THERMAL STRESS ANALYSIS OF UNIDIRECTIONAL COMPOSITE MATERIAL BY FEM MODELING

By

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Presented to the Faculty of the Graduate School of

The University of Texas at Arlington in Partial Fulfillment

of the Requirements

for the Degree of

MASTER OF SCIENCE IN MECHANICAL ENGINEERING

THE UNIVERSITY OF TEXAS AT ARLINGTON

DECEMBER 2016

Acknowledgements

I express my gratitude towards Dr. Andrey Beyle for his support and encouragement. He has always been there to solve any problem that I faced during my research work.

I am grateful to my family, for their belief and confidence in me. Without them I would not be where I am today.

I want to thank my lab-mates Himanshu Raut, Akashsingh Thakur, Pavan Agarwal and Aditya Dandekar for their help and support.

November 29, 2016

Abstract

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BY FEM MODELING

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The University of Texas at Arlington, 2016

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Micromechanics of composite is study of complex mechanism of stress and strain transfer between fiber and matrix within a composite as well as distributions of physical fields inside heterogeneous medium. It was used successfully in predicting accurately physical properties like effective stiffness, effective linear thermal expansion of heterogeneous materials like composite. However, when it comes to strength characteristics of composite especially in transversal direction it failed to predict more accurate results.

This thesis is focused on thermally induced stress analysis of unidirectional composite by finite element method. Thermal stresses which are dangerously ignored factor in mechanics of composite usually occur due to difference in coefficient of thermal expansion between matrix and fiber within a composite. The distinction between other studies and this thesis is single cell is considered in most of the studies. But, this study not only focus on single cell but also takes into account effect of neighboring cells on the stress analysis of that cell. The results from the study is then compared with analytical

models for single cell also for infinitely big regular array of inclusions in matrix. This study is useful in finding more accurate strength characteristics compared to studies done in the past.

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Chapter 1

INTRODUCTION

1.1 Background

Composite materials are being used by mankind for generations, even thought beginning of the composite materials are unknown. Throughout history we can recite use of composite material. For example, the people in Egypt used mud bricks reinforced with straw, Medieval swords were crafted with more than one material [1]. In the late 1800s canoe builders were using Kraft paper by gluing them with shellac to form paper laminates. second half of the 20th century was most important era for composites, it became popular in spacecraft and military aircraft to improve the structural performance of the aircrafts.

Composites can be defined as a combination of two materials in which one of the material is called the reinforcing phase, which is a load carrying component, and it's usually in the form of Fibers, sheets or particles. It is surrounded by the other material call the matrix phase, which transfers the applied loads to the reinforcement.

Composites are of two types natural and artificially-made composites. Examples of natural composites are wood consists of cellulose fibers in a lignin matrix [2]. The artificially-made composites are divided into many subgroups on the basis of the type of matrices and reinforcement, which include ceramic matrix composites (CMCs), metal matrix composites (MMCs), intermetallic matrix composites (IMCs), carbon-carbon composites (CCCs) and polymer matrix composites (PMCs). In this work the focus is mainly on polymer matrix composites (PMCs). The polymer matrix is then subdivided into two groups, thermoset or a thermoplastic. The classification based on fibers types is short-fiber and continuous-fiber composites while considering polymer matrix composites

(PMCs). Continuous-fiber composites that offer the best mechanical properties with high performance such as Carbon, Boron, Kevlar or glass fibers. These composites are often utilized in special applications like aircraft components and also in automobile industries some times where advanced properties of fiber are fully used in those applications.

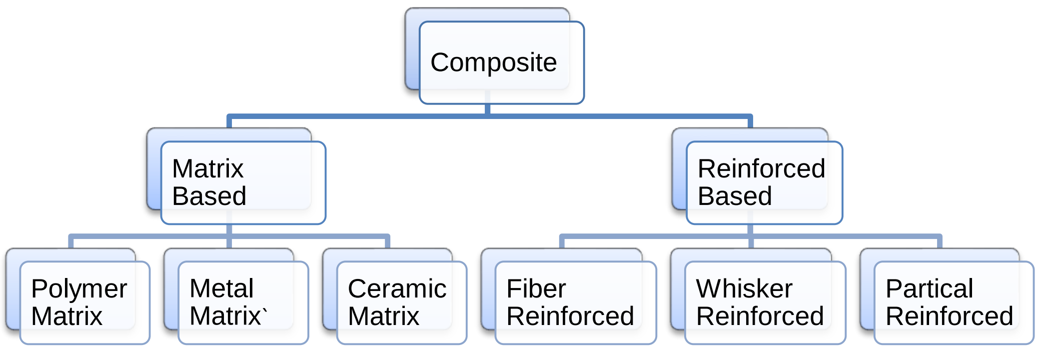


Figure 1.1 Classification of composites

Composites have so many advantages over conventional materials. They are considered as an alternative to conventional metal because composites are superior in design consideration to conventional materials because it has superior specific strength i.e. strength to weight ratio. Also it has a better corrosion resistance, high-impact strength, low thermal conductivity, design flexibility and better elevated temperature properties can be achieved by combining two or more distinctly different materials. One of the main problems in composites is because its consists of more than one material which has different thermal properties. Possibility of thermal stress resulting from temperature changes can occur within the composites. Usually, these stresses are due to a mismatch of coefficient of thermal expansion (CTE) of the composite constituents.

1.2 Mechanics of Composites

Composite materials are not homogenous in nature. Most of the materials are isotropic like other engineering materials, their physical behavior is different. This affects physical properties of composites, which can vary with location and orientation of the principal axes. These materials are called as anisotropic material that means their properties change in direction. In Mechanics of Composites the analysis of stress-strain relationship of these anisotropic material is complicated because of the it involves of many parameters like the type of reinforcement, reinforcement volume fraction, interface strength, and properties of the constituents all can influence the stress distribution in composites. The mechanical behavior of composites is evaluated on both microscopic and macroscopic scale to take into account inhomogeneity.

Analysis of a composite material depend upon a particular characteristic and behavior requirement of the composite under study. When local failures like breaking or failure of the fiber and/or matrix or debonding between fiber and matrix are on the focus, the analysis is performed at the fiber and matrix level and reformed to as the **Micromechanics** of composites. Micromechanics attempts to quantify the interactions of fiber and matrix on a microscopic scale on par with the diameter of a single fiber. On the other hand, when we consider Micromechanics, it treats composites as homogeneous materials, with mechanical properties representative of the laminate as a whole

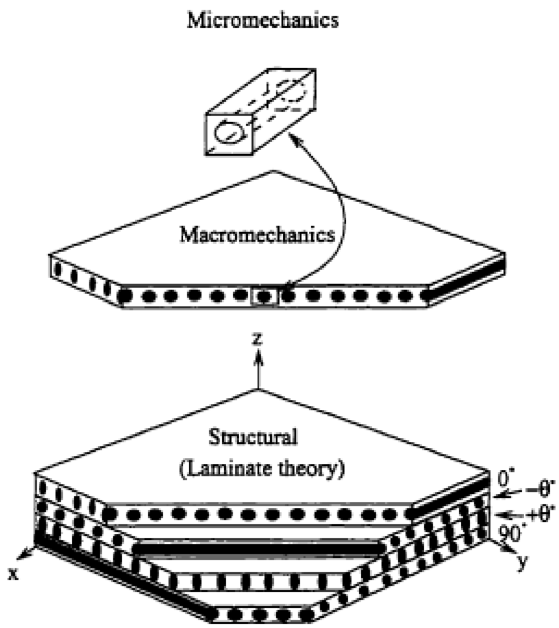


Figure 1.2 Mechanics theory of composite materials

In Macromechanics composites can be considered as homogeneous anisotropic materials with average properties, the local failure mechanisms are not taken into account. Instead, failure criteria are expressed in terms of average stresses and overall lamina strength values. The overall behavior of a laminate is analyzed as a function of lamina properties and stacking sequence. This approach is convenient for analysis of overall stress of composites.

Micromechanics have become an important means of understanding the mechanical behavior of the composite material. Certain assumptions are taken into account like idealized packing of the fiber in the matrix. This type of analysis is performed by introducing representative unit cell and then applying uniform strain boundary condition to analyze the material properties of the composite. This analysis is particularly helpful when focus is on the study of properties such as strength, structure toughness, fatigue life, local plastic and viscoplastic deformations, and local stress concentrations. These properties cannot be inferred from averaged characteristics of the composite. The analyses in this study is based on such Micromechanics approach.

Micromechanical approach is advantageous to Macromechanics because while deriving the effective material properties arises from the fact that many composites are formed of layers in addition to being anisotropic. A micromechanical approach can be used to find out the effective material properties for the composite. A macroscopic analysis on the other hand requires that the effective material properties be known before the analysis may be performed. Usually, effective properties are a function of the configuration of the individual layers, in a macroscopic analysis a different layup is a completely different composite whereas the micromechanical analysis may still be performed by simply changing the orientation of the layers.

1.3 Thermal stresses in composites

Thermal residual stresses are inherent to any composite structure and are internal stress within the structure that forms during curing process is done on composite. Their formation arises from the difference in coefficient of thermal expansion of the fiber and the matrix. This difference in coefficients of thermal expansion between fiber and matrix when considered with temperature step during the fabrication of the composite can cause complex mechanisms of differential shrinkage in the composite. This shrinkage results in thermal residual stresses, which are not to be ignored and must be taken into account in any stress analysis of the composite structure. These stresses will act on both a fiber-matrix interface and on a ply-to-ply scale within a laminate. The presence of residual stress hinders the strength potential of composites.

Thermal stress distribution due to thermal loading is very important factor while dealing with the performance of composite materials. This becomes crucial when either the coefficients of thermal expansion (CTE) of the composite constituents are far apart in magnitude which is called CTE mismatch or the working temperature is high enough to induce thermal stresses within composite. This difference in CTE causes different expansion or contraction in composite constituents even if there is a uniform temperature change.

Thermal residual stresses can influence the behavior of a composite in various ways. considering these thermal residual stresses either on a micro or macro-scale is very important issue while studying thermal stress distribution.

On a microscopic scale, when considering a fiber embedded within a matrix in the single cell model, when the temperature changes while cooling from the melting temperature of the matrix to room temperature can lead to shrinking of the matrix onto the fiber. This enhances the adhesion of the matrix on the fiber.

On a macroscopic scale, the coefficient of thermal expansion is entirely depending on the ply orientation. In this case because there are different layers built up in composite, thermal residual stresses occur due to different ply orientation within the composite. The thermal stresses acting are depends on materials used as well as layup of composite its very important for integrity of the composite.

1.4 Motivation

Thermal stresses are the most ignored factor in mechanics of composites. This can cause the failure of fiber-matrix interface. So it is necessary to find out its influence on composite. Micromechanics has been a widely adopted approach for the prediction of effective properties of a composite material. Most methods in micromechanics of materials are based on continuum mechanics. Continuum mechanics includes few important studies like various mathematical expressions including rules of mixtures by Voigt and Reuss, and mean field methods by Eshelby leading to the derivation of semi-empirical formulae by Halpin and Tsai, as models for the calculation of approximate effective properties of composites.

While this micromechanics approaches have been shown to be more or less accurate in calculating effective properties of composites but when it comes to prediction of strength characteristics micromechanics approach was not successful. Especially in transversal direction of composite where it is weaker. So it is very important to figure out stress distribution along transversal direction to find out accurate strength characteristics of a composite.

1.5 Objectives

The primary objective of this study is to observe behavior of unidirectional composites with transversely isotropic fiber or completely isotropic fiber, combined with isotropic matrix subject to temperature changes. This type of study has been done in the past, but this study is not only focuses on the behavior of the single unit cell of the composite but also takes in to account on the effect of the neighboring cell or neighboring fiber on the cell under study when composite is under thermal loading.

In composite material fiber volume ratio, or fiber volume fraction, is the percentage of fiber volume in the entire volume of a fiber-reinforced composite material. The objective of the study is also to find out the effect of fiber volume percentage on strength of the composite wile considering both single cell model and multi cell model. The models used in this study are square and hexagonal unit cell along with coaxial cylindrical cell to figure out effects of thermal loading.

Chapter 2

Literature review

2.1 Micromechanical modeling

This section contains some introductory methods used to determine the influence of mechanical properties of the fiber and matrix on effective properties of composite materials. As micromechanical analysis of fiber reinforced composite materials has become an important to understand the behavior of these materials. Analysis of composites is done by introducing representative unit cell and then by applying uniform strain boundary condition analysis of the material properties of the composite is done. For this first step an assumption of ideal arrangement of reinforcing fiber in the matrix is considered in this type of analysis. For, unidirectional fiber reinforced composites, the square packing unit cell and hexagonal unit cell is considered by most researchers. Also during early research coaxial cylindrical unit cell is also by several researchers

.

A cylindrical unit cell shown in figure below is consists of a cross-section with many concentric cylinders. The inner cylinder is fiber and outer cylinder is matrix. The cylinders have different radii, but the ratio of the fiber radius and matrix radius is the same for all cylinders. Therefore, here fiber volume fraction is the same for each concentric cylinders. Also gap between two cylinder models is filled by smaller cylindrical unit. This gap is filled by small cylinders with same volume fraction as big cylinders. But, this model doesn’t fill all the space so, this model is not a representation of is not realistic representation compared to square model or hexagonal model.

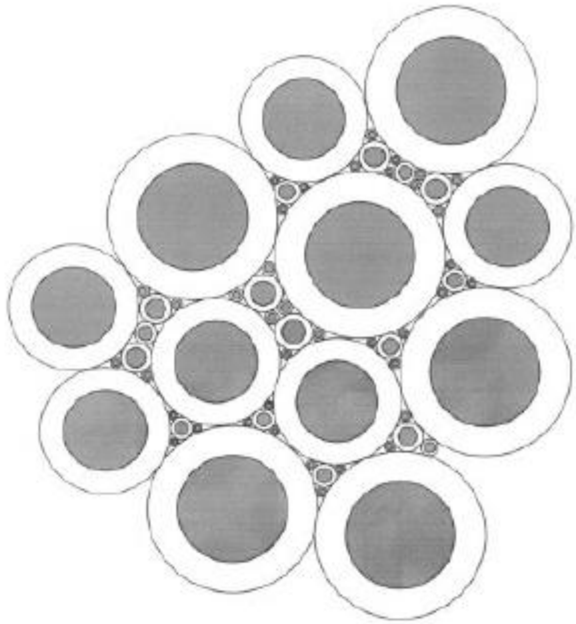


Figure 2.1 Concentric cylindrical assemblage (CCA). [13]

Hashin and Rosen [4] used concentric cylindrical assemblage (CCA) model which included the gaps between cylinders. They assumed an artificial displacement or stress field in their study before finding out analytical solutions. Because of this assumption distribution of stresses in this model is different form that of square cell or hexagonal cells.

