INVESTIGATION OF SIDE WALL EFFECTS ON AN INWARD SCRAMJET INLET AT MACH NUMBER 8.6

by

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2 Timothy 4:7 - I have fought the good fight to the end; I have run the race to the finish; I have kept the faith.
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ABSTRACT

INVESTIGATION OF SIDE WALL EFFECTS ON AN INWARD SCRAMJET INLET AT MACH NUMBER 8.6

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The University of Texas at Arlington, 2013

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Experimental and computational studies were conducted to evaluate the performance of a scramjet inlet as the side cowl length is changed. A slender inward turning inlet with a total length of 304.8 mm, a span of 50.8 mm with a top and a bottom compression ramp each at 11.54 deg and a compression ratio of 4.79 was used. The side cowl lengths were of 0, 50.8 and 76.2 mm. The UTA Hypersonic Shock Tunnel facility was used in the reflected mode to test the inlet model. The model was instrumented with nine piezoelectric pressure transducers, for static and total pressure measurements. A wedge was mounted at the rear of the inlet to accommodate a Pitot pressure rake. The driven tube was instrumented with three pressure transducers. Two of them were used to measure the incident shock wave speed, and a third one was used for stagnation pressure measurements during a test. Furthermore, a Pitot probe was installed below the model to measure the impact pressure on each run, this reading along with the driven sensor readings, allowed us for the calculation of freestream properties. During the experiments, nominal stagnation enthalpy of 0.67 MJ/kg and stagnation pressure of 3.67 MPa were achieved.
Freestream conditions were Mach number 8.6 and Reynolds number of 1.94 million per m. Test times were 300–500 $\mu$s. Numerical simulations using RANS with the Wilcox $K-\omega$ turbulence model were performed using ANSYS Fluent. The results from the static pressure measurements presented a good agreement with CFD predictions. Moreover, the uniformity at the inlet exit was achieved within experimental accuracy. The experiments showed that the cowl length has a pronounced effect on the inlet pressure distribution but a minor effect on the exit flow Mach number. The numerical results confirmed these trends and showed that a complex flow structure is formed in the cowl-ramp corners; a non-uniform transverse shock structure was found to be related to the cowl leading edge position.
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<td>$A$</td>
<td>Area</td>
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<tr>
<td>$a$</td>
<td>Acoustic speed</td>
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<td>$a_{1i}, \ldots, a_{15i}$</td>
<td>JANAF thermodynamic coefficients</td>
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<tr>
<td>$C$</td>
<td>Chapman-Rubesin factor</td>
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<tr>
<td>$c_p$</td>
<td>Specific heat at constant pressure</td>
</tr>
<tr>
<td>$CR$</td>
<td>Inlet contraction ratio</td>
</tr>
<tr>
<td>$d_d$</td>
<td>Unsupported diameter of the diaphragm plate</td>
</tr>
<tr>
<td>$e$</td>
<td>Specific internal energy</td>
</tr>
<tr>
<td>$G$</td>
<td>Gibbs function</td>
</tr>
<tr>
<td>$g$</td>
<td>Gibbs function per unit of mole</td>
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<td>$G_K, Y_K, S_K$</td>
<td>Generation, diffusion and dissipation terms of $K$ in $K-\omega$ turbulence model, respectively</td>
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<tr>
<td>$g_v$</td>
<td>Equivalence factor in shock tubes with area change</td>
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<td>$G_\omega, Y_\omega, S_\omega$</td>
<td>Generation, diffusion and dissipation terms of $\omega$ in $K-\omega$ turbulence model, respectively</td>
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<td>$h$</td>
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<td>$J_+, J_-$</td>
<td>Riemann invariants</td>
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<tr>
<td>$K$</td>
<td>Turbulent kinetic energy in $K-\omega$ turbulence model</td>
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<tr>
<td>$k$</td>
<td>Thermal conductivity</td>
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<tr>
<td>$L_t$</td>
<td>Inlet throat height</td>
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<td>$L_x$</td>
<td>Inlet throat distance from leading edge</td>
</tr>
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<td>$L_y$</td>
<td>Capture height</td>
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<tr>
<td>$M$</td>
<td>Mach number</td>
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<tr>
<td>$M_s$</td>
<td>Mach number of the incident shock wave, in the laboratory frame</td>
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<tr>
<td>$M_r$</td>
<td>Mach number of the reflected shock wave, in the shock wave frame</td>
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<tr>
<td>$M_{ri}$</td>
<td>Mach number of the reflected shock wave at contact surface, in the shock wave frame</td>
</tr>
<tr>
<td>$M_{ti}$</td>
<td>Mach number of the transmitted shock wave at contact surface, in the shock wave frame</td>
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<tr>
<td>$MW$</td>
<td>Molecular weight</td>
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<tr>
<td>$n, \vec{n}$</td>
<td>Number of moles, scalar and vector respectively</td>
</tr>
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\begin{itemize}
\item \textit{NE} \hspace{1cm} \text{Number of elements}
\item \textit{NS} \hspace{1cm} \text{Number of species}
\item \textit{Pr} \hspace{1cm} \text{Prandtl number}
\item \textit{Pr}_{\text{turb}} \hspace{1cm} \text{Turbulent Prandtl number}
\item \textit{p} \hspace{1cm} \text{Pressure}
\item \textit{\dot{q}} \hspace{1cm} \text{Heat flux}
\item \textit{r} \hspace{1cm} \text{Total enthalpy ratio}
\item \textit{R} \hspace{1cm} \text{Gas constant of air}
\item \textit{\Re} \hspace{1cm} \text{Universal constant of gases}
\item \textit{Re} \hspace{1cm} \text{Reynolds number}
\item \textit{s} \hspace{1cm} \text{Specific entropy}
\item \textit{s_d} \hspace{1cm} \text{Diaphragm score depth}
\item \textit{t} \hspace{1cm} \text{Time}
\item \textit{T} \hspace{1cm} \text{Temperature}
\item \textit{t_d} \hspace{1cm} \text{Diaphragm plate thickness}
\item \textit{TOF} \hspace{1cm} \text{Time of flight}
\item \textit{u, v, w} \hspace{1cm} \text{Flow velocity components in a cartesian system}
\item \textit{\bar{u}, \bar{v}, \bar{w}} \hspace{1cm} \text{Mean velocity component in a Favre-averaged form}
\item \textit{u', v', w'} \hspace{1cm} \text{Velocity fluctuations in a cartesian system}
\item \textit{u'', v'', w''} \hspace{1cm} \text{Velocity fluctuations in a cartesian system in a Favre-averaged form}
\item \textit{u_r} \hspace{1cm} \text{Speed of the reflected shock wave, in the laboratory frame}
\item \textit{u_{ri}} \hspace{1cm} \text{Speed of the reflected shock wave at contact surface, in the laboratory frame}
\item \textit{u_s} \hspace{1cm} \text{Incident shock wave speed, in the laboratory frame}
\item \textit{u_{ti}} \hspace{1cm} \text{Speed of the transmitted shock wave at contact surface, in the laboratory frame}
\item \textit{x, y, z} \hspace{1cm} \text{Cartesian coordinate system}
\item \textit{x_1, x_2, x_3} \hspace{1cm} \text{Cartesian coordinate system}
\item \textit{x_{ni}} \hspace{1cm} \text{Molar fraction in a chemically reactive mixture}
\end{itemize}
Greek Letters

- \( \beta \): Shock wave angle
- \( \beta_1 \): Shock wave angle in region 1, used in the inlet design
- \( \beta_2 \): Shock wave angle in region 2, used in the inlet design
- \( \gamma \): Ratio of specific heats
- \( \delta \): Boundary layer thickness
- \( \delta^* \): Boundary layer displacement thickness
- \( \delta_i^* \): Boundary layer displacement thickness at inlet throat
- \( \delta_{ij} \): Kronecker delta
- \( \epsilon_m \): Eddy viscosity
- \( \theta \): Flow deflection angle
- \( \lambda \): Lagrange multiplier, scalar and vector respectively
- \( \mu \): Dynamic viscosity
- \( \nu \): Kinematic viscosity
- \( \omega \): Specific dissipation rate in \( K-\omega \) turbulence model
- \( \rho \): Density
- \( \sigma_u \): Ultimate tensile stress of a material
- \( \tau \): Shear stress

Subscripts

- \( 0 \): Free stream static conditions
- \( A \): Denotes upstream conditions, in oblique shock wave calculations
- \( B \): Denotes downstream conditions, in oblique shock wave calculations
- \( e \): Boundary-layer edge conditions
- \( t \): Total conditions
- \( ref \): Reference pressure, 1.0 atm
- \( wall \): Denotes wall conditions

Superscripts

- \( init \): Initial quantity of atoms of a given element
- \( * \): Throat conditions
CHAPTER 1
INTRODUCTION

In the United States, supersonic combustion research began with the works of Ferri [1,2] and Dugger [3] in the early 1960s. These works concluded that hypersonic vehicles flying over Mach 6 using scramjets would produce lighter mechanical and thermal loads, and a higher pressure recovery than ramjets. Furthermore, they suggested the use of airbreathing propulsion in launch vehicles, which could decrease the cost of operation and increase the payload of their missions. The first successful flight of a scramjet engine, however, was performed by Russia, in 1991, when a rocket-boosted axisymmetric dual-mode scramjet achieved Mach 6 [4]. To date, numerous investigations were conducted regarding these engines. A complete description of research activities and flight test programs around the world can be found elsewhere [5].

Although the scramjet idea has been around for 50 years, practical utilization is still a matter of research. For instance, extensive studies have been carried out in inlet starting characteristics [6–8], boundary-layer transition [9] thermal protection systems [10], inlet flow uniformity [11–13], supersonic mixing and so on. From an engine performance standpoint, the interest in scramjets lies on the fact that they can provide a higher specific impulse at hypersonic speeds compared to rocket engines. Figure 1.1 shows a schematic view of a scramjet. In a typical scramjet, the flow is decelerated by the inlet, which transforms the high kinetic energy of the flow into enthalpy. In the isolator-combustor region, further compression occurs and fuel is
added so it is ideally completely mixed with air at the burner inlet but remains supersonic. Finally, the combustion products are expanded in the nozzle.

A typical scramjet thermodynamic cycle is depicted in Fig. 1.2. In this cycle, the working fluid (air) is compressed adiabatically from stations 0 to 2 by the inlet with a series of shock waves. The isolator, station 2 to 3, serves to provide a further compression necessary for appropriate combustion and to prevent the back pressure from the combustor from choking the inlet flow, thereby causing unstart. In the combustor, stations 3 to 4, the heat is added to the flow at constant pressure and the combustion products are expanded adiabatically from stations 4 to 10.

1.1 Inlet-Related Issues

Amongst others, the function of a scramjet inlet is to decelerate and compress the air for further reaction in the combustor. The inlet must maintain adequate mass flow for the entire mission, in order to meet the thrust requirement. It is desirable that the inlet be light weight and that its geometry be capable of producing a uniform flow in an appropriate state to permit an efficient mixing and subsequent combustion. The auto-ignition limits of a hydrogen-air mixture, for example, correspond to pressures of 25–100 kPa and temperatures of 1000–2000 K [14]. The inlet should satisfy these requirements through a substantial part of the trajectory.
1.1.1 Starting

From engine cycle analysis, one can conclude that high contraction ratios CR are desirable for achieving a high engine overall efficiency. Indeed, practical inlets have CRs very close to the physical limit, namely, the isentropic line [6]. However, at low altitudes, during the acceleration phase, the maximum allowable CR is reduced due to the low flight Mach number. Such situation can potentially lead to inlet unstart if no precautions are taken. Unstart is an unsteady phenomenon due to the failure of the engine to process the ingested flow. It begins with shock train formation that travels upstream through the engine, since not all the mass flow ingested is being exhausted [15]. In a typical unstart, a bow shock wave is formed in front of the engine inlet, bringing the flow inside the engine to subsonic.

The Kantrowitz quasi-one-dimensional flow model was used as the basis for obtaining high CRs in early inlet design. This model imposes that a normal shock
is be formed in the inlet throat when the CR is maximum for a given flight Mach number $M_0$, choking the mass flow. With this assumption, the limiting area ratio can be obtained by [16]:

$$\frac{A_0}{A_2}_{KZ} = [2(\gamma_0 + 1)]^{-\frac{\gamma_0+1}{2(1-\gamma_0)}} \left[ \frac{2 + (\gamma_0 - 1)M_0^2}{2\gamma_0M_0^2 - (\gamma_0 - 1)} \right]^{1/2} \left\{ \frac{\gamma_0 - 1}{2} \left[ \frac{2 + (\gamma_0 - 1)M_0^2}{2\gamma_0M_0^2 - (\gamma_0 - 1)} \right] + 1 \right\}^{-\frac{\gamma_0+1}{2(1-\gamma_0)}} \tag{1.1}$$

where $\gamma_0$ is the ratio of specific heats, at free stream conditions. In Fig. 1.3, one can see the variation of the maximum CR for several Mach numbers, according to the Kantrowitz theory.

The Kantrowitz limit proved to be very conservative and it limits the compression ratio to intolerable values in terms of cycle efficiency. Also, it is obvious that the isentropic line, which is the CR theoretical upper limit, obtained by an isentropic compression, does not represent the real case with the presence of boundary layers, dissipating flow energy and increasing entropy and even shock waves. This isentropic line can be obtained via:

$$\frac{A_0}{A_2}_{IS} = \left[ M_0 \left( \frac{\gamma_0 + 1}{2} \right)^{\frac{\gamma_0+1}{\gamma_0(\gamma_0-1)}} \left( 1 + \frac{\gamma_0 - 1}{2} M_0^2 \right)^{-\frac{\gamma_0+1}{\gamma_0(\gamma_0-1)}} \right]^{-1} \tag{1.2}$$

and is also shown in Fig. 1.3.

Thus, for practical applications, for a fixed geometry inlet, the CR limit is usually estimated based on experimental data gathered from several sources, the CR can be estimated with the trend relationship [17]:

$$\frac{A_0}{A_2}_{Exp} = \left( 0.05 - \frac{0.52}{M_0^2} + \frac{3.65}{M_0^4} \right)^{-1} \tag{1.3}$$

As one can see, the practical maximum inlet CR is very close to isentropic line, making unstarting a serious issue for the inlet designer.
In addition, the potential for unstart is caused not only by high CRs but it could be caused by an increase in combustor pressure and temperature, which decreases the mass flow rate inside that component. If there is a mismatch between this mass flow and that which the inlet is providing, there will be unstart in the absence of flow control mechanisms.

To circumvent unstart, several options were presented in the open literature to date. For instance, a port could be used to isolate the inlet from the flow [18] during the early flight phase at low Mach number. A second one was to adopt a sidewall compression to allow flow spillage of part of the compressed flow [19]. Also, variable throat area can be used to increase mass flow.
1.1.2 Air Chemistry

At hypersonic speeds, temperature changes are pronounced due to flow deceleration in the inlet. Indeed, the fluid properties are very different from the perfect gas assumption. For instance, the ratio of specific heats $\gamma$ and the specific heats themselves can vary throughout the entire body of a hypersonic vehicle. Therefore, the flow analysis becomes more complex and relies heavily on computational solutions due to the complexity of the equations.

The high temperatures can produce the so-called real gas effects in air [20]. At about 800 K, we have the excitation of vibrational modes of O$_2$ and N$_2$. When the temperature is raised to 2000 K, dissociation of O$_2$ starts and ends close to 4000 K. At this temperature, dissociation of N$_2$ also becomes significant. Near 9000 K, N$_2$ almost no longer exists. From 2000 to 9000 K, NO molecules are also formed, along with ions O$^+$, N$^+$ and e$^-$. 

Note that at temperatures below 800 K, the air is usually assumed as calorically perfect gas, described by $p = \rho RT$, in which $c_p$ and $\gamma$ are constants. For temperatures above 800 K and lower than 2000 K, statistical quantum mechanics allows us to estimate the energy fraction that goes to the vibrational mode of O$_2$ and N$_2$. In fact, at these temperatures, air can be treated as a thermally perfect gas, which can also be described by $p = \rho RT$, while $c_p$ and $\gamma$ are functions of temperature only.

When dissociation occurs, the formation of new species takes place in the air mixture as mentioned above. The air composition is then a function of both pressure and temperature. Moreover, if air is subject to fast changes in the properties, there is chemical non-equilibrium, where chemical reaction kinetics are crucial to the evaluation of these properties. In order to evaluate the thermochemical state of air under such conditions, several finite rate mechanisms were described in the literature [21]. Simplifying assumptions can be made under certain circumstances.
The first is known as equilibrium where the reactions occur much faster than the characteristic flow speed. The second is the frozen conditions where the composition is assumed to be constant throughout the fluid process.

1.1.3 Viscous Effects

Another aspect of hypersonic flow is that viscous effects are more pronounced due to the high temperature and the potential occurrence of shock-wave/boundary-layer interactions. For high speed inlet design, boundary-layer analysis becomes of paramount importance since high contraction ratios are desirable but the large boundary-layer displacement can obstruct mass ingestion thereby promoting the possibility of unstart. Also, the kinetic energy dissipation in the thermal load must be accounted in the engine design.

In cases where the boundary layer is fully transitioned, the heat transfer rate and skin friction can be as large as three times the laminar case [22]. The transition from laminar to turbulent transition is still a matter of research. It has been shown that high speed flow transition is very sensitive to external disturbances, freestream turbulence and wall temperature. Therefore, it is generally accepted that trustworthy data can be only obtained from flight tests. In trying to detect boundary-layer transition, several ways have been reported such as the use of heat transfer gages, thermocouples, skin friction gauges [23], or even photodiodes [24].

