STEADY STATE AND TRANSIENT ANALYTICAL MODELING OF NON-UNIFORM CONVECTIVE COOLING OF A MICROPROCESSOR CHIP DUE TO JET-IMPINGEMENT

by

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Abstract

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Heat removal from microprocessor chips with multiple regions of dynamic heat generation remains a critical technological challenge. Excessive temperature rise is undesirable for performance as well as reliability. Jet impingement cooling has been widely investigated as a potential thermal management technique due to the capability of localized cooling and of dynamically following the heat generation distribution. A jet offers large local convective heat transfer coefficient, for which theoretical models and correlations have been proposed for a variety of scenarios. However, not much work exists on using this information to determine the resulting temperature distribution. This work addresses this need by developing analytical steady state and transient heat transfer models that account for the spatial variation in convective heat transfer coefficient and for spatially non-uniform heat flux. The solution is derived in the form of an infinite series, the coefficients of which are determined by solving a set of algebraic equations. Temperature rise predicted by the models are found to be in excellent agreement with finite-element simulations, while offering faster computation time and easier integration with design and performance optimization tools used in microelectronics. The analytical model is used for predicting temperature rise in a variety of scenarios to examine interesting optimization problems such as the cooling of multiple

hotspots with a single jet, determining the optimal location of a jet, etc. Results presented here may facilitate improved thermal design and real-time performance optimization of microprocessor chips.

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Chapter 1

INTRODUCTION

Cooling of a microprocessor chip is an important technological problem that has attracted significant research over past several decades [1–4]. Heat generated during transistor operation on a chip must be conducted through the chip and package, and rejected to the ambient in order to maintain the microprocessor temperature below an acceptable threshold. Thermal management directly affects device performance, as the mobility of charge carriers deteriorates at higher temperatures [5]. Device and package reliability is also adversely affected by high temperatures. Most modern microprocessor chips are multi-core in nature, and include several other power-intensive blocks such as Graphics Processing Units (GPUs) on the same substrate. This results in multiple regions of high power density, or hotspots, on the chip. Further, hotspots also shift dynamically, depending on the nature of microprocessor load, thereby presenting significant thermal management challenges. Specifically, it is difficult to reduce peak temperature rise on a hotspot using a passive thermal management technique that does not specifically address the hotspot location and the dynamic changes in power dissipation on the chip.

Natural convective cooling may be sufficient for very low power chips. At higher powers, air cooling is employed, typically by attaching a metal heat sink to the chip via a thermal interface material and heat spreader, and providing air flow over the heat sink [6]. Heat removal is also often carried out using a heat pipe or vapor chamber [7,8], particularly in space-constrained applications such as laptops. Single-phase and two-phase liquid cooling offer much larger heat transfer coefficients than air cooling. Much research has also been carried out for investigating liquid cooling for thermal management of higher-power chips [9]. These include liquid flow through microchannels, either in the heat sink, or on the back of the microprocessor chip itself, jet impingement on the chip backside [10], thin film liquid cooling utilizing electrowetting-on-discharge (EWOD) [11], etc. Both implementation and modeling of liquid-based cooling are more complicated than air cooling.

A laminar liquid jet impinging on the backside of a chip offers very large local convective heat transfer coefficients in the vicinity of the impingement spot. This cooling approach offers several advantages such as spatially directed cooling, and rapid temporal response, and thus has been extensively studied. Key challenges in this approach include management of vapor formation due to boiling, laminar fluid delivery and exit, and dynamic hotspot management. Several papers have demonstrated the experimental implementation of this approach, often employing a chip with resistive heating and temperature sensors for mimicking an actual microprocessor chip. Synthetic air jets impinging on such a thermal test die have been shown to result in significant reduction in thermal resistance [12]. A method has been developed for three-dimensional visualization of single and multijet arrays using micron resolution particle image velocimetry [13]. Experiments have been carried out to study the effect of jet impingement of alumina–water based nanofluids for a range of physical parameters such as Reynolds number, Prandtl number and volume fraction [14]. Cu–water nanofluid jet array impingement has been reported to result in 6.8% heat transfer enhancement [15]. In comparison with a sizable literature on experimental investigation, there is relatively lesser work done on theoretical modeling of jet impingement based cooling of microprocessors.

A key parameter to consider in such a modeling effort is the spatial variation of convective heat transfer coefficient due to the impinging jet. Correlations for different shapes and flow conditions have been developed through experiments and theoretical modeling. Typically, the heat transfer coefficient is the highest in the vicinity of the impingement spot, and reduces farther away. A number of heat transfer regions have been identified, in which heat transfer correlations have been developed. Analytical development of correlations for local Nusselt number for single phase free surface circular liquid jets has been carried out [16]. By combining experimental results and theoretical solutions of jet impingement boundary layer, the impinging jet has been shown to hydrodynamically evolve through four distinct regions: stagnation zone, boundary layer, viscous similarity, developing turbulence and fully turbulent [17].

While such models help understand the fundamental nature of heat transfer in an impinging jet, such models have not been sufficiently translated into tools for predicting temperature distribution in the presence of an impinging jet. Modeling the spatial variation in convective heat transfer coefficient due to an impinging jet presents significant analytical difficulties that are not present when the heat transfer coefficient is uniform [18]. Some work exists where spatially varying convective heat transfer has been accounted for in fins [19], heat generating slab [20], cylinder [21] and sphere [22] using a variant of Fourier series expansion method, but there is a lack of such work for jet impingement cooling of microprocessors.

In this work, a theoretical model is developed for predicting the steady-state and transient temperature distribution on a microprocessor chip in presence of spatially varying convective cooling due to jet impingement. A series solution is derived, and it is shown that the coefficients in this series can be determined by solving a set of coupled algebraic equations. The transient problem is solved by combining this approach with the Laplace transform technique. The resulting solutions are shown to agree well with finite-element simulation results. The models are used for analyzing the effect of jet cooling on thermal performance of microprocessor chips. Several interesting optimization problems, such as jet placement and jet fluid distribution are analyzed using the model, demonstrating the capability of rapid computation of temperature rise in a microprocessor with spatial and dynamic variation of convective cooling and heat generation.

Chapter 2

DERIVATION OF TEMPERATURE DISTRIBUTION IN STEADY STATE

Figure 2.1 shows a schematic of the geometry of a microprocessor chip of dimensions *a*×*b*×*c*, with spatially varying heat flux on the bottom face and spatially varying convective heat transfer coefficient *h*(*x*,*y*) on the top face due to impingement of one or multiple jets.

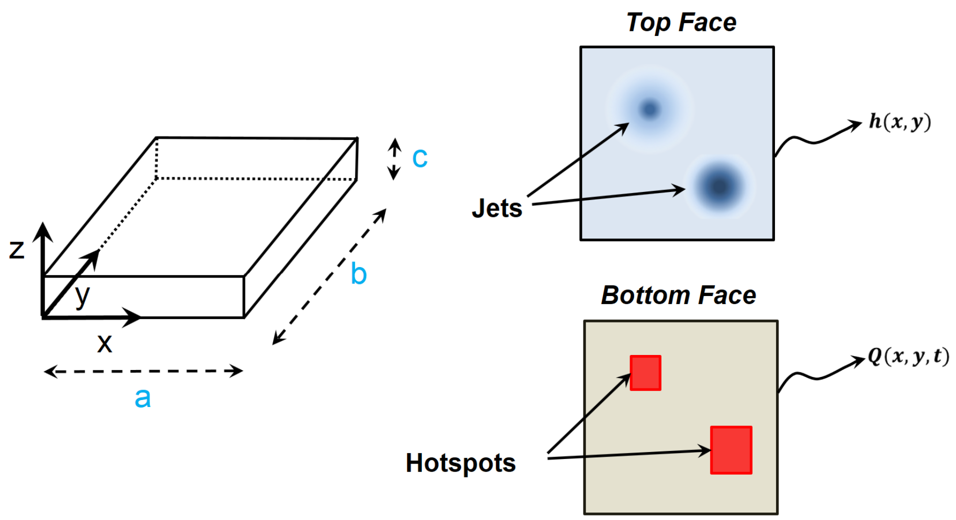

Figure 2.1 Schematic of the geometry

The steady state problem is considered in this section. In general, thermal conductivity is assumed to be orthotropic. In this case, the governing energy equation for the temperature field is given by

$$k_x \frac{\partial^2 T}{\partial x^2} + k_y \frac{\partial^2 T}{\partial y^2} + k_z \frac{\partial^2 T}{\partial z^2} = 0 \qquad (1)$$

The temperature field *T*(*x*,*y*,*z*) satisfies the following boundary conditions given by

$$\frac{\partial T}{\partial x} = 0 \qquad \text{at } x = 0,\ a \qquad (2)$$

$$\frac{\partial T}{\partial y} = 0 \qquad \text{at } y = 0,\ b \qquad (3)$$

$$k_z \frac{\partial T}{\partial z} + q(x, y) = 0 \qquad \text{at } z = 0 \qquad (4)$$

$$k_z \frac{\partial T}{\partial z} + h(x, y) \cdot T = 0 \qquad \text{at } z = c \qquad (5)$$

In this case, the solution for the temperature field may be written as the following two-variable Fourier series

$$T(x, y, z) = C_{00}(z) + \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} C_{nm}(z) \cos\left(\frac{n\pi x}{a}\right) \cos\left(\frac{m\pi y}{b}\right) \qquad (6)$$

Note that the double summation in equation (6) and subsequent equations in Chapters 2 and 3 excludes the case where *n* and *m* are both zero, since that term is being considered separately. Equation (6) satisfies equations (2) and (3). Functions $C_{00}(z)$ and $C_{nm}(z)$ must be chosen so as to satisfy the remaining equations. To proceed, the given heat flux distribution $q(x,y)$ is expressed as a Fourier cosine series in two variables [23,24]

$$q(x, y) = P_{00} + \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} P_{nm} \cos\left(\frac{n\pi x}{a}\right) \cos\left(\frac{m\pi y}{b}\right) \qquad (7)$$

where

$$P_{00} = \int_{y=0}^{b} \int_{x=0}^{a} q(x, y)\, dx dy \qquad (8)$$

$$P_{nm} = \frac{\delta_{nm}}{a\,b} \int_{y=0}^{b} \int_{x=0}^{a} q(x,y) \cos\left(\frac{m\pi y}{b}\right) \cos\left(\frac{n\pi x}{a}\right) dx\, dy \tag{9}$$

$$\delta_{\mathrm{nm}} = \begin{cases} 4 & n \neq 0, m \neq 0 \\ 2 & \mathit{otherwise} \end{cases} \tag{10}$$

Substituting equation (6) in equation (1) results in

$$C''_{00}(z) + \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \cos\left(\frac{n\pi x}{a}\right) \cos\left(\frac{m\pi y}{b}\right) \left[C''_{nm}(z) - \tilde{k}_x \left(\frac{n\pi}{a}\right)^2 C_{nm}(z) - \tilde{k}_y \left(\frac{m\pi}{b}\right)^2 C_{nm}(z) \right] \tag{11}$$

Here, $\tilde{k}_x = \frac{k_x}{k_z}$ and $\tilde{k}_y = \frac{k_y}{k_z}$ are the thermal conductivity ratios that account for orthotropy in thermal conduction, if present.

