STRUCTURAL ANALYSIS AT AIRCRAFT CONCEPTUAL DESIGN STAGE

by

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Abstract

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In the past 50 years, computers have helped by augmenting human efforts with tremendous pace. The aircraft industry is not an exception. Aircraft industry is more than ever dependent on computing because of a high level of complexity and the increasing need for excellence to survive a highly competitive marketplace. Designers choose computers to perform almost every analysis task. But while doing so, existing effective, accurate and easy to use classical analytical methods are often forgotten, which can be very useful especially in the early phases of the aircraft design where concept generation and evaluation demands physical visibility of design parameters to make decisions [39, 2004]. Structural analysis methods have been used by human beings since the very early civilization. Centuries before computers were invented; the pyramids were designed and constructed by Egyptians around 2000 B.C, the Parthenon was built by the Greeks, around 240 B.C, Dujiangyan was built by the Chinese. Persepolis, Hagia Sophia, Taj Mahal, Eifel tower are only few more examples of historical buildings, bridges and monuments that were constructed before we had any advancement made in computer aided engineering. Aircraft industry is no exception either. In the first half of the 20th century, engineers used classical method and designed civil transport aircraft such as Ford Tri Motor (1926), Lockheed Vega (1927), Lockheed 9 Orion (1931),

Douglas DC-3 (1935), Douglas DC-4/C-54 Skymaster (1938), Boeing 307 (1938) and Boeing 314 Clipper (1939) and managed to become airborne without difficulty.

Evidencing, while advanced numerical methods such as the finite element analysis is one of the most effective structural analysis methods; classical structural analysis methods can also be as useful especially during the early phase of a fixed wing aircraft design where major decisions are made and concept generation and evaluation demands physical visibility of design parameters to make decisions. Considering the strength and limitations of both methodologies, the question to be answered in this thesis is: How valuable and compatible are the classical analytical methods in today's conceptual design environment? And can these methods complement each other?

To answer these questions, this thesis investigates the pros and cons of classical analytical structural analysis methods during the conceptual design stage through the following objectives:

Illustrate structural design methodology of these methods within the framework of Aerospace Vehicle Design (AVD) lab's design lifecycle.

Demonstrate the effectiveness of moment distribution method through four case studies. This will be done by considering and evaluating the strength and limitation of these methods. In order to objectively quantify the limitation and capabilities of the analytical method at the conceptual design stage, each case study becomes more complex than the one before.

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Notations

Abbreviations

AVDS	=	Aerospace Vehicle Design Synthesis
CAE	=	Computer Aided Engineering
CMH	=	Composite Materials Handbook
CD	=	Conceptual Design
DBS	=	Data Base System
DD	=	Detail Design
DF	=	Distribution Factor
DOF	=	Degree of Freedom
FAA	=	Federal Aviation Administration
FARS	=	Federal Aviation Regulations
FEA	=	Finite Element Analysis
FEM	=	Finite Element Method
FT/C/M	=	Flight Test/Certification/Manufacturing
I/AI	=	Incident/Accident Investigation
KBS	=	Knowledge Base System
O	=	Operations
PD	=	Preliminary Design
MDM	=	Moment Distribution Method
ROM	=	Reduced Order Modeling

Chapter 1

Introduction

1.1 Research Initiation and Motivation

As the condition of the aircraft market starts recovering from the recession of recent years [29, 2000] and the concerns over the environment and our ever growing consumption of oil rises to its highest level, and with state regulations aimed at making future emission and oil consumption reductions, it is possible that the current historically established fixed wing transportation projects and aircraft configuration becomes unacceptable to function in tomorrow's unknown environmental and political circumstances [5, 2001]. Gary Coleman writes:

> "These driven forces have led to design environments to explore a variety of different aircraft configurations in the pursuit of better performance and efficiency" [7, 2007].

The rising cost of jet fuel has seriously affected the profitability of all industry airline carriers. A one-cent increase in the price of a gallon of fuel translates into an additional $25 million annual cost for American Airlines, according to reports obtained from 2011. For additional perspective,

> "By removing just one pound of weight from each aircraft in American's fleet would save more than 11,000 gallons of fuel annually. If 100 pounds of unnecessary weight was removed across the fleet, it would save more than 1.1 million gallons of fuel each year" [10, 2011].

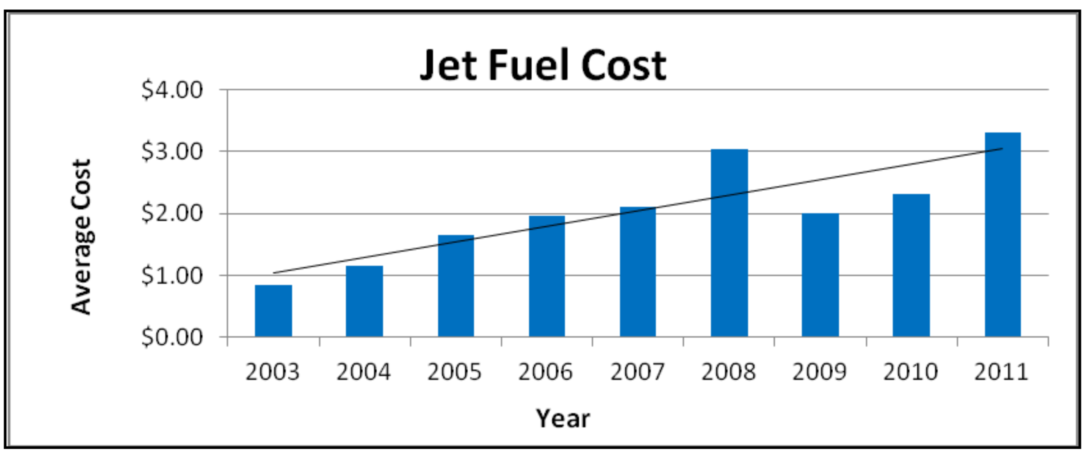


Figure 1-1 Average fuel cost per year [10, 2010]

For that reason, today's fixed wing aircraft industry is more than ever in need for excellence to survive a highly dynamic and competitive marketplace. As the result more customized design and configurations are offered and need to be evaluated [3, 2009]. This industry situation creates a need in the market to use light weight material and alternative light weight designs (configurations) to save cost and make profit without compromising safety enhancements. This makes the highly stiff and low density composites the highly regarded choice. The advancement of composite materials in the last few decades are motivation for the future of airframe structural application as they provide enhanced strength and stiffness to weight ratios. Some examples are some of the latest Boeing products; Boeing 777 and 787. The Boeing 777 uses composites on secondary structures such as wing trailing edge and fairings. The Boeing 787 goes as far as using composite for most of the fuselage and wings.

In the early design stages fundamental decisions are made regarding material selection, structural configuration through a choice of structural analysis methods. The reason it is important to make these decisions at this stage is because:

> "the experience of manufacturers from many industries has shown that 85-90% of the total time and cost of product development is defined in the early stages of product development, this is when 5-10% of project time and cost have been expended" [11, 2000].

To goal of this thesis is to investigate the structural toolbox during the conceptual design phase through comparing Finite Element Analysis method with Moment Distribution Method. Considering the strength and limitations of both methodologies, the question to be answered in this thesis is: How valuable and compatible is the moment distribution method in today's conceptual design environment?? And can moment distribution method and FEA complement each other?

1.2 Basic Definitions

This section includes references to definitions used in this thesis. All the information in this section will be discussed in more detail in the following chapters. These definitions are based on published documents, industry and academic contacts. These terms may be defined differently in other documents based on the context of those documents.

1.2.1 Aircraft:

Aircraft includes lighter than air-craft and heavier than air-craft. Balloons and blimps are lighter than air-craft. Heavier than air-craft are either Rotary or fixed wing. The focus of this thesis in on fixed wing aircraft.

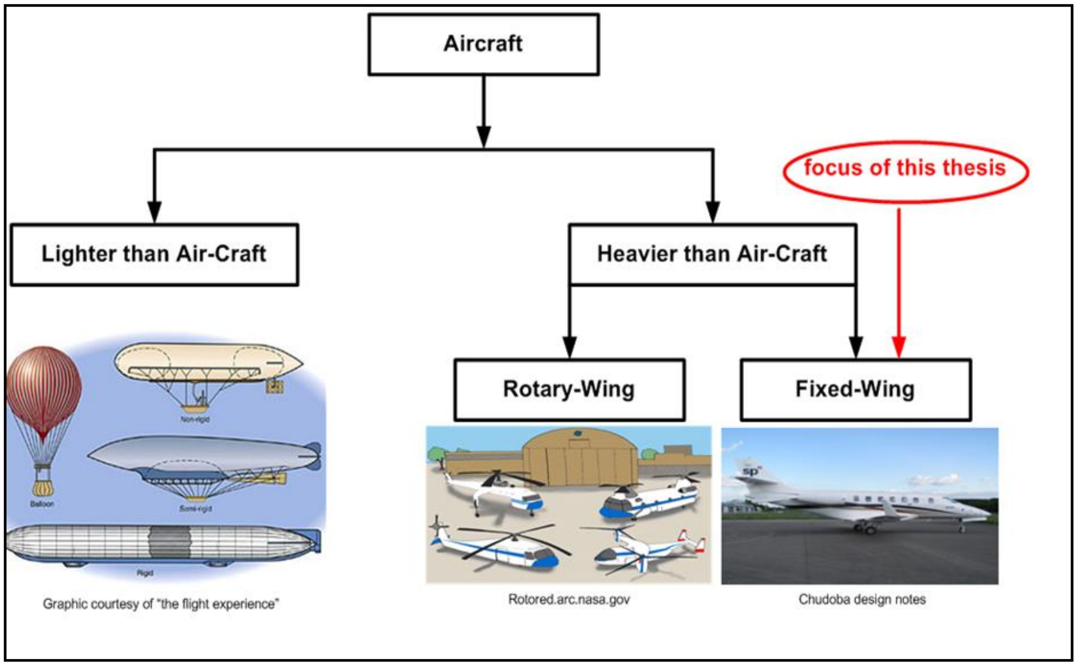


Figure 1-2 Aircraft classification

1.2.2 Design Life Cycle Segments:

The AVD lab uses the term "design lifecycle" to illustrate the life span or life cycle of a fixed wing aircraft, right after the missions and goals for that vehicle have been defined [8, 2010]. The AVD lab divides this life cycle into six continuous phases, which are shown in Figure below:

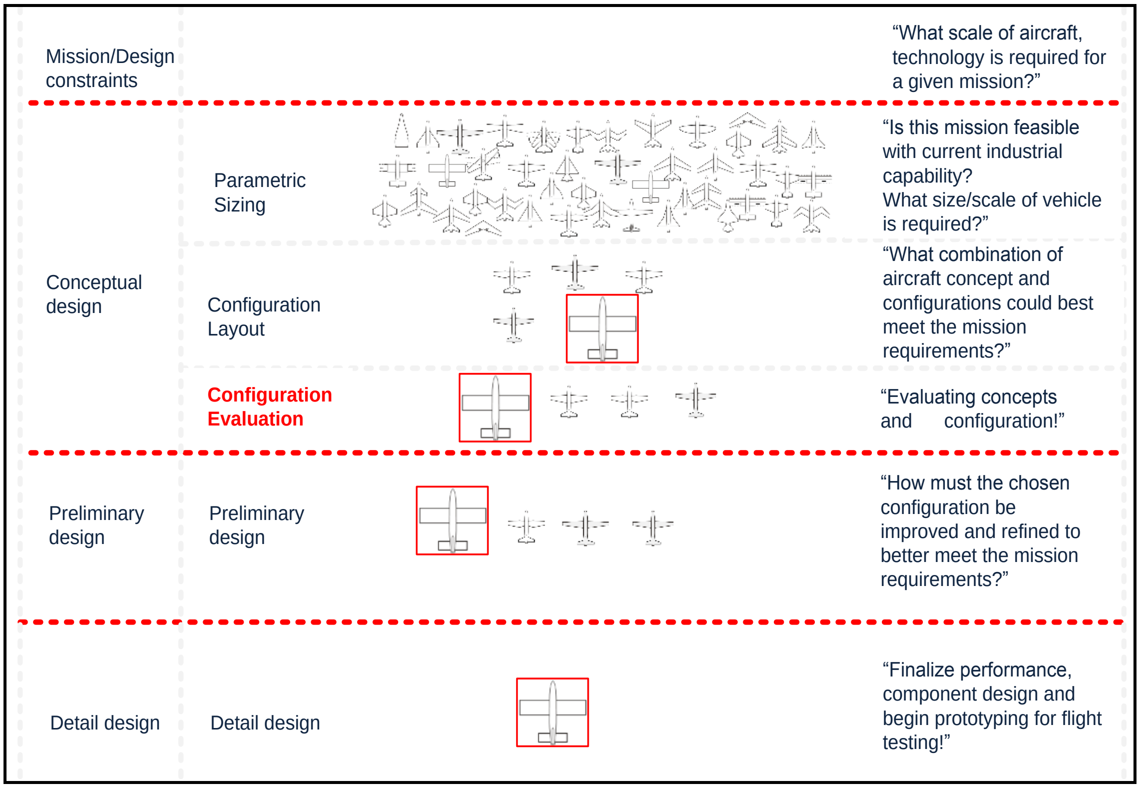


Figure 1-3 Design life cycle segments

Conceptual Design (CD): The Conceptual Design (CD) phase is the first phase in the design process. This stage refers to the time frame that is between mission and goal selection and the most realistic and practical design solutions and concepts. The CD phase itself is further broken down into; parametric sizing (PS), configuration layout (CL) and Configuration evaluation (CE). According to Chudoba:

> "During the PS stage, the goal of a designer is to converge on a specific aircraft design when having visualized the solution space. The aircraft match point is characterized by size required, resulting weight, and power required. The selected match point or converged point design can then be further evaluated through the later CL and CE steps" [4, 2006].

For more detailed information about the PS stage see reference [5, 2001] and [25, 2010]. Based on the parametric sizing results, an initial layout of major aircraft components (i.e., fuselage, wing, propulsion system, etc.) will be determined, "sized" and they will be further analyzed and studied during configuration evaluation (CE) stage [7, 2007]. After the designer has identified the solution space through the PS stage, and once the most feasible aircraft configurations have been selected during the CL stage, conceptual evaluation stage proceeds with multi-disciplinary analysis and studies. This includes but is not limited to: aerodynamics, aeromechanics, structural sizing, propulsion, etc. The goal of this stage is to determine and verify some of the gross aircraft design parameters such as dimensions, weight, engine size, etc for each concept variation [4, 2006][7, 2007]. This thesis concentrates on the structural analysis method during the conceptual evaluation stage of fixed wing aircraft. This is done with particular emphasis on quantifying the pros and cons of these methods for structural analysis during the conceptual evaluation stage, Figure 1-4.

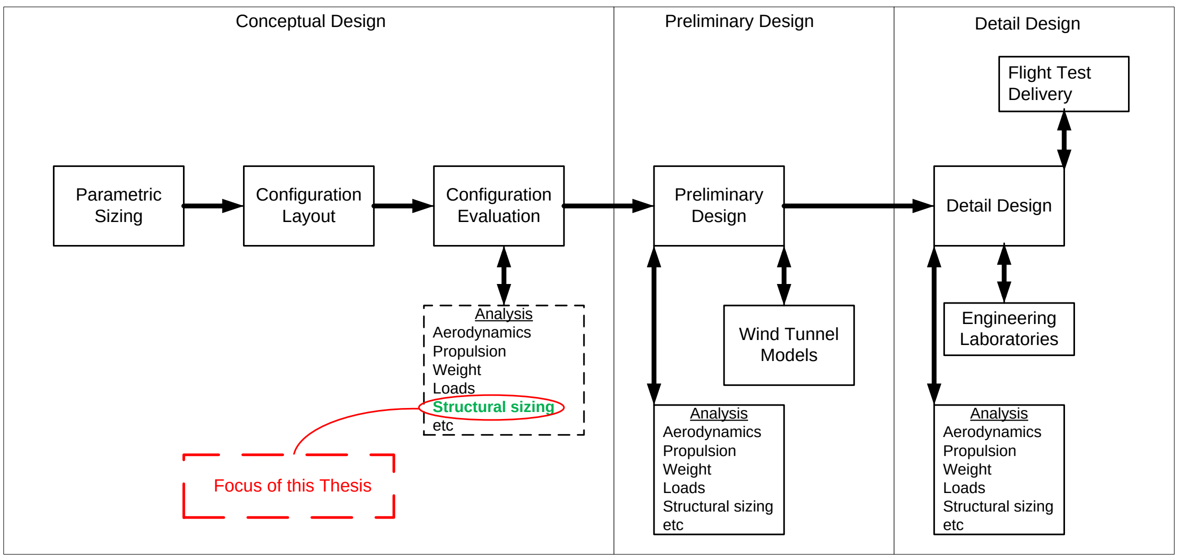


Figure 1-4 Design cycle segments

Preliminary Design (PD): The Preliminary Design (PD) stage starts when major configuration arrangements are expected to remain unchanged and only minor design parameter changes are expected towards improving upon an already selected design concept. At this stage of the life cycle, a design concept is further evaluated and analyzed in order validate and develop efforts that were made at the CD stage. It is during the Preliminary Design stage that the concept becomes frozen (see figure 5). This is confirmed based on the more objective and detailed configuration evaluations.

Detail Design (DD): During the Detail Design (DD) stage the designers become more focused with detail and exact analysis. At this stage, the design will be broken down into more detail parts. The more detailed components and dimensions are also considered during the DD stage. For example during the conceptual and preliminary design stage we studied the wing-box as whole system, but during the DD stage the wing box will be broken down into more detail component such as spars, ribs and skin. Each part and its miscellaneous details will be analyzed separately.

Flight Test / Certification/ Manufacturing (FT/C/M): The goal during this stage is to prove viability of the fixed wing design. This is not to say that these requirements were not considered previously. A designer should always make design decisions based upon all the required guidelines, and this includes manufacturability, inspection and certification. The goal at this stage is to verify detail design product. Amen Omoragbon writes:

> “This is done by proving vehicles airworthiness and by demonstrating the performance promised to the costumers “[8, 2010].

Operations (O): Once the flight testing stage and all certification requirements have been completed, the fixed wing flight vehicle is sent to the costumers. Design changes at this stage come in a form of liaison engineering and up or down grades that can influence cost [8, 2010].

Incident / Accident Investigation (I/AI): This stage starts from the time the first test flight is started until the end of the operation stage. Incidents and accidents can always produce vital design knowledge. These new design information can further improve future designs [8, 2010][9, 2008].

Lifecycle simulation shows concept feasibility and optimum configuration selection throughout the lifecycle, however; this can also be done by simulating these design phases at the conceptual design stage. The big benefit in doing extra analysis at the conceptual design stage is that it helps increase upfront knowledge generation. Having more information and design freedom at the early design stage should accelerate design response time and at the same time increase reliability of decisions made before it is too late [9, 2008][8, 2010]. Oza writes:

> “Basically, this methodology helps avoid “fires” in the early design stage, while the cost for design change is minimum, rather than to put out “fires” in later design stage, where the cost for design change is much higher” [9, 2008].

Figure 1-5, illustrates the relationship between lifecycle time and knowledge, cost, design freedom and structural analysis.

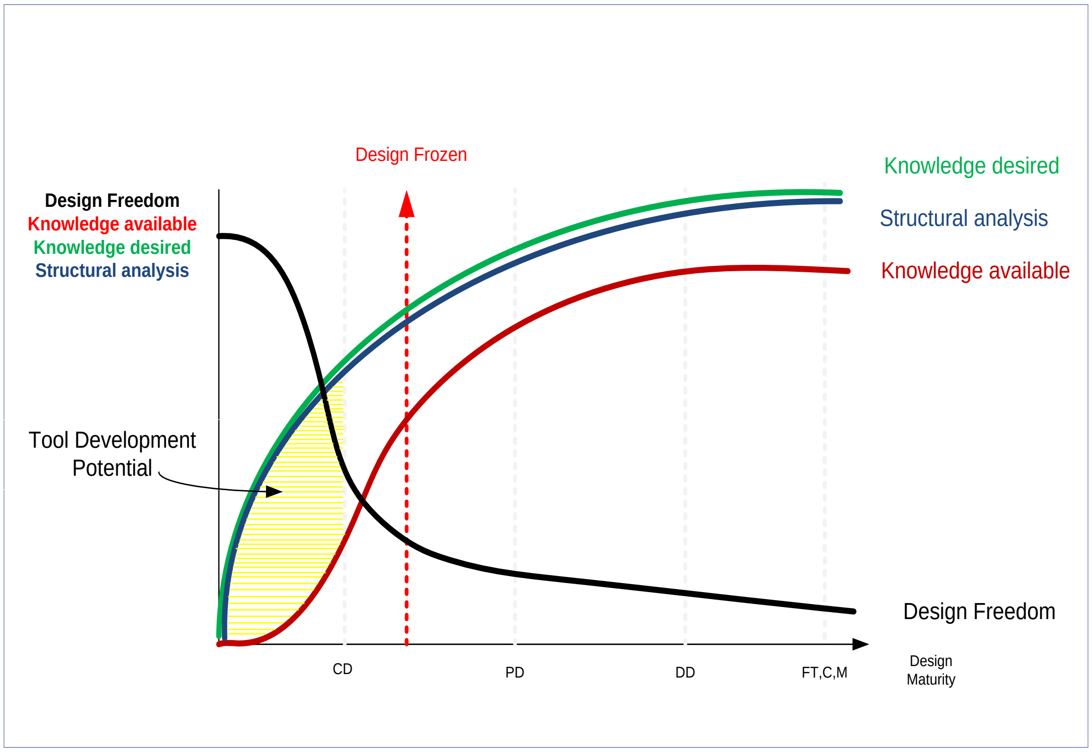


Figure 1-5 Product lifecycle parameters

1.2.3 Structural Analysis and Sizing:

Structural analysis is the determination of load effects on a structure. Given a structure subjected to external loads, it's the determination of internal loads and displacement. The goal of structural analysis is to verify that an "unsafe" structural failure does not occur. This is demonstrated through figure 1-6. Structural sizing is to determine structural layout, material and appropriate cross sections, thickness based on structural analysis results. This practice is demonstrated through the composite design approach.

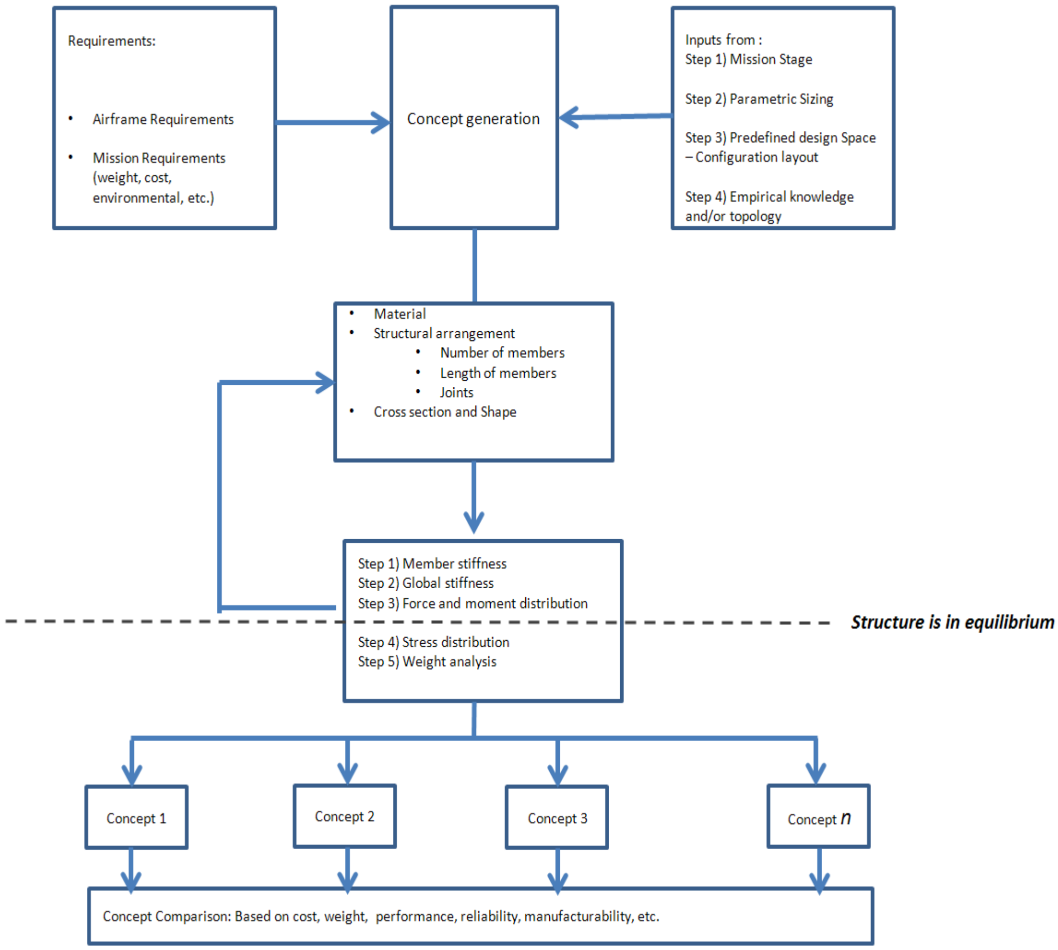
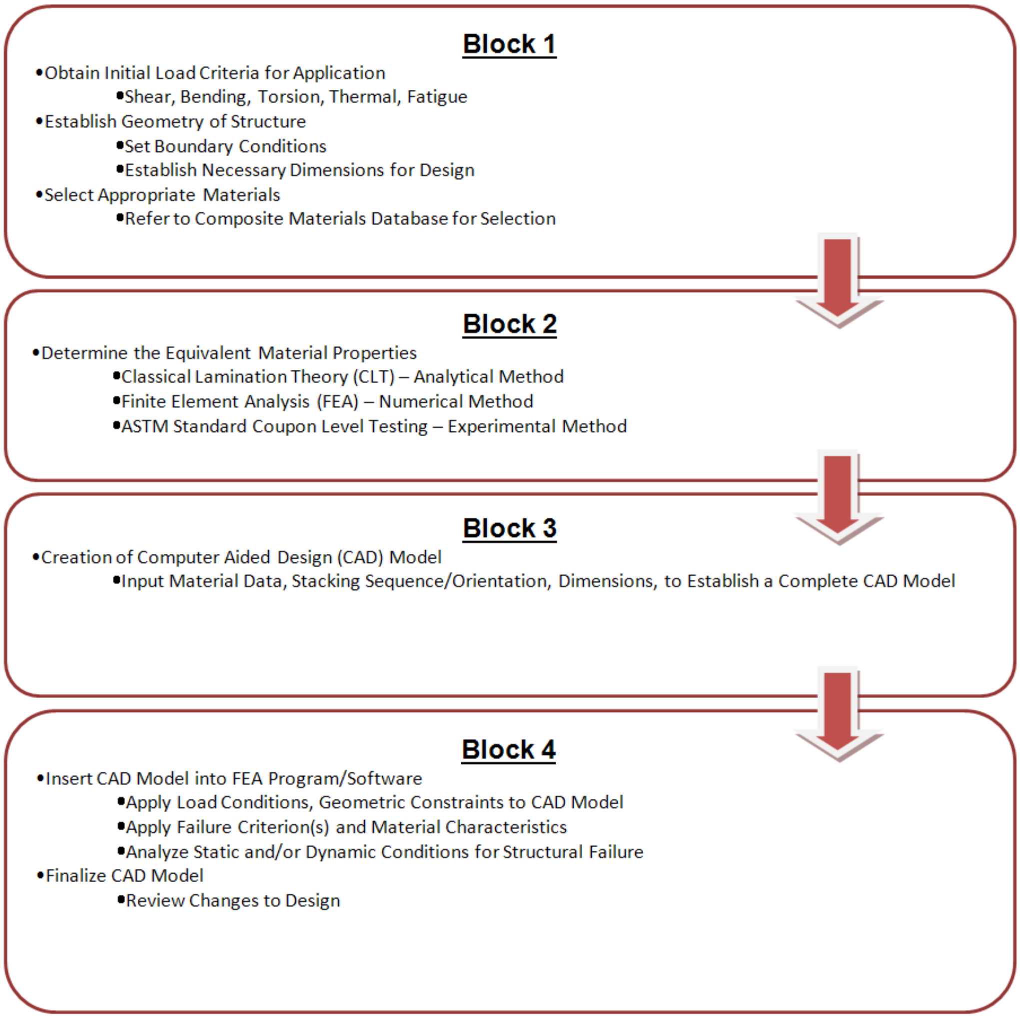


Figure 1-6 Structural design approach using FEA at CD stage

In order to design a composite structure, once the structure has reached a state of equilibrium, the approach illustrated in diagram above, can be further broken down into more detailed steps for a composite structure.

Table 1-1 Composite Building Block Approach [32, 2012]



Given that the structural component is made from composites or an isotropic material (both defined in section 1.2.5), the structural analysis task will not change. It always assumes that the behavior of the structure can be predicted through analysis that is based upon suitable analysis methods, loads, material properties and other boundary conditions. These are explained next.

1.2.4 Classification of Structures:

Structures are divided into two major groups based on evaluating the external reactionary forces and solving for internal stresses in the structure. They can be determinate or indeterminate.

1.2.4.1 Determinate structures:

If it is possible to determine the internal forces and moments (stresses) and reactionary forces of structural members using the statically equations of equilibrium alone, then that structure is defined as a determinate structure. Also, if we can only solve for the reactionary forces and moments using equations of equilibrium, then the structure is an externally determinate structure. In order for a structure to be classified as a determinate structure, the structure has to be externally and internally determinate [18, Kalani][33, Kharagpur][23, 1989].

1.2.4.2 Indeterminate structures:

An indeterminate structure, on the hand is a structure that has 'too Complex' to use a free body diagram to solve it. If it becomes impossible to determine forces (stresses) and reactionary forces and moments of structural members using the statistical equations of equilibrium alone, that structure is externally and internally an indeterminate structure. If the reactionary forces of structure can be solved using equations of equilibrium but it is not possible to solve for the internal forces and moments, then that structure is externally determinate but internally indeterminate. To be classified as an indeterminate structure, the structure can be externally or internally determinate or externally and internally indeterminate [18, Kalani][33, Kharagpur][23, 1989].

1.2.5 Classification of materials:

Structures of materials are made from atoms. There are about 100 different known types of atom in the universe. These atoms can form millions of different matters ranging from the air we breathe to metals and advanced composites that are used to make high rising buildings, bridges, cars and airplanes. Based on their atomic structures materials can be grouped into five categories: metals, polymers, ceramics, glasses and hybrids. Hybrids are composites materials that are made from the combination of more than one material; and in this thesis we often refer to them as unconventional materials. Materials are selected based on the application of the material and also the process. Unidirectional (UD) composites (defined below) have become ideal material in the aerospace industry as they provide high strength to weight ratios and exceptional corrosion resistance compared to most aluminum alloys (Aluminum Lithium alloys offer exceptional corrosion resistance properties).

1.2.5.1 Conventional Materials:

Based on their atomic structure and chemical compounds, conventional materials are grouped into: metals, ceramics, polymers, elastomers and glasses [20, 2007]. The metallic materials are largely used within the airplane design industry and are a combination of metallic elements. Ceramics on the other hand are usually a combination of metallic and nonmetallic elements. Polymers unlike the Ceramics and metallic materials are made from non-metallic elements and include plastics and rubber materials. Polymers are usually organic compounds based on carbon, hydrogen, oxygen, etc. Glasses are non-crystalline materials, with the exception of ceramics, they are awfully brittle and they respond less to plastic behavior than other materials. Elastomers are polymers, except their elongation is much higher than the common polymer [20, 2007].

