MATHEMATICAL PROGRAMMING APPROACHES FOR LAND USE PLANNING THAT LIMITS URBAN SPRAWL

by

PIYUSH KUMAR

Presented to the Faculty of the Graduate School of

The University of Texas at Arlington in Partial Fulfillment

of the Requirements

for the Degree of

DOCTOR OF PHILOSOPHY

THE UNIVERSITY OF TEXAS AT ARLINGTON

December 2013

To my family, teachers and friends

ACKNOWLEDGEMENTS

as long as I was at UTA. It was a source of strength that helped me through all of the challenges of studying in the US as an international student.

In the end, I cannot find words to express my appreciation for my parents and siblings, who were always there for me. Even though I had to spend so much time away from them to do my PhD, they never complained and were always encouraging me to keep going forward. I will always be indebted to all my family members who have been cheering for me and without whom, this accomplishment would lose quite a lot of its sheen.

<div align="right">November 22, 2013</div>

ABSTRACT

MATHEMATICAL PROGRAMMING APPROACHES FOR LAND USE
PLANNING THAT LIMITS URBAN SPRAWL

Piyush Kumar, Ph.D.

The University of Texas at Arlington, 2013

Supervising Professor: Dr. Jay Rosenberger

Sprawl has a detrimental effect on quality of life and the environment. For example, it increases our dependence on vehicles for daily tasks. This leads to increased vehicular pollution. It also increases traffic, which results in people spending more time on the road. With dwindling resources and increasing population, it becomes necessary that we manage sprawl. Ewing et al. [1] defined factors to measure sprawl in the present urban structure. The measures are divided into four broad categories, which are density factors, mixed use factors, streets factor, and centers factors. These measures address various aspects that contribute to sprawl. For example, in streets factors, the local street density affects sprawl. A balance between street density and the size of the land plots would determine the smoothness of traffic and the population density. These measures are then adapted for use in future planning of metro areas.

In this research, we develop a mixed integer linear programming (MILP) model to optimize land usage subject to sprawl constraints, which are based upon the adapted sprawl measures. For a typical data-set of a city of moderate size, the

MILP becomes too large for a commercial solver. In addition, all of the measures given by Ewing et al. [1] are incorporated in the MILP as constraints as opposed to an objective function.

The MILP has a special structure; that is the constraints containing quadratic variables in the MILP that can be isolated from the rest of the model. We use Benders decomposition to attempt to solve it. Due to the enormity of the problem and the fact that the planner might be unaware of the initial bounds for the various constraints, we create a scatter plot between the total land use suitability and the violations for land mixed use bounds under varying bounds on certain constraints.

An orthogonal design of three factors is used to build the scatter plot. Land use objective value and land mixed use violations form the X-axis and Y-axis on the scatter plot. The factors used in orthogonal design are gross population density, density gradient between the central census tract and the census tracts around it, and the lower bound on the commercial activity, which determines the Central Business District (CBD). Since the normal mathematical model is intractable, the goal is to provide the planner with a tool to determine what would be a suitable land use assignment. The planner can use this tool to analyze how various factors affect the land use objective value and the violations on the land mixed use constraint.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

LIST OF TABLES

CHAPTER 1

Introduction

1.1   History of Urban Planning

According to Catanese and Snyder [2], the earliest known examples of urban planning were by the Sumerians of Assyria. Their cities included fortresses and marketplaces for populations of 3000-5000 that lived in them. The unifying features of the cities were the tall buildings called ziggurats, which had the dual role of being both temples and observatories. A notable example of this city was Babylon in present day Iraq. Similarly, there were other civilizations, like the Egyptian, Indus valley, and Chinese, that began building cities. The common characteristic among all of these ancient cities was that they were all built along great rivers, which afforded them advantages with regards to transportation and defense.

The first example of zoning in cities was in the first century A.D. in Rome when Augustus established a 70-foot height limit. Rome grappled with the problems of overcrowding and transportation when its estimated population grew from 250,000 to 2,000,000 residents. To tackle these problems, Romans started building roads and military cities.

All of these developments in the ancient world established a pattern in which cities are now built. There are four layers in the pattern, which are:

- *Physical base*: The visible form of the city, like the roads, buildings, parks, etc. This was illustrated by the rectangular pattern of the street systems.

- *Political base*: This is the base that gives the city its *meaning*. The planner must determine how the city must be designed around the political base. For

example, the ancient cities were built around fortresses where the rulers of the land resided.

- *Economic base*: The planner located various centers of commerce in the city, such as the marketplaces.

- *Social base*: The planner allowed for open spaces or centers where the residents may assemble and socialize.

The Medieval ages leading up to the Renaissance primarily led to cities and towns being built for defense with very limited population. The Renaissance shifted the focus of urban planning to artistic style rather than function. Notable planners during this period included Leonardo and Michelangelo.

With the industrial revolution, there was the need to bring raw materials to factories and finished products to market areas. Thus, the cities needed streets, railways, shipping lanes without which the industrial revolution would have been impossible. Increased commerce and manufacturing led to congestion, new safety hazards, and air and water pollution. As the central areas became more crowded, the wealthy began moving into the suburbs. The invention of the automobile only served to hasten and promote this migration. This phenomenon was marked as an early form of urban sprawl.

1.2 Planning Method

According to Catanese and Snyder [2], the major components for urban planning process are:

- *Problem Diagnosis*: A planner must identify which problems afflict the present city and then, define them in specific terms. However, the problem diagnosis depends on the individual planner's perspective on definitions of various norms, ideologies, and standards. *Descriptive statistics* is used extensively to describe

a problem, such as means, medians, ranges, ratios, etc. An important source of information at this stage for the planner is the U.S. Bureau of Census. The information available there includes but is not limited to census blocks, census tracts, cities, counties, standard metropolitan statistical areas, etc. In case the available information does not fit the needs of the planner, then he/she must use *survey research methods* to generate specific information.

- *Goal Articulation*: After identifying the problems, specific goals must be set as to what extent the problem has to be resolved. The challenge lies in translating the verbal goals into operational objectives.

- *Prediction and Projection*: The planner must determine the time span of the project. For example, California water plans required 20 years of lead time. Hence, to plan a new city or to redevelop an old one, future projections of the population growth and trade are required, since they have a direct effect on the services in the city.

- *Alternative Development*: At this stage, the planner develops alternatives to the original plans. If the situation is simple, such as developing a business corridor, the planner has already been given a location and does not have many competing factors. But if the situation is complex and involves many different aspects, such as environmental planning, transportation and public facilities, housing, historic preservation, spatial distribution, and land use planning, then it is necessary that the planner develop multiple options.

- *Feasibility Analysis*: Even though the model inherently accounts for constraints, such as size and availability of land, finance, etc., the planner must also ask whether the alternatives are feasible on other vague constraints, such as organizational or political acceptability. These constraints are not present in the

model since they are subjective in nature and hence, require the planner's attention.

- *Evaluation*: The planner must now decide which alternative to implement based on the relative impact of the decisions made in each alternative. *Cost-Benefit analysis* is used often at this stage.

- *Implementation*: Two factors vital to success at this stage are political commitment and clearly defined goals.

The method given above is very general. There are now specific techniques and tools that modern planners use to build cities. We examine *land-use suitability analysis*, which is a tool that identifies the most suitable places for locating future land uses [3]. According to the authors, the evolution of land use suitability techniques through history can be divided into five stages. These stages are:

- *Early hand drawn sieve mapping*: As early as 1912, planners drew maps of various topographical features of the land by hand. These maps were then combined together to recommend changes in land use. These maps were a precursor to the modern land-use suitability analysis.

- *Advancement in literature*: According to Steinitz et al. [4], *overlay techniques* were improved after the publication of an article by Jacqueline Tyrwhitt in *Town and Country Planning*. She gave an example of four maps (relief, hydrology, rock types, and soil drainage) drawn on transparent paper to the same scale. Those were then integrated into one map, which provided a unified interpretation of the four different land characteristics [4]. As time passed, more maps were drawn as more attributes for the land were identified. This posed a problem since there was a limit to what may be feasible to draw by hand.

- *Computer-assisted overlay mapping*: In 1963, Harvard became one of the first places to use computers to help draw overlay maps. The trend continued to

surge as computers became more powerful, and the techniques to draw the maps became more sophisticated.

- *Redefinition of spatial data and multi-criteria evaluation*: At this stage, *spatial data*, which describes the various attributes of the land in quantifiable terms, was used as an input to optimization models. The two approaches used were boolean logic and a fuzzy approach, both offering distinct sets of pros and cons. Since there are conflicting objectives when planning a city, researchers introduced decision making models where multiple criteria were evaluated. Moreno and Seigel [5] provides an application of multi-criteria evaluation via an impact analysis for the building of a highway corridor in Colorado. To identify the land-use allocation criteria and suitability weights, a modified *Delphi* method is used. After preparing maps for each factor, they are combined to form a composite map to find a corridor with minimal environmental impact.

- *Replicating expert knowledge in the process*: In the current state, there is extensive use of Artificial Intelligence (AI) in order to find an optimal solution. Techniques like genetic algorithms, artificial life, genetic programming, and hybrid intelligent systems are being used [3]. These techniques though, are experimental, complicated, and too expensive to be considered for everyday planning. A more detailed discussion will follow in the next section.

1.3   Sprawl

As we have seen from the history of urban planning, the rise of sprawl as an issue has its roots in the Industrial Revolution. There is no consensus in the literature as to the definition of sprawl, which shows how difficult measure sprawl quantitatively is. There are some characteristics that are common among the many attempts to define sprawl in the literature. Those are:

- Unplanned and scattered development.

- Low population density.

- High reliance on automobiles.

- Locations outside of the metro area.

Here, we primarily concentrate on sprawl in the context of the United States. Delafons [6] attributes the U.S. system of urban planning to be influenced by "prairie psychology." The development in the U.S. has taken place with the following basic mindset:

- Virtually unlimited supply of land.

- Land accessible to everyone and the rights of ownership protected by the U.S. Constitution.

- No intervention with market driven growth.

- Planners who do not question the need for development.

- Inherent distrust towards the government and hence, minimal public review of the policies that are already in place.

From a historical perspective, the outward development was driven by increased accessibility to commercial centers in the heart of cities since 1830 via transportation [7]. As cities became more congested, sanitary conditions worsened and green space was lost, and the wealthy found it necessary to relocate to more suitable places [8]. With the arrival of street cars and subsequently automobiles, the working population in the cities moved to open lands outside of the cities and started travelling to work. Federal aid for the highway systems in the U.S. also aided sprawl. In the post war period, a number of policies were enacted that served to increase the rate of sprawl. These are:

- Federal subsidies for housing and highways.

- Accessible auto loans and easy credit.

- Federal mortgage guarantees and tax policies affording easier home ownership.

All of these social and institutional factors combined to aid urban sprawl. The reasons why sprawl is a cause for concern are:

- *Loss of Open Space*: The pace of development in the U.S. has not been proportional to the rate of population growth. For example, in the metropolitan area of Cleveland, the amount of developed area increased whereas the population decreased [9]. Even though the early suburbs were dense enough to be served via mass transit systems, the present suburbs only have 2-5 lots per acre, and hence, it becomes necessary that the residents drive automobiles for transportation. The urban population density has decreased by 23 percent between 1970 to 1990 [10]. A contributing factor towards that has also been the increased size of housing. For example in Massachusetts, the average house size was 800 square feet in 1950, 1,500 square feet in 1970, and 2,190 square feet in 1998, even though the size of the households have been shrinking from 3.67 in 1940 to 2.58 in 2010 [11, 12]. Larger suburban houses consume more energy and produce more pollution too. This pattern can be seen all over the U.S. Between 1970 and 1990, the population size of Chicago only grew by 4 percent, but the residential land development increased by 46 percent [13]. Loss of open space is also a major contributor in prime farmland being lost to development. A study by the American Farmland Trust measured the rate of loss of farming land to sprawl and put the figure at 1 million acres per year [14]. With the loss of agricultural farm lands and increasing population, more land would have to be brought under cultivation, which would only mean more environmental problems.

- *Air Quality*: Low density and discontinuous development makes automobile use mandatory. It leads people to drive more, and hence, it burns more fossil

fuels leading to increases in greenhouse gases and air pollution. Between 1960 and 1990, the percentage of people working outside their counties increased by 200 [15]. The average American driver spends 443 hours each year driving [15]. The spread out development discourages alternate means of mobility like walking, cycling, or mass transit [16]. Increased usage of vehicles leads to traffic congestions. An average driver spends 51 hours a year stuck in traffic [17]. This is an issue of both quality of life and health. Automobiles are responsible for one-third to one-half of carbon monoxide, nitrogen oxides (NO_X) and volatile organic compounds (VOC) pollution, which are health hazards [18]. Hence, increased usage of automobiles leads to degrading air quality.

- *Climate Change*: According to the U.S. Environmental Protection Agency (EPA) report, greenhouse gas emissions by light vehicles (transportation for general population, which is separate from trade oriented heavy traffic) account for 62 percent of the total emissions in the transportation sector [19]. Clearing land for highways, residential areas, and service areas due to sprawl lead to the destruction of green cover. The loss of green cover leads to the destruction of carbon sinks (i.e., areas of forests that are large enough to absorb significant amounts of carbon dioxide). Scientists from the Intergovernmental Panel on Climate Change (IPCC) predict that climate change will lead to reduced fresh water supplies, loss of fisheries and biodiversity, unpredictable wind patterns among other negative impacts [20].

