_n}{a_2}\right)^{k_2-1} + c_2^{\,2k_2} \cdot \left(\frac{a_2}{r_n}\right)^{k_2+1} \right)$$

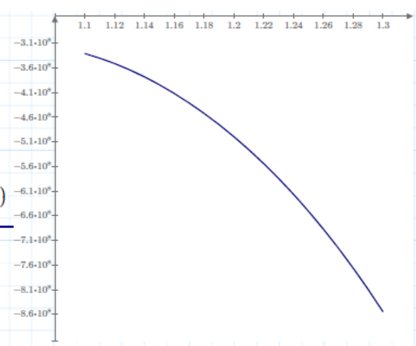
$$\sigma2_{\theta_n} := (q_1) \cdot \frac{c_2^{\,k_2+1} \cdot k_2}{1-c_2^{\,2k_2}} \cdot \left(\left(\frac{r_n}{a_2}\right)^{k_2-1} + \left(\frac{a_2}{r_n}\right)^{k_2+1} \right) - \left(\frac{q_2 \cdot k_2}{1-c_2^{\,2k_2}}\right) \cdot \left(\left(\frac{r_n}{a_2}\right)^{k_2-1} + c_2^{\,2k_2} \cdot \left(\frac{a_2}{r_n}\right)^{k_2+1} \right)$$

$$u2_{r_n} := \left(\frac{q_1}{E2_\theta}\right) \cdot \frac{c_2^{\,k_2+1} \cdot a_2}{1-c_2^{\,2k_2}} \cdot \left((k_2 - \nu2_\theta) \left(\frac{r_n}{a_2}\right)^{k_2} + (k_2 - \nu2_\theta) \left(\frac{a_2}{r_n}\right)^{k_2} \right) - \left(\frac{q_2 \cdot a_2}{E2_\theta \cdot \left(1-c_2^{\,2k_2}\right)}\right) \cdot \left((k_2 - \nu2_\theta) \left(\frac{r_n}{a_2}\right)^{k_2} + (k_2 + \nu2_\theta) \cdot c_2^{\,2k_2} \cdot \left(\frac{a_2}{r_n}\right)^{k_2} \right)$$

$$\sigma2_{\theta_0} := (q_1) \cdot \frac{c_2^{\,k_2+1} \cdot k_2}{1-c_2^{\,2k_2}} \cdot \left(\left(\frac{r_0}{a_2}\right)^{k_2-1} + \left(\frac{a_2}{r_0}\right)^{k_2+1} \right) - \left(\frac{q_2 \cdot k_2}{1-c_2^{\,2k_2}}\right) \cdot \left(\left(\frac{r_0}{a_2}\right)^{k_2-1} + c_2^{\,2k_2} \cdot \left(\frac{a_2}{r_0}\right)^{k_2+1} \right) = -3.308 \cdot 10^8 \; Pa \qquad F2_{s1} := \left| \frac{\tau2_\theta}{\sigma2_{\theta_0}} \right| = 6.348$$

$$\sigma2_{\theta_{20}} := (q_1) \cdot \frac{c_2^{\,k_2+1} \cdot k_2}{1-c_2^{\,2k_2}} \cdot \left(\left(\frac{r_{20}}{a_2}\right)^{k_2-1} + \left(\frac{a_2}{r_{20}}\right)^{k_2+1} \right) - \left(\frac{q_2 \cdot k_2}{1-c_2^{\,2k_2}}\right) \cdot \left(\left(\frac{r_{20}}{a_2}\right)^{k_2-1} + c_2^{\,2k_2} \cdot \left(\frac{a_2}{r_{20}}\right)^{k_2+1} \right) = -8.551 \cdot 10^8 \; Pa \qquad F2_{s2} := \left| \frac{\tau2_\theta}{\sigma2_{\theta_{20}}} \right| = 2.456$$





$$F2_{s3} := \left| \frac{\tau2_r}{q_1} \right| = 5.12$$

$$F2_{s4} := \left| \frac{\tau2_r}{q_2} \right| = 0.933$$

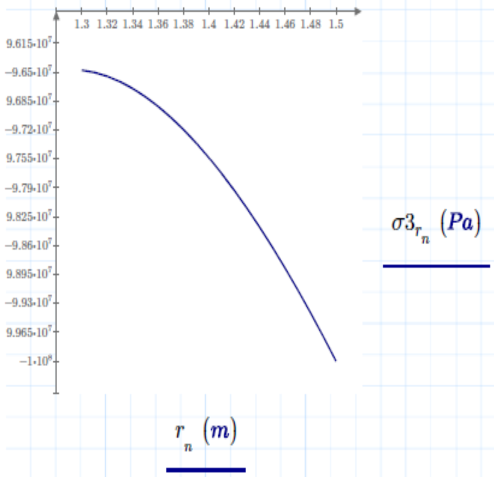
$n := 0\,..\,20$

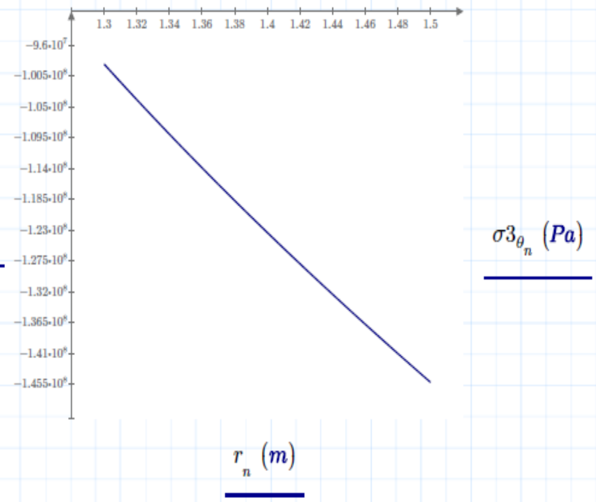
$r_n := (1.3 + 0.01\,(n))\; m$

$$\sigma 3_{r_n} := (q_2)\cdot\frac{c_3^{k_3+1}}{1-c_3^{2k_3}}\cdot\left(\left(\frac{r_n}{a_3}\right)^{k_3-1}-\left(\frac{a_3}{r_n}\right)^{k_3+1}\right)+\left(\frac{q_3}{1-\left(c_3^{2k_3}\right)}\right)\cdot\left(-\left(\frac{r_n}{a_3}\right)^{k_3-1}+c_3^{2k_3}\cdot\left(\frac{a_3}{r_n}\right)^{k_3+1}\right)$$

$$\sigma 3_{\theta_n} := (q_2)\cdot\frac{c_3^{k_3+1}\cdot k_3}{1-c_3^{2k_3}}\cdot\left(\left(\frac{r_n}{a_3}\right)^{k_3-1}+\left(\frac{a_3}{r_n}\right)^{k_3+1}\right)-\left(\frac{q_3\cdot k_3}{1-c_3^{2k_3}}\right)\cdot\left(\left(\frac{r_n}{a_3}\right)^{k_3-1}+c_3^{2k_3}\cdot\left(\frac{a_3}{r_n}\right)^{k_3+1}\right)$$

$$u3_{r_n} := \left(\frac{q_2}{E3_\theta}\right)\cdot\frac{c_3^{k_3+1}\cdot a_3}{1-c_3^{2k_3}}\cdot\left(\left(k_3-\nu 3_\theta\right)\left(\frac{r_n}{a_3}\right)^{k_3}+\left(k_3-\nu 3_\theta\right)\left(\frac{a_3}{r_n}\right)^{k_3}\right)-\left(\frac{q_3\cdot a_3}{E3_\theta\cdot\left(1-c_3^{2k_3}\right)}\right)\cdot\left(\left(k_3-\nu 3_\theta\right)\left(\frac{r_n}{a_3}\right)^{k_3}+\left(k_3+\nu 3_\theta\right)\cdot c_3^{2k_3}\cdot\left(\frac{a_3}{r_n}\right)^{k_3}\right)$$





$$\sigma 3_{\theta_0} := (q_2)\cdot\frac{c_3^{k_3+1}\cdot k_3}{1-c_3^{2k_3}}\cdot\left(\left(\frac{r_0}{a_3}\right)^{k_3-1}+\left(\frac{a_3}{r_0}\right)^{k_3+1}\right)-\left(\frac{q_3\cdot k_3}{1-c_3^{2k_3}}\right)\cdot\left(\left(\frac{r_0}{a_3}\right)^{k_3-1}+c_3^{2k_3}\cdot\left(\frac{a_3}{r_0}\right)^{k_3+1}\right)$$

$$\sigma 3_{\theta_{20}} := (q_2)\cdot\frac{c_3^{k_3+1}\cdot k_3}{1-c_3^{2k_3}}\cdot\left(\left(\frac{r_{20}}{a_3}\right)^{k_3-1}+\left(\frac{a_3}{r_{20}}\right)^{k_3+1}\right)-\left(\frac{q_3\cdot k_3}{1-c_3^{2k_3}}\right)\cdot\left(\left(\frac{r_{20}}{a_3}\right)^{k_3-1}+c_3^{2k_3}\cdot\left(\frac{a_3}{r_{20}}\right)^{k_3+1}\right)$$

$$F3_{s1} := \left|\frac{\tau 3_\theta}{\sigma 3_{\theta_0}}\right| = 32.41$$

$$F3_{s2} := \left|\frac{\tau 3_\theta}{\sigma 3_{\theta_{20}}}\right| = 22.032$$

$$F3_{s3} := \left|\frac{\tau 3_r}{q_2}\right| = 1.347$$

$$F3_{s4} := \left|\frac{\tau 3_r}{q_3}\right| = 1.3$$

(vi) **3 2 1**

$a_0 \equiv 0.9\ m \quad a_1 \equiv 1.1\ m \quad a_2 \equiv 1.3\ m \quad a_3 \equiv 1.5\ m \quad q_0 \equiv 0\ GPa \quad q_3 \equiv 100\ MPa$

$c_1 \equiv \frac{a_0}{a_1} = 0.818 \quad c_2 \equiv \frac{a_1}{a_2} = 0.846 \quad c_3 \equiv \frac{a_2}{a_3} = 0.867$

$E3_\theta \equiv 61.8\ GPa \quad E2_\theta \equiv 194\ GPa \quad E1_\theta \equiv 582\ GPa \quad E4_\theta \equiv 0\ GPa$

$E3_r \equiv 13.5\ GPa \quad E2_r \equiv 8.1\ GPa \quad E1_r \equiv 8.1\ GPa$

$\nu 3_\theta \equiv 0.253 \quad \nu 2_\theta \equiv 0.321 \quad \nu 1_\theta \equiv 0.31 \quad \nu 4_\theta \equiv 0 \quad \tau 3_\theta \equiv 3.2\ GPa \quad \tau 2_\theta \equiv 4.4\ GPa \quad \tau 1_\theta \equiv 2.1\ GPa$

