

INVESTIGATING STUDENT ENGAGEMENT IN MATHEMATICAL  
CONVERSATION: TEACHER QUESTIONS  
ELICITING STUDENT  
RESPONSES

by

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Presented to the Faculty of the Graduate School of  
The University of Texas at Arlington in Partial Fulfillment  
of the Requirements  
for the Degree of

DOCTOR OF PHILOSOPHY

THE UNIVERSITY OF TEXAS AT ARLINGTON

May 2014

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## Acknowledgements

I am indebted to all the individuals who participated in this study. I especially want to acknowledge the generosity and courage of the two teachers who welcomed me into their classrooms to study the questions they asked to include their students in mathematical discourse.

I would like to thank my dissertation committee for all the shared knowledge and valuable conversations. Special thanks to Dr. Ann Cavallo, who chaired my committee. I will always be forever grateful for your expertise and guidance. Thank you for spending the hours mentoring my writing skills. Thank you, Dr. Adrienne Hyle for all your thoughtful advice and encouragement throughout my doctoral studies. Thank you, Dr. James Hardy for all your encouragement from the beginning of my doctoral process.

I would like to thank my friends and family who provided support and encouragement through completion of this work. I want to thank my parents, Dr. Elliott and Inge Johnson for caring for my children when I was in class or studying and for always giving an encouraging word throughout the process. I want to thank Diane Bunker for your friendship and support during this educational journey. I want to especially thank my husband and best friend, Raymond, my daughter, Rebecca, and my son, Caleb for the continual understanding and encouragement over the years.

Last, but certainly not least, I want to thank my Lord Jesus Christ for blessing me with the strength and perseverance to complete this enriching journey.

April 21, 2014

Abstract

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Mathematics educators advocate that K-12 teachers should create classrooms where students are engaged in conversation about mathematical ideas. However to achieve these goals, it is important that teachers understand how to engage students in discussion. The purpose of this study was to identify teacher questions and student responses in a problem solving environment and examine how teachers used questioning to engage students in conversation. The findings provide descriptions of the dialogue that transpires between teachers and student as mathematical ideas are developing in the learning process. This study contributes to the mathematical research of constructivist learning theory by providing information on the dialogue between teachers and students in the mathematics learning process. This study also analyzed possible links between teachers' motivation for teaching and their teaching practices, to further knowledge on the influence of motivational theory.

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## Chapter 1

### Design of the Study

Influential educational theories in mathematics, including general constructivist (Piaget, 1965) and social constructivist theories (Vygotsky, 1978) support the philosophy that active participation in mathematical discussion is an important part of students' learning processes (Ernst, 1996; Davis & Maher, 1997; Watson & Chick, 2001; Voigt, 2007). This philosophy espouses that teachers must provide opportunity for students to formulate ideas and concepts in their own minds through experiential learning, rather than the teachers simply giving students information to learn. By allowing opportunity for student discovery, classroom discussions, problem solving activities, and group work, the teacher is promoting students formulation of understandings. This philosophy therefore, supports limited or no direct instruction through the traditional lecture format.

Discussion of concepts and ideas being learned with peers and the teacher, also known as classroom discourse, is consistent with constructivist theory and has been shown to promote student learning (Ball & Friel, 1991). Accordingly, discussion makes apparent how students' understandings are constructed and exchanged in the classroom (Maher & Alston, 1990). Discussion enables students to share, create, and justify meanings for their ideas and discoveries as well as build their understanding of mathematical concepts. Listening to discussion about mathematics may provide both teachers and students with insights into students' reasoning, learning, and problem solving, facilitating mathematical communication in the classroom. Furthermore, conversations in which mathematical ideas are explored from multiple perspectives can help the participants make connections and develop different ways of representing the same mathematical idea (NTCM, 2000).

National Council of Teachers of Mathematics (2000) contends that the classroom should be an environment that encourages problem-solving, expression of the students' ideas, presentation of convincing arguments, and where developing an approach to thinking about mathematics is valued over rote memorization. This contention emphasizing active learning versus passive, rote memorization has been supported in the research literature (Pimm, 1987; Maher & Alston, 1990; Hoffman, Breyfogle, & Dressler, 2009; Cengiz, Kline, & Grant, 2011). Learning theories supporting constructivist teaching focus on how learners construct mathematical understandings. Von Glaserfeld (1990) provides a perspective of constructivism as a theory of students' knowing, rather than a theory of students' knowledge, which holds as paramount the establishment of environments in which students have opportunity to connect conceptual practice to everyday life. Constructivist theories contend that there is a social aspect important to students' learning and advocate that teachers be more attentive to student thinking by insisting on the use of pedagogy that endorses active thinking by students (Ernest, 1996; Voigt, 2007).

As teachers incorporate constructivist theories by using practices to support student learning in classroom discussions, they may differ in the way they approach, interpret, and respond to its challenges. One area in which teachers may differ in their use of constructivist practices is teacher motivation. Teacher motivation is described in the literature as *goal orientation* with respect teaching approach. Butler (2007), Retelsdorf, Butler, Streblov, & Schiefele (2010), and Netsche, Dickhauser, Fasching, & Dresel (2011) have attempted to express differences in teaching approach by observing different goal orientations a teacher may hold in classroom settings.

Goal orientation arises from achievement goal theory, which deals with the purposes or reasons individuals focus on an achievement task such as teaching, and the

standards they use to evaluate their task performance (Pintrich, 2000). In regards to teaching, Butler (2007) identified four classes of achievement goal orientation: a) mastery goal orientation (focusing on learning and developing professional competence), b) ability approach goal orientation (focusing on demonstrating superior professional skills), c) ability-avoidance goal orientation (focusing on avoiding the demonstration of inferior professional skills), and d) work-avoidance goal orientation (focusing to get through the day with little effort). Butler also suggests that goals may impact how teachers react to challenges of implementing constructivist practice in the classroom.

The teachers' goal orientation may be related to the extent to which they use mathematical discussion. In constructivist classrooms, teachers allow students to drive the discussion, while facilitating and managing the discourse to promote all students' thinking and engagement (Yackel & Cobb, 1996). Constructivist theory is consistent with teachers' creating a supportive environment where students feel free to share their mathematical ideas; agreeing to transfer some control of the mathematical direction of the discourse to the class of students; preparing interesting tasks for students that elicit discussion, and; asking probing questions that provoke students' reasoning. On the other hand, the roles of the students focus on developing the abilities to engage in advanced mathematical reasoning and to verbalize their reasoning to others (Ilaria, 2009). The teacher nurtures students' thinking by giving them the opportunity to accept responsibility of describing their work, justifying their claims, and answering with a contemplative response to the explanation of others. Though research exists on teachers' goal orientations in teaching in general terms, new research needs to explore possible relationships between teachers' goal orientations and their constructivist practices in the classroom, specifically in regards to facilitating classroom discourse.

## 1.1 Statement of the Problem

With the development in education of the 21st century skills like problem solving and critical thinking, teachers must incorporate instruction which fosters the students' abilities to engage in sophisticated mathematical reasoning and to articulate this reasoning to others (Darling-Hammond, 2006). Discussion between students and teachers becomes an instructional practice by which teachers encourage students to become active participants in the learning process. While a range of pedagogical responses are possible, teachers' questions can be useful for encouraging this discussion and promoting students' reasoning (Van Zee & Minstrell, 1997). Typically, teachers' questions have been planned in advance as dictated by their lesson plans and objectives, which may be given to the teachers in their curriculum or have been self-identified. Using these objectives, teachers may script their part of the classroom discourse, which does not allow for open discussion led by the students (Manouchehri, 2007). Using constructivist practices teachers allow student discussion and questioning that may be beyond the scripted questions and/or objectives. The teacher spontaneously makes decisions on whether or not to incorporate certain student-initiated questions and responses in the discussion. The teacher's ability to continue a conversation based upon student responses can be described as an improvisational move, which is made "on the fly in response to unanticipated developments in the discourse" (Dick & Springer, 2006, p.106). Since leading a constructivist-based discussion requires the teacher to have adequate content and pedagogical knowledge, Dick and Springer contend that the most demanding challenge for a teacher in a discourse rich classroom is the improvisational move. Therefore, useful pedagogical knowledge for teachers would necessitate having understanding of the types of questions that engage students in mathematical

conversations and having skill and knowledge of how to scaffold learning and support continued discourse when students generate such questions.

Clearly, teacher facilitated classroom discussion requires the use of questioning techniques. Research has described frameworks for categorizing teachers' questions, providing guidelines or techniques for asking productive questions, and illustrating how productive norms in the classroom are established by the use of questioning (O'Connor & Michaels, 1993; Van Zee & Minstrel, 1997; Martino & Maher, 1999; Mewborn & Huberty, 1999; Stylianou & Blanton, 2002; White, 2003; Goos, 2004). For example, Cotton (1989) summarized the research done on types of questions and explained this dualistic system. There are two general types of teacher questions: low-level and high-level. Low level questions are also called closed, direct, knowledge, and recall questions. From research, when questions require students to recall specific knowledge from text or teacher's notes, they are low level questions. On the other hand, high-level questions are open-ended, interpretive, evaluative, inquisitive, inferential, and synthesis-based. These questions require students to elaborate on the information so responses are not always immediate. In her research, Goos (2004) emphasized the need for teachers to allow "wait time" for responses from students. When teachers allow processing time students have opportunity to respond using higher levels of thinking.

Substantial research has addressed how to start and end a discussion (Van Zee & Minstrel, 1997; Stylianou & Blanton, 2002; White, 2003; Soucy McCrone, 2005) but studies of the nature of the actual discussion that transpires between teachers and students is sparse. Specifically, the literature has incomplete information on the types of questions that are useful in engaging students' reasoning in mathematics classrooms and the types of responses these questions conjure in the students (Ilaria, 2009). This

information will contribute to the knowledge base in mathematics education on teacher questioning and discourse that may promote deep level student learning.

## 1.2 Orienting Theoretical Frameworks

The environments selected for this study are classrooms where the teachers with two opposing goal orientations towards teaching, mastery versus ability, are participating in discussion about mathematics. These opposing teacher goals were selected to provide a potential contrast in questioning usage in the two environments. The importance of mathematical discussion in supporting students' understanding is established in many theories of learning such as constructivist theory, sociocultural theory, and linguistic theory (Noddings, 1990; Forman, 1996; Sfard, 2008). In order to frame this study, the theoretical lens investigates the benefits of teacher questioning and classroom discussion within Piaget's constructivist theory of learning, Vygotsky's zones of proximal development, and more recent theories concerning sociomathematical norms and the significance of participation in mathematical activities (Yackel & Cobb, 1996). It is posed that evaluating teacher questions may incorporate a blend of background theories of learning and a review of each of the learning theories provides foundational research that has been conducted in this area.

Constructivist theories of learning promote active participation of students in the learning process indicating a significant change away from the traditional, teacher-centered instructional mode in classroom. According to Piaget (1995), the "role is less that of a person who gives 'lessons' and is rather that of someone who organizes situations that will give rise to curiosity and solution-seeking in the child" (p. 731). These ideas transitioned from the role of the teacher as disseminating information to the students to the teacher who builds students' mental models by implementing appropriate mathematical situations that require students to think (Cobb & Steffe, 2011). These

theories feature the teacher as crucial to helping students formulate understandings of mathematics in their own minds, and not being the focal point of instruction in the classroom. The following analysis portrays how teacher questions designed toward helping students understand mathematics are consistent with constructivist theories of learning, particularly those of Piaget (1952) and Vygotsky (1978).

Piaget (1952) submits a theory of how intellect develops requires that learners progress through mental functioning processes: assimilation  $\leftrightarrow$  *disequilibrium*  $\rightarrow$  accommodation  $\rightarrow$  equilibrium. Assimilation is the process by which a person takes in new experiences and information through the senses. Disequilibrium is when new information and/or experiences conflict with what is known and may not readily make sense to the learner, also known as cognitive conflict. Learners in disequilibrium often need to go back and assimilate more information in the attempt to make sense of new information/experiences toward understanding. When new understanding is achieved (“ah-ha” moment), it is known as accommodation. Learners have now gained new understandings of what was assimilated and can now adjust cognitive structures to “accommodate” this new knowledge. Equilibrium is the balance between assimilation and accommodation. Mathematics teaching needs to match how students learn by promoting these mental processes. To do so, teachers first provide experiences for students to assimilate new mathematics ideas and concepts. Students best assimilate new information through direct experiences, without first being “told” how things work. Teachers should allow students to experience “disequilibrium” or cognitive conflict as they work to understand; which also promotes motivation to learn and helps them reorganize existing cognitive structures. Providing active, direct experiences for students to assimilate new mathematical ideas promotes reasoning skills. Effective teachers have a clear understanding of what students already know and use this understanding in



planning new experiences for students to assimilate. Teachers therefore know if students are ready to assimilate the new ideas to be investigated and in most cases, know how to build in disequilibrium toward motivating the “need to know” among students. Teachers can then use questions to guide student thinking toward helping them achieve accommodation or equilibrium (new concept understanding). Questioning and having students verbalize their thinking is one way for the teacher to gain knowledge of student thinking as they progress from assimilation through disequilibrium to accommodation (Ilaria, 2009). Thus questioning is critical for teachers to use in eliciting students thinking, and better understanding their reasoning.

Questions are ways for teachers to measure the extent to which students are assimilating knowledge and to guide them through disequilibrium to foster growth in understanding. Piaget emphasized four criteria necessary for individuals to progress to higher levels of intellectual development and reasoning ability, 1) experience, including physical and logical-mathematical, 2) social interaction, 3) maturation, and 4) resolving disequilibrium to equilibrium. Phillips (1981) describes the knowledge that an individual abstracts from physical objects as physical experience. The individual plays or works with an object and consequently, engages in an abstraction process that conveys knowledge about that particular object. During a logical-mathematical experience, the individual constructs relationships between objects. Further, social interaction implies that the knowledge is acquired through interactions with people. Maturation affects an individual's ability to learn depending on his genetic growth at the time. Finally, the resolving of disequilibrium to equilibrium integrates with the other three factors to encourage growth in understanding.

Several themes are the foundation for Vygotsky's theory of social constructivist learning as derived from the work of Goos (2004), Cobb and Bauserfield (1996), and Van

Oers (1996). The mental phenomenon of learning focuses on the process of growth and change, rather than the end product. In addition, social interaction is critical to promoting individuals' learning and intellectual development. Mental processes are communicated by signs and tools, such as algebraic symbols or verbal language, also supporting the theory that learning is socially constructed. Vygotsky described the zone of proximal development in accounting for intellectual development, in which transformation from social phenomena to psychological phenomena occurs (Goos, 2004). Vygotsky's (1978) zone of proximal development is considered the mental distance between a child's problem-solving capability when working alone (actual developmental level) and what can be accomplished with the assistance of someone more skilled and/or knowledgeable (potential developmental level).

Vygotsky's zone of proximal development posits that learning can occur through attentively planned instructional tasks (Sfard, 2003). In addition, students learn by experiencing problems beyond their current level of ability (potential developmental level), and when an individual with more capability or knowledge, such as a teacher, guides the learning process, scaffolding learning with appropriately designed questions (Sfard, 2003). When students are able to work at the potential developmental level without the assistance of more capable peers or the instructor, the potential level becomes their new "actual" developmental level. Thus, teachers promote more advanced levels of intellectual development and skill among students by providing them with problems beyond their current abilities, at a level that allows them to achieve new skills and understandings with guidance from a teacher or more capable peer. Teachers guide students through the zone of proximal development from actual to potential development through scaffolding, where once accomplished, the potential developmental level becomes the new actual developmental level. This process requires knowing students'

potential developmental levels, using effective questioning, and exchanging of discourse with students to understand their progress in reasoning. The exchange between teacher questioning and student responses elicits student reasoning and helps the teacher move students toward independence.

Social norms are the established classroom culture that defines the way students interact with the teacher and each other (Ilaria, 2009). The inquiry classroom can promote student participation by encouraging social norms like: students collaborating to solve problems, students viewing mistakes as a natural part of the learning process, students sharing strategies and explaining their thinking, and students solving problems using a variety of strategies and representations (Cheval, 2009). Yackel and Cobb (1996) describe social norms further and define sociomathematical norms as “normative aspects of mathematics discussion specific to students’ mathematical activity” (p. 461). These norms define acceptable mathematical explanations and justifications for a mathematics classroom. The following example clarifies the difference between social and sociomathematical norms. The understanding that students are expected to explain their solutions and their ways of thinking is a social norm, whereas the understanding of what counts as an acceptable *mathematical* explanation is a sociomathematical norm. Students and the teacher within their own environment define what would be an acceptable mathematical conversation (Yackel & Cobb, 1996). Teacher questions can allow students to have opportunities to present new concepts, ideas, and ways of solving problems in addition to those presented by the teachers; but at the same time, they communicate their expectations for acceptable mathematical arguments. The students’ responsibilities are communicated as they craft their ideas verbally with the teacher and the other students. The resulting discourse communicates socially and mathematically acceptable arguments for that classroom. With the constructivist-based social and

sociomathematical norms, students can engage in sense-making and justification responses to questions, thereby promoting active participation in learning.

Sfard (1998) describes learning foundationally in terms of two metaphors, acquisition and participation. A common historical view of learning views the acquisition of knowledge as something that can be accumulated by the active or passive participation of the learner (Ilaria, 2009). However, some contemporary educational research alludes to the participation metaphor as a shift that considers 'knowing' as the process of becoming a member of an established community (Sfard, 1998). When knowledge is defined as process instead of a product, communication and language become mathematical activities that coincide with learning (Sfard, 2008).

Lambert and Cobb (2003) reviewed studies on the participatory metaphor of learning that emphasized talking and writing as aspects of doing mathematics. If mathematical talk is to be a priority in the classroom, teachers need to encourage the development of abstract mathematical reasoning. For example, teachers can provide supporting discourse by arranging the introduction of mathematical words and definitions and helping students' articulate meanings. However, the practice of talking about mathematics and negotiating mathematical meaning is neither a focal point nor typical in the mathematics classroom (Lambert & Cobb, 2003). Consequently, few classrooms offer opportunities for students to mathematically communicate or communicate to mathematically learn. Therefore, students' active participation and promotion of their construction of understanding could be promoted by teachers utilizing questions (Bennett, 2010).

The constructivist learning theories of Piaget and Vygotsky emphasize social interaction is important for learning. Accordingly, students' active participation and social exchange of ideas among teachers and students in mathematics would foster

development of reasoning and sense-making (Yackel & Cobb, 1996). These psychological learning theories support the contention that students learn best when actively participating in the learning process through direct experiences and social discourse, including teacher questioning. However, such mathematical activity in the classroom may be influenced by the teachers' personal beliefs and goal orientations for teaching (Bulter, 2007). For example, teachers motivated by *mastery* goals for teaching could measure their competence for asking questions in the classroom by how successful their prior questioning session was or their perceived ability to ask questions. While teachers who are *ability approach* motivated might assess their questioning ability by how they did relative to another teacher. Teachers who are incorporating the discussion into their instructional strategies confront issues with their students' initial abilities with mathematical concepts and their students' perceptions of their teachers' and parents' expectations and the teacher's own communicating abilities and mathematical knowledge to name a few (Schoenfeld, 1987; Cobb & Yackel, 2003; White, 2003), Teachers will be continually adjusting and learning as they seek to bring understanding to their students through questioning. Butler (2007) found that teachers identified with mastery goals were more apt to ask for help in improving their teaching practices and environment. Whereas, teachers with the ability approach orientation did not necessarily ask for help when confronting difficulties. Using social interaction with teacher questioning is a complex undertaking; the progression of incorporating dialogue into the student learning process is a learning process for the teacher, as well as the students. The motivational goals of teachers may have an impact on the success of the whole learning process.

Although there are various learning theories in mathematics education, constructivist theories expect the students be active in their learning and teachers guide

the process. In constructivist teaching, teachers' roles do not simply convey information; rather students actively engage in the process of acquiring knowledge. Through social interaction in the classroom and the teachers' participation in the learning process, teachers who practice constructive teaching utilize strategies to elicit and understand their students' thinking. When questions are used strategically by the teacher, sociomathematical norms are established in the classroom. Teachers are able to evaluate the students' thoughts. With this information, teachers can provide students the opportunity to grapple with cognitively challenging problems as they guide students through the process of assimilation and accommodation in order to understand the problems. Questioning and discourse promotes reasoning and intellectual development through social interaction. Teacher questioning accesses students' mathematical reasoning, may help promote disequilibrium, and provides needed information for scaffolding, toward new understandings. In addition, questioning, requiring student engagement consistent with constructivist theories, promotes student-centered learning.

### 1.3 Purpose of the Study

The purpose of this descriptive study was to observe classroom discourse during problem solving in mathematics to explore teacher questioning and student responses, and to examine the nature of the interactive discourse in this setting. Classroom discourse was examined to identify the ways in which teachers' questions engage students in mathematical conversation. During this process, teachers' questions and students' responses were described and categorized. This study focused on revealing and describing types of questions teachers use to engage students in mathematical discourse. For example, what types of questions help students take part in the conversation in solving mathematics problems? What means do teachers use to better understand how students are thinking and reasoning? If teachers' questions elicit

students' mathematical reasoning, what is the nature of the teachers' questions and students' responses? These and related questions were the focus of this research.

The specific purposes of this study were:

1. To describe and identify themes on the types of questions teachers ask during classroom discourse and determine the frequency each type of question is asked.
2. To describe the types of student reasoning each identified type of question tends to evoke by analyzing and coding the students' responses.
3. To describe the extent and nature, and identify possible patterns in the types of teacher questions that elicits student engagement in mathematical reasoning.

#### 1.4 Methods

Specific data was collected from selected teacher and students through responses and observation to respond to the research questions. After receiving a brief description of the researcher, the following sections detail data needs, sources, collection and analysis.

##### *1.4.1 The Researcher*

As a mathematics teacher for 25 years in junior high school through college undergraduate level, my desire is that my students are successful through their education and on into their careers. As a beginning teacher in junior high and high school, I was fortunate in having mentor teachers who were devoted to educating each individual student. Their passions were infectious and nurturing. Consequently, I became intrigued on how to best educate students in mathematics.

Through the years, I was a part of several teams of teachers who worked together devising methods and curriculums to enhance the learning process. When the graphing calculators became popular in the nineties, I was trained on the use of them,

developed curriculums, and used the calculators in my lessons for discovery. In addition, I was part of the team who developed an Algebra I program for at-risk teenagers. A key motivator for me is teaching mathematics in ways that will enhance students learning of mathematics

During the last couple of years, I have been teaching mathematics education courses at a local university. The courses are designed to develop and strengthen the thinking skills and communication of mathematical concepts of undergraduate students seeking admission into the K-8 teacher certification program. The courses are set up in a problem-solving format where students discuss ideas and form new understandings in a collaborative environment. Students work in groups as they progress in learning the mathematics concepts in the curriculum. As I facilitate the activities, questions dominate my communication with my students. Most intriguing to me is the nature and process of using questions to lead and challenge students to engage and seek clarity in understanding the mathematical concepts. Therefore, teacher questioning in mathematics is the focus of my dissertation research.

#### *1.4.2 Data Needs*

The data needed to complete this study was gathered in two phases. First, the study identified the goal orientation for teaching mathematics among teachers in the study sample as mastery or ability-approach goal orientation. These two opposing orientations were selected for use to identify teachers who focus on mastery learning (learning for the sake of learning) and therefore would be expected to use more constructivist-based procedures such as group work and questioning; and to identify teachers who focus on ability or performance and therefore may use less constructivist-based teaching procedures. A glance into the classrooms of the two teachers granted a look at the nature of the discussion in their respective environments. Second, data was



collected via classroom observation, focusing on the mathematical discourse that transpires between teachers in the sample and their students during two problem-solving class sessions. The teachers' questions and responses to their students along with the responses of the students were observed and audio/video recorded. The mathematical discourse of the teachers with their students was both qualitatively and quantitatively described.

#### *1.4.3 Data Sources*

The settings for data collection were precalculus high school classrooms of teachers with particular goal orientations for teaching, determined by survey to be either mastery or ability-approach. Precalculus was selected because in these classrooms, teachers have the challenge of preparing students for advanced levels of mathematics by helping them establish their understanding of Algebra I and II. In addition to the understandings of the algebras, teachers bridge the gap to calculus by working with students on concepts in trigonometry. The teachers were drawn from private schools in the Dallas-Ft. Worth area and represent convenience sampling due to the accessibility and availability of these teachers. If a sample population is believed to be a representation of a given population, many researchers take advantage of populations that are convenient to access (Gall, Gall, & Borg, 2007).

#### *1.4.4 Data Collection*

Permission was obtained from private school administrators in the area to conduct this study with their teachers. Upon approval, teachers were contacted by email and in person and asked to participate in the study. An online demographic survey was collected that gathered information on the teachers such as years of teaching experience, degree, gender, and ethnicity. Upon collection of demographic data the study took place in two phases.

*1.4.4.1 Phase One: Identifying Teachers' Goal Orientation.* The teachers were given a survey, which is a version of Retelsdorf et al. (2011) scale for teachers' goal orientations for teaching. Teachers rated responses to 16 statements (see Appendix A) on a four point Likert-type scale defined as 1 being 'not true at all' and 4 being 'absolutely true'. Each subscale consists of four items describing mastery orientation, ability-orientation, ability-avoidance orientation, and work avoidance. When this survey was used in previous research, the Cronbach's alpha was reported between .68 and .78 (Butler, 2007; Retelsdorf et al., 2011).

Data collected on the goal orientation survey was used to identify teachers' specific goal orientations. A teacher who scored highest in their mastery orientation was selected, and a teacher who scored highest in ability-approach orientation was selected to participate in this study. The two identified teachers had similar years of teaching mathematics and similar educational backgrounds, as both were certified in secondary level mathematics. Two classes of each teacher where problem solving was the structure of the session were identified for observation and data collection for the second part of this study on questioning, student reasoning, and discourse.

*1.4.4.2 Phase Two: Teacher Questioning, Responses, and Student Discourse.* The selected classroom teachers were observed with attention focused on the questions asked and the students' responses to teacher questions. To facilitate observational data collection, each class session was video-taped. Field notes were taken on the set up of the classroom, basic patterns of interaction between teachers and their students, and the goals of the lesson. The recordings were transcribed, and the field notes supplemented the transcription as to the happenings in the classroom.

#### *1.4.5 Data Analysis*

To accomplish the purpose of this study, videotape recordings of the classroom setting and the field notes were used to perform the analysis. The video recordings of both teacher and students in the class setting provided several advantages. The classroom activity was permanently recorded in both the visual and audio formats. Transcripts recorded of the verbal dialogue, and videotape recordings allowed for repeated viewing. Finally, field notes added additional information for analysis.

The focus of data analyses was on the types of teacher questions and student responses indicating mathematical reasoning. While viewing the videotapes, conversations were transcribed, identified, and described for more in-depth analysis as explained in Powell, Francisco, and Maher (2003). These researchers developed a methodology “to investigate the nature of teacher intervention in the growth of student mathematical ideas” (p. 422) by viewing videotaped classroom data. This design was employed in the current study because of the intended concentration on the ways teachers question students and how the students respond.

After transcribing the teacher-student conversations in the data, discourse was coded according to exchanges focused on teacher questions and student responses. The coding started with a predetermined list of teacher question codes from Speiser, Maher, and Walter (2001) and Ilaria (2009) as an initial framework. From these previous studies, the teacher question codes were summarized and are shown in Table 1.1 (Ilaria, 2009). These codes represent the types of questions discussed in the literature that connected questions to student thinking and reasoning. After coding the data for teacher questions, calculations were made on the number of teacher questions in each category. These totals provided baseline information used to describe the types of questions teachers asked and how often these types of questions were asked. Additional codes were

developed as revealed by the data collected. This procedure served to respond to the first purpose of this study: *To describe and identify themes on the questions teachers ask during classroom discourse and determine the frequency each type of question is asked.*

Table 1.1 Initial Teacher Question Codes

Code	Description
T(r)	Teacher asks a student to consider an old idea.
T(d)	Teacher asks a student to contribute to the emerging discourse.
T(c)	Teacher asks the student to clarify his/her statements or ideas.
T(j)	Teacher asks the student to justify his/her statements or ideas.
T(con)	Teacher confirms the student and teacher both agree on what has been done or said.
T(f)	Teacher follows the student's idea or suggestion.

The second purpose of this study was: To describe the types of student reasoning each identified type of questions tended to evoke by analyzing and coding the students' responses. To respond to this purpose, the student conversations and responses were examined from the transcriptions of discourse during classroom observations. The students' dialogue was transcribed in response to the teachers' questions, and the responses coded. The initial codes shown in Table 1.2 were generated by Ilaria (2009), and provided baseline codes for this study. If student responses were repeated and could not be described by the initial codes, new categories were developed. After completing the coding, calculations were made on the number of times each code appeared in classroom discourse.

Table 1.2 Initial Codes for Student Responses

Code	Student Response
S(ta)	Thinking aloud
S(pb)	Proof-building
S(ans)	Answer
S(c)	Clarification
S(con)	Confirmation
S(a)	Attunement
S(qs)	Questions students
S(seek)	Seeking feedback
S(dnc)	Non-contribution

To provide reliability/validity check of data, two mathematics teachers in the mathematics department were asked to code the transcripts using the initial codes and to compare their coding with the researcher's coding to determine level of agreement.

The third purpose of this study was: *To describe the extent and nature, and identify possible patterns in the types of teacher questions teachers use to engage students in mathematical conversation and elicit mathematical reasoning.* After completing the coding for teacher questions and student responses, data analysis tabulated the frequency and type of each student response to each type of teacher question. This analysis revealed information on the types of student responses evoked by various teachers' questions. The data were examined for possible patterns in questions that engage students in conversation.

### 1.5 Significance of the Study

Teacher quality has been found to be the single most important factor related to student achievement (Darling-Hammond, 2000; Rice, 2002). A few of the popular measures researchers use to assess teacher quality are subject knowledge, pedagogical content knowledge, years of experience, level of certification, and degree level (Wayne & Youngs, 2003). Besides looking at *who* the teachers are (e.g., experience), this study

examined *what* the teachers do in the classroom. This study described the discourse and identified characteristics of this discourse that lead to student learning. Further, this discourse was analyzed through the lens of teachers' different motivational goal orientations. This information is important to know because delineating the skills, actions, and dispositions required of teachers to create environments that support useful discussions supports the effectiveness of teachers, pre-service teachers, and administrators in their pursuit in promoting student learning.

### *1.5.1 Theory*

This study adds to the discussions in educational research pertaining to constructivist learning theories with respect to the student. As classroom discourse was investigated, the themes of the teacher questions give important insights to the process guiding students through the learning process. In the environment of problem solving, the identification of student responses reveals actual scenarios of student thinking as the learning process takes place.

The literature indicates that motivational theory is a powerful factor that intervenes with teachers' classroom practice. However, the connection of goal orientation to the nature teacher questioning, student responses, and classroom discourse is not yet well defined. This study contributes to the knowledge base on motivational theory by determining possible differences in questioning and discourses between teachers and students with mastery orientation teachers as compared to those with ability goal orientation teachers. The nature of this interaction furthers the knowledge on how instructional practices may encourage and facilitate students verbal sharing of their thinking processes.

### *1.5.2 Implications to Practice*

This study identified specific characteristics of teacher questions and questioning themes. The questioning patterns of the observed teachers provide information for mathematics teachers and educators on the dialogue that occur in classroom discussions, and perhaps lead to greater student learning. Teacher questions elicit student responses. Therefore, teachers may better understand the types of student feedback they can typically expect when specific types of questions are asked. With a better understanding of question types teachers will be prepared to more conscientiously employ specific types of questions in their classrooms to elicit the desired types of responses and reasoning from students and more effectively lead students through the learning process. With knowledge on questioning teachers gain insight on student reasoning as they publically share their thinking. Describing the interaction of teacher questions and student responses furthers understanding on ways to engage students in mathematical conversation/discourse.

The results of this study, help teachers and administrators gain new knowledge on how specific teachers with particular goal orientations for teaching may conduct their mathematical classroom discussions. The patterns revealed according to teachers' goal orientations could provide a direction for professional development and teacher mathematics education programs. Mastery or ability goal orientations may be addressed through self-reflection and evaluation in these in-service programs. Educational programs may be designed to guide teachers in questioning, setting appropriate motivational goals, and improving upon their teaching, thus enhancing student mathematics learning.

### 1.5.3 Research

NTCM has clearly articulated guidelines for what should be covered in productive mathematical discussions but it has not given teachers specific guidance on how to go about creating effective discussions (Lampert, Rittenhouse, & Crumbaugh, 1996). This study will systematically analyze and identify patterns between what teacher asks and how students respond. As prior research has suggested categories of questions for teachers, a detailed level of coding questions and responses will be one outcome of this study to articulate specific, rich descriptions of question types and anticipated responses in actual classrooms. Finally, this study seeks to describe how questions types are used by the teachers seeking to engage students in mathematical discussions and how the questions may foster the learning process.

This research aimed to add to the relatively new research pertaining to how teacher goal orientations for teaching might influence mathematics' teacher instruction practices. While researchers are identifying avenues of equipping teachers to be highly effective in the mathematics classroom, this study furthers existing information in the literature by identifying patterns according to goal orientations of the teachers with respect to their questioning and discourse patterns. Knowledge of teachers' goal orientation may reveal the context that promotes a productive learning environment between teacher and student.

## 1.6 Reporting

Chapter One has been designed to set the stage for this study, namely -- background, problem, research questions, methods, and significance. Chapter Two presents an extensive review of the literature to include the students' roles in the learning, the discussion in the mathematics classroom, questions used by teachers in the classroom, and discussion in education. Chapter Three describes in detail the research



methods including justification for the research design, a description of the population and sample, and the procedures for data collection and analysis. In Chapter Four, the data are presented through the codes of the actual dialogue in the classroom giving voices to the teachers and students. In Chapter Five, the results of the data analysis are presented. Finally, Chapter Six provides a summary of the findings and conclusions, presents recommendations for practice and future research, and closes with final thoughts about the study.

## Chapter 2

### Literature Review

This study is informed by the growing body of research associated with how teachers question students and the ways students respond to the questions in mathematics education settings. The study is based on literature on discourse, conversation, and communication in students learning. This section begins by describing the vision of teacher questions in the mathematics classroom, as well as the need for teacher questions in mathematical conversations and the types of questions teachers ask. This section then provides information on learning outcomes associated with encouraging student thinking in the mathematics classroom and reviews research on the role of discourse in the classroom.

#### 2.1 Teacher Questions

Teacher questions are studied as an essential component of effective teaching. Glenn (2001) defines effective teaching as “qualities that benefit students, improve instruction, and help an organization run more smoothly” (pg. 19). The value of focusing on teacher questions is that they are a foundational unit underlying most pedagogies of effective teaching (Gall, 1970). Reynolds and Muijs (1999) reviewed 50 years of educational research pertaining to effective teaching of mathematics. They reported that effective teachers were inclined to ask more “process questions”, which ask for explanations, although the majority of questions asked by teachers were “product questions,” asking for a single response. Related research indicates that asking good questions are a part of effective teaching by keeping students involved and monitoring their understanding (Franke et al., 2001). Thus, teacher questions that entice students to engage in mathematical conversation and relevant thought process involved in such conversation are important for productive teaching and learning in the classroom.

## 2.2 Need for Teacher Questions

As teachers attempt to incorporate mathematical discussion in their classrooms, they may encounter students who have limited ability to talk about mathematics. Kitchen (2004) conducted a study in a high-poverty, rural school analyzing the potential obstacles to productive discussion in a mathematics classroom. He found that students who have greater mathematical knowledge are the main influences or “voices” in the discussion. A majority of students are hesitant to use higher order thinking when prompted by teacher questions and/or share mathematical ideas (Kitchen, 2004).

If students have never experienced a classroom where discussion is valued, they may have difficulty contributing ideas in classroom discussions. Often students believe that the reason for the discussion is to give the teacher the opportunity to measure their mathematical knowledge. Consequently, they would rather condense their contributions to simple statements that they know are correct (Lubienski, 2000). Establishing a risk-free environment between teachers and students where students feel comfortable articulating their ideas is an essential first step toward enabling a discussion-filled classroom (Mewborn & Huberty, 1999; NCTM, 2000).

The acceptable and respected activity during mathematical discourse is defined as sociomathematical norms (Yackel & Cobb, 1996). Through continuous student and teacher interactions, sociomathematical norms are created and adapted. Through teacher questioning, student explanations of their understandings in mathematics can be enacted. Therefore, an important part of promoting student participation is arbitrating productive norms dealing with mathematical argumentation as an essential part of classroom discussion. Yackel and Cobb (1996) argue teachers who are guided by goals and beliefs and focused on creating active mathematical learning experiences are pivotal in determining the norms for student activity and the quality of mathematical learning in

the classroom. In other words, such teachers are responsible for helping students understand what counts as an acceptable mathematical explanation of their solutions and their ways of thinking.

