

AN INVESTIGATION OF BAND-LIMITED FLUID DAMPER CONFIGURATION
USING LUMPED PARAMETER MODELING

by

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Presented to the Faculty of the Graduate School of
The University of Texas at Arlington in Partial Fulfillment
of the Requirements
for the Degree of

MASTER OF SCIENCE IN MECHANICAL ENGINEERING

THE UNIVERSITY OF TEXAS AT ARLINGTON

December 2015

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Acknowledgements

I would like to express my sincere gratitude to my thesis adviser Dr. Robert Woods for the continuous support of my studies and related researches, for his patience, motivation, and knowledge. His guidance helped me in the research and writing of this thesis.

My sincere gratitude also goes to Dr. D. A. Hullender, who has been a great source of knowledge and help in this thesis.

Besides, I would like to thank my thesis committee: Dr. Robert Woods, D. A. Hullender, and Dr. B. P. Wang, for their valuable comments and suggestions, but also for the hard question which incited me to widen my research from different perspectives.

I am thankful to the University of Texas Arlington for providing me this great opportunity and facilities for my graduate studies.

I thank my friends, here at UTA and elsewhere in the world, for their support and accompany.

Finally, I would like to thank my family, my parents and other relatives for supporting me materially and spiritually. If is not for them, I would not have achieve this today.

November 24, 2015

Abstract

AN INVESTIGATION OF BAND-LIMITED DAMPER FLUID CONFIGURATION
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The University of Texas at Arlington, 2015

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Vehicle suspension system is one of the key factors related to the vehicle dynamic characteristic. The fine tuning of suspension system between vehicle handling and ride comfort is not only a trade-off between these two aspects, but also, has its impact on vehicle safety and durability.

For an optimal damper performance, the damper needs to generate sufficient damping force at maneuvering frequency (< 2 Hz); and it also needs to generate less damping force, which allow spring to filter the high frequency noise (> 2 Hz) generated by vehicle travels along rough road condition at speed. In order to achieve such performance, a lumped parameter model of Band-Limited Damper is proposed. Frequency response of both Conventional Damper and Band-Limited Damper are studied and compared. Also, Lumped Parameter Quarter Car Models are used to compare the system frequency response of the one with Conventional Damper to the ones with Band-Limited Damper. In addition, performance index of two type of systems are calculated by using a generated road profile input. Based on the comparative studies, suggestions and conclusion are made.

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Chapter 1

Introduction

1.1 Lumped Parameter Quarter Car Model

For vehicle suspension dynamic characteristic research and development opposes, simulation often carry out before investing effort on building prototype. However, vehicle suspension system is a complex system having multiple degrees of freedom [1, 2]. To simply the simulation process, lumped parameter quarter car model often used to analyze the vibration and control problem [3]. For typical lumped parameter quarter car model, the value of sprung mass, unsprung mass cannot be arbitrarily alter. Although tire stiffness can vary by choosing different type of tire, its range is limited. Also, the selection of main spring stiffness is based on the natural frequency of the sprung mass, thus, the main spring stiffness is also a fix value. As a result, damping coefficient become the only variable which can be alter for suspension tuning [6].

The inputs to the quarter car model are the road irregularities, which is a collection of random elevation changes along the length the road. When vehicle travels along the road at a certain speed, the irregularities become an excitation source with a certain frequency to the suspension system.

1.2 Generating Road Profile

Simplified form of road input, such as step input, sine waves etc. are often used for preliminary researches or comparative studies of vehicle suspension dynamic characteristics. However, due the random natural of elevation changes at road surface, the irregularities of road surface cannot be describe mathematically. Therefore, it is more accurate to model the road profile as stochastic process [7]. And, there are various methods to model the road profile as stochastic process for simulation purposes [5, 7, 8].

1.3 Thesis Outline

Chapter 2 gives a detail descriptions of the lumped parameter models of both conventional damper and band-limited damper. Then, frequency responses of both type of dampers are compared in this chapter. Follow by a comparative study between the quarter car models consists both type of damper.

Chapter 3 gives the procedures for generating road profile accords to ISO 8608 Standard, which is an international standard of different road roughness profile. A road profile is generated for quantitative measure in next chapter.

In Chapter 4, performance indexes of quarter car models consist both types of damper are calculated by using the same road profile as input.

In Chapter 5 conclusion and suggestions are made.

Chapter 2

Investigation of Band-Limited Fluid Damper

2.1 Lumped Parameter Quarter Car Model

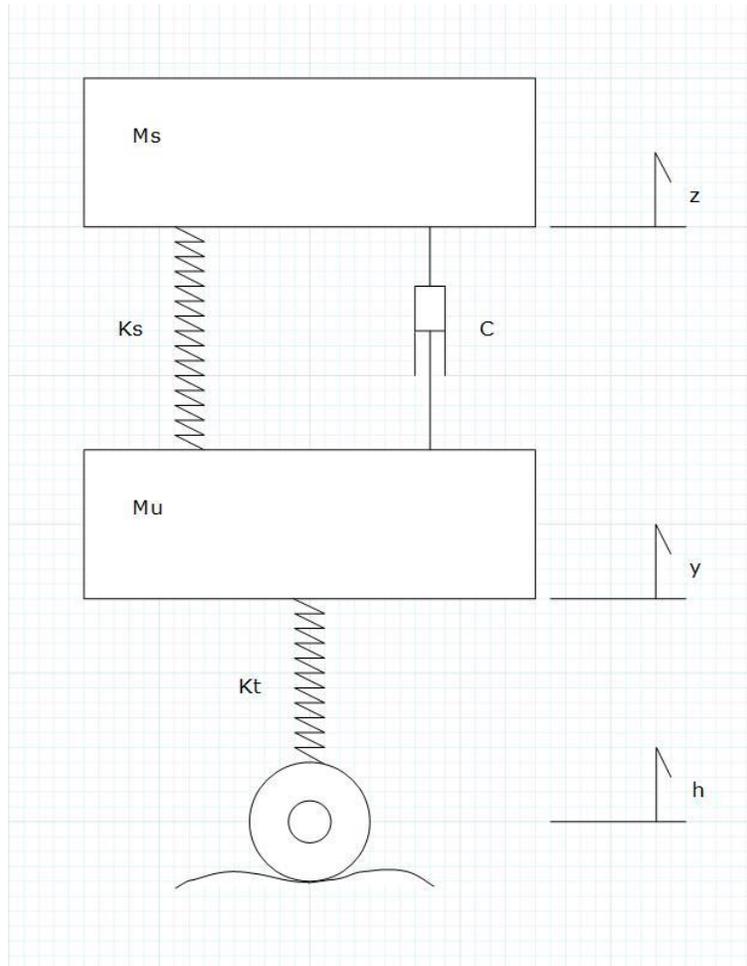


Figure 2–1 Lumped Parameter Quarter Car Model

Figure 2-1 shown a lumped parameter quarter car model consists with a sprung mass, M_s , which counts for a quarter of the total sprung mass of vehicle, an unsprung mass, M_u , representing the total weight of wheel, tire, brake and other associated components. K_s , as the main spring rate, K_t , as the tire stiffness, and C , as damping coefficient. Vertical displacement of sprung mass, unsprung mass, z and y are measure

from the static equilibrium position. Here, sprung mass, unsprung mass, main spring rate and tire stiffness are considered as fixed parameters, leaving damping coefficient as variable.

Consider the Newton's second law of motion, the equations of motion of the components in the lumped parameter quarter car model are as follow,

For sprung mass,

$$M_s \ddot{z} - F_1 - F_2 = 0 \quad (2.1)$$

For main spring,

$$F_1 = K(y - z) \quad (2.2)$$

For damper,

$$F_2 = C(\dot{y} - \dot{z}) \quad (2.3)$$

For unsprung mass,

$$M_u \ddot{y} + F_1 + F_2 - F_3 = 0 \quad (2.4)$$

For tire,

$$F_3 = K_t(h - y) \quad (2.5)$$

As shown in equation 1.3, the damper force is proportional to the vertical speed difference between sprung mass and unsprung mass. In next section, lumped parameter model of conventional and band-limited damper will be discussed.

2.2 Lumped Parameter Model of Conventional Damper and Band-Limited Damper

2.2.1 Lumped Parameter Model of Conventional damper

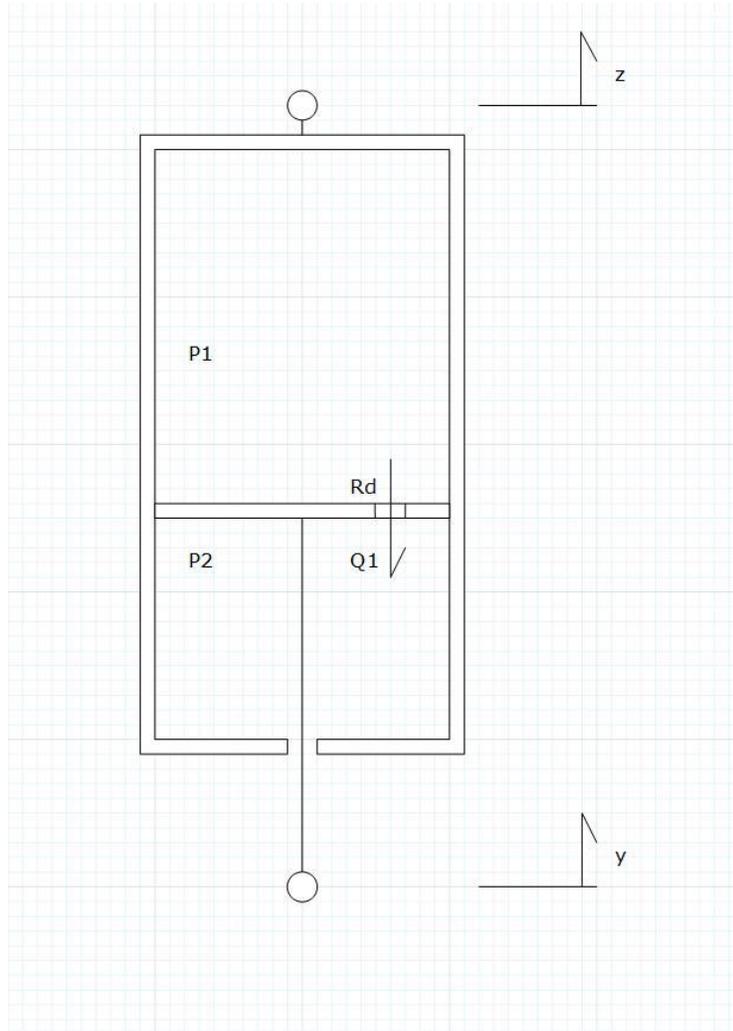


Figure 2–2 Lumped Parameter Model of Conventional Damper

Figure 2-2 shows the lumped parameter model of a typical conventional passive fluid damper. P_1 is the compression chamber pressure, P_2 is the rebound chamber pressure, and R_d is the orifice resistance. The top end of the compression chamber is connected to the sprung mass, and the end of piston rod is connected to the unsprung mass, adversely, top end displacement is z and piston rod end displacement is y . Piston