$k_1 \equiv \sqrt{\frac{E1_\theta}{E1_r}} \quad k_2 \equiv \sqrt{\frac{E2_\theta}{E2_r}} \quad k_3 \equiv \sqrt{\frac{E3_\theta}{E3_r}} = 2.14 \quad \tau 3_r \equiv 0.13\ GPa \quad \tau 2_r \equiv 0.10\ GPa \quad \tau 1_r \equiv 0.09\ GPa$

$$\alpha_1 \equiv 2 \cdot \left(\frac{k_1}{E1_\theta}\right)\left(\frac{{c_1}^{k_1}}{1-{c_1}^{2k_1}}\right) = \left(5.499 \cdot 10^{-12}\right)\frac{1}{Pa} \quad \beta_1 \equiv \frac{1}{E1_\theta}\left(\nu 1_\theta - (k_1)\frac{\left(1+{c_1}^{2k_1}\right)}{\left(1-{c_1}^{2k_1}\right)}\right) - \frac{1}{E2_\theta}\left(\nu 2_\theta + (k_2)\frac{\left(1+{c_2}^{2k_2}\right)}{\left(1-{c_2}^{2k_2}\right)}\right) = -5.413 \cdot 10^{-11}\ \frac{1}{Pa}$$

$$\alpha_2 \equiv 2 \cdot \left(\frac{k_2}{E2_\theta}\right)\left(\frac{{c_2}^{k_2}}{1-{c_2}^{2k_2}}\right) = \left(2.767 \cdot 10^{-11}\right)\frac{1}{Pa} \quad \beta_2 \equiv \frac{1}{E2_\theta}\left(\nu 2_\theta - (k_2)\frac{\left(1+{c_2}^{2k_2}\right)}{\left(1-{c_2}^{2k_2}\right)}\right) - \frac{1}{E3_\theta}\left(\nu 3_\theta + (k_3)\frac{\left(1+{c_3}^{2k_3}\right)}{\left(1-{c_3}^{2k_3}\right)}\right) = -1.565 \cdot 10^{-10}\ \frac{1}{Pa}$$

$$\alpha_3 \equiv 2 \cdot \left(\frac{k_3}{E3_\theta}\right)\left(\frac{{c_3}^{k_3}}{1-{c_3}^{2k_3}}\right) = \left(1.113 \cdot 10^{-10}\right)\frac{1}{Pa} \quad \beta_3 \equiv \frac{1}{E3_\theta}\left(\nu 3_\theta - (k_3)\frac{\left(1+{c_3}^{2k_3}\right)}{\left(1-{c_3}^{2k_3}\right)}\right) = -1.125 \cdot 10^{-10}\ \frac{1}{Pa}$$

$$q_2 \equiv \frac{((q_3)\ (\alpha_3))}{\left(\frac{(\alpha_1)\ (\alpha_2)}{\beta_1} - \beta_2\right)}\left(\frac{a_3}{a_2}\right) = \left(8.36 \cdot 10^{7}\right) Pa \quad q_1 \equiv \frac{((-q_2)\ (\alpha_2))}{\beta_1}\left(\frac{a_2}{a_1}\right) = \left(5.05 \cdot 10^{7}\right) Pa$$

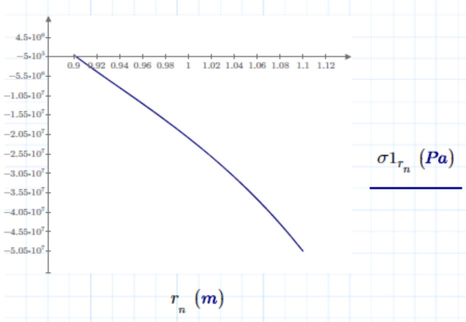
$n := 0 \,..\, 20$

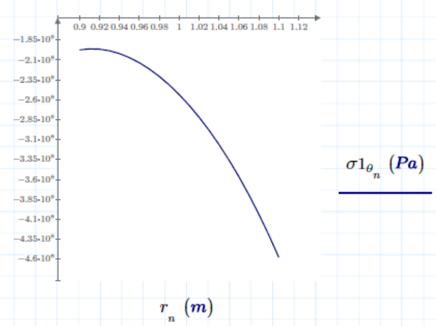
$r_n := (0.9 + 0.01\ (n))\ m$

$$\sigma 1_{r_n} := (q_0) \cdot \frac{{c_1}^{k_1+1}}{1-{c_1}^{2k_1}} \cdot \left(\left(\frac{r_n}{a_1}\right)^{k_1-1} - \left(\frac{a_1}{r_n}\right)^{k_1+1}\right) + \left(\frac{q_1}{1-\left({c_1}^{2k_1}\right)}\right) \cdot \left(-\left(\frac{r_n}{a_1}\right)^{k_1-1} + {c_1}^{2k_1} \cdot \left(\frac{a_1}{r_n}\right)^{k_1+1}\right)$$

$$\sigma 1_{\theta_n} := (q_0) \cdot \frac{{c_1}^{k_1+1} \cdot k_1}{1-{c_1}^{2k_1}} \cdot \left(\left(\frac{r_n}{a_1}\right)^{k_1-1} + \left(\frac{a_1}{r_n}\right)^{k_1+1}\right) - \left(\frac{q_1 \cdot k_1}{1-{c_1}^{2k_1}}\right) \cdot \left(\left(\frac{r_n}{a_1}\right)^{k_1-1} + {c_1}^{2k_1} \cdot \left(\frac{a_1}{r_n}\right)^{k_1+1}\right)$$

$$u1_{r_n} := \left(\frac{q_0}{E1_\theta}\right) \cdot \frac{{c_1}^{k_1+1} \cdot a_1}{1-{c_1}^{2k_1}} \cdot \left((k_1 - \nu 1_\theta)\left(\frac{r_n}{a_1}\right)^{k_1} + (k_1 - \nu 1_\theta)\left(\frac{a_1}{r_n}\right)^{k_1}\right) - \left(\frac{q_1 \cdot a_1}{E1_\theta \cdot \left(1-{c_1}^{2k_1}\right)}\right) \cdot \left((k_1 - \nu 1_\theta)\left(\frac{r_n}{a_1}\right)^{k_1} + (k_1 + \nu 1_\theta) \cdot {c_1}^{2k_1} \cdot \left(\frac{a_1}{r_n}\right)^{k_1}\right)$$





$$\sigma 1_{\theta_0} := (q_0)\cdot\frac{c_1^{k_1+1}\cdot k_1}{1-c_1^{2k_1}}\cdot\left(\left(\frac{r_0}{a_1}\right)^{k_1-1}+\left(\frac{a_1}{r_0}\right)^{k_1+1}\right)-\left(\frac{q_1\cdot k_1}{1-c_1^{2k_1}}\right)\cdot\left(\left(\frac{r_0}{a_1}\right)^{k_1-1}+c_1^{2k_1}\cdot\left(\frac{a_1}{r_0}\right)^{k_1+1}\right)=-1.975\cdot 10^8\ \mathbf{Pa}$$

$$F1_{s1} := \left|\frac{\tau 1_\theta}{\sigma 1_{\theta_0}}\right| = 10.631$$

$$\sigma 1_{\theta_{20}} := (q_0)\cdot\frac{c_1^{k_1+1}\cdot k_1}{1-c_1^{2k_1}}\cdot\left(\left(\frac{r_{20}}{a_1}\right)^{k_1-1}+\left(\frac{a_1}{r_{20}}\right)^{k_1+1}\right)-\left(\frac{q_1\cdot k_1}{1-c_1^{2k_1}}\right)\cdot\left(\left(\frac{r_{20}}{a_1}\right)^{k_1-1}+c_1^{2k_1}\cdot\left(\frac{a_1}{r_{20}}\right)^{k_1+1}\right)=-4.576\cdot 10^8\ \mathbf{Pa}$$

$$F1_{s2} := \left|\frac{\tau 1_\theta}{\sigma 1_{\theta_{20}}}\right| = 4.59$$

$n := 0..20$

$$F1_{s3} := \left|\frac{\tau 1_r}{q_1}\right| = 1.782$$

$r_n := (1.1+0.01\ (n))\ \mathbf{m}$

$$\sigma 2_{r_n} := (q_1)\cdot\frac{c_2^{k_2+1}}{1-c_2^{2k_2}}\cdot\left(\left(\frac{r_n}{a_2}\right)^{k_2-1}-\left(\frac{a_2}{r_n}\right)^{k_2+1}\right)+\left(\frac{q_2}{1-\left(c_2^{2k_2}\right)}\right)\cdot\left(-\left(\frac{r_n}{a_2}\right)^{k_2-1}+c_2^{2k_2}\cdot\left(\frac{a_2}{r_n}\right)^{k_2+1}\right)$$

$$\sigma 2_{\theta_n} := (q_1)\cdot\frac{c_2^{k_2+1}\cdot k_2}{1-c_2^{2k_2}}\cdot\left(\left(\frac{r_n}{a_2}\right)^{k_2-1}+\left(\frac{a_2}{r_n}\right)^{k_2+1}\right)-\left(\frac{q_2\cdot k_2}{1-c_2^{2k_2}}\right)\cdot\left(\left(\frac{r_n}{a_2}\right)^{k_2-1}+c_2^{2k_2}\cdot\left(\frac{a_2}{r_n}\right)^{k_2+1}\right)$$

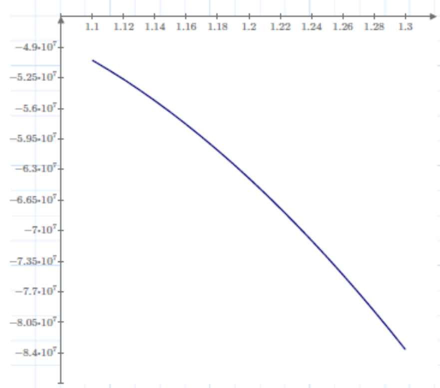
$$u2_{r_n} := \left(\frac{q_1}{E2_\theta}\right)\cdot\frac{c_2^{k_2+1}\cdot a_2}{1-c_2^{2k_2}}\cdot\left((k_2-\nu 2_\theta)\left(\frac{r_n}{a_2}\right)^{k_2}+(k_2-\nu 2_\theta)\left(\frac{a_2}{r_n}\right)^{k_2}\right)-\left(\frac{q_2\cdot a_2}{E2_\theta\cdot\left(1-c_2^{2k_2}\right)}\right)\cdot\left((k_2-\nu 2_\theta)\left(\frac{r_n}{a_2}\right)^{k_2}+(k_2+\nu 2_\theta)\cdot c_2^{2k_2}\cdot\left(\frac{a_2}{r_n}\right)^{k_2}\right)$$

$$\sigma 2_{\theta_0} := (q_1)\cdot\frac{c_2^{k_2+1}\cdot k_2}{1-c_2^{2k_2}}\cdot\left(\left(\frac{r_0}{a_2}\right)^{k_2-1}+\left(\frac{a_2}{r_0}\right)^{k_2+1}\right)-\left(\frac{q_2\cdot k_2}{1-c_2^{2k_2}}\right)\cdot\left(\left(\frac{r_0}{a_2}\right)^{k_2-1}+c_2^{2k_2}\cdot\left(\frac{a_2}{r_0}\right)^{k_2+1}\right)=-1.635\cdot 10^8\ \mathbf{Pa}$$

$$F2_{s1} := \left|\frac{\tau 2_\theta}{\sigma 2_{\theta_0}}\right| = 26.91$$

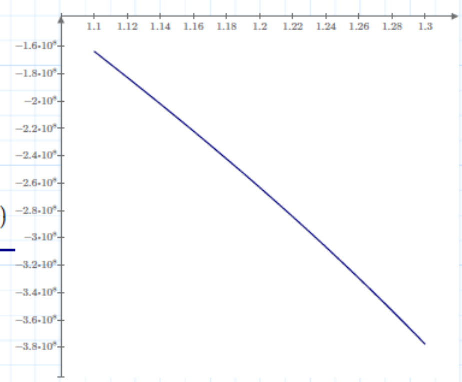
$$\sigma 2_{\theta_{20}} := (q_1)\cdot\frac{c_2^{k_2+1}\cdot k_2}{1-c_2^{2k_2}}\cdot\left(\left(\frac{r_{20}}{a_2}\right)^{k_2-1}+\left(\frac{a_2}{r_{20}}\right)^{k_2+1}\right)-\left(\frac{q_2\cdot k_2}{1-c_2^{2k_2}}\right)\cdot\left(\left(\frac{r_{20}}{a_2}\right)^{k_2-1}+c_2^{2k_2}\cdot\left(\frac{a_2}{r_{20}}\right)^{k_2+1}\right)=-3.779\cdot 10^8\ \mathbf{Pa}$$

$$F2_{s2} := \left|\frac{\tau 2_\theta}{\sigma 2_{\theta_{20}}}\right| = 11.644$$