One of the first steps in creating the productive norms of mathematical argumentation is devising an environment in which students feel free to express their mathematical ideas. With this initial step, one of the greatest challenges for teachers emerges as they give more control to the students and incorporate students' ideas into lessons (Manouchehri & Enderson, 1999; Soucy McCrone, 2005). Teachers need to establish a learning environment where each student feels safe to speak and express ideas, or they will fail to achieve the needed level of engagement for productive mathematical learning. Teachers may have difficulty implementing active mathematical discourse in classroom teaching for various reasons. The first difficulty may be due to lack of preparation to effectively use questioning in pre-service or in-service teacher education (Van Zoest & Enyart, 1998). At the in-service level, it is posited that teachers tend to maintain teaching in the way they were taught, which may be direct instruction, if they do not receive continued encouragement and support to understand the instructional demands the interactive mathematics classroom (Van Driel, Bulte, & Verloop, 2007). Further, an interactive classroom that embeds consistent questioning and discourse is often steered by student ideas; situations which may be unnerving to teachers as they no longer have full control of happenings in the classroom. However, as teachers encourage students to investigate and discuss mathematical ideas in small groups, they will gain the important benefit of understanding of how students' mathematical knowledge is developing (Knott, Sriraman, & Jacob, 2008). The challenges for teachers are to listen to the students' ideas, determine how to have the students explain their ideas, and use the students' ideas, discussion, and approaches to the material as primary guides for

their teaching. In this process, teachers ensure students are developing deep conceptual and mathematically accurate understandings of the material. To be effective with questioning, teachers need strong content knowledge and pedagogical skill in many possible mathematics curriculum topics, and need to be well apprised of their students' cultural behaviors (White, 2003).

In orchestrating an active mathematics learning environment as described, teachers must be qualified to discuss numerous related or supporting concepts based on students' responses and actions in the classroom (Yackel, 2002). Interactive teaching through questioning requires teachers to be flexible, responsive, and able to quickly adapt to students' verbal explanations and the mathematics discussed in the classroom (Manouchehri, 2007; Himmelberger & Schwartz, 2007). Teachers successful in using interactive teaching are typically confident in their own mathematical knowledge and are able to easily guide open-ended conversations as they occur. Rich teacher-student discussions involve teachers choosing related tasks, identifying the nature of questions needed, communicating purposeful questions, and all the while, growing the communicative competence of the students (Soucy McCrone, 2005). Student participation is critical to the process and it is most effective when all students participate and become involved in the conversation (Manouchehri & Enderson, 1999). Teachers who use interactive questions motivate students to engage in mathematical thinking and reasoning and arrange learning occasions that challenge students at all levels of understanding (NCTM, 2000).

Mathematical discussion reveals students' understandings and appropriate use of mathematics language as well as interpretations of meanings. Such language use and interpretations are revealed in interactive learning and can be addressed by teachers if not well understood by students. Kotsopoulos (2007) describes the lack of understanding

of mathematical language and interpretation of that language as two types of interference between discussing and understanding mathematics. Teacher-talk interference happens when too many mathematical terms are used, and student-talk interference develops from students who use everyday language rather than appropriate mathematics language. The mathematical understandings of students are hindered by both types of interference.

Two recent studies present specific suggestions for establishing discussion in classroom with in-depth discourse. Staples and Colonis (2007) emphasize aspects of *sharing* and *collaborating* in discussion. Sharing is characterized by students' expression of ideas and teachers valuing the ideas. Collaboration is students sharing ideas with other students, and then building upon each other's ideas. This process extends students' line of thinking in mathematics and problem solving. During sharing, Staples and Colonis encourage teachers to use students' ideas to ignite comments by other students, ask students for alternative ideas, and connect ideas together for the students. During collaboration, they suggest teachers build upon students' ideas, generate discussion about these ideas, and expose connections through additional student input.

Truxaw and DeFranco (2007) propose an inductive model of teaching in conjunction with a mixture of two types of discussion to further understanding. In one type of discussion, univocal, teachers ask questions and provide feedback from the teacher's point of view. A second type of discussion, dialogic, is give-take communication where students are an integral part of creating meaning to this discussion. Truxaw and DeFranco (2007) provide an example in an 8<sup>th</sup> grade algebra class where dialogic discussion is immediately applied, followed by the univocal discussion. The teacher revisits the students' initial thoughts to bring shared meaning of the problem for the students. Then the teacher guides students from their specific case

to a more generalized theory in algebra to advance the students' understanding. This description characterizes an inductive model of teaching that analyzes a selected conversation and determines that the teacher used questioning and feedback to maneuver students through recursive, inductive cycles rather than in linear steps (Truxaw & DeFranco, 2007). To elucidate the contrast, recursive inductive cycles are discussions that bring meaning to concepts whereas linear steps are just following a process, such as identifying the effects of transformations on the graph of a parent function.

### 2.3 Questioning Categories

In productive questioning teachers encourage mathematical discussion while not extinguishing students' original thoughts. Harris (2000) describes several purposes of questions where teachers can use questions for "checking understanding, starting a discussion, inviting curiosity, beginning an inquiry, determining student's prior knowledge, and stimulating critical thinking." Furthermore, when teachers design questions, they should consider their form, content, and purpose (Manouchehri & Lapp, 2003).

Some areas of the literature on using questions in education focus on correlating the types of questions that teachers ask with the anticipated or desired student responses. Cotton (1989) examined nearly fifty documents on questioning prior to 1989. Subsequently, he reports that based on the types of students' responses each of the questions were designed to evoke, researchers placed questions into higher and lower cognitive domain categories. While types of questions in the lower cognitive domain are regarded as facts, closed, recall, direct, and knowledge; those in the higher cognitive domain are open-ended, inquiring, interpretive, evaluative, and synthetic in nature. Although the two-levels of questions form a foundation for analysis, researchers continue to identify more specific and detailed information to expand these definitions for the categories of questions. These studies focused on what the expected student response

would be corresponding to the format of the teacher question and not the actual student response.

Woolfolk (1989) uses the lower and higher cognitive domains in describing questions and proposes categorizing questions into convergent questions with one correct answer and divergent questions with many possible answers. Cunningham (1987) publishes an extended list of questions where convergent and divergent questions are each separated into low and high subcategories. Based on the cognitive level of the students' responses, low-convergent questions would ask students to convey information by comparing, contrasting, or explaining. High-convergent questions would ask students to defend their reasoning and draw conclusions. Additionally, low-divergent questions might ask students to solve a problem in a different manner, and high-convergent questions might encourage students to elaborate, point out implications, or do open predicting. With the definition of these questioning levels, researchers find that teachers invite students to cognitively think about and process information in different ways.

In mathematics education, researchers have also formed categories of questions focusing on the intended student responses. Hiebert and Wearne (1993) identify four types of questions: recall, describe strategy, generate problem, and examine underlying features when they examine the types of questions asked by teachers in second grade mathematics classrooms. While conducting a study dealing with language in the classroom, Barnes (1990) cites three categories of questions that transpire in the mathematics classroom: factual by recalling facts, reasoning in putting together a logical argument, and open-based not using reasoning. Vacc (1993) uses this framework of these three types of questions in her own analysis. She concludes that teachers asking factual questions will find out the specific facts their students know, but teachers who ask questions in the open category will attain information about their students' understanding.



In these studies, the researchers generally aim to identify the types of responses they expected the questions to extract from the students.

Hierarchy is another categorization design for teacher questions. Bloom's taxonomy, where questions are labeled from simple to complex cognitive objectives is the most widely used hierarchy (Woolfolk, 1998). From observations in the classroom, Wolf (1987) introduces a different hierarchy, which addresses the selection of challenging questions. This hierarchy expands Bloom's six levels of questions and includes five more categories at the higher levels of thinking. They are comprised of: inference questions asking students to go beyond immediately available information; interpretation questions asking students to fill in missing information and consequences of information; transfer questions asking students to think about what can be predicted and tested; questions about hypotheses asking students to think about what can be predicted and tested, and; reflective questions asking students to ponder how they know what they know. Table 2.1 shows a summary of the teacher questions in the literature and the type of student responses that each is expected to elicit.

Table 2.1 Summary of Teacher Questions in Literature (Ilaria, 2009)

Questions of Comparison Type	Author		
	Cotton	Cunningham	Woolfolk
Dualist	Convergent: Low- predictable transfer of information High- encourage reasoning Divergent: Low- think of alternative way to do something High- encourage creative thinking	Low: recall information High: manipulate information to create an answer	Convergent: one right answer Divergent: many answers
	Vacc		Hiebert and Wearne
Category	Factual- name specific information Reasoning- develop one or more logically organized response Open- have a wide range of possible answers	Recall- give known Information Describe strategy- explain solutions Generate problems- extend thinking to new areas Examine underlying features- generalize ideas	
	Bloom		Wolf
Hierarchical	Knowledge- recalling information Comprehension- demonstrating understanding of information Application- use information to solve a problem Analysis- making references Synthesis- divergent, original thinking Evaluation- judge the merit of ideas		
	Interpretation- understand consequences of information Inference- go beyond available information Hypothesis- predictive thinking Transfer- take knowledge to new areas Reflective- explaining how you know		

Table 2.1 summarizes the types of teacher questions defined by their intended student response: low and high cognitive levels, categorical types, and hierarchical student thinking levels. The researcher who defined the specific question types is identified. The current study extends this work by analyzing the actual student responses to teacher questions in two teachers' classrooms.

#### 2.4 Initiating Student Thinking

Encouraging discussion in the mathematics classroom is an avenue for students to make their thinking explicit. Divulging student thinking is an approach for initiating conversation in the classroom. In addition, this communication of student thinking affords teachers and students the opportunity to discuss, organize, evaluate, and refine their understanding of mathematical concepts.

Teachers pose questions to students with a goal of affecting their thinking (Cazden, 2001). Steele (2003) suggests that teachers should avoid hindering the students' construction of knowledge by rushing to tell students concepts they do not know. Students constructing the knowledge themselves allows for better understanding. Steele (2003) describes, "I give them the opportunity to think. I am silent. I wait. I listen. I encourage them to test their ideas. I encourage them to talk to each other. I wait. I listen." (p. 59). Giving students time to think about mathematical ideas allows for development of their ideas so timing of the teachers' questions becomes an important component in the learning process.

As a part of a larger longitudinal study, two elementary age students were studied while they solved the same problem at two different time periods (Martino & Maher, 1999). Martino and Maher found how timing affects students' understandings and report that teachers should refrain from asking questions while students are developing

ideas. After the initial processing is complete, teachers can use probing questions to provide insights into the students' thinking.

Teacher questions can encourage students' to justify their reasoning verbally (Cazden, 2001). Dann, Pantozzi, and Steencken(1995) examined teacher questions in a seventh grade classroom where students were investigating ideas in combinatorics. To help students broaden their ideas and justify their conclusions, the teacher asked questions to promote student interaction with the teacher and other students. Additionally, Ilaria (2009) analyzed the questions and responses of teachers and students in math secondary classrooms that were student-centered allowing the student's ideas to guide the discussion. The teacher' questions are initiating, inviting, supporting, and revisiting in nature while the dialogue was exchanged between the students and teacher. These types of questions encouraged students to share ideas, to have multiple-sourced ideas, to foster student-to-student discussions, and to develop concepts over time (Ilaria, 2009).

Researchers advocate that teachers should avoid giving explicit suggestions or hints to students because this action may be counterproductive in allowing students to build their own understandings. In a study involving proof making of ninth grade geometry students advocates teachers' natural practice of "suggestion" as a form of questioning(Herbst, 2002). For example, suggestion allows teachers to ask interesting questions that lead students to make and prove conjectures, in place of providing explicit hints (Herbst, 2002). Ideally, teachers utilize questions to lead students to draw their own conclusions.

Furthering the research on the purposes of teacher questions, Davis (1997) warns against using teacher questions with the intention of evaluating the correctness of the students' explanations. Questions should supply understanding of the students'

reasoning. While teachers listen to their students, the questions target elaboration, clarification, and explanation among the students rather than following a logical predefined sequence. Suurtamm and Vezina (2010) reported in their study that teachers concluded that they could use student errors as ways to explore ideas that lead to better understanding, rather than merely signals of misunderstanding. Inoue (2011) was a part of a study that supported teaching by implementing inquiry-based lessons. When the students were developing deeper understandings of the concepts, the teachers were encouraged to listen to student conversation and then guide the conversation by interjecting questions. The teachers used guiding questions, such as “Which of these makes sense?”, “Why do you like this?”, “Tell me why you disagree?” (Inoue, 2011).

## 2.5 Discourse in Classroom

A large body of research in mathematics education seeks to identify how teachers can use questions as a way to create an environment that encourages rich meaningful discussion. While the need for and difficulties of teacher questions are described by the research discussions in the first section of the literature review, the focus shifts now to certain dynamics of mathematical discourse. Therefore, this study aimed to inform work bridging the gap between establishing mathematical discourse and the actual teacher questions.

A valuable aspect of mathematics discourse is that students have to be responsible for sharing their own understandings. Mathematics discussions are more beneficial when teachers use questions to show students how to use their ideas and understandings to bring deeper meaning (Manouchehri & St. John, 2006). In addition, if all students are involved in sharing, teacher questions can have a positive influence on the students’ mathematical thinking (White, 2003). In White’s study, the teacher’s questions followed four patterns: 1) asking students what they initially noticed and how to

solve a problem; 2) how they arrived at the answer; 3) to share solution strategies, and; 4) to communicate with other students. The study took place at different urban magnet schools in Washington, D.C. where the questions of two third grade teachers were analyzed. When the questions valued the students' ideas and thinking, teachers were able to develop productive classroom discourse with their students (White, 2003).

Hufford-Ackles, Fuson, and Sherin (2004) present a specific description of community from a year-long study of one teacher in an urban classroom of Latino students. Questioning, explaining mathematical thinking, source of mathematical ideas, and responsibility for learning were components of this particular community. These four factors enabled the teacher to move the students toward a student-centered environment. Questioning acted as a key component, allowing students' ideas to be made a public part of the discourse and where students could elaborate on their work.

The belief of shifting responsibility to the students through questions is further viewed in a study about undergraduate students' understanding in a one-year discrete mathematics course. Stylianou and Blanton (2003) focused on one teacher shifting the responsibility of construction of mathematical ideas to the students. By using questions that request clarification, justification, and elaboration from the students, they were drawn into the discussion. Requests for summarizing followed and established what was spoken or learned by the students. The summarizing stage was referred to as reconceptualization where students' ideas were rephrased into culturally concise terms. Finally, elaboration by the teacher followed students' original idea in this community in order to lead the students toward desired mathematical goals.

Inquiry-based mathematics enables students to learn to speak and act mathematically by engaging in mathematical discussion and solving new or unfamiliar problems (Goos, 2004). Teacher questioning is a method of moving the classroom

environment toward an inquiring community. In an Australian school, teachers used questions to make public students' ideas, to make their statements more explicit, and prompt student reflection (Goos, 2004). Mewborn and Huberty (1999) also cite that teachers can insert follow-up questions after student responses, ask other students to restate one student's ideas, or investigate by inviting alternate methods for solving a problem.

When analyzing the communication to the teachers, a couple of studies describe another discourse strategy allowing teachers to develop discourse with their students. Dick and Springer (2006) define revoicing "as a move made when one person repeats, summarizes, rephrases, translates, or recasts the contribution of another participant in the discourse"(p. 107). By bringing students into the process of intellectual socialization, O'Connor and Michaels (1993) demonstrate how incorporating revoicing enables teachers to coordinate academic tasks. Several purposes are served by using revoicing statements. By clarifying statements to the whole class, these statements can be evaluated for correctness of ideas. When voice is given to the contributions of quiet students, they can be brought into discussions. As teachers use revoicing, they extend evaluation of statements to the students and alter the expectations of the classroom discussions (Dick & Springer, 2006). Consequently, students will be brought into the mathematical discourse as they respond to other students' statements. Although revoicing is not a specific type of question, it is similar to questioning in that it brings students into the mathematical discussion.

Questions have been used to support progress in mathematical discourse, or reflective discourse. Cobb and his colleagues (1997) describe reflective discourse as a model where previous student and teacher actions become explicit objects of discussion. Teachers in this model incorporate questions to guide students into discussion after the

action and exploration phases. The goal of the discussion is to allow students to step back and reorganize the work they have already done. Additionally, questions are a tool to create discussion.

Research further defines explicit teacher actions with respect to dialogue that can construct a community of communication. Lobato, Clarke, and Ellis (2005) provide initiating and eliciting as two such teacher actions. Initiating is defined as the set of teaching actions that serve the function of stimulating students' mathematical constructions via the introduction of new mathematical ideas into classroom conversation (p. 110). Eliciting is a teacher action intended to ascertain how students interpret the information introduced by the teacher (p. 111). These terms were identified by data from a single ninth grade student in an after-school teaching experiment to redefine traditional teacher-centered classroom actions of telling. With this viewpoint, questions are valuable in the initiating and eliciting processes when the teachers' questions enable student to explain, share, discuss, and justify their understanding of mathematics.

## 2.6 Summary

The topic of teacher questions has been extensively analyzed through various viewpoints. Researchers have suggested categorizing teachers' questions with several frameworks, identified techniques, or ground rules for asking productive questions, and have illuminated how productive norms can be promoted through questioning in the classroom. The importance of the literature is supported with the statement, "students do not automatically begin talking about mathematics in a meaningful way simply because they are presented with appropriate tasks or are placed together in groups and told 'talk to each other'"(Rittenhouse, 1998, p.169). Therefore, this study focused on connecting questions to establishing mathematical conversation. The teacher is 'steppin in', as coined term conveys by Rittenhouse (1998), to bring mathematical understanding and



competence to the students. This study analyzed and described the moment when the teacher 'steps in' to the work and thinking of students by observing the questions teachers ask to establish mathematical discussion. Furthermore, the study delineated the types of teacher questions that elicit students' reasoning and the resulting responses specific question types evoke among students.

## Chapter 3

### Methodology

The purpose of this chapter is to describe my data collection activities and the analytic processes used in this study to explore the nature of mathematical discourse between teacher and students. Specifically, I examined teacher questions and student responses as the teacher brings about student thinking through discussion. A qualitative design was chosen because a qualitative study uses inquiry to help understand and explain the meaning of social phenomena in the natural setting with as little disruption as possible (Merriam, 1998). This inductive mode of inquiry supports the development of themes through collected data analysis. The samples are often small and purposefully selected (Yin, 2003). This type of study also seeks to describe the phenomenon with respect to individuals with specific motivations. Observations and video recordings are important in gathering detailed data to give a firsthand encounter account of the phenomenon of interest (Merriam, 1998). The following sections detail the researcher's position, the data needs for the study, data sources that were identified and used, collection of the data, methods for analyzing the data, research criteria, and timeline.

#### 3.1 The Researcher

As a mathematics teacher, it is my vision that students in mathematics classes have the opportunity to learn how express their thinking processes. This verbalization helps students understand the art and logic of mathematics rather than view mathematics as completing memorized algorithms. I have experienced the rewards of fostering this learning process while teaching students in high school mathematics classes and also in undergraduate mathematics courses.

Currently, I teach mathematics courses for future elementary education teachers. In the effort to encourage these future teachers to think and communicate basic

mathematical concepts, the instructional method is problem solving in small groups, and I am the facilitator. I use questioning to identify and initiate the students' thinking.

Consistent with theory, in using this questioning process, students take ownership of their learning and understand the mathematical concepts being presented. The aim is for these new teachers to take this process of constructivism in learning mathematics into their own classrooms in the future.

Over the past ten years, I have been teaching junior and senior level mathematics classes at a private school. To prepare the students for college, classroom teaching was consistent with the constructivist philosophy, focusing on active student learning, collaborative group work, questioning, and student-centered problem solving. For homework students would watch videos or read material that described and illustrated a particular mathematics algorithm. Then in class, the students would solve problems using the algorithms in the homework and verbally discussing the mathematical concepts. In teaching I guided the questioning process but did not control the direction of the discussion. While I taught these classes, I found challenges in teaching those students who were used to a traditional teacher-lead, expository-based classroom from previous school years. I also spent time helping other teachers learn to use questions to help students verbalize their mathematical reasoning.

During the past couple of years, I have become interested in how teachers enable students to talk about mathematics and share mathematical reasoning. As teachers start using more questions with different levels of reasoning, the dialogue is likely to change. However, it is yet unknown how such changes in classroom discourse will occur and evolve. It is also important to reveal how to help teachers make the transition from implementing no or largely infrequent sharing among students about

mathematical concepts and problems, to allowing students to guide the discussion and classroom learning with their thinking.

### 3.2 Data Needs

The data needed for this study were twofold. First, I needed to identify the motivational goals for teaching of the sample. Second, I needed to document how the teachers actually used questions to foster dialogue in classroom activities that elicit student's thinking.

### 3.3 Data Sources

In eight private high schools in the Dallas/Fort Worth area precalculus teachers who had similar years of teaching experience and common levels of educational background were asked to participate in this study. The teachers from these private schools represent a convenience sample due to accessibility and availability of these teachers to me. In many research studies, researchers use populations that are most available and convenient to approach (Gall et al., 2007).

### 3.4 Data Collection

Data was collected in two phases. Phase One was survey administration in which teacher responses indicated their motivational goals for teaching. Classroom observations were conducted in Phase Two, focusing on teacher questioning and student responses.

#### 3.4.1 Phase One

The teachers were given a survey that is a version of the Retelsdorf et al. (2011) scale for teachers' goal orientations for teaching (shown in Appendix A). Teachers rated responses to 16 statements on a four point Likert-type scale defined as 1 being "not true at all" and 4 being "absolutely true". Each subscale consists of four items describing 1) mastery orientation, 2) ability-orientation, 3) ability-avoidance orientation, and 4) work

avoidance. The average of the four items in each category was tabulated. A teacher who scored highest average in their *mastery orientation* was selected, and a teacher who scored highest average in *ability-approach orientation* was selected to participate in this study. Mastery orientation means the individual will define and evaluate competence relative to the particular tasks demands or prior outcomes and attribute outcomes to effort. In contrast, ability-approach orientation refers to the individual's tendency to define and evaluate competence relative to others and attribute outcomes to ability (Butler, 2007). The ability-avoidance orientation and work avoidance orientations were not the focus of this study and therefore not used. The survey was solely used for screening purposes of the larger sample of teachers to identify those with the opposing master orientation and ability-approach orientation for Phase Two of this study. The survey results also provided one means of description of teachers selected for observation in this study.

After receiving IRB approval, through used emails or phone calls to contact the eight private high schools that identified themselves as "college preparatory" schools in their mission statements and provide information to their constituents and prospective families that support graduates' college and career success. Five of the administrators responded favorably with willingness to participate in the study. I gathered a list of five pre-calculus teachers from the administrators and contacted these teachers by email. I requested their participation in the study by completing the online survey through a program known as "Survey Monkey". The survey began with a voluntary participation consent statement on the first page so the teachers could decide if they wanted to participate in the survey. Out of the five teachers contacted, three teachers completed the survey. The final page of the survey included a statement asking permission to be

contacted by email and discussed that two classroom observations would be video recorded. The three teachers positively responded.

Two teachers were selected for the classroom observations. One teacher's highest average for teaching motivation was in the category of mastery-orientation, and the other teacher's highest average was in ability-approach orientation. A table of the participant's averages in each category is found in Appendix B. The identity of the two teachers is protected throughout the study through the use of pseudonyms for their names.

#### *3.4.2 Phase Two*

I conducted classroom observations which were supported by video and audio recordings to obtain an accurate account of the kinds of questions teachers and students asked and to provide evidence of student mathematical thinking. I met with each teacher and principal prior to the observations and explained my aim to observe a problem solving class during two different class periods. I collected parent permission forms from each of the students for recording approval. I visited the teachers in their classrooms at least once before the observations with video and audio recordings were conducted. Each teacher was asked to identify two class sessions in which the same classroom of students would be problem solving.

Two forty-five minute sessions of each classroom were videotaped and audio recorded during the months of October through December that yielded a majority of data for this study. When the observations occurred, a camera and several voice recorders placed throughout the rooms recorded the dialogue and happenings during these class sessions. I collected field notes describing the overall environment of the classroom in order to account for events beyond the view of the camera. In addition, I created field notes of the worksheets being completed with the contained problems. The purpose of

the field notes was to supplement video and audio recordings and provide the researcher with journal recordings of the events captured on video. When the classroom recordings were transcribed, the field notes provided an additional written summary of the happenings in the classroom.

### 3.5 Data Analysis

Based on the study's research questions, the overall goals of data analyses were to: 1) identify what types of questions teachers asked and how often they asked them; 2) identify what types of student reasoning these questions tended to evoke, and; 3) describe how specific questions allow students to share their reasoning.

The first step in analyzing types of teacher questions and student reasoning is gaining a strong sense of the data (Powell, Francisco, & Maher, 2003). Accordingly, in-depth analyses were done by viewing the videotapes, identifying and describing the conversations, and transcribing the conversations. After transcribing the teacher and student conversations from collected data, transcribed data was analyzed for coding teacher questions and student responses. Conversations that did not pertain to teacher questioning were not coded, but their content was recorded to bring understanding to the continuum of data.

To ensure reliability in coding the data in this study, two mathematics instructors at the collegiate level served as an expert review panel. These reviewers were given the codes for the teachers and students (identified by researcher along with three different sections from the transcripts). They were asked to coded transcripts of two lessons and identify agreement and non-agreement with the codes assigned. Any discrepancy in codes assigned to teacher questions and student responses were discussed among the review panel to reach agreement of 95% in the final coding system to be used in data

analyses. Chapter four presents the final list of codes used in this study on teacher questionings and student responses.

### *3.5.1 Teacher Coding*

The process of developing teacher codes from the transcribed data began by using a predetermined list of teacher codes from Ilaria (2009) as an initial framework. Teacher's words were examined for the types of questions asked when teacher and students were involved in conversation. The discourse encompassing the questions was analyzed as to how the questions guided its progression.

Table 3.1 aligns the findings of student thinking studies in the literature with the initial teacher question codes used in this study. The literature cites teacher question types that promote certain kinds of thinking among students. To organize these data from the literature, Table 3.1 shows the teacher questions codes: retracing, discursive, clarifying, justifying, confirmation, and following as they are supported by the literature.



Table 3.1 Teacher Codes aligned with Literature Review

Code	Description	Authors	Association to Teacher Code
T-r	Teacher asks a student to consider an old idea.	White (2003)	Teacher asking a student to use prior knowledge
T-d	Teacher asks a student to contribute to the emerging discourse.	Van Zee & Minstrell (1997)	Teacher using reflective toss to give students responsibility for thinking
		White (2003)	Teacher promoting student to student interaction
T-c	Teacher asks the student to clarify his/her statements or ideas.	Stylianou & Blanton (2002)	Teacher seeking clarification, elaboration, or justification
		O'Connor & Michaels (1993)	Teacher uses re-voicing to clarify academic content
		Goos (2004)	Teacher seeks clarification, elaboration, or justification from individual students
T-j	Teacher asks the student to justify his/her statements or ideas.	Dann, Pantozzi, & Steekncken (1995)	Teacher leads students to explain and justify their ideas
		Martino & Maher (1999)	Teacher uses questions in order to help students build arguments
		Inoue (2011)	Teacher seeks clarification, elaboration, or justification from students
		Lobato, Clark, & Ellis (2005)	Teacher initiating student conversation in order to ascertain student understanding
T-con	Teacher confirms the student and teacher both agree on what has been done or said.	O'Connor & Michaels (1993)	Teacher uses revoicing to acknowledge students' responses and grant student opportunity to confirm teacher's intervention
		Stylianou & Blanton (2002)	Teacher affirming a student's idea
T-f	Teacher follows the student's idea or suggestion.	Mewborn & Huberty (1999)	Teacher interjects a follow-up question to help students revise their thinking

Inductive analysis of the data during each class lesson initiated the development of additional categories for teacher questions by the meticulous line-by-line evaluation of the transcriptions for each cohesive segment of discourse. This process revealed that at times there were teacher utterances that matched two or more coding categories. Although the codes that emerged for Ilaria (2009) provided a starting point for the data, the final codes used in the current study mirror the codes Ilaria identified only in name. After continual and repetitive reading of the transcripts for each instance of teacher questions, the final teacher question codes emerged. These codes are explicitly defined and described in chapter four.

After the defining of the codes the analysis of data revealed the number of iterations for the specific teacher codes. Any teacher question that received more than one code and could not be separated was eliminated from the final analysis, however there were a minimal number of statements in this category. If a teacher's utterance had two codes and could be divided into two phrases, the separated dialogue was linked to the corresponding appropriate code. The totals are discussed in chapter five to address the research goal of identifying the types of questions teachers asked and how often each type of question was used in classroom teaching.

### *3.5.2 Student Coding*

For student coding, Ilaria's (2009) student response codes were used as a starting point. These codes laid the groundwork for the second goal of this study on the types of student responses each category of teacher questions evoke. Table 3.2 presents Ilaria's initial codes also used in this study, along with their meanings.

Table 3.2 Initial codes for Student Responses

Code	Student Response
S(ta)	Thinking aloud
S(pb)	Proof-building
S(ans)	Answer
S(c)	Clarification
S(con)	Confirmation
S(a)	Attunement
S(qs)	Questions students
S(seek)	Seeking feedback
S(dnc)	Non-contribution

This phase of coding concentrated on conversations and inspections of student responses.

With codes defined, the number of student codes observed in the transcripts in each category was tabulated. Any student response that received more than one code and could not be separated was eliminated from the final analysis. The number of statements eliminated in this category was minimal. If a student's utterance had two codes and could be divided into two phrases, the separated dialogue was linked to the appropriate code. The totals are discussed in chapter five to address the research goal of identifying the student's responses to questions teachers asked and how often each was used.

### 3.5.3 Teacher Questions and Student Response Relationship.

A table was generated after the coding for both the teacher and students. This table determined the frequency of each student response to each type of teacher question. The tallying of the student responses that immediately followed a teacher's question supplied evidence of specific elicited responses. The table can only partially address the third research question, which focused on the types of student responses each type of teacher question tended to evoke.

In analyzing the data, representative conversations were selected from each of the observed teachers' classrooms for additional qualitative description on how teacher questions may produce mathematical conversation and engage student thinking. In the first teacher's classroom (Classroom A), the class session transcripts for the two observed class periods were subdivided into three parts: the checking of homework, the lesson, and the lesson practice. In the second teacher's classroom (Classroom B), the class session transcripts for two days are divided into two parts: the class review problem solving and the activity for acquiring the new concepts. These divisions provided several exchanges where the teachers were asking questions and the students were responding.

The initial divisions of overall class time periods were then subdivided into smaller parts. In Classroom A the homework review addressed each individual problem assigned. The lesson portion was divided by the different problems that were discussed. During practice time at the end of class, an *exchange* accounted for each discussion the teacher engaged in whether with a student or group of students. In the Classroom B, the problem solving review for individual students to work was divided into the different conversations of the teacher with a student or groups of students. When the whole class came together to review the answers, additional divisions grew from the original five problems assigned. The conversations of the teacher with one student or a group of students within the solving process of one problem defined an *exchange*. As the class moved into the investigation activity, each problem formed an *exchange*. Patterns were identified in the data to determine questions that engaged students in conversation. Themes were subsequently developed that characterized how teachers in the study used questions to foster student conversation.

#### 3.5.4 Developing Questioning Themes

The questioning themes occurred based on the dialogue between the teachers and the students. When the student or students were being asked by the teacher to share their thinking, the themes of *exploring* and *involving* were identified. If reviewing information from previous knowledge was the focus of the questions, the theme of *connecting* was the label given to the question exchange.

In Classroom B there were instances when students were engaged in direct conversation. During these exchanges, the teacher used questions in different ways to encourage discussion between students. These exchanges were characterized by the theme of *supporting*.

With the characterization of the themes that emerged from the initial data analysis, *exploring*, *involving*, *connecting*, and *supporting*, the entire data set was examined and discourse exchanges labeled as they pertained to each theme. The *exploring* theme described the dialogues between the teacher and students in the small or large group where the questions were used to encourage students to verbalize their thinking. The *involving* theme described the dialogue of several students with the teacher in which students' thinking was being shared with the group. Both of these themes permeated throughout both Classrooms A and B. The themes of *connecting* and *supporting* were not as prevalent in the data. The *connecting* theme defined conversations where the questions were designed to review previous mathematical thinking or ideas. The theme of *supporting* described teachers' questions when conversations were focused on students sharing their thinking with each other. Creation of these themes helped generate a narrative describing the conversations that characterized how each teacher used questions to elicit student thinking. The narrative

revealed the types of teacher questions that occurred as consistent with each theme, as well as the predominant student responses within these questioning themes.

### 3.6 Research Criteria

This study implemented checks for internal validity and reliability to help ensure fidelity to the implemented qualitative techniques and data analyses. Regarding validity, as a teacher I recognize the value I place on student-led conversations as I consistently implement such conversations in my own classroom. In this study, I first internally clarified my own ideals with respect to mathematical discourse and focused on separating myself from my own propensity toward student-led conversations. This separation allowed me to become the non-participant observer and data collector on the happenings of another teacher's classroom. To further increase the validity of the findings, the data collection took place over a period of time (Merriam, 1998). I attended Classroom A three times and Classroom B four times. I collected data and recorded the class activities in both classroom settings as they occurred during the second and third observations for the teacher of Classroom A and the third and fourth observations for the teacher of Classroom B. In addition, the collection of data from multiple sources provided corroborating evidence and provided triangulation. Triangulation using multiple sources of data confirmed emerging findings (Denzin, 1970). In this study, the use of the surveys, video and audio recordings, non-participant observations, and field notes provided multiple sources of data collection for a robust account of classroom teaching and learning exchanges central to this study. Finally, the expert panel members and I collaborated on the coding as it emerged from the narrative, transcribed data. All of these sources provided a copious amount of data as desired in a study of this nature thereby becoming worthy of analysis (Yin, 2003).

To address study reliability, the initial authority of the researcher was established through the discussion of my background, qualifications, and experiences. The study implemented well-established research methods and triangulation of multiple forms of data. The procedures implemented by the researcher were described by detailing how data was collected, how categories were derived, and how decisions were made throughout the study. Consistent with Merriam (1998), authentication of the findings was strengthened by the detailed account describing the researcher's pathway of analysis.

As a qualitative study, external validity was addressed through transferability as opposed to generalizability. The use of triangulation and dense description of the research methods enhanced transferability in this study (Creswell, 2007). In addition, the study included rich descriptions of the research context and the assumptions that were central to this study. Lincoln and Guba (1985) asserted, "the person who wishes to 'transfer' the results to a different context is then responsible for making judgment of how sensible the transfer is" (p. 298). Thus the reading of the qualitative description of this study determines its transferability.

### 3.7 Timeline

Data collection was conducted from September 2013 through November 2013. Online surveys were conducted in September of 2013 to teachers from schools that had agreed to participate in the study. Observations began in October of 2013 and continued until December of 2013. The observations were conducted at the participants' school sites during school hours.

### 3.8 Summary

This chapter presented a thorough description of the methods used in this study. At the beginning, I discussed the researcher's position, the data needs, the data sources, and the collection of the data in two phases. I described how the data would be

analyzed. Finally, I included steps taken to establish validity and reliability of the study. In Chapter Four, the codes and the specific description of the teachers' questions and student responses will give voices to the mathematical discussions that took place in the classrooms.



## Chapter 4

### Results

The codes developed from the data for the teacher questions and student responses are described in this chapter. Each of the codes was given a viable definition and several descriptive examples from each of the observed settings. This chapter provides a description of the classes and information on each pre-calculus teacher who participated in the study. The names given to the two the teachers are pseudonyms to protect their identities.

Classroom A was a mathematics classroom in an affluent private suburban high school. Twenty honors precalculus students worked on two topics dealing with quadratics during two 45 minute sessions: completing the square and synthetic division. The students were seated in four rows with a depth of five persons. They were attentive during class and kept their conversations to a minimum except when the teacher called on them. Their teacher, Amber, was a veteran teacher of 28 years who was a certified mathematics teacher and had completed her masters in mathematics education about 15 years ago. Her motivation for teaching was identified by the survey as ability-approach orientation. The principal described her as “one of his best mathematics teachers”.

I observed Classroom A three times and recorded two class sessions. The class sessions contained two main focuses: time to disseminate information and time to practice skills. The practice problems came from a packet that the teacher had compiled for each unit. Initially, students completed a warm-up on potential standardized test questions and returned it to the teacher. Homework was checked next and feedback was given to students if they specifically asked for feedback from the teacher. During most of the class time, Amber described mathematical concepts or ideas pertaining to the daily subject to her students who dutifully took notes. She used a white board at the front of

the room. As her lecture progressed, she would interject questions directed to the students in general. At the end of the class, the students were given time to practice completing mathematical problems using process that had been modeled that day. Amber walked around the room and offered feedback pertaining to the homework problems.

Classroom B was a mathematics classroom in an affluent private suburban high school. Twenty Pre-AP precalculus students worked on the following topics during two 45 minute sessions: solving with exponential equations, investigating structure of the unit circle, solving trigonometric equations, and proving trigonometric identities. In the first session, the class solved five exponential equation word problems for 25 minutes and investigated the unit circle for 20 minutes. During the second observation, they solved the trigonometric equations for 30 minutes and spend 15 minutes proving the identities. The students were seated in five rows with a depth of four persons. Their teacher, Brandi, was a veteran teacher of 28 years who was certified and had completed her masters in mathematics education. Her motivation for teaching was identified by the survey as mastery orientation. She was highly praised by her administrator as a teacher who engaged her students by involving them in the lesson.

While students turned in homework, class time began with a warm-up of practice problems disseminated to the students. Five-sixths of the class time, Brandi walked around the classroom while the students worked on problems either individually or in small groups. As she made herself available to her students, she either asked or answered questions of her students. Sometimes, she used a document camera projector to communicate the steps of problems by having students show and project their own work or by writing the problems' steps as students were called upon or volunteered to dictate their work. About one-sixth of the time the teacher directly provided information

to the students. Students were free to ask questions of Brandi or fellow students and interject ideas into the discussion at any time. Brandi gave students review practice problems to discuss and solve during class. She randomly called on students to respond to questions.