Christensen and Lo [5] used a three--phase cylinder model in order to determine the effective transverse shear modulus of a unidirectional composite. The model they used is called 3 phase model or embedded cell model in which a cylindrical fiber surrounded by a concentric cylindrical matrix is embedded in an effective homogeneous medium which has same properties as effective properties of composites. The solution for this model is dependent on average formulation of all materials along with satisfactory boundary conditions.

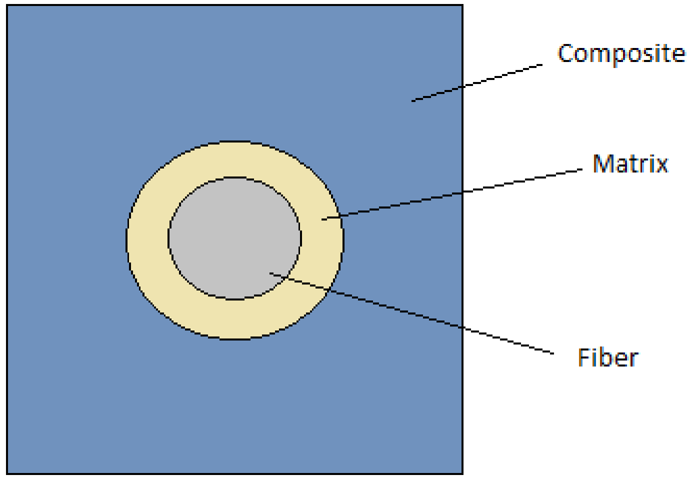


Figure 2.2 Embedded Cell model

The most rigorous model of unidirectional fiber reinforced material was developed by Van Fo Fy [8,9] during sixties. This model was reviewed Sendeckyj in volume 2 of his book 'Mechanics of composite materials'. This model takes into account the regular lattice of parallel cylindrical inclusions in the matrix. It gives double periodic stress distribution. The integral characteristics of this distribution are corresponding to integral values of the applied loads.

Composite are manufactured at elevated temperature. Because the thermal expansion coefficients of the fibers do not match with matrix, the coefficient of thermal expansion mismatch (CTE) occurs. As a result, thermal residual stresses generate in both the fibers and the matrix when the composite cools down to room temperature. While these residual stresses essentially do not affect the composite stiffness, but when we consider its influence on composite strength then it becomes an important issue. This amount entirely depends on the temperature difference and on the thermal–mechanical properties of the fiber as well as matrix. It is, thus, very important to establish a relationship of effect thermal residual stresses into the composite strength formulae. There are several researches have been made to understand relation ship between thermal stress and its effects on composites.

Schapery [6] in his paper he obtained expression for overall thermal expansion coefficient. He used thermo-elastic extreme principle using thermomechanical properties of fiber and matrix. This study was done under assumption that composite is elastic in behavior. Schapery was successful in obtaining expression for longitudinal thermal expansion coefficient of composite. This study was not as accurate for thermal stresses than mechanical stresses.

You et al. [12] in their study that focused on thermal analysis of transversely isotropic fiber-reinforced composites with an interphase. This interphase was considered as inhomogeneous. With generalized plane strain assumption. They derived generalized equation for composite with thermal loading, describing its stress and deformation. They compared various material and different volume fractions while investigating. They concluded that thermal stresses make more impact on interphase and fiber or matrix.

Shokrieh et al. [7] in their paper studied thermal residual stresses in fiber reinforced composite. They used Representative volume elements (RVE) to conduct finite element analysis of the composite. They used different types of loading on cylindrical, square and hexagonal models. They studied and compared mono fiber model and compared it with You's analytical model. Then they proposed a multi fiber model similar to this study. This approach was helpful in finding out effects on neighboring cell thermal stresses of composite. They calculated stresses on interface between fiber and matrix. They concluded by saying that single model can not be chosen as most of them have different results, and we can't choose one typical model.

Chen et al. [11] in their study developed advanced boundary element approach to study effects of neighboring fiber in multi cell model with interphase present. They also used Finite element approach along with boundary element model which is based on elastic theory. They studied effects of interphase and material properties on Transversal young's modulus of then composite. They concluded by proving that new boundary element model is precise in analysis of multi cell models.

Chapter 3

Modeling approach

Finite element analysis has become very important tool available to an engineer for use in design analysis. The finite element method is one of the most general procedures for attacking complex analysis problems. This research focused on the finite element modeling of thermal stress analysis within the composite. Micromechanical composite model was used into the finite element framework. This model has the capability to analyze a number of different composite types, which increases the flexibility of the analysis. But, the most important steps in using the finite element method is to make an appropriate choice for the idealization of the problem and correctly interpret the results. [19]

In this study micromechanical material model was developed by ANSYS Workbench 17.0, a large commercially available finite element code. ANSYS provides the analyst with the ability to native capabilities through user defined materials and element libraries with the help of Ansys tools provided in workbench. The only requirement for these user defined materials is that provision of all the information must be fulfilled by user to achieve the appropriate solution by Ansys workbench [Ansys reference].

3.1 Materials Used and their Properties

Composite consists of two main constituents, first is matrix phase and second reinforcing fiber. The thermos-mechanical properties of both these materials is very important for effective properties of composite materials. The materials studied in this research are two matrix materials and two fiber materials.

Matrix

The matrix properties determine the resistance of the composites to most of the degradative processes that eventually cause failure of the structure. These processes include impact damage, delamination, water absorption, chemical attack, and high-temperature creep. Thus, the matrix is typically the weak link in the composite structure. The matrix phase of commercial composites can be classified as either thermoset or thermoplastic.

1. **Epoxy Resin**

Epoxies are thermosetting resins and are available in a variety of viscosities from liquid to solid. There is verity of epoxies available commercially because of their advantage over other thermosetting resins such as, high strength and modulus, low levels of volatiles, excellent adhesion, low shrinkage, good chemical resistance, and ease of processing. Their major disadvantages of epoxies are brittleness and the reduction of properties while moisture is present. The processing or curing of epoxies is slower than polyester resins. Curing temperatures vary from room temperature to approximately 180 °C. The most common cure temperatures range between 250° and 350 °F (120–180 °C).

2. **Polyether Ether Ketone (PEEK)**

Polyether ether ketone, better known as PEEK, is a high temperature thermoplastic as compared to epoxies which are thermosetting resins. This aromatic ketone material offers outstanding thermal and combustion characteristics and resistance to a wide range of solvents and proprietary fluids that is why its very famous alternative to epoxies especially in aerospace applications. PEEK is usually reinforced with glass or carbon fibers.

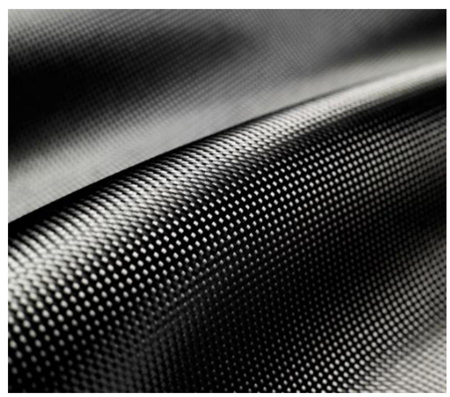
Fiber

The continuous reinforcing fibers of advanced composites are responsible for their high strength and stiffness. The most important fibers usually used most of the times are glass, graphite, and aramid fibers.

1. **Fiberglass**

The vast majority of all fibers used in the composites industry are glass. Glass fibers when compared to fiber like carbon or aramid fiber has a relatively low stiffness. The advantage of using glass fiber is its tensile strength is higher with the other fibers compared to its lower cost. The low cost and adequate properties makes glass fiber commercially most used in low cost applications. graphite fibers are important for applications where stiffness or weight are very important factors

Glass Fabric



Carbon Fabric

Figure 3.1 Glass and Carbon Fabrics

2. **Carbon fiber**

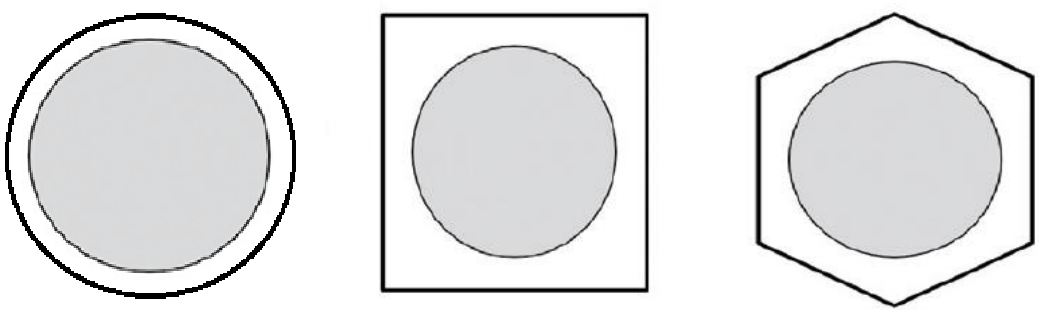
Carbon fiber are most widely used high performance applications. Carbon fibers are very stiff and strong, 3 to 10 times stiffer than glass fibers. Also they are lighter when compared with glass fibers for same volume of material. But, they do not possess a good impact-resistant and also have problem of galvanic corrosion in contact with metal. carbon fiber is popular in aircraft industry applications as they are used to produce primary fuselage, wing structures and flight controls etc.

Material	E-Glass fiber	Carbon fiber (T300)	Carbon fiber (P100)	Epoxy Resin [matrix	PEEK matrix
Density (g/cc)	2.54	1.8	1.78	1.15	1.32
E1 (GPa)	72.4	230	796.4	3.5	3.95
E2 (GPa)	72.4	8	7024	3.5	3.95
V12	0.21	0.256	0.2	0.39	0.3931
V23	0.21	0.3	0.4	0.39	0.3931
G12 (GPa)	30	27.3	6.89	1.26	1.418
α1 (/˚C) *10^-6	5	-0.41	-1.404	70	194.4
α1 (/˚C) *10^-6	5	10.08	7.8	70	194.4

Table 3.1 Material properties

3.2 Representative volume element

Composites often deal with thermal or mechanical loading, a representative volume element (RVE) is used in those cases. According to Nemat-Nasser and Hori [13], RVE is a material volume which statistically representation of the infinitesimal material. While creating micro-mechanical models consideration of the size of that cell is very essential for finite element analysis. RVE is very accurate about presenting real stress and strain acting on the system. Simulation if RVE provides thermal or mechanical behavior of the composites.



Cylindrical Unit cell Square Unit cell Hexagonal Unit cell

Figure 3.2 Representative volume elements

The Three representative volume elements have been proposed in (Liu and Chen, 2002) for the study of fiber reinforced composites. They are shown in figure 3.1 above first one is cylindrical unit cell and then square and hexagonal unit cell respectively. It is based on the 3-D elasticity theory. These models are used in finding out stresses acting on the matrix and fiber interphase as well as load transfer between fiber and matrix during mechanical or thermal loading.

3.3 Cylindrical model

3.3.1 Analytical model

This is coaxial cylindrical model, which is used for the derivation of the effective elastic properties of unidirectional composites. Similar model is used by Christensen [5]. This model is applied for thermal stresses analysis.

As it is an axially symmetrical problem.

Practically it is special case of Lamé problem. The equation of equilibrium is:

$$\sigma_\theta = \sigma_r + r\frac{d\sigma_r}{dr} \tag{1}$$

Axial stress is not uniform but the averaged stress depends on the selected version of the plane strain state problem; in all cases:

$$\langle\sigma_z\rangle = \frac{2\pi\int_a^b \sigma_z^M r dr + 2\pi\int_0^a \sigma_z^I r dr}{\pi b^2} \tag{2}$$

Here *a* and *b* –radius of the fiber and radius of the cell, correspondingly.