$\sigma 2_{r_n}\ (\mathbf{Pa})$

$r_n\ (\mathbf{m})$



$\sigma 2_{\theta_n}\ (\mathbf{Pa})$

$r_n\ (\mathbf{m})$

$$F2_{s3} := \left|\frac{\tau 2_r}{q_1}\right| = 1.98$$

$$F2_{s4} := \left|\frac{\tau 2_r}{q_2}\right| = 1.196$$

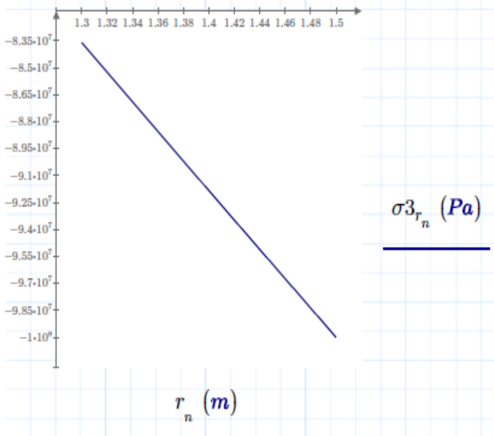
$n := 0..20$

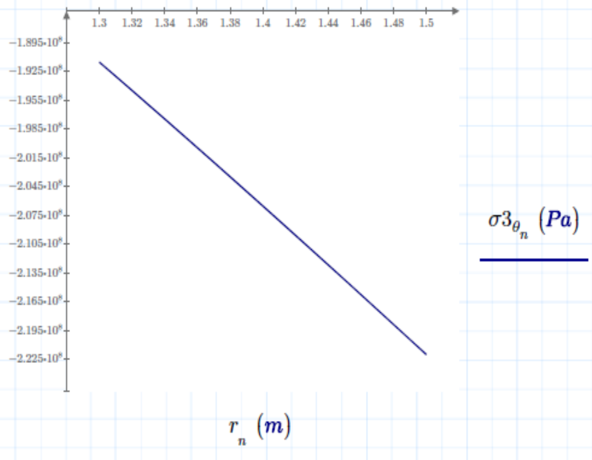
$r_n := (1.3+0.01\ (n))\ \mathbf{m}$

$$\sigma3_{r_n} := (q_2)\cdot\frac{c_3^{k_3+1}}{1-c_3^{2k_3}}\cdot\left(\left(\frac{r_n}{a_3}\right)^{k_3-1}-\left(\frac{a_3}{r_n}\right)^{k_3+1}\right)+\left(\frac{q_3}{1-\left(c_3^{2k_3}\right)}\right)\cdot\left(-\left(\frac{r_n}{a_3}\right)^{k_3-1}+c_3^{2k_3}\cdot\left(\frac{a_3}{r_n}\right)^{k_3+1}\right)$$

$$\sigma3_{\theta_n} := (q_2)\cdot\frac{c_3^{k_3+1}\cdot k_3}{1-c_3^{2k_3}}\cdot\left(\left(\frac{r_n}{a_3}\right)^{k_3-1}+\left(\frac{a_3}{r_n}\right)^{k_3+1}\right)-\left(\frac{q_3\cdot k_3}{1-c_3^{2k_3}}\right)\cdot\left(\left(\frac{r_n}{a_3}\right)^{k_3-1}+c_3^{2k_3}\cdot\left(\frac{a_3}{r_n}\right)^{k_3+1}\right)$$

$$u3_{r_n} := \left(\frac{q_2}{E3_\theta}\right)\cdot\frac{c_3^{k_3+1}\cdot a_3}{1-c_3^{2k_3}}\cdot\left(\left(k_3-\nu3_\theta\right)\left(\frac{r_n}{a_3}\right)^{k_3}+\left(k_3-\nu3_\theta\right)\left(\frac{a_3}{r_n}\right)^{k_3}\right)-\left(\frac{q_3\cdot a_3}{E3_\theta\cdot\left(1-c_3^{2k_3}\right)}\right)\cdot\left(\left(k_3-\nu3_\theta\right)\left(\frac{r_n}{a_3}\right)^{k_3}+\left(k_3+\nu3_\theta\right)\cdot c_3^{2k_3}\cdot\left(\frac{a_3}{r_n}\right)^{k_3}\right)$$





$$\sigma3_{\theta_0} := (q_2)\cdot\frac{c_3^{k_3+1}\cdot k_3}{1-c_3^{2k_3}}\cdot\left(\left(\frac{r_0}{a_3}\right)^{k_3-1}+\left(\frac{a_3}{r_0}\right)^{k_3+1}\right)-\left(\frac{q_3\cdot k_3}{1-c_3^{2k_3}}\right)\cdot\left(\left(\frac{r_0}{a_3}\right)^{k_3-1}+c_3^{2k_3}\cdot\left(\frac{a_3}{r_0}\right)^{k_3+1}\right)$$

$$\sigma3_{\theta_{20}} := (q_2)\cdot\frac{c_3^{k_3+1}\cdot k_3}{1-c_3^{2k_3}}\cdot\left(\left(\frac{r_{20}}{a_3}\right)^{k_3-1}+\left(\frac{a_3}{r_{20}}\right)^{k_3+1}\right)-\left(\frac{q_3\cdot k_3}{1-c_3^{2k_3}}\right)\cdot\left(\left(\frac{r_{20}}{a_3}\right)^{k_3-1}+c_3^{2k_3}\cdot\left(\frac{a_3}{r_{20}}\right)^{k_3+1}\right)$$

$$F3_{s1} := \left|\frac{\tau3_\theta}{\sigma3_{\theta_0}}\right| = 16.708$$

$$F3_{s2} := \left|\frac{\tau3_\theta}{\sigma3_{\theta_{20}}}\right| = 14.412$$

$$F3_{s3} := \left|\frac{\tau3_r}{q_2}\right| = 1.555$$

$$F3_{s4} := \left|\frac{\tau3_r}{q_3}\right| = 1.3$$

Some solutions for selected order (**3 2 1)** of compound layers with different thicknesses:

(A) Thickness **10**, **20**, **30 cm**

$a_0 \equiv 0.9\ m \quad a_1 \equiv 1\ m \quad a_2 \equiv 1.2\ m \quad a_3 \equiv 1.5\ m \quad q_0 \equiv 0\ GPa \quad q_3 \equiv 100\ MPa$

$c_1 \equiv \frac{a_0}{a_1} = 0.9 \quad c_2 \equiv \frac{a_1}{a_2} = 0.833 \quad c_3 \equiv \frac{a_2}{a_3} = 0.8$

$E3_\theta \equiv 61.8\ GPa \quad E2_\theta \equiv 194\ GPa \quad E1_\theta \equiv 582\ GPa \quad E4_\theta \equiv 0\ GPa$

$E3_r \equiv 13.5\ GPa \quad E2_r \equiv 8.1\ GPa \quad E1_r \equiv 8.1\ GPa$

$\nu 3_\theta \equiv 0.253 \quad \nu 2_\theta \equiv 0.321 \quad \nu 1_\theta \equiv 0.31 \quad \nu 4_\theta \equiv 0 \quad \tau 3_\theta \equiv 3.2\ GPa \quad \tau 2_\theta \equiv 4.4\ GPa \quad \tau 1_\theta \equiv 2.1\ GPa$

$\tau 3_r \equiv 0.13\ GPa \quad \tau 2_r \equiv 0.10\ GPa \quad \tau 1_r \equiv 0.09\ GPa$

$k_1 \equiv \sqrt{\frac{E1_\theta}{E1_r}} \quad k_2 \equiv \sqrt{\frac{E2_\theta}{E2_r}} \quad k_3 \equiv \sqrt{\frac{E3_\theta}{E3_r}} = 2.14$

$$\alpha_1 \equiv 2.\left(\frac{k_1}{E1_\theta}\right)\left(\frac{c_1^{k_1}}{1-c_1^{2k_1}}\right) = (1.433\cdot 10^{-11})\ \frac{1}{Pa} \quad \beta_1 \equiv \frac{1}{E1_\theta}\left(\nu 1_\theta - (k_1)\ \frac{(1+c_1^{2k_1})}{(1-c_1^{2k_1})}\right) - \frac{1}{E2_\theta}\left(\nu 2_\theta + (k_2)\ \frac{(1+c_2^{2k_2})}{(1-c_2^{2k_2})}\right) = -5.696\cdot 10^{-11}\ \frac{1}{Pa}$$

$$\alpha_2 \equiv 2.\left(\frac{k_2}{E2_\theta}\right)\left(\frac{c_2^{k_2}}{1-c_2^{2k_2}}\right) = (2.484\cdot 10^{-11})\ \frac{1}{Pa} \quad \beta_2 \equiv \frac{1}{E2_\theta}\left(\nu 2_\theta - (k_2)\ \frac{(1+c_2^{2k_2})}{(1-c_2^{2k_2})}\right) - \frac{1}{E3_\theta}\left(\nu 3_\theta + (k_3)\ \frac{(1+c_3^{2k_3})}{(1-c_3^{2k_3})}\right) = -1.158\cdot 10^{-10}\ \frac{1}{Pa}$$

$$\alpha_3 \equiv 2.\left(\frac{k_3}{E3_\theta}\right)\left(\frac{c_3^{k_3}}{1-c_3^{2k_3}}\right) = (6.983\cdot 10^{-11})\ \frac{1}{Pa} \quad \beta_3 \equiv \frac{1}{E3_\theta}\left(\nu 3_\theta - (k_3)\ \frac{(1+c_3^{2k_3})}{(1-c_3^{2k_3})}\right) = -7.385\cdot 10^{-11}\ \frac{1}{Pa}$$

$$q_2 \equiv \frac{((q_3)\ (\alpha_3))}{\left(\frac{(\alpha_1)\ (\alpha_2)}{\beta_1} - \beta_2\right)}\left(\frac{a_3}{a_2}\right) = (7.969\cdot 10^{7})\ Pa \quad q_1 \equiv \frac{((-q_2)\ (\alpha_2))}{\beta_1}\left(\frac{a_2}{a_1}\right) = (4.171\cdot 10^{7})\ Pa$$