- *Water Supply*: As a result of sprawl, suburban areas are now characterized by wide streets, long driveways, large lawns, and huge houses. All of these structures have impervious surfaces that lead to rainwater runoff. The runoff from the surface of urban and suburban areas are contaminated from roofs, roads, building sites, and chemically treated lawns [13]. For example, according

to the U.S. EPA, the majority of wildlife and bird poisonings are a result of lawn chemicals [21]. Moreover, wetlands and forests filter rainwater absorb flood waters and supply lakes and rivers that are our primary source of drinking water [22]. Sprawl leads to the destruction of the wetlands and forests, and hence, it impedes nature's ability to provide clean water.

- *Habitat and Wildlife*: Encroachment of suburban areas near forests and wildlife habitats inevitably leads to less room for wild animals to roam. According to Lowy [23], the human injuries from bear, alligator, and cougar attacks in 1990s have been the highest in American history. With humans living in close proximity to wildlife, overpopulation of certain species may occur. For example, the population of deer in New Jersey has doubled between 1983-2003 [24].

- *Ecosystem Services*: An *ecosystem service* refers to the role natural systems such as forests, wetlands, rivers, and estuaries play towards sustaining human life. Environmental economists have calculated the fiscal worth of the services provided by nature. In Massachusetts, the Charles river basin saves the state $18 million in flood protection [13]. The protected lands surrounding Quabbin and Wachusett reservoirs cleanses as much water as a $180 million filtration plant would [13]. The land desirable for development are the ones that are clear and flat or gently sloping. Ironically, these types of lands are suitable for farmland and floodplains. Developing these lands leads to loss of farmland for local agriculture and loss of important ecological systems.

- *Human Health Impacts*: Sprawl causes both air and water pollution as we have seen above. Air pollution leads to a variety of respiratory diseases, such as asthma, bronchitis, etc. Water pollution caused by surface runoff poses a serious risk to human health as well. A study of New Jersey drinking water found that the water supply was contaminated with arsenic, radium, mercury,

VOCs, pesticides, and toxic by-products as a result of sprawl induced runoff [25]. Sprawl also fosters sedentary lifestyles that lead to obesity, hypertension, heart diseases among other negative health effects. A health promotion study, joint venture by the *American Journal of Health Promotion* and the *American Journal of Public Health*, examined research on over 200,000 people living in 448 counties. They concluded that "U.S. adults living in sprawling counties weigh more, are more likely to be obese, and are more likely to suffer from high-blood pressure than their counterparts in compact counties [26]."

With all of the issues surrounding sprawl, there have been past attempts to estimate the costs associated with it. Even given how disputed the definition of sprawl is itself, the costs of sprawl also do not give rise to any consensus in the planning community. One of the more significant studies done on the costs of sprawl was by Robert Burchell et al. [27, 28]. Burchell et al. [27, 28] divided the costs into five major categories:

- Public and private capital and operating costs.
- Transportation and travel costs.
- Land/natural habitat preservation.
- Quality of life.
- Social issues.

The study defined more than 40 measures (one third of which were positive) that divided into these five categories. For example, in capital costs alone, if sprawl goes unchecked, then the U.S. would spend $143.2 billion every year on services, and the revenues would only be $99.4 billion, leading to a deficit of $43.7 billion annually [28]. Given all of the negative impacts of sprawl, the authors felt it necessary to provide tools to planners that would enable them to design cities/downtowns that would be walk-able and transit oriented.

Despite all of the concerns with sprawl, the literature is still inadequate on how to measure sprawl quantitatively and use those measures in land use planning. In our research, we develop a mathematical optimization model for either designing and developing a new area or redeveloping a previously developed area. In particular, the model strives to minimize the negative effects of sprawl by managing various parameters that were derived from the Transportation Research Board report by Ewing et al. [1].

In the next chapter, we present a literature review of the research done for measuring and optimizing sprawl, as well as decomposition methods, and quadratic assignment problems. Then, we discuss the model that we are proposing for optimizing sprawl, and we develop methodologies that would be appropriate for solving our model. Finally, we present the computational results to show the efficacy of decomposition methods and heuristics in solving the model.

CHAPTER 2

Literature Review

2.1   Land Use Optimization

Most literature on land use optimization takes at least one or more aspects that affect sprawl into consideration. These considerations may range from managing *peak run off*, to air quality, to travelling costs. The term that is frequently associated with sustainable land use planning is *smart growth*. Smart growth is a term used for judicious stewardship of natural resources to prevent urban sprawl. Hence, to differentiate between the literature of simple land use allocation and sprawl, we decided that the papers that do not mention sprawl or sustainability explicitly as one of their objectives will be discussed in this section.

The GIS-based land-use suitability analysis has been used to solve an array of problems. For example, it has been used in ecological models for defining land suitability (in this case, habitat for animal and plant species [29, 30]), geological preference [31], suitability of land for agricultural use [32, 33], environmental impact evaluation [5], site selection for facilities location [34, 35], and regional planning [36]. There is also a significant part of the literature that is concerned with simultaneous optimization of land use assignment and transportation with the focus on minimizing sprawl i.e. minimizing travelling cost [37, 38, 39, 40] . Moore and Gordon [41] extend the integration of land use and transportation to include environmental applications as well. Another area is optimizing the land use allocation problem with respect to economic activities [37, 42, 43, 44]. Increasing popularity of the concept of sustainability has led to research focusing on sustainable spatial optimization of land use

allocation [45, 46, 47]. All of the papers cited above account for only some measures that affect sprawl. The rationale here is that, since the measures are inter-dependent, increasing compactness would automatically lead to minimization of sprawl. This is not ideal, since it is not known to what degree other sprawl measures would be reduced, as they are not accounted for in the model.

Most literature on land use allocation uses integer programming (IP). IP involves a decision variable of whether a particular activity should be allotted to a site or not [48]. *Land use* is the utilization of the land and its resources by humans for a specific purpose. *Land use suitability analysis* searches for the best site for an intended land use based on various characteristics of the land. The assumption here is that the area is subdivided into a set of basic units of observation [49]. The basic units of observation are referred to as *land pieces* or *cells*. Then, the sites are assigned a suitability factor for each category of land use, which indicates how suitable a land piece is for a particular land use. Modern land use suitability analysis depends on Geographic Information Systems (GIS) to form overlay maps. The reason a Decision Support System (DSS) has not been added to GIS is that, there is no general consensus in the literature on a suitable spatial analytic tool. It is partly due to the fact that most of the models are based on different underlying assumptions. For example, according to Malczewski [48], the model assumption is that the planner will interact closely with it to evaluate the various solutions. This assumption requires that the model be relatively simple and fast even if the solution is suboptimal. In this case, an LP method applied to a simplified version of the model would suffice. On the other hand, if the assumption is that the model must be a black box and must produce an output for a given set of input, the choice of a spatial analytic tool would be completely different. In this case, the model would be very detailed, and hence, the

algorithm used would be complex and relatively slower, such as a Genetic algorithm (GA), Cellular Automata (CA), etc.

2.1.1 Linear and Integer Programming Techniques

Implementation of Linear Programming (LP) to solve land use suitability problems started with multi-criteria decision making (MCDM) techniques. MCDM involves defining a relationship between the input and output maps. The technique combines the geographical information and the planner's preferences to provide alternative decision options. After assigning weights to each objective and combining them into a single equation, the problem would be solved by using standard LP/IP [50, 51]. Moore and Gordon [37] use an LP model for dividing economic activities over the planning area. They focus on how to assign the activities to a physical site in an iterative manner.

The problems associated with MCDM methods/LP are [48]:

- The input data to the GIS multi-criteria evaluation procedures are inaccurate, imprecise, and ambiguous.

- Standardization of the criterion have no suitable theoretical or empirical justification [52].

- Different multi-criteria evaluation rules generate different land use suitability patterns [53], and hence, it is difficult to choose the best method in a given circumstance.

2.1.2 Artificial Intelligence Methods

Because of the large sizes of the allocation problems, the focus has been, for the most part, on heuristic algorithms. The downside to heuristics is that they do not guarantee optimal solutions, though most of the time, the solution/the set

of solutions is near optimal [34]. A variety of meta-heuristic techniques, such as simulated annealing, GAs [54], artificial neural networks [55], CA [56], etc. are used in combination with GIS for optimization of land use allocation.

As we observed before, the assumptions of the input data being precise is not realistic. Within the context of complex factors involved in land use suitability analysis, it is difficult to provide accurate numerical data. Since fuzzy logic techniques have sets without clearly defined boundaries, and partial membership of elements is allowed, it works well with the imprecise input data given. Wang [57] proposes a method of representing fuzzy information in GIS, which leads to the formation of a fuzzy suitability rating system. Banai et al. [58] and Jiang et al. [52] combine a fuzzy membership function with MCDM to develop GIS-based land use suitability methods.

A plethora of research has been conducted to test the applicability of artificial neural networks for land use suitability analysis techniques [59, 60, 55]. Sui [55] uses and compares a back propagation network to measure the suitability of land pieces for development against a traditional map overlay modelling technique. However, there are several problems with neural networks [49], which include the following:

- With a complex algorithm, it is unclear as to what makes an optimal structure. The structure of neural networks might be too complex for an urban planner. The planner would not be able to make any modifications to the solutions since he/she does not know how it would affect the objective function.

- Neural networks require training with sample sets before they are applied to the actual model. The network might *overtrain*, which simply means that the network only memorizes the solutions during training, and thus, performs poorly on other real data sets.

Significant papers that use evolutionary algorithms, such as Gas, to optimize the multi-objective (linear or nonlinear) land use allocation problem include Brookes [61], Fotakis and Sidiropoulos [62], Holzkamper and Seppelt [63], Pereira and Duckstein [29], Matthews et al. [64, 65], Los [38], Manson [66], Xiao et al. [67], Gabriel et al. [68], and Zhang and Bian [69] among others. Zhou and Civco [59] uses a combination of neural networks and a GA for solving land use suitability model. Matthews et al. [65] compares GA to traditional deliberative methods. They report that the GA methods are capable of delivering a range of options, along with cost-benefit analysis for each such option. Manson [66] shows that evolutionary algorithms give better solutions in general than traditional methods, such as a weighted sum method. Pereira and Duckstein [29] use GA along with GIS-based multi-criteria evaluation methods. There are research papers that explore land use optimization with simulated annealing [70, 71, 72].

CA explicitly deals with a cell and its contiguous neighbours. CAs have been used to simulate urban development, change in land use, freeway traffic, and the spread of fires [56, 73]. CA does not take a global view of the model but examines regional or local interactions to build system complexity. Wu [74] integrates CA with GIS and multi-criteria evaluation. Recently, the CA method has been extended to include the addition of agents and non-local search methods [75, 76].

Jonsson [77] and Vold [40] develop an integrated land use-transportation framework, which is then optimized by implementing response surface methodology. The recommendations made, as a result of the optimization, are for government policy. Ouyang and Lam [39] use heuristics to solve the joint optimization problem of land use allocation and transportation network design. They use an activity based model, which determines the resources needed to accommodate the maximum population, while satisfying user utility constraints. The models above strive to generate multi-

16

ple solutions instead of just one. Hence, these models depend heavily on heuristic techniques.

2.2 Measurement and Optimization of Sprawl

In recent years, sprawl has increasingly come under focus in not just the United States, but in other countries around the world, such as China and Brazil. In these countries, urban sprawl is fuelled by the combination of rapid economic growth and large populations. A large number of publications have come out that focus on sprawl as an issue in countries apart from the U. S. [78, 79, 80, 81, 82, 83, 84, 85].

There is a variety of research that chooses one or more aspects of sprawl to manage. Urban sprawl is minimized from the standpoint of preservation of forests and farmland [62, 63, 64, 70, 71, 72, 86]. Attempts to minimize sprawl by suggesting changes in policies at the government level have been made in the past [78, 87, 88, 89]. Stone Jr. [90] measures how sprawl affects air quality. Feitosa et al. [91] focuses on designing inclusive cities from an economic standpoint in Brazil.

Gabriel et al. [68] take a multi-objective approach to controlling sprawl in land development. They do so by taking into account objectives from the perspective of the government, planners, environmentalists, conservationists, and land developers. The intention of the authors is to balance the trade-off between the different objective functions. The paper employs linear and quadratic objective functions, subject to polyhedral and binary constraints, to come up with a Quadratic Mixed Integer Program (QMIP). Spatially, the various objectives considered in the paper are maximizing compactness, minimizing impervious area, protection of certain environmentally sensitive areas, and maximizing the total value of the developed land pieces. The authors solve an example with 913 undeveloped and 4837 developed cells using XPRESS-MP solver. After relaxing some constraints, the computations become

17

tractable for the problem. The measures given in [68] do not cover many measures of sprawl, such as the existence of Central Business Districts (CBDs) and Population Centers (PCs). In addition, there is an overlap in the objectives of compactness and impervious surfaces. When maximizing compactness (that is, developing land pieces in the immediate neighbourhood), it automatically reduces the total impervious surface area. Another point of interest is that the paper does not describe any special algorithm to deal with the model.

Stewart et al. [92] suggests that there should be a close interaction between the model and the planner at the evaluation stage. Hence, the model must be fast, provide detailed feedback to the stakeholders/planners, generate a range of solutions, and be flexible in terms of input data. The paper considers three objectives, which are:

- *The number of clusters for each land use*: This measures the degree of fragmentation. The existence of clusters of single land use would require the use of automobiles to travel from residential areas to service areas. Hence, the requirement is for a holistic mix of land uses.

- *The relative magnitude of the largest cluster for each land use*: The goal here is to minimize the existence of multiple clusters. The rationale is that one big cluster is better than a number of clusters, since one big cluster would mean that the other clusters would automatically be smaller.

- *Compactness of land uses*: A compact area of a single land use is preferable to a long thread-shaped cluster.

