SOLVING THE RUBIK'S CUBE WITH PARALLEL PROCESSING

By

ASLESHA NARGOLKAR

Presented to the Faculty of the Graduate School of

The University of Texas at Arlington in Partial Fulfillment

Of the Requirements

For the Degree of

MASTER OF SCIENCE IN COMPUTER SCIENCE ENGINEERING

THE UNIVERSITY OF TEXAS AT ARLINGTON

May 2006

ACKNOWLEDGEMENTS

I would like to express my sincere thanks to my Dr. Ishfaq Ahmad for his continuous support, guidance and encouragement through out the course of my research. It was because of him that I was able to do research on this topic and complete my thesis. I would also like to thank Patrick Mcguian for all his help in running the experiments on the DPCC cluster.

I also owe my thanks to my mother, father, brother and my family for their support and encouragement. The same is true for all my friends who remained behind me and encouraged me throughout my Masters study.

November 21, 2005

ABSTRACT

PARALLEL PROCESSING A RUBIK'S CUBE

Aslesha Nargolkar, M.S.

The University of Texas at Arlington, 2006

Supervising Professor: Dr. Ishfaq Ahmad

This thesis investigates parallel processing techniques for solving the 3 x 3 x 3 Rubik's Cube. We explore various state-space search based algorithmic approaches to optimally solve the Cube. The parallel processing approach is based on IDA* using a pattern database as the underlying heuristic because of its well established effectiveness. The parallel algorithm is an extension of the Michael Reid algorithm which is sequential. The parallel algorithm exhibits good speedup and scalability. Nearly 150 random as well as symmetrical cube configurations were tested for the experiments on sequential and parallel implementations. The proposed parallel algorithm using master-slave type of load balancing proves efficient in terms of time as well as memory resources while yielding an optimal solution to find the state of a Rubik's cube. Parallel processing helps in solving a Cube with initial cube configurations having solutions at a higher depth level

in the search tree. Various comparative results are provided to support the efficiency of the parallel implementation.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

LIST OF TABLES

CHAPTER 1

INTRODUCTION

Improving an algorithm to solve a Rubik's cube requires a thorough analysis of past efforts in both multi-processor algorithms and sequential solutions to the problem. With the extensive experimentation carried out, it became clear that Manhattan distance heuristic, as was used in our first approach, was not the best approach. During the course of the research work and the experimentation done, it became clear that pattern databases as modified by Michael Reid [26] is perhaps the best existing heuristic for this problem. The use of pruning and symmetry improve the pattern database and hence the performance of the algorithm. Also, parallelization of the search would maximize the success of the algorithm being used while reducing the time required in finding a solution for larger depths (above 18). Hence all the necessary issues related to the parallel approach like the load balancing techniques and the task distribution and scalability would need to be considered. Although the heuristic used is improved, the algorithm used for searching remains the same, IDA*. The experiments carried out show the number of nodes generated and the time taken for the most difficult known configuration: the super flip configuration. This is a 24-depth configuration.

1.1 The 3x3x 3 Cube

Rubik's cube is considered to be one of the most famous combinatorial puzzles of its time. Erno Rubik of Hungary invented it in the late 1970s. The standard version of

this cube consists of a 3 X 3 X 3 cube (Figure 1.1), with a different color on each face: red, blue, green, yellow, white and orange. The faces are divided into 9 squares each, which can be scrambled by rotating a row (or column) at a time. It is divided into three 3×3×1 slices along each of *x*, *y*, and *z* axes such that each slice can be rotated a quarter turn clockwise or counterclockwise with respect to the rest of the cube. The slices divide each face of the Cube into 9 facelets. The slices also divide the Cube into $3^3 = 27$ pieces, or sub cubes, 26 of which are visible. In its goal state, each of the six sides has its own color. Each visible piece has one, two or three facelets that are a unique subset of the colors, which makes it distinct from the others. This preserves the orientation of the piece as it changes position as the slices are rotated.

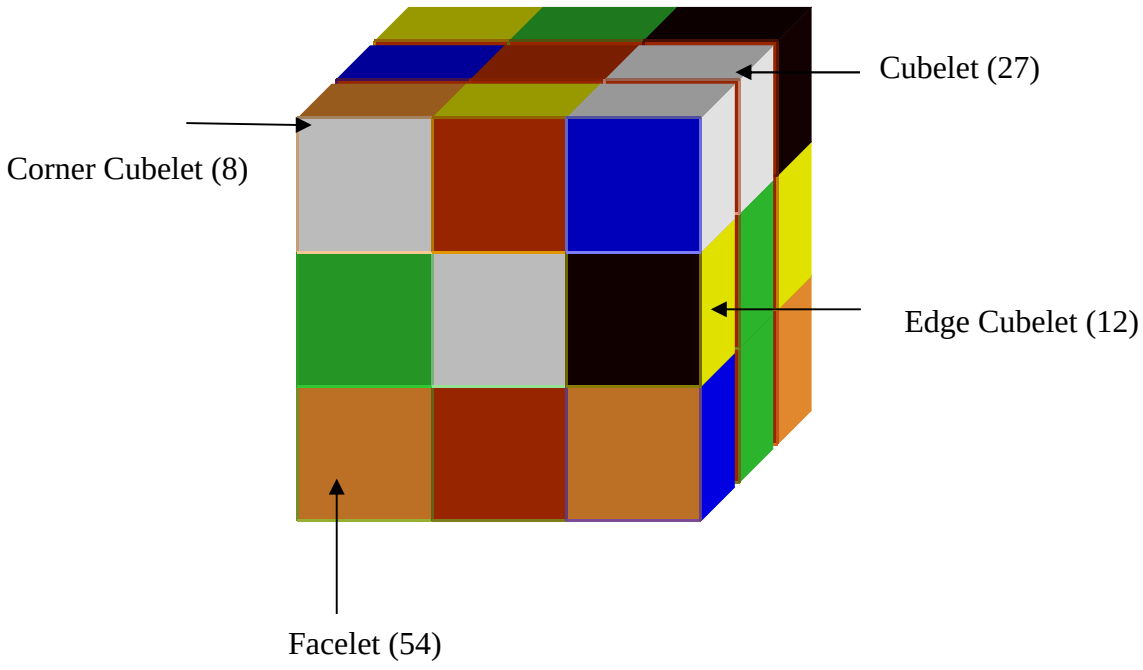


Figure 1.1: The Rubik's Cube
(Note the numbers in bracket denote the number of those pieces in a 3x3 Cube)

1.1.1 Different pieces of the Cube

Of the 27 sub cubes, or *cubies*, 26 are visible, and of those only 20, the 8 *corner pieces* with three facelets and the 12 *edge pieces* with two facelets, actually move. The corner pieces only move to other corner positions, or *cubicles*, and edge pieces only move to other edge cubicles. The center pieces with a single facelet in the center of each face merely rotate. The corner pieces can be twisted any of three ways. Likewise, the edge pieces can be flipped either of two ways. Thus, the corner pieces have three orientations each, and the edge pieces have two orientations each.

1.1.2 Singmaster notation

Singmaster notation devised by David Singmaster [2] describes a sequence of moves. Clockwise turns of the six outer layers, or slices, are denoted by the capital letters where F is front, B is back, U is up, D is down, L is left and the right face is denoted by an R:

<F, B, U, D, L, R> which are 90 degrees clockwise face moves

<F2, B2, U2, D2, L2, R2> which are 180 degrees clockwise face moves

<F', B', U', D', L', R'>which are 270 degrees clockwise i.e. 90 degrees anticlockwise face moves

If a subsequence of moves is repeated then it is listed in parenthesis with an exponent equal to the number of repetitions. Singmaster notation is also used to label each of the cubies and cubicles. Lowercase italicized letters given in clockwise order of

how the two or three sides intersect at a given corner or edge are used to describe a cubie. Uppercase is used to denote cubicles. Since there are either two or three possibilities depending on which side is listed first, the first letter is given in order of *U, D, F, B, R, L*. While the cubicles never change orientation, the cubies do. So this precedence in labeling cubies is with respect to the goal state. Thus *URF* and not *RFU* would be used to denote the upper front right-hand cubicle. The cubie corresponding to that cubicle would be denoted by *urf* if its orientation has not changed from the goal state and as either *rfu* or *fur* if it had. The orientation is determined by keeping track of how the pieces move from cubicle to cubicle with respect to the initial state. If an R move was applied to the goal state, then *u* facelet of the *urf* cubie would move to the *b* facelet of the *UBR* cubicle. Thus, with $u \rightarrow b$, $r \rightarrow r$, and $f \rightarrow u$, the corner piece would then become *fur*.

1.1.3 The Metrics

To reach the goal state from any random configuration, a sequence of moves is applied. The minimum number of these moves required to solve the Cube is known as the *distance* from the goal state while the maximum distance is the *diameter* of the Cube. These moves or distances are measured using either the Half Turn Metric (HTM) or the Quarter Turn Metric (QTM). In the first metric, only half turns are allowed while in the quarter turn metric, only quarter turns are allowed. Such a sequence of moves used to get to the goal state is known as a macro operator or a macro.

The Cube can be represented by a graph in which a vertex represents a unique state and an edge represents a transition between adjacent states. Since there are 18

elementary moves, there are 18 possible adjacent states. The branching factor is considered to be the ratio of the number of nodes explored at a given depth (d+1) over the number of nodes explored at a depth d.

1.1.4 The Super flip position

Cube solving has been proven to generally take less than 20 moves. But the diameter of the cube is said to be the super flip position which is known to take 24 moves. [26] It is not maximally distant from the start position. It is also known as 12-flip, all-flip, all-edges-flipped.

1.1.5 Solving the Cube

This puzzle is challenging to solve because of the large number of different states that can be reached from any configuration. The goal state in this puzzle is the state with all the squares on each side of the cube having the same color. In addition, large memory requirements and computational resources are required to solve this problem, which is also the reason why it has been labeled as a very difficult problem. Although several algorithms have been successfully developed to solve the Rubik's cube on sequential machines, they all face the same problem of limited memory and computational requirements, especially when trying to find an optimal solution and when we consider the required amount of time for those algorithms to find a solution. Therefore, parallelizing the algorithm seems to be promising in terms of reducing execution time and having enough resources to find a solution.

The Rubik's cube has also been an interesting research problem in the Artificial Intelligence area. Researchers had reached a consensus that an informed searching

methodology is the most feasible way to find a solution in such a vast search space. The fact that finding a solution to this puzzle will require a large memory and better computational resources has made it difficult to run efficiently on sequential machines, thus parallelizing the search is the next best choice along with reducing the computational complexity. It is however true that we need to ensure that we make the best of the scalability and all other available resources of the parallel hardware. Since each cube configuration leads to a large number of states, the search space unfolds in the shape of a tree, so parallelizing can be done by handing over the branches of the tree to separate processors for performing search independently. One such architecture that could fulfill all our requirements regarding memory and power resources is the cluster of workstations.