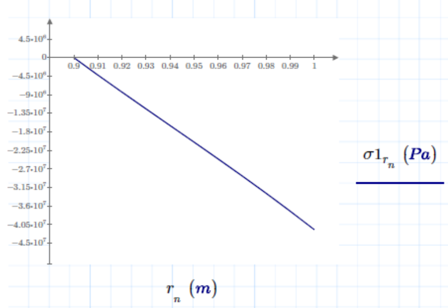
$n := 0..10$

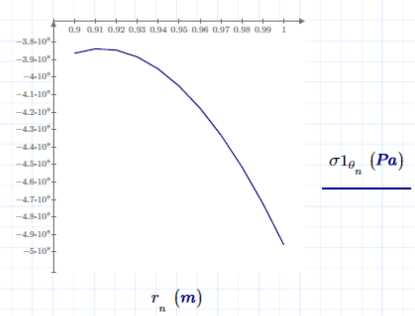
$r_n := (0.9 + 0.01\ (n))\ m$

$$\sigma 1_{r_n} := (q_0)\cdot\frac{c_1^{k_1+1}}{1-c_1^{2k_1}}\cdot\left(\left(\frac{r_n}{a_1}\right)^{k_1-1} - \left(\frac{a_1}{r_n}\right)^{k_1+1}\right) + \left(\frac{q_1}{1-(c_1^{2k_1})}\right)\cdot\left(-\left(\frac{r_n}{a_1}\right)^{k_1-1} + c_1^{2k_1}\cdot\left(\frac{a_1}{r_n}\right)^{k_1+1}\right)$$

$$\sigma 1_{\theta_n} := (q_0)\cdot\frac{c_1^{k_1+1}\cdot k_1}{1-c_1^{2k_1}}\cdot\left(\left(\frac{r_n}{a_1}\right)^{k_1-1} + \left(\frac{a_1}{r_n}\right)^{k_1+1}\right) - \left(\frac{q_1\cdot k_1}{1-c_1^{2k_1}}\right)\cdot\left(\left(\frac{r_n}{a_1}\right)^{k_1-1} + c_1^{2k_1}\cdot\left(\frac{a_1}{r_n}\right)^{k_1+1}\right)$$

$$u1_{r_n} := \left(\frac{q_0}{E1_\theta}\right)\cdot\frac{c_1^{k_1+1}\cdot a_1}{1-c_1^{2k_1}}\cdot\left((k_1-\nu 1_\theta)\ \left(\frac{r_n}{a_1}\right)^{k_1} + (k_1-\nu 1_\theta)\ \left(\frac{a_1}{r_n}\right)^{k_1}\right) - \left(\frac{q_1\cdot a_1}{E1_\theta\cdot(1-c_1^{2k_1})}\right)\cdot\left((k_1-\nu 1_\theta)\ \left(\frac{r_n}{a_1}\right)^{k_1} + (k_1+\nu 1_\theta)\cdot c_1^{2k_1}\cdot\left(\frac{a_1}{r_n}\right)^{k_1}\right)$$





$$\sigma1_{\theta_0} := (q_0)\cdot\frac{c_1^{\,k_1+1}\cdot k_1}{1-c_1^{\,2k_1}}\cdot\left(\left(\frac{r_0}{a_1}\right)^{k_1-1}+\left(\frac{a_1}{r_0}\right)^{k_1+1}\right)-\left(\frac{q_1\cdot k_1}{1-c_1^{\,2k_1}}\right)\cdot\left(\left(\frac{r_0}{a_1}\right)^{k_1-1}+c_1^{\,2k_1}\cdot\left(\frac{a_1}{r_0}\right)^{k_1+1}\right)=-3.864\cdot10^8\ Pa \qquad F1_{s1}:=\left|\frac{\tau1_\theta}{\sigma1_{\theta_0}}\right|=5.435$$

$$\sigma1_{\theta_{10}} := (q_0)\cdot\frac{c_1^{\,k_1+1}\cdot k_1}{1-c_1^{\,2k_1}}\cdot\left(\left(\frac{r_{10}}{a_1}\right)^{k_1-1}+\left(\frac{a_1}{r_{10}}\right)^{k_1+1}\right)-\left(\frac{q_1\cdot k_1}{1-c_1^{\,2k_1}}\right)\cdot\left(\left(\frac{r_{10}}{a_1}\right)^{k_1-1}+c_1^{\,2k_1}\cdot\left(\frac{a_1}{r_{10}}\right)^{k_1+1}\right)=-4.959\cdot10^8\ Pa \qquad F1_{s2}:=\left|\frac{\tau1_\theta}{\sigma1_{\theta_{10}}}\right|=4.235$$

$n:=0..20$

$r_n:=(1+0.01\,(n))\ m$

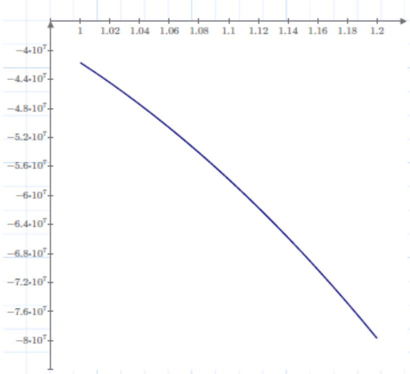
$$\sigma2_{r_n} := (q_1)\cdot\frac{c_2^{\,k_2+1}}{1-c_2^{\,2k_2}}\cdot\left(\left(\frac{r_n}{a_2}\right)^{k_2-1}-\left(\frac{a_2}{r_n}\right)^{k_2+1}\right)+\left(\frac{q_2}{1-\left(c_2^{\,2k_2}\right)}\right)\cdot\left(-\left(\frac{r_n}{a_2}\right)^{k_2-1}+c_2^{\,2k_2}\cdot\left(\frac{a_2}{r_n}\right)^{k_2+1}\right)$$

$$\sigma2_{\theta_n} := (q_1)\cdot\frac{c_2^{\,k_2+1}\cdot k_2}{1-c_2^{\,2k_2}}\cdot\left(\left(\frac{r_n}{a_2}\right)^{k_2-1}+\left(\frac{a_2}{r_n}\right)^{k_2+1}\right)-\left(\frac{q_2\cdot k_2}{1-c_2^{\,2k_2}}\right)\cdot\left(\left(\frac{r_n}{a_2}\right)^{k_2-1}+c_2^{\,2k_2}\cdot\left(\frac{a_2}{r_n}\right)^{k_2+1}\right)$$

$$u2_{r_n} := \left(\frac{q_1}{E2_\theta}\right)\cdot\frac{c_2^{\,k_2+1}\cdot a_2}{1-c_2^{\,2k_2}}\cdot\left((k_2-\nu2_\theta)\left(\frac{r_n}{a_2}\right)^{k_2}+(k_2-\nu2_\theta)\left(\frac{a_2}{r_n}\right)^{k_2}\right)-\left(\frac{q_2\cdot a_2}{E2_\theta\cdot\left(1-c_2^{\,2k_2}\right)}\right)\cdot\left((k_2-\nu2_\theta)\left(\frac{r_n}{a_2}\right)^{k_2}+(k_2+\nu2_\theta)\cdot c_2^{\,2k_2}\cdot\left(\frac{a_2}{r_n}\right)^{k_2}\right)$$

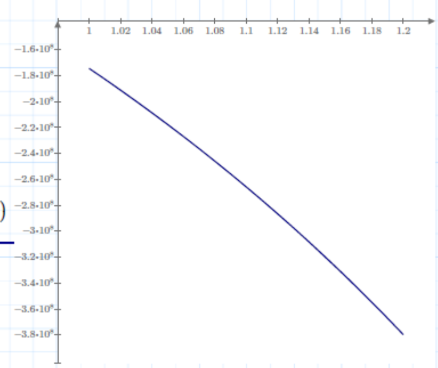
$$\sigma2_{\theta_0} := (q_1)\cdot\frac{c_2^{\,k_2+1}\cdot k_2}{1-c_2^{\,2k_2}}\cdot\left(\left(\frac{r_0}{a_2}\right)^{k_2-1}+\left(\frac{a_2}{r_0}\right)^{k_2+1}\right)-\left(\frac{q_2\cdot k_2}{1-c_2^{\,2k_2}}\right)\cdot\left(\left(\frac{r_0}{a_2}\right)^{k_2-1}+c_2^{\,2k_2}\cdot\left(\frac{a_2}{r_0}\right)^{k_2+1}\right)=-1.744\cdot10^8\ Pa \qquad F2_{s1}:=\left|\frac{\tau2_\theta}{\sigma2_{\theta_0}}\right|=25.232$$

$$\sigma2_{\theta_{20}} := (q_1)\cdot\frac{c_2^{\,k_2+1}\cdot k_2}{1-c_2^{\,2k_2}}\cdot\left(\left(\frac{r_{20}}{a_2}\right)^{k_2-1}+\left(\frac{a_2}{r_{20}}\right)^{k_2+1}\right)-\left(\frac{q_2\cdot k_2}{1-c_2^{\,2k_2}}\right)\cdot\left(\left(\frac{r_{20}}{a_2}\right)^{k_2-1}+c_2^{\,2k_2}\cdot\left(\frac{a_2}{r_{20}}\right)^{k_2+1}\right)=-3.798\cdot10^8\ Pa \qquad F2_{s2}:=\left|\frac{\tau2_\theta}{\sigma2_{\theta_{20}}}\right|=11.584$$