In a trajectory corresponding to high Reynolds numbers, which is the case in the airbreathing corridor, it is commonly assumed for design purposes that the boundary layer is fully transitioned to the turbulent regime in the first stations. With that in mind, estimations of the flow properties within the boundary layer can only be made with the use of a turbulence model. Among them, the most common model types are the eddy viscosity model, where the turbulence is accounted for by
adding an additional term to the transport coefficients. The short length requirement of inlets can produce large adverse pressure gradients, potentially causing the boundary layer to separate. If a separation zone is formed, it creates additional shock waves, elevated heat transfer in reattachment points, high acoustic loads, and adds aerodynamic contraction to the inlet [17]. Even for cases in which the flow separation is two-dimensional, the flow structure can be very complex. For instance, a shock induced separation, as depicted in Fig. 1.4, can form a recirculation zone creating compression and expansion waves near that region. If the geometry permits, three-dimensional flow separations can also take place, producing very complex flow structures. Of special interest is the case of three-dimensional flows around sidewalls of two-dimensional inlets. When the oblique shock wave from the inlet ramp intersects the boundary layer, transverse pressure gradients appear and generate cross flows. Shock waves impinge the sidewall and interact with the boundary layer.

Figure 1.4. Flow structure schematics for a shock-induced two-dimensional boundary layer separation [25].
1.2 Review and Objectives

From the above, due to the existence of numerous and yet not completely described phenomena related to the hypersonic flight, the ability to control the inlet flow has sparked continued research interest. Flow control techniques have been used from subsonic to hypersonic regime, and several devices have been studied, such as micro-vortex generators (MVGs) to mitigate shock-wave/boundary-layer interactions [26], and magnetohydrodynamic (MHD) actuators [27]. Mechanical actuators, however, are simpler to design, construct, and model, in order to predict their effects.

In the present investigation, an inward turning inlet was adopted where some spillage is allowed, with variable sidewall geometry, to avoid unstart while ensuring flow uniformity and good pressure recovery. Assessment of the flow quality produced by this configuration is needed since the shocks formed by the side cowls and their interactions with the main shocks generate a complex three-dimensional flow field. In two-dimensional inlets, the sidewalls represent added internal flow path, contributing to boundary-layer growth. A schematic of the side spillage is shown in Fig. 1.5.

The adoption of an inward turning geometry for the inlet is justified by the fact that it minimizes booster dynamics effects on the inlet performance caused by slight variations of the angle of attack, such as mass capture and pressure at the combustor entrance. Moreover, a symmetric ramp system can yield a structure with low products of inertia, which makes any development of the control system easier. This is preferable also when spin-stabilized flight is used [28]. Another advantage of this type of configuration is that it uses a shorter length for compression than two-dimensional asymmetric geometries. This reduces not only the structural weight but reduces also the flow path, therefore the boundary-layer thickness and the related displaced mass capture. Based on similar arguments to those stated above, an inward inlet was adopted for the Hypersonic International Flight Research and Experimen-
In this flight test program, the intent was to make a captive scramjet engine test in the acceleration phase from Mach 5.5 to 8.5. The test time was estimated to be 7 to 9 seconds. Finally, another aspect that is advantageous for this kind of inlet is the absence of type IV Edney shock-shock interaction in the cowl region which is present in asymmetric designs and constitutes a major heat protection problem. This shock-shock interaction is formed when an oblique shock wave intersects a bow shock wave, the heat transfer rate in the stagnation point can destroy the cowl structure in seconds without a robust heat protection. Rather, in this kind of geometry, the flow can present types I or II shock-shock interaction. In type I, weak shock waves generated by high Mach numbers or by a slight turn of the forebody intersect as described in Fig. 1.6(a). The interaction is so that the flow in all regions remains supersonic. Since the incoming flow is processed differently in the upper and lower regions, a slip line is formed between regions R3 and R4, creating a region of concentrated vorticity. In type II Edney interaction, see Fig. 1.6(b), the shock waves in the forebody are such that the flow downstream the intersection
forms a subsonic region, R5. If type II interaction is formed, it would produce a substantial loss in the mass capture and pressure recovery of the inlet.

Concerning the sidewall interaction in internal flow fields, Fisher [29] presented an experimental study at Mach 3.05, evidencing some characteristics of the three-dimensional flow in regions near the sidewalls. In that study, the author used total pressure rakes and surface flow visualization using pigmented oil. He suggested the existence of a pair of streamwise vortices that propagates downstream. He also pointed out that these vortices are essentially due to the interaction of sidewall boundary layers with oblique shock waves. A more specific investigation, even though computational, was carried out by Gaitonde and Shang [30]. Two symmetric 15-deg. ramps were mounted on a flat plate, see Fig. 1.7. A free-stream Mach 4 flow formed a crossing shock wave system which interacts with the flat plate boundary layer, see Fig. 1.8. The authors concluded that four main regimes are present in such a flow: i) a separated boundary layer due to the ramps pressure gradient; ii) a shear layer formed by a vortex interaction flow and separated boundary layer; iii) a flow entrainment with high energy exchange near the regions affected by the vortex; iv) and on the center of the channel, the entire structure separates forming a pair
of secondary vortices. Zheltovodov and Maksimov [31] conducted experimental and numerical analysis of this same configuration, and angles of 7, 11 and 15-deg., at Mach numbers of 3.92 and 5, with significant refinement of the numerical modeling of the turbulent separated flow. But they concluded that numerical data fails to match experimental results as the shock strength grows. As a recommendation, they suggested a deeper analysis of the flow topology for these cases along with more accurate skin friction and heat transfer measurements in order to improve their turbulence model. The double fin configuration was also studied at hypersonic speeds. The work of Knight [32] details numerical simulations of such configurations at free stream Mach number of 8.3. He verified the existence of the basic features encountered in the supersonic flow case, namely, the main structure of the flow is a pair of counter-rotating vortices. The propagated flow is a complex system of shock waves and expansion fans that interacts with the flat plate boundary layer.

From the above, it is known that all the interactions caused by the sidewall geometry with possible boundary-layer separation and formation of vortices can cause a substantial loss of inlet compression efficiency, namely, flow distortion and total
pressure loss. Furthermore, the knowledge of the scramjet inlet flow field can only be attained by experiments and then used to validate numerical codes. The present work is an attempt to give some experimental data concerning the effects of side walls on the inlet performance at hypersonic Mach numbers, using the UTA’s Hypersonic Shock Tunnel (HST). However, the reader should bear in mind that is nearly impossible to simulate all flight parameters of a typical scramjet trajectory in ground facilities, which is due to the required high stagnation enthalpies and Reynolds numbers.

The main goals of the present research are presented below:

1) Design and construction of a modular scramjet inlet with instrumentation;

2) Perform experiments with free stream Mach number of 8.6 and stagnation enthalpy of 0.67 MJ/kg, take surface static pressure measure-
ments along the center line of the model and obtain the Mach number
distribution at inlet exit;

3) Compare the experimental data with numerical simulation, using
the commercial code ANSYS Fluent [33], and support experimental
analysis.
CHAPTER 2
INLET DESIGN AND GEOMETRY

The convergent inlet is composed of two symmetric ramps with a baseline geometry taken as a 12 deg. ramp with a 101.6 mm (4 in) entrance height and 50.8 mm (2 in) span. The design velocity was 2 km/s at an altitude of 30 km. The shock wave position and flow properties were calculated using the chemical equilibrium flow assumption and the viscous correction was applied using the Cebeci-Bradshaw algorithm [34], considering an over speed of 3 km/s, at the design altitude. Both trajectory points are in the airbreathing corridor which corresponds to dynamic pressures from 0.2 to 1.0 kPa [35].

2.1 Shock Wave System

The schematic for calculating the properties of regions behind the shock wave system formed on the inlet is shown in Fig. 2.1. The aim of this design is to obtain the correct position of the throat $L_x$ and size $L_t$, from given turning angle $\theta$, and capture height $L_y$.

From simple trigonometric relations, one can find:

$$L_y - L_t = L_x \tan \theta$$  \hspace{1cm} (2.1)

and,

$$L_x = \frac{L_y}{\tan \beta_1} + \frac{L_t}{\tan (\beta_2 - \theta)}$$  \hspace{1cm} (2.2)

thus,

$$L_t = L_y \frac{1/ \tan \theta - 1/ \tan \beta_1}{1/ \tan (\beta_2 - \theta) + 1/ \tan \theta}$$  \hspace{1cm} (2.3)
2.1.1 Equilibrium Oblique Shock Waves

Consider an oblique shock wave with an angle $\beta$ and flow deflection $\theta$, with $A$ denoting upstream and $B$ denoting downstream conditions, and $An$ and $Bn$ for their normal components, respectively. The pressure $p$, temperature $T$, density $\rho$, specific enthalpy $h$ and velocity $u$ can be found from a control volume analysis of the flow across this oblique shock wave, which yields for mass conservation [20]:

$$\rho_A u_{An} = \rho_B u_{Bn}$$  \hspace{1cm} (2.4)

for momentum:

$$p_A + \rho_A u_{An}^2 = p_B + \rho_B u_{Bn}^2$$  \hspace{1cm} (2.5)

and for energy:

$$h_A + \frac{u_{An}^2}{2} = h_B + \frac{u_{Bn}^2}{2}$$  \hspace{1cm} (2.6)

where,

$$u_{An} = u_A \sin \beta$$  \hspace{1cm} (2.7)

and,

$$u_{Bn} = u_B \sin(\beta - \theta)$$  \hspace{1cm} (2.8)
Since the tangential component of the velocity is unaltered one can use the continuity equation to obtain:

\[ \rho_B \tan(\beta - \theta) = \rho_A \tan \beta \quad (2.9) \]

Moreover, the gas state equation is:

\[ h_B = h_B(p_B, \rho_B) \quad (2.10) \]

The numerical procedure is started by an estimate of the density ratio, from perfect gas oblique shock relations in Appendix A:

\[ \frac{\rho_B}{\rho_A} = \epsilon \quad (2.11) \]

Thus, using Eq. (2.9):

\[ \tan(\beta - \theta) = \frac{\tan \beta}{\epsilon} \quad (2.12) \]

Applying the trigonometric identity:

\[ \tan(\beta - \theta) = \frac{\tan \beta - \tan \theta}{1 + \tan \beta \tan \theta} \quad (2.13) \]

one obtains:

\[ \tan \theta \tan^2 \beta - (\epsilon - 1) \tan \beta + \epsilon \tan \theta = 0 \quad (2.14) \]

Solving for \( \beta \) and choosing the weak solution:

\[ \beta = \tan^{-1} \left[ \frac{\epsilon - 1 - \sqrt{(\epsilon - 1)^2 - 4 \epsilon \tan^2 \theta}}{2 \tan \theta} \right] \quad (2.15) \]

Therefore, with Eq. (2.7), we can calculate:

\[ u_{An} = u_A \sin \beta \quad (2.16) \]

and, with Eq. (2.8):

\[ u_{Bn} = \frac{u_{An}}{\epsilon} \quad (2.17) \]
which permits us to obtain the post shock pressure via Eq. (2.5):

\[ p_B = p_A + \rho_A u_{An}^2 - \rho_B u_{Bn}^2 \]  

(2.18)

The computed specific enthalpy downstream the shock can be obtained via Eq. (2.6):

\[ h_{B\text{comp}} = h_A + \frac{u_{An}^2}{2} - \frac{u_{Bn}^2}{2} \]  

(2.19)

The iteration uses the secant method to find the correct \( \epsilon \) and continues until the criterion below is satisfied:

\[ |h_B - h_{B\text{comp}}| \leq 10^{-8} \]  

(2.20)

With the procedure described above and implemented in a Matlab code, listed in Appendix F, we were able to obtain the flow properties at inlet regions 0, 1 and 2, for the flight speeds of 2 and 3 km/s, see Tables 2.1 and 2.2, respectively.

Table 2.1. Calculated properties assuming equilibrium condition, for 2 km/s flight speed.

<table>
<thead>
<tr>
<th>Region</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure (Pa)</td>
<td>1171.9</td>
<td>6033</td>
<td>20698</td>
</tr>
<tr>
<td>Temperature (K)</td>
<td>226.7</td>
<td>408.5</td>
<td>597.1</td>
</tr>
<tr>
<td>Mach number</td>
<td>6.62</td>
<td>4.74</td>
<td>3.73</td>
</tr>
</tbody>
</table>

Table 2.2. Calculated properties assuming equilibrium condition, for 3 km/s flight speed.

<table>
<thead>
<tr>
<th>Region</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure (Pa)</td>
<td>1171.9</td>
<td>10640</td>
<td>48049</td>
</tr>
<tr>
<td>Temperature (K)</td>
<td>226.7</td>
<td>548.3</td>
<td>896.7</td>
</tr>
<tr>
<td>Mach number</td>
<td>9.94</td>
<td>6.27</td>
<td>4.35</td>
</tr>
</tbody>
</table>
Finally, given that our design options are $L_y = 2$ in (50.8 mm) and $\theta = 12$ deg, and flight speed of 2 km/s at 30 km, we have: $\beta_1 = 18.7$ deg, $\beta_2 = 21.8$ deg, $L_t = 0.34$ in (8.46 mm) and $L_x = 7.82$ in (19.86 mm).

2.2 Viscous Correction

The Navier-Stokes equations are very complex and difficult to solve even with numerical methods. To circumvent this problem, it is common to adopt the reduced boundary layer equations. In this approach, the flow within the boundary layer is treated separately from the outer inviscid flow. Some of the assumptions are that in the layer between the surface and the edge flow, the pressure is constant across the normal from the surface and the no-slip condition is satisfied at the wall. As stated before, the boundary-layer analysis in this work used the Cebeci-Smith [34] formulation and algorithm, assuming a fully-transitioned boundary layer and frozen properties in region 1, according to the nomenclature in Fig. 2.1. The edge conditions were calculated from the inviscid, equilibrium air, shock-wave analysis discussed in previous section, but at an overspeed of 3 km/s at 30 km of altitude.

The objective of inlet viscous correction is to assure that the captured mass flow rate calculated in the inviscid case is also obtained in real conditions. This is because hypersonic boundary layers can be thick enough to produce problems in maintaining supersonic combustion and starting, due to the required large contraction ratio for scramjets. The calculated displacement thickness $\delta^*$ is used to account for the “displaced mass” from the baseline geometry, see Fig. 2.2. Thus, $L_{t,corr.} = L_t + \delta^*/\cos \theta$.

The steady boundary-layer equations for a two-dimensional turbulent flow without body forces, neglecting terms involving density fluctuations can be written as [34]:

19
Figure 2.2. Viscous correction scheme applied to the original ramp.

\[
\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial x}(\rho v) = 0
\]  

(2.21)

\[
\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = -\frac{dp}{dx} + \frac{\partial}{\partial y} \left( \mu \frac{\partial u}{\partial y} - \rho u'v' \right)
\]  

(2.22)

\[
\rho u \frac{\partial h_t}{\partial x} + \rho v \frac{\partial h_t}{\partial y} = \frac{\partial}{\partial y} \left[ k \frac{\partial T}{\partial y} - c_p \rho \frac{T'v'}{T} + u \left( \mu \frac{\partial u}{\partial y} - \rho u'v' \right) \right]
\]  

(2.23)

with the eddy viscosity model defined by:

\[
-u'v' = \epsilon_m \frac{\partial u}{\partial y}
\]  

(2.24)

and,

\[
-T'v' = \epsilon_m \frac{\partial T}{Pr_t \frac{\partial y}{y}}
\]  

(2.25)

We used a modified Falkner-Skan transformation for compressible flows:

\[
d\eta = \sqrt{\frac{\mu_e}{\nu_e x}} \frac{\rho}{\rho_e} dy
\]  

(2.26)

\[\text{The Falkner-Skan transformation was originally applied for incompressible flows past wedges [36,37].}\]
and,
\[
\psi(x, y) = (\rho \mu u_e x)^{\frac{1}{2}} f(x, \eta)
\]  
(2.27)
to reduce the boundary-layer equations, Eqs. (2.21) to (2.23), into a system of non-linear ODEs.

With the condition of the continuity equation to be satisfied,
\[
\rho u = \frac{\partial \psi}{\partial y}
\]  
(2.28)
and,
\[
\rho v = -\frac{\partial \psi}{\partial x}
\]  
(2.29)
one can summarize the transformed momentum and energy equation as:
\[
(b f'')' + m_1 f f'' + m_2 \left[ c - f'' \right] = x \left( f' \frac{\partial f'}{\partial x} - f'' \frac{\partial f}{\partial x} \right)
\]  
(2.30)
and,
\[
(l r' + d f' f'')' + m_1 f r' = x \left( f' \frac{\partial g}{\partial x} - r' \frac{\partial f}{\partial x} \right)
\]  
(2.31)
with,
\[
b = C \left( 1 + e_m^+ \right)
\]  
(2.32)
where the Chapman-Rubesin factor is given by:
\[
C = \frac{\rho \mu}{\rho_e \mu_e}
\]  
(2.33)
and,
\[
c = \frac{\rho_e}{\rho}
\]  
(2.34)

\[
d = \frac{C u_e^2}{h_{te}} \left[ 1 - \frac{1}{Pr \epsilon_m^+} \left( 1 - \frac{1}{Pr_{turb}} \right) \right]
\]  
(2.35)

\[
l = \frac{C}{Pr} \left( 1 + \epsilon_m^+ \frac{Pr}{Pr_{turb}} \right)
\]  
(2.36)
\[m_1 = \frac{1}{2} \left[ 1 + m_2 \frac{x}{\rho_e \mu_e} \frac{d}{dx} (\rho_e \mu_e) \right] \quad (2.37)\]

\[m_2 = \frac{x}{u_e} \frac{du_e}{dx} \quad (2.38)\]

and,

\[\nu_e = \frac{\mu_e}{\rho_e} \quad (2.39)\]

The total enthalpy ratio is given by:

\[r = \frac{h_t}{h_{te}} \quad (2.40)\]

The Reynolds number at each station was calculated by:

\[Re_x = \frac{\rho_e u_e x}{\mu_e} \quad (2.41)\]

where the viscosity is given by Sutherland’s law for air:

\[\mu = 1.45 \times 10^{-6} \frac{T^{1.5}}{T + 110.39} \quad (2.42)\]

where \(T\) is in K and \(\mu\) is in kg/(m \cdot s). The term \(\epsilon_m^+\) accounts for the eddy viscosity and is given by:

\[\epsilon_m^+ = \frac{\rho}{\mu} \epsilon_m \quad (2.43)\]

The Prandtl number assumed \(Pr = 0.71\) and the turbulent Prandtl number \(Pr_{turb} = 0.9\).