A specially designed genetic algorithm is used to solve the resulting constrained nonlinear combinatorial programming problem. The objective functions are similar to the measures given by Ewing et al. [1], but they are generic as far as sprawl is concerned. Similar to Gabriel et al. [68], the paper does not account for the existence of

18

PCs or CBDs, which form naturally in an urban setting even if the planner does not intend for them.

Zielinska et al. [45, 93] develop an optimization model that minimizes perhaps the most accurate model of sprawl in the current literature. The objective functions given in the paper are:

- Minimization of open space development.

- Redevelopment of inner areas if economically feasible.

- Minimization of incompatibility between land uses.

- Minimization of distance to already developed areas.

The model employs a constraint to limit the population density. The authors suggest that having density as an objective function might result in an unsustainable solution. The paper employs a Branch-and-Bound method to solve the resulting model. They do not consider the factors that affect sprawl like mixed use development, population density, and degrees of centering.

Given the nature of the problem, which involves combining the disciplines of urban planning and optimization, it is challenging for researchers to be an expert in both. Most attempts at optimizing land use allocation models have been made by researchers outside of the field of optimization. Zielinska et al.[45] made one of the more significant attempts at designing a sustainable land use model for urban planning, and the authors belong to the department of geography. The authors of Riveira et al. [71] were a multidisciplinary team, which combined agriculture and modern technology. Holzkamper and Seppelt [63] combined the disciplines of environmental research and software design to form a GUI for land use planning tools.

The performance of the models was not reported explicitly in a large part of the literature. Zielinska et al. [45] and Stewart et al. [92] solve the model for a 20by-20 matrix. The maximum solution time was found to be 85 seconds with 978

decision variables in [45]. It is also noted by Stewart et al. [92] that the computational time increased quadratically with the problem size. Gabriel et al. [68] solve the optimization problem with 913 undeveloped land parcels and 4837 developed land parcels. It is a QMIP with 3500 variables (mostly binary) and over 23,000 constraints. However, the computational time to solve it is unreported.

Some of the attempts to measure sprawl quantitatively are Ewing et al. [1], Galster et al. [94], and Malpezzi [95]. We find that the current most comprehensive framework to quantify and measure sprawl is constructed by Ewing et al. [1]. Hence, we primarily focus on their measures and interpret them in a way that is suited to future land use planning. Ewing et al. [1] include 22 measures that are broadly divided into four categories, which are:

- Residential density.

- Neighbourhood mix of homes, jobs, and services.

- Strength of centers, such as business districts.

- Accessibility to the street network.

As we observed previously, the literature on land use optimization takes into account only a part of each of these measures.

2.3 Decomposition Methods

Decomposition methods solve large scale problems by breaking them into several smaller subproblems, along with a master problem. *Dantzig-Wolfe decomposition* for linear programming with angular block structure [96, 97], started the trend of decomposition of large optimization problems [98]. Some of the decomposition methods are dual methods, primal cutting plane methods, delayed column generation, Benders decomposition. Decomposition methods are suitable for situations that require analytical solutions of dynamical systems that are characterized by Adomian [99]:

20

- No linearization or weak nonlinearity assumptions.

- Closure approximations.

- Perturbation theory.

- Restrictive assumptions on stochasticity.

Decomposition methods have been used in a wide variety of applications ranging from multi-commodity distribution network design [100], to locomotive and car assignment problems [101, 102, 103], to large-scale water resource management [104]. However, according to the literature, decomposition methods have never been used to solve a land use suitability problem.

2.3.1   Benders Decomposition

Benders decomposition [105], named after Jacques F. Benders is a decomposition method to solve mixed-variable programming problems. Since its inception, it has been used in a variety of applications. For example, Ghotboddini et al. [106] uses a Benders decomposition approach to solve multi-objective cellular manufacturing system problems. It involves increasing efficiency in small-to-medium lot size production environments. Gendron et al. [107] employs Benders decomposition to solve an integer program with non-linear constraints. The nonlinearity of the model was taken care of with Benders decomposition. Benders decomposition involves splitting the mathematical model into a master problem and a single or multiple subproblems. The complicating variables and the associated constraints are put in the subproblem. Hence, the master problem which is only a subset of the original problem is easier to solve. First, the master problem is solved to get a solution. Then, after fixing certain variables in the subproblem, we solve it and generate cuts that penalize the master problem objective function. Benders decomposition is an iterative procedure where

the master problem and the subproblems are solved multiple times to arrive at an optimal solution.

Given the following mixed integer programming formulation,

$$\max \quad z = c^T x + d^T y \tag{2.1}$$

$$\text{subject to:} \tag{2.2}$$

$$Ax + By \leq b$$

$$y \in R_+^{n_1}, x \in R_+^{n_2} \tag{2.3}$$

where c and d are associated cost column vectors of appropriate size for continuous variable vector x and integer variable vector y, respectively. Matrices A, B, and D and vectors b and e have appropriate dimensions. The above formulation can be rewritten as follows,

$$\max \quad z_1 = c^T x + \mu(x) \tag{2.4}$$

$$\text{subject to:} \tag{2.5}$$

$$Ax \leq b$$

$$x \geq 0 \tag{2.6}$$

where $\mu(x)$ is the maximum value of the subproblem which is as follows,

$$\max \quad z_2 = d^T y \tag{2.7}$$

$$\text{subject to:}$$

$$By \leq b - Ax \tag{2.8}$$

$$y \geq 0 \tag{2.9}$$

Let $u \in R^{m_1}$ be the dual variable associated with constraint 2.8. From duality theory, the dual problem for the model (2.7)-(2.9) can be written as follows,

22

$$\min \quad u^T(b - Ax) \tag{2.10}$$

$$\text{subject to:} \tag{2.11}$$

$$B^T u \geq d$$

$$u \geq 0 \tag{2.12}$$

We can see that the feasible space of the subproblem is independent of the optimal values of x chosen in the master problem. Let $F = \{u \mid Bu \geq d, u \geq 0\}$. The presumption is that F is nonempty, since F being empty would imply that the primal subproblem is either infeasible or unbounded. Hence, F is considered to be composed of extreme points u^p $(p \in P)$ and extreme rays r^q $(q \in Q)$. Hence, the Benders's reformulated master problem becomes as follows,

$$\max \quad c^T x + \mu \tag{2.13}$$

$$\text{subject to:} \tag{2.14}$$

$$Ax \leq b$$

$$\mu \leq u^p(b - Ax) \qquad p \in P \tag{2.15}$$

$$r^q(b - Ax) \leq 0 \qquad q \in Q \tag{2.16}$$

$$x \in X \tag{2.17}$$

The drawback of Benders decomposition method is that the number of extreme points and extreme rays is quite large. To address this drawback, delayed constraint generation is used. The master problem and the dual subproblem are solved iteratively, until the termination criteria is reached. The criteria for stopping Bender decomposition is that the upper bound from the subproblem and the lower bound are sufficiently close.

2.4   Quadratic Assignment Problems

Koopmans et al. [43] introduced the concept of the Quadratic Assignment Problems (QAP) to model the problem of locating economic activities. The location of the activities depends upon the locations of other facilities in the neighbourhood. Afterwards, QAPs were used to model a variety of different problems. For example, Steinberg [108] uses QAPs to minimize backboard wiring in electronic circuits. Burkard et al. [109] and McCormick [110] use a QAP to design typewriter keyboards and control panels. Heffley [111] suggests that assigning runners to a relay team transforms into a QAP.

QAP has several formulations which are used most often for the various applications, such as integer linear programming, mixed integer linear programming (MILP), graph formulation, etc. [112].

The methods to solve QAPs can be divided into two categories, which are:

- *Exact algorithms*: Significant exact methods to solve QAPs are branch-and-bound or dynamic programming [113, 114, 115]. There has been some research into using Benders decomposition algorithm along with some heuristics to solve QAPs [116, 117]. Miranda et al. [116] uses a Benders decomposition algorithm to solve a motherboard design issue.

- *Heuristic algorithms*: There is an abundance of research in solving QAP using heuristics. Heuristics can be divided into three categories, which are constructive, limited enumeration, and improvement methods. Even under these categories, most of the research is done in improvement methods. Before the introduction of meta-heuristics, such as simulated annealing, GAs, etc., the heuristics were customized for each problem [112].

As we observed, the majority of the research tends towards heuristic algorithms for QAPs. That is the trend we observed for land use optimization. All the quadratic formulations in land use suitability models were solved with meta-heuristics.

2.5   Contribution

In this research, we develop a mixed integer linear programming model for land use optimization. The objective of the model is to maximize suitability while constraining sprawl. The constraints are constructed from the measures of sprawl as given in Ewing et al. [1]. The rationale here is that various features of a metro area, such as population centers, business districts, distance to services, etc. are always present. Hence, instead of ignoring some or all of these, and maximizing suitability alone, the measures are accounted for and managed at planning level.

The contributions of this research include:

- *A comprehensive treatment of sprawl measures*: From the literature survey, we concluded that no one has accounted for measuring and minimizing sprawl clearly and comprehensively. Rather, the focus has been on sustainability, which focuses on the larger context of the land use problem. We believe it over-complicates the model since destruction of farmland, pollution, and discontinuous development are a result of urban sprawl, and not the cause/characteristics. Hence, if sprawl itself is focused upon, and minimized, then all the other symptoms of sprawl, like pollution and low density development, would diminish.

- *Restricting the sprawl measures*: Most of the research focuses on incorporating the measures in objective functions. But, as noted by Zielinska et al. [45], if population density is included as an objective function, then either maximizing or minimizing it would go against the principles of sustainable development. For example, maximizing population density would lead to overcrowding and

minimizing population density would lead to sprawl. Hence, our model includes all the significant measures of sprawl as constraints in the model. This allows the planner to quickly perform sensitivity analysis. It also enables the planner to generate a range of solutions based on the manipulation of the parameters.

- *Use of decomposition methods*: The literature is completely devoid of research that employs decomposition methods to solve large QMIPs for land use allocation, even though decomposition methods have been used extensively in other areas that involve large-scale problems. We develop a land use model with sprawl constraints and customized decomposition methods to solve it.

- *Method for solving the dual problem for constraints between quadratic and linear variables*: If any linear programming problem only has quadratic variables and the corresponding fixed linear variables, then we devised a method to obtain the optimal values for the corresponding dual variables without solving the problem with any linear programming technique e.g. simplex method. This saves the memory overhead required for building a sparse matrix for the mathematical model and solving it using a commercial software. Although given the sparse matrix, finding the dual variables involved searching for values in a four dimensional matrix which is very slow given a linear index search method.

CHAPTER 3

Sprawl Considerations in Land Use Optimization

3.1   Problem Description

According to the literature, there are various land use allocation models that deal with different aspects, such as agricultural, watersheds, sprawl, etc. However, there is a common theme among all such models in that they aspire to maximize the perceived utility by assigning a land use to each land piece.

The planner's objective is to develop all or part of a metro area. We assume that the planner has already assigned a suitability value to each land piece, which, in our model, varies from -10 to 10 depending on the fitness of the land pieces towards a land use. In our model, we consider eight different land uses, which are High Industrial (HI), High Commercial (HC), High Industrial Residential (HIR), High Residential (HR), Low Commercial (LC), Low Industrial (LI), Low Industrial Residential (LIR), and Low Residential (LR). The planner also has the future population and trade growth projections. The aim is to plan the area in such a way that it naturally resists sprawling in the future. To achieve this target, the planner must find a balance between population growth and services in the area. If he/she fails to do so, then the sprawl would naturally occur as we have observed from history. The planner controls the upper and lower bounds for the parameters given in the model. By changing those, the planner gets information about how the model behaves under different conditions. In some cases, the bounds also depend upon the demands of the market. In others, the bounds must be controlled to manage sprawl.

3.1.1 Measures of Sprawl

The paper by Ewing et al. [1] was funded by Smart Growth America. The objective of the study is to characterize sprawl and relate it with a wide set of outcomes. Based on the characterization of sprawl, the authors select four characteristics of sprawl, which are:

- Low Development Density.

- Segregated Land Uses.

- Lack of Significant Centers.

- Poor Street Accessibility.

Various outcomes, such as vehicle ownership, air quality, commute times, commute mode, etc. are analysed to check how they relate to the four characteristics. The study sample includes 101 of the largest metropolitan statistical areas in 1990 in the U. S. Within this sample, the metro areas are measured on the various quantifiable sprawl factors. The various sprawl factors are then combined into four categories via *Principal Component Analysis (PCA)*. PCA is an analytic technique that extracts a small number of factors from a large pool of correlated variables that represent the common variance in the data.

The four categories of sprawl factors are:

- *Density Factor*: It includes seven variables, four of which were measured from data by the U. S. Bureau of Census. The assumption here is that the census tracts that include low population density areas, such as rural tracts, deserts, etc., are not included. These factors deal with the population density in the metro areas and their distribution. Population Centers (PCs) are areas with local density maxima. They serve as a focal point for social activities, and hence, are considered important.

- *Mix Factor*: These factors are included to ensure a good mix of land uses in a compact area. Sprawl is characterized by long commuting time. For example, the principle behind measuring the percentage of residents within 1 mile of an elementary school is to minimize travelling. Hence, there should be a good mix of services and residences in an area.

- *Centers Factors*: According to Ewing et al. [1], metropolitan centers are considered a hub of concentrated activities that allow multi-purpose trip making, alternate modes of transport, and a sense of place in a metro. Centers may be either residential or commercial. They include six factors, which are density gradient, coefficient of variation of population density across census tracts, etc., percentage of metropolitan population less than 3 miles from the CBD, percentage of population more than 10 miles from the CBD, percentage of the population relating to centers or subcenters within the same metropolitan statistical area (MSA), and the ratio of the weighted density of population centers within the same MSA to the highest density center to which a metro relates.