$\sigma2_{r_n}\ (Pa)$

$r_n\ (m)$



$\sigma2_{\theta_n}\ (Pa)$

$r_n\ (m)$

$$F2_{s3}:=\left|\frac{\tau2_r}{q_1}\right|=2.398$$

$$F2_{s4}:=\left|\frac{\tau2_r}{q_2}\right|=1.255$$

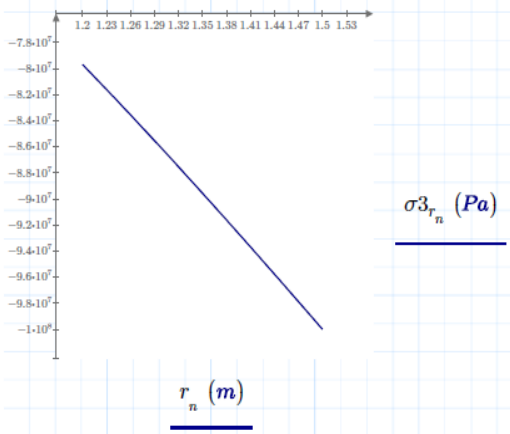
$n:=0..30$

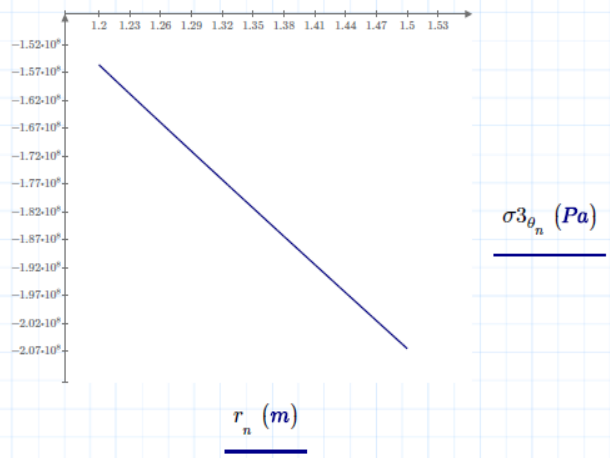
$r_n:=(1.2+0.01\,(n))\ m$

$$\sigma3_{r_n} := (q_2)\cdot\frac{c_3^{k_3+1}}{1-c_3^{2k_3}}\cdot\left(\left(\frac{r_n}{a_3}\right)^{k_3-1}-\left(\frac{a_3}{r_n}\right)^{k_3+1}\right)+\left(\frac{q_3}{1-\left(c_3^{2k_3}\right)}\right)\cdot\left(-\left(\frac{r_n}{a_3}\right)^{k_3-1}+c_3^{2k_3}\cdot\left(\frac{a_3}{r_n}\right)^{k_3+1}\right)$$

$$\sigma3_{\theta_n} := (q_2)\cdot\frac{c_3^{k_3+1}\cdot k_3}{1-c_3^{2k_3}}\cdot\left(\left(\frac{r_n}{a_3}\right)^{k_3-1}+\left(\frac{a_3}{r_n}\right)^{k_3+1}\right)-\left(\frac{q_3\cdot k_3}{1-c_3^{2k_3}}\right)\cdot\left(\left(\frac{r_n}{a_3}\right)^{k_3-1}+c_3^{2k_3}\cdot\left(\frac{a_3}{r_n}\right)^{k_3+1}\right)$$

$$u3_{r_n} := \left(\frac{q_2}{E3_\theta}\right)\cdot\frac{c_3^{k_3+1}\cdot a_3}{1-c_3^{2k_3}}\cdot\left(\left(k_3-\nu3_\theta\right)\left(\frac{r_n}{a_3}\right)^{k_3}+\left(k_3-\nu3_\theta\right)\left(\frac{a_3}{r_n}\right)^{k_3}\right)-\left(\frac{q_3\cdot a_3}{E3_\theta\cdot\left(1-c_3^{2k_3}\right)}\right)\cdot\left(\left(k_3-\nu3_\theta\right)\left(\frac{r_n}{a_3}\right)^{k_3}+\left(k_3+\nu3_\theta\right)\cdot c_3^{2k_3}\cdot\left(\frac{a_3}{r_n}\right)^{k_3}\right)$$





$$\sigma3_{\theta_0} := (q_2)\cdot\frac{c_3^{k_3+1}\cdot k_3}{1-c_3^{2k_3}}\cdot\left(\left(\frac{r_0}{a_3}\right)^{k_3-1}+\left(\frac{a_3}{r_0}\right)^{k_3+1}\right)-\left(\frac{q_3\cdot k_3}{1-c_3^{2k_3}}\right)\cdot\left(\left(\frac{r_0}{a_3}\right)^{k_3-1}+c_3^{2k_3}\cdot\left(\frac{a_3}{r_0}\right)^{k_3+1}\right)=-1.556\cdot 10^8\ Pa$$

$$\sigma3_{\theta_{30}} := (q_2)\cdot\frac{c_3^{k_3+1}\cdot k_3}{1-c_3^{2k_3}}\cdot\left(\left(\frac{r_{30}}{a_3}\right)^{k_3-1}+\left(\frac{a_3}{r_{30}}\right)^{k_3+1}\right)-\left(\frac{q_3\cdot k_3}{1-c_3^{2k_3}}\right)\cdot\left(\left(\frac{r_{30}}{a_3}\right)^{k_3-1}+c_3^{2k_3}\cdot\left(\frac{a_3}{r_{30}}\right)^{k_3+1}\right)=-2.066\cdot 10^8\ Pa$$

$$F3_{s1} := \left|\frac{\tau3_\theta}{\sigma3_{\theta_0}}\right| = 20.565$$

$$F3_{s2} := \left|\frac{\tau3_\theta}{\sigma3_{\theta_{30}}}\right| = 15.492$$

$$F3_{s3} := \left|\frac{\tau3_r}{q_2}\right| = 1.631$$

$$F3_{s4} := \left|\frac{\tau3_r}{q_3}\right| = 1.3$$

(B) Thickness **10**, **15**, **35 cm**

$a_0 \equiv 0.9\ m \qquad a_1 \equiv 1\ m \qquad a_2 \equiv 1.15\ m \qquad a_3 \equiv 1.5\ m \qquad q_0 \equiv 0\ GPa \qquad q_3 \equiv 100\ MPa$

$c_1 \equiv \frac{a_0}{a_1} = 0.9 \qquad c_2 \equiv \frac{a_1}{a_2} = 0.87 \qquad c_3 \equiv \frac{a_2}{a_3} = 0.767$

$E3_\theta \equiv 61.8\ GPa \qquad E2_\theta \equiv 194\ GPa \qquad E1_\theta \equiv 582\ GPa \qquad E4_\theta \equiv 0\ GPa$

$E3_r \equiv 13.5\ GPa \qquad E2_r \equiv 8.1\ GPa \qquad E1_r \equiv 8.1\ GPa$

$\nu 3_\theta \equiv 0.253 \qquad \nu 2_\theta \equiv 0.321 \qquad \nu 1_\theta \equiv 0.31 \qquad \nu 4_\theta \equiv 0 \qquad \tau 3_\theta \equiv 3.2\ GPa \qquad \tau 2_\theta \equiv 4.4\ GPa \qquad \tau 1_\theta \equiv 2.1\ GPa$

$k_1 \equiv \sqrt{\frac{E1_\theta}{E1_r}} \qquad k_2 \equiv \sqrt{\frac{E2_\theta}{E2_r}} \qquad k_3 \equiv \sqrt{\frac{E3_\theta}{E3_r}} = 2.14$

$\tau 3_r \equiv 0.13\ GPa \qquad \tau 2_r \equiv 0.10\ GPa \qquad \tau 1_r \equiv 0.09\ GPa$

$$\alpha_1 \equiv 2.\left(\frac{k_1}{E1_\theta}\right)\left(\frac{c_1^{k_1}}{1-c_1^{2k_1}}\right) = \left(1.433\cdot 10^{-11}\right)\frac{1}{Pa} \qquad \beta_1 \equiv \frac{1}{E1_\theta}\left(\nu 1_\theta - (k_1)\,\frac{\left(1+c_1^{2k_1}\right)}{\left(1-c_1^{2k_1}\right)}\right) - \frac{1}{E2_\theta}\left(\nu 2_\theta + (k_2)\,\frac{\left(1+c_2^{2k_2}\right)}{\left(1-c_2^{2k_2}\right)}\right) = -6.401\cdot 10^{-11}\,\frac{1}{Pa}$$

$$\alpha_2 \equiv 2.\left(\frac{k_2}{E2_\theta}\right)\left(\frac{c_2^{k_2}}{1-c_2^{2k_2}}\right) = \left(3.416\cdot 10^{-11}\right)\frac{1}{Pa} \qquad \beta_2 \equiv \frac{1}{E2_\theta}\left(\nu 2_\theta - (k_2)\,\frac{\left(1+c_2^{2k_2}\right)}{\left(1-c_2^{2k_2}\right)}\right) - \frac{1}{E3_\theta}\left(\nu 3_\theta + (k_3)\,\frac{\left(1+c_3^{2k_3}\right)}{\left(1-c_3^{2k_3}\right)}\right) = -1.122\cdot 10^{-10}\,\frac{1}{Pa}$$

$$\alpha_3 \equiv 2.\left(\frac{k_3}{E3_\theta}\right)\left(\frac{c_3^{k_3}}{1-c_3^{2k_3}}\right) = \left(5.774\cdot 10^{-11}\right)\frac{1}{Pa} \qquad \beta_3 \equiv \frac{1}{E3_\theta}\left(\nu 3_\theta - (k_3)\,\frac{\left(1+c_3^{2k_3}\right)}{\left(1-c_3^{2k_3}\right)}\right) = -6.323\cdot 10^{-11}\,\frac{1}{Pa}$$

$$q_2 \equiv \frac{((q_3)\ (\alpha_3))}{\left(\frac{(\alpha_1)\ (\alpha_2)}{\beta_1} - \beta_2\right)}\left(\frac{a_3}{a_2}\right) = \left(7.201\cdot 10^{7}\right) Pa \qquad q_1 \equiv \frac{((-q_2)\ (\alpha_2))}{\beta_1}\left(\frac{a_2}{a_1}\right) = \left(4.419\cdot 10^{7}\right) Pa$$

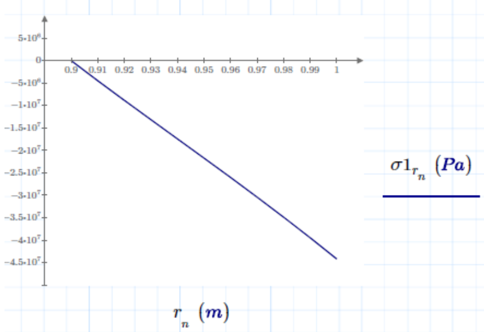
$n := 0..10$

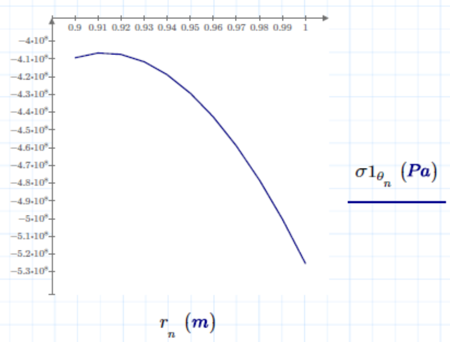
$r_n := (0.9 + 0.01\ (n))\ m$

$$\sigma 1_{r_n} := (q_0)\cdot\frac{c_1^{k_1+1}}{1-c_1^{2k_1}}\cdot\left(\left(\frac{r_n}{a_1}\right)^{k_1-1} - \left(\frac{a_1}{r_n}\right)^{k_1+1}\right) + \left(\frac{q_1}{1-\left(c_1^{2k_1}\right)}\right)\cdot\left(-\left(\frac{r_n}{a_1}\right)^{k_1-1} + c_1^{2k_1}\cdot\left(\frac{a_1}{r_n}\right)^{k_1+1}\right)$$