Radial and circumferential strains are connected by equation of strain compatibility

$$\varepsilon_r = \varepsilon_\theta + r\frac{d\varepsilon_\theta}{dr} \tag{3}$$

followed from Cauchy equations connecting strains and displacements in axisymmetric problem.

$$
\begin{aligned}
&\varepsilon_r = \alpha_T \Delta T + \frac{1}{E_T}\sigma_r - \frac{\nu_{TT}}{E_T}\sigma_\theta - \frac{\nu_{LT}}{E_L}\sigma_z;\\
&\varepsilon_\theta = \alpha_T \Delta T - \frac{\nu_{TT}}{E_T}\sigma_r + \frac{1}{E_T}\sigma_\theta - \frac{\nu_{LT}}{E_L}\sigma_z;\\
&\varepsilon_z = \alpha_L \Delta T - \frac{\nu_{TL}}{E_T}\sigma_r - \frac{\nu_{TL}}{E_T}\sigma_\theta + \frac{1}{E_L}\sigma_z = \Lambda = const;\\
&\frac{\nu_{LT}}{E_L} = \frac{\nu_{TL}}{E_T};
\end{aligned}
\tag{4}
$$

$$
\begin{aligned}
&\alpha_T^{*} = \alpha_T + \nu_{LT}\alpha_L; E_T^{*} = \frac{E_T}{1-\nu_{LT}\nu_{TL}}; \frac{\nu_T^{*}}{E_T^{*}} = \frac{\nu_{TT}+\nu_{LT}\nu_{TL}}{E_T};\\
&\nu_T^{*} = \left(\nu_{TT}+\nu_{LT}\nu_{TL}\right)\left(1-\nu_{LT}\nu_{TL}\right);
\end{aligned}
\tag{5}
$$

Substitution of constitutive law (4) into equation of strain compatibility (5) gives

$$
\begin{aligned}
&\frac{1}{E_T^{*}}\sigma_r - \frac{\nu_{TT}^{*}}{E_T^{*}}\sigma_\theta = -\frac{\nu_{TT}^{*}}{E_T^{*}}\sigma_r + \frac{1}{E_T^{*}}\sigma_\theta +\\
&\alpha_T^{*} r\frac{d\Delta T}{dr} - \frac{\nu_{TT}^{*}}{E_T^{*}} r\frac{d\sigma_r}{dr} + \frac{1}{E_T^{*}} r\frac{d\sigma_\theta}{dr};
\end{aligned}
$$

Let's substitute equation of equilibrium into the last equation

$$
\alpha_T^{*} E_T^{*} r\frac{d\Delta T}{dr} + 3r\frac{d\sigma_r}{dr} + r^2\frac{d^2\sigma_r}{dr^2} = 0;
\tag{6}
$$

For uniform temperature field

$$
\begin{aligned}
&3r\frac{d\sigma_r}{dr} + r^2\frac{d^2\sigma_r}{dr^2} = 0;\\
&\sigma_r = C_1 + C_2 r^{-2};
\end{aligned}
$$

Circumferential stresses are

$$\sigma_\theta = \sigma_r + r\frac{d\sigma_r}{dr} = \frac{d}{dr}(r\sigma_r) = \frac{d}{dr}\left(C_1 r + \frac{C_2}{r}\right) = C_1 - C_2 r^{-2};$$

Stresses in the transversal plane for transversally isotropic (monotropic) cylindrical fiber and in isotropic matrix are described for the axially-symmetrical case as

$$\begin{aligned} \sigma_r &= C_1 + C_2\frac{1}{r^2}; \\ \sigma_\theta &= C_1 - C_2\frac{1}{r^2} \end{aligned} \tag{7}$$

Let's consider 3 models: coaxial cylinder 2-phase model, model with small concentration of fibers and coaxial cylinder 3-phase model. Let's denote matrix by upper index "M", fiber by upper index "I" (inclusion), and composite by upper index "C". It is obvious that to avoid singularities or unbalanced stresses in thermal expansion only, it must be

$$\begin{aligned} C_2^I &= 0; \\ C_1^C &= 0; \end{aligned} \tag{8}$$

From the condition of continuity of radial stresses on the boundary fiber-matrix $r = a$, where *a* – is the radius of fiber, we have a relationship valid for all three models:

$$C_1^I = C_1^M + C_2^M\frac{1}{a^2} \tag{9}$$

For applying conditions of continuity of radial and axial displacements, it is necessary first to find the strains:

$$\begin{aligned} \varepsilon_\theta &= \frac{u_r}{r} = \alpha_T\Delta T + \frac{\sigma_\theta}{E_T} - \frac{\sigma_r\nu_{TT}}{E_T} - \frac{\sigma_z\nu_{LT}}{E_L}; \quad \frac{\nu_{LT}}{E_L} = \frac{\nu_{TL}}{E_T}; \\ \varepsilon_z &= \frac{du_z}{dz} = \alpha_L\Delta T + \frac{\sigma_z}{E_L} - \frac{(\sigma_r + \sigma_\theta)\nu_{TL}}{E_T} = \alpha_L\Delta T + \frac{\sigma_z}{E_L} - \frac{C_1\nu_{TL}}{E_T} = \alpha_L\Delta T + \frac{\sigma_z}{E_L} - \frac{C_1\nu_{LT}}{E_L}; \end{aligned} \tag{10}$$

Here index “L” denotes longitudinal direction, index “T” denotes transversal direction. Conditions of continuity of radial and axial displacements can be replaced by conditions of continuity of circumferential and axial strains. In combination of the condition of plane strain state

$$\varepsilon_z^I = \varepsilon_z^M = \varepsilon_z^C = \Lambda = const \tag{11}$$

the unknown constant Λ can be found from the condition of zero axial force in linear thermal expansion

$$F_z = \langle \sigma_z \rangle \pi b^2 = 2\pi \int_a^b \sigma_z^M r dr + 2\pi \int_0^a \sigma_z^I r dr = 0 \tag{12}$$

Here *b* is the outer radius of matrix tube in the model. From the constitutive law (10) and lane strain condition (11) it is follows that the axial stresses are constant inside each phase and condition of zero axial force is simplified

$$\begin{aligned} &\sigma_z^I c + \sigma_z^M (1-c) = 0; \\ &\sigma_z^C = 0 \end{aligned} \tag{13}$$

Continuity of axial strains gives:

$$\alpha_L^C \Delta T = \alpha_L^I \Delta T + \frac{\sigma_z^I}{E_L^I} - \frac{C_1^I \nu_{LT}^I}{E_L^I} = \alpha^M \Delta T + \frac{\sigma_z^M}{E^M} - \frac{C_1^M \nu^M}{E^M} = \Lambda; \tag{14}$$

Continuity of circumferential strains on the boundary fiber-matrix gives:

$$\alpha_T^I \Delta T + \frac{C_1^I \left(1-\nu_{TT}^I\right)}{E_T^I} - \sigma_z^I \frac{\nu_{LT}^I}{E_L^I} = \alpha^M \Delta T + \frac{C_1^M \left(1-\nu^M\right)}{E^M} - \frac{C_2^M \left(1+\nu^M\right)}{E^M} \frac{1}{a^2} - \sigma_z^M \frac{\nu^M}{E^M}; \tag{15}$$

Three models are different only in the condition on the outer boundary of composite cell. In the model of small concentration of fibers, it is supposed that the stresses in matrix are distributed similarly to the situation “cylindrical pore in infinite medium subjected to internal pressure”, i.e.

$$C_1^M = 0 \tag{16a}$$

In 2-phase coaxial cylinder model, the condition is zero radial stresses on outer boundary of matrix:

$$C_1^M + C_2^M \frac{1}{b^2} = 0 \tag{16b}$$

In 3-phase coaxial cylinder model, the conditions are continuity of radial stresses and continuity of circumferential strains

$$\begin{aligned} &C_1^M + C_2^M \frac{1}{b^2} = C_2^C \frac{1}{b^2}; \\ &\alpha^M \Delta T + \frac{C_1^M \left(1-\nu^M\right)}{E^M} - \frac{C_2^M \left(1+\nu^M\right)}{E^M}\frac{1}{b^2} - \sigma_z^M \frac{\nu^M}{E^M} = \alpha_T^C \Delta T - \frac{C_2^C \left(1+\nu_{TT}^C\right)}{E_T^C}\frac{1}{b^2}; \end{aligned} \tag{16c}$$

Solution of corresponding systems of equations gives values of constants, distributions of stresses and effective linear thermal expansion coefficients. Before solution, let’s reorganize the system of equations. From (11) and (10)

$$\begin{aligned} &\sigma_z^I = \left(\Lambda - \alpha_L^I \Delta T\right) E_L^I + C_1^I \nu_{LT}^I = \left(\Lambda - \alpha_L^I \Delta T\right) E_L^I + \left(C_1^M + C_2^M \frac{1}{a^2}\right)\nu_{LT}^I; \\ &\sigma_z^M = \left(\Lambda - \alpha^M \Delta T\right) E^M + C_1^M \nu^M \end{aligned} \tag{17}$$

Here relationship (9) was taken into account. Substitution of (17) into (13) gives

$$\begin{aligned} &\left(\Lambda - \alpha_L^I \Delta T\right) E_L^I c + \left(C_1^M + C_2^M \frac{1}{a^2}\right)\nu_{LT}^I c + \left(\Lambda - \alpha^M \Delta T\right) E^M \left(1-c\right) + C_1^M \nu^M \left(1-c\right) = 0; \\ &or \\ &\Lambda = \frac{\left[\alpha_L^I E_L^I c + \alpha^M E^M \left(1-c\right)\right]\Delta T - \left(C_1^M + C_2^M \frac{1}{a^2}\right)\nu_{LT}^I c - C_1^M \nu^M \left(1-c\right)}{E_L^I c + E^M \left(1-c\right)} \end{aligned} \tag{18}$$

Let's use notations:

$$\begin{aligned}
&\Upsilon = \frac{\left[\alpha_L^I E_L^I c + \alpha^M E^M \left(1-c\right)\right]}{E_L^I c + E^M \left(1-c\right)};\\
&\Xi = \frac{\nu_{LT}^I c + \nu^M \left(1-c\right)}{E_L^I c + E^M \left(1-c\right)};\\
&\Psi = \frac{\nu_{LT}^I c}{E_L^I c + E^M \left(1-c\right)};\\
&\Lambda = \Upsilon \Delta T - C_1^M \Xi - C_2^M \Psi \frac{1}{a^2}
\end{aligned} \tag{19}$$

As a result, (17) can be rewritten as

$$\begin{aligned}
&\sigma_z^I = \left(\Upsilon - \alpha_L^I\right) E_L^I \Delta T + C_1^M \left(\nu_{LT}^I - \Xi E_L^I\right) + C_2^M \frac{1}{a^2}\left(\nu_{LT}^I - \Psi E_L^I\right);\\
&\sigma_z^M = \left(\Upsilon - \alpha^M\right) E^M \Delta T + C_1^M \left(\nu^M - \Xi E^M\right) - C_2^M \Psi \frac{1}{a^2} E^M
\end{aligned} \tag{20}$$

Substitution (20) into (10) for both phases gives:

$$\begin{aligned}
&\varepsilon_\theta^I = \left(\alpha_T^I + \nu_{LT}^I \alpha_L^I - \nu_{LT}^I \Upsilon\right)\Delta T + \frac{\left(C_1^M + C_2^M \frac{1}{a^2}\right)\left(1 - \frac{\left(\nu_{LT}^I\right)^2 E_T^I}{E_L^I} - \nu_{TT}^I\right)}{E_T^I}\\
&+\nu_{LT}^I \left[C_1^M \Xi + C_2^M \frac{1}{a^2}\Psi\right];\\
&\varepsilon_\theta^M = \left[\alpha^M \left(1+\nu^M\right) - \nu^M \Upsilon\right]\Delta T + \frac{C_1^M\left[1 - \nu^M - \left(\nu^M\right)^2 + \nu^M \Xi E^M\right]}{E^M} - \frac{C_2^M \frac{1}{r^2}\left(1+\nu^M\right)}{E^M}\\
&+\nu^M C_2^M \Psi \frac{1}{a^2};
\end{aligned} \tag{21}$$

Where plane strain state thermoelastic characteristics can be introduced (denoted by upper index "□ "

$$\alpha_T^{\square I} = \alpha_T^I + \nu_{LT}^I \alpha_L^I;$$
$$E_T^{\square I} = \frac{E_T^I}{1-\nu_{LT}^I \nu_{TL}^I} = \frac{E_T^I}{1-\frac{\left(\nu_{LT}^I\right)^2 E_T^I}{E_L^I}} \qquad (22)$$

As a result, the circumferential strains can be rewritten as:

$$\varepsilon_\theta^I = \left(\alpha_T^{\square I} - \nu_{LT}^I \Upsilon\right)\Delta T + \left(C_1^M + C_2^M \frac{1}{a^2}\right)\left(\frac{1}{E_T^{\square I}} - \frac{\nu_{TT}^I}{E_T^I}\right) + \nu_{LT}^I\left[C_1^M \Xi + C_2^M \frac{1}{a^2}\Psi\right];$$
$$\varepsilon_\theta^M = \left(\alpha^{\square M} - \nu^M \Upsilon\right)\Delta T + C_1^M\left(\frac{1}{E^{\square M}} - \frac{\nu^M}{E^M} + \nu^M \Xi\right) - \frac{C_2^M\left[\frac{1}{r^2}\left(1+\nu^M\right) - \nu^M E^M \Psi \frac{1}{a^2}\right]}{E^M} \qquad (23)$$

Substitution into condition of continuity of circumferential strains on the boundary fiber-matrix gives instead of (15) the following expression:

$$\left[\left(\alpha_T^{\square I} - \alpha^{\square M}\right) - \left(\nu_{LT}^I - \nu^M\right)\Upsilon\right]\Delta T + \left(C_1^M + C_2^M \frac{1}{a^2}\right)\left(\frac{1}{E_T^{\square I}} - \frac{\nu_{TT}^I}{E_T^I}\right) +$$
$$\left(\nu_{LT}^I - \nu^M\right)\left[C_1^M \Xi + C_2^M \frac{1}{a^2}\Psi\right] = C_1^M\left(\frac{1}{E^{\square M}} - \frac{\nu^M}{E^M}\right) - \frac{C_2^M\left(1+\nu^M\right)}{a^2 E^M} \qquad (24)$$

This equation containing only two unknown constants has to be combined with one of equations (16) to have the final solution of the problem in the framework of corresponding model.

