SIMULATION OF INDETERMINATE MULTIPLE, SIMULTANEOUS IMPACT AND CONTACT FOR A FLEXIBLE MULTIBODY SYSTEM

by

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For amma, pappa, and aks

whose sacrifices cannot be quantified.

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ABSTRACT

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This work presents a method for understanding the impact behaviour of a flexible body undergoing multiple, simultaneous contacts. A continuous model is used with an event-driven function in MATLAB, which detects the collisions. The flexible body is defined as a system of particles, having inter-particle forces in terms of spring, damper coefficients. Equations of motion for such a Flexible MultiBody system are determined and then solved for different phases.

In the method presented, the indeterminate nature of equations of motion encountered, during impact, and contact for flexible body are examined. Constraint forces are determined during the different phases of an impact to address the equations. These techniques are applied to a planar model of an elliptical body, which is dropped freely under the effect of gravity and collision occurs at the ground determined. A simulation is presented demonstrating the behaviour of the body during impact, and contact with the ground with the proposed method.

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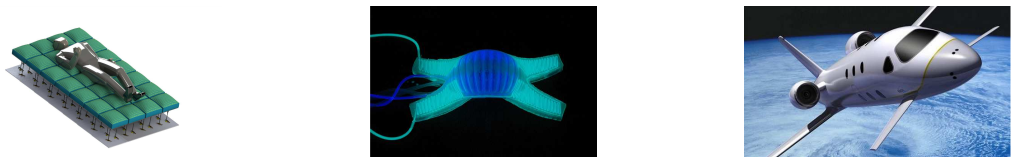
CHAPTER 1

INTRODUCTION

1.1 Overview

Flexible Multi-body dynamics is the study of modelling and analysis of *flexible*, or *deformable* bodies. A *flexible* body is defined as a system which undergoes significant deformation under external forces. The significant part of modern engineering design is the analysis and prediction of the dynamic behaviour and performance of physical systems, which are in general very complex and difficult to analyse. In most cases they consist of a large number of components, also called as *bodies*, acting together as a *single entity* [1]. This *single entity*, or *system*, is termed as a Multi-body System and can be a consolidation of rigid bodies, flexible bodies, or a combination of flexible-rigid bodies.The physical properties for a single body can be defined in the system and a mathematical model can be constructed which represents the idealization of an actual physical system.

It is classically known that any body, can be broken down to finitely small division having the same characteristics of the body. Similarly, a body in the Multi-body System, can be further divided into smaller entities to determine a more accurate behaviour of the individual body. This subdivided entities for a body are termed as particles, and the individual body behaves as a *system of particles*. These particles ,together, retain the essential features of the body. A Flexible Multi-body System(FMS) may consist of elastic and rigid components which are connected by joints and/or forces, defined by different methods. And, since the displacements of bodies in this system are not completely independent of each other, any external force would cause a *ripple* effect through the system. Examples of such system , where the deformation of the body has a significant effect on the system dynamics [2], are ground and space vehicles, mechanisms, robotics, space structures, and precision machines.



(a) Smart Bed (b) Robotics (c) Space Vehicles

Figure 1.1: Applications

The objective of this research is to demonstrate the dynamic behaviour of a flexible body, modelled as a *system of particles* also referred as a *flexible multi-body system* in the literature. The framework model used in this work is a system designed based on the *finite segment approach*, wherein a system is divided into multiple bodies. In this work, we concentrate on analysing the constraint forces, required to handle the impact, and contact phases of a flexible multi-body system during a collision. The post impact-behaviour of the system is demonstrated by the parameters used to define the system initially.

1.2 System of Particles

We consider a system of particles to define a *flexible body.* the advantage of doing so it that every particle can be individually treated in a system. This system is defined as follows:

- n particles
- m_i, is the mass of each particle
- q_i, are the variables defining the position

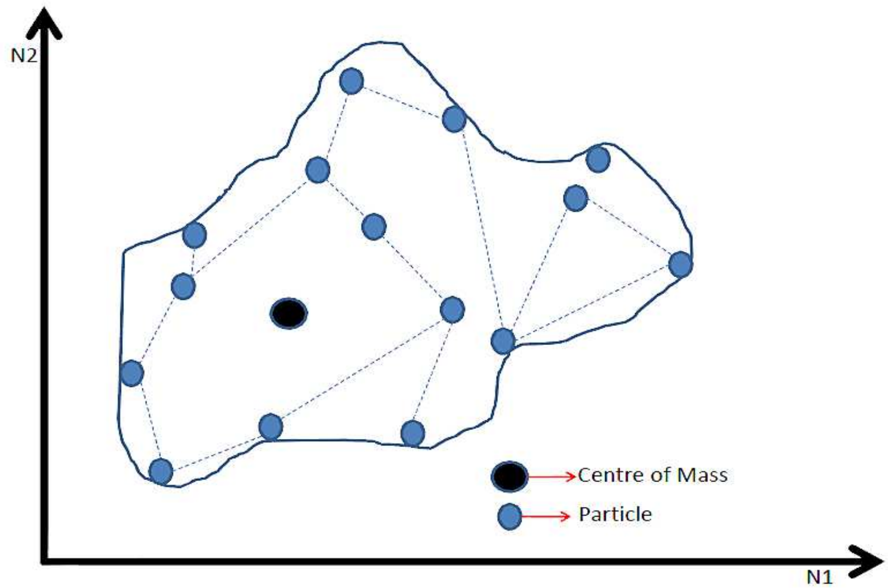


Figure 1.2: System of Particles

There are two types of forces acting on the *system of particles*:

- External Forces
- Internal Forces

The different types of internal forces are spring, damper, molecular, etc. and the external forces are due to the influence of gravity, friction, and the constraint forces.