$$\sigma 1_{\theta_n} := (q_0)\cdot\frac{c_1^{k_1+1}\cdot k_1}{1-c_1^{2k_1}}\cdot\left(\left(\frac{r_n}{a_1}\right)^{k_1-1} + \left(\frac{a_1}{r_n}\right)^{k_1+1}\right) - \left(\frac{q_1\cdot k_1}{1-c_1^{2k_1}}\right)\cdot\left(\left(\frac{r_n}{a_1}\right)^{k_1-1} + c_1^{2k_1}\cdot\left(\frac{a_1}{r_n}\right)^{k_1+1}\right)$$

$$u1_{r_n} := \left(\frac{q_0}{E1_\theta}\right)\cdot\frac{c_1^{k_1+1}\cdot a_1}{1-c_1^{2k_1}}\cdot\left((k_1-\nu 1_\theta)\left(\frac{r_n}{a_1}\right)^{k_1} + (k_1-\nu 1_\theta)\left(\frac{a_1}{r_n}\right)^{k_1}\right) - \left(\frac{q_1\cdot a_1}{E1_\theta\cdot\left(1-c_1^{2k_1}\right)}\right)\cdot\left((k_1-\nu 1_\theta)\left(\frac{r_n}{a_1}\right)^{k_1} + (k_1+\nu 1_\theta)\cdot c_1^{2k_1}\cdot\left(\frac{a_1}{r_n}\right)^{k_1}\right)$$





$$\sigma1_{\theta_0} := (q_0)\cdot\frac{c_1^{k_1+1}\cdot k_1}{1-c_1^{2k_1}}\cdot\left(\left(\frac{r_0}{a_1}\right)^{k_1-1}+\left(\frac{a_1}{r_0}\right)^{k_1+1}\right)-\left(\frac{q_1\cdot k_1}{1-c_1^{2k_1}}\right)\cdot\left(\left(\frac{r_0}{a_1}\right)^{k_1-1}+c_1^{2k_1}\cdot\left(\frac{a_1}{r_0}\right)^{k_1+1}\right) = -4.094\cdot10^8\ Pa \qquad F1_{s1} := \left|\frac{\tau1_\theta}{\sigma1_{\theta_0}}\right| = 5.13$$

$$\sigma1_{\theta_{10}} := (q_0)\cdot\frac{c_1^{k_1+1}\cdot k_1}{1-c_1^{2k_1}}\cdot\left(\left(\frac{r_{10}}{a_1}\right)^{k_1-1}+\left(\frac{a_1}{r_{10}}\right)^{k_1+1}\right)-\left(\frac{q_1\cdot k_1}{1-c_1^{2k_1}}\right)\cdot\left(\left(\frac{r_{10}}{a_1}\right)^{k_1-1}+c_1^{2k_1}\cdot\left(\frac{a_1}{r_{10}}\right)^{k_1+1}\right) = -5.254\cdot10^8\ Pa \qquad F1_{s2} := \left|\frac{\tau1_\theta}{\sigma1_{\theta_{10}}}\right| = 3.997$$

$n := 0..15$

$$F1_{s3} := \left|\frac{\tau1_r}{q_1}\right| = 2.037$$

$r_n := (1+0.01\ (n))\ m$

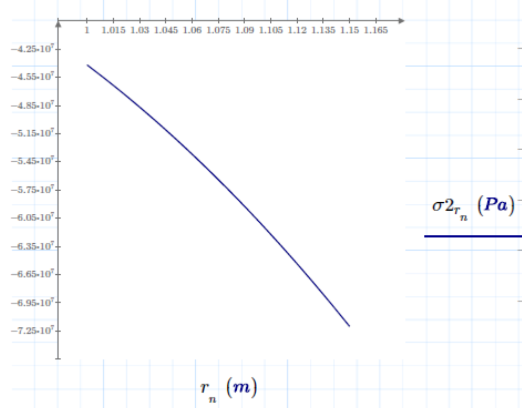
$$\sigma2_{r_n} := (q_1)\cdot\frac{c_2^{k_2+1}}{1-c_2^{2k_2}}\cdot\left(\left(\frac{r_n}{a_2}\right)^{k_2-1}-\left(\frac{a_2}{r_n}\right)^{k_2+1}\right)+\left(\frac{q_2}{1-\left(c_2^{2k_2}\right)}\right)\cdot\left(-\left(\frac{r_n}{a_2}\right)^{k_2-1}+c_2^{2k_2}\cdot\left(\frac{a_2}{r_n}\right)^{k_2+1}\right)$$

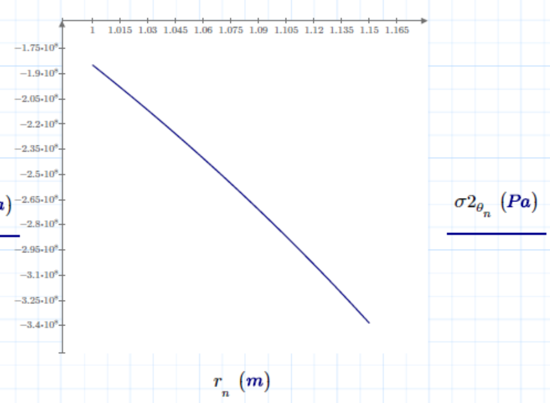
$$\sigma2_{\theta_n} := (q_1)\cdot\frac{c_2^{k_2+1}\cdot k_2}{1-c_2^{2k_2}}\cdot\left(\left(\frac{r_n}{a_2}\right)^{k_2-1}+\left(\frac{a_2}{r_n}\right)^{k_2+1}\right)-\left(\frac{q_2\cdot k_2}{1-c_2^{2k_2}}\right)\cdot\left(\left(\frac{r_n}{a_2}\right)^{k_2-1}+c_2^{2k_2}\cdot\left(\frac{a_2}{r_n}\right)^{k_2+1}\right)$$

$$u2_{r_n} := \left(\frac{q_1}{E2_\theta}\right)\cdot\frac{c_2^{k_2+1}\cdot a_2}{1-c_2^{2k_2}}\cdot\left((k_2-\nu2_\theta)\left(\frac{r_n}{a_2}\right)^{k_2}+(k_2-\nu2_\theta)\left(\frac{a_2}{r_n}\right)^{k_2}\right)-\left(\frac{q_2\cdot a_2}{E2_\theta\cdot\left(1-c_2^{2k_2}\right)}\right)\cdot\left((k_2-\nu2_\theta)\left(\frac{r_n}{a_2}\right)^{k_2}+(k_2+\nu2_\theta)\cdot c_2^{2k_2}\cdot\left(\frac{a_2}{r_n}\right)^{k_2}\right)$$

$$\sigma2_{\theta_0} := (q_1)\cdot\frac{c_2^{k_2+1}\cdot k_2}{1-c_2^{2k_2}}\cdot\left(\left(\frac{r_0}{a_2}\right)^{k_2-1}+\left(\frac{a_2}{r_0}\right)^{k_2+1}\right)-\left(\frac{q_2\cdot k_2}{1-c_2^{2k_2}}\right)\cdot\left(\left(\frac{r_0}{a_2}\right)^{k_2-1}+c_2^{2k_2}\cdot\left(\frac{a_2}{r_0}\right)^{k_2+1}\right) = -1.847\cdot10^8\ Pa \qquad F2_{s1} := \left|\frac{\tau2_\theta}{\sigma2_{\theta_0}}\right| = 23.816$$

$$\sigma2_{\theta_{15}} := (q_1)\cdot\frac{c_2^{k_2+1}\cdot k_2}{1-c_2^{2k_2}}\cdot\left(\left(\frac{r_{15}}{a_2}\right)^{k_2-1}+\left(\frac{a_2}{r_{15}}\right)^{k_2+1}\right)-\left(\frac{q_2\cdot k_2}{1-c_2^{2k_2}}\right)\cdot\left(\left(\frac{r_{15}}{a_2}\right)^{k_2-1}+c_2^{2k_2}\cdot\left(\frac{a_2}{r_{15}}\right)^{k_2+1}\right) = -3.386\cdot10^8\ Pa \qquad F2_{s2} := \left|\frac{\tau2_\theta}{\sigma2_{\theta_{15}}}\right| = 12.994$$





$$F2_{s3} := \left|\frac{\tau2_r}{q_1}\right| = 2.263$$

$$F2_{s4} := \left|\frac{\tau2_r}{q_2}\right| = 1.389$$

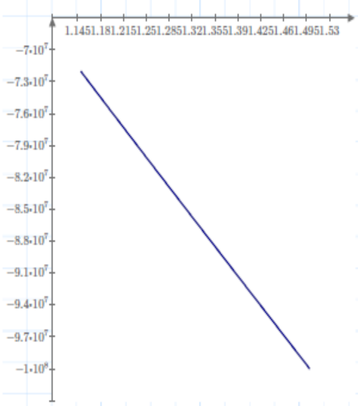
$n := 0..35$

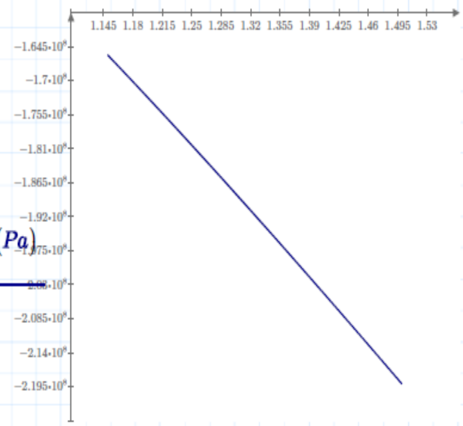
$r_n := (1.15+0.01\ (n))\ m$

$$\sigma3_{r_n} := (q_2)\cdot\frac{c_3^{k_3+1}}{1-c_3^{2k_3}}\cdot\left(\left(\frac{r_n}{a_3}\right)^{k_3-1}-\left(\frac{a_3}{r_n}\right)^{k_3+1}\right)+\left(\frac{q_3}{1-\left(c_3^{2k_3}\right)}\right)\cdot\left(-\left(\frac{r_n}{a_3}\right)^{k_3-1}+c_3^{2k_3}\cdot\left(\frac{a_3}{r_n}\right)^{k_3+1}\right)$$

$$\sigma3_{\theta_n} := (q_2)\cdot\frac{c_3^{k_3+1}\cdot k_3}{1-c_3^{2k_3}}\cdot\left(\left(\frac{r_n}{a_3}\right)^{k_3-1}+\left(\frac{a_3}{r_n}\right)^{k_3+1}\right)-\left(\frac{q_3\cdot k_3}{1-c_3^{2k_3}}\right)\cdot\left(\left(\frac{r_n}{a_3}\right)^{k_3-1}+c_3^{2k_3}\cdot\left(\frac{a_3}{r_n}\right)^{k_3+1}\right)$$

$$u3_{r_n} := \left(\frac{q_2}{E3_\theta}\right)\cdot\frac{c_3^{k_3+1}\cdot a_3}{1-c_3^{2k_3}}\cdot\left((k_3-\nu3_\theta)\left(\frac{r_n}{a_3}\right)^{k_3}+(k_3-\nu3_\theta)\left(\frac{a_3}{r_n}\right)^{k_3}\right)-\left(\frac{q_3\cdot a_3}{E3_\theta\cdot\left(1-c_3^{2k_3}\right)}\right)\cdot\left((k_3-\nu3_\theta)\left(\frac{r_n}{a_3}\right)^{k_3}+(k_3+\nu3_\theta)\cdot c_3^{2k_3}\cdot\left(\frac{a_3}{r_n}\right)^{k_3}\right)$$