Model of small concentration of fibers:

Combination of (24) and (16a) gives:

$$C_1^M = 0;$$
$$C_2^M = -\frac{a^2\left[\left(\alpha_T^{\square I} - \alpha^{\square M}\right) - \left(\nu_{LT}^I - \nu^M\right)\Upsilon\right]}{\left[\frac{\left(1+\nu^M\right)}{E^M} + \left(\frac{1}{E_T^{\square I}} - \frac{\nu_{TT}^I}{E_T^I}\right) + \Psi\left(\nu_{LT}^I - \nu^M\right)\right]}\Delta T \qquad (25)$$

Maximal in absolute values stresses in matrix are on the boundary with fiber

$$\sigma_r\left(a\right)=-\sigma_\theta\left(a\right)=\frac{\left[\left(\alpha^{M}-\alpha_T^{I}\right)-\left(\nu^M-\nu_{LT}^I\right)\frac{\alpha_L^I E_L^I c+\alpha^M E^M\left(1-c\right)}{E_L^I c+E^M\left(1-c\right)}\right]}{\left[\frac{\left(1+\nu^M\right)}{E^M}+\left(\frac{1}{E_T^{I}}-\frac{\nu_{TT}^I}{E_T^I}\right)+\frac{\nu_{LT}^I c}{E_L^I c+E^M\left(1-c\right)}\left(\nu_{LT}^I-\nu^M\right)\right]}\Delta T;$$

$$\sigma_z=\left(\frac{\alpha_L^I E_L^I c+\alpha^M E^M\left(1-c\right)}{E_L^I c+E^M\left(1-c\right)}-\alpha^M\right)E^M\Delta T+ \tag{26}$$

$$\frac{\left[\left(\alpha_T^{I}-\alpha^{M}\right)-\left(\nu_{LT}^I-\nu^M\right)\frac{\alpha_L^I E_L^I c+\alpha^M E^M\left(1-c\right)}{E_L^I c+E^M\left(1-c\right)}\right]}{\left[\frac{\left(1+\nu^M\right)}{E^M}+\left(\frac{1}{E_T^{I}}-\frac{\nu_{TT}^I}{E_T^I}\right)+\frac{\nu_{LT}^I c}{E_L^I c+E^M\left(1-c\right)}\left(\nu_{LT}^I-\nu^M\right)\right]}\frac{\Delta T\nu_{LT}^I E^M c}{E_L^I c+E^M\left(1-c\right)}$$

Von Mises equivalent stress is calculated according standard formula:

$$\sigma_{VM}=\sqrt{\frac{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2}{2}} \tag{27}$$

Or because here $\sigma_2=-\sigma_1$

$$\sigma_{VM}=\sqrt{3(\sigma_r)^2+(\sigma_z)^2} \tag{28}$$

Effective axial linear thermal expansion coefficient is calculated as:

$$\alpha_L^C=\frac{\Lambda}{\Delta T}=\frac{\left[\alpha_L^I E_L^I c+\alpha^M E^M\left(1-c\right)\right]}{E_L^I c+E^M\left(1-c\right)}+$$

$$\frac{\nu_{LT}^I c\left[\left(\alpha_T^{I}-\alpha^{M}\right)-\left(\nu_{LT}^I-\nu^M\right)\frac{\left[\alpha_L^I E_L^I c+\alpha^M E^M\left(1-c\right)\right]}{E_L^I c+E^M\left(1-c\right)}\right]}{\left[\frac{\left(1+\nu^M\right)}{E^M}+\left(\frac{1}{E_T^{I}}-\frac{\nu_{TT}^I}{E_T^I}\right)\right]\left[E_L^I c+E^M\left(1-c\right)\right]+\nu_{LT}^I c\left(\nu_{LT}^I-\nu^M\right)} \tag{29}$$

Effective transversal linear thermal expansion coefficient is calculated as:

$$\alpha_T^C = \frac{\varepsilon_\theta^M(b)}{\Delta T} = \alpha^{\square M} - \nu^M \frac{\left[\alpha_L^I E_L^I c + \alpha^M E^M (1-c)\right]}{E_L^I c + E^M (1-c)} +$$

$$\frac{c\left[\left(\alpha_T^{\square I} - \alpha^{\square M}\right) - \left(\nu_{LT}^I - \nu^M\right)\dfrac{\left[\alpha_L^I E_L^I c + \alpha^M E^M (1-c)\right]}{E_L^I c + E^M (1-c)}\right]\left[\left(1+\nu^M\right) - \dfrac{\nu^M E^M \nu_{LT}^I}{E_L^I c + E^M (1-c)}\right]}{1+\nu^M + E^M\left(\dfrac{1}{E_T^{\square I}} - \dfrac{\nu_{TT}^I}{E_T^I}\right) + \dfrac{\nu_{LT}^I c E^M}{E_L^I c + E^M (1-c)}\left(\nu_{LT}^I - \nu^M\right)} \qquad (30)$$

Two-phase coaxial cylinder model:

Combination of (24) and (16b) gives:

$$\left[\left(\alpha_T^{\square I} - \alpha^{\square M}\right) - \left(\nu_{LT}^I - \nu^M\right)\Upsilon\right]\Delta T + C_2^M \frac{1}{a^2}(1-c)\left(\frac{1}{E_T^{\square I}} - \frac{\nu_{TT}^I}{E_T^I}\right) +$$

$$\left(\nu_{LT}^I - \nu^M\right) C_2^M \frac{1}{a^2}\left[-c\Xi + \Psi\right] = -C_2^M \frac{1}{a^2} c\left(\frac{1}{E^{\square M}} - \frac{\nu^M}{E^M}\right) - \frac{C_2^M\left(1+\nu^M\right)}{a^2 E^M};$$

$$C_2^M \frac{1}{a^2} = -\frac{\left(\alpha_T^{\square I} - \alpha^{\square M}\right) - \left(\nu_{LT}^I - \nu^M\right)\Upsilon}{\dfrac{1+\nu^M}{E^M} + c\left(\dfrac{1}{E^{\square M}} - \dfrac{\nu^M}{E^M}\right) + (1-c)\left(\dfrac{1}{E_T^{\square I}} - \dfrac{\nu_{TT}^I}{E_T^I}\right) + \left(\nu_{LT}^I - \nu^M\right)\left[-c\Xi + \Psi\right]}\Delta T =$$

$$= -\frac{\left(\alpha_T^{\square I} - \alpha^{\square M}\right) - \left(\nu_{LT}^I - \nu^M\right)\dfrac{\left[\alpha_L^I E_L^I c + \alpha^M E^M (1-c)\right]}{E_L^I c + E^M (1-c)}}{\dfrac{1+\nu^M}{E^M} + c\left(\dfrac{1}{E^{\square M}} - \dfrac{\nu^M}{E^M}\right) + (1-c)\left(\dfrac{1}{E_T^{\square I}} - \dfrac{\nu_{TT}^I}{E_T^I}\right) + \dfrac{\left(\nu_{LT}^I - \nu^M\right)^2 (1-c)}{E_L^I c + E^M (1-c)}}\Delta T$$

$$C_1^M = -C_2^M \frac{1}{a^2} c \qquad (31)$$

Maximal in absolute values stresses in matrix are on the boundary with fiber

$$
\sigma_r(a) = C_1^M + C_2^M \frac{1}{a^2} = C_2^M \frac{1}{a^2}(1-c) =
$$

$$
(1-c)\frac{\left(\alpha^{M} - \alpha_T^{I}\right) - \left(\nu^M - \nu_{LT}^I\right)\dfrac{\left[\alpha_L^I E_L^I c + \alpha^M E^M (1-c)\right]}{E_L^I c + E^M (1-c)}}{\dfrac{1+\nu^M}{E^M} + c\left(\dfrac{1}{E^{M}} - \dfrac{\nu^M}{E^M}\right) + (1-c)\left(\dfrac{1}{E_T^{I}} - \dfrac{\nu_{TT}^I}{E_T^I}\right) + \dfrac{\left(\nu_{LT}^I - \nu^M\right)^2 (1-c)}{E_L^I c + E^M (1-c)}}\Delta T
$$

$$
\sigma_\theta(a) = C_1^M - C_2^M \frac{1}{a^2} = -C_2^M \frac{1}{a^2}(1+c) =
$$

$$
-(1+c)\frac{\left(\alpha^{M} - \alpha_T^{I}\right) - \left(\nu^M - \nu_{LT}^I\right)\dfrac{\left[\alpha_L^I E_L^I c + \alpha^M E^M (1-c)\right]}{E_L^I c + E^M (1-c)}}{\dfrac{1+\nu^M}{E^M} + c\left(\dfrac{1}{E^{M}} - \dfrac{\nu^M}{E^M}\right) + (1-c)\left(\dfrac{1}{E_T^{I}} - \dfrac{\nu_{TT}^I}{E_T^I}\right) + \dfrac{\left(\nu_{LT}^I - \nu^M\right)^2 (1-c)}{E_L^I c + E^M (1-c)}}\Delta T
\tag{32}
$$

By further simpilfication

$$
\sigma_z^M = \left(\Upsilon - \alpha^M\right)E^M \Delta T + C_1^M\left(\nu^M - \Xi E^M\right) - C_2^M \Psi \frac{1}{a^2} E^M =
$$

$$
\left(\Upsilon - \alpha^M\right)E^M \Delta T - C_2^M \frac{1}{a^2}\left[c\left(\nu^M - \Xi E^M\right) + \Psi E^M\right] =
$$

$$
\left[\frac{\alpha_L^I E_L^I c + \alpha^M E^M (1-c)}{E_L^I c + E^M (1-c)} - \alpha^M\right] E^M \Delta T
$$

$$
-C_2^M \frac{c}{a^2}\left[\left(\frac{\nu^M E_L^I c}{E_L^I c + E^M (1-c)} + (1-c)\frac{\nu_{LT}^I}{E_L^I c + E^M (1-c)} E^M\right)\right] =
$$

$$
\left[\frac{\alpha_L^I E_L^I c + \alpha^M E^M (1-c)}{E_L^I c + E^M (1-c)} - \alpha^M\right] E^M \Delta T +
$$

$$
c\frac{\left(\alpha_T^{I} - \alpha^{M}\right) - \left(\nu_{LT}^I - \nu^M\right)\dfrac{\left[\alpha_L^I E_L^I c + \alpha^M E^M (1-c)\right]}{E_L^I c + E^M (1-c)}}{\dfrac{1+\nu^M}{E^M} + c\left(\dfrac{1}{E^{M}} - \dfrac{\nu^M}{E^M}\right) + (1-c)\left(\dfrac{1}{E_T^{I}} - \dfrac{\nu_{TT}^I}{E_T^I}\right) + \dfrac{\left(\nu_{LT}^I - \nu^M\right)^2 (1-c)}{E_L^I c + E^M (1-c)}}\Delta T\left[\frac{c\nu^M E_L^I + (1-c)\nu_{LT}^I E^M}{E_L^I c + E^M (1-c)}\right]
$$

Effective axial linear thermal expansion coefficient

$$\alpha_L^C = \frac{\Lambda}{\Delta T} = \Upsilon - \frac{C_1^M \Xi + C_2^M \Psi \dfrac{1}{a^2}}{\Delta T} = \Upsilon - \frac{C_2^M}{a^2}\frac{\Psi - c\Xi}{\Delta T} = \frac{\left[\alpha_L^I E_L^I c + \alpha^M E^M (1-c)\right]}{E_L^I c + E^M (1-c)} +$$

$$\frac{\left(\alpha_T^{\Box I} - \alpha^{\Box M}\right) - \left(\nu_{LT}^I - \nu^M\right)\dfrac{\left[\alpha_L^I E_L^I c + \alpha^M E^M (1-c)\right]}{E_L^I c + E^M (1-c)}}{\dfrac{1+\nu^M}{E^M} + c\left(\dfrac{1}{E^{\Box M}} - \dfrac{\nu^M}{E^M}\right) + (1-c)\left(\dfrac{1}{E_T^{\Box I}} - \dfrac{\nu_{TT}^I}{E_T^I}\right) + \dfrac{\left(\nu_{LT}^I - \nu^M\right)^2 (1-c)}{E_L^I c + E^M (1-c)}}\left[\frac{c(1-c)\left(\nu_{LT}^I - \nu^M\right)}{E_L^I c + E^M (1-c)}\right] \tag{33}$$

Effective transversal linear thermal expansion coefficient is calculated as:

$$\alpha_T^C = \frac{\varepsilon_\theta^M (b)}{\Delta T} = \alpha^{\Box M} - \nu^M \Upsilon + \frac{C_1^M}{\Delta T}\left(\frac{1}{E^{\Box M}} - \frac{\nu^M}{E^M} + \nu^M \Xi\right) - \frac{C_2^M \dfrac{1}{a^2}\left[c\left(1+\nu^M\right) - \nu^M E^M \Psi\right]}{\Delta T E^M} =$$

$$\alpha^{\Box M} - \nu^M \frac{\left[\alpha_L^I E_L^I c + \alpha^M E^M (1-c)\right]}{E_L^I c + E^M (1-c)} - \frac{C_2^M}{\Delta T}\frac{1}{a^2}\left\{c\left(\frac{1}{E^{\Box M}} + \nu^M \Xi\right) + \frac{c - \nu^M E^M \Psi}{E^M}\right\} =$$

$$\alpha^{\Box M} - \nu^M \frac{\left[\alpha_L^I E_L^I c + \alpha^M E^M (1-c)\right]}{E_L^I c + E^M (1-c)} - \frac{C_2^M}{\Delta T}\frac{c}{a^2}\left\{\left(\frac{1}{E^{\Box M}} + \frac{\nu^M (1-c)\left(\nu^M - \nu_{LT}^I\right)}{E_L^I c + E^M (1-c)}\right) + \frac{1}{E^M}\right\} =$$

$$\alpha^{\Box M} - \nu^M \frac{\left[\alpha_L^I E_L^I c + \alpha^M E^M (1-c)\right]}{E_L^I c + E^M (1-c)} +$$

$$c\left[\frac{1}{E^{\Box M}} + \frac{\nu^M (1-c)\left(\nu^M - \nu_{LT}^I\right)}{E_L^I c + E^M (1-c)} + \frac{1}{E^M}\right]\frac{\left(\alpha_T^{\Box I} - \alpha^{\Box M}\right) - \left(\nu_{LT}^I - \nu^M\right)\dfrac{\left[\alpha_L^I E_L^I c + \alpha^M E^M (1-c)\right]}{E_L^I c + E^M (1-c)}}{\dfrac{1+\nu^M}{E^M} + c\left(\dfrac{1}{E^{\Box M}} - \dfrac{\nu^M}{E^M}\right) + (1-c)\left(\dfrac{1}{E_T^{\Box I}} - \dfrac{\nu_{TT}^I}{E_T^I}\right) + \dfrac{\left(\nu_{LT}^I - \nu^M\right)^2 (1-c)}{E_L^I c + E^M (1-c)}}$$

Three-phase coaxial cylinder model:

Combination of (24) and (16c) gives:

$$\left[\left(\alpha_T^{\square I}-\alpha^{\square M}\right)-\left(\nu_{LT}^{I}-\nu^{M}\right)\Upsilon\right]\Delta T+\left(C_1^M+C_2^M\frac{1}{a^2}\right)\left(\frac{1}{E_T^{\square I}}-\frac{\nu_{TT}^{I}}{E_T^{I}}\right)+$$

$$\left(\nu_{LT}^{I}-\nu^{M}\right)\left[C_1^M\Xi+C_2^M\frac{1}{a^2}\Psi\right]=C_1^M\left(\frac{1}{E^{\square M}}-\frac{\nu^M}{E^M}\right)-\frac{C_2^M\left(1+\nu^M\right)}{a^2E^M};$$

$$C_1^M+C_2^M\frac{1}{b^2}=C_2^C\frac{1}{b^2};$$

$$\alpha^M\Delta T+\frac{C_1^M\left(1-\nu^M\right)}{E^M}-\frac{C_2^M\left(1+\nu^M\right)}{E^M}\frac{1}{b^2}-\sigma_z^M\frac{\nu^M}{E^M}=\alpha_T^C\Delta T-\frac{C_2^C\left(1+\nu_{TT}^C\right)}{E_T^C}\frac{1}{b^2};$$

Effective transversal Young modulus can be found from three-phase model as it described by Christensen but effective linear thermal expansion of composite is exactly the same as in two-phase model. Which means that for our particular application the analysis of this model can be omitted.