1.3 Literature Review

Based on the survey done in [2, 3] different methods have been analysed referred for this work, and also the past research work done in the analysis of flexible bodies. Some of the basic computer aided methods are studied and further moderations were made to available approaches to determine a efficient solution. As per the definition, flexible bodies undergo deformation under the influence of external forces, and regain its shape when these forces are removed.

In the last two decades, research in the field of *machine dynamics* has led to great advancement in tackling the problem of analysis for flexible bodies, with single, and multiple contacts. Based on this research methods have been formulated in approximation to the rigid body methodologies, which are in use extensively:

- **Discontinuous: Impulse-Momentum based**
- **Continuous: Constrain Forces**

1.3.1 Discontinuous scheme

This scheme is more prominently used in analysis the rigid bodies undergoing collision[4, 5, 6]. Based on the **Law of Conservation of Momentum**, and using it to obtain the Impulse-Momentum relation, impact forces are computed to obtain the post-impact velocities of bodies. This approach can be modified and be adapted to be used in the study of flexible bodies as well. Since a body undergoes deformation only under external influence,until a collision or external force is applied to a flexible body, it behaves as a rigid body.

Unlike rigid bodies, there is no constraint acting on the body avoiding any kind of deformation. Also the post-impact velocities computed for a rigid body are based on an assumed value for coefficient of restitution. In the work demonstrated in [7], a numerical and experimental value of coefficient of restitution was adapted to examine the validity.

This scheme is termed discontinuous because of the jump in the bodies velocities and reaction forces[2].

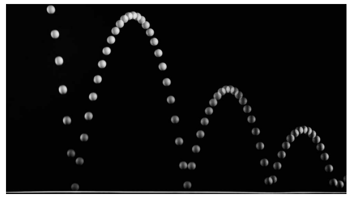


Figure 1.3: Discontinuous Scheme

1.3.2 Continuous Scheme

Unlike the discontinuous scheme the force model used does not cause sudden discontinuity in the systems velocity. Another difference with the discontinuous scheme is that the impulse-momentum relation was algebraically used to iterate the post-impact velocities. But, for the continuous scheme the governing equation of the body is integrated over the duration of contact. The post-impact behaviour of a body is governed by the system definition and the reaction forces during collision. The relationship for the coefficient of restitution can be examined experimentally and analytically in this scheme, thereby leading to less approximations.

Although this scheme is impressive in theory, but practical implementation causes inefficiency due to dependence on the parameters defining structural flexibility.

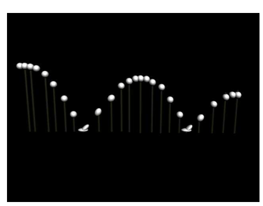


Figure 1.4: Continuous Scheme

CHAPTER 2

BACKGROUND

2.1 Overview

Dynamics of a flexible body is a very complex phenomena and has been extensively been the research focus for many academics. As discussed in section 1.3, there are various methods to approach the solution for this analysis. Mainly these methods are classified into two type; continuous and discontinuous based on the integration scheme. The former method are preferred because of the continuity in the solution, and over the fact that the latter has restrictions for flexible body consideration. In this work, we discuss the constrain forces required to derive a continuous flow for the collision response.

The integrator used in the design of this model is the **MATLAB**'s, **ode45** module which has an inbuilt event function, schemed to capture the collision with the ground and process the constrain forces as per the requirement. Any time the event function is triggered, the contact point, time of contact, and the state variables for that time are extracted to define the system. The integration is not terminated in this case, since a continuous approach is demonstrated.

There are various ways in which the usage of constrained dynamics is theorized to be applied for flexible bodies[1, 8, 9]. Here we compute the constraint forces, in the absence of friction due to the ground in contact. Since, multiple impact, and contact points can occur simultaneously during collision, each particle is dealt with individually.

2.2 Approach

The flexible model defined in this system is a *system of particles*, behaving to be a flexible body. We modify the *finite segment method*, where a flexible body comprises a system of particles are connected by spring,and damper linkages. This method uses a continuous scheme to study the collision response of a flexible body.