1.2.5.1 Unconventional Materials:

Composites: Are made from a combination of reinforcement material (fibers) and matrix materials. Composites are generally classified into three categories based on the orientation, type and geometry of the reinforcement material. They are unidirectional (UD) fiber reinforced composites, fabric/cloth fiber reinforced composites, hybrid (or mix) fiber reinforced composites, short fiber reinforced composites, short and randomly distributed fiber reinforced composites. Fibers can be carbon, glass, aramid, boron, graphite, alumna, etc. The matrix is either thermoset or thermoplastic. Table 1-1 has a widespread list of different fibers and matrix (or resins) material.

Table 1-2 List of Common Fibers and Resins [1, 2011]

Fibers	Matrix
Alumina	Bismaleimide
Aramid	Cyanate Ester
Boron	Epoxy
Carbon	Flourocarbon
D-Glass	Phenolic
E-Glass	Polyamide-imide
Graphite	Polybenzimidazole
Lithium	Polytheretherketone
Polybenzothiazole	Polyetherimide
Silicon	Polyethersulfone
Silicon Carbide	Polyimide
S-Glass	Polyphenylene
Titanium	Silicone
Tungsten	Thermoplastic Polyster

1.2.6 External and Internal loads:

The forces acting on a body are either external or internal. For every action there is an equal and opposite reaction. This is Newton's third law of motion, and this law best describes the relationship between external and internal loads. If forces are exerted by another body they are external loads (i.e. Gravity) and the forces acting to keep the body together are internal loads. The concentration of these internal forces per unit area is defined as stress distribution.

1.2.6.1 External Loads:

Before any structural system or component can be designed, it is important to have a basic understanding of the loads that will be forced on that structure during its operation and life cycle. During the life cycle of an aircraft, it is subjected to different types of loading conditions. This includes, but is not limited to the weight of the aircraft, wind, snow, landing and maneuver loads. These loading conditions are often illustrated through V-N Diagrams. External loads in general are divided into Static or dead loads (aircraft seats), dynamic loads or live loads (wind, snow) and cyclic loads or repeated loads. According to Roark's Formula for stress and strain [21, 2002], different types of loads are explained below:

> "Short Time Static Loading: These are constant loads throughout the life time of the structure. As the name suggests these types of loads are forced and increased gradually in a "short period" of time to a maximum value, and is not reapplied. In testing, load is applied gradually until the specimen breaks. This time frame is usually less than a few minutes" [21, 2002].

> "Longtime Static Loading: Unlike the short time static loading conditions, here maximum loading condition is maintained during the lifecycle of a structure for a longer time frame. Stress corrosion cracking and the creep of a material is some examples of longtime static loading. These properties are determined by maintaining a test specimen for a sufficient time frame under an environmental condition similar to the anticipated service condition" [21, 2002].

For example, according to ASTM G47, the standard test method for determining susceptibility to stress corrosion cracking of aluminum alloy products; the length of exposure is between 10 to 40 days (depending on the grain orientation) to a magnitude of stress of about 103 MPa in order to identify signs of cracking and corrosion.

> "Cyclic Loading: these are repeated loads. They can vary from few cycles to million cycles. During material testing, few cycle conditions are usually exposed to larger forces. Test specimens that are exposed to "Many times" repetition loading conditions are (usually) exposed to lower forces" [21, 2002].

> "Dynamic loading: unlike the static loads, dynamic forces are not constant and can change when acting on a structure. Consequently, no part of a structure that is exposed

to dynamic loading could be considered in a state of equilibrium and thus rate of change of momentum of the parts must be considered [26, 2002]. Generally there are two types of dynamic loading conditions. In one in which the body has imposed upon it a particular kind of motion involving known accelerations, and second one is impact. As the result of which sudden loading may be considered a special case".

For more detail see Reference [26, 2002].

"Support Reactions: Support reactions as the name suggests are reactionary forces exerted on the body by the supports and joints. Unlike the other external forces discussed, support and connections are reaction forces" [21, 2002].

1.2.6.2 Internal Loads

Once a body is subjected to a system of external loads (forces, moments or both), the reaction is either acceleration of the body or development of internal loads to balance the external loads. These resisting forces are the internal loads or stress resultants. The maximum number of stress resultants is six, which includes three orthogonal forces (shear and axial forces) and three orthogonal moments (bending moment and torsion), Figure 1-7.

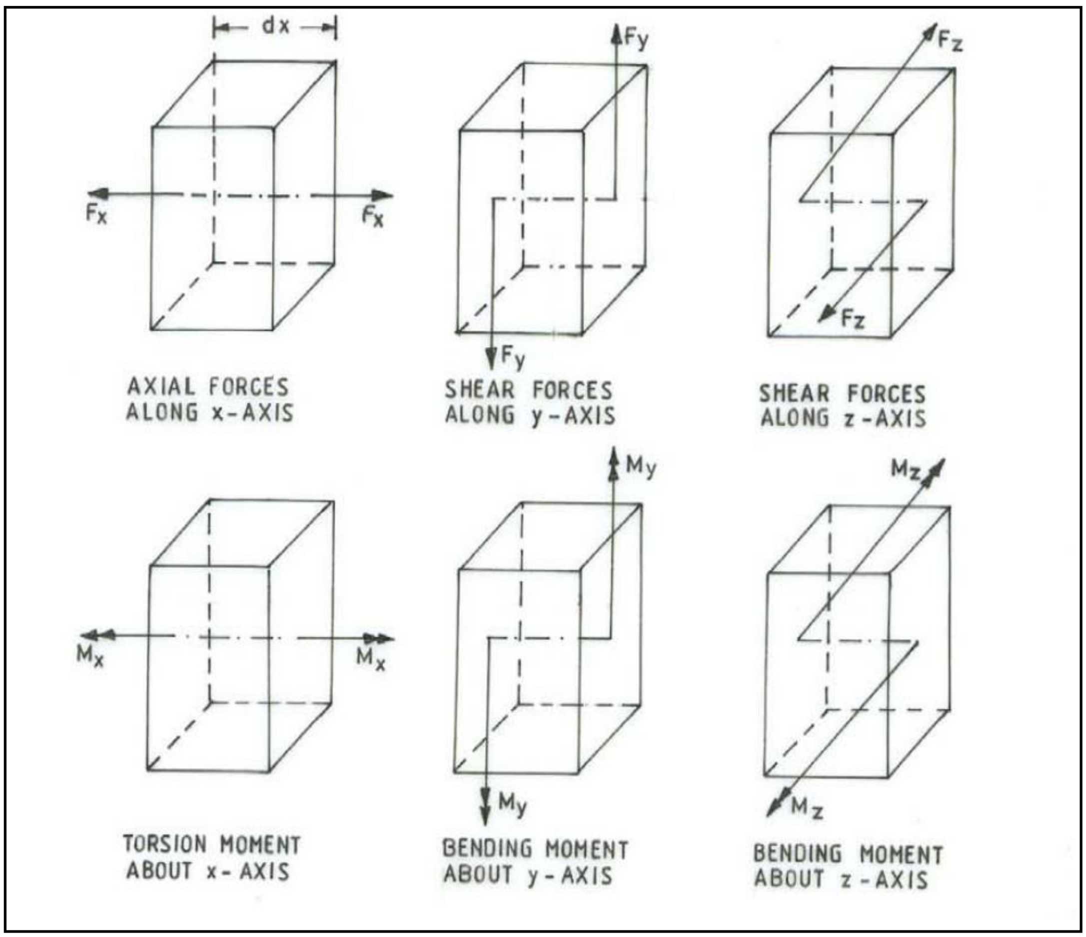


Figure 1-7 Six internal forces and on an element [17, Kalani]

1.2.7 Mechanical Properties:

Properties of materials include their physical, burning behavior, exposure to humidity, cosmetic, thermal, etc and their mechanical characteristics. Mechanical properties are the elastic and strength properties of the material. These properties are estimated according to approved testing standards.. This section will be explained in more detail in chapter 3 of this thesis.

1.2.8 Methods of Structural Analysis:

Previously we divided structures into two major categories. This was based on evaluating for external reactionary forces and internal stresses in the structure. If the reactionary forces and internal stress distribution can be solved by equations of equilibrium alone, then such structures are determinate structures. Depending on the shape (trusses, cables, or a simple beam) of a determinate structure, the methods of analysis are the method of joints, the method of sections or the direct method. There are however generally two different methods of structural analysis for statically indeterminate structures [18, Kalani][33, Kharagpur]. One approach is the force or flexibility approach. The second approach is the displacement or stiffness method. The difference between the two is that they result in two different unknowns. In one the internal forces are the unknown, and in the other displacement is the unknown.

Force Method is also identified as the flexibility method. The reason it is called the forced method is because unlike the displacement method, in this method; the forces are what we are solving for and are treated as the “unknown”. The flexibility method of analysis or the force method was originally developed by J. Maxwell in 1864 and O.C. Mohr in 1874 [16, 2006].

In the displacement method on the other hand displacement is the unknown. Moment Distribution method by Hardy Cross [12,1930], method of Successive Approximations by Calisev [16,2006], Relaxation method by Southwell [13,1940] and Slope Deflection Method by Wilson and Maney [16,2006] are all examples of analytical classical displacement methods of analysis [36, Brun]. The Finite Element Methodology has become the modern displacement method of analysis and was in fact developed (direct stiffness or matrix method) based on all the earlier classical displacement matrix methods of analysis [16,2006].

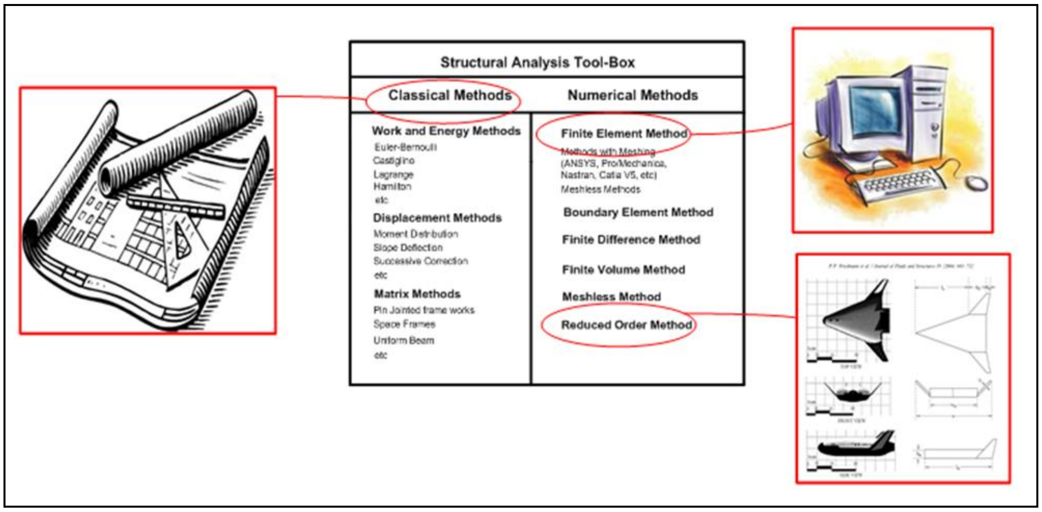


Figure 1-8 Structural analysis toolbox

In “Elementary of matrix analysis of structures” [27, Kardestuncer], Kardestuncer explains why he believes the displacement method of analysis has historically been the preferred choice. According to him it is because the force method of analysis was not suitable for matrix and computer programming. As he puts it, in the force method of analysis the choice of redundant is never exclusive.

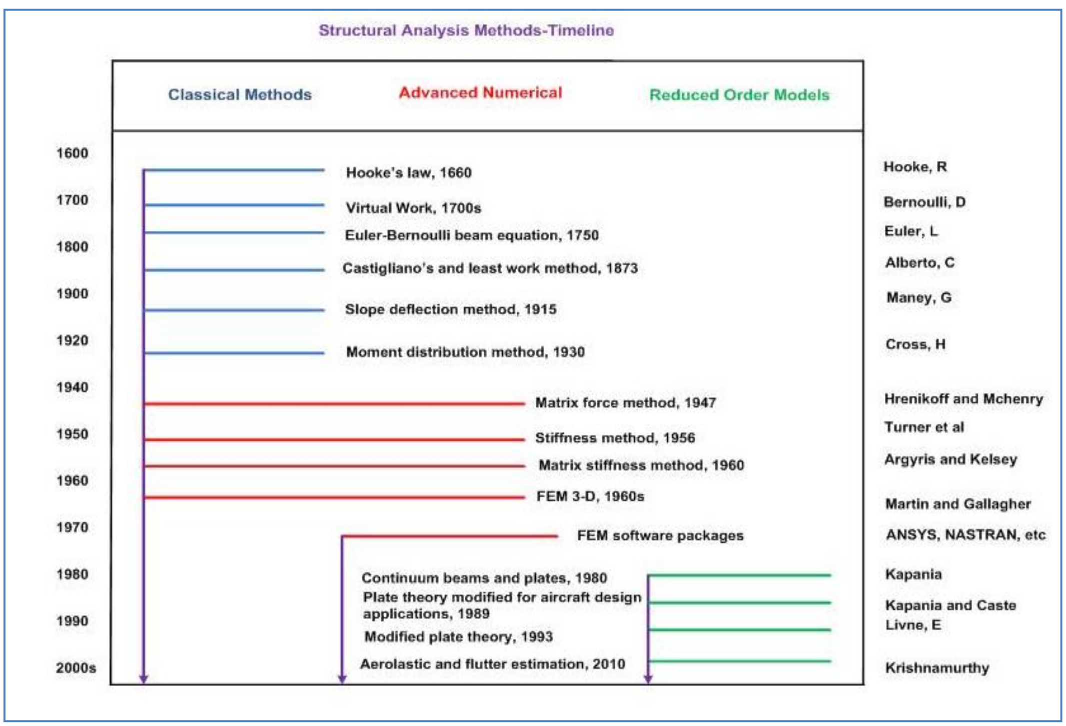


Figure 1-9 Structural analysis methods

1.3 Background

On the importance of early analysis, Ullman writes:

> "The experience of manufacturers in many industries has shown that 85-90% of the total time and cost of product development is defined in the early stages of product development, when only 5-10% of project time and cost have been expended" [28,1992].

This is mainly because in the early concept stages, fundamental decisions are made regarding structural arrangements, cross sections, materials and manufacturing process. This is why the decisions made during the conceptual design stage are very important. Therefore, it makes enormous sense to perform analysis as early as possible. This moves structural analysis forward into the conceptual design stage, where in fixing poor design, material selection and manufacturing process selections, changes are much easier and more economical to make [11, 2000].

In the past five decades, the calculation breakthrough started by the advancement of fast computers has been available through computer aided engineering (CAE) packages to provide designers and engineers with accurate and quick results [23, 1989]. Today's aircraft industry is no different; in fact the aerospace industry has become highly addicted to CAE. While the permits of using CAE during all design stages is supported in this thesis, accurate and often easy to use classical methods are still useful, especially during the conceptual design stage of fixed wing aerospace vehicle development.

Finite Element Analysis (FEA) methodology is one of the most important CAE tool. This CAE tool can model and used to study the static and dynamic response of airframe structures in great detail. Before the advancement of FEA, engineers and designers used classical methods and managed to become airborne without difficulty. These advanced numerical methods are based upon classical methods that have evolved from analytical methods to easy to use digitized numerical methods.

Structural design methods have been used by human beings since early civilization. Centuries before computers were invented; the standing strong pyramids were designed and constructed by the Egyptians around 2000 B.C, the Parthenon was built by the Greeks, around 240 B.C, Dujiangyan was built by the Chinese and Persepolis constructed by the Persians 2500 years ago. Today we see countless complex structures such as houses, buildings, bridges and aircraft constructed before we had any advancement in computer programs for various numerical methods. We witness countless historical monuments still standing strong amid rains and earthquakes. Hagia Sophia, Taj Mahal, Eifel tower are only few examples. It only makes sense that the builders of these amazing monuments understood and used most of basic principles of structural design. These historical houses, buildings, bridges and aircrafts produced from early 1900s to late 1940s, were constructed before we had any advancement in computer aided engineering packages for various numerical methods.

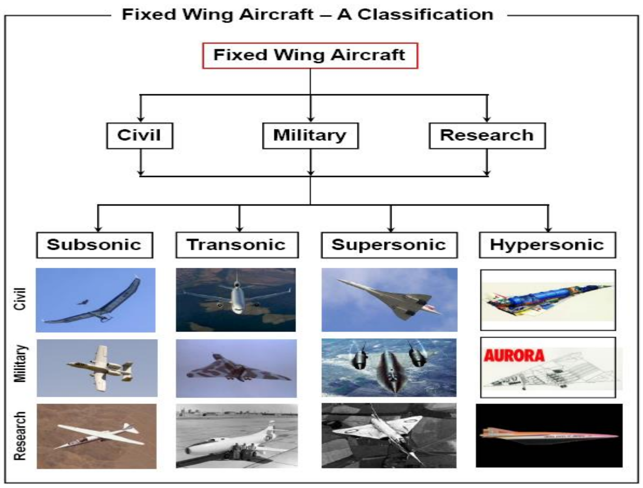


Figure 1-10 Fixed wing aircraft classification

Classical structural analysis methods were used to design these structures to absorb various forces. These classical methods are often forgotten and hardly used these days and as mentioned, the aircraft industry is not an exception here either. Structural designers and engineers utilize Finite Element Analysis to perform almost all structural analysis tasks. But while Finite Element Analysis is one of the most effective structural analysis methods; classical analytical methods can also be as useful especially during the early phase of a fixed wing aircraft design where major decisions are made.

1.4 Problem Description

The advancement of modern numerical methods (e.g. FEM) has given the engineers more accurate and faster answers. Today, problems of highly complex structural arrangements that were tediously and practically impossible to calculate in the past are now easy to solve thanks to the advancement of these high fidelity computer aided engineering methods. Michael Niu writes:

> “For airframe structures, the number of redundancies is of the order of thousands and the solution of such problems by analytical methods for solving highly intermediate structures is extremely tedious and is, indeed, not feasible; computer analysis, such as Finite Element Modeling (FEM) method are the only reasonable method to use in these cases [23, 1989].

The structural specialist and designers have chosen to implement such high fidelity methods to satisfy the need for excellence and accuracy. While the FEA is the method of choice when detailed results and accurate results are required, the relevance of detailed FEA as a CAE tool during the conceptual design stage may face some challenges. The task of the conceptual designer during the conceptual evaluation stage is to evaluate alternative concepts and configuration at the early design lifecycle to arrive at a solution. Youhua Liu in “Efficient Methods for Structural Analysis of Built-Up Wings” has described the inappropriateness of detailed FEA as the tool of CAE during the conceptual design stage. Youhua writes:

> “For complex structures composed of large number of components, a detailed FEA involves huge number of degrees of freedom, and needs large amount of CPU time and computation capacity, which makes the cost to high” [11, 2000].

As the result of this problem, other possible replacements were investigated by Youhua and others. One of those methods is the Reduced Order Modeling (ROM). During the conceptual design stage, accurate and still effective to use reduced order modeling can be very useful. Because of the tremendous computation times needed for detailed FEA to help compute structural sizing, it’s often more satisfactory to use a reduced order modeling of a complex structure. The procedure of modeling a structure by reducing the degree of complexity and solving by analytical and numerical methods is commonly known as Reduced Order Modeling (ROM) [6, 2010]. There are various ways in which a structure can become less complex [14, 1954][15, 1937][30, 1990][31, 2010].

In one method, the designer simplifies the structure into simple and well known structural cross sections such as a prismatic beams, plate, or shell models in order to simulate the more complex structure [1.15]. For conceptual evaluation of various configurations, vast work has been done by Lovejoy and Kapania [88, 1994][89, 1994]. This includes more than 300 references on static and dynamic behavior of reduced order model plates. The theories behind these structural analyses are: classic plate theory, first order shear, higher order shear deformation and energy methods. These theories worked fine for thin plates. Other structural analysis models were built by Giles [90, 1995] and Tizzi [91, 1997]. These latest methods were applicable to thicker plate sections, but they did not consider the primary structural arrangements (spars, ribs, etc) of aircraft main sections (wings, fuselage, etc). However; Liu extended the works of Kapania and Singhvi [11, 2000] by using the Rayleigh Ritz and applied lagrange’s equations to obtain stiffness and mass matrices of structural components. To demonstrate the effectiveness of these structural analysis methods, the results and time to setup and run the models were compared with FEA method.

Similar to FEA, these methods were proven to be a very helpful and effective aid as we tackle structural problems that require complex structural arrangements and can be extremely time consuming to solve these problems using an analytical method. However; similar to FEA this numerical models may prevent the designers from doing the critical thinking that is required in understanding absolutely how the model behaves under the effect of changes made to certain or combination of design variable and parameter. These capabilities are specifically important during the conceptual design stage where concept generation and evaluation demands physical visibility of design parameters.

Up to 1940s, analytical methods were used to help design airplanes. Unlike the FEA, these methods allowed the designer to examine structural response to changes based on various design parameters [34, 2004]. The Maxwell Mohr, least work, slope deflection, and moment distribution methods have all been employed, but among them moment distribution method became the most popular rapidly practiced way to solve structural frames [37, 1961]. H. A Williams of Stanford writes this about all the other analytical methods:

> “The laborious computations involved together with the tendency for small errors to accumulate, have discouraged their use” [35, 1956].

By the late 1930s, one of the most popular classical analytical displacement methods became the moment distribution method invented by Hardy Cross. By this time in history moment distribution method had become the analytical method of choice and very popular method among architects, airplane designers and structural engineers [35, 1997][37, 1961][38, 2001].

Given that, the question to be answered in this thesis is: How valuable and compatible is the moment distribution method in today’s conceptual design environment? And can FEA and moment distribution method complement each other?

1.5 Research Objective and Approach

The major goal of this thesis is to investigate the pros and cons of analytical structural analysis methods during the conceptual design stage through the following objectives:

1) Illustrate structural design methodology of these methods within the framework of Aerospace Vehicle Design (AVD) lab's design lifecycle.

2) Demonstrate the effectiveness of moment distribution method through four case studies. This will be done by considering and evaluating the strength and limitation of these methods. In order to objectively quantify the limitation and capabilities of the analytical method at the conceptual design stage, each case study becomes more complex than the one before.

This thesis approaches this design problem with the particular goal to explore efficient methods for structural analysis at the conceptual design stage, such that accurate results can be achieved. Granted, that the accuracy and reliability of the results always depends on comprehensiveness of modeling and the experience of the engineer in modeling the methods. The results from the two approaches will be compared, analyzed and quantified.

1.6 Research Organization

Table 1-3 Research Organization

Chapter	Task	Deliverables
One	Introduction	1.1. Motivation
		1.2 Basic Description
		1.3 Background
		1.4 Problem Description
		1.5 Research Objectives and Strategy
		1.6 Research Organization
Two	Design Process	2.1 Introduction
		2.2 Conceptual Design Stage
		2.3 Preliminary Design Stage
		2.4 Detail Design Stage
		2.5 Summary
Three	Structural Design at Conceptual Design Stage	3.1 Introduction
		3.2 External Load Analysis
		3.3 Material and Process
		3.4 Design and Modeling
		3.5 Structural Analysis Toolbox
		3.6 Summary
Four	Structural Analysis Methods	4.1 Brief History of Structural Analysis methods
		4.2 Deflection Method
		4.3 Moment Distribution Method
		4.4 Case Study One: MDM without sidesway vs. FEA
		4.5 Comparison of Analytical Approach vs. FEA
		4.6 From Classic Analytical methods to Advanced Numerical Methods
		4.7 Summary
Five	Structural Analysis of Indeterminate Frames with Sidesway	5.1 Introduction
		5.2 Case Study two: Analysis of Single Indeterminate Frame with Sidesway vs FEA
		5.3 Case Study three: Analysis of double interminate Frame with Sidesway vs FEA
		5.4 Case Study Four: Analysis of Structural Concept vs FEA
		5.5 FEA + Moment Distribution Method = A New Approach
		5.6 Summary
Six	Conclusion	6 Conclusion and Recommendation

Chapter 2

Design Process

2.1 Introduction

Human beings have been designing products for thousands of years but still there is no one ultimate product development and design process defined. This is because we always need to come up with new, cost-effective and high quality products and, thus; change the design process. Data shows that 85% of the problems with products are as the result of poor and understudied conceptual design process [28, 1992][39, 2004].

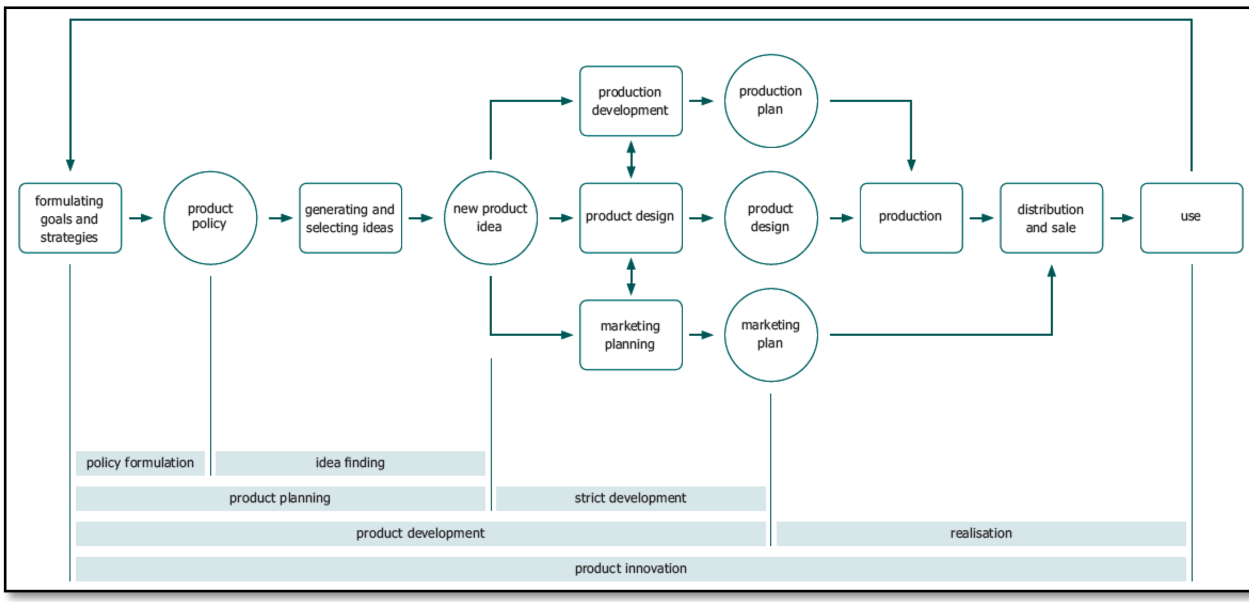


Figure 2-1 Phases of the product innovation process [40, 1995]

Design process can generally be divided into three different phases called Conceptual Design, Preliminary Design and Detail Design. These design phases can be separated by very fine lines; sometimes, the phases may be overlapping. Figure 2-2 below is a diagram of the design process.

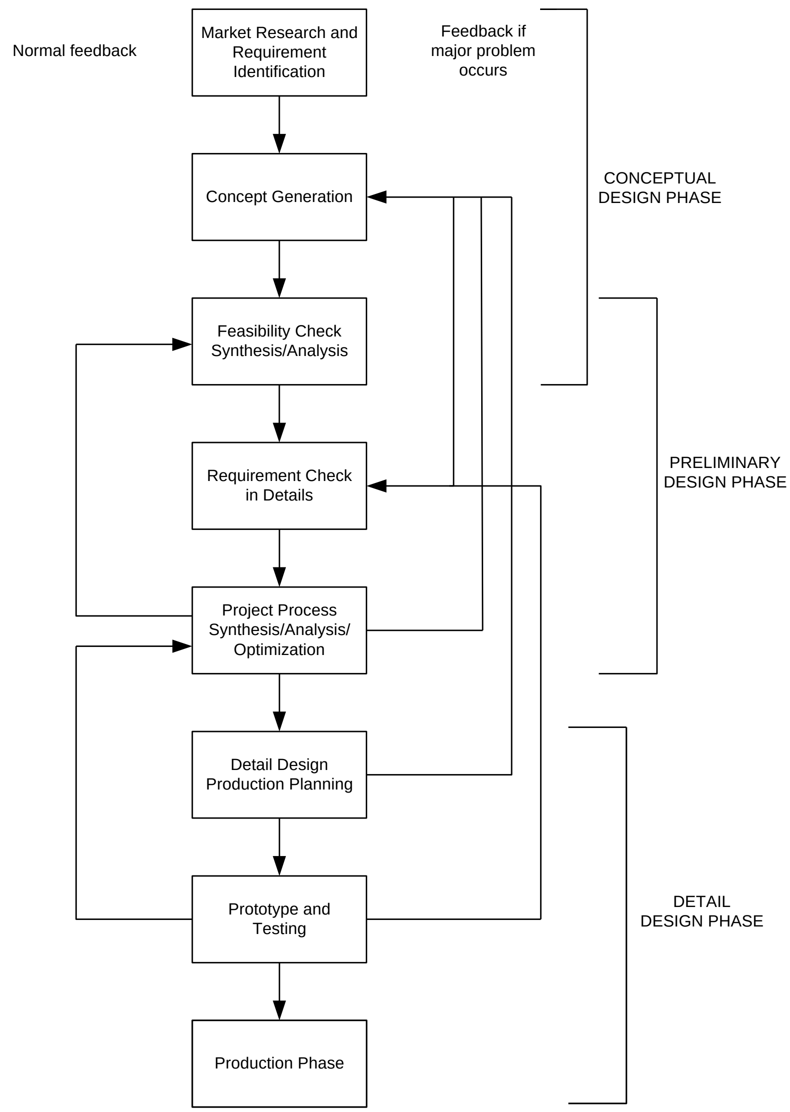


Figure 2-2 Design process [28, 1992][39, 2004]

In the conceptual design phase various concepts can be generated based on specific mission(s), and out of these concepts, the most promising concepts are finalized and evaluated during the preliminary design process. In the detail design process, only one concept is finalized for manufacturing, production and sale. From the three design phases, the conceptual design stage has been regarded to be the most important since every analysis and study done during the later phases is based on the concepts generated in the conceptual design phase [5, 2001][41, 1999][42, 2002][43, 1995].

2.2 Conceptual Design Phase

The Conceptual Design (CD) phase is the first phase of the design process. Conceptual design stage always follows the mission selection. Conceptual Design cannot start without a specific mission or goal. It is based on the mission of the projects, that ideas are developed. These ideas mainly define how the product will function, how it will look etc. The end goal of conceptual design phase is to ‘flesh out’ numerous design concepts by proportional and comparative evaluation (“apples vs. apples”) based on certain design requirement(s) to a point where they can be evaluated more objectively during the preliminary and detail design stage. [43, 1995][42, 2002][39, 2004][44, 2004].

Conceptual design phase requires design(s) studies and analysis to be sufficiently detailed, as design errors made during conceptual design phase can never be rectified by good detail design [45, 1998]. Data shows that more than often the errors during production phase are the outcome of poor and incompatible conceptual design [41, 1999].