3.3.2 FEM model

The whole ANSYS simulation procedure can be broadly divide in to 3 steps preprocessing, solution and post processing. Preprocessing involves model geometry with elements and meshing procedure along with material properties. Solution step involves all the lo ads and boundary conditions used during the simulation and final step is post-processing which covers all the results obtained during simulation.

Geometry

. The geometrical model of cylindrical unit cell is created using Solidworks V16 and then is transferred to Ansys workbench 17.0 for further simulation There are symmetrical aspects to this unit cell model. It is understandable to use quarter model instead of complete single cell. But, in this study a full model is used to get more detailed

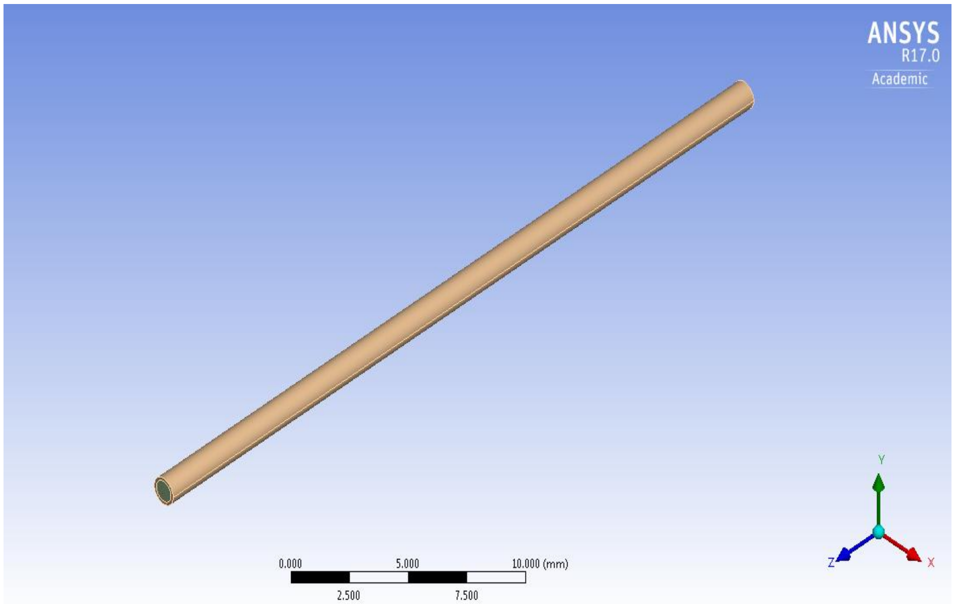


Figure 3.3 Cylindrical Unit cell model

results. The unit cell of cylindrical model is shown in Fig 3.2 inner circle representing fiber and outer circle representing matrix phase. The fiber volume fraction **(C),** which is percentage of volume of fiber in composite**,** in terms of the fiber radius (ri) and outer radius (rC), is given as

$$C = (ri/rC)^2$$

This ratio is very important part of the composite as it has direct impact to all the properties including strength properties of the composite. Although theoretically this faction can be from 0 to 100 percentage but it's usually between 50 to 70 percent for most of the composites that are used in applications. So, Ansys models created here are with volume fraction of 50, 60 and 70 percentages. All these models have different inner diameter of 0.7979, 0.874 and 0.941 mm respectively.

FEM Meshing

In this simulation element used in Ansys during meshing is Solid186, which is used for modeling of 3-D structures. Solid186 consists of 20 nodes having three degree of freedom per node in the x, y, and z directions [10]. This element supports plasticity, hyper-elasticity, large deflection, creep and stress stiffening. The simulation considered in this thesis as linear elastic in behavior.

For the purpose of this study, surface areas of all models between fiber and matrix are considered to be perfectly bonded. The bonding method involves merging of coincident key points of neighboring volumes. The ANSYS uses merge command to merge two surfaces which in contact with each other.

Boundary Conditions

The boundary condition used for these simulations is called is 6 DOF system. In this simulation the 4 points that are representing all boundary conditions as shown in fig 3.4. Point A is fixed point in all coordinates so it can't move in any of the direction. On the other hand, point B, is only fixed in one direction which is Z axis and point C is has restriction on X and Y axis movements. This system is used to prevent any errors that can occur due to excessive constrains which might hinder the accuracy of the results.

The last boundary condition is -200°C temperature of whole body with surrounding temperature of 0°C. This study covers the temperature effects on thermal stresses of composites the difference of 200°C between unit cell and surrounding is ideal for this type of study because, it is very close to extreme conditions that composites have to go through. The results of cylindrical model with both analytical and simulation results are discussed in detail later in chapter 4.

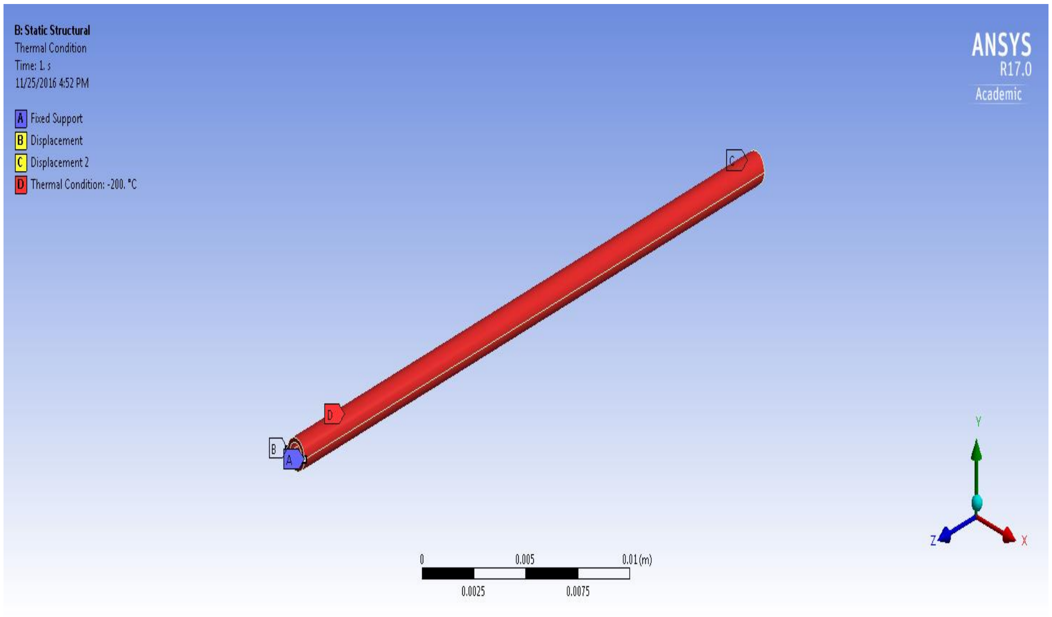


Figure 3.4 Boundary conditions used for Simulation

3.4 Square and Hexagonal model

3.4.1 Unit cell model

The square and hexagonal unit cell models are similar to coaxial cylindrical model. The geometry of square unit cell model has been created in solid works V16 and transferred to Ansys workbench 17.0 for simulation. The square model is shown in figure 3.5. the inner circle is fiber and outer square is matrix phase. The volume of matrix and fiber is also similarly calculated to have typical volume fraction for these models. It is also having 3 models with 50, 60 and 70 percentage of fiber volume fraction. With dimensions of square with 1mm length and width. And fiber with diameter similar to cylindrical model. They are 0.7979, 0.874 and 0.941 mm respectively.

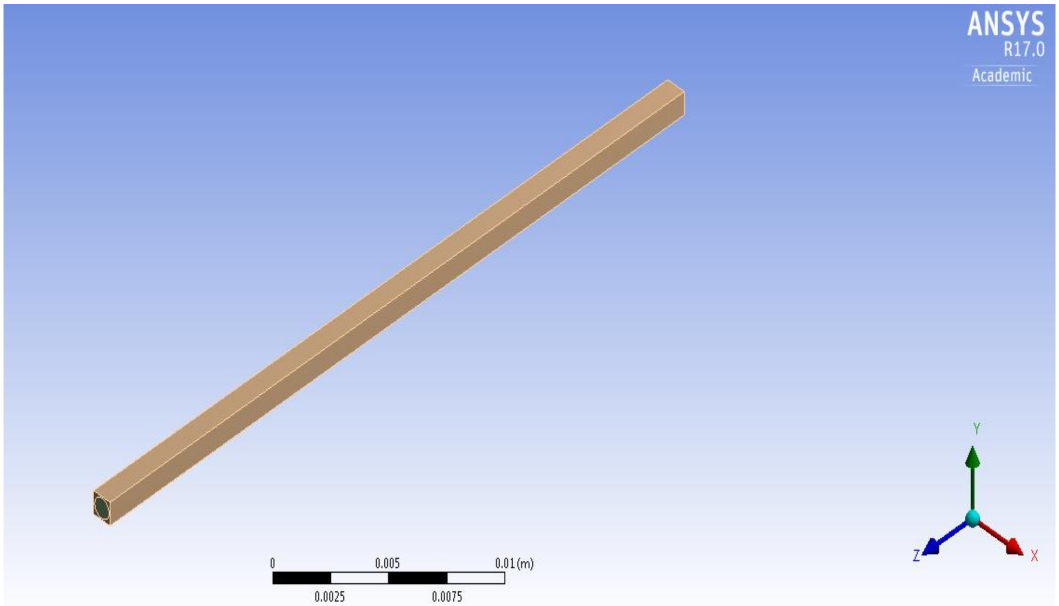


Figure 3.5 Square unit cell model

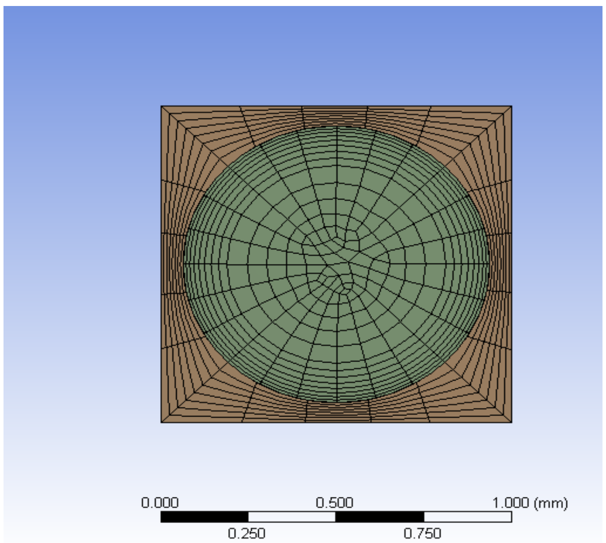


Figure 3.6 Cross sectional view of square unit cell with mesh

As for hexagonal model with same procedure model are created. The dimensions of this model is 0.6203 mm for its 6 sides because it is homogenous hexagon. But diameter of fiber is same for this model to have fixed volume fraction of 50,60 and 70 percent.

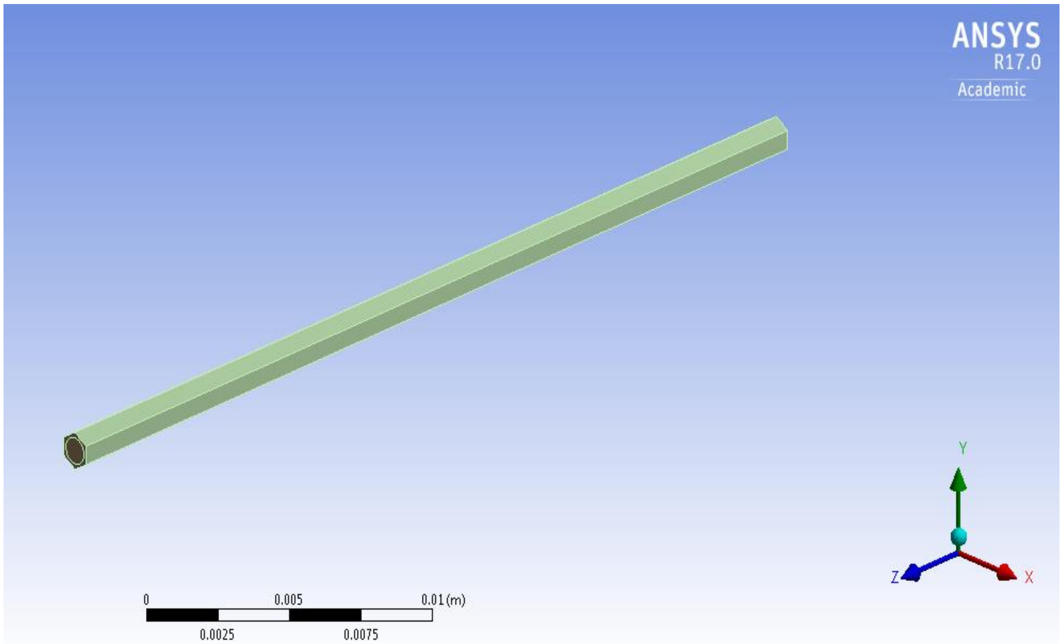


Figure 3.7 Hexagonal unit cell model

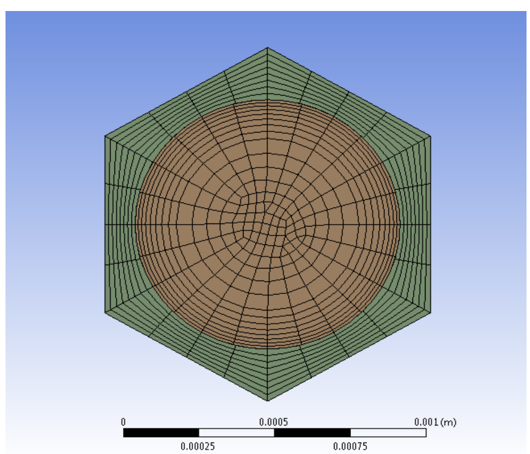


Figure 3.8 Cross sectional view of hexagonal unit cell with mesh

After detail description of geometry information about meshing and boundary conditions that are used in this models are very important. The square and hexagonal models both have same meshing element as cylindrical model which is solid 185 with eight nodes. And contact type between fiber and matrix is 'bonded' for both cases. And boundary condition is also similar to cylindrical unit with constrains of 6 DOF system with one fixed point, second point with Z axis restriction and third point with X and Y axis movement restriction. Also temperature load is acting o the entire body. The composite temperature is at -200°C with surrounding temperature at 0°C.