As a result, $C_{00}(z)$ and $C_{nm}(z)$ may be written as

$$C_{00}(z) = A_{00} + B_{00}\, z \tag{12}$$

$$C_{nm}(z) = A_{nm} \exp(\lambda_{nm} z) + B_{nm} \exp(-\lambda_{nm} z) \tag{13}$$

where $\lambda_{nm}^2 = \tilde{k}_x \left(\frac{n\pi}{a}\right)^2 + \tilde{k}_y \left(\frac{m\pi}{b}\right)^2$.

Using the boundary condition at z=0, equations (12) and (13) may be simplified to

$$C_{00}(z) = A_{00} - \frac{P_{00}}{k_z}\, z \tag{14}$$

$$C_{nm}(z) = A_{nm} \left(\exp(\lambda_{nm} z) + \exp(-\lambda_{nm} z) \right) + \frac{P_{nm}}{\lambda_{nm} k_z} \exp(-\lambda_{nm} z) \tag{15}$$

These expressions involve several unknowns – A_{00} and A_{nm} – which may be obtained using the boundary conditions at $z=c$. Substituting equations (14) and (15) in (5) results in

$$\begin{aligned}
&A_{00}h(x,y)+\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}A_{nm}\,k_z\,\lambda_{nm}\left(\exp(\lambda_{nm}c)-\exp(-\lambda_{nm}c)\right)\cos\left(\frac{n\pi x}{a}\right)\cos\left(\frac{m\pi y}{b}\right)\\
&+\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}A_{nm}\left(\exp(\lambda_{nm}c)+\exp(-\lambda_{nm}c)\right)h(x,y)\cos\left(\frac{n\pi x}{a}\right)\cos\left(\frac{m\pi y}{b}\right)=\\
&P_{00}\left(1+\frac{c\,h(x,y)}{k_z}\right)+\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}P_{nm}\left(1-\frac{h(x,y)}{\lambda_{nm}k_z}\right)\exp(-\lambda_{nm}c)\cos\left(\frac{n\pi x}{a}\right)\cos\left(\frac{m\pi y}{b}\right)
\end{aligned} \tag{16}$$

In order to extract the unknowns A_{00} and A_{nm}, equation (16) is integrated in x and y, resulting in a linear algebraic equation involving the unknown coefficients

$$\begin{aligned}
&A_{00}\int_{y=0}^{b}\int_{x=0}^{a}h(x,y)dxdy+\\
&\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}A_{nm}\left(\exp(\lambda_{nm}c)+\exp(-\lambda_{nm}c)\right)\int_{y=0}^{b}\int_{x=0}^{a}h(x,y)\cos\left(\frac{n\pi x}{a}\right)\cos\left(\frac{m\pi y}{b}\right)dxdy\\
&P_{00}ab+P_{00}\frac{c}{k_z}\int_{y=0}^{b}\int_{x=0}^{a}h(x,y)dxdy-\\
&\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\frac{P_{nm}}{k_z\lambda_{nm}}\exp(-\lambda_{nm}c)\int_{y=0}^{b}\int_{x=0}^{a}h(x,y)\cos\left(\frac{n\pi x}{a}\right)\cos\left(\frac{m\pi y}{b}\right)dxdy
\end{aligned} \tag{17}$$

Also, equation (16) is multiplied by $\cos\left(\frac{i\pi x}{a}\right)\cos\left(\frac{j\pi y}{b}\right)$ for i=0,1,2,..N and j=0,1,2,..M, except when i and j are both zero, and then integrated in x and y. Several terms drop out due to orthogonality of the eigenfunctions. For each i and j,

$$A_{00}\,f_{ij}+A_{ij}\,e_{ij}+\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}A_{nm}\,d_{ijnm}=P_{00}u_{ij}+P_{ij}\,v_{ij}+\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}P_{nm}\,w_{ijnm} \tag{18}$$

where the various terms are given by

$$f_{ij} = \int_0^a \int_0^b h(x,y)\cos\left(\frac{i\pi x}{a}\right)\cos\left(\frac{j\pi y}{b}\right) dx\,dy \qquad (19)$$

$$e_{ij} = k_z\,\lambda_{ij}\left(\exp(\lambda_{ij}c) - \exp(-\lambda_{ij}c)\right) N_{x,i} N_{y,j} \qquad (20)$$

$$d_{ijnm} = \left(\exp(\lambda_{nm}c) + \exp(-\lambda_{nm}c)\right)\int_0^a \int_0^b h(x,y)\cos\left(\frac{i\pi x}{a}\right)\cos\left(\frac{j\pi y}{b}\right)\cos\left(\frac{n\pi x}{a}\right)\cos\left(\frac{m\pi y}{b}\right) dx dy \qquad (21)$$

$$u_{ij} = \frac{c}{k_z} f_{ij} \qquad (22)$$

$$v_{ij} = \frac{\exp(-\lambda_{ij}c)}{k_z\,\lambda_{ij}\left(\exp(\lambda_{ij}c) - \exp(-\lambda_{ij}c)\right)} e_{ij} \qquad (23)$$

$$w_{ijnm} = \frac{-\exp(-\lambda_{nm}c)}{k_z\lambda_{nm}\left(\exp(\lambda_{nm}c) + \exp(-\lambda_{nm}c)\right)} d_{ijnm} \qquad (24)$$

The norms $N_{x,i}$ and $N_{y,j}$ in equation (19) are given by [25]

$$N_{x,i} = \int_0^a \cos^2\left(\frac{i\pi x}{a}\right) dx = \begin{cases} a & \mathrm{i}=0 \\ a/2 & \text{otherwise} \end{cases} \qquad (25)$$

$$N_{y,j} = \int_0^b \cos^2\left(\frac{j\pi y}{b}\right) dy = \begin{cases} b & \mathrm{j}=0 \\ b/2 & \text{otherwise} \end{cases} \qquad (26)$$

Equation 17 and 18 represent a set of (N+1)×(M+1) equations in (N+1)×(M+1) variables, which can be solved using matrix inversion to complete the solution for the steady-state temperature field.

Chapter 3

DERIVATION OF TRANSIENT TEMPERATURE DISTRIBUTION

The governing transient energy equation is similar to equation (1), and includes a transient term as follows:

$$k_x \frac{\partial^2 T}{\partial x^2} + k_y \frac{\partial^2 T}{\partial y^2} + k_z \frac{\partial^2 T}{\partial z^2} = \rho c_p \frac{\partial T}{\partial t} \qquad (27)$$

subject to boundary conditions given by equations (2), (3) and (5). There is a small change in the boundary condition at *z=0* to account for the transient variation in heat flux

$$k_z \frac{\partial T}{\partial z} + q(x,y,t) = 0 \qquad \text{at } z = 0 \qquad (28)$$

In addition, zero temperature rise is assumed at the initial time.

$$T = 0 \qquad \text{at } t = 0 \qquad (29)$$

The Laplace transform method is used to solve for the transient temperature field. Laplace transform of equation (27) results in

$$\tilde{k}_x \frac{\partial^2 \overline{T}}{\partial x^2} + \tilde{k}_y \frac{\partial^2 \overline{T}}{\partial y^2} + \frac{\partial^2 \overline{T}}{\partial z^2} = \frac{s}{\alpha_z} \overline{T} \qquad (30)$$

where $\tilde{k}_x = \frac{k_x}{k_z}$, $\tilde{k}_y = \frac{k_y}{k_z}$ and $\alpha_z = \frac{k_z}{\rho c_p}$ is the thermal diffusivity using thermal conductivity in the *z* direction.

The boundary conditions are transformed to

$$\left.\frac{\partial \overline{T}}{\partial x}\right|_{x=0,a} = 0 \tag{31}$$

$$\left.\frac{\partial \overline{T}}{\partial y}\right|_{y=0,b} = 0 \tag{32}$$

$$k_z \left.\frac{\partial \overline{T}}{\partial z}\right|_{z=0} + \overline{q}(x,y,s) = 0 \tag{33}$$

$$k_z \left.\frac{\partial T}{\partial z}\right|_{z=c} + h(x,y)\overline{T} = 0 \tag{34}$$

$\overline{q}(x,y,s)$ in equation (33) is the Laplace transform of the applied heat flux $q(x,y,t)$.

The transient temperature distribution is determined using a similar approach as chapter (2). $\overline{q}(x,y,s)$ is first expanded as follows

$$\overline{q}(x,y,s) = \overline{P}_{00}(s) + \sum_{n=0}^{\infty}\sum_{m=0}^{\infty} \overline{P}_{nm}(s)\cos\left(\frac{n\pi x}{a}\right)\cos\left(\frac{m\pi y}{b}\right) \tag{35}$$

where the coefficients $\overline{P}_{00}(s)$ and $\overline{P}_{nm}(s)$ are found similar to the coefficients P_{00} and P_{nm} in chapter 2.

$$\overline{P}_{00}(s) = \int_{x=0}^{a}\int_{y=0}^{b} \overline{q}(x,y,s)\,dy\,dx \tag{36}$$

$$\overline{P}_{nm}(s) = \frac{\delta_{nm}}{a\,b}\int_{x=0}^{a}\int_{y=0}^{b} \overline{q}(x,y,s)\cos\left(\frac{m\pi y}{b}\right)\cos\left(\frac{n\pi x}{a}\right)dy\,dx \tag{37}$$

The temperature solution is given by

$$\overline{T}(x,y,z,s)=\overline{C}_{00}(z,s)+\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\overline{C}_{nm}(z,s)\cos\left(\frac{n\pi x}{a}\right)\cos\left(\frac{m\pi y}{b}\right) \tag{38}$$

where, using the governing energy equation and the boundary condition at z=0, it is found that

$$\overline{C}_{00}(z,s)=\overline{A}_{00}\left(e^{\sqrt{\frac{s}{\alpha_z}}z}+e^{-\sqrt{\frac{s}{\alpha_z}}z}\right)+\frac{\overline{P}_{00}(s)}{k_z\sqrt{\frac{s}{\alpha_z}}}e^{-\sqrt{\frac{s}{\alpha_z}}z} \tag{39}$$

$$\overline{C}_{nm}(z,s)=\overline{A}_{nm}\left(e^{\overline{\lambda}_{nm}z}+e^{-\overline{\lambda}_{nm}z}\right)+\frac{\overline{P}_{nm}(s)}{k_z\,\overline{\lambda}_{nm}}e^{-\overline{\lambda}_{nm}z} \tag{40}$$

Here, the eigenvalues $\overline{\lambda}_{nm}$ are given by

$$\overline{\lambda}_{nm}^2=\tilde{k}_x\left(\frac{n\pi}{a}\right)^2+\tilde{k}_y\left(\frac{m\pi}{b}\right)^2+\frac{s}{\alpha_z} \tag{41}$$