The following figure is called as Ullman’s Design Paradox, [28, 1992] and it highlights the importance of conceptual design and analysis at early design stage. Graph shows; as we move to more detailed design phases, design flexibility drastically decreases and the cost of design increase (consequently design change) significantly. Hence we can say that the correctness of conceptual design is highly desired to avoid further complications, frustration and waste of money [28, 199][39, 2004].

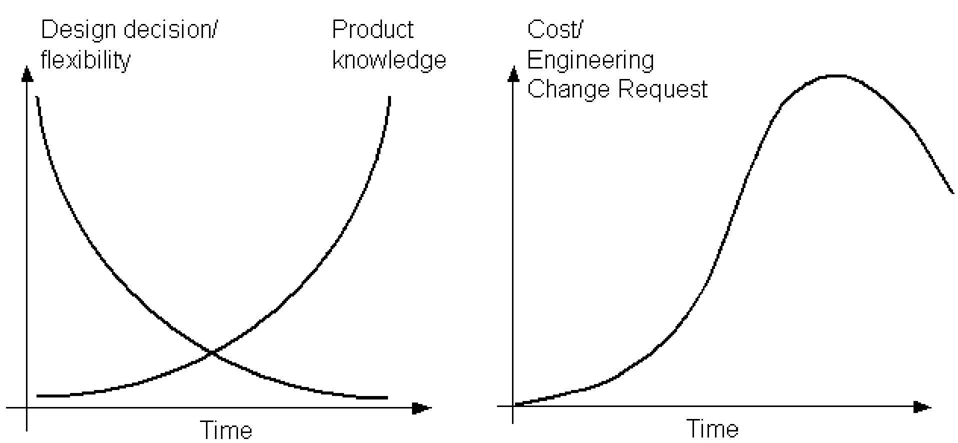


Figure 2-3 Ullman diagram [28, 1992]

The Conceptual Design phase can be further broken down into; parametric sizing (PS), configuration layout (CL) and Configuration evaluation (CE).

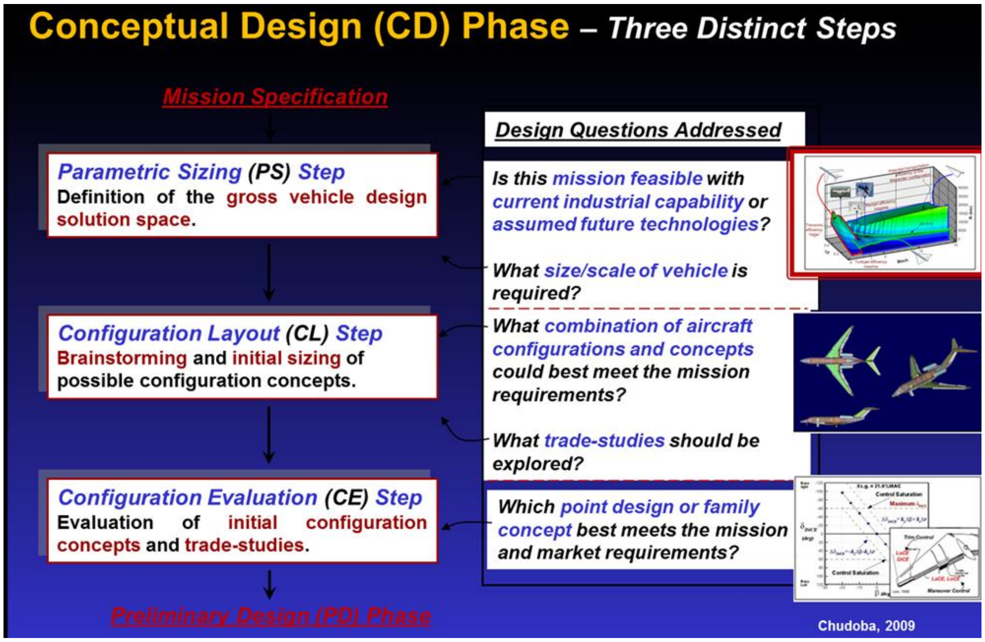


Figure 2-4 Conceptual design phases

During the PS stage, the goal of a designer is to converge on a specific aircraft design when having visualized the solution space. These aircraft match points are often identified by size required, resulting weight, and power required. The selected match point or converged point design can then be further evaluated through the later CL and CE steps [4, 2006]. For more detailed information about the Parametric Sizing stage, see reference [5, 2001] and [25, 2011].

Based on the parametric sizing results, an initial external layout of major aircraft components (i.e., fuselage, wing, propulsion system, etc.) will be determined and further evaluated and studied during configuration evaluation stage [7, 2007]. Once the solution space (PS step) is identified, studied, and pre-defined design space and relevant configuration and concept is selected (CL step), conceptual evaluation step proceeds with multidisciplinary analysis that includes: aerodynamics, aeromechanics, structural sizing, propulsion, etc in order to determine and verify the aircraft gross design parameters such as dimensions, structural arrangements, weight, cost, etc for each configuration and concept permutation [4, 2006][7, 2007]. This thesis concentrates on the structural analysis during the conceptual evaluation stage of fixed wing aircraft. This is done with particular emphasis on quantifying the pros and cons of these methods for structural analysis during the conceptual evaluation stage.

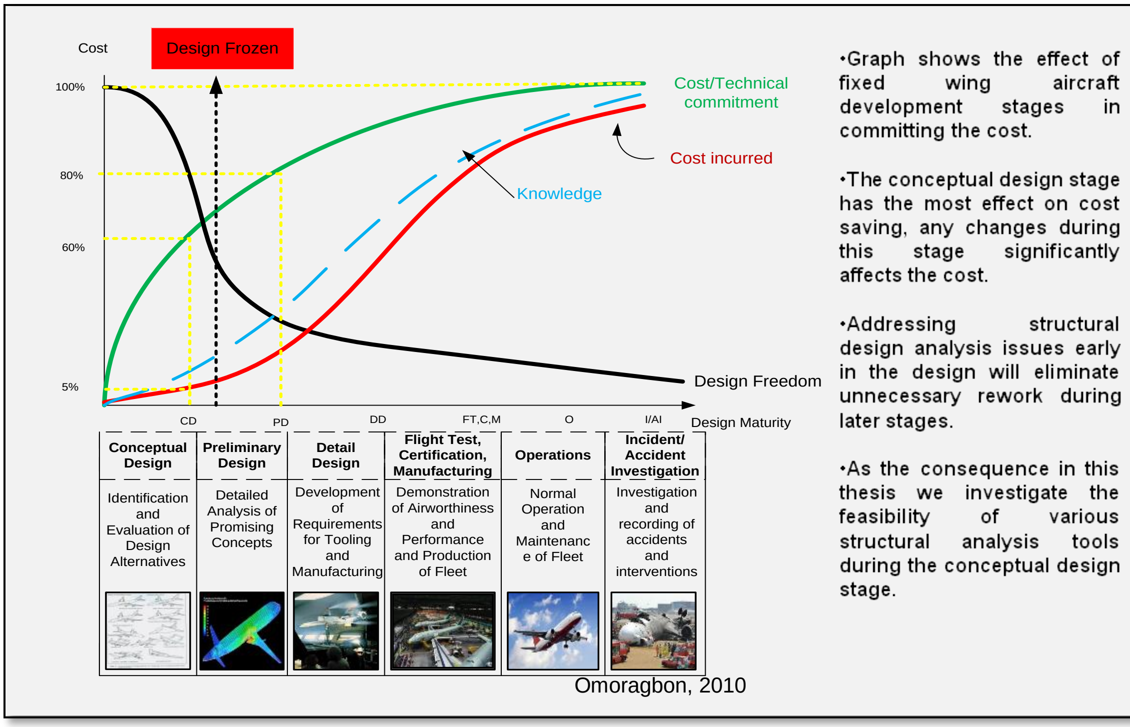


Figure 2-5 Design life cycle

During conceptual design phase of a fixed wing aircraft, the structural designers are responsible to evaluate and come up with preliminary dimensions, cross sections, structural arrangement of main load carrying components and material through comparative evaluation [41, 1999][46,1997][47, 2011][44, 2004].

The intent of structural designer at this stage is to help “flesh out” design concepts to the point where they can be evaluated in more details at the preliminary and detail design stages. Hendri Syamsudin writes:

> “This is to help improve early design decisions by utilizing structured concept evaluation and decision making on process or critical airframe parameters” [48, 2009].

This is achieved through structural analysis of various concepts at the conceptual evaluation stage. The behavior and performance of the design concepts under a certain load(s), material(s), boundary condition and predefined design space are tested, in order to help “flesh out” the most promising structural arrangements.

The predefined design space is what sets apart the work of the structural designer at various (CD, PD and DD) design stages. Initially, the shape of the predefined design space (or the external layout) is based upon the aerodynamic constraints, which are determined initially at the parametric sizing and configuration layout stage. Consequently, the structural analysis at the conceptual design stage is focused on primary and first order structural arrangements which outlines the shape of the structural configuration of an aircraft's main sections [44, 2004][49, 2012]. The main sections of a typical fixed wing aircraft include the wing, tail section and the fuselage. These main sections of an aircraft are subjected to major aerodynamic loads and are often defined as the primary structure.

For example, the primary structural components of a delta wing-box includes: spars, ribs, skin, etc. Each of these components can be created from different materials and processes with very different stiffness and strength properties (at different costs). Each component can have a different cross section (Circular, I-beam, Rectangular, etc) and lengths. Different cross sections have different surface inertias, and for that reason different stiffness properties. The combination of these items can change the structural performance of each concept. To select the most efficient (i.e. best strength to weight ratio, best aerodynamic performance, etc) structural arrangement, one has to evaluate the concepts before anyone of the concepts can be ruled out. Figure (below) illustrates various structural arrangements of a delta wing box.

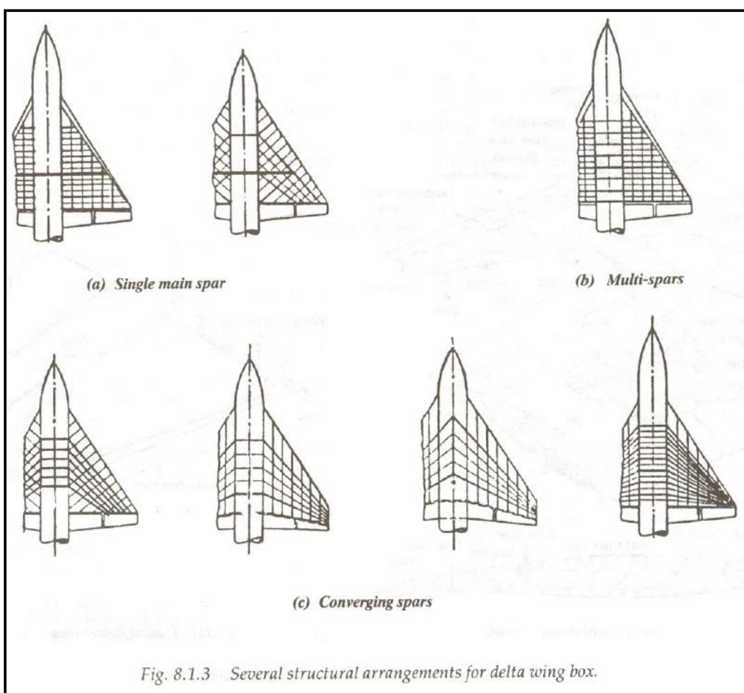


Figure 2-6 Delta wing box [50, 2012]

The predefined structural arrangements are often based on preliminary human knowledge (experience), topology optimization methods or both. In the recent years, topology optimization has become the method of choice by major aircraft manufacturers in order to determine the most efficient way to distribute loads. Given a predefined design space, topology optimization is used to identify optimum material distribution without depending on designer's prior knowledge [41, 2012].

To better explain this, let us assume that the figure below represents a predefined design space of an aircraft wing-box:

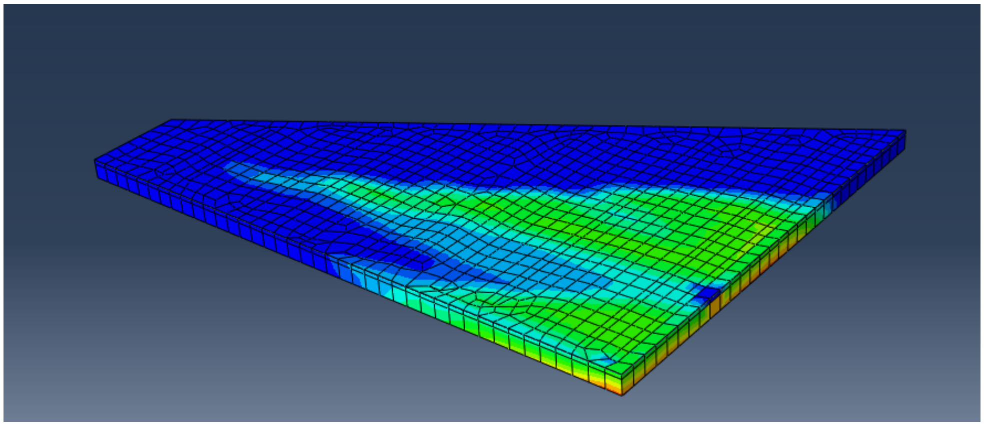


Figure 2-7 Wing box pre-defined design space [75, 2013]

Based on the loading conditions, materials and boundary condition, topology method will determine the most efficient load path.

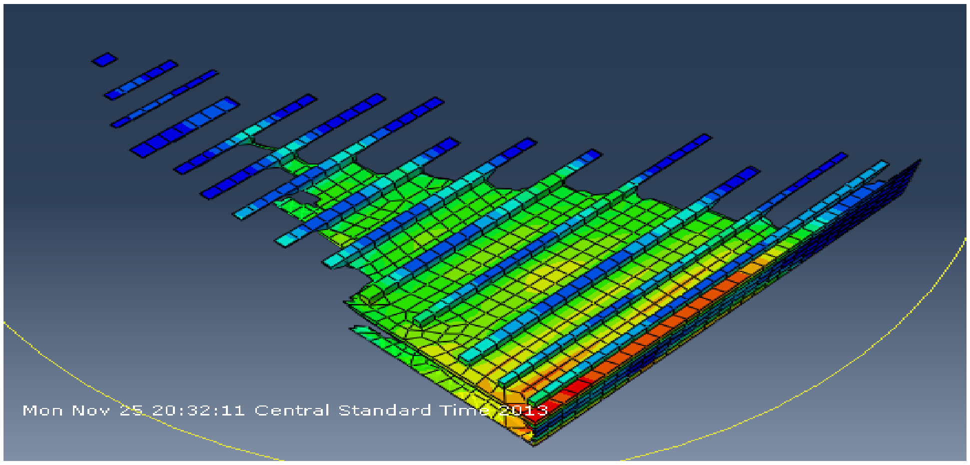


Figure 2-8 Topology results [75, 2013]

> “Simply, in a formulation of the topology optimization problem, the artificial material is defined to have variable material density and an associated variable stiffness. Associating one material density variable to each finite element in a design space and taking E as the specific stiffness of an isotropic material, a design description that each finite element in the design space model to become either a void “ρ=0” or material “ρ=1” is achieved” [49, 2013].

The combination of Topology technology and experience can provide a good initial starting point, to start with the first structural concept. This initial design permits the designer to model structural components (Material selection, cross section, component arrangements, ect) and start the initial structural analysis. As it was mentioned before, at this stage the main sections of the aircraft are analyzed as a single body. Structural analysis of different configurations and materials are started based on finite design parameters at the conceptual evaluation stage. Structural analysis criteria and structural design parameters at conceptual design stage will be discussed in more detail in the chapter 3 of this thesis.

2.3 Preliminary Design Phase

The conceptual design phase is very hard to conclude with complete satisfaction. Once all the feasible concepts have been identified, preliminary design phase takes over the concept refinement and concepts are analyzed and evaluated in more details [41, 1999][42, 2002][46,1997][39, 2004]. The preliminary design stage starts when major configuration and structural arrangements are expected to remain unchanged, however; minor changes are always expected towards improving the design concept. At this stage the design concept is further evaluated and analyzed to validate and further develop any assumptions and analysis that were made during the CD stage [44, 2004]. It is during the Preliminary Design stage that the concept becomes definite based on detailed configuration evaluations, customer and structural requirements. This is often referred to as a design concept becoming “frozen”. During the preliminary design stage, the single body structural system can also be broken down into individual components and analyzed and designed separately. Also at the preliminary design stage, structural efforts now includes other loading conditions such as dynamic loading, fatigue life and cyclic loading conditions [44, 2004][48, 2011]. Once the concept is “frozen”, further changes should not change the overall configuration [41, 1999]. Minor changes can constantly fine-tune the configuration layout, structural arrangements, choice of material and the structural designers work towards maturation of the selected design concept. The end goal of the preliminary design phase is to get ready for the detail design and establish confidence that the design is certifiable, manufacturble and can be completed in the given time frame and cost margin [4, 2006][46, 1997].

2.4 Detail Design Phase

The detail design phase does not involve any decision making regarding the basic configuration, structural arrangements or material selection. The decisions made in previous phases are frozen and at it is during the detail design stage that the designers become more focused with all the details and exact analysis. At this stage, the design will be broken down into more detail parts.

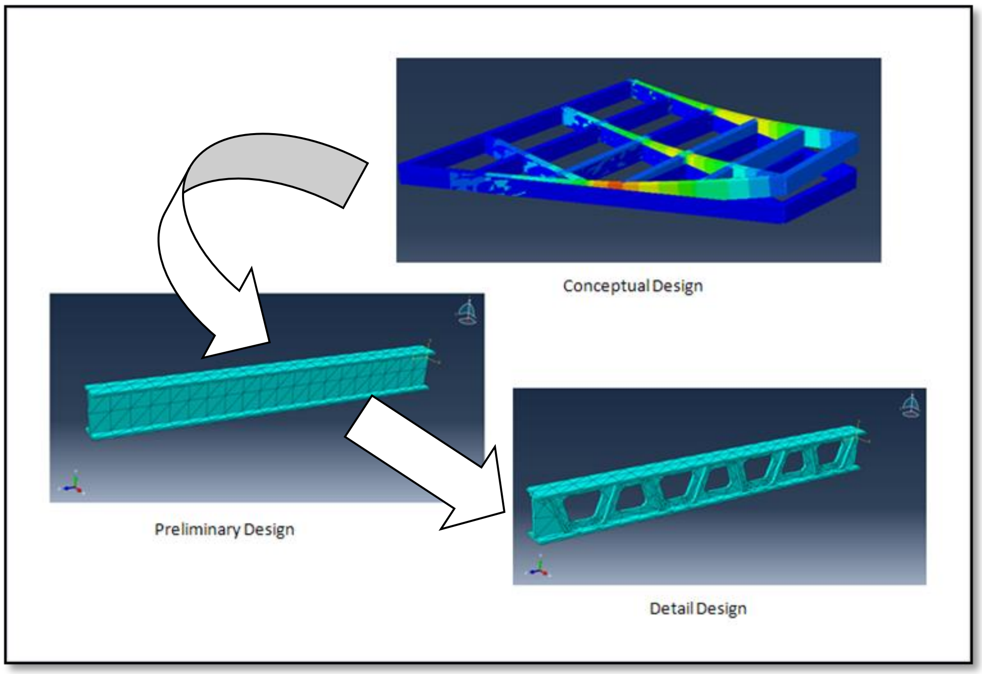


Figure 2-9 Structural sizing during different design stages

The most detailed components, hardware and detailed dimensions are considered during the detailed design stage. During the detail design stage, this specific component is selected and further evaluated and analyzed. At this stage of analysis, the structural designer is concerned with the details of the design: the type (tritangent, variable, etc) and size of fillets, rounds, holes, rib thickness, chambers, bolts, etc. However, the objective is the same, to come up with the most efficient structural (strength to weight ratio) design. Higher fidelity FEA tools are used to optimize each member as much as it is possible. Figure below illustrates the kind of development and progress that takes place at the detail design stage. At this stage structural analysis includes linear and non-linear structural analysis, fatigue, bolt-loads, etc. Figure 2.10 shows an example of detail design being done on a wing-box rib component.

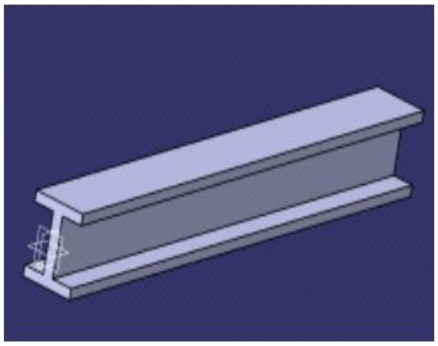
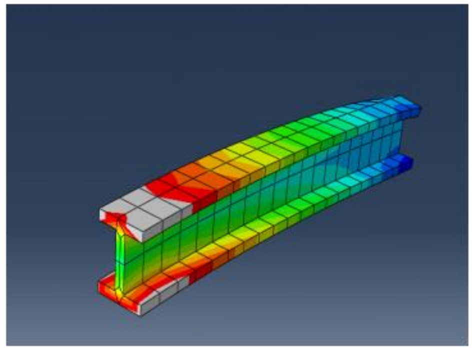
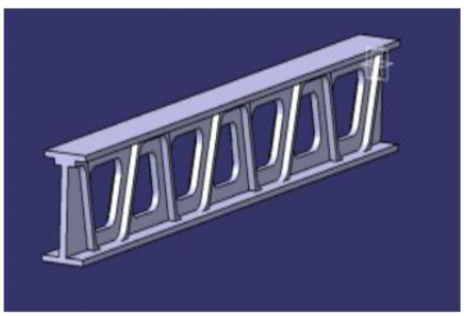
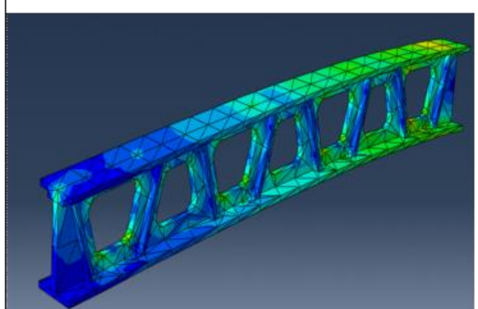
Pre-Detail Design		
I-beam Sized Per Preliminary Requirements	Stress Analysis	Resluts
		Max Stress: 78000 (Psi) Mas Deformation: 4 inches Volume: 200 in^3 Weight: 20.4 lbs Cost of production: $145
Detail Design		
Detail Design of the Spar	Stress Analysis	Results
		Max Stress: 68000 (Psi) Mas Deformation: 4.2 inches Volume: 175 in^3 Weight: 17.8 lbs Cost of production: $245

Figure 2-10 Structural design at detail design stage

By the end of this phase the design group will come up with a matured design ready for production. Detailed CAD models and drawings with actual manufacturing data and specification, such as precise dimensions and tolerances are produced [41, 1999][28, 1992][43, 1995].

During this phase the designers are concerned about ‘exact numbers’, including exact radius of corner pocket, rivet diameter locations of the holes for fasteners, machining time, etc. Thus the hardware, and other detailed components not considered during previous phases will be covered during this stage. As the result of this the detailed drawing, process specifications will be put together and taken to manufacturing and production [4, 2006][41, 1999].

2.5 Summary

In this chapter we learned that, similar to other design processes; fixed wing aircraft design process can generally be divided into three different phases called Conceptual Design, Preliminary Design and Detail Design. The predefined design space is what sets apart the work of the structural designer at various (CD, PD and DD) design stages. Initially, the shape of the predefined design space or the external layout is based upon the aerodynamic constraints, which are determined initially at the parametric sizing and configuration layout stage. For that reason, the structural analysis at the conceptual design stage is focused on primary and first order structural arrangements which outline the shape of the structural configuration of an aircraft's main sections (wing, fuselage, etc). While, during the preliminary and detail design stage the designer becomes more focused with detail and exact analysis. Here, the concept can be broken down into more detail parts.

The intent of structural design at the conceptual design stage is to help "flesh out" design concepts to the point where they can be evaluated in more details at the preliminary and detail design stages. The critical design parameters are: structural layout, boundary and load conditions, material properties, cross section and geometry of the structural components. Structural analysis criteria and requirements at conceptual design and the design parameters will be discussed in more detail in the chapter 3.

Chapter 3

Structural Design at Conceptual Design Stage

3.1 Introduction

In the previous chapter we learned that the main goal of structural design at the conceptual evaluation stage is to help "sort out" design concepts to a point where they can be evaluated in more details at the preliminary and detail design stages. This was achieved through analyzing the structural performance and integrity of different design concepts.

Structural integrity of a structural system is defined as a structural component or system such as an aircraft that exists in an undamaged condition. So, if an aircraft loses its structural integrity it will fail and breakdown into pieces. A structure needs to be strong and stiff enough to withstand specific loading conditions in which it is designed to operate [28, 2004]. There are some main drivers that play an important role in maintaining the structural integrity of an aircraft. These main drivers are: the external loading conditions (design loads), type of material and structural arrangement (design and boundary conditions). Due to economic drivers, the goal is to find the optimal balance between the weight of the vehicle and structural integrity. This means that the structural integrity should be achieved with minimum possible weight increase, since any excess weight has negative effect of the performance of the aircraft.

3.2 External Load Analysis

In chapter one, structural analysis was defined as the determination of load effects on a structure. Given a structure subjected to external loads, structural analysis is the determination of internal moments and forces, displacement, structural failure, etc. Therefore, before any part of the aircraft structural system or component can be structurally analyzed and sized, one must learn about the loads that will be forced on the aircraft system and each component during the operation and life cycle of the aircraft.

During the life cycle of an aircraft, the aircraft is subjected to different types of loads. The fuselage for example must be designed to withstand weight of the fuselage, loads, passenger and seating arrangements, etc, during maneuvering flight conditions, emergency landing, etc. These loads come from many sources and can range from major forces such as the weight of the aircraft, wind, snow, power plant forces, landing, launching and maneuvers, to more detail loads such as track, clamp and bolt forces. It is the “aerodynamic” or “a load” group task to come up with the external forces from the flow of air around the airplane surfaces during maneuvers (aerodynamic forces), aircraft inertia, clamp loads, etc. The final results of these groups can include axial forces, moments and distributed forces applied at the main components such as wings, fuselage, tail section, etc. During the conceptual design stage the loading condition is based on static aerodynamic loading condition, however; during the preliminary design stage it expands into dynamic loads, airframe life, etc. [44, 2004].

In the early years of aircraft design, loads were estimated for main structural sections of an aircraft mainly using hand-book calculations. Today, advanced numerical methods such as computational fluid dynamics (CFD) tools play an important role to estimate aerodynamic loads. Table 3.1 shows the time frame of various Analytical, Semi-empirical, Empirical and Numerical methods for calculating aerodynamic loads. Wind tunnel measurements are often used for situations where loads are difficult to predict.

Table 3-1 Method of Calculating Aerodynamic Loads [4, 2006]

Method	Investigator	Year	Method	Investigator	Year	Method	Investigator	Year
Lifting Line Theory	Prandtl	1921	RAE Standard Method	RAE Staff	1940	Vortex Lattice Method	Falkner	1943
Swept-Wing Theory	Busemann	1935	Hoerner	Hoener	1951	Panel Method	Hess	1962
Swept Lifting Line Theory	Weissinger	1942	DATCOM	Hoak	1960	Finite Defference Method	Adam	1975
Low Aspect Ratio Wing Theory	Jones	1946	ESDU	RAE	1963	Finite Element Method	Chung	1978
Loading Function method	Multhopp	1950	Schemenski	Schemenski	1973	Finite Volume Metgod	Rizzi	1973
Modified Lifting Line Method	kuchemann	1952	Missle DATCOM	Vukelich	1981	Spectral Methods	Gottlieb	1977
etc			etc			etc		

Load estimation is a very critical area because errors or a wrong assumptions may result in an over engineered heavy structure or an un-certifiable weak structure. As the result, national aviation authorities specify design standards in order to regulate aircraft airworthiness and safety. The Federal Aviation Administration (FAA) prescribes Federal Aviation Regulations (FAR). FARSs are grouped into different section within the Code of Federal Regulations (CFR). For Normal, acrobatic and commuter category aircraft, the regulations and direction on loads are illustrated in the CFR 14 part 23 and for transport category fixed wing airplanes; the regulations and direction on loads are described in CFR 14 part 25 through operation constraints. Many of these requirements are defined in terms of “load factors” [52, 2012].

In general, load factor is defined as the ratio of specified force acting on an aircraft divided by the gross weight of the aircraft. This specified force includes but is not limited to: aerodynamic forces, inertia forces, and ground force or water reactions. Assuming the angle of attack is not large, in a straight normal flight, Load factor is the wing lift that supports the weight of the aircraft. In the load factor equation, “n” is the load factor, “W” is the gross weight of the aircraft; “L” is the aerodynamic force perpendicular to longitudinal axis.

$$n = \frac{L}{W}$$

Load factors are sometimes expressed in “G’s”. As it was mentioned, during an un-accelerated and normal straight flight condition the wing supports the weight of the aircraft, as the result load factor is always 1, however; this value can increase during flight maneuvers and turbulent air conditions as additional aerodynamic forces are imposed. These higher values are also identified through load factors. At lower speed the load factor is constrained by the maximum Coefficient of Lift alone, but as the load factor increases as the results of higher speeds the restriction is specified by FAR Part 25 [53, 2003]. Similar to maneuver loads, loads associated with gusts and turbulent conditions at different airspeeds are also fully described in FAR part 25.341. This change in load factor verses airspeed is shown through V-n diagrams. This diagram may also be referred to as the V-g, Vgn or Vg-Vn diagram [54, 2013].

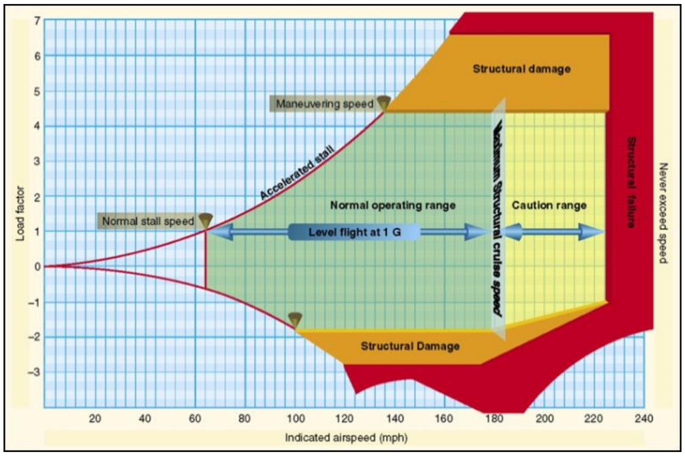


Figure 3-1 V-n diagram [54, 2013]

V-n diagrams can change for different aircraft configurations, however; the aim of the V-n diagram remains to be the same for all aircraft. The V-n diagram always defines the operation envelope or the flight limitations for a specific flight vehicle design. Structural speaking, it is a summary of an airplane's load limitation and design loads. These FAA established load limitations or design loads are "limit loads" and "ultimate loads".