1.2 Background

A standard 3 X 3 X 3 Rubik's cube can have 4.3252×10^{19} different states from a given configuration. In our initial approach, to reduce large computational complexity involved in finding a solution to this puzzle, we had used a two dimensional flat representation obtained by opening the cube along its edges. This representation can then be converted into an array representation with each color being represented by a numerical value.

The search algorithm used was the **Iterative Deepening A* (IDA*) algorithm** which is an optimal, memory bounded heuristic search. A heuristic function provides an estimate of the distance of a particular state from a goal state. The heuristic used was the one Richard Korf (1997) [1, 5] had suggested. It was a modified version of the

Manhattan heuristic used in the sliding puzzle where the Manhattan distance of only the edge cubes was calculated.

The performance of the search can be improved by using parallel hardware. But the choice of heuristic is another main factor affecting the performance of the algorithm. Heuristics functions should be chosen properly to avoid inherent problems pertaining to high time complexity. If there is no overestimation of depth then the proposed method is guaranteed to improve the performance of IDA* algorithm.

1.3 Organization of the thesis

The rest of the thesis is organized into different chapters, starting with Chapter 2 where all the standard optimal algorithms are explained. This same chapter also explains the standard IDA* algorithm along with the IDA* algorithm used with pattern databases. The actual method of implementation with the experiment results is given in Chapters 3, 4, 5. The thesis is concluded with Chapter 6.

CHAPTER 2

STANDARD ALGORITHMS

Michael Reid's optimal cube solver [26] is based on ideas developed by Herbert Kociemba [24] and Professor Richard Korf [5]. This is supposed to be a practical implementation of "God's Algorithm" [7] for Rubik's Cube. He has made some improvements in the Korf's method.

2.1 God's Algorithm

God's algorithm [7] uses a lookup table that is keyed over the various instances of the problem. Rather than giving the solution, it gives the minimum number of moves remaining to reach the goal state. Then to solve a particular instance one simply chooses the next move that is one closer to the goal state. The lookup table can be further refined to only store the number of moves left modulo 3, which only requires 2 bits per entry. This is sufficient in picking the next move since any adjacent move is one less, the same or one more towards the solution.

2.2 Basic tree searches

This can be thought of as tree in which the root node is the initial state, and each level below is the nodes explored at that given depth. The nodes in the tree need not be unique. Pruning techniques can be used to reduce the likelihood of encountering duplicates.

2.2.1 Uniformed searches

Given the extreme number of state, 4.3×10^{19}, uninformed searches such as BFS (breadth first search) or DFS (depth first search) turn out to be impractical. While BFS a slow and methodical search that is guaranteed to produce an optimal solution, it doesn't work since all the states must be kept in memory, which clearly exceeds the memory limit of on any conventional computer.

DFS fails for several reasons. First of all, there is no guarantee it won't run into some infinite cycle. To correct this one can limit the depth to which it explores. However, then there is no guarantee it will find the goal state. To further correct the problem, one can employ an incremental DFS, in which one incrementally increases the depth being explored. But the advantage of DFS over BFS is that it can potentially find the solution quite quickly if it happens to select nodes that lead to the solution. It also has the advantage of only having a linear memory complexity that is $O(b\,d)$ where b is the asymptotic branching factor and d in this case is the depth of the search. The disadvantage of DFS used without iterative deepening is that it is not optimal in that there is no guarantee it will choose the shortest path, nor is it complete in that there is no guarantee it will explore the whole graph.

2.2.2 Informed Searches

Informed searches like best-first search keeps a queue of the nodes left to explore and orders them by some evaluation function that attempts to select the best

nodes to search first. Two versions of best-first search base the evaluation function either on the distance incurred or estimated distance remaining.

2.2.2.1 Greedy Best-First Search

Greedy best-first search orders the nodes in the queue based on an estimate of the distance remaining to the goal state. As the greedy estimate becomes accurate, this algorithm becomes more similar to God's algorithm. To increase the likelihood of the solution found to be optimal, it helps if the estimate never overestimates the distance remaining. Such an estimate is known as an admissible heuristic. This increases the chances that the shortest path will be found.

Greedy search is also a way of directing a standard DFS search. It chooses the next node to explore based on some greedy estimate of the distance remaining to the goal state. However, this approach cannot be used in conjunction with iterative deepening since that would force the tree to be explored up to a given depth. As such it suffers from the same limitations of DFS in that it is neither optimal nor complete. However, it is more likely to find the solution before iterative deepening DFS.

2.2.2.2 Uniform cost

Uniform cost best-first search, known more simply as uniform cost, orders the elements in the queue by the distance from the initial state. Here the distance for the Rubik's Cube is simply based on the number moves from the initial state, which makes this uniform cost equivalent to BFS. However, if one were to favor some moves over others and weight them accordingly, then uniform cost would still produce an optimal solution, i.e. a path to the goal with minimum weight.

2.2.2.3 A*

A* combines the two best-first searches of uniform cost with the greedy search using an admissible heuristic to produce an optimal search that is more efficient than standard BFS, by setting its heuristic to be the sum of the distance from the initial state and is admissible estimate of the distance to the goal state. The basic idea behind the proof of optimality is that by keeping track of distance one has come in addition to how far one has to go; one will never choose a path longer than necessary.

2.3 Iterative Deepening A*

Iterative-Deepening A*, also known as IDA*, was introduced by Richard Korf in 1985 [4]. It has since become the dominate method to solve this class of permutation puzzles to which Rubik's Cube belongs. Where A* fails, IDA* tends to succeed given strong enough admissible heuristic.

*2.3.1 Standard IDA**

IDA* is simply A* with iterative deepening. This reduces the memory complexity from $O(b^d)$ to $O(b\,d)$. It does this by keeping track of the maximum depth explored from the last iteration and incrementally increases it. By doing so, it no longer needs to store all the states in memory. The optimality of A* still applies, so IDA* also gives an optimal solution. The Manhattan distance is less useful as a heuristic for the Cube since if one were to total the number of moves necessary to place each cubie, one

must divide that total by eight since eight cubies are moved for each rotation of a slice of the Cube. This results in a very low heuristic value.

2.3.2 Problem with this approach

Although IDA* is the best fit to solve the Rubik's cube, as it was previously discussed in this paper, the search space size remains one of the main problems to solve in order to obtain an optimal solution. Therefore, we need to optimize IDA*, by reducing the search space.

Initially the algorithm was tested for obtaining solutions up to depths 12. But Korf mentions in his paper "Finding optimal solutions to Rubik's Cube using pattern databases" [5], that at depth over 15 are real significant moves and that any problem can generally be solved in nearly 18 moves. For the same purpose of experimentation of his approach, he had generated ten solvable instances of Rubik's Cube, one solvable in 16 moves, three 17 and six requiring 18 moves. For all further studies related to the Rubik's Cube solution these 10 instances were used as standard. We also carried out experiments on our sequential program [6] with these input configurations to get the time required and the number of nodes generated. These programs were run in the cluster of workstations at UTA, where every job submitted is assigned a dedicated processor.

All programs were running for more than 50 hours of processing at depth 14 itself. The programs were writing output every 20 minutes, indicating what depth they are working on, and the number of the iteration they are in, making sure they were not stuck in an infinite loop. The program was also modified to reduce the amount of

memory needed to store every node and therefore decreasing the possibility of memory problems. The program was extensively tested with depths up to 13, and the program reported the optimal solutions for each configuration of the cube that was tested. However, the results show that the technique used in the program may not be the most effective. The same is illustrated in the following charts:

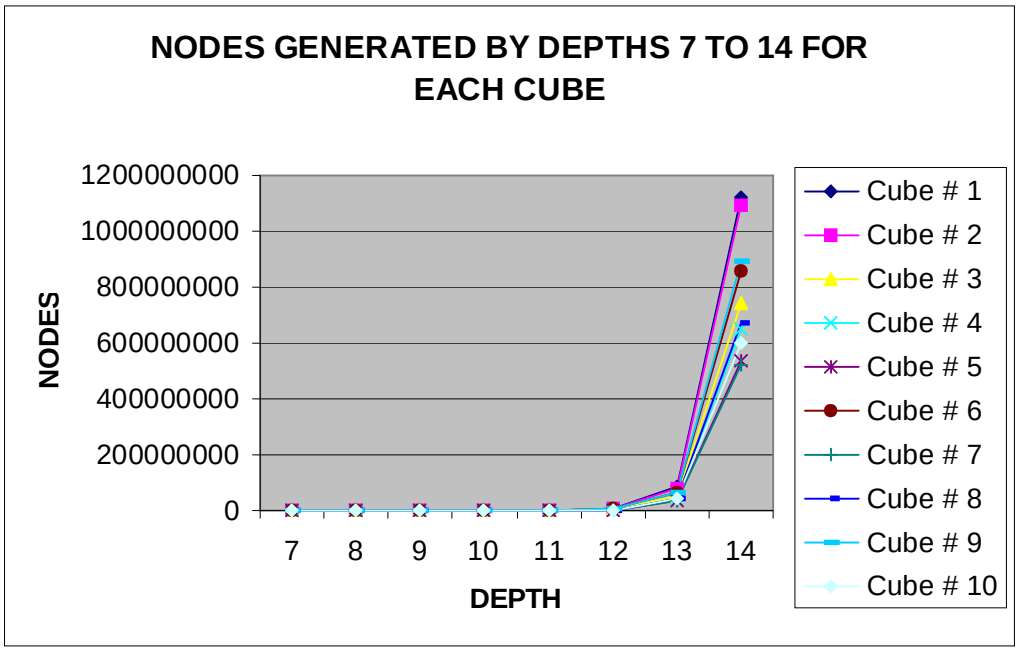


Figure 2.1: Number of nodes generated Vs Depth

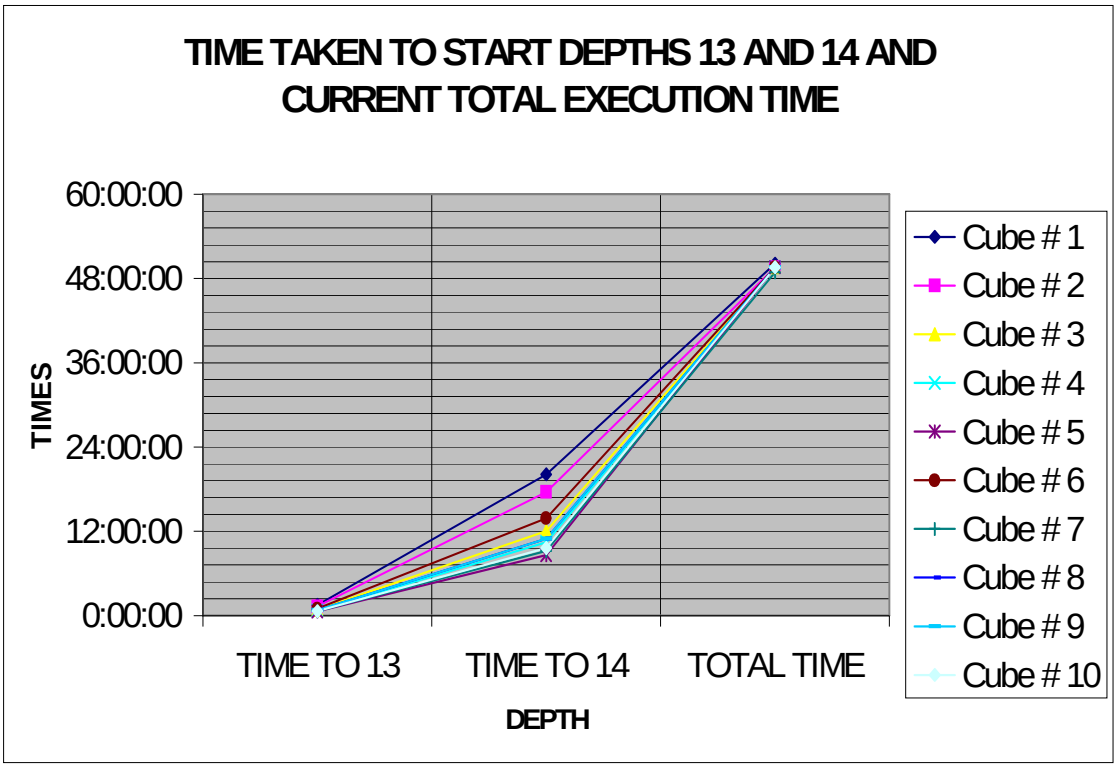


Figure 2.2: Execution time Vs Depth

2.3.3 Solution: IDA with Pattern Databases*

The idea of using large tables of precomputed optimal solution lengths for subparts of a combinatorial problem was first proposed by (Culberson and Schaeffer, 1996) [1]. They studied these pattern databases in the context of the 15-puzzle, achieving significant performance improvements over simple heuristics like Manhattan distance [5], and applied it to the Rubik's cube. The key idea is to take a subset of the goals of the original problem, precompute and store the exact number of moves needed to solve these sub goals from all possible initial states. Then the exact solution to the sub goals is used as a lower bound heuristic for an IDA* search of the original problem.

The idea behind the pattern database was to avoid the runtime computation of the heuristic, at the expense of the extra memory storage. Since we have a large computational engine in the form of a cluster we don't need to worry about the extra runtime computation.

2.4 Thistlethwaite's algorithm

By restricting the type of moves used, different subgroups could be formed. If only half moves were allowed then the subgroup will be <L2, R2, F2, B2, U2, D2>. There could be some more nested subgroups in this, like <L2, R2, F, B, U, D> formed by applying half turns only to the right and the left faces so that no edges can be flipped or disoriented. These subgroups help since they have a comparatively smaller order. Similarly, the corner orientations can be further fixed by restricting the half turns for the front and the back by the subgroup <L2, R2, F2, B2, U, D>. The goal state or the identity can be seen as a subgroup of these successively nested subgroups.

In the early 1980's, the English mathematician Morwen B. Thistlethwaite devised an algorithm [7] with the use of these nested subgroups in a lookup table. His solution progressed from one nested subgroup to the other until the goal state. There were tables for every stage and at every stage to move from one subgroup to the next; the properties of the subgroup leading to the factors composing the two nested subgroups were used for indexing in the tables. This algorithm gave the first known diameter of the Cube as 52. With further improvements, he had improved the best known upper bound to 45.

Table 2.1 – Moves per stage in the Thistlethwaite's algorithm

Stage:	**1**	**2**	**3**	**4**	**Total**
Original algorithm:	7	13	15	17	52
Improved algorithm:	7	13	15	15	50
Best possible:	7	10	13	15	45

2.5 Kociemba's algorithm

Herbert Kociemba of Germany [24] developed an algorithm that to date is the algorithm that produces the best near optimal solutions in the shortest amount of time. It typically runs in a few minutes whereas finding an optimal for the same state may take far longer. Kociemba came upon the idea of only using <L2 R2, F2, B2, U, D> as an intermediate nested subgroup. Using IDA* with this sub group for his pattern database instead of lookup tables, he then searched for a position in it. By keeping track of the maneuvers to get there and subsequently using IDA* again to search from that position in the sub group to the goal state, he obtained a near optimal solution by combining the two sets of maneuvers.

2.6 Korf's algorithm

In his 1997 paper [5], Korf used the eight corners as one sub problem and two sets of six of the twelve edges for the two others, which completely covers the Cube in

that any of the 20 movable pieces is in one of these three sets. For his heuristic he examined the eight or six cubies in one of these smaller problems (smaller in the sense of far fewer states) and looked up the remaining number of moves to solve that sub problem. To create an admissible heuristic, he took the maximum of the three.

Michael Reid [26] improved this approach first suggested by Korf using the pattern database to find optimal solutions for the Rubik's Cube. This particular pattern database produced a better heuristic than the ones based off of corners and edges used by Korf. In fact IDA* ran twenty times faster for Reid on a slower computer using less memory. The main concept here is that pattern databases need not be based off subset of puzzle pieces, but more generally, can be based off any subgroup of the appropriate order.

2.7 Reid's algorithm

Michael Reid [26] uses distances to the intermediate position: <U, D, F2, R2, B2, L2> for the pattern database. If there is a group G and a subgroup H, then for each element g from G the set {a*g | a belongs to H} is called a right coset of H. Each scrambled cube can be seen as a permutation with attached orientations. Coordinates represent cosets; each coset usually consists of many permutations. Equivalent cubes have the same structure but the number of moves necessary to solve them is the same. Equivalence is defined with permutations. For each cube there are up to 48 equivalent cubes, because the cube has 48 symmetries including reflections. Every closet in the subgroups is characterized by corner orientation, edge orientation, location of the four U-D slice edges. Since this gives a lot of configurations, symmetry is used. Also, 2

coordinates edge and location are combined into a single coordinate and it is divided by the 16 symmetries. A BFS is done in the coset-space to calculate the distances to be stored.

CHAPTER 3

PARALLEL PROCESSING

3.1 Algorithm

The parallel implementation aims to save time as well as memory resources while doing the extensive search to the goal state. In the parallel implementation, every processor searches separately to get a solution to the cube. Since every processor will need a copy of the pattern databases and to avoid communication to pass this whole structure to every processor, the initialization part of these databases is done by every processor. The communication is further lessened with every processor having its own copy of the cube configuration and initial variables to begin the processing. In this parallel implementation, every processor searches the same tree but the iterative deepening A* algorithm runs on every processor with different search limits. The work distribution is done using a master-slave technique where every processor communicates with the master, or processor 0, for the current search limit that it has to search with. This assures that no processor is left without any work. Though processor 0 has to communicate with all the remaining processors, it does take part in computation too.

The algorithm terminates when goal state is found by one of the processors with an optimal path. When a processor searching along a particular branch

of the tree comes across the goal node, it sends the goal found information first to processor 0 and then to the others. This goal found declaration is done in an asynchronous manner. Before beginning the search, every processing element waits for the goal found declaration using an asynchronous receive. Once the node is found an exit message is sent to all the processors by the processor which finds the goal using a non-blocking send call. Upon the receipt of the goal found declaration every processor exits the communication world if the goal found is optimal and the processor which found the goal prints the goal along with the steps to reach the goal before exiting the communication world itself.

The different search limits help in finding the solution to the configuration faster than the serial implementation because an increase in the search limit means more pattern subgroup matches from the pruning tables or pattern databases. And these could mean a faster and shorter search at most of the times. Another factor that aids in faster search is the symmetry, a symmetrical configuration is observed to be running faster than an asymmetrical one.

3.2 Performance metrics

The behavior or performance of a parallel program depends on various variables like the number of processors, the size of the data being used for the experiments, the interprocessor communications limit and also the available memory among the others. We have experimented with different numbers of processors but the size of data remains the same for any input given. As far as the memory is concerned,

the use of UTA clusters has helped us look over this concern but the communication still needs to be taken care of.

Three different metrics are commonly used to measure performance: execution time, efficiency and speedup. A parallel program's execution time or wall-clock time as it is referred to is an obvious performance measure. This time is the time elapsed from when the first processor starts executing a problem to when the last processor completes execution. The relative efficiency and relative speedup are two performance metrics which are independent of the problem size.

3.2.1 Execution time

The *execution time* is made up of three components: computation time, idle time and communication time. Computation time is the time spent performing computations on the data whereas communication time is the time taken for processes to send and receive messages and the time spent by process to wait for data from other processors is the idle time.

The *computation time* depends on the problem size and the specifics of the processor. Ideally it is the ratio of time required for the serial algorithm and the number of processing elements but might be different for different algorithms. The *communication time* includes the latency which is the time required for initialization of the communication. The other time included in the communication time is the actual time taken to send a message of a particular length. This time depends on the message length and the physical bandwidth. The *idle time* is the time when the processor is neither communicating nor is it computing. This is the reason we always try to

minimize the processors idle time with proper load balancing and efficient co ordination of processor computation and communication.

The performance of the parallel algorithm with respect to this metric is mentioned in the chapter of results.

3.2.2 Efficiency

The relative efficiency is defined as

$$T1/ (P*Tp),$$

Where **T1** is the execution time on one processor and **Tp** is the execution time on **P** processors whereas the absolute efficiency is defined by making the time T1 as the execution time on a processor of the fastest sequential algorithm. It is sometimes possible to get efficiencies greater than 1. The efficiency values are calculated with varying numbers of processors for comparison purposes.

3.2.3 Speedup

Relative speedup is defined as T1 / Tp, where T1 is the execution time on one processor and Tp is the execution time on P processors whereas the absolute speedup is defined by making the time T1 the execution time on a processor of the fastest sequential algorithm. In certain conditions, a speedup of greater than P can be achieved.

3.2.4 Serial fraction

According to the Amdahl's Law, the speedup of a parallel program is effectively limited by the number of operations which must be performed sequentially; this is known as the *serial fraction, F*. This serial fraction is calculated as:

$$F = (1/\text{Speedup} - 1/P)/ (1-1/P)$$

Amdahl's law tells that the serial fraction F places a severe constraint on the speedup as the number of processors increase. Since most parallel programs contain a certain amount of sequential code, a possible conclusion of Amdahl's Law is that it is not cost effective to build systems with large numbers of processors. It is expected that the parallel implementation has a small serial fraction. A larger load imbalance results in a larger F and thus problems not apparent from speedup or efficiency can be identified. Communication and synchronisation overhead tends to increase as the number of processors increases. An increasing serial fraction may suggest a smaller grain size. A serial fraction tending to zero would help in achieving an ideal speedup while a value towards 1 will suggest hardly any speedup.