Since the boundary conditions were defined for no wall transpiration:

\[f (x, 0) = 0 \quad (2.44)\]

and no slip:

\[f' (x, 0) = 0 \quad (2.45)\]
Compatibility with edge conditions yields:

\[ r(x, \eta_e) = 1 \] (2.46)

and,

\[ f'(x, \eta_e) = 1 \] (2.47)

Another boundary condition was the cold wall, with a specified constant temperature \( T_{wall} \). Thus the total enthalpy ratio \( w \) at the wall is given by:

\[ r(x, 0) = c_p T_{wall}/h_{te} \] (2.48)

After solving these equations, the displacement thickness was calculated via:

\[ \delta^* = \int_0^\delta \left( 1 - \frac{\rho u}{\rho_e u_e} \right) dy = \frac{x}{\sqrt{Re_x}} \int_0^\infty (c - f') d\eta \] (2.49)

Similarly, the momentum thickness can be evaluated by the expression:

\[ \theta = \int_0^\delta \frac{\rho u}{\rho_e u_e} \left( 1 - \frac{u}{u_e} \right) dy = \frac{x}{\sqrt{Re_x}} \int_0^\infty f'(1 - f') \ d\eta \] (2.50)

Another important output is the skin friction coefficient which is obtained by:

\[ c_f = \frac{\tau_{wall}}{\frac{1}{2} \rho_e u_e^2} = \frac{2 C_{\eta=0} f''_{\eta=0}}{\sqrt{Re_x}} \] (2.51)

Finally, the Stanton number at each station was evaluated by means of:

\[ St_x = \frac{\dot{q}_{wall}}{\rho_e u_e (h_{wall} - h_e)} = \frac{C_{\eta=0} r'_{\eta=0}}{Pr \sqrt{Re_x} (1 - r'_{\eta=0})} \] (2.52)

The original Cebeci-Bradshaw code for solving Eqs. (2.30) and (2.31), with boundary conditions given by Eqs. (2.44) to (2.48), was written in Fortran IV and was translated to Matlab by the author. Therefore, a brief description will be provided. More detailed explanation can be found in Ref. 34. The program uses a finite-difference solution of the two-dimensional boundary-layer equations with the
Figure 2.3. Skin friction coefficient as function of the distance, for a uniform flow over a flat plate. Free stream conditions are $M_0 = 4$, $u_0 = 1390 \text{ m/s}$ and $p_0 = 2500 \text{ Pa}$.

block elimination method. With the edge properties of the flow provided, an initial guess is obtained by solving the incompressible case. Then the coefficients of the linearized momentum and energy equations are calculated and the code solves the resulting linear system, and, finally, the desired outputs are calculated.

The inputs are the Mach number, temperature, pressure, and the fluid properties and the boundary condition type, specified wall temperature or heat flux, and their values. The outputs are the calculated parameters: displacement thickness, momentum thickness, skin friction coefficient and the Stanton number, at each sta-
Figure 2.4. Calculated displacement thickness variation.

tion. For the sake of completeness, a test case defined by a flat plate in a Mach 4 flow was used to validate the translated code against the Van-Dries II expression for the skin friction coefficient distribution, see Fig. 2.3. In this test case, adiabatic and cold wall conditions were compared and good agreement was achieved with less than 6% of deviation for this coefficient, when evaluated at the flat plate end.

With the geometric and flow data obtained using the procedures outlined in Section 2.1, we were able to analyze how the key boundary layer parameters vary with the surface temperature $T_{wall}$ at the trajectory point of 3 km/s at 30 km of altitude. Comparisons between the displacement thickness, heat flux and skin friction coefficient are provided for three different wall temperature values of 300, 1000 and
2000 K in Figs. 2.4 to 2.6 and Table 2.3. As one can see, as the wall temperature increases, from 300 to 2000 K, the displacement thickness can increase more than 50% at the end of the inlet, from 1.59 to 2.48 mm. This corresponds to nearly 30% of the throat height obtained in Section 2.1. Moreover, as the wall temperature is decreased, the heat flux can increase from 426.2 to 841.8 kW/m² for the cases of 2000 and 300 K, respectively. One can realize that any heat protection mechanism is benefited if higher temperatures are allowed in the outer layers of the structure. Finally, the skin friction shows a slight variation in comparison with the previous parameters, varying from 0.00165 to 0.00143, decreasing from 300 to 2000 K.

Figure 2.5. Heat flux distribution along the ramp, in tangential coordinate.
Figure 2.6. Skin friction variation as a function of the tangential distance from the leading edge to throat.

Table 2.3. Summary of boundary layer properties at the end of ramps.

<table>
<thead>
<tr>
<th>Wall temperature (K)</th>
<th>Displacement thickness (mm)</th>
<th>Skin friction coefficient</th>
<th>Wall heat flux (kW/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>2.43</td>
<td>0.00143</td>
<td>426</td>
</tr>
<tr>
<td>1000</td>
<td>1.98</td>
<td>0.00157</td>
<td>665</td>
</tr>
<tr>
<td>300</td>
<td>1.59</td>
<td>0.00165</td>
<td>841</td>
</tr>
</tbody>
</table>
From the above results, to account for the displaced mass flux due to viscous effects, the calculated displacement thickness at the end of the ramp was used as a relief parameter in the throat height, as depicted in Fig. 2.2. As a design option, a wall temperature of 1000 K was assumed. This corresponds to a correction of less than 0.5 deg in the original ramp angle of 12 deg.

2.3 Test Article

The resulting geometry is depicted in Figs. 2.7, 2.8, 2.9 and 2.10. In Fig. 2.7, a side view shows the configuration adopted, the entrance and the throat heights, and the rake position. The model consists of three main parts: an instrumented upper ramp, a lower ramp and the rake mechanism. A wedge was chosen to hold the total pressure rake at the exit. The upper ramp was closed by two side walls, two plates at the top and a rear wall that allows the transducers wiring to go out from the model. The transducers positions are shown in Fig. 2.8. Also, in that figure one can see the position of the side cowls. A rendering of the CAD model used for construction assembled in test section can be seen in Fig. 2.9. Actual pictures of the machined model are shown in Fig. 2.10. This model has a total length of 304.8 mm and a span of 50.8 mm with the compression at 11.54 degrees. The throat height is 21.22 mm, thus CR = 4.79. The rake probes used 3.175 mm diameter stainless steel tubes in order to minimize perturbations on the inlet exit.
Figure 2.7. Side view of the model.

Figure 2.8. Instrumented ramp top view.
Figure 2.9. Rendering of model inside test section.

Figure 2.10. Model photographs. Top right: model in test section with impact pressure probe. Bottom right: model alone. Bottom left: Pitot rake. Top left: closeup of inlet model.
CHAPTER 3
NUMERICAL MODELING OF INLET FLOW

3.1 Current Commercial Codes for Hypersonic Regime

Computational fluid dynamics (CFD) can provide reasonable data for several problems. However, in the hypersonic regime, the correct choice of a code requires some previous knowledge about the phenomena to be investigated. This is because, up to date, no powerful CFD code exists that is able to describe the entire physics in a hypersonic vehicle. Indeed, for high Mach numbers, CFD tools tend to be more specialized, aiming for specific problems like external or internal flows, flow with finite rate chemistry, etc.

Available commercial codes properly validated for cases in hypersonic speeds are: GASP [38], CFD++ [39] and Fluent [33]. The next paragraphs give short descriptions about them and their usage in high speed aerodynamics.

Aerosoft’s GASP [38] is a multi-block, structured or unstructured, solver for the steady or unsteady Reynolds-averaged Navier-Stokes (RANS). It has options to solve parabolized Navier-Stokes (PNS), thin-layer Navier-Stokes (TLNS), the Euler equations and incompressible Navier-Stokes equations. Turbulence models included are: the Baldwin-Lomax, the Spalart-Allmaras one-equation, $K-\epsilon$, Wilcox $K-\omega$ with compressibility correction,$^1$ $K$–length, SST model for $K-\omega$, Reynolds stress model, detached eddy simulation (DES) based on the Spalart-Allmaras (SA) model and SST models, and supports user-input intermittency values for transition modeling.

$^1$Without the compressibility correction, the $K-\omega$ model gives poor results for cold walls as the Mach number is increased in the hypersonic regime [40]
GASP also permits six DOF motion modeling, finite-rate chemistry, non-equilibrium thermodynamics, steady and unsteady time integration. Applications of GASP have been reported in ramjet and scramjet combustors [41–44], launch vehicles, re-entry vehicles [45], reacting flows [46], and subsonic, supersonic and hypersonic aircrafts [47].

Metacomp Technologies’s CFD++ [39] can solve steady and unsteady, compressible and incompressible flows, reacting flows, multiphase flows, conjugate heat transfer, porous media, and so on. Large eddy simulation (LES) models and hybrid LES/RANS models are also available. A method allows for the compressibility correction for turbulence production and dissipation. The code is structured/unstructured, and can have 4th-order accuracy in time. Also a finite-volume method is implemented, the code features a second-order total variation diminishing (TVD) polynomial for spatial discretization. In this code, viscous fluxes calculations use the same TVD polynomial and inviscid fluxes are calculated by the Harten, Lax and Van Leer scheme. The code allows the use of various elements such as hexahedral, triangular prism, pyramid and tetrahedral in 3D, quadrilateral and triangular elements in 2D, and line elements in 1D. CFD++ has been validated in the high-speed regime, according to several works [48–50].

ANSYS Fluent [33] is capable of solving the steady/unsteady, 2D/3D Navier-Stokes equations with a pressure-based or a density-based solvers. The latter one is used for high-speed flows. ANSYS Fluent runs in all flow types from low subsonic to hypersonic, laminar or turbulent flows, and ideal or real gases. It is a second-order finite volume solver that uses unstructured meshes, which can be triangular, quadrilateral, tetrahedral, hexahedral, etc. Several turbulence models are present in the code, these include $K-\epsilon$ and $K-\omega$ and Reynolds stress models. The code emphasizes the shear stress transport (SST) turbulence model, because, according
to the developer, it offers advantages for non-equilibrium turbulent boundary layer flows and heat transfer estimations. ANSYS Fluent also has capabilities for laminar-to-turbulent transition. In addition, ANSYS Fluent has large- and detached-eddy simulation (LES and DES) models, and a scale-adaptive simulation (SAS) model is a highlight. Applications in hypersonic regime have been reported for external or internal, reactive or non-reactive flow fields [11,51–53].

3.2 Reynolds-averaged Navier-Stokes (RANS)

The RANS equations are the most widely used when dealing with turbulent flows. This is due to their low computational cost and versatility. In conventional Reynolds decomposition, flow properties are decomposed in a fluctuating variable, specified by the prime superscript, and in a time average of this property, specified by an overbar. For a Cartesian coordinate system [54]:

\[
\begin{align*}
    u &= \overline{u} + u' \\
    v &= \overline{v} + v' \\
    w &= \overline{w} + w' \\
    \rho &= \overline{\rho} + \rho' \\
    p &= \overline{p} + p' \\
    h &= \overline{h} + h' \\
    T &= \overline{T} + T' \\
    h_t &= \overline{h_t} + h'_t
\end{align*}
\] (3.1)

However, for compressible flows, where the density changes, it is convenient to use Favre-averaged variables, defined by:

\[
\begin{align*}
    \tilde{u} &= \frac{\rho u}{\rho} \\
    \tilde{v} &= \frac{\rho v}{\rho} \\
    \tilde{w} &= \frac{\rho w}{\rho} \\
    \tilde{h} &= \frac{\rho h}{\rho} \\
    \tilde{T} &= \frac{\rho T}{\rho} \\
    \tilde{h}_t &= \frac{\rho h_t}{\rho}
\end{align*}
\] (3.2)

Thus, the fluctuating components of these new variables are given by:

\[
\begin{align*}
    u &= \tilde{u} + u'' \\
    v &= \tilde{v} + v'' \\
    w &= \tilde{w} + w'' \\
    \rho &= \tilde{\rho} + \rho'' \\
    p &= \tilde{p} + p'' \\
    h &= \tilde{h} + h'' \\
    T &= \tilde{T} + T'' \\
    h_t &= \tilde{h}_t + h''_t
\end{align*}
\] (3.3)

Applying the mass-weighted variables into Navier-Stokes equations, one can get for the continuity equation:

\[
\frac{\partial \tilde{\rho}}{\partial t} + \frac{\partial}{\partial x_i} (\tilde{\rho} \tilde{u}_i) = 0
\] (3.4)
for the momentum equation:

$$\frac{\partial}{\partial t}(\rho \tilde{u}_i) + \frac{\partial}{\partial x_j}(\rho \tilde{u}_i \tilde{u}_j) = -\frac{\partial \bar{p}}{\partial x_i} + \frac{\partial}{\partial x_j}(\tau_{ij} - \rho u''_i u''_j)$$  \hspace{1cm} (3.5)$$

and for the energy equation:

$$\frac{\partial}{\partial t}(\rho h_t) + \frac{\partial}{\partial x_j}(\rho \tilde{u}_j \tilde{h}_t + \rho u''_j h''_t - k \frac{\partial T}{\partial x_j}) = \frac{\partial \bar{p}}{\partial t} + \frac{\partial}{\partial x_j}(\tilde{u}_j \tau_{ij} + u''_i \tau_{ij})$$  \hspace{1cm} (3.6)$$

where,

$$\tau_{ij} = \mu \left( \frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i} - \frac{2}{3} \delta_{ij} \frac{\partial \tilde{u}_l}{\partial x_l} \right) + \mu \left( \frac{\partial u''_i}{\partial x_j} + \frac{\partial u''_j}{\partial x_i} - \frac{2}{3} \delta_{ij} \frac{\partial u''_l}{\partial x_l} \right)$$  \hspace{1cm} (3.7)$$

3.3 Wilcox $K-\omega$ Turbulence Model

The Wilcox $K-\omega$ model is a robust model that handles with adverse pressure gradients well and it is integrated with the laminar sublayer [55]. This is a two-equation turbulence model where the length scale is obtained by solving two coupled transport equations, one for the turbulent kinetic energy (TKE), $K$, and other for the specific dissipation rate (SDR), $\omega$. For a proper utilization of this model, the near wall grid must be refined enough to result in a $y^+ \leq O(1)$, where:

$$y^+ = \frac{yu_*}{\nu}$$  \hspace{1cm} (3.8)$$

and,

$$u_* = \sqrt{\frac{\tau_{\text{wall}}}{\bar{p}}}$$  \hspace{1cm} (3.9)$$

The turbulent kinetic energy is obtained from [33]:

$$\frac{\partial (\bar{p} \tilde{K})}{\partial t} + \frac{\partial (\bar{p} \tilde{K} \tilde{u}_j)}{\partial x_j} = \frac{\partial}{\partial x_j} \left( \Gamma_K \frac{\partial \tilde{K}}{\partial x_j} \right) + G_K - Y_K + S_K$$  \hspace{1cm} (3.10)$$

and the specific dissipation rate from:

$$\frac{\partial (\bar{p} \tilde{\omega})}{\partial t} + \frac{\partial (\bar{p} \tilde{\omega} \tilde{u}_j)}{\partial x_j} = \frac{\partial}{\partial x_j} \left( \Gamma_\omega \frac{\partial \tilde{\omega}}{\partial x_j} \right) + G_\omega - Y_\omega + S_\omega$$  \hspace{1cm} (3.11)$$
where the terms $G_K$, $Y_K$ and $S_K$ represents the generation, diffusion and dissipation of $K$, respectively. Similarly, the corresponding terms for $\omega$ are $G_\omega$, $Y_\omega$ and $S_\omega$. See Appendix D for details about this turbulence model implementation in ANSYS Fluent.

3.4 ANSYS Fluent Setup

The steady Navier-Stokes equations were solved with using ANSYS Fluent, Academic Research, Release 14.0 [33]. The mesh domain followed the geometry discussed in Section 2.3, with addition of a lateral free stream part to capture some effects of the spillage, see Fig. 3.1. For saving computation time, just a quarter of the inlet geometry was modeled.

The unstructured mesh was generated using the ANSYS meshing tool, with tetrahedral elements. The coarse, medium and fine meshes had 205,427, 343,561 and 503,450 nodes respectively. A very fine mesh was achieved with 855,657 nodes. In this case, the element size was fixed at 0.5 mm at wall faces with a maximum allowable element size of 2 mm in inner regions; the growth rate per layer of element was set up to 1.4. With this configuration, we achieved about 12 nodes to describe the boundary layer, estimated in Section 2.2, see Fig. 3.2. This mesh generated a total number of 4,877,898 elements. A mesh independence study was conducted by observing the $y^+$ value along the centerline of the ramp, see Fig. 3.3. The $y^+$ value of the order of unity shows that a reasonable resolution was found for the turbulent boundary layer.