- *Streets Factors*: Street networks in an metro area form a network, which may be dense or sparse depending on the geography and planning of the area. There is no information available regarding degree of connectedness or curvature of street networks. Hence, the authors use the information about block lengths to generate sprawl measures. The principle behind this is that if the block lengths are larger, then the street network is sparse, so the metro area is spread out. However, one issue was that large rural tracts distorted the average of the block lengths. Three factors were included in this category, which are percentage of small blocks, average block size in square miles, and percentage of small blocks (< 0.01 square miles).

3.1.2 Assumptions

Prior to the assignment of land uses to land pieces, the planner must decide on the scope of the various measures to be taken into account. Since there are multiple conflicting objectives, to make our problem relatively tractable, we assume the following:

- The distance between the various land pieces is calculated *a priori*. The distance between two land pieces is calculated from the center of the first land piece to the center of the latter. The measure of land mixed use variable LM_i is inversely dependent on the distance between the two land pieces under consideration. If the distances are to be calculated dynamically, the equation (3.10), and consequently equation (3.11) would cease to be linear. Since our focus is on developing a linear model, we calculate the distances beforehand.

- Each land piece is assigned to one and only one census tract. Census tracts are meant to be territorial units that are homogeneous with respect to factors like population characteristics, living conditions, etc. These imply that census tracts are developed after the population has settled. But in case of future planning, the planner may rely on clear geographical boundaries to divide the planning area into census tracts.

- The census tracts are known *a priori* and are determined by the planner.

- The model accounts for only one Central Business District (CBD). The planner decides the center of the CBD *a priori* and then develops a potential set of CBDs and inputs them into the model. The model ultimately selects one of the potential CBDs in the solution.

- The model assigns each land piece under consideration to one particular land use. If a land piece has a pre-existing land use, it can simply be removed from consideration or included in the model as a hard constraint.

- The density at the center of the planning area is the density of the census tract, which includes the central coordinates of the planning area.

- Single family dwellings belong in low density residential spaces.

- The census tracts that include low population density areas, such as rural tracts, deserts, etc. are not included.

- For each land piece, the planner determines an *area of influence*, which is the set of surrounding land pieces within a specified distance from the land piece. This is based on the assumption that the land pieces that are apart for more than 1 mile have negligible effect on the mixed use factor of each other.

3.2 Land Use Model

3.2.1 Sets

The following is a description of the sets used in the model. Let,

- C be the set of different land uses (indexed by j).

- N be the set of land pieces in the planning area (indexed by i).

- CT be the set of census tracts in the planning area (indexed by k).

- CBD be the set of potential central business districts in the planning area (indexed by k).

- For each set census tract or CBD, $k \in CT \cup CBD$, let N_k be the set of land pieces in k.

- For each land piece $i \in N$, let N_i be the set of land pieces that fall under the sphere of influence of land piece i.

3.2.2 Parameters

The parameters used in the model are:

- S_{ij} = Suitability factor for each land piece $i \in N$ assigned to land use category $j \in C$.

- U_j, L_j = Upper and lower bounds on the number of land pieces that can be assigned to each category of land use $j \in C$.

- U_{PC}, L_{PC} = Upper and lower bounds on the mean density in the cells considered to be in Population Centers.

- U_{MD}, L_{MD} = Upper and lower bound for the mean density for each Census Tract $k \in CT$.

- U_{DG}, L_{DG} = Upper and lower bounds for the density gradient between census tracts.

- U_{CV}, L_{CV} = Upper and lower bound for the coefficient of variation of the planning area.

- U_{Mix}, L_{Mix} = Upper and lower bound for land mix factor.

- ρ_j = An estimated population density for a land use type $j \in C$.

- A_i= The area of each land piece $i \in N$.

- $AF_{j\hat{j}}$= Attraction factor for each pair of land use type $j, \hat{j} \in C$, which describes the desirability of having the land use types near each other.

- τ_j = A constant for the level of commercial activity in each land use category $j \in C$.

- L_{CBD} = Lower bound for the level of commercial activity in the CBD.

- i_{CBD} = The land piece chosen by the planner around which the CBD is built.

- i_k = Central land piece of any set of land pieces N_k.

- $d_{i_o k}$ = Distance between the land piece at the center of the planning area to the land piece at the center of census tract $k \in CT$.

- $d_{i\hat{i}}$= Distance between the centers of two land pieces $i, \hat{i} \in N$.

- ρ_o= Estimated density at the center of the metro area.

3.2.3 Variables

The variables used in the model are:

- Let the binary variable x_{ij} be defined such that,

$$
x_{ij} =
\begin{cases}
1, & \text{if land piece } i \in N \text{ is assigned land use } j \in C, \\
0, & \text{otherwise.}
\end{cases}
$$

- Let the binary variable y_k be defined such that,

$$
y_k =
\begin{cases}
1, & \text{if a set of land pieces } k \text{ is selected to be the CBD,} \\
0, & \text{otherwise.}
\end{cases}
$$

- Let the binary variable $x_{ij\hat{i}\hat{j}}$ be defined such that,

$$
x_{ij\hat{i}\hat{j}} =
\begin{cases}
1, & \text{if land piece } i \in N \text{ and land piece } \hat{i} \in N \text{ are assigned} \\
   & \quad \text{land uses } j \in C \text{ and } \hat{j} \in C, \text{ respectively,} \\
0, & \text{otherwise.}
\end{cases}
$$

- Let the variable w_{ij} be defined such that,

$$
w_{ij} =
\begin{cases}
1, & \text{if land piece } i \text{ is assigned category } j \text{ and the 1-mile} \\
   & \quad \text{radius around it has density} \geq 850 \text{ people per square mile,} \\
0, & \text{otherwise.}
\end{cases}
$$

- GPD = Gross population density.
- $\rho_{k_1 k_2}$= Covariance of density between two census tracts $k_1, k_2 \in CT$.
- ρ_k= Mean population density for a census tract $k \in CT$.
- ρ_o= Mean population density of the census tract at the center of the planning area.

- b_k = Density gradient of census tract $k \in CT$ with respect to the census tract at the center of the planning area.

- LM_i = Measure of the level of mixed land use around land piece $i \in N$.

- $\bar{\rho}$ = Mean population density of all census tracts.

3.2.4 Mathematical Programming Model

$$\max \quad z_{LP} = \sum_{i \in N} \sum_{j \in C} S_{ij} x_{ij} \tag{3.1}$$

subject to:

$$\sum_{j \in C} x_{ij} = 1 \qquad\qquad \forall i \in N \tag{3.2}$$

$$U_j \geq \sum_{i \in N} x_{ij} \geq L_j \qquad\qquad \forall j \in C \tag{3.3}$$

$$GPD = \frac{\sum_{i \in N} \sum_{j \in C} x_{ij} \rho_j A_i}{\sum_{i \in N} A_i} \tag{3.4}$$

$$U_{PD} \geq GPD \geq L_{PD} \tag{3.5}$$

$$U_{PC} \geq \frac{\sum_{i \in N} \sum_{j \in C} \rho_j A_i w_{ij}}{\sum_{i \in N} A_i} \geq L_{PC} \qquad\qquad \forall k \in PC \tag{3.6}$$

$$M \cdot \sum_{j \in C} w_{ij} \geq \sum_{\hat{i} \in N_i} \sum_{j \in C} \rho_j A_{\hat{i}} x_{\hat{i}j} - 850 \times \sum_{\hat{i} \in N_i} A_{\hat{i}} \qquad\qquad \forall i \in N \tag{3.7}$$

$$850 \times \sum_{j \in C} A_i w_{ij} - \sum_{j \in C} \rho_j A_i x_{ij} \leq 0 \qquad\qquad \forall i \in N \tag{3.8}$$

$$x_{ij} \geq w_{ij} \qquad\qquad \forall i \in N, j \in C \tag{3.9}$$

$$LM_i = \sum_{\hat{i} \in N_i} \left(\frac{\sum_{j \in C} \sum_{\hat{j} \in C} x_{ij\hat{i}\hat{j}} (S_{ij} + S_{\hat{i}\hat{j}}) AF_{j\hat{j}}}{d_{i\hat{i}}} \right) \qquad\qquad \forall i \in N \tag{3.10}$$

$$U_{Mix} \geq LM_i \geq L_{Mix} \tag{3.11}$$

$$\rho_k = \frac{\sum_{i \in N_k} \sum_{j \in C} \rho_j A_i x_{ij}}{\sum_{i \in N_k} A_i} \qquad\qquad \forall k \in CT \tag{3.12}$$

$$U_{MD} \geq \rho_k \geq L_{MD} \qquad\qquad \forall k \in CT \tag{3.13}$$

$$\rho_{k_1 k_2} = \frac{\sum_{i \in N_{k_1}} \sum_{j \in C} \sum_{\hat{i} \in N_{k_2}} \sum_{\hat{j} \in C} \rho_j \rho_{\hat{j}} A_i A_{\hat{i}} x_{ij\hat{i}\hat{j}}}{(\sum_{i \in N_{k_1}} A_i)(\sum_{i \in N_{k_2}} A_i)} \quad \forall k_1, k_2 \in CT$$

(3.14)

$$m(U_{CV}^2 + 1) \sum_{k \in CT} \rho_{kk} \geq \sum_{k_1 \in CT} \sum_{k_2 \in CT} \rho_{k_1 k_2} \geq m(L_{CV}^2 + 1) \sum_{k \in CT} \rho_{kk} \quad (3.15)$$

$$b_k = \frac{-1}{d_{i_o k}} \ln \frac{\rho_k}{\rho_o} \quad \forall k \in CT \setminus k_o$$

(3.16)

$$\rho_o \exp^{-d_{i_o k} U_{DG}} \geq \rho_k \geq \rho_o \exp^{-d_{i_o k} L_{DG}} \quad \forall k \in CT \setminus i_o$$

(3.17)

$$\sum_{i \in N_k} \sum_{j \in C} \tau_j x_{ij} \geq L_{CBD} \cdot y_k \quad \forall k \in CBD$$

(3.18)

$$\sum_{k \in CBD} y_k = 1 \quad (3.19)$$

$$x_{ij} \geq x_{ij\hat{i}\hat{j}} \quad \forall i, \hat{i} \in N; i > \hat{i}; j, \hat{j} \in C$$

(3.20)

$$x_{\hat{i}\hat{j}} \geq x_{ij\hat{i}\hat{j}} \quad (3.21)$$

$$x_{ij\hat{i}\hat{j}} \geq x_{ij} + x_{\hat{i}\hat{j}} - 1 \quad (3.22)$$

3.2.5   Model Justification

3.2.5.1   Deterministic Land use suitability optimization

Equation (3.1) maximizes the overall utility value for assigning land uses to land pieces. Equation (3.2) ensures that each land piece is assigned exactly one land use. Equation (3.3) provides the upper and lower bounds for the total number of land pieces that may have a particular land use. These equations alone represent a

classical linear programming approach to optimizing a land use suitability problem. Now, we add constraints to manage sprawl.

3.2.5.2 Density Factors

The following is a description of various measures for population density as given in Ewing et al. [1]. We have also described whether and how these measures are included in the model.

- *Gross population density*: Equations (3.4) and (3.5) allow the planner to control the gross population density of the population within certain bounds.

- *Percentage of population living at densities less than* 1500 *persons per square mile*: A density less than 1500 persons per sq mi. is referred to as low suburban density. Since the population density of each land use is already set at a certain level, the equations (3.3) and (3.5) automatically constrain this measure. However, we do not explicitly include it in the model.

- *Percentage of population living at densities greater than* 12,500 *persons per square mile*: A density above 12,500 persons per square mile generally supports mass transport systems. For the reasons stated above, this measure is only implicitly included via the constraints (3.3) and (3.5).

- *Estimated density at the center of the metro area derived from a negative exponential density function*: Ewing et al. [1] estimates the density at the center of the metropolitan area and the density gradient after fitting a negative exponential density function to the data points that include densities of census tracts versus the distance from the center to those census tracts. The central coordinates are determined by taking into consideration the extreme edges of the planning area, which may lead to biased center coordinates if there are a

significant number of already developed cells. This measure is included using the constraint sets (3.16) and (3.17).

- *Gross population density of urban lands*: The following measure was derived from the U. S. Department of Agriculture's Natural Resources Inventory (NRI) by separating the population based on whether they are located in urban, or built up land, as opposed to being in sub-urban or rural areas. Every land use has a fixed population density and there are bounds on how many land pieces we can have for a particular land use (3.3). This implicitly controls the population density of the urban lands.

- *Weighted average lot size in square feet for single family dwellings*: Since low residential land use is already bounded by the planner's specifications (3.3), the value is controlled implicitly.

- *Weighted density of all population centers within a metro area*: Population centers as described by Ewing et al. [1] are any 9 grid cell areas that satisfy a population density threshold of 850 persons per sq mi. The rationale here is that any land piece is a part of a population center if the area surrounding the land piece has density greater than 850 persons per square mile. By controlling the total density of the population centers using equations (3.6), (3.7) and (3.8), the measure is automatically bounded.

3.2.5.3 Mix Factor

The various measures for Mix Factor as given in Ewing et al. [1] are as follows:

- *Percentage of residents with businesses within certain blocks of their homes.*
- *Percentage of residents with satisfactory neighbourhood shopping within 1 mile.*
- *Percentage of residents with a public elementary school within 1 mile.*
- *Job-resident balance.*

38

- *Population-serving job-resident balance.*

- *Population-serving job mix.*

All of the measures given above are for metro areas that have already been developed. We substitute these measures with another model that has extensively been used in the literature. Attraction factor A_{ij} refers to whether it is desirable to have the land uses closer together or farther apart. For example, it is not desirable to have high industrial and high residential land uses adjacent to each other, and yet it is desirable that there be a good mix of job-resident balance. Hence the land uses cannot be too far apart either. It varies between 0 to 1, depending on whether the land uses should be further apart or closer together.