> Per CFR25.301: "Strength requirements are specified in terms of limit loads (the maximum loads to be expected in service) and ultimate loads (limit loads multiplied by prescribed factors of safety)." [95, 2013]

Limit loads are usually defined as the highest loads a structure is designed to safely carry in its lifecycle, however; the ultimate loads are limit loads multiplied to a safety factor. Based on the type of aircraft, the factor of safety can vary from 1.2 to 1.75 [53, 2013]. Unlike the limit loads (that the aircraft structure is designed to always carry), the ultimate load factor is the highest load the airplane can withstand without structural failure. Permanent deformation is allowed, however; no actual failure of the major carrying components should ever take place. If a structure is designed based on a specific ultimate design load, exceeding that ultimate design load factor should cause structural failure [36, 1991].

> Per CFR25.305: "The structure must be able to support limit loads without detrimental permanent deformation. At any load up to limit loads, the deformation may not interfere with safe operation." [95, 2013]

> Per CFR25.305:"The structure must be able to support ultimate loads without failure for at least 3 second. However, when proof of strength is shown by dynamic tests simulating actual load conditions, the 3 second limit does not apply. Static tests conducted to

ultimate load must include the ultimate deflections and ultimate deformations induced by the loading.” [95, 2013]

3.3 Material and Process

It goes without saying that the demand for lighter, stronger, easier to maintain, cheaper to produce structural elements is very important in the aerospace design environments. In the early design stage, the designer is not only required to explore the most efficient structure configuration, but; this has to be done based on researching conventional and unconventional material and process. Different materials have different mechanical and physical properties and mechanical properties distribute and absorb loads differently. These materials can be divided into five categories: metals, polymers, ceramics, glasses and hybrids. Materials that are mostly used in the primary and secondary structures of fixed wing aircraft are divided into two major categories: metallic and hybrids.

The metallic material application has historically been the major part of fixed wing aircraft applications, however; the advancements of light weight and stronger composite (Hybrids) materials in the recent year’s offers better strength (and stiffness) to weight ratio compared to metallic materials [1, 2011]. It should be remembered that the use of composite materials can come with challenges such as higher material cost, higher manufacturing costs, and inability to predict structural failure, inspection and repair costs, etc. The higher material and process cost could eventually outweigh the fuel cost saving due to lower weight [48, 2009].

Consequently, during the early design stage, the material selection process is based not only on the physical and mechanical properties, but it is also based on other important considerations as well. This includes, but is not limited to lower acquisition and operation cost, manufacturability and manufacturing costs, certification and reliability and life cycle cost, corrosion and stress corrosion cracking resistance, non-destructive evaluation methods [55, 2001]. A summary of these considerations have been broken down by Hendri Syamsudin and are illustrated in table 3.2 [48, 2009]:

Table 3-2 Material Selection Consideration [48, 2009]

Acquisition, manufacturing, certification and lifecycle cost; incomplete understanding of failure mechanism and their interaction; technology risks; and the state of material supplier base.
Increasingly, airframe manufacturer are using an integrated product development approach that considers such factors as predictability, cost, non-destructive evaluation methods and criteria and repair and maintenance issues; and involves airlines and suppliers from the outset of the development process
Commercial aircraft are built and operated on a global basis with international teaming of manufacturers and suppliers. The competitive pressure of cost will continue to influence the selection criteria for the application of new material and processing technology
With the increased emphasis on affordability, it is a possibility that less new material may be developed. On the other hand, robust and cost effective processing methods as well as compliance with environmental regulation will become paramount issues to provide lower cost.

Once a material is selected, structural analysis research can begin based on material properties and load and boundary conditions. At this stage it is important to use material properties that are based on credible and statistically based sources. This information is often gathered from material handbooks or from the suppliers or a company's own (often proprietary) data base. When we are developing a model for structural analysis, it is essential to input the material allowable. The allowable can be gathered through coupons testing, looking up the information online, or through the material suppliers. Either approach would be suitable if the process that these properties are gathered meets the Federal Aviation Regulations (FARS). This is because according to FAR, material strength properties must be based on enough test results to establish allowable values on a statistical basis. More on the requirement is established in the aircraft airworthiness certification requirements; CS25.613 and FAR25.613. Metallic Material Properties Development and Standardization Handbook (MMPDS) is recognized internationally as a reliable source of metallic mechanical property data and it is considered acceptable by Federal Aviation Administration (FAA) to be compliant with FAR requirements. In the recent years, composite material handbooks are being developed as a reliable source for composite material mechanical properties too.

Following describes the material selection process used, according to the Metallic Materials Properties Development and Standardization Handbook. The information can be identified by referring to each specific material property table:

1) Select material Class (Steel, Aluminum, Magnesium, Titanium, etc)

2) Select material Sub-class (1000, 2000, 3000, 4000, etc)

3) Select member (7075, 6061, 2195, etc)

4) Select table, material specification, form and temper (AMS 4472, Plate, T82)

5) Select Thickness (0.6-1.499 inches)

6) Based on airworthiness requirements, select A basis or B basis

7) Select grain direction (to be conservative, we use the LT direction)

8) Select the following properties:

   Ultimate Tensile Strength (UTS)

   Tensile Yield Strength (YS)

   Poisson's Ratio

   Elastic Modulus (E)

9) Identify the total elongation

10) Based on the total elongation, UTS and Elastic Modulus, solve for the plastic strain at UTS. By solely using plastic strain at YS (0) and UTS makes the estimation conservative (see figure 3.2).

Table 3-3 Typical Properties of Aluminum Plates [56, 2012]

	Alloy	specification	Temper	uts (KSI)	ys (KSI)	Cy (KSI)	Density (l	elongatio	Modulus (	YS/Densit	uts/densi	E/density	
1	2014	4028	T62	67	59	62	0.101	6	10.7	584.1584	663.3663	105.9406	
2	2014	4029	T651	67	59	61	0.101	6	10.7	584.1584	663.3663	105.9406	
3	2024	QQ-A-250/4	T351	63	42	45	0.1	8	10.7	420	630	107	
4	2024	QQ-A-250/4	T351	61	40	43	0.1	8	10.7	400	610	107	
5	2024	QQ-A-250/4	T851	63	56	56	0.1	5	10.7	560	630	107	
6	2027	4213	T351	67	46	47	0.101	14	10.6	455.4455	663.3663	104.9505	
7	2050	4413	T84	73	67	69	0.098	7	10.9	683.6735	744.898	111.2245	
8	2098	4327	T82	73	67	69	0.097	6	10.9	690.7216	752.5773	112.3711	
9	2099	4458	T86	72	62	63	0.095	5	11.3	652.6316	757.8947	118.9474	
10	2124	4101	T851	66	57	57	0.1	5	10.4	570	660	104	
11	2195	4472	T82	82	76	75	0.098	5	10.8	775.5102	836.7347	110.2041	
12	2219	QQ-A-250/30	T851	62	46	48	0.103	8	10.5	446.6019	601.9417	101.9417	
13	2219	QQ-A-250/30	T87	64	51	53	0.103	6	10.5	495.1456	621.3592	101.9417	
14	2519	MIL-DTL-46192	T87	68	58	60	0.102	7	10.5	568.6275	666.6667	102.9412	
15	2624	4473	T39	71	58	61	0.1	9	10.3	580	710	103	
16	5052	4015	H32	31	22	23	0.097		10.1	226.8041	319.5876	104.1237	
17	5083	4056	O	40	18	18	0.096	16	10.2	187.5	416.6667	106.25	
18	5083	ASTM B928	H321	44	28		0.096	12	10.2	291.6667	458.3333	106.25	
19	5086	ASTM B209	O	35	14	14	0.096		10.2	145.8333	364.5833	106.25	
20	5086	ASTM B209	H34	44	33	34	0.096		10.2	343.75	458.3333	106.25	
21	5086	ASTM B209	H112	35	16	16	0.096	10	10.2	166.6667	364.5833	106.25	
22	5454	QQ-A-250/10	H32	36	24	24	0.097	8	10.2	247.4227	371.134	105.1546	
23	5456	QQ-A-250/9	O	42	19	19	0.096	16	10.2	197.9167	437.5	106.25	
24	6061	4026	T451	30	16	16	0.098	16	9.9	163.2653	306.1224	101.0204	
25	6061	4027	T651	42	35	36	0.098	6	9.9	357.1429	428.5714	101.0204	
26	7010	4205	T7451	72	62	63	0.102	6	10.2	607.8431	705.8824	100	
27	7010	4204	T7651	76	66	67	0.102	6	10.2	647.0588	745.098	100	
28	7049	4200	T7351	74	65		0.103	8	10.1	631.068	718.4466	98.05825	
29	7050	4050	T7451	74	64	66	0.102	9	10.3	627.451	725.4902	100.9804	
30	7050	4201	T7651	76	66	68	0.102	8	10.3	647.0588	745.098	100.9804	
31	7055	4206	T7751	89	85	86	0.103	8	10.4	825.2427	864.0777	100.9709	
32	7056	4407	T7651	83	78	78	0.102	10	10.4	764.7059	813.7255	101.9608	
33	7075	4045	T651	78	68	71	0.101	6	10.3	673.2673	772.2772	101.9802	
34	7075	4044	T62	74	67	70	0.101	7	10.3	663.3663	732.6733	101.9802	
35	7075	4078	T7351	69	57	56	0.101	8	10.3	564.3564	683.1683	101.9802	
36	7075	4315	T7651	72	62	62	0.101	6	10.3	613.8614	712.8713	101.9802	
37	7075	4049	T651	76	66	65	0.101	7	10.3	653.4653	752.4752	101.9802	
38	7075	4048	T62	72	64	68	0.101	9	10.3	633.6634	712.8713	101.9802	
39	7075	4316	T7651	68	58	57	0.101	6	10.3	574.2574	673.2673	101.9802	
40	7150	4306	T6151	84	77	77	0.102	9	10.2	754.902	823.5294	100	
41	7150	4252	T7751	84	77	81	0.102	8	10.3	754.902	823.5294	100.9804	
42	7255	4463	T7751	91	86	88	0.103	7	10.3	834.9515	883.4951	100	
43	7449	4250	T7651	83	77	80	0.103	8	10.3	747.5728	805.8252	100	
44	7449	4299	T7951	88	84	85	0.103	7	10.4	815.534	854.3689	100.9709	
45	7475	4090	T651	78	68	68	0.101	9	10.2	673.2673	772.2772	100.9901	
46	7475	4089	T7651	70	59	59	0.101	8	10.2	584.1584	693.0693	100.9901	
47	7475	4204	T7351	70	60	61	0.101	9	10.3	594.0594	693.0693	101.9802	

Table 3-4 A Material Database Samples [56, 2012]

Table 3.7.9.0(b_2). Design Mechanical and Physical Properties of 7075 Aluminum Alloy Sheet and Plate

Specification	AMS 4045 and AMS-QQ-A-250/12[a]													
Form	Plate													
Temper	T651													
Thickness, in.	0.250-0.499		0.500-1.000		1.001-2.000		2.001-2.500		2.501-3.000		3.001-3.500		3.501-4.000	
Basis	A	B	A	B	A	B	A	B	A	B	A	B	A	B
Mechanical Properties:														
F_{tu}, ksi:														
L	77	79	77	79	76	78	75	77	71	73	70	72	66	68
LT	78	80	78	80	77	79	76	78	72	74	71	73	67	69
ST	...	...	...	...	...	...	70[b]	71[b]	66[b]	68[b]	65[b]	67[b]	61[b]	63[b]
F_{ty}, ksi:														
L	69	71	70	72	69	71	66	68	63	65	60	62	56	58
LT	67	69	68	70	67	69	64	66	61	63	58	60	54	56
ST	...	...	...	...	...	...	59[b]	61[b]	56[b]	58[b]	54[b]	55[b]	50[b]	52[b]
F_{cy}, ksi:														
L	67	69	68	70	66	68	62	64	58	60	55	57	51	52
LT	71	73	72	74	71	73	68	70	65	67	61	64	57	59
ST	...	...	...	...	...	...	67	70	64	66	61	63	57	59
F_{su}, ksi: L & LT	43	44	44	45	44	45	44	45	42	43	42	43	39	41
F_{bru}[c] ksi:														
L & LT (e/D = 1.5)	117	120	117	120	116	119	114	117	108	111	107	110	101	104
L & LT (e/D = 2.0)	145	148	145	148	143	147	141	145	134	137	132	135	124	128
F_{bry}[c], ksi:														
L & LT (e/D = 1.5)	97	100	100	103	100	103	98	101	94	97	89	93	84	87
L & LT (e/D = 2.0)	114	118	117	120	117	120	113	117	109	112	104	108	98	103
e, percent (S-Basis):														
LT	9	...	7	...	6	...	5	...	5	...	5	...	3	...
E, 10^3 ksi	10.3													
E_c, 10^3 ksi	10.6													
G, 10^3 ksi	3.9													
μ	0.33													
Physical Properties:														
ω, lb/in.^{3}														
C, K, and α														

Revised: Apr 2008, MMPDS-04, Item 05-14
a Mechanical properties were established under MIL-QQ-A-250/12.
b Caution: This specific alloy, temper, and product form exhibits poor stress-corrosion cracking resistance in this grain direction. It corresponds to an SCC resistance rating of D, as indicated in Table 3.1.2.3.1(a).
c Bearing values are "dry pin" values per Section 1.4.7.1. See Table 3.1.2.1.1.

11) Convert data to true stress and true strain. Figure below illustrates how for non-linear analysis, the full plasticity curve is sometimes not considered, but to save time; instead the tangent line that connects YS to UTS is used.

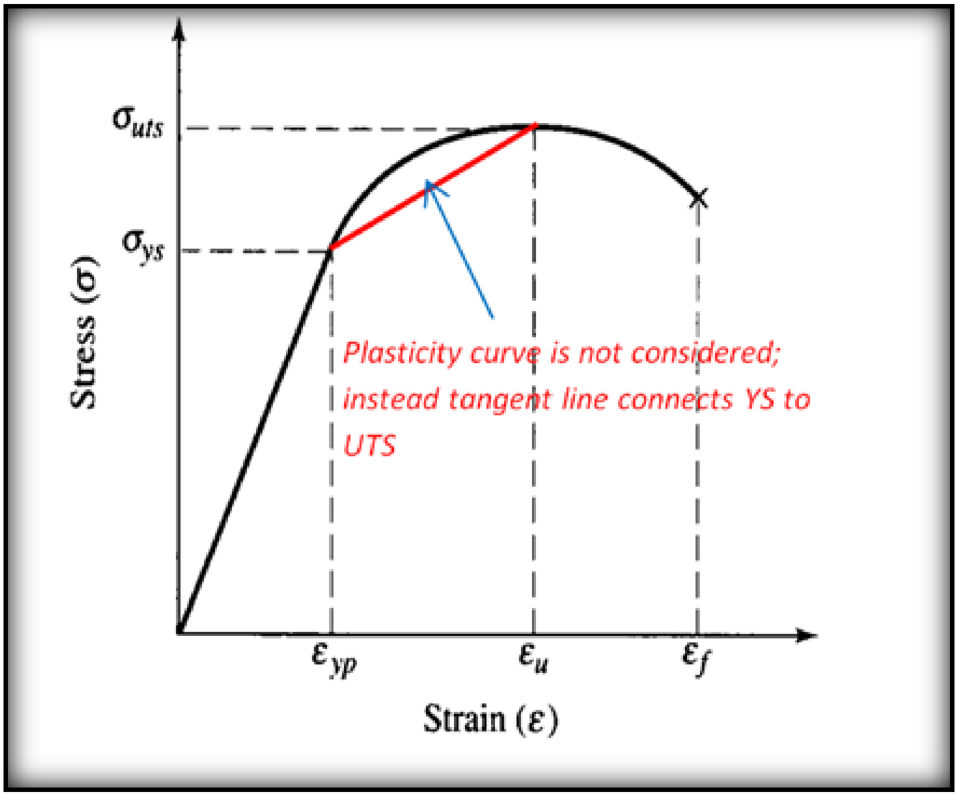


Figure 3-2 Typical stress-strain curve [57, 2013]

3.4 Design and modeling

The basis of doing structural analysis is to have a structural model. Having a model setup and ready for analysis is as essential as having design loads, as they are both pre-requisites to starting structural analysis. Analysis cannot begin without loads or a predefined structural arrangement. As it was mentioned in the previous chapter; the predefined structural arrangements of structural components within a structural system are often based on preliminary human knowledge (experience), topology optimization methods or both. The structural components that make the structural system can be prismatic, non-prismatic members or both. The joints connecting these members can be fixed (i.e. welded) or free to rotate in different directions. Aircraft joints are mainly fixed joints and are riveted, bolted, welded because they are high stress and fatigue areas [58, 2009].

As it was said before; the intent of structural design at the conceptual design stage is to evaluate design concepts through evaluation of major design parameters. These main design parameters include:

configuration arrangement of main carrying components, cross sections and materials. For example, each of these components in a structural system can be created from a different cross section (Circular, I-beam, Rectangular, etc) and lengths. Different cross sections have different surface inertias. The combination of these design parameters can change the structural performance of each concept. To select the most efficient (i.e. best strength to weight ratio, best aerodynamic performance, etc) structural arrangement, one has to evaluate the effect of design parameters toward the concept. The effect of design parameters is evaluated through structural analysis.

Modeling a design on the other hand is the process of defining the parts, shapes and dimensions on a piece of paper or computer aided design software. Modeling can be surface, solid or wire-frame. Surface modeling is a collection of surfaces and not applicable for primary structures [59, 2013]. Solid modeling is the method of defining a part as solid or structural system made from non-prismatic and prismatic members, however; wire-frame modeling is method of connecting prismatic members together. Wire-frame modeling is the best of the three because it can limit the design parameters to the primary conceptual design variables and it is also done in a much quicker time frame. Whether the modeling is done on a piece of paper or in auto-CAD, wire-frame modeling is done much faster as the members are all prismatic.

At the conceptual design stage, it is possible to model a structural system from both prismatic and non-prismatic components, however; the use of a non-prismatic component adds indefinite design variables into the conceptual evaluation stage. The author believes that the initial baseline structural system should be modeled with prismatic members until the designer has gained a mastery of how loads are distributed.

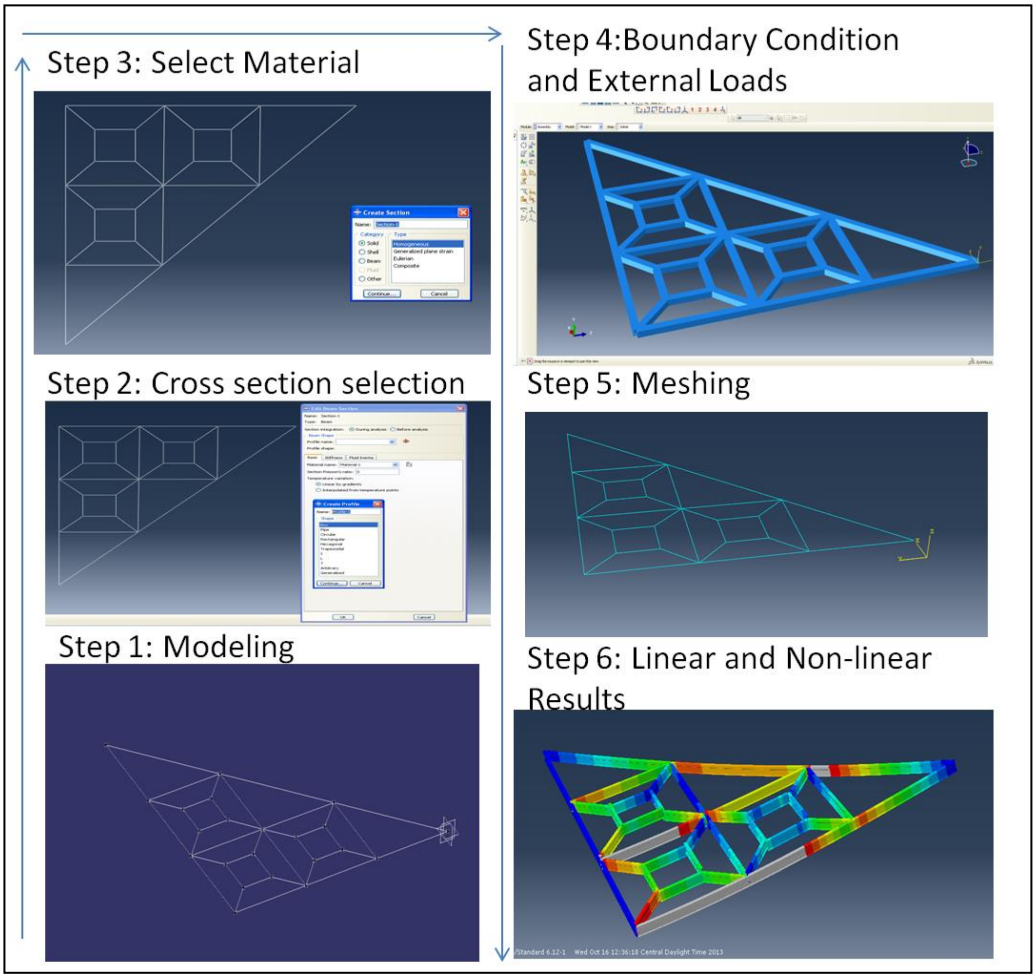


Figure 3-3 Wire frame FEA modeling of a delta wing-box

3.5 Structural Analysis Toolbox

As it was said before, aircraft industry has become dependent on CAE tools more than ever before. One of the most valuable and most used structural analysis CAE advance numerical tools is the Finite Element Analysis (FEA) methodology. Yet, before the advancement of FEA, engineers and designers used classical and analytical methods and managed to become airborne without difficulty. Figure 3.4 shows number of fixed wing aircraft that were designed before the advancements of CAE tools.

Figure 3-4 American aviation from 1903-1945 [61, 2013]

These advanced numerical methods are the result of classical methods that over the years evolved and are now used in advanced numerical packages. Some of these classical methods are often forgotten and hardly used in the aircraft industry anymore. Table 3.2 shows various structural analysis methods.

Table 3-5 Structural Analysis Methods

Displacement Methods	Forces Methods	Energy Methods
• The slope deflection Methods (Beams, frames with sidesway and without sidesway) • Moment Distribution Methods (Beams, frames and multi frames with sidesway and without sidesway, • Stiffness Method: Matrix & FEA Methods	• Used in Beams and frames • Three moment equations	• Principle of superposition, strain Energy • Castigliano's Theorems • Theorem of Least Work • Virtual Work • Engresser's Theorem

Before knowing which structural analysis method to use, it is important to identify the classification of the structural model. Structural models are divided into two major groups. They are either determinate or they are indeterminate. If it is possible to determine the internal forces (stresses) and reactionary forces and moments of structural members using the statically equations of equilibrium alone, then that structure is a determinate structure. Also, if we can only solve for the reactionary forces and moments using equations of equilibrium, then the structure is an externally determinate structure. To be classified as a determinate structure, the structure must be externally and internally determinate [18, Kalani][33, Kharagpur].

An indeterminate structure, on the hand is a structure that is a lot more complex than a determinate structure. Now, if it becomes impossible to determine forces (stresses) and reactionary forces and moments of structural members using the statistical equations of equilibrium, then that structure is externally and internally an indeterminate structure. If the reactionary forces and moments of structural system can be solved using equations of equilibrium but it is not possible to solve for the internal loads, then that structure is externally determinate but internally indeterminate. To be classified as an indeterminate structure, the structure can be externally or internally determinate or externally and internally indeterminate [18, Kalani][33, Kharagpur]. Other differences between determinate and indeterminate structures are shown in table 3.6.

Table 3-6 Classification of Structures [62, 2013]

S. No.	Determinate Structures	Indeterminate Structures
1	Equilibrium conditions are fully adequate to analyse the structure.	Conditions of equilibrium are not adequate to fully analyse the structure.
2	Bending moment or shear force at any section is independent of the material property of the structure.	Bending moment or shear force at any section depends upon the material property.
3	The bending moment or shear force at any section is independent of the cross-section or moment of inertia.	The bending moment or shear force at any section depends upon the cross-section or moment of inertia.
4	Temperature variations do not cause stresses.	Temperature variations cause stresses.
5	No stresses are caused due to lack of fit.	Stresses are caused due to lack of fit.
6	Extra conditions like compatibility of displacements are not required to analyse the structure.	Extra conditions like compatibility of displacements are required to analyse the structure along with the equilibrium equations.

At the conceptual design stage, the main sections of an aircraft structure (Wing, Fuselage, etc) could be complex enough that statistical equations of equilibrium alone are not good enough to analyze the structure. As the result methods of analysis ought to be methods that can be used to solve determinate and indeterminate designs.

In the next chapter we will briefly look at the history of structural analysis; from 3000 BC to its modern development. The emphasis would be the displacement methods of analysis as moment distribution method is a displacement method. The goal is to explore classical analytical methods for structural analysis at the conceptual design stage. This will be done by considering and evaluating the strength and limitation of a displacement analytical method vs. Finite Element Analysis. This evaluating will be done over four case studies. To objectively evaluate the strength and limitation of the classical analytical methods vs. Finite Element Analysis, these case studies will progress from simple designs to more complex structural designs.

3.6 Summary

In the previous chapter we learned that the intent of structural design at the conceptual design stage was to help evaluate and flesh-out design concepts to the point where they can be evaluated in more details at the preliminary and detail design stages. This is done to “help improve early design decisions by utilizing structured concept evaluation and decision making on process or critical airframe parameters [48, 2009]. In this chapter we learned that these parameters play an important role in maintain the structural integrity of an aircraft.

These main structural design parameters are: the External loading conditions (design loads), type of material and structural arrangement (design and boundary conditions). We also learned that structural analysis cannot begin without loads or a predefined structural arrangement. The predefined structural arrangements of structural components within a structural system are often based on preliminary human knowledge (experience), topology optimization methods or both. The parameters that define the configuration include: configuration arrangement of main carrying components, cross sections and materials. Different materials have different mechanical and physical properties and mechanical properties (i.e stiffness) distribute and absorb loads differently. During the early design stage, the material selection process is also based on operation cost, manufacturability and manufacturing costs, certification and reliability and life cycle cost, corrosion and stress corrosion cracking resistance, non-destructive evaluation methods, etc as well. Once the loading requirements, pre-defined design space and design parameters have been identified, structural analysis can begin.

At the end of this chapter we looked into various methods of structural analysis and how at the conceptual design stage, the main sections of an aircraft structure (Wing, Fuselage, etc) could be complex enough that statistical equations of equilibrium alone are not adequate to analyze the structure. For that reason, the methods of analysis ought to be methods that can be used to solve determinate and indeterminate structural designs.

In the next chapter we will provide a brief historical overview of displacement structural analysis methods and FEA. This will be done by considering and evaluating the strength and limitation of a displacement analytical method vs. Finite Element Analysis. This evaluating will be done over 4 case studies starting from chapter 4 and ending in chapter 5. To objectively evaluate the strength and limitation of the classical analytical method vs. Finite Element Analysis, these case studies will progress from simple designs to more complex structural designs.

Chapter 4

Structural Analysis Methods

4.1 A Brief History of Structural Analysis (Displacement) Methods

Best known for discovering gravity, in 1687 Isaac Newton publishes the laws of motion that described the relationships between force(s) acting on an object and the object intending to respond to that force, successfully illustrating the relationship between motion of bodies based upon a system of external forces. Isaac Newton published this theory in his book Philosophiae Naturalis Principia Mathematica marking a turning point in how we understand the classical mechanics and modern physics [63, 2003]. However; the development of classical mechanics can go as far back as the 4th century BC and Aristotelian physics. Famously Isaac Newton once said:

> “If I have been able to see a little farther than some others, it was because I stood on the shoulder of giants” [64, 2010].

Table 4.1 provides a brief history of some of the contributions made from Imhotep (3000 BC) to medieval period (477-1492) and the Renaissance.

Human understanding of “physics” during the pre-renaissance era may have been a lot different than how it is understood today, but nevertheless it contributed to the later discoveries. Martin Heidegger writes this about Aristotle Physics:

> “Aristotelian ‘physics’ is different from what we mean today by this word, not only to the extent that it belongs to antiquity whereas the modern physical sciences belong to modernity, rather above all it is different by virtueof the fact that Aristotle’s ‘physics’ is philosophy, whereas modern physics is a positive science that presupposes a philosophy…This book determines the warp and woof of the whole western thinking , even at that place where it, as modern thinking, appears to think as odds with ancient thinking. But opposition is invariably comprised of a decisive, and often even perilous, dependence. Without Aristotle’s physics there would have been no Galileo” [65, 1991].

Table 4-1 Brief History of Structural Design During Pre-renaissance Era [64, 2010]

Name	Year	Description
Imhotep	2600 BC	Often credited to be the first structural engineer. Designed the step pyramid of Sakkara. Similar work does exist from 3000 BC and the Archaic period.
Hamurrabi	1750 BC	Identified detailed rules and penalties to improve the saftey of structural architects and homes.
Pythagoras	523 BC	Reported to have coined the term *Mathematics* and *Philosopher.*
Aristotle	384-322 BC	Credit having written in ore than 25 different fileds of knkowledge, including physics and mathematics.
Ptolemy	356-323 BC	Established the largest library of the ancient world, containing 700,000 scrolls. Many translated and by the Arabs and Persians [96, 2013]
Euclid	315-250 BC	First professor of geometry in Alexandria.
Archimedes	287-212 BC	Famous Mathematician and Physicist. Introduced the concept of center of gravity and considered by many as the founder of mechanics.
Marcus Vituvius Poll	70-25 BC	Roman Architect and artillery engineer, wrote books on architecture.
Sun Tzu	400	Famous chinese mathematician, authored "The Mathematical Classic of Sunzi", provides detailed multiplication and division algorithm methods. It has been showsn by lam Lay Yong that Al-Khawarizmi's methods are very similar to Sub Tzu's earlier work.
Isidore of Miletus & Anthemius of Tralles	532-537	Famous Hagia Sophia, a Byzantine structure was built by orders of Emperor Justinian I. The structure combined the Romans bascilca and central plan of a sum reinforced done, to withstand earthquakes and the weight of the structure. Isiodre nephew, later, introduced the new dome design that can be seen today in Istanbul, Turkey.
Li Chun	600	Anji Bridge world's oldest spandrel arch bridge was made from stine. This bridge is still standing 1400 years after it was built.
Al-Khwarizmi	780-850	Famous Persian mathematician, adopted indian numbering system. Developed Algebra through systematic approach to solving linear and quadratic equations.
Avicenna	980-1037	In his book of healing, he help develop his theory of motion. Was the first to describe that motion was the result of inclination transferred to the object by a thrower and that projectile motion in vaccum would not ease [97, 2005]
Leonardo da Vinci	1452-1519	Explored various concepts of mechanics. Studies the strength of structural materials through physical testing. Studied the effects of external loads on different beams and columns.

Before Galileo and Isaac Newton, Avicenna and Jean Buridan’s efforts also helped pave the way in the area of classical mechanics. In the 11 Century, Avicenna worked on and developed a detailed and correct theory of motion [67, 2005], and this may have helped pave the way for concepts such as inertia, momentum and acceleration [68, 2013]. Other works in the area of mechanics of bodies were also done by Hibat Allah Abul-Barakat Al-Baghdaadi [69, 1970], Leonardo Da Vinci, Al-Birjandi [70, 2001].