A thermally perfect gas assumption along with the Wilcox $K-\omega$ turbulence model was applied to evaluate the flow properties inside the inlet. The solver option was density based and the Green-Gauss cell based method was applied to calculate
Figure 3.1. Example of computational domain for simulations.

gradients. Roe approximation to the numerical flux was also used. The CFL number was varied from 0.2 at the start of the computations to 0.5 near convergence.

The boundary conditions were type pressure-far-field for the free stream regions, pressure outlet for the exit plane, symmetry for the planes of symmetry, and non-slip, cold wall at room temperature at the walls.

Figure 3.2. Near wall mesh.
Figure 3.3. $y^+$ variation along centerline.
CHAPTER 4

UTA SHOCK TUNNEL FACILITY

The UTA Hypersonic Shock Tunnel facility is depicted in Figs. 4.1, 4.2 and 4.3, and its details will be described below. This facility can be operated with a cold driver gas or with a shock-induced, detonation-driver, the later to reach high enthalpies amenable for hypersonic flight paths [56, 57].

Figure 4.1. Tunnel CAD representation.
4.1 Driver Section

The driver section is a 4043 steel tube with an internal diameter of 152.4 mm and a length of 3.0 m, with a wall thickness of 25.4 mm, see Fig. 4.4. At one end, outside the laboratory room, it has a closed semi-spherical cap and at the other, it has a 482.6 mm diameter flange with 8 holes for 50.8 mm (2 in) bolts. The flanges have two o-ring grooves for sealing purposes. Buna-n o-rings of 0.25 in diameter round cross section were used on all them. Also, silicone grease was used for obtaining a good seal. To permit longitudinal movement such as for diaphragm replacement, the driver is attached to a hydraulic jack, shown in Fig. 4.5. A manual hydraulic pump is used to actuate the jack. The hydraulic system has a four-way valve to allow the working fluid from the pump to reach the jack and a switch for choosing which sense
the tunnel will move. To lock the tunnel position and absorb the recoil produced by each test, two 50.8 mm (2 in) pins must be placed in the thrust stand, at positions also shown in Fig. 4.5.

Usually the driver uses air as the working gas, but it can be filled with different gases. The maximum allowable pressure of this tube is 41.4 MPa (6000 psi). Also, two spark plug housings and connections are placed on the outside part of this section for detonation driver utilization.

4.2 Double Diaphragm Section (DDS)

The DDS separates the driver and the driven tubes and accommodates two diaphragms, one at each end, see Fig. 4.6. This section works with half the driver pressure and its made of 4043 steel as well. The section is 114.3 mm long and has a diameter of 482.6 mm. Each experiment is initiated by venting this section by a remotely operated diaphragm valve. The cut sides of the diaphragms were always
placed facing the driven section. Moreover, the DDS must be tightened enough to avoid leaks of the pressurized air. Care was taken also to ensure the o-rings surfaces are lubricated.

4.2.1 Diaphragms

The proper opening of the DDS diaphragms is very important to a reliable test. Premature or partial rupture of them can denigrate the formation of the shock wave and shape of contact surface as well. For a good rupture, an X-letter was scored on one side of the diaphragm. The necessary score depth is dependent on the differential pressure desired, type of material and plate thickness. The cross section of the cuts is also important. The work of Ben-Dor [60] can be used to estimate the rupture pressure for a given score depth, which is given by:

\[
p_{\text{rupture}} = \sigma_u \left[ 4.2 \frac{t_d}{d_d} \left( \frac{t_d - s_d}{t_d} \right)^{2.2} \right] \pm 7.5\%
\]

(4.1)
where $\sigma_u$ is the ultimate tensile stress of the material, $t_d$ is the plate thickness, $d_d$ is the unsupported diameter of the plate and $s_d$ is the score depth.

The expression above is valuable for a preliminary estimate of the needed score depth but slight variations of material properties and usage of different end mills to make the scores can produce unacceptable variations of the rupture pressure so the proper scores were determined experimentally. Figure 4.7 shows a scored diaphragm before and after a burst. An improper diaphragm rupture is characterized by missing petals or a non-ruptured region. The lost petals can travel in a high speed and could damage the tube section or even the test article.

Previous runs in the shock tunnel used Gauge 10 (3.42 mm) hot-rolled 1000 series steel plates for diaphragm materials, but in this case, several petals were lost due to the reflected shock wave. Bello and Lu [61] then concluded that cold-rolled 1000 series steel plates of Gauge 7 (4.55 mm thick) were more appropriate due their softness and thickness for driver pressures as high as 34.5 MPa (5000 psi). The score depths were 1.524 mm (0.06 in) and they were machined by CNC to obtain a very precise cut, since a manual milling machine would not be capable of keeping constant
cut properties, mainly due to improper cooling of the surface [62]. The CNC was operated with a 3/8 in mill and a speed of 1181 rpm and feed of 2.36 in/min, for drilling the four holes. After that, an end mill of 90° angle was placed to cut the scores with the desired depth, with 879 rpm and feed rate of 1.76 in/min. The end mill diameter was varied depending on the score depth, ranging from 1/4 to 1/2 in. The total time for all these processes was about 1 1/4 h. The technical drawing of a typical set of diaphragms is shown in Fig. 4.8, for a score depth of 1.524 mm (0.06 in).

The choice of the right diaphragm score was done after single diaphragm bursting tests when only one scored diaphragm was placed in the driven side and a non-scored plate on the driver side of the DDS, with this section filled up to observation of rupture. The results of these tests are shown in Fig. 4.9 the rupture pressure for several diaphragm scores and two gauges tested are plotted according to Eq. (4.1).
4.3 Driven Section

The driven section, Fig. 4.10, also has a bore of 152.4 mm but is 8.22 m long, with wall thickness of 25.4 mm. The maximum operating pressure is also 41.4 MPa (6000 psi). The driven section consists of 3 sections each 2.74 m long. Each tube has 482.6 mm diameter flanges with 8 holes for 50.8 mm (2 in) bolts at both ends. The first section, close to the DDS, has two 1/4 in tubes connected for the hydrogen and oxygen lines, used in the detonation-driven mode. This section and the middle section each has two housings for pressure sensors in the upper surface. The third section, close to the throat, has three housings for the pressure sensors used for taking shock tube properties. This section also has one 3/4 NPT connection for the vacuum
system and a 1/2 NPT connection for the 175 psi line. Each of this lines has one open-close hand ball valves and each one MUST BE CLOSED for the tests.

Each tube of the driven section is supported at two places by a circular adjustable cradle and fixed to an A-frame support with wheels. Four 1/2 in bolts are used to adjust the height of each supported place and four additional 1/2 in bolts are used to adjust the desired height. Two more bolts allow lateral alignment of these sections. The driven tube can be charged with any gas or combinations of gases. In this investigation, however, only air was used. Each time before a test, the driven tube was evacuated up to at least 7 mbar, with a dedicated vacuum pump, type Boekel HYVAC 14, which took about 15 min to reach this pressure. After pumping
down, the tube was remotely filled to the desired pressure using the dry air from a 175 psi line.

4.4 Interchangeable Throat Section (ITS)

The driven section is connected to the ITS at one end. The 4043 steel ITS is shown in Fig. 4.11. The ITS has a total length of 254 mm and it has a flange of 482.6 mm diameter, 127 mm long with 8 holes for 2 in bolts placement. Moreover, in one end it has a convergent part that fits into the internal diameter of the driven section, which is 76.2 mm long and is 151.4 mm in diameter, not captured in Fig. 4.11. At the other end, the ITS features a tailored profile for the throat locker, described in the following subsection.
4.4.1 Throat Locker

The throat locker is depicted in Fig. 4.12. The part is made also of 4043 steel and is 149.9 mm long. It has a maximum external diameter of 190.5 mm and a minimum internal diameter of 88.9 mm. Two 1/2 in bolts are attached to this section in order to ease twist, for a proper lock. From the unlock position to the locked position, a 45 deg rotation is necessary, the lock position is indicated by an arrow on its surface that matches with another arrow in the ITS. Also, for an easier handling of that part, silicone-based spray lubricant was used on both internal and external surfaces.

4.4.2 Throat Insert

The throat insert assembly is shown in Fig. 4.13, on one side it has two metal rings of 58.42 mm diameter, one ring has cutters to improve the diaphragm burst
profile and the other one has a convergent profile. A 0.254 mm thick mylar diaphragm is placed between these two rings and the set, detailed in Fig. 4.14, is held against the throat by a third threaded ring also shown in Fig. 4.14. At the other end of the throat insert, the throat part is placed, partially shown in Fig. 4.13. This part has a total length of 146 mm and a maximum diameter of 58.42 mm, the internal geometry is a convergent–divergent profile with a minimum diameter of 14.5 mm, and a divergent half angle of 7.5 degrees. Another part is used to hold the throat in the throat insert, far left in Fig. 4.13. This part is 139.7 mm long with maximum diameter of 76.2 mm. It matches with the throat part at one side, and has threads on the other, so a proper flange can be attached, which connects the whole driven tube to the nozzle. The assembly shown in Fig. 4.13 is placed and set inside the ITS by means of the throat locker. In order to connect the ITS to the divergent flange, the throat insert must be locked by 10-32 screws and then a 190.5 mm diameter flange is connected in the end of this part. This flange is also connected to the throat locker by four 7/16 in black oxide bolts, see Fig. 4.15. Four grade 5, 1/2 in bolts are used to tighten this flange to the divergent flange.
4.5 Conical Divergent Nozzle

The conical divergent nozzle is 2.1 m long, see Fig. 4.16. It has a starting diameter of 57.15 mm and has an expansion half-angle of 7.5 degree. It is connected to a final wood flange that adjusts the end diameter to the test section diameter. The nozzle has seven pressure taps used for previous investigations of the nozzle characteristics of this tunnel [57].

4.6 Test Section

The test section, also shown in Fig. 4.16, is 610 mm long and it has a maximum external diameter of 440 mm. It is a semi-free jet type and ends with a convergent conical diffuser, that contracts from 381 to 310 mm. Thus, it decelerates the flow.
before entering into the diffuser tube. Currently, three 1/2 in holes are present for model attachment if necessary, two of them in the upper surface and the third one in the lower part of the section. The test section has two access ports of 230 mm in diameter that can be used for the 203.2 mm diameter BK7 optical windows for flow visualization.

4.7 Diffuser and Dump Tank

Also shown in Fig. 4.16, another section follows the test section to accommodate the sting for Pitot probes and to be used as feedthrough for the instrumentation needed; the sting mounting section has five 101.6 mm diameter ports, is 610 mm long, and is connected to the diffuser near the wall. The diffuser is a 4 m long, 305 mm in diameter aluminum pipe that connects the sting mounting section to the dump tank. The dump tank is a 4.25 m³ vessel, 1.68 m in diameter and a height of 2.13 m, see Fig. 4.17. It has a 508 mm access port in order to allow cleaning. A safety valve also exists to limit the pressure rise from the test gases, otherwise this overpressure could damage the instrumentation or even tunnel parts, see Fig. 4.18. The tank is connected to the vacuum pump by means of a 2.6 m long, 3 in NTP Schedule 40 pipe. The vacuum pump is a two stage BOC Edwards E2M40 with an EH-250 booster. To
protect this pump from overpressure, a gate valve is used to shut off the connection between the pump and the dump tank. THIS MUST BE DONE BEFORE EACH TEST.

4.8 Subsystems

4.8.1 Pneumatic System

For using the hypersonic shock tunnel, a pneumatic system is used to control the diaphragm valves used for high pressure air, to control the pneumatic actuators
used in the detonation mode of operation, to control pneumatic valves in the vacuum board and to drive the pressure booster, see Fig. 4.20. In the low pressure line, 1.2 MPa (175 psi) compressed dry air is used. 24 V three-way solenoid valves control the low pressure lines, and they are remotely operated from the control room.

Five diaphragm valves are used for filling the driver and DDS, isolating the yellow 6000 psi storage sphere and venting the driver and DDS. In the detonation part, six pneumatic actuators are used to control the lines of fuel, oxygen, air, vacuum, transducer and vent.

The vacuum board houses the two vacuum gauges, four pneumatic actuators, and four solenoid valves and two excess flow valves. Using this board, it is possible to choose the vacuum transducer to be used, and to choose whether line it is measuring, driven tube or test section.
4.8.2 Auxiliary Board

The HST pneumatic system requires all the hand valves on the main and auxiliary boards to be in a proper position. The main board is located near the supersonic wind tunnel, while the auxiliary board is adjacent to the driver section of the HST, see Fig. 4.19.

4.8.3 Haskell Pump

The Haskell pump, model 55696 two-stage booster pump, is used to further compress the air from the high pressure line 14.5 MPa (2100 psi) to up to 41.4 MPa (6000 psi). It operates with a minimum pressure of 689.5 kPa (100 psi) and it is controlled from the control room. The setup necessary for the proper operation of this pump is shown in Table E.1, from Appendix E, for the case of simply filling the yellow sphere, and for the occasion of a test, in Table E.2, also in Appendix E.
Figure 4.20. Pneumatic system diagram.
4.9 Electrical Control System

The HST is remotely operated from the control board located in the control room, see Fig. 4.21. This board is powered by a 24 V supply and has several switches and displays. The displays are for driver, driven (INOP.) and DDS pressure, driver temperature and vacuum pressures. The on/off switches operate the solenoid valves, consequently they are proper labeled. However, for the sake of completeness, each switch and correspondent function is described in Table 4.1. The electrical connection diagram is shown in Fig. 4.22 for the switches wiring connections and in Fig. 4.23 for the display board wiring, both were adapted from Ref. [56].
Table 4.1. Functions of control room display switches

<table>
<thead>
<tr>
<th>Switch</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>M.SW.</td>
<td>Master switch</td>
</tr>
<tr>
<td>DRIVEN TUBE VACUUM SWITCH</td>
<td>vacuum level reading</td>
</tr>
<tr>
<td>HASK. PUMP</td>
<td>Haskell pump operation</td>
</tr>
<tr>
<td>DRVR. VENT</td>
<td>Driver relief</td>
</tr>
<tr>
<td>DRVR.</td>
<td>Driver filling</td>
</tr>
<tr>
<td>1(left)</td>
<td>vacuum transducer 1, driven tube vac.</td>
</tr>
<tr>
<td>2(left)</td>
<td>vacuum transducer 2, driven tube vac.</td>
</tr>
<tr>
<td>2</td>
<td>vacuum transducer 2, test section vac.</td>
</tr>
<tr>
<td>1</td>
<td>vacuum transducer 1, test section vac.</td>
</tr>
<tr>
<td>SPHE ISO.</td>
<td>for yellow sphere control</td>
</tr>
<tr>
<td>DIAP.VENT</td>
<td>vents DDS</td>
</tr>
<tr>
<td>DIAP.CHGE.</td>
<td>fill DDS, DRVR. must be ON</td>
</tr>
<tr>
<td>DRVN.</td>
<td>INOP., used before to control driven vac. pump</td>
</tr>
<tr>
<td>VAC.</td>
<td>INOP., used before to control driven vac. pump</td>
</tr>
<tr>
<td>DRVN. AIR</td>
<td>fills driven using the 175 psi line</td>
</tr>
<tr>
<td>DRVN. VENT</td>
<td>vents the driven tube</td>
</tr>
<tr>
<td>$H_2$</td>
<td>fills detonation driven with hydrogen</td>
</tr>
<tr>
<td>TRANS</td>
<td>open the line for the vac. transducer</td>
</tr>
<tr>
<td>AIR</td>
<td>INOP., fills detonation driven with air</td>
</tr>
<tr>
<td>$O_2.$</td>
<td>fills detonation driven with oxygen</td>
</tr>
<tr>
<td>VAC.</td>
<td>INOP., used before to control the vac. in det. sec.</td>
</tr>
<tr>
<td>VENT</td>
<td>vents detonation section</td>
</tr>
<tr>
<td>DET. DRIVER PRESS. SWITCH</td>
<td>vacuum level reading</td>
</tr>
<tr>
<td>RED BUTTON</td>
<td>INOP., operates driver spark plugs</td>
</tr>
</tbody>
</table>
Figure 4.22. Electronic control system schematic, adapted from Ref. [56].
Figure 4.23. Electronic display connections, adapted from Ref. [56].
CHAPTER 5

MODELING OF A GASEOUS, NON-REACTIVE DRIVER SHOCK TUBE

The high stagnation enthalpy and high dynamic pressure required for simulation of hypersonic flow make shock tunnel facilities the most cost effective tools for this purpose. The necessity of shock tunnel flow modeling is mainly due to the difficulty in measuring the stagnation and free stream properties in a very short test time. The UTA shock tunnel consists of a high pressure section (driver section) separated by means of a double diaphragm section from a low pressure section (driven section); when the diaphragms burst, compression waves are propagated toward the driven tube. Since each compression wave increases the local speed of sound, the compression wave train tends to coalesce into a shock wave. This shock wave propagates toward the driven tube increasing the gas pressure and temperature. Meanwhile, an unsteady expansion wave propagates into the driver. Once the shock wave reaches the throat, it is reflected. The reflected shock wave then increases the pressure and the temperature of the driven gas even more, generating the stagnation conditions of the nozzle flow. Similarly, the rarefaction wave is reflected off the closed end and propagates downstream towards the driven tube. When the reflected shock wave interacts with the contact surface, a Mach wave, a shock wave or a rarefaction wave can be reflected back, characterizing the tailored, over-tailored and under-tailored modes, respectively.
5.1 History

Shock tubes were introduced by Paul Vieille [63] in 1899. The shock tube equation, which relates the driver-to-driven pressure ratio and the shock wave pressure ratio, was presented first by Schardin [64], in 1932. In the United States, work in shock tubes was initiated by Prof. W. Bleakney at Princeton University [65], in 1942, for calibration of crystal pressure gauges. Applications of shock tubes are wide, from shock front formation studies, shock wave collision, rarefaction, refraction, shock-resistant structures, chemical kinetics, combustion, detonation, condensation, dissociation, ionization, electrical conductivity, radiation and heat transfer, to cite only a few examples.

