ANALYTICAL EVALUATION OF FREQUENCY REUSE FOR OFDMA CELLULAR NETWORKS

by

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ABSTRACT

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NETWORKS

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In this thesis, the goal is to arrive at tractable general expressions for probability of coverage and achievable rate for a typical mobile user. For this purpose, the grid network model is replaced with the Poisson Point Process model. Later, the scenario has been made narrower and then, it is assumed that the power of the interference signals has exponential distribution, which has helped to greatly simplify the formulas. Then, to evaluate the effect of frequency reuse on the probability of coverage and the average achievable rate in the new model, these results have been simulated. Frequency reuse had a positive effect on the probability of coverage, but a negative effect on the achievable rate. In the simulations, results of two cases, one with noise and one neglecting the noise were compared. Unlike the case with Rayleigh distribution, here noise has a significant effect and cannot be neglected, as their difference mounted to 6 dB at some points.

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CHAPTER 1

INTRODUCTION

1.1 Introduction

Interference is an important problem that affects the quality of service for multi-cellular systems; its primary source in the OFDMA networks being the inter cell interference. With universal frequency resource reuse, and no macro diversity or fast power control, the high level of inter cell interference at the cell borders weakens the signals' strength.

The literature for interference mitigation has considered three approaches for dealing with the inter cell interference issue: 1) inter cell interference randomization, 2) inter cell interference cancellation, and 3) inter cell coordination/avoidance [6].

The inter cell interference randomization tries to randomize the interfering signals. It uses a pseudo random cell scrambling or a cell specific interleaving to let a specific processing approach at the mobile terminal suppress the interference. Since this scheme does not decrease the average interference level, the cell edge user performance does not improve in this approach [6].

The inter-cell-interference cancellation aims to suppress the interference at the receiver, for which it uses either a spatial processing approach using multiple antennas or detection/subtraction of the interference. There are certain downfalls to this approach. The techniques used for this approach require high computational complexity and specific capabilities for the receiver. Also, the gain in signal strength is limited because only a few dominant interferers can be effectively cancelled. In addition, some mobile users might only experience small interference which makes the interference cancellation with reasonable computational complexity largely ineffective [6].

Inter cell interference coordination (ICIC) is a strategy that aims to improve the performance of the network, by minimizing the experienced interference in the network through allocation of time and frequency resources in each cell in a coordinated manner [3].

1.2 Scope and Organization of Thesis

This thesis is organized into five more chapters. Chapter 2,"Frquency Reuse" introduces the concept of frequency reuse for inter cell interference coordination and its types and their specifications. Chapter 3, "Poisson Point Process" gives the basics of Poisson Point Processes and why and how they are used to model cellular networks. Chapter 4, "System Model", gives the specifications of the scenario that is used in the rest of the thesis, and also gives the necessary information for the Rician fading used in the rest of the thesis. Chapter 5, "Achievable Rate", gives the general and special case formulas for the achievable rate of a typical user in this specific scenario for network and discusses the effect of frequency reuse on it. Simulations accompany these formulas to make the concept clearer.

In Chapter 6, "Probability of Coverage", the desired general and special case formulas are derived for the probability of coverage and also the effect of noise has been evaluated. The effect of frequency reuse on the probability of coverage is also discussed, with the necessary formula being derived. After these formulas, simulations are done based on them with some specific parameters, to further explain the changes in different cases.

Finally, Chapter 7, "Conclusions and future work" summarizes the results achieved for the different cases in the previous chapters and discusses the possible work that can be done to extend the work presented in this thesis.

CHAPTER 2

FREQUENCY REUSE

2.1 Introduction

Frequency reuse is the process of using the same radio frequencies on radio transmitter sites within a geographic area, which are separated by sufficient distance to cause minimal interference with each other. Frequency reuse allows for a dramatic increase in the number of customers that can be served (capacity) within a geographic area on a limited amount of radio spectrum (limited number of radio channels).

The frequency band allocated for a cellular system can be reused with different clusters. A cluster is the configuration of cells over which the complete frequency band is divided, and this configuration of cells is repeated over and over. The frequency reuse factor is defined as 1 over the number of cells in the cluster of the system. Valid clusters are those that result in 6 cells with the same frequency of a particular cell located at an equal distance from it.

2.2 Reuse Factor in Frequency Reuse

Several cluster shapes with different frequency reuse factors are considered, among them is the case for 1-Cell Frequency Reuse Cluster (Frequency Reuse Factor = 1); which means that the whole band of frequency of used in a cell and reused in each of the adjacent cells. Because the same frequency is used in all cells, high interference occurs in this system making it impractical. Note that there are 6 cells with the same frequency band around each cell [26].

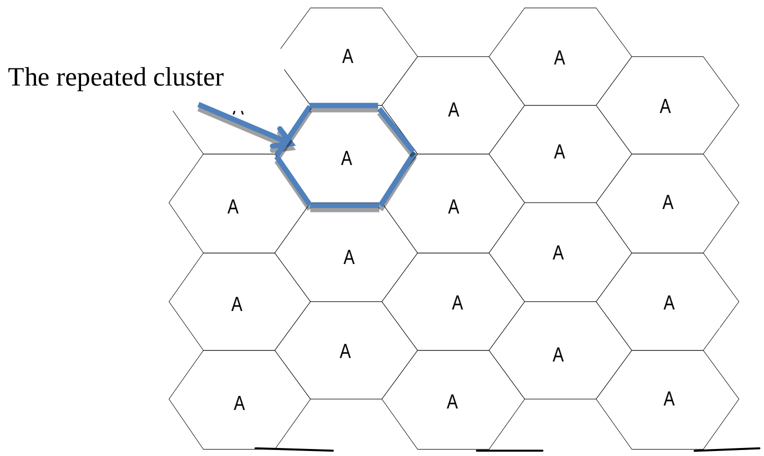

Figure 2.1: Cluster for frequency reuse factor 1.

Another case is the case using 3-Cell Frequency Reuse Cluster (Frequency Reuse Factor = 3); where the allocated band is divided into 3 bands (possibly with equal bandwidth) and the three sub-bands are reused in an alternating fashion. No neighboring cells have the same frequency in this configuration resulting in it being the cluster with the least number of cells that provides practical frequency reuse [26].

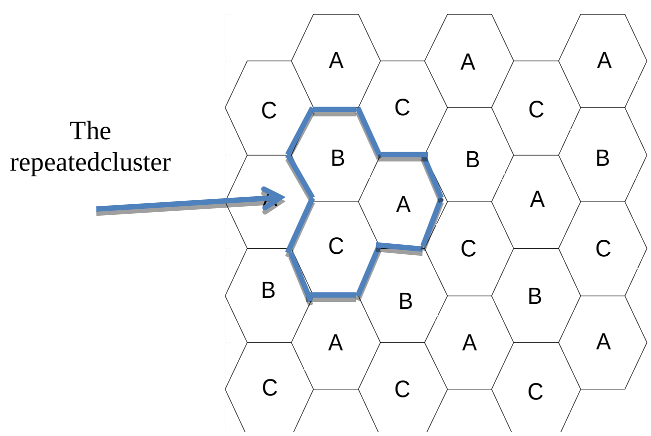

Figure 2.2: Cluster for frequency reuse factor 3.

Reusing frequencies by dividing the allocated band by a specific integer number of cells and assigning each cell one of the partitions, and then repeating the assignment over and over produces a tradeoff between network capacity and reception quality as the capacity of network decreases with the number of divisions of the spectrum over the cells (cell-reuse factor), but cells with similar frequency allocations are located further away, resulting in lower interference [25].

Reusing all sub-carriers in all cells, i.e., reuse with factor one (reuse-1), typically gives the best performance in terms of the overall system throughput. However, cell-edge users may suffer severely in this scheme, because the data throughput of users at cell-edge zones is very interference- sensitive. On the other hand, higher order reuse schemes, such as a reuse factor of three, can be used to limit the interference and improve the throughput of cell, but this will cause a huge loss of bandwidth and bring down significantly the overall throughput [24].

There are only some cluster sizes that if repeated they will be able to cover the complete region. The following formula gives the possible cluster sizes, N, where i and j are natural numbers:

$$N=i^2+ij+j^2 \tag{2.1}$$

The practical cluster sizes are generally 4, 7, and 12. Other cluster sizes either cause too much interference (such N= 1) or waste system resources by insuring a very low interference level that is much lower than the maximum acceptable value (such as N=27) [26].

Frequency reuse is limited by the SIR_{min} (minimum Signal–to-Interference Ratio), which is itself determined by the modulation characteristics of the radio system. To find the co-channel cells, move i cells along any chain of hexagons; turn counter- clockwise 60 degree; move j cells along the chain that lies on this new heading [25].

CHAPTER 3

POISSON POINT PROCESS

3.1 Introduction

The Poisson Point Process is a spatial generalization of the Poisson Process; and a Poisson Process can be characterized by two properties: I) the numbers of points (events) in disjoint intervals are independent, and II) these intervals have a Poisson distribution. A Poisson Point Process can also be defined using these two properties, and it is characterized by the intensity, λ,, with the following relationship:

$$P_r[\xi(B_i) = k_i, 1 \leq i \leq n] = \prod_i e^{-\lambda\|B_i\|} \frac{(\lambda\|B_i\|)^{k_i}}{k_i!}, \tag{3.1}$$

for any disjoint bounded subsets $B_1, \dots, B_n$; and non-negative integers $k_1, \dots, k_n$, and where $\|.\|$ denotes the Lebesgue measure.