As we observe from the definition of LM_i given by equation (9), it is a quadratic variable that involves multiplying two integer variables x_{ij} and $x_{\hat{i}\hat{j}}$. Hence, to convert LM_i to a linear variable, we introduced the variable $x_{\hat{i}\hat{i}\hat{j}\hat{j}}$. The relationship between x_{ij}, $x_{\hat{i}\hat{j}}$, and $x_{\hat{i}\hat{i}\hat{j}\hat{j}}$ is described by equations (3.20), (3.21), and (3.22).

Land mixed use factor LM_i is calculated for each land piece $i \in N$. Equation (3.10) and (3.11) bounds the value of land mixed use factor. It is necessary to obtain a holistic mix of land uses and avoid formations of clusters. If LM_i does not have a lower bound, then there is no constraint to stop the formation of clusters. If LM_i does not have an upper bound on it, then it might lead to a compact but very haphazard assignment of land uses, which is unsustainable.

3.2.5.4 Centers Factors

The various measures for Centers Factors as given in Ewing et al. [1] are as follows:

- *Coefficient of variation of population density across census tracts*: Coefficient of variation is given as standard deviation divided by mean density. This measure

is not linear. Hence, we use algebraic techniques to reduce this to a linear equation. Equation set (3.14) and (3.15) bounds the value of this measure.

- *Density gradient*: Density gradient is defined by Ewing et al. [1] as rate at which the density declines with distance from the center of the metro area estimated with a negative exponential density function. The decline in density is directly proportional to the centering of the metropolitan area. Equations (3.12), (3.13), (3.16), and (3.17) bound the value of density gradient by controlling both the density gradient and the mean density of the census tracts.

- *Percentage of metropolitan population less than 3 miles from the CBD*: To determine this measure, the first task is to determine the location and size of the CBD. According to our assumptions, we have a given location of the CBD from the planner and a potential set of CBDs from which the model can choose. Equations (3.18) and (3.19) determine which CBD is selected and ensure that it has sufficient commercial activity. Since the total population density is bounded within the planning area, and compactness as a factor is already included in mixed use equations (3.10) and (3.11), this measure is only included implicitly. The metropolitan population density would naturally tend to be higher near the CBD.

- *Percentage of metropolitan population more than 10 miles from the CBD*: As above, not including these two measures explicitly helps to make our model computationally simpler but does not adversely affect the model's ability to manage sprawl.

- *Percentage of the metropolitan population relating to centers within the same MSA*: This measure is directly controlled by bounds on high residential and low residential land uses (3.3).

- *Ratio of the weighted density of population centers within the same MSA to the highest density center to which a metro relates*: Since the ratio is nonlinear in nature, bounding it in the model would lead to unnecessary complications. Consequently, this measure is not included in the model.

3.2.5.5 Streets Factors

The various measures for Streets Factors as given in Ewing et al. [1] are as follows:

- *Approximate average block length in urbanized portion of the metro*: Since the size of the land pieces is decided by the planner *a priori*, and the number of land pieces for each land use is already bounded, we do not include this measure explicitly in the model.

- *Average block size in square miles (excluding blocks greater than 1 square mile)*: For the reasons stated above, this measure is not included in the model.

- *Percentage of small blocks (less than 0.01 square mile)*: This measure is not included in the model.

3.2.6 Summary

The following table summarizes how each measure of sprawl as given by Ewing et al. [1] was incorporated into our model.

Table 3.1: Summary of measures of sprawl and corresponding interpretation in the mathematical model

Measures of sprawl	Ewing et al. [1]	Interpretation
Centers Factors	Coefficient of variation of population density across census tracts	(3.14)
	Density gradient (rate of decline of density with distance from the center of the metro area)	(3.16)
	Percentage of population < 3 miles from CBD	Implicit
	Percentage of population > 10 miles from CBD	Implicit
	Percentage of population relating to centers within the same MSA	Implicit
	Ratio of weighted density of population centers to highest density in the same MSA	Not included
Density Factor	Gross population density in persons per square miles (PSM)	(3.4)
	Percentage of population living at density < 1500 PSM	Implicit
	Percentage of population living at density > 12,500 PSM	Implicit

Table 3.1– Continued

Measures of sprawl	Ewing et al. [1]	Interpretation
Density Factor	Estimated density at the center of the metro area derived from negative exponential density function	Assumed to be the census tract which is at the center
	Gross population density of urban lands	Implicit
	Weighted average lot size in square feet for single family dwellings	Implicit
	Weighted density of all population centers (local density maxima) within a metro area	(3.6)
Mix Factor	Percentage of residents with businesses within certain blocks of their homes	Implicit
	Percentage of residents with satisfactory neighbourhood shopping within 1 mile	Implicit
	Percentage of residents with schools within 1 mile	Implicit
	Job-resident balance	Implicit
	Population-serving job mix	Implicit
	Population serving job resident balance	(3.10)
Streets Factors	Approximate average block length in urbanized portion of the metro	Pre-determined

Table 3.1– Continued

Measures of sprawl	Ewing et al. [1]	Interpretation
Streets Factors	Average block size in square miles (excluding blocks > 1 square mile)	Pre-determined
	Percentage of small blocks (< 0.01 square mile)	Pre-determined

CHAPTER 4

Algorithms and Results

Due to the enormity of the scope of the research problem, we present a Benders decomposition approach to solving the given research problem. Since obtaining a globally optimal solution in a limited time is not feasible yet, the focus is to provide relevant information to the planner so that he/she may choose a solution from a given set of solutions, which may satisfy the concrete and abstract set of requirements.

The data-set used in the experiments was provided by the Urban planning department at The University of Texas at Arlington. The data-set is for the city of Leander, Texas. It has 7632 land pieces, each with a size of 150 feet by 150 feet. The suitability factors for each land piece were provided for eight different categories. This results in a total of 8^{7632} possible land use assignments.

The number of variables in the current sprawl formulation is $2 \times |N| \times |C| + |CT|^2 + \sum_{i \in N} \sum_{\hat{i} \in N_i} n_{i\hat{i}} \times |C|^2 + |CT|$. For a 7632 land piece problem with 8 land use categories (divided into 5 census tracts), the number of variables is roughly 4 million, and the number of rows exceeds 10^{10}. Since such a large matrix cannot be solved by a commercial solver (e.g. CPLEX), and due to the large number of quadratic variable constraints, a Benders decomposition method was chosen to solve this assignment problem.

4.1   Benders Decomposition applied to the MILP

There are a number of reasons for choosing Benders decomposition to solve the current problem.

As stated previously, the MILP rapidly becomes too large for CPLEX to handle as the number of land pieces increase, especially when the variables and constraints associated with land mixed use (3.10), (3.11) and (3.20)-(3.22) are included. For each neighbor of a land piece i with assigned land use j, we have a corresponding quadratic variable for the neighboring land piece $\hat{i}$ with assigned land use $\hat{j}$. Hence, for each neighboring land piece, we have 64 quadratic variables, and each quadratic variable has 3 constraints linking $x_{ij\hat{i}\hat{j}}$, x_{ij}, and $x_{\hat{i}\hat{j}}$.

Since the relationship between the quadratic variables $x_{ij\hat{i}\hat{j}}$ and x_{ij} is clearly defined, the solution to the primal subproblem is already known. The solution for the primal subproblem is then used to construct the dual objective function without pushing the subproblem model into CPLEX. The dual objective function is constructed using duality theory from linear programming.

A large number of constraints have to have their bounds specified manually by the planner. In the beginning, the planner would have difficulty ascertaining a reasonable range on the upper and lower bounds on the constraints. If the bounds are too tight, it would run into infeasibility. If the planner starts with very relaxed bounds, it would take a significant amount of time to determine a feasible set of bounds that would adhere to the planner's requirements and minimize sprawl. To overcome this problem, the subproblem was constructed so as to minimize the violation of the bounds on land mixed use constraints.

A central composite design is used to design the experiment so that the results may help the planner decide what the bounds on various constraints should be. The three factors chosen for the design are the gross population density, the density gradient across census tracts, and the lower bound on commercial density in the central business district.

A central composite design is used to design the experiment so that the results may lead the planner to decide what the optimal bounds on various constraints should be. The three factors chosen for the design are:

- *Gross Population Density*: Gross population density directly affects an optimal land use assignment. With higher population density bounds, categories with higher density are chosen.

- *Density Gradient between Census Tracts*: Gross population density however does not control how the population density is spread across the planning area. It only accounts for the total density. Hence, the density gradient across census tracts is chosen to control how the density should be distributed. Higher density gradient bounds means the planning area becomes more monocentric (i.e. high population density in the middle and starts falling off as we move outwards).

- *Lower Bound on Commercial Activity for CBD*: The lower bound on the commercial activity that decides which central business district would be selected is considered to control how dense the commercial area at the center of the planning area can be. This factor only has a lower bound and no upper bound. Hence, we only increase the values for the lower bound over its default value from the relaxed MILP.

There were two methods that were explored in using Benders decomposition, which are as follows:

- *Single Benders Cut*: A single optimal solution is generated from the master problem, and that solution is used to generate the Benders cut.

- *Multiple Benders Cuts*: We obtained multiple solutions from solving the master problem by using the CPLEX solution pool property. Given certain parameters on time limit and MIP gap, CPEX generated a pool of solutions that were then

used to generate multiple Benders cuts. The termination criteria was evaluated for all the cuts that were generated.

4.1.1 Variables for Master Problem and Subproblem

The variables for the master problem and subproblem are given as follows:

- θ = The upper bound on the subproblem objective function.
- S_i^L = Violation of the lower bound L_{Mix} for land piece $i \in N$.
- S_i^U = Violation of the upper bound U_{Mix} for land piece $i \in N$.
- $\bar{x}$ = Fixed value of the variable from the master problem solution.
- π_i^L = Dual variable corresponding to constraints (4.5).
- π_i^U = Dual variable corresponding to constraints (4.6).
- μ_{ij}^I = Dual variable corresponding to constraints (4.7).
- $\mu_{i\hat{j}}^{II}$ = Dual variable corresponding to constraints (4.8).
- $\mu_{ij\hat{i}\hat{j}}^{III}$ = Dual variable corresponding to constraints (4.9).

The complicating variables in sprawl formulation are the quadratic variables in the land mixed use constraints. Hence, the constraints from the original MILP that are handled in the subproblem are (3.20)–(3.22), (3.10), and (3.11).

4.1.2 Master Problem

The master problem now becomes the following:

$$\text{max} \qquad z_{LP} = \sum_{i \in N} \sum_{j \in C} S_{ij} x_{ij} + \theta \qquad (4.1)$$

subject to: (3.2) - (3.9), (3.12) (3.19)

$$\theta \leq -L \sum_{i \in N} \bar{\pi}_i^L + U \sum_{i \in N} \bar{\pi}_i^U + \sum_{i \in N} \sum_{j \in C} \bar{\mu}_{ij}^I x_{ij} + \sum_{\hat{i} \in N} \sum_{\hat{j} \in C} \bar{\mu}_{\hat{i}\hat{j}}^{II} x_{\hat{i}\hat{j}} \qquad (4.2)$$

$$+ \bar{\mu}_{ij\hat{i}\hat{j}}^{III} \cdot (1 - \sum_{\hat{i} \in N} \sum_{\hat{j} \in C} x_{\hat{i}\hat{j}} - \sum_{i \in N} \sum_{j \in C} x_{ij})$$

$$\forall i, \hat{i} \in N; i > \hat{i}; j, \hat{j} \in C$$

$$\theta \text{ is free}$$

4.1.3 Primal Subproblem

The formulation of the primal subproblem is given below:

$$\max \quad -\sum_{i \in N} S_i^L - \sum_{i \in N} S_i^U \tag{4.3}$$

subject to:

$$\omega_{ij\hat{i}\hat{j}} = \left(\frac{x_{ij\hat{i}\hat{j}}(S_{ij} + S_{\hat{i}\hat{j}})AF_{j\hat{j}}}{d_{i\hat{i}}} \right) \tag{4.4}$$

$$-S_i^L - \sum_{j \in C} \sum_{\hat{i} \in N_i} \sum_{\hat{j} \in C} \omega_{ij\hat{i}\hat{j}} x_{ij\hat{i}\hat{j}} \leq -L \qquad \forall i \in N \tag{4.5}$$

$$-S_i^U + \sum_{j \in C} \sum_{\hat{i} \in N_i} \sum_{\hat{j} \in C} \omega_{ij\hat{i}\hat{j}} x_{ij\hat{i}\hat{j}} \leq U \qquad \forall i \in N \tag{4.6}$$

$$x_{ij\hat{i}\hat{j}} \leq \bar{x}_{ij} \qquad \forall i, \hat{i} \in N; i > \hat{i}; j, \hat{j} \in C \tag{4.7}$$

$$x_{ij\hat{i}\hat{j}} \leq \bar{x}_{\hat{i}\hat{j}} \tag{4.8}$$

$$-x_{ij\hat{i}\hat{j}} \leq 1 - \bar{x}_{ij} - \bar{x}_{\hat{i}\hat{j}} \tag{4.9}$$

$$x_{ij\hat{i}\hat{j}}, S_i^L, S_i^U \geq 0 \qquad \forall i, \hat{i} \in N; i > \hat{i}; j, \hat{j} \in C$$

Observe that with a solution from the master problem $\bar{x}$, the primal subproblem yields solutions with integer values for x.