#### 4.1 Teacher Questions

To respond to the first research question of this research, the codes from a study conducted in student-centered classrooms by Ilaria (2009) were applied as the initial codes. After several readings of the transcripts, the codes slightly varied from that reported in Ilaria's study. Thus, using Ilaria's codes as a starting point, new teacher questioning categories emerged.

The definition and identification of a "question" often became clouded because of the progression of the dialogue in the mathematics classroom. Mathematical research presents questions as being a frequent mode of teacher-student interaction (Harris, 2000). While reading through the transcripts, it was evident that teachers were inquiring about the knowledge of the students. However, the utterances of the teacher were not always in interrogative form. If any teachers' statements were seeking a response of some nature from the students, they were categorized as questions. Therefore, the categories for the teacher questions are as follows: *teacher confirmation, teacher confirmation of student, suggestion, following, procedural, initiative, retracing, repeating, explanation, clarification, and justification*. The suggestion and justification questions were only observed in Classroom B. The rest of the questions were observed in both classrooms.

##### 4.1.1 Teacher Confirmation Questions

Teacher confirmation questions are those that check for students' agreement with teachers. Both teachers were observed using teacher confirmation questions in

statements they made asking students to identify if they agree with what was being presented by the teacher. If the teacher sought agreement from a specific student or the whole class about ideas stated in the discussion, the teacher's statement was coded in this category, and titled: *teacher confirmation* or T-cont. The teacher verifying that all students involved were "on the same page" is a primary generalization of this category.

In the Classroom A, Amber instructed the students on how to solve quadratic equations by completing the square. She allowed the students to complete several practice problems. At the end of one of the problems, the students questioned how to

simplify a radical of the form,  $\sqrt{\frac{-68}{9}}$ .

- |              |                |               |   |
|--------------|----------------|---------------|---|
| 34:18        | Student 9      | S-seek        | So is the fraction -68 so 4 and 17 over 3 ... so I didn't know if you could do it like that take it out of the fraction? Oh you have to leave it as a fraction... |
| <b>34:23</b> | <b>Teacher</b> | <b>T-cont</b> | <b>Oh, That's what I did. I could make this over 3 and just all over one fraction. You see what I am saying?</b>  |
| 34:28        | Student 9      | S-ans         | Yes   |
| 34:29        | Student 10     | S-seek        | Can I do this? (pointing to paper)  |
| 34:29        | Teacher        | T-cons        | If you want to.   |
| 41:01        | Student 10     | S-seek        | Can we take out this 9?   |
| 41:02        | Teacher        | T-p           | Yes, there is your denominator.   |
| 41:10        | Student 10     | S-seek        | You would do the square root of 17 over 3?  |
| 41:26        | Teacher        | T-p           | Yes   |
| 41:29        | Student 10     | S-seek        | Oh my, is that correct?   |

41:33	Teacher	T-i	Has anyone else gotten to number 11?(To the whole class)
41:35	Students	S-ans	yes... no..
<b>41:38</b>	<b>Teacher</b>	<b>T-cont</b>	<b>Okay, now be careful. I didn't say it because I didn't have one like that. On that, you were taking. It was the square root of -68/9 right?</b>
<b>41:53</b>	<b>Teacher</b>	<b>T-cont</b>	<b>Negative, is that what yall got?</b>
41:56	Students	S-con	Yes

On the three highlighted questions in the transcript, Amber seeks agreement from one student in the first question and from the class in the last two questions. In Amber's first question, the student asks for feedback on steps of computation dealing with the square root. She interjects her ideas and searches for agreement. On the last of her two questions, she seems to sense the class might be struggling with a certain problem in the practice problems. As she attempts to help all the students advance through the steps of computation, she continues to ask questions to achieve consensus from the students before she proceeds.

During a different lesson, Amber introduced the remainder theorem as she focuses on the application of synthetic division. She employed the mathematical term *remainder* in her dialogue.

<b>6:45</b>	<b>Teacher</b>	<b>T-cont</b>	<b>So okay, the remainder theorem let me illustrate. Ya'll know what the remainder is right?</b>
7:15	Teacher	T-p	What is a remainder?
7:50	Student 2	S-ans	It's the last thing when you do that little chart thing.

7:52 Teacher T-d Okay it is when you divide; it's what's left over. So the remainder theorem says, (writing on the board), we'll do number 1 together. They want me to evaluate the polynomial at 2. So let's talk about if I didn't say anything how would you evaluate  $f(2)$ . What is one way you would evaluate  $f(2)$ ?

In Amber's question, she desires confirmation of students' agreement or disagreement as to whether they understand the vocabulary. She could proceed to the next idea if her feedback revealed "everyone on the same page".

In Classroom B, Brandi had the students complete some review problems in which a trigonometric expression,  $\frac{\tan x + \sin x}{2 \tan x}$ , is being simplified. In this dialogue, she and the students focused on the numerator which in the previous dialogue had been written in terms of sines and cosines.

21:10 Teacher T-p Oh no, I totally lost Student 4? Student4

21:15 Student 4 S-ans Yes

**21:16 Teacher T-cont You're okay with sine over cosine plus  $\sin x \cos x$  over cosine  $x$ .**

21:25 Student 4 S-ta Other than the fact that you have now separated something which you couldn't separate before because you added

21:30 Teacher T-p Wait, Wait, I didn't separate something

21:32 Student 4 S-ta You multiplied it but

**21:35 Teacher T-cont I added these two pieces and making fractions. Okay, Student 4.**

21:44 Student 4 S-ta You couldn't do that

Brandi seeks agreement from an individual student who is struggling with understanding the steps of getting a common denominator. Her questions are interrogative statements where she requests the student's agreement with what he does understand.

An example from Brandi's classroom portrayed the students reviewing and solving exponential word problems as she walked around the room giving feedback. The students solved the following problem. "A company is investing 18 million dollars and hopes to have 25 million dollars in eight years. What should be the percentage rate if the money is compounded annually?"

04:15 Teacher T-e Compounded annually means?

04:25 Student 2 S-ans Compounded annually means once a year

04:30 Student 6 S-ta Oh so it would be one.

05:21 Teacher T-c I can just raise that to the "1/8" ?

**05:40 Teacher T-cont Are you with me?**

05:41 Student 3 S-con Oh!

While Brandi discusses the problem with a couple of students, she pursues a strategy with them. As she progresses, she questions to see if they are following her thinking.

#### *4.1.2 Student Confirmation Question*

Student Confirmation Questions are similar to teacher confirmation questions, however in this case the teacher conveys agreement with the student in her statement. This question category identifies the bi-directional agreement between the teacher and the students. The teacher's statement may not be interrogative in nature but the teacher indicates she is working to make sure everyone involved "is on the same page." If the teacher establishes agreement with the student statement or provides an indication of

hearing the student statement, the teacher's statement is coded *student confirmation* or T-cons.

In Classroom A, the students and Amber were solving some practice problems that involve completing the square. In the process of solving, the class needed to simplify a larger square root,  $\sqrt{432}$ .

12:12	Teacher	T-p	So one way is just to come off to the side and say 2 times what. Did anyone do their list or am I going to have to do it? (no response)
12:25	Teacher	T-p	So you take 432 divided by 2.(teacher using calculator)
12:29	Student 3	S-ans	Just divide by 4 and then
12:32	Teacher	T-p	So you just divided by 4 and what did you get?
12:34	Student 3	S-ans	432 divided and then 3
12:36	Teacher	T-p	So 432 divided by 4 is.(typing)
12:41	Student 4	S-ans	It is 108
<b>12:42</b>	<b>Teacher</b>	<b>T-cons</b>	<b>Alright</b>
12:44	Student 4	S-ans	It is 144 times 3
13:00	Teacher	T-c	It is 144 times 3?
<b>13:03</b>	<b>Teacher</b>	<b>T-cons</b>	<b>That is why I usually do not jump to 4 since she found a bigger one than with 4, right? You may have done it in steps right?(Addressing Student 3)</b>
13:13	Student 3	S-c	I thought you were just factoring like
13:15	Teacher	T-p	But what we're looking for is a perfect square.



And she found this one and this would be three times 144 all over 18.

As Amber communicates with the whole class, she conveys agreement with three students. With the first statement, she provides agreement to an individual student's comment in order to progress through the problem. In the last two questions, she communicates agreement with two students' analysis of the multiple factors employed in simplifying the radical.

Amber sought to invite students into verbalizing their understanding of a multiplication factor in a lesson pertaining to synthetic division.

13:55	Teacher	T-r	Is 2 a factor of 6?
13:58	Students	S-ans	Yes
14:00	Teacher	T-c	How do you know? (pause)
14:12	Teacher	T-r	Is 2 a factor of 7?
14:14	Students	S-ans	No
<b>14:15</b>	<b>Teacher</b>	<b>T-cons</b>	<b>No</b>
14:16	Teacher	T-p	If 2 goes into 6 what is the remainder?
14:17	Student 7	S-ans	Zero
<b>14:18</b>	<b>Teacher</b>	<b>T-cons</b>	<b>Zero</b>
14:19	Teacher	T-p	2 does not divide into 7 and there is a remainder of?
14:21	Student 7	S-ans	one

Amber asks several questions to probe the students' knowledge of factors. When she receives positive feedback measuring the computational knowledge of the definition, she is able to progress through the lesson.

In Classroom B, Brandi asked a student to respond with the simplified expression to a trigonometric expression  $\frac{\tan x + \sin x}{2 \tan x}$ . In this dialogue, she asked for feedback as to the sum of two trigonometric fractions.

- |              |                |               |  |
|--------------|----------------|---------------|--|
| 20:02        | Teacher        | T-p           | I got number 22 and Student 3 that's you. Do you have any idea what I have when I add these two fractions together?(pointing to the board) |
| 20:15        | Student 3      | S-ans         | Sin2x/Cosx   |
| 20:20        | Teacher        | T-c           | Sin2x/cosx so sinx plus sinx is?   |
| 20:44        | Student 4      | S-ans         | 2sinx  |
| <b>20:45</b> | <b>Teacher</b> | <b>T-cons</b> | <b>There you go 2sinx. I need to 2 in front. 2sinx ahh cosx (writes, reducing the resulting fraction)</b>                                  |

The student initially answers incorrectly and later responds with a correct response.

Brandi confirms the student by affirming the verbal response.

In this next example, Brandi was still walking around the room giving feedback. A different group of students was trying to solve this problem. "A company is investing 18 million dollars and hopes to have 25 million dollars in eight years. What should be the percentage rate if the money is compounded annually?"

- |       |           |       |   |
|-------|-----------|-------|---|
| 05:42 | Teacher   | T-r   | I have an exponent on the variable. So how do I get rid of?                       |
| 06:03 | Teacher   | T-f   | (looking at student's paper) So you are trying to bring the exponent down? Right? |
| 06:05 | Student 5 | S-con | nod   |
| 06:10 | Teacher   | T-s   | So undo that  |
| 06:12 | Student 4 | S-con | Ohh, Ohh!   |

**06:15 Teacher T-cons So the light bulb goes on!**

06:18 Student 4 S-ta So you can just raise it to the “1/8” power.

**06:20 Teacher T-cons Nice, nice**

Brandi uses the moment to encourage a student when he is successful in completing a problem. In her statements, she provides positive feedback pertaining to the student’s solving process. The student in this example continues to add more to the discussion.

By the teachers asking confirmation questions, teachers acquire a method of informal assessment. During a mathematical discussion, the *teacher confirmation* category indicates teachers questioning students for agreement while they evaluate the students’ understanding of the mathematics. On the other hand, the *student confirmation* category reveals teachers agreeing with the students’ responses as they provide them with feedback about their understanding. If students gage their thinking as being understood, they can verbally extend their mathematical thoughts. Both forms of confirmation supply the participants with an informal assessment of their understanding in the continuing discussion.

#### 4.1.3 Suggestion Question

The *suggestion question* is when teachers desire students to move in a new direction from a current conversation. To do so, teachers provide questions to lead students down a new path of thinking. Teachers’ statements in this category were coded as a suggestion question or T-s, which was observed when the teacher added an idea not previously discussed in the conversation. Working to initiate discussions, teachers’ suggestion questions were based on one of two objectives: the students’ ongoing discussion, or the new direction of the mathematical discussion.

Classroom A observations during the two class sessions did not reveal an example of the suggestion questions. In Classroom B during the introductory warm-up

activity, the students individually worked on simplifying the trigonometric expression,

$\frac{1-\tan^2x}{1+\cot^2x}$ , and Brandi walked around the class answering questions.

- |              |                |            |   |
|--------------|----------------|------------|---|
| 08:24        | Student 5      | S-seek     | Will this not cross out?  |
| 08:26        | Teacher        | T-f        | if I do cross it out, I will be back to what I started with<br>which is this(pointing)  |
| 08:28        | Student 5      | S-con      | Oh, yah.  |
| <b>08:30</b> | <b>Teacher</b> | <b>T-s</b> | <b>Okay, I am going to foil the top and foil the bottom<br/>and see if I get anywhere or did I just create a<br/>nightmare?</b> |
| 08:37        | Student 5      | S-con      | Okay  |

The student tries to find a method of simplifying. Brandi suggests factoring the numerator and denominator, if possible, as another avenue of simplifying.

During a different lesson, Brandi called on different students to share their process of solving the word problems with the entire class. This problem was mentioned in the confirmation section while she was working with individual students. In this example, the students were asked to read the problem. She wrote the problem down then it was projected on the screen for the class to view.

- |       |           |        |  |
|-------|-----------|--------|--|
| 13:33 | Teacher   | T-p    | Okay, next one-- #9 is (student's name).<br>Can you set it up for us?  |
| 13:43 | Student 9 | S-ans  | 25 million   |
| 13:46 | Teacher   | T-cont | Okay I am going to write 25 since everything is in<br>millions, right? |
| 13:48 | Student 9 | S-ans  | Yes  |
| 13:49 | Teacher   | T-c    | So...  |

13:50	Student 9	S-ta	well
<b>14:00</b>	<b>Teacher</b>	<b>T-s</b>	<b>So did you log both sides?</b>
14:05	Student 9	S-ans	No
14:07	Teacher	T-e	Why not?
14:10	Student 9	S-pb	There's no variable in the exponent
14:12	Teacher	T-cons	There's no variable in the exponent
14:13	Teacher	T-c	so how did you have to get rid of the eighth power?
14:17	Student 9	S-ans	Ah... divide 18 on the other side and root 8

The student struggles initially in verbalizing how she solved the problem. Brandi's suggestion encourages the dialogue to continue by presenting possible methods of solving exponential equations. After the initial push, the student expands on her idea.

The reason for *suggestion* questions may be to aid the students in moving forward with the mathematics. Occasionally, students are not able to progress toward a solution, reach a conceptual understanding, or develop an explanation in their thinking. Teachers sometimes use this category of questions to insert additional information into the students' thinking processes. Essentially, teachers may be trying to reach the goal of the lesson or problem, so they ask this type of question to allow students to contemplate the teachers' ideas with respect to the current discussion topic.

#### 4.1.4 Following Question

In an environment of mathematical discourse, following questions were observed when teachers responded to students' statements and thoughts in a reciprocating dialogue. Teachers' questions were founded frequently on something the students said or did. Thus, if teachers' questions immediately followed and directly related to ideas of students' statements or actions, the question's code was termed as *following* or T-f.

In the first setting, the class checked their homework assignments. The answers were posted on the front board. While checking her paper, a student inquired about how much would be taken off her grade. She stated her only error was one number in the answer.

- |             |                |            |  |
|-------------|----------------|------------|--|
| 8:35        | Teacher        | T-p        | Alright here we go on your quadratic formula.<br>(Answers on board) Where there some that gave you problems? |
| 8:50        | Student 1      | S-seek     | What about if you just got the outside number of the radical wrong?  |
| <b>8:52</b> | <b>Teacher</b> | <b>T-f</b> | <b>So you...</b>   |
| 8:56        | Student 1      | S-ans      | Everything else was right but that number  |
| 9:04        | Teacher        | T-i        | I would mark it for sure. Does anyone else have one?   |

Amber analyzes the student's paper trying to identify the mistake. She follows the student's work visually as she verbally provides feedback.

During the next class session, Amber demonstrated an example of *following*. While the class practiced using synthetic division to find factors of a polynomial, a student interjected an observed pattern. He communicated that his idea would increase efficiency of finding factors. When he interjected the idea earlier, the class discussion went in another direction. So Amber brought the idea back up for discussion during the individual practice time.

- |              |                |            |  |
|--------------|----------------|------------|--|
| <b>28:19</b> | <b>Teacher</b> | <b>T-f</b> | <b>Ok, let's see his theory then. His theory was it was prime it would never divide.</b> |
| 28:38        | Student 4      | S-c        | By whole numbers not fractions   |
| 28:36        | Student 1      | S-ta       | The first one didn't work, it was a 2  |

28:38	Teacher	T-cons	Yes, the first one ended in a 2 which is prime but it factored.
28:40	Teacher	T-cons	That falsified your theory. It was a good thought
28:48	Teacher	T-cont	See one example can disprove it. One example can't prove it but one example can disprove a thought. And I do that all the time, when yall will ask me hey does this the same if I have (writing on the board) Is that just 8 squared minus 4 squared?

Amber follows the basics of his idea by restating its components. She even gives him the opportunity to give feedback to her analysis.

Classroom B contained more examples of Brandi following the students' thinking. As the individual students simplified five practice trigonometric problems, a student sought feedback from the teacher.

05:39	Teacher	T-s	I think that might work better for you than doing that squaring thing though I could be wrong.
<b>05:42</b>	<b>Teacher</b>	<b>T-f</b>	<b>Where are you going from here?</b>
05:44	Student 6		(Writing)
05:46	Teacher	T-p	You're going to get "tan2x +" what in the middle?
<b>05:50</b>	<b>Teacher</b>	<b>T-f</b>	<b>Your terms don't cancel.</b>
05:55	Student 6	S-ta	plus 2(as he is writing)
05:57	Teacher	T-cons	but I like how you are thinking.

Brandi observes the steps on his paper, follows his thinking, and comments on his choices. Her questions allow her to communicate to the student her paths of analysis.

In the review part of the class period in Classroom B, a student read and solved the following problem, "A person is setting up an annuity fund. He would like to have

\$120,000 in the fund after 30 years at 4.5% interest compounding monthly. How much should he put into the fund each month to reach the goal?" The students took the information and substituted values into a formula to find the answer.

09:35 Teacher T-cons Some of your banks, I would never borrow money from ... (in passing to same student)

09:37 Teacher T-cons Like, "no way José" (pointing to a student paper)

**09:48 Teacher T-f MMM, what did you do? Did you divide by 12? Yes you did. Did you raise it to the**

09:52 Student 2 S-ans 30

09:54 Teacher T-cont times 12, right?

As Brandi stops at this student's desk, she identifies what she thinks the student has done. The teacher questions him on his actions, and he replies verbally.

The teachers' reasons for implementing the *following* questions may be to utilize the students' ideas to progress toward a solution, a mutual agreement, or a goal. The focus is on understanding the students' thinking and moving forward using the students' ideas. The teachers probe with questions in order to comprehend the viewpoints of the students.

#### *4.1.5 Procedural Question*

*Procedural question* defines teachers directing students' actions. Teachers frequently have an orchestrating role for deciding what students will do during a lesson. Their procedural questions do not always pertain to the mathematical procedures students perform when solving a problem but the directing of students' actions in the classroom. When the directions given by the teachers to the students were related to a specific mathematical task, teachers' questions were coded as *procedural* or T-p.



In Classroom A, Amber answered questions over the student's homework in the first setting. The homework entailed solving equations using the quadratic formula.

- 9:55 Student 2 S-seek Can you do number 13?
- 9:58 Teacher T-e Okay let's do number 13. What did you not get right on it?
- 10:02 Student 2 S-ta I think I just didn't factor out as far as it could go.
- 10:05 Teacher T-p Okay on this one (teacher writing on the board)**  
**What do you always have to have before you start using the quadratic formula?**
- 10:07 Teacher T-p What is the first step?**
- 10:10 Student 2 S-ans Equals zero

After Amber receives a student's feedback, she decides to approach the problem's review by inquiring as to the sequential steps used in solving.

In Classroom A, the present lesson reviewed the process of the synthetic division.

- 9:37 Teacher T-p It's a process an easy process. It is still a process. How do we do this?**
- 9:57 Student 5 S-ans you bring down the negative 1.
- 9:58 Teacher T-cons Yes, you bring down the negative 1.
- 10:00 Teacher T-p So if I am going to write it down for synthetic division, (writing on the board)**  
**This is called synthetic division. I help you build our case. You bring down first column. Okay, does anybody remember what you did next?**
- 10:17 Students 5 and 6 S-ans Multiply the 2 by negative 1.

22:22	Teacher	T-p	Okay, so I multiply (pointing) =2. Where do I write it?
10:24	Student 6	S-ans	Below the zero
10:26	Teacher	T-p	Yes, in the next column. Ya'll do remember. You multiply, record in next column, and this is the part that a few people maybe not you would get mixed up. What do I do next?
10:40	Student 7	S-ans	add

Using the fact that the students have done the process in the previous year, Amber asks the students to recall the ordered steps. She proceeds to ask a couple of procedural questions to complete the entire problem.

In Classroom B, the students have finished solving the warm-up trigonometric equations,  $\sqrt{2}\sin x \cos x - \cos x = 0$  and  $2\cos^2 x = 1$ , for the interval  $[0, 2\pi]$ .

12:23	Teacher	T-p	(Addressed the whole class) How many answers did you get to number 4? Ah how about number 4?
12:35	Students	S-ans	4 answers
12:37	Teacher	T-p	4 answers?
12:39	Students	S-ans	yah, yah
12:42	Teacher	T-p	I got 2. Okay I got 4.
12:43	Student 3	S-ans	Good.
12:46	Teacher	T-f	Okay you just removed it(to a student about his paper)
13:00	Student 1	S-seek	When do you have to do all 4 quadrants?
13:02	Teacher	T-p	If you have to unsquare something, by square

rooting it.

13:04	Student 1	S-a	Okay
<b>13:05</b>	<b>Teacher</b>	<b>T-p</b>	<b>No, is the square root is already in the problem? But if you had to unsquare it, by square rooting.</b>
13:09	Student 1	S-a	Okay
<b>13:12</b>	<b>Teacher</b>	<b>T-p</b>	<b>So on 5 on the back one, how many answers do you get?(to whole class)</b>
13:15	Students	S-ans	4
<b>13:17</b>	<b>Teacher</b>	<b>T-p</b>	<b>(nods affirmatively) So the moral to the story is that you always get 4?</b>
13:24	Students	S-ans	No
<b>13:28</b>	<b>Teacher</b>	<b>T-p</b>	<b>Right, darn I was trying to make a generic rule</b>

Brandi asks questions about the completed review problems so students will have the opportunity to generalize an idea pertaining to the number of solutions in a trigonometric equation.

In Classroom B, the students investigated the unit circle. They completed a worksheet containing questions using Wikki Stix to replicate the radius and visualize its relationship to a radian.

<b>20:38</b>	<b>Teacher</b>	<b>T-p</b>	<b>The bottom of your sheet as it is facing this way. It should definitely fit. I cannot (helping a student with compass)</b>
21:03	Student 5	S-seek	So at the very bottom, touching the bottom?
<b>21:04</b>	<b>Teacher</b>	<b>T-p</b>	<b>(nods)</b>
21:08	Student 1	S-ta	So why am I incapable of drawing?
<b>21:12</b>	<b>Teacher</b>	<b>T-p</b>	<b>So have a friend at your table draw the</b>

circle at the bottom.

21:14 Student 3 S-seek 2 1/2?

**21:20 Teacher T-p Lord have mercy, are you between 1 and 2 centimeters?**

21:21 Student 6 S-ans No

**21:22 Teacher T-p Centimeters?**

21:23 Student 6 S-seek Inches

**21:24 Teacher T-p Okay, let's do it in centimeters. Our Wiki Stixs are not long enough.**

21:32 Students S-con Ahh

21:35 Student 1 S-con Centimeters! 1 and 2 centimeters (rustle)

21:41 Student 3 S-seek Is that the radius or is that the diameter?

**21:45 Teacher T-p I just want the pulley wooly thing in between 1 and 2 cm and it's only because the Wikki Stixs are not long enough which I know some of you are going to be challenged (pause)**

**22:14 Teacher T-p Now it says put a dot in the center of the circle.**

22:18 Student 8 S-seek Oh, I don't know where

**22:20 Teacher T-p Okay I know there should be an indentation from where your compass was. There should be an indentation there. Put a P at the center of the circle.**

22:32 Student 1 S-con Oh

**22:35 Teacher T-p Is that not what is says? (Pause)**

Brandi uses these procedural questions to keep all students advancing through the activity.

*Procedural* questions are used by teachers to help with classroom management and to aid students in their thinking. The teachers often work to make sure the mathematics is clearly and properly displayed to the class. In addition, by using procedural questions, teachers may be motivating students to continue with the mathematical thinking process. Besides a management tool, teachers convey actions the students must take to complete a mathematical process. This category of questions can help bring organization to students' ideas so the communication process of the mathematical thinking will be enhanced.

#### 4.1.6 Initiating Question

The teachers' questions were coded as *initiating* or T-i, if the teachers explicitly addressed the whole class or a specific student not engaged in the conversation immediately before being addressed. Interacting questions are implemented when a specific student who is not a part of the ongoing conversation is asked to join.

In the Classroom A, the class studied a unit on solving quadratic equations. During previous class periods, they examined how both factoring and quadratic formula are methods of solving quadratic equations.

- |       |         |     |  |
|-------|---------|-----|--|
| 15:22 | Teacher | T-i | <b>So let's, what are we finding? We've been doing this. What are we finding? (pause)</b>  |
| 15:45 | Teacher | T-i | <b>What we are finding, we are solving this, what is it? Geographically what are we finding? (pause)</b>   |
| 16:02 | Teacher | T-p | The x-intercepts that is what we are finding. Did you realize when you take and are solving something by, you can either factor it, you can do the quadratic |

formula

Amber addresses the whole class and inquires as to what they are actually doing when solving these problems. One of her questions attempts to receive a response from a symbolic perspective. When she does not obtain any responses, she asks the same question from a graphical perspective.

In Classroom A during a discussion about the remainder theorem, Amber initially started the discussion by asking the class about the definition of the remainder. She attempted to draw the students into a conversation concerning the definition of a factor.

<b>13:40</b>	<b>Teacher</b>	<b>T-i</b>	<b>Oh that's nothing (teacher wipes off the mark) Now we are going to do the first ones. We are going to evaluate it using the remainder theorem. Which says, the very last column says the answer. Alright now then how do I know if 2 is a factor of 6? (pause)</b>
<b>13:55</b>	<b>Teacher</b>	<b>T-i</b>	<b>is 2 a factor of 6?</b>
13:58	Student 5	S-ans	Yes
14:00	Teacher	T-c	How do you know? (pause)
<b>14:12</b>	<b>Teacher</b>	<b>T-i</b>	<b>Is 2 a factor of 7? (looking a another student)</b>
14:14	Student 10	S-ans	No
14:15	Teacher	T-cons	No

Amber starts by directing her question to the whole class then eventually engages two different students as to their thoughts pertaining to the definition of a factor.

In Classroom B as Brandi walked around the classroom, she gave feedback to students while they worked on their warm-up review problems. The student in which she communicated was working on simplifying,  $\frac{1+\tan^2x}{1+\cot^2x}$ .

<b>04:04</b>	<b>Teacher</b>	<b>T-i</b>	<b>(moves on) You stopped?</b>
04:14	Student 5	S-ans	No
04:16	Teacher	T-f	Your answer?
04:17	Teacher	T-e	Oh, oh, oh, you squared it because you felt like it?
04:21	Teacher	T-r	Okay, if I square that, let me go back to algebra a minute (writes on the board), if I square that, what will I end up with?
04:38	Student 5	S-ans	(writes)
04:46	Teacher	T-cont	So I can't just randomly start squaring things cause I want to, Notecard 67 says that $\sec^2x$ equals (writes $1 + \tan^2x$ ) Right?
05:00	Student 5	S-ans	Yes

Brandi tries to motivate the student back into the engagement of finding a solution.

In Classroom B, students worked on an activity investigating radians on the unit circle. At this point in the conversation, students wrapped the Wikki Stixs around their circles and tried to determine the relationship between a radian and the radii of the circles.

<b>30:23</b>	<b>Teacher</b>	<b>T-i</b>	<b>Okay does anyone besides, student's name, cause she already go a piece of candy. Does anyone besides student's name, know how many it is going to take? Student 8, how many?</b>
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30:37	Student 8	S-ans	6 and a little more
30:39	Teacher	T-cons	6 and a little bit more, 6 and a little bit more. Let me know
30:43	Student 11	S-ans	6.28
30:47	Teacher	T-cons	Yeah, Yeah! So it's related to
30:48	Student 5	S-ans	$2\pi$

Brandi tries to engage other students in answering her question. A student had already answered a question so she calls on another student by name to get a response.

The intent of teachers using *initiating* questions may be to promote the involvement of other students. Instead of having a conversation between one student and teacher, the teacher encourages other students to voice their thinking. Teachers can employ these questions to foster involvement of other students so the teachers are not the focus of the dialogue.

#### 4.1.7 Retracing Question

When teachers interjected a specific idea mentioned in a conversation prior to that point, the teachers' questions was coded as a *retracing* question or T-r. The retracing question focuses on how teachers build on concepts or ideas as they progress through a unit of study. Teachers often refer back to previous discussions in order to provoke thinking about a problem solving process or a solution through retracing questions.

In Classrooms A, the class had just completed the synthetic division process. Amber questioned the students about taking the quotient and writing it back into polynomial form.



21:58	Teacher	T-r	It will be n to the 3 <sup>rd</sup> . Looking at board) So this will be the coefficient of n to the 3 <sup>rd</sup> , n cubed. This is the coefficient of n squared. This is the coefficient of n. This is the constant and this is the remainder. Do yall remember?
22:15	Teacher	T-r	Let me show you something (Writing on the board). 2 divided into 7. How did you write your answer?
22:20	Teacher	T-rep	What did you say your answer was?
22:23	Student 4	S-ans	3 remainder of 1
22:26	Teacher	T-cont	You could say 3 remainder 1.(writing on the board)
22:29	Teacher	T-p	What is another way you could state your answer?  (Pause)
22:38	Teacher	T-r	<b>You remember this (writing) you take this and write it over that?</b>
22:51	Teacher	T-cont	3 ½ is the answer and isn't that true?

Amber reminds the students that they had done the process previously. She inquires if they remember writing the quotient as a polynomial and the remainder as a fraction of the divisor.

In Classroom B, Brandi roamed around the room and gave feedback to students as they worked on their warm-up review problems. A student worked on simplifying,

$\frac{1+\tan^2x}{1+\cot^2x}$  as she approached his desk.

04:04	Teacher	T-i	(moves on) You stopped?
04:14	Student 5	S-ans	No

04:16	Teacher	T-f	Your answer?
04:17	Teacher	T-e	Oh, oh, oh, you squared it because you felt like it?
<b>04:21</b>	<b>Teacher</b>	<b>T-r</b>	<b>Okay, if I square that, let me go back to algebra a minute (writes on the board), if I square that, what will I end up with?</b>
04:38	Student 5		(writes)
04:46	Teacher	T-cont	So I can't just randomly start squaring things cause I want to, Notecard 67 says that $\sec^2 x$ equals (writes $1 + \tan^2 x$ ) Right?
05:00	Student 5	S-ans	Yes

Brandi analyzes the student's simplification process. She questions him on his decision to use a particular strategy. Then she reviews the implications of his choice by reviewing a specific algebra property.

In the review part of a class in Classroom B, a student read the current problem, "A person is setting up an annuity fund. He would like to have \$120,000 in the fund after 30 years at 4.5% interest compounding monthly. How much should he put into the fund each month to reach the goal?" The students took the information and substituted values into a formula in order to find the answer. Brandi stopped at a student's desk to examine what work she has completed.

09:52	Student 2	S-ans	30
09:54	Teacher	T-cont	times 12, right?
09:56	Student 2	S-seek	times 12?
<b>09:58</b>	<b>Teacher</b>	<b>T-r</b>	<b>That's what n is. That is just your annual percentage rate. You are still getting charged per month.</b>

10:02 Student 2 S-seek Well?

**10:05 Teacher T-r So if I take the annual percentage rate and divide it by 12 don't I get my monthly percentage rate?**

10:15 Student 2 S-con Yes

Brandi asks the student to think about what annual percentage rate represents and how monthly investments affect the answer. She encourages the student to review the terms and processes from previous lessons. Consequently, the student might understand why 12 is part of the formula.

The evident use of *retracing* questions was for teachers to allow review of an idea by the students. When teachers lead students back to the idea, students have the opportunity to reorganize their mathematical thinking. Teachers using retracing questions also encourage students to connect previous mathematical concepts with the topic being investigated. As a result, these questions give students numerous opportunities to examine their thinking and grow their understanding.

#### *4.1.8 Repeat Question*

When teachers ask students to repeat their previous statement for bringing clarity or importance to the speech, teachers' questions are coded *repeat* or T-rep. Repeat questions are used when teachers notice that not all students may have heard another students' statement, or when some are distracted at different times during the lesson. If students' statements are important to the scaffolding of the concept of the lesson, teachers' using repeat questions may help re-focus attention.

The repeat question was observed in Classroom A. Amber reviewed how the dividend, divisor, remainder, and quotient are symbolically displayed specifically, seven divided by two.

22:15	Teacher	T-r	Let me show you something (Writing on the board). 2 divided into 7. How did you write your answer?
22:19	Student 4	S-ans	(Mumble)
<b>22:20</b>	<b>Teacher</b>	<b>T-rep</b>	<b>What did you say your answer was?</b>
22:23	Student 4	S-ans	3 remainder of 1

When the student responds to the retracing question, Amber does not hear his response. She asks him to repeat his answer so the class can be aware of the reply.

While everyone in the classroom is part of the discussion, not all students are consistently engaged. Retracing questions allow teachers to present opportunities for more student involvement. In addition, the teachers may be asking students to repeat a response to emphasize the importance of verbalized ideas. The *repeating* questions serve as a tool to help teachers guide the direction and emphasis during class discussion.

#### 4.1.9 Explanation Question

Explanation questions were observed when students were asked to explain their thinking coded as *explanation* or T-e. In explanation questions, teachers ask students to verbalize what they are doing when they are writing their mathematical steps. If the students have not made their mathematical thinking public yet, teachers use explanation questions to prompt them to verbalize their thoughts.

In Classroom A, Amber and the students practiced completing the square to solve quadratic equations. A student read one of the problems.

21:40	Student 5	S-ans	$x^2 - 12x - 10 = 0$
21:43	Teacher	T-p	Okay step 1 move the 10 to the other side.(Writing on the board) So it is x squared minus 12x equals

10. What number do I take  $\frac{1}{2}$  of and square it?

21:45	Students	S-ans	12
21:58	Teacher	T-p	Half of 12 is 6. Square it. It is 36. So we can write the left-hand side as something squared.  What is that?
22:11	Student 7	S-ans	$x - 6$
22:12	Teacher	T-e	<b><math>x - 6</math>, When would you have a plus? Right here (pointing to the sign between the x and 6)</b>
22:16	Student 3	S-ans	If it's positive
22:18	Teacher	T-c	If what's positive?
22:20	Student 3	S-ans	12

While the class works through the completing the square, Amber inquires about the sign of the factored binomial. At that moment, the student only dictates an answer. She pursues an explanation by asking the student to verbalize the determination of the positive sign in factoring.

In Classroom B the students complete some reviewed problems in which a trigonometric expression,  $\frac{\tan x + \sin x}{2 \tan x}$ , was being simplified. In this dialogue the students and Brandi discussed how the “two” was affected by the division during the simplification process.

24:40	Teacher	T-i	So I so hope I pick your number but Student 6 it is you. When you flipped it, simplified, and did all that, what did you end up with?
24:49	Student 6	S-ans	I plus $\cos x$ all divided by 2
24:51	Teacher	T-cons	Yes, that is the answer.

24:53	Student 15	S-seek	What??
24:53	Student 15	S-c	I thought you had to go down further.
24:56	Teacher	T-f	I think what she did when she flipped it (looking at a student's paper), my guess it she got sine over cosine, she was able to cross out the cosines.
25:03	Teacher	T-f	You are multiplying the fractions at that point so you can cross them off. You are not allowed to cancel inside of a binomial.
25:16	Teacher	T-cons	And I think that was what Student 4 was trying to say. When I was trying to add fractions, he was saying you can't cancel. You are right.
25:28	Teacher	T-p	I can't just go and say oh the tanx here cancel with the tanx here. You are not allowed to do that. So when I flipped it, I am allowed to cancel the sine and cosine because it is multiplication.
25:43	Teacher	T-p	Now where is that 2? Now where is that 2?
25:50	Student 4	S-ans	Still there
<b>25:51</b>	<b>Teacher</b>	<b>T-e</b>	<b>Why?</b>
25:52	Student 6	S-ta	Cause it will not cancel

Brandi follows the thinking of several students on the problem. The students struggle with why the “two” remains in the answer. When she questions the students about the location of the “two”, one student responds with an exact answer. She proceeds to ask the student who responded to make his thoughts public.