3.2 Previous Models

In order to achieve higher spectral efficiency, especially for the dense cellular networks in the urban areas that are under the most strain, techniques are needed to reduce inter cell interference, and in order to do so, tractable models are needed that accurately model the inter cell interference. But such models are still unavailable despite decades of research, and this has thwarted the development and implementation of techniques for reducing the inter cell interference [2].

The Wyner model has been commonly used to model other-cell interference, but it is typically one dimensional and its gain assignment is highly inaccurate unless there is a very large amount of interference averaging over space. For cellular systems that use orthogonal multiple access techniques, such as in LTE (Long Term Evolution), the SINR (Signal to Interference and Noise Ratio) values vary dramatically over a cell. Thus, the Wyner model and related mean-value approaches are inaccurate; nevertheless it has been used to evaluate the capacity of multi-cell systems, even up to the present.

Another approach has been to consider a single interfering cell, or two interfering cells. In the latter case at least the SINR varies depending on the position of the user and possibly fading, but it is still highly idealized since it neglects most of the sources of interference in the network [2].

Another approach has been a 2-D network of base stations on a regular hexagonal lattice, or a little simpler, a square lattice. By considering very limited cases, such as a small number of interfering base stations for a fixed user on the "worst-case" location, the cell corner, tractable analysis can be achieved. The obtained analysis is very pessimistic and does not provide much guidance to the performance of most users in the system, since it generally does not give tractable expressions for random user locations. Thus, complex time-consuming simulations are used to arrive at more general results that can provide some guidance into typical SINR or the probability of outage/coverage over the entire cell. There are issues that make this approach an unfavorable one. They include its repeatability and transparency issues, being onerous to construct and run, and their rare capability to inspire optimal or creative new algorithms or designs. It is also important to realize that although widely accepted, grid-based models are themselves highly idealized and may be increasingly inaccurate for the heterogeneous and ad hoc deployments common in urban and suburban areas, where cell radii vary considerably due to differences in transmission power, tower height, and user density. For example, picocells are often inserted into an existing cellular network in the vicinity of high-traffic areas, and short-range femtocells may be scattered in a haphazard manner throughout a centrally planned cellular network [2].

The authors in [2] have addressed these long-standing problems by introducing an additional source of randomness: the positions of the base stations. Instead of assuming they are placed deterministically on a regular grid, they model their location as a homogeneousPoisson Point Process of intensity λ. Such an approach for BS (Base Station) modeling has been considered as early as 1997, but the key metrics of coverage, such as SINR distribution

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intensity λ. Such an approach for BS (Base Station) modeling has been considered as early as 1997, but the key metrics of coverage, such as SINR distribution and the rate have not been studied. The main advantage of this approach is that the base station positions are all independent which allows substantial tools from stochastic geometry to be used. Although BSs' are not independently placed in practice, the results here can in principle be generalized to point processes that model repulsion or minimum distance, such as determinantal or Matern processes. The mobile users are scattered about the plane according to some independent homogeneous point process with a different intensity, and they communicate with the nearest base station while all other base stations act as interferers [2], [9].

The grid model, as expected, is more optimistic in terms of coverage probability than the results based on the actual deployment. This is primarily due to the minimum distance between the interfering base stations and the typical edge user, resulting in well-defined fixed-sized tiers of interference, with the outlying tiers much less important to calculating the overall performance due to the exponentially decaying nature of the path loss [1].

CHAPTER4

SYSTEM MODEL

The OFDMA cellular downlink network model used here consists of base stations (BSs) that are arranged according to some homogeneous Poisson Point Process (PPP) Φ of intensity λ in the Euclidean plane. In this model mobile users are located according to some independent stationary point process. It is assumed that each mobile user is associated with the closest base station, resulting in coverage areas shown in the figure 4.1 [2].

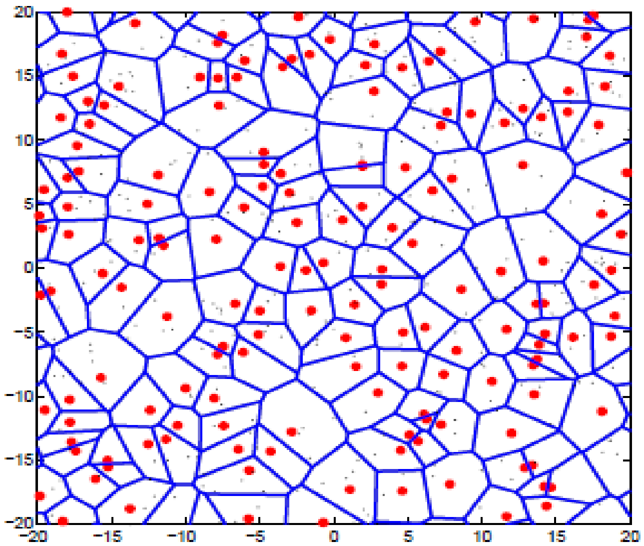

Figure 4.1: Poisson distributed base stations and mobile users, with each mobile user associated with the nearest base station [2].

The figure 4.2 shows the actual base station deployment for an actual real-world network, and from the figure the PPP-model deployment above looks more representative of the real-world network below than the square grid networks that have been used so far. The main disadvantage of the Poisson model is that because of the independence of the PPP, BSs may in some cases be located very close together but with a significant coverage area, i.e. this model might give an artificially high probability of nearby interfering base stations than the case is in the real world. But this model also has two strong advantages: it includes different cell sizes and shapes, and the network extends indefinitely in all directions, thus there would not have be edge effects. [2]

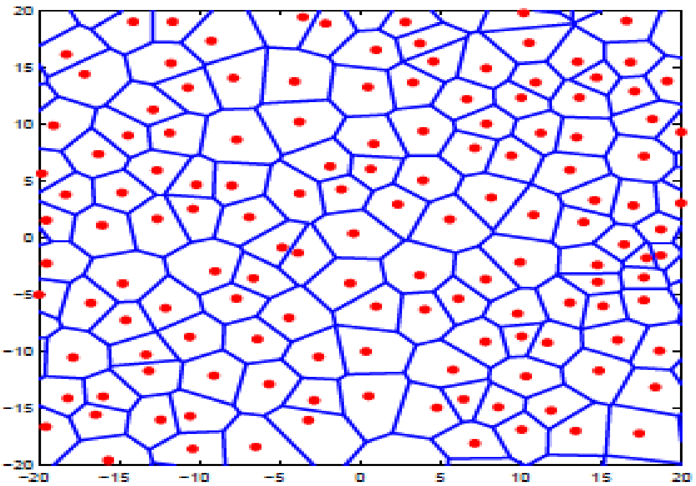

Figure 4.2: A 40x40 km view of a current base station deploymentby a major service provider in a relatively flat urban area [2].

Intuitively, one must expect the Poisson model to give pessimistic results compared to a planned deployment due to the strong interference generated by nearby base stations. The grid model is clearly an upper bound since a perfectly regular geometry is in fact optimal from a coverage point of view [17].

In this thesis, the standard power loss propagation model is used with path loss exponent $\alpha>2$. It is assumed, unless otherwise noted, that the tagged base station and tagged user experience Rician fading. In this case, the received power at a typical node a distance r from its base station is$Pg_y r^{-a}$ where the random variable $g_y$follows a non-central chi-squared distribution with its degree of freedom, k, being 2. The interference power follows a general statistical distribution g_i that includes fading, shadowing and any other desired random effects. Simpler expressions result when g_i also follows specific distributions and these are given as especial cases. All results are for a single transmit and single receive antenna, although future extensions to multiple such antennas are clearly desirable [2].

The interference power at the typical receiver, I_r, is defined as sum of the received powers from all base stations other than the home base station. Because of using OFDMA, there is no same-cell interference. The noise power is assumed to be additive and constant with value σ^2 but no specific distribution is assumed in general. The SNR is defined to be the received SNR at a distance of r=1. All analysis is for a typical mobile node, which is permissible in a homogeneous PPP by Slivnyak's theorem, which states that for a Poisson point process the reduced Palm measure is equal to the original measure, where $P^{!o}$is the reduced Palm measure. So in a PPP, a point can be added at any location without changing the properties of the underlying node distribution. This follows from the independence properties of PPP [2],[3].

It is assumed that all the base stations transmit with an equal power P. The Signal to Noise Ratio (SINR) is given as

$$SINR = \frac{P g_y r^{-\alpha}}{\sigma^2 + P I_r} \tag{4.1}$$

In equation (4.1), for an interfering BS set Z,

$$I_r = \sum_{z \in Z} g_z R_z^{-\alpha}. \tag{4.2}$$

In the expressions given above, the assumption was made that the nearest base station to the mobile y is at a random variable distance r [1].

In the PPP model, the definitions of edge users and interior users are different from those assumed in a typical square grid model. In a grid model of networks, there are constant concentric circles defined around the central base station, and edge users and interior users are recognized by whether their geographical position follows inside or outside of these circles. But in the present model, edge users and interior users are identified depending on the SINR of the users. Users with SINR less than a pre-determined threshold, T_{FR}, are regarded as edge users. Likewise, users with SINR larger than T_{FR} are considered to be interior users [1].