4.1.4 Dual Subproblem

The formulation of the dual subproblem is given below:

$$
\min \quad -L \sum_{i \in N} \pi_i^L + U \sum_{i \in N} \pi_i^U + \sum_{i \in N} \sum_{j \in C} \mu_{ij}^I \bar{x}_{ij} \tag{4.10}
$$

$$
+ \sum_{\hat{i} \in N} \sum_{\hat{j} \in C} \mu_{\hat{i}\hat{j}}^{II} \bar{x}_{\hat{i}\hat{j}} + \mu_{ij\hat{i}\hat{j}}^{III} \cdot (1 - \sum_{\hat{i} \in N} \sum_{\hat{j} \in C} \bar{x}_{\hat{i}\hat{j}} - \sum_{i \in N} \sum_{j \in C} \bar{x}_{ij})
$$

subject to:

$$
\pi_i^L \leq 1 \qquad\qquad\qquad \forall i \in N
$$

$$
\tag{4.11}
$$

$$
\pi_i^U \leq 1 \qquad\qquad\qquad \forall i \in N
$$

$$
\tag{4.12}
$$

$$
\omega_{ij\hat{i}\hat{j}} \cdot (\sum_{\hat{i} \in N_i} \pi_{\hat{i}}^U - \sum_{\hat{i} \in N_i} \pi_{\hat{i}}^L) + \mu_{ij}^I + \mu_{\hat{i}\hat{j}}^{II} - \mu_{ij\hat{i}\hat{j}}^{III} \geq 0 \tag{4.13}
$$

$$
\pi_i^U, \pi_i^L, \mu_{ij}^I, \mu_{\hat{i}\hat{j}}^{II}, \mu_{ij\hat{i}\hat{j}}^{III} \geq 0 \qquad\qquad \forall i \in N, \hat{i} \in N_i, j, \hat{j} \in C
$$

4.1.5 Benders Decomposition Algorithm

The data points are all the various combinations of values of the three factors, which are gross population density, density gradient, and the lower bound on commercial activity for CBD as shown in Table (4.1).

Data: Suitability Factors for the land area, census tract and relaxed
   parameter values

Result: Land use objective and land use mix violations for all data points

Initialization: Create vector of non-zero values for coefficients of quadratic
variables in the subproblem.;

while *For all the data points* **do**

    if *1st pass* **then**

        Use the relaxed bounds;

    else

        Use the data points created from solving relaxed bounds;

    end

    STOP = FALSE;

    Set $\theta = 0$;

    while *STOP = FALSE* **do**

        Solve the master problem (4.1.2);

        From the optimal solution/solution pool of the master problem,
   generate single/multiple Benders cuts by solving the subproblem
   (4.1.4);

        if *Violation = 0 OR θ = Violation* **then**

            STOP = TRUE;

        else

            Send the cut to master problem;

        end

    end

end

Algorithm 1: Single/Multiple Benders Cut

4.1.6 Solving the Subproblem

The quadratic variable $x_{ij\hat{i}\hat{j}}$ is dependent on two binary variables, x_{ij} and $x_{\hat{i}\hat{j}}$. There are only 4 possible combinations for the two binary variables. The values of assignment vector x_{ij} come from solving the master problem (4.1.2). Using properties of duality theory and the solution from the primal problem, we came up with the following solution for the dual subproblem (4.1.4). The cut as calculated below is unique. Although during our experiments, any other combinations of the values still yielded the same values on the coefficients of the cut. All the 4 cases are listed as follows:

- **Case I**: If $x_{ij} = 0$ and $x_{\hat{i}\hat{j}} = 0$, then

$$\mu_{ij}^{I} = -\min(\omega_{ij\hat{i}\hat{j}}(\textstyle\sum_{\hat{i}\in N_i} \pi_{\hat{i}}^{U} - \sum_{\hat{i}\in N_i} \pi_{\hat{i}}^{L}), 0), \quad \mu_{\hat{i}\hat{j}}^{II} = 0, \quad \mu_{ij\hat{i}\hat{j}}^{III} = 0.$$

- **Case II**: If $x_{ij} = 0$ and $x_{\hat{i}\hat{j}} = 1$, then

$$\mu_{ij}^{I} = 0, \quad \mu_{\hat{i}\hat{j}}^{II} = -\min(\omega_{ij\hat{i}\hat{j}}(\textstyle\sum_{\hat{i}\in N_i} \pi_{\hat{i}}^{U} - \sum_{\hat{i}\in N_i} \pi_{\hat{i}}^{L}), 0), \quad \mu_{ij\hat{i}\hat{j}}^{III} = 0.$$

- **Case III**: If $x_{ij} = 1$ and $x_{\hat{i}\hat{j}} = 0$, then

$$\mu_{ij}^{I} = -\min(\omega_{ij\hat{i}\hat{j}}(\textstyle\sum_{\hat{i}\in N_i} \pi_{\hat{i}}^{U} - \sum_{\hat{i}\in N_i} \pi_{\hat{i}}^{L}), 0), \quad \mu_{\hat{i}\hat{j}}^{II} = 0, \quad \mu_{ij\hat{i}\hat{j}}^{III} = 0.$$

- **Case IV**: If $x_{ij} = 1$ and $x_{\hat{i}\hat{j}} = 1$, then

$$\mu_{ij}^{I} = -\min(\omega_{ij\hat{i}\hat{j}}(\textstyle\sum_{\hat{i}\in N_i} \pi_{\hat{i}}^{U} - \sum_{\hat{i}\in N_i} \pi_{\hat{i}}^{L}), 0), \quad \mu_{\hat{i}\hat{j}}^{II} = 0,$$
$$\mu_{ij\hat{i}\hat{j}}^{III} = \max(\omega_{ij\hat{i}\hat{j}}\pi, 0).$$

4.2 Experimental Setup

The goals of the experiment are to create an efficient frontier between land use objective and land mixed use violation and to obtain the best possible solution in a limited time.

Initially, a problem with very relaxed bounds on these three factors is solved. Only the land mixed use factor is limited. Once we obtain an optimal solution with

relaxed bounds, the values of the factors described above are calculated. Based on these values, the central composite design is constructed. The variation on the values were obtained by running multiple experiments on a 400 land piece problem (a subset of the Leander, TX data-set) to determine which changes would significantly affect the objective value of the problem without leading to infeasibility.

Characteristics of the experimental setup are as follows:

- *Time limit*: The time limit on the CPLEX optimizer for the master problem is 30 minutes, and the time limit on Benders algorithm overall is 5 hours.

- *MIP Emphasis*: The parameter in CPLEX for MIP emphasis was set to feasibility instead of optimality for the master problem.

- *Termination criteria*: If the subproblem objective value i.e the violation of the land mixed use constraints is 0 or the gap between subproblem objective value and master problem parameter is less then 0.1, then the Benders decomposition algorithm terminates.

- *Solution pool limit*: The maximum total number of solutions allowed in the master problem solution pool is 5.

- *Maximum iterations for Benders algorithm*: The maximum number of iterations allowed for Benders algorithm was set to 5000. However, in practice, this termination criteria was never met, and the maximum number observed was 200.

- *Design for observations*: An orthogonal 3 factorial design was used to collect observations and form the scatter plot as shown in figure 4.1.

- *Solution pool population parameter*: The MIP gap allowed for a solution to be a part of the master problem solution pool is 0.1%.

- *Candidates for CBD*: The number of candidates to be evaluated for the CBD is 5. All of the candidates form a concentric circle with increasing radius. The

center of the concentric circles is the land piece chosen by the planner, which becomes the origin for all the candidates for CBD.

- *Number of neighbors*: The number of blocks from a central land piece inside which all the surrounding land pieces are considered to be its neighbors is 2. If the number of blocks under consideration is larger, the problem size increases rapidly.

- *Bounds on land mixed use*: The lower and upper bound for land mixed use is set to be 2 and 3 respectively. The land mixed use values should not be either too high or too low so as to strike a balance between various land uses present within the sphere of influence.

- *Bounds on land use categories*: The lower and upper bounds for land use categories is set to be 0 % and 100 % respectively.

- *Changes in factors for the orthogonal design*: Gross population density was varied by 45 people per square mile. Density gradient was varied by 20 units (it is a unit-less measure). Lower bound for the CBD was varied by 1.2 units (level of commercial activity is a unit-less measure as well).

- *Upper and lower bounds for factors in orthogonal design*: Given the value of gross population density for the planning area from solving the relaxed model, the upper and lower bound is obtained by adding and subtracting 5% to it respectively. For density gradient, the upper and lower bound were obtained by adding and subtracting 10 units from the average gradient value.

4.3 Results

Table (4.1, 4.2, 4.3) has $3^3 = 27$ data points, which is as a result of all possible unique combinations of three factors at three different levels. 0 denotes the default value as calculated in a relaxed MILP. + and − denotes the increase and decrease of the values of the factors from their respective default values. ++ denotes twice the increase over the default. 0 denotes that the master problem was infeasible.

For changes in gross population density (GPD) in figure (4.1), we observe that an increase in GPD results in a decrease of the land use objective value but an increase in the land mixed use violations. By contrast, a decrease in GPD results only in a slightly worse land use objective value but a large increase in mixed use violations compared to the increase in GPD. This behavior leads us to conclude that GPD bounds should be higher than the relaxed value if it should be changed. Also, given the variation in GPD was 45 people per sq. mile, we conclude that smaller changes would have less effect on the values of land use objective value and mixed use violations.

For changes in density gradient between census tracts (DG) in figure (4.1), we observe that decrease in DG results in a large decrease in the land use objective value and an increase in land mixed use violations. A decrease in values for the bounds on DG results in only a slight increase in land mixed use violations but a relatively larger decrease in the land use objective value. This leads us to conclude that the MIP is very sensitive to the values of DG bounds, and hence, should be not changed. But once the decision to change the bounds has been taken, it does not make a very large difference whether the values of the bounds go up or down.

As we observe from Table (4.1, 4.2, 4.3), changes in the lower bound on commercial activity of the CBD does not affect the land use objective value or the land

mixed use violations. We conclude then that the planner can significantly increase the lower bound L_{CBD}.

From Table (4.1, 4.2), we observe that a simultaneous decrease in values for the bounds on GPD and an increase in values for the bounds on DG leads to infeasibility in the model. Furthermore, from Table (4.3), we observe that all of the infeasible points occur because of changes in DG.
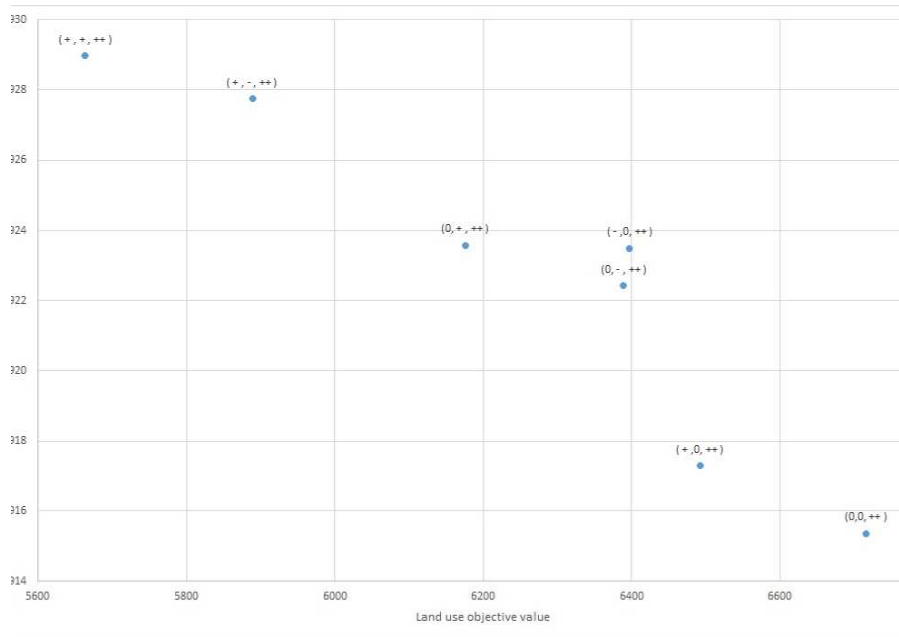

Figure 4.1. Scatter plot between Land Use Objective value and Land Mixed Use Violations from Table 4.1.