In Classroom B, the students and Brandi checked off the review problems completed at the beginning of class. The problem read, "How many years will it take \$8,000 to be worth \$16,000 if the money is compounded monthly with an annual percentage rate of 9%?" A student following Brandi's request verbally explained how he substituted the numbers into the formula.

12:20	Teacher	T-p	So I got #20 so (student's name) which he is not here. So #20 again so it likes (student's name) today. So #4 which should be (another student name). Set it up for me.
12:45	Student 8	S-ans	16,000( student reads problem)
12:53	Teacher	T-r	So 9% divided by 12 cause it's per month.
12:58	Student 8	S-ans	(finishes reading)
13:02	Teacher	T-r	12P then you did your log magic. Did you have to log both sides?
13:03	Student 8	S-ans	Yea
<b>13:04</b>	<b>Teacher</b>	<b>T-e</b>	<b>Why?</b>
13:05	Student 8	S-c	Cause you had to bring the 12P down.
13:10	Teacher	T-f	Cause you got a variable in the exponent which take log ... which is what you did, what did you end up with anyway?
13:20	Student 8	S-ans	7.73
13:22	Teacher	T-cons	Years! Awesome, you get to be first person to try Christmas candy.(throws candy to student)

After the student reads the choice for his substitutions, Brandi proceeds into solving the resulting equation. She inquires as to “why” he did his algorithm in the manner of taking the” log of both sides”.

#### 4.1.10 Clarification Question

The *clarification question* category is similar to explanation by requiring the students’ thinking to be explained. However, in this situation, the students have already verbalized ideas. When teachers seek information from students about a particular explanation or idea the student previously verbally explained, the questions are coded as *clarification* or T-c.

In the first setting, Amber introduced the difference between completing the square on a quadratic equation where  $a$  on  $x^2$  is one and is greater than one. She started by inquiring about the differences in a couple of equations.

25:20	Teacher	T-i	Actually this method is really not bad once you get use to what you have to do. It is almost as fast as the quadratic formula. Now, look at 12, 14, 16, and 18. Look at what is different. What is different?
25:45	Students	S-ans	Coefficient
<b>25:47</b>	<b>Teacher</b>	<b>T-c</b>	<b>Coefficient where?</b>
25:48	Student 4	S-ans	On the $x^2$
25:50	Student 3	S-c	The a
25:51	Teacher	T-cons	Right, the a in that quadratic formula has to be a 1 to complete the square.

The students identify what is different about the equations but they are general in their verbal description. Amber asks them to be more specific in their explanation.



In Classroom A during a discussion about the remainder theorem, Amber asked about the definition of the remainder. She tried to draw the students into a conversation defining a remainder in the division algorithm.

13:40	Teacher	T-r	Oh that's nothing (teacher wipes off the mark) Now we are going to do the first ones. We are going to evaluate it using the remainder theorem. Which says, the very last column says the answer. Alright now then how do I know if 2 is a factor of 6? (pause)
13:55	Teacher	T-r	is 2 a factor of 6?
13:58	Student 5	S-ans	Yes
<b>14:00</b>	<b>Teacher</b>	<b>T-c</b>	<b>How do you know? (pause)</b>
14:12	Teacher	T-r	Is 2 a factor of 7?
14:14	Student 10	S-ans	No
14:15	Teacher	T-cons	No
14:16	Teacher	T-p	If 2 goes into 6 what is the remainder?
14:17	Student 7	S-ans	Zero
14:18	Teacher	T-cons	Zero

Initially, Amber attempts to get the students to explain why they thought two was a factor of six. The students respond with the verbal agreement, "yes". Consequently, she encourages the students to verbally explain their thinking pertaining to definition of a factor.

In Classroom B the students worked to add trigonometric fractions. Brandi asked them to complete the following sum:  $\frac{\sin x}{\cos x} + \frac{\sin x}{\cos x}$ .

20:02	Teacher	T-p	I got number 22 and Student 3 that's you. Do you have any idea what I have when I add these two fractions together?
20:15	Student 3	S-ans	$\sin 2x / \cos x$
<b>20:20</b>	<b>Teacher</b>	<b>T-c</b>	<b><math>\sin 2x / \cos x</math> so <math>\sin x</math> plus <math>\sin x</math> is?</b>
20:44	Student 4	S-ans	$2\sin x$
20:45	Teacher	T-cons	There you go $2\sin x$ . I need to 2 in front. $2\sin x$ ahh $\cos x$ (writes, reducing the resulting fraction)

The students struggle with adding fractions with trigonometric functions so Brandi gives them fractions with common denominators to add. A student contributes his answer to the sum. She utilizes a clarification question to encourage the student to evaluate his thought process.

In Classroom B, students engaged in solving a word problem. This particular problem was used as an example in the confirmation section. The problem stated, "A company is investing 18 million dollars and hopes to have 25 million dollars in eight years. What should be the percentage rate if the money is compounded annually?"

13:46	Teacher	T-cont	Okay I am going to write 25 since everything is in millions, right?
13:48	Student 9	S-ans	Yes
<b>13:49</b>	<b>Teacher</b>	<b>T-c</b>	<b>So...</b>
13:50	Student 9	S-ta	well
14:00	Teacher	T-s	So did you log both sides?
14:05	Student 9	S-ans	No
14:07	Teacher	T-e	Why not?
14:10	Student 9	S-pb	There's no variable in the exponent

14:12	Teacher	T-cons	There's no variable in the exponent
<b>14:13</b>	<b>Teacher</b>	<b>T-c</b>	<b>so how did you have to get rid of the eighth power?</b>
14:17	Student 9	S-ans	Ah... divide 18 on the other side and root 8

Brandi tries to elicit the student's thinking process. As the student starts to respond, Brandi clarifies the student's thinking by asking her to respond to questions pertaining to her solving process.

Utilizing both the explanation and clarification of questions is a means by which the teachers may draw out the thinking of one individual student. These categories ask a student to verbalize their mathematical thoughts. With explanation questions, students are requested to vocalize their thinking because they are executing mathematics without talking. For the clarification questions described earlier, students have verbalized some thinking but the teacher is seeking more specific details. Because students will reason about the mathematics they are doing, teachers' understanding can benefit with the employment of these types of questions.

#### 4.1.11 Justification Question

A justification question is when teachers request the students to demonstrate a form of a proof, provide an example, or build an argument for supporting reasoning, coded as *justification* or T-j. Justification questions may be used when a description of students' mathematical action is inadequate in helping the teacher understand the students' thinking. Justification encourages students to explain what aspects of the mathematical situation made their actions relevant and valid. Teachers' justification questions occasionally seek reasoning for how or why certain students' actions in solving problems were legitimate. This category defines a deeper level of students' thinking

compared to explanation or clarification. The basic form of these questions often comes begins with “why”.

The justification question category was exhibited once in the analysis of the transcripts. In Classroom B, students engaged in solving a word problem. The problem was used in examples in the confirmation and clarification sections. The problems stated, “A company is investing 18 million dollars and hopes to have 25 million dollars in eight years. What should be the percentage rate if the money is compounded annually?”

13:46	Teacher	T-cont	Okay I am going to write 25 since everything is in millions, right?
13:48	Student 9	S-ans	Yes
13:49	Teacher	T-c	So...
13:50	Student 9	S-ta	well
14:00	Teacher	T-r	So did you log both sides?
14:05	Student 9	S-ans	No
14:07	Teacher	T-e	Why not?
14:10	Student 9	S-pb	There's no variable in the exponent
14:12	Teacher	T-cons	There's no variable in the exponent
14:13	Teacher	T-e	so how did you have to get rid of the eighth power?
14:17	Student 9	S-ans	Ah... divide 18 on the other side and root 8
<b>14:24</b>	<b>Teacher</b>	<b>T-j</b>	<b>So just because?</b> What did you do that she did not do? (Pause)
14:35	Teacher	T-e	Like she 8 rooted it, did you do something different?
14:37	Student 10	S-ans	well
14:40	Teacher	T-f	What?

14:41 Student 3 S-ta Raised it to the  $\frac{1}{8}$  power.  
 14:42 Teacher T-cons Oh is that what you did?  
 14:43 Students S-con Oh yah

Brandi already received feedback from another student about how to solve the problem. When finished recording the student's work, she notices another student has solved it differently. She moves her attention to the other student. She asks him to justify what he did and compare it to the present algorithm.

This category allows teachers to analyze the students' thinking at a deeper level. The intent of the teacher for asking a justification question is to have students verbally support their mathematical thinking. While the clarification questions ask the students to respond to their actions or statements, the justification questions go further by also asking students to prove their thinking. Since justification often enables students to support their understandings from an external logical perspective rather than an internal individual one, teachers' reason for asking these questions may be to give students opportunity to base their reasoning in mathematics (Ilaria, 2009).

A summary of the codes for the teacher questions in this study is listed in Table 4.1.

Table 4.1 Codes for Teacher Questions

Teacher Questions	Definitions
Confirmation-t	teachers seek agreement from a specific student or the whole class about ideas stated in the discussion
Confirmation-s	teachers establish agreement with the student or extends an indication of following the student response
Suggestion	teachers add an idea not previously discussed in the conversation
Following	teachers' questions immediately follow and directly relate to ideas of a students' statement or action
Procedural	directions given by the teachers to the students are related to a specific mathematical task

Table 4.1 - continued

Initiating	teachers explicitly address the whole class or a specific student not engaged in the conversation immediately before being addressed
Retracing	teachers' questions interject a specific idea mentioned in a conversation prior to that point
Repeat	teachers ask the students to repeat their previous statement for bringing clarity or importance to the speech
Explanation	teachers ask students to verbalize what they are doing when they are writing their mathematical steps
Clarification	teachers desire more information from students about a particular explanation or idea the student previously explained verbally
Justification	teachers request the students to demonstrate a form of a proof, to provide an example, or to build an argument for supporting reasoning

#### 4.2 Student Responses

When students participate in the classroom discourse, a conversation between students and teachers will transpire. In constructivist learning theory, students' ideas and thoughts help guide teachers in the direction of new learning episodes, rather than following a strictly prescribed teacher-made lesson plan. In constructivist classrooms, understanding the verbal dialogue of the students becomes an integral part of the learning process. Therefore, the second question of this study was to describe the students' dialogue in the mathematical discussion.

This research question seeks to identify teacher questions that engage students in mathematical conversation. When the students communicate mathematical ideas, either with the teacher or with other students, the codes for student responses identify characteristics of a mathematical conversation and their relationship to the teachers' questions. This section identifies and describes the student responses in this study. The response categories are *thinking aloud*, *proof building*, *answer*, *clarification*, *seeking*, *confirmation*, *questions student*, and *non-contribution*.

#### 4.2.1 Thinking Aloud Response

Students' speaking publicly about mathematics with no justification component was coded as *thinking aloud response* or S-ta. The transcripts revealed that students sometimes responded to teacher questions by expressing their thinking processes. The thinking aloud response category emerged with a focus on the students' thinking.

In Classroom A, Amber answered questions over the student's homework on solving equations using quadratic formula.

9:55	Student 2	S-seek	Can you do number 13?
9:58	Teacher	T-e	Okay let's do number 13. What did you not get right on it?
<b>10:02</b>	<b>Student 2</b>	<b>S-ta</b>	<b>I think I just didn't factor out as far as it could go.</b>
10:05	Teacher	T-p	Okay on this one (teacher writing on the board) What do you always have to have before you start using the quadratic formula?
10:07	Teacher	T-p	What is the first step?
10:10	Student 2	S-ans	Equals zero
10:12	Teacher	T-cont	Yah, we have to have zero on one side even when you factor right? So we're going to subtract 6. (Pause
10:20	Teacher	T-cont	So you've had the quadratic formula before. So mark what a, b, and c are. But this is a formula you should already have memorized but if you do not, it will need to be done by tomorrow. I am not going to give it to you for your quiz. You can

kind of hum Mrs.(previous teacher) song quietly.  
 ( writes on the board) So let do that would be a  
 negative b pulse or minus ...b2 ... 4ac.. all over  
 2a. Is that what you did so far?

11:05 Teacher T-cont So 6 ... you are going to have to ... you got 2  
 negatives makes a plus so 36 plus Now you  
 have to take 36 times 11 Get that right? You got  
 36 times 11(on the calculator) You got 396 over  
 18.

**11:54 Student 2 S-ta I put in a wrong number. I accidentally put in  
 a for b.**

As the students check their work, Amber responds to questions from the students. If a  
 student had a particular question on a problem, she works the problem and seeks  
 feedback on what misunderstandings the student may have. The student responds  
 verbally as to what she did incorrectly.

In Classroom B, the students and Brandi worked to simplify the trigonometric  
 expression  $\frac{\tan x + \sin x}{2 \tan x}$ . As she and the students dialogued, they took the numerator and  
 put it into the form,  $\frac{\sin x}{\cos x} + \frac{\sin x \cos x}{\cos x}$ .

21:06 Teacher T-cont No, No, I say you factor out a sinx with tanx is sinx  
 over cosx Ahh there we go.

21:08 Student 2 S-a Yeah(as teacher reduces fraction)

21:10 Teacher T-p Oh no, I totally lost Student 4? Student4

21:15 Student 4 S-ans Yes

21:16 Teacher T-cont You're okay with sine over cosine plus sinxcosx



21:25	Student 4	S-ta	<b>Other than the fact that you have now separated something which you couldn't separate before because you added</b>
21:30	Teacher	T-p	Wait, Wait, I didn't separate something
21:32	Student 4	S-ta	<b>You multiplied it but</b>
21:35	Teacher	T-cont	I added these two pieces and making fractions. Okay, Student 4, Student 4
21:44	Student 4	S-ta	<b>You couldn't do that</b>
21:45	Teacher	T-e	Wait Time out. When did I say you could not do that?
21:49	Student 4	S-pb	You said when you are adding or subtracting them, you cannot multiply or cancel them.
21:51	Student 13	S-pb	She is just trying to get the common denominator. She is multiplying the numerator by sin and the denominator by sin which equals 1.

A student is frustrated with not being able to understand how Brandi is adding two trigonometric fractions in her demonstration of the review problem. He expresses his perceptions of the mathematical thinking as he views the problem simplification process.

In Classroom B, the participants investigated the unit circle. In the problem numbered seven on their worksheet, the intention was to algebraically show the connection between the formula for the circle's circumference and the number of radians that coincide with the circle.

36:27	Teacher	T-p	Okay, what does number 6 say? Okay using the other color we did prove algebraically that. Okay go to #7, we already number 6. (reading number 7) Prove that the circumference in
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radians of any circle. So write the formula for circumference. It's on #7.

- 37:00 Student 10 S-seek Okay, did I do this right?
- 37:14 Teacher T-s Now you are solving for r?
- 37:18 Student 5 S-ta Okay it is not the area but circumference.**
- 37:21 Teacher T-c You are solving for r. What is r? Wait, Wait! Time out! What did we say r was going to be on every unit circle?
- 37:28 Students S-ans 1 unit

The students solve for the radius in the circumference formula. A student verbally utters that he recognizes he is using the area formula instead of the circumference.

#### 4.2.2 Proof-Building Response

In proof-building responses, students respond to teacher questions by communicating their thinking process, but they also include a “why” component. Students speaking publicly about mathematics including evidence of justification for their mathematical thinking were coded as proof-building response or S-pb. Differentiating between the two responses was a fine distinction, but the crucial determinant for proof-building was the students providing a reason or reasons for their thinking in responses.

In Classroom A the students along with Amber practiced finding binomial factors of polynomials using the synthetic division algorithm. A specific problem asked, “Is  $b - 7$  a factor of the polynomial  $b^4 - 8b^3 - b^2 + 62b - 37$ ?”

- 19:35 Student 5 S-seek Do we need state why?
- 19:37 Teacher T-p No sometimes we don't really answer the question. Yes we did that work but we didn't

answer the question.

19:40 Student 7 S-seek So if the last number is a prime can we just say  
it is not a factor?

19:45 Teacher T-c You're trying to skip the process here?

19:50 Student 7 S-pb **You know that that will multiply by this and not  
get a zero when subtracted.**

As a student identifies whether a binomial is a factor of the polynomial, he discovers a pattern with respect to the constant of the polynomial. He questions Amber about his idea. She interprets his idea as a means of eliminating the longer process. He responds with why mathematically he thinks his idea is correct.

In Classroom B as mentioned in the previous section, Brandi and students simplify a trigonometric expression  $\frac{\tan x + \sin x}{2 \tan x}$ . As they dialogued, they took the numerator and put it into the form,  $\frac{\sin x}{\cos x} + \frac{\sin x \cos x}{\cos x}$ .

21:10 Teacher T-p Oh no, I totally lost Student 4? Student 4

21:15 Student 4 S-ans Yes

21:16 Teacher T-cont You're okay with sine over cosine plus  $\sin x \cos x$

21:25 Student 4 S-ta Other than the fact that you have now separated  
something which you couldn't separate before  
because you added

21:30 Teacher T-p Wait, Wait, I didn't separate something

21:32 Student 4 S-ta You multiplied it but

21:35 Teacher T-cont I added these two pieces and making fractions.  
Okay, Student 4, Student 4

21:44 Student 4 S-ta You couldn't do that

21:45	Teacher	T-e	Wait Time out. When did I say you could not do that?
21:49	Student 4	S-pb	<b>You said when you are adding or subtracting them, you cannot multiply or cancel them.</b>
21:51	Student 13	S-pb	<b>She is just trying to get the common denominator. She is multiplying the numerator by sin and the denominator by sin which equals 1.</b>

One of the students does not understand the process of obtaining a common denominator. He articulates his perception of what is mathematically happening. Another student enters the conversation and expresses differently what Brandi is doing. He included how and why in his understanding the common denominator was found.

The students from a classroom lesson in Classroom B reviewed and solved exponential word problems. While Brandi requested one student to verbally express his findings, all the students solved the following problem. "A company is investing 18 million dollars and hopes to have 25 million dollars in eight years. What should be the percentage rate if the money is compounded annually?"

13:33	Teacher	T-p	Okay, next one-- #9 is (student's name). Can you set it up for us?
13:43	Student 9	S-ans	25 million
13:46	Teacher	T-cont	Okay I am going to write 25 since everything is in millions, right?
13:48	Student 9	S-ans	Yes
13:49	Teacher	T-c	So...
13:50	Student 9	S-ta	well

14:00	Teacher	T-s	So did you log both sides?
14:05	Student 9	S-ans	No
14:07	Teacher	T-e	Why not?
<b>14:10</b>	<b>Student 9</b>	<b>S-pb</b>	<b>There's no variable in the exponent</b>
14:12	Teacher	T-cons	There's no variable in the exponent
14:13	Teacher	T-c	so how did you have to get rid of the eighth power?
14:17	Student 9	S-ans	Ah... divide 18 on the other side and root 8

Brandi asks the student to convey the algorithmic process he used. He is not articulating what he did so she prompts him with a possible solution step. After he negates her suggestion, she desires to know “why”. He responds with his thinking in the mathematical process.

During mathematical discourse, teachers and students dialogue about mathematical concepts or ideas. Students may desire to converse with the teachers or other students so they take the time to verbalize their thoughts. This desire could be internally or externally motivated, but either way, their thoughts are articulated in an unorganized or organized manner. The thinking aloud responses express their mathematical thinking, whereas, the proof-building responses include the mathematical justification component to the utterance.

#### *4.2.3 Answer Response*

In *answer response*, students contribute to one of the following: a short or closed-ended recall response, a fact or piece of information, no explanation for how they arrived at the answer, or no justification for why it is correct. These responses were coded as *answer response* or S-ans. Answer response is dialogue between students and teachers that may not convey mathematical thinking. The teachers' questions, the

students' understanding, or the students' ability to communicate reasoning at the time of the response are factors affecting the verbalizing of thinking.

In Classroom A, Amber and students practiced completing the square to solve quadratic equations. A student read one of the problems.

21:23	Teacher	T-p	And so we're going to. It takes me longer to tell you problem number 1 then the rest because we have to write out the instructions so we are all on the same page. Alright another one, let's do mmm. Let's jump to number 6. That says...
21:40	Student 5	S-ans	$x^2 - 12x - 10 = 0$
21:43	Teacher	T-p	Okay step 1 move the 10 to the other side.(Writing on the board) So it is x squared minus 12x equals 10. What number do I take ½ of and square it?
21:45	Students	S-ans	12
21:58	Teacher	T-p	Half of 12 is 6. Square it. It is 36. So we can write the left-hand side as something squared. What is that?
22:11	Student 7	S-ans	$x - 6$
22:12	Teacher	T-e	$x - 6$ , When would you have a plus? Right here (pointing to the sign between the x and 6)
22:16	Student 3	S-ans	If it's positive
22:18	Teacher	T-c	If what's positive?
22:20	Student 3	S-ans	12
22:21	Teacher	T-p	If the 12 is positive you'll have a plus right here(pointing) the middle term determines the middle

sign. Okay now to get rid of the square, we take the square root. Remember plus or minus. Okay, is 46 a perfect square?

**22:35 Student 3 S-ans No**

22:37 Teacher T-p Does it have a perfect square in it?

22:42 Teacher T-p It is 2 times 23. 4 does not divide into it. I don't think so. I think we're done except for, what's my last step?

**22:52 Student 3 S-ans Move the 6.**

This example displays Amber progressing through a problem using a particular algorithm.

The students provide input to complete each step.

In Classroom A the lesson reviewed the process of the synthetic division.

9:37 Teacher T-p It's a process an easy process. It is still a process. How do we do this?

**9:57 Student 5 S-ans you bring down the negative 1.**

9:58 Teacher T-cons Yes, you bring down the negative 1.

10:00 Teacher T-p So if I am going to write it down for synthetic division, (writing on the board) This is called synthetic division. I help you build our case. You bring down first column. Okay, does anybody remember what you did next?

**10:17 Students 5 and 6 S-ans Multiply the 2 by negative 1.**

10:22 Teacher T-p Okay, so I multiply (pointing) =2. Where do I write it?

**10:24 Student 6 S-ans Below the zero**

10:26 Teacher T-p Yes, in the next column. Ya'll do remember. You multiply, record in next column, and this is the part that a few people maybe not you would get mixed up. What do I do next?

**10:40 Student 7 S-ans add**

As Amber leads and questions the students through the division algorithm, several students reply to each question naming the sequential steps of the procedure.

In Classroom A, Amber introduced the difference between completing the square on quadratic equations where  $a$  on  $x^2$  is one and where  $a$  is not one. She started by inquiring about differences in a couple of equations.

26:12 Teacher T-p Okay number 3, what one word did you end up with Student 4?

**26:25 Student 4 S-ans  $\sec^2 x$**

26:27 Teacher T-p Ahh that is not what I ended up with. What did you get?(speaking to another student)

**26:33 Student 15 S-ans Tanx**

26:35 Teacher T-cons That is what I ended with. You are not just going to write  $\tan x$  on your paper. We are going to work that out.

**26:45 Student 1 S-ans I got tan squared x**

The students give answers to the questions in the warm-up practice.

In Classroom B the class investigated how the length of a circle's radius relates to the circumference of the circle.



30:23	Teacher	T-i	Okay does anyone besides, student's name, cause she already go a piece of candy. Does anyone besides student's name, know how many it is going to take? How many?
<b>30:37</b>	<b>Student 8</b>	<b>S-ans</b>	<b>6 and a little more</b>
30:39	Teacher	T-cons	6 and a little bit more, 6 and a little bit more. Let me know
<b>30:43</b>	<b>Student</b>	<b>S-ans</b>	<b>6.28</b>
30:47	Teacher	T-cons	Yeah, Yeah! So it's related to
<b>30:48</b>	<b>Student 5</b>	<b>S-ans</b>	<b><math>2\pi</math></b>

When the students take segments, the length of a circle's radius, and lay them on the circle's circumference, they observe how many of these lengths wrap around a circle.

The number is quoted by several students in different mathematical formats.

The *answer* response was a frequent response observed in the transcripts and was a response that did not require verbalizing mathematical thinking. The students may have been responding in this manner because the teachers' questions only expected a small amount of information. These responses were progressive parts of a process or the final answer.

#### 4.2.4 Clarification Response

*Clarification responses* were observed when students contributed answers to teachers' questions during a discussion, but the teachers wanted students to provide further information; coded as *clarification response* or S-c. This type of response is described by students articulating more information to a previous statement without justifying their thinking.

In Classroom A, Amber introduced the difference between completing the square on quadratic equations where  $a$  on  $x^2$  is one and where  $a$  is not one. She started by inquiring about differences in a couple of equations.

25:20	Teacher	T-i	Actually this method is really not bad once you get use to what you have to do. It is almost as fast as the quadratic formula. Now, look at 12, 14, 16, and 18. Look at what is different. What is different?
25:45	Students	S-ans	Coefficient
25:47	Teacher	T-c	Coefficient where?
25:48	Student 4	S-ans	On the $x^2$
<b>25:50</b>	<b>Student 3</b>	<b>S-c</b>	<b>The a</b>
25:51	Teacher	T-cons	Right, the $a$ in that quadratic formula has to be a 1 to complete the square. Oh, no. Okay, here we go. I'm going to copy this real quick.(says to a student)

Several students answer Amber's question which is followed by another teacher question. A student responds, but a third student clarifies this response with a more detailed description.

During the next observed class session, another student responded with a clarification response. While the class practiced using synthetic division to find factors of a polynomial, a student interjected an observed pattern. He communicated that his idea would increase efficiency of finding factors. When he brought the idea forth earlier in the class period, the class discussion went in another direction. So Amber brought the idea back up for discussion during the individual practice time.

- 28:19 Teacher T-f Ok, let's see his theory then. His theory was it was prime it would never divide.
- 28:38 Student 4 S-c By whole numbers not fractions**
- 28:36 Student 1 S-ta The first one didn't work, it was a 2
- 28:38 Teacher T-cons Yes, the first one ended in a 2 which is prime but it factored.

After Amber described her interpretation of the student's thinking, he responds by clarifying his thoughts. He defines the numbers of which he was referring.

In Classroom B, Brandi and the students were simplifying a trigonometric expression  $\frac{\tan x + \sin x}{2 \tan x}$ . In the dialogue, she and the students found the answer. A student inquired about the last simplification step of the expression,  $\frac{\sin x(1 + \cos x)}{\cos x} \cdot \frac{\cos x}{2 \sin x}$ .

- 23:45 Teacher T-i Did you get it?(student 3)
- 23:50 Teacher T-f Where did the 2 go?
- 24:03 Teacher T-p That is what I keep asking yall. The 2 disappears automatically. It shouldn't go anywhere. The 2 doesn't cancel. The sine does.
- 24:05 Student 3 S-seek Shouldn't the two cancel with the sine?
- 24:09 Teacher T-p The 2 doesn't cancel the sine does.
- 24:12 Student 3 S-c I thought you canceled so there is just one sine left? So the 2 goes away.**
- 24:14 Student 10 S-ta it is just for sine squared, right?
- 24:15 Teacher T-cons Yes

The student's question pertains to the sine and 2 being divided by common factors. Brandi answers the question but the student is not satisfied with her answer. He elaborates his question by verbally describing that only one sine remained.

In the review part of a class session in Classroom B, a student read the problem he has solved. “A person is setting up an annuity fund. He would like to have \$120,000 in the fund after 30 years at 4.5% interest compounding monthly. How much should he put into the fund each month to reach the goal?” The students took the information and substituted values into a formula in order to find the answer.

15:17	Teacher	T-p	Okay, #3... lucky, #9 – that would be nobody. #16 is (student’s name)
15:28	Student 12	S-ans	1200 (reads the rest of problem)
15:38	Teacher	T-c	Okay why over 12?
15:42	Student 12	S-ans	Cause its monthly
15:48	Teacher	T-f	Raised to the
15:50	Student 12	S-ans	12 * 30
15:55	Teacher	T-p	This is where some of you went wrong. You didn’t raise the exponent to the monthly times the number of years since you thought annual percent rate yearly. They are always annual percent rates, APR. Okay so it’s always annual % rate so you then have to raise it to the monthly in number of years okay on the bottom.
16:22	Student 12	S-ans	... over 0.045
16:22	Teacher	T-cont	Okay, you probably have one of those fancy pants calculators and put all of that in at one time, right?
16:29	Student 12	S-ans	yes
16:34	Teacher	T-c	How did you get it to other side?
<b>16:35</b>	<b>Student 12</b>	<b>S-c</b>	<b>divided</b>
16:39	Teacher	T-f	Divided so you ended up with a monthly payment.

16:42 Student 12 S-ans \$128

As a student communicates his substitution and solving process, Brandi questions his thinking in the process of solving. He divulges that he had divided in order to finish solving for the monthly payment.

If students respond to the questioner with additional information from their initial response, they may give a *clarification* response. This kind of response may be in reaction to a questioner who communicated dissatisfaction with the previous utterance. In addition, these clarification responses may be used by students to rephrase the original statement because the questioner has incorrectly interpreted it. The result of teachers or students questions not receiving feedback or receiving mistaken feedback may prompt students to provide further information.

#### 4.2.5 Seeking Response

In *seeking response* students request feedback from the teachers with requests coded as *seeking response* or S-seek. The students may ask the teacher for verification of their answers. In addition, observed during mathematical discussion was that students expressed the perception that they could not attain further progress with problem solving. It was noted that to make progress, students requested assistance from the teacher by seeking response.

In Classroom A, Amber instructed the students on how to solve quadratic equations by completing the square. She allowed students to complete several practice problems. At the end of one of the problems, the students questioned how to simplify a radical of the form,  $\sqrt{\frac{-68}{9}}$ .

34:18 Student 9 S-seek So is the fraction -68 so 4 and 17 over 3 ... so  
I didn't know if you could do it like that take it out

of the fraction? Oh you have to leave it as a fraction...

34:23 Teacher T-cont Oh, That's what I did. I could make this over 3 and just all over one fraction. You see what I am saying?

34:28 Student 9 S-ans Yes

**34:29 Student 10 S-seek Can I do this?(pointing to paper)**

34:29 Teacher T-cons If you want to.

**41:01 Student 10 S-seek Can we take out this 9?**

41:02 Teacher T-p Yes, there is your denominator.

**41:10 Student 10 S-seek You would do the square root of 17 over 3?**

41:26 Teacher T-p Yes

**41:29 Student 10 S-seek Oh my, is that correct?**

A student asks Amber if her idea is allowed and Amber responds. Then the student proceeds through her algorithm by questioning her steps in order to verify accuracy.

In Classroom A, Amber discussed how to symbolically write a divisor of a polynomial dividend in synthetic division as a factor of the dividend.

14:58 Teacher T-p We are going to know if we get zero as a remainder. I am going to put a 7 right here. I don't know how you want to think about it. It's the value that makes it zero. I see a negative 7 so I will put a 7. Or I'll put the opposite. The other way is if they say add 7 that is what you always put there.

**15:58 Student 8 S-seek Why is it that you put the opposite?**

16:00 Teacher T-p Well, (pause) it's because this is a factor and this was a root. This is a factor and it is always x minus whatever the root is.

16:08 Student 8 S-con Okay

As Amber writes the answer in symbolic notation, she changes a sign on the seven from a negative to a positive value. A student questions why the sign was changed.

In Classroom B, Brandi and the students worked to simplify a trigonometric expression  $\frac{\tan x + \sin x}{2 \tan x}$ . In the dialogue, two students inquired about the last simplification

step of the expression,  $\frac{\sin x(1 + \cos x)}{\cos x} \cdot \frac{\cos x}{2 \sin x}$ .

20:02 Teacher T-p I got number 22 and Student 3 that's you.  
Do you have any idea what I have when I add these two fractions together?

20:15 Student 3 S-ans  $\sin 2x / \cos x$

20:20 Teacher T-c  $\sin 2x / \cos x$  so  $\sin x$  plus  $\sin x$  is?

20:44 Student 4 S-ans  $2 \sin x$

20:45 Teacher T-cons There you go  $2 \sin x$ . I need to 2 in front.  $2 \sin x$   
ahh  $\cos x$  (writes, reducing the resulting fraction)

**20:50 Student 6 S-seek How did you?**

**20:51 Student 2 S-seek You just crossed those out? How did you?**

20:53 Student 6 S-con Ohh

21:06 Teacher T-cont No, No, I say you factor out a  $\sin x$  with  $\tan x$  is  $\sin x$  over  $\cos x$  Ahh there we go.

21:08 Student 2 S-con Yeah (as teacher reduces fraction)

Some students did not achieve the mathematically correct answers. They evaluate their simplification processes with the Brandi's work. This analysis initiates questions from students as to what fraction rules permits reducing the fractions in that manner.

In Classroom B during another lesson, the class participated in an activity using Wikki Stix to investigate the relationship between the radius of a circle and angles measured in radians. Brandi directed the students to problem four and read its directions.

26:30	Teacher	T-p	We are on #4. You all are working together. Draw a vertical tangent line from Q to the top of your card stock.
<b>26:50</b>	<b>Student 15</b>	<b>S-seek</b>	<b>I don't know if my radius is completely horizontal</b>
26:52	Teacher	T-cons	It's okay
<b>26:53</b>	<b>Student 15</b>	<b>S-seek</b>	<b>Are you sure?</b>
26:54	Student 7	S-ans	draw a tangent

A student is seeking Brandi's confirmation on a step in the directions.

The intent of a seeking response may be for students to receive information from the teacher. If students have reached a point of needing help, they ask teachers to assist in moving them forward in their thinking. In addition, they may be searching for validation in the correctness of their actions or thinking. In the classroom, an instinctive response is to anticipate answers from the perceived expert in the classroom, the teacher.

#### *4.2.6 Questions Student Response*

In cases observed where students were seeking feedback from other students their utterances are coded as *questions student response* or S-qs. In mathematical discourse from the transcripts of this study, students were observed to not always seek help from the teacher, but from other students instead.



In Classroom B the students have solved  $3\sin x = 2\cos^2 x$  in their homework from the previous day. Some of the students had questions on how to solve the problem. They had reached the point where  $2\sin^2 x + 3\sin x - 2 = 0$  and did not know how to proceed.

34:59	Teacher	T-s	Good, so here's what I see when I see a squared, $x^2$ , an x, and a number. That is a trinomial. I am going to have to factor that. That is a let y = problem; If I write it in the right order, it would be (writing) $2y^2$
35:02	Student 7	S-con	It is the same thing as 62!
35:05	Student 4		Hush
35:08	Teacher	T-s	and $3\sin x$ , no no, drop that sine, y minus 2
<b>35:12</b>	<b>Student 4</b>	<b>S-qs</b>	<b>why did you do that? (looking at another student)</b>
35:18	Teacher	T-p	I let y = $\sin x$ . What does that factor into?
35:28	Student 6	S-pb	We can factor it easier.
35:33	Students	S-ans	$(2y - 1) (y + 2)$

Brandi gives the students a suggestion on how to proceed. A student inquires of the student next to him as to purpose of the suggestion.

The objective of the *questions student* responses may be the students seeking information to continue working. Students may have insufficient understanding of the mathematics being discussed and seek help from other students. They may enter the conversation because they have offered to help, or have analyzed the mathematics and can provide clarity to the situation. It is posed from these findings that students who are confident in their own abilities and if teachers provide opportunities for student dialogue, the students may request feedback from other students.

#### 4.2.7 Confirmation Response

Students were observed verbalizing agreement with a previous statement, which was coded as *confirmation response* or S-con. Teachers desire confirmation in order to move forward in a lesson or apprise the students of productive progress. In the teacher question codes, the confirmation question category described conversation from the teachers' perspective. In the environment of reciprocated dialogue, student must also express agreement so there is unity for the conversation to move forward.

In the first setting, Amber discussed how to symbolically write a divisor of a polynomial dividend in synthetic division as a factor of the dividend.

14:58 Teacher T-p We are going to know if we get zero as a remainder. I am going to put a 7 right here. I don't know how you want to think about it. It's the value that makes it zero. I see a negative 7 so I will put a 7. Or I'll put the opposite. The other way is if they say add 7 that is what you always put there.

15:58 Student 8 S-seek Why is it that you put the opposite?

16:00 Teacher T-p Well, (pause) it's because this is a factor and this was a root. This is a factor and it is always x minus whatever the root is.

**16:08 Student 8 S-con Okay**

The student asks Amber a question about the notation. She gives a procedural answer in which the student positively responds.

In Classroom B Brandi reviewed the process of simplifying a couple of trigonometric expressions. The class worked on simplifying  $\frac{\tan x + \sin x}{2 \tan x}$ .

18:29	Teacher	T-p	I feel like I need to "RandInt" so (student's name) doesn't answer all of them. Yah, student's name says no please then you'll pick me and I really don't want to be picked today. Is that what you are saying to me? Okay, ya'll are sitting in different seats so you need to know where your seat is. Cause like now Miss(student's name) you are now number 2. Okay, I didn't pick you yet. Hold on, I have not even touched the denominator.
18:48	Student 4	S-ta	You have to multiply all that out though
18:52	Teacher	T-p	Yes I know. Hold on to your britches. I am not even touching the denominator yet.
18:55	Student 4	S-c	Okay, when you multiply sine by cosine You didn't multiply the sine over cosine
19:09	Teacher	T-p	No, No, all I did was find a common denominator for sine
19:16	Student 4	S-c	Okay, you need to do that for the other one too
19:18	Students	S-ta	No you do not, it already has the denominator
19:19	Teacher	T-p	No
<b>19:20</b>	<b>Student 1</b>	<b>S-con</b>	<b>No you don't cause it already has the denominator.</b>

The class substituted tangent  $x$  for sine  $x$  divided by cosine  $x$  in the numerator. They are now getting a common denominator for the sum in the numerator. Student 4 is having trouble understanding the process. While the dialogue progresses as Brandi and students are trying to understand his difficulty, another student communicates

understanding by confirming the statements of other students and Brandi. His statement repeats what others have said.