3.4.2 Multi cell model

The studies that are conducted in the past were focused on single unit cell model no matter whether it's square or hexagonal model. Composite material has more complicated and rigorous models. They usually consist regular lattice of infinite number of

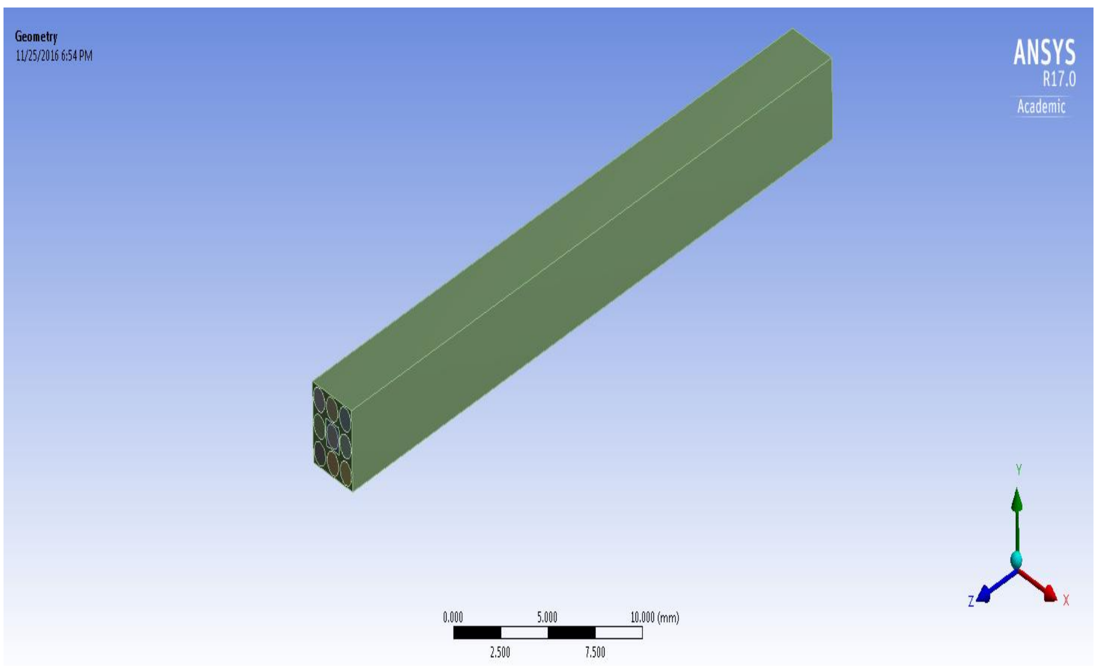


Figure 3.9 square multi cell model

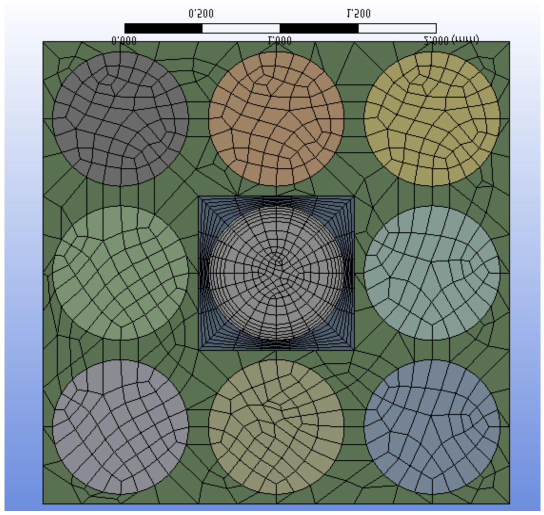


Figure 3.10 Cross sectional view of square multi cell with mesh

fiber. This model which is more realistic model to study composite is impossible to create because it has infinite number of fibers. To study the model which will be an improvement to single cell model. The multi cell model is created on solid works v16. This multi cell model is created to understand the role of all neighboring cell on that cell.

Square multi cell model consists of 9 fibers are arranged in 3x3 formations as a reinforcement. The matrix phase consists of 3 mm width and 3 mm of height with symmetric inclusions in 3x3 formations. Fiber have same diameter as single cell models to keep volume fraction of 50, 60 and 70 which is same as single cell models.

Hexagonal multi cell model as oppose to square multi cell model has very different geometry in this model middle hexagonal cell which has same dimensions as unit cell is surrounded by 6-unit cell models sharing one side in common between them. The outer hexagonal unit cell are just fraction of a whole unit cell which can fit into 3 mm width and 3 mm height That is why it consist of partial fibers. This model is created to consider impact immediate surrounding cell on a composite material. As per all the

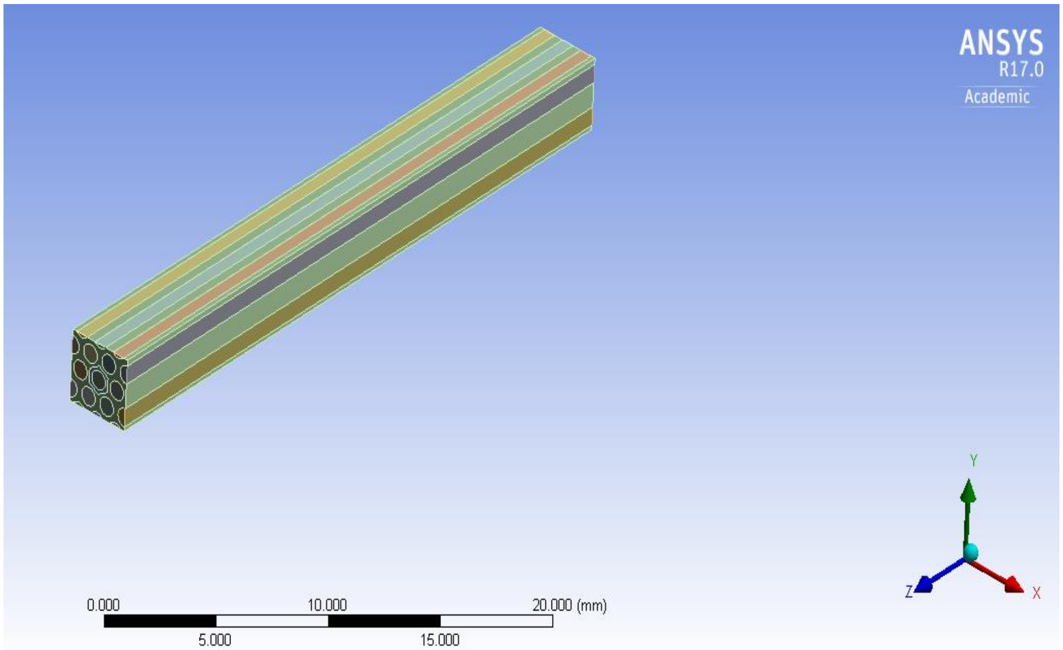


Figure 3.11 **Hexagonal multi cell model**

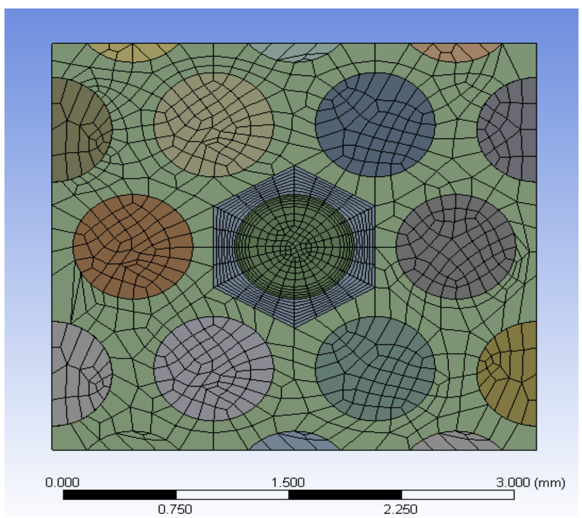


Figure 3.12 Cross sectional view of Hexagonal Multi cell with mesh

boundary conditions, meshing and element types are kept same as with single cell models. To figure the difference between results of single and multi cell models

Chapter 4

Results

The strength characteristics of the composite is depending on the stresses acting on the matrix as matrix is weaker than fiber. The simulation with thermal loading of all models is presented here with all the results for cylindrical, square and hexagonal unit cell models. The table 4.1, 4.2 and 4.3 represents values of radial stresses and hoop stresses acting on the interface as well as outer corners of the unit cell. If we compare the cylindrical model with square and hexagonal model, we can identify few differences between them.

C		50%	60%	70%
Radial stress at boundary of fiber and matrix	Analytical	-20.475	-16.099	-11.871
	Fem	-21.289	-16.936	-12.065
Hoop stress at boundary of fiber and matrix	Analytical	61.424	64.396	67.269
	Fem	61.318	65.390	66.678
Hoop stress at corner of matrix	Analytical	40.949	48.297	55.398
	Fem	38.248	46.093	54.032

All values are in MPa

Table 4.1 Result table of Cylindrical model

While comparing cylindrical model which is almost a theoretical model. Because in realistic case composites are not made up of single fiber cell but it is surrounded by other infinite cells. In case of cylindrical unit cell even if there are few unit cell models surrounding that cell. There are always going to be few gaps between them. This case of cylindrical model is not a practical representation of the composite material.

C		50%		60%		70%	
		Max	Min	Max	Min	Max	Min
Radial stress at boundary of fiber and matrix	FEM	-8.82	-26.89	-4.82	-24.31	0.941	-21.52
Hoop stress at boundary of fiber and matrix	FEM	70.306	56.264	73.443	58.530	77.506	59.4632
Hoop stress at corner of matrix	FEM	76.65	-4.284	79.936	-5.45	90.687	-5.5147

All values are in MPa

Table 4.2 Result table of Square model

C		50%		60%		70%	
		Max	Min	Max	Min	Max	Min
Radial stress at boundary of fiber and matrix	FEM	-14.63	-25.98	-8.712	-23.62	-4.24	-20.731
Hoop stress at boundary of fiber and matrix	FEM	65.025	57.806	69.751	59.342	74.518	61.115
Hoop stress at corner of matrix	FEM	61.263	-1.489	72.091	-1.961	80.249	-2.054

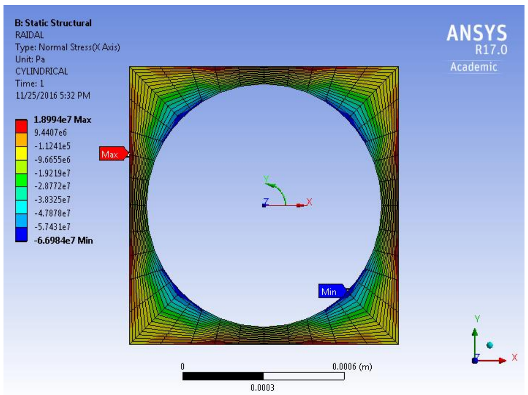
All values are in MPa

Table 4.3 Result table of Hexagonal model

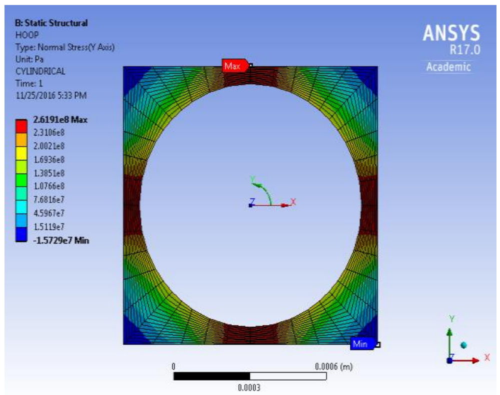
In case of square and hexagonal model there aren't any gaps between two adjacent unit cells. This is the reason that for micromechanics study of the composite square and Hexagonal models are used extensively for the research. Here for comparison point of view results from cylindrical model are also given in the table

While comparing cylindrical model with square and hexagonal model, consider a case of 60 percentage fiber volume fraction, radial stress for cylindrical model is -16 MPa for analytical case and -16.98MPa for Fem model. In case of square it varies between -4. 8 MPa to -24.31 MPa also for hexagon it's -8.71 MPa to -23.63 MPa. These values clearly show that in case of square and hexagonal model stress value varies along the inclusion between fiber and matrix and it is no a constant value like in cylindrical model.

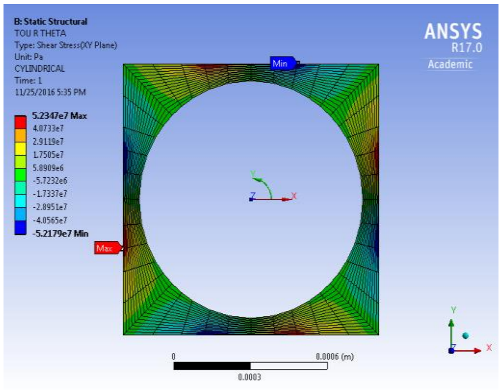
As Matrix is weaker constituent in composites, the stresses acting on matrix phase is very important in these cases. The figures 4.1 shown below is consists of radial, hoop and shear stresses acting on square unit cell of the E-glass Epoxy composites.