Table 4.1. Result from 2500 land pieces data set with single Benders cut

Data Point	Land use objective	Violation	Gross population density	Density Gradient	L_{CBD}
1	6716	4915.36	0	0	0
2	6716	4915.36	0	0	+
3	6716	4915.36	0	0	++
4	6176	4923.58	0	+	0
5	6176	4923.58	0	+	+
6	6176	4923.58	0	+	++
7	6389	4922.42	0	-	0
8	6389	4922.46	0	-	+
9	6389	4922.43	0	-	++
10	6492	4917.3	+	0	0
11	6492	4917.3	+	0	+
12	6492	4917.3	+	0	++
13	5664	4928.97	+	+	0
14	5664	4928.97	+	+	+
15	5664	4928.97	+	+	++
16	5889	4927.75	+	-	0
17	5889	4927.75	+	-	+
18	5889	4927.75	+	-	++
19	6397	4923.49	-	0	0
20	6397	4923.49	-	0	+
21	6397	4923.49	-	0	++
22	Infeasible		-	+	0
23	Infeasible		-	+	+
24	Infeasible		-	+	++
25	Infeasible		-	-	0
26	Infeasible		-	-	+
27	Infeasible		-	-	++

Table 4.2. Result from 2500 land pieces data set with multiple Benders cut

Data Point	Land use objective	Violation	Gross population density	Density Gradient	L_{CBD}
1	6716	4912.82	0	0	0
2	6716	4912.82	0	0	+
3	6716	4912.86	0	0	++
4	6181	4923.48	0	+	0
5	6182	4923.50	0	+	+
6	6181	4923.48	0	+	++
7	6384	4922.37	0	-	0
8	6384	4922.37	0	-	+
9	6384	4922.37	0	-	++
10	6484	4917.16	+	0	0
11	6484	4917.16	+	0	+
12	6484	4917.17	+	0	++
13	5671	4929.745	+	+	0
14	5671	4929.745	+	+	+
15	5671	4929.745	+	+	++
16	5887	4928.29	+	-	0
17	5887	4928.29	+	-	+
18	5887	4928.29	+	-	++
19	6398	4923.53	-	0	0
20	6398	4923.53	-	0	+
21	6398	4923.53	-	0	++
22	Infeasible		-	+	0
23	Infeasible		-	+	+
24	Infeasible		-	+	++
25	Infeasible		-	-	0
26	Infeasible		-	-	+
27	Infeasible		-	-	++

Table 4.3. Result from 7600 land pieces data set with single Benders cut

Data Point	Land use objective	Violation	Gross population density	Density Gradient	L_{CBD}
1	24733	15264	0	0	0
2	24733	15264	0	0	+
3	24733	15264	0	0	++
4	Infeasible		0	+	0
5	Infeasible		0	+	+
6	Infeasible		0	+	++
7	Infeasible		0	-	0
8	Infeasible		0	-	+
9	Infeasible		0	-	++
10	23837	15264	+	0	0
11	23836	15264	+	0	+
12	23837	15264	+	0	++
13	Infeasible		+	+	0
14	Infeasible		+	+	+
15	Infeasible		+	+	++
16	Infeasible		+	-	0
17	Infeasible		+	-	+
18	Infeasible		+	-	++
19	17120	15264	-	0	0
20	17120	15264	-	0	+
21	17120	15264	-	0	++
22	Infeasible		-	+	0
23	Infeasible		-	+	+
24	Infeasible		-	+	++
25	Infeasible		-	-	0
26	Infeasible		-	-	+
27	Infeasible		-	-	++

CHAPTER 5

Future Research

To discuss areas of future research, we first have to understand the limitations of the current research.

5.1   Limitations

In our experiments, the time taken to solve the master problem was negligible, whereas the time to generate the Benders cut was very large. For example, for the 2500 dataset, the time taken to solve the master problem was 0 minutes, whereas the time taken to generate a single Benders cut was close to 200 minutes. The reason for the large time consumption is that while solving the subproblem, we generate the violations, create the dual subproblem objective coefficients, and then recombine them to form Benders cut. In all of these steps, the index for the variables depend on four dimensions, which are land piece i, land piece $\hat{i}$, land use category j and land use category $\hat{j}$. Given the large matrix for the subproblem, the time consumed while searching for the index of the location of the variable values is quite large. Since all of the coding was done in the C programming language, which has very limited libraries available for optimizing index search, it is impossible to reduce the time to solve the subproblem without writing an optimized search algorithm for indexing or implementation of map libraries from C++/C#.

There are a number of factors that affect the land use objective value. Even using 3 factors over 3 different levels yields 27 different observation points. Hence, as the number of factors and levels increase, the number of observation points increase

exponentially. Thus, given the amount of time it takes to solve the subproblem, there is a virtual limit on the number of factors that can be incorporated into the experiment.

Given the multi-objective nature of the problem, it is very difficult to formulate a model that would give the planner a single solution that satisfies all of the requirements. Hence, there is no single decision making framework that does not require a lot of input from the planner.

Land use mix factor is only computed over a limited area around a land piece. As the number of neighbors increases, the number of quadratic variables also increases rapidly. Given our current algorithm, it would take a substantially long time to solve a single iteration if a certain land piece is influenced by every other land piece in the planning area. Hence, a limit to the number of neighbors of a land piece affects our ability to produce a more holistic land use solution.

There is an interaction between gross population density and density gradient between census tracts. The effects of that interaction are more pronounced when the gross population density reaches either extremes for the planning area. For example, if the gross population density of the planning area is bounded to be as low as possible, then it is not possible for there to be a steep density gradient between different census tracts, whereas if the gross population density is bounded at an average value, then the census tracts are allowed have sharply varying densities.

5.2   Future Research

As stated above, the time taken to search over the four dimensions is very long. One solution is to form a sparse four dimensional matrix but that would be very expensive memory-wise. Hence, the goal is to find a way to calculate the values which is efficient with respect to both computations and memory. It would reduce

the time to generate the Benders cut. This would also enable the inclusion of more quadratic variables.

As seen from figure 4.1, there is some interaction between gross population density and density gradient, and the effect on the land use objective value from the lower bound on the CBD is not significant. Hence, in future research, the various constraints have to be studied to isolate quasi-independent factors that can then be used in the orthogonal design.

5.3   Summary and Conclusion

Urban sprawl is genuine problem in all the major cities of the world. Controlling urban sprawl would make the cities sustainable and a pleasant place to live. Given the various sprawl factors defined by Ewing et al. [1], we formulated a mixed integer linear programming (MILP) model for urban land use assignment with the focus on controlling urban sprawl. The MILP model was then solved using Benders decomposition. The subproblem was solved using a deterministic method that employed properties from duality theory instead of solving it using a commercial solver. Since the problem has a number of factors affecting urban sprawl, the sprawl constraints were introduced as bounds instead of putting them in the objective function. We then create a scatter plot between land use objective values and land mixed use violations to assist the planner in analyzing the effects of certain factors like gross population density, density gradient between census tracts, and the lower bound on the commercial activity for assigning a CBD on the land use assignment.

In conclusion, the scatter plot would allow the planner to analyze the effects of various factors on the land use objective value and land mixed use violations. This would assist the planner in determining what the best assignment for the given land area should be. Although there are limitations to the decision framework presented

63

above, we believe that in the future, incorporating more factors in the orthogonal design, and a more efficient procedure for solving the subproblem would make this framework significantly helpful for an urban planner who wishes to focus on controlling urban sprawl.

REFERENCES

[1] R. Ewing, R. Pendall, and D. Chen, "Measuring sprawl and its impact: Technical report," Smart Growth America, Washington DC, Tech. Rep., 2002.

[2] A. J. Catanese and J. C. Snyder, Eds., *Introduction to Urban Planning*. New York: McGraw-Hill, 1979.

[3] M. G. Collins, F. R. Steiner, and M. J. Rushman, "Land-use suitability analysis in the United States: historical development and promising technological achievements." *Environmental Management*, vol. 28, no. 5, pp. 611–21, Nov. 2001.

[4] C. Steinitz, P. Parker, and L. Jordan, "Hand drawn overlays: their history and prospective uses," *Landscape Architecture*, vol. 66, no. 5, pp. 444–455, 1976.

[5] D. Moreno and M. Seigel, "A gis approach for corridor siting and environmental impact analysis," in *GIS/LIS'88 Proceedings from the Third Annual International Conference*, San Antonio, Texas, 1988, pp. 507–514.

[6] *Land-use Controls in the United States*. MIT Press, 1962.

[7] S. B. J. Warner, *Streetcar Suburbs: The Process of Growth in Boston*, 2nd ed. Cambridge, MA: Harvard University Press, 1978.

[8] J. R. Stilgoe, *Borderland: Origins of the American Suburb*. Yale University Press, 1988.

[9] F. K. Benfield, D. R. Matthew, and D. D. T. Chen, "Once there were greenfields: How urban sprawl is undermining america's environment, economy, and social fabric," Natural Resources Defense Council with Surface Transportation Policy Project, New York, Tech. Rep., 1999.

[10] O. D. of Land Conservation and Development, "Indicators of urban sprawl," 1992. [Online]. Available: www.uoregon.edu/p̃ppm/landuse/sprawl

[11] K. Breunig, "Losing ground: At what cost? changes in land use and their impact on habitat, biodiversity, and ecosystem services in massachusetts," MA: Mass Audubon, Tech. Rep., 2003.

[12] U. S. Bureau of Census, "American community survey," 2010. [Online]. Available: www.census.gov

[13] D. C. Soule, Ed., *Urban Sprawl: A Comprehensive Reference Guide*. Greenwood Press, 2006.

[14] "Farming on the edge," American Farmland Trust, Tech. Rep., 1998.

[15] R. Jackson and C. Kochtitzky, "Creating a healthy environment: The impact of the built environment on public health," Sprawl Watch Clearinghouse, Washington DC, Tech. Rep., 2001.

[16] R. Dunphy and K. Fisher, "Transportation congestion, and density: New insights," Transportation Research Board, Tech. Rep. Record no. 1552, 1996.

[17] D. Howlett, "Study finds traffic congestion bad and getting worse," USA Today, October 1 2003.

[18] J. B. Stephenson, "The federal government could help communities better plan for transportation that protects air quality," Testimony before the Committee on environmental and public works, 2002, u.S. Senate.

[19] "Greenhouse gas emissions from the u.s. transportation sector," U.S. Environmental Protection Agency, Tech. Rep., March 2006.

[20] I. P. on Climate Change, "Second assessment report: Summary for policy makers," New York, Tech. Rep., 1995.

[21] M.-A. R. Council, "The facts about lawn chemicals," 2004. [Online]. Available: www.marc.org

[22] T. Doggett, "Suburban sprawl blocks water, worsens u.s. drought," Reuters News Service, August 29.

[23] J. Lowy, "Predator attacks escalate as americans encroach on wildlife habitat," National Geographic News, August 27 2001.

[24] "Final report of the new jersey comparative risk project," New Jersey department of environmental protection, Trenton, NJ, Tech. Rep., March 2003.

[25] D. Pringle, "Environmentalists launch anti-sprawl blitz," Garden state clean water action, 2002.

[26] M. Fox, "Urban sprawl makes americans fat," Reuters News Service, August 29 2003.

[27] R. Burchell, "The costs of sprawl-revisited," National Academy Press, Washington DC, Tech. Rep., 1998.

[28] ——, "The costs of sprawl-2000," National Academy Press, Washington DC, Tech. Rep., 2002.

[29] J. M. C. Pereira and L. Duckstein, "A multiple criteria decision making approach to gis-based land suitability evaluation," *International Journal of Geographical Information Systems*, vol. 7, no. 5, pp. 407–424, 1993.

[30] R. Store and J. Kangas, "Integrating spatial multi-criteria evaluation and expert knowledge for gis-based habitat suitability modelling," *Landscape and Urban Planning*, vol. 55, no. 2, pp. 79–93, 2001.

[31] *Geographic Information Systems for Geoscientists: Modeling with GIS.* Pergamon Press, 1994.

[32] J. C. Cambell, J. Radke, J. T. Gless, and R. M. Whirtshafter, "An application of linear programming and geographic information systems: Cropland allocation in antigue," *Environment and Planning*, vol. A 24, pp. 535–549, 1992.

[33] S. Kalogirou, "Expert systems and gis: an application of land suitability evaluation," *Computers, Environment and Urban Systems*, vol. 26, no. 2-3, pp. 89–112, 2002.

[34] *GIS and Decision Making*. Geneva: UNITAR, 1993.

[35] R. L. Church, "Geographical information systems and location science," *Computers and Operations Research*, vol. 29, no. 6, pp. 541–562, 2002.

[36] *Multicriteria Analysis and Geographical Information Systems: An Application to Agricultural Land Use in Netherlands*. Kluwer Academic Publishers, 1990.

[37] J. E. Moore II and P. Gordon, "A sequential programming model of urban land development," *Socio-economic Planning Sciences*, vol. 24, no. 3, pp. 199–216, 1990.

[38] M. Los, "Simultaneous optimization of land use and transportation," *Regional Sciences and Urban Economics*, vol. 8, pp. 21–42, 1978.

[39] L. Q. Ouyang and W. H. K. Lam, "An activity based land use and transportation optimization model," *Journal of the Eastern Asia Society for Transportation Studies*, vol. 8, 2009.

[40] A. Vold, "Optimal land use and transport planning for the greater oslo area," *Transportation Research Part A*, vol. 39, pp. 548–565, 2005.

[41] J. E. Moore II and T. J. Kim, "Mill's urban system models: Perspective and template for lute (land use/transport/environment) applications," in *7th World Transportation Conference SIG-1*, Sydney, NSW, Australia, 1995.

[42] *Studies in the Economics of Transportation*. Yale University Press, 1956.

[43] T. C. Koopmans and M. Beckmann, "Assignment problems and the location of economic activities," *Econometrica*, vol. 25, pp. 23–76, 1957.

[44] E. S. Mills, "Markets and efficient resource allocation in urban areas," *Swedish Journal of Economics*, vol. 74, pp. 100–113, 1972.

[45] A. L. Zielinska, R. Church, and P. Jankowski, "Sustainable urban land use allocation with spatial optimization," in *8th ICA Workshop on Generalisation and Multiple Representation*, 7-8 July 2005.

[46] D. P. Ward, A. T. Murray, and S. R. Phinn, "An optimized cellular automata approach for sustainable urban development in rapidly urbanizing regions," in *Proceedings of the 4th International Conference on GeoComputation*, 1999.

[47] ——, "Integrating spatial optimization and cellular automata for evaluating urban change," *The Annals of Regional Science*, vol. 37, pp. 131–148, 2003.

[48] J. Malczewski, "Gis-based land-use suitability analysis: A critical review," *Progress in Planning*, vol. 62, pp. 3–65, 2004.

[49] *GIS and Multicriteria Decision Analysis*. New York: Wiley, 1999.

[50] J. Aerts, "Spatial decision support for resource allocation: Integration of optimization, uncertainty analysis and visualization techniques," Ph.D. dissertation, Faculty of Science, University of Amsterdam, 2002.

[51] J. T. Diamond and J. R. Wright, "Design of an integrated spatial information system for multiobjective land use planning," *Environment and Planning*, vol. B 15, no. 2, pp. 205–214, 1988.