Substituting equation (38) in equation (34) results in,

$$\begin{aligned}
&A_{00}\left\{k_z\sqrt{\frac{s}{\alpha_z}}\left(e^{\sqrt{\frac{s}{\alpha_z}}c}-e^{-\sqrt{\frac{s}{\alpha_z}}c}\right)+\left(e^{\sqrt{\frac{s}{\alpha_z}}c}+e^{-\sqrt{\frac{s}{\alpha_z}}c}\right)h(x,y)\right\}+\\
&\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}A_{nm}\left\{\begin{array}{l}k_z\;\overline{\lambda}_{nm}\left(e^{\overline{\lambda}_{nm}c}-e^{-\overline{\lambda}_{nm}c}\right)\\+\left(e^{\overline{\lambda}_{nm}c}+e^{-\overline{\lambda}_{nm}c}\right)h(x,y)\end{array}\right\}\cos\left(\frac{n\pi x}{a}\right)\cos\left(\frac{m\pi y}{b}\right)=\\
&\overline{P}_{00}(s)\left\{e^{-\sqrt{\frac{s}{\alpha_z}}c}-\frac{1}{k_z\sqrt{s/\alpha_z}}e^{-\sqrt{\frac{s}{\alpha_z}}c}\,h(x,y)\right\}+\\
&\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\overline{P}_{nm}(s)\left\{1-\frac{1}{k_z\,\overline{\lambda}_{nm}}\right\}e^{-\overline{\lambda}_{nm}c}\,h(x,y)\cos\left(\frac{n\pi x}{a}\right)\cos\left(\frac{m\pi y}{b}\right)
\end{aligned} \tag{42}$$

Similar to chapter 2, integration of equation in x and y results in

$$
\begin{aligned}
&-\overline{A}_{00}k_z\sqrt{\frac{s}{\alpha_z}}\left(e^{\sqrt{\frac{s}{\alpha_z}}c}-e^{-\sqrt{\frac{s}{\alpha_z}}c}\right).a.b-\overline{A}_{00}\left(e^{\sqrt{\frac{s}{\alpha_z}}c}+e^{-\sqrt{\frac{s}{\alpha_z}}c}\right)\int_{y=0}^{b}\int_{x=0}^{a}h(x,y)dxdy-\\
&\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\overline{A}_{nm}\left(e^{\overline{\lambda}_{nm}c}+e^{-\overline{\lambda}_{nm}c}\right)\int_{y=0}^{b}\int_{x=0}^{a}h(x,y)\cos\left(\frac{n\pi x}{a}\right)\cos\left(\frac{m\pi y}{b}\right)dxdy=\\
&-\overline{P}_{00}(s)e^{-\sqrt{\frac{s}{\alpha_z}}c}ab+\frac{\overline{P}_{00}(s)}{k_z\sqrt{s/\alpha_z}}e^{-\sqrt{\frac{s}{\alpha_z}}c}\int_{y=0}^{b}\int_{x=0}^{a}h(x,y)dxdy+\\
&\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\frac{\overline{P}_{nm}(s)}{k_z\overline{\lambda}_{nm}}e^{-\overline{\lambda}_{nm}c}\int_{y=0}^{b}\int_{x=0}^{a}h(x,y)\cos\left(\frac{n\pi x}{a}\right)\cos\left(\frac{m\pi y}{b}\right)dxdy
\end{aligned}
\tag{43}
$$

Also, equation (42) is multiplied by $\cos\left(\frac{i\pi x}{a}\right)\cos\left(\frac{j\pi y}{b}\right)$ for i=0,1,2,..N and j=0,1,2,..M, except when i and j are both zero, and then integrated in x and y. Simplification using principle of orthogonality similar to chapter 2 results in

$$
\overline{A}_{00}\,\overline{f}_{ij}+\overline{A}_{ij}\,\overline{e}_{ij}+\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\overline{A}_{nm}\,\overline{d}_{ijnm}=\overline{P}_{00}(s)\overline{u}_{ij}+\overline{P}(s)_{ij}\overline{v}_{ij}+\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\overline{P}_{nm}\,\overline{w}_{ijnm} \tag{44}
$$

where the various terms are given by

$$
\overline{f}_{ij}=\left(e^{\sqrt{\frac{s}{\alpha_z}}c}+e^{-\sqrt{\frac{s}{\alpha_z}}c}\right)\int_0^a\int_0^b h(x,y)\cos\left(\frac{i\pi x}{a}\right)\cos\left(\frac{j\pi y}{b}\right)dx\,dy \tag{45}
$$

$$
\overline{e}_{ij}=k_z\,\overline{\lambda}_{ij}\left(e^{\overline{\lambda}_{ij}c}-e^{-\overline{\lambda}_{ij}c}\right)N_{x,i}N_{y,j} \tag{46}
$$

$$
\overline{d}_{ijnm}=\left(e^{\overline{\lambda}_{nm}c}+e^{-\overline{\lambda}_{nm}c}\right)\int_0^a\int_0^b h(x,y)\cos\left(\frac{i\pi x}{a}\right)\cos\left(\frac{j\pi y}{b}\right)\cos\left(\frac{n\pi x}{a}\right)\cos\left(\frac{m\pi y}{b}\right)dxdy \tag{47}
$$

$$\bar{u}_{ij} = \frac{-e^{-\sqrt{\frac{s}{\alpha_z}}c}}{k_z\sqrt{\frac{s}{\alpha_z}}\left(e^{\sqrt{\frac{s}{\alpha_z}}c}+e^{-\sqrt{\frac{s}{\alpha_z}}c}\right)}\bar{f}_{ij} \tag{48}$$

$$\bar{v}_{ij} = \frac{e^{-\bar{\lambda}_{ij}c}}{k_z\ \bar{\lambda}_{ij}\left(e^{\bar{\lambda}_{ij}c}-e^{-\bar{\lambda}_{ij}c}\right)}\bar{e}_{ij} \tag{49}$$

$$\bar{w}_{ijnm} = \frac{-e^{-\bar{\lambda}_{nm}c}}{k_z\ \bar{\lambda}_{nm}\left(e^{\bar{\lambda}_{nm}c}+e^{-\bar{\lambda}_{nm}c}\right)}\bar{d}_{ijnm} \tag{50}$$

This set of linear algebraic equations can be solved using matrix inversion similar to chapter 2. The solution for the temperature distribution in the Laplace domain represented by equation (38) must be inverted for the final solution. Due to the complicated nature of the solution, Laplace inversion is carried out using the de Hoog's quotient difference method algorithm [26] as implemented by Hollenbeck [27].

Chapter 4

RESULTS AND DISCUSSION

The impingement of a cooling jet on a surface produces a spatially varying convective heat transfer coefficient. While the precise nature of the convective heat transfer coefficient depends on a number of parameters related to the fluid flow in the jet [17], such as turbulence, convective heat transfer due to an impinging jet is often modeled using representative functions that produce large values of the convective heat transfer coefficient in and close to the jet region, which decline to lower values farther out. In this work, in order to demonstrate the capability of the analytical model to compute the temperature distribution for spatially varying convective heat transfer, the following expression is used for *h(r)* based on past work [20]:

$$h(r) = h_{\max}\left[\frac{1 - R\tanh\left(\gamma\left(\frac{r}{d} - \frac{3}{2}\right)\right)}{1+R}\right] \qquad (51)$$

where r is the distance away from the jet center and $R = \frac{h_{\max} - h_{\min}}{h_{\max} + h_{\min}}$. In this definition, d refers to the jet diameter, and h_{max} and h_{min} refer to the convective heat transfer coefficient at the jet impingement location and far away from the jet respectively. The parameter γ is representative of the width of the h vs r curve.

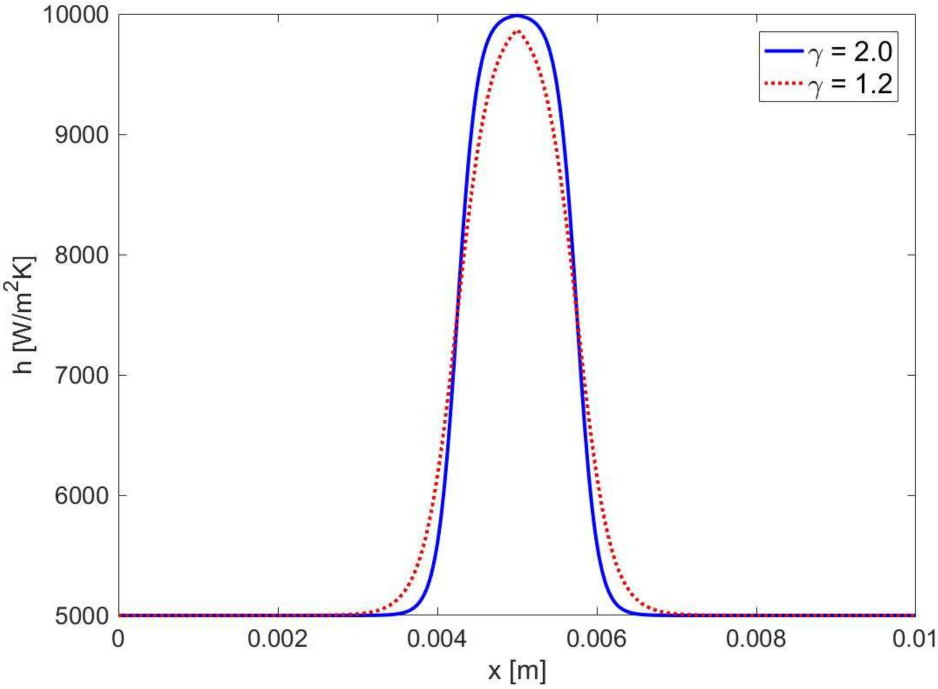

Figure 4.1 Convective heat transfer coefficient function

Figure 4.1 plots h as a function of r for two values of γ, showing that the value of γ can be used to modulate the region of influence of the impinging jet. In general, the lower the value of γ, the larger is the region of influence due to the jet. While the purpose of the present work is not to analyze convective heat transfer due to an impinging jet, equation (51) provides a convenient model for convective heat transfer coefficients for jet impingement with which to study the analytical models in chapters 2 and 3 for predicting steady state and transient temperature rise due to the spatially varying convective heat transfer.

Although the models are derived assuming orthotropic thermal conductivity, the rest of the paper assumes a constant thermal conductivity in all directions, as is usual for Silicon and other commonly used materials for substrates of microelectronic chips. The analytical model described in chapters 2 and 3 are validated by comparison with finite-element simulations. Finite-element simulations are carried out in a commercial software in which the geometry of the microprocessor chip, including the desired power map is modeled and meshed with around 750000 elements. Power dissipation is modeled as a heat flux boundary condition on one face, whereas jet cooling is modeled with a spatially varying convective heat transfer coefficient on the other face. Grid-independence of the finite-element simulation is ensured. The finite-element simulation serves to provide a validation of the analytical model.