$$\sigma3_{\theta_0} := (q_2)\cdot\frac{c_3^{k_3+1}\cdot k_3}{1-c_3^{2k_3}}\cdot\left(\left(\frac{r_0}{a_3}\right)^{k_3-1}+\left(\frac{a_3}{r_0}\right)^{k_3+1}\right)-\left(\frac{q_3\cdot k_3}{1-c_3^{2k_3}}\right)\cdot\left(\left(\frac{r_0}{a_3}\right)^{k_3-1}+c_3^{2k_3}\cdot\left(\frac{a_3}{r_0}\right)^{k_3+1}\right)=-1.658\cdot10^{8}\ Pa$$

$$\sigma3_{\theta_{35}} := (q_2)\cdot\frac{c_3^{k_3+1}\cdot k_3}{1-c_3^{2k_3}}\cdot\left(\left(\frac{r_{35}}{a_3}\right)^{k_3-1}+\left(\frac{a_3}{r_{35}}\right)^{k_3+1}\right)-\left(\frac{q_3\cdot k_3}{1-c_3^{2k_3}}\right)\cdot\left(\left(\frac{r_{35}}{a_3}\right)^{k_3-1}+c_3^{2k_3}\cdot\left(\frac{a_3}{r_{35}}\right)^{k_3+1}\right)=-2.191\cdot10^{8}\ Pa$$

$$F3_{s1} := \left|\frac{\tau3_\theta}{\sigma3_{\theta_0}}\right| = 19.299$$

$$F3_{s2} := \left|\frac{\tau3_\theta}{\sigma3_{\theta_{35}}}\right| = 14.608$$

$$F3_{s3} := \left|\frac{\tau3_r}{q_2}\right| = 1.805$$

$$F3_{s4} := \left|\frac{\tau3_r}{q_3}\right| = 1.3$$

(C) Thickness **2.5**, **3.75**, **8.75 cm**

$a_0 \equiv 1.35\ m$ $\quad a_1 \equiv 1.375\ m$ $\quad a_2 \equiv 1.4125\ m$ $\quad a_3 \equiv 1.5\ m$ $\quad q_0 \equiv 0\ GPa$ $\quad q_3 \equiv 100\ MPa$

$c_1 \equiv \frac{a_0}{a_1} = 0.982$ $\quad c_2 \equiv \frac{a_1}{a_2} = 0.973$ $\quad c_3 \equiv \frac{a_2}{a_3} = 0.942$

$E3_\theta \equiv 61.8\ GPa$ $\quad E2_\theta \equiv 194\ GPa$ $\quad E1_\theta \equiv 582\ GPa$ $\quad E4_\theta \equiv 0\ GPa$

$E3_r \equiv 13.5\ GPa$ $\quad E2_r \equiv 8.1\ GPa$ $\quad E1_r \equiv 8.1\ GPa$

$\nu 3_\theta \equiv 0.253$ $\quad \nu 2_\theta \equiv 0.321$ $\quad \nu 1_\theta \equiv 0.31$ $\quad \nu 4_\theta \equiv 0$ $\quad \tau 3_\theta \equiv 3.2\ GPa$ $\quad \tau 2_\theta \equiv 4.4\ GPa$ $\quad \tau 1_\theta \equiv 2.1\ GPa$

$\tau 3_r \equiv 0.13\ GPa$ $\quad \tau 2_r \equiv 0.10\ GPa$ $\quad \tau 1_r \equiv 0.09\ GPa$

$k_1 \equiv \sqrt{\frac{E1_\theta}{E1_r}}$ $\quad k_2 \equiv \sqrt{\frac{E2_\theta}{E2_r}}$ $\quad k_3 \equiv \sqrt{\frac{E3_\theta}{E3_r}} = 2.14$

$$\alpha_1 \equiv 2.\left(\frac{k_1}{E1_\theta}\right)\left(\frac{c_1^{k_1}}{1-c_1^{2k_1}}\right) = \left(9.326 \cdot 10^{-11}\right)\frac{1}{Pa} \qquad \beta_1 \equiv \frac{1}{E1_\theta}\left(\nu 1_\theta - (k_1)\frac{\left(1+c_1^{2k_1}\right)}{\left(1-c_1^{2k_1}\right)}\right) - \frac{1}{E2_\theta}\left(\nu 2_\theta + (k_2)\frac{\left(1+c_2^{2k_2}\right)}{\left(1-c_2^{2k_2}\right)}\right) = -2.882 \cdot 10^{-10}\ \frac{1}{Pa}$$

$$\alpha_2 \equiv 2.\left(\frac{k_2}{E2_\theta}\right)\left(\frac{c_2^{k_2}}{1-c_2^{2k_2}}\right) = \left(1.91 \cdot 10^{-10}\right)\frac{1}{Pa} \qquad \beta_2 \equiv \frac{1}{E2_\theta}\left(\nu 2_\theta - (k_2)\frac{\left(1+c_2^{2k_2}\right)}{\left(1-c_2^{2k_2}\right)}\right) - \frac{1}{E3_\theta}\left(\nu 3_\theta + (k_3)\frac{\left(1+c_3^{2k_3}\right)}{\left(1-c_3^{2k_3}\right)}\right) = -4.658 \cdot 10^{-10}\ \frac{1}{Pa}$$

$$\alpha_3 \equiv 2.\left(\frac{k_3}{E3_\theta}\right)\left(\frac{c_3^{k_3}}{1-c_3^{2k_3}}\right) = \left(2.685 \cdot 10^{-10}\right)\frac{1}{Pa} \qquad \beta_3 \equiv \frac{1}{E3_\theta}\left(\nu 3_\theta - (k_3)\frac{\left(1+c_3^{2k_3}\right)}{\left(1-c_3^{2k_3}\right)}\right) = -2.666 \cdot 10^{-10}\ \frac{1}{Pa}$$

$$q_2 \equiv \frac{((q_3)\ (\alpha_3))}{\left(\frac{(\alpha_1)\ (\alpha_2)}{\beta_1} - \beta_2\right)}\left(\frac{a_3}{a_2}\right) = \left(7.057 \cdot 10^{7}\right) Pa \qquad q_1 \equiv \frac{((-q_2)\ (\alpha_2))}{\beta_1}\left(\frac{a_2}{a_1}\right) = \left(4.805 \cdot 10^{7}\right) Pa$$

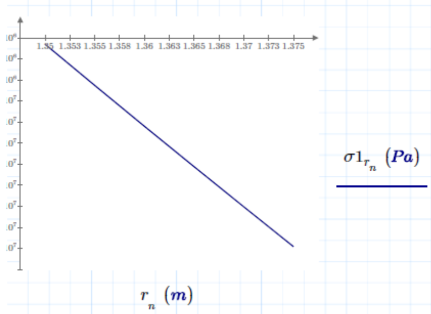
$n := 0..5$

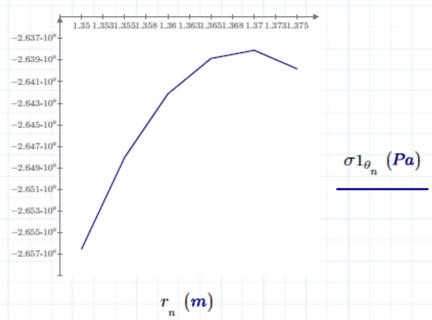
$r_n := (1.35 + 0.005\ (n))\ m$

$$\sigma 1_{r_n} := (q_0) \cdot \frac{c_1^{k_1+1}}{1-c_1^{2k_1}} \cdot \left(\left(\frac{r_n}{a_1}\right)^{k_1-1} - \left(\frac{a_1}{r_n}\right)^{k_1+1}\right) + \left(\frac{q_1}{1-\left(c_1^{2k_1}\right)}\right) \cdot \left(-\left(\frac{r_n}{a_1}\right)^{k_1-1} + c_1^{2k_1} \cdot \left(\frac{a_1}{r_n}\right)^{k_1+1}\right)$$

$$\sigma 1_{\theta_n} := (q_0) \cdot \frac{c_1^{k_1+1} \cdot k_1}{1-c_1^{2k_1}} \cdot \left(\left(\frac{r_n}{a_1}\right)^{k_1-1} + \left(\frac{a_1}{r_n}\right)^{k_1+1}\right) - \left(\frac{q_1 \cdot k_1}{1-c_1^{2k_1}}\right) \cdot \left(\left(\frac{r_n}{a_1}\right)^{k_1-1} + c_1^{2k_1} \cdot \left(\frac{a_1}{r_n}\right)^{k_1+1}\right)$$

$$u1_{r_n} := \left(\frac{q_0}{E1_\theta}\right) \cdot \frac{c_1^{k_1+1} \cdot a_1}{1-c_1^{2k_1}} \cdot \left((k_1 - \nu 1_\theta)\left(\frac{r_n}{a_1}\right)^{k_1} + (k_1 - \nu 1_\theta)\left(\frac{a_1}{r_n}\right)^{k_1}\right) - \left(\frac{q_1 \cdot a_1}{E1_\theta \cdot \left(1-c_1^{2k_1}\right)}\right) \cdot \left((k_1 - \nu 1_\theta)\left(\frac{r_n}{a_1}\right)^{k_1} + (k_1 + \nu 1_\theta) \cdot c_1^{2k_1} \cdot \left(\frac{a_1}{r_n}\right)^{k_1}\right)$$





$$\sigma1_{\theta_0} := (q_0)\cdot\frac{c_1^{k_1+1}\cdot k_1}{1-c_1^{2k_1}}\cdot\left(\left(\frac{r_0}{a_1}\right)^{k_1-1}+\left(\frac{a_1}{r_0}\right)^{k_1+1}\right)-\left(\frac{q_1\cdot k_1}{1-c_1^{2k_1}}\right)\cdot\left(\left(\frac{r_0}{a_1}\right)^{k_1-1}+c_1^{2k_1}\cdot\left(\frac{a_1}{r_0}\right)^{k_1+1}\right)=-2.657\cdot10^9\ Pa$$

$$F1_{s1} := \left|\frac{\tau1_\theta}{\sigma1_{\theta_0}}\right| = 0.791$$

$$\sigma1_{\theta_5} := (q_0)\cdot\frac{c_1^{k_1+1}\cdot k_1}{1-c_1^{2k_1}}\cdot\left(\left(\frac{r_5}{a_1}\right)^{k_1-1}+\left(\frac{a_1}{r_5}\right)^{k_1+1}\right)-\left(\frac{q_1\cdot k_1}{1-c_1^{2k_1}}\right)\cdot\left(\left(\frac{r_5}{a_1}\right)^{k_1-1}+c_1^{2k_1}\cdot\left(\frac{a_1}{r_5}\right)^{k_1+1}\right)=-2.64\cdot10^9\ Pa$$

$$F1_{s2} := \left|\frac{\tau1_\theta}{\sigma1_{\theta_5}}\right| = 0.796$$

$n := 0..15$

$$F1_{s3} := \left|\frac{\tau1_r}{q_1}\right| = 1.873$$

$r_n := (1.375 + 0.0025\ (n))\ m$

$$\sigma2_{r_n} := (q_1)\cdot\frac{c_2^{k_2+1}}{1-c_2^{2k_2}}\cdot\left(\left(\frac{r_n}{a_2}\right)^{k_2-1}-\left(\frac{a_2}{r_n}\right)^{k_2+1}\right)+\left(\frac{q_2}{1-\left(c_2^{2k_2}\right)}\right)\cdot\left(-\left(\frac{r_n}{a_2}\right)^{k_2-1}+c_2^{2k_2}\cdot\left(\frac{a_2}{r_n}\right)^{k_2+1}\right)$$

$$\sigma2_{\theta_n} := (q_1)\cdot\frac{c_2^{k_2+1}\cdot k_2}{1-c_2^{2k_2}}\cdot\left(\left(\frac{r_n}{a_2}\right)^{k_2-1}+\left(\frac{a_2}{r_n}\right)^{k_2+1}\right)-\left(\frac{q_2\cdot k_2}{1-c_2^{2k_2}}\right)\cdot\left(\left(\frac{r_n}{a_2}\right)^{k_2-1}+c_2^{2k_2}\cdot\left(\frac{a_2}{r_n}\right)^{k_2+1}\right)$$