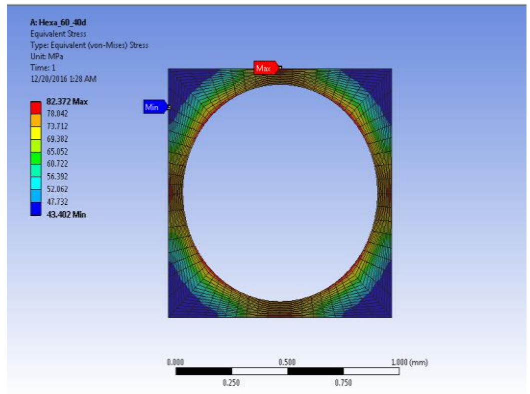


Radial stresses acting on square unit cell



Hoop stress acting on square unit cell

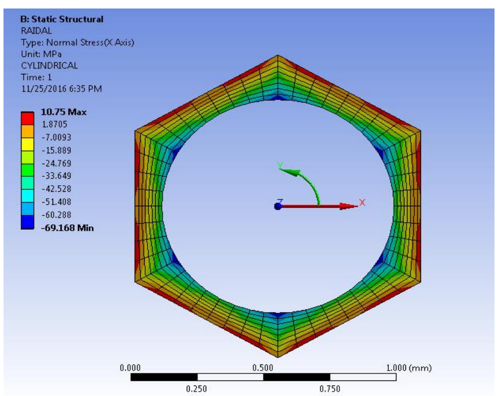


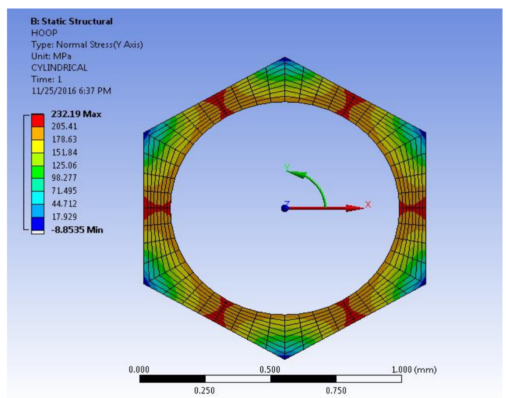


Shear stress acting on square unit cell   Von-Mises stress acting on square unit cell

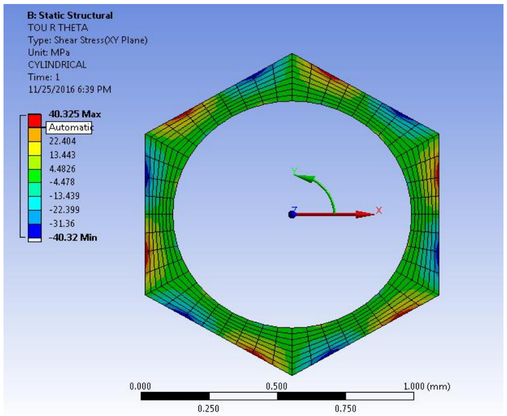
Figure 4.1 **Stresses acting on Square Unit Cell E-glass Epoxy composite**

The figures 4.2 shown below is consists of radial, hoop and shear stresses acting on Hexagonal unit cell of the composites.

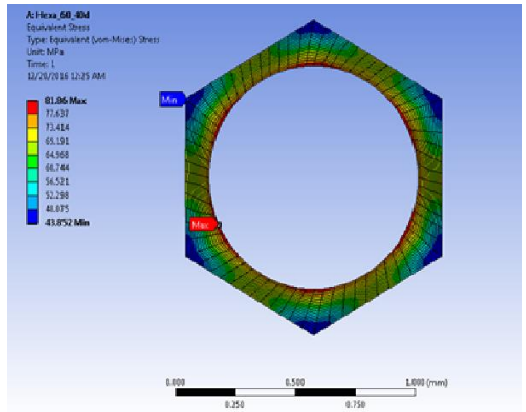




Radial stress acting on Hexagonal unit cell   Hoop stress acting on Hexagonal unit cell



Shear stress acting on Hexagonal unit cell



Von-Mises stress acting on Hexagonal unit cell

Figure 4.2. **Stresses acting on Hexagonal Unit Cell E-glass Epoxy composite**

The figure 4.1 and 4.2 are representation of stress distribution of the stresses acting on the matrix in case of single cell models due to thermal loading as compared to Figure 4.3 and 4.4 are representation of stress distribution of the stresses acting on the matrix in case of multi cell models.

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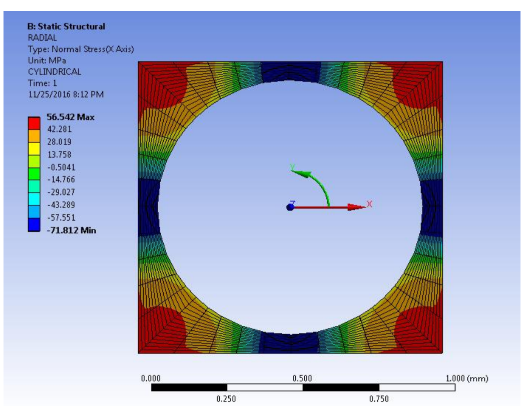
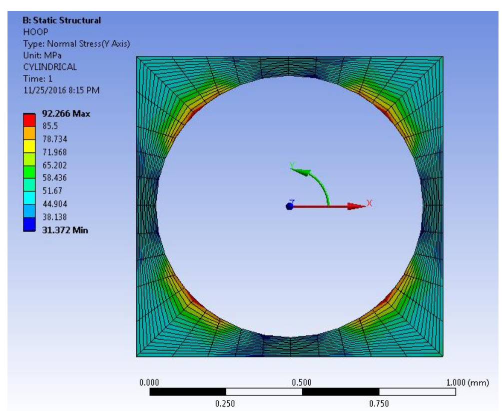
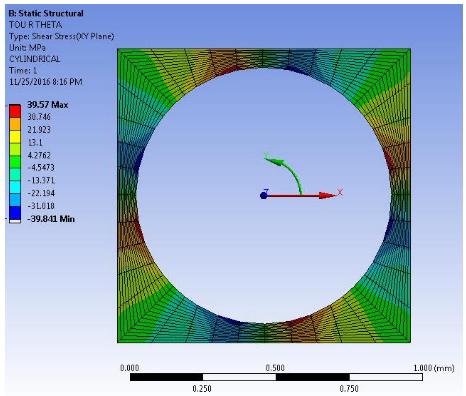


Radial stress acting on square multi cell



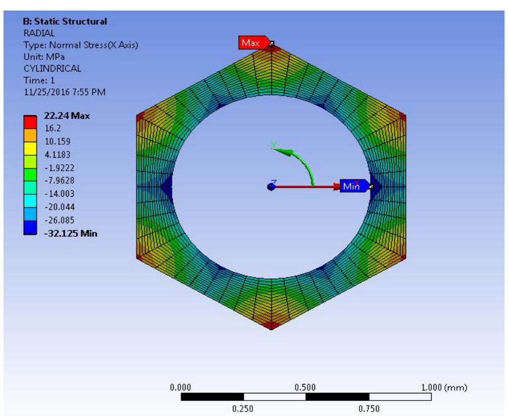
Hoop stress acting on square multi cell



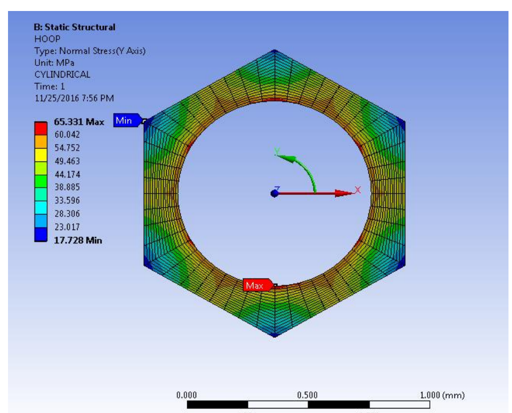
Shear stress acting on square Multi cell

Figure 4.3 **Stresses acting on square multi cell E-glass Epoxy composite**

The figures 4.4 shown below is consists of different stresses acting on Hexagonal Multi cell of the E-glass Epoxy composites.

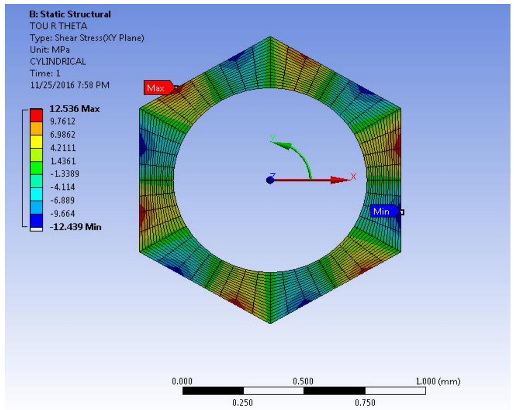


Radial stress acting on Hexagonal Multi cell



Hoop stress acting on Hexagonal Multi cell

Looking at single cell and multi cell models it can be seen that distribution of stresses even though uniform but stresses like radial stress or Von-Mises stress is



Shear stress acting on Hexagonal Multi cell

Figure 4.4 **Stresses acting on Hexagonal multi cell E-glass Epoxy composite**

different in both the cases. The reason behind that is effect of neighboring fiber on that unit cell. That is why stresses acting on multi cell model is different than that of single cell model.

The stresses like Von-Mises stress is very important for engineering point of view. It is helpful in determining whether material will withstand the loading condition or will it fail. Von-Mises stress is a combination of stresses (like radial stress, axial stress and shear stress) acting on the material. So, from the results of both single cell model and multi cell models we can see that the single cell approach, if used by designer to determine its strength, might not be as accurate because, stress acting on multi cell model is different than that of single cell. Which might be helpful in increasing accuracy of the result.

It is important to find values of stress acting on the outer faces of the matrix. Figure below represents the variation of stress along outer edges single cell and multi cell model.

Single cell vs multi cell model

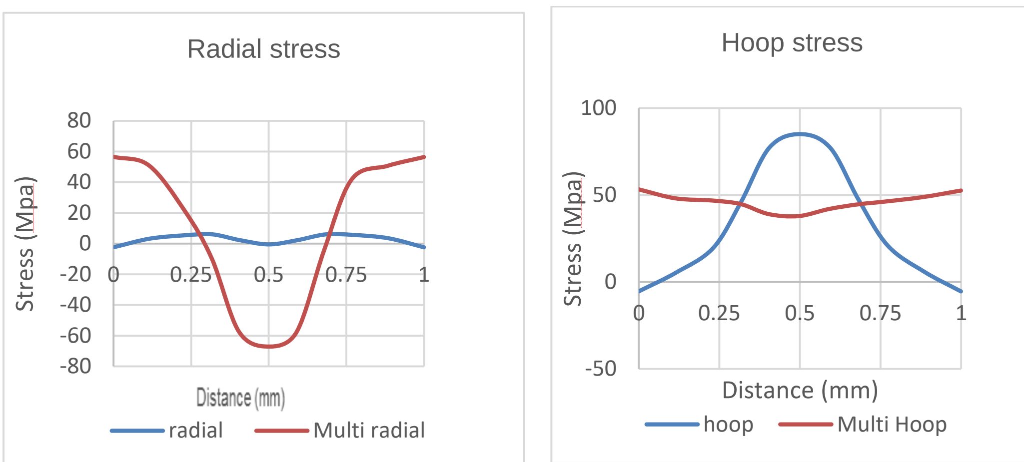


Figure 4.5 Stress comparison of square single cell vs multi cell model

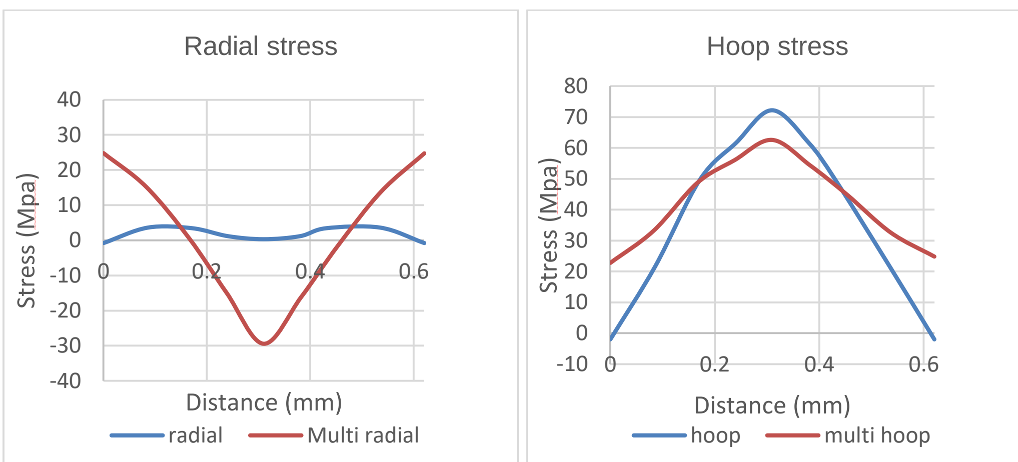
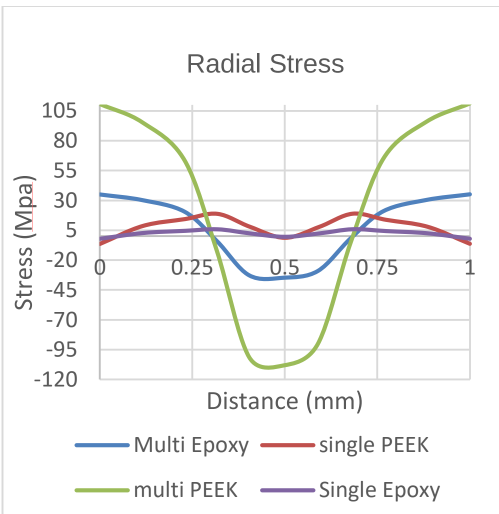


Figure 4.6 Stress comparison of Hexagonal single cell vs multi cell model

In the comparison between single cell model and multi cell model, Because of the boundary condition used during simulation composite is expanding i.e. matrix is also expanding along with fiber. The radial stress acting on outer surface of the composite should be zero in case of single cell model. But a comparison with multi cell system for both square and hexagonal model, reveals that it is not the case. The distribution of hoop stress is also changing as it depends on the value of radial stress. So, there is a variation of stress acting along the outer edge of the matrix. Which will definitely hinder the accuracy of the results.