4.1 Rician Fading

In the present scenario, the network experiences Rician fading. Rician fading is a stochastic model for radio propagation that occurs when one of the paths, typically a line-of-sight signal, is much stronger than the others. In Rician fading the amplitude gain is characterized by a Rician distribution given below:

$$f(x) = \frac{2(K+1)x}{\Omega} \exp\left(-K - \frac{(K+1)x^2}{\Omega} \right) I_0\left(2\sqrt{\frac{K(K+1)}{\Omega}} x \right) \qquad (4.3)$$

where $I_0(.)$ is the 0th order modified Bessel function of the first kind, given by

$$I_0(x) = \frac{1}{\pi} \int_0^{\pi} \exp\left(x.\cos(\theta)\right) d\theta, \qquad (4.4)$$

K is the ratio between the power in the direct path and the power in the other, scattered, paths. Ω is the total power from both paths, and is defined as $\Omega = \nu^2 + 2\sigma^2$.

In Rician fading networks, the received signal power has non-central chi square distribution with its degree of freedom, k, being 2:

$$f_x(x; k, \lambda') = \frac{1}{2} e^{-(x+\lambda')/2} \left(\frac{x}{\lambda'}\right)^{\frac{k}{4}-1/2} I_{\frac{k}{2}-1}\left(\sqrt{\lambda' x}\right) = \frac{1}{2} e^{-(x+\lambda)/2} I_0\left(\sqrt{\lambda' x}\right) \qquad (4.5)$$

where $I_0(x)$ Is the modified Bessel function of the first kind, and $\lambda' = \sum_{i=1}^{k} \left(\frac{\mu_i}{\sigma_i}\right)^2$ being the non-centrality parameter. In [10], the following approximation has been obtained for the Bessel function of zero degree as given by:

$$J_0(x) \approx \frac{1}{7} + \frac{2}{7}\cos\left(x \sin\left(\frac{2\pi}{7}\right)\right) + \frac{2}{7}\cos\left(x \sin\left(\frac{4\pi}{7}\right)\right) + \frac{2}{7}\cos\left(x \sin\left(\frac{6\pi}{7}\right)\right) \qquad (4.6)$$

with its error being $-2J_{14}(x)$.

As the first modified Bessel function is related to the Bessel function by the following formula,

$$I_\alpha(x) = i^\alpha J_\alpha(ix) . \tag{4.7}$$

Then, the approximation for $I_0\left(\sqrt{\lambda x}\right)$ can be given by

$$I_0\left(\sqrt{\lambda x}\right) \approx \frac{1}{7} + \frac{2}{7}\cosh\left(\sqrt{\lambda x}\sin\left(\frac{2\pi}{7}\right)\right) + \frac{2}{7}\cosh\left(\sqrt{\lambda x}\sin\left(\frac{4\pi}{7}\right)\right) + \frac{2}{7}\cosh\left(\sqrt{\lambda x}\sin\left(\frac{6\pi}{7}\right)\right) \tag{4.8}$$

Considering the received signal power h to be in the range [0~1]; and the product of λx will have values ranging from 0 to 10, the approximation given above has values close to the much simpler formula given below:

$$I_0\left(\sqrt{\lambda x}\right) \approx \frac{1}{7} + \sinh\left(\left(1 + \frac{\lambda x}{5}\right) \times 0.815\right). \tag{4.9}$$

Thus the original distribution for non-central chi-square function can be written as:

$$f_x(x;k,\lambda') = \frac{1}{2}e^{-(\lambda' + x)/2} I_0\left(\sqrt{\lambda' x}\right) \approx \frac{1}{2}e^{-\lambda'/2}e^{-x/2}\left\{\frac{1}{7} + sinh\left(\left(1 + \frac{\lambda' x}{5}\right) \times 0.815\right)\right\} \tag{4.10}$$

Given that $\sinh(t) = \frac{e^t - e^{-t}}{2}$, this final result can be given by:

$$f_x(x;k,\lambda') \approx A\frac{1}{2}e^{-\frac{\lambda'}{2}}e^{-\frac{x}{2}}\{\frac{1}{7} + \frac{1}{2}\left[exp\left[\left(1 + \frac{\lambda' x}{5}\right) \times 0.815\right] - exp\left[-\left(1 + \frac{\lambda' x}{5}\right) \times 0.815\right]\right]$$

$$= Ae^{-\frac{\lambda'}{2}}e^{-\frac{x}{2}}\left\{\frac{1}{14} + \frac{1}{4}\left[e^{0.815}e^{\frac{0.815\,\lambda' x}{5}}\right] - \frac{1}{4}\left[e^{-0.815}e^{-\frac{0.815\,\lambda' x}{5}}\right]\right\}. \tag{4.11}$$

A is used here to make $\int_0^\infty f_x(x;k,\lambda')\,dx = 1$, hold for different values of λ'.

CHAPTER 5

ACHIEVABLE RATE

There are scenarios where the desired goal might be to provide peak data rates for interference-limited edge users. In this section the mean data rate achievable over a cell is discussed. Specifically the mean rate is computed in units of nats/Hz (1 bit = ln(2) = 0.693 nats) for a typical user where adaptive modulation/coding is used so each user can set their rate such that they achieve Shannon bound for their instantaneous SINR, I.e. ln(1+SINR). Interference is treated as noise, which means the true channel capacity is not achieved [15]. In general, almost any type of modulation, coding, and receiver structure can be easily treated by adding a gap approximation to the rate expression, i.e. $\tau \to \ln(1 + SINR/G)$, where $G \geq 1$ is the gap. Naturally, multi-antenna transmission could further increase the rate.

5.1 Achievable Rate, General Case

The average ergodic rate of a typical mobile user and its associated base station in the downlink is obtained as given below.

$$\tau(\lambda, \alpha) \triangleq E[ln(1 + SINR)] = \int_{r>0} e^{-\pi\lambda r^2} \int_{t>0} E\left(ln\left(1 + \frac{P g_y r^{-\alpha}}{\sigma^2 + PI_r}\right)\right) 2\pi\lambda r dr$$

$$= \int_{r>0} e^{-\pi\lambda r^2} \int_{t>0} P\left[ln\left(1 + \frac{P g_y r^{-\alpha}}{\sigma^2 + PI_r}\right) > t\right] dt\ 2\pi\lambda r dr$$

$$= \int_{r>0} e^{-\pi\lambda r^2} \int_{t>0} P\left[g_y > \frac{r^\alpha}{P}(\sigma^2 + PI_r)(e^t - 1)\right] dt\ 2\pi\lambda r dr, \qquad (5.1)$$

where,

$$P\left[g_y > \frac{r^\alpha}{P}(\sigma^2 + PI_r)(e^t - 1)\right] = Ae^{-\frac{\lambda\prime}{2}}[1/7e^{\frac{-r^\alpha\sigma^2}{2P}(e^t-1)}E\{e^{-\frac{r^\alpha}{2}I_r(e^t-1)}$$

$$+\frac{e^{0.815}}{4}\frac{e^{\frac{-r^{\alpha}\sigma^2}{P}(e^t-1)\left(\frac{1}{2}-\frac{0.815}{5}\lambda'\right)}}{\frac{1}{2}-\frac{0.815}{5}\lambda'}E\left\{e^{-r^{\alpha}I_r(e^t-1)\left(\frac{1}{2}-\frac{0.815}{5}\lambda'\right)}\right\}$$

$$-\frac{e^{-0.815}}{4}\frac{e^{\frac{-r^{\alpha}\sigma^2}{P}(e^t-1)\left(\frac{1}{2}+\frac{0.815}{5}\lambda'\right)}}{\frac{1}{2}+\frac{0.815}{5}\lambda'}E\left\{e^{-r^{\alpha}I_r(e^t-1)\left(\frac{1}{2}+\frac{0.815}{5}\lambda'\right)}\right\}, \qquad (5.2)$$

and,

$$\mathrm{E}\left\{e^{-\mu r^{\alpha}I_r(e^t-1)}\right\} = L_{I_r}\left(\mu r^{\alpha}(e^t-1)\right)$$

$$= exp\left(-2\pi\lambda\int_0^{\infty}\left(\int_r^{\infty}\left(1-e^{-m(e^t-1)r^{\alpha}gv^{-\alpha}}\right)vdv\right)f(g)dg\right). \qquad (5.3)$$