2.3 Methods

In this section we discuss the methods employed in this work to model the constraint forces. The impact, and contact of any body are together referred as the collision response for a body. For a body free-falling under the influence of gravity, the collision is detected using an *event* function in MATLAB[6]. When collision is detected on any point in a body, we construct a method to determine the forces acting on that particle. For simplicity we shall divide the review of these methods individually:

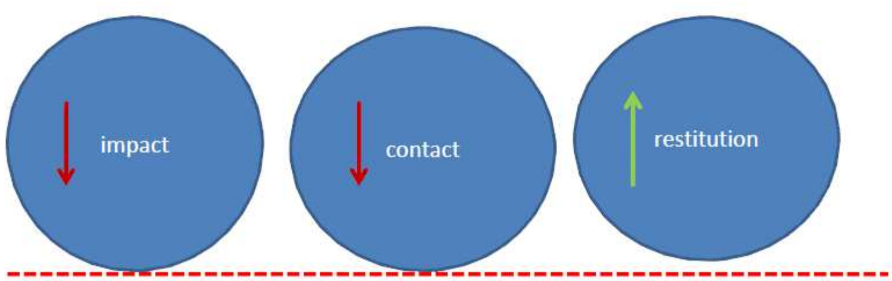


Figure 2.1: Collision Response

2.3.1 Impact Modelling

Impact is a phenomena occurring on a body undergoing motion, when a sudden obstacle is encountered. This force is applied instantaneously resulting in the change in velocity of both the bodies. Using this to our advantage, we consider one of the bodies to be stationary and does not move or undergo any kind of deformation under external influence.

When a flexible body impacts a rigid surface, such as the one explained before, there is a certain discontinuity observed in the velocity of the flexible body. Since, we consider that the rigid surface does not deform, the only change observed is in the case of flexible body. This impact force is also referred as the *impulse* force acting on a body for a short period of time.

Since the internal forces of the body are considered to stabilize the body to retain its shape under free fall, these forces are not disturbed until collision is detected. Hence, the impulse force is directly related to the change in momentum alone.

2.3.2 Contact Modelling

Once the *Impact* phase is completed and the body has reached to a resting position, internal forces get disturbed due to the Impact force. These forces now contribute for the motion of the body further into the rigid surface defined. Since, our model aims to avoid any penetration into ground during collision, these internal forces and the acceleration due to gravity, needs to be constrained to avoid penetration of the contact point.

This constraint acting at the contact point is the Contact force modelled to ensure that the body does not penetrate into the surface. Due to the impulse initially acting on the flexible body, there is a change in the internal energy of the system, which is lost due to the damping effect in the body. The contact force is active until the body has lost the energy incurred due to the collision.

We consider the time at which the energy dissipation is completed to be the end of compression of the body during collision, after which the body restitutes back.

2.3.3 Restitution

As the word suggests, restitution is to regain a previously described state. In this phase the body completes its compression and starts moving away from the collision surface. This restitution of the flexible body occurs completely due to the energy stored in the flexible body due to the collision. Since certain amount of energy is lost due to damping during collision, the body does not reach the same position before collision. This process continues until the energy disturbance due to collision is significant enough to resitute a body away from the collision surface.

CHAPTER 3

MODEL DEFINITION

In this chapter, we shall discuss the design of a *Flexible Multi-body System* using the approach defined earlier. This system is parametrized based on particles distributed to behave as a flexible body. Since, the system is flexible in nature there are internal forces that hold the system together. Although these forces are initially defined, they become active only under the influence of external force applied.

3.1 Planar Model

A planar model, is defined as per figure **??**. This *Flexible Multi-body system*, consists of 22 particles on the perimeter of a ball, and 1 particle at the geometric centre of the body. The radius of the ball is given as LO, which is also the rest length of the spring defined between the centre and the particles at the perimeter. This model is defined in a frame **T**.

Each particle in the system is positioned using the state vector for the particle, defined by indeterminate variables q_i, in the body attached frame **T**. This frame is located at the geometric centre of the body, which is defined in the **N** frame as:

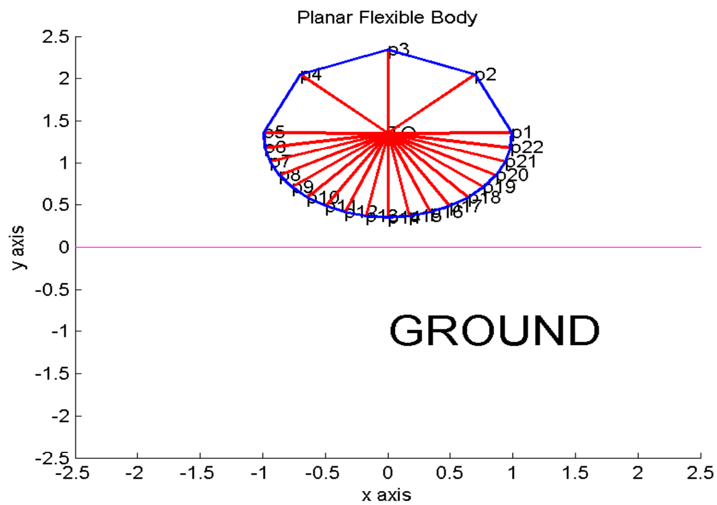


Figure 3.1: Planar Model: 23 particles

$$\mathbf{P}_{NT} \;=\; q_1\;\mathbf{N}_1 \;+\; q_2\;\mathbf{N}_2 \tag{3.1}$$

and the angular velocity of the body is:

$$\boldsymbol{\omega}_{NT} \;=\; q_3\;\mathbf{N}_3 \tag{3.2}$$

From equation 3.1 the velocity of the flexible body can be derived as:

$$\mathbf{V}_{NT} \;=\; \frac{d\;(\mathbf{P}_{NT})}{dt} \;=\; \dot{q}_1\;\mathbf{N_1} \;+\; \dot{q}_2\;\mathbf{N_2} \tag{3.3}$$

the acceleration is given by:

$$\mathbf{A}_{NT} \;=\; \frac{d\;(\mathbf{V}_{NT})}{dt} \;=\; \ddot{q}_1\;\mathbf{N_1} \;+\; \ddot{q}_2\;\mathbf{N_2} \tag{3.4}$$

and the angular acceleration is given from Equation 3.2

$$\boldsymbol{\alpha}_{NT} \;=\; \frac{d\;(\boldsymbol{\omega}_{NT})}{dt} \;=\; \dot{q}_3\;\mathbf{N}_3 \tag{3.5}$$