Material selection



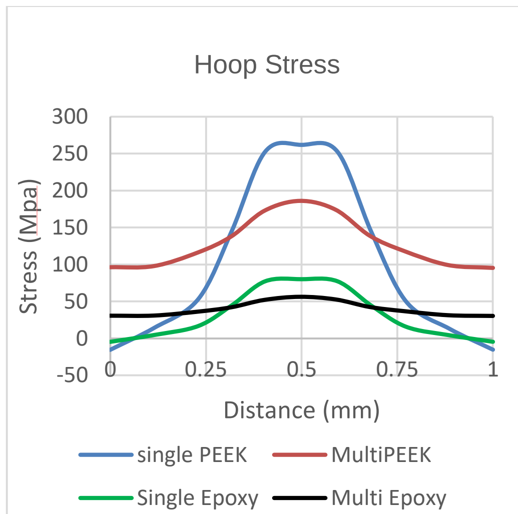


Figure 4.7 Stress comparison between PEEK vs Epoxy for square model

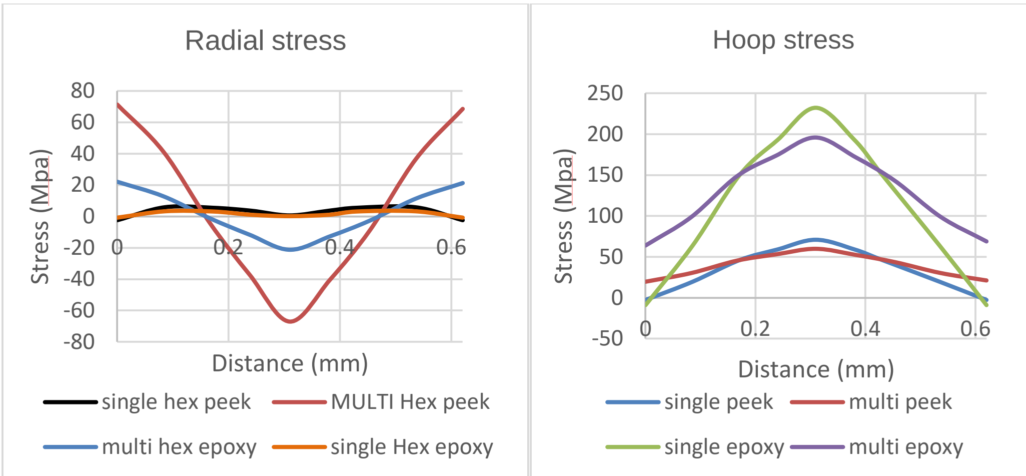


Figure 4.8 Stress comparison between PEEK vs Epoxy for hexagonal model

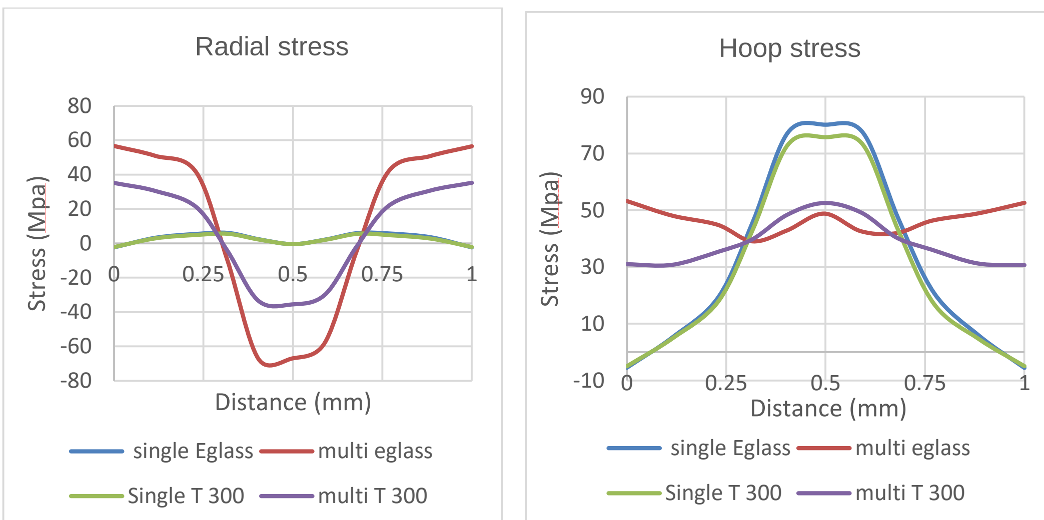


Figure 4.9 Stress comparison between E-glass vs CF T-300 for square model

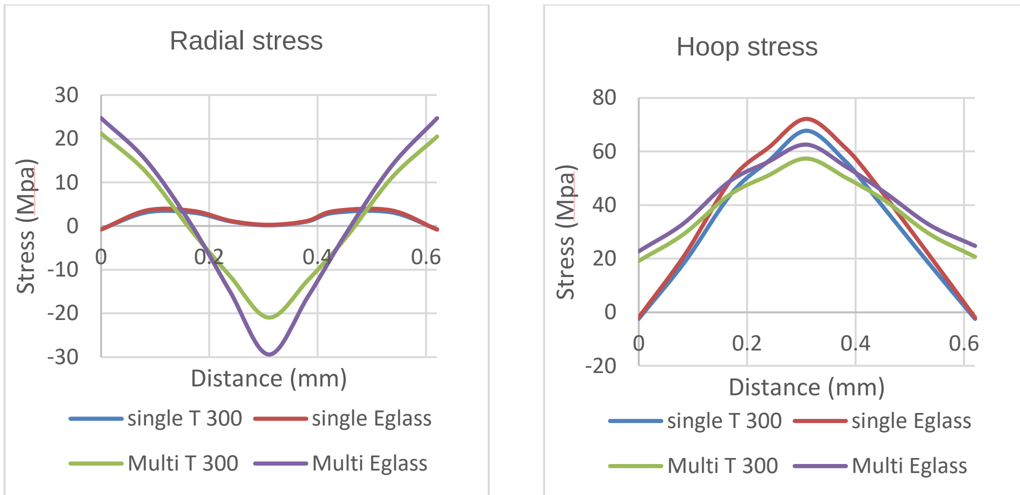


Figure 4.10 Stress comparison between E-glass vs CF T-300 for hexagonal model

Materials used as a matrix are usually fall into two categories thermoplastic and thermosets. Epoxy resin is a thermoset matrix with decent material properties. Compared to epoxy PEEK has better tensile strength and can work in high temperatures. But, in transversal direction, as shown in figure above for both square and hexagonal models. In case of multi cell model values of both epoxy and PEEK are different due to effects of neighboring cells. stresses acting on PEEK are higher than epoxy which can not be seen by single cell model.

In case of fibers, comparison between carbon fiber and glass fiber has been done. In theses cases we observed that stresses acting on carbon fiber are less than glass fibers. This is because CTE mismatch between carbon fiber and matrix is lower than between glass fiber and matrix.

Volume Concentration

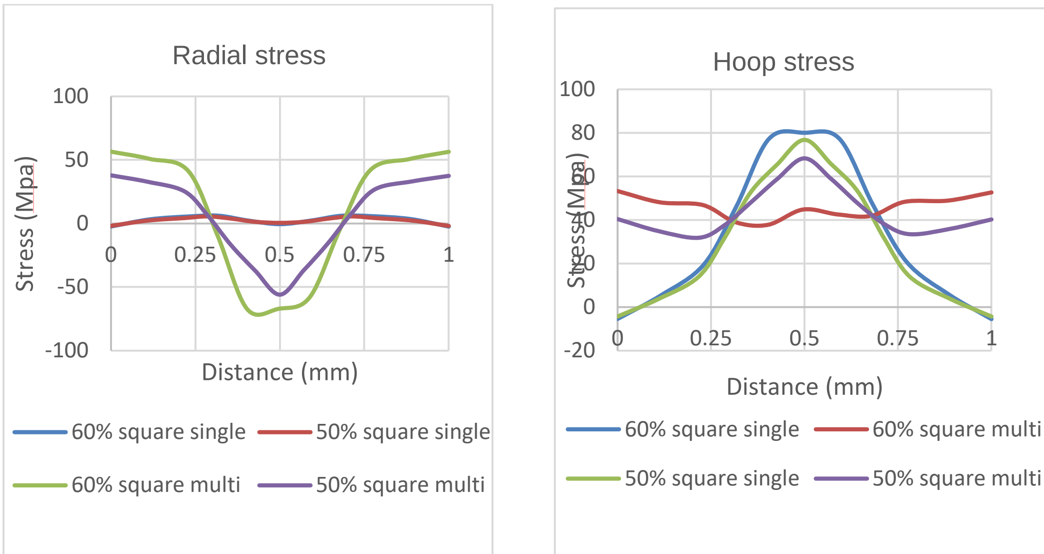


Figure 4.11 Stress comparison between 50% vs 60% Volume fraction of square model

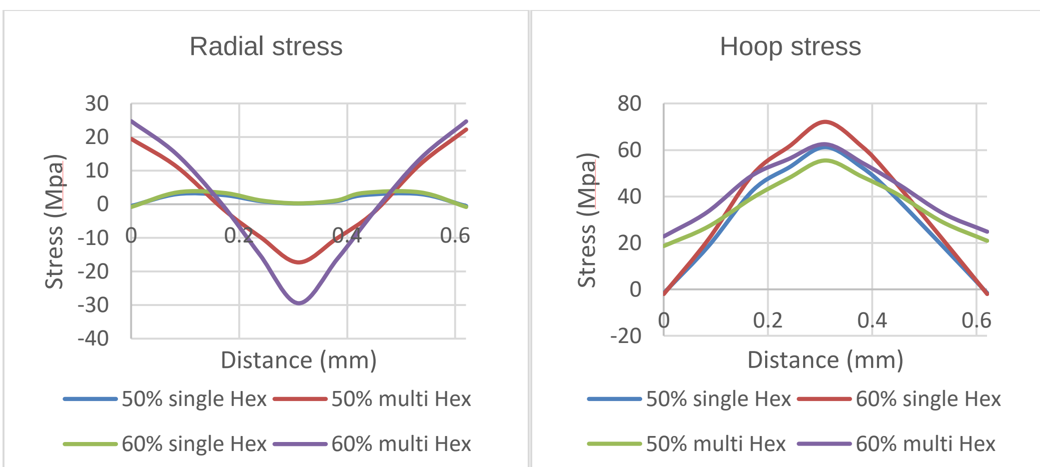


Figure 4.12 Stress comparison between 50% vs 60% Volume fraction of Hexagonal model

The comparison of stresses acting on a same composite with different fiber volume concentration (50% and 60%) is shown above. The figure above for both Hexagonal and Square models are given. It can be said that stresses for both the model are increasing with stress concentration. It is obvious because with increase in fiber concentration and expansion due to boundary condition applied for the simulation. Matrix will try to expand along with fiber but, Coefficient of linear expansion of matrix is higher that fiber. So, fiber will resist the expansion of matrix which will result in higher stresses on matrix.

Chapter 5

Conclusion

5.1 Summary and Conclusion

In this paper effects of thermally induced stresses acting in transversal plane on the unidirectional fiber reinforced composites are studied by means of finite element modeling. Particularly When it comes to transversal direction of composites, they are weak especially matrix is weaker than fibers. This study focuses on the stress generated due to thermal loading and its effects on the composites. For this study Representative volume elements were used which consist of cylindrical, square and hexagonal unit cells. Results form the cylindrical models are then compared with its analytical solution. A further objective was to see effects of neighboring fibers on the stresses acting on the cell. As the next step, to study effects of neighboring cell Square and Hexagonal multi cell models are created. The results from these multi cell models are then compared with unit cell models.

The conclusions from this study are as follows-

The comparison of cylindrical model with its analytical solution yields that the Finite element approach is well with in the margin of error and can be used to find out the stresses acting on the system.

In the comparison between single and multi cell model, in case of both square and hexagonal cells, it was observed that because of the effects of neighboring fibers stress distribution was changed for both models. This affects the estimation of strength characteristics of composite. So, use of multi cell model will be helpful to yield more accurate results compared to unit cell models.

In comparison between materials used in composite, in case of matrix, comparison between PEEK and epoxy resin has been done. Even though PEEK is a very good choice because it has high tensile strength but, from the results it is clear that while working under high temperature thermal stresses acting on PEEK in transversal direction are higher than Epoxies. In case of fiber, comparison between E-glass and Carbon fiber (T300), stresses acting on carbon fiber in transversal direction are lesser than that of E-glass. It is because CTE mismatch between matrix and fiber is less in case of carbon fiber than glass fiber.

In case of Volume concentration of the fiber, as boundary condition here dictates that, composite is expanding, but because of CTE mismatch matrix will expand faster than fiber. If the volume concentration of fiber is higher, for the same loading condition, the stress acting on the system will increase because of more resistance from fiber to expand and more stress will generate from fiber on matrix.

5.2 Future work

This study is based on linear elastic behavior of composites, but study on nonlinear behaviors of composite is very important issue. The stress strain diagram of polymeric matrix is usually nonlinear. Here study was considered as elastic. But Viscoelastic effect is significant at very high temperature and moisture condition and it can affect matrix as it is weaker constituent in composites. So for high temperature study viscoelasticity of composites should be considered.

In this paper focus was on thermal loading and its effect on composites but, in most of the cases, there is a combination of mechanical loading and thermal loading. This combined loading effects can further current study that has been done in this thesis.

In this study we assumed a uniform temperature distribution throughout composite material., in some extreme cases, we can see some temperature gradient present in composites. In those cases, temperature distribution is non uniform even on a scale of single unit cell. So a non uniform temperature distribution can improve this study.

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BIOGRAPHICAL STATEMENT

Gopal Gulwani completed his Bachelor in engineering degree in mechanical engineering from Shivaji University, India in 2013. While there, he conducted research in ‘Life Cycle Cost estimation of an Industrial Product through Reliability Analysis’. He also presented a technical paper on 'Pulse Tube Refrigerators' while he was there. He attended the University of Texas at Arlington for his Master of Science degree in mechanical engineering from January 2015 to December 2016. During his studies at UTA, he conducted research under the guidance of Dr. Andrey Beyle on ‘effects of thermal stresses on composite materials’. He graduated in December 2016. The author is strongly interested in Finite element analysis and Structural Mechanics.