area is A , and compression chamber's control volume is V_1 . Consider the compression stroke, piston move upwards, $P_1 > P_2$, damper fluid flow from compression chamber to rebound chamber as Q_1 's positive direction. Then, the amount of damping force generated can be calculated by multiply the pressure difference to the piston area. The fluid capacitance equations of compression chamber during compression stroke are as follow, For compression chamber fluid capacitance,

$$Q_{in} - Q_{out} - \dot{V}_1 = \frac{V_1}{\beta} \dot{P}_1 \quad (2.6)$$

For volumetric flows,

$$Q_{in} = 0 \quad (2.7)$$

$$Q_{out} = \frac{P_1 - P_2}{R_D} \quad (2.8)$$

For compression chamber control volume, and its differentiated form,

$$V_1 = A(z - y) \quad (2.9)$$

$$\dot{V}_1 = A(\dot{z} - \dot{y}) \quad (2.10)$$

Substitute equations (2.7, 2.8, 2.9, and 2.10) into equation (2.6), by assuming damper fluid is incompressible $\beta \rightarrow \infty$, following equation is obtained,

$$\frac{P_1 - P_2}{R_D} = A(\dot{z} - \dot{y}) \quad (2.11)$$

For damping force,

$$F_D = A(P_1 - P_2) \quad (2.12)$$

Also, as pervious stated in equation (2.3), $F_2 = F_D$, it shown that the damper coefficient is determined by two physical values of the damper. In other words, the damping coefficient is a constant.

$$C = A^2 R_D \quad (2.13)$$

2.2.2 Lumped Parameter Model of Band-Limited Damper

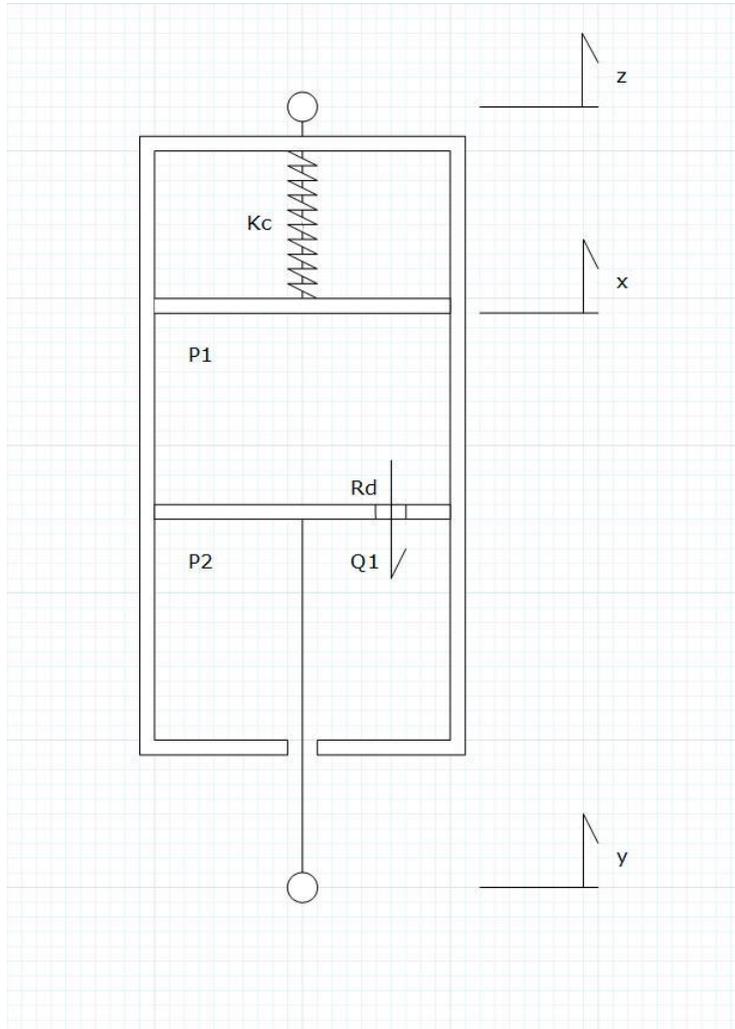


Figure 2-3 Lumped Parameter Model of Band-Limited Passive Damper

Figure 2-3 shows the lumped parameter model of band-limited passive damper. The main difference between the conventional damper and band-limited damper is the band-limited damper has a spring loaded piston on the upper part of compression chamber. Now, the compression chamber is also double as an accumulator, which will give the damper a 'Low Pass Filter' like dynamic characteristic. Similarly to the conventional damper, compression chamber and rebound chamber pressure, respectively, are P_1 and

P_2 . R_d is the orifice resistance, positive direction of Q_1 is flow from compression chamber to rebound chamber. Since the band-limited damper is install the same way as conventional damper, z and y marks the positive direction of sprung mass and unsprung mass displacement. The spring loaded piston has spring rate, K_c , and it has the same area as the damper piston area, A , and x marks the displacement of the piston. Note that, pressure above the spring loaded is set to be ambient pressure.

Since the lumped parameter the band-limited damper is linear, it can be divide into upper and lower part, then analyze them separately. The upper part consists the compression chamber and spring loaded piston; the lower part is similar to the conventional fluid damper, with only one difference, the upper end of compression chamber is now the spring loaded piston.

For the upper part, at the spring loaded piston,

$$F_2 = K_c(x - z) \quad (2.14)$$

For force balance at the spring loaded piston, then differentia,

$$P_1 A - F_2 = 0 \quad (2.15)$$

$$\dot{P}_1 A - \dot{F}_2 = 0 \quad (2.16)$$

Substitute equation (2.14) into equation (2.15), then differentia,

$$P_1 A - K_c(x - z) = 0 \quad (2.17)$$

$$\dot{P}_1 A - K_c(\dot{x} - \dot{z}) = 0 \quad (2.18)$$

Substitute equation (2.16) into (2.18),

$$\frac{\dot{F}_2}{K_c} = \dot{x} - \dot{z} \quad (2.19)$$

For the lower part, similar to the conventional damper,

$$Q_{in} - Q_{out} - \dot{V}_1 = \frac{V_1}{\beta} \dot{P}_1 \quad (2.20)$$

$$Q_{in} = 0 \quad (2.21)$$

$$Q_{out} = \frac{P_1 - P_2}{R_D} \quad (2.22)$$

$$V_1 = A(x - y) \quad (2.23)$$

$$\dot{V}_1 = A(\dot{x} - \dot{y}) \quad (2.24)$$

Substitute equation (2.21, 2.22, 2.23, and 2.24) into (2.20) and assume β is large,

$$\frac{P_1 - P_2}{R_D} + A(\dot{x} - \dot{y}) = 0 \quad (2.25)$$

For damper force,

$$F_2 = A(P_1 - P_2) \quad (2.26)$$

With equation (2.3) in mind, equation (2.25) can be re-write as follow,

$$\frac{F_2}{A^2 R_D} = \dot{y} - \dot{x} \quad (2.27)$$

Superposition equation (2.19) and equation (2.27),

$$\frac{\dot{F}_2}{K_C} + \frac{F_2}{A^2 R_D} = (\dot{x} - \dot{z}) + (\dot{y} - \dot{x}) \quad (2.28)$$

Then,

$$\frac{A^2 R_D}{K_C} \dot{F}_2 + F_2 = A^2 R_D (\dot{y} - \dot{z}) \quad (2.29)$$

As previously found as equation (2.13), equation (2.29) can be re-write as follow,

$$\frac{C_C}{K_C} \dot{F}_2 + F_2 = C_C (\dot{y} - \dot{z}) \quad (2.30)$$

Laplace transform equation (2.30), becomes,

$$F_2(s) = \frac{C_C}{\frac{s}{\omega_c} + 1} (\dot{y} - \dot{z}) \quad , \text{where } \omega_c = \frac{K_C}{C_C} \quad (2.31)$$

As equation (2.31) shown, the equivalent damping coefficient has a 'Low Pass Filter' like behavior, thus, the damper will generate less damping force at higher frequency, where ω_c is the 'Cut-Off' frequency of the 'Low Pass Filter'.

In next section, a comparative studies aim to demonstrate the different dynamic characteristics of both type of damper, and the systems consists with these type of damper are shown.

2.3. Comparative Studies of Frequency Responses

2.3.1 Frequency Response of Conventional Damper and Band-Limited Damper

In order to demonstrate the different dynamic characteristic, first to write transfer functions of both type of dampers, which can be defined as an equivalent damping coefficient.

For conventional damper, the equivalent damping coefficient is the same as its original value,

$$C_{equivalent}(s) = C \quad (2.32)$$

For band-limited damper,

$$C_{equivalent}(s) = \frac{C}{\frac{s}{\omega_c} + 1} \quad (2.33)$$

Then, plot the transfer function (2.32, 2.33) for the frequency response of both type of damper, shown as Figure 2-4.