Aerospace application of shock tubes was first noted by Hertzberg in 1951 [66], by the advent of a shock tunnel. The shock tunnel uses the high pressure/high enthalpy air produced in a shock tube as a gas generator for the stagnation chambers of supersonic and hypersonic wind tunnels. This air is expanded in a divergent nozzle to high Mach numbers.

5.2 Reflected Mode of Operation

Shock tube flow can be studied as an application of unsteady, one-dimensional, inviscid gasdynamics [67]. Initially, two quiescent gases are separated by a diaphragm. According to the usual nomenclature, see Fig. 5.1(a), the initial driver gas condition is designated as region 4 while the initial driven gas is designated as region 1. After diaphragm burst, a shock wave forms instantly and moves toward region 1, at a constant speed \(-u_s\). The disturbed driven gas is indicated by region 2. A rarefaction wave propagates into region 4, forming region 3. A contact surface separates the two gases initially separated by the diaphragm, the contact surface is
assumed to be an impervious surface moving at a constant speed into the driven tube. The reflected shock wave propagates backwards into region 2, forming region 5, depicted in Fig. 5.1(b). An effective way to represent the shock tube/tunnel physics is the $x$-$t$ diagram, shown in Fig. 5.2, where each region described above is located in space and time. Regions 6 to 10, also shown in Fig. 5.2, are related with shock-wave/contact surface collisions and shall be described in Section 5.3.

Mathematical treatment of the rarefaction wave which travels towards the driver is often done by the method of characteristics (MOC). With the assumptions that the flow is one-dimensional, unsteady, inviscid, isentropic and the gas is perfect, we obtain from the conservation equations [67]:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x} (\rho u) = 0 \quad (5.1)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + \frac{1}{\rho} \frac{\partial p}{\partial x} = 0 \quad (5.2)$$

Figure 5.1. Shock tube scheme in the reflected mode. (a) incident shock wave. (b) reflected shock wave.
where for a perfect gas,

\[ p = \rho RT \quad (5.4) \]

\[ h_t = \frac{\gamma}{\gamma - 1} RT + \frac{1}{2} u^2 \quad (5.5) \]

\[ a = (\gamma RT)^{1/2} = (\gamma p/\rho)^{1/2} \quad (5.6) \]

In the above equations, \( t \) and \( x \) stands for time and axial coordinates, respectively, \( \rho \) is the gas density, \( u \) is the velocity, \( p \) is the pressure, \( h_t \) is the total enthalpy, \( R \) is the gas constant, \( \gamma \) is the ratio of specific heats, \( T \) is temperature and \( a \) is the acoustic speed.

With the isentropic flow assumption, the relations in App. B, yield that \( p \propto \rho^\gamma \) and \( \rho \propto a^{2/(\gamma - 1)} \). Therefore, one can reduce Eqs. (5.1) through (5.3) to:

\[ \frac{\partial a}{\partial t} + u \frac{\partial a}{\partial x} + \frac{\gamma - 1}{2} a \frac{\partial u}{\partial x} = 0 \quad (5.7) \]

\[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + \frac{2}{\gamma - 1} a \frac{\partial a}{\partial x} = 0 \quad (5.8) \]
These two equations are coupled. However, they can be manipulated to yield, after some algebra [67]:

\[
A \frac{\partial^2 J_\pm}{\partial t^2} + 2B \frac{\partial^2 J_\pm}{\partial x \partial t} + C \frac{\partial^2 J_\pm}{\partial x^2} = D
\]  

(5.9)

where,

\[
J_\pm = u \pm \frac{2a}{\gamma - 1}
\]

(5.10)

and,

\[
A = \left( \frac{\partial J_\pm}{\partial x} \right)^2
\]

(5.11)

\[
B = -\frac{2}{3 - \gamma} \frac{\partial J_\pm}{\partial x} \left( \frac{\partial J_\pm}{\partial t} + \frac{\gamma - 1}{2} J_\pm \frac{\partial J_\pm}{\partial x} \right)
\]

(5.12)

\[
C = \frac{\gamma + 1}{3 - \gamma} \frac{\partial J_\pm}{\partial t} \left( \frac{\partial J_\pm}{\partial t} + 2 \frac{\gamma - 1}{\gamma + 1} J_\pm \frac{\partial J_\pm}{\partial x} \right)
\]

(5.13)

\[
D = \frac{\gamma^2 - 1}{2(3 - \gamma)} \left( \frac{\partial J_\pm}{\partial x} \right)^3 \left( \frac{\partial J_\pm}{\partial t} + J_\pm \frac{\partial J_\pm}{\partial x} \right)
\]

(5.14)

Also, the condition for Eq. (5.9) to be hyperbolic is satisfied. Thus, MOC can be applied:

\[
B^2 - AC = \left[ \frac{\gamma - 1}{3 - \gamma} \left( \frac{\partial J_\pm}{\partial t} + J_\pm \frac{\partial J_\pm}{\partial t} \right) \right]^2 > 0
\]

(5.15)

Hyperbolic equations present wave behavior. Therefore lines exist along which the solution is constant. In this particular case, one can show that:

\[
J_\pm = u \pm \frac{2a}{\gamma - 1} \text{(Riemann invariants)}
\]

(5.16)

are constant in lines (characteristic lines) where:

\[
\frac{dx_\pm}{dt} = u \pm a
\]

(5.17)

Through regions 4 and 3, \( J_+ \) is invariant since all the '+' characteristic lines in region 3 originate on region 4. Consequently, since \( u_4 = 0 \):

\[
2 \frac{a_4}{\gamma_4 - 1} = u_3 + 2 \frac{a_3}{\gamma_4 - 1}
\]

(5.18)
From the isentropic relations, the pressure ratio between the regions 4 and 1 is given by:

$$\frac{p_4}{p_3} = \left(\frac{a_4}{a_3}\right)^{\frac{2\gamma_4}{\gamma_4-1}}$$  (5.19)

Although viscous effects and diffusion contribute to decelerate the contact surface and to mix the gases from the regions 2 and 3, in the ideal case, we can assume the dynamic equilibrium and uniformity of the contact surface. Therefore, we can state that:

$$p_3 = p_2$$  (5.20)

Moreover, physical compatibility results in:

$$u_2 = u_3$$  (5.21)

Therefore, using Eq. (5.20):

$$\frac{p_4}{p_1} = \frac{p_4}{p_3} \frac{p_3}{p_1} = \frac{p_4}{p_3} \frac{p_2}{p_1}$$  (5.22)

The shock wave moving toward the driven tube modifies region 1 properties; the new properties of the perturbed gas, region 2, can be found by applying the thermodynamic relations through a normal shock wave for a perfect gas, see Appendix A. In particular, we can write:

$$\frac{p_2}{p_1} = \frac{2 \gamma_1 M_s^2 - (\gamma_1 - 1)}{\gamma_1 + 1}$$  (5.23)

where $M_s = u_s/a_1$.

Further,

$$\frac{a_2}{a_1} = \left[1 + \frac{2\gamma_1}{\gamma_1 + 1} (M_s^2 - 1)\right]^{1/2} \left[\frac{2 + (\gamma_1 - 1)M_s^2}{(\gamma_1 + 1)M_s^2}\right]^{1/2}$$  (5.24)

The enthalpy in region 2 can be calculated with:

$$\frac{h_2}{h_1} = \left(\frac{a_2}{a_1}\right)^2$$  (5.25)
Furthermore, the induced flow speed in region 2 can be evaluated via:

\[ u_2 = \frac{2}{\gamma_1 + 1} \left( \frac{M_s^2 - 1}{a_1} \right) \]  

(5.26)

Finally, using Eqs. (5.18) through (5.26), we can find the so-called ideal shock tube equation:

\[ \frac{p_4}{p_1} = \frac{2 \gamma_1 M_s^2 - (\gamma_1 - 1)}{\gamma_1 + 1} \left[ \frac{1}{1 - \frac{\gamma_1 - 1}{\gamma_1 + 1} \frac{M_s^2 - 1}{a_1 a_4 M_s^2}} \right]^{\frac{2\gamma_4}{\gamma_4 - 1}} \]  

(5.27)

The shock tube equation relates the incident shock wave Mach number with the driver-to-driven pressure ratio \( p_4/p_1 \). The variation of \( p_4/p_1 \) is depicted in Fig. 5.3, for \( 1 \leq M_s \leq 2.5 \), for air and helium as driver gases. Note that for the same Mach number, \( M_s \), the required driver pressure is much lower for helium than air. This is a consequence of the dependence of \( M_s \) on the driver gas speed of sound. Indeed, this clarifies why low molecular weight gases such as helium or hydrogen are used to enhance the performance of shock tunnels in the reflected mode. However, while hydrogen has an even lower molecular weight, safety issues have prevented its use.

Assuming a total reflection of the generated shock wave at the end of the driven tube, with no refraction and no leaks through the throat \( u_5 = 0 \). One can also calculate the conditions of region 5, formed when the this reflected shock wave strikes region 2. In this case, we adopt the reflected shock wave reference frame to use the normal shock wave relations in Appendix A. Let us consider that this wave is moving in a velocity of \( u_r \), with the respect to the laboratory frame. Thus, the flow in region 2 is moving towards a standing shock wave with velocity \( u_2 + u_r \), since \( u_5 \) is neglected, the perturbed flow in region 5 is moving with respect to the reflected shock with a velocity equal to \( u_r \). With these considerations on mind, one can use the velocity ratio, Eq. (A.3), from Appendix A, as:

\[ \frac{u_2 + u_r}{u_r} = \frac{\gamma_1 + 1}{2} \frac{M_r^2}{1 + \frac{2\gamma_1 - 1}{\gamma_1} M_r^2} \]  

(5.28)
Figure 5.3. Incident Mach number variation with driver-to-driven pressure, for air and helium as driver gases, at room temperature.

with \( M_r = \frac{u_2 + u_r}{a_2} \), since \( M_2 = \frac{u_2}{a_2} \):

\[
\frac{u_2 + u_r}{u_r} = \frac{M_r}{M_r - M_2}
\]  \hspace{1cm} (5.29)

Combining Eqs. (5.28), (5.29) and (5.26), one can solve for the reflected Mach number to get:

\[
M_r = \left( \frac{\gamma_1 M_s^2 - \frac{\gamma_1 - 1}{2}}{1 + \frac{\gamma_1 - 1}{2} M_s^2} \right)^{1/2}
\]  \hspace{1cm} (5.30)

which permits us to calculate the flow properties in region 5, \( p_5, \rho_5, T_5, h_5 \), using the relationships in Appendix A. For instance, we can relate the pressures of regions 5 and 2 via:

\[
\frac{p_5}{p_2} = \frac{2 \gamma_1 M_r^2 - (\gamma_1 - 1)}{\gamma_1 + 1}
\]  \hspace{1cm} (5.31)
Thus one can get:

\[
\frac{p_5}{p_1} = \frac{p_5}{p_2} \frac{p_2}{p_1} \quad (5.32)
\]

Further since,

\[
\frac{h_5}{h_2} = \left( \frac{a_5}{a_2} \right)^2 = \left[ 1 + \frac{2 \gamma_1}{\gamma_1 + 1} (M_r^2 - 1) \right] \frac{2 + (\gamma_1 - 1)M_r^2}{(\gamma_1 + 1)M_r^2} \quad (5.33)
\]

One can get the enthalpy in region 5 using:

\[
\frac{h_5}{h_1} = \frac{h_5}{h_2} \frac{h_2}{h_1} \quad (5.34)
\]

We use Eqs. (5.30) to (5.34) along with Eqs. (5.23), (5.25) and (5.27), to calculate the stagnation conditions, pressure and enthalpy in region 5, as functions of the initial driver-to-driven pressure ratio, \( p_4/p_1 \), see Figs. 5.4(a) and (b). It is worth noting in these graphs the steep variation of stagnation properties with \( p_4/p_1 \) for the helium-driven shock tube. Meanwhile, the air-driven shock tube shows the limitation of this gas to provide high enthalpies and pressures in region 5, at driver and driven low initial temperatures, by simply adjusting the \( p_4/p_1 \) value. In fact, at high driver-to-driven pressure ratios, the gain in stagnation pressure and enthalpy tends to diminish for an increase of \( p_4/p_1 \).

5.3 Collisions Between Reflected Shock Wave and Contact Surface

One-dimensional refraction of the reflected shock wave onto the contact surface can produce three possible conditions [12,68]. A shock wave or a rarefaction wave can be reflected back, both causing premature termination of the uniform conditions of region 5. If certain conditions of region 2 and 3 are matched, however, the contact surface may become soft (or transparent) to the reflected shock and only a Mach wave is reflected back, thus no disturbances reach region 5. This last case is the so-called tailored mode. See schemes in Fig. 5.5. Compared to the first two cases, the tailored mode can produce a test time of one order of magnitude higher [68].
Figure 5.4. Stagnation conditions variation with driver-to-driven pressure ratio, for the reflected mode, for air and helium as driver gases. (a) Pressure and (b) stagnation ratios.

5.3.1 Tailored Condition

For the calculation of the initial conditions of the driver and driven gases required to generate a Mach wave after a head-on collision of the reflected shock wave and the gas interface, we follow the nomenclature of Fig. 5.5. As in Section 5.2, we assume that the contact surface is in dynamic equilibrium. Thus,

\[ u_6 = u_7 \]

(5.35)

and,

\[ p_6 = p_7 \]

(5.36)

With the shock wave relations in Appendix A, we can relate the properties after the transmitted shock wave. In particular,

\[
\frac{u_3 - u_7}{a_3} = \left[ \frac{2}{\gamma_4(\gamma_4 - 1)} \right]^{1/2} \left( \frac{p_7}{p_3} - 1 \right) \left[ \left( \frac{\gamma_4 + 1}{\gamma_4 - 1} \right) \frac{p_7}{p_3} + 1 \right]^{-1/2}
\]

(5.37)
Figure 5.5. Possible outcomes from a shock wave/contact surface one-dimensional collision. (a) a shock wave is reflected (over-tailored), (b) the reflection is a rarefaction wave (under-tailored), and (c) an acoustic wave is reflected (tailored). Adapted from Minucci [12].

Since we assume the flow is stagnant in region 5, \( u_5 = 0 \), the conditions of the gas which is processed by the shock wave reflected in the contact surface can be also related. Therefore,

\[
\frac{u_6}{a_5} = \left[ \frac{2}{\gamma_1 (\gamma_1 - 1)} \right]^{1/2} \left( \frac{p_6}{p_5} - 1 \right) \left[ \frac{\gamma_1 + 1}{\gamma_1 - 1} \right] \left( \frac{p_6}{p_5} - 1 \right) \left( \frac{p_6}{p_5} + 1 \right)^{-1/2}
\]  

(5.38)

For the case when a rarefaction wave is reflected, one can use the unsteady rarefaction wave model, previously described, along with the isentropic relationships (Appendix B) to yield:

\[
\frac{u_6}{a_5} = \frac{2}{\gamma_1 - 1} \left[ 1 - \left( \frac{p_6}{p_5} \right)^{\frac{\gamma_1 - 1}{2\gamma_1}} \right]
\]  

(5.39)
Ideally, there are cases when a Mach wave is reflected so that the contact surface completely stops, \( u_6 = u_7 = 0 \). Furthermore, invoking the analogy of the contact surface and a piston, one can conclude that in cases when a shock wave is reflected, the contact surface is moving towards the nozzle. On the other hand, a rarefaction wave is formed when the contact surface is running towards the driver.