The particles defined in the system are mass points having a mass of m_i. The total mass of the flexible body is given by:

$$\sum_{i=1}^{n=23} m_i \; = \; M_T \tag{3.6}$$

3.2 Internal Forces

As discussed earlier, the flexible body is defined as a *System of Particles* in a modified **Finite Segment Method**. This approach has particles, or *mass points* distributed in a system interacting with spring, and damper forces. Since we have established the fact that, a body does not deform in the absence of external influence, we can clearly state that the internal forces do not cause any deformation, until external influence arouses.

The particles have an initial distance of say r_{ij}, which is the rest length of the springs. Based on **Hooke's Law**, deformation in the length of the spring, causes a force proportional to the change in original length. The proportionality is resolved using a spring constant, $k_i j$

$$\mathbf{f}_{s_{ij}} \; = \; \frac{k_{ij} \; (\; r_{ij} \; - |\mathbf{P}_{ij}| \;) \; \mathbf{P}_{ij}}{|\mathbf{P}_{ij}|} \tag{3.7}$$

where, $\mathbf{f}_{s_{ij}}$ is the spring force , and k_{ij} is the spring constant between i^{th}, and j^{th} particle.

To dissipate the potential energy added to the system due to compression of springs, dampers are included. These damping forces, are proportional to the *relative velocity* between two particles, and the proportionality is resolved by a damping coefficient, c_{ij}

$$\mathbf{f}_{d_{ij}} \; = \; c_{ij} \; (\; \mathbf{V}_j \; - \; \mathbf{V}_i \;) \tag{3.8}$$

where, $\mathbf{f}_{d_{ij}}$ is the damping force , and c_{ij} is the damping coefficient between i^{th}, and j^{th} particle.

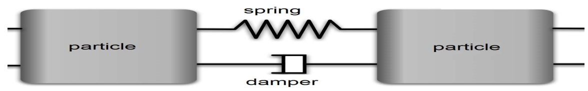


Figure 3.2: Internal forces

3.3 Initial Conditions

Initial conditions are parameters needed to define the flexible body. These constants are determined based on the material properties for the same. Since, the system is considered to be stable when no external forces act on it, the internal forces should completely negate each other. External forces such as gravity act on the body, due to which the state variables of the system vary. The system parameters, can be described in different methods [10]. We use an approximation to determine the spring, and damping coefficients.

A simple mass-spring-damper model has a governing equation given by:[11]

$$m\,\ddot{\mathbf{x}} + c\,\dot{\mathbf{x}} + k\,x = -\,m\,g \tag{3.9}$$

where the damped natural frequency, $\boldsymbol{\omega}_d$, is

$$\boldsymbol{\omega}_d = \frac{\sqrt{4\,k\,m - c^2}}{2\,m} \tag{3.10}$$

From equation 3.9, 3.10, we can define the $\boldsymbol{\omega}_d$ for the inter particle forces in the flexible body to be

$$\boldsymbol{\omega}_d = \frac{\sqrt{4\,k\,m - c^2}}{2\,m} \tag{3.11}$$

and, for the body to be flexible in nature we need an underdamped solution for which $\boldsymbol{\omega}_d > 0$ or

$$\boldsymbol{\omega}_d = (4\, k\, m - c^2) > 0 \tag{3.12}$$

We define the spring and damper coefficients in the *System of Particles*, based on equation 3.12.

3.4 Equation of Motion

The generalized governing equation of motion for the system of particles is given by:

$$M\, \ddot{\mathbf{q}} + C\, \dot{\mathbf{q}} + K\, \mathbf{q} + g\, (\mathbf{q}) = \mathbf{F} \tag{3.13}$$

where, $\mathbf{F}$ is the set of constraint forces acting on the particles in contact during collision. The other parameters in the equation 3.13 have their general meaning as follows:

- M, is the mass matrix
- C, is the damping coefficient matrix
- K, is the spring constant matrix
- g, is the gravity acting on the system
- $\ddot{\mathbf{q}}$, is the generalized acceleration of the system
- $\dot{\mathbf{q}}$, is the generalized velocity of the system
- $\mathbf{q}$, is the generalized co-ordinate of the system

3.5 Energy

The energy of the system is a sum of the individual components of potential and kinetic energy in the system of particles.

$$U = \sum_{i=1}^{n} K.E. + \sum_{i=1}^{n} P.E. \tag{3.14}$$

Potential Energy of the system is the energy due to the position of the system or the configuration of its element and is divided into the spring and gravitational potential[12].

$$P.E. \; = \; P.E._{spring} \; + \; P.E._{gravitational} \tag{3.15}$$

The total energy of the system is conserved because of the type of collision, but due to the damping in the system, there is energy dissipated due to the work done by the damper. Hence the total energy of the system is:

$$U \; = \; K.E._{total} \; + \; P.E._{total} \; - W_{damping} \tag{3.16}$$

where, $W_{damping}$ is the work done by the damper to dissipate energy.