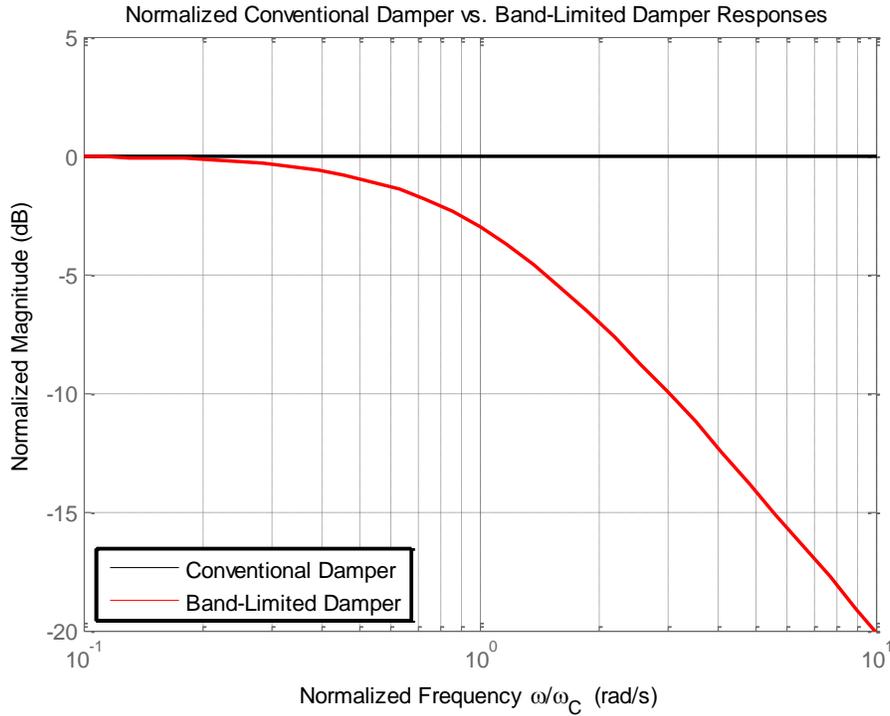


Figure 2–4 Frequency Responses of Conventional Damper vs Band-Limited Damper

From Figure 2-4, it can be observed that at the higher input frequency ($\omega > \omega_c$), the band-limited damper generates less damping force, while the conventional damper does not have this characteristic. In section 2.3.2, the effects of the cut-off frequency, ω_c on the system will be study.

2.3.2 Frequency Response of Quarter Car Models with Conventional and Band-Limited Damper

To compare the dynamic characteristic of systems using different type of damper, consider two quarter car models, one use conventional damper, as shown in section 2.1, for the other one model, the conventional damper is replace by a band-limited damper. Then, properties listed in Table 2.1 are used to construct the quarter car models.

Table 2-1 List of Sample System Properties

Sprung mass, kg	$M_s = 400$
Unsprung mass, kg	$M_u = 40$
Tire stiffness, N/m	$K_t = 150000$
Spring rate, N/m	$K_s = 21000$
Conventional Damping coefficient, N*s/m	$C_s = 1500$

Note that, the damping coefficient of band-limited damper is set to be equal to the damping coefficient of conventional damper, as such, the location of the cut-off frequency become the only contributing factor of different system behavior. Also, cut-off frequency and natural frequency of sprung mass can be link mathematically by introducing following equation,

$$R = \frac{\omega_C}{\omega_N} \tag{2.34}$$

Then, use the properties from Table 2.1, frequency responses for sprung mass, Figure (2-5), and for unsprung mass, Figure (2-6) are generated.

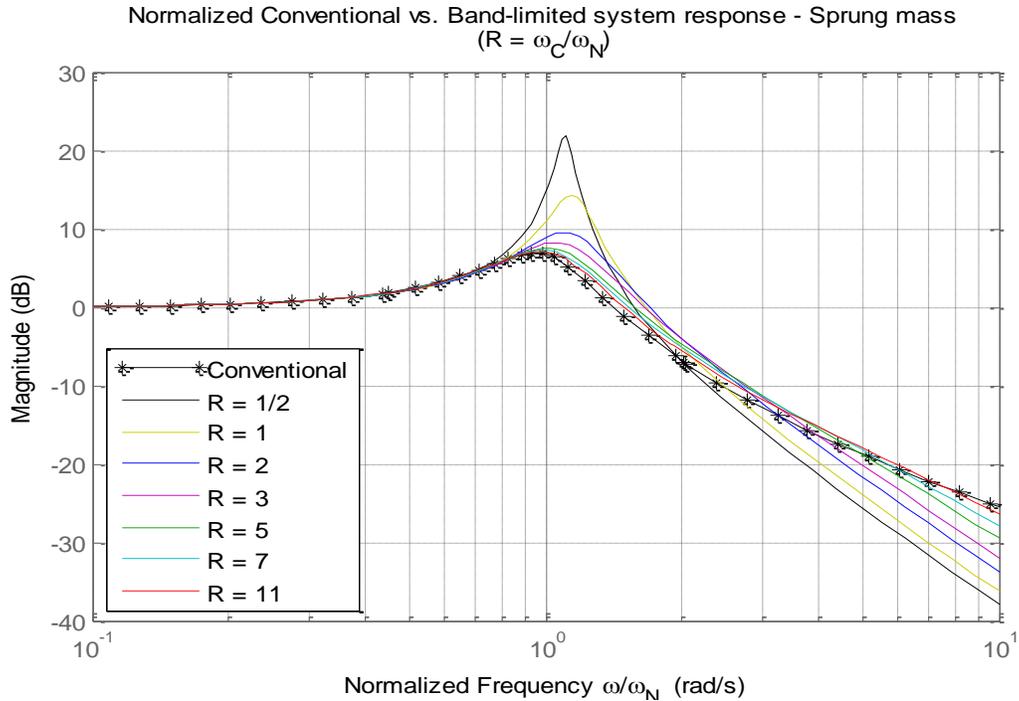


Figure 2-5 Normalized System Frequency Response – Sprung Mass

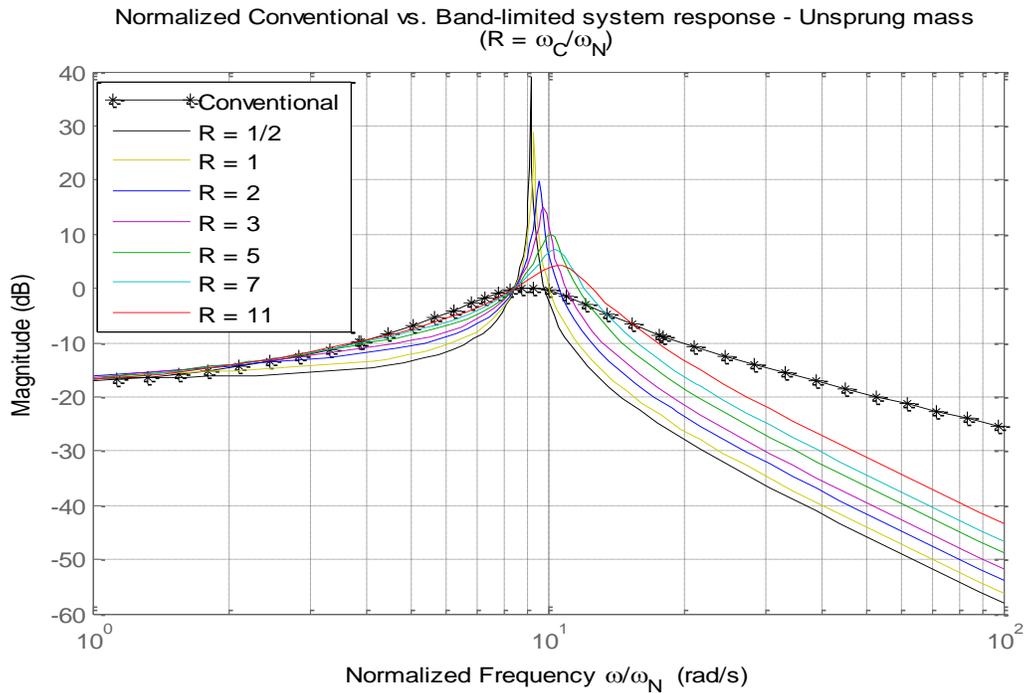


Figure 2-6 Normalized System Frequency Response – Unsprung Mass

Figure 2-5 and Figure 2-6 shows how both sprung and unsprung mass respond to different cut-off frequency ω_c in a quarter car model simulation. As stated in equation (2.31), in the band-limited damper, the spring rate of piston spring, K_c and damping coefficient C_c donates ω_c . In this section, with a fixed C_c values, ω_c vary with different ω_c values only.

From Figure 2-5, system with band-limited damper excites more than the ones with conventional damper when ω_c is less than, or close to ω_N . Or, in other words, when a softer piston spring is in use, the band limited damper have a lower equivalent damping coefficient than conventional damper. Same effects shown in Figure 2-6, too.

In this chapter, for comparative studies purpose, the damping coefficient was set to be a fixed value. However, when designing a Band-limited damper for a specific vehicle,

both C_c and K_c can be select in accordance to the performance requirement of that particular vehicle.

2.3.3. An Example of two Quarter Car Models Consist Conventional damper and Band-limited Damper

Consider two sample quarter car models, one model consists a conventional damper, the other consists a band-limited damper, using the listed properties from Table 2-2 below.

Table 2-2 List of Sample System Properties

Sprung mass, kg	$M_s = 400$
Unsprung mass, kg	$M_u = 40$
Tire stiffness, N/m	$K_t = 150000$
Spring rate, N/m	$K_s = 21000$
Conventional Damping coefficient, N*s/m	$C_s = 1500$
Band-Limited damper spring rate, N/m	$K_c = 150000$
Band-Limited damper damping coefficient, N*s/m	$C_c = 1500$

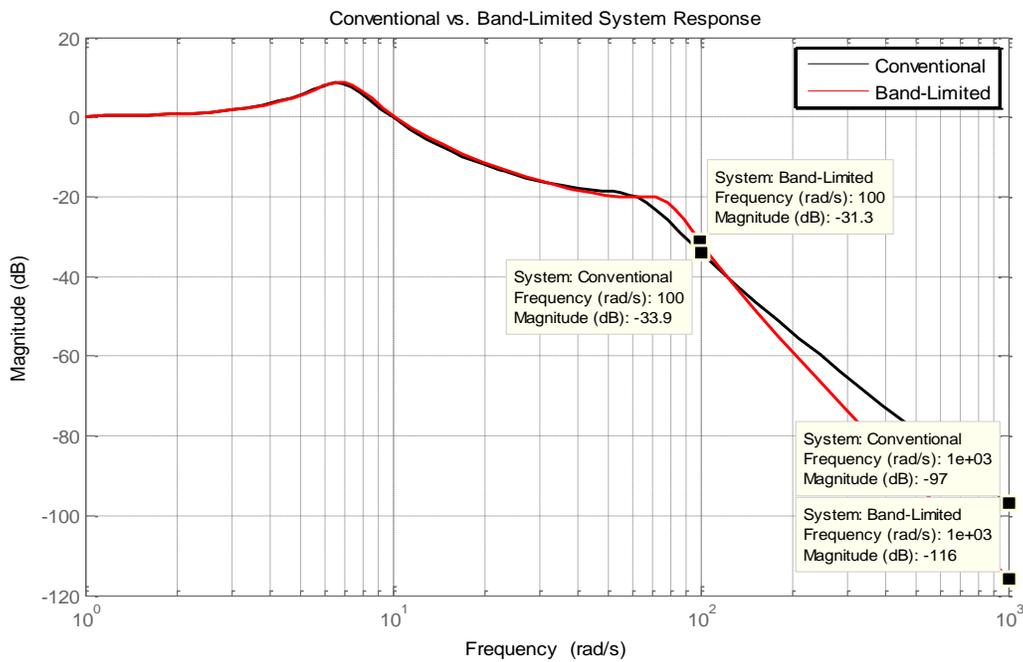


Figure 2-7 Frequency Response Quarter Car Model Consist Different Damper

Note that, in Figure 2-7, the gain of the model using band-limited damper has a stiffer negative slope than the one using conventional damper at higher frequency, or -84.7dB/dec compared to -63.1dB/dec. Thus, Band-Limited Damper generate less damping force at higher input frequency.