With Eq. (5.36) and, since \( p_3 = p_2 \),

\[
\frac{p_7}{p_3} = \frac{p_6}{p_3} = \frac{p_6 p_5}{p_5 p_3} = \frac{p_6 p_5}{p_5 p_2} \tag{5.40}
\]

If a Mach wave is reflected, \( p_6 = p_5 \), thus:

\[
\frac{p_7}{p_3} = \frac{p_5}{p_2} \tag{5.41}
\]

Now, with Eqs. (5.41) and (5.35), Eq. (5.37) can be rewritten as:

\[
\frac{u_3 - u_6}{a_3} = \left[ \frac{2}{\gamma_4 (\gamma_4 - 1)} \right]^{1/2} \left( \frac{p_5}{p_2} - 1 \right) \left[ \left( \frac{\gamma_4 + 1}{\gamma_4 - 1} \right) \frac{p_5}{p_2} + 1 \right]^{-1/2} \tag{5.42}
\]

But, for the initial shock, which was reflected at the end of driven tube, we get:

\[
\frac{u_2 - u_5}{a_2} = \left[ \frac{2}{\gamma_1 (\gamma_1 - 1)} \right]^{1/2} \left( \frac{p_5}{p_2} - 1 \right) \left[ \left( \frac{\gamma_1 + 1}{\gamma_1 - 1} \right) \frac{p_5}{p_2} + 1 \right]^{-1/2} \tag{5.43}
\]

Since we consider that \( u_5 = 0, u_2 = u_3 \), and at tailored condition the contact surface stops, \( u_6 = u_7 = 0 \), Eqs. (5.42) and (5.43) can be manipulated to yield:

\[
\frac{a_2}{a_3} = \frac{\gamma_1}{\gamma_4} \left[ \frac{1 + \frac{\gamma_1 + 1}{2\gamma_4} (p_5/p_2 - 1)}{1 + \frac{\gamma_4 + 1}{2\gamma_1} (p_5/p_2 - 1)} \right]^{1/2} \tag{5.44}
\]

Furthermore, \( a_2/a_3 \) can be written as:

\[
\frac{a_2}{a_3} = \frac{a_2 a_1 a_4}{a_1 a_4 a_3} \tag{5.45}
\]

From Eq. (5.19) and (5.20) one can write:

\[
\frac{a_2}{a_3} = \frac{a_2}{a_1} \left( \frac{a_4}{a_1} \right)^{-1} \left( \frac{p_4}{p_3} \right)^{\frac{\gamma_4 - 1}{2\gamma_4}} = \frac{a_2}{a_1} \left( \frac{a_4}{a_1} \right)^{-1} \left( \frac{p_4 / p_3}{p_1 / p_1} \right)^{\frac{\gamma_4 - 1}{2\gamma_4}} \tag{5.46}
\]
Finally, one can relate the incident shock Mach number $M_s$ with the required ratio $a_4/a_1$ which produces the tailored condition. We use Eqs. (5.46), (5.44), (5.31), (5.30), (5.27), (5.24) and (5.23) to obtain:

\[
\sqrt{\left(\frac{1}{2} (\gamma_1 - 1) M_s^2 \right) + 1} \left[ \frac{\left(-\gamma_1+2\gamma_1 M_s^2+1\right) \left(1-\frac{(\gamma_4-1)(M_s^2-1)}{\gamma_1+1}M_s\right)}{(\gamma_1+1)\left(2\gamma_1(M_s^2-1)+1\right)} \right] = \frac{a_4}{a_1} \left[ \frac{1}{2} (\gamma_1 - 1) \left(\frac{\gamma_1}{\gamma_1(M_s^2-1)} + 1\right) \right]^{1/2}
\]

which can only be solved numerically.

This last relation along with Eq. (5.27) allowed us to obtain the required ratio $a_4/a_1$ as a function of $p_4/p_1$ for two different driver gases, helium and air, see Fig. 5.6. Note that at room temperature, $T = 300$ K, $a_4/a_1$ is equal to 1.0 for air and 2.93 for helium.

Now it is possible to get the stagnation properties at the tailored condition. Since,

\[
\frac{p_6}{p_1} = \frac{p_5}{p_1} = \frac{p_5 p_2}{p_2 p_1}
\]

and,

\[
\frac{h_6}{h_1} = \frac{h_5}{h_1} = \frac{h_5 h_2}{h_2 h_1}
\]

Eqs. (5.31) through (5.34) permit us to obtain $p_6/p_1$ and $h_6/h_1$. The reflected Mach number $M_r$ is calculated via Eq. (5.30).

By relating the incident Mach number $M_s$ with the driver-to-driven pressure ratio $p_4/p_1$ with Eq. (5.27), one can study the variation of the conditions at region 6 with $p_4/p_1$, see Fig. 5.7(a) and (b), for pressure and enthalpy, respectively. Interest-
ing enough, the variation is almost linear for both $p_6/p_1$ and $h_6/h_1$. And, both gases showed very close values within the studied range of $p_4/p_1$. However, one must keep in mind that at tailored condition the temperature of the air in the driver must be several times than when helium is adopted.

5.3.2 Under-tailored Mode

When we have the under-tailored condition, Eqs. (5.37) and (5.39), can be applied (also with $u_6 = u_7$) to yield:

$$\left[ \frac{2}{\gamma_4(\gamma_4 - 1)} \right]^{1/2} \left( \frac{p_7}{p_3} - 1 \right) \left( \frac{\gamma_4 + 1}{\gamma_4 - 1} \right) \left( \frac{p_7}{p_3} + 1 \right)^{-1/2} = \frac{u_3}{a_3} + \frac{a_5}{a_3 \gamma_1 - 1} \left[ 1 - \left( \frac{p_6}{p_5} \right)^{\frac{\gamma_1-1}{2\gamma_1}} \right]$$

(5.50)
Figure 5.7. Stagnation conditions variation with driver-to-driven pressure ratio, for the tailored mode, for air and helium as driver gases. (a) Pressure and (b) stagnation ratios.

The under-tailored mode, however, means termination of shock tube stagnation conditions by the arrival of an expansion wave. Therefore, no further analysis of this mode is of interest.

5.3.3 Over-tailored or Equilibrium Interface Mode

For the over-tailored condition, one can obtain from Eqs. (5.37) and (5.38), with the condition $u_6 = u_7$:

$$
\left[ \frac{2}{\gamma_4 (\gamma_4 - 1)} \right]^{1/2} \left( \frac{p_7}{p_3} - 1 \right) \left[ \left( \frac{\gamma_4 + 1}{\gamma_4 - 1} \right) \frac{p_7}{p_3} + 1 \right]^{-1/2} = \\
\frac{u_3}{a_3} - \frac{a_5}{a_3} \left[ \frac{2}{\gamma_1 - 1} \right]^{1/2} \left( \frac{p_6}{p_5} - 1 \right) \left[ \left( \frac{\gamma_1 + 1}{\gamma_1 - 1} \right) \frac{p_6}{p_5} + 1 \right]^{-1/2}
$$

(5.51)

Since $p_3 = p_2$,

$$\frac{p_7}{p_3} = \frac{p_7 p_5}{p_5 p_3} = \frac{p_7 p_5}{p_5 p_2}
$$

(5.52)
and with $p_6 = p_7$ one can obtain:

$$
\frac{2}{\gamma_4(\gamma_4 - 1)}^{1/2} \left( \frac{p_6 p_5}{p_5 p_2} - 1 \right) \left[ \frac{\gamma_4 + 1}{\gamma_4 - 1} \frac{p_6 p_5}{p_5 p_2} + 1 \right]^{-1/2} = \frac{u_3}{a_3} - \frac{a_5}{a_3} \left[ \frac{2}{\gamma_1 - 1} \right]^{1/2} \left( \frac{p_6}{p_5} - 1 \right) \left[ \frac{\gamma_1 + 1}{\gamma_1 - 1} \frac{p_6}{p_5} + 1 \right]^{-1/2} \quad (5.53)
$$

also,

$$
\frac{a_5}{a_3} = \frac{a_5}{a_2 a_1 a_4} \quad (5.54)
$$

where $a_5/a_2$ and $a_2/a_1$ are calculated via Eqs. (5.33) and (5.24), respectively. $a_4/a_3$ can be manipulate with Eqs. (5.19) and (5.22). $a_1/a_4$ is given and we assume that $M_s$ is also a given parameter. Thus,

$$
\frac{a_5}{a_3} = \left\{ \frac{1 + 2\gamma_1}{\gamma_1 + 1} \left( M_r^2 - 1 \right) \left[ \frac{2 + (\gamma_1 - 1) M_r^2}{(\gamma_1 + 1) M_r^2} \right] \right\}^{1/2} \left\{ \frac{1 + 2\gamma_1}{\gamma_1 + 1} \left( M_s^2 - 1 \right) \left[ \frac{2 + (\gamma_1 - 1) M_s^2}{(\gamma_1 + 1) M_s^2} \right] \right\}^{1/2} \left( \frac{p_4/p_2}{p_1} \right)^{\frac{\gamma_1 - 1}{2\gamma_4}} \quad (5.55)
$$

with $p_4/p_1$ given by Eq. (5.27), $p_2/p_1$ by Eq. (5.23) and $M_r$ by Eq. (5.30). Also,

$$
\frac{u_3}{a_3} = \frac{u_2 a_1 a_4}{a_1 a_4 a_3} \quad (5.56)
$$

where, $u_2/u_1$ is given by Eq. (5.26).

One can solve Eq. (5.53) to obtain $p_6/p_5 = p_7/p_5$ and thus, calculate the properties in regions 6 and 7, such as the Mach number of shock wave reflected on the contact surface:

$$
M_{ri} = \sqrt{1 + \frac{\gamma_1 + 1}{2\gamma_1} \left( \frac{p_6}{p_5} - 1 \right)} \quad (5.57)
$$

The induced speed in region 6 can be calculated with:

$$
\frac{u_6}{a_5} = \frac{2}{\gamma_1 + 1} \frac{M_r^2 - 1}{M_{ri}} \quad (5.58)
$$

The sound speed is given by in this region:

$$
\frac{a_6}{a_5} = \left( \frac{h_6}{h_5} \right)^{1/2} = \left\{ \frac{2 + 2\gamma_1}{\gamma_1 + 1} (M_r^2 - 1) \left[ \frac{2 + (\gamma_1 - 1) M_r^2}{(\gamma_1 + 1) M_r^2} \right] \right\}^{1/2} \quad (5.59)
$$
On the driver gas, the transmitted shock wave Mach number can be expressed by:

\[ M_{ti} = \sqrt{1 + \frac{\gamma_4 + 1}{2\gamma_4} \left( \frac{p_7}{p_3} - 1 \right)} \]  

(5.60)

and

\[ \frac{a_7}{a_3} = \left( \frac{h_7}{h_3} \right)^{1/2} = \left\{ 1 + \frac{2\gamma_4}{\gamma_4 + 1} (M_{ti}^2 - 1) \right\} \left[ \frac{2 + (\gamma_4 - 1)M_{ti}^2}{(\gamma_4 + 1)M_{ti}^2} \right]^{1/2} \]  

(5.61)

The reflection of the shock wave in the driven tube end generates region 8, see Figs. 5.2 and 5.8. The properties in this region can be obtained via the reflected Mach number:

\[ M_r = M_{r,n=1} = \left( \frac{\gamma_1 M_{ri}^2 - \frac{\gamma_1 - 1}{2}}{1 + \frac{\gamma_1 - 1}{2} M_{ri}^2} \right)^{1/2} \]  

(5.62)

Eqs. (5.51) through (5.62) permit us to obtain all the properties in a shock tube after one shock wave/contact surface interaction. For the case of \( n > 1 \) of such interactions, one can show, according to the nomenclature in Fig. 5.8(b), that these same equations can be modified for \( n \) interactions. One can notice that regions \( 3n, 3n + 1 \) and \( 3n + 2 \) are calculated in interaction \( n - 1 \) and the unknown regions are \( 3n + 3 \) and \( 3n + 4 \). Thus, Eq. (5.53) can be adapted as:

\[ \frac{u_{3n+1}}{a_{3n+1}} - \frac{a_{3n+2}}{a_{3n+1}} \left[ \frac{2}{\gamma_1 - 1} \right]^{1/2} \left( \frac{p_{3n+3}}{p_{3n+2}} - 1 \right) \left[ \left( \frac{\gamma_1 + 1}{\gamma_1 - 1} \right) \frac{p_{3n+3}}{p_{3n+2}} + 1 \right]^{-1/2} = \]  

(5.63)

which we must solve for \( p_{3n+3}/p_{3n+2} \). Therefore, the Mach number of the reflected shock wave resulted from the \( n \)th contact surface interaction is given by:

\[ M_{ri,n} = \sqrt{1 + \frac{\gamma_1 + 1}{2\gamma_1} \left( \frac{p_{3n+3}}{p_{3n+2}} - 1 \right)} \]  

(5.64)

The induced speed in region \( 3n + 3 \) can be calculated with:

\[ \frac{u_{3n+3}}{a_{3n+2}} = \frac{2}{\gamma_1 + 1} \frac{M_{ri,n}^2 - 1}{M_{ri,n}} \]  

(5.65)
Figure 5.8. Nomenclature for multiple shock wave/contact surface collisions. (a) for two collisions, \( n = 2 \). (b) for \( n \) collisions.

The sound speed is given by:

\[
\frac{a_{3n+3}}{a_{3n+2}} = \left( \frac{h_{3n+3}}{h_{3n+2}} \right)^{1/2} = \left\{ 1 + \frac{2\gamma_1}{\gamma_1 + 1} (M_{ri,n}^2 - 1) \left[ 2 + (\gamma_1 - 1)M_{ri,n}^2 \right] \right\}^{1/2} \tag{5.66}
\]

On the driver gas, since \( p_{3n+4} = p_{3n+3} \), the transmitted shock wave Mach number can be expressed by:

\[
M_{ti,n} = \sqrt{1 + \frac{\gamma_4 + 1}{2\gamma_4} \left( \frac{p_{3n+4}}{p_{3n+1}} - 1 \right)} \tag{5.67}
\]

and,

\[
\frac{a_{3n+4}}{a_{3n+1}} = \left( \frac{h_{3n+4}}{h_{3n+1}} \right)^{1/2} = \left\{ 1 + \frac{2\gamma_4}{\gamma_4 + 1} (M_{ti,n}^2 - 1) \left[ 2 + (\gamma_4 - 1)M_{ti,n}^2 \right] \right\}^{1/2} \tag{5.68}
\]

The shock wave is reflected again in the driven tube and generates region \( 3n + 5 \).

One can obtain the properties on that region with:

\[
M_{r,n} = \left( \frac{\gamma_1 M_{ri,n}^2 - \frac{\gamma_1 - 1}{2}}{1 + \frac{\gamma_1 - 1}{2} M_{ri,n}^2} \right)^{1/2} \tag{5.69}
\]
Finally, in Section F.1, of Appendix F, one can find a computer code, in Matlab language, for calculation of the shock tube properties for the equilibrium interface mode, assuming perfect gases in driven and driver. All mathematical development are the same as the described in this section and in the previous one, Section 5.2.

5.3.4 Isentropic Compression Approximation for the Equilibrium Interface Mode

There are cases when a few shock-wave/contact surface interactions are produced, in general three to four [12], until a Mach wave is resulted from contact surface interaction with the shock wave. This equilibrium state provides the stagnation condition for the test gas. However, such a condition will end by the arrival of either the contact surface or the first expansion wave reflected in the driver section.

It was pointed out by Copper [69] that the compression from region 5 to the final equilibrium state can be modeled as an isentropic process. Copper realized that successive shock wave/contact surface interactions decreases the contact surface speed in such a way that the most part of the entropy gain occur in the first reflected shock wave (which produces region 5). In fact, it is possible to calculate the entropy gain between regions 5 and 2 via Eqs. (A.9) and (A.11) from Appendix A. From the first relation we get:

\[
\frac{p_5}{p_2} = \left( \frac{\gamma_1 + 1}{2} \right)^{\frac{1}{\gamma_1 - 1}} M_r^{\frac{2\gamma_1}{\gamma_1 - 1}} \left[ 1 + \frac{\gamma_1 - 1}{2} M_r^2 \right]^{-\frac{1}{\gamma_1 - 1}} \left[ \frac{\gamma_1 - 1}{2} + \gamma_1 M_r^2 \right]^{-\frac{1}{\gamma_1 - 1}}
\]

and the second relation gives:

\[
s_5 - s_2 = -R \ln \left( \frac{p_5}{p_2} \right)
\]

On the other hand, from conditions 1 to 2 we have that:

\[
\frac{p_2}{p_1} = \left( \frac{\gamma_1 + 1}{2} \right)^{\frac{1}{\gamma_1 - 1}} M_s^{\frac{2\gamma_1}{\gamma_1 - 1}} \left[ 1 + \frac{\gamma_1 - 1}{2} M_s^2 \right]^{-\frac{1}{\gamma_1 - 1}} \left[ \frac{\gamma_1 - 1}{2} + \gamma_1 M_s^2 \right]^{-\frac{1}{\gamma_1 - 1}}
\]
and,

\[ s_2 - s_1 = -R \ln \left( \frac{p_2}{p_1} \right) \quad (5.73) \]

Since at \( T_1 = 298 \) K and \( p_1 = 1 \) bar, \( s_1 = 6870.4 \) J/(kg \cdot K), we can evaluate the variation of the entropy gain between regions 5 and 2, with Eqs. (5.70) through (5.73) along with Eq. (5.30), as a function of the incident Mach number \( M_s \). Thus, one can find that the maximum value of \((s_5 - s_2)/s_2\) is only 2.56%, at \( M_s = 9.28 \). This supports the assumption that the compression beyond region 5 can modeled as an isentropic process, as stated before.

5.4 Real Gas Effects in Shock Tubes

Since high stagnation temperatures can be achieved in a shock tube, the perfect gas assumption will be no longer valid, making the results from previous section inaccurate. In fact, although the stagnation pressure can be estimated with a perfect gas assumption, the stagnation temperature and, consequently, the freestream properties, will present a larger deviation from the real gas case.

Despite the fact that a full chemical kinetic mechanism would be the closest model for calculation of chemical and thermodynamic state of the air inside a shock tube, equilibrium flow assumption has be proven adequate for such purposes [12, 70, 71]. Therefore, in this section, the shock tube flow was modeled using a similar equilibrium development shown by Minucci [12] in order to estimate important flow properties.