Figure 4.2 presents a comparison for a steady-state case, where two hotspots of sizes 1mm by 1mm, each generating 10 W/mm^2 heat in a 10mm×10mm microprocessor die, which is being cooled by a single jet impinging on the die backside at the location corresponding to the first hotspot. The convective heat transfer parameters of the impinging jet are h_{max}=60000 W/m^{2}K, h_{min}=5000 W/m^{2}K, γ=2, and d=0.5 mm. Colorplots for the analytical model and finite-element simulation results shown in Figure 4.2(a) indicate very close agreement between the two, less than 4.8 % over the entire microprocessor die. The close agreement between the two is further illustrated in Figure 4.2(b), which show the variation of temperature along two lines passing through the centers of the two hotspots.

The typical computation time for computing temperature at a point of interest using the analytical model is around 5 seconds, compared to 1-2 minutes for the finite-element simulation depending on the level of convergence desired. Note that this is in addition to time taken for setting up the geometry and creating a mesh for the simulation. A further computational advantage is observed for transient cases, where the finite-element simulation takes much longer due to need for timestepping and convergence at each time step.

In order to validate the transient model presented in chapter 3, a computation is carried out to predict temperature as a function of time for a two-hotspot case, where the first hotspot remains on between t=0s and t=0.1s, and the second hotspot stays on afterwards. For ease of computation, a two-dimensional geometry is considered for transient computations. Figure 4.3(a) compares the computed temperature at the center of the first hotspot as a function of time with finite-element simulations. The two are found to be in very good agreement through the computation period, with a worst-case deviation of less than 2.4%. The first hotspot temperature rises during its active duration up to 0.1s, and reduces afterwards. Figure 4.3(b) plots the temperature as a function of x at t=0.25s, at which the first hotspot of size 1mm by 1mm and centered at 2.5mm has switched off while the second hotspot of the same size and centered at x=7.5mm is active. As expected, the peak temperature occurs at the second hotspot, and there is very good agreement between analytical computations and finite-element simulation results, with a worst-case deviation of less than 1.6%.

The very good agreement of temperature computed using the models presented in chapters 2 and 3 with finite-element simulation results provide validation of these analytical models. In comparison with finite-element simulations, these models offer faster computational time. The analytical nature of the solution also offers greater ease of integration with other computations related to the microprocessor chip, which might make possible to predict and monitor the temperature of the chip in real time. Finally, the analytical solution enables parametric study of the effect of various parameters on the temperature field on the chip, which is cumbersome and time-consuming to perform through finite-element simulations.

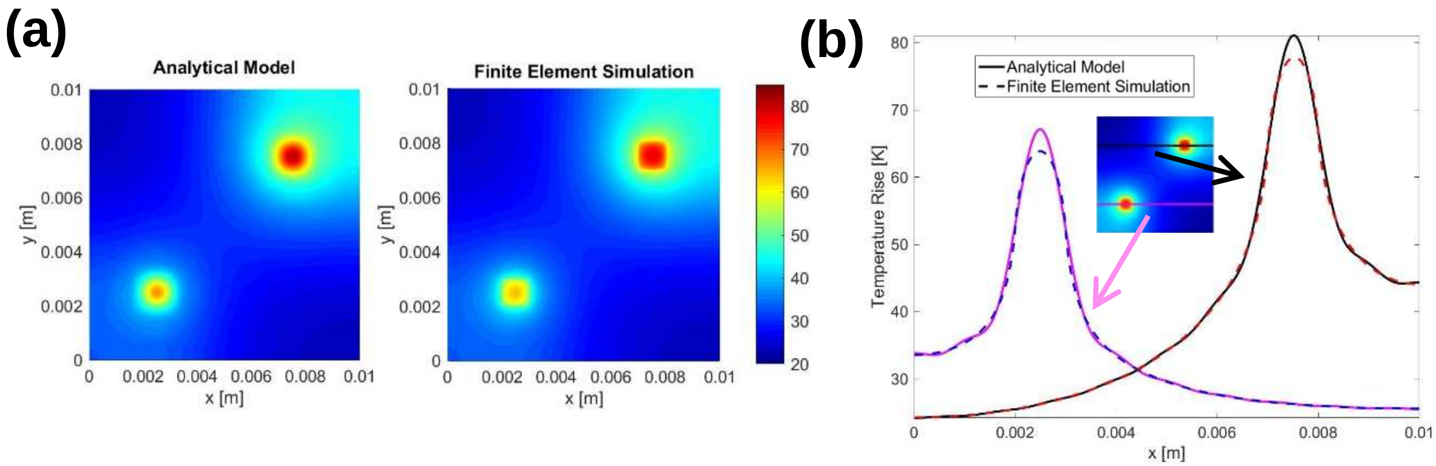

Figure 4.2 Comparison of the steady-state analytical model results with finite-element simulations

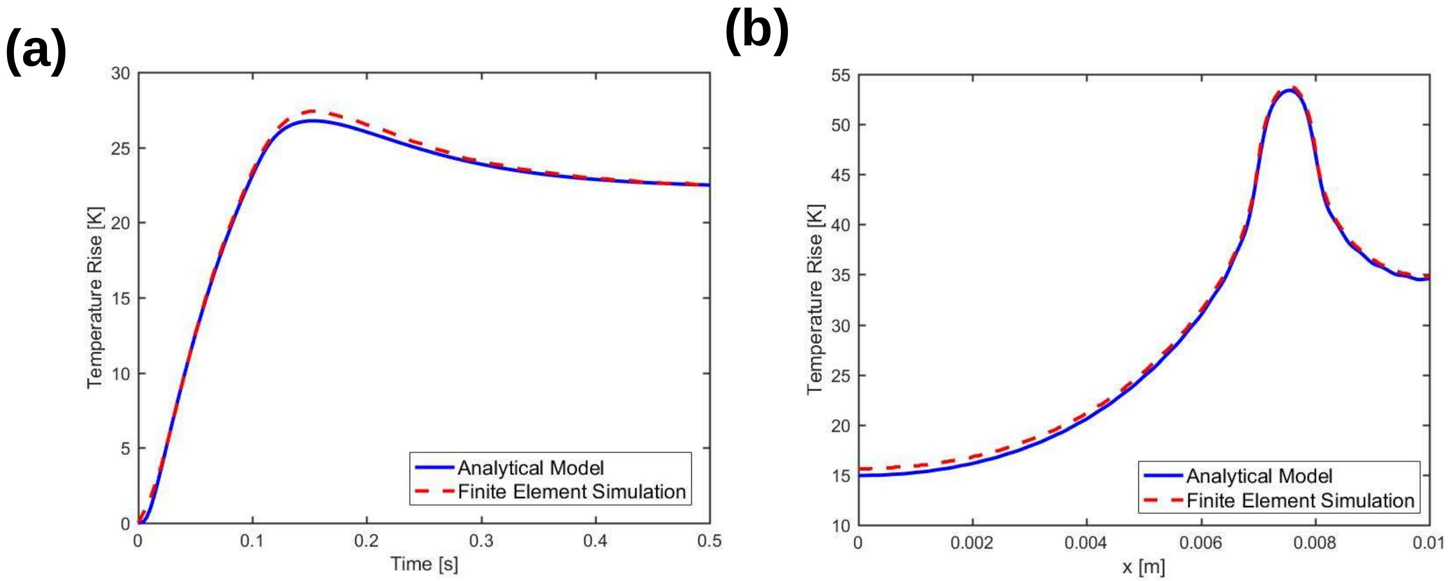

Figure 4.3 Comparison of transient analytical model with finite element simulations

Since the analytical solution for temperature is obtained in terms of an infinite series, the coefficients of which must be determined by solving a set of coupled algebraic equations, it is important to determine the minimum number of eigenvalues required to ensure reasonable accuracy of the analytical model. In general, the larger the number of eigenvalues, the larger is the set of algebraic equations such as equation (18) and thus the greater is the computational effort. Figure 4.4 plots the computed temperature as a function of space with varying numbers of eigenvalues in each spatial direction. For comparison, results from a finite-element simulation are also shown. Two hotspots of size 1 mm by 1 mm, each generating 10 W/mm^2 heat in a 10mm×10mm microprocessor die, which is being cooled by a single jet impinging on the die backside at the center of the microprocessor chip. Figure 4.4(a) shows the power map and figure 4.4(b) shows the heat transfer coefficient distribution for this computation. Figure 4.4(c) shows convergence of the computed temperature distribution towards the finite-element simulation curve as the number of eigenvalues increase. For this specific problem, Figure 4.4(c) shows that around 40 eigenvalues are needed in each spatial direction for reasonable accuracy of better than 0.5%. For a three-dimensional microprocessor, this corresponds to around 1681 unknown coefficients to determine, which is still not particularly challenging, since robust methods for solving much larger sets of algebraic equations exist and are commonly used.

The analytical model is then used for analyzing a number of thermal optimization problems related to the jet impingement cooling of a microprocessor. Figure 4.5 shows the computed temperature distribution for a number of cases with the same power, but increasing heat generation density in a single hotspot located at the center of the microprocessor, which is being cooled by a jet impinging on the backside, along at the center of microprocessor. In each case, the background heat generation in the remainder

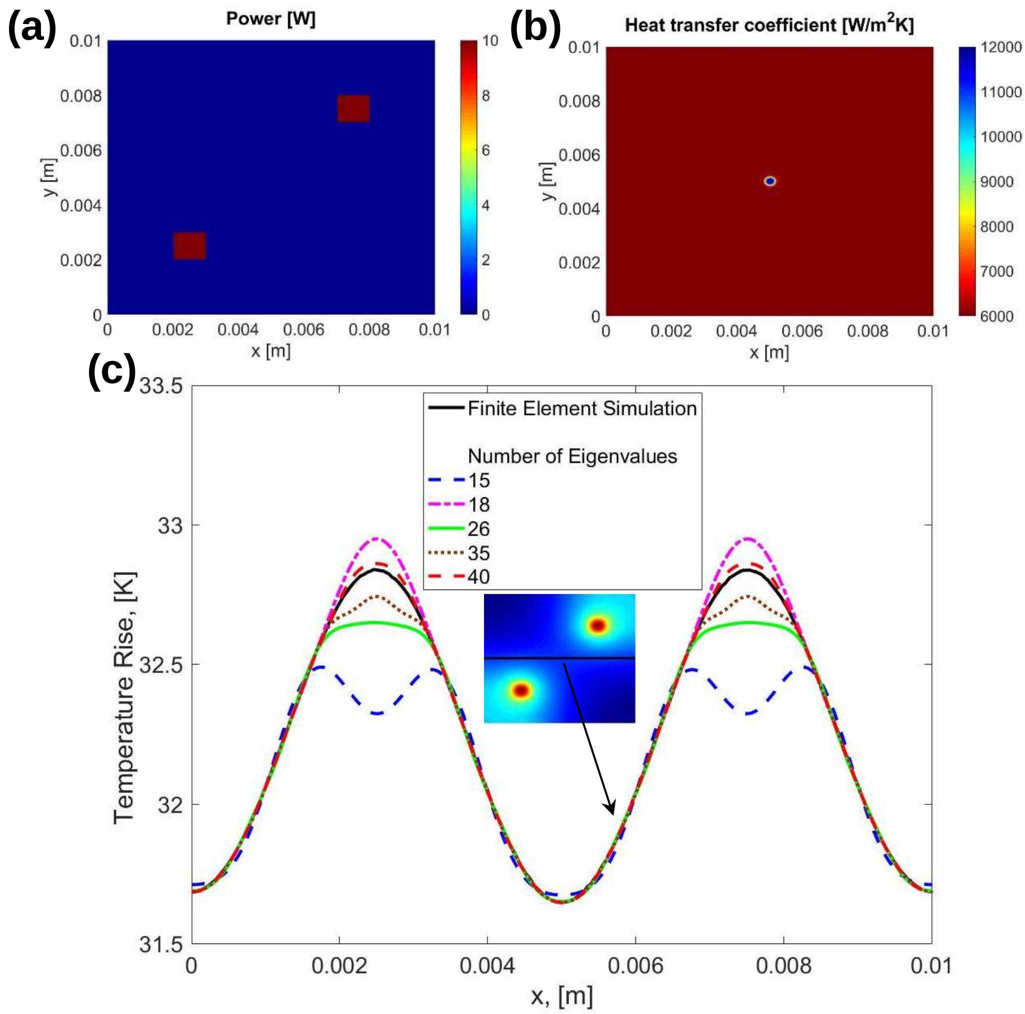

Figure 4.4 (a) Power dissipation map (b) Heat transfer coefficient distribution (c) Resulting temperature rise as a function of *x* for multiple number of eigenvalues