[52] H. Jiang and J. R. Eastman, "Application of fuzzy measures in multi-criteria evaluation in gis," *International Journal of Geographic Information Science*, vol. 14, pp. 173–184, 2000.

[53] S. J. Carver, "Integrating multi-criteria evaluation with geographical information systems," *International Journal of Geographic Information Systems*, vol. 5, no. 3, pp. 321–339, 1991.

[54] *Spatial Evolutionary Modeling*. Oxford: Oxford Press, 2001.

[55] D. Z. Sui, "Integrating neural networks with gis for spatial decision making," *Operational Geographer*, vol. 11, no. 2, pp. 13–20, 1993.

[56] M. Batty, Y. Xie, and Z. Sun, "From cells to cities," *Environment and Planning*, vol. B 21, pp. 31–48, 1994.

[57] F. Wang, "Improving remote sensing image analysis through fuzzy information representation," *Photogrammetric engineering and remote sensing*, vol. 56, no. 8, pp. 1163–1168, 1990.

[58] R. Banai, "Fuzziness in geographic information systems: Contributions from the analytic hierarchy process," *International Journal of Geographical Information Systems*, vol. 7, pp. 315–329, 1993.

[59] J. Zhou and D. L. Civco, "Using genetic learning neural networks for spatial decision making in gis," *Photogrammetric Engineering and Remote Sensing*, vol. 11, pp. 1287–1295, 1996.

[60] R. H. Gimblett, G. L. Ball, and A. W. Guise, "Autonomous rule generation and assessment for complex spatial modeling," *Landscape and Urban Planning*, vol. 30, pp. 13–26, 1994.

[61] C. J. Brookes, "A parameterized region-growing programme for site allocation on raster suitability maps," *International Journal of Geographic Information Science*, vol. 11, pp. 375–396, 1997.

[62] D. Fotakis and E. Sidiropoulos, "A new multi-objective self-organizing optimization algorithm (mosoa) for spatial optimization problem," *Applied Mathematics and Computation*, vol. 218, no. 9, pp. 5168–5180, 2012.

[63] A. Holzkamper and R. Seppelt, "A generic tool for optimising land-use patterns and landscape structures," *Environmental Modeling & Software*, vol. 22, pp. 1801–1804, 2007.

[64] K. B. Matthews, A. R. Sibbald, and S. Craw, "Implementation of a spatial decision support system for rural land use planning: Intergrating geographic

information system and environmental models with search and optimisation algorithms," *Computers and Electronics in Agriculture*, vol. 23, pp. 9–26, 1999.

[65] K. B. Matthews, K. Buchan, A. R. Sibbald, and S. Craw, "Combining deliberative and computer-based methods for multi-objective land-use planning," *Agricultural Systems*, vol. 87, pp. 18–37, 2006.

[66] S. M. Manson, "Agent based dynamic spatial simulation of land use/cover change in the yucatan peninsula," in *Fourth International Conference on Integrating GIS and Environmental Modeling*, Banff, Canada.

[67] N. Xiao, D. A. Bennett, and M. P. Armstrong, "Using evolutionary algorithms to generate alternatives for multiobjective site-search problems," *Environment and Planning*, vol. A 34, no. 4, pp. 639–656, 2002.

[68] S. A. Gabriel, J. A. Faria, and G. E. Moglen, "A multiobjective optimization approach to smart growth in land development," *Socio-economic Planning Sciences*, vol. 40, pp. 212–248, 2006.

[69] H. H. Zhang and L. Bian, "Simulating multi-objective spatial optimization allocation of land use based on the integration of multi-agent system and genetic algorithm," *International Journal of Environmental Research*, vol. 4, no. 4, pp. 765–776, 2010.

[70] J. Bos, "Zoning in forest management: A quadratic assignment problem solved by simulated annealing," *Journal of Environmental Management*, vol. 37, pp. 127–145, 1991.

[71] I. S. Riveira, R. C. Maseda, and D. M. Barros, "GIS-based planning support system for rural land-use allocation," *Computers and Electronics in Agriculture*, vol. 63, pp. 257–273, 2008.

[72] L. Xiaoli, Y. Chen, and L. Daoliang, "A spatial decision support system for land use structure optimization," *WSEAS Transactions on Computers*, vol. 8, no. 3, March 2009.

[73] K. Clarke and L. Gaydos, "Loose-coupling a cellular automaton model and gis: Long term urban growth prediction for san francisco and washington/baltimore," *International Journal of Geographical Information Science*, vol. 12, no. 7, pp. 699–714, 1998.

[74] F. Wu, "Simland: a prototype to simulate land conversion through the integrated gis and ca with ahp-derived transition rules," *International Journal of Geographic Information Science*, vol. 12, pp. 63–82, 1998.

[75] A. Ligtenberg, A. K. Bregt, and R. van Lammeren, "Multi-actor-based land use modelling: Spatial planning using agents," *Landscape and Urban Planning*, vol. 56, no. 1-2, pp. 21–33, 2001.

[76] M. Batty, Y. Xie, and Z. Sun, "Modeling urban dynamics through gis-based cellular automata," *Computers, Environment and Urban Systems*, vol. 23, pp. 205–233, 1999.

[77] R. D. Jonsson, "Analysing sustainability in a land use and transport system," *Journal of Transport Geography*, vol. 16, pp. 28–41, 2008.

[78] A. Anas and D. Pines, "Anti-sprawl policies in a system of congested cities," *Regional Science and urban Economics*, vol. 38, no. 5, pp. 408–423, 2008.

[79] F. Jiang, S. Liu, H. Yuan, and Q. Zhang, "Measuring urban sprawl in beijing with geo-spatial indices," *Journal of Geographical Sciences*, vol. 17, no. 4, pp. 469–478, 2007.

[80] X. Hongjie, "Toward a "compact settlement": A sustainable development way of settlements for chinese city," in *International Conference on Management and Service Science*, 2009.

[81] B. Saikia, "Urban sprawl and its periphery- a case study of guwahati city and its periphery in assam (in north-east india)," in *Society of Interdisciplinary Business Research (SIBR) 2011 Conference on Interdisciplinary Business Research*, 2011.

[82] S. Z. Shahraki, D. Sauri, P. Serra, S. Modugno, F. Seifolddini, and A. Pourahmad, "Urban sprawl pattern and land-use change detection in yazd, iran," *Habitat International*, vol. 35, no. 4, pp. 521–528, 2011.

[83] K. Muller, C. Steinmeir, and M. Kuchler, "Urban growth along motorways in switzerland," *Landscape and Urban Planning*, vol. 98, no. 1, pp. 3–12, 2010.

[84] X. Tong, X. Zhang, and M. Liu, "Detection of urban sprawl using a genetic algorithm-evolved artificial neural network classification in remote sensing: A case study in jiading and putuo districts of shanghai, china," *International Journal of Remote Sensing*, vol. 31, no. 6, pp. 1485–1504, 2010.

[85] X. J. Yu and C. N. Ng, "Spatial and temporal dynamics of urban sprawl and two urban-rural transects: A case study of guangzhou, china," *Landscape and Urban Planning*, vol. 79, no. 1, pp. 96–109, 2007.

[86] A. Q. Thompson and L. S. Prokopy, "Tracking urban sprawl: Using spatial data to inform farmland preservation policy," *Land Use Policy*, vol. 22, pp. 194–202, 2009.

[87] A. Anas and H. J. Rhee, "Curbing excess sprawl with congestion tolls and urban boundaries," *Regional Science and Urban Economics*, vol. 36, no. 4, pp. 510–541, 2006.

[88] H. S. Banzhaf and N. Lavery, "Can the land tax help curb urban sprawl? evidence from growth patterns in pennsylvania," *Journal of Urban Economics*, vol. 67, no. 2, pp. 169–179, 2010.

[89] S. Habibi and N. Asadi, "Causes, results and methods of controlling urban sprawl," *Procedia Engineering*, vol. 21, pp. 133–141, 2011.

[90] B. Stone Jr, "Urban sprawl and air quality in large us cities," *Journal of Environmental Management*, vol. 86, no. 4, pp. 688–698, 2008.

[91] F. Feitosa, Q. B. Le, and P. L. G. Vlek, "Multi-agent simulator for urban segregation (masus): A tool to explore alternatives for promoting inclusive cities," *Computers, Environment and Urban Systems*, vol. 35, no. 2, pp. 104–115, 2011.

[92] T. J. Stewart, R. Janssen, and M. V. Herwijnen, "A genetic algorithm approach to multiobjective land use planning," *Computers and Operations Research*, vol. 31, pp. 2293–2313, 2004.

[93] A. Ligmann-Zielinska and R. Church, "Spatial optimization as a generative technique for sustainable multiobjective land-use allocation," *International Journal of Geographical Information Science*, vol. 22, no. 6, pp. 601–622, 2008.

[94] G. Galster, R. Hanson, M. R. Ratcliffe, H. Wolman, S. Coleman, and J. Freihage, "Wrestling sprawl to the ground: Defining and measuring an elusive concept," *Housing Policy Debate*, vol. 12, no. 4, pp. 681–717, 2001.

[95] S. Malpezzi, "Estimates of the measurement and determinants of urban sprawl in u.s. metropolitan area," University of Wisconsin Center for Urban Land Economic Research, Tech. Rep. 99-06, 1999.

[96] G. B. Dantzig and P. Wolfe, "Decomposition principle for linear programs," *Operations Research*, vol. 8, no. 1, pp. 101–111, 1960.

[97] ——, "The decomposition algorithm for linear programs," *Econometrica*, vol. 29, no. 4, pp. 767–778, 1961.

[98] *Relaxation and Decomposition Methods for Mixed integer nonlinear programming.* Birkhauser Verlag, 2005.

[99] G. Adomian, "A Review of the Decomposition method in Applied Mathematics," *Journal of Mathematical Analysis and Applications*, vol. 135, no. 2, pp. 501–544, Nov. 1988.

[100] A. M. Geoffrion and G. W. Graves, "Multicommodity distribution system design by benders decomposition," *Management Science*, vol. 20, pp. 822–844, 1974.

[101] J.-F. Cordeau, F. Soumis, and J. Desrosiers, "A benders decomposition approach for the locomotive and car assignment problem," *Transportation Science*, vol. 34, pp. 133–149, 2000.

[102] J. Cordeau, F. Soumis, and J. Desrosiers, "Simultaneous assignment of locomotives and cars to passenger trains," *Operations Research*, vol. 49, pp. 531–548, 2001.

[103] M. Florian, G. Guerin, and G. Bushel, "The engine scheduling problem on a railway track," *INFOR*, vol. 14, pp. 121–128, 1976.

[104] X. Cai, D. C. McKinney, L. S. Lasdon, and D. W. Walkins Jr, "Solving large nonconvex water resources management models using generalized benders decomposition," *Operations Research*, vol. 49, pp. 235–245, 2001.

[105] J. Benders, "Partitioning procedures for solving mixed variables programming problems," *Numerische Mathematik*, vol. 4, pp. 238–252, 1962.

[106] M. Ghotboddini, M. Rabbani, and H. Rahimian, "A comprehensive dynamic cell formation design: Benders' decomposition approach," *Expert Systems with Applications*, vol. 28, pp. 2578–2488, 2011.

[107] B. Gendron, R. Garroppo, G. Nencioni, M. Scutella, and L. Tavanti, "Benders decomposition for a location-design problem in green wireless local area networks," *Electronic Notes in Discrete Mathematics*, vol. 41, pp. 367–374, 2013.

[108] L. Steinberg, "The backboard wiring problem: A placement algorithm," *SIAM Review*, vol. 3, pp. 37–50, 1961.

[109] R. E. Burkard and J. Offermann, "Entwurf von schreibmaschinentastaturen mittels quadratischer zuordnungsprobleme," *Z Oper. Res.*, vol. 21, pp. B121–B132, 1977.

[110] *Human Factors Engineering.* McGraw-Hill, 1970.

[111] D. R. Heffley, "Assigning runners to a relay team," *Optimal Strategies in Sports*, pp. 169–171, 1977.

[112] E. M. Loiola, N. M. M. de Abreu, P. O. Boaventura-Netto, P. Hahn, and T. Querido, "A survey for the quadratic assignment problem," *European Journal of Operational Research*, vol. 176, no. 2, pp. 657–690, 2007.

[113] K. Spiliopoulos and S. Sofianopoulou, "An optimal tree search method for the manufacturing systems cell formation problem," *European Journal of Operational Research*, vol. 105, no. 3, pp. 537–551, 1998.

[114] M. O. Ball, B. K. Kaku, and A. Vakhutinsky, "Network-based formulations of the quadratic assignment problem," *European Journal of Operational Research*, vol. 104, no. 1, pp. 241–249, 1998.

[115] N. Christofides and E. Benvant, "An exact algorithm for the quadratic assignment problem," *Operations Research*, vol. 37, no. 5, pp. 760–768, 1989.

[116] G. Miranda, H. P. L. Luna, G. R. Mateus, and R. P. M. Ferreira, "A performance guarantee heuristic for electronic components placement problems including thermal effects," *Computers and Operations Research*, vol. 32, pp. 2937–2957, 2005.

[117] L. Kaufman and F. Broeckx, "An algorithm for the quadratic assignment problem using bender's decomposition," *European Journal of Operational Research*, vol. 2, pp. 204–211, 1978.

BIOGRAPHICAL STATEMENT

Piyush Kumar was born in Ludhiana, India, in 1984. He received his B.Tech degree from Dr. B.R Ambedkar National Institute of Technology, Jalandhar, Punjab, India, in 2007, his M.S. and PhD degrees from The University of Texas at Arlington in 2009 and 2013, respectively, all in Industrial engineering. In 2012, he joined a software company E2open as software engineer, working on supply chain optimization and planning algorithms. His current research interests include linear mixed integer programming and modeling.