5.2 Achievable Rate, Rayleigh Interference Power

Here for the special case when the interference power experiences Rayleigh fading and shadowing is neglected, the expression can be significantly simplified for the achievable rate. The achievable rate of a typical randomly located mobile user experiencing exponential interference with mean m can be obtained as shown below following the formula for the achievable rate theorem for the general case:

$$\tau(\lambda,\alpha) = \int_{r>0} e^{-\pi\lambda r^2}\int_{t>0} P\left[g_y > \frac{r^{\alpha}}{P}(\sigma^2+PI_r)(e^t-1)\right]dt\; 2\pi\lambda r dr, \qquad (5.4)$$

where,

$$P\left[g_y > \frac{r^{\alpha}}{P}(\sigma^2+PI_r)(e^t-1)\right] = Ae^{-\frac{\lambda'}{2}}[1/7e^{\frac{-r^{\alpha}\sigma^2}{2P}(e^t-1)}E\{e^{-\frac{r^{\alpha}}{2}I_r(e^t-1)}$$

$$+\frac{e^{0.815}}{4}\frac{e^{\frac{-r^{\alpha}\sigma^2}{P}(e^t-1)\left(\frac{1}{2}-\frac{0.815}{5}\lambda'\right)}}{\frac{1}{2}-\frac{0.815}{5}\lambda'}E\left\{e^{-r^{\alpha}I_r(e^t-1)\left(\frac{1}{2}-\frac{0.815}{5}\lambda'\right)}\right\}$$

$$-\frac{e^{-0.815}}{4}\frac{e^{\frac{-r^{\alpha}\sigma^2}{P}(e^t-1)\left(\frac{1}{2}+\frac{0.815}{5}\lambda'\right)}}{\frac{1}{2}+\frac{0.815}{5}\lambda'}E\left\{e^{-r^{\alpha}I_r(e^t-1)\left(\frac{1}{2}+\frac{0.815}{5}\lambda'\right)}\right\}]. \qquad (5.5)$$

But for this case, the expected value of $e^{-r^{\alpha}I_r(e^t-1)\left(\frac{1}{2}+\frac{0.815}{5}\lambda'\right)}$ can be written as

$$\mathrm{E}\left\{e^{-\mu r^{\alpha}I_r(e^t-1)}\right\} = L_{I_r}\left(\mu r^{\alpha}(e^t-1)\right) =$$

$$exp\left(-2\pi\lambda\int_r^{\infty}\left(1-\frac{m}{m+\mu(e^t-1)(r/v)^{\alpha}}\right)vdv\right) = \exp\left(-2\pi\lambda\int_r^{\infty}\left(\frac{1}{1+\frac{m}{\mu(e^t-1)}\left(\frac{v}{r}\right)^{\alpha}}\right)vdv\right)$$

$$= exp\left(-\pi\lambda r^2 \left(\frac{\mu(e^t-1)}{m}\right)^{2/\alpha} \int_{\left(\frac{\mu(e^t-1)}{m}\right)^{-2/\alpha}}^{\infty} \frac{1}{1+u^{\alpha/2}} du\right)$$

$$= exp\left(-\pi\lambda r^2 \rho(T,\alpha,\mu)\right), \tag{5.6}$$

and,

$$\rho(T,\alpha,\mu) = \left(\frac{m}{(e^t-1)\mu}\right)^{-\frac{2}{\alpha}} \int_{\left(\frac{m}{(e^t-1)\mu}\right)^{\frac{2}{\alpha}}}^{\infty} \frac{1}{1+u^{\frac{\alpha}{2}}} du. \tag{5.7}$$

In the terms written above, a change of variable $u = (v/r)^2 \left(\frac{m}{\mu(e^t-1)}\right)^{2/\alpha}$ has been used.

The results given above are for exponentially distributed interference power but general distributions could be handled as well following the approach of obtaining the probability of coverage in the general case and techniques from [16]. In this section, the performance of the model will be shown when the noise is not considered, and also for non-zero modeling.

With further simplification, for the case of no noise, the average rate is given as

$$\tau(\lambda,\alpha) = \int_{r>0} e^{-\pi\lambda r^2} \int_{t>0} P\left[g_y > \frac{r^\alpha}{P}(\sigma^2 + PI_r)(e^t-1)\right] dt\, 2\pi\lambda r dr, \tag{5.8}$$

where

$$P\left[g_y > \frac{r^\alpha}{P}(\sigma^2 + PI_r)(e^t-1)\right] = Ae^{-\frac{\lambda\prime}{2}}[1/7E\{e^{-\frac{r^\alpha}{2}I_r(e^t-1)}$$

$$+\frac{e^{0.815}}{4}\frac{1}{\frac{1}{2}-\frac{0.815}{5}\lambda\prime}E - \frac{e^{-0.815}}{4}\frac{1}{\frac{1}{2}+\frac{0.815}{5}\lambda\prime}E\left\{e^{-r^\alpha I_r(e^t-1)\left(\frac{1}{2}+\frac{0.815}{5}\lambda\prime\right)}\right\}], \tag{5.9}$$

where the expected value in the expression above can be written as

$$\mathrm{E}\left\{e^{-\mu r^\alpha I_r(e^t-1)}\right\} = exp\left(-\pi\lambda r^2 \left(\frac{\mu(e^t-1)}{m}\right)^{2/\alpha} \int_{\left(\frac{\mu(e^t-1)}{m}\right)^{-2/\alpha}}^{\infty} \frac{1}{1+u^{\alpha/2}} du\right)$$

$$= exp\left(-\pi\lambda r^2 \rho(T,\alpha,\mu)\right), \tag{5.10}$$

and,

$$\rho(T,\alpha,\mu) = \left(\frac{m}{(e^t-1)\mu}\right)^{-2/\alpha} \int_{\left(\frac{m}{(e^t-1)\mu}\right)^{2/\alpha}}^{\infty} \frac{1}{1+u^{\alpha/2}} du. \tag{5.11}$$

Thus,

$$\tau(\lambda,\alpha) = Ae^{-\frac{\lambda'}{2}}\int_{t>0}\int_{r>0} e^{-\pi\lambda r^2}\ [1/7E\{e^{-\frac{r^\alpha}{2}I_r(e^t-1)}$$

$$+\frac{e^{0.815}}{4}\frac{1}{\frac{1}{2}-\frac{0.815}{5}\lambda'}E\left\{e^{-r^\alpha I_r(e^t-1)\left(\frac{1}{2}-\frac{0.815}{5}\lambda'\right)}\right\}$$

$$-\frac{e^{-0.815}}{4}\frac{1}{\frac{1}{2}+\frac{0.815}{5}\lambda'}E\left\{e^{-r^\alpha I_r(e^t-1)\left(\frac{1}{2}+\frac{0.815}{5}\lambda'\right)}\right\}]2\pi\lambda r dr dt. \quad (5.12)$$

5.3 Achievable Rate with Frequency Reuse

The desirable increase in coverage with increasing δ, the reuse factor, has to be balanced against the fact that each cell can only use $1/\delta$th of the available frequencies. Any increase in coverage from frequency reuse is paid for by decrease in the overall sum rate in the network. The following general result can be given for average rate with frequency reuse.

If δ frequency bands are randomly allocated to the cells, the average rate of a typical mobile user in a downlink is obtained as follows: Because the average rate of a typical mobile user in this case is $\frac{1}{\delta}E[ln(1+SINR)]$,

$$\tau(\lambda,\alpha,\delta) = \frac{1}{\delta}\int_{r>0} e^{-\pi\lambda r^2}\int_{t>0} E\left(ln\left(1+\frac{Pg_y r^{-\alpha}}{\sigma^2+PI_r}\right)\right)2\pi\lambda r dr$$

$$= \frac{1}{\delta}\int_{r>0} e^{-\pi\lambda r^2}\int_{t>0} P\left[g_y > \frac{r^\alpha}{P}(\sigma^2+PI_r)(e^t-1)\right]dt\ 2\pi\lambda r dr. \quad (5.12)$$

$$P\left[g_y > \frac{r^\alpha}{P}(\sigma^2+PI_r)(e^t-1)\right] = Ae^{-\frac{\lambda'}{2}}[1/7e^{\frac{-r^\alpha\sigma^2}{2P}(e^t-1)}E\{e^{-\frac{r^\alpha}{2}I_r(e^t-1)}\}$$

$$+\frac{e^{0.815}}{4}\frac{e^{\frac{-r^\alpha\sigma^2}{P}(e^t-1)\left(\frac{1}{2}-\frac{0.815}{5}\lambda'\right)}}{\frac{1}{2}-\frac{0.815}{5}\lambda'}E\left\{e^{-r^\alpha I_r(e^t-1)\left(\frac{1}{2}-\frac{0.815}{5}\lambda'\right)}\right\},$$

$$-\frac{e^{-0.815}}{4}\frac{e^{\frac{-r^\alpha\sigma^2}{P}(e^t-1)\left(\frac{1}{2}+\frac{0.815}{5}\lambda'\right)}}{\frac{1}{2}+\frac{0.815}{5}\lambda'}E\left\{e^{-r^\alpha I_r(e^t-1)\left(\frac{1}{2}+\frac{0.815}{5}\lambda'\right)}\right\} \quad (5.13)$$

and,

$$\mathrm{E}\left\{e^{-\mu r^\alpha I_r(e^t-1)}\right\} = L_{I_r}\left(\mu r^\alpha(e^t-1)\right) = exp\left(\frac{-\pi\lambda r^2\left(\frac{\mu(e^t-1)}{m}\right)^{2/\alpha}}{\delta}\int_{\left(\frac{\mu(e^t-1)}{m}\right)^{-2/\alpha}}^{\infty}\frac{1}{1+x^{\alpha/2}}dx\right). \quad (5.14)$$

For the case of no noise, the value given by equation (5.13) is maximized for δ=1. This can be generalized to the case of non-zero noise.