of the chip is changed in order to maintain the total power dissipation in the chip. Colorplots in Figure 4.5(a) for four cases of 0, 100, 500 and 1000 W/cm^2 dissipation in the hotspot show that as more and more heat is dissipated in a small region of the chip, the peak temperature rise increases significantly. This is also shown in Figure 4.5(b)

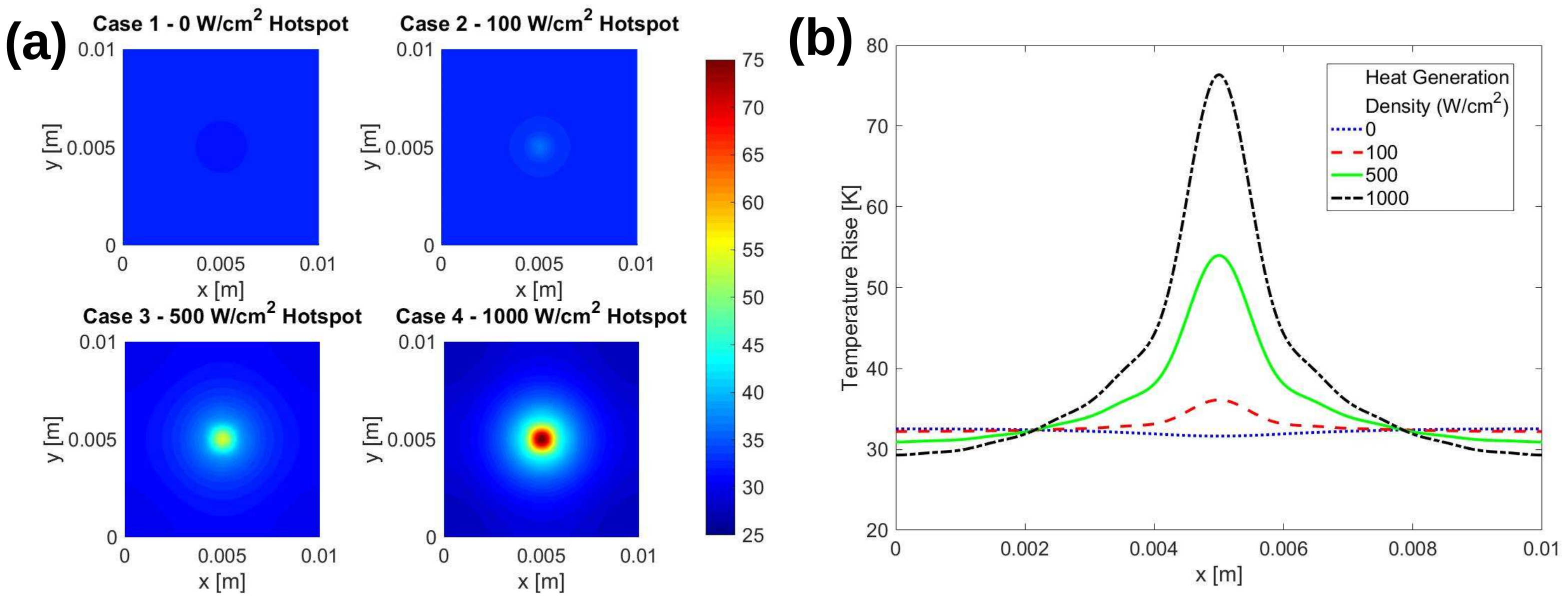

Figure 4.5 Effect of localization of heat generation

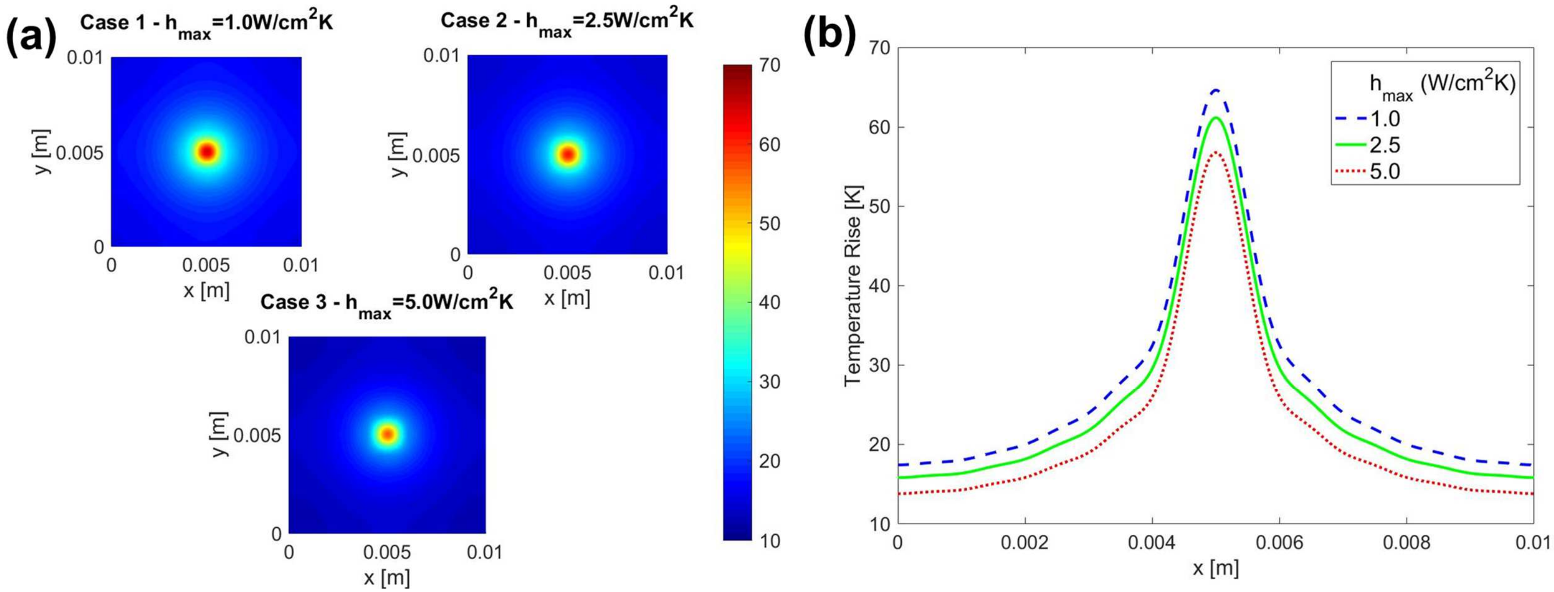

Figure 4.6 Effect of changing the peak heat transfer coefficient of the impinging jet

where the temperature rise is plotted along the centerline of the chip on the heat-generating face for the four cases. For a single 1000 W/cm^2 hotspot in the center, Figure 4.6 presents the effect of changing h_{max}. As expected, increasing the value of h_{max} results in lower peak temperatures.

Figure 4.7 examines the effect of varying the jet diameter on the temperature distribution in the case of a single hotspot at the center of the chip. The jet also impinges at the center of the chip on the back face. Colorplots in Figure 4.7(a) show minimal effect of increasing the jet diameter on peak temperature in the chip. This is also shown in the lineplots in Figure 4.7(b) that show less than 5.3% change in peak temperature rise in changing the jet diameter from 0.1mm to 1.0mm. This may be a useful insight in the design of jet impingement cooling, since a lower diameter jet will consume much lesser cooling fluid without a dramatic impact on peak temperature.

The effect of the location of the jet is examined next. Figure 4.8 presents computed temperature fields for a single hotspot in the center of the die being cooled by a single jet. Four cases with varying location of the jet impingement are computed. In case 1, the jet impingement corresponds to the center of the hotspot. In other cases, the jet progressively moves farther away from the center of the hotspot, thereby increasing the offset distance. As shown in the colorplots in Figure 4.8(a) and lineplots in Figure 4.8(b), Case 1 results in the lowest temperature as expected due to the alignment of heat dissipation with the cooling jet. Peak temperature rise in the chip becomes progressively larger as the jet moves farther away from the center of the hotspot. For Cases 2 through 4, the temperature plot in Figure 4.8(b) shows a local depression in the temperature field at the location where the jet impinges on the chip.