Starting from the conservation laws through the incident shock wave, as depicted in Fig. 5.1(a) of Section 5.2, and recalling that these laws are constructed in the shock wave reference frame, for mass conservation we have:

\[ \rho_1 u_s = \rho_2 (u_s - u_2) \quad (5.74) \]
Momentum:
\[ p_1 + \rho_1 u_s^2 = p_2 + \rho_2 (u_s - u_2)^2 \]  \hspace{1cm} (5.75)

Energy:
\[ h_1 + \frac{u_s^2}{2} = h_2 + \frac{(u_s - u_2)^2}{2} \]  \hspace{1cm} (5.76)

Now, since the gas is treated as real, we write the equation of state:
\[ h_2 = h(p_2, \rho_2) \]  \hspace{1cm} (5.77)

This system of equations can be solved by iteration, in a similar procedure described by Anderson [20]. With the initial guess of \( \rho_2/\rho_1 \), obtained from the shock relations for the perfect gas model, at the \( j \)th iteration, we have:
\[ \rho_2 = (\rho_2/\rho_1)_j \rho_1 \]  \hspace{1cm} (5.78)

and, with Eq. (5.74):
\[ u_s - u_2 = u_s/ (\rho_2/\rho_1)_j \]  \hspace{1cm} (5.79)

which through Eq. (5.75) permits us to obtain the post shock pressure:
\[ p_2 = p_1 + \rho_1 u_s^2 - \rho_2 (u_s - u_2)^2 \]  \hspace{1cm} (5.80)

Finally the computed specific enthalpy downstream the shock can be obtained via Eq. (5.76):
\[ h_2 = h_1 + \frac{u_s^2}{2} - \frac{(u_s - u_2)^2}{2} \]  \hspace{1cm} (5.81)

The iteration uses the secant method until the criterion below is satisfied:
\[ |h(p_2, \rho_2) - h_2| < 10^{-8} \text{J/kg} \]  \hspace{1cm} (5.82)

The quantity \( h(p_2, \rho_2) \) and other thermodynamic properties are obtained via the Gibbs free energy minimization, described in Appendix C and calculated with help of the Matlab routine listed in Section F.12 of Appendix F.
As stated before, the incident shock propagates until the end of the driven
section and reflects, changing in direction, toward the driver, modifying the state 2,
see Fig. 5.1(b) in Section 5.2. In this case the conservation equations are represented
in the reflected shock wave reference frame. Therefore, for mass conservation we have:

\[ \rho_2 (u_2 + u_r) = \rho_5 u_r \quad (5.83) \]

Similarly, for the momentum:

\[ p_2 + \rho_2(u_2 + u_r)^2 = p_5 + \rho_5 u_r^2 \quad (5.84) \]

and, for the energy:

\[ h_2 + \frac{(u_2 + u_r)^2}{2} = h_5 + \frac{u_r^2}{2} \quad (5.85) \]

with the equation of state:

\[ h_5 = h(p_5, \rho_5) \quad (5.86) \]

Again, with the properties of region 2 already known, we can use them to build a
numerical procedure such as the previously described for the incident shock wave,
we start by guessing \( \rho_5/\rho_2 \), via perfect gas calculations, and thus:

\[ \rho_5 = (\rho_5/\rho_2) \rho_2 \quad (5.87) \]

With Eq. (5.83),

\[ u_r = \frac{u_2}{(\rho_5/\rho_2) \rho_2} \quad (5.88) \]

one can calculate the pressure after the reflected shock via Eq. (5.84):

\[ p_5 = p_2 + \rho_2(u_2 + u_r)^2 - \rho_5 u_r^2 \quad (5.89) \]

and, for the enthalpy with Eq. (5.85):

\[ h_5 = h_2 + \frac{(u_2 + u_r)^2}{2} - \frac{u_r^2}{2} \quad (5.90) \]
Finally, the iteration uses the secant method until the criterion below is satisfied:

$$|h(p_5, \rho_5) - h_5| < 10^{-8}\text{J/kg}$$

(5.91)

In Section 5.3.1 we saw that, at tailored condition, a Mach wave is reflected back after the collision between the contact surface and the reflected shock from the end of driven tube. Therefore, properties of region 5 and 6 are identical. However, this is not true for the under and over-tailored cases, and if real gas effects are taken in account, the thermodynamic equilibrium of air is also assumed.

In the over-tailed case, we have, according to the nomenclature in Fig. 5.5(a), that the conservation equations through the transmitted shock wave can be written as [12]:

$$\rho_3 (u_3 + u_{ti}) = \rho_7 (u_{ti} + u_7)$$

(5.92)

for mass,

$$p_3 + \rho_3 (u_3 + u_{ti})^2 = p_7 + \rho_7 (u_{ti} + u_7)^2$$

(5.93)

for the momentum, and,

$$h_3 + \frac{(u_3 + u_{ti})^2}{2} = h_7 + \frac{(u_{ti} + u_7)^2}{2}$$

(5.94)

for the energy. With the equation of state:

$$h_7 = h(p_7, \rho_7)$$

(5.95)

For the reflected shock wave, one can get for continuity:

$$\rho_5 (u_{ri} - u_5) = \rho_6 (u_{ri} - u_6)$$

(5.96)

for mass,

$$p_5 + \rho_5 (u_{ri} - u_5)^2 = p_6 + \rho_6 (u_{ri} - u_6)^2$$

(5.97)
for the axial momentum, and,

\[ h_5 + \frac{(u_{ri} - u_5)^2}{2} = h_6 + \frac{(u_{ri} - u_6)^2}{2} \]  \hspace{1cm} (5.98)

for the energy. With the equation of state:

\[ h_6 = h(p_6, \rho_6) \]  \hspace{1cm} (5.99)

In the case of the equilibrium interface mode (or over-tailored mode), one must solve simultaneously Eqs. (5.100) through (5.107). For the case of multiple interactions with the contact surface, the same equations can be applied using the nomenclature of Fig. 5.8. Thus, for the transmitted shock wave:

\[ \rho_{3n+1} (u_{3n+1} + u_{ti}) = \rho_{3n+4}(u_{ti} + u_{3n+4}) \]  \hspace{1cm} (5.100)

\[ p_{3n+1} + \rho_{3n+1}(u_{3n+1} + u_{ti})^2 = p_{3n+4} + \rho_{3n+4}(u_{ti} + u_{3n+4})^2 \]  \hspace{1cm} (5.101)

\[ h_{3n+1} + \frac{(u_{3n+1} + u_{ti})^2}{2} = h_{3n+4} + \frac{(u_{ti} + u_{3n+4})^2}{2} \]  \hspace{1cm} (5.102)

\[ h_{3n+4} = h(p_{3n+4}, \rho_{3n+4}) \]  \hspace{1cm} (5.103)

For the reflected wave,

\[ \rho_{3n+2}(u_{ri} - u_{3n+2}) = \rho_{3n+3}(u_{ri} - u_{3n+3}) \]  \hspace{1cm} (5.104)

\[ p_{3n+2} + \rho_{3n+2}(u_{ri} - u_{3n+2})^2 = p_{3n+3} + \rho_{3n+3}(u_{ri} - u_{3n+3})^2 \]  \hspace{1cm} (5.105)

\[ h_{3n+2} + \frac{(u_{ri} - u_{3n+2})^2}{2} = h_{3n+3} + \frac{(u_{ri} - u_{3n+3})^2}{2} \]  \hspace{1cm} (5.106)

\[ h_{3n+3} = h(p_{3n+3}, \rho_{3n+3}) \]  \hspace{1cm} (5.107)

where \( n \) is the interaction number.

One must also keep in mind that the shock wave/contact surface interactions end when a Mach wave is reflected back. The computer code for solving the equilibrium interface mode with equilibrium air is listed in Section F.2 of Appendix F.
5.5 Effect of Driver-to-Driven Area Change

The modeling of the unsteady expansion as described in Section 5.2, relates to constant diameter shock tubes. For the shock tunnels with driver-to-driven area contractions, however, a different approach has been used to predict flows in Regions 3 and 4. Alpher and White [72] described theoretical and experimental studies of the effects on shock tube flows of a monotonic convergence at the diaphragm section, for ideal gases. Their results showed agreement over a wide range of Mach numbers, even at high Mach numbers where the shock formation processes are complex.

The scheme for the flow with area change is shown in Fig. 5.9. The diaphragm rupture is followed by an unsteady isentropic expansion similar to the constant bore case from the state 4 to state 3a. Then, a steady nozzle flow is formed in the convergent region 3b’, where the flow is sonic. Finally, the flow in 3b is subjected to a further unsteady expansion to state 3.

Without further details, we can write:

\[
\frac{p_4}{p_1} = \frac{1}{g_v} \left[ 1 - 2 \frac{2}{\gamma_1 + 1} \frac{M_5^2 - 1}{a_4 \gamma_4 + 1} \frac{-\gamma_4 + 1}{2 \gamma_4} \frac{2 \gamma_4 M_5^2 - (\gamma_1 - 1)}{\gamma_1 + 1} \right] (5.108)
\]

The equivalence factor \( g_v \) is given by the relation:

\[
g_v = \left[ \frac{a_4 \gamma_4 + 1}{\gamma_4} \frac{2 + (\gamma_4 - 1) M_{3a}^2}{2 + (\gamma_4 - 1) M_{3a}^2} \right]^{\frac{\gamma_4}{2(\gamma_4 - 1)}} (5.109)
\]

The Mach number in the region 3a can be found from the nozzle area-Mach relation, given that \( M_{3a} \) is sonic.

\[
\frac{A_4}{A_1} = \frac{1}{M_{3a}^2} \left( \frac{2 + (\gamma_4 - 1) M_{3a}^2}{\gamma_4 + 1} \right)^{\frac{\gamma_4}{2(\gamma_4 - 1)}} M_{3a}^2 (5.110)
\]

The above three equations can be solved by iteration. Given the area ratio \( A_4/A_1 \), the Mach number in region 3a is obtained by Eq. (5.110). Since we have \( M_{3a} \), the equivalence factor is directly given by Eq. (5.109). Then, given the pressure ratio \( p_4/p_1 \), one can get the incident Mach number, \( M_s \), using Eq. (5.108).
5.6 Test Time in Shock Tunnels

Basically, from all above, the duration of the run time depends on the shock wave Mach number and the length of the low pressure section, the mode of operation, under-tailored, tailored or over-tailored. This time can be a few hundred microseconds for the under-tailored and over-tailored modes and few milliseconds for the tailored condition. This time is limited by either the reflected leading edge of the rarefaction wave from the driver gases, which is the case for non-optimal shock tube lengths, as the HST itself, or by the arrival of the contact surface, which comes first.

Other contributors for reduction of the shock tunnel run time are [68]: i) boundary-layer growth which diminishes the useful diameter of the driven tube; ii) interactions between the reflected wave and the boundary-layer itself; iii) instabilities and non-uniformity of the contact surface; iv) combustion in the contact surface, caused by mixtures like H$_2$/air or H$_2$/O$_2$.

Figure 5.9. Flow scheme suggested for shock tubes with area contractions.
CHAPTER 6

NOZZLE FLOW

Determining the free stream conditions is of paramount importance for shock tunnel tests. These properties are the Mach number, static temperature, static pressure, Reynolds number and velocity, just to cite a few. Except for the static pressure, which can be measured, all other properties are estimated.

As stated in Section 5.6, the test time begins with the arrival of a rarefaction wave and terminates with either the arrival of the contact surface or the rarefaction wave reflected at the driver. Despite the fact that the test times in shock tunnels are very short, after the unsteady nozzle starting process, the nozzle expansion is modeled as a steady process.

6.1 Real Gas Effects

Since flows in shock tunnels can present a wide range of reservoir temperatures, the calorically perfect gas assumption is not always valid. As mentioned before, high temperatures can start real gas effects in air [20]. Thus, capable of producing excitation of vibrational modes of $O_2$ and $N_2$, dissociation of $O_2$ and $N_2$, formation of NO molecules, and ultimately formation of $O^+$, $N^+$ ions and $e^−$.

These real gas effects require time to develop. However, and due to the pulsed nature of shock tunnels, non-equilibrium chemistry can become a problem in the hypersonic nozzle flow. That means the reaction times are of the order of the time taken for the test flow to completely pass through the nozzle. Vibrational and rotational excitation introduces the relaxation times. By comparing the reaction or
relaxation time with the nozzle flow time, we can define two flow distinctions: frozen and equilibrium. In a frozen situation, the reaction times (or excitation times) are several orders of magnitude greater than the nozzle flow time, so the reaction rates are considered zero. This implies that the chemical composition throughout the nozzle is the same as in the reservoir and the flow is said to be frozen, meaning that the chemical reactions are frozen. On the other hand, when the reaction times are significantly lower than the nozzle flow time, the flow is considered to be in equilibrium, and the reaction rates are taken as infinite. In this case, the chemical composition of air varies at each point of the nozzle. Finally, a third situation is when the nozzle flow time and the reaction times are comparable, leading to a complex way to evaluate the air chemical composition. Several kinetic mechanisms have been proposed to describe this case though [21].

Stuessy [56] presented the characterization of the nozzle flow in the UTA’s Hypersonic Shock Tunnel showing that frozen chemistry can be applied at low enthalpies (less than 1 MJ/kg) such as those of the present investigation. Recently, Leamon [59] utilized the same the assumption for characterization of test conditions, also at low enthalpies, using thin-film RTDs. For higher enthalpies (in general, higher than 4 MJ/kg), obtained with the detonation driven shock tunnel technique, Bello [61] also considered frozen chemistry in the nozzle.

6.2 Condensation

Air condensation may also arise in the expansion of low enthalpy flows [73]. The outcome is a two-phase flow that is non-repeatable and contaminated, causing faulty test conditions.

In fact, under certain pressure and stagnation conditions, air can condense as a consequence of the expansion cooling. Thus, low stagnation temperatures are of
concern, the minimum stagnation temperature to avoid saturation in the test section can be estimated from the relation [74]:

\[
\frac{370}{T_{t0}} \left(1 + \frac{M_0^2}{5}\right) = 4.7 + 3.5 \log_{10} \left(1 + \frac{M_0^2}{5}\right) - \log_{10} p_{t0}
\]  

where \( T_{t0} \) is the stagnation temperature in K and \( p_{t0} \) is the stagnation pressure in bar. In the present investigation, we had typical values of free stream Mach number \( M_0 = 8.6 \), and \( p_{t0} = 36.7 \) bar so that the minimum required stagnation temperature was 797 K, according to Eq. (6.1), which is below 923 K which was a typical value for the stagnation temperatures for our tests.

Although not observed in our tests, condensation can be delayed due to the high velocities and low densities in the hypersonic nozzle [75]. This is called supersaturation and, similar to the supercooling phenomena in liquids, the air remains in a gaseous state even beyond the saturation line.

6.3 Modeling of Nozzle Flow

6.3.1 Frozen Chemistry Nozzle

For simplicity, the expansion can be assumed as an isentropic, quasi-one-dimensional and inviscid flow. The process to find the freestream properties consists first in determining the free stream Mach number. Since in each test a Pitot probe was used to measure the impact pressure at the test section and the stagnation pressure was also read in the end of driven section, using the Rayleigh-Pitot formula (see Appendix A) one can solve numerically for the free stream Mach number, \( M_0 \):

\[
\frac{p_{pitot}}{p_{t0}} = \left(\frac{\gamma_{t0} + 1}{2}\right)^{\frac{\gamma_{t0} - 1}{\gamma_{t0} + 1} M_0^{\frac{2}{\gamma_{t0} - 1}}} \left[1 + \frac{\gamma_{t0} - 1}{2} M_0^2\right]^{-\frac{\gamma_{t0}}{\gamma_{t0} - 1}} \left[\frac{\gamma_{t0} - 1}{2} + \gamma_{t0} M_0^2\right]^{-\frac{1}{\gamma_{t0} - 1}}
\]

(6.2)

where the subscript \( t0 \) means total conditions and 0 stands for free stream conditions.
Other properties were obtained using isentropic relationships, Appendix B.

Thus, the free stream temperature

\[ T_0 = \frac{T_{t0}}{1 + \frac{\gamma-1}{2}M_0^2} \]  \hspace{1cm} (6.3)

and, the density

\[ \rho_0 = \frac{\rho_{t0}}{(1 + \frac{\gamma-1}{2}M_0^2)^{\frac{1}{\gamma-1}}} \]  \hspace{1cm} (6.4)

Since \( a_0 = \sqrt{\gamma_{t0}R_{t0}T_0} \), the free stream velocity

\[ u_0 = M_0a_0 \]  \hspace{1cm} (6.5)

With the above, the unit Reynolds number

\[ Re_x = \frac{\rho_0u_0}{\mu_0} \]  \hspace{1cm} (6.6)

can be calculated with, the Sutherland’s law applied for the dynamic viscosity of air:

\[ \mu_0 = 1.716 \cdot 10^{-5} \left( \frac{T_0}{273.15} \right)^{3/2} \frac{383.55}{T_0 + 110.4} \]  \hspace{1cm} (6.7)

where \( T_0 \) is in K and \( \mu_0 \) is in kg/(m·s).

6.3.2 Equilibrium Chemistry Nozzle

Firstly, let us consider that no Pitot measurement was taken. In this case, the process to find the free stream properties consists first in determining the conditions in the throat (station \(*\)), which is sonic. Then these conditions are used in an isentropic expansion through the nozzle (station 0). In the convergent section of the nozzle the conservation relations reduce to:

\[ h^* + \frac{u^{*2}}{2} = h_{t0} \]  \hspace{1cm} (6.8)

for energy. Since the flow is isentropic,

\[ s^* = s_{t0} \]  \hspace{1cm} (6.9)
From the throat to the test section, the conservation equations yield:

\[ \rho_0 u_0 A_0 = \rho^* u^* A^* \]  
(6.10)

for mass, and for energy:

\[ h_0 + \frac{u_0^2}{2} = h^* + \frac{u^*^2}{2} \]  
(6.11)

with the isentropic assumption,

\[ s_0 = s^* \]  
(6.12)

Initial guesses for solving Eqs. (6.8) to (6.12) are taken from the perfect gas relations for isentropic expansion, see Appendix B. Now, since the Pitot pressure was measured in each test, consider a centered streamline that goes through the free stream region and downstream of the bow shock wave formed in front of the Pitot probe, see Fig. 6.1. If we denote \( ds0 \) the region downstream the normal shock, one can get from the conservation equations:

\[ \rho_{ds0} u_{ds0} = \rho_0 u_0 \]  
(6.13)

for mass,

\[ p_{ds0} + \rho_{ds0} u_{ds0}^2 = p_0 + \rho_0 u_0^2 \]  
(6.14)

for momentum, and for energy:

\[ h_{ds0} + \frac{u_{ds0}^2}{2} = h_0 + \frac{u_0^2}{2} \]  
(6.15)

The free stream flow can be related to the stagnation condition as follows,

\[ h_0 + \frac{u_0^2}{2} = h_{st0} \]  
(6.16)

and,

\[ s_0 = s_{st0} \]  
(6.17)
Figure 6.1. Flow stations for calculating free stream conditions.