CHAPTER 4

PROBLEM STATEMENT

In this chapter, we discuss the various methods applied to compute the constrain forces during the collision. The iterative scheme for computation is discussed further in sections 4.2 & 4.3.

4.1 Collision

An event in which two or more bodies come in surface contact and thereby exert forces on each other is termed as collision. In this work, we study the collision response of a flexible body defined as a system of particles. Collision between bodies is a short lived phenomena and involves change in the velocities of the bodies, before and after. Due to external forces the *Kinetic Energy* of the system is disturbed. The change in *Kinetic Energy* is a deciding factor to classify the type of collision. It is very important to note that, every collision follows the **"Law of Conservation of Momentum"**.

Collision response in this work is divided into two phases, *Impact*, and *Contact*. During the Impact phase, the velocity of the particle undergoing collision is discontinued to ensure there is no penetration into the colliding surface[13]. Even though we discontinued the velocity of particle during the Impact, the velocities of other particles in the system still have a velocity, which is active due to the presence of internal forces. Due to internal forces, there is a deceleration observed in other particles of the system. From this assumption, we can state that this effect propagates through the system as a *ripple effect*.

While this is happening, a non-impulsive contact force is acting on the particle in contact. This phase is defined as the *Contact*, during which force acts normal to the collision surface. This *Contact force*, is valid under the following conditions:

- Contact exists between two bodies.
- The velocity of the system is not in the positive direction of the normal of colliding surface.

We define the collision response into phases to clearly explain the analysis of constrain forces. As per the generalized equation of motion for single particle is given by:

$$m\,\dot{\mathbf{V}} \;+\; \mathbf{f}_d \;+ \mathbf{f}_s \;+\; \mathbf{f}_g \;=\; \mathbf{f} \tag{4.1}$$

where, $\mathbf{f}$ is the constraint component acting on an individual particle and the other parameters in the equation 4.1 have their meaning as follows:

- m, is the mass of a particle
- $\dot{\mathbf{V}}$, is the acceleration of a particle
- $\mathbf{f}_d$, is the damping force acting on a particle
- $\mathbf{f}_s$, is the spring force acting on a particle
- $\mathbf{f}_g$, is the gravitational force acting on a particle

A flexible body has various types of forces acting internally and externally. Spring, and damper forces are defined as internal forces which are proportional to the deformation of inter-particle distance, and the relative velocities respectively. While on the other hand external forces are due to gravity and the constraints applied on collision. During free fall under the influence of gravity, there are no constraint forces defined for the system and the only external force acting on the body is due to gravity. Since, we assume that the body does not deform until a collision occurs, the contribution of spring and damper forces are zero until then. These internal forces of the system become active when there is deformation in the body, due to collision. In the following sections, we shall discuss more about how the modelling of constraint forces is done in these two phases.

4.2 Impact Constraint Forces

In this work, we have divided the collision response into phases. The modelling of constraint forces during the impact phase are discussed and derived in this section. From equation 4.1 we understand that during free fall there is no constraint force acting on the body. To understand the phase of Impact we shall divide the equation 4.1, and understand the contributions individually.

A body dropped under the influence of gravity has certain acceleration which contributes to its momentum. From Newtons's 2^{nd} law, we can state that the change in momentum of a stationary or moving object is always conserved.

$$\textbf{force} = mass\ \textbf{acceleration} \tag{4.2}$$

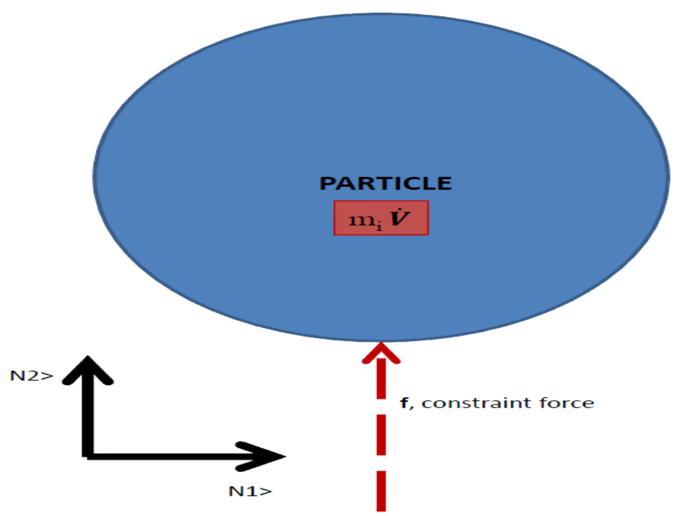


Figure 4.1: Impact Constraint Forces

Impact force is a normal force acting on the particle in contact causing the velocity of the particle tend to zero, as shown in Fig. ??. To compute such a normal force acting we first need to understand the impulse acting on the particle. From equation 4.1,

$$\mathbf{J}_i \;=\; \int_t^{t+\epsilon} \mathbf{f}_i \; dt \tag{4.3a}$$

$$\mathbf{J}_i \;=\; \int_t^{t+\epsilon} m_i \; \dot{\mathbf{V}}_{\mathbf{i}} \; dt \tag{4.3b}$$

where, $\mathbf{J}_i$ is the impulse acting on the particle.