Most modern microprocessor chips contain multiple computation and data storage blocks, each of which has its own power dissipation characteristics. The discrete

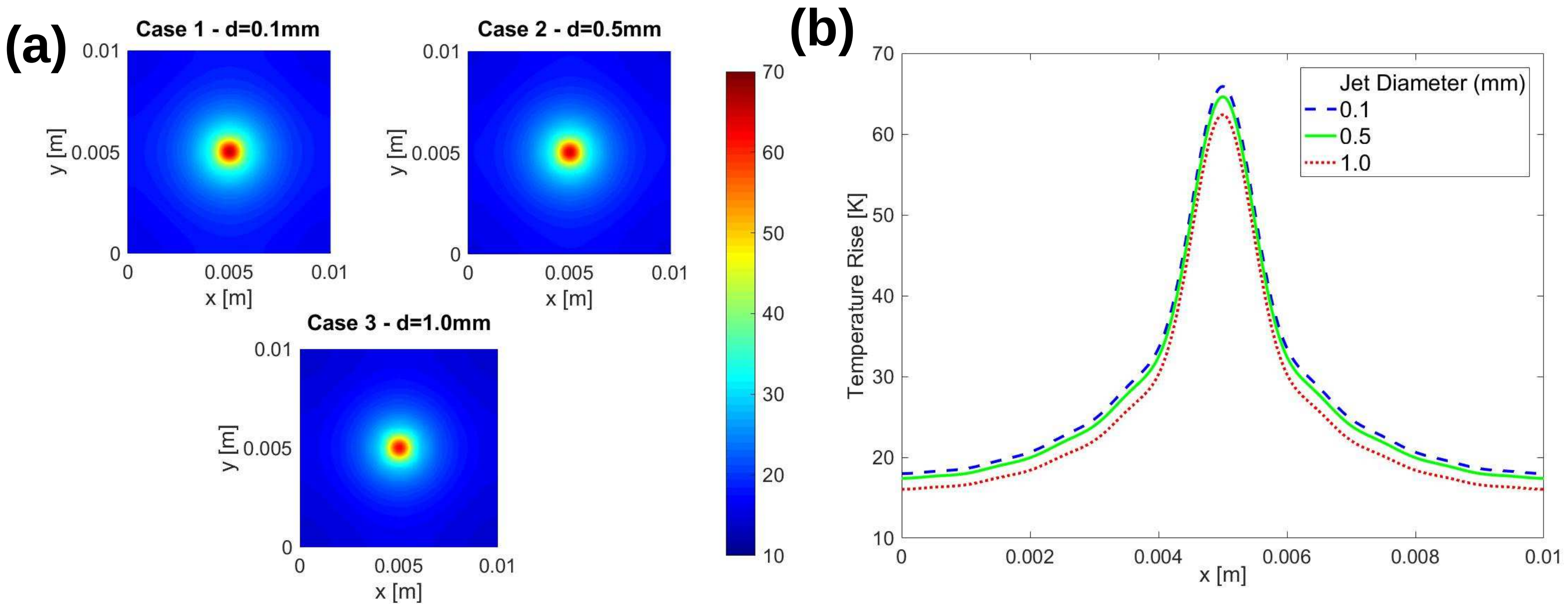

Figure 4.7 Effect of jet diameter in cooling effectiveness

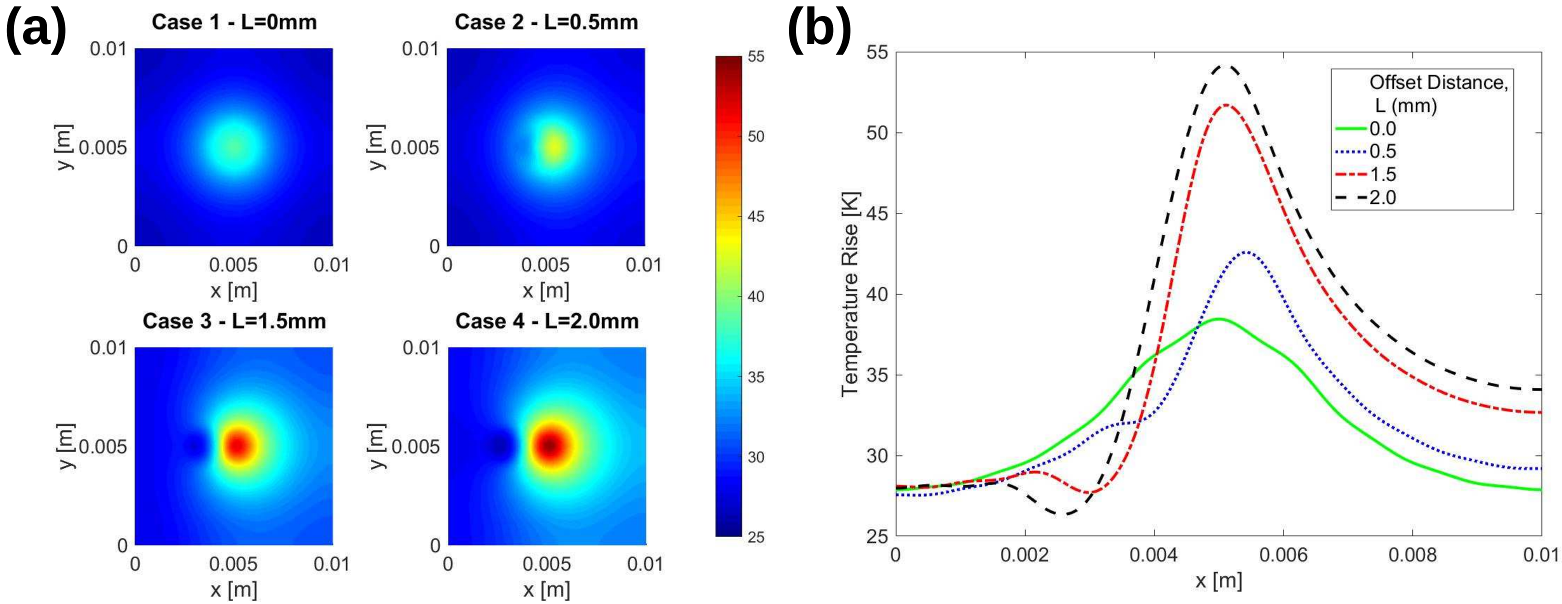

Figure 4.8 Effect of changing the jet impingement location relative to the center of the hotspot

power dissipation in each block is usually modeled through electrical simulations [28], and measured through voltage and current measurements. In general, the power map is significantly distributed due to the presence of multiple computation and data storage blocks, each of which dissipates power at different rates [29]. In case the chip is being cooled by multiple jets with different coolant flowrates, the variation of the convective heat transfer coefficient in space can also be complicated. In order to demonstrate the efficacy of the analytical model in such complicated scenarios, the temperature is computed when there are ten blocks of unequal sizes and heat dissipation rates on the chip, along with ten cooling jets of different characteristics and at different locations. In such a case, the resulting temperature field is not intuitive at all, and must be computed carefully in order to account for the effects of various parameters. Figure 4.9 presents the heat dissipation map, variation of the convection heat transfer coefficient in space, and the resulting temperature field computed by the analytical model. As expected, the temperature field is quite complicated and not intuitive. For example, the peak temperature shown in Figure 4.9(c) does not correspond to the block with the highest power. This happens because of the relatively small size of the block, which increases the power density, and because that block is not being effectively cooled by any of the cooling jets. Such quantification enabled by the analytical model is critical for a variety of optimization tasks, such as power distribution, distribution of cooling jets, as well as the allocation of coolant flow among the jets.

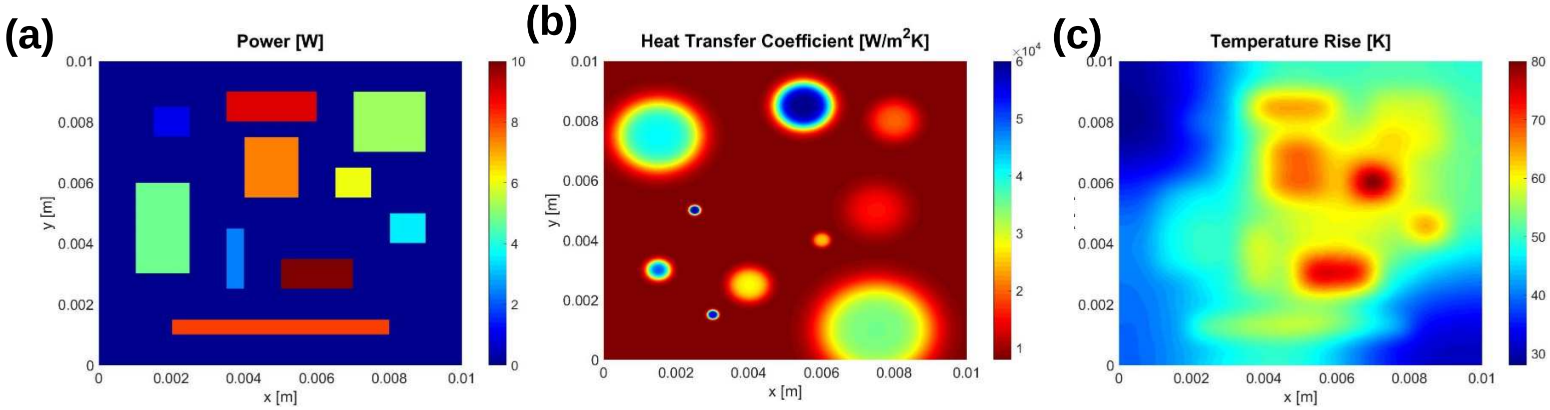

Figure 4.9 (a) Power dissipation map, (b) Heat transfer coefficient distribution, and (c) resulting temperature field due to a complicated power map comprising ten heat dissipation regions with varying shapes and powers being cooled by ten impinging jets with varying locations and strengths

The analytical models presented here can also be used for optimization of transient thermal performance of jet-cooled microprocessors. An illustration is provided through a scenario summarized in Figure 4.10. Here, two hotspots of equal strength 5 W/cm and size 1mm are centered at 2.5mm and 7.5mm respectively. The first hotspot is active from *t*=0s to *t*=0.1s, whereas the second hotspot becomes active at *t*=0.1s and

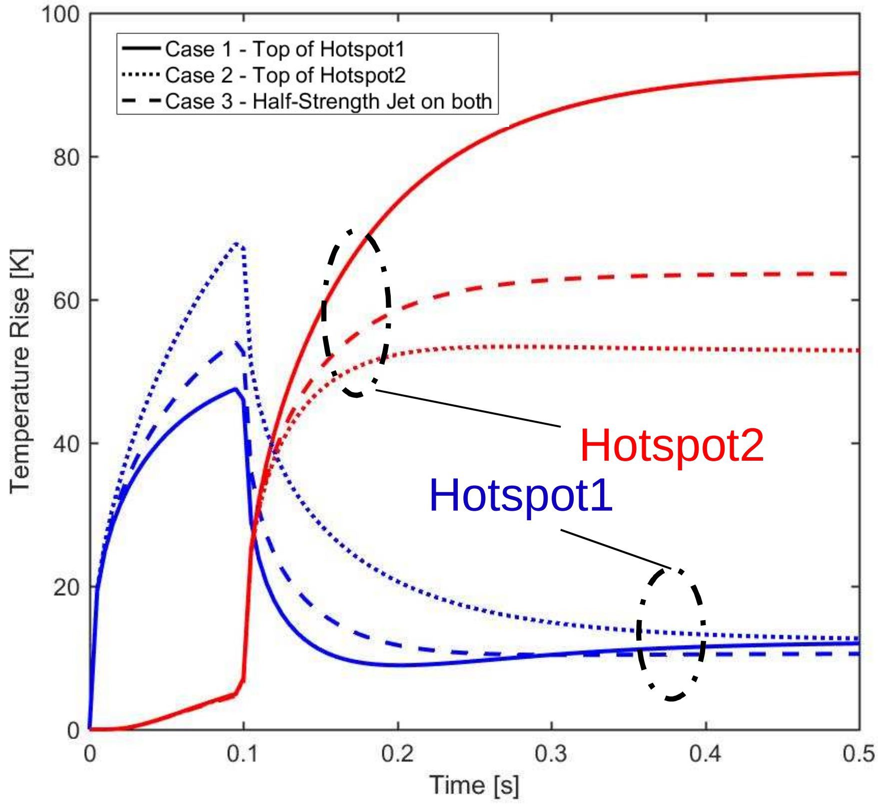

Figure 4.10 Temperature as a function of time for a transient process involving two hotspots of equal magnitude, one of which is active between t = 0 s and t = 0.1 s and the other is active afterwards.

remains active afterwards. Three jet configurations are considered. In the first case, the jet is located directly at the first hotspot, in which case, Figure 4.10 shows reduced peak temperature of the first hotspot compared to the other two cases. However, due to the absence of direct cooling of the second hotspot, temperature of the second hotspot is significantly greater than the other two cases. These findings are reversed for Case 2, where a single jet impinges on top of the second hotspot. A compromise between these two extremes is shown in Case 3, where two equal strength jets impinge on each of the hotspots, due to which both hotspot temperatures are relatively lower and are between the extremes observed in Cases 1 and 2. Note that the strength of the two jets in Case 3 are determined assuming the same total coolant flowrate as Cases 1 and 2. Because the jet is most effective close to where it impinges, the splitting up of a single large jet into multiple smaller jets offers better thermal performance and ability to cool more hotspots.