Chapter 3

Generating Random Road Profile As Stochastic Process

3.1 ISO 8608 road surface roughness classification

Various organizations have classified the road roughness profile for analytical purposes. The International Organization of Standardization (ISO) provides ISO 8608, a uniform method of reporting measured vertical surface profile data from streets, roads, highways and off-road terrain. The main objective of this chapter is therefore to develop road surface profile in accordance with ISO 8608 [5,8].

From Figure 3-1, The ISO 8608 classified the road surface profile into eight categories labelled from A to H, A being the smoothest surface, and H being the roughest surface. As Shown in Table 3-1 below, each category is defined by a range of artificial Power Spectral Density (PSD).

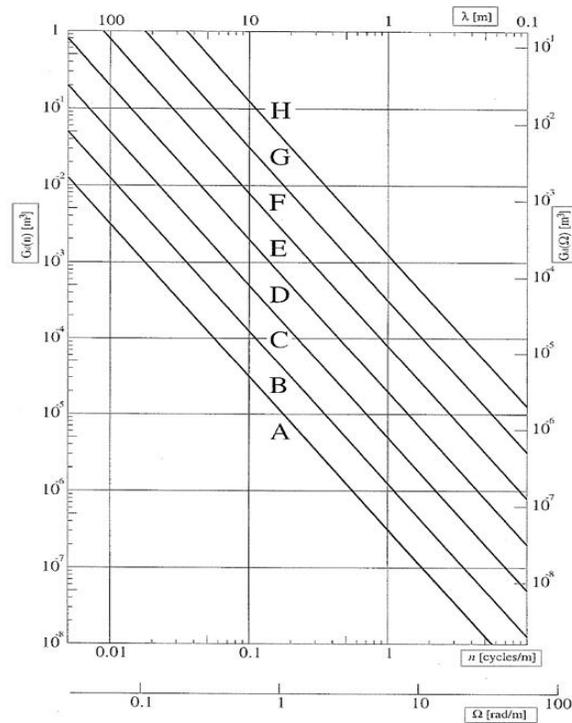


Figure 3–1 ISO 8608 Standard Classification of Roughness

Table 3-1 ISO 8608 Standard Classification of Roughness

Degree of roughness expressed in terms of spatial frequency units, n				
Road class	k	$G_d(n_0)[10^{-6}m^3]$		
		Lower limit	Geometric mean	Upper limit
A – B	3	--	2^4	2^5
B – C	4	2^5	2^6	2^7
C – D	5	2^7	2^8	2^9
D – E	6	2^9	2^{10}	2^{11}
E – F	7	2^{11}	2^{12}	2^{13}
F - G	8	2^{13}	2^{14}	2^{15}
G - H	9	2^{15}	2^{16}	2^{17}
H +	--	2^{17}	--	--

$n_0 = 0.1 \text{ cycle/m}$

For on-site data collecting and analysis purpose, the identification of the class of roughness of a real road profile is first to calculate the PSD of the profile in correspondence of n_0 , then comparing the calculated PSD with the ISO standard for class determination. For simulations purpose, the ISO 8608 defines the road roughness profile as following equation (3.1)

$$G_d(n) = G_d(n_0) \left(\frac{n}{n_0}\right)^{-2} \quad (3.1)$$

Where $G_d(n_0)$ value is picked from Table 3-1 according to the desired road class, and the conventional value of spatial frequency is $n_0 = 0.1 \text{ cycles/m}$.

3.2 Generating Road Profile

In accordance with the ISO 8608 road roughness surface classification, it is possible to generate a road roughness profile from a stochastic representation. The procedure is explained in the third chapter of paper "The vibrations induced by surface irregularities in road pavements – a MATLAB® approach" by M. Agostinacchio, D. Ciampa and S. Olita, School of Engineering, University of Basilicata [5].

By following the procedure described in this paper, the PSD function at a defined spatial frequency n , within a frequency band Δn , can be express as equation (3.2),

$$G_d(n) = \lim_{\Delta n \rightarrow 0} \frac{\psi_x^2}{\Delta n} \quad (3.2)$$

Where ψ_x^2 is the mean square value of the component of the signal for the defined spatial frequency, n ?

Since the vertical displacement PSD is defined in terms of spatial frequency n , the road profile signal becomes a sequence of uniformly spaced elevation points. Suppose the length of the road is L , within the spatial frequency domain, the frequency is evenly spaced with an interval $\Delta n = 1/L$. Suppose the sampling interval (Tire Contact Patch) is B , then the maximum theoretical sampling spatial frequency is $n_{max} = 1/B$ and the maximum effective is half of the n_{max} , or $n_{eff} = 1/2B$. The generic spatial frequency value n_i can be represent as $i \cdot \Delta n$, as i varying from 0 to $N = n_{max} / \Delta n$. As a result, equation (3.2) can be rewrite in discrete form (3.3),

$$G_d(n_i) = \frac{\psi_x^2(n_i, \Delta n)}{\Delta n} = \frac{\psi_x^2(i \cdot \Delta n, \Delta n)}{\Delta n} \quad (3.3)$$

In the paper, the author defines the road profile through a harmonic function as equation (3.4),

$$h(x) = A_i \cos(2\pi \cdot n_i \cdot x + \varphi) = A_i \cos(2\pi \cdot i \cdot \Delta n \cdot x + \varphi) \quad (3.4)$$

Where A_i is the amplitude, n_i is the spatial frequency and φ is the random phase angle between 0 and 2π . Then, the mean square value of the $h(x)$ becomes (3.5),

$$\psi_x^2 = \frac{A_i^2}{2} \quad (3.5)$$

Combined both equation (3.3) and (3.5), yields (3.6),

$$G_d(n_i) = \frac{\psi_x^2(n_i)}{\Delta n} = \frac{A_i^2}{2 \cdot \Delta n} \quad (3.6)$$

Equation (3.4) can be rewritten using a known PSD of vertical displacement, yields (3.7),

$$h(x) = \sum_{i=0}^N \sqrt{2 \cdot \Delta n \cdot G_d(i \cdot \Delta n)} \cdot \cos(2\pi \cdot i \cdot \Delta n \cdot x + \varphi_i) \quad (3.7)$$

Then, substituting (3.1) into (3.7), results equation (3.8), which is the equation used for generating an artificial random road surface profile accords to ISO 8608 standard.

$$h(x) = \sum_{i=0}^N \sqrt{\Delta n} \cdot 2^k \cdot 10^{-3} \cdot \left(\frac{n_0}{i \cdot \Delta n}\right) \cdot \cos(2\pi \cdot i \cdot \Delta n \cdot x + \varphi_i) \quad (3.8)$$

3.3 MATLAB simulation of ISO 8608 Road Profile

To simulate sample road profiles, consider road length equal to 250 m, road condition from very good (ISO A-B class, $k = 3$), to good (ISO B-C class, $k = 4$), average (ISO C-D class, $k = 5$), and poor (ISO D-E class, $k = 6$). The lower bound of spatial frequency was considered as $1/L = 0.004\text{m}^{-1}$, and upper bound of spatial frequency was consider to be $1/B = 20\text{m}^{-1}$, where $B = 0.05\text{m}$ (Contact patch).

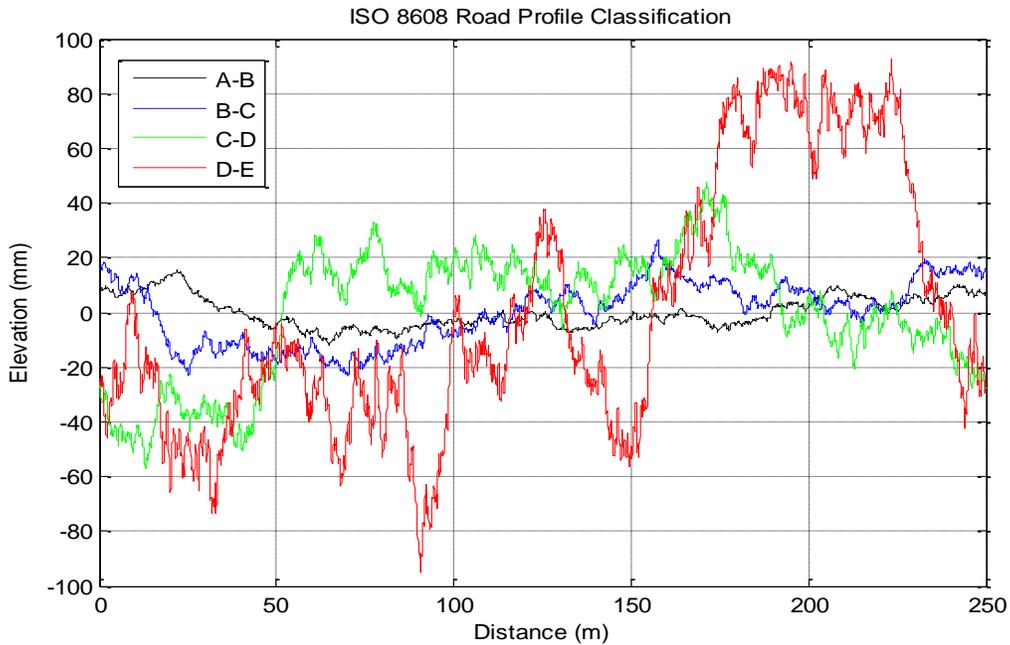


Figure 3–2 Simulated Road Profile accords to ISO 8608 Standard

Figure 3-2 shows the simulated road profile from very good (ISO A-B class, $h_{max} = \pm 15mm$), to good (ISO B-C class, $h_{max} = \pm 25mm$), average (ISO C-D class, $h_{max} = \pm 50mm$), and poor (ISO D-E class, $h_{max} = \pm 100mm$).