During *Impact* we apply a constrain force to discontinue the velocity of the colliding particle P_i to zero.

$$\mathbf{V}_i(t+\epsilon) \;=\; 0 \tag{4.4}$$

By evaluating equation 4.3b and substituting equation 4.4,

$$\mathbf{J}_i \;=\; m_i \; \mathbf{V}_i(t+\epsilon) \;-\; m_i \; \mathbf{V}_i(t) \tag{4.5a}$$

$$\mathbf{J}_i \;=\; -\, m_i \; \mathbf{V}_i \; (t) \tag{4.5b}$$

The impulse computed in equation 4.5b, is aaplied on the colliding particle over a time-step, ΔT, to estimate a *Impact Constraint Force* acting on the particle

$$\mathbf{f}_{est_i} \;=\; \frac{\mathbf{J}_i}{\Delta T} \tag{4.6a}$$

$$\mathbf{f}_{est_i} \;=\; -\,\frac{m_i \; \mathbf{V}_i(t)}{\Delta T} \tag{4.6b}$$

Hence, the Impact force acting on the colliding particle can be estimated to be

$$\boxed{\mathbf{F}_i \cdot \; \mathbf{N}_2 \;=\; \mathbf{f}_{est_i} \;\cdot\; \mathbf{N}_2} \tag{4.7}$$

4.3 Contact Constraint Forces

During the Impact modelling for the flexible body, we discontinued the velocity of the particle in contact to zero (equation 4.4). Once the particle is at rest the forces acting on the particle are due to the springs and dampers connected to it along with the gravitational force acting on it as shown in Fig. **??**

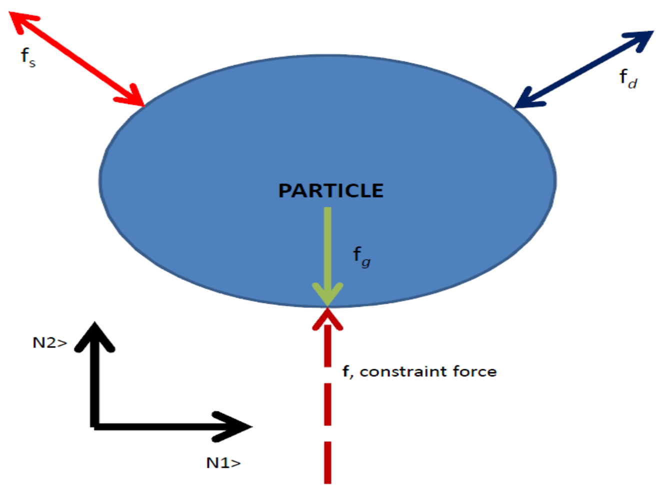


Figure 4.2: Contact Constraint Forces

The force acting during this phase of collision is termed as *Contact Constraint Force*, which is a non-impulsive force acting, normal to the collision surface, on the particle to avoid further penetration into the ground due to internal, and gravitational forces. From Fig. **??**,

$$\mathbf{f}_i = \sum_{j=1}^{n} \mathbf{f}_{s_{ij}} + \sum_{j=1}^{n} \mathbf{f}_{d_{ij}} + \mathbf{f}_{g_i} \tag{4.8}$$

Equation 4.8, is the estimated value of *Contact Constraint Force* acting on an individual particle, at rest after impact, to avoid further penetration into the collision surface. Hence the *Contact Constraint Force* is:

$$\boxed{\mathbf{F}_i \cdot \mathbf{N}_2 = \mathbf{f}_i \cdot \mathbf{N}_2} \tag{4.9}$$

This *Constraint Force* acts on the particle until contact exists with the collision surface. Once the contact ceases to exist, the force disappears too.

CHAPTER 5

RESULTS AND CONCLUSION

In this chapter we shall discuss the major results obtained for this work. These results mentioned are for the planar model discussed.

5.1 Simulation

The Planar model used in this work was dropped freely under the influence of gravity and the collision response was analysed for the system. As discussed earlier, the *Flexible Multi-body* system undergoes Impact, and Contact phase when collision occurs. The constraint forces for both the phases were computed and a simulation was generated to visually understand the behaviour.