Corollary 1: The average rate of a typical mobile user $\tau(\lambda, \alpha, \delta)$ is maximized for δ=1. This can be shown by putting $r^2 \rightarrow \delta y$ in the theorem above, since it can then be seen that the integrand decreases with δ.

In the case with frequency reuse also, no noise consideration can lead to significant simplification in the formula for computing the average rate.

$$\tau(\lambda, \alpha, \delta) =$$

$$\frac{1}{\delta} A e^{-\frac{\lambda'}{2}} \int_{t>0} \int_{r>0} e^{-\pi\lambda r^2} \, [1/7 E\{e^{-\frac{r^\alpha}{2} I_r (e^t - 1)} + \frac{e^{0.815}}{4} \frac{1}{\frac{1}{2} - \frac{0.815}{5}\lambda'} E\left\{e^{-r^\alpha I_r (e^t - 1)\left(\frac{1}{2} - \frac{0.815}{5}\lambda'\right)}\right\} -$$

$$\frac{e^{-0.815}}{4} \frac{1}{\frac{1}{2} + \frac{0.815}{5}\lambda'} E\left\{e^{-r^\alpha I_r (e^t - 1)\left(\frac{1}{2} + \frac{0.815}{5}\lambda'\right)}\right\}] 2\pi\lambda r dr dt, \quad (5.15)$$

where

$$\mathrm{E}\left\{e^{-\mu r^\alpha I_r (e^t - 1)}\right\} = L_{I_r}\left(\mu r^\alpha (e^t - 1)\right) = exp\left(\frac{-\pi\lambda r^2 \left(\frac{\mu(e^t-1)}{m}\right)^{2/\alpha}}{\delta} \int_{\left(\frac{\mu(e^t-1)}{m}\right)^{-2/\alpha}}^{\infty} \frac{1}{1 + x^{\alpha/2}} dx\right). \quad (5.16)$$

So now the rate can be written as

$$\tau(\lambda, \alpha, \delta) = A e^{-\frac{\lambda'}{2}} \int_{t>0} \{1/7 (\frac{1}{\delta + \sqrt{e^t - 1}\left(\frac{\pi}{2} - \mathrm{atan}(\frac{1}{\sqrt{e^t - 1}})\right)}) + 56.47 \, (\frac{1}{\delta + \sqrt{0.02(e^t - 1)}\left(\frac{\pi}{2} - \mathrm{atan}(\frac{1}{\sqrt{0.02(e^t - 1)}})\right)})$$

$$-1/9 (\frac{1}{\delta + \sqrt{1.98(e^t - 1)}\left(\frac{\pi}{2} - \mathrm{atan}\left(\frac{1}{\sqrt{1.98(e^t - 1)}}\right)\right)})\} dt. \quad (5.17)$$

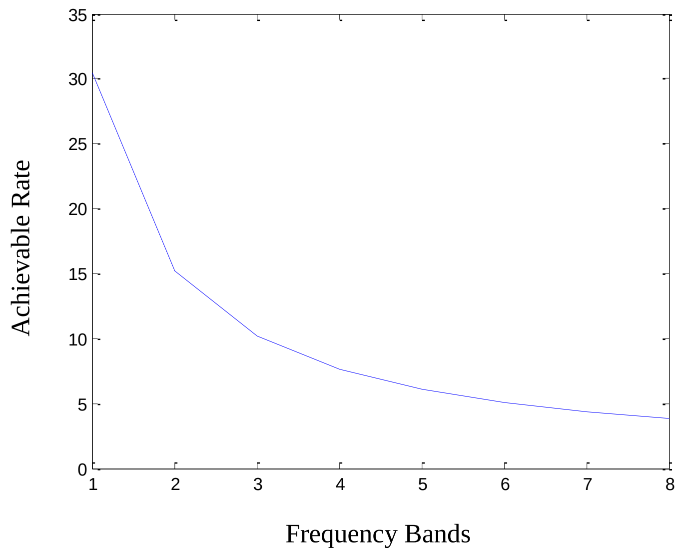

Figure 5.1: Average rate of a typical user with no noise for Poisson distributed base station locations. The average rate is maximized when the cells use the same frequency and hence the complete bandwidth.

Figure 5.2 shows the average rate as a function of the number of frequency bands for two different values for the path loss exponent. For the average rate like the probability of coverage, the density of base stations does not affect the results.

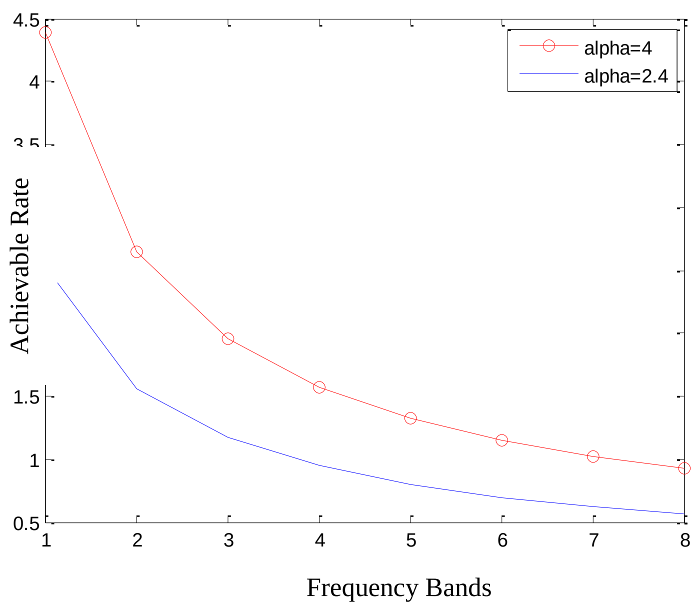

Figure 5.2: Average rate of a typical user with no noise for Poisson distributed base station locations with two different path loss exponents.

CHAPTER 6

PROBABILITY OF COVERAGE

In the previous chapter, formulas were obtained for the achievable rate for the present system model, but in most scenarios it is also necessary to provide high probability of coverage. Coverage probability can be thought of equivalently as: (i) the probability that a random user can achieve a target SINR T, (ii) the average fraction of users who at any time achieve SINR T, or (iii) the average fraction of the network area that is in “coverage” at any time [2].

6.1 Probability of Coverage, General Case

In this part of the thesis, the expressions that calculate the probability of coverage for the present scenario that experiences Rician fading will be obtained. First the Probability Density Function (PDF) of *r*, the distance to the nearest base station, must be obtained. Assuming that each user communicates with the closest base station; other base stations cannot be closer than this distance, r. Thus, the PDF of r can be derived as P[r>R]=P[No BS closer than R] $= e^{-\lambda\pi R^2}$. Therefore, the CDF for r is $P[r \le R] = F_r(R) = 1 - e^{-\lambda\pi R^2}$ and the PDF can be found as in [2] to be

$$f_r(r) = \frac{dF_r(r)}{dr} = e^{-\lambda\pi r^2} 2\pi\lambda r. \qquad (6.1)$$

Conditioning on the nearest base station being at a distance r from the typical user, the probability of the coverage averaged over the plane is [2]:

$$P_c = E_r\left[P[SINR > T|r]\right] \int_{r>0} P[SINR > T|r] f_r(r)\,dr$$

$$\overset{a}{\Rightarrow} \int_{r>0} P\left[\frac{P g_y r^{-\alpha}}{\sigma^2 + PI_r} > T|r\right] e^{-\pi r^2 \lambda} 2\pi\lambda r dr \ = \int_{r>0} e^{-\pi r^2} P\left[g_y > \frac{T}{P} r^{\alpha}(\sigma^2 + PI_r)|r\right] 2\pi\lambda r dr. \qquad (6.2)$$

The distribution for r is given above, and step (a) follows directly from implementing the distribution for r.

$$P\left[g_y > \frac{T}{P} r^{\alpha}(\sigma^2 + PI_r)|r\right] = E_{I_r}\left[\left[g_y > \frac{T}{P} r^{\alpha}(\sigma^2 + PI_r)|r, I_r\right]\right]$$

$$= E_{I_r}[A\{e^{-\frac{\lambda'}{2}}[\frac{2}{14} e^{-\frac{T}{2P} r^{\alpha}(\sigma^2+PI_r)} + \frac{1}{4}\frac{e^{0.815}}{\frac{1}{2} - 0.163\lambda} e^{\left(-\frac{1}{2}+0.163\lambda'\right)\frac{T}{2P} r^{\alpha}(\sigma^2+PI_r)}$$

$$-\frac{1}{4} e^{-0.815} \frac{1}{\frac{1}{2}+0.163\lambda} e^{-\left(\frac{1}{2}+0.163\lambda'\right)\frac{T}{2P} r^{\alpha}(\sigma^2+PI_r)}]\}|r]$$

$$= Ae^{-\frac{\lambda'}{2}}\{\frac{1}{7} e^{-\frac{T}{2P} r^{\alpha}\sigma^2} L_{I_r}\left[\frac{T}{2} r^{\alpha}\right]$$

$$+\frac{1}{4} e^{0.815} \frac{e^{\left(-\frac{1}{2}+0.163\lambda'\right)\frac{T}{P} r^{\alpha}\sigma^2}}{\frac{1}{2} - 0.163\lambda} L_{I_r}\left[\left(\frac{1}{2} - 0.163\lambda'\right) Tr^{\alpha}\right]$$