4.3 Performance evaluation of a parallel algorithm

When the actual performance of a parallel program differs from the predictions, it is necessary to check for the unaccounted overhead and speedup anomalies. The reasons for unaccounted-for overhead are as follows:

- *Load imbalances:* Computation and communication imbalances among the processors can affect the performance of the algorithm.
- *Replicated computation:* The deficiencies in the implementation can be pointed out by the disparities between the observed and predicted times.
- *Tool/Algorithm mismatch:* Inefficiencies can be introduced in the code due to incorrect tool or libraries used in the implementation.
- *Competition for bandwidth:* The total communication costs may be increased by concurrent communications, competing for bandwidth.

CHAPTER 4

IMPLEMENTATION

A solved cube is represented as (12 edges, 8 corners)

UF UR UB UL DF DR DB DL FR FL BR BL UFR URB UBL ULF DRF DFL DLB DBR

An example of scrambled cube is:

UL BD LB UF FL FD UR RF DR BU LD BR BDL FRU BRD RFD BLU URB FUL FLD

Reid [26] uses "0" if the twist does not change, "1" for a clockwise twist and "2" for an anti-clockwise twist. In this way we can add orientations in a simple way. For example, F(URF).c = UFL and F(URF).o = 1 in the table above tells us that the corner at position URF is replaced by the corner at position UFL and that the orientation of the corner which moves to the position URF is increased by 1 when performing a F move. A coordinate or also a tuple of several coordinates represent cosets corresponding to some subgroup H (if we use a tuple of coordinates, the corresponding subgroup H is the intersection of the subgroups defining the single coordinates). A coordinate itself or an index computed from two or three coordinates define the position in the pruning table.

In this position Reid stores the number of moves which are necessary to bring the cube back to the subgroup H. Because the goal state is always included in H, the number of moves stored in the pruning table is always a lower bound for the number of moves to bring the cube back to the goal state. This is essential to make the algorithm work

To reduce memory size, Reid actually does not store the number of moves but only the number of moves modulo 3. This is possible because each face turn changes the number of moves only by -1, 0 or 1. So when a face turn is applied it is easy to keep track of the number of moves, this number for the initial state also can be reconstructed with the table mod 3: From the initial state try which one of the 18 face turns decreases the number modulo 3. Repeat this until you have reached the goal state and count the number of moves you needed to do so.

4.1 Use of Symmetry

In terms of three dimensional geometry, symmetry performs a transformation upon some solid figure, in this case a cube, using rotation, reflection or inversion, such that the resulting solid occupies the same space it did before the transformation. This transformation may correspond to an actual physical movement of the solid as is the case with rotation or it may not as is the case with reflection and inversion. These symmetries do help in improving the algorithm by reducing the number of nodes generated. This observation is evident form the results.

4.2 Creation of pattern database

The table is generated in a breadth-first "forward-search" manner. Depth 0 is stored at the position of the goal state and all 18 moves are applied to this state. At the corresponding positions depth 1 is stored. In the next pass the 18 moves are applied to all states corresponding to those positions in the pruning table which have an entry 1. And the process continues. If there are not many empty entries left in the pruning table, we flip to "backward search". We apply the 18 moves to all permutations which belong to empty entries and look if the result is a permutation which has a entry corresponding to the depth d of the last pass. In this case we fill the entry with d+1. In this way we save a considerable amount of time when generating the tables.

4.3 Pseudo Code for the parallel implementation

```
1)  Initialization of the cube
2)  Initialization of the pattern databases
3)  Expand the initial node
4)  if(master)
5)          distribute the initial search limit to each of the processor
6)          Repeat
7)                  Receive request from one of the processors
8)                  Check if goal reached
9)                  If(goal reached)
10)                         Send termination message to other processors and stop
11)                 Else
12)                         Send next work to the requesting processor
13)                 End if
14)                 Search tree with the current search limit
15)         Until solution found or end of tree
16) Else
17)         Receive the search limit from the master
18)         Check for the Exit tag
19)          If (exit tag)
20)                 Stop
21)          End if
```

22) Search the tree with the new search limit
23) If (goal reached)
24) Send termination message to the master
25) Else
26) Request for new search limit.

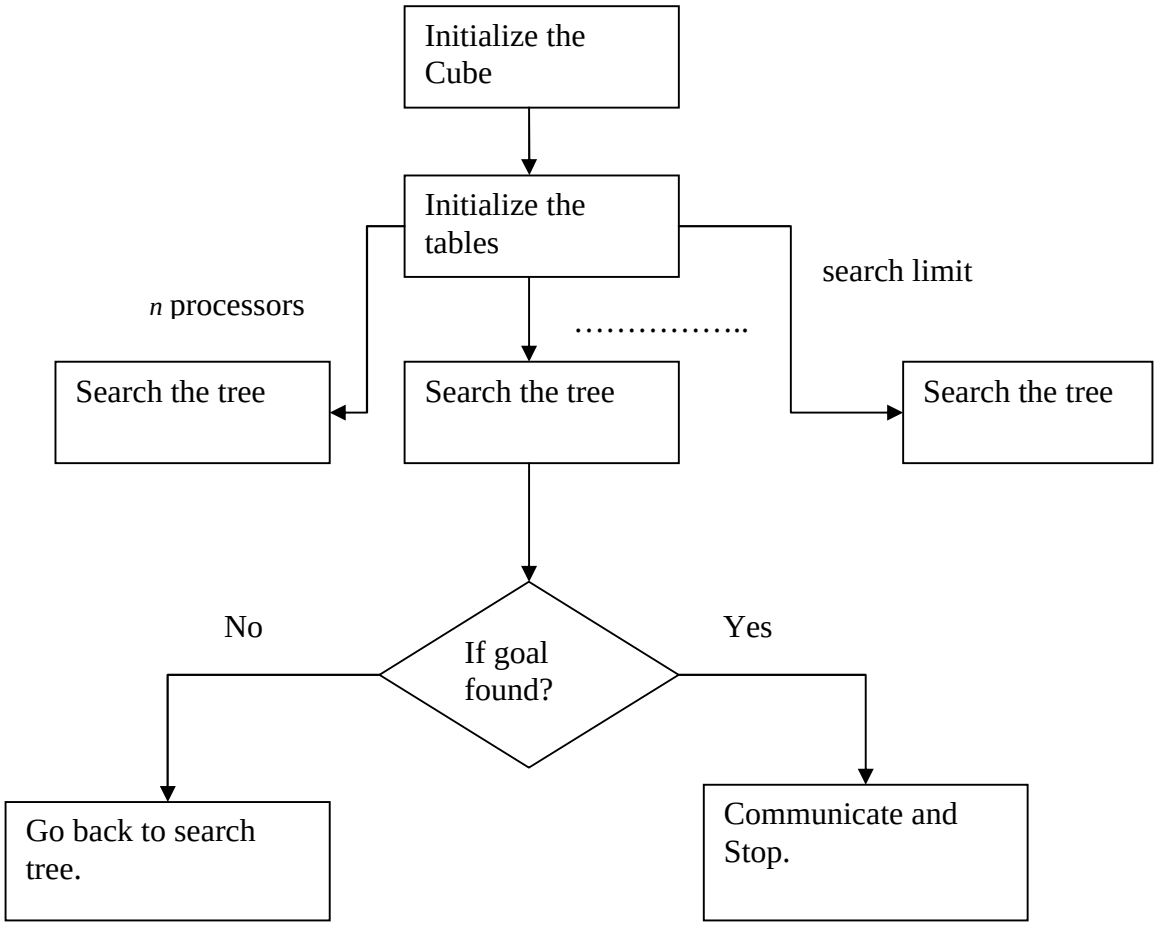


Fig 4.1 Data Flow Diagram

CHAPTER 5

EXPERIMENTAL METHODOLOGY

We ran several experiments in the pursuit of our research. We wanted to confirm that the algorithm we have chosen is the best fit. The program was executed for several randomly generated test cases at varied depths. The graphs below for sequential as well as parallel implementation are drawn for the average times of five configurations of every depth.

5.1 Results Analysis

The aim was to carry out to extensive experimentations for different cube configurations at various depths. The plots for the same are plotted in this chapter. All these programs were submitted as batch programs on the UTA DPCC cluster using MPI with C. For the sequential algorithm, configurations of depth 12-22 are tested while results for parallel implementations are from depth 19 onwards because such higher depths require more time and hence we need to parallelize them.

Section 5.1.1 shows graphs of execution times for the sequential implementation, and section 5.1.2 for the parallel implementation and the comparison of the two is done in section 5.1.3.

5.1.1 Results for the sequential implementation

Figures 5.1, 5.2, 5.3 are plotted to show the average time spent to get to a solution of that particular depth. The average time is calculated for five configurations of each depth, the times for which are listed in the corresponding tables. Figure 5.3 presents the average time Vs depth plot for all the depth from 12-22 whereas figures 5.1 and 5.2 are two parts of figure 5.3.

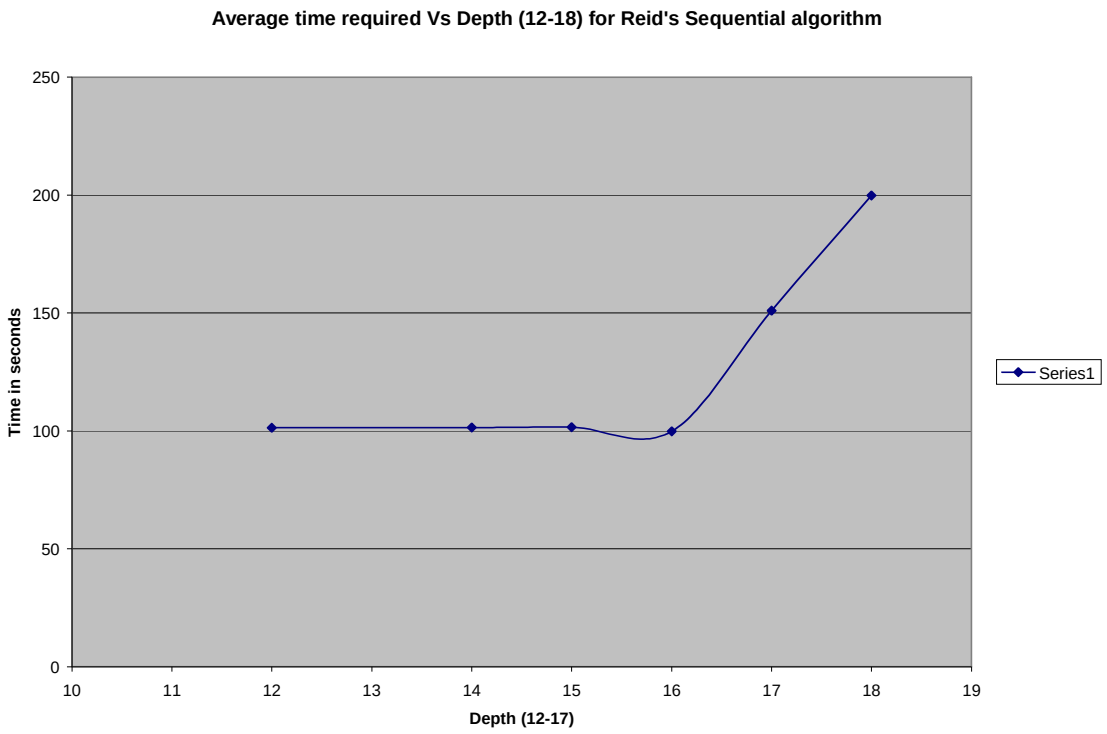


Fig 5.1: Average time Vs Depth (12-18) for Reid's sequential implementation

Table 5.1- Data for average time Vs depth (12-18) for Reid's sequential implementation