5.1.1 FreeFall

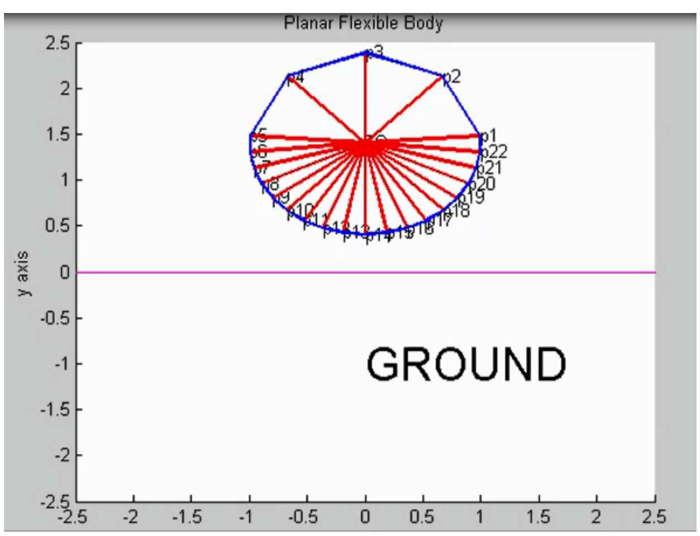


Figure 5.1: Planar Model under Free-Fall

As shown in Fig. 5.1, the *Flexible Multi-body* is dropped freely under the influence of gravity completely. There are no constraint forces acting on the system in this case and the body continues to drop until it touches the ground defined.

5.1.2 Impact Phase

Once collision is detected for the system, an Impact Constraint Force acts on the colliding particle and discontinues the velocity of that particle to zero. This constraint is an Impulsive force acting for a short time-step.

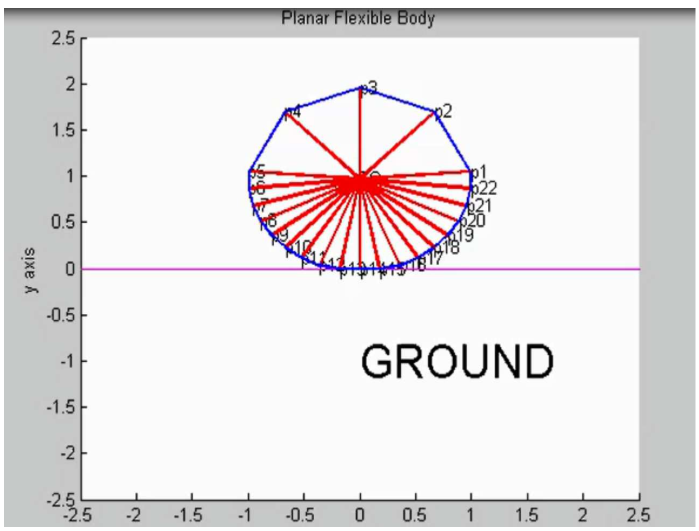
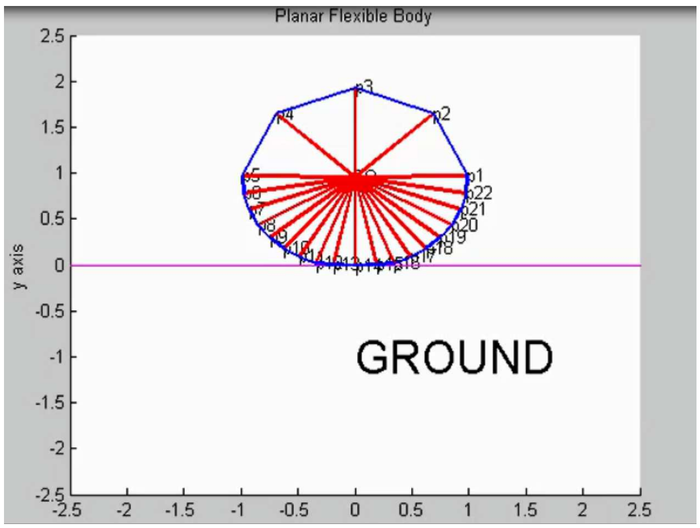


Figure 5.2: Planar Model, Impact phase

As shown in Fig. 5.2, the velocities of the colliding particles are discontinued to zero, and the system does not penetrate into the ground defined.

5.1.3 Contact Phase

The Constraint forces, acting in this phase nullify the effect of any kind of forces acting on the colliding particles so that there is no further penetration of the system into the ground. Basically, the idea of applying a Constraint Contact Force is to keep the colliding particles at rest until complete compression occurs and the body is ready to restitute back. The Contact Constraint Force is a non-impulsive force acting on the particles until the contact exists.



5.2 Colliding Points

The points or *particles* coming colliding with the ground defined are the Colliding Points in the *System of Particles*. In this work, the scheme constraints the penetration of the *Flexible Body* into the ground defined. This is achieved by the Constraint Forces acting on the body. Figure 5.3, shows that, there is no penetration of the Colliding points into the ground. The first graph in the figure shows that, the body is unstable when initially dropped under free-fall, this is because of the assumed arbitrary values of the spring and damper constants defining the system. From the second part of the graph, we can conclude that all Colliding Points, do not penetrate into the ground defined and due to energy loss of the system the body does not attain the initial height and gradually comes to rest.