Sample road profile is generated in the same way for quantitative analysis in next chapter.

Chapter 4

Suspension Optimization

In this chapter, an example is given to demonstrate vehicle's different dynamic characteristic when using conventional damper vs. band-limited damper.

In order to more accurately simulate a vehicle travels along a sample road profile, a sample road profile needs to be generated. Two quarter car models are considered as two test vehicles, and since the purpose of this example is to show the effects on dynamic characteristic by using different damper, all the lumped parameters are set to be the same, leaving the damper as the only variable. Then, use the generated sample road profile as system input, as quarter car model 'run' along the length of road profile, the elevation changes becomes the excitation source to the quarter car model with a frequency correlated to vehicle's velocity. For suspension performance evaluation, dimensionless term – Performance Index is introduced [6]. Performance index needs to consider for both ride and handling.

The ride performance index is the root mean square of the sprung mass/g ratio, g as the gravitational acceleration, the math representation of the ride performance index becomes,

$$PI_{ride} = \sqrt{\frac{\int_0^{T_f} \left(\frac{\ddot{z}}{g}\right)^2}{T_f}} \quad (4.1)$$

The handling performance index is the root mean square of the tire contact force/total weight ratio, then the math representation of the handling performance index is as follow,

$$PI_{Handling} = \sqrt{\frac{\int_0^{T_f} \left(\frac{F_h}{W_{total}}\right)^2}{T_f}} \quad (4.2)$$

For optimal suspension performance, we look for a damper setting which can minimize the both of this values. Note that, the smallest value for ride performance index is 0, and the smallest value for handling performance index is 1.

Use the system properties listed in Table 4-1, 4-2, vehicle is set to 'run' along the road profile at $V = 36 \text{ km/h}$, or 10 m/s , a set of performance index is calculated, as shown in Table 4-3.

Table 4-1 List of System Properties of Quarter Car Model w/ Conventional Damper

Sprung mass, kg	$M_s = 400$
Unsprung mass, kg	$M_u = 40$
Tire stiffness, N/m	$K_t = 150000$
Spring rate, N/m	$K_s = 21000$
Conventional Damping coefficient, N*s/m	$C_s = 1500$

Table 4-2 Band-Limited Damper Properties

Band-Limited damper spring rate, N/m	$K_c = 250000$
Band-Limited damper damping coefficient, N*s/m	$C_c = 2500$
Cut-off frequency, rad/s	$\omega_c = 100$

Table 4-3 Calculated Performance Indexes

	PI_{ride}	PI_{handling}
QCM w/ Conventional Damper	0.90	1.29
QCM w/ Band-Limited Damper	1.19	1.15

Based on the results listed in Table 4-3, shows that the band-limited damper can alter the vehicle's dynamic characteristic with different combination of K_c and C_c according to the vehicle performance requirement. Further research on tuning suspension with band-limited damper needs to be done in future work.

Chapter 5

Result and Conclusions

This Thesis purposed a passive band limited fluid damper configuration for automotive suspension. Lumped parameter models of both conventional and band-limited damper are shown, frequency responses are generated, and frequency response of both type of damper are compared. Frequency response of quarter car model consist both type of damper are also shown and compared.

Random road profile is generated in accordance with ISO 8608 Standard. A study on the dynamic characteristic of quarter car models consist different damper is carried out by using an ISO 8608 road profile as input.

The band-limited damper demonstrated a 'Low Pass Filter' like dynamic characteristics. That is, the damper can generate damping force at low input frequency for vehicle maneuvering; at high input frequency cause by the road surface irregularity, the damper generates less damping force compare to conventional damper. As shown in equation (2.31), damping coefficient, C and piston spring constant, K_c donates the cut-off frequency, ω_c at which frequency, the damper begins to generate less damping force. That means, cut-off frequency can be select according to vehicle performance requirement with combination of C and K_c . Subsequently, the band limited damper gives automotive engineer more room for suspension tuning. Thus, with the proper suspension tuning, the vehicle suspension with band-limited damper can provides good road holding without compromising ride comfort.

Appendix A

MATLAB® Files Used for This Thesis

Main Script

```
% An Investigation of Band-Limited Fluid Damper Configuration
% Using Lumped Parameter Modeling
% Thesis adviser: Dr. Woods
% Student: Ye Hong

close all; clear all; clc
% Knowns
Ms = 400;           % Sprung mass, kg
Mu = 40;           % Unsprung mass, kg
Kt = 150000;       % Tire stiffness, N/m
Ks = 21000;        % Spring rate, N/m
Cs = 1500;         % Ideal Damping coefficient, N*s/m
Kc = 50000;        % Band-Limited damper spring rate, N/m
Cc = 3000;         % Band-Limited damper damping coefficient, N*s/m

% OP - Model options: OP = 1, Quarter car model
%                               OP = 2, Normalized natural frequency quarter
care model
OP = 2;

% Normalization
Rm = Mu/Ms          % Mass ratio
Rk = Kt/Ks          % Spring rate ratio
zeta = Cs/(2*sqrt(Ks*Ms)) % Damping ratio
omegaN = sqrt(Ks/Ms) % Sprung mass natural frequency
omegaC = Kc/Cc      % Damper cut-off frequency

%% Conventional vs. Band-Limited Damper Responses
% Inputs
% Cs - Damping coefficient, N.s/m
% Cc - Band-limited dumping coefficient, N.s/m
% Kc - Band-limited spring constant, N/m
% OP - Model options: OP = 1, Quarter car model
%                               OP = 2, Normalized natural frequency quarter
care model

% Outputs
% CC - Conventional damper transfer function
% CB - Band-limited damper transfer function

disp('Conventional vs. Band-Limited Damper Responses')
[ CC,CB ] = DR( Cs,Cc,Kc,OP );

%% Conventional vs. Band-Limited Damper System Responses
% Inputs
% Ms - Sprung mass, kg
% Mu - Unsprung mass, kg
% Ks - Suspension stiffness, N/m
% Cs - Dumping coefficient, N.s/m
% Kt - Tire stiffness, N/m
```

```

% Outputs
% zeta - Damping ratio
% omegaN - Natural Frequency rad/s
% omegaC - Cut-off Frequency, rad/s

% Conventional vs. Band-Limited Damper System Responses - Sprung Mass
disp('Conventional vs. Band-Limited System Responses - Sprung Mass')
[ zeta,omegaN,omegaC ] = SRs( Ms,Ks,Cs,OP );

% Conventional vs. Band-Limited Damper System Response - Unsprung mass
disp('Conventional vs. Band-Limited System Response - Unsprung
mass')
[ zeta,omegaN,omegaC,Rm,Rk ] = SRu( Ms,Mu,Ks,Cc,Kt,OP );

%% Quarter Car Model
% Inputs
% Ms - Sprung mass, kg
% Mu - Unsprung mass, kg
% Ks - Suspension stiffness, N/m
% Kc - Damper spring rate, N/m
% Cc - Dumping coefficient, N.s/m
% Kt - Tire stiffness, N/m
% OP - Model options: OP = 1, Quarter car model
%
% OP = 2, Normalized natural frequency quarter
care model

% Outputs
% A, B, C, D, 4 matrices for linear state space model
% Generate frequency reponse of quarter-car model from inputs (h), to
% outputs, (body travel, body velocity, wheel travel, wheel velocity,
% suspension deflection and body acceleration).

% Quarter Car Model w/ Conventional Damper
disp('Quarter Car Model w/ Conventional Damper')
[ A,B,C,D,QCM ] = QCM( Ms,Mu,Ks,Cs,Kt,OP );

% Quarter Car Model w/ Band-Limited Passive Damper
disp('Quarter Car Model w/ Band-Limited Damper')
[ AI,BI,CI,DI,QCMI ] = QCMI( Ms,Mu,Ks,Cc,Kc,Kt,OP );
% Compare System Responses
% Conventional Damper
C1 = [ 1 0 0 0];
D1 = 0;
qcm = ss(A,B,C1,D1);
% Band-Limited Damper
CI1 = [ 1 0 0 0 0];
DI1 = 0;
qcmi = ss(AI,BI,CI1,DI1);

Case = OP;

```

```

switch Case
    case 1
        bodemag(qcm,'k',qcmi,'r',{1,1000}); grid;
        legend('Conventional','Band-Limited')
        title('Conventional vs. Band-Limited System Response')
    case 2
        bodemag(qcm,'k',qcmi,'r',{0.1,100}); grid;
        legend('Conventional','Band-Limited')
        xlabel('Normalized Frequency')
        title('Normalized Conventional vs. Band-Limited System
Response')
end

%% Road Profile
% Inputs
% B - Sampling interval, m
% L - Length of profile, m
% nL - Lower boundary of spatial frequency, m^-1
% nU - Upper boundary of spatial frequency, m^-1
% k - ISO 8608 road profile classification
% Plot - Plot options: Plot = 0 for no plot;
%         Plot = 1 for plot;

% Outputs
% n_max - Maximum theoretical sampling spatial frequency, m^-1
% n_eff - Maximum effective sampling spatial frequency, m^-1
% Delta_n - Discretized spatial frequency spacing, m^-1
% N - Maximun number possible sampling size
% Road - Road signal

disp('Road Profile')
Bx = 0.05; % Sampling interval, m
L = 250; % Length of profile, m
nL = 1/L; % Lower boundary of spatial frequency, m^-1
nU = 1/Bx; % Upper boundary of spatial frequency, m^-1
k = 5; % ISO 8608 road profile classification
Plot = 1; % Plot options

% Plot different roughness profiles
figure
kk = 3:1:6; % ISO 8608 road profile classification
line = ['k','b','g','r'];
for i = 1:1:length(kk)
    [ n_max,n_eff,Delta_n,N,Road ] = ISO8608k( Bx,L,nL,nU,kk(i),0 );
    xx = Road(1,:);
    hh = Road(2,:);
    plot(xx,hh*1000,line(1,i)); hold on;
end
grid; hold off;
xlabel('Distance (m)'); ylabel('Elevation (mm)');
title('ISO 8608 Road Profile Classification');