In Classroom B during another lesson, the class participated in an activity using Wikki Stix to investigate concepts dealing with the unit circle.

21:20	Teacher	T-p	Lord have mercy, are you between 1 and 2 centimeters?
21:21	Student 6	S-ans	No
21:22	Teacher	T-p	Centimeters?
21:23	Student 6	S-seek	Inches
21:24	Teacher	T-p	Okay, let's do it in centimeters. Our Wiki Stix are not long enough.
<b>21:32</b>	<b>Students</b>	<b>S-con</b>	<b>Ahh, okay</b>
<b>21:35</b>	<b>Student 1</b>	<b>S-con</b>	<b>Centimeters! 1 and 2 centimeters (rustle)</b>

The students construct circles with radii of one to two inches. They have trouble arranging the circle on the page. After Brandi clarifies the unit of measurement, the students come to the consensus that centimeters will correct the problem.

The confirmation responses supply a signal to teachers when the students are ready to advance. Often, the utterance is merely an "okay". The communication of consensus among the participants is important for the teachers guiding the discussion.

#### *4.2.8 Non-contribution Response*

In the transcripts it was observed that students did not participate in the current conversation, and their utterances were coded *non-contribution response* or S-nc. The transcripts revealed students not entering the conversation. When working to encourage mathematical discourse, the teachers in this study had students who did not respond to their questions.

In Classroom A the students worked on homework problems and completed the square in order to solve quadratic equations. The equation has a fractional coefficient on the “x” term.

- 43:57 Student 11 S-seek What if you have a fraction as b?
- 43:59 Teacher T-p That is fine, it is  $20/9$ . So that  $\frac{1}{2}$  of it. What is  $\frac{1}{2}$  of  $20/9$ ?
- 44:10 Student 11 S-nc I don't know I am really bad at fractions.**
- 44:15 Student 12 S-nc That's what the calculator is for.**
- 44:32 Teacher T-p What is  $\frac{1}{2}$  of  $20/9$ ?
- 44:45 Student 5 S-ans  $10/9$  ...no...
- 44:48 Student 11 S-ans  $20/18$ . This is hard.
- 44:51 Teacher T-cons This is a worksheet not an easy sheet.

Some students sought help from Amber since they claim not to be comfortable working with fractions. When she asks them to complete a multiplication problem with fractions, they express not having the knowledge to complete the skill and looks for another means of multiplying the fractions.

In Classroom B, Brandi reviewed the process of simplifying a couple of trigonometric expressions. The class worked on simplifying  $\frac{\tan x + \sin x}{2 \tan x}$ .

- 19:22 Teacher T-r Student's name(student 4) I am taking a commercial break for you. If I trying to add  $2/3$  plus  $3$ , oh you say it is  $3$  and  $2/3$ . (Laughter from class) Okay but you had to get a common denominator. Okay I am with you. If you had to get a common denominator, what would it be?

19:39 Student 4 S-ans 3

19:41 Teacher T-p So this is 9 over 3. And I didn't have to change that one. You closed your eyes like you do not believe me. Okay Mrs.(Teachers Name) do your magic.

**19:45 Student 4 S-nc Never mind. Ignore me.**

19:49 Teacher T-p Okay, I won't ignore you but I will pretend that I am. Okay so my common denominator is  $\cos x$ . What do I have even if I'm not touching this  $2 \tan x$  yet? (students murmuring)

19:54 Teacher T-p What do I have left on the top?

Student 4 struggles to understand the process of getting common denominators and summing the trigonometric fractions. After several attempts through input from students and review from Brandi, the student chose to leave the conversation.

In Classroom B the class investigated the relationship between radii and radians. The students defined radius and diameter but they still had not verbalized the relationship with the radian.

25:10 Teacher T-e The radius is half the diameter. I got a bigger question here, cause she said it was half the diameter and yet on #3 it does not say radius. It says radian. So

25:28 Student 1 S-ta Because it is looking at the plural of radius.

25:32 Teacher T-p No

25:35 Student 12 S-ans Radii is the plural of radius.

25:41 Teacher T-e So why did she use radian when clearly she meant radius

25:49 Student S-nc well, I don't know

11

25:51 Teacher T-p Well, that is what we are about to find out. So now we are going to step 4. (reading) How is a radius related to radians?

The student does not express understanding of how to answer Brandi's questions and articulates the lack of understanding. He does not even include possible answers.

Students may deliver a non-contribution response because they do not desire to engage in the conversation. The reasons for non-contribution may be numerous, however within the context of classroom discourse, it is the student's choice to respond or not respond. Their choice not to respond may be a consequence of not knowing what to say due to a lack of knowledge about how to engage in the conversation. Literature has validated the difficulty of eliciting dialogue from students (Soucy McCrone, 2005).

Table 4.2 displays the definitions to the final codes for the student responses in this study.

Table 4.2 Summary of Student Responses across Classes

Student Responses	Definitions
Thinking Aloud	students speak publicly about mathematics with no justification component
Proof-Building	students speak publicly about mathematics including evidence of justification for their mathematical thinking
Answer	students contribute to one of the following: a short or closed-ended recall response, a fact or piece of information, no explanation for how they arrived at the answer, or no justification for why it is correct
Clarification	students articulate more information to a previous statement without justifying their thinking
Seeking	students request feedback from the teachers
Questions Student	students are seeking feedback from other students

Table 4.2 - continued

Confirmation	students indicate agreement with a previous statement
Not Contributing	students do not participate in the current conversation

#### 4.3 Summary

The purpose of this chapter was to present the codes for the teacher questions and student responses giving a voice to the discussion between the teachers and students. The next chapter presents the analysis of these codes and a description of questioning themes developed by the dialogue. The main goal of the next chapter is to present a description of the nature of the mathematical discussion when teachers question students in order to elicit their thinking.



## Chapter 5

### Teacher Question and Student Response Relationship

The previous chapter described the types of questions that the two teachers asked in their classrooms during problem-solving activities and categorized the student responses to reveal communication patterns of both groups. In this chapter, the relationship between the questions and responses is examined to describe how the teachers engaged their students in mathematical conversation and elicited their thinking. The frequency of each category of questions and responses provide a depiction of how often the teachers and students communicated in a specific form.

After viewing the frequency of each teacher question and student response code, the relationship between a particular teacher question and a particular student response was analyzed by identifying which responses followed specific questions. The number of times a specific student response followed a teacher question was also noted. These comparisons provide quantitative descriptions of the possible relationships between teacher questions and the extent to which they may encourage student thinking. The analyses then excerpts discourse exchanges from each setting and develops questioning themes. These themes provide a description of student talk and how it may have been encouraged by teacher questioning.

#### 5.1 Teacher Question Frequency

The frequency of teacher questions provides an indication of how often teachers asked a particular type of question as described in Chapter Four. Table 5.1 presents frequency counts for each type of teacher question in each classroom setting and as the total in the study. The total number of questions asked by Amber was 127, and Brandi asked 297 both during two 45 minute class periods. The topics in each setting were different therefore a comparison is not made between the two settings.

Table 5.1 Teacher Question Frequency/Percentage

<i>Teacher Questions</i>	<i>Classroom A</i>		<i>Classroom B</i>		<i>Total</i>	
<i>Confirmation-t</i>	18	14.17%	26	8.75%	44	10.38%
<i>Confirmation-s</i>	22	17.32%	36	12.12%	58	13.68%
<i>Suggestion</i>	0	0.00%	20	6.73%	20	4.72%
<i>Following</i>	4	3.15%	41	13.80%	45	10.61%
<i>Procedural</i>	60	47.24%	98	33.00%	158	37.26%
<i>Initiating</i>	8	6.30%	26	8.75%	34	8.02%
<i>Retracing</i>	5	3.94%	17	5.72%	22	5.19%
<i>Repeat</i>	1	0.79%	2	0.67%	3	0.71%
<i>Explanation</i>	2	1.57%	8	2.69%	10	2.36%
<i>Clarification</i>	7	5.51%	22	7.41%	29	6.84%
<i>Justification</i>	0	0.00%	1	0.34%	1	0.24%

As shown in Table 5.1, the most frequent question in the data is a *procedural* question that solicits for specific mathematical tasks from the students. There were 158 of this type of question which is 37% of all questions. Teachers use these questions to direct classroom activities whether it is a discussion, an investigation, or a group or individual problem solving. Truxaw and DeFranco (2007) in their research envision the teacher directing discussions so students can build connections and cultivate mathematical understanding. Furthermore, Vygotsky's perspective on learning views the teacher as the expert in the room, directing students to perform a task that helps them move forward with or demonstrates their thinking (Sfard, 2003). The *procedural* questions are one mechanism by which teachers may assist student learning.

In both classroom settings, the teachers were primary directors of student learning. In Classroom B one-third of Brandi's questions were procedural, while almost half of Amber's questions are in this category. Amber devoted about half of the class time using problems from a worksheet and presenting information on how to proceed through a mathematical algorithm such as synthetic division. When she asked for student input as she moved through the solving process, her format yielded itself to this type of question. Brandi spent about one fifth of the class time dictating mathematical information to be put on the students' notecards. During the rest of the time, her activities compelled students to process the information through either investigating or problem solving. On the other hand, in a mathematical research study focusing on student-centered classrooms, the students guided the direction of the class activity by their thinking (Ilaria, 2009). The teachers incorporated procedural questions less than 20% of the time (Ilaria, 2009). The teachers' choice of classroom activity may affect the extent of teachers' use of procedural questions.

The second most frequent question as observed in Table 5.1 was the *confirmation* questions that solicit agreement among the participants. There are 102 of this type of question which is 24% of all questions. With these questions, the teachers maintain consistent agreement feedback to the students and check for the students and teachers to be united in comprehension of what is being stated in the conversation. Education groups support the agreement between participants during mathematical discussions (NTCM, 2000). Ilaria (2009) found confirmation between all participants was a key to moving the conversation forward while not leaving anyone behind. Since there were several participants in these dialogues, the teacher confirmation questions were observed both from the perspectives of the teachers providing feedback on their own thinking and of students' thinking.

In Classroom A, the confirmation questions along with the procedural questions were used 79% of the time. So most of class time, Amber used one of these two types. In examining the breakdown, there were 40 confirmation questions used 31% of the time. Within the 40 confirmation questions, Amber confirmed the student's thinking about 17% of the time; and sought student agreement 14% of the time. Thus, both confirmation types were about equally applied. Amber used these question types to keep the class together as they worked through the different problems.

In Classroom B the procedural and confirmation questions were used about 50% of the time. The confirmation questions, identified as used 21% of the time, were subdivided into the two categories: teacher agreement at 9% and agreement with students at 12% of the class time. During problem solving time, there were several one-to-one conversations between Brandi and a student as the students individually tried to solve the problems. As the students were working, she incorporated these questions to encourage students to advance in the lesson and in their thinking.

The third most common type of question was the *following* questions, or questions created upon the students' statement or action (see Table 5.1). There were 45 of this type of question or 11% of all questions. Mathematics education research validates teachers listening to students' ideas and then inquiring based on those ideas. In informal mathematics environments, Abdi (2009) determined that teachers asked following questions while leading problem solving sessions. When teachers are trying to build connections and encourage more input based on students thoughts, *following* questions are important factors during classroom discussions (Staples & Colonis, 2007).

The teachers in this study asked *following* questions with differing frequencies. Brandi used following questions 14% of the time during the two observed class periods with this type ranked as the second most frequently used question type. She had

students solve problems then she used these opportunities to view students' thinking in either written or verbal forms. She asked following questions to identify what students were mathematically doing. On the other hand, Amber only incorporated following questions 3% of the time, which ranked as one of the least frequently used in her classroom. The teacher was working and figuring out the problems along with the students. The disparity in the percentages may arise from either different learning approaches (meaning versus ability based) or may simply be an artifact of the different lesson formats. Further research would be needed to clarify these patterns.

The fourth most common type of question was the *clarification* and *explanation* questions that sought more information from the students. There were 39 of this type of question which is 9% of all questions. While interpreting the students' mathematical statements, explanations, and descriptions can be difficult for teachers, integrating clarifying questions can be a tool for the teachers in alleviating some of the frustration (Manouchehri, 2007). Dick and Springer (2006) suggest teachers ask "revoicing" questions for students to clarify their thinking. Whether it be clarifying previously voiced ideas or explaining non-verbal ideas, the students asked this type of question were given the chance to verbally express their thinking.

The percentages of the use of clarification questions and explanation questions by the teachers were similar in both settings. Amber asked clarification questions about 5% of the time whereas Brandi asked this question type about 6% of the time. These questions encouraged students to elucidate on their previous mathematical statement. In addition, Amber asked explanation questions 2% of the time and Brandi asked them 3% of the time. In both situations, students were working problems on their papers. The teacher was either viewing the students' work or hearing their stated symbolic answer.

As a result, there was little intentionality on the part of the teachers to have the student's verbalize their thinking during the mathematical process.

The fifth most common type of question was the *initiating* questions, or questions inviting other students who were not in conversation to share their thinking. There were 34 of this type of question which was 8% of all questions. Another method of including students in a discussion is by teachers inviting the students to share their thinking (Manouchehri & Enderson, 1999). In addition, studies have shown that teachers including students in the conversation positively affect mathematical thinking (White, 2003). These studies provide groundwork for the existence of this category of teacher questioning.

When comparing the frequency of *initiating* of questions in Classrooms A and B, there were more of these questions asked in Classroom B. Classroom A had 8 initiating questions, which was 6% of the total questions. Six of those questions were directed at the whole class to bring them into a conversation either at the beginning of the lesson or at a deviation point in the conversation. Amber may be including these questions to capture the focus of all participants by inviting them to answer and to follow her path. Classroom B had 26 initiating questions which was 9% of the total questions. Twenty of the questions were directed at an individual student throughout the class period. Brandi was directly inviting specific students to share their thoughts on particular ideas or solution processes. Although both Amber and Brandi incorporated the use of the initiating questions in different ways, both worked toward inclusion of the students in the class discussion.

The sixth most common type of question was *retracing* questions, or questions reviewing concepts previously discussed in earlier meetings. There were 22 of this type of question which was 5% of all questions. Pirie and Kieren (1994) introduce an idea of

folding back by reviewing past concepts in order for students to develop a deeper understanding of a mathematical concept. If teachers reconsider previous thought processes through questions, students' initial thinking can change, and their developing thoughts can form connections as a discussion progresses. The teachers using this question type have opportunities for individual students along with all participants to build a stronger foundation in achieving a thorough understanding of mathematical concepts and problems.

In comparing the frequency of *retracing* questions asked Classrooms A and B, they were essentially the same. Amber used these questions 4% of the time whereas Brandi used them 6% of the time. Both teachers took advantage of a few opportunities to bring in previous ideas when they were seeking to bring understanding to the present ideas.

The seventh most common type of question was *suggestion* questions, or questions asked by teachers to inject ideas into the conversation. As shown in Table 5.1, there were 20 of this type of question which is 5% of all questions. These types of questions can help lead students in a direction toward understanding, but at the same time, give them the opportunities to draw their own conclusions (Herbst, 2002). Furthermore, instead of the teacher focusing on the errors in students' statements, these questions can redirect the thinking in a course leading to understanding (Falle, 2003). Suggestion questions thus move class discourse in a positive direction as teachers help students in their mathematical thinking.

Suggestion questions were only used by Brandi. Usually, the students were expressing information or their thinking. In most of the examples, Brandi responded with a question that guided the students' thoughts in a particular direction. Students may have been struggling with responding to a question, and the teacher responded with a

question that focused their thinking down a particular path. The suggestion type of question may enable the teacher to help students by leading their thoughts in the desired direction.

The least frequently used questions in both settings were the *repeating* and *justification* questions (Table 5.1). When teachers asked students to repeat what they had previously said, *repeating* questions were demonstrated which appeared three times or 1% of the time in this study. The *justification* questions asking students to provide proof of thinking appeared only once in this study. Repeating questions were not in the literature, but they were often a part of verbal dialogue between several participants. If everyone is to be a part of the discussion, these utterances become bring an integral component in reaching that goal. Justification questions are an avenue to encourage students to pursue talking about their mathematical thinking (Van Zee & Minstrell, 1997).

When comparing these two types, *repeating* and *justification* questions in each setting, there was a minimal difference. With repeating questions, both settings used the questions less than one percent of the time. Because these are classroom settings, these question types may result from the large group discussion where students may not be speaking loud enough for others to hear or where the other students may not be actively listening to the speaker. In contrast, these question types take more class time, and perhaps were not frequently used for that reason. The justification questions were only identified once in Classroom B. Brandi was attempting to help students acknowledge that there were two possible thinking processes for the problem. The question presented was intended to ask the student to verbally compare the two problem solving methods.



## 5.2 Student Response Frequency

The frequency of student codes identifies the type of responses students were giving to teacher questions. During a mathematical conversation, the quantity of student responses addressed how many times students were communicating in a specific form. In addition, frequency of student response codes supplied evidence on teachers' ability to prompt and reveal the students' thinking. The total number of student responses from Classroom A was 91, and was 263 from Classroom B. The frequency and percentage of student response has been shown in Table 5.2.

Table 5.2 Student Response Frequency/Percentage

Student Responses	Classroom A		Classroom B		Total	
	Frequency	Percentage	Frequency	Percentage	Frequency	Percentage
Thinking Aloud	5	5%	27	10%	32	9%
Proof-Building	1	1%	6	2%	7	2%
Answer	60	66%	111	42%	171	48%
Clarification	3	3%	14	5%	17	5%
Questions Student	0	0%	7	3%	7	2%
Confirmation	1	1%	34	13%	35	10%
Seeking	21	23%	56	21%	77	22%
Not Contributing	0	0%	8	3%	8	2%

As shown in Table 5.2, in both Classrooms A and B, the frequency of teacher responses was larger than the students' responses. The ratio of teacher questions to student responses was 127 to 91 in Classroom A. This proportion reduces to approximately 7 student responses for every 10 teacher questions. The ratio was more evenly divided in Classroom B where the teacher questions to student responses were 297 to 263. Even though teacher responses were still predominant, the ratio simplifies to about 9 student responses for every 10 teacher questions. This ratio confirms the

characterization of both settings where the teachers were leading the class activities and discourse, with more exchanges in conversation occurring in the mastery oriented classroom.

The mathematical education literature on teacher questions advocates teachers asking questions that require students to share their thinking about mathematics (Van Zee & Minstrell, 1997; Falle, 2003; Cazden, 2001). Teachers can encourage students to share the reasonableness of a claim, offer justifications for their solutions, or engage with each other's justifications (Cengiz, Kline, & Grant, 2011). However, there is little research that actually connects the student responses with the teacher questions. Therefore this study's analysis seeks to describe the links that exist.

The most frequent student response was the *answer response* where students gave a short recall response containing a fact or piece of information (Table 5.2). There were 171 of this type which makes up 48% of all responses. Overall, this type of questions was the most common response by students, occurring either because students conformed to a learned habit of providing rote responses, or by answering the type of question asked.

The finding that *answer* responses were the most frequently observed type in both settings indicates students were often replying to questions with brief and explicit statements containing small amounts of information. In Classroom A these student responses encompassed 66% of the total. The answer responses along with seeking responses were used 90% of the time. In Classroom B these two question types total 42% of all responses. These types were not as prevalent in Classroom B as they were in Classroom A because there were three other types of responses in Classroom B that constituted 86% of the total: thinking aloud, confirmation, and seeking.

The second most frequent student response was the *seeking response* or responses where the student requested feedback from the teacher (Table 5.2). There were 77 of this type, which is 22% of all responses. In comparing Classrooms A and B these responses were given about 21% and 23% of the time, respectively. Since both classrooms had this response type as second in rates of use, the students expected the teachers to “impart” or directly give information during the dialogue. While Vygotsky’s perspective on learning views the teacher as the expert in the room, students will acknowledge this role and ask for guidance during struggles (Sfard, 2003). Therefore, students may not have had the mathematical understanding to extend thinking so they requested information specifically from the teacher.

The third most frequent student response was the *confirmation response* where student agreement is expressed. While 35 of the responses were confirmation in nature, this number is 10% of all responses (Table 5.2). In the constructivist model of learning, understanding of the initial ideas will generate more productive thinking when learners interact among themselves bringing about accomplishments and agreements, not just receiving knowledge (Confrey & Maloney, 2006). This response type represented a manner in which students could verbally signal concurrence with participants.

The students in Classroom B interjected with *confirmation* responses more often than those in Classroom A. They were observed responding 34 times with the confirmation response type as compared to only once in Classroom A. This higher frequency in Classroom B may be due to Brandi structuring class time for students to first work individually followed by a dialogue between her and the students. In doing so, the objective of seeking consensus between students resulted in bringing their thinking in line with all who had worked a particular problem.

The fourth most frequent student response was the *thinking aloud* response where students verbally revealed their thinking. There were a total of 32 thinking aloud responses where nine percent represented the comparison to all responses (see Table 5.2). These responses provided verbal evidence of the students' thinking for the teachers to observe. Classroom B supplied these responses 10% of the time as compared to 5% in Classroom A.

The fifth most frequent student response was the *clarification* responses. The responses enabled students to give more specifics to a previous response without justifying their thinking. There were 17 of this type of response which was about five percent of all responses. The students in Classrooms A and B employed this type of response in the discussion almost with almost the same ratios of 3% and 5%, respectively.

The thinking aloud and clarification responses elicited the deepest level of student thinking. While making students' active involvement and participation central to learning, constructivism encourages the teachers to focus on the students' strengths and resources they bring to the task (Confrey & Maloney, 2006). When teachers listen to the students' mathematical thinking by encouraging thinking aloud and clarification, they can assist students in traversing difficulties and progressing to new levels of thinking.

The least frequent of the student responses was comprised of three categories: *proof-building*, *questions student*, and *noncontributing* (Table 5.2). Proof-building responses allowed students to give reasons for their mathematical thinking. In the questions student response type, the students queried another student during the discussion yielding a response. Noncontributing responses denote that the student did not participate in the conversation. In these response categories, there were a total of 22 responses, which represent 6% of all the responses.

Classroom B contributed to all three of these question type categories. As the students solved and discussed the various review problems, they were given opportunities to respond with their mathematical ideas and give why they supported those ideas. Discourse between students was observed as they discussed their questions and ideas among their peers. As in any conversation, students had the choice of not engaging in the dialogue because of lack of motivation or lack of knowledge pertaining to the subject. All of these responses become a part of the classroom environment that encouraged mathematical thinking. Students' taking ownership of mathematical activities, justifying mathematical ideas, and collaborating in their work are important to establish when promoting conditions for fostering the students' mathematical reasoning (Francisco & Maher, 2005). This ownership was more evident in Classroom B through the exchange of the three response types as compared to Classroom A.

### 5.3 Teacher Question and Student Response Relationship

The frequencies of relationships between teacher questions and student answers in Classrooms A and B reveal how the teachers engaged students in mathematical discourse. The association between the teacher question and student responses was explored to illustrate the kinds of questions that elicited student thinking through dialogue. Classroom A and B are described separately because of the disparity between them in total number of student responses and in teaching approach. The number of student responses following a teacher question was clearly distinguished. In Classroom A there were 76 responses, whereas in Classroom B there were 187. Tables 5.3 and 5.5 presents the frequency in which a student response immediately followed a teacher question. Tables 5.4 and 5.6 shows the percentage of times a student response immediately followed a teacher question.

Table 5.3 Number of Times a Student Response Immediately Followed Amber's Question

Teacher Questions	Student Responses							
	Thinking Aloud	Proof-Building	Answer	Clarification	Questions Student	Confirmation	Seeking	Not Contributing
Confirmation-t	1	-	2	-	-	4	-	-
Confirmation-s	-	-	1	-	-	1	2	-
Suggestion	-	-	-	-	-	-	-	-
Following	1	-	2	-	-	1	-	-
Procedural	-	-	36	-	-	1	8	1
Initiating	1	-	3	-	-	-	1	-
Retracing	-	-	2	-	-	-	-	-
Repeat	-	-	-	1	-	-	-	-
Explanation	1	-	1	-	-	-	-	-
Clarification	-	1	4	-	-	-	-	-
Justification	-	-	-	-	-	-	-	-

Note: Dashes signal that student responses did not immediately follow teacher's question.

Table 5.4 Percentage of Times the Student Response Immediately Followed Amber's Questions

Teacher Questions	Student Responses							
	Thinking Aloud	Proof-Building	Answer	Clarification	Questions Student	Confirmation	Seeking	Not Contributing
Confirmation-t	14%	-	29%	-	-	57%	-	-
Confirmation-s	-	-	25%	-	-	25%	50%	-
Suggestion	-	-	-	-	-	-	-	-
Following	25%	-	50%	-	-	25%	-	-
Procedural	-	-	80%	-	-	1%	18%	1%
Initiating	20%	-	60%	-	-	-	20%	-
Retracing	-	-	100%	-	-	-	-	-
Repeat	-	-	-	100%	-	-	-	-
Explanation	50%	-	50%	-	-	-	-	-
Clarification	-	20%	80%	-	-	-	-	-
Justification	-	-	-	-	-	-	-	-

Note: Dashes signal that student responses did not immediately follow teacher's question

Table 5.5 Number of Times a Student Response Immediately Followed a Brandi's Question

Teacher Questions	Student Responses							
	Thinking Aloud	Proof-Building	Answer	Clarification	Questions Student	Confirmation	Seeking	Not Contributing
Confirmation-t	4	1	2	-	-	8	1	-
Confirmation-s	1	-	4	1	1	-	5	1
Suggestion	2	-	4	-	1	4	3	-
Following	-	-	11	1	-	4	4	-
Procedural	6	-	30	2	-	10	15	1
Initiating	-	-	18	-	-	1	-	4
Retracing	-	-	8	-	-	2	2	-
Repeat	-	-	-	2	-	-	-	-
Explanation	3	2	1	1	-	-	-	1
Clarification	1	-	10	2	-	2	-	-
Justification	-	-	-	-	-	-	-	-

Note: Dashes signal that student responses did not immediately follow teacher's question

Table 5.6 Percentage of Times the Student Response Immediately Followed Brandi's Question

Teacher Questions	Student Responses							
	Thinking Aloud	Proof-Building	Answer	Clarification	Questions Student	Confirmation	Seeking	Not Contributing
Confirmation-t	25%	6%	13%	-	-	50%	6%	-
Confirmation-s	8%	-	31%	8%	8%	-	38%	8%
Suggestion	14%	-	29%	-	7%	29%	21%	-
Following	-	-	55%	5%	-	20%	20%	-
Procedural	9%	-	47%	3%	-	16%	23%	2%
Initiating	-	-	78%	-	-	4%	-	17%
Retracing	-	-	67%	-	-	17%	17%	-
Repeat	-	-	-	100%	-	-	-	-
Explanation	38%	25%	13%	13%	-	-	-	13%
Clarification	7%	-	67%	13%	-	13%	-	-
Justification	-	-	-	-	-	-	-	-

Note: Dashes signal that student responses did not immediately follow teacher's question

By observing Tables 5.3 through 5.6 in both classrooms, the teacher questions resulted in more than one student response. With some questions, there was more than one common student response. In Classroom A only 24% of the cells were represented by student responses that immediately followed the teacher's questions. In addition, 80% of these types of student responses were found in seven of the cells. The variance in the type of response from a specific teacher question was minimal so predicting responses may be easier in most cases. In Classroom B, student responses that immediately followed a teacher's question complete only 48% of the cells, but 13 cells make 80% of these types of student responses. For most questions, there was a degree of variation in the response to each question.

Certain teacher questions and student responses lacked connections. For example, clarification questions should have elicited clarification responses but this pattern was not observed. The answer response was the most frequent response 67% of the time. In Classroom A, the answer response was elicited 80% of the time, and the clarification response was zero. Therefore, if the teacher asked a student to clarify a previous response, most students' responses about 70% of the time were short recall answers.

The most frequent responses to the *explanation questions* were the *thinking aloud*, *answer*, and *proof building* responses (Tables 5.3 through 5.6). In Classroom A, the thinking aloud and answers responses were equally dispersed, with two total. When Amber asked the explanation question twice, the response was an answer once and the other was thinking aloud. The students articulated their thoughts half the time. If she did receive the recall answer, she could respond with another question. Meanwhile, there were a total of eight explanation questions in Classroom B. The responses were more dispersed in Classroom B. The thinking aloud response was given 38% of the time and



followed by the thinking aloud response 25% of the time. The answer, clarification, and not contributing responses were least in number and occurred with equal frequency. When Brandi expressed an explanation question, the students responded with their thoughts a majority of the time. This type of question seemed to have a positive effect in eliciting student responses most of the time.

The most frequent response to *initiating questions* was *answer responses*. The responses were 60% and 78% of all responses, respectively in both classrooms (Tables 5.4 and 5.6). The questions might have encouraged responses from the students but further questioning may have encouraged sharing of their thinking. In Classroom A, thinking aloud and seeking were additional responses at 20% of all responses, so students were joining the conversation. However, Classroom B received 17% of all responses from students who did not voluntarily contribute. Therefore, there were those students who, even when invited, did not enter the dialogue.

The most frequent response to *retracing questions* was the *answer responses*. There were ten *retracing* questions in the classrooms, which occurred 70% of the time. When the teachers asked the students to recall a previous mathematical idea, the response was usually a short recall statement.

The most frequent response to the *confirmation questions* where the teacher was seeking confirmation from the students was the *confirmation response*. There were 12 responses which was over 50% of all responses of the confirmation question. This finding indicates that mutual understanding was reached by the student and teacher, because the response matched the intent of the question.

The most frequent response to confirmation questions where teachers indicate agreement was *seeking response* from students. If the students asked a seeking question after confirmation of their thinking, they may have needed assistance in their

thinking to continue in the discussion. At 41% of responses to this question type, the seeking question was asked seven times. In both classrooms, the answer response was second at 29% of responses to this question. After receiving confirmation from the teacher, the student may have included additional information to the discussion. Classroom B showed tolerance for more diverse student responses, including the data disclosing four other student responses with smaller percentages than the previously discussed responses.

The most frequent response to the *procedural questions* was the *answer responses*. There were a total of 66 answer responses encompassing 61% of all responses to procedural questions. The students were most likely to answer with a short recall statement in response to this type of teacher questions. Students in both classrooms also responded 23 times with seeking responses which was 21% of the responses to this type of question. When the students searched for more information from the teacher's previous procedural response, they were likely seeking additional help. As in the previous question category, Classroom B had a diversity of responses spread over six types of responses.

The most frequent responses to the *suggestion questions* were the *answer responses*, *confirmation responses*, and *seeking responses* at 29%, 29%, and 21% of all responses, respectively. During the observations Amber did not use a suggestion question. Consequently, all investigation of these response types took place in Classroom B. After Brandi posed the suggestion questions, the students would respond with a short recall answer, agree to her new direction, or seek more information from her.

The most frequent responses to the *following questions* were the *answer responses*. There were 13 answer responses which represented 54% of all responses. Most of the examples came from Brandi's questions. The first setting had four total

responses to following questions, whereas the second setting had a total of 20. The other responses with lesser frequencies were the *seeking*, *confirmation*, and *thinking aloud* responses. When the teacher asked a question pertaining to the students' statements or actions, a short recall answer might have been students adding one more piece of information to their prior action. In addition, the students may not have been able input more information to their thinking or were unsure of the next thinking process. The secondary responses convey students who needed more information, were unsure of their thinking, or needed time to process their thinking.

The last two categories, *repeat* and *justification questions*, were represented only a few times in the observations. The repeat questions only had three responses, and all of them were responded to by *clarification responses*. Basically, in all situations the teacher asked the students to repeat their previous statement. Finally, the *justification* question which was used once by the teacher in Classroom A was not observed as connected to any responses. Because students find it difficult to talk about mathematics, additional teacher justification questions would be needed help them draw conclusions about the nature of generated student responses. From the four observations, the teachers rarely used a justification question as a mode of eliciting student thinking.

#### 5.4 Teachers Questions and Student Responses

When students were communicating their thinking, the various student responses to each teacher question did not portray a thorough description. The number of total student responses was limited especially in Classroom A, with only 76 responses to teacher questions. Examining the data in Tables 5.3, 5.4, 5.5, and 5.6 reveals limited correspondence occurred between the teacher questions and student responses. By analyzing the various student responses, *thinking aloud* and *proof-building* were the options in which students could readily express their thinking. Proof-building responses

were only observed four times as shown in the Tables 5.3 – 5.6. When focusing on the numbers of responses, thinking-aloud responses were associated most frequently to the categories of procedural and confirmation teacher questions with six responses in each category. Yet, the explanation and confirmation questions were followed by thinking aloud responses 40% and 15% of the time, respectively. The procedural questions only yielded thinking aloud responses 6% of the time. With these discrepancies and limited examples of student responses following teacher questions, engaging students to express their thinking in mathematical discourse is a more complicated process that just asking specific types of questions.

When teachers encourage students to verbalize their thinking and move forward in their thinking, the analysis of student responses portrays a limited picture. In Tables 5.3, 5.4, 5.5, and 5.6, the justification questions did not have any student responses because these questions were limited in the observations. The initiating questions elicit *answer*, *thinking aloud*, and *seeking* responses in Classroom A and *answer*, *confirmation*, and *non-contributing* responses in Classroom B. The explanation questions only describe 2.5% of all questions represented on the table. Since there was such a small number of teacher explanation questions and justification questions asked, further analysis describes how teachers used questions to engage students into verbalizing their thinking.

### 5.5 Themes of Questioning

In order to develop questioning themes, each classroom was subdivided into smaller sections pertaining to the natural divisions of instruction taking place during the class period. The first set of divisions was characterized by the on-going activities whether problem solving or checking homework. Smaller divisions resulted from specific conversations or specific problems completed. Within the sub-division, four themes of

teacher questioning appeared with respect to the student responses. The first three themes, *connecting*, *involving*, and *exploring*, occurred in both classrooms. The last theme, *supporting*, was observed only in the Classroom B.

#### 5.5.1 *Connecting*

The *connecting* theme was defined as the teacher using questions to enable students to reconsider their previously studied concepts or ideas to have a better understanding of present ideas. When students were sharing and discussing ideas to mathematically problem solve, the teachers were guiding the process. Teachers made sure the paths for the mathematical ideas were valid and agreement between the participants was actualized. In order to build the mathematical ideas of the students, teachers took students back to previous concepts or ideas studied. They questioned students as to their understanding of the foundational ideas. In order to move forward in the learning, correct understanding and thinking is crucial for the students to proceed (Baxter and Williams, 2010).

#### 5.5.2 *Involving*

The *involving* theme is defined as the teacher using questions to include other members of the class to become participants in the classroom discourse. When students' thinking was verbally communicated during a class discussion, their reasoning became a focus of the information discussed. The teacher used this thinking to progress in two different ways. The teacher continued to question the student to foster more thinking, or the teacher could bring in additional voices into the dialogue. When the other students shared their thinking in the classroom discourse, the discussion could focus on the substance of and the agreement with all submitted student ideas.

### 5.5.3 Exploring

The *exploring* theme is defined as the teacher using questions to enable student thinking to be a part of the classroom discourse. When students are asked to discuss a mathematical idea or concepts, they do not just spontaneously start talking about mathematics in a productive manner (Rittenhouse, 1998). In exploring, the teachers assumed the responsibility of encouraging the students to communicate their thinking. In exploring, teachers' questions engaged the students to verbally communicate their thoughts.

### 5.5.4 Supporting

The *supporting* theme is defined as the teacher using questions to encourage conversation between students about mathematical thinking. Students were participating in student-to-student conversation when, without teacher urging, they introduced new ideas or questions into the discussion. The teacher accepted the role of determining if ideas and mathematical arguments were correct when students entered the conversation uninvited by the teacher. Many times students engaged each other in dialogue by questioning in each other. These actions of the students can produce students who think mathematically independently (Francisco & Maher, 2005).

## 5.6 Discussion of Questioning Themes

An important purpose of this study was to describe the extent and nature, and identify possible patterns in the types of teacher questions that elicit student engagement in mathematical reasoning. To achieve this purpose the conversations in each classroom were subdivided into segments of separate conversations or exchanges. The questioning themes were developed from each of these exchanges. The themes centered on how each teacher in the exchange drew out the students' reasoning and engaged them in mathematical dialogue. In Classroom A, exchanges from the

homework review, the lesson practice, and the completing homework were chosen for analysis. Amber was completing problems either from review or new concepts. In Classroom B, discourse exchanges were determined to occur from solving word problems and working on an investigation. When analyzing the discourse exchanges, multiple examples were labeled.

### 5.6.1 Classroom A

This exchange took place at the beginning of the class period. The 16 homework answers were written on the front white board. They were the solutions to quadratic equations solved using the quadratic formula. Amber inquired about questions on the students' homework. Student 2 asked the teacher to complete the steps to solve the following equation,  $9x^2 - 11 = 6x$ .