It was not until the late 1400s that continuous concepts of mechanics were tested and explored in more details through scientific experiments. In the late 1400s, Leonardo da Vinci explored beams, columns and strength of various materials. In the 1600s Galileo performed numerous experiments trying to first understand and then describe mathematical rules for the motion of objects. One of his famous experiments was the famous dropping of two cannonballs with different weights, but from the same distance. This experiment showed that both objects hit the ground at the same time and proved an error in Aristotle’s belief that speed of fall is proportional to weight [64, 2010][71, 2013]. These experiments help prove one of the basic foundations of classical mechanics; the theory of acceleration of motion. However, some of these basic conclusions were faulty as well. For example, in his final book, Galileo wrote on the topic of mechanical properties based on the strength of cantilevered beams [64, 2009]. He concluded that stress did not vary throughout the beam, while today we know this theory of him cannot be correct.

Figure 4-1 Galileo's beam [64, 2010]

It wasn’t until 1687, when isaac Newton published “Philosophiae Natural Principia Mathematica” (often referred to as “the Principia”) that for the first time Newton’s laws help describe the relationships between force acting on an object and the response of an object to that force. These Newton’s laws of motion are described in this book [72, Newton]. As mentioned before, these laws help shape the foundation of classical mechanics. Third law of motion illustrates the relationship between external forces and stress. This means if the forces are exerted by another body they are external loads (i.e. Gravity) and the forces acting to keep the body together are internal loads. The concentration of these internal forces per unit area is what we define as stress distribution. In 1687, another famous physicist Robert Hooke had also apparently discovered these fundamental principles, and claimed that Newton had stolen from him, however; this was not widely accepted and is not what made Robert Hooke famous.

Instead, in the 1660 Robert Hooke came up with the law of elasticity. This is a principle that states that a finite force has to be applied to extend or compress a spring by a certain displacement. The constant factor in this force/displacement relationship is denoted by stiffness or rigidity of an object in resisting displacement. The results of Hooke's experiments were published in 1687 in a paper called De Potentia Restitutiva. This was also the first published paper, where elastic properties are discussed [64, 2010].

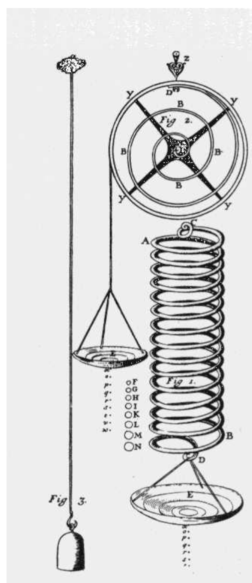


Figure 4-2 The setup experiment by Robert Hooke [64, 2010]

Hooke's work was continued through Jacob Bernoulli and Leonard Euler and others. In the early 1700s, Jacob Bernoulli continued investigating beam deflection and stiffness through analysis of elastic flexure of a beam. In mid 1700s, Leonard Euler introduced analytical methods as a replacement to Newton's geometrical methods, and was able to get the exact solutions for deflections of the cantilever beam problem and buckling load of a column. In the late 1700s and early to mid-1800s, Coulomb, Navier, Lame, Clapyron and de saint-Venant were among great contributors that help further develop the theory of elasticity. By this time in history, this theory could be stated as the relationship between forces applied at certain locations of component and displacements occurring as the result of this external force(s) thought the component. This method was also known as the displacement method of analysis. Later, in mid 1800s, an alternative approach was introduced. In this method the internal forces were treated as the unknowns, and compatibility equations were written for displacements (and rotations), and in return magnitude of force is solved from continuity requirements. It has been shown by Kardestuncer [73, 1974] and others [74, 2007][33, Kharagpur] why force methods were not as widely used as the displacement method of analysis.

In the displacement method of analysis equations were written for six displacement components by implementing Hooke's law and through satisfying equations of equilibrium. In 1862 Alfred Clebsch, in his book [76, 1862] used the displacement method for linear analysis of a 3D truss. In Clebsch's model, the ends of the truss bars were free to rotate at the joints, to make this work; assumptions need to be made that displacements were small enough and the joints were not rigid joints. But by this time in history, constructions of tall buildings and railways had started and these real world structures were made from columns and beams that were connected with rigid and stiff joints [38, 2001]. So, in 1880, Heinrich Manderla was able to solve for this issue by considering rigid joints and translating bending moments from one beam or column to another beam or column [78, 1880][16, 2006]. In 1892, this method was now published and improved by Otto Mohr [79, 1892]. Otto Mohr assumed that the displacements at the nodes or joints are small enough that the bending moment does not induce to the side-sway or displacement of the joints. But in reality, free joints always can side-sway, especially in large building and bridge structural frames.

4.2 Deflection Method

To tackle this side-sway problem, in 1915, Wilson and Maney came up with the slope deflection method. Similar approach was published by Axel Bendixen in 1914 [80, 1915]. Unlike, the earlier method, this method considered all joints to be rigid so much that the angles between members that meet each other at the joints remain unchanged once force is applied. Bruhn writes:

> “In this method the rotation at the joints are treated as the unknown. So for example, for a member bounded by two end joints, the end member can be expressed in terms of the end rotations. Furthermore for static equilibrium the sum of the end moments on the members meeting at a joint must be equal to zero. Once the unknown joint rotations are found the end moments can be computed from the deflection method” [36, 1997].

The slope deflection method was widely used and the method of choice prior to 1930, but its popularity begins to wane in favor of the moment distribution method [37, 1961]. Bendixen, Wilson and Maney did not take advantage of the iterative method. This was the major difference, also very little attention had been paid to the practical hand solution of final set of equations [76, 2006].

4.3 Moment Distribution Method

In 1922, Calisev published an iterative approach for frames that can side-sway [81, 1923]. This method was very similar to the deflection method in how the joint rotations were treated, but instead of setting up the full set of equations, he solved a series of equations with only one known each time. He arbitrarily “locked” all joints, calculated the fixed end moments and summed the moments of the members for each joint. Once this process was completed for all joints, the joint with the highest fixed end moment was “unlocked”. From that, he solved for the moments and joint rotations that are transferred to the neighboring joints. Then, these new distributed moments are added to the moment summation at those joint. This procedure was the repeated for all joints. As the iteration is continued a second and third time for all joints, the imbalance in joints continues to reduce as the accuracy of the end moments and rotations increases [16, 2006].

A similar method was published in 1930 by Hardy Cross [82, 1930], with the major difference was that Hardy Cross did not think that he needs to calculate joint rotations, he instead carried over the unbalance moments at joints in proportion to the stiffness of the connecting beams. This method was called the Hardy Cross method or Moment Distribution Method. By late 1930s this became the method of choice and very popular method among architects, airplane designers and structural engineers [36, Bruhn][37, 1961][38, 2001]. Mainly because the demand to build multistory structural frames had risen, and the development of new materials had made it vital to come up with a method of analysis that combines reasonable accuracy with faster solutions. The methods until 1930s required tedious longhand calculations to solve rigid frame indeterminate structures. Hardy Cross discovered that he could bypass adjusting rotations to get the moment balance at each node. This was accomplished by distributing the unbalanced moment while unlocking one joint at a time and keeping all the other joints temporary fixed [38, 2001][16, 2006]. In order describing the moment distribution method, Bruhn famously writes:

> "In the cross method each member of a structure is assumed in a definite restrained state. Continuity of the structure is thus maintained but the statics of the structure are unbalanced. The structure is then gradually released from its arbitrary assumed restrained state according to definite laws of continuity and statics until every part of the structure rests in its true state of equilibrium [36, 1997]".

In description of the moment distribution method, Hardy Cross himself writes [12, 1930]:

> The method of moment distribution method is this: (a) Imagine all joints in the structure held so that they cannot rotate and compute the moments at the ends of the members for this condition; (b) at each joint distribute the unbalanced fixed end moment among the connecting members in proportion to the constant for each member defined as "stiffness"; (c) multiply the moment distributed to each member at a joint by the carry over factor at the end of the member; (d) distribute these moments just "carried over"; (e) repeat the process until the moment to be carried over are small enough to be neglected; (f) add all moments – fixed end moments, distributed moments, moments carried over at the end of each member to obtain the true moment at the end.

Once this process was ended, one could use the static equations of equilibrium to treat each member as a determinate structure and solve for the reactionary forces at each joint. Following describes some of the definitions used in the moment distribution method according to Sanks [37, 1961]:

> "Fixed End Moment: This moment is one which would exist at the end of a moment if its end were fixed against rotation. Fixed end moments can also be the result of deflection of one joint with respect to another" [37, 1961].
>
> These equations are provided in works of Bruhn and Sanks.
>
> "Stiffness: stiffness is that moment which is required to rotate one end of a member through an angle of 1 radian. The stiffness is designated by K and equals EI/L for a fixed prismatic member. For non-prismatic mebers, stiffness can be calculated analytically, determined experimentally or found from charts and tables" [37, 1961].

These charts and tables are provided in Appendix A of Sanks book.

> "Carry Over factor: if one end of a member is rotated by an applied moment while the other end is held fixed, some moment is induced at the fixed end. The ratio of the moment at the fixed end to the moment at the rotated end is called the carry over factor. The value of the carry over factor is 0.5 for prismatic members, for non-prismatic members, the carry over factor (just like stiffness) must be determined analytically and or experimentally" [37, 1961].

"Distribution factor: when a joint composed of several rigidity connected members rotates, moments are induced into the members. The proportion of the total moment that is induced into each member is the distribution factor" [37, 1961].

"Subscripts: are used for identification. The first letter indicates the end to which the moment, stiffness or carry over factor applies, and the second letter indicates the member" [37, 1961].

The general principals and a brief comparison of Moment Distribution Method and Finite Element Analysis are expressed through case study number one. In this example, we will solve for the fixed end moments, reactions, bending moment diagram and compare these hand calculations results with the proposed FEA modeling Method. At the same time, we will discuss the design/analysis transparency that is produced through this classical analytical method.

4.4 Case Study One: Indeterminate Structure without Side-sway vs. FEA

This is the study of an indeterminate structure, without side-sway. In this problem we assume I and E to be fixed, and length of all members to be the same (4m).

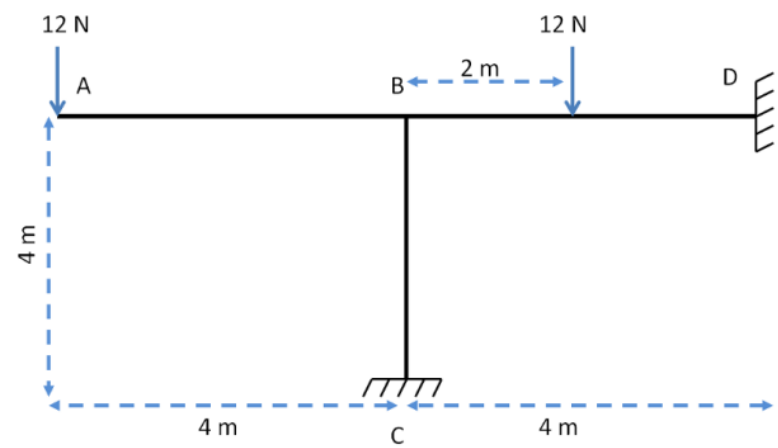


Figure 4-3 Simple indeterminate frame without side-sway

Step 1) Find stiffness:

$K_{BD} = \frac{EI}{4} = 0.25\ EI$

$K_{BC} = \frac{EI}{4} = 0.25\ EI$

Step 2) Find Distribution Factors:

$DF_{BA} = DF_{AB} = DF_{DB} = DF_{CB} = 0$

At the fixed supports the distribution factor is zero because when we arbitrarily release a fixed support (joints D and C here), all imbalance moment goes into the connecting members (BC and BD) and not the joint, and therefore; the distribution factor is zero. The cantilever beam stiffness factor is zero since joint A is free to displace. The only joint that can arbitrarily be released is joint B. Therefore, we can start by adding the stiffness of the members attached to Joint B, in order to compute the stiffness at joint B.

$\Sigma K_B = K_{BD} + K_{BC} = 0.5\ \mathrm{EI}$

$DF_{BD} = \frac{0.25\ \mathrm{EI}}{0.5\ \mathrm{EI}} = 0.5$

$DF_{BC} = \frac{0.25\ \mathrm{EI}}{0.5\ \mathrm{EI}} = 0.5$

Step 3) Find End moments:

$M_{BA} = (-12)(4) = -48\ \mathrm{N.m}$, $M_{AB} = 0\ \mathrm{N.m}$,

$M_{BD} = 6\ \mathrm{N.m}$, $M_{DB} = -6\ \mathrm{N.m}$,

$M_{BC} = 0\ \mathrm{N.m}$, $M_{CB} = 0\ \mathrm{N.m}$,

The next step in the solution is to solve for the moments at the joints using the moment distribution method. We go straight to joint B.

The unbalanced moment in joint B is $(-48\ \mathrm{N.m} + 6\ \mathrm{N.m})$: $-42\ \mathrm{N.m}$. This joint is balanced by distributing $[-(-42\ \mathrm{N.m}) = 42\ \mathrm{N.m}\ ]$ to member BC and BD. The unbalanced moment (42 N.m) is now distributed based on the value of distribution factor.

$\text{To BC} = (42)(0.5) = 21\ \mathrm{N.m}$ and The Carry over to C is $10.5\ \mathrm{N.m}$

$\text{To BD} = (42)(0.5) = 21\ \mathrm{N.m}$ and The Carry over to D is $10.5\ \mathrm{N.m}$

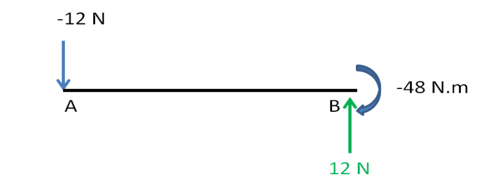
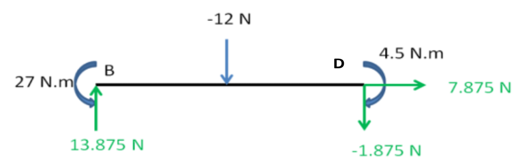
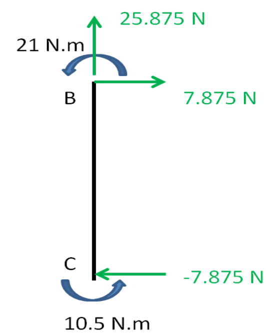
Since joints D and C are rigid (as the result $DF_{DB} = DF_{CB} = 0$), an unbalanced moment is not distributed back. Now we can compute the final end moments:

$BA = -48\ \mathrm{N.m}, \ \ AB = 0$

$BD = 6 + 21 = 27\ \mathrm{N.m}, \ \ DB = -6 + 10.5 = 4.5\ \mathrm{N.m}$

$BC = 0 + 21 = 21\ \mathrm{N.m}, \ \ CB = 10.5 = 10.5\ \mathrm{N.m}$

Table 4-2 Case Study One: Solving for Reactionary Forces

Equations of Statics	FBD
$R\text{-}B_y = -(-12) = \underline{12\ N}$	
$\Sigma M_D = 0$, $(4.5+27-24+(4\ R\text{-}D_y))=0$, $R\text{-}D_y = \underline{-1.875\ N}$, $R\text{-}B_y = -(-1.875-12) = \underline{13.875\ N}$	
$\Sigma M_C = 0$, $(21+10.5+(4\ R\text{-}C_x))=0$, $R\text{-}C_x = \underline{-7.875\ N}$, $R\text{-}B_x = -(-7.875) = \underline{7.875\ N}$ $R\text{-}B_y = \underline{12 + 13.875} = 25.875\ N$	

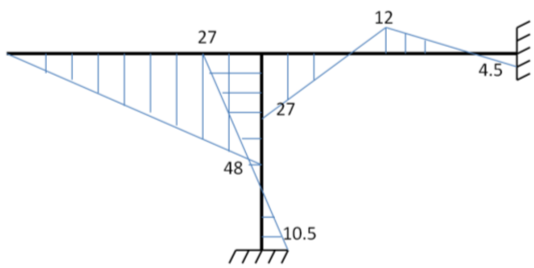


Figure 4-4 Bending moment diagram

Note that, the designer is actually solving the moments and forces based on each specific design parameters. For example, the designer can see how would changing the elasticity (material), inertia (cross section) vs. change in length of a specific member can affect distribution factor and how that change in the distribution factor effects the moment distribution and forces throughout each step, as the design parameters are visible during each one of these step. This transparency to the design/analysis procedure provides the greatest advantage of the analytical methods and is often referred to as "Sanity Check". Once the moments at joints are determined, the equations of statics can be used to solve for the reactions at each joint.

To help explain this more precisely, let us assume that designer wants to reduce the Reactionary force C_x in case study one. The first thing and easiest option that comes to mind is to decrease or increase the length of BC member, as R-C_x is; $(-(M_{bc} + M_{cb})$/length of BC). Now, the moment distribution method can show us the exact effect of that increase or decrease of the length through the distribution factor. The change in length results in a new Distribution Factor which now distributes the moments differently along members BD and BC. The designer can see if BD is decreased from 4 to 2 meters, the K_{BD} now becomes 0.50EI (instead of 0.25EI) and therefore, DF_{BD} now becomes: $(\frac{0.5EI}{0.5EI+0.25EI} = 0.667)$, and DF_{BC} instead becomes $\left(\frac{0.25EI}{0.75EI}\right) = 0.333$. As the result of this change (DF_{BD} becomes larger) in the distribution factor, higher moments are induced from joint B to member BC. Therefore, moments M_{BC} and M_{CB} will increase. The combination of higher moments and smaller BC length results in a higher reactionary force at C_x. As we can see, at this stage the designer has a perfect physical feasibility of each specific design parameters and how they can change the distribution factor and how the distribution factor (as it provides a direct relationship between the design parameters) can directly change the moments and forces (Figure 4.5).

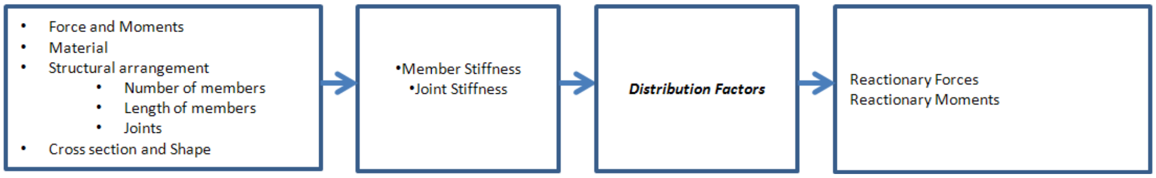


Figure 4-5 Importance of distribution factor

Next we will verify the results of case study one with the analytical method (moment distribution method) by comparing it to Finite Element Analysis method. This will also serve as a self-checking approach that also demonstrates the accuracy of the Finite Element Method.

Step1 – Design: We use wire-frame modeling to design the frame in CatiaV5. The reason we can use wire frame modeling is because the members are all assumed to be prismatic. A prismatic member is a member that has a fixed “E” and “I” throughout its length.

Modeling starts by identifying the location of the joints in space and the length of each member connecting the joints. To do this in Catia V5, instead of the “Part Design” work bench, we apply the “Generative Shape Design” Work bench.

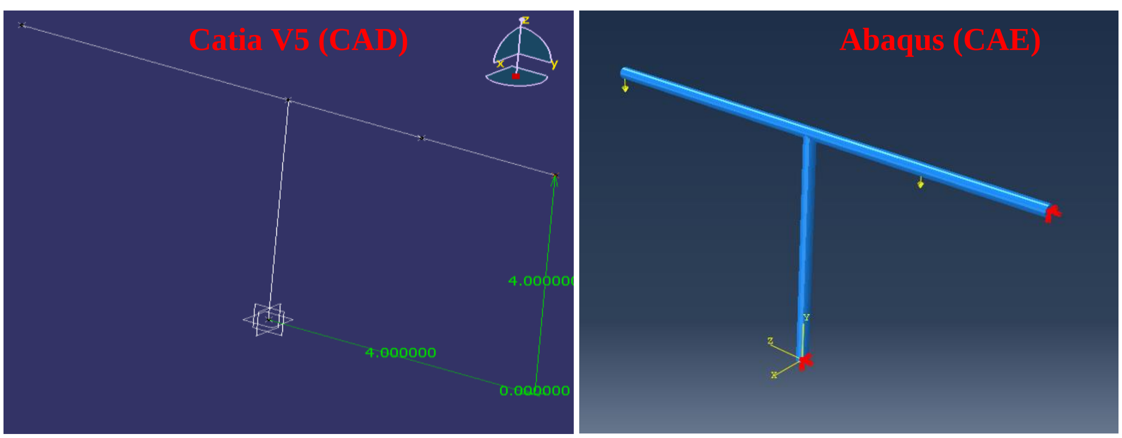


Figure 4-6 Case study one: CAD and CAE models

Step 2 – Modeling using FEM (Abaqus): Once the frame drawing is made in Catia V5, we import the “step” file into FEA tool (Abaqus) and we assign cross section(s), material properties, boundary conditions, mesh and loading condition.

Step 3 – Results: We can compare the results from the analytical approach with the FEA results. For FEA results references, see appendix C.

Table 4-3 Case Study One Results (FEA vs. MDM)

Joint Moments	Moment Distribution Method (N.m)	Wire Frame Modeling in Abaqus (N.m)	Accuracy of the proposed FEA Method (Error %)
M_{BA}	-48	-48	0
M_{AB}	0	0	0
M_{BD}	27	26.9665	0.124074074
M_{DB}	4.5	4.3921	2.397777778
M_{BC}	21	21.0335	-0.15952381
M_{CB}	10.5	10.4591	0.38952381
Reactionary Forces	FBD (N)	Wire Frame Modeling in Abaqus (N)	Accuracy of the proposed FEA Method
R_{Dy}	-1.875	-1.83965	1.885333333
R_{Dx}	7.875	7.87315	0.023492063
R_{Cy}	25.8775	25.87315	0.01680997
R_{Cx}	-7.875	-7.87315	0.023492063

The FEA results verify the correctness of the analytical approach. The FEA results (by comparison to the accuracy of the analytical approach) are also self-checking and demonstrate the accuracy of the proposed FEA.

4.5 Comparison of Analytical Approach vs. FEA

In this chapter we got a brief historical overview of structural analysis methods. We demonstrated through an example the effectiveness of a classical analytical method in the analysis of an indeterminate structure (without side-sway). It was shown how the design parameters such as length of beam, cross section, mechanical properties are physically visible throughout the calculations hence the designer exactly knows how to approach the appropriate solution. It was described how by changing the design parameters (elasticity, inertia and length of a specific member), the designer can directly affect moment distribution throughout each step, as these design parameters were visible during each one of these steps. The designer gets a result and feedback at each stage based on the design variables. Figure 4-7, has combined the moment distribution method into the structural analysis method as it was explained in chapter 3 and put into practice in case study one. This graph illustrated the structural evaluation method at the conceptual design stage using moment distribution method.

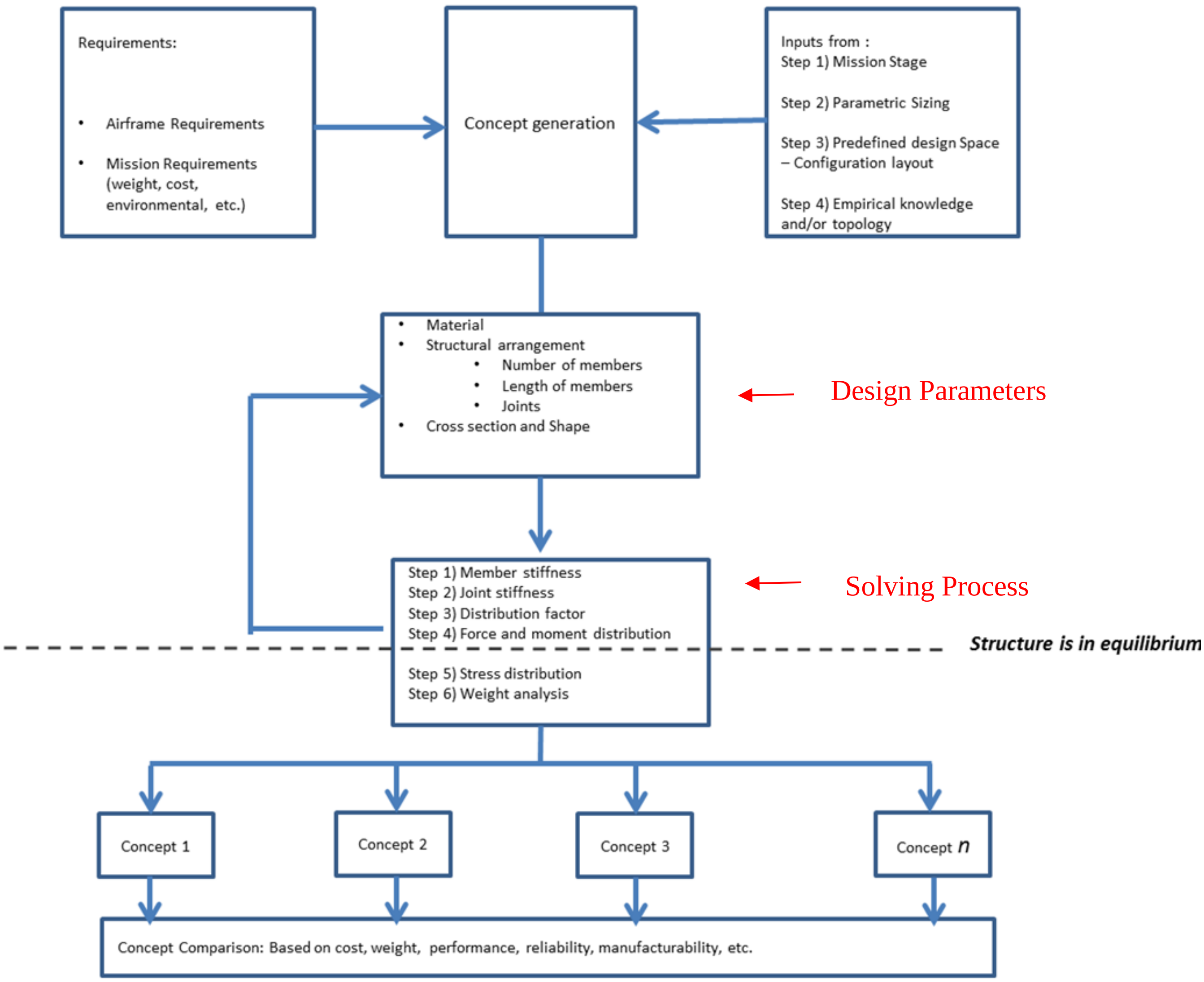


Figure 4-7 Moment distribution method at CD stage

The transparency to the design/analysis procedure provides the greatest advantage of the analytical method, which provides a greater feel for design and sound engineering judgment. These capabilities are specifically important during the conceptual design stage where concept generation and evaluation demands physical visibility of design parameters to make decisions. Also in this chapter, a wire-frame modeling finite element analysis method was used to confirm the results and at the same time demonstrated the accuracy of wire-frame modeling. Example demonstrated that FEM can be a great and fast tool when used appropriately. Additional structural analysis results such as dynamic simulation and plasticity results can also be computed using this FEA method.

However; in the FEM approach, after we created the model in a CAD tool and completed the pre-processing (applied cross section to prismatic member, properties, loads, boundary condition, etc), we arrived to the solution (post processing) without physical visibility of how each design parameters can change the results.

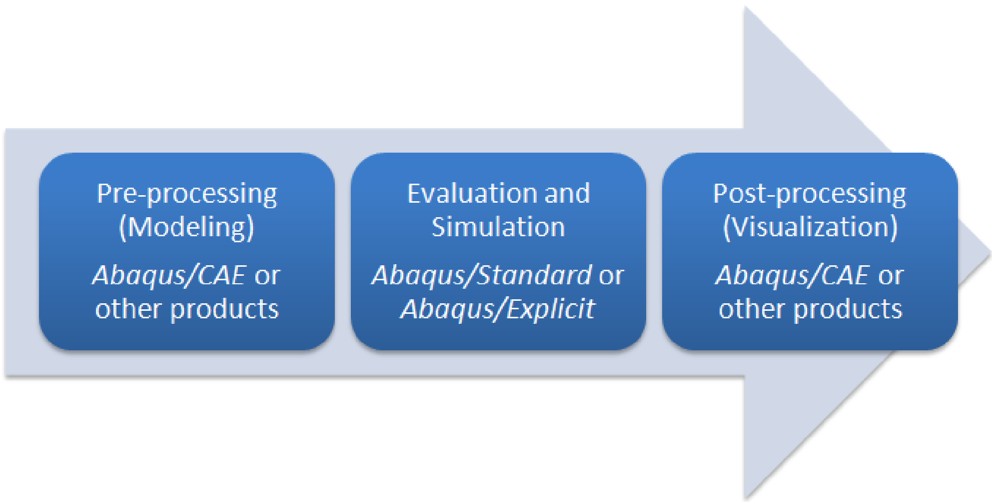


Figure 4-8 FEA process [98, 2012]

In order to understand how each of these design parameters effect the final design, the designer must go through the procedure and change the parameters and run the analysis again. This is the reason why the FEA method is often referred to as a “black box” [34, 2004]. In the FEA solving process (“black box”), the structure is modeled by using small units, also referred to as the finite elements. Stress and deformation will be determined once the structure reaches the equilibrium state. Following Table demonstrates the steps.

Step 1) Determine Material properties, boundary conditions and apply loads.

Step 2) Divide the structural system into an equivalent system of finite elements or meshes.

Step 3) A displacement function is made within each element of the structural system.

Step 4) Stress vs. Strain relationships: $\sigma = E\varepsilon$.

Step 5) Element stiffness and mass matrix is derived based on nodal forces and moments once the equilibrium conditions are satisfied.

Step 6) Stiffness equations of are elements are combined and the global stiffness and mass matrices are produced.

Step 7) Solving for the stress and/or strain.

Following diagram has combined the FEA approach into the structural analysis method as it was explained in chapter 3 and demonstrated through case study one. This graph illustrated the structural evaluation method at the conceptual design stage using finite element analysis method.

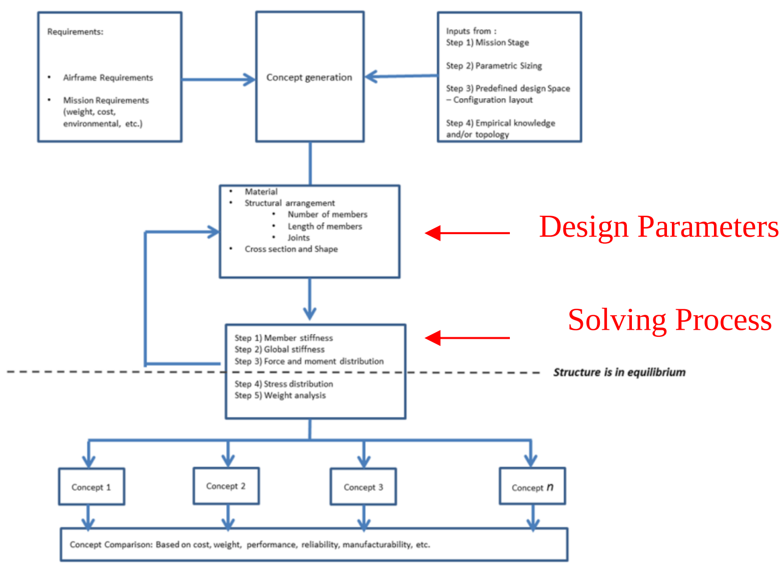


Figure 4-9 FEA approach at CD stage

Another difference (see figure below) between the FEA and Moment distribution approach is in an additional step that exists in the moment distribution method. That is in the approach that is taken to solve for the Distribution Factor (DF).