$$-\frac{1}{4} e^{-0.815} \frac{e^{-\left(\frac{1}{2}+0.163\lambda'\right)\frac{T}{P} r^{\alpha}\sigma^2}}{\frac{1}{2}+0.163\lambda} L_{I_r}\left[\left(\frac{1}{2} + 0.163\lambda'\right) Tr^{\alpha}\right]\}. \quad (6.3)$$

where$L_{I_r}(s)$ is the Laplace transform of the random variable I_r evaluated at s based on the distance to the closest base station from the origin, is defined and used as below [2]:

$$L_{I_r}(s) = E_{I_r}[e^{-sI_r}] = E_{\Phi,g_i}\left[exp\left(-s\sum_{i\in\Phi\backslash\{b_o\}} g_i R_i^{-\alpha}\right)\right]$$

$$= E_{\Phi,\{g_i\}}\left[\prod_{i\in\Phi\backslash\{b_o\}} exp(-sg_i R_i^{-\alpha})\right] \stackrel{a}{=} E_{\Phi}\left[\prod_{i\in\Phi\backslash\{b_o\}} E_g[exp(-sg_i R_i^{-\alpha})]\right]$$

$$= exp\left(-2\pi\lambda\int_r^{\infty}\left(1 - E_g[exp(-sgv^{-\alpha})]\right)vdv\right) \quad (6.4)$$

where (a) follows from the i.i.d. (identically and independently distributed) distribution of g_i and its further independence from the point process F , and the last step follows from the probability generating functional (PGFL) of the PPP, which states for some function f(x) that

$$E[\prod_{x\in\Phi} f(x)] = exp\left(-\lambda\int_{R^2}(1 - f(x))dx\right) \quad (6.5)$$

The integration limits are r and ¥ since the closest interferer is at least at a distance r. Let f(g) denote the PDF of g. Using $s = \left(mTr^{a}\right)$, m being one of the elements of the set $\{1/2, \left(\frac{1}{2} - 0.163\lambda\right), \left(\frac{1}{2} + 0.163\lambda\right)\}$, and changing the integration order gives

$$L_{I_r}(mTr^{\alpha}) = exp\left(-2\pi\lambda\int_r^{\infty}\left(1 - E_g[exp(-mTr^{\alpha}gv^{-\alpha})]\right)vdv\right)$$

$$= exp\left(-2\pi\lambda\int_r^{\infty}\left(1 - \int_0^{\infty}[exp(-mTr^{\alpha}gv^{-\alpha})f(g)dg]\right)vdv\right)$$

$$= exp\left(-2\pi\lambda\int_0^{\infty}\left(\int_r^{\infty}\left(1 - e^{-mTr^{\alpha}gv^{-\alpha}}\right)vdv\right)f(g)dg\right). \quad (6.6)$$

Thus, the coverage probability can be obtained as:

$$p_c(T,\lambda,\alpha) = 2\pi\lambda\int_{r>0} Are^{-\pi\lambda r^2}\,[e^{-\frac{\lambda\prime}{2}}\{\frac{1}{7}e^{-\frac{T}{2P}r^{\alpha}\sigma^2}L_{I_r}\left[\frac{T}{2}r^{\alpha}\right] +$$

$$+\frac{1}{4}e^{0.815}\frac{1}{\frac{1}{2}-0.163\lambda'}e^{\left(-\frac{1}{2}+0.163\lambda\prime\right)\frac{T}{P}r^{\alpha}\sigma^2}L_{I_r}\left[\left(\frac{1}{2}-0.163\lambda'\right)Tr^{\alpha}\right]$$

$$-\frac{1}{4}e^{-0.815}\frac{1}{\frac{1}{2}+0.163\lambda\prime}e^{-\left(\frac{1}{2}+0.163\lambda\prime\right)\frac{T}{P}r^{\alpha}\sigma^2}L_{I_r}\left[\left(\frac{1}{2}+0.163\lambda'\right)Tr^{\alpha}\right] \quad (6.7)$$

<u>6.2 Probability of Coverage, Rayleigh Interference Power</u>

The statistical properties of a communication environment are often characterized by the nonfading component. The statistics of this line of sight signal's envelope tends to follow a Rician distribution [18]. There is usually no direct path component in the interference signals from co-channel cells at the receiving location, and the Nakagami m-distribution is versatile in characterizing the statistics of their envelopes. The Nakagami m-distribution can be reduced to a Rayleigh distribution for m=1 [19]. For the special case when the interference power experiences Rayleigh fading and shadowing is neglected, the expression for the probability of coverage can significantly be simplified. The probability of coverage of a typical randomly located mobile user experiencing exponential interference with mean m can be obtained as:

$$E_{I_r}(e^{-sI_r}) = exp\left(-2\pi\lambda\int_r^{\infty}\left(1 - E_{g_i}[exp(-sg_iv^{-\alpha})]\right)vdv\right)$$

$$= exp\left(-2\pi\lambda\int_r^{\infty}\left(1 - \frac{m}{m+sv^{-\alpha}}\right)vdv\right) = exp\left(-2\pi\lambda\int_r^{\infty}\left(\frac{1}{1+\frac{m}{T\mu}\left(\frac{v}{r}\right)^{\alpha}}\right)vdv\right)$$

$$= exp\left(-2\pi\lambda\left(\frac{m}{T\mu}\right)^{-2/\alpha}\int_{\left(\frac{m}{T\mu}\right)^{2/\alpha}}^{\infty}\left(\frac{1}{1+u^{\alpha/2}}\right)du\right) = exp\left(-r^2\pi\lambda\rho(T,\alpha)\right) \quad (6.8)$$

and,

$$\rho(T,\alpha,\mu) = \left(\frac{m}{T\mu}\right)^{-2/\alpha} \int_{\left(\frac{m}{T\mu}\right)^{2/\alpha}}^{\infty} \left(\frac{1}{1+u^{\alpha/2}}\right) du. \quad (6.9)$$

In the expressions written above, $s = \mu T r^{\alpha}$, and also the change of variable

$u = (v/r)^2 \left(\frac{m}{T\mu}\right)^{2/\alpha}$ has been used.

Thus, the probability of coverage in this case can be written as:

$$p_c(T,\lambda,\alpha) = 2\pi\lambda \int_{r>0} Are^{-\pi\lambda r^2}\, [e^{-\frac{\lambda\prime}{2}}\{\frac{1}{7} e^{-\frac{T}{2P}r^{\alpha}\sigma^2} exp\left(-r^2\pi\lambda\rho(T,\alpha,1/2)\right)$$

$$+\frac{1}{4}e^{0.815}\frac{1}{\frac{1}{2}-0.163\lambda'} e^{\left(-\frac{1}{2}+0.163\lambda\prime\right)\frac{T}{P}r^{\alpha}\sigma^2} exp\left(-r^2\pi\lambda\rho\left(T,\alpha,\frac{1}{2}-0.163\lambda'\right)\right)$$

$$-\frac{1}{4}e^{-0.815}\frac{1}{\frac{1}{2}+0.163\lambda'} e^{-\left(\frac{1}{2}+0.163\lambda'\right)\frac{T}{P}r^{\alpha}\sigma^2} exp\left(-r^2\pi\lambda\rho\left(T,\alpha,\frac{1}{2}+0.163\lambda'\right)\right)\}]dr. \quad (6.10)$$

Figure 6.1 shows the effect of the value of the path loss exponent. PPP model gives a more accurate result for lower path loss exponents. The first reason is that the PPP models distant interferers, while the grid model does not, and with lower path loss exponents, the interference of far base stations is more significant. The other reason comes from the disadvantage of the PPP model. The effect of artificially high probability of nearby and dominant interfering base stations is less corrupting with lower path loss exponents, because a dominant base station contributes to a lower fraction of the total interference due to the lower attenuation of non-dominant interferers.

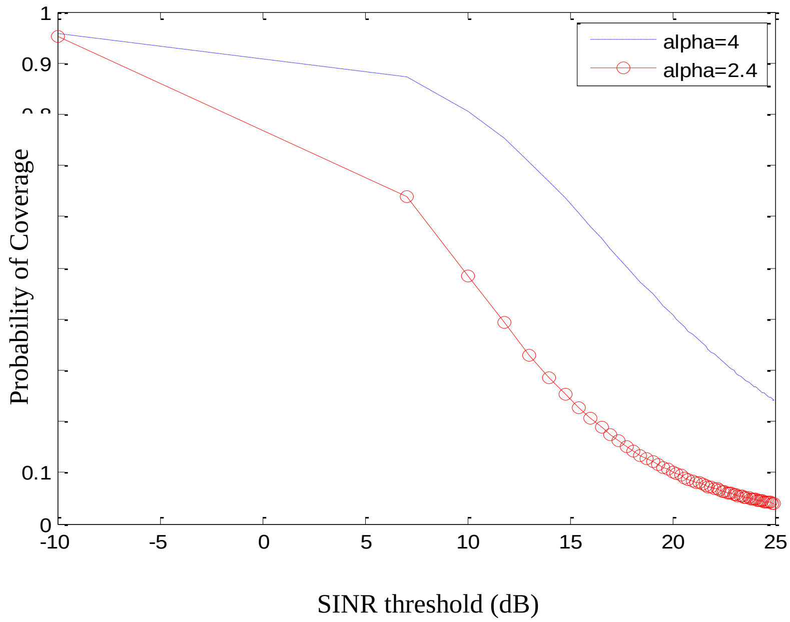

Figure 6.1: Probability of coverage for two different values for the path loss exponent (no noise).