#### Exploring Theme

9:55	Student 2	S-seek	Can you do number 13?
9:58	Teacher	T-e	Okay let's do number 13. What did you not get right on it?
10:02	Student 2	S-ta	I think I just didn't factor out as far as it could go.
10:05	Teacher	T-p	Okay on this one (teacher writing on the board) What do you always have to have before you start using the quadratic formula?
10:07	Teacher	T-p	What is the first step?
10:10	Student 2	S-ans	Equals zero
10:12	Teacher	T-cont	Yah, we have to have zero on one side even when you factor right? So we're going to subtract 6. (Pause
10:20	Teacher	T-cont	So you've had the quadratic formula before. So

mark what a, b, and c are. But this is a formula you should already have memorized but if you do not, it will need to be done by tomorrow. I am not going to give it to you for your quiz. You can kind of hum Mrs.(previous teacher) song quietly. (writes on the board) So let do that would be a negative b pulse or minus ... $b^2$  ...  $4ac$ .. all over  $2a$ . Is that what you did so far?

11:05 Teacher T-cont So 6 ... you are going to have to ... you got 2 negatives makes a plus so 36 plus (writing on the board) Now you have to take 36 times 11 Get that right? You got 36 times 11(on the calculator) You got 396 over 18.

11:54 Student 2 S-ta I put in a wrong number. I accidentally put in a for b.

Amber responded to the student's inquiry at the beginning with a question. She asked his perception of the actual error in his work. The student responded and concluded that his simplification process may be the issue. When she finished listening to his idea, she decided to start at the beginning of the problem. As the dialogue continued, she actually questioned him about the first step. He established what form the problem should be in so that values could be substituted into the quadratic formula. While he was analyzing his work, she proceeded to complete the steps of substitution. During this process, she asked him for agreement along the way. He eventually explained his mistake.

The dialogue from the first exchange continued. Amber shifted to question the whole class. Student 2 ascertained the inaccuracy in the homework problem and left the



conversation. At this point, the problem took on the form,  $x = \frac{6 \pm \sqrt{396}}{18}$ . On the white board, Mary proceeded to simplify the fraction by reducing the radical.

*Involving Theme*

11:56	Teacher	T-r	Okay so 396 plus 36.(writing on the board) So now this kind of. We don't ever. We learned this last year... if there are any perfect squares, how does one determine if there is a perfect square?(pause) You need to know the factors, right?
12:12	Teacher	T-p	So one way is just to come off to the side and say 2 times what. Did anyone do their list or am I going to have to do it? (no response)
12:25	Teacher	T-p	So you take 432 divided by 2.(teacher using calculator)
12:29	Student 3	S-ans	Just divide by 4 and then
12:32	Teacher	T-p	So you just divided by 4 and what did you get?
12:34	Student 3	S-ans	432 divided 4 and then 3
12:36	Teacher	T-p	So 432 divided by 4 is... (typing on calculator)
12:41	Student 4	S-ans	it is 108
12:42	Teacher	T-cons	Alright
12:44	Student 4	S-ans	It is 144 times 3
13:00	Teacher	T-c	It is 144 times 3?
13:03	Teacher	T-cons	That is why I usually do not jump to 4 since she

found a bigger one than with 4, right? You may have done it in steps right?(Addressing Student 3)

- 13:13 Student 3 S-c I thought you were just factoring like
- 13:15 Teacher T-p But what we're looking for is a perfect square. And she found this one and this would be three times 144 all over 18.
- 13:20 Teacher T-p So you can pull out from underneath the square root . The square root of 144 which is 12. A mistake that I see quite often is we pull the wrong one out. Make sure you're pulling out the square root of 144. Anybody know what else we can do?
- 13:46 Student 5 S-ans Reduce
- 13:47 Teacher T-c How? How do I reduce?
- 13:49 Student 5 S-c The 12 and the 18.
- 13:50 Teacher T-cont Yah, but you have to make sure isn't one along the way. (Writing on the board) Every one of those needs to be reduced by. I can divide 6 into each one of those right?
- 14:11 Teacher T-cont So you get I plus or minus 2 square root of 3 all over 3 is that what I wrote down for 13?

Amber asked a review question about how to find perfect square factors of a number and tried to include the class in the discussion. A couple of students responded during the dialogue as to which numbers they would try. The issue on "how to find the largest

perfect square factor of a number” became the main focal point of the discussion during times 12:12 to 13:15. Amber and one student started by dividing 432 by small factors and continued dividing by progressively larger ones. She asked the students what 432 divided by two equals. A second student entered the conversation and interjected the idea of dividing by the perfect square of four. Amber followed the idea with questions focusing on that factor. As the dialogue proceeded, the student realized that dividing by three results in a larger perfect square as its quotient. Amber included utterances that confirm the students’ responses achieving forward progress in the conversation. Finally, the participants came to the conclusion that dividing by the smaller factor results in quickly finding the largest perfect square.

When the radical had been reduced, Amber invited more students to be involved in the discussion. She used a procedural question to discover the student’s idea as to the next step in simplifying the fraction. A student responded with a one-word answer then she asked for clarification on the idea. The student replied with brief response which is straight to the point. In order to confirm the student understanding of simplifying the binomial in the numerator, the teacher’s questions elaborated on the student’s answer and asked for student confirmation.

In this next exchange, Amber had just finished solving two quadratic equations using completing the square on the white board. The students followed Amber and solved the problems simultaneously on their papers. Amber proceeded to choose  $n^2 = -93 - 2n$  to solve.

*Connecting Theme*

23:56	Teacher	T-p	Okay I need a zero. I like to start with the first one positive. (Writing on the board) So I am going to move $n^2$ over. The $2n$ over and the 93
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over. Okay, well I should have left 93 over there. Actually, I just wanted to move the  $2n$  over. So then I have step one done. I had the 93 on the right hand side. Now you take  $\frac{1}{2}$  of 2, 1 and square it so you add 1. So what goes in the parentheses?

- 24:28 Student 5 S-ans  $n + 1$
- 24:29 Teacher T-cons So  $n+1$  correct.
- 24:30 Teacher T-cont And we get  $- 92$ .
- 24:32 Teachers T-r Okay we have a negative. In algebra II and algebra I, it was panic time. No more right?
- 24:39 Teacher T-p We're going to get a what in our answer?
- 24:46 Student 6 S-ans  $i$
- 24:47 Teacher T-cons  $i$  so lets go and
- 24:48 Teacher T-p it doesn't matter which step you move that  $i$ .  
square root of 92, are there any perfect squares in 92?
- 24:54 Student 6 S-ans 4
- 24:55 Teacher T-c 4, it's 4 times what?
- 24:57 Students S-ans 23
- 24:59 Teacher T-cons Okay, 4 times 23 (writing on the board). So we have, if I take the square root of 4 out. You'll do plus or minus  $2i$  square root of 23. So the last one you move the 1 over so  $-1$  plus or minus  $2i$  square root of 23.

Amber proceeded to work through the completing the square algorithm. She asked some procedural questions along the way pertaining to specific steps. At 24:30, Amber, knowing that the answer to the problem was an imaginary solution, includes a review question. She asked the students to remember how radicals with imaginary numbers are simplified. She may have been trying to show how their knowledge of imaginary numbers applied to the situation.

The following exchange took place near the end of the class period. The students were working homework problems solving quadratic equations by completing the square. A couple of students were completing this algorithm with a group. At that moment, they were unable to move forward through the problem,  $9\left(x^2 + \frac{20}{9}x\right) = 14$ .

*Supporting Theme*

43:57	Student 11	S-seek	What if you have a fraction as b?
43:59	Teacher	T-cons	That is fine, it is 20/9.
44:02	Teacher	T-p	So that ½ of it. What is ½ of 20/9?
44:10	Student 11	S-nc	I don't know I am really bad at fractions.
44:15	Student 12	S-nc	That's what the calculator is for.
44:32	Teacher	T-p	What is ½ of 20/9?
44:45	Student 5	S-ans	10/9 ...no...
44:48	Student 11	S-ans	20/18. This is hard.
44:51	Teacher	T-cons	This worksheet is not an easy sheet.

Bell rings

One of the students asked Amber how to take half of a fraction. After being included in the on-going dialogue, Amber validated that the initial fraction was correct. She asked the students verbally how to symbolically complete the arithmetic. Quickly, two students responded with non-contributing answers, not being comfortable with fractions and not

wanting to do it without a calculator. Eventually, one of the students did answer the question after Amber repeated her question for a third time. As the students wrote out answers on their papers, they reflected verbally on their computed answers. Finally, while Amber agreed with the problems' difficulty, the final bell rang. Even in this example when the group's negative dialogue jeopardized their advancement, Amber's questions helped support the eventual progress.

The following exchanges took place during a different class period when the participants were using synthetic division to factor polynomials and solve polynomial equations. In this exchange, Amber worked through a problem with the class. The algorithm utilized the remainder theorem to find a polynomial function value of a specific independent value. Then she and the class tackled a problem in which synthetic division was applied to identify factors of a polynomial.

*Connecting Theme*

13:40	Teacher	T-r	Now we are going to do the first ones. We are going to evaluate it using the remainder theorem. Which says, the very last column says the answer. Alright now then how do I know if 2 is a factor of 6? (pause)
13:55	Teacher	T-r	is 2 a factor of 6?
13:58	Students	S-ans	Yes
14:00	Teacher	T-c	How do you know? (pause)
14:12	Teacher	T-r	Is 2 a factor of 7?
14:14	Students	S-ans	No
14:15	Teacher	T-cons	No
14:16	Teacher	T-p	If 2 goes into 6 what is the remainder?

14:17	Student 7	S-ans	Zero
14:18	Teacher	T-cons	Zero
14:19	Teacher	T-p	2 does not divide into 7 and there is a remainder of?
14:21	Student 7	S-ans	one
14:29	Teacher	T-p	So if it divides evenly, So when you divide and you get a zero right here (pointing to the board) , you have found a factor. So these next ones and we won't do all of them. We'll do enough that you got it from 7 to 26.

Amber reviewed the definition of a factor with the students by asking recall questions. In order to aid the students' understanding as to why there has to be a zero in the last row and column of the synthetic division structure, she inquired about the students' previous understanding of factors of natural numbers. She tried connecting the division of natural numbers with their factors and the division of polynomials with their factors. The similarities with respect to remainders could have helped with the student's current thinking.

The next exchange was divided into two parts. Amber continued to work another example where a value was identified as not being a factor of the polynomial. A student asked her about an idea he had. She acknowledged his thinking, but she did not seek to follow it to a resolution until later in the class period. The present example read, "Is  $b - 7$  a factor of the polynomial  $b^4 - 8b^3 - b^2 + 62b - 37$ ?"

#### *Exploring Theme*

19:40	Student 7	S-seek	So if the last number is a prime can we just say it is not a factor?
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19:45	Teacher	T-c	You're trying to skip the process here?
19:50	Student 7	S-pb	You know that that will multiply by this and not get a zero when subtracted.
19:58	Teacher	T-f	If you're saying that if this is always a prime...I don't know. I've never thought about it. (pause) I don't what to make that statement, yet. What we'll do tomorrow, you'll have all your homework. We'll look and see if that is a true statement. That is a good thought. We'll have all those examples and I will do all the evens. And we'll see if you can draw that conclusion. That is a good observation. Did yall here what he said? He said if the last number is prime, can we always assume that it is not? Well, we have two that are not prime numbers and they are not.
20:50	Student 7	S-ta	Well I'm saying if it is prime, it will not be.
20:52	Teacher	T-f	I still don't want to say yes because I haven't thought of it that way. Okay we are doing our investigations here to see if we can do it that way.

*Involving Theme*

28:19	Teacher	T-f	Ok, let's see his theory then. His theory was it was prime it would never divide.
28:38	Student 7	S-c	By whole numbers not fractions



28:36	Student 1	S-ta	The first one didn't work, it was a 2
28:38	Teacher	T-cons	Yes, the first one ended in a 2 which is prime but it factored.
28:40	Teacher	T-p	That falsified your theory. It was a good thought
28:48	Teacher	T-cont	See, one example can disprove it? One example can't prove it but one example can disprove a thought. And I do that all the time, when yall will ask me hey is this the same if I have (writing on the board)

In the first part of the scene from 19:40 to 20:52, Amber finished working the problem on the board. After hearing the student's idea which identifies a possible pattern (i.e. the constant term of the polynomial being a prime number) alleviating the time-consuming algorithm, she asks him to clarify his intentions for its use. He subsequently verbalizes his thinking pertaining to the pattern and why the algorithm can be bypassed. Amber follows his thinking by questioning using deductive reasoning. The transpiring dialogue allows both participants to explore the student's thinking.

In the second part of the exchange at 28:19, the students were individually working on their homework. As Amber walks around the room giving students feedback, she involves all the students in the earlier conversation about the prime constant. She asks them to verbalize their observations of the student's idea at 19:58 and 28:19. One of the students is able to find a counterexample to the idea from the homework therefore the idea is negated. Then Amber brings closure to the conversation by giving confirmation of the student's thinking and seeking confirmation of the students' understanding her thoughts.

### 5.6.2 Classroom B

The class period had just begun, and the students were individually simplifying trigonometric expressions and solving trigonometric equations from five review problems. Several discussions between different students and Brandi defined the first few scenes of the class period. Brandi had dialogued with a student at this point about his decisions on how to simplify the expression,  $\frac{1+\tan^2x}{1+\cot^2x}$ .

#### Connecting Theme

04:04	Teacher	T-i	(moves on) You stopped?
04:14	Student 5	S-ans	No
04:16	Teacher	T-f	Your answer?
04:17	Teacher	T-e	Oh, oh, oh, you squared it because you felt like it?
04:21	Teacher	T-r	Okay, if I square that, let me go back to algebra a minute(writes on the board), if I square that, what will I end up with?
04:38	Student 5		(writes)
04:46	Teacher	T-cont	So I can't just randomly start squaring things cause I want to, Notecard 67 says that $\sec^2x$ equals (writes $1 + \tan^2x$ ) Right?
05:00	Student 5	S-con	Yes (writing on paper)
05:04	Teacher	T-cont	I threw you a bone on that didn't I?
05:10	Teacher	T-cons	I think this looks pretty good. (pause)

Brandi started by using questions to explore what the student was thinking when he simplified the fraction. She noticed that he squared the fraction and proceeded to inquire why he made that choice. When he did not answer, she asked him to review the process

of squaring a binomial and mentioned there are appropriate times to square values. After being reminded of the distribution process in squaring, he reexamines the simplification process. He recognized a different pathway of action and began to simplify the fraction again. Throughout the exchange he responds verbally only once; he communicated with Brandi through his written work. She acknowledged his thinking in the immediate dialogue by affirming his work.

This next exchange begins as the previous one ended. Brandi moves to the next student's desk and examines her paper. She is trying to simplify the trigonometric

fraction,  $\frac{\tan x + \sin x}{2 \tan x}$ .

*Exploring Theme*

05:20	Teacher	T-r	Okay, if you don't know what to do what did I tell you to do if you are clueless?
05:35	Student 6	S-ans	put it in terms of sines and cosines
05:39	Teacher	T-s	I think that might work better for you than doing that squaring thing though I could be wrong.
05:42	Teacher	T-f	Where are you going from here?
05:44	Student 6		(Writing)
05:46	Teacher	T-f	You're going to get "tan <sup>2</sup> x +" what in the middle?
05:50	Teacher	T-p	Your terms don't cancel.
05:55	Student 6	S-ta	plus 2(as he is writing)
05:57	Teacher	T-cons	but I like how you are thinking.
06:04	Teacher	T-s	I just now thought of something. (writes on the

board) I am just thinking ... If I am going along with your thinking ways to cancel, why don't I just multiply by this...

- 06:23 Teacher T-cont and if I do it on top and I have to do it on bottom, are you with me?
- 06:30 Student 6 S-con Yes
- 06:31 Teacher T-s (writing on board) then I'll get ...which equals something like this... Notecard 67... You still have to multiply this.. I don't know I just went out on a limb for a moment.

Brandi again was exploring the options of simplification with the next student. She recognized he was trying to square the fraction and used a question to remind him of what to revert to if all else fails. Additionally, through her questioning, she identified the errors. Her final strategy of exploration with him was to suggest multiplying the numerator and denominator by the conjugate of the numerator. They both followed the suggestion with her working on the board and him on his paper. She left the conversation by encouraging him to continue simplifying what had resulted.

As Brandi shifted to the next group of students, the next exchange began. The group of two students was working on simplifying the same fraction as in the previous exchange.

*Involving Theme*

- 08:24 Student 5 S-seek Will this not cross out?
- 08:26 Teacher T-s if I do cross it out, I will be back to what I started with which is this (pointing)
- 08:28 Student 5 S-con Oh, yah.

08:30	Teacher	T-s	Okay, I am going to foil the top and foil the bottom and see if I get anywhere or did I just create a nightmare?
08:37	Student 5	S-ans	Okay
08:50	Student 9	S-seek	Can I do that?(to teacher) Can I put those in there?
08:52	Teacher	T-f	What you can do is, you can factor out what each one of these has in common(pointing to student's paper)
08:54	Teacher	T-cont	like you can take that word out but you can't just put your parentheses just randomly there. No.
09:08	Teacher	T-i	(moves on) Did you and another student's name get number 1?
09:10	Student 10	S-nc	Nope
09:12	Student 11	S-c	I got number 1. But I just made up my own thing.
09:20	Teacher	T-c	What happened to the 2?
09:25	Teacher	T-f	If I take something that is in the denominator and I move it to the top, how did you get it to the numerator? How did I get it up to the top?
09:36	Student 12	S-con	Ahhh
09:40	Teacher	T-p	by the reciprocal
09:46	Student 12	S-ta	changing it to cot x(writing)
10:05	Teacher	T-f	Here's what I hear you saying, (writing on the paper) 2 and the reciprocal of this is cosecant but the 2 is still on top

10:14	Student 13	S-con	Ahh
10:15	Teacher	T-cont	Are you with me?
10:16	Student 13	S-con	Yes
10:17	Teacher	T-f	You flipped it is now on the bottom (pointing to student's paper)
10:18	Teacher	T-cont	But I like how you are thinking leaving it in the cotangent cause I think that it made it easier... I just saw something different

Brandi started exploring the simplification process with two students by following their work and suggesting outcomes with their choices. While the discussion proceeded, she invited more students to join the discussion at 9:08 by inquiring about their thinking with respect to simplifying this fraction. One student chose not to join the discussion since he did not have an answer. Another student responded more confidently and declared she had the answer, but she applied her own approach. Brandi then followed the student's work and questioned some of her steps. A couple more students joined in the dialogue from 9:25 to 10:16. In the ensuing discussion, the group discovered an additional method of simplification for the fraction.

As class time continued, Brandi explored different students' thinking and involved students in the dialogue. After about 15 minutes, she began a whole class review of the five problems. She had one student share how she had simplified the first problem,  $\frac{\tan x + \sin x}{2 \tan x}$ . Brandi was about to call on another student to contribute his answer to problem two when a different student uttered an objection to the simplification process for number one. So the exchange had begun.

*Connecting Theme*

18:48	Student 4	S-ta	You have to multiply all that out though
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18:52	Teacher	T-p	Yes I know. Hold on to your britches. I am not even touching the denominator yet.
18:55	Student 4	S-c	Okay, when you multiply sine by cosine You didn't multiply the sine over cosine
19:09	Teacher	T-p	No,No, all I did was find a common denominator for sine
19:16	Student 4	S-c	Okay, you need to do that for the other one too
19:18	Students	S-ta	No you do not, it already has the denominator
19:19	Teacher	T-p	No
19:20	Student 1	S-con	No you don't cause it already has the denominator.
19:22	Teacher	T-r	Student's name(student 4) I am taking a commercial break for you. If I trying to add $2/3$ plus 3, oh you say it is 3 and $2/3$ . (Laughter from class) Okay but you had to get a common denominator. Okay I am with you. If you had to get a common denominator, what would it be?
19:39	Student 4	S-ans	3
19:41	Teacher	T-p	So this is 9 over 3. And I didn't have to change that one. You closed your eyes like you do not believe me. Okay Mrs.(Margaret) do your magic.
19:45	Student 4	S-nc	Never mind. Ignore me.
19:49	Teacher	T-p	Okay, I won't ignore you but I will pretend that I am. Okay so my common denominator is $\cos x$ . What do I have even if I'm not touching

this 2 tanx yet? (students murmuring)

19:54 Teacher T-p What do I have left on the top?

The student was struggling with understanding the sum of a mixed number and a fraction,  $\frac{\sin x}{\cos x} + \sin x$ . Brandi had projected on the board its sum after a common denominator was found. The student entered the discussion stating what he thought should be done. Brandi replied by explaining what she was writing. He protested and continued to verbalize his idea. Another student interjected and supported Brandi's idea. Then Brandi reviewed the algorithm obtaining a common denominator with rational numbers and making the connection with the trigonometric fractions. The student's frustration grew, and he stopped engaging in the problem for the moment.

The next exchange began where the previous one ended. Brandi invited non-participating student to enter the dialogue by asking him the answer to a sum of trigonometric fractions,  $\frac{\sin x}{\cos x} + \frac{\sin x}{\cos x}$ .

#### *Involving Theme*

20:02 Teacher T-p I got number 22 and Student 3 that's you. Do you have any idea what I have when I add these two fractions together?

20:15 Student 3 S-ans  $\sin 2x / \cos x$

20:20 Teacher T-c  $\sin 2x / \cos x$  so  $\sin x$  plus  $\sin x$  is?

20:44 Student 4 S-ans  $2\sin x$

20:45 Teacher T-cons There you go  $2\sin x$ . I need to 2 in front.  $2\sin x$   
ahh  $\cos x$  (writes, reducing the resulting fraction)

#### *Supporting Theme*

20:50 Student 6 S-seeK How did you?

20:51 Student 2 S-ans You just crossed those out.



20:52 Student 2 S-see How did you?  
 20:53 Student 6 S-con Ohh  
 21:06 Teacher T-cont No, No, I say you factor out a sinx with tanx is  
 sinx over cosx Ahh there we go?  
 21:08 Student 2 S-con Yeah(as teacher reduces fraction)

A student responded incorrectly to Brandi's question. He added the two "like" trigonometric terms and placed the 2 in the wrong place. After she restated the algebraic sum, she asked him to clarify his answer. Another student entered the dialogue and contributed the correct answer. When the participants in the dialogue came to agreement, a different student entered the discussion seeking understanding on the reducing of a fraction,  $\frac{\frac{\sin x + \sin x \cos x}{\cos x}}{\frac{2 \sin x}{\cos x}}$ , from problem one. He and another student were analyzing that problem. Brandi demonstrated the process from the projector and allowed the students to continue their thinking. A verbal confirmation from one of the students was voiced. This exchange exhibited how Brandi orchestrated bringing participants into the discussion and allowing students who were having their own private discussion to enter the main discussion.

After the numerous utterances from various people in the previous scene, Brandi noticed the student who had some trouble understanding previous thinking was not following the discussion. She started discussing the process of simplifying the following

fraction,  $\frac{\frac{\sin x + \sin x \cos x}{\cos x}}{\frac{2 \sin x}{\cos x}}$ .

*Exploring Theme*

21:10 Teacher T-i Oh no, I totally lost Student 4? Student4  
 21:15 Student 4 S-ans Yes  
 21:16 Teacher T-cont You're okay with sine over cosine plus sinxcosx

21:25	Student 4	S-ta	Other than the fact that you have now separated something which you couldn't separate before because you added
21:30	Teacher	T-p	Wait, Wait, I didn't separate something
21:32	Student 4	S-ta	You multiplied it but
21:35	Teacher	T-cont	I added these two pieces and making fractions. Okay, Student 4, Student 4
21:44	Student 4	S-ta	You couldn't do that
21:45	Teacher	T-c	Wait Time out. When did I say you could not do that?

*Supporting Theme*

21:49	Student 4	S-pb	You said when you are adding or subtracting them, you cannot multiply or cancel them.
21:51	Student 13	S-pb	She is just trying to get the common denominator. She is multiplying the numerator by $\sin$ and the denominator by $\sin$ which equals 1.
22:02	Teacher	T-cons	Thank you Student 13 for saying it differently. And going off dimly for Student 4. So now, just write it like this $\sin x$ plus $\sin x \cos x$ all over $\cos x$ .

Sensing the frustration of the student, Brandi invited him into the discussion. During the next two minutes, she explored his thinking and his perceptions of the class' discussion obtaining a common denominator with the trigonometric terms. While she asked questions seeking agreement and clarification from him, he verbally thought aloud and tried to build a proof for his ideas. Finally, a different student entered the dialogue and

built a verbal argument explaining the algorithm in alternative words. The exchange then closed as Brandi confirmed agreement with the student's final statement and affirmed the frustrated his pursuit for reaching understanding.

In the next exchange, the class discussion was in the final step of simplifying problem one,  $\frac{\tan x + \sin x}{2 \tan x}$ . The final simplified fraction contained a two in the denominator. Several students did not have a two in their fraction, or they had a two in the numerator. Brandi asked a specific student to come into the conversation and share his thinking.

*Exploring Theme*

23:45	Teacher	T-i	Did you get it?(student 3)
23:50	Teacher	T-f	Where did the 2 go?
24:03	Teacher	T-p	That is what I keep asking yall. The 2 disappears automatically. It shouldn't go anywhere. The 2 doesn't cancel. The sine does.
24:05	Student 3	S-seek	Shouldn't the two cancel with the sine?
24:09	Teacher	T-p	The 2 doesn't cancel the sine does.
24:12	Student 3	S-c	I thought you canceled so there is just one sine left? So the 2 goes away.
24:14	Student 10	S-ta	it is just for sine squared, right?
24:15	Teacher	T-cons	Yes

After the initial involving question, Brandi notices the two missing in the student's work. She asks the student where the missing number went. The student dialogues with her as to the algorithm which eliminated the two. When a second student recognizes the error in the thinking, she interjects the correct algebraic interpretation. Bringing consensus to the immediate dialogue, Brandi agrees with the second student's analysis.

In the final highlighted exchange of the class period, the class had finished discussing the review problems. A couple of students had a question on a homework problem dealing with solving a trigonometric equation. So before they handed in the assignment, Brandi discussed the correct algorithm to incorporate. She posed using the substitution of the variable  $y$  in order to simplify the process of solving  $2\sin^2x + 3\sinx - 2 = 0$ .

*Supporting Theme*

35:08	Teacher	T-s	and $3\sinx$ , no no, drop that sine, $y$ minus 2
35:12	Student 4	S-qs	why did you do that?
35:18	Teacher	T-p	I let $y = \sinx$ . What does that factor into?
35:28	Student 6	S-pb	We can factor it easier.
35:33	Students	S-ans	$(2y - 1)(y+2)$
35:42	Student 7	S-ta	Wait, that is the same, no, yes it is.
35:50	Teacher	T-cont	So $1/2$ and $-2$ ; but those are not the final answers, Right?
35:58	Students	S-ans	No
36:02	Teacher	T-p	That's the sine equaling $1/2$ and the sine equaling $-2$ ; well, that is easy, sine will never be less than $-1$ so sine cannot be $-2$ ; so where is sine positive?
36:12	Student 4	S-ans	$\pi/6$
36:20	Teacher	T-p	What finger do you have to pull back on the hand trick?
36:26	Students	S-ans	second
36:40	Teacher	T-p	What is $\pi/6$ called in the second quadrant?

36:52 Student 5 S-ans  $5\pi/6$

When Brandi works through the steps of the algorithm, the class indicates each sequential step. Concurrently, a student questions as to the purpose of the steps using substitution. Another student responds to the question by explaining why it was incorporated. During this exchange, Brandi directs the flow of the discussion of the featured problem but allows students to dialogue about questioning in their thinking.

In the next exchange, a second class period had begun. The class was working on five review word problems on exponential growth. Each individual student was working on the problems although a couple of students were exchanging ideas among themselves. Brandi was moving around the room and conversing with them about their thinking. The students were substituting values from the problem into a formula and algebraically solving for the unknown value. The problem read, "Eighteen million dollars is invested for eight years and compounded annually. It yields 25 million dollars at the end of the eight years. What percentage rate was applied to this investment?"

*Involving and Connecting Themes*

04:15	Teacher	T-e	Compounded annually means?
04:25	Student 2	S-ans	Compounded annually means once a year
04:30	Student 6	S-ta	Oh so it would be one.
05:21	Teacher	T-e	I can just raise that to the "1/8" ?
05:40	Teacher	T-cont	Are you with me?
05:41	Student 3	S-con	Oh!
05:42	Teacher	T-r	I have an exponent on the variable. So how do I get rid of?
06:03	Teacher	T-f	(looking at student's paper) So you are trying to bring the exponent down? Right?

06:05	Student 5	S-con	nod
06:10	Teacher	T-s	So undo that
06:12	Student 4	S-con	Ohh, Ohh!
06:15	Teacher	T-cons	So the light bulb goes on!
06:18	Student 4	S-ta	So you can just raise it to the "1/8" power.
06:20	Teacher	T-cons	Nice, nice
06:21	Teacher	T-cont	Okay so you logging it which takes the exponent and brings it down, right?
06:45	Teacher	T-p	That's why we log something cause we have a variable in the exponent position. Do you have a variable in the exponent position?
07:00	Student 11	S-ans	No
07:05	Teacher	T-r	No so what do you do to undo that?

Brandi utilizes the group of students working together to question different students to involve them in the discussion. She also asks different students questions to strengthen their thinking with respect to different steps in the solving process. The posed questions focus on word definitions and algebraic solving processes. When students do not respond to questions, she reviews the algebra involved in solving for a variable base raised to a power and for a variable in the exponent position through her questions. She compares the two processes so the distinctions and connections could bring clarity to their thinking.

The next exchange had Brandi directing a review with the whole class over the five problems. She called on one student to read his formula with the substituted values. The problem read, "How many years will it take \$8,000 to be worth \$16,000 if the money is compounded monthly with an annual percentage rate of 9%?"

## Exploring Themes

12:20	Teacher	T-p	So I got #20 so (student's name) which he is not here. So #20 again so it likes (student's name) today. So #4 which should be (another student name). Set it up for me.
12:45	Student 8	S-ans	$16,000 = 8,000 \left(1 + \frac{.09}{12}\right)^{12t}$ (formula with substituted values)
12:53	Teacher	T-r	So 9% divided by 12 cause its per month.
12:58	Student 8	S-ans	(finishes reading)
13:02	Teacher	T-r	12P then you did your log magic. Did you have to log both sides?
13:03	Student 8	S-ans	Yea
13:04	Teacher	T-e	Why?
13:05	Student 8	S-c	Cause you had to bring the 12P down.
13:10	Teacher	T-f	Cause you got a variable in the exponent which take log ... which is what you did, what did you end up with anyway?
13:20	Student 8	S-ans	7.73
13:22	Teacher	T-cons	Years! Awesome, you get to be first person to try Christmas candy.(throws candy to student)

After writing the formula to be projected for the class to view, Brandi solves for the unknown variable. As she is solving, she asks a student to respond to the specific procedure used to move the variable from the exponent position. After he agrees with her step, she explores why he chose that particular action by asking him to verbalize his thinking.

The next exchange had the whole class reviewing the word problems with Brandi. They had completed their discussion over problem one and moved on to the next one. A student read her formula with the substituted values to the class. Brandi projected it so all participants can see the process. This particular problem was observed in an earlier exchange where the students were working in a small group. The problem read, "A company is investing 18 million dollars and hopes to have 25 million dollars in eight years. What should be the percentage rate if the money is compounded annually?"

*Exploring Theme*

13:33	Teacher	T-p	Okay, next one-- #9 is (student's name). Can you set it up for us?
13:43	Student 9	S-ans	$25,000,000 = 18,000.000(1 + r)^8$
13:46	Teacher	T-cont	Okay I am going to write 25 since everything is in millions, right?
13:48	Student 9	S-con	Yes
13:49	Teacher	T-c	So...
13:50	Student 9	S-ta	well
14:00	Teacher	T-s	So did you log both sides?
14:05	Student 9	S-ans	No
14:07	Teacher	T-e	Why not?
14:10	Student 9	S-pb	There's no variable in the exponent
14:12	Teacher	T-cons	There's no variable in the exponent
14:13	Teacher	T-c	so how did you have to get rid of the eighth power?
14:17	Student 9	S-ans	Ah... divide 18 on the other side and root 8
14:24	Teacher	T-j	So just because? What did you do that she did



not do? (Pause)

*Involving Theme*

14:35	Teacher	T-i	Like she 8 rooted it, did you do something different?(directs at another student)
14:37	Student 10	S-ans	well
14:40	Teacher	T-f	What?
14:41	Student 3	S-ans	Raised it to the $1/8$ power.
14:42	Teacher	T-f	Oh is that what you did?
14:43	Students	S-con	Oh yah
14:45	Teacher	T-p	Oh but that is another way to do it
14:46	Students	S-ans	Yes
14:48	Teacher	T-cont	Right, that's another way to do it
14:54	Student 8	S-pb	You still need to multiply that by 100 to make that a percent
14:56	Teacher	T-f	What?
14:58	Teacher	T-cons	Yes, that is the final answer. I just wanted you to see there are 2 ways to do it. You can either 8th root it or change it to the $1/8$ power. You should get the same answer.

Brandi writes the formula read by the student on the projection scene. She tries to get the student to talk about how the algorithm progressed. The student initially is not verbally expressive. After several types of questions, the student begins to elaborate more. After a conclusion to the problem is found, Brandi involves other students through a dialogue about the two solving patterns exhibited among the students. She identifies one student's process and questions other students on how theirs compares to the

example student. As seen in other exchanges, Brandi explores the algorithm with one student and then brings more students into the conversation by trying to conjure agreement within the group.

This next exchange portrayed the student and teacher dialogue when the student was asked to share his answers to number three of the review. Brandi chose a student to read his formula for the problem which read, ““A person is setting up an annuity fund. He would like to have \$120,000 in the fund after 30 years at 4.5% interest compounding monthly. How much should he put into the fund each month to reach the goal?”

*Exploring Theme*

15:17	Teacher	T-p	Okay, #3... lucky, #9 – that would be nobody. #16 is (student's name)
15:28	Student 12	S-ans	$120,000 = R \left[ \frac{\left(1 + \frac{0.045}{12}\right)^{12 \cdot 30} - 1}{\frac{0.045}{12}} \right]$
15:38	Teacher	T-c	Okay why over 12?
15:42	Student 12	S-ans	Cause its monthly
15:48	Teacher	T-f	Raised to the
15:50	Student 12	S-ans	12 *30
15:55	Teacher	T-p	This is where some of you went wrong. You \ didn't raise the exponent to the monthly times the number of years since you thought annual percent rate yearly. They are always annual percent rates,APR. Okay so it's always annual % rate so you then have to raise it to the monthly in number of years okay on the bottom.
16:22	Student 12	S-ans	... over 0.045

16:22	Teacher	T-cont	Okay, you probably have one of those fancy pants calculators and put all of that in at one time, right?
16:29	Student 12	S-con	yes
16:34	Teacher	T-c	How did you get it to other side?
16:35	Student 12	S-c	divided
16:39	Teacher	T-f	Divided so you ended up with a monthly payment.
16:42	Student 12	S-ans	\$128
16:45	Teacher	T-cont	That's what I got. That's a lot better than \$1090. You were going to make me pay (threw candy to student)

After writing the formula to be projected for the class to view, Brandi solves for the unknown variable. As she is solving, she asks him why 12 is in the formula. After his response, she responds with agreement and continues to finish the rest of solving process with his assistance. She uses confirmation questions during the last one-third of the dialogue, making sure all participants have come to the same answer and understand the specific substitutions.

After the class completed the review word problems, they worked in groups on an investigation activity exploring the characteristics of a unit circle. Initially, they were asked to cut a Wikki Stix into parts, each representing the length of the circle's radius. To help students see the correspondence between the circle's radius and the measure of a radian, they were instructed to lay the pieces sequentially on the circumference of the circle. Within the activity's instructions, questions guided their thinking to lead them to specific conclusions.

*Involving Theme*

24:56	Teacher	T-p	Did you write that okay #3 on the paper. NO, NO, on the packet, you are writing that. We just answered #3.
25:08	Student 2	S-ans	The general statement
25:10	Teacher	T-e	The radius is half the diameter. I got a bigger question here, cause she said it was half the diameter and yet on #3 it does not say radius. It says radian. So
25:28	Student 1	S-ta	Because it is looking at the plural of radius.
25:32	Teacher	T-p	No
25:35	Student 12	S-ans	Radii is the plural of radius.
25:41	Teacher	T-e	So why did she use radian when clearly she meant radius
25:49	Student 11	S-nc	well, I don't know
25:51	Teacher	T-p	Well, that is what we are about to find out. So now we are going to step 4. (reading) How is a radius related to radians?

To keep students moving forward through the activity, Brandi reads the questions and requests responses from the students. Several of them bring their input into the discussion when she incorporates her own questions pertaining to the meaning of a particular student's response. She uses this dialogue to highlight key ideas to in turn focus their thinking toward the concept of the next question on the activity.

In this final exchange, the students were still working in their groups on the investigation. They were working on problem four which focused on the linearity of the

radian measure. Brandi guided the class to focus on a specific question. The dialogue of the group ensued.

### *Supporting Theme*

26:30	Teacher	T-p	We are on #4. You all are working together. Draw a vertical tangent line from Q to the top of your card stock.
26:50	Student 15	S-seek	I don't know if my radius is completely horizontal
26:52	Teacher	T-cons	It's okay
26:53	Student 15	S-seek	Are you sure?
26:54	Student 7	S-ans	draw a tangent
26:56	Student 10	S-c	You can have it like that side
27:00	Student 15	S-ans	Not on the left

Brandi supports the dialogue of the students by keeping them focused and contributing agreement. The resulting conversation describes the students' actions as they work through problem four.