```

```

legend('A-B','B-C','C-D','D-E')

% Plot B-C roughness profiles for Performance Indices
[ n_max,n_eff,Delta_n,N,Road ] = ISO8608k( Bx,L,nL,nU,k,Plot );

%% Performance Indices
% Inputs
% A,B,C,D - State variable matrices
% Ms - Sprung mass, kg
% Mu - Unsprung mass, kg
% Kt - Tire stiffness, N/m
% x0 - Initial state
% V - Vehicle speed, km/h
% dt - Time steps size,s
% tmax - Maximum simulation time, s
% Road - Road profile

% Outputs
% R1 - Instantaneous Sprung mass acc. vs gravatational acc.
% PIr - Performance index - ride
% R2 - Instantaneous normal force - total weight vs total weight
% PIh - Performance index - handling

% Inputs
V = 100; % Vehicle speed, km/h
dt = 0.005; % Time steps size
tmax = 5; % Maximum simulation time
Road; % Road profile

% Performance Indices for QCM w/ Conventional Damper
disp('Performance Indices for QCM w/ Conventional Damper')
x0 = [0 0 0 0]';
[ Rr,PIr,Rh,PIh ] = PIM( A,B,C,D,Ms,Mu,Kt,x0,Road,V,dt,tmax );
PIr
PIh

% Performance Indices for QCM w/ Band-Limited Damper
disp('Performance Indices for QCM w/ Band-Limited Damper')
x0I = [0 0 0 0 0]';
[ RrI,PIrI,RhI,PIhI ] =
PIM( AI,BI,CI,DI,Ms,Mu,Kt,x0I,Road,V,dt,tmax );
PIrI
PIhI

Frequency Response of Dampers
function [ CC,CB ] = DR( Cs,Cc,Kc,OP )
% Conventional vs. Band-Limited Damper Response

% An Investigation of Band-Limited Fluid Damper Configuration
% Using Lumped Parameter Modeling

```

```

% Thesis adviser: Dr. Robert L. Woods
% Student: Ye Hong
% Date: 11/30

% Inputs
% Cs - Damping coefficient, N.s/m
% Cc - Band-limited dumping coefficient, N.s/m
% Kc - Band-limited spring constant, N/m
% OP - Model options: OP = 1, Quarter car model
%                               OP = 2, Normalized natural frequency quarter
care model

% Outputs
% C - Conventional damper transfer function
% CI - Band-limited damper transfer function

Case = OP;
switch Case
    case 1
        CC = tf(Cs,1);
        CB = tf(Cc,[Cc/Kc 1]);
        bodemag(CC,'k',CB,'r',{1 1000}); grid;
        legend('Conventional Damper','Band-Limited Damper',3)
        title('Conventional Damper vs. Band-Limited Damper Responses')
% -----
% -----
    case 2
        omegaC = Kc/Cc;
        CC = tf(Cs/Cs,1);
        CB = tf(Cc/Cc,[omegaC/omegaC 1]);
        bodemag(CC,'k',CB,'r'); grid
        axis([0.1,10,-20,5])
        xlabel('Normalized Frequency \omega/\omega_C')
        ylabel('Normalized Magnitude')
        legend('Conventional Damper','Band-Limited Damper',3)
        title('Normalized Conventional Damper vs. Band-Limited Damper
Responses')
    end
end
%
=====
===
end

```

System Frequency Response – Sprung Mass

```

function [ zeta,omegaN,omegaC ] = SRs( Ms,Ks,Cc,OP )
% Conventional vs. Band-limited system Response - Sprung mass

% An Investigation of Band-Limited Fluid Damper Configuration
% Using Lumped Parameter Modeling
% Thesis adviser: Dr. Robert L. Woods
% Student: Ye Hong

```

```

% Date: 11/30

% Inputs
% Ms - Sprung mass, kg
% Ks - Suspension stiffness, N/m
% Cs - Damping coefficient, N.s/m
% OP - Model options: OP = 1, Quarter car model
%                               OP = 2, Normalized natural frequency quarter
care model

% Outputs
% zeta - Damping ratio
% omegaN - Natural Frequency rad/s
% omegaC - Cut-off Frequency, rad/s

Case = OP;
switch Case
    case 1
% Calculate damping ratio zeta & natural frequency omegaN
    zeta = Cc/(2*sqrt(Ks*Ms)); % Damping ratio
    omegaN = sqrt(Ks/Ms); % rad/s

% Conventional damper spring system
    NUMc = [2*zeta*omegaN,omegaN^2];
    DENc = [1,2*zeta*omegaN,omegaN^2];
    SYSc = tf(NUMc,DENc);
    damp(SYSc);
    figure
    bode(SYSc,'k-*)
    hold on

% Band-limited damper spring system
% Vary omegaC in terms of omegaN
    R = [1/2,1,2,3,5,7,11]; % Ratio of omegaC/omegaN
    omegaC = R*omegaN; % rad/s
% Square root of K*M
    srKM = sqrt(Ks*Ms);

% Transfer function & bode plot
    line = ['k','y','b','m','g','c','r'];
    for i = 1:length(omegaC)
        NUM1 =
    [(Cc*omegaC(i)*omegaN+srKM*omegaN^2),srKM*omegaC(i)*omegaN^2];
        DEN1 =
    [srKM,srKM*omegaC(i),(Cc*omegaC(i)*omegaN+srKM*omegaN^2),...
        srKM*omegaC(i)*omegaN^2];
        SYS1 = tf(NUM1,DEN1);
        damp(SYS1);
        bodemag(SYS1,line(1,i),{1e0 1e2});
        hold on
        disp('-----')
    end
--');

```

```

end
hold off
grid;
titles{1} = 'Normalized vs. Band-limited system response - Sprung
mass';
titles{2} = '(R = \omega_C/\omega_N)';
title(titles);
legend('Conventional','R = 1/2','R = 1',...
      'R = 2','R = 3','R = 5','R = 7','R = 11',3);

case 2
% Calculate damping ratio zeta & natural frequency omegaN
zeta = Cc/(2*sqrt(Ks*Ms)); % Damping ratio
omegaN = 1; % rad/s

% Conventional damper spring system
NUMc = [2*zeta*omegaN,omegaN^2];
DENc = [1,2*zeta*omegaN,omegaN^2];
SYSc = tf(NUMc,DENc);
damp(SYSc);
figure
bode(SYSc,'k-*)
hold on

% Band-limited damper spring system
% Vary omegaC in terms of omegaN
R = [1/2,1,2,3,5,7,11]; % Ratio of omegaC/omegaN
omegaC = R*omegaN; % rad/s
% Square root of K*M
srKM = sqrt(Ks*Ms);

% Transfer function & bode plot
line = ['k','y','b','m','g','c','r'];
for i = 1:length(omegaC)
    NUM1 =
    [(Cc*omegaC(i)*omegaN+srKM*omegaN^2),srKM*omegaC(i)*omegaN^2];
    DEN1 =
    [srKM,srKM*omegaC(i),(Cc*omegaC(i)*omegaN+srKM*omegaN^2),...
      srKM*omegaC(i)*omegaN^2];
    SYS1 = tf(NUM1,DEN1);
    damp(SYS1);
    bodemag(SYS1,line(1,i),{0.1 10});
    hold on
    disp('-----');
end
hold off
grid;
xlabel('Normalized Frequency \omega/\omega_N')
titles{1} = 'Normalized Conventional vs. Band-limited system
response - Sprung mass';
titles{2} = '(R = \omega_C/\omega_N)';
title(titles);

```

```

    legend('Conventional','R = 1/2','R = 1',...
          'R = 2','R = 3','R = 5','R = 7','R = 11',3);
end
end

```

System Frequency Response – Sprung Mass

```

function [ zeta,omegaN,omegaC,Rm,Rk ] = SRu( Ms,Mu,Ks,Cc,Kt,OP )
% Conventional vs. Band-limited system Response - Unsprung mass

% An Investigation of Band-Limited Fluid Damper Configuration
% Using Lumped Parameter Modeling
% Thesis adviser: Dr. Robert L. Woods
% Student: Ye Hong
% Date: 11/30

% Inputs
% Ms - Sprung mass, kg
% Mu - Unsprung mass, kg
% Ks - Suspension stiffness, N/m
% Cc - Dumping coefficient, N.s/m
% Kt - Tire stiffness, N/m
% OP - Model options: OP = 1, Quarter car model
%                               OP = 2, Normalized natural frequency quarter
care model

% Outputs
% zeta - Damping ratio
% omegaN - Natural Frequency rad/s
% omegaC - Cut-off Frequency, rad/s

Case = OP;
switch Case
    case 1
% Calculate damping ratio zeta & natural frequency omegaN
    zeta = Cc/(2*sqrt(Ks*Ms)); % Damping ratio
    omegaN = sqrt(Ks/Ms); % rad/s

% Mass and spring rate ratio
    Rm = Mu/Ms; % Mass ratio
    Rk = Kt/Ks; % K ratio

% Conventional damper spring system
    NUMc = [2*zeta*omegaN/Rm,omegaN^2/Rm];
    DENC = [1,2*zeta*omegaN/Rm,(1+Rk)*omegaN^2/Rm];
    SYSc = tf(NUMc,DENC);
    damp(SYSc);
    figure
    bode(SYSc,'k-*)
    hold on