For subsonic flow from downstream of the normal shock to the Pitot probe surface – assumed to be a stagnation point – the process can be considered as an isentropic compression. Thus, the conservation equations lead us to:

$$h_{Pitot} = h_{ds0} + \frac{u_{ds0}^2}{2} = h_{t0}$$ \hspace{1cm} (6.18)

For energy. And, for entropy:

$$s_{Pitot} = s_{ds0}$$ \hspace{1cm} (6.19)

with,

$$s_{Pitot} = s(p_{Pitot}, h_{Pitot})$$ \hspace{1cm} (6.20)

One can numerically solve Eqs. (6.13) to (6.20), with initial guesses taken from the perfect gas assumption.
CHAPTER 7
INSTRUMENTATION

7.1 Model Instrumentation

Given the short test time, in the order of a few hundreds microseconds, piezoelectric pressure transducers are the most appropriate types of pressure gages, due to their short rise time. In these transducers, preloaded ceramic plates, separated from the environment by a diaphragm, are used as the sensing elements [76], see Fig. 7.1. Once under stress, the ceramic generates charge. However, besides the good electrical insulation of the ceramic housing, the charge discharges rapidly, following a RC-circuit charge exponential decay. Thus, piezoelectric transducers can only measure dynamic pressure. The high impedance output is converted by a built-in integrated circuit amplifier (ICP) to a low impedance output, that can be easily read, see Fig. 7.2 for details. This conversion avoids the use of low-impedance cables and allows the output signal to be transmitted by long cables with minimum noise interference.

The model was instrumented with nine piezoelectric pressure transducers, see Fig. 7.3 regarding their positions. Three type PCB 103B02 were located at the upper ramp, see Table 7.1 for specifications, for low static pressure measurements. Three type 111A21 were located at inner positions where higher static pressures are expected, refer to Table 7.2 for general characteristics of these sensors. Moreover, the inlet exit was instrumented with a total pressure rake composed of three PCB 111A21 to assess the flow on the exit plane. Furthermore, a Pitot probe was installed below the model with a PCB 111A21 in order to measure the impact pressure during
Figure 7.1. Piezoelectric pressure sensor construction [76].

Figure 7.2. Example circuit of a piezoelectric sensor system [76].

each run. This reading along with the driven sensor readings, allowed the calculation of freestream properties. Table 7.3 shows the transducer inventory for the model. Figure 7.5 shows a typical trace of a piezoelectric transducer.
Table 7.1. General specifications of PCB 103B02 [77].

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measurement Range</td>
<td>3.33 psi</td>
</tr>
<tr>
<td>Sensitivity</td>
<td>1500 mV/psi</td>
</tr>
<tr>
<td>Maximum Dynamic Pressure</td>
<td>250 psi</td>
</tr>
<tr>
<td>Resolution</td>
<td>0.02 mpsi</td>
</tr>
<tr>
<td>Rise Time</td>
<td>$\leq 25 \mu s$</td>
</tr>
<tr>
<td>Output Impedance</td>
<td>$\leq 100 \Omega$</td>
</tr>
</tbody>
</table>

Table 7.2. General specifications of PCB 111A21 [78].

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measurement Range</td>
<td>100 psi</td>
</tr>
<tr>
<td>Sensitivity</td>
<td>50 mV/psi</td>
</tr>
<tr>
<td>Maximum Dynamic Pressure</td>
<td>1000 psi</td>
</tr>
<tr>
<td>Resolution</td>
<td>2 mpsi</td>
</tr>
<tr>
<td>Rise Time</td>
<td>$\leq 2 \mu s$</td>
</tr>
<tr>
<td>Output Impedance</td>
<td>$\leq 100 \Omega$</td>
</tr>
</tbody>
</table>

Figure 7.3. Location of pressure transducers.
Figure 7.4. Typical trace of a piezoelectric transducer.

Table 7.3. Model pressure transducers information.

<table>
<thead>
<tr>
<th>TYPE</th>
<th>SN</th>
<th>Sensitivity (mV/psi)</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>PCB103B02</td>
<td>5676</td>
<td>1455</td>
<td>ramp center line</td>
</tr>
<tr>
<td>PCB103B02</td>
<td>5679</td>
<td>1475</td>
<td>ramp center line</td>
</tr>
<tr>
<td>PCB103B02</td>
<td>5673</td>
<td>1472</td>
<td>ramp center line</td>
</tr>
<tr>
<td>PCB111A21</td>
<td>15891</td>
<td>50.47</td>
<td>inner part, center line</td>
</tr>
<tr>
<td>PCB111A21</td>
<td>15915</td>
<td>51.75</td>
<td>inner part, center line</td>
</tr>
<tr>
<td>PCB111A21</td>
<td>15914</td>
<td>51.31</td>
<td>inner part, center line</td>
</tr>
<tr>
<td>PCB111A21</td>
<td>15920</td>
<td>51.88</td>
<td>rake</td>
</tr>
<tr>
<td>PCB111A21</td>
<td>15919</td>
<td>51.1</td>
<td>rake</td>
</tr>
<tr>
<td>PCB111A21</td>
<td>15917</td>
<td>50.28</td>
<td>rake</td>
</tr>
<tr>
<td>PCB111A21</td>
<td>15916</td>
<td>51.51</td>
<td>Pitot probe</td>
</tr>
</tbody>
</table>

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7.2 HST Instrumentation

7.2.1 Pressure Transducers

During the present investigation, the end of the HST driven section features three type PCB111A23 pressure transducers, see Table 7.4 for general specifications, and this inventory is listed in Table 7.5. Two of them are separated by 1.37 m and they are used to estimate the incident shock wave speed by time-of-flight (TOF) method. The final one nearest the nozzle measures the reservoir pressure and is used to trigger the data acquisition system (DAQ). This transducer is positioned just a few centimeters from the throat.

<table>
<thead>
<tr>
<th>Measurement Range</th>
<th>10,000 psi</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sensitivity</td>
<td>0.5 mV/psi</td>
</tr>
<tr>
<td>Maximum Dynamic Pressure</td>
<td>15,000 psi</td>
</tr>
<tr>
<td>Resolution</td>
<td>200 mpsi</td>
</tr>
<tr>
<td>Rise Time</td>
<td>≤ 1.5 µs</td>
</tr>
<tr>
<td>Output Impedance</td>
<td>≤ 100 Ω</td>
</tr>
</tbody>
</table>

Table 7.4. General specifications of PCB 111A23 [79].

<table>
<thead>
<tr>
<th>TYPE</th>
<th>SN</th>
<th>Sensitivity (mV/psi)</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>PCB111A23</td>
<td>15931</td>
<td>0.5095</td>
<td>driven tube</td>
</tr>
<tr>
<td>PCB111A23</td>
<td>9709</td>
<td>0.5095</td>
<td>driven tube</td>
</tr>
<tr>
<td>PCB111A23</td>
<td>15932</td>
<td>0.5095</td>
<td>stagnation</td>
</tr>
</tbody>
</table>

Table 7.5. Tunnel pressure transducers information.
7.2.2 Thin Film RTDs

Resistance temperature detectors are sensor elements that show resistance variation with temperature. In general, RTDs are fabricated with materials like platinum or nickel due to their resistance to contamination and corrosion, providing a good repeatability of measurements. In thin-film RTDs, a thin-film of metallic element is placed on a flat ceramic tube with an appropriate housing for the lead wires. The output of an RTD is a resistance change that can be conditioned with a Wheatstone bridge and the temperature is then determined with the calibration curve. Although RTDs were originally made for temperature measurements, they are used as heat flux meters in pulsed facilities. This is because in these facilities thermal equilibrium is never reached between the free stream and the probe, and appropriate thin film thickness, in general from 0.01 to 0.1 µm [80], must be provided so the resistance variation can be measured. These sensors were constructed for HST characterization at low enthalpies [59, 81, 82].
Each of the thin-film RTD gauges needs to be static and dynamically calibrated to obtain the two key sensor properties, the temperature coefficient of resistance, \( \alpha_R = \frac{1}{R_0} \frac{\Delta R}{\Delta T} \), and the thermal product, \( \sqrt{\rho c k} \), where \( R_0 \) is the initial resistance of the sensing element and \( \Delta R \) its variation with the temperature \( T \). The material density is \( \rho \), the specific heat is \( c \) and \( k \) is the thermal conductivity. During the static calibration, the probes are immersed in glycerin due to its high boiling point and low gas solubility [59]. This process is used to obtain \( \alpha_R \). After that, dynamic calibration determines the thermal product of the RTDs with a transient double electrical discharge [59, 81].

Once the calibration is done, the voltage output from a test must be conditioned to obtain the exact heat flux. One method for calculating heat fluxes under unsteady conditions is the Cook-Felderman algorithm [82, 83]:

\[
\dot{q}_s(t_n) = \frac{\sqrt{\rho c k}}{\sqrt{\pi \alpha_R V_o}} \left\{ \frac{V(t_n)}{\sqrt{t_n}} + \sum_{i=1}^{n-1} \left[ \frac{V(t_n) - V(t_i)}{\sqrt{t_n - t_i}} - \frac{V(t_n) - V(t_{i-1})}{\sqrt{t_n - t_{i-1}}} \right] \right\} \right. \\
\left. + \frac{\sqrt{\rho c k}}{\sqrt{\pi \alpha_R V_o}} \left\{ \sum_{i=1}^{n-1} \left[ 2 \frac{V(t_i) - V(t_{i-1})}{\sqrt{t_n - t_i} + \sqrt{t_n - t_{i-1}}} \right] + \left[ \frac{V(t_n) - V(t_{n-1})}{\sqrt{\Delta t}} \right] \right\} 
\] (7.1)

where \( \dot{q}_s \) is the heat flux, \( V_o \) is the bias voltage and \( V \) is the voltage reading. The algorithm uses a discrete set \( t_0, ..., t_i, ..., t_n \) in time, with a constant step of \( \Delta t \).

As pointed out by Leamon [59], most common problems using thin-film RTDs are the large dependence of these sensors on the substrate surface preparation and on the ceramic production. Also, difficulties can arise due to improper lead wire connection, self heating, fragility, and sensitivity to vibrations.

### 7.2.3 Ionization Gages

For sufficiently high shock wave Mach numbers, in general greater than 4.5, high post-shock temperature is enough to increase air ionization and conductivity so
that ionization probes can be used [84]. A typical ion probe is depicted in Fig. 7.6. When the high-temperature air reaches the probe, the circuit closes and capacitor discharges. The response time of these gauges is about 1 $\mu$s and it is dependent on the shock strength and on the distance between the copper wires. These gauges have been used to measure shock and detonation wave speeds, and also to detect contact surface arrival.

7.2.4 Schlieren Apparatus

The UTA Aerodynamics Research Center (ARC) also has an entire schlieren system for flow visualization, originally acquired for the Supersonic Wind Tunnel [26]. This system is a Z-type configuration\(^1\), described in Fig. 7.7. The light source is a Nasun WFL604MR161D3W 3 W white LED, 150 lumens. The condenser is an Edmund Optics 43-593 50 mm diameter and 40 mm focal length. The slit is an

\(^1\)See Ref. 85 for further details about this technique.
Edmund Optics iris diaphragm type 57-583, 53 mm diameter. Two parabolic mirrors type Edmund Optics 32-071-533 152.4 mm (6 in) diameter with a focal length of 914.4 mm (36 in). A common razor blade is then used as spatial filter at the second mirror focal point. Also, a double-convex lens Edmund Optics 45-171 is used to adjust the image position to be captured by the camera, given the short space existent for placement of the schlieren tables. The camera is a Lavision Imager Intense 1101024, originally used for a particle image velocimetry (PIV) system. The camera is capable of taking two images with a 500 ns interframing time, an exposure time of 500 ns and 1.43 Mpxls resolution.

7.2.5 Force Balance

The challenge of measuring forces in pulsed facilities is that steady state is never reached due to the short test time. However, force measurements in the UTA hypersonic shock tunnel have been possible with the development of a stress wave
The drag balance uses flexible piezo-polymer film (Measurement Specialties LDT0-028K) for measuring dynamic strain. These gages can achieve an impressive frequency response of the order of 1 GHz, about 10 times higher than common strain gages.

The theory behind the stress wave force measurement is that once the transfer function $g(t)$ of the force balance system is known, a deconvolution process may be applied to reconstruct the input signal, which is the applied force. Let us consider that for a given dynamic load $u(t)$, we have that the output strain $y(t)$ is given by [87]:

$$y(t_n) = \sum_{m=0}^{n} g(t_m) u(t_n - t_m) \Delta t_m$$  \hspace{1cm} (7.2)

with $n, m = 0, 1, 2, \ldots$. Note that this is the discrete convolution of functions $u(t)$ and $g(t)$.

In order to obtain the transfer function, Vadassery [58] used a thrust stand to accommodate the force balance; a steel wire was attached to the balance in one end, and on the other, known weights were attached; fast wire cutting simulated a negative step load, the transfer function being the first derivative of the output signal. Another method described for obtaining $g(t)$ was to use an impulse hammer (PCB 086C01), simulating an impulse the author could directly obtain the transfer function from the response signal.

Once the transfer function is obtained, the strain gage response is used to obtain the force load history by deconvolving Eq. (7.2) with a convenient method, whether by Fourier transform [86] or in time domain [88].
7.2.6 Data Acquisition System (DAQ)

The PCB sensors were powered by a 24 to 27 VDC, 2 to 20 mA constant-current supply, provided from a signal conditioner type PCB 481A02, which featured 16 channels, programmable gains of 1, 10 or 100, with BNC input/output. Data is recorded using a DAQ composed of a NI PXIe-1065 Express Chassis with two TB-2709 8-channel modules, capable of sampling at 2.5 MHz per channel and configured to take a total of 200 ksamples. The trigger for this DAQ was set up to occur with a 200 mV output from the stagnation pressure transducer signal. See Fig. 7.8 regarding the Labview block diagram used for the tests.
Figure 7.8. Block diagram used in DAQ.
CHAPTER 8

RESULTS

This chapter discusses the results obtained in experimental and numerical investigations of the scramjet inlet, at hypersonic Mach number 8.6 and specific stagnation enthalpy of 0.67 MJ/kg. We also present shock tube and free stream flow conditions. The objective was to evaluate the performance of a scramjet inlet as the side cowl length is changed. The slender inward turning inlet had opposing compression ramps at 11.54 deg and CR = 4.79. With geometry described in Chapter 2, the model was used with side cowl lengths of 0, 50.8 and 76.2 mm. Refer to Table 8.1, regarding the nomenclature used throughout this chapter.

Table 8.1. Nomenclature of inlet configurations.

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Cowl Length (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>–</td>
</tr>
<tr>
<td>B</td>
<td>50.8</td>
</tr>
<tr>
<td>C</td>
<td>76.2</td>
</tr>
</tbody>
</table>

8.1 Hypersonic Shock Tunnel Conditions

During this investigation, the hypersonic shock tunnel was operated in the reflected mode, using dry air as driver gas. The maximum driver pressure achieved was 35.85 MPa (5200 psi) and the minimum was 31.02 MPa (4500 psi), the driven initial pressure was kept in 1.4 atm (20 psi). Operation of the tunnel was performed remotely, with the control boards setup as described in Appendix E. The three pressure sensors installed in the driven section were used to measure the shock wave speed and reservoir pressure during the tests. Typical traces from these three driven tube sensors can be observed in Fig. 8.1.
Figure 8.1. Typical traces from the pressure sensors installed in the driven section.

For each test, it were collected the driven initial pressure and temperature, and the time-of-flight of the incident shock wave between the two driven pressure sensors placed for this purpose, $p_{2(1)}$ and $p_{2(2)}$. These data were used to calculate the shock wave speed and then used as inputs for calculation of stagnation flow properties during each test. The procedure for this calculation was described in Chapter 5 for the reflected mode of operation, and the code used is listed in Appendix F. The measured stagnation pressure was used as well to improve accuracy when calculating the free stream conditions. Also in Fig. 8.1, one can notice that the incident shock wave arrival is a very clear steep signal, similar is the reflected shock wave only observed in the stagnation pressure trace $p_5$. In this figure, one can also see the
arrival of the leading edge of the unsteady expansion wave produced in the diaphragm opening and which is reflected back at the end of the driver tube. Although this is an expansion wave, the reader should note that the contact surface arrival is not perceived by pressure measurements, so the expansion wave is propagated in the driver gas, initially at very high pressure. This is the reason why this wave is characterized by a pressure rise and not a pressure drop.

With the calculated stagnation conditions, test section conditions were obtained via the procedure described in Chapter 6, for a frozen chemistry flow. To do this, it was also necessary to measure the impact pressure for each test. Thus, using the Rayleigh–Pitot formula one can solve numerically for the free stream Mach num-
ber and then find all other free stream flow properties. A typical trace of the impact probe can be see in Fig. 8.2. In this figure, one can see the starting shock arrival, with a strength of about 5.5 psi (0.38 bar), followed by an unsteady expansion and then the test gas is characterized by a ‘plateau’ in this sensor trace at 13.1 ms that lasts less than 1 