### 5.7 Relationship between Questions and Themes

When analyzing the themes as they looped through the class periods, there were some evident patterns. As shown in the featured exchanges, the themes occurred over varying lengths of time and in different sequences. When coding for themes in the data, Amber's questions exhibited ten instances of *exploring*, five instances of *involving*, five instances of *connecting*, and one instance of *supporting*. Brandi's questioning accounted for 22 instances of *exploring*, 17 instances of *involving*, and three instances of *connecting*, and five instances of *supporting*. In both classrooms, supporting theme was predominately found at the end of the class period. Whereas exploring and involving themes accounted for about 70% of the themes during the first part of the class periods.

The connecting theme was found at different times during the class periods. While the questioning themes varied in sequential order, the teacher questions overlapped within them. Within each theme, the kinds of teacher questions and the percentages of the utilization of teacher questions for each setting were determined. Each type of teacher question was compared in its usage in the various themes. Table 5.7 presented the percentages of utilization in each of the settings. Consequently, a description followed revealing how the teacher questions in each setting support each questioning theme.

Table 5.7 Percentages of Teacher Questions within each Theme

Teacher Questions	Exploring		Involving		Connecting		Supporting	
	1st Setting	2nd Setting	1st Setting	2nd Setting	1st Setting	2nd Setting	1st Setting	2nd Setting
Confirmation-t	13%	14%	23%	12%	12%	17%	0%	15%
Confirmation-s	10%	11%	10%	9%	22%	8%	0%	45%
Suggestion	0%	10%	0%	5%	0%	4%	0%	5%
Following	13%	16%	0%	15%	0%	4%	0%	0%
Procedural	41%	23%	55%	15%	44%	25%	100%	25%
Initiating	10%	5%	5%	18%	4%	0%	0%	0%
Retracing	0%	5%	3%	5%	12%	17%	0%	0%
Repeat	0%	0%	0%	2%	2%	0%	0%	0%
Explanation	5%	4%	0%	8%	0%	13%	0%	0%
Clarification	8%	10%	5%	9%	4%	8%	0%	10%
Justification	0%	1%	0%	2%	0%	4%	0%	0%

### 5.7.1 Questions and Exploring Theme

The *exploring* theme provided the opportunity for students to voice their thinking verbally to the other participants. The teacher questions could encourage a student to make their ideas public in both the whole class and small group discussion. When the teacher intervened in the small groups, it was a method of listening to what the individual was thinking. In terms of the whole class, the teacher enabled the large group to hear each other's thoughts.

The procedural and *confirmation* questions had the highest percentage in both settings. Within this theme, the exchanges had high percentages of these types of questions occurring at 41% and 33 % in Classroom A, respectively and 23% and 25% in Classroom B, respectively (Table 5.7). The procedural question directed the students to perform additional mathematical tasks in order to yield additional processes for the students to explain. The confirmation questions allowed the student to voice agreement or disagreement with the public thinking. The student could provide support for agreement or could share their thinking about their disagreement in turn discussing a different approach to the thinking. The observed classrooms both were more teacher-directed than student-directed so the amount of teacher questions with initial direction for the discussions was greater than other categories.

The *explanation*, *clarification*, and *justification* teacher questions were the categories which align most closely with the *exploring* theme as far as the intentions of the categories were described. In the classrooms, they were observed at 13% and 15% of the time, respectively. The most straight forward questions for eliciting the students' thinking were the explanation questions. While not having to prove the ideas, the students were asked to share the basic mathematical thinking for public evaluation. Clarification questions implored the student to speak using more mathematical language

in their explanation. Then the justification questions sought a mathematical argument from the students for their thinking. All of these questions asked for the students to voice mathematical thinking which were possible the intentions of the exchanges in the exploring discourse.

In this study when the teacher question categories of *explanation*, *clarification*, and *justification* were tallied for usage in the classroom, the three combined question categories ranked third along with the following questions in both settings. This ranking was preceded by the usage of the procedural and *confirmation* questions. Amber used these three teacher questions only used 13% of the time, and Brandi used them 15% of the time. Incorporating these questions into the discussion allowed the students' thinking to directly impact the course of the discussion.

#### 5.7.2 Questions and the Involving Theme

The *involving* theme supplied the opportunity for students to voice their thinking verbally to each other. The teacher questions encouraged multiple students to input their thinking in order to present alternative ideas to current thinking and bring justification of why a specific student idea was correct. Whether the groups were large or small, the teacher initiated involvement and orchestrated the various student ideas as they became part of the dialogue.

The two question categories of *initiating* and *confirming* paralleled this theme in their goals. Initiating questions directly asked students who were not a part of the discussion to join. The teacher could be addressing one or multiple students. In Classroom B, the teacher called on specific students by name to give their answer. As noted in the previous theme, confirmation questions allowed students to join by expressing their views with the current thinking in the discussion. Both of these



categories supported the theme which brought opportunity for multiple students to share their thinking in the current discussion.

The ranking of these categories were different in the two classrooms. Classroom B had the highest rank for *confirmation* questions, first at being used 21% of the time and the *initiating* questions, second at 18% of the time during the inviting theme scenes (Table 5.7). Brandi identified different methods of algebraically solving problems with her questioning of different students. On the other hand, Classroom A identified a high frequency of procedural questions and a limited number of initiating questions. Consequently, Amber asked the initiating questions only 5% of the time and ranked third in use for this theme. She relied on the confirmation questions to invite students into the conversation 33% of the time.

*Following* questions were used by both teachers in the *exploring* and *involving* themes. They are based on the students' ideas and were an approach to support the students as they tried to articulate their thoughts. While building on the student's ideas, the teacher could steer the student in a desired direction or help the student articulate their actions or ideas. In Classroom A, Amber used these questions 13% of the time in the exploring theme exchanges which ranked third after the procedural and confirmation questions. But in Classroom B, Brandi used these questions about 15% of the time in both the exploring and involving theme scenes. Brandi's inquiry with following questions ranked third to procedural and confirmation questions in both themes. These questions can aid students as they were trying to voice their thinking.

### 5.7.3 Questions and the Connecting Theme

The *connecting* theme enables the opportunity for students and teachers to review previously learned mathematical ideas in order for students to strengthen their current thinking. The teacher questions focused students' thinking on fundamental

mathematical ideas and enabling them to be part of a public discussion. In the small and large group setting, the teacher observed a need for a foundational mathematical concept or idea to aid in the students' present understanding. She could ask students to recall to this concept or idea to make connections to the ongoing discussion.

The most direct question to revisit ideas in a discussion is by asking *retracing* teacher questions. By referring to previously discussed or encountered ideas, the teacher focused students on earlier concepts or ideas in order to help them solve a problem or determine an error in their thinking. Amber used these questions to bridge similar concepts from old problems to the present problems being solved. In addition, Brandi used the questions with fractions for the students to work with the more complex fraction. These questions challenge student conceptions and cause deeper student understanding by having them reformulate their arguments to the present problem.

In the *connecting* theme scenes, the *procedural* and *confirmation* questions ranked first and second being used 44% and 34% of time, respectively in Classroom A and used jointly at 25% of the time in Classroom B (see Table 5.7). However, the *retracing* questions ranked third in both settings. The teachers asked them 12% of the time in Classroom A and 17% of the time in Classroom B.

#### 5.7.4 Questions and the Supporting Theme

The *supporting* theme provides for discussion among the students. The teacher questions encouraged the students to form a dialogue between them. The teacher's desire would be to be a part of the verbal conversation as little as possible. While letting students present their ideas, which may or may not be productive, the teacher must determine if the conversation taking place is constructive for reaching the mathematical goals. This theme was observed in the small group setting in this study. Generating independent student discussion in the large group would be more difficult for possible

reasons of student confidence in their mathematical ideas and organization of student talk.

The exchanges of the supporting theme were few in number in this study. The first setting has one example during which Amber used a procedural question in order to keep the students moving forward in their discussion. In Classroom B, Brandi encouraged the students to dialogue between each other by using *confirmation* and *suggestion* questions 65% of the time (see Table 5.7). The suggestion teacher questions allow the teacher direct along a pathway of thinking without just stating the answer. In this theme, students need to feel confident in their own dialogue in order to keep the discussion moving.

### 5.8 Summary

The purpose of this chapter was to present an analysis of the teacher questions, student responses, and their interrelationships. The first two parts of the chapter identified the frequencies and patterns within the types of teacher questions and within the types of student responses. A correspondence between the teacher question and student responses was described. In order to further analyze this relationship, smaller exchanges within larger segment of the class time were identified with respect to the type of mathematical discussion represented. Finally the types of teacher questions were portrayed within the themes. Chapter Six provides a summary of the study, conclusions, recommendations, and a summary.

## Chapter 6

### Summary of the Study, Study Implications, Chapter Summary, Final Thoughts

Classroom discourse enables students to discuss mathematical ideas and concepts with their teacher and peers and has been shown to increase student learning (White 2003; Hoffman, Breyfogle, & Dressler, 2007; Cengiz, Kline, & Grant, 2011). Constructivist learning theory advocates that teachers need to grant time for students to mentally interact with material and problems they are learning to more fully develop new mathematical concepts and ideas (Ball & Friel, 1991). This study sought to describe two teachers, one self-identified as motivated in teaching toward mastery orientation and the other toward ability-approach orientation, as they involved students in classroom mathematical discourse. The questioning themes, teacher questions, and student responses found in this study provide inform educators of the nature of discourse that takes place through teacher questioning. Moreover, this study provides new information on relationships between teacher questions and student responses, particularly those that elicit constructivist learning in the students' minds, as verbalized in classroom discourse. While most studies describe the kinds of questions and actions teachers can incorporate (White, 2003; Manouchehri & Lapp, 2003; Bennett, 2010), this study extends this work by describing the questions teachers ask to promote students thinking, the responses students give, and the nature of the correspondence between the two.

#### 6.1 Summary of Participants

This study involved in-depth observation and analysis of teacher questioning and student responses of two teachers currently employed in private K–12 college preparation schools. The teachers are pre-calculus teachers selected after an achievement goal survey scoring of ability approach and mastery goal orientations were identified. The teacher in Classroom A was given the pseudonym Amber, and was a

veteran teacher of 28 years. Amber's identified motivation in teaching ability approach, indicating her goal was to focus on demonstrating superior professional skills. Her classroom exhibited her desire for the appearance of an organized environment. In Classroom B, the teacher Brandi had the same qualities as Amber in years of experience and level of education. Her identified motivation for teaching was mastery oriented, indicating her goal was on learning and developing professional competence. Her classroom activities were less teacher-centered compared to Amber's classes. However, she did spend about ten minutes in one class period dictating notes for her students' notecards pertaining to the homework problems.

## 6.2 Summary of Methods

After the two teachers were selected based on the survey results, the teachers were asked to choose two class times in which I could observe and document the dialogue between the teacher and students. I asked to view a class activity where the students were problem solving, and I would be analyzing the conversation in the classroom. Amber invited me into the same classroom of students at two different class meeting times. The mathematical algorithms completed during the class time were algorithms that had been introduced and practiced in Algebra II the previous year. A majority of her questions asked for student's input as to what they thought was the next step in an algorithm. Brandi's invitation was to observe two class periods with the same classroom of students were problem solving on concepts that had been discussed the previous week. The students individually solved five problems, and then in a large group discussion, were asked to share their conclusions. While they worked the problems, students did converse with each other, and the teacher provided feedback to individuals or groups of students. This summary describes the classroom environments during the time the observations with video and audio recordings were conducted.

### 6.3 Summary of the Findings and Conclusions

This study aimed to examine how teacher questions may engage students in mathematical conversation. With focus on this goal, three research questions framed the structure and analyses of this study. The first question sought to determine the types of questions two teachers asked in problem solving activities. The findings identified 11 questions asked of which nine were observed in both classroom settings. Data analysis determined the frequency of each type of teacher question and the types were listed according to frequency of use from greatest to least: *procedural*, *confirmation* while agreeing with student statements, *confirmation* while asking for student agreement, *following*, *initiating*, *clarification*, *retracing*, *suggestion*, *explanation*, *repeat*, and *justification*. These teacher questions were used during problem solving pertaining to quadratic, exponential, trigonometric expressions, and equations with a minor focus on quadratic functions.

Both Amber and Brandi asked *procedural* and *confirmation* questions more frequently than the other types of questions. Amber used these question types a majority of the time. Her decision to work the problems with the students allowed the questions to easily become procedural in nature. Brandi used these questions about half the class meeting time. She waited to work the problems with the students in the large group and allowed the students to elicit their own initial thoughts on the problems. Allowing students time to think and process thoughts can foster an environment of student understanding (McTighe & Wiggins, 2013). Amber's procedural questions took little processing time because most of them were asking for recalled student answers. She asked her students to remember the definitions and the steps pertaining to an algorithm. Her questions did not encourage an environment where students had to be thoughtful about their answers.

On the other hand, Brandi gave her students' time to process concepts and ideas through review problems at the beginning of the two observed class periods.

The *procedural* questions are typically used in the classroom environment to keep discussion moving forward or for giving directions to a specific mathematical task. In a study of a student centered classroom, Ilaria (2009) describes procedural questions as being used about a fifth of the time. Teachers are essentially orchestrating the behaviors and contributions of each participant in the discussion as it unfolds. The procedural questions act as guides for student actions in the discussion but need not control the discussion (Maher, 2005). In Classroom A the procedural questions were identified as comprising 50% of the teacher questions, thus indicating that these questions did control much of the discussion.

In both observed classrooms, students were encouraged to talk about the mathematical activities that were the focus of the day's lesson. Confirmation questions were used to encourage dialogue among the students. The frequency of Brandi's use of confirmation questions was equivalent to her procedural questions. In Ilaria's (2009) study of the student-centered classroom, confirmation questions were used most frequently among other types of questions. While agreement in a discussion is important, these questions can bring a sense of closure and motivation for the participants and encourage them to proceed in the discussion (Knott, Sriraman, & Jacob, 2008). Both Brandi and Amber asked confirmation questions to affirm student's answers and to bring participants to agreement.

Although Amber had a traditional teacher-centered classroom, she did encourage students to talk. She used a few *initiating* questions mainly as a method of eliciting the entire group of students into a discussion of a topic. At times she asked a student to clarify a previous response by providing more details. In addition, she

incorporated some *retracing* questions to make connections with previously studied algorithms that were aligned by definition with the present mathematical idea. When the students were doing or checking their homework, she initially started following the student's thinking by observing their actions on their paper and forming a few questions. These examples display where she was to some extent enabling her students to not only talk but to communicate their thinking.

When observing Amber as she interacted with her class, she only asked a few *explanation* and *clarification* questions. *Justification* questions were never observed. It is difficult to have students verbalize their thoughts for public analysis as it can be uncomfortable and intimidating for the students. Helping students develop the willingness to share their thinking through the teachers' questions takes knowledge, experience, and encouragement (Manouchehri & St. John, 2006). To be successful, teachers must remove themselves from the central role of analyzing mathematical concepts and ideas and invite greater student participation (Nathan & Knuth, 2003). Amber maintained control of the direction of the discussion by completing the problems alongside the students and asking a majority of *procedural* questions. Students were never individually invited to share their thinking. The one example of student proof building was initiated by the student and his curiosity.

In Brandi's classes where I observed the mathematical discussion, she did incorporate a variety of types of questions. In fact, she asked twice as many questions as Amber. Even though half her questions were *procedural* and *confirmation* in nature, she included *following*, *initiating*, *clarifying*, *retracing*, *suggesting*, *explaining* and *justifying*. When she was working with individual students, she used her following, retracing, and suggesting questions. She tried to understand their thinking on the problem, connect their understanding to prior knowledge, and suggest alternative pathways for understanding



mathematical concepts or ideas. Her initiating questions and clarifying questions were mainly observed in the whole group setting. She made attempts to encourage students to share their thinking, including what was written on their papers, and/or to verbally explain their reasoning to the class.

Although Brandi used many types of questions, she did not ask many *explanation* questions and asked only one *justification* question. Her justification question did not receive a response from the targeted student. The student may not have known the answer or did not want to vocalize his thinking for a variety of reasons, including possible anxiety associated with publicly expressing ideas or supplying incorrect “answers” and thinking processes. Furthermore, when Brandi questioned students in the large group, she generated random numbers as a method of selecting students to share their answers and their analysis process. My observations of the student responses indicated they were not always willing or eager to present their work. As supported in the literature, it takes time and effort for teachers to build a foundation of comfort among students in establishing for explaining and justification types of questions in the classroom (Nathan & Knuth, 2003). Although this study did not entail observing classes throughout a school year, perhaps Brandi would establish a necessary level of comfort and find more students willing to answer the explaining and justification questions as the year progresses if she continued using her questions.

The motivational goals for teaching could have some influence on the mathematical discussions in these two classrooms. Butler (2007) identified that students with mastery motivation for learning view difficult tasks as challenges where outcomes are measured by effort; and students with ability-approach motivation for learning view difficult tasks and outcomes as identifying lack of ability. Brandi was identified a teacher with a mastery orientation, and Amber was identified as a teacher with ability-approach

motivational goals for teaching. Brandi asked a variety of questions and engaged in more dialogue with her students. She may have seen the task of developing a social norm where students shared their thinking as a challenge to overcome with effort. The number of questions asked by Brandi was double the number of Amber's questions. Amber may be less aggressive in questioning students since she might have perceived the task as difficult. She asked confirmation and procedural questions, which took less verbalizing of student thinking to answer. Further, she was observed showing concern for her students as she helped them complete mathematical tasks with low difficulty level. During both observations, she was working algorithms that the students had done in Algebra II. She may not have wanted her students to encounter too difficult a task, which may explain her use of mainly lower level question types.

The second research question sought to determine student responses that were evoked from the identified types of teacher questions. The findings identified eight student responses of which six were observed in both settings. The frequencies of the responses from most observed to least observed were: *answer, seeking, confirmation, thinking aloud, clarification, not contributing, questions students, and proof building*. These responses were evident in the small and large group discussions where the teacher was questioning students.

The *answer, seeking, and confirmation* student responses were the most frequently identified in both classrooms. This finding supports previous research which reported that answer response was probably a favorite and safe response for students even in a student-centered setting (Hannel, 2003; Ilaria, 2009). The answer, seeking, and confirmation responses by definition did not have to come from the student's own thinking and were often simply reciting something previously memorized or recalling a specific fact. To engage the students in higher level thinking and in vocalizing their

thinking processes, teachers would need to follow up these lower level question types with *explanation, clarification, initiating, following, or suggestion* questions.

The clarification and *thinking aloud* responses provided a forum for the students to share their mathematical thinking. Brandi's student-centered activities during class achieved more responses in these categories than Amber's more teacher-centered activities. Brandi's types of questions went beyond low level *procedural* questions. Her questioning at the higher levels of clarification and thinking aloud brought about more dialogue among students, in turn yielding more opportunities for students to voice their thinking. On the other hand, Amber's predominant use of *procedural* questions was typically followed by short recall answers.

Students' choosing not to answer a teacher question was observed when the teacher asked a variety of teacher questions. Brandi received all the *non-contributing* responses observed in the study. She posed questions like initiating, suggesting, and following that directed a response from an individual student. Several times students did not want to respond or did not know the answer. The opportunities for student responses was greater in Brandi's class than in Amber's class just by the number of teacher questions asked in each classroom. The expectation of teachers receiving a *non-contributing* response could be greater if they expected students' to verbalize their thinking.

Neither classroom had many examples of *proof building* responses. Thus both teachers neglected to use this technique to hear their students' mathematical thinking, which is an important aspect of mathematical reasoning. Brandi had asked a *justification* question and none of the students responded. Thus, though Brandi's classroom was more student-centered, the lack in using these higher level questions necessary to elicit students' mathematical thinking actually indicates that both settings were teacher-

centered to differing degrees. Though both classrooms incorporated teacher questioning the evidence on the types of questions indicates they are actually on a continuum of a teacher-centered classroom to student-centered classroom. Brandi's classroom was not fully, but scaled more toward a student-centered classroom by her use of questioning types that encouraged the verbalization of students' thinking; compared to Amber's classroom where lower level questions were used that did only minimally elicited verbalization of students' thinking.

The third research question of this study sought to determine the extent and ways these teachers' questions engaged students in mathematical conversation. The focus was on how the teachers elicited students' mathematical reasoning. The analysis of this question was dualistic in nature. First, I identified and ascertained the frequency of student responses to the teacher questions in each setting. The second part of the analysis described how specific questions engaged students to share their mathematical reasoning through dialogue by identifying question themes.

Even though Amber's use of many types of questions was limited and patterns were sparse, some of her questions were followed by a few responses that exhibited student thinking. Most of her questions were followed by answer responses. She was asking questions pertaining to the steps of an algorithm while she herself worked the problems on the front board. She did have a few examples of asking initiating, explaining, or *retracing* questions when no one answered, sometimes rephrasing or changing the type of question in order to get a response. A mathematical discussion is bi-directional in nature, therefore teachers have to consistently enable students to share their mathematical analysis and students have to be willing and confident in sharing their thinking (Knott, Sriraman, & Jacob, 2008). Amber's classroom demonstrated the need

for teacher persistence in the use of questions to engage students in mathematical discussion.

Amber's questions did result in some of examples of students' thinking aloud and one example of the student's proof building. Most of these examples came from one exchange during one of her classes. The primary student involved in the discussion had identified a possible pattern in solving a problem. He was seeking Amber's approval in the employing of his mathematical idea. He was motivated to share his thinking by exhibiting thinking aloud and *proof building* responses to her questions. Her questions ranged from following his thinking to asking for more explanations. She even used *initiating* questions to move more students into the discussion so they could contribute their thoughts. These findings indicate that Amber did have opportunity to encourage student thinking but she was not observed encouraging this kind of dialogue with the majority of her questions.

The lack of patterns with resulting student responses from most teacher questions was more evident with Brandi's questions. Many question types such as *confirmation*, *following*, and *initiating* questions had many different student responses ranging from answering, thinking aloud, confirming, to *seeking*. However, two response patterns did surface. The procedural questions resulted predominately in answer and *seeking* responses where the student was giving a minimal response or looking for the teacher to aid in the thinking process. Brandi's questions of clarification and explanation revealed student responses of thinking aloud, clarification, and proof building. Although there were only a few examples, these questions did result in the students sharing their mathematical thinking.

The second part of the analysis which identified four questioning themes: *supporting*, *connecting*, *exploring*, and *involving*. A majority of the exchanges identified in

each classroom setting were characterized by the exploring and involving themes. The connecting and supporting themes were represented with fewer examples.

In Classroom A, which from the data represented a more traditional classroom, there were about half the number of teacher questions and student responses as compared to the Classroom B, which incorporated somewhat more student-centered activities. The limited exchanges, the teacher explaining procedures, and the large number of procedural and *confirmation* questions in Classroom A narrowed the identification of number of exchanges identifying particular themes. When Amber was not asking questions, she spent time explaining the process for completing algorithms. When she worked the problems along with the students, she dominated the conversation. The total number of exchanges during the two observations was 21 as opposed to Classroom B, which had 47 total exchanges. This study revealed an example through the questioning themes that the number of teacher-centered activities in Classroom A limited the amount of opportunities to hear the students' thinking.

The *supporting* theme is primarily students discussing mathematical ideas among themselves with little teacher involvement. A student-centered study that analyzed students' engagement in verbalizing mathematical thinking, Ilaria (2009) identified about one-fourth of the total conversations were primarily between students. Classroom B was identified with the supporting exchange in about one-tenth of the conversations. When this class was problem solving, the students were not necessarily working in groups. The opportunities to observe the student discourse were limited. The few examples were of supporting exchanges occurred in the whole group discussion when a student volunteered to answer other students' questions.

Although in Classroom A the exchanges that elicited the *connecting* theme contained several *retracing* questions, these questions followed the *procedural* and

*confirmation* questions in frequency. Classroom B showed this same relationship between *connecting* theme and *retracing* questions. The use of the retracing questions enabled the teachers to connect mathematical concepts and ideas in the observed discourse exchanges. This finding supports the research on incorporating students' background knowledge to elicit productive mathematical classroom discourse (White, 2003). Both teachers interjected retracing questions during the connecting theme exchanges so students would refer back to previous knowledge to help encourage verbalization of student thinking and strengthen student understanding.

The *exploring* theme contains questions dealing with following and clarifying student's thinking as well as suggesting possible avenues of thought. *Procedural* and *confirmation* were the most frequent questions observed in Classroom B, with following, suggesting, and *clarifying* questions followed closely in use. In exploring theme, Brandi was encouraged to ask questions that probed and elaborated on a particular student's thinking. As individual students were asked to explain their thinking through a series of questions, the following and clarifying questions gave the teacher an avenue to make the argument personal to each student's ideas as well as communicate the ideas to the other students. Suggestion questions aid students who are hampered by uncertainty on which direction to proceed with their thoughts. The continued use of questions by teachers to reveal students' thinking can build students' confidence in their own ideas and consequently, foster discussion (Manouchehri & St. John, 2006). Brandi used the exploring theme to foster the discussion with a student and build the student's confidence in his thinking.

The *involving* theme makes various students' thinking public and brings several students into the discussion. In Classroom B, *involving* questions allowed progress in mathematical thinking which was enhanced by the number of students contributing to the

ideas. In addition, the *explanation* and *clarification* questions allowed other students to be a part of the dissecting and summarizing of the students' thinking. This process creates learning opportunities for students to engage in more sophisticated reasoning (Weber, Maher, Powell, & Lee, 2008). Thus, it can be posited that in Classroom B the interchange between various students encouraged students to mathematically reason.

#### 6.4 Study Implications

The specific teacher questions and questioning themes described in this study inform mathematics teachers of the characteristics involved in classroom discussions of these two participating teachers. As this study focused on two problem-solving classrooms, it identified who the teachers are, what the teachers did in the classroom, and similarities and differences in their teacher questions along with their student responses. Furthermore, the questioning themes described in this study provide awareness as to how certain types of questions could be used to engage students in mathematical conversation.

##### 6.4.1 Informing Theory

This study's findings contribute to educational research on questioning to promote discourse toward fostering constructivist learning in mathematics classrooms. As classroom discourse was investigated, the exploring, involving, connecting, and supporting themes of the teacher questions give important insights to the process guiding students through the learning process. For instance, in the environment of problem solving, eight student responses revealed actual scenarios of student thinking as the learning process commenced and carried through to fruition.

The different themes reported in this research demonstrate the mental functioning processes described by Piaget. Exploring theme showed the teacher asking questions of a student. Students working out the problem and then hearing the teacher's



question represents their assimilation of the information. While students were in disequilibrium, the teacher asked clarification, explanation, retracing, procedural, suggestion, and justification questions attempting to bring about accommodation of the mathematical concepts or ideas. Students achieved equilibrium at the conclusion of the dialogue between the teacher and student. However in this study, there were examples where students' achievement of equilibrium was not immediately observed, but became evident in later conversations. The involving theme also follows the mental functioning process and replicates it with several students trying to reach equilibrium. In integrating both Piaget and Vygotsky's theories of learning, as students' are in disequilibrium, teacher questioning scaffolds the learning toward resolving disequilibrium to achieve equilibrium (Piaget, 1965) and toward moving from actual to potential intellectual development (Vygotsky, 1978). The thinking of several students and teacher questions reciprocate each other to move through the mental processes to reach a level of understanding or equilibrium. The connecting theme is similar in nature to involving theme. But the teacher relies on retracing questions to build on previous mathematical concepts to bring the disequilibrium stage in the students to accommodation instead of using the different student thinking conveyed in their responses. The final theme of supporting views the teacher questions guiding the discussion but they are not the focus of the dialogue. The students are attempting to become autonomous in their dialogue as they themselves work through the mental processes.

The motivational theory helped to identify one distinction between the two teachers in the study. Although both teachers had teacher-centered classrooms, Brandi who was identified as having mastery goal orientation toward teaching did incorporate relatively more student-centered activities in her classroom. Her problem solving activity enabled her to ask more questions and acquire more types of student responses like

thinking aloud, clarification, and proof building. Amber whose average was highest in ability-approach motivation used more teacher-centered problem solving with her students. The two settings provide examples of specific behaviors supporting the differences in the research pertaining to mastery and ability approach goals. Mastery goals prefer challenging tasks and associate outcomes with effort (Butler, 2007). Brandi asked twice as many questions as Amber did during the entire observation. She had more variety in her types of questions. She encouraged the students to share their thinking during the problem solving of the warm-up review problems. Although she had limited examples of supporting themes, she allowed dialogue between the students. The environment of the classroom encouraged students to take risks in sharing their ideas with the entire class. In contrast, ability goals interpret difficult tasks as low ability and connect outcomes to ability (Butler, 2007). Amber had instructional activities with low difficulty levels for her students. She worked the problems with them and asked questions requiring concise recall responses. In both observations, the students had studied those specific algorithms the previous school year and thus were familiar with the material. If the students answered the procedural questions, their responses were “correct” about 90% of the time. The environment in Amber’s classroom protected students against being “put on the spot”. The students could volunteer to share mathematical thinking but they were never expected to respond. The result of this protective behavior however, does not promote the discourse needed to promote student thinking and move them toward higher cognitive levels as described by Piaget and Vygostky.

This study focused on the self-identified characteristics of motivational theory with respect to these two teachers. Longitudinal observations of traditional settings throughout a school year could be a subject of further research since this study’s limited

observation took place in the first half of the year. In future research, each classroom's sociomathematical norms could be identified and monitored for change throughout the year as the familiarity and expectations between teacher and students develops. The frequency in the use of the different question types could change, and students may be more comfortable to share their mathematical thinking. Further, teacher interviews could ask their motivations for asking their questions and choosing the particular instructional activities. Conclusions related to teachers' motivational intentions may be strengthened with extended observation and with the inclusion of personal perspective.

#### *6.4.2 Implications to Practice*

The descriptions of the discourse presented two teacher-centered teachers asking questions and using problem solving activities to generate opportunities for students to communicate their mathematical thinking. These teachers along with the student-centered teachers in the research (Ilaria, 2009) illustrate a continuum of change in teacher questions and student responses from teacher-centered to student-centered. Being aware of the teacher questions may allow teachers to use the question types revealed in this study in their own classrooms to elicit particular types of discourse among students. Teachers may use the findings of this study to become more cognizant of the numbers and types of questions they are asking, achieving better understanding of the types of questions that will elicit certain types of student responses. For example, procedural questions will elicit less verbalizing of student thinking. In contrast, to understand the students' thinking teachers would use explanation, clarification, and justification questions, and focus on using them with higher frequency. In addition, administrators can use the question analysis presented in this study to specify types of questions with frequencies that will help teachers quantify the dialogue and support teachers in their work to elicit and understand students' thinking.

The relationship between teacher questions and student responses did provide some direct insight into questions that provided particular responses. In a majority of the cases, the most frequent student response was the answer response, but there were some relationships that may help teacher questions engage students in mathematical conversation. The thinking aloud responses followed the suggestion, initiating, and explanation questions most of the time. It should be noted that in these classrooms half of the explanation questions were followed by thinking aloud responses. In addition, when teachers are incorporating student-centered discussion, they need to expect some students who will initially not want to contribute to the conversation (Goos, 2004). Brandi was asking students to independently think and problem solve. She received all of the non-contributing student responses and had to continue questioning after that response. In the more teacher centered classroom, the non-contributing response was never encountered. Becoming aware of the nature of the mathematical discussion as it relates to the teacher questions provides the opportunity for teachers to better understand the process and progression of their students' learning.

From the results of this study, teachers and administrators gain new knowledge on how specific teachers with particular goal orientations for teaching may conduct their mathematical classroom discussions. The pattern of the ability-approach linked with the teacher-centered and the mastery approach with the more student-centered classroom could provide a direction for professional development and teacher mathematics education programs. Mastery or ability goal orientations may be addressed through self-reflection and evaluation in these in-service programs. Educational programs can bring teacher awareness as to the consequences of certain motivations, thus improving on the potential dialogue of the student thinking that occurs in mathematics classrooms.

### *6.4.3 Informing Research*

NTCM has clearly articulated guidelines for what should be covered in productive mathematical discussions while not communicating to teachers how to insure that students are learning (Lampert, Rittenhouse, & Crumbaugh, 1996). Research has contributed ideas on roles and tasks of teachers during mathematical discourse (Staples & Colonis, 2007; Truxaw & Franco, 2007; Kotsopoulos, 2007). This study supports the arguments of other researchers by finding similar statements within the analysis. Teachers should ask questions that probe, extend or elaborate upon previously voiced student thinking or ask them to justify their ideas (Dann, Pantozzi, & Steekcken, 1995; Davis, 1997; Van Zee & Minstrell, 1997; Martino & Maher, 1999; Steele, 2003). Herbst (2002) referred to suggestion questions as a way for teachers to assist students by providing information to the student. Initiating questions are a way to bring numerous students into the conversation and used in scenes where students are encouraged by the teacher to get involved (Ilaria, 2009)

The questioning themes identified in this study are aligned with the research. The exploring theme allowed one student to share thinking with the assistance of teacher questioning. Goos (2004) described how the teacher questions and student responses can be used as an agent to bring a student's thinking through to sense-making. In addition, when several students are involved in discussion, there is a positive influence on the student mathematical thinking (White, 2003). In the involving themes, several students shared their ideas. As a result, students were able to clarify, explain, and verbalize concepts and ideas of others and themselves. The process of bringing more students into the conversation and making a student's reasoning the object of discussion creates learning opportunities for students to engage in more sophisticated reasoning (Weber, Maher, Powell, & Lee, 2008). When multiple students shared their thinking, they

were able to help each other through debate and confirmation to reach conclusions about the problems they were solving.

The supporting theme occurred when students engaged in conversations among themselves rather than focusing on dialogue with the teacher. Supporting questions in a student-centered environment allowed students to be responsible for their own ideas while the teacher ensured they understood each other and making valid arguments (Ilaria, 2009). The classrooms of this study had limited number of these exchanges. The sociomathematical norms in the two classrooms could have influenced the amount of independent discussion between the students. The teachers did attempt to steer the discussion in directions that they felt were appropriate while still having students generating discussion. Further, the teachers encouraged students to respond to other student questions. Giving students the freedom to collaborate with each other and to view their mistakes as a learning process encourages the discussion of student thinking among themselves (Knott, Sriraman, & Jacob, 2008; Cheval, 2009). Students are challenged to verbalize with each other to develop mathematical concepts or ideas.

The connecting theme initiated student thinking when teachers ask questions that had students return to their previous ideas. Both teachers asked retracing questions to refer to previously studied mathematical concepts that were related to the present discussion. The interweaving of information both old and new helps students to progress to higher levels of thinking (Goos, 2004). Questions can be used as one method to encourage this type of thinking (Lambert & Cobb, 2003; Manouchehri, 2007; Himmelberger & Schwartz, 2007). Teachers could ask retracing questions to strengthen their student's understanding of a particular mathematical concept or idea.

This research extends the previous research by analyzing the correlation between what teachers asked and how students actually responded. Most research

categorized questions by the types of responses students were expected to give. A research study recently described how students actually responded in an environment which was designed as student-centered (Ilaria, 2009). He identified the same themes, exploring, initiating (involving), supporting, and connecting themes; and his teacher questions and student codes were similar to this study. He had one extra student response, attunement (Ilaria, 2009). This study analyzed teacher-centered environments where students were responding to teacher questions. The percentages of actual use of teacher questions and students responses had differences within the student and teacher-centered classrooms. For example, the procedural questions in a teacher-centered classroom were asked 55% of the time. In the student-centered classroom, the teachers asked procedural questions about 20% of the time (Ilaria, 2009). In addition, students in Ilaria's (2009) study responded about 10% of the time with proof building statements. In this study, the responses were used 1.5% of the time. The proof building responses were identified more often in the student-centered classroom. To be able to vocalize student's thinking, teachers in the teacher-centered classroom would need to ask more explanation, clarification, and justification questions and less procedural questions.

Based on this study, an area of further research is to identify several mathematics classrooms the sociomathematical norms where students are expected to share their thinking and to examine the teacher questioning and discussion in the classroom environment. A possible research question could be: To what extent do teacher questions, as identified in this study and according to the themes, promote an environment of discussion? By examining how questions contributed to the classroom environment, the study could determine if non-questioning factors, such as students writing on the board or working in groups, is sufficient to engage students in

mathematical conversation or if teacher questions play an important role in creating conversation.

## 6.5 Chapter Summary

Questioning used in the education process traces back to Socrates in historical accounts. In Plato's *The Republic*, Socrates used a series of strategic questions to help his student come to understand the concept of justice (Tienken, Goldberg, & DiRocco, 2009). Even today, teachers at all levels of K–16 education still use questions as one method of helping students develop productive thinking skills and understand concepts and ideas. Consequently, students at all ages can use their responses to become more actively involved in their learning (Marksberry, 1979).

The overall significance of this study was distinguishing the manner in which mathematics teachers interact with their students when they are teaching. Attaining insight into student understanding is an important part of teaching (Yackel & Cobb, 1996; Sfard, 2008). Typically, teachers give written assessments in order to collect information about student knowledge. However, written assessments do not fully provide teachers with insight into the development of student thinking and understanding as it is occurring (Lubienski, 2000). The assimilation–accommodation process of students' mathematical learning is not viewed by the teachers with written work. Conversation is one way teachers can elicit information about student understanding (White, 2003). However, as demonstrated in this study, the nature of engaging students in mathematical discussion is a complex process. Students will not simply share their thinking when teachers ask the right questions, such as clarification, explanation, or justification. Therefore, knowing the research and asking the appropriate questions for a desired type of response from students is only a part of the process of becoming adept at promoting conversations in their classrooms. Teachers need to be open to student discourse and build student



confidence as they practice and experience classroom teaching with mathematical discussions first hand. By doing so, they will build a sociomathematical norm where students are expected and freely participated in explaining and justifying their thinking (Bennett, 2010).