```

```

% Vary omegaC in terms of omegaN
R = [1/2,1,2,3,5,7,11]; % Ratio of omegaC/omegaN
omegaC = R*omegaN; % rad/s

% Square root of K*M
srKM = sqrt(Ks*Ms);

% Transfer function & bode plot
line = ['k','y','b','m','g','c','r'];
for i = 1:length(omegaC)
    NUM1 =
    [(Cc*omegaC(i)*omegaN+srKM*omegaN^2),srKM*omegaC(i)*omegaN^2];
    DEN1 =
    [Rm*srKM,Rm*srKM*omegaC(i),(Cc*omegaC(i)*omegaN+(1+Rk)*...
    srKM*omegaN^2),(1+Rk)*srKM*omegaC(i)*omegaN^2];
    SYS1 = tf(NUM1,DEN1);
    damp(SYS1);
    bodemag(SYS1,line(1,i),{1 1000});
    hold on
    disp('-----');
end
hold off
grid;
titles{1} = 'Conventional vs. Band-limited system response -
Unsprung mass';
titles{2} = '(R = \omega_C/\omega_N)';
title(titles);
legend('Conventional','R = 1/2','R = 1',...
'R = 2','R = 3','R = 5','R = 7','R = 11',3);

case 2
% Calculate damping ratio zeta & natural frequency omegaN
zeta = Cc/(2*sqrt(Ks*Ms)); % Damping ratio
omegaN = 1; % rad/s

% Mass and spring rate ratio
Rm = Mu/Ms; % Mass ratio
Rk = Kt/Ks; % K ratio

% Conventional damper spring system
NUMc = [2*zeta*omegaN/Rm,omegaN^2/Rm];
DENC = [1,2*zeta*omegaN/Rm,(1+Rk)*omegaN^2/Rm];
SYSc = tf(NUMc,DENC);
damp(SYSc);
figure
bode(SYSc,'k-*')
hold on

% Vary omegaC in terms of omegaN
R = [1/2,1,2,3,5,7,11]; % Ratio of omegaC/omegaN
omegaC = R*omegaN; % rad/s

```

```

% Square root of K*M
srKM = sqrt(Ks*Ms);

% Transfer function & bode plot
line = ['k','y','b','m','g','c','r'];
for i = 1:1:length(omegaC)
    NUM1 =
[(Cc*omegaC(i)*omegaN+srKM*omegaN^2),srKM*omegaC(i)*omegaN^2];
    DEN1 =
[Rm*srKM,Rm*srKM*omegaC(i),(Cc*omegaC(i)*omegaN+(1+Rk)*...
srKM*omegaN^2),(1+Rk)*srKM*omegaC(i)*omegaN^2];
    SYS1 = tf(NUM1,DEN1);
    damp(SYS1);
    bodemag(SYS1,line(1,i),{1 100});
    hold on
    disp('-----');
end
hold off
grid;
xlabel('Normalized Frequency \omega/\omega_N')
titles{1} = 'Normalized Conventional vs. Band-limited system
response - Unsprung mass';
titles{2} = '(R = \omega_C/\omega_N)';
title(titles);
legend('Conventional','R = 1/2','R = 1',...
'R = 2','R = 3','R = 5','R = 7','R = 11',3);
end
end

```

Quarter Car Model with Conventional Damper

```

function [ A,B,C,D,QCM ] = QCM( Ms,Mu,Ks,Cc,Kt,OP )
% Quarter Car Model w/ Conventional Damper

% An Investigation of Band-Limited Fluid Damper Configuration
% Using Lumped Parameter Modeling
% Thesis adviser: Dr. Robert L. Woods
% Student: Ye Hong
% Date: 11/30

% Inputs
% Ms - Sprung mass, kg
% Mu - Unsprung mass, kg
% Ks - Suspension stiffness, N/m
% Cc - Conventional suspension dumping, N.s/m
% Kt - Tire stiffness, N/m
% OP - Model options: OP = 1, Quarter car model
% OP = 2, Normalized natural frequency quarter
care model

```

```

% Outputs
% A, B, C, D, 4 matrices for linear state space model
% Generate frequency response of quarter-car model from inputs, (r) to
% outputs, (body travel, body velocity, wheel travel, wheel velocity,
% suspension deflection and body acceleration).

Case = OP;          % Model options
switch Case
  case 1
    % State-space matrices
    A = [
        0      1      0      0
       -Ks/Ms -Cc/Ms      Ks/Ms  Cc/Ms
         0      0      0      1
        Ks/Mu   Cc/Mu  -(Ks+Kt)/Mu -Cc/Mu ];

    B = [
         0      0      0      Kt/Mu ]';

    C = [
         1      0      0      0
         0      1      0      0
       -Ks/Ms -Cc/Ms      Ks/Ms  Cc/Ms
         0      0      1      0
         0      0      0      1
        Ks/Mu   Cc/Mu  -(Ks+Kt)/Mu -Cc/Mu
         1      0      -1      0];

    D = [ 0 0 0 0 0 0 0 0 ]';

    QCM = ss(A,B,C,D);
    QCM.StateName = {'z (m)'; 'zd (m/s)'; ...
                    'y (m)'; 'yd (m/s)'};
    QCM.InputName = {'h'};
    QCM.OutputName = {'z (m)'; 'zd (m/s)'; 'zdd (m/s^2)'; ...
                     'y (m)'; 'yd (m/s)'; 'ydd (m/s^2)'; ...
                     'sd (m)'};

% Plot the frequency response of the quarter-car model from inputs,
(h)
% to outputs, sprung mass position, sprung mass velocity.
figure
bodemag(QCM({'z (m)', 'zd (m/s)'}, 'h'), 'k', {1 1000}); grid
title(['Gain from road profile (h) '...
      'to sprung mass position (z) and velocity (zd)'])

% Plot the frequency response of the quarter-car model from inputs,
(h)
% to outputs, unsprung mass position, unsprung mass velocity.
figure
bodemag(QCM({'y (m)', 'yd (m/s)'}, 'h'), 'k', {1 1000}); grid
title(['Gain from road profile (h) '...
      'to unsprung mass position (z) and velocity (yd)'])

```

```

% Plot the frequency response of the quarter-car model from inputs,
(h)
% to outputs, sprung mass acceleration, unsprung mass acceleration.
figure
bodemag(QCM({'zdd (m/s^2)','ydd (m/s^2)'},'h'),'k',{1 1000}); grid
titles{1} = 'Gain from road profile (h)';
titles{2} = 'to sprung mass acc. (zdd) and unsprung mass acc.
(ydd)';
title(titles)

% Plot the frequency response of the quarter-car model from inputs,
(h)
% to outputs, suspension travel.
figure
bodemag(QCM({'sd'}),'h'),'k',{1 1000}); grid
title('Gain from road profile (h) to suspension travel (sd)')

case 2
% Normalization
wN = sqrt(Ks/Ms); % Sprung mass natural frequency

% State-space matrices
A = [
0 1 0 0
-Ks/Ms/wN^2 -Cc/Ms/wN Ks/Ms/wN^2 Cc/Ms/wN
0 0 0 1
Ks/Mu/wN^2 Cc/Mu/wN -(Ks+Kt)/Mu/wN^2 -Cc/Mu/wN ];

B = [
0 0 0 Kt/Mu/wN ]';

C = [
1 0 0 0
0 1 0 0
-Ks/Ms/wN^2 -Cc/Ms/wN Ks/Ms/wN^2 Cc/Ms/wN
0 0 1 0
0 0 0 1
Ks/Mu/wN^2 Cc/Mu/wN -(Ks+Kt)/Mu/wN^2 -Cc/Mu/wN
1 0 -1 0];

D = [ 0 0 0 0 0 0 0 ]';

QCM = ss(A,B,C,D);
QCM.StateName = {'z (m)';'zd (m/s)';...
'y (m)';'yd (m/s)'};
QCM.InputName = {'h'};
QCM.OutputName = {'z (m)';'zd (m/s)';'zdd (m/s^2)';...
'y (m)';'yd (m/s)';'ydd (m/s^2)';...
'sd (m)'};

% Plot the frequency response of the quarter-car model from inputs,
(h)
% to outputs, sprung mass position, sprung mass velocity.

```

```

figure
bodemag(QCM({'z (m)', 'zd (m/s)'}, 'h'), 'k', {0.1 100}); grid
xlabel('Normalized Frequency \omega/\omega_N')
title(['Gain from road profile (h) '...
      'to sprung mass position (z) and velocity (zd)'])

% Plot the frequency response of the quarter-car model from inputs,
(h)
% to outputs, unsprung mass position, unsprung mass velocity.
figure
bodemag(QCM({'y (m)', 'yd (m/s)'}, 'h'), 'k', {0.1 100}); grid
xlabel('Normalized Frequency \omega/\omega_N')
title(['Gain from road profile (h) '...
      'to unsprung mass position (z) and velocity (yd)'])

% Plot the frequency response of the quarter-car model from inputs,
(h)
% to outputs, sprung mass acceleration, unsprung mass acceleration.
figure
bodemag(QCM({'zdd (m/s^2)', 'ydd (m/s^2)'}, 'h'), 'k', {0.1 100}); grid
xlabel('Normalized Frequency \omega/\omega_N')
titles{1} = 'Gain from road profile (h)';
titles{2} = 'to sprung mass acc. (zdd) and unsprung mass acc.
(ydd)';
title(titles)

% Plot the frequency response of the quarter-car model from inputs,
(h)
% to outputs, suspension travel.
figure
bodemag(QCM({'sd'}, 'h'), 'k', {0.1 100}); grid
xlabel('Normalized Frequency \omega/\omega_N')
title('Gain from road profile (h) to suspension travel (sd)')
end
end

```

Quarter Car Model with Band-Limited Damper

```

function [ A,B,C,D,QCMI ] = QCMI( Ms,Mu,Ks,Cc,Kc,Kt,OP )
% Quarter Car Model w/ Band-Limited Damper

% An Investigation of Band-Limited Fluid Damper Configuration
% Using Lumped Parameter Modeling
% Thesis adviser: Dr. Robert L. Woods
% Student: Ye Hong
% Date: 11/30

% Inputs
% Ms - Sprung mass, kg
% Mu - Unsprung mass, kg
% Ks - Suspension stiffness, N/m