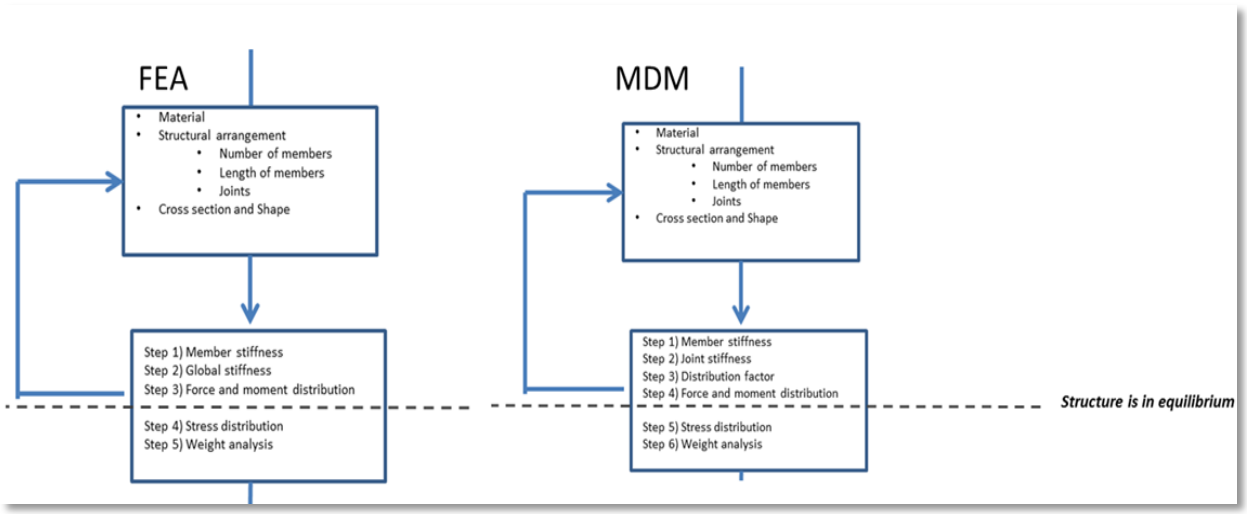


Figure 4-10 Solving process in MDM vs. FEA

As it was mentioned before, distribution factor is important because based on flexural stiffness (EI/L) of the members connected at a joint and the stiffness of the joint; it determines the magnitude of the moments carried, induced and reactionary forces created by each member.

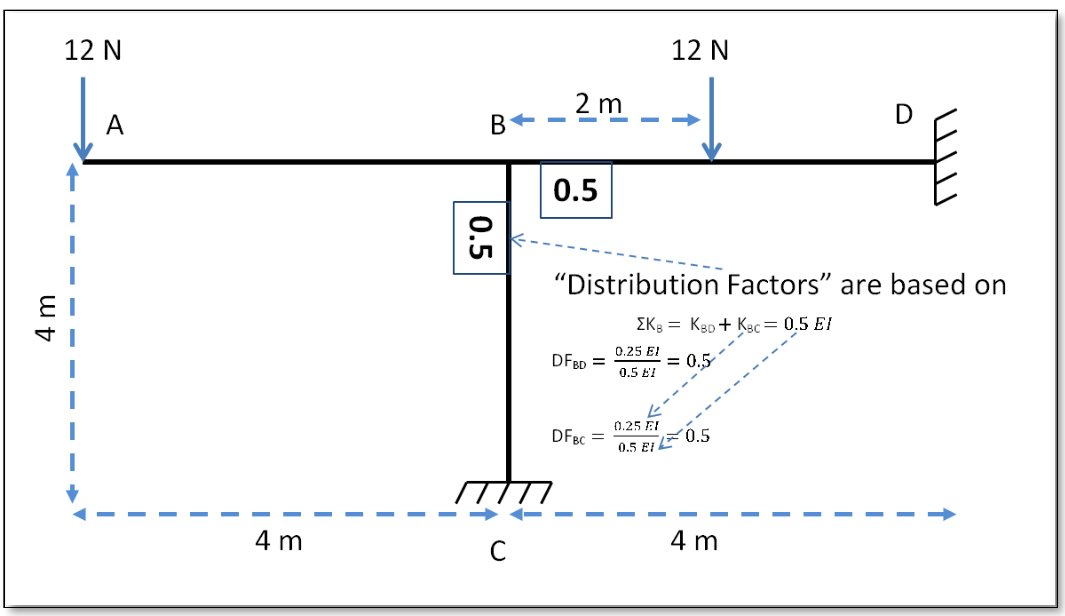


Figure 4-11 Distribution factor vs. design parameters

Therefore, at one side of the process (post processing side) the designer can view the reactionary forces and nodal moments and change those through changing the distribution factor (before the solution process). The design parameters such as length of beam, cross section, mechanical properties are physically visible throughout the calculations as they directly change the distribution factor and consequently; nodal moments and force. Distribution factors denote through a fixed factor how the design parameters interrelate. The higher the factor the more moments is induced and carried over to the joints.

Case study one demonstrated a brief overview of Analytical method vs. FEM, nevertheless; main sections of an aircraft structure (Wing, Fuselage, etc) are a lot more complex than the example that was demonstrated here. These sections are often composed of tens of thousands of different parts with different shapes, lengths, materials and boundary conditions. In the next chapter we will demonstrate the effectiveness of the analytical methods as structural complexity increases.

4.6 Modern Evolution of FEA

Before we go to the next chapter we end this chapter by providing a brief modern historical overview (see table 4-4) of the development of Finite element analysis. We start from where we left off; the stiffness matrix formulations. Following the invention of moment distribution method and by late 1930s a new trend of setting up matrix forms of equations had just begun. In 1934 and 1938 A.R Collar and W.J Duncan published the first papers representing an introduction of the matrix notation and formulation system that has been used until today [83, 1934][84, 1960]. Soon after, the matrix methods of analysis contained element stiffness equations, for purposes of complex structural analysis calculations. Very soon after, the newly invented high speed electronic computers became adapted to solve complicated structures using the stiffness method, making it possible to solve for the solution of millions of equations in a matter of seconds.

Table 4-4 Modern Evolution of FEA [85, 2013]

Name	Year	Description
Hrennikoff	1941	used a lattice of line elements representing bars and beams for stress calculations
McHenry	1943	Used a method very similar to the one used by Hrennikoff
Courant	1943	For the first time propsed a solution of stress calculation through tiangular subregions representing a full region
Levy	1947	Developed the force method, but in 1953 presented an alternative (Stiffness method) that became suitable to be used by digital computers.
Argyris and Kelsey	1954	developed matrix structural analysis methods by using energy principles
Turner at el	1956	They derived stiffness matrices for two-dimensional problems that became the direct stiffness method
Clough	1957	Working under Turner (at the time the head of Boeing's structural dynamics unit), modeled the beam problem (a year before had modeled according to levy) based on Ritz type analysis.The phrase Finite Element Analysis was introduced.
Martin	1961	Finite Element Analysis method extended to 3-D problems
Argyris	1964	Additional three dimensional elements were covered by Argyris
Archer	1965	Dynamic analysis was now considered
Belytschko	1976	Contributed with considering solutions to non-linear dynamic analysis
IBM	1990	IBM released windows operating system and integration of graphical user interface into the software

4.7 Summary

In chapter three of this thesis we went over the structural design requirements and parameters at the conceptual design stage. In this chapter we discussed through literature and a case study how the design parameters and design requirements interrelate in order to analyze a structural concept. We also (very briefly) covered the history behind the displacement methods of analysis and showed just how by the late 1930s moment distribution method had become the method of choice and very popular method among architects, airplane designers and structural engineers.

The general principals and a brief comparison of Moment Distribution Method and Finite Element Analysis were expressed through case study number one. In this example, we solved for the fixed end moments, reactions, bending moment diagram and compare these hand calculations results with the proposed FEA modeling Method. Then we compared the solving process of each method, and by comparing each detailed steps, we illustrated the design/analysis transparency that is produced through this moment distribution method. Case study one demonstrated a brief overview of Analytical method vs. FEM, nevertheless; main sections of an aircraft structure (Wing, Fuselage, etc) are a lot more complex than the example that was demonstrated in this chapter.

In the next chapter we will demonstrate the effectiveness of the analytical methods as structural complexity increases and structures side-sway.

Chapter 5

Structural Analysis of Complex Indeterminate Structures

5.1 Introduction

In this chapter three case studies are presented to demonstrate the capabilities of moment distribution method vs. FEA modeling in analyzing fewer complexes to highly complex structures. Each case study becomes more complex than the one before. This approach is taken to objectively quantify the limitation and capabilities of the analytical method at the conceptual design stage.

By late 1930s the demand to built complex structural systems had risen to a level which made it clear that further improvements need to be made to the latest structural analysis methods. Therefore, further improvements were also made to moment distribution method by Grinter of College of Texas A&M [86, 1933] and Southwell and coworkers [87, 1935]. As it was mentioned previously, Hardy Cross did not consider joint translation and secondary moments due to deflection. The improvements made by Grinter, Southwell and others extended the moment distribution method to frames undergoing “side-sway”[78, 2006]. It is important to note that there were also other problems would not be solved using the hardy cross’s method. Eaton writes:

Chief among them were: 1) Methods of constructing curves of maximum moments and 2) methods of constructing curves of maximum shears [38, 2001]. This limitation did not hurt the popularity of the hardy cross’s method as most indeterminate structures were of the continuous rigid frames and suited to be analyzed using the moment distribution method [37, 1961].

5.2 Case study 2: Analysis of single Indeterminate frame with side-sway

The reality of most indeterminate structural systems is that the joints will deflect and as the result the effect of the joint translation and deflection must be accounted for. Method of superposition is used in moment distribution method, to solve for this problem. But before, we use this method we can first solve for the member stiffness, joint stiffness and distribution factors.

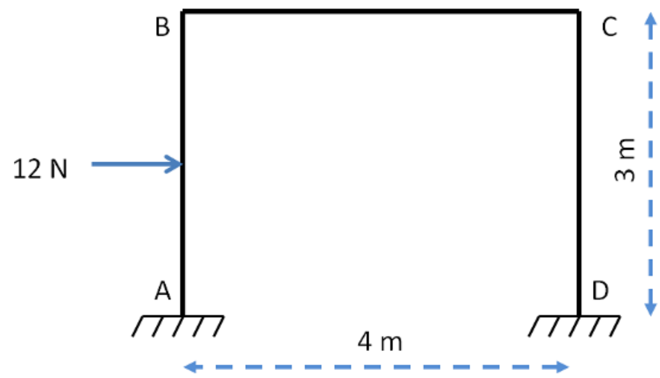


Figure 5-1 Single indeterminate frame with side-sway

The procedure to calculate these data is no different from the approach we implemented in case study one. We first solve for the member stiffness.

$K_{BA} = K_{AB} = \frac{EI}{3}$, $K_{BC} = K_{CB} = \frac{EI}{4}$, $K_{CD} = K_{DC} = \frac{EI}{3}$

Then we solve for the distribution factors. Note that since members Nodes A and D are fixed supports the distribution factor is zero because if we arbitrarily release a fixed support (joints A and D here), all imbalance moment goes into the connecting members (AB and DC) and not the joint, and therefore; the distribution factor is zero.

$DF_{AB}=0$, $DF_{DC}=0$,

Node B distribution factor:

$$\Sigma K_B = K_{BA} + K_{BC} = \frac{EI}{3} + \frac{EI}{4} = \frac{7EI}{12}$$

$$DF_{BA} = \frac{\frac{EI}{3}}{\frac{7EI}{12}} = 0.57, \ DF_{BC} = \frac{\frac{EI}{4}}{\frac{7EI}{12}} = 0.43$$

Node C distribution factor:

$$\Sigma K_C = K_{CB} + K_{CD} = \frac{EI}{4} + \frac{EI}{3} = \frac{7EI}{12}$$

$$DF_{CD} = \frac{\frac{EI}{3}}{\frac{7EI}{12}} = 0.57, \ DF_{CB} = \frac{\frac{EI}{4}}{\frac{7EI}{12}} = 0.43$$

At this stage, the designer can modify the distribution factor by using different materials, cross sections and element size, or can wait and make adjustments based on outcome of the analytical method. Next, superposition method is used when the structural frame side-sway. It is to construct a system of superimposed loadings to replicate of the side-sway frame. Since the load is applied on member AB of this frame, we know that Node B and C will displace in the direction that the force is applied. According to the superposition method, we arbitrarily fix node C and prevent nodes B and C from side-sway. By doing this we solve for the moments for the arbitrary constrained structure, then using static equations of equilibrium we then solve for the horizontal reactionary forces at A and D, and from those forces we solve for the reactionary force at C. But before we go further and solve for a “constant K”, we will solve for the fixed end moments of the restricted scenario.

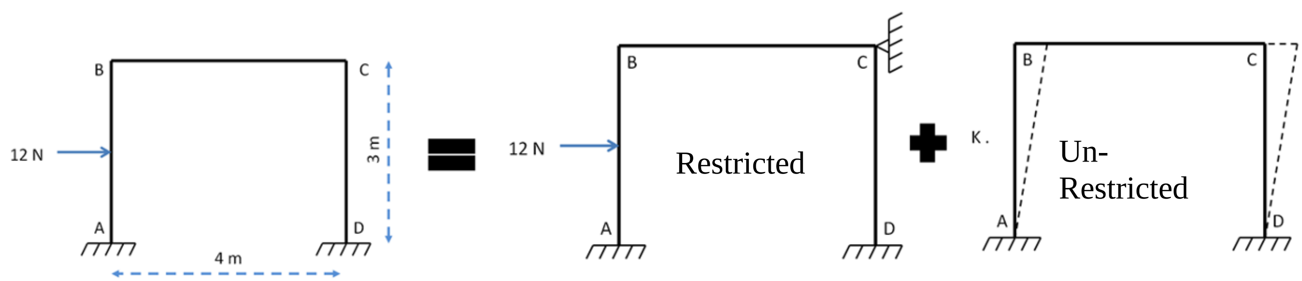


Figure 5-2 Superposition approach

$M_{AB} = \frac{12(1.5)(1.5^2)}{3^2} = 4.5\ \text{N.m},$ $M_{BA} = -4.5\ \text{N.m}$

$M_{BC} = M_{CB} = M_{DC} = M_{DC} = 0$

Finding moments in the restrained (the frame with the arbitrary support) frame using moment distribution method:

Table 5-1 Moment Distribution Iteration for Case Study 2 (unrestricted)

Node B: +4.5 is released BA=(4.5)(0.57)=2.565 BC=(4.5)(0.43)=1.935	 1.28 carried over to node A 0.96 carried over to node C
Node C: -0.96 is released CB=(-0.96)(0.43)= -0.412 CD=(-0.96)(0.57)=-0.5472	 -0.206 carried over to B -0.2736 carried over to D
Back to Node B: +0.206 is released BA=(0.206)(0.57)=0.12 BC=(0.206)(0.43)=0.088	 0.06 carried over to node A 0.044 carried over to node C
Back to Node C: -0.044 is released CB=(-0.044)(0.43)=-0.02 CD=(-0.044)(0.57)=-0.025	 -0.01 carried over to node B -0.0125 carried over to node D

The process of moment distribution can be continued until the carried over moments become as small as possible. Next we add all the moments at the nodes:

Table 5-2 Moment Distribution Summation for Case Study 2 (unrestricted)

AB	BA	BC	CB	CD	DC
4.5 1.28 0.06	-4.5 2.565 0.12	1.935 -0.206 0.088 -0.01	0.96 -0.412 0.044 -0.02	-0.5472 -0.025	-0.2736 -0.0125
5.84	-1.815	1.807	0.5722	-0.5722	-0.286

Next using FBD we find out the Reactions in the horizontal direction (R_{Ax} and R_{Dx}).

$R_{Ax}=\frac{5.84-1.815}{3}+\frac{(12)*(1.5)}{3}$ =7.341 N

$R_{Dx}=\frac{-0.5722-0.286}{3}=-0.286$ N, Reaction at the arbitrary joint becomes: $12-7.341+0.286\ +R=0$,

$R=-4.945$ N, so after we solve for the reactionary force at the arbitrary support, we know that if we apply a force in the opposite direction of R, we will have a certain resistance. We will call that K and it is solved by $kF=R$. The common practice is that instead of applying force F, an arbitrary displacement can be applied instead. The arbitrary side-sway is Δ' and it can be any value (as K is a constant) [87, 1935][37, 1961].

$FEM_{AB}=-\frac{6EI\Delta'}{L^2}$, $FEM_{AB}=FEA_{BA}$=-100 N.m , $FEM_{BC}=FEM_{CB}$=0, $FEM_{DC}=FEM_{CD}$=-100 N.m

Since the relative displacements should be the same, the magnitude of both fixed end moments are the same for both members. Now we start the moment distribution iteration:

Table 5-3 Moment Distribution Iteration for Case Study 2 (restricted)

Node B: +100 is released BA=(100)(0.57)=57 BC=(100)(0.43)=43	 28.5 is carried over to node A 21.5 is carried over to node C
Node C: 21.5-100=-78.5, hence 78.5 is released CB=(78.5)(0.43)= 33.7 CD=(78.5)(0.57)= 44.8	 17 is carried over to B 22.4 is carried over to D
Node B: -17 is released BA=(-17)(0.57)=-9.7 BC=(-17)(0.43)=-7.3	 -4.85 is carried over to node A -3.65 is carried over to node C
Node C: +3.65 is released CB=(3.65)(0.43)= 1.56 CD=(3.65)(0.57)= 2.08	 0.78 carried over to B 1.04 carried over to D
Node B: -0.78 is released We can stop here since the value of -0.78 is less than 1% of the original moment	

Table 5-4 Moment Distribution Summation for Case Study 2 (Restricted)

M_{AB}	M_{BA}	M_{BC}	M_{CB}	M_{CD}	M_{DC}
-100	-100	0	0	-100	-100
28.5	57	43	21.5	44.8	22.4
-4.85	-9.7	17	33.7	2.1	1.04
	0.7	-7.3	-3.65		
		0.78	1.56		
-76.35	-52.7	53.48	53.11	-53.1	-76.56

Next using FBD we solve for the Reactions in the horizontal direction (R_{Ax} and R_{Dx}) for the unrestrained (or un-restricted) frame:

$R_{Ax}=\left(\frac{-76.35-52.7}{3}\right) = -43.01$, $R_{Dx}=\frac{-53.1-76.56}{3} = -43.22$

F=-43.01-43.22=-86.23

$K= \frac{R}{F} = \frac{4.945}{-86.23} = -0.0573$

Then the moments from the restricted and un-restricted scenario are added.

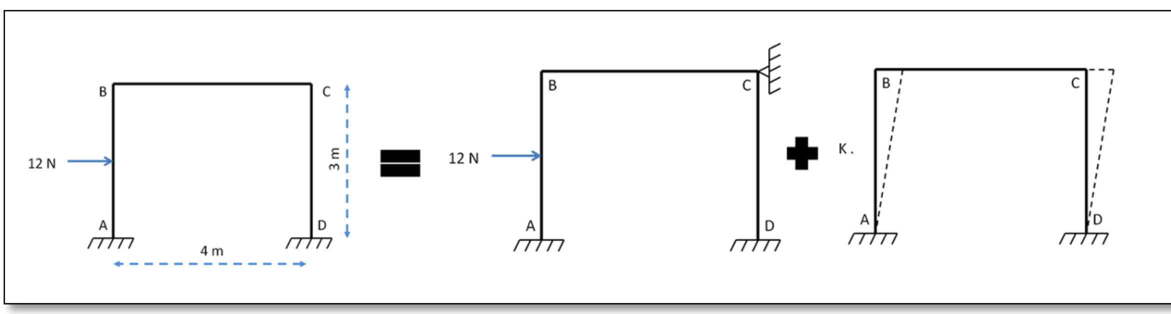


Figure 5-3 Case study two: superposition approach

Table 5-5 Case Study 2: MDM vs. FEA Results

Joint Moments	MDM (N.m)	FEA (N.m)	Error %
M_{AB}	$(5.84) + (-0.057)(-76.3) = 10.21$	10.23	0.2%
M_{BA}	$(-1.815) + (-0.057)(-52.7) = 1.204$	0	NA
M_{BD}	$(1.807) + (-0.057)(53.5) = -1.25$	0	NA
M_{DB}	$(0.572) + (-0.057)(53.11) = -2.45$	0	NA
M_{BC}	$(-0.572) + (-0.057)(-53.1) = 2.45$	0	NA
M_{CB}	$(-0.286) + (-0.057)(-76.56) = 4.10$	4.10	0%

What is also noticed is that, in some of the FEA results, the nodal moments are zero. The reason for that is that the opposite nodal moments in these specific nodes have canceled each other out. The major disadvantage of the moment distribution method is that side-sway requires a second set of calculations and as the number of frames increases, this method becomes even more time consuming. This is demonstrated through Case study three.

5.3 Case Study 3: Analysis of Multi Indeterminate Frame with Side-sway

In this case study (similar to a problem solved by [33, Kharagpur], but with different design parameters) the problem becomes more complex. In this example, 2 axial loads are applied at nodes B and C. As the result of these two side loads, joints D and E side-sway, therefore; the summation of moments needs to be done on three different scenarios.

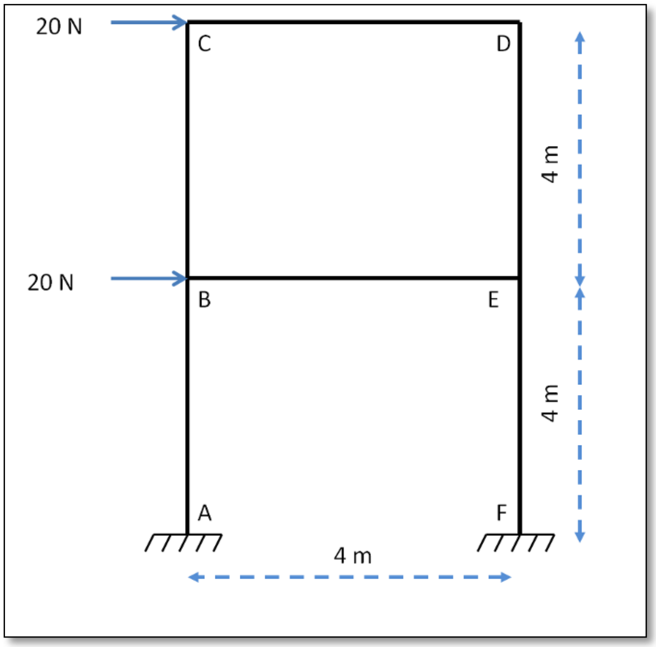


Figure 5-4 A multi indeterminate frame with side-sway

Firstly, the stiffness of each member is solved for and from that the total stiffness at each node is calculated:

$K_{BA} = K_{AB} = \frac{EI}{4}$, $K_{BC} = K_{CB} = \frac{EI}{4}$, $K_{CD} = K_{DC} = \frac{EI}{4}$, $K_{DE} = K_{ED} = \frac{EI}{4}$, $K_{EB} = K_{BE} = \frac{EI}{4}$, $K_{EF} = K_{FE} = \frac{EI}{4}$

At the fixed supports the distribution factor is zero because as the fixed supports are arbitrarily released (joints A and F here), all imbalance moment goes into the connecting members (AB and FE) and not the joint, and therefore; the distribution factors become zero.

$DF_{AB}=0$, $DF_{FE}=0$,

Node B distribution factor:

$$\Sigma K_B = K_{BA} + K_{BC} + K_{BE} = \frac{EI}{4} + \frac{EI}{4} + \frac{EI}{4} = \frac{3EI}{4}$$

$$DF_{BA} = \frac{\frac{EI}{4}}{\frac{3EI}{4}} = 0.333,\ DF_{BC} = \frac{\frac{EI}{4}}{\frac{3EI}{4}} = 0.333,\ DF_{BE} = \frac{\frac{EI}{4}}{\frac{3EI}{4}} = 0.333,$$

Node C distribution factor:

$$\Sigma K_B = K_{CB} + K_{CD} = \frac{EI}{4} + \frac{EI}{4} = \frac{2EI}{4}$$

$DF_{CB} = \frac{\frac{EI}{4}}{\frac{2EI}{4}} = 0.5$, $DF_{CD} = \frac{\frac{EI}{4}}{\frac{2EI}{4}} = 0.5$

Node D distribution factor:

$\Sigma K_D = K_{DC} + K_{DE} = \frac{EI}{4} + \frac{EI}{4} = \frac{2EI}{4}$

$DF_{DC} = \frac{\frac{EI}{4}}{\frac{2EI}{4}} = 0.5$, $DF_{DE} = \frac{\frac{EI}{4}}{\frac{2EI}{4}} = 0.5$

Node E distribution factor:

$\Sigma K_E = K_{EF} + K_{EB} + K_{ED} = \frac{EI}{4} + \frac{EI}{4} + \frac{EI}{4} = \frac{3EI}{4}$

$DF_{EF} = \frac{\frac{EI}{4}}{\frac{3EI}{4}} = 0.333$, $DF_{EB} = \frac{\frac{EI}{4}}{\frac{3EI}{4}} = 0.333$, $DF_{ED} = \frac{\frac{EI}{4}}{\frac{3EI}{4}} = 0.333$,

Starting with MDM for case 1, at first fixed end moments are solved for, but; since the external forces are axial forces at nodes B and C, there is no bending moment induced as the result of these two forces. Consequently, this step can be skipped and MDM can start with CASE 2 (not to be confused with case study two) condition (see figure 5-5).

CASE 2: Moment distribution for sideways Δ'_1 at beam CD. An arbitrary side way be $\Delta'_1 = \left(\frac{26.66}{EI}\right)$ is completed. As the result of this arbitrary side-sway, the fixed end moment in column CB and DE become:

$M_{BC} = \left(\frac{6EI\Delta'}{L^2}\right) = \left(\frac{6EI}{16}\right) X \left(\frac{26.66}{EI}\right) = 10$ N.m, $M_{CB} = 10$ N.m

$M_{ED} = M_{DE} = 10$ N.m

Again, at this step, any value can be chosen, but 26.66 was used, so that the fixed end moment becomes a more rounded out value (10) and easier to solve.

Step 2: superposition:

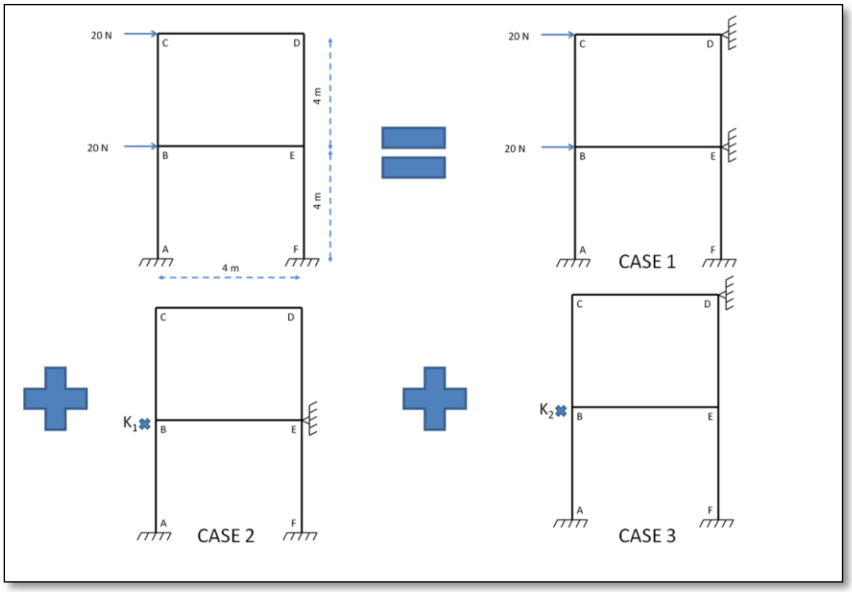


Figure 5-5 Superposition Approach for CASE 2

Table 5-6 Moment Distribution Iteration for CASE 2 (Part1)

Node B: Distribute -10 BA=(-10)(0.33)=-3.33 BE=(-10)(0.33)=-3.33 BC=(-10)(0.33)=-3.33	 -1.65 carried over to A -1.65 carried over to E -1.65 carried over to C
Node C: Distribute –(10-1.65) CB=(-8.35)(0.5)=--4.2 CD=(-8.35)(0.5)=-4.2	 -2.1 carried over to B -2.1 carried over to D
Node D: Distribute –(10-2.1) DC=(-7.9)(0.5)=-4 DE=(-7.9)(0.5)=-4	 -2.0 carried over to C -2.0 carried over to E
Node E: Distribute –(10-1.65-2) ED=(-6.35)(0.33)=-2.1 EB=(-6.35)(0.33)=-2.1 EF=(-6.35)(0.33)=-2.1	 -1.05 carried over to D -1.05 carried over to B -1.05 carried over to F

Table 5-7 Moment Distribution Iteration for CASE 2 (Part 2)

Node B: Distribute –(-2.1-1.05)	
BA=(3.1)(0.33)=1	0.5 carried over to A
BE=(3.1)(0.33)=1	0.5 carried over to E
BC=(3.1)(0.33)=1	0.5 carried over to C
Node C: Distribute –(-2+0.5)	
CB=(1.5)(0.5)=0.75	0.375 carried over to B
CD=(1.5)(0.5)=0.75	0.375 carried over to D
Node D: Distribute –(-1.05+0.375)	
DC=(0.675)(0.5)=0.3375	0.17 carried over to C
DE=(0.675)(0.5)=0.3375	0.17 carried over to E
Node E: Distribute –(0.5+0.17)	
ED=(-0.67)(0.33)=-0.22	-0.11 carried over to D
EB=(-0.67)(0.33)=-0.22	-0.11 carried over to B
EF=(-0.67)(0.33)=-0.22	-0.11 carried over to F
Node B: Distribute –(0.375-0.11)	
BA=(-0.265)(0.33)=-0.088	-0.044 carried over to A
BE=(-0.265)(0.33)=-0.088	-0.044 carried over to E
BC=(-0.265)(0.33)=-0.088	-0.044 carried over to C
Node C: Distribute –(0.17-0.044)	
CB=(0.126)(0.5)=-0.063	-0.032 carried over to B
CD=(0.126)(0.5)=-0.063	-0.032 carried over to D
Node D: Distribute –(-0.11-0.032)	
DC=(0.142)(0.5)=0.071	0.035 carried over to C
DE=(0.142)(0.5)=0.071	0.035 carried over to E
Node E: Distribute –(-0.044+0.035)	
ED=(0.0085)(0.33)=0.003	0.0015 carried over to D
EB=(0.0085)(0.33)=0.003	0.0015 carried over to B
EF=(0.0085)(0.33)=0.003	0.0015 carried over to F

Table 5-8 Summation of Moments for CASE 2

AB	BA	BC	CB	CD	DC	DE	ED	EB	BE	EF	FE
-1.65	-3.33	10	10	-4.2	-2.1	10	10	-1.65	-3.33	-2.1	-1.05
0.5	1	-3.33	-1.65	-2	-4	-4	-2	-2.1	-1.05	-0.22	-0.11
-0.044	-0.088	-2.1	-4.2	0.75	0.375	-1.05	-2.1	0.5	1	0.003	0.0015
		1	0.5	0.17	0.3375	0.3375	0.17	-0.22	-0.11		
		0.375	0.75	-0.063	-0.032	-0.11	-0.22	-0.044	-0.088		
		-0.088	-0.044	0.035	0.071	0.071	0.035	0.003	0.0015		
		-0.032	-0.063			0.0015	0.003				
-1.194	-2.418	5.825	5.293	-5.308	-5.3485	5.25	5.888	-3.511	-3.5765	-2.317	-1.1585