While the choice of which cooling configuration to use depends on the overall thermal management objective, the analytical model presented here provides the capability for rapid computation of temperature rise due to a variety of thermal dissipation and spatially varying convective cooling of the microprocessor chip. Having studied the effect of various parameters on the cooling profile, raises the question of optimization i.e. which configuration of the parameters offers the best possible cooling. As mentioned earlier, which cooling configuration to be used depends on the overall thermal management objective. Here is an effort to try and optimize a few of those parameters to offer an improved cooling configuration.

With given amount of flow rate, *hmax* can be well known. With given several number of jets having maximum heat transfer coefficient *hmax* each, not much choices exist in jet flow parameters and the following optimization algorithm helps to obtain the best possible thermal performance. If the number of hotspots are same as number of jets, it becomes clear to address each of the hotspot with each jet because of the localized nature of cooling jets. Similarly, if the number of hotspots are less than number of jets, each hotspot can be addressed with jet and additional jets can be used to address hotspots of high strength. The main optimization question arises when the resources are finite i.e. number of hotspots are more than number of jets. To demonstrate the usefulness of optimization algorithm developed, two cases are considered here in which one of the cases, the power on each hotspot is constant whereas in the other case, the power is variable. Without using any kind of optimization and addressing the first hotspots with jets results in higher temperature rise.

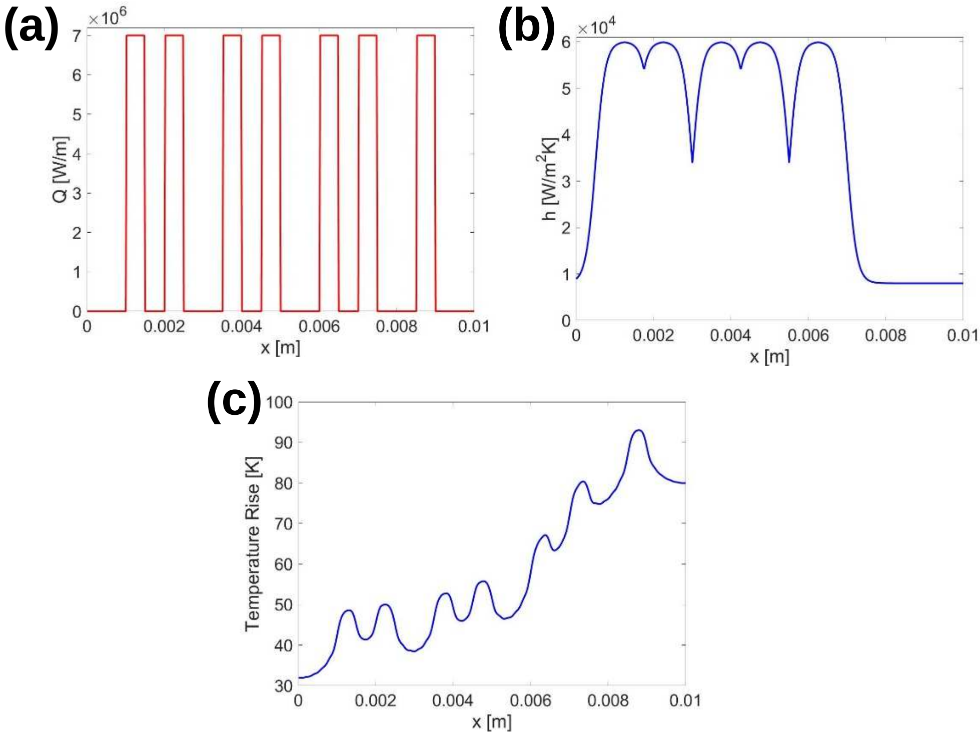

Figure 4.11 (a) Heat flux distribution (b) Convective heat transfer coefficient distribution (c) Temperature distribution of seven hotspots of same strength cooled by five jets of same strength

As for example, in figure 4.11(a) seven hotspots of equal strength of 70,000 W/cm each at varied location are shown. Five jets of equal *hmax* of 6 W/cm^{2}K addresses the first five hotspots as shown in plot of the heat transfer coefficient versus *x* in figure 4.11(b) and as a result of it, the max temperature rise goes to around 93 K as shown in the figure 4.11(c). Using the optimization tool, it can be seen that the hotspots on the edges produces higher temperature rise compared to the hotspots in the center to the microprocessor chip. This is due to the adiabatic boundary conditions on the sides of the microprocessor chip. Thus, as a result if we decide to address the hotspots on the edges, the max temperature rise can be reduced to 62 K. This finding can be seen in figure 4.12(b) where there are same seven hotspots as the case above in which same strength five jets address the hotspots on the edges as shown in the figure. This finding is the

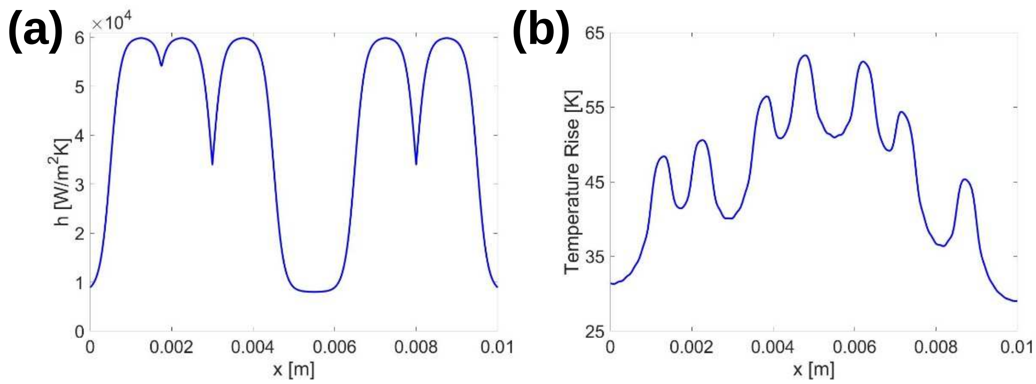

Figure 4.12 (a) Convective heat transfer coefficient distribution (c) Temperature distribution of seven hotspots of same strength cooled by five jets of same strength on the hotspots on the edges

one of the best possible solutions if there are not enough resources to split the jets. If splitting of jets is considered, based on the number of hotspots and jets, the optimization algorithm developed, splits the jets in all possible configurations. As for example, given seven hotspots and five jets, only two configurations are possible i.e. to split one jet into three smaller equal parts each or split two of the jets into two smaller equal parts each.

Note that the strength of the split jets is determined assuming the same total coolant flowrate as of a full-strength jet. As for example if a full-strength jet of flowrate *Q* splits into two jets then the strength of each split jet is *Q*/2. In such a case, the maximum heat transfer coefficient becomes $hmax/\sqrt{2}$ as heat transfer coefficient is proportional to the square root of coolant flow rate. The results for these cases are as shown in figure 4.13. As shown in figure 4.13(a), one jet splits up into three equal parts to make seven jets in total. The jets with higher strengths are addressed to the hotspots on the edges as they

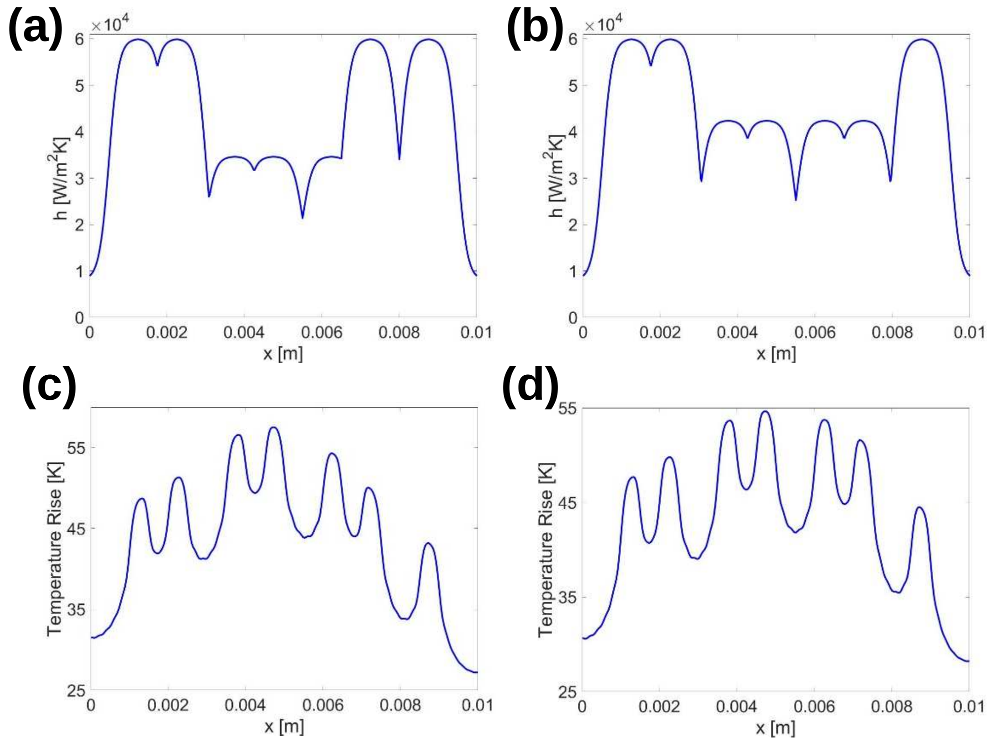

Figure 4.13 (a) Convective heat transfer coefficient distribution for one jet splits into three (b) Convective heat transfer coefficient distribution for two jets splits into two each (c) Resulting temperature distribution of due for one jet splits into three (d) Resulting temperature distribution of due for two jet splits into two each

tend to generate more heat as discussed previously. The temperature rise plots are as shown in figure 4.13(c), in which the maximum temperature rise can be seen as 57.5 K. If two of the jets splits up into 2 equal parts each, the convective heat transfer coefficient profile looks as shown in figure 4.13(b) and the temperature rise is as shown in figure 4.13(d). Maximum temperature rise in this case is 54.6 K. Hence, using the optimization, the max temperature rise can be reduced from 93 K to 54.6 K.