It can be seen from the formula for the probability of coverage that when noise is negligible, the probability of coverage does not depend on the base station density λ. This means that increasing the number of base stations does not affect the probability of coverage, because the increase in signal power is exactly counter-balanced by the increase in interference power. Figure 6.2 shows the probability of coverage for two different values of λ and two different values of α. Since each pair of curves that have the same λ will have the same curve, only two curves can be seen. Figure 6.3 shows the four curves, since two of them that have λ=0.125 are lifted by an increase of 1 in the vertical axis.

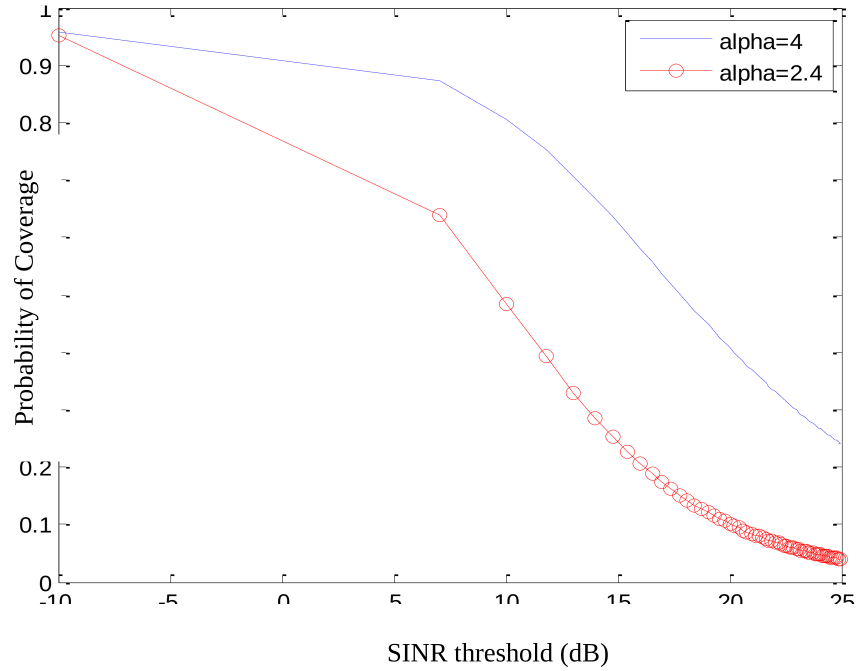

Figure 6.2: Probability of coverage for two different values for each of path loss exponent and base station density.

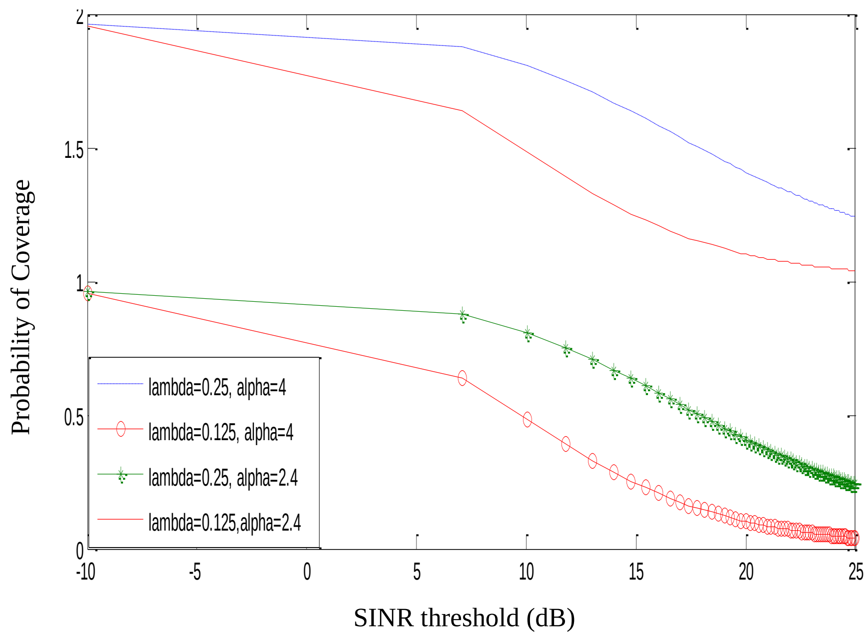

Figure 6.3: Probability of coverage for two different values for each of path loss exponent and base station density. In this figure, the two repeated curves are shifted for clearance.

6.3 Probability of Coverage and Noise Considerations

In most modern cellular networks thermal noise is not an important consideration. It can generally be neglected in the cell interior because it is very small compared to the desired signal power (high SNR), and also at the cell edge because the interference power is typically so much larger (high INR). But as figure 6.4 shows, for a network modeled by Rician fading, and its interference experiencing Rayleigh fading, there is a recognizable gap difference between the two cases with SNR = 10 and very large SNR.

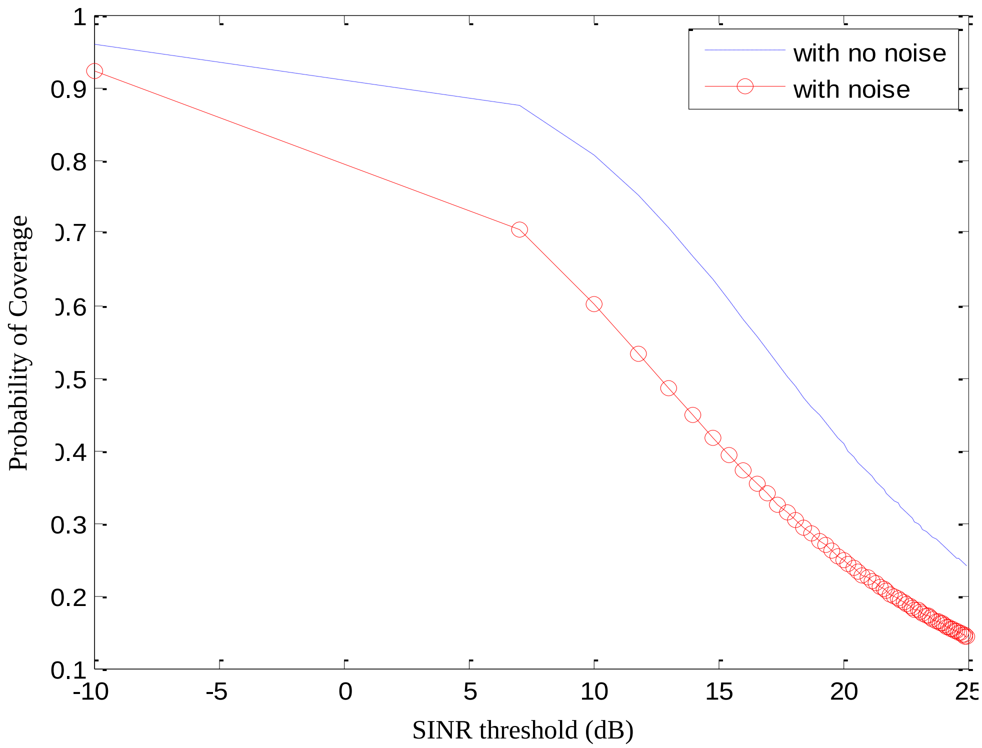

Figure 6.4: A comparison of the probabilities of coverage with noise and without noise.

Figure 6.5 compares the probability of coverage for different values of path loss exponents, noise is also taken into consideration. It can be seen that for both values of path loss exponent, 2.4 and 4, the case considering noise has a lower probability of coverage, also the difference by considering or neglecting the noise in the evaluation is more significant for α=4. As it was opined out earlier that noise cannot be neglected in the Rician fading, in figure 6.6 probability of coverage is computed for two different values for the base station density. Unlike the case where noise is neglected, here different base station densities result different values for the probability of coverage.

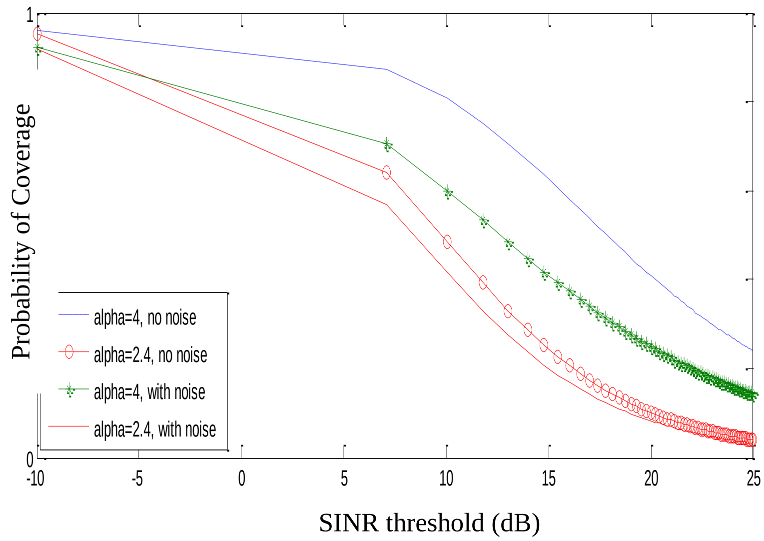

Figure 6.5: Effect of noise with two different path loss exponents.