Next by using FBD, reactionary forces are determined:

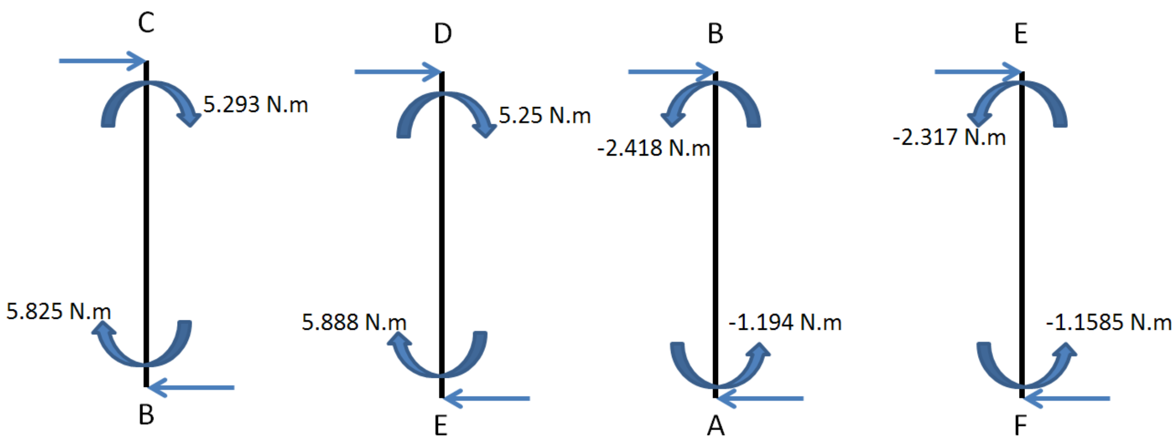


Figure 5-6 Case study 3: Solving for the reactionary forces

Resultant at C = $(5.293 + 5.825)/(4) = 2.66$,   Resultant at D = $(5.25 + 5.888)/(4) = 2.75$

Resultant at A = $(-1.194 - 2.418)/(4) = -0.92$, Resultant at F = $(-2.317 - 1.1585)/(4) = -0.87$

Then the same approach is now used for CASE 3 (when member BE beam sides-ways). For simplicity, side-sway is allowed to be:

$M_{AB} = \left(\frac{6EI\Delta'}{L^{\wedge}2}\right) = \left(\frac{6EI}{16}\right) X \left(\frac{26.66}{EI}\right) = 10$ N.m, $M_{BA} = 10\ \mathrm{N.m}$

$M_{EF} = M_{FE} = 10\ \mathrm{N.m}$

For the complete moment distribution iteration of CASE 3, see Appendix A. Similar to CASE 2, all the moments are added together and based on the end moments of each member, the reactionary forces are computed:

Table 5-9 Moment Summation for CASE 3

AB	BA	BC	CB	CD	DC	DE	ED	EB	BE	EF	FE
10	10	-3.33	-1.65	0.825	0.41	-0.2	-0.1	-1.65	-3.33		10
-1.65	-3.33	0.41	0.825	-0.1	-0.2	-1.36	-2.72	-2.72	-1.36	-2.72	-1.36
0.15	0.31	0.31	0.15	-0.025	-0.012	0.69	-0.16	0.15	0.31	-0.16	-0.08
0.015	0.03	-0.012	-0.025	0.343	0.69	-0.08	0.042	-0.16	-0.08	-0.02	-0.01
		0.03	0.015	-0.179	-0.09	0.085	-0.02	0.015	0.03		
		-0.09	-0.179	0.042	0.085	-0.01	0.343	-0.02	-0.01	10	
8.515	7.01	-2.682	-0.864	0.906	0.883	-0.875	-2.615	-4.385	-4.44	7.1	8.55

Resultant at C = $\frac{-2.682-0.864}{4} = -0.8865$, Resultant at D = $\frac{-2.615-0.875}{4} = -0.873$

Resultant at A = $\frac{8.515+7.01}{4} = 3.88$,   Resultant at F = $\frac{8.55+7.1}{4} = 3.91$

Based on scenario of CASE 2 and CASE 3, the superposition constants can be calculated:

K2(2.65+2.78)+K3(-0.88-0.88)= 20

K2(-0.92-0.87)+K3(3.88+3.91)= 20+20

K2=5.78 and K3=6.46

Next, by visiting the superposition scenario and one can solve for the actual moments at the nodes:

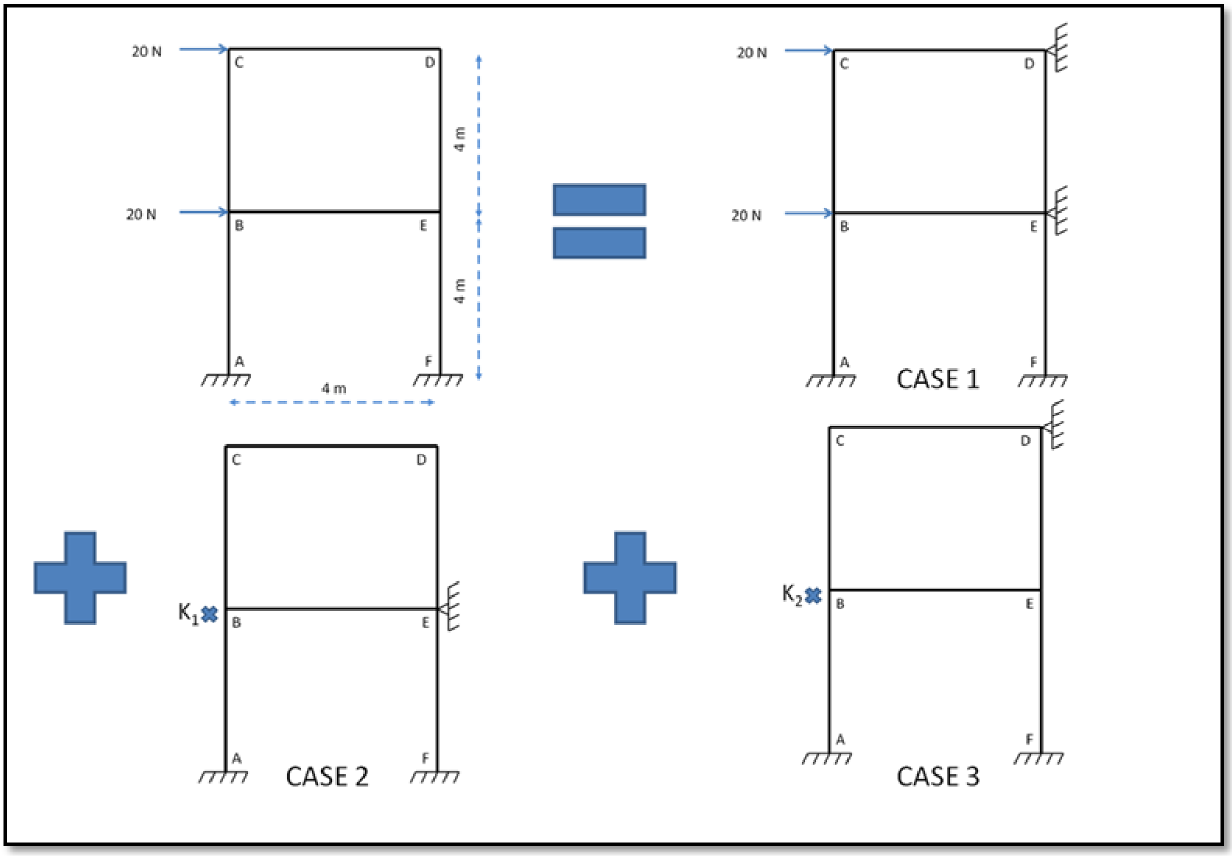


Figure 5-7 Case study three: superposition approach

CASE 1 did not induce any moments from the two axial forces, but case 2 and 3 induced the following results:

5.78	+ 6.46	=
-1.194	8.515	48.10558
-2.418	7.01	31.30856
5.825	-2.682	16.34278
5.293	-0.864	25.0121
-5.308	0.906	-24.8275
-5.3485	0.883	-25.2102
5.25	-0.875	24.6925
5.888	-2.615	17.13974
-3.511	-4.385	-48.6207
-3.5765	-4.44	-49.3546
-2.317	7.1	32.47374
-1.1585	8.55	48.53687

So for example: (5.78 x -1.194) + (6.46 x 8.515) = -6.9 + 55 = **48.1**

Table 5-10 Case Study Three Results: MDM vs. FEA Results (Frames With Side-Sway)

Joint Moments	Moment Distribution Method (N.m)	Wire Frame Modeling in Abaqus (N.m)	Accuracy of the proposed FEA Method (Error %)
M_{BA}	48.1	48.07	0.062370062
M_{AB}	31.31	31.95	-2.044075375
M_{BC}	16.34	16	2.080783354
M_{CB}	25.01	na	na
M_{CD}	-24.82	na	na
M_{DC}	-25.21	na	na
M_{DE}	24.69	na	na
M_{ED}	17.14	16.01	6.592765461
M_{EB}	-48.62	-47.94	1.398601399
M_{BE}	-48.93	-47.95	2.00286123
M_{EF}	32.47	31.92	1.693871266
M_{FE}	48.53	48.04	1.009684731

The results reflect the accuracy of the methods. But the major issue with the analytical approach remains to be that as the problem becomes more complex the problem becomes a lot more time consuming to solve. At this stage we know that this method can be used to solve complex structures confined to the solution of bending moments, but; to utilize it without using numerical tools such as Matlab or GNU Octave, assumptions needs to be made so the structural components do not side-sway. To do this we may neglect the effect of joint translations and secondary moments due to deflections and axial loads. This case study will also be done in order to demonstrate how these assumptions may affect the final results. This approach has been used by Bruhn E.F [36, 1997] and H. A Williams of Stanford University [35, 1956]. This was done to illustrate how moment distribution can be used to study different sections of fixed wing aircraft (See Fig 5.8).

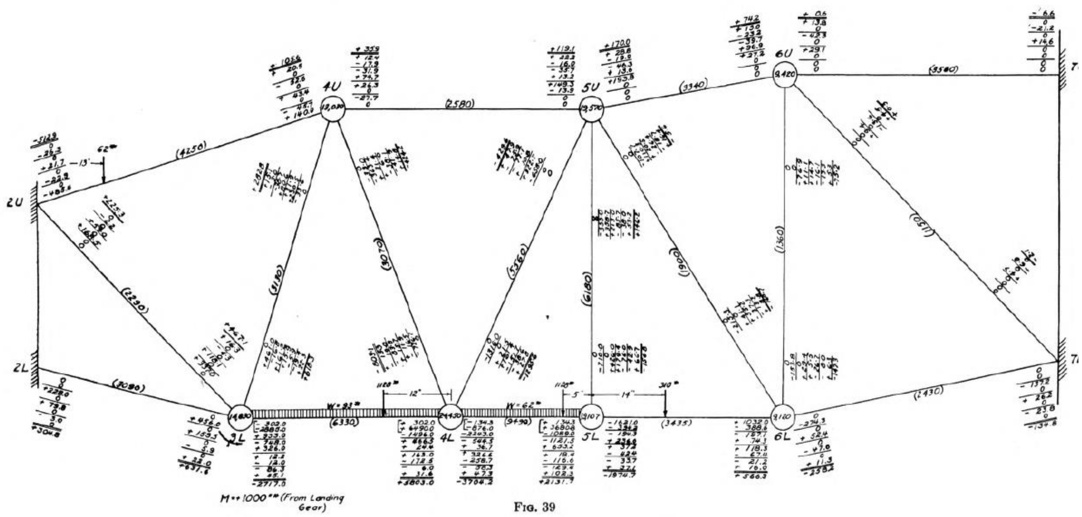


Figure 5-8 MDM Calculations [35, 1956]

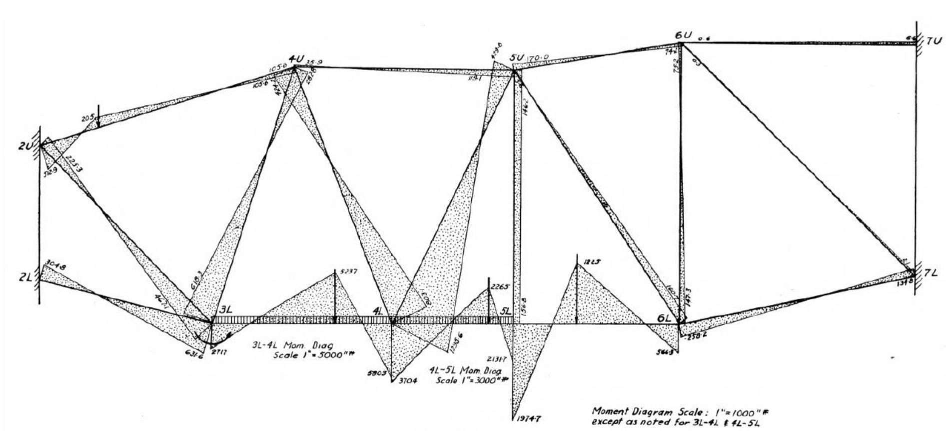


Figure 5-9 Moment diagram [35, 1956]

5.4 Case study 4: Analysis of Fuselage Truss without Side-sway

Figure 5.10, is the front portion of a fuselage and similar to Buhn's example problem 8 [36, 1997]. But unlike the problem used by Burhn, in this case study, E and I have been remained fixed. The design loads are different as well. Two moments are applied (positive) around the X-axis due to the eccentricity of Wing-spars and applied at Nodes B and A. As Bruhn and Williams [35, 1956] had previously done, in these case studies (due to their complex nature of the model) the effect of joint translation and secondary moments due to side-sway and axial loads have been neglected. These results will then be compared with Finite Element Analysis model. FEA method does not neglect the effect of joint translation and secondary moments due to side-sway and axial loads. First Step in this Analysis is to compute member length and cross section. In this problem the cross sections are all the same, therefore; the inertia or geometrical stiffness is also the same for all members.

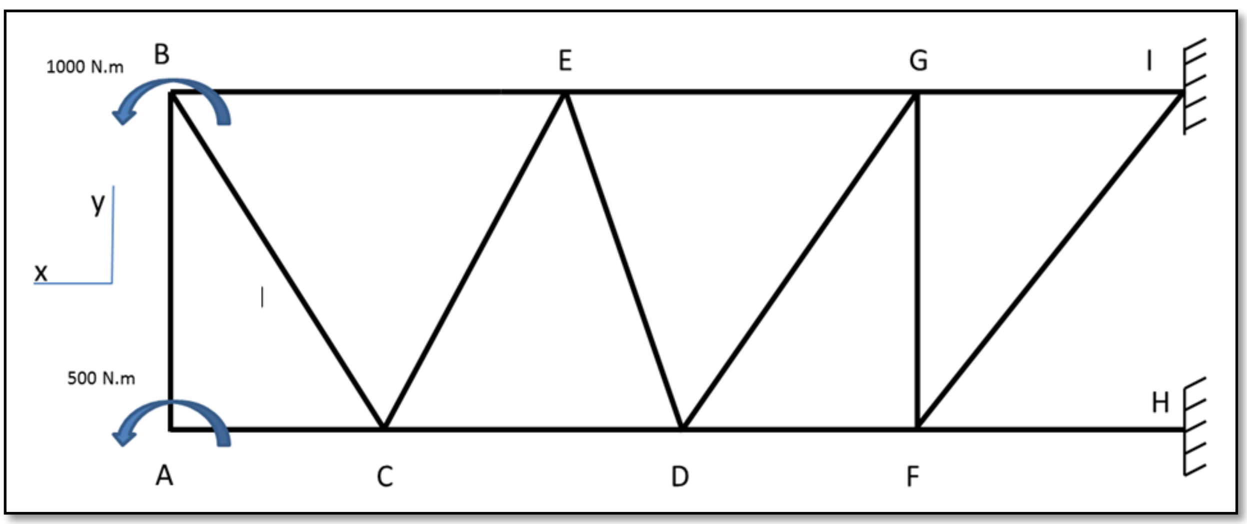


Figure 5-10 Front portion of fuselage truss [36, 1997]

We solve for the stiffness of each member based on the elasticity of the material, cross section of the prismatic beam and the length of the beam. The results are illustrated in the table 5-11:

Table 5-11 Design Parameters and Member Stiffness

Member	Length	I		Member Stiffness=EI/L X 1000
BE	41.25	1	1000	24.24
EG	38.75	1	1000	25.81
GI	34	1	1000	29.41
AC	19.25	1	1000	51.95
CD	30	1	1000	33.33
DF	30	1	1000	33.33
FH	34	1	1000	29.41
AB	34.5	1	1000	28.99
ED	35.4	1	1000	28.25
FG	34.5	1	1000	28.99
BC	39.5	1	1000	25.32
CE	41	1	1000	24.39
DG	46	1	1000	21.74
FI	48.5	1	1000	20.62

Stiffness at joint is then calculated based on the stiffness of the members meeting at their connecting joint. For example, joint stiffness at $\Sigma K_A = K_{AC} + K_{AB} = 51.95 + 28.98 = 80.93$

Table 5-12 Joint Stiffness

Joint	Stiffness
A	80.93356
B	78.54439
C	134.9881
D	116.6544
E	102.6877
F	112.3492
G	105.9429

Next step is to solve for the Distribution factors. For example, $DF_{BE} = \frac{24.24}{78.54} = 0.309$

Table 5-13 Case Study 4 Distribution Factors

Distribution Factors			
BE	0.309	EB	0.236
BA	0.369	AB	0.358
BC	0.322	CB	0.188
AC	0.642	CA	0.385
CE	0.181	EC	0.238
CD	0.247	DC	0.286
ED	0.275	DE	0.242
EG	0.251	GE	0.244
DG	0.186	GD	0.205
DF	0.286	FD	0.297
FG	0.258	GF	0.274
FI	0.184	IF	0.000
FH	0.262	HF	0.000
GI	0.278	IG	0.000

Once the distribution factors for all members are computed, the fixed end moments can be solved. The process until the fixed end moments are determined is the same for any type of structure. Whether the structures can side-sway or not, no matter the complexity of the loading condition, the process until now the fixed end moments are determined, are the same for all structures.

Next step is to solve for the fixed end moments. In this problem, due to eccentricity of the moments generated from the spars, 1000 N.m external moment is produced at Joints B and 500 N.m is produced at joint A. Using the fixed end moments and distribution factors, the end moments (neglecting the effect of joint translation and secondary moments due to deflection) are determined.

Table 5-14 Case Study 4 Design Parameters and Distribution Factors

Member	Length	I		Member Stiffness=EI/L X 1000	Joint Stiffness		Distribution Factors			
BE	41.25	1	1000	24.24			BE	0.308646	EB	0.236079
EG	38.75	1	1000	25.81			BA	0.369033	AB	0.35814
GI	34	1	1000	29.41			BC	0.32232	CB	0.187546
AC	19.25	1	1000	51.95			AC	0.64186	CA	0.384834
CD	30	1	1000	33.33			CE	0.180684	EC	0.237519
DF	30	1	1000	33.33			CD	0.246935	DC	0.285744
FH	34	1	1000	29.41			ED	0.275092	DE	0.242156
AB	34.5	1	1000	28.99	A	80.93	EG	0.25131	GE	0.243588
ED	35.4	1	1000	28.25	B	78.54	DG	0.186355	GD	0.205197
FG	34.5	1	1000	28.99	C	134.99	DF	0.285744	FD	0.296694
BC	39.5	1	1000	25.32	D	116.65	FG	0.257995	GF	0.273596
CE	41	1	1000	24.39	E	102.69	FI	0.183522	IF	0
DG	46	1	1000	21.74	F	112.35	FH	0.261789	HF	0
FI	48.5	1	1000	20.62	G	105.94	GI	0.277619	IG	0

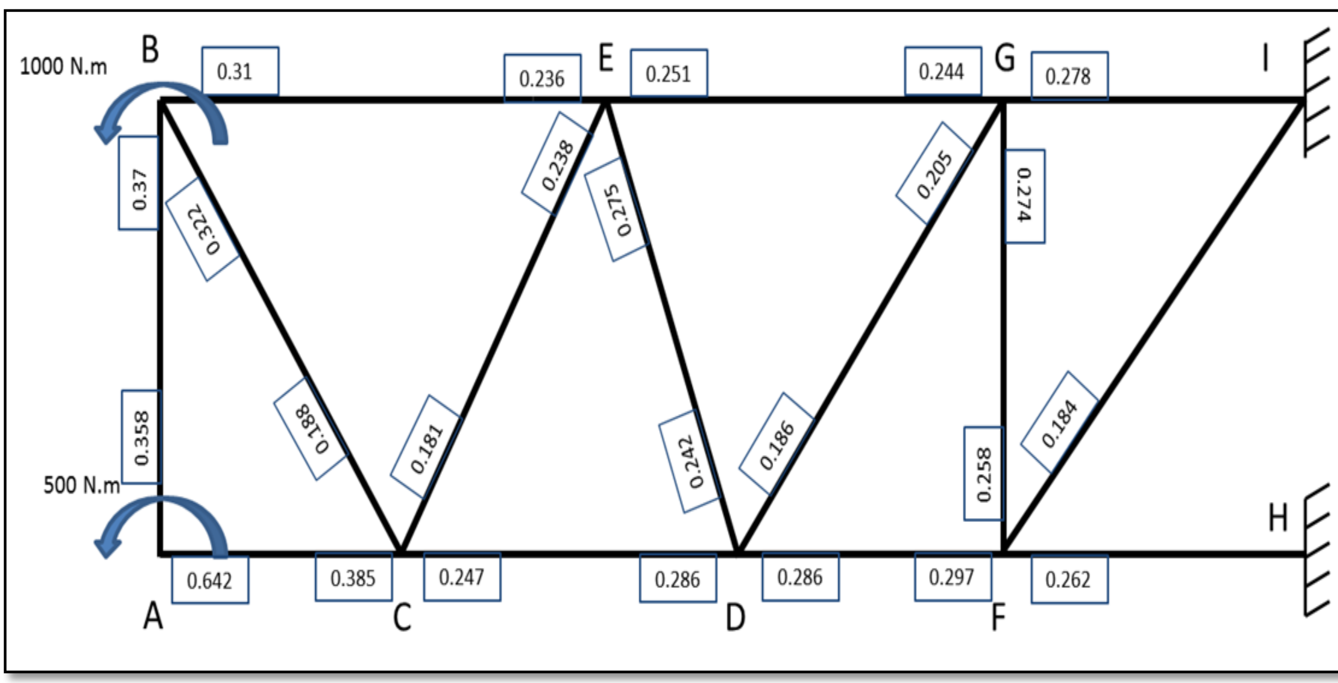


Figure 5-11 Frame distribution factors

Table 5-15 Case Study 4: Moment Distribution Iteration (Part 1)

Node A: 500, so we distributed -500 AC=(-500)(0.64)=-320 AB=(-500)(0.36)=-180	-160 carried over to C -90 carried over to B
Node B: 1000-90=910, We distribute -910 BA=(-910)(0.37)=-337 BC=(-910)(0.32)=-291 BE=(-910)(0.31)=-282	-169 carried over to A -146 carried over to C -141 carried over to E
Node C: -160-146=-306, We distribute 306 CA=(306)(0.384)=118 CB=(306)(0.187)=57 CE=(306)(0.18)=55 CD=(306)(0.24)=73	59 carried over to A 28.5 carried over to B 22.5 carried over to E 36.5 carried over to D
Node D: 36.5, We distribute -36.5 DC=(-36.5)(0.285)=-10.4 DE=(-36.5)(0.242)=-9 DG=(-36.5)(0.187)=-7 DF=(-36.5)(0.285)=-10.4	-5.2 carried over to C -4.5 carried over to E -3.5 carried over to G -5.2 carried over to F
Node E: -141+22.5-4.5=-123, We distribute 123 EB=(123)(0.236)=29 EC=(123)(0.237)=29 ED=(123)(0.27)=33 EG=(123)(0.25)=31	14.5 carried over to B 14.5 carried over to C 16.5 carried over to D 15.5 carried over to G
Node F: -5.2, We distribute 5.2 FD=(5.2)(0.296)=1.5 FG=(5.2)(0.258)=1.3 FI=(5.2)(0.183)=1 FH=(5.2)(0.262)=-1.4	0.75 carried over to D 0.65 carried over to G 0.5 carried over to I 0.7 carried over to H
Node G: -3.5+15.5+0.5=12.5, We distribute -12.5 GE=(-12.5)(0.24)=-3.1 GD=(-12.5)(0.205)=-3 GF=(-12.5)(0.273)=-3.2 GI=(-12.5)(0.277)=-3.2	-1.5 carried over to E -1.5 carried over to D -1.6 carried over to F -1.6 carried over to I

Part 2 of this process is in Appendix B. All the moments are then combined and compared with the FEA simulation results. Note, that the FEA simulation (as it was illustrated in case study 2 and 3) does not neglect the effect of joint translation and secondary moments due to side-sway and axial loads.

Table 5-16 Case Study 4 MDM Results

Moment Distribution Results - neglecting the effects of joint translation and secondary moments due to deflection								
AB	-320.5		ED	30.1		FH	2.2
BA	-430		DE	5.3		HF	1.1
AC	-191		EG	33.1		GI	-3
CA	-7		GE	14.3		IG	-1.5
BE	-285.8		DG	-11.5			
EB	-118.6		GD	-8			
BC	-282.5		DF	-13.15			
CB	-99		FD	-4.7			
CE	71.2		FG	0.6			
EC	54.9		GF	-1.95			
CD	65.8		FI	1.6			
DC	22.1		IF	0.8			

Table 5-17 MDM (without side-sway) vs. FEA (with side-sway) Results

Joint Moments	Moment Distribution Method (N.m)	Wire Frame Modeling in Abaqus (N.m)
M_{BA}	-430	-333
M_{AB}	-321	-250
M_{BC}	-283	-662
M_{CB}	-99	-270
M_{CD}	66	165
M_{DC}	22	-700
M_{DE}	5	-168
M_{ED}	30	162
M_{EB}	-118.6	-584
M_{BE}	-285.8	-662
M_{EG}	33.1	657
M_{GE}	14.3	-526
M_{CA}	-7	-602
M_{AC}	-191	-555
M_{CE}	71.2	247
M_{EC}	54.9	-235
M_{DG}	-11.5	289
M_{GD}	-8	-267
M_{DF}	-13.15	580
M_{FD}	-4.7	-702
M_{FG}	0.8	-86
M_{GF}	-1.95	86
M_{FI}	1.6	272
M_{IF}	0.8	370
M_{FH}	2.2	516
M_{HF}	1.1	544
M_{GI}	-3	208
M_{IG}	-1.5	585

Per hypothesis, the effects of joint translation and secondary moments were neglected, however; the FEA procedure does not make this assumption. The comparison of results illustrates the inaccuracy of the MDM when the effect of joint translation and secondary moments are neglected. What is noticed is how the results are becoming more inaccurate as the nodal moments are moving further away from the fixed end moments. This is especially significant when the moments are applied so far away from the fixed constraints. This was also evident in case study two. Looking back at case study 2, these changes were quite noticeable:

Table 5-18 Case Study 2 Results

Joint Moments	MDM (N.m)	FEA (N.m)	Error %
M_{AB}	$(5.84) + (-0.057)(-76.3) = 10.21$	10.23	0.2%
M_{BA}	$(-1.815) + (-0.057)(-52.7) = 1.204$	0	NA
M_{BD}	$(1.807) + (-0.057)(53.5) = -1.25$	0	NA
M_{DB}	$(0.572) + (-0.057)(53.11) = -2.45$	0	NA
M_{BC}	$(-0.572) + (-0.057)(-53.1) = 2.45$	0	NA
M_{CB}	$(-0.286) + (-0.057)(-76.56) = 4.10$	4.10	0

Note, the difference between (-0.286) and (4.10), this shows that the nodal moment from the side-sway is around 14 times higher than results from the arbitrary restrained scenario. Based on such a high inaccuracy in results it is recommended that when using moment distribution method, the effect of joint translation and secondary moments should not be neglected.

Now, the main reason for neglecting the effect of joint translation and secondary moments due to deflection and axial loads was to save computation time, otherwise it would be extremely tedious to use the moment distribution method (with side-sway) and superposition approach on a complicated structure that is made from more than few joints. This illustrates why advanced numerical methods, such as Finite Element Analysis (FEM) methods are the reasonable method to use in these cases.

5.5 FEA + Moment Distribution Method = A New Approach

It was illustrated in chapter 4 and in this chapter, through all case studies that that steps 1, 2 and 3 of the moment distribution method are the same for any type of structure. Whether the structure side-sways or not, no matter the complexity of the loading condition, this three process is the same for all structures. Therefore, not only all these steps can be used for any type of structure, but as it is explained in this section, the designer can use these steps as an accumulation with FEA.

The combination of Steps 1, 2 and 3 with FEA, provides a transparency to the design/FEA procedure. Also, from these “three steps”, step 3 or obtaining the distribution factor was created by Hardy Cross and has yet to be implemented in the latest FEA software [92, 2012][93, 2013][94, 2012][74, 2009]. “Distribution factor” will be a unique additional accumulating step to FEA methodology that has historically been used as part of the moment distribution approach.

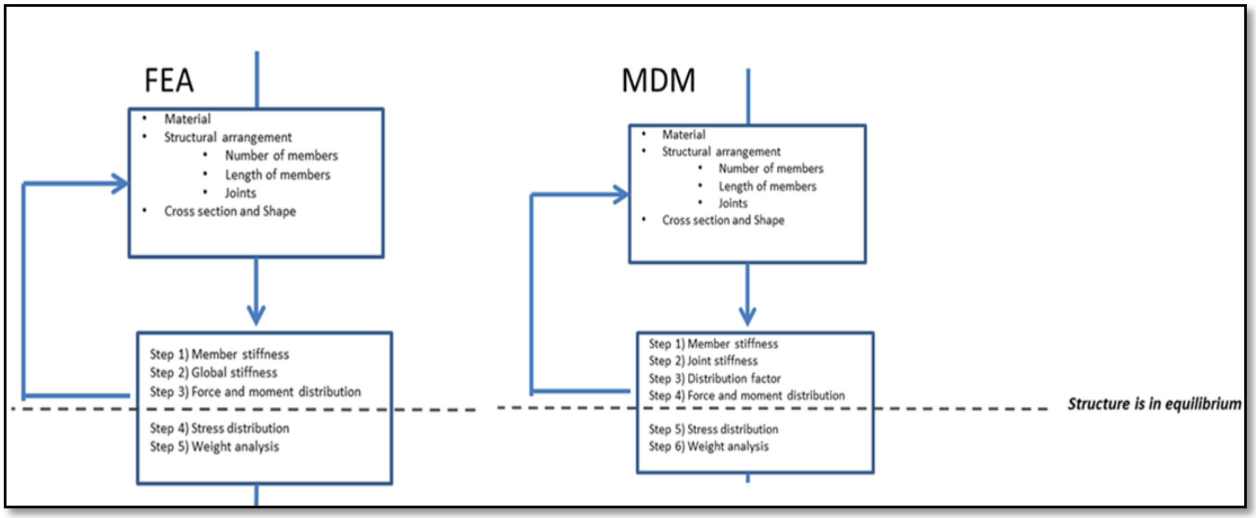


Figure 5-12 FEA vs. MDM solving process

As it was mentioned before; distribution factor is important because based on flexural stiffness (EI/L) of the members connected to a specific joint stiffness; it determines the magnitude of the moments carried and reactionary forces that are created through a combination of various design parameters. It was demonstrated in case study one that design parameters such as length of beam, cross section, mechanical properties are physically visible through calculating distribution factor. A distribution factor pinpoints how the design parameters interrelate, as it provides a constant relationship between the design parameters. The higher the factor the more moments is induced and carried over to the joints. Also, the summing up of the distribution factors of the members connecting to a joint always adds up to the constant value of one.