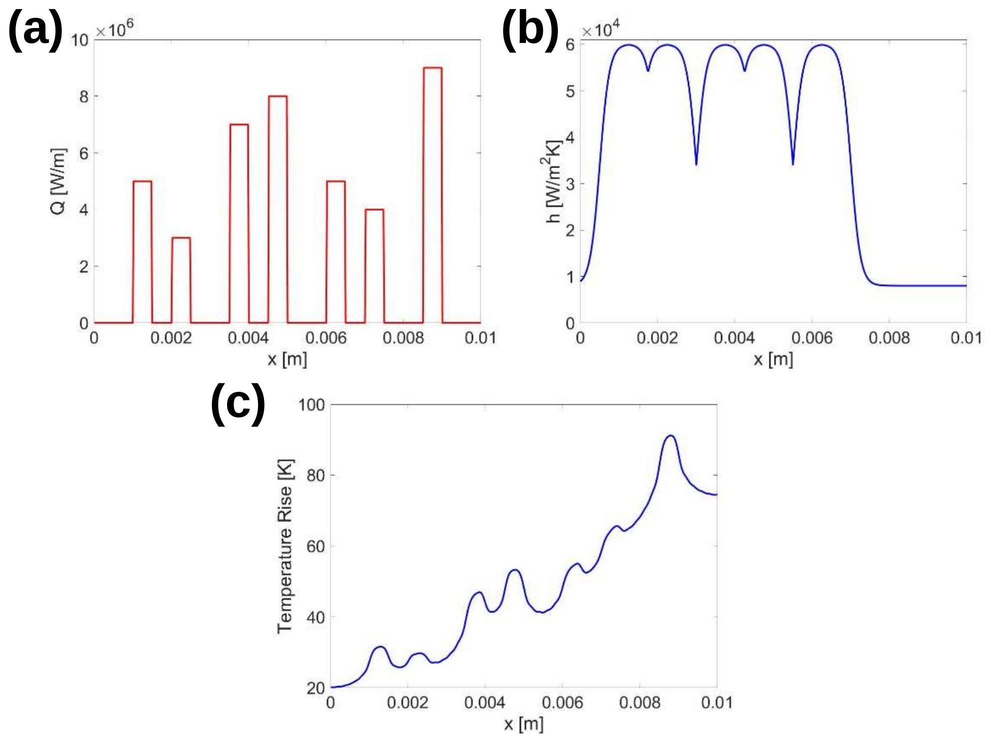

Figure 4.14 (a) Heat flux distribution (b) Convective heat transfer coefficient distribution (c) Temperature distribution of seven hotspots of different strength cooled by five jets of same strength

For the case in which power of each of the hotspots are different, addressing jets with higher power is beneficial as an obvious result. As for example, seven hotspots with variable power as shown in figure 4.14(a) are being cooled by five jets of equal strength of *hmax* of 6 W/cm^{2}K addressing the first five hotspots as a baseline case and in which the maximum temperature rise goes to around 91.1 K as seen in figure 4.14(c). If the jets

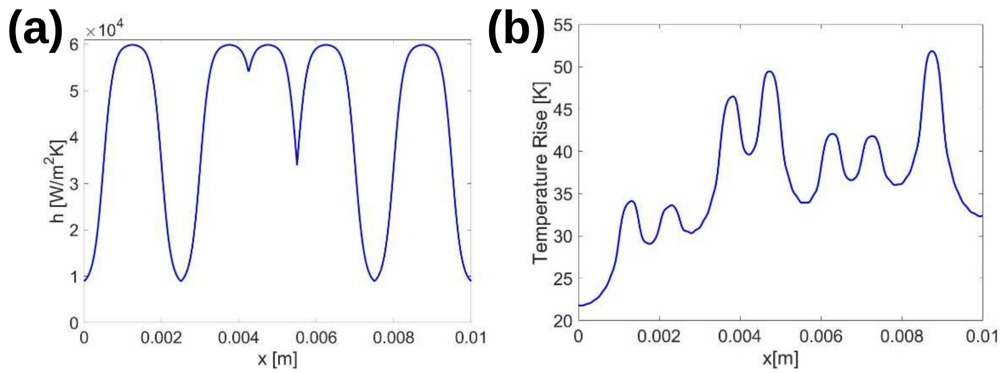

Figure 4.15 (a) Convective heat transfer coefficient distribution (c) Temperature distribution of seven hotspots of different strength cooled by five jets of same strength on the hotspots of the maximum strength

address the hotspots with maximum power it can be seen that the maximum temperature rise reduces to around 51.8 K as seen in temperature distribution plot in figure 4.15(b).

Same as the case with constant power hotspots, if jets splits and all the hotspots are addressed the maximum temperature rise can be further reduced. The results for these cases are as shown in figure 4.16. As shown in figure 4.16(a), one jet is splits up into three equal parts to make seven jets in total. The jets with higher strengths are addressed to the hotspots with high power. The temperature rise plots are as shown in figure 4.16(c), in which the maximum temperature rise can be seen as 51.4 K. If two of the jets are splits up into two equal parts each, the convective heat transfer coefficient in figure 4.16(d). Maximum temperature rise in this profile looks as shown in figure 4.16(b) and the temperature distribution plot is as shown case is 50 K. Hence using the optimization, the max temperature rise can be reduced from 91.1 K to 50 K.

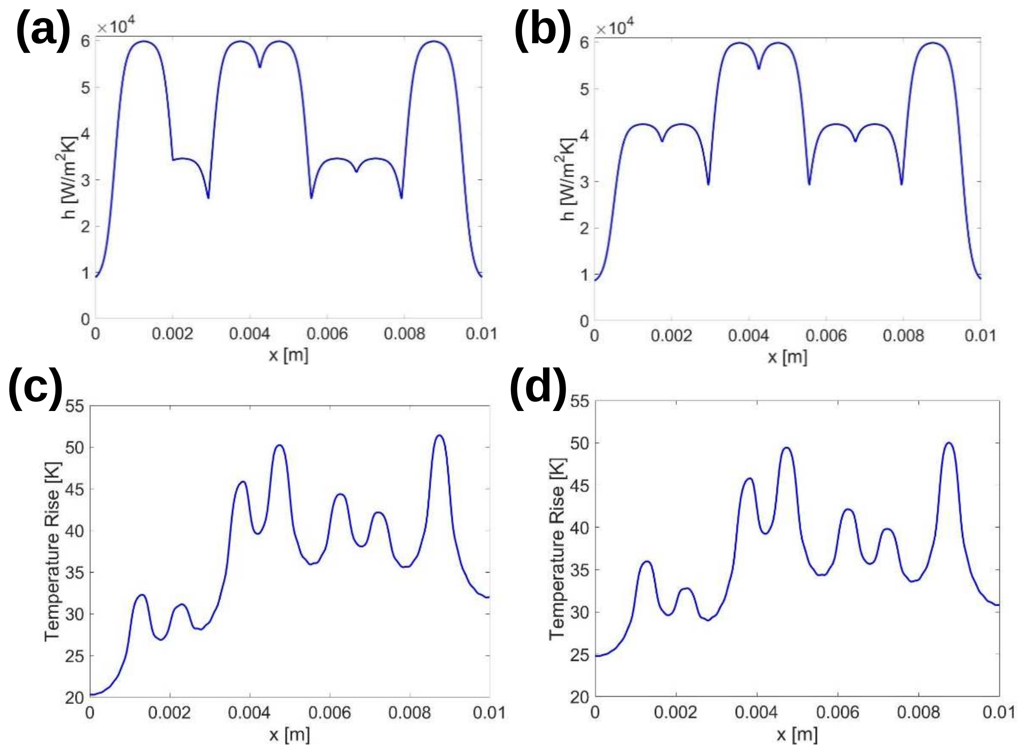

Figure 4.16 (a) Convective heat transfer coefficient distribution for one jet splits into three (b) Convective heat transfer coefficient distribution for two jets splits into two each (c) Resulting temperature distribution of due for one jet splits into three (d) Resulting temperature distribution of due for two jet splits into two each

The effects of using optimization tool is summarized in table 1. For the case with constant power hotspots, the maximum temperature without using optimization tool was 93.0 K which is reduced to 54.6 K using optimization tool which is around 41% of the temperature reduction. Similarly, for the case with variable power hotspots, the maximum temperature without using optimization tool was 91.2 K which is reduced to 50.0 K using optimization tool which is around 45% of the temperature reduction.

Table 4-1 Effects of Optimization Algorithm

	ΔT Without optimization (K)	ΔT With optimization (K)	% reduction
Constant Heat Flux	93.0	54.6	41 %
Variable Heat Flux	91.1	50.0	45 %

Chapter 5

CONCLUSIONS

The cooling of microprocessor chips is a significantly important technological problem. Since heat dissipation in multicore microprocessor chips is highly distributed and dynamic, the localized cooling offered by jet impingement is very attractive. The analytical models presented here offer a robust technique for rapid computation of steady state and transient temperature rise in a microprocessor chip with spatially varying heat dissipation and convective cooling due to an impinging jet. The model has been validated against finite-element simulations and has been shown to be effective for analysis of a variety of optimization problems that may be encountered in jet impingement based thermal management design.

A key limitation of the model presented here is that it does not account for time-varying convective cooling. This could arise, for example, in an actively cooled chip where the location of the jet can be modulated in time. Development of a model for such a scenario is considerably complicated and is an important direction for future work

Appendix A

Nomenclature

a,b,c	Dimensions
A00, B00, Anm, Bnm	Coefficients in the temperature field solution
C00, Cnm	Fourier series coefficients for temperature field
Cp	Heat capacity
d	Jet diameter
h	Convective heat transfer coefficient
k	Thermal conductivity
N	Norms
P00, Pnm	Fourier series coefficients for heat flux field
q	Heat flux
r	Distance away from jet center
s	Laplace parameter
t	time
T	Temperature rise
α	Thermal diffusivity
γ	Parameter representative of the width of the *h* vs *r* curve
λ	Eigenvalue
ρ	Density

<u>Subscripts:</u>

max	*Maximum*
min	*Minimum*
x,*y*,*z*	Rectilinear coordinates

Overbars represent variables in Laplace domain.

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Swapnil Dharamshi Luhar was born in Gujarat, India. He received his Bachelor of Engineering in Mechanical Engineering from Gujarat Technological University, Ahmedabad, India in the year 2014. After working for more than a year, he started his Master of Science from University of Texas at Arlington in Fall 2015. He started his research work related to jet impingement cooling of a microprocessor chip in Jan 2016. While pursuing his Master's, he received Graduate Teaching Assistantship by maintaining 4.0 GPA throughout. Swapnil received his Master of Science in Mechanical Engineering from University of Texas at Arlington in May 2017.

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