Depth	Time in seconds for five configurations					Average time in seconds
	Conf 1	Conf 2	Conf 3	Conf 4	Conf 5	
12	103	101	101	102	100	101.3
14	100	103	104	100	100	101.4
15	103	104	100	100	101	101.6
16	99	101	100	99	100	99.8
17	315	104	104	118	114	151
18	379	166	157	155	142	199.8

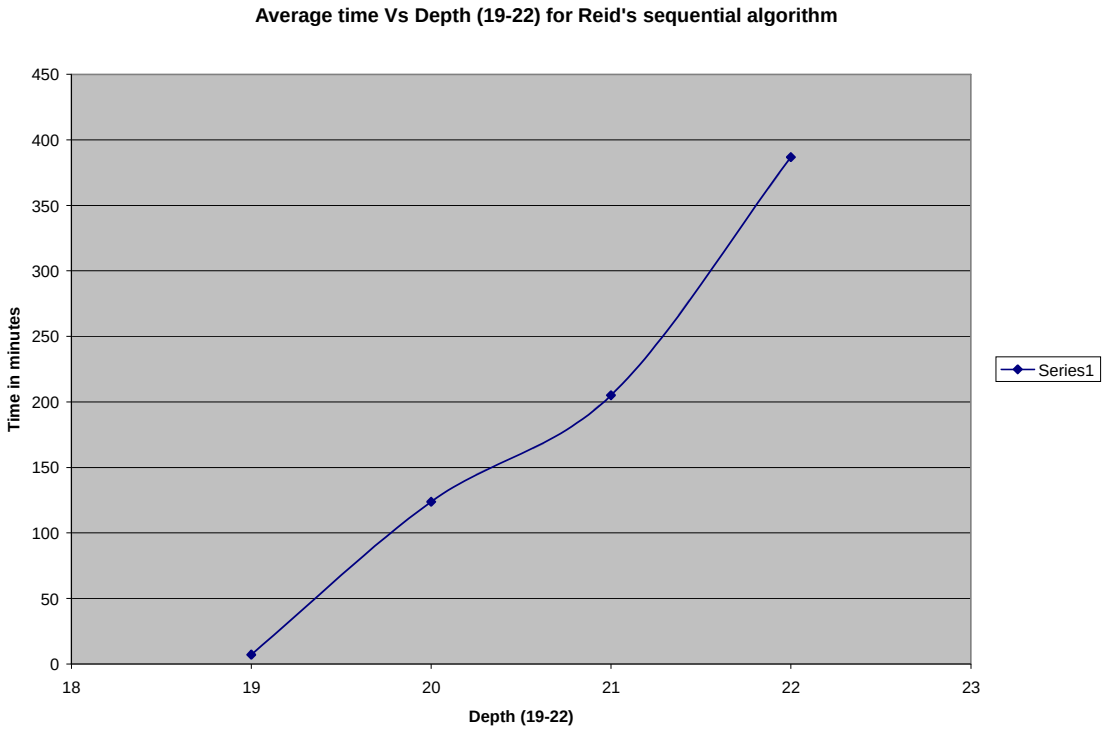


Fig 5.2: Average time Vs Depth (19-22) for Reid's sequential implementation

Table 5.2- Data for average time Vs depth (19-22) for Reid's sequential implementation

Depth	Time in minutes for five configurations					Average time in minutes
	Conf 1	Conf 2	Conf 3	Conf 4	Conf 5	
19	17.38	5.28	3.71	9.56	3.95	7.12
20	3.23	36.48	88.53	255	236	123.85
21	32.51	223.6	27.05	277.36	464.61	205.03
22	97.66	676.16	-	-	-	386.91

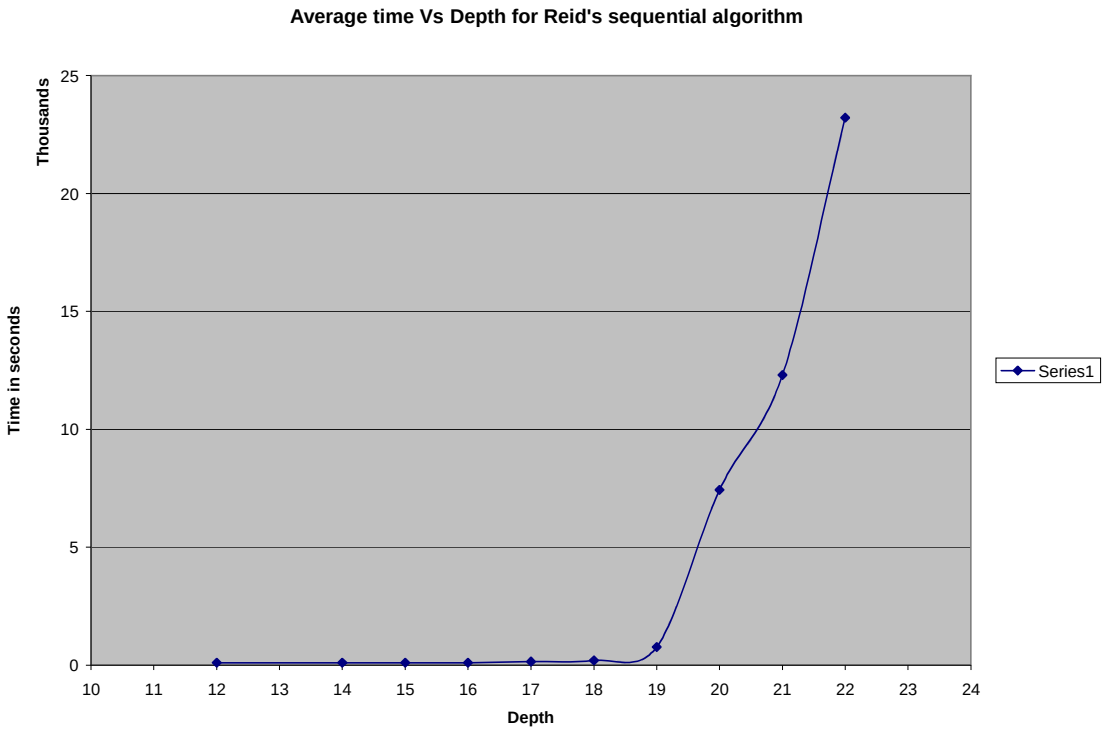


Fig 5.3: Average time Vs Depth for Reid's sequential implementation

5.1.2 Results for the parallel implementation

The configurations tested for serial implementation were all tested for the parallel implementation too. Table 6.3 presents the times for five configurations for every depth form 19-22 and the average time in minutes.

Table 5.3- Data for average time Vs depth (19-22) for the parallel implementation using np=8

Depth	**Time in minutes for five configurations**					**Average time in minutes**
	Conf 1	**Conf 2**	**Conf 3**	**Conf 4**	**Conf 5**	
19	8.38	2.36	2.08	1.68	3.86	3.38
20	1.93	19.48	47.2	124.93	107.5	60.24
21	24.2	160.64	3.13	180.06	213.93	116.39
22	307.5	-	-	-	-	307.05

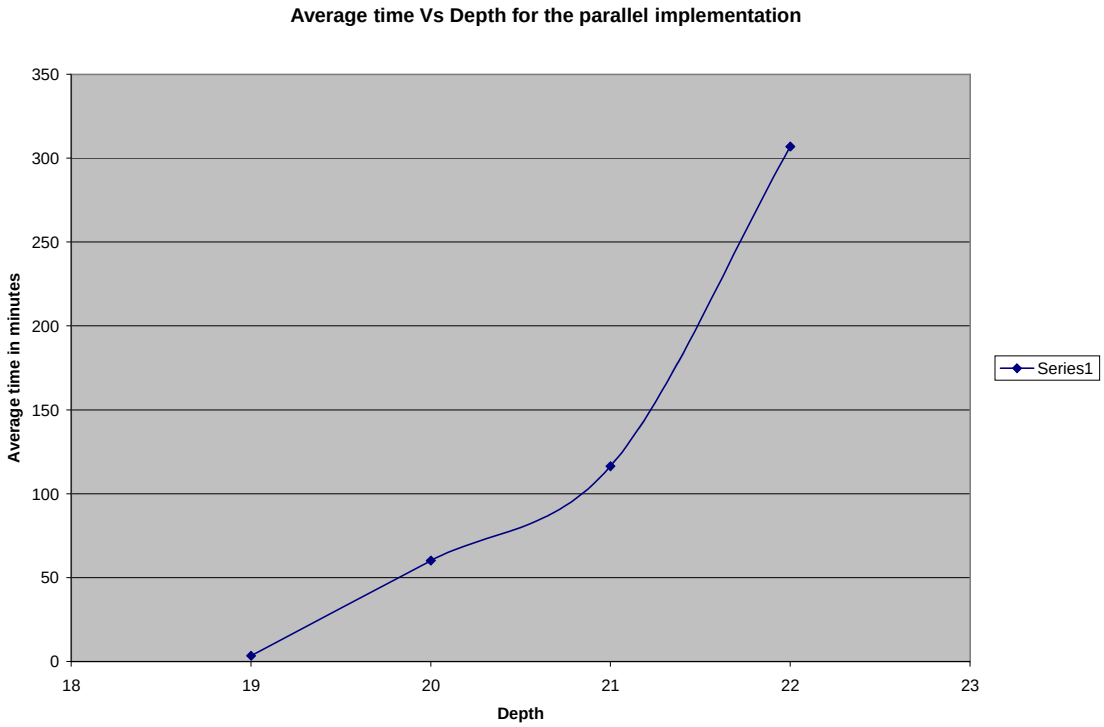


Fig 5.4: Average time Vs Depth for the parallel implementation with np=8

5.1.3 Comparisons

It can be observed that out of the depths for which the program was run, it has started taking considerable time after depth equal to 19. A parallel programs execution time or wall-clock time as it is referred to be an obvious performance measure. This time is the time elapsed from when the first processor starts executing a problem to when the last processor completes execution. The graphs show some unpredictable results as far as the relation between time taken and the number of processors is concerned. One of the reasons for this peculiarity is the solution found by that particular processor for the given configuration being different. As mentioned above since the processors search the tree with different search limits for the IDA*, the solution found could be different. But the most important factor here is the time gained while doing a parallel search and hence the speedup achieved, which is considerable in most of the cases.