```

```

% Cc - Band-limited dumping coefficient, N.s/m
% Kc - Band-limited spring constant, N/m
% Kt - Tire stiffness, N/m
% OP - Model options: OP = 1, Quarter car model
%                               OP = 2, Normalized natural frequency quarter
care model

% Outputs
% A, B, C, D, 4 matrices for linear state space model
% Generate frequency response of quarter-car model from inputs, (r) to
% outputs, (body travel, body velocity, wheel travel, wheel velocity,
% suspension deflection and body acceleration).

Case = OP;           % Model options
switch Case
    case 1
% State-space matrices
A = [    0    1            0    0    0
      -Ks/Ms  0            Ks/Ms  0    1/Ms
           0    0            0    1    0
          Ks/Mu  0    -(Ks+Kt)/Mu  0    -1/Mu
           0   -Kc            0    Kc   -Kc/Cc];

B = [    0    0            0    Kt/Mu    0]';

C = [    1    0            0    0    0
       0    1            0    0    0
      -Ks/Ms  0            Ks/Ms  0    1/Ms
           0    0            1    0    0
           0    0            0    1    0
          Ks/Mu  0    -(Ks+Kt)/Mu  0    -1/Mu
           1    0            -1    0    0];

D = [0 0 0 0 0 0 0]';

QCMDI = ss(A,B,C,D);
QCMDI.StateName = {'z (m)'; 'zd (m/s)'; ...
                  'y (m)'; 'yd (m/s)'; ...
                  'F2 (N)'};
QCMDI.InputName = {'h'};
QCMDI.OutputName = {'z (m)'; 'zd (m/s)'; 'zdd (m/s^2)'; ...
                   'y (m)'; 'yd (m/s)'; 'ydd (m/s^2)'; ...
                   'sd (m)'};

% Plot the frequency response of the quarter-car model from inputs,
(h)
% to outputs, sprung mass position, sprung mass velocity.
figure
bodemag(QCMDI({'z (m)', 'zd (m/s)'}, 'h'), 'k', {1 1000}); grid
title(['Gain from road profile (h) '...
      'to sprung mass position (z) and velocity (zd)'])

```

```

% Plot the frequency response of the quarter-car model from inputs,
(h)
% to outputs, unsprung mass position, unsprung mass velocity.
figure
bodemag(QCMI({'y (m)', 'yd (m/s)'}, 'h'), 'k', {1 1000}); grid
title(['Gain from road profile (h) '...
      'to unsprung mass position (z) and velocity (yd)'])

% Plot the frequency response of the quarter-car model from inputs,
(h)
% to outputs, sprung mass acceleration, unsprung mass acceleration.
figure
bodemag(QCMI({'zdd (m/s^2)', 'ydd (m/s^2)'}, 'h'), 'k', {1 1000}); grid
titles{1} = 'Gain from road profile (h)';
titles{2} = 'to sprung mass acc. (zdd) and unsprung mass acc.
(ydd)';
title(titles)

% Plot the frequency response of the quarter-car model from inputs,
(h)
% to outputs, suspension travel.
figure
bodemag(QCMI({'sd'}, 'h'), 'k', {1 1000}); grid
title('Gain from road profile (h) to suspension travel (sd)')

case 2
% Normalization
wN = sqrt(Ks/Ms); % Sprung mass natural frequency

% State-space matrices
A = [
      0 1 0 0 0 0
     -Ks/Ms/wN^2 0 Ks/Ms/wN^2 0 1/Ms/wN
      0 0 0 1 0 0
     Ks/Mu/wN^2 0 -(Ks+Kt)/Mu/wN^2 0 -1/Mu/wN
      0 -Kc 0 Kc -Kc/Cc];

B = [ 0 0 0 Kt/Mu/wN 0]';

C = [
      1 0 0 0 0 0
      0 1 0 0 0 0
     -Ks/Ms/wN^2 0 Ks/Ms/wN^2 0 1/Ms/wN
      0 0 1 0 0 0
      0 0 0 1 0 0
     Ks/Mu/wN^2 0 -(Ks+Kt)/Mu/wN^2 0 -1/Mu/wN
      1 0 -1 0 0 0];

D = [0 0 0 0 0 0]';

QCMI = ss(A,B,C,D);

```

```

QCMI.StateName = {'z (m)';'zd (m/s)';...
                 'y (m)';'yd (m/s)';...
                 'F2 (N)'};
QCMI.InputName = {'h'};
QCMI.OutputName = {'z (m)';'zd (m/s)';'zdd (m/s^2)';...
                  'y (m)';'yd (m/s)';'ydd (m/s^2)';...
                  'sd (m)'};

% Plot the frequency response of the quarter-car model from inputs,
(h)
% to outputs, sprung mass position, sprung mass velocity.
figure
bodemag(QCMI({'z (m)','zd (m/s)'},'h'),'k',{0.1 100}); grid
xlabel('Normalized Frequency \omega/\omega_N')
title(['Gain from road profile (h) '...
       'to sprung mass position (z) and velocity (zd)'])

% Plot the frequency response of the quarter-car model from inputs,
(h)
% to outputs, unsprung mass position, unsprung mass velocity.
figure
bodemag(QCMI({'y (m)','yd (m/s)'},'h'),'k',{0.1 100}); grid
xlabel('Normalized Frequency \omega/\omega_N')
title(['Gain from road profile (h) '...
       'to unsprung mass position (z) and velocity (yd)'])

% Plot the frequency response of the quarter-car model from inputs,
(h)
% to outputs, sprung mass acceleration, unsprung mass acceleration.
figure
bodemag(QCMI({'zdd (m/s^2)','ydd (m/s^2)'},'h'),'k',{0.1 100}); grid
xlabel('Normalized Frequency \omega/\omega_N')
titles{1} = 'Gain from road profile (h)';
titles{2} = 'to sprung mass acc. (zdd) and unsprung mass acc.
(ydd)';
title(titles)

% Plot the frequency response of the quarter-car model from inputs,
(h)
% to outputs, suspension travel.
figure
bodemag(QCMI({'sd'},'h'),'k',{0.1 100}); grid
xlabel('Normalized Frequency \omega/\omega_N')
title('Gain from road profile (h) to suspension travel (sd)')
end

end
end

```

ISO 8608 Road Profile Standard

```
function [ n_max,n_eff,Delta_n,N,Road ] = ISO8608k( B,L,nL,nU,k,Plot )
```

```

% Generate random road from PSD according to ISO 8608 k-method

% Frequency Dependent Shock Simulation
% Thesis adviser: Dr. Woods
% Student: Ye Hong
% Date: 9/18

% Inputs
% B - Sampling interval, m
% L - Length of profile, m
% nL - Lower boundary of spatial frequency, m^-1
% nU - Upper boundary of spatial frequency, m^-1
% k - ISO 8608 road profile classification
% Plot - Plot options: Plot = 0 for no plot;
%          Plot = 1 for plot;

% Outputs
% n_max - Maximum theoretical sampling spatial frequency, m^-1
% n_eff - Maximum effective sampling spatial frequency, m^-1
% Delta_n - Discretized spatial frequency spacing, m^-1
% N - Maximum number possible sampling size
% Road - Road signal

% Generate Elevation (mm) vs. Distance (m) plot

%%
% Maximum theoretical sampling spatial frequency n_max
n_max = 1/B;

% Maximum effective sampling spatial frequency n_eff
n_eff = n_max/2;

% Discretized spatial frequency spacing
Delta_n = 1/L;

% Number of iterations
N = L/B;

% Spatial frequency band
ni = nL:Delta_n:nU;

% Amplitude (from 0 to Amp_i)
Amp_i = 1e-3*2^k*sqrt(Delta_n)*0.1./ni;

% Random phase angle
phi = 2*pi*rand(size(ni));

% Abscissa variable x
x = 0:B:L;

```

```

% Road profile (From -Ampi/2 to Ampi/2)
h = zeros(size(x));
for j=1:length(x)
    h(j) = sum( Ampi/2.*cos(2*pi*x(j).*ni + phi) );
end
Road = [ x; h ];

% Plot road profile
if Plot == 1
    figure
    plot( x,h*1000,'k' ); grid
    xlabel('Distance (m)'); ylabel('Elevation (mm)');
    title('ISO 8608 Road Profile Classification - k');
end
if Plot == 0
    disp('Set Plot = 1 for road profile plot');
end

end

```

Performance Index

```

function [ R1,PIr,R2,PIh ] = PIM( A,B,C,D,Ms,Mu,Kt,x0,Road,V,dt,tmax )
% Performance index - Ride & Handling

% An Investigation of Band-Limited Fluid Damper Configuration
% Using Lumped Parameter Modeling
% Thesis adviser: Dr. Robert L. Woods
% Student: Ye Hong
% Date: 11/30

% Inputs
% A,B,C,D - State variable matrices
% Ms - Sprung mass, kg
% Mu - Unsprung mass, kg
% Kt - Tire stiffness, N/m
% x0 - Initial state
% V - Vehicle speed, km/h
% dt - Time steps size,s
% tmax - Maximum simulation time, s
% Road - Road profile

% Outputs
% R1 - Instantaneous Sprung mass acc. vs gravatiational acc.
% PIr - Performance index - ride
% R2 - Instantaneous normal force - total weight vs total weight
% PIh - Performance index - handling

g = 9.81; % Gravitational acceleration, m/s^2

% State-space equations

```

```

qcm = ss(A,B,C,D);

% simulation conditions
rx = Road(1,:); rh = Road(2,:); % Seperate road profile data
dx = rx(2) - rx(1); % Spacial step for input data
v = V/3.6; % Converts km/h to m/s
hd = [0 diff(rh)/dt]; % Road profile velocity
t = 0:dt:tmax; % Time steps to record output
x = v*t; % Space steps to record output
u = interp1(rx,hd,x); umf = 1; % simulation input

% Simulate quarter car model
s = lsim(qcm,u*umf,t,x0);
z = s(:,1); % Sprung mass position
zd = s(:,2); % Sprung mass velocity
zdd = s(:,3); % Sprung mass acceleration
y = s(:,4); % Unsprung mass position
yd = s(:,5); % Unsprung mass velocity
ydd = s(:,6); % Unsprung mass acceleration
sd = s(:,7); % Suspension deflection

% Calculate performance index for ride
R1 = zdd./g;
PIr = rms(R1);

% Calculate performance index for handling
W = (Mu+Ms).*g;
Fh = Kt.*sd;
R2 = Fh./W;
PIh = rms(R2);

end

```

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Biographical Information

Ye earned his Bachelor of Science in Mechanical Engineering from The Ohio State University, Columbus, Ohio. He worked as an adviser on OSU/Honda R&D Americas joint project in advising an undergraduate design team. The project was on optimizing the performance of localized corrosion rate measurement tool. His research area includes solid mechanics, simulation & modeling, system dynamics, and numerical optimization.