## 6.6 Final Thoughts

This study has been important to me because as I have developed as a teacher over the past 25 years, I have desired to understand how to encourage my students' thinking and have been mentored by more experienced teachers to improve my questioning strategies. This basic question and response model in this study presents a glimpse of the discussion patterns present in two teacher-centered classrooms. The kinds of questions and responses in the student centered classroom (Ilaria, 2009) were similar in nature to the teacher-centered, but the frequencies were different. In the student-centered environment, teacher asked more justifications questions and received more proof-building responses. The students were not comfortable with these responses but the expectation of the social norms encouraged students try to at least think aloud response (Ilaria, 2009). In this study, the expectation for certain student responses were different. In the Classroom A setting, the teacher asked initiating questions to the whole class and if an individual volunteered they could answer. In Classroom B, volunteering responses were accepted but the teacher initiated discussion with individual students with the expectation of explaining their thinking. The type and frequency of teacher questions and their intended audience can be key factors in defining the sociomathematical norms of the particular classroom.

Mathematics educators and administrators can support teachers in becoming more comfortable with mathematical discussions in the classroom. As shown in the study, eliciting student thinking through discourse is a complex, dynamic, and challenging

task for even experienced mathematics teachers. From the pre-service perspective, mathematics educators should provide pre-service teachers the opportunities to observe and analyze teachers who know the benefits of and practice conversation in the classroom. As for the future mathematics teachers, a support system with mentoring and professional development for them must help and encourage them achieve an environment where students are sharing their mathematical thinking (Ilaria, 2009). The support would include the teacher awareness through reflection of the type of teacher questions asked and the type of student responses. In addition, the dialogue between mathematics teachers and administration would include the goals for the level of eliciting student thinking with respect to the specific mathematical concepts. For example, when studying the quadratic equations as in the first setting of this study, will the student thinking involve using the algorithm of solving of these types of equations, or will it include why does the algorithm work with these equations? Teachers can use guiding questions which expect students to respond with explanations or justifications (Inoue, 2011). Setting and developing the discourse expectations can enhance the verbalization of student thinking and thus promote better and more sophisticated levels mathematics learning needed in our schools.

Appendix A  
Teacher Permission and Survey

## Engaging Students in Mathematical Conversation: Teachers' Questions Eliciting Students Responses

### INFORMED CONSENT

UT Arlington - Informed Consent Document

#### PRINCIPAL INVESTIGATOR

Glenda Mitchell  
Educational Policy and Leadership  
Phone: 817-201-2583  
Email: gmitchel@uta.edu

#### FACULTY ADVISOR

Ann M.L. Cavallo, Ph.D.  
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Professor of Science Education  
Co-Director, UTeach Arlington  
Program Coordinator, AP Science and Mathematics  
Email: cavallo@uta.edu

#### TITLE OF PROJECT

Engaging Students in Mathematical Conversation: Teachers' Questions Eliciting Students Responses

#### INTRODUCTION

You are being asked to participate in a research study investing discussion between teachers and students during mathematical problem solving sessions. Your participation is voluntary. Refusal to participate or discontinuing your participation at any time will involve no penalty. Please ask questions if there is anything you do not understand.

#### PURPOSE

The purpose of this study is to identify teacher questions and student responses in a problem solving environment and how teachers used questioning to engage students in conversation.

#### DURATION

After completing this 10 minute survey, a couple of teachers will be asked to participate in 2 classroom observations, each observation will last one class period.

#### NUMBER OF PARTICIPANTS

Two teachers with their students in one of their precalculus classes will be asked to participate in the recordings.

#### PROCEDURES

The procedures which will involve you as a research participant include:

1. answering the questions on a 10 minute survey.
2. having two problem solving sessions observed and video and audio recorded during one of your precalculus classes. (This step will be completed if you are one of the two chosen teachers.)

The classes will be video and audio recorded. The recordings will then be transcribed, which means they will be typed exactly as they were recorded, word-for-word, by the researcher. The recordings will be destroyed after transcription.

#### POSSIBLE BENEFITS

The findings will provide actual descriptions of the dialogue that transpires between teacher and student as mathematical ideas are developing. In turn, it will seek to contribute to the mathematical research of constructivist learning theory as dialogue between teachers and students commence.

#### POSSIBLE RISKS/DISCOMFORTS

There are no perceived risks or discomforts for participating in this research study. Should you experience any discomfort please inform the researcher, you have the right to quit any study procedures at any time at no consequence.

#### COMPENSATION

There is no compensation offered for participation in this study.

#### ALTERNATIVE PROCEDURES

There are no alternative procedures offered for this study. However, you can elect not to participate in the study or quit at any time at no consequence.

#### VOLUNTARY PARTICIPATION

Participation in this research study is voluntary. You have the right to decline participation in any or all study procedures or quit at any time at no consequence.

#### CONFIDENTIALITY

Every attempt will be made to see that your study results are kept confidential. The results of the survey will be used to determine which teachers to include in the observations. Then the results will be deleted. A copy of this signed consent form and all data and transcriptions collected from this study will be stored in room 484 of Pickard Hall at UTA for at least three years after the end of this research. The results of this study may be published and/or presented at meetings without naming you as a participant. Additional research studies could evolve from the information you have provided, but your information will not be linked to you in anyway; it will be anonymous. Although your rights and privacy will be maintained, the Secretary of the Department of Health and Human Services, the UTA Institutional Review Board (IRB), and personnel particular to this research have access to the study records. Your records will be kept completely confidential according to current legal requirements. They will not be revealed unless required by law. The IRB at UTA has reviewed and approved this study and the information within this consent form. If in the unlikely event it becomes necessary for the Institutional Review Board to review your research records, the University of Texas at Arlington will protect the confidentiality of those records to the extent permitted by law.

**CONTACT FOR QUESTIONS**

Questions about this research study may be directed to Glenda Mitchell at 817-201-2583 or gmitchel@uta.edu or Ann Cavallo at cavallo@uta.edu . Any questions you may have about your rights as a research participant or a research-related injury may be directed to the Office of Research Administration; Regulatory Services at 817-272-2105 or regulatoryservices@uta.edu.

As a representative of this study, I have explained the purpose, the procedures, the benefits, and the risks that are involved in this research study:

 **8/2/13**  
Signature and printed name of principal investigator or person obtaining consent Date

**\* 1. CONSENT**

By clicking accept below, you confirm that you are 18 years of age or older and have read or had this document read to you. You have been informed about this study's purpose, procedures, possible benefits and risks, and you have received a copy of this form. You have been given the opportunity to ask questions before you sign, and you have been told that you can ask other questions at any time.

You voluntarily agree to participate in this study. By accepting this form, you are not waiving any of your legal rights. Refusal to participate will involve no penalty or loss of benefits to which you are otherwise entitled. You may discontinue participation at any time without penalty.

- Accept
- Decline

Next

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Engaging Students in Mathematical Conversation: Teachers' Questions Eliciting Students Responses

**Section I**

Please respond to each of the questions below by checking your answer or filling in the blank.

**1. What is the highest level of education that you have completed? Select only one.**

- Bachelor's Degree
- Master's Degree
- Sixth Year Degree
- Doctoral Degree
- Other (please specify)

**2. Currently, what type of teaching certificate do you hold?**

- Initial
- Provisional
- Professional
- Other (please specify)

**3. How many years have you taught in any school?**

**4. List the courses you are currently assigned to teach.**

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Engaging Students in Mathematical Conversation: Teachers' Questions Eliciting Students Responses

**Section II: Directions**

Please respond to the following sentences. Read each sentence carefully and respond to them honestly. There are no right or wrong answers.

**1. It is important to me as a teacher that I learn something new about myself in class.**

Not True At All      Somewhat True      True      Absolutely True

                

**2. It is important to me as a teacher that I can avoid extra tasks.**

Not True At All      Somewhat True      True      Absolutely True

                

**3. It is important to me as a teacher that I impress my students with my knowledge.**

Not True At All      Somewhat True      True      Absolutely True

                

**4. It is important to me as a teacher that I do not fail with difficult tasks.**

Not True At All      Somewhat True      True      Absolutely True

                

**5. It is important to me as a teacher that my abilities as a teacher are recognized and appreciated.**

Not True At All      Somewhat True      True      Absolutely True

                

**6. It is important to me as a teacher that my experiences in the classroom motivate me to learn more about how to be a good teacher.**

Not True At All      Somewhat True      True      Absolutely True

                

**7. It is important to me as a teacher that I have as few disruptive students as possible.**

Not True At All      Somewhat True      True      Absolutely True



Not True At All      Somewhat True      True      Absolutely True

**8. It is important to me as a teacher that I give a really good account of myself in the class.**

Not True At All      Somewhat True      True      Absolutely True

                

**9. It is important to me as a teacher that I do not make a fool of myself in front of the students.**

Not True At All      Somewhat True      True      Absolutely True

                

**10. It is important to me as a teacher that I get through the day with as little effort as possible.**

Not True At All      Somewhat True      True      Absolutely True

                

**11. It is important to me as a teacher that I have plenty of interesting activities to keep me busy throughout the day.**

Not True At All      Somewhat True      True      Absolutely True

                

**12. It is important to me as a teacher that I am able to use teaching materials from previous years and therefore do not have to prepare so much.**

Not True At All      Somewhat True      True      Absolutely True

                

**13. It is important to me as a teacher that I do not make a mess of things in any of my classes.**

Not True At All      Somewhat True      True      Absolutely True

                

**14. It is important to me as a teacher that preparing for a class allows me to gain a deeper understanding of the actual topic I am to teach.**

Not True At All      Somewhat True      True      Absolutely True

                

**15. It is important to me as a teacher that I am able to overcome difficult situations in my class.**

Not True At All      Somewhat True      True      Absolutely True

                

**16. It is important to me as a teacher that no one notices when I do not understand something.**

Not True At All      Somewhat True      True      Absolutely True

                

 

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Appendix B  
Teacher Survey Results

	Teaching Motivation Survey Average			
Teachers	Mastery	Ability-Approach	Ability-Avoidance	Work-Avoidance
Amber	2	3.5	2	2
Brandi	3.5	1.25	2	2

Appendix C  
Parent Permission Letter

Dear Parents and Students:

My name is Glenda Mitchell and I am a mathematics instructor at UT Arlington. I am involved in the education process of preparing students to be future teachers. I am currently working on my dissertation for a doctoral degree in K16 Educational Leadership and Policy Studies at the University of Texas – Arlington. Currently, mathematics educators advocate that teachers should create classrooms where students are engaged in conversation about mathematical ideas. Although, to achieve these goals, it is important that teachers understand how to engage students in discussion. The purpose of this study is to identify teacher questions and student responses in a problem solving environment and how teachers use questioning to engage students in conversation.

In collaboration with your child's precalculus teacher, I am planning on video recording two problem solving activities in your child's math classroom. The findings will provide actual descriptions of the dialogue that transpires between teacher and students as mathematical ideas are developing. This activity poses no known safety concerns.

The information collected on the dialogue of teacher and students will be summarized, interpreted, and used to help improve teaching and learning of mathematics. The overall findings may also be presented at national and international conferences and/or published in scholarly journals in order to communicate insights for improving mathematics education on a broader scale.

Be assured that all information gathered will be held in strict confidence, and no names or other identifying information will be used in the analysis and reports of findings from this study. Once the video recordings of the dialogue have been analyzed, they will be destroyed. The findings of this research will be available for your review upon your request.

If you do not wish for your child to be video or audio recorded during the class time, then I will have your child in the room but not a part of the video recording. Your child will not be able to verbally interact with the discussion but he/she will be given time after the class to ask questions. I do want your child to learn the concepts for the daily activity.

If you agree or do not agree to allow me to video record your child during the two class periods for the purposes of this study, please indicate your choice by signing on the following (attached) page in the appropriate blanks. If you do not return the consent form, your child will not be a part of the video or audio recording during the class time. Please respond to this email address ([gmitchel@uta.edu](mailto:gmitchel@uta.edu)) or phone Glenda Mitchell (817-201-2583) if you have any questions about the study.

Thank you for your consideration.

Sincerely,

Glenda Mitchell  
Doctoral Student at UTA

**APPROVED**

AUG 13 2013

Institutional Review Board

Please fill in the following and return to your child's precalculus teacher as soon as possible if you choose **to consent**:

I hereby **give** consent to my child being recorded during their classroom activities for your research purposes. I understand that all information collected in this study will be held in strict confidence. I also understand that I may choose to have my child withdrawn from the study at any time.

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Parent Signature	Printed Name	Date
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Child's Name	Child's School	Child's Teacher
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Please fill in the following and return to your child's precalculus teacher as soon as possible if you choose **not to consent**:

I hereby **do not give** consent to my child being recorded during their classroom activities for your research purposes. I understand that he/she will be in the classroom but not seen on the recording nor verbally interacting in the session. After the recording, he/she may ask questions concerning the concepts investigated, if he/she needs clarification.

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Parent Signature	Printed Name	Date
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Child's Name	Child's School	Child's Teacher
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**APPROVED**

AUG 13 2013

Institutional Review Board

Appendix D  
Principal Permission Letter



Dear (Principal's Name);

My name is Glenda Mitchell and I am a mathematics instructor at UT Arlington. I am involved in the education process of preparing students to be future teachers. I am currently working on my dissertation for a doctoral degree in K16 Educational Leadership and Policy Studies at the University of Texas – Arlington. Currently, mathematics educators advocate that teachers should create classrooms where students are engaged in conversation about mathematical ideas. Although, to achieve these goals, it is important that teachers understand how to engage students in discussion. The purpose of this study is to identify teacher questions and student responses in a problem solving environment and how teachers use questioning to engage students in conversation. The findings will provide actual descriptions of the dialogue that transpires between teachers and students as mathematical ideas are developing. In turn, it will seek to contribute to the mathematical research of constructivist learning theory as dialogue between teachers and students commences. This theory espouses that teachers provide opportunities for students to formulate ideas and concepts in their own minds through experiential learning, rather than the teachers simply giving students information to learn. Finally, while this study will analyze the link between teachers' motivations for teaching and their teaching practices, it will seek to further knowledge on the influence of motivational theory with respect to the teacher.

Since my interest is with private Christian school education, I am trying to secure a sample population of precalculus teachers among private Christian schools in the DFW area. Participants would go online to Survey Monkey to complete the Teacher's Goal Orientation Questionnaire which takes about ten to fifteen minutes. From the teachers surveyed, two of the teachers will be asked to further participate in two classroom observations of their questioning and discussion practices with their students during problem solving activities. The information from the observation and the video recording of the class session will be transcribed and analyzed. Video recordings will be destroyed after they are transcribed. Neither, the teacher, his/ her students, nor your institution will be identified in this study. Participation in both the questionnaire and classroom observations is completely voluntary, and participants may withdraw from involvement in the study at any time without explanation or repercussion. There will be no compensation for participation in this study.

Once all the data have been collected, analyzed, and documented, I will be happy to share a summary of my findings upon your request. This study will also conform to the ethical research guidelines of UTA, which serve as protection for both the participants and the researcher.

Please complete the attached permission form for the principal and return it to:

Glenda Mitchell  
[gmitchel@uta.edu](mailto:gmitchel@uta.edu) or  
FAX: (817)459-4687

The parent/student permission form is also attached. I will send them out after the teacher has agreed to participate in the study. Thank you for your cooperation in allowing me to complete this study.

Sincerely,

Glenda Mitchell  
Doctoral Student at University of Texas at Arlington

## References

- Abdi, Y. (2007). *Analysis of teachers' discourse moves in an open-ended problem solving environment*. (Order No. 3274175, Rutgers The State University of New Jersey - New Brunswick). *ProQuest Dissertations and Theses*, , 243. Retrieved from <http://ezproxy.uta.edu/docview/304806173?accountid=7117>. (304806173).
- Aguirre, J., & Speer, N. M. (1999). Examining the relationship between beliefs and goals in teacher practice. *The Journal of Mathematical Behavior*, 18(3), 327-356. doi: 10.1016/S0732-3123(99)00034-6
- Ball, D. L., Lubienski, S. T., & Mewborn, D. S. (2001). Research on teaching mathematics: The unsolved problem of teachers' mathematical knowledge. *Handbook of Research on Teaching*, 4, 433-456.
- Ball, D. L., & Friel, S. N. (1991). Implementing the "professional standards for teaching mathematics": Improving, not standardizing, teaching. *The Arithmetic Teacher*, 39(1), 18-22. Retrieved from <http://www.jstor.org/stable/41194896>
- Barnes, D. (1990). 3 Oral language and learning. *DOCUMENT RESUME ED 318 076 CS 507 150*, 41. Retrieved from <http://files.eric.ed.gov/fulltext/ED318076.pdf#page=47>
- Baxter, J. A., & Williams, S. (2010). Social and analytic scaffolding in middle school mathematics: Managing the dilemma of telling. *Journal of Mathematics Teacher Education*, 13(1), 7-26.
- Bennett, C. A. (2010). "It's hard getting kids to talk about math": Helping new teachers improve mathematical discourse. *Action in Teacher Education*, 32(3), 79-89. Retrieved from

<http://search.ebscohost.com/login.aspx?direct=true&db=eric&AN=EJ915855&site=ehost-live>; <http://www.rowmaneducation.com/Journals/ATE/Index.shtml>

- Borko, H. (2004). Professional development and teacher learning: Mapping the terrain. *Educational Researcher*, 33(8), 3-15.
- Butler, R. (2007). Teachers' achievement goal orientations and associations with teachers' help-seeking: Examination of a novel approach to teacher motivation. *Journal of Educational Psychology*, 99, 241-252.
- Cazden, C. B. (2001). The language of teaching and learning. *The language of teaching and learning*. Retrieved from <http://orange.schoolwires.com/cms/lib7/NJ01000601/Centricity/Domain/28/Classroom%20Discourse.pdf>
- Cengiz, N., Kline, K., & Grant, T. J. (2011). Extending students' mathematical thinking during whole-group discussions. *Journal of Mathematics Teacher Education*, 14(5), 355-374.
- Cheval, K. (2009, January 1). Beginning the Year in a Fifth-Grade Reform-Based Mathematics Classroom: A Case Study of the Development of Norms. ProQuest LLC.
- Clausen, M. (2002). *Quality of education: A problem of perspective?*. Munster: Waxmann.
- Cobb, P. & Bauersfeld, H. (1996). Introduction: The coordination of psychological and sociological perspectives in mathematics education. In Steffe, L., Neshier, P., Cobb, P., Goldin, G., & Greer, B. (Eds.), *Theories of mathematical learning* (pp. 21-50). Mahwah, NJ: Erlbaum.
- Cobb, P., Boufi, A., McClain, K., & Whitenack, J. (1997). Reflective discourse and collective reflection. *Journal for research in mathematics education*, 28, 258-277.

- Cobb, P., & Steffe, L. P. (2011). The constructivist researcher as teacher and model builder. *A Journey in Mathematics Education Research*, (pp. 19-30). Springer: Netherlands. doi: 10.1007/978-90-481-9729-3\_3
- Cobb, P., & Yackel, E. (1996). Constructivist, emergent, and sociocultural perspectives in the context of developmental research. *Educational Psychologist*, 31(3-4), 175-190.
- Confrey, J., & Maloney, A. (2006). From constructivism to modeling. In The Second Annual Meeting of Middle East Teachers of Science, Mathematics, and Computing, Abu Dhabi, UAE.
- Cotton, K. (1989). *Expectations and student outcomes*. Portland, OR: Northwest Regional Educational Laboratory.
- Creswell, J. (2007). *Qualitative inquiry and research design: Choosing among five approaches*. Thousand Oaks, CA: Sage Publications.
- Cunningham, R. (1987). What kind of question is that? In W. W. Wilem (Eds.), *Questions, questioning techniques, and effective teaching* (pp. 67-94). Washington, DC: National Education Association.
- Dann, E., Pantozzi, R., & Steencken, E.(1995). "Unconsciously learning something": A focus on teacher questioning. (Ed.), *Psychology of Mathematics Education* (pp.276 – 284). Columbus, Ohio: ERIC Clearinghouse for Science, Mathematics, and Environmental Education.
- Darling-Hammond, L. (2000). How teacher education matters. *Journal of Teacher Education*, 51(3), 166-173.
- Darling-Hammond, L. (2006). Constructing 21st-century teacher education. *Journal of Teacher Education*, 57(3), 300-314.

- Davis, B. (1997). Listening for differences: An evolving conception of mathematics teaching. *Journal for Research in Mathematics Education*, 355-376.
- Davis, R. B., & Maher, C. A. (1997). How students think: The role of representations. In English, L. (Ed.), *Mathematical Reasoning: Analogies, Metaphors, and Images* (pp. 93-116). Mahwah, NJ: Erlbaum.
- Denzin, N. K. (2012). Triangulation 2.0. *Journal of Mixed Methods Research*, 6(2), 80-88.
- Dick, T., & Springer, G. T. (2006). Making the right (discourse) moves: Facilitating discussions in the mathematics classroom. *Mathematics teacher*, 100(2), 105-109.
- Ernest, P. (1996). Varieties of constructivism: A framework for comparison. In Steffe, L., Neshier, P., Cobb, P., Goldin, G., & Greer, B. (Eds.), *Theories of mathematical learning* (pp. 335-350). Mahwah, NJ: Erlbaum.
- Falle, J. (2004). Let's Talk Maths: A Model for Teaching to Reveal Student Understandings. *Australian Senior Mathematics Journal*, 18(2), 17-27.
- Floden, R. (1997). Reforms that call for teaching more than you understand. *Teaching and its Predicaments*, In Burbules, N. & Hansen, D. (Eds.), *Teaching and its Predicaments* (pp. 11-28). Boulder: Westview.
- Forman, E. A. (1996). Participation in classroom practice: Implications of sociocultural theory for educational reform. In Steffe, L., Neshier, P., Cobb, P., Goldin, G., & Greer, B. (Eds.), *Theories of mathematical learning* (pp. 115-130). Mahwah, NJ: Erlbaum.
- Francisco, J. M., & Maher, C. A. (2005). Conditions for promoting reasoning in problem solving: Insights from a longitudinal study. *The Journal of Mathematical Behavior*, 24(3), 361-372.

- Franke, M., Carpenter, T., Levi, L., & Fennema, E. (2001). Capturing teachers' generative change: A follow-up study of professional development in mathematics. *American Educational Research Journal*, 38(3), 653-689. Retrieved from <http://www.jstor.org/stable/3202495>
- Gall, M. D. (1970). The use of questions in teaching. *Review of educational research*, 707-721.
- Gall, M., Gall, J., & Borg, W. (2007). *Educational research: An introduction* (8th ed.). Boston: Pearson.
- Glenn, R.E. (2001). What teachers need to be. *Education Digest*, 67(1), 19 – 22.
- Goos, M. (2004). Learning mathematics in a classroom community of inquiry. *Journal for Research in Mathematics Education*, 35(4), 258-291.
- Hannel, G. I. (2003). *Highly effective questioning: Developing seven steps of critical thinking*. Phoenix, AZ: Hannel Educational Consulting.
- Harris, R. L. (2000). Batting 1,000: Questioning techniques in student-centered classrooms. *Clearing House*, 74(1), 25-26.
- Herbst, P. G. (2002). Engaging students in proving: A double bind on the teacher. *Journal for Research in Mathematics Education*, 33(3), 176-203.
- Hiebert, J., & Wearne, D. (1993). Instructional tasks, classroom discourse, and students' learning in second-grade arithmetic. *American educational research journal*, 30(2), 393-425.
- Himmelberger, K. S., & Schwartz, D. L. (2007). It's a home run! Using mathematical discourse to support the learning of statistics. *Mathematics Teacher*, 101(4), 250-256.
- Hoffman, B. L., Breyfogle, M. L., & Dressler, J. A. (2009). The power of incorrect answers. *Mathematics Teaching in the Middle School*, 15(4), 232-238.

- Hufford-Ackles, K., Fuson, K. C., & Sherin, M. G. (2004). Describing levels and components of a math-talk community. *Journal for Research in Mathematics Education*, 35(2), 81-116.
- Ilaria, D. R. (2009). *Teacher questions that engage students in mathematical conversation* (Doctoral Dissertation). Retrieved from ProQuest Dissertations and Theses. (304990530).
- Inoue, N. (2011). You asked open-ended questions now what? Understanding the nature of stumbling blocks in teacher inquiry lessons. *The Mathematics Educator*, 20(2), 10-23. Retrieved from <http://www.eric.ed.gov/contentdelivery/servlet/ERICServlet?accno=EJ923721>
- Kitchen, R. S. (2004). Challenges associated with developing discursive classrooms in high-poverty, rural schools. *MATHEMATICS TEACHER-WASHINGTON THEN RESTON VA-*, 97(1), 28-33.
- Kotsopoulos, D. (2007). It's like hearing a foreign language. *Mathematics Teacher*, 101(4), 301-305.
- Knott, L., Sriraman, B., & Jacob, I. (2008). A Morphology of Teacher Discourse in the Mathematics Classroom. *The Mathematics Educator*, 11(1/2), 89-110.
- Lampert, M., & Cobb, P. (2003). Communication and language. In Kilpatrick, J., Martin, G., & Shifter, D. (Eds.), *A research companion to principles and standards for school mathematics* (pp. 237-249). Reston, VA: NCTM.
- Lampert, M., Rittenhouse, P., & Crumbaugh, C. (1996). Agreeing to disagree: Developing sociable mathematical discourse. In Olson, D. & Torrance, N. (Eds.), *The handbook of education and human development: New models of learning, teaching, and schooling* (pp. 731-764). Oxford, England: Blackwell.



- Lincoln, Y. & Guba, E., (1985). *Naturalistic Inquiry*. New York: Sage Publications.
- Lobato, J., Clarke, D., & Ellis, A. B. (2005). Initiating and eliciting in teaching: A reformulation of telling. *Journal for research in mathematics education*, 101-136.
- Lubienski, S. T. (2000). Problem solving as a means toward mathematics for all: An exploratory look through a class lens. *Journal for Research in Mathematics Education*, 454-482.
- Maher, C. A. (2005). How students structure their investigations and learn mathematics: Insights from a long-term study. *The Journal of Mathematical Behavior*, 24(1), 1-14.
- Maher, C. A., & Alston, A. (1990). Teacher development in mathematics in a constructivist framework. In Davis, R., Maher, C., & Noddings, N. (Eds.), *Constructivist Views on the Teaching and Learning of Mathematics* (p. 147-165). Reston, VA: NCTM. Retrieved from <http://www.jstor.org/stable/749918>
- Manouchehri, A. (2007). Inquiry-Discourse Mathematics Instruction. *Mathematics Teacher*, 101(4), 290-300.
- Manouchehri, A., & Enderson, M. C. (1999). Promoting mathematical discourse: Learning from classroom examples. *Mathematics Teaching in the Middle School*, 4(4), 216-222.
- Manouchehri, A., & Lapp, D. A. (2003). Unveiling student understanding: The role of questioning in instruction. *MATHEMATICS TEACHER-WASHINGTON THEN RESTON VA-*, 96(8), 562-573.
- Manouchehri, A., & St John, D. (2006). From classroom discussions to group discourse. *Mathematics Teacher*, 99(8), 544-552.
- Marksberry, M. L. (1979). Student Questioning: An Instructional Strategy. *Educational Horizons*, 57(4), 190-95.

- Martino, A. M., & Maher, C. A. (1999). Teacher questioning to promote justification and generalization in mathematics: What research practice has taught us. *The Journal of Mathematical Behavior*, 18(1), 53-78. doi: 10.1016/S0732-3123(99)00017-6
- Merriam, S. B. (1998). *Qualitative research and case study applications in education*. San Francisco, CA: Jossey-Bass.
- McTighe, J. & Wiggins, G. P. (2013). *Essential questions: Opening doors to student understanding*. Association for Supervision and Curriculum Development.
- Mewborn, D. & Huberty, P. (1999). Questioning your way to the standards. *Teaching Children Mathematics*, 6(4), 226-232.
- Nathan, M. J., & Knuth, E. J. (2003). A study of whole classroom mathematical discourse and teacher change. *Cognition and instruction*, 21(2), 175-207.
- National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: NCTM.
- Nitsche, S., Dickhäuser, O., Fasching, M. S., & Dresel, M. (2011). Rethinking teachers' goal orientations: Conceptual and methodological enhancements. *Learning and Instruction*, 21(4), 574-586.
- Noddings, N. (1990). Constructivism in mathematics education. In Davis, R., Maher, C., & Noddings, N. (Eds.), *Constructivist Views on the Teaching and Learning of Mathematics* (pp. 7-18). Reston, VA: NCTM. Retrieved from <http://www.jstor.org/stable/749909>
- O'Connor, M. C., & Michaels, S. (1993). Aligning academic task and participation status through revoicing: Analysis of a classroom discourse strategy. *Anthropology and Education Quarterly*, 24, 318-318.
- Piaget, J. (1952). *The Origins of Intelligence in Children*. (M. Cook, Trans. 2nd ed.). New York: International Universities Press. (Original work published 1936).

- Piaget, J. (1965). Educational Psychology in Context. In Marlowe, J. & Canestrari, A. (Eds.), *Educational psychology in context: Readings for future teachers* (pp.98-106). Thousand Oaks, CA: Sage.
- Piaget, J. (1995). Comments on Mathematical Education. In Gruber, H. & Voneche, J. (Eds.) *The essential Piaget: An interpretive reference and guide* (pp.726 -732). New York, NY: Rowan & Littlefield.
- Pimm, D. (1987). *Speaking mathematically: Communication in Mathematics Classrooms*. London: Routledge & Kegan Paul.
- Pintrich, P. (2000). Multiple goals, multiple pathways: The role of goal orientation in learning and achievement. *Journal of Educational Psychology*, 92(3), 544.
- Pirie, S. E., & Kieren, T. E. (1994). Beyond metaphor: Formalising in mathematical understanding within constructivist environments. *For the learning of Mathematics*, 39-43.
- Phillips, J. L. (1981). *Piaget's theory: A primer*. San Francisco: WH Freeman.
- Powell, A., Francisco, J., & Maher, C. (2003). An analytical model for studying the development of learners' mathematical ideas and reasoning using videotape data. *The Journal of Mathematical Behavior*, 22(4), 405-435.
- Retelsdorf, J., Butler, R., Streblov, L., & Schiefele, U. (2010). Teachers' goal orientations for teaching: Associations with instructional practices, interest in teaching, and burnout. *Learning and Instruction*, 20(1), 30-46.
- Retelsdorf, J., & Günther, C. (2011). Achievement goals for teaching and teachers' reference norms: Relations with instructional practices. *Teaching and Teacher Education*, 27(7), 1111-1119.
- Reynolds, D., & Muijs, D. (1999). The effective teaching of mathematics: A review of research. *School Leadership & Management*, 19(3), 273-288.

- Rheinberg, F. (1988). Motivation and learning activities: How research could proceed. *International Journal of Educational Research*, 12(3), 299-306. doi: 10.1016/0883-0355(88)90007-9
- Rice, K. (2002). Surviving your first year in the classroom. *Middle Ground*, 10(1), 43-45.
- Richardson, V. & Placier, P. (2001). Teacher change. In Richardson, V. (Ed.), *Handbook of research on teaching* (pp. 905-947). Washington, DC: American Educational Research Association.
- Rittenhouse, P. S. (1998). The teacher's role in mathematical conversation: Stepping in and stepping out. In Lampert, M & Blunk, M.L. (Eds.), *Talking mathematics in school: Studies of teaching and learning* (pp.163-189). Cambridge: Cambridge University Press.
- Schoenfeld, A. H. (1987). Cognitive Science and Mathematics Education. In Schoenfeld, A. H. (Ed.), *What's all the fuss about metacognition* (p.189). Hillsdale, NJ: Erlbaum.
- Sfard, A. (1998). On two metaphors for learning and the dangers of choosing just one. *Educational Researcher*, 27(2), 4-13.
- Sfard, A. (2003). Balancing the unbalanceable: the NTCM standards in light of theories of learning mathematics. In Kilpatrick, J., Martin, G.W., & Schifter, D. (Eds.), *A research companion to principles and standards for school mathematics* (pp. 353-392). Reston, VA: NCTM.
- Sfard, A. (2008). Thinking as communicating: Human development, the growth of discourses, and mathematizing. Cambridge, UK: Cambridge University Press.
- Soucy McCrone, S. (2005). The development of mathematical discussions: An investigation in a fifth-grade classroom. *Mathematical Thinking and Learning*, 7(2), 111-133.

- Speiser, R., Maher, C. A., & Walter, C. N. (2001). *Proceedings of the annual meeting of the North American chapter of the international group for the psychology of mathematics education (Snowbird, Utah, October 18-21, 2001)*. Volume 1 [and] Volume 2. Columbus, OH: ERIC.
- Staples, M. & Colonis, M. M. (2007). Making the most of mathematical discussions. *Mathematics Teacher*, 101(4), 257-261.
- Steele, A. (2003). Just get out of the way. *The Mathematics Educator* 13(2), 58 -59.
- Stylianou, D. A., & Blanton, M. L. (2002). Sociocultural factors in undergraduate mathematics: The role of explanation and justification. Paper presented at the *Proceedings of the Second International Conference on the Teaching of Mathematics, Crete, Greece*.
- Suurtamm, C., & Vézina, N. (2010). Transforming pedagogical practice in mathematics: Moving from telling to listening. *International Journal for Mathematics Teaching & Learning*.
- Thompson, P. (1996). Imagery and the development of mathematical reasoning. In Steffe, L., Neshier, P., Cobb, P., Goldin, G., & Greer, B. (Eds.), *Theories of mathematical learning* (pp. 267-284). Mahwah, NJ: Erlbaum.
- Tienken, C. H., Goldberg, S., & Dirocco, D. (2009). Questioning the questions. *Kappa Delta Pi Record*, 46(1), 39-43.
- Truxaw, M. & DeFranco, T. (2007). Lessons from mr. laron: An inductive model of teaching for orchestrating discourse. *Mathematics Teacher*, 101(4), 268-272.
- Vacc, N. (1993). Questioning in the mathematics classroom. *Arithmetic Teacher*, 88 – 91.

- Van Driel, J. H., Bulte, A. M., & Verloop, N. (2007). The relationships between teachers' general beliefs about teaching and learning and their domain specific curricular beliefs. *Learning and Instruction, 17*(2), 156-171.
- Van Oers, B. (1996). Learning mathematics as a meaningful activity. In Steffe, L., Neshier, P., Cobb, P., Goldin, G., & Greer, B. (Eds.), *Theories of mathematical learning* (pp. 91-113). Mahwah, NJ: Erlbaum.
- Van Zee, E., & Minstrell, J. (1997). Using questioning to guide student thinking. *The Journal of the Learning Sciences, 6*(2), 227-269.
- Van Zoest, L. R., & Enyart, A. (1998). Discourse, of Course: Encouraging Genuine Mathematical Conversations. *Mathematics Teaching in the Middle School, 4*(3), 150-57.
- Voigt, J. (2007). Negotiation of mathematical meaning in classroom processes: Social interaction and learning mathematics. In Steffe, L., Neshier, P., Cobb, P., Goldin, G., & Greer, B. (Eds.), *Theories of mathematical learning* (pp. 21-50). Mahwah, NJ: Erlbaum.
- von Glasersfeld, E. (1990). Environment and communication. *Transforming children's mathematics education: International perspectives*, 30-38.
- Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes*. Cambridge, MA: Harvard University Press.
- Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education*, 458-477.
- Yackel, E. (2002). What we can learn from analyzing the teacher's role in collective argumentation. *The Journal of Mathematical Behavior, 21*(4), 423-440.
- Yin, R. K. (2003). *Case Study Research Design and Methods*. Thousand Oaks, CA: Sage.

- Watson, J. M., & Chick, H. L. (2001). Factors influencing the outcomes of collaborative mathematical problem solving: An introduction. *Mathematical Thinking and Learning*, 3(2/3), 125-173.
- Wayne, A., & Youngs, P. (2003). Teacher characteristics and student achievement gains: A review. *Review of Educational Research*, 73(1), 89-122.
- Weber, K., Maher, C., Powell, A., & Lee, H. S. (2008). Learning opportunities from group discussions: Warrants become the objects of debate. *Educational Studies in Mathematics*, 68(3), 247-261.
- White, D. (2003). Promoting productive mathematical classroom discourse with diverse students. *Journal of Mathematical Behavior*, 22(1), 37-53.
- Wolf, D. P. (1987). The art of questioning. *Journal of State Government*, 60(2), 81-85.
- Woolfolk, A. (1998). *Educational psychology* (2<sup>nd</sup> ed.). Boston, MA: Allyn and Bacon.

### Biographical Information

Prior to completing a Doctor of Philosophy in Educational Leadership and Policy Studies at the University of Texas at Arlington, Glenda Mitchell completed a Master of Science in Mathematics at the same university. She has worked as a high school mathematics teacher for 20 years in public schools. She now works as a lecturer in the Mathematics Department at University of Texas at Arlington and teaches mathematics classes for future K-8 teachers. Her research interests include teacher training, effective instructional methodology, and educational motivations. Her current plans are to continue training future teachers and to help affect change in mathematics education.