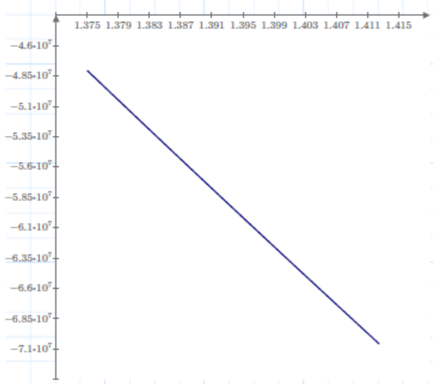
$$u2_{r_n} := \left(\frac{q_1}{E2_\theta}\right)\cdot\frac{c_2^{k_2+1}\cdot a_2}{1-c_2^{2k_2}}\cdot\left((k_2-\nu2_\theta)\left(\frac{r_n}{a_2}\right)^{k_2}+(k_2-\nu2_\theta)\left(\frac{a_2}{r_n}\right)^{k_2}\right)-\left(\frac{q_2\cdot a_2}{E2_\theta\cdot\left(1-c_2^{2k_2}\right)}\right)\cdot\left((k_2-\nu2_\theta)\left(\frac{r_n}{a_2}\right)^{k_2}+(k_2+\nu2_\theta)\cdot c_2^{2k_2}\cdot\left(\frac{a_2}{r_n}\right)^{k_2}\right)$$

$$\sigma2_{\theta_0} := (q_1)\cdot\frac{c_2^{k_2+1}\cdot k_2}{1-c_2^{2k_2}}\cdot\left(\left(\frac{r_0}{a_2}\right)^{k_2-1}+\left(\frac{a_2}{r_0}\right)^{k_2+1}\right)-\left(\frac{q_2\cdot k_2}{1-c_2^{2k_2}}\right)\cdot\left(\left(\frac{r_0}{a_2}\right)^{k_2-1}+c_2^{2k_2}\cdot\left(\frac{a_2}{r_0}\right)^{k_2+1}\right)=-8.904\cdot10^8\ Pa$$

$$F2_{s1} := \left|\frac{\tau2_\theta}{\sigma2_{\theta_0}}\right| = 4.942$$

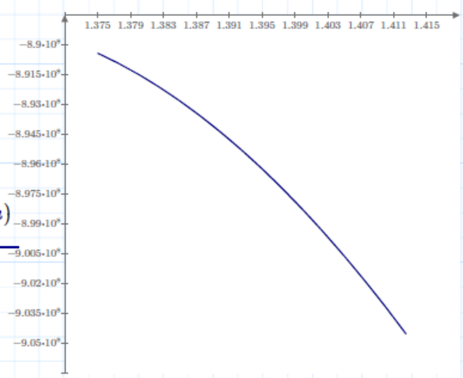
$$\sigma2_{\theta_{15}} := (q_1)\cdot\frac{c_2^{k_2+1}\cdot k_2}{1-c_2^{2k_2}}\cdot\left(\left(\frac{r_{15}}{a_2}\right)^{k_2-1}+\left(\frac{a_2}{r_{15}}\right)^{k_2+1}\right)-\left(\frac{q_2\cdot k_2}{1-c_2^{2k_2}}\right)\cdot\left(\left(\frac{r_{15}}{a_2}\right)^{k_2-1}+c_2^{2k_2}\cdot\left(\frac{a_2}{r_{15}}\right)^{k_2+1}\right)=-9.045\cdot10^8\ Pa$$

$$F2_{s2} := \left|\frac{\tau2_\theta}{\sigma2_{\theta_{15}}}\right| = 4.864$$



$\sigma2_{r_n}\ (Pa)$

$r_n\ (m)$



$\sigma2_{\theta_n}\ (Pa)$

$r_n\ (m)$

$$F2_{s3} := \left|\frac{\tau2_r}{q_1}\right| = 2.081$$

$$F2_{s4} := \left|\frac{\tau2_r}{q_2}\right| = 1.417$$

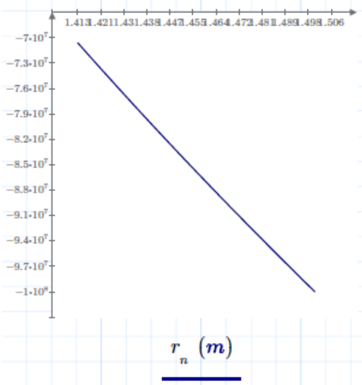
$n := 0..35$

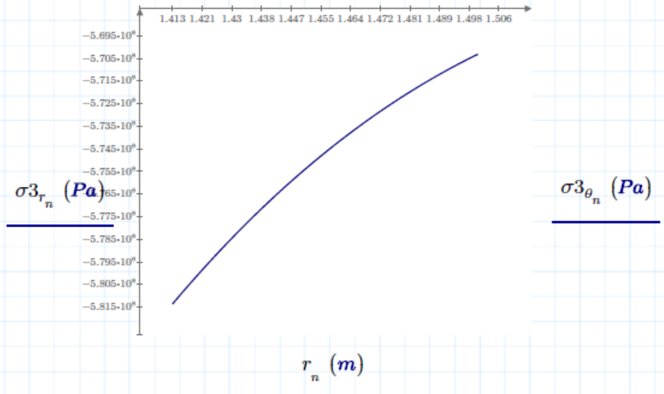
$r_n := (1.4125 + 0.0025\ (n))\ m$

$$\sigma 3_{r_n} := (q_2)\cdot\frac{c_3^{\,k_3+1}}{1-c_3^{\,2k_3}}\cdot\left(\left(\frac{r_n}{a_3}\right)^{k_3-1}-\left(\frac{a_3}{r_n}\right)^{k_3+1}\right)+\left(\frac{q_3}{1-\left(c_3^{\,2k_3}\right)}\right)\cdot\left(-\left(\frac{r_n}{a_3}\right)^{k_3-1}+c_3^{\,2k_3}\cdot\left(\frac{a_3}{r_n}\right)^{k_3+1}\right)$$

$$\sigma 3_{\theta_n} := (q_2)\cdot\frac{c_3^{\,k_3+1}\cdot k_3}{1-c_3^{\,2k_3}}\cdot\left(\left(\frac{r_n}{a_3}\right)^{k_3-1}+\left(\frac{a_3}{r_n}\right)^{k_3+1}\right)-\left(\frac{q_3\cdot k_3}{1-c_3^{\,2k_3}}\right)\cdot\left(\left(\frac{r_n}{a_3}\right)^{k_3-1}+c_3^{\,2k_3}\cdot\left(\frac{a_3}{r_n}\right)^{k_3+1}\right)$$

$$u3_{r_n} := \left(\frac{q_2}{E3_\theta}\right)\cdot\frac{c_3^{\,k_3+1}\cdot a_3}{1-c_3^{\,2k_3}}\cdot\left((k_3-\nu 3_\theta)\left(\frac{r_n}{a_3}\right)^{k_3}+(k_3-\nu 3_\theta)\left(\frac{a_3}{r_n}\right)^{k_3}\right)-\left(\frac{q_3\cdot a_3}{E3_\theta\cdot\left(1-c_3^{\,2k_3}\right)}\right)\cdot\left((k_3-\nu 3_\theta)\left(\frac{r_n}{a_3}\right)^{k_3}+(k_3+\nu 3_\theta)\cdot c_3^{\,2k_3}\cdot\left(\frac{a_3}{r_n}\right)^{k_3}\right)$$





$$\sigma 3_{\theta_0} := (q_2)\cdot\frac{c_3^{\,k_3+1}\cdot k_3}{1-c_3^{\,2k_3}}\cdot\left(\left(\frac{r_0}{a_3}\right)^{k_3-1}+\left(\frac{a_3}{r_0}\right)^{k_3+1}\right)-\left(\frac{q_3\cdot k_3}{1-c_3^{\,2k_3}}\right)\cdot\left(\left(\frac{r_0}{a_3}\right)^{k_3-1}+c_3^{\,2k_3}\cdot\left(\frac{a_3}{r_0}\right)^{k_3+1}\right)=-5.814\cdot 10^{8}\ \boldsymbol{Pa}$$

$$\sigma 3_{\theta_{35}} := (q_2)\cdot\frac{c_3^{\,k_3+1}\cdot k_3}{1-c_3^{\,2k_3}}\cdot\left(\left(\frac{r_{35}}{a_3}\right)^{k_3-1}+\left(\frac{a_3}{r_{35}}\right)^{k_3+1}\right)-\left(\frac{q_3\cdot k_3}{1-c_3^{\,2k_3}}\right)\cdot\left(\left(\frac{r_{35}}{a_3}\right)^{k_3-1}+c_3^{\,2k_3}\cdot\left(\frac{a_3}{r_{35}}\right)^{k_3+1}\right)=-5.703\cdot 10^{8}\ \boldsymbol{Pa}$$

$$F3_{s1} := \left|\frac{\tau 3_\theta}{\sigma 3_{\theta_0}}\right| = 5.504$$

$$F3_{s2} := \left|\frac{\tau 3_\theta}{\sigma 3_{\theta_{35}}}\right| = 5.611$$

$$F3_{s3} := \left|\frac{\tau 3_r}{q_2}\right| = 1.842$$

$$F3_{s4} := \left|\frac{\tau 3_r}{q_3}\right| = 1.3$$

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Biographical Information

Salman Bakhrayba earned his Bachelor's degree from Manipal Institute of Technology Manipal, India in May 2015. He then decided to pursue his Master's degree in Mechanical Engineering from the University of Texas at Arlington in Fall'15. He has been involved in research in areas like Finite Element Analysis, Mechanics of Materials, Structure Design and Energy Harvesting. Salman plans to pursue his career in the path where his knowledge and skills are fully utilized.