The behavior or performance of a parallel program depends on various variables like the number of processors, the size of the data being used for the experiments, the interprocessor communications limit and also the available memory among others. We have experimented with different numbers of processors but the size of data remains the same for any input given. As far as the memory is concerned, the use of UTA cluster has helped us look over this concern but the communication still needs to be taken care of.

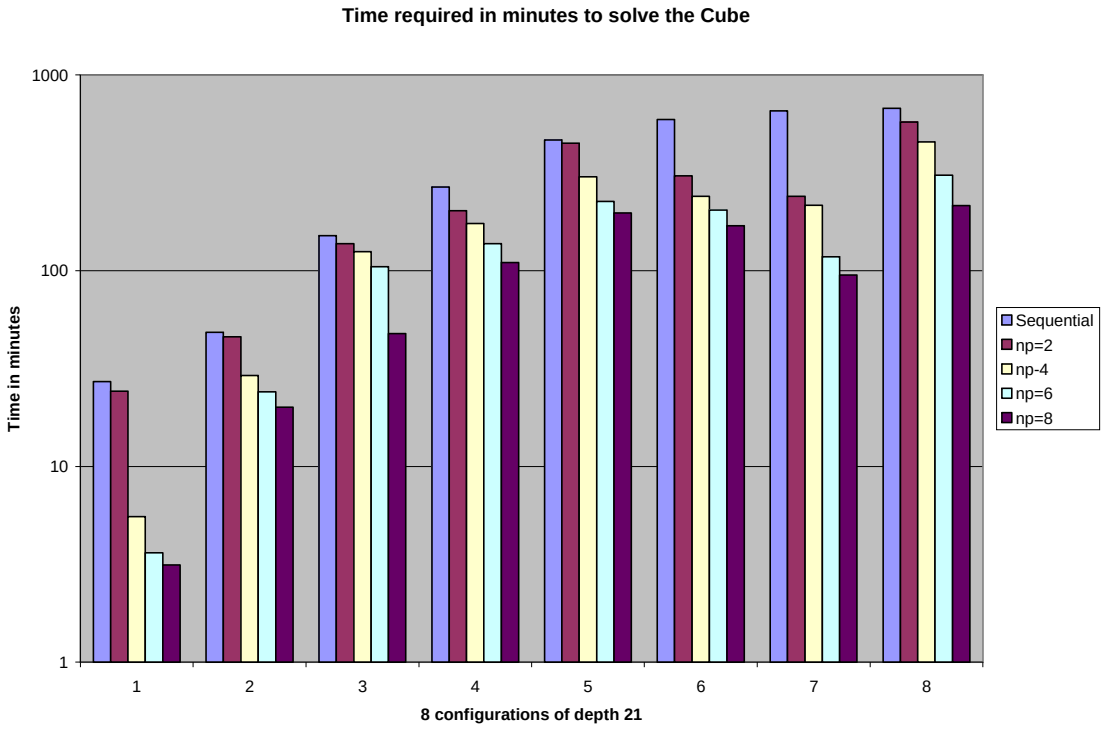


Fig 5.5: Comparisons of time in minutes required to solve the cube for 8 random cubes of depth 21

The chart in fig.5.5 shows comparisons for execution times in solving a position created by applying a sequence of moves to a solved cube. These times are as required in obtaining an optimal solution to solve the Cube whereas the chart in fig.5.6 is a similar graph but showing execution times in solving the inverse of the configurations used in the fig 5.5. Both the graphs show that there is a definite time efficiency obtained in solving the Cube or inverse in a parallel manner. The speedup achieved is even better when the number of processors being as large as 8.

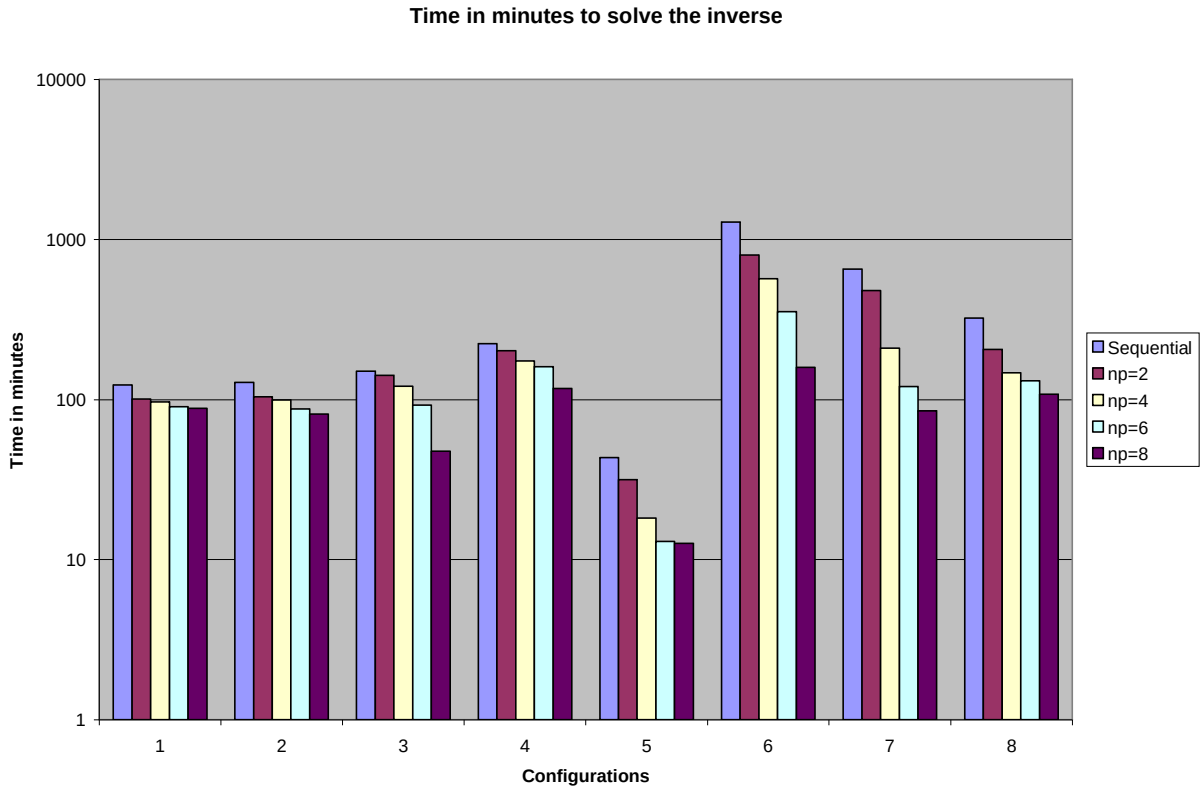


Fig 5.6: Comparisons of time in minutes required to solve the inverse for 8 random cubes of depth 21

5.1.4 Comparisons of results for the super flip configuration

Another challenge in solving Rubik's cube is the "Super flip" configuration, in which the edges are flipped in place. This configuration is said to provide a solution after 24 moves and is supposedly the hardest one. Below is the comparison of times required to solve this configuration using the sequential approach and the amount of time saved by using the parallel implementation.

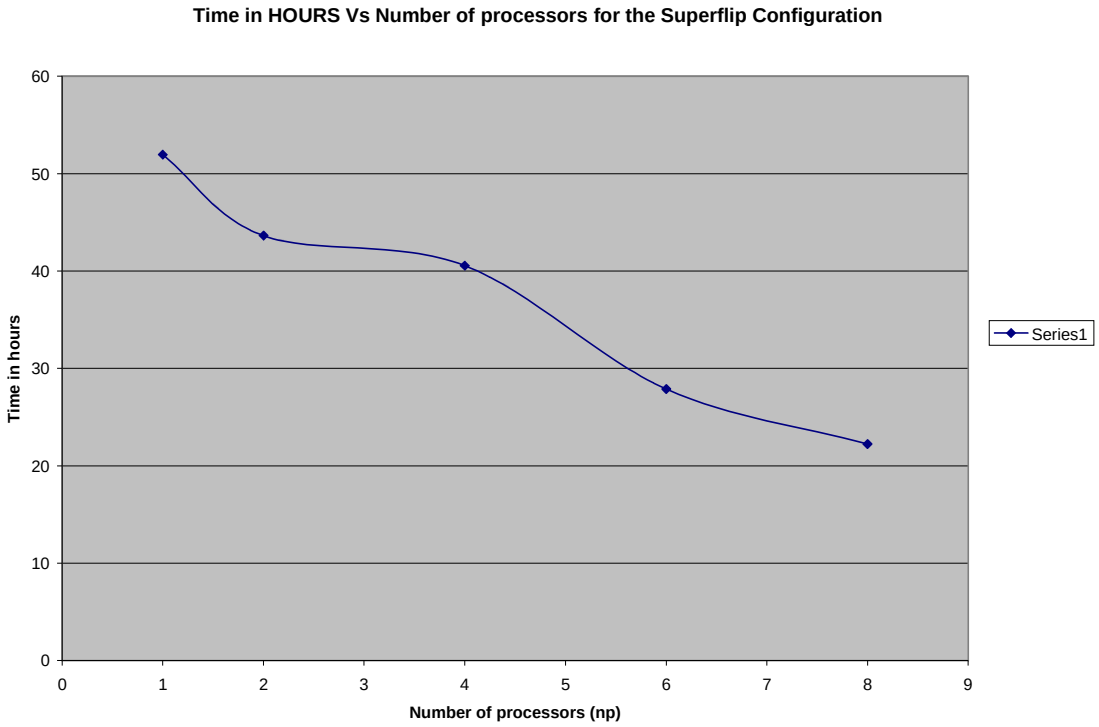


Fig 5.7: Time in hours required to solve the super flip position Vs Number of processors used

Table 5.4- Data for Time in hours Vs Number of processors required to solve the Super Flip position

Algorithm	**Time required to solve the Cube in hours**
Sequential	51.94972
np=2	43.63417
np=4	40.55639
np=6	27.87778
np=8	22.23417

Note: np= Number of processors used in the parallel implementation.

CHAPTER 6

CONCLUSIONS

Although a lot of work has been done in order to achieve an optimal solution for the Rubik's cube, finding an optimal solution has been a difficult task. After researching the Rubik's puzzle and its solutions using heuristics search, it is clear that along with speed, memory efficiency is the most important factor to be looked at. IDA* fits the criteria of a search algorithm providing an optimal solution without larger space complexity. However in searching for the best heuristic to solve the cube, we incorporated the pattern database approach to achieve an optimized solution to the Rubik's cube in the shortest time.