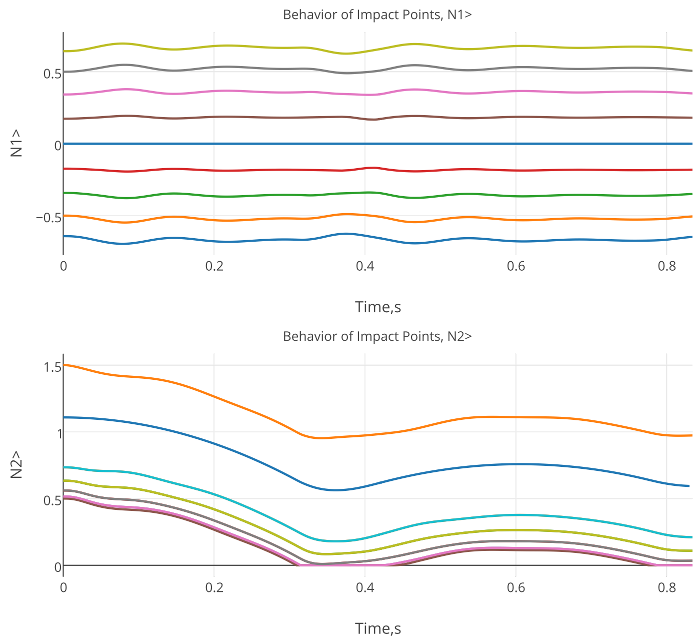


Figure 5.3: Colliding Points

5.3 Energy of System

From the Section 3.5 we know that the total Energy of the System is given by Equation 3.16. Intuitively, we can conclude that the total energy of a Flexible body after collision should not increase and should either decrease or remain at a constant value. Based on the collision response, following results are expected to the energies defining the system:

- **Potential Energy(Spring)**: Should be at a constant value until collision occurs and increases after collision until the contact phase exists.
- **Potential Energy(Gravity)**: As the body continues to drop under free-fall, the height of the ball from the ground decreases, thereby we can conclude that this value should decrease until collision occurs and thereafter increase.

- **Kinetic Energy**: Since the body gains velocity under free-fall, the Kinetic Energy of the system increases until collision occurs, and thereafter decreases until the contact phase exists.

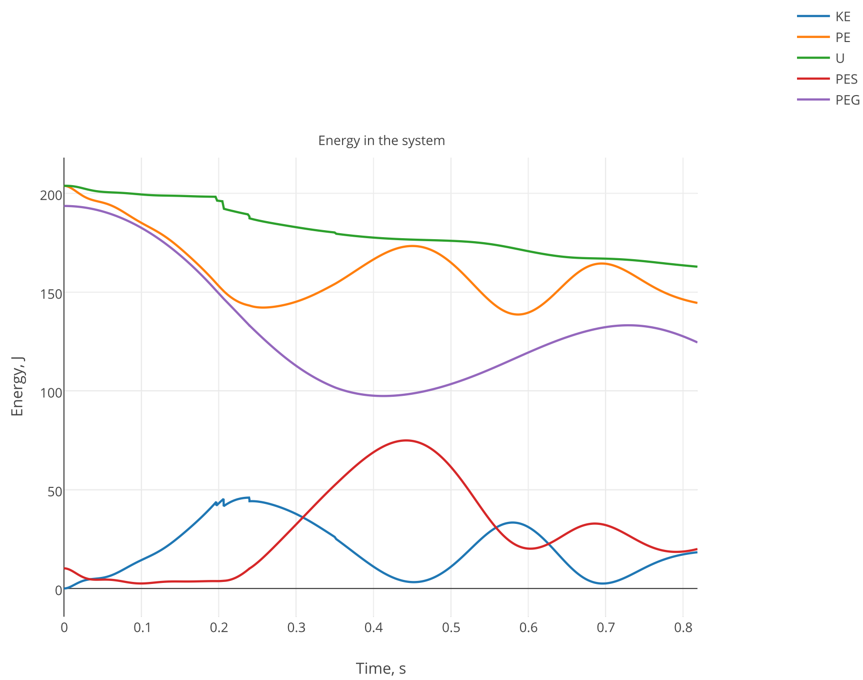


Figure 5.4: Energy of System

The desired behaviour for the energies of the system as mentioned are similar to the observed results as shown in Figure 5.4.

5.4 Conclusion

This work provides a Continuous Iterative scheme to evaluate the collision response for a *Flexible Multi-body System* using Constraint Modelling. This approach gives us a better understanding to the behaviour of a *Flexible Multi-body* during collision response. By considering the body to be a *System of Particles* we can create any arbitrary shape representing a body and analyse its behaviour dynamically. By analysing a body to be a *system of particles* we can efficiently solve this problem and can demonstrate a real-world scenario.

5.5 Future Work

There is a lot of scope in this area that can be implemented on different kinds of Multi-body systems. By modifying the system of particles, we can construct 3D models based on any desired arbitrary shape and the iterative scheme mentioned in this work can be used to solve for the collision response of the same. To design a real world problem frictional force can be considered to resolve the problem. Using the proposed methodology we can design a *flexible* as well as a rigid body.

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BIOGRAPHICAL STATEMENT

Rohit Vijay Katti was born in Thane, India, in 1988. He received his B.E. degree from visvesvaray Technological University, India, in 2010, in Mechanical Engineering. From 2010 to 2012, he was an Associate Developer at DELMIA, Dassault Systemes, India, in the Machining domain. He joined University of Texas at Arlington in 2013 Spring for his M.S. degree in Mechanical Engineering and graduated in the Spring of 2015. His current research area varies in Robotics, Multibody Systems, and Computational Geometry.