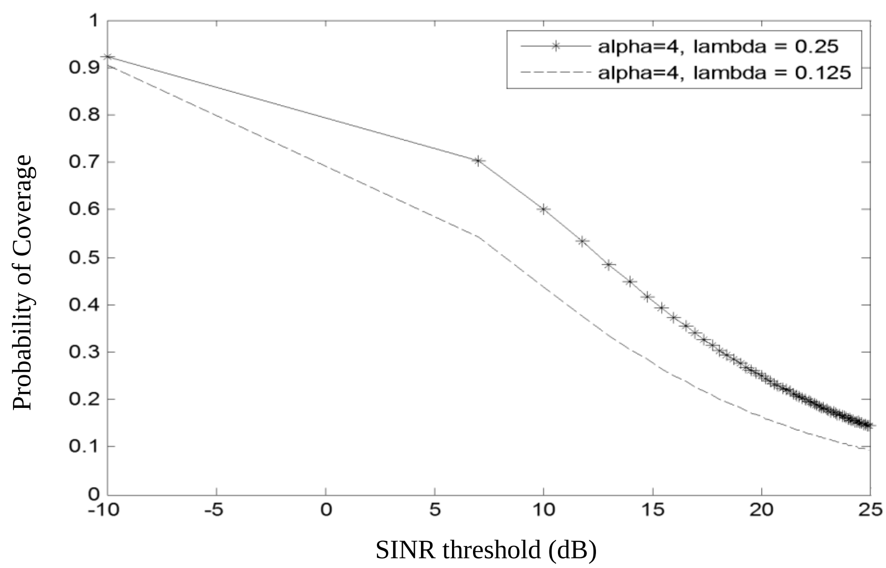

Figure 6.6: The density of base stations in the network changes the probability of coverage in cases that the noise is not neglected.

6.4 Probability of Coverage with Frequency Reuse

For interference-limited networks a common way to increase the probability of coverage is to reduce the number of interfering base stations. This can be done statically through a planned and fixed frequency reuse pattern and/or cell sectoring, or more adaptively via a reduced duty cycle in time, fractional frequency reuse, dynamic bandwidth allocation, or other approaches. More sophisticated interference cancellation/suppression approaches can also be used, potentially using multiple antennas [22],[13].

In this part, the formulas for probability of coverage with frequency reuse are obtained, and the effect of using frequency reuse is studied. If δ frequency bands are randomly allocated to the cells, where δ is the reuse factor, then the coverage probability with exponentially distributed interference power is equal to

$$p_c(T,\lambda,\alpha,\delta) = 2\pi\lambda \int_{r>0} Are^{-\pi\lambda r^2} [e^{-\frac{\lambda\prime}{2}}\{\frac{1}{7} e^{-\frac{T}{2P}r^\alpha\sigma^2} exp\left(-r^2\pi\lambda\rho\left(T,\alpha,\frac{1}{2}\right)/\delta\right) +$$

$$+\frac{1}{4}e^{0.815}\frac{\exp\left((-\frac{1}{2}+0.163\lambda')\frac{T}{P}r^\alpha\sigma^2\right)1}{\frac{1}{2}-0.163\lambda'} exp\left(-r^2\pi\lambda\rho\left(T,\alpha,\frac{1}{2}-0.163\lambda\right)/\delta\right)$$

$$-\frac{1}{4}e^{-0.815}\frac{\exp(-\left(\frac{1}{2}+0.163\lambda\prime\right)\frac{T}{P}r^\alpha\sigma^2)}{\frac{1}{2}+0.163\lambda\prime} exp\left(-r^2\pi\lambda\rho\left(T,\alpha,\frac{1}{2}+0.163\lambda\right)/\delta\right)\}]dr. \quad (6.11)$$

Since the interfering base stations which transmit in the same frequency band are a thinned version of the original PPP, and since a thinned version of a PPP is again a PPP, the rest of the proof exactly follows the process for the probability of coverage in the case when the interference is experiencing Rayleigh fading [2].

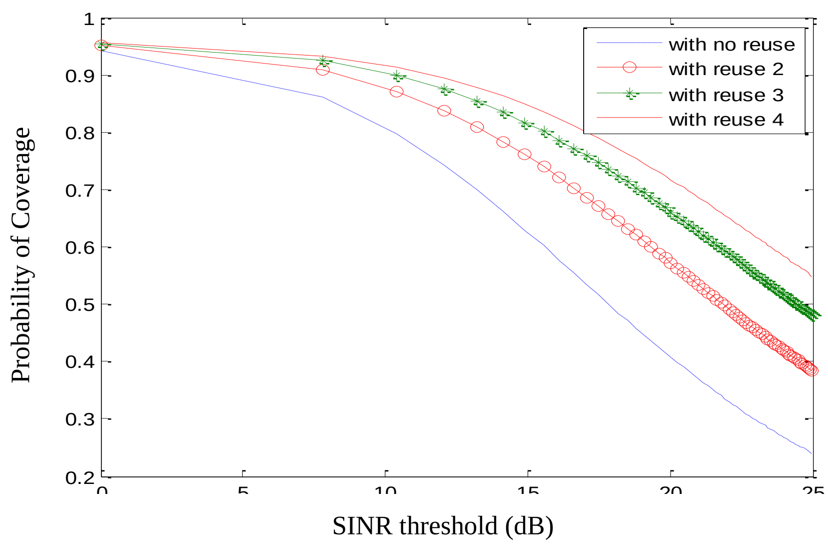

Figure 6.7:Probability of coverage for frequency reuse factors 2, 3, and4, and no reuse. Lower spatial reuse (higher frequency reuse factor) leads to better coverage performances.

It can be seen from (6.11) that as the number of frequency bands increases to infinity, a coverage limit is reached which depends only on the noise power. Frequency planning in actual cellular networks is a complex optimization problem that depends on the specific geography and data loads; it is often performed heuristically by the service provider.

Figure 6.8 also compares the “with noise” and “without noise” scenarios with and without frequency reuse.

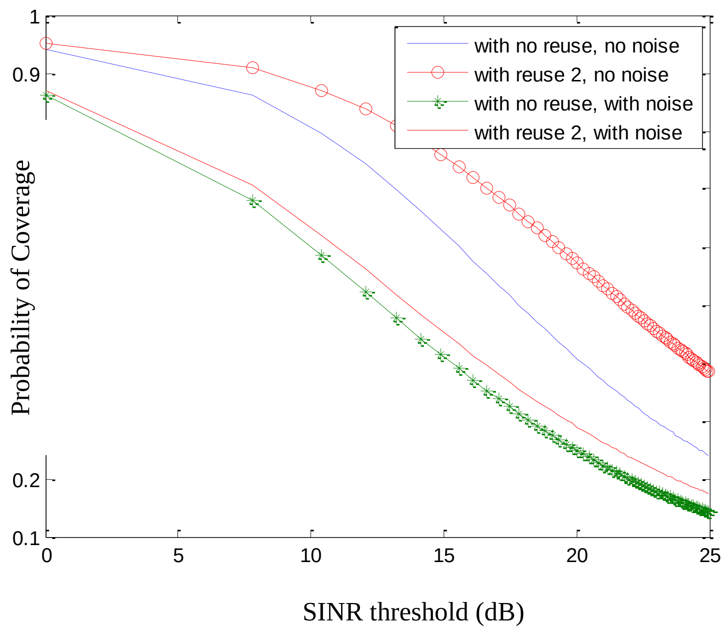

Figure 6.8: Effect of frequency reuse, involving the effect of noise.

CHAPTER 7

CONCLUSIONS AND FUTURE WORK

7.1 Conclusions

In this thesis, general and tractable formulas for probability of coverage and achievable rate have been obtained using the Poisson Point Process model for the network instead of the grid model. The implied randomness of the placement of the Base Stations in the former allows deriving the desired formulas. Then, a narrower scenario is considered, assuming that the interference in the network has an exponential distribution. Then, the formulas for the case when frequency reuse has been applied have been derived, and observed through simulations. Frequency reuse can improve the performance of the network regarding the probability of coverage, since it reduces the number of interfering base stations. On the other hand, in chapter 6, deriving the formulas for the achievable rate using frequency reuse, both through the formulas and simulations it could be seen that the achievable rate decreases with frequency reuse, the maximum achievable rate being for the case with no reuse. For both metrics, it has also been shown that unlike the case with Rayleigh fading networks, noise has a considerable effect on the results and cannot be neglected.

7.2 Future Work

Authors in [1] have evaluated the probability of coverage and achievable rate for interior users and edge users separately, for the case when the interference experiences Rayleigh fading, the same can be done for the system model considered. Also the uplink scenario and the extension of the present results to heterogeneous networks can be considered. The background scenario for this thesis was based on assuming the interference is experiencing Rayleigh fading, but there are other important fadings that can be considered.

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