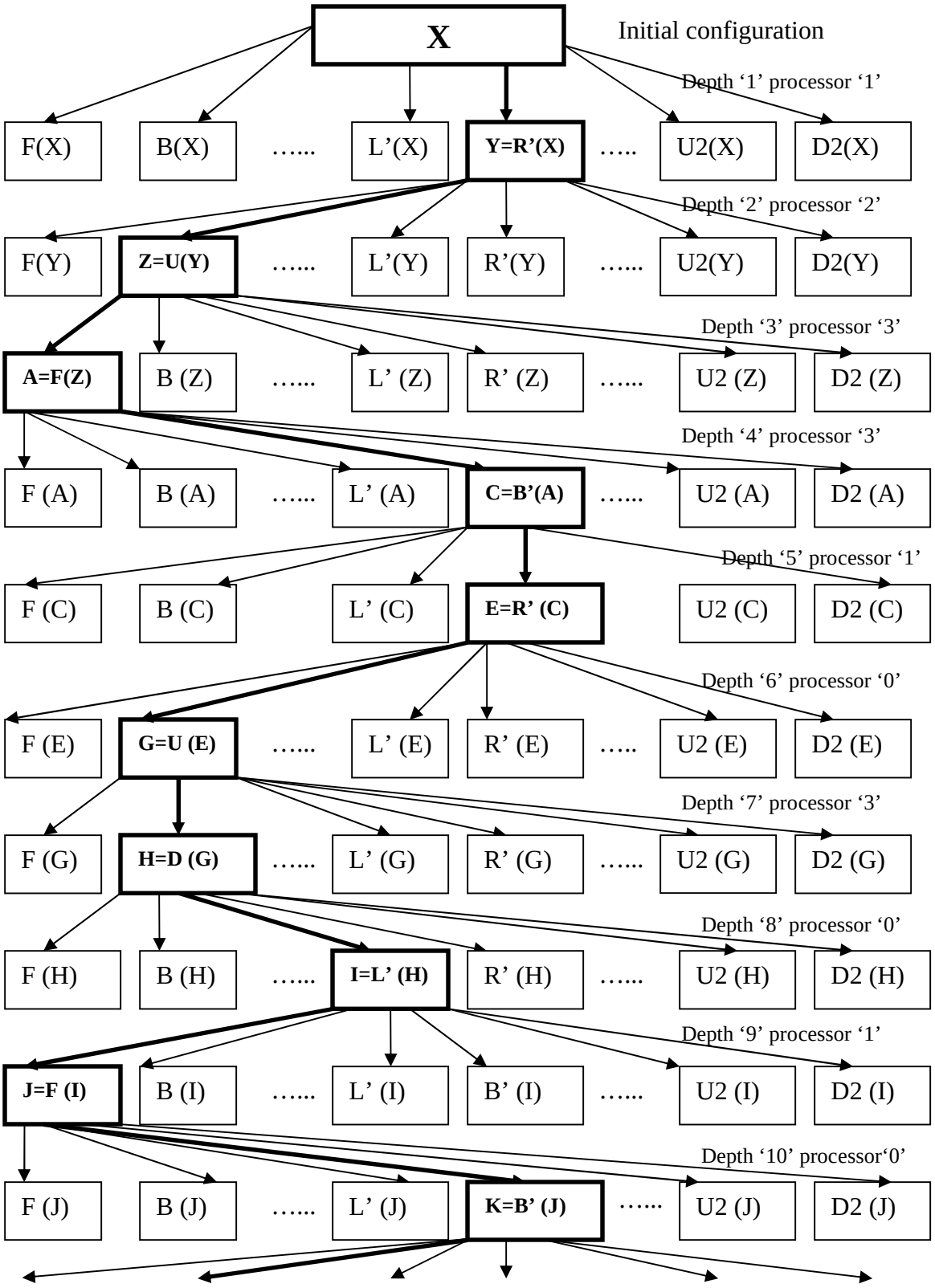
But this approach uses a static pattern database; improvement can be done to this algorithm by using disk storage for the database. This will eliminate the time for initializing the pattern database for every run. Its performance is inversely proportional to the memory requirements. Thus populating the pattern database with numerous patterns would help in improving the performance. Also the symmetry factor does seem to help pruning since the results show that the number of nodes generated for any depth are large in case of asymmetry.

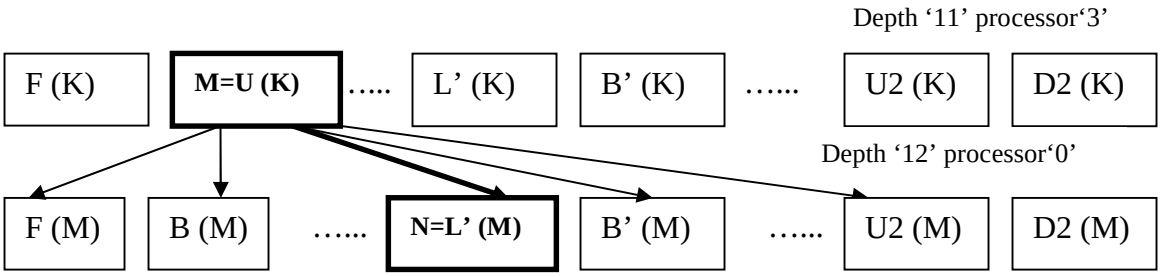
Along with these improvements that we plane to make in this algorithm, the other main challenge in solving Rubik's cube is the "Super flip" configuration, in which the edges are flipped in place. This configuration is said to provide a solution after 24 moves and is supposedly the hardest one. We plan to solve this configuration using the improved pattern database approach.

Parallel processing is used to address the high computation effort required by this particular problem at greater depths. This will further help in boosting the performance. The long term objective of this project is to use a grid for any other puzzle solving problem including the Rubik's Cube.

APPENDIX A

SEARCH TREE SIMULAITON


X
Initial configuration
Depth ‘1’ processor ‘1’
F(X)
B(X)
…...
L’(X)
Y=R’(X)
…..
U2(X)
D2(X)
Depth ‘2’ processor ‘2’
F(Y)
Z=U(Y)
…...
L’(Y)
R’(Y)
…...
U2(Y)
D2(Y)
Depth ‘3’ processor ‘3’
A=F(Z)
B (Z)
…...
L’ (Z)
R’ (Z)
…...
U2 (Z)
D2 (Z)
Depth ‘4’ processor ‘3’
F (A)
B (A)
…...
L’ (A)
C=B’(A)
…...
U2 (A)
D2 (A)
Depth ‘5’ processor ‘1’
F (C)
B (C)
L’ (C)
E=R’ (C)
U2 (C)
D2 (C)
Depth ‘6’ processor ‘0’
F (E)
G=U (E)
…...
L’ (E)
R’ (E)
…..
U2 (E)
D2 (E)
Depth ‘7’ processor ‘3’
F (G)
H=D (G)
…...
L’ (G)
R’ (G)
…...
U2 (G)
D2 (G)
Depth ‘8’ processor ‘0’
F (H)
B (H)
…...
I=L’ (H)
R’ (H)
…...
U2 (H)
D2 (H)
Depth ‘9’ processor ‘1’
J=F (I)
B (I)
…...
L’ (I)
B’ (I)
…...
U2 (I)
D2 (I)
Depth ‘10’ processor‘0’
F (J)
B (J)
…...
L’ (J)
K=B’ (J)
…...
U2 (J)
D2 (J)



In the following tree simulation assume:

Initial configuration:
X = RB RD FD FR BU BL LF LU BD UR DL FU FUL FLD FDR FRU BLU BUR BRD BDL

The path to the goal node comprises of the following intermediate configurations be:

Depth 1:
Y=DB RD FD FR RU BL LF LU UB BR DL FU LUB FLD FDR LFU RBU RUF BRD BDL
Depth 2:
Z= DL FD FR RB BU BD LF LU RD UR BL FU DLB FDR FRU DFL ULF BUR BRD UBL
Depth 3:
A= RU FD FR RB DR BD LF LU LD UB BL FU FLD FDR FRU RBU BDL LFU BRD UBL
Depth 4:
C= RU FD UF RB DR BD LB LU LD UB RF FL FLD UFR RDB RBU BDL LFU LUB DRF
Depth 5:
E= RU RF UF RB DR LD LB LU FD UB BD FL RUF RFD RDB RBU LDF LFU LUB LBD
Depth 6:
G= RF UF RB RU DR LD LB LU FD UB BD FL RFD RDB RBU RUF LDF LFU LUB LBD
Depth 7:
H= RF UF RB RU LU DR LD LB FD UB BD FL RFD RDB RBU RUF LFU LUB LBD LDF
Depth 8:
I= RF UF RB UB LU DR LD FL FD LB BD RU RFD RDB FRU UBL LFU DLB BUR LDF

Depth 9:
J= BL UF RB UB DF DR LD FL FR UL BD RU BLU RDB FRU BDL DRF FUL BUR LDF
Depth 10:

K= BL UF UR UB DF DR DB FL FR UL BR DL BLU UFR URB BDL DRF FUL FLD DBR

Depth 11:

M= UF UR UB BL DF DR DB FL FR UL BR DL UFR URB BDL BLU DRF FUL FLD DBR

Depth 12:

N= UF UR UB UL DF DR DB DL FR FL BR BL UFR URB UBL ULF DRF DFL DLB DBR = **Goal State**

APPENDIX B

PROGRAM EXECUTION MODULE

```
Processors received  =  4
Script running on host node9.cluster
PBS NODE FILE
node9.cluster
node8.cluster
node7.cluster
node6.cluster
Start time: 0.158430 secs
using quarter turn metric
using symmetry reductions
only finding one solution

initializing transformation tables
initializing distance table ... this will take several minutes
distance     positions     (quotient)
     0q            1     (       1)
     1q            4     (       1)
     2q           34     (       3)
     3q          312     (      24)
     4q         2772     (     185)
     5q        24996     (    1633)
     6q       225949     (   14708)
     7q      2017078     (  130032)
     8q     17554890     ( 1124165)
     9q    139132730     ( 8868078)
    10q    758147361     (48182278)
    switching to backwards searching
    11q   1182378518     (75087495)
    12q    117594403     ( 7498528)
    13q        14072     (    1279)

Getting cube (Ctrl-D to exit):
RU UF LF LU RB RD LB DB FR BU FD DL RUF LDF LFU LUB RDB RBU LBD
RFD
asymmetric position

Hello world from node: process 0 of 4
depth  4q completed  (              18 nodes,              0 tests)
solfoud:0
  Start time: 0.107394 secs)

Hello world from node: process 1 of 4
```

depth 1q completed (12 nodes, 0 tests)
  (time: 108.921634 secs)solfoud:0
  Start time: 0.056627 secs)

Hello world from node: process 2 of 4
depth 2q completed (18 nodes, 0 tests)
  (time: 108.921944 secs)solfoud:0

Hello world from node: process 0 of 4
depth 6q completed (18 nodes, 0 tests)

Hello world from node: process 0 of 4
depth 8q completed (18 nodes, 0 tests)

Hello world from node: process 1 of 4
depth 5q completed (18 nodes, 0 tests)
  (time: 108.922071 secs)solfoud:0

Hello world from node: process 0 of 4
depth 10q completed (45 nodes, 0 tests)

Hello world from node: process 1 of 4
depth 9q completed (18 nodes, 0 tests)
  (time: 108.922195 secs)solfoud:0

Hello world from node: process 2 of 4
depth 7q completed (18 nodes, 0 tests)
  (time: 108.922364 secs)solfoud:0