This transparency to the design/analysis procedure provides the greatest advantage of the distribution factor, which provides a greater feel for design and sound engineering judgment. These capabilities are specifically important during the conceptual design stage where concept generation and evaluation demands physical visibility of design parameters to make decisions. This new methodology is illustrated in figure 5-13 (next page).

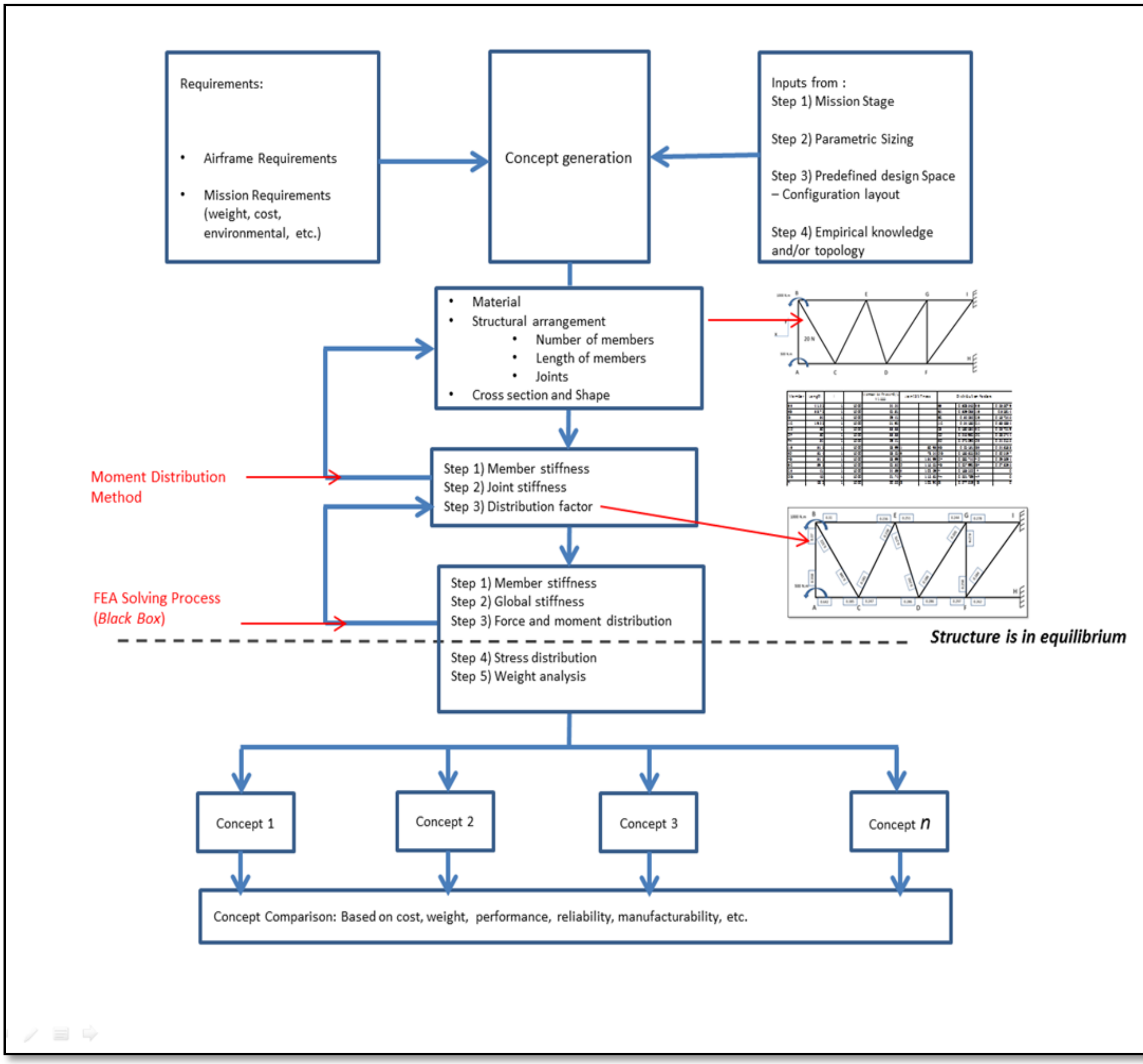


Figure 5-13 New structural analysis method for the CD stage

5.6 Summary

In this chapter three case studies were presented to demonstrate the capabilities of moment distribution method vs. FEA modeling by studying less complicated to more complex indeterminate structures that side-sway. Each case study became more complex than the one before to better quantify the limitation and capabilities of the moment distribution method at the conceptual design stage.

It was shown that that moment distribution method can effectively be used to compute end moments of a single and double indeterminate frame with side-sway. However, it was shown through solving case studies 2 and 3 that the major disadvantage of the moment distribution method is that side-sway requires a second set of calculations and as the number of frames increases, this method can becomes even more time consuming and tedious as the structure becomes more complex. This was due to the time consuming nature of superposition approach. So in order to utilize moment distribution method without using numerical tools such as Matlab or GNU Octave, assumptions needs to be made so the structural components do not side-sway. To do this, in case study four we may neglect the effect of joint translations and secondary moments due to deflections and axial loads.

The result illustrated the inaccuracy of the MDM when the effect of joint translation and secondary moments are neglected. The errors became especially significant when the moments are applied so far away from the fixed constraints. This was also confirmed through case study two as well.

Finally, it was confirmed through all case studies that steps 1, 2 and 3 of the moment distribution method are the same for any type of structural system. Whether the structure side-sway or not, and independent from the complexity of the loading condition, these steps are the same for all structures. Certainly, these steps of the moment distribution method can be used as an accumulation with FEA. This new approach was illustrated in figure 5-13. This is mainly because “Distribution Factor” will be a unique additional accumulating step to FEA. With it, distribution factor provides a transparency to the design/FEA procedure as it provides a greater feel for design and sound engineering judgment. These capabilities are specifically important during the conceptual design stage where concept generation and evaluation demands physical visibility of design parameters to make decisions.

Chapter 6

Conclusion and Recommendation

The goal of this thesis was to investigate the structural toolbox during the conceptual design phase through comparing Finite Element Analysis method with Moment Distribution Method. Considering the strength and limitations of both methodologies, the question to be answered in this thesis was: How valuable and compatible is the moment distribution approach in today's conceptual design environment? And can these methods complement each other?

As a pre-requisite, this research presents complete aircraft design phases in chapter 2 and structural design criteria requirements at the CD stage in chapter 3, in order to identify design solution space and design parameters at the conceptual design stage. To introduce the structural analysis tool-box, in the first part of chapter four of this thesis, a brief historical review of structural analysis tools is provided and supported by literature review. In this chapter the benefits of moment distribution method over the other previous analytical methods is explained and supported by literature. In order to answer the objective questions, moment distribution method and finite element method are compared in order to quantify their limitations and strengths during the Conceptual Design Phase. Four case studies were presented to demonstrate the capabilities of Moment Distribution Method and FEM. Each case study becomes more complex than the one before, in order to objectively quantify the limitation and capabilities of the analytical method (moment distribution method) at the conceptual design stage.

Case study one: was a basic indeterminate structural system, as this structure is constrained and does not side-sway. In this problem I and E were assumed to be constant. It was proven how the design parameters such as length of beam, cross section, mechanical properties are physically visible throughout the calculations of the distribution factor, hence; the designer exactly knew how to approach the appropriate solution. While, in the FEA approach, after the model was created (in a CAD tool) and completed the pre-processing (applied cross section to prismatic member, properties, loads, boundary condition, etc), we arrived to the solution (post processing) without physical visibility of design parameters. It was shown, that in order to understand how each of these design parameters effect the final design, the designer must go through the procedure and change the parameters and run the analysis again.

Case study 2: was the analysis of single indeterminate frame with side-sway. The reality of most indeterminate structural systems is that the joints will deflect and as the result the effect of the joint translation and deflection must be accounted for. This was shown through case study 2. Method of superposition was used in combination with moment distribution method, to solve for the effects of side-sway. It was illustrated that the procedure to calculate member stiffness, joint stiffness and distribution factors are as similar to case study one (without side-sway) and no extra effort (or steps) was taken to solve for these data. While, the FEA effort was the same as case study one, the major disadvantage of the moment distribution method for this type of indeterminate structure was that side-sway requires a second set of calculations (and as the number of frames increases). So this method naturally becomes more time consuming as the number of deflected joints increases. The effect of this added complexity was demonstrated through Case study three.

Case study 3: In this case study, the indeterminate frame becomes multi frame and more complex. In this example, 2 axial loads are applied at nodes B and C. As the result of these two side loads, joints D and E side-sway, hence; the summation of moments needs to be done on three different scenarios (per-superposition approach). While, the FEA approach remained the same, the MDM approach became a lot more time consuming. As the problem becomes more complex, using the analytical approach the problem becomes a lot more time consuming to solve. To utilize the moment distribution method without using numerical tools such as Matlab or GNU Octave, assumptions were made and supported through literature [36, 1997][35, 1956], so that the structural joints do not side-sway. Therefore, the effects of joint translations and secondary moments due to side-sway and axial loads have to be neglected.

Case Study 4: While, the FEA effort remained to be the same as the other case studies, using the analytical approach, compared to case study two and three, this complex problem was now a lot less time consuming, since the superposition approach was no longer implemented. However, the results showed huge inaccuracy as the effects of joint side-sway and secondary moments were no longer accounted for. This became very significant when the moments are applied so far away from the fixed constraints and fixed end moments, or as the number of joints and length of beams increase. This proved advanced numerical methods, such as Finite Element Analysis (FEM) methods are the reasonable method to use in these situations.

Through the combination of the lessons learned from all four case studies, a unique conclusion was reached. This conclusion answered the other hypothesis behind this thesis: Can Moment Distribution Method and FEA complement each other?

It was shown by the end of chapter five that moment distribution and FEA method can be combined in order to take advantage of FEAs faster problem solving process and Moment distributions greater physical visibility. It was illustrated through all case studies that steps 1, 2 and 3 of the moment distribution method are the same for any type of structure. Whether the structure side-sway or not, no matter the complexity of the loading condition, these three steps are the same for all structures. Therefore, not only all these steps can be used for any type of structure (confined to the solution of bending moment, with the exception of curves [38, 2001][37, 1961]), but; now the designer can use these steps as an accumulation with FEA.

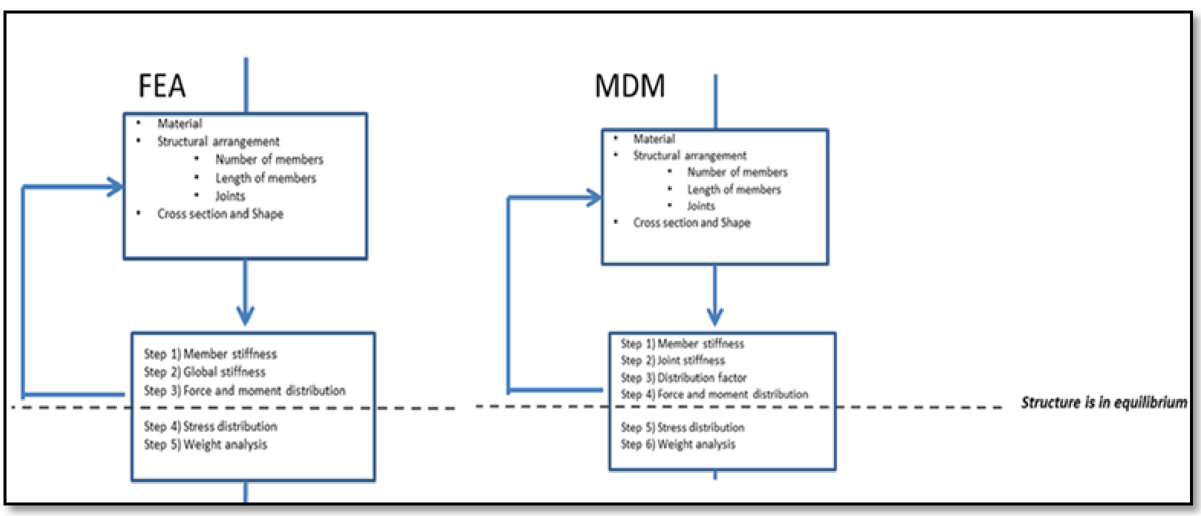


Figure 6-1 Solving process in FEA vs. MDM

The combination of Steps 1, 2 and 3 with FEA, provides a transparency to the design/FEA procedure. Also, from the “three steps”, the step 3 (obtaining distribution factor) created by Hardy Cross is currently not used in FEA software [92, 2012][93, 2013][94, 2012][74, 2009]. This step 3 is the “distribution factor” step, and will be a unique additional accumulating step in structural analysis methodology described in chapter 3.

Distribution factor is an important addition in utilizing FEA at the conceptual design stage. Design parameters such as length of beam, cross section, mechanical properties are physically visible through calculating distribution factor. The higher the distribution factor the more moments is induced and carried over to the joints. This transparency to the design/analysis procedure provides the greatest advantage of the distribution factor, which provides a greater feel for design and engineering judgment. These capabilities are specifically important during the conceptual design stage, as concept design and evaluation demands physical visibility of design parameters and how they interact. The new methodology was illustrated in figure 5-13.

Appendix A

Case Study 3: CASE 4 MDM Iteration

Table A-1 MDM CASE 4 Iteration Process

Node B: Distribute -10	
BA=(-10)(0.33)=-3.33	-1.65 carried over to A
BE=(-10)(0.33)=-3.33	-1.65 carried over to E
BC=(-10)(0.33)=-3.33	-1.65 carried over to C
Node C: Distribute –(10-1.65)	
CB=(-8.35)(0.5)=–4.2	-2.1 carried over to B
CD=(-8.35)(0.5)=-4.2	-2.1 carried over to D
Node D: Distribute –(10-2.1)	
DC=(-7.9)(0.5)=-4	-2.0 carried over to C
DE=(-7.9)(0.5)=-4	-2.0 carried over to E
Node E: Distribute –(10-1.65-2)	
ED=(-6.35)(0.33)=-2.1	-1.05 carried over to D
EB=(-6.35)(0.33)=-2.1	-1.05 carried over to B
EF=(-6.35)(0.33)=-2.1	-1.05 carried over to F
Node B: Distribute –(-2.1-1.05)	
BA=(3.1)(0.33)=1	0.5 carried over to A
BE=(3.1)(0.33)=1	0.5 carried over to E
BC=(3.1)(0.33)=1	0.5 carried over to C
Node C: Distribute –(-2+0.5)	
CB=(1.5)(0.5)=0.75	0.375 carried over to B
CD=(1.5)(0.5)=0.75	0.375 carried over to D
Node D: Distribute –(-1.05+0.375)	
DC=(0.675)(0.5)=0.3375	0.17 carried over to C
DE=(0.675)(0.5)=0.3375	0.17 carried over to E
Node E: Distribute –(0.5+0.17)	
ED=(-0.67)(0.33)=-0.22	-0.11 carried over to D
EB=(-0.67)(0.33)=-0.22	-0.11 carried over to B
EF=(-0.67)(0.33)=-0.22	-0.11 carried over to F
Node B: Distribute –(0.375-0.11)	
BA=(-0.265)(0.33)=-0.088	-0.044 carried over to A
BE=(-0.265)(0.33)=-0.088	-0.044 carried over to E
BC=(-0.265)(0.33)=-0.088	-0.044 carried over to C
Node C: Distribute –(0.17-0.044)	
CB=(0.126)(0.5)=-0.063	-0.032 carried over to B
CD=(0.126)(0.5)=-0.063	-0.032 carried over to D
Node D: Distribute –(-0.11-0.032)	
DC=(0.142)(0.5)=0.071	0.035 carried over to C
DE=(0.142)(0.5)=0.071	0.035 carried over to E
Node E: Distribute –(-0.044+0.035)	
ED=(0.0085)(0.33)=0.003	0.0015 carried over to D
EB=(0.0085)(0.33)=0.003	0.0015 carried over to B
EF=(0.0085)(0.33)=0.003	0.0015 carried over to F

Appendix B

Case Study 4: MDM Iteration and Summation

Table B-1 Case Study 4: Moment Iteration (Part 2)

Node A: -169+59=-110, so we distributed 110	
AC=(110)(0.64)=70	35 carried over to C
AB=(110)(0.36)=40	20 carried over to B
Node B: 28.5+14.5+20=63, We distribute -63	
BA=(-63)(0.37)=-23	-11.5 carried over to A
BC=(-63)(0.32)=-20	-10 carried over to C
BE=(-63)(0.31)=-20	-10 carried over to E
Node C: -5.2+14.5-10=-0.7, We distribute 0.7	
CA almost 0	0 carried over to A
CB almost 0	0 carried over to B
CE almost 0	0 carried over to E
CD almost 0	0 carried over to D
Node D: 16.5+0.75-1.5=14.25, We distribute -14.25	
DC=(-14.25)(0.285)=-4	-2 carried over to C
DE=(-14.25)(0.242)=-3.75	-2 carried over to E
DG=(-14.25)(0.187)=-2.5	-1.5 carried over to G
DF=(-14.25)(0.285)=-4	-2 carried over to F
Node E: -1.5-10-2=-13.5, We distribute 13.5	
EB=(13.5)(0.236)=3.4	1.7 carried over to B
EC=(13.5)(0.237)=3.4	1.7 carried over to C
ED=(13.5)(0.27)=3.6	1.8 carried over to D
EG=(13.5)(0.25)=3.6	1.8 carried over to G
Node F: -1.5-2=-3.5, We distribute 3.5	
FD=(3.5)(0.296)=1	0.5 carried over to D
FG=(3.5)(0.258)=0.8	0.4 carried over to G
FI=(3.5)(0.183)=0.6	0.3 carried over to I
FH=(3.5)(0.262)=0.8	0.4 carried over to H
Node G: -1.5+1.8+0.4=0.7, We distribute -0.7	
GE almost 0	0 carried over to E
GD almost 0	0 carried over to D
GF almost 0	0 carried over to F
GI almost 0	0 carried over to I

Appendix C

FEA Results of All Case Studies

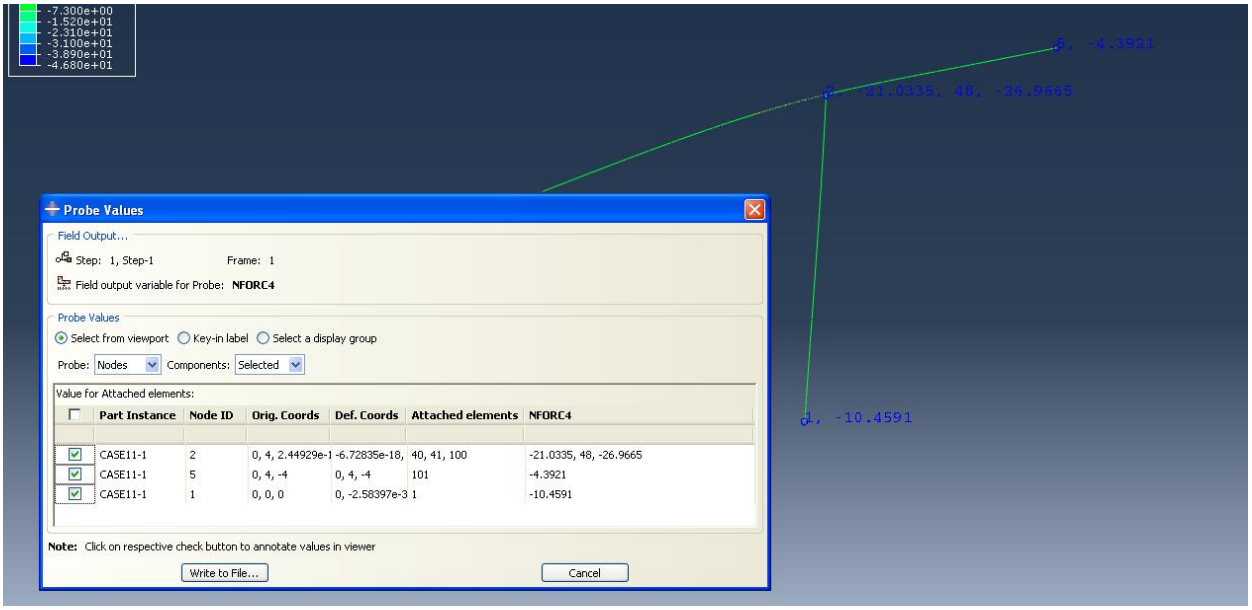


Figure C-1 Case study one FEA results (nodal moments)

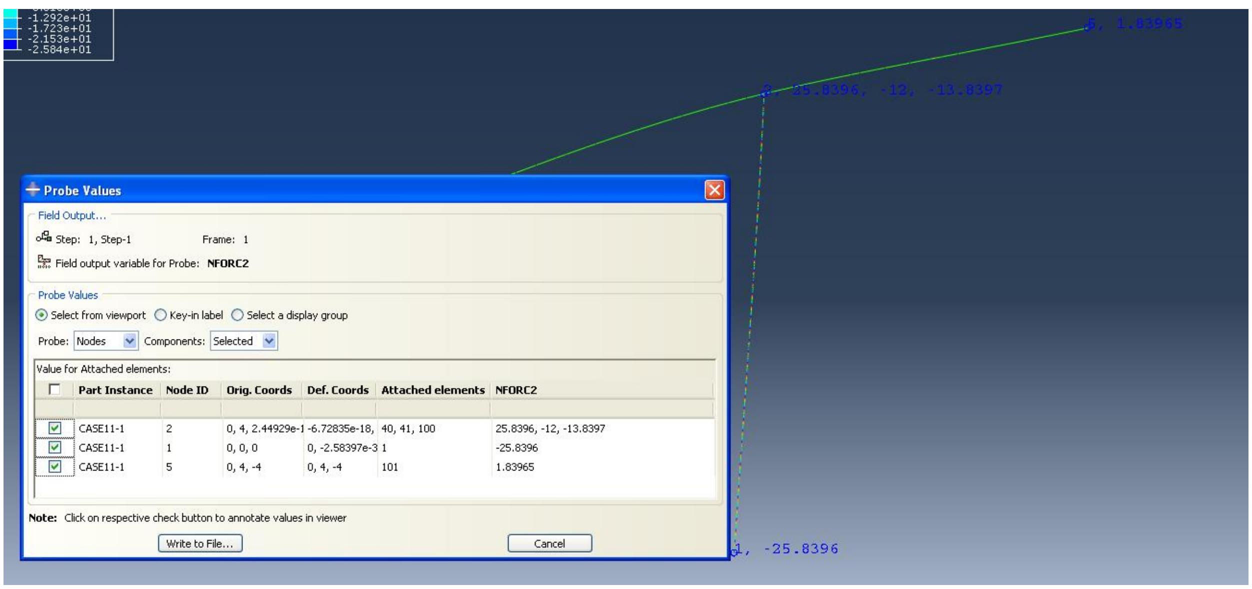


Figure C-2 Case study one FEA results (nodal forces)

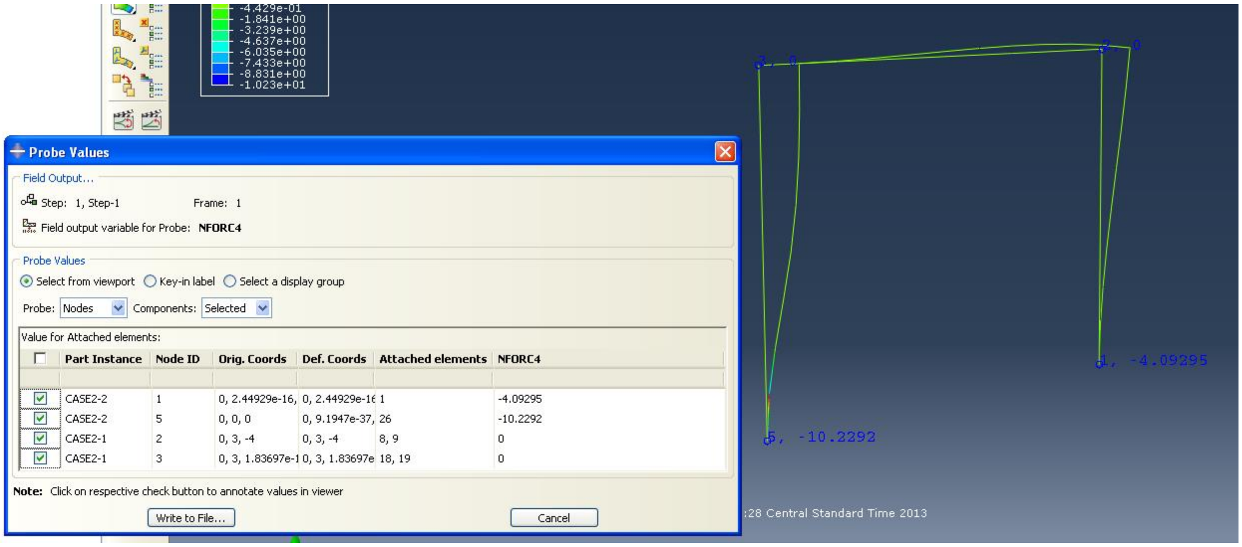
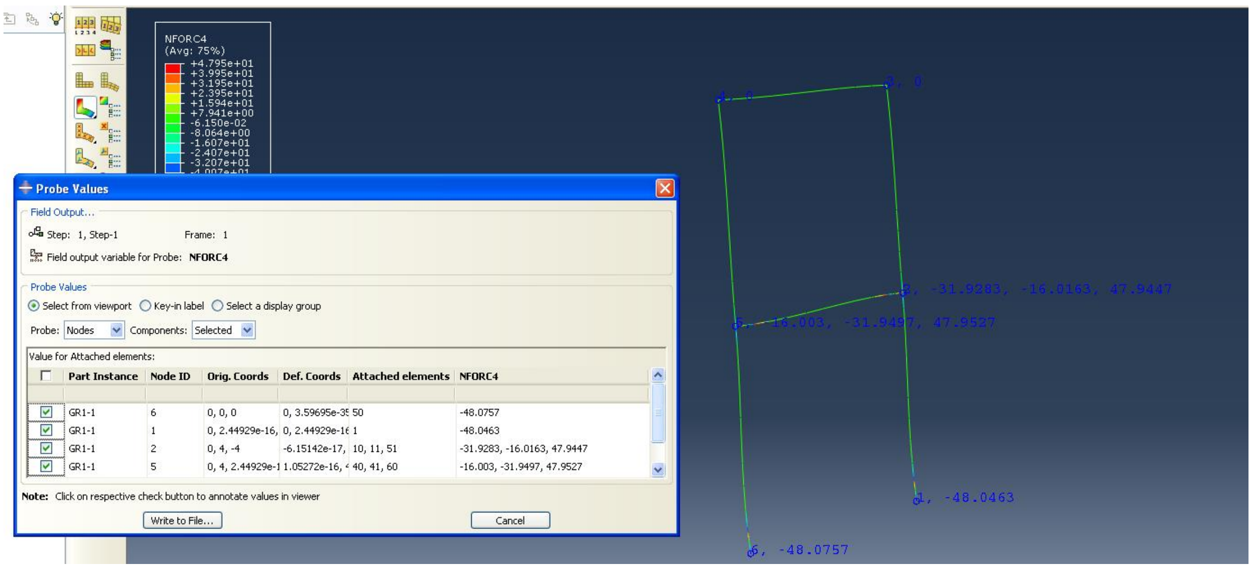


Figure C-3 Case study two FEA results (nodal moments)



FigureC-4 Case study three FEA results (nodal moments)

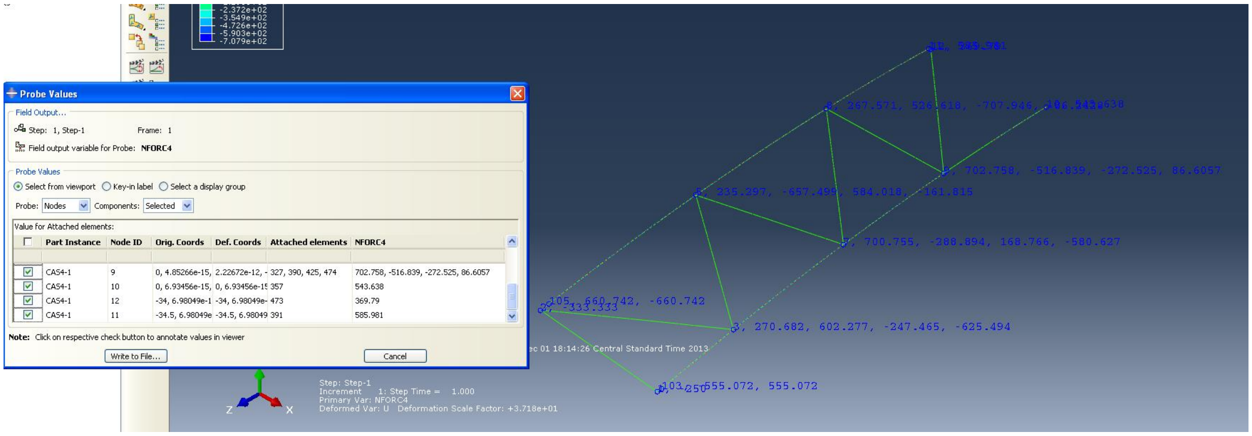


Figure C-5 Case study four FEA results (nodal moments)

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Biographical Information

Reza Mansouri was born in Pullman, which is a small town in the state of Washington, located 8 miles west of Moscow, Idaho in 1980. He grew up in Shiraz, Iran and Sheffield, UK. In the city of Shiraz, he completed his secondary education and passed a flight school exam to become a pilot. It was during his flying as a student pilot that he fell in love with airplanes and decided that he eventually was going to design airplanes.

After completing a year of flight school, he came to Texas to study aerospace engineering. He began his carrier at North Lake community college and obtained his associate of science degree in 2003. The next year, he transferred to the University of Texas at Arlington (UTA). He obtained his BS in Aerospace Engineering degree in 2007. After earning his Aerospace Engineering degree, in the following year he started his master's career and in spring of 2009 he joined Dr. Chudoba's Aerospace Vehicle Design (AVD) lab to continue pursuing his dream to design airplanes.

As a member of the AVD lab, he studied classical structural analysis methods and latest advanced numerical methods. In 2011, he started working at Weber Aircraft as research engineer, while continuing to work on his MS Thesis.

This thesis is the result of Reza's work toward combining a classical structural analysis method with FEA methodology. This new combination provides a transparency to the design/analysis procedure, which is the greatest advantage of the classical methods. His future plans include continuing working as a R&D engineer for Zodiac Aerospace to design the next generation aircraft seat structures and continue working with the AVD lab at the University of Texas at Arlington to help design the next generation of fixed wing aircraft.