Hello world from node: process 1 of 4
depth 11q completed (126 nodes, 0 tests)
  (time: 108.922577 secs)solfoud:0

Hello world from node: process 0 of 4
depth 12q completed (1,693 nodes, 2 tests)
solfoud:0
  Start time: 0.006165 secs)

Hello world from node: process 3 of 4
depth 3q completed (18 nodes, 0 tests)
  (time: 108.930303 secs)solfoud:0

Hello world from node: process 1 of 4
depth 13q completed (12,513 nodes, 10 tests)

(time: 108.932165 secs)solfoud:0

Hello world from node: process 0 of 4
depth 14q completed (101,902 nodes, 96 tests)

Hello world from node: process 1 of 4
depth 15q completed (778,574 nodes, 446 tests)
(time: 109.466568 secs)solfoud:0

Hello world from node: process 0 of 4
depth 16q completed (6,192,097 nodes, 3,280 tests)

Hello world from node: process 1 of 4
depth 17q completed (50,023,079 nodes, 21,866 tests)
(time: 143.674447 secs)solfoud:0

Hello world from node: process 0 of 4
depth 18q completed (413,618,183 nodes, 153,812 tests)
solfoud:0

Hello world from node: process 1 of 4
F U R U' L' U' R2 U' R F' R' F B' R U' L D' L' (19q*, 18f)

Rank 1 found the solution after time2: 414.972172

Fri Oct 28 15:33:51 CDT 2005

APPENDIX C

PATTERNS USED FOR EXPERIMENTATION

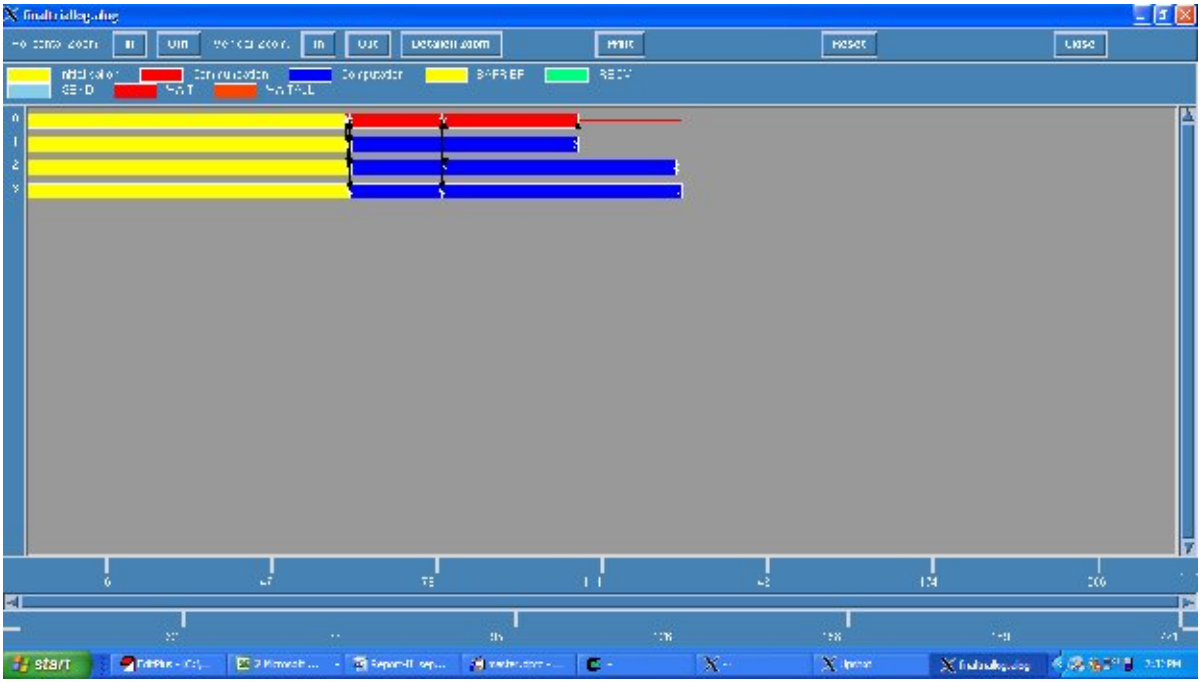
Depth	Patterns
12	F2 D F2 D2 L2 U L2 U' L2 B D2 R2
12	D' L' R' B F D' U' L' R' B' F' U'
12	L U' B F' L D' U' R B F' U' R
12	F2 D F2 D2 L2 U L2 U' L2 B D2 R2
13	F' U' L' B' F' R' U' D B R2 U B R
13	F R D F2 R' U' D B L' R' F' D' R'
14	D2 L D' B' D L' D2 U2 R' U F' U' R U2
14	R2 U' R2 F' R U R' F' L' D' B' L2 F U
14	F' U' F2 L' U B' L F U' R' F' L F2 R2
14	L B2 D R B' F D' L' R D' U F' R2 U'
14	U' R' F R D2 F D2 R F' R U' R2 F' U2
14	F2 U' R2 F2 R F D2 U L B2 D2 B' D' L'
14	B' R' D F' D2 F R2 D2 F' U2 F R' D F
15	U' R2 F2 U2 L' D2 B' L2 U' L2 D2 L U2 F' U2
16	F2 D' R2 D' L' U' L' R B D' U B L F2 L U2
16	B2 D B U L R' B' F' R' B' R' D U' L' U' L
16	L U B2 L2 D2 B U B' D' R' B' F' U2 B2 U R
16	D' L' F2 L2 B F D R F L' F' R2 U2 F2 L' U'
16	R2 B2 U' R U2 B' R B' L F D F2 L2 D' L B
17	U B2 F2 D' R L U' R L D B' F' U' R L B F

17	L2 D' F D F' R2 D' U' F' L R B D2 L' D' L U
17	R2 D B2 R U' F L' R2 D U' B' R F2 L' U' B' U'
17	D' F D' L B D2 F2 U R B' U R2 F D' R F U2
18	U L2 U R2 B R' B2 L2 F D' B' L U2 B R' U B' U2
18	U L2 U R2 B R' B2 L2 F D' B' L U2 B R' U B' U2
19	U' R2 U2 B2 D' R2 F2 U' R' F' U2 L' B F' R L2 F D' U'
19	U' L2 D' B2 D L2 U' L' B' D U' R' B' U F D F2 R' U
19	B2 F2 U B2 F2 L2 R2 U' L2 R F L' F' R B R' F D U'
19	L' U2 R B2 R' F2 R B' F D' F' D U' B2 U2 R F2 U2 R
19	D2 L2 D U' F2 U L' F2 D' L B' R' U' F' R D B L2 R2

APPENDIX D

PROFILING OUTPUT

The profiles of the MPI program is obtained using a profiling software named upshots. To see the profile, an alog file is created and is then displayed using the logviewer program. The profiles show the time distribution and the communication across the processors.



REFERENCES

[1] Culberson, J.C., and Schaeffer, J., “Searching with Pattern Databases,” Proceedings of the 11th conference of the Canadian society for the computational study of intelligence, Springer Verlag, 1996.

[2] David Singmaster Notes on Rubik’s magic cub, Hillside, New Jersey, 1980.

[3] Keith, H.R., “Solving Rubik’s Cube,” NE, pp. 43-207.

[4] Korf, R.E., “Depth-First Iterative-Deepening: An Optimal Admissible Tree Search,” Artificial Intelligence, 1985, Vol. 27, pp. 97-109.

[5] Korf, Richard. “Finding optimal solutions to Rubik's Cube using pattern databases”, Proceedings of the Fourteenth National Conference on Artificial Intelligence (AAAI-97), Providence, RI, pp. 700-705, July 1997

[6] Study Report titled “Implementing an IDA* search on a cluster of workstations to solve a Rubik’s cube” by Sameer Abhyankar and Bimal Tandel. University of Texas at Arlington.

[7] Thesis report by Joe Fowler, University of Colorado in Boulder,1995

[8] Culberson, Joseph C. and Schaeffer, Jonathan. Searching with Pattern Databases. Canadian Conference on AI, 1996, pp. 402-416.

[9] Culberson, Joseph C. and Schaeffer, Jonathan.. Efficiently searching the 15-puzzle. Technical report, Department of Computer Science, University of Alberta, 1994.

[10] Hecker, David and Banerji, Ranan. The Slice Group in Rubik's Cube. Mathematics Magazine, Vol. 58, No. 4, pp. 211-218, September 1985.

[11] Joyner, David. Adventures in group theory: Rubik's Cube, Merlin's Machine, and Other Mathematical Toys. Johns Hopkins University Press, April 2002.

[12] Korf, Richard; Reid, Michael; and Edelkamp, Stefan. Time complexity of Iterative-Deepening A*, Artificial Intelligence, Vol. 129, No. 1-2, pp. 199-218, June 2001.

[13] Singmaster, David. Notes of the Rubik's Magic Cube. Enslow, 1981.

[14] Butler, Gregory. Fundamental Algorithms for Permutation Groups. Springer-Verlag, 1991.

[15] Egner, Sebastian and Püschel, Markus. Solving Puzzles related to Permutation Groups. Proceedings of the International Symposium on Symbolic and Algebraic Computing (ISSAC), 1998

[16] Korf, Richard. Sliding-tile puzzles and Rubik's Cube in AI research, IEEE Intelligent Systems, pp. 8-12, November 1999.

[17] Korf, Richard. Depth-first iterative-deepening: An optimal admissible tree search, Artificial Intelligence, Vol. 27, No. 1, pp. 97-109, 1985

[18] Rotman, Joseph J. An Introduction to the Theory of Groups, 4th Edition. Springer-Verlag, 1995.

[19] Russell, Stuart and Norvig, Peter. Artificial Intelligence, A Modern Approach. Prentice-Hall, 1995.

[20] Schönert, Martin et. al. Cube Lover's Index by Date. http://www.math.rwth-aachen.de/~Martin.Schoenert/Cube-Lovers/

[21] http://jeays.net/rubiks.htm

[22] http://www.cs.princeton.edu/~amitc/Rubik/solution.html

[23] http://www.rubiks.com, Rubik/ Seven towns, 1998

[24] http://home.t-online.de/home/kociemba/cube.htm

[25] http://www.math.ucf.edu/~reid/Rubik/

[26] http://www.math.rwthachen.de/~Martin.Schoenert/CubeLovers/Mark_Longridge__Superflip_24q.html

BIOGRAPHICAL INFORMATION

Miss.Aslesha Nargolkar is born on 26th April 1982 in the city of Pune, Maharashtra in India. She completed her high school education in Muktangan English School in 1997 and in 2003 she graduated with a Bachelors degree in Computer Engineering from the Pine University in Maharashtra, India with a first class. She started pursuing her Masters degree in Computer Science and Engineering in fall'2003. She did research in "Paralllel Processing the Rubik's Cube "under the supervision of Dr. Ahmad. In future she plans to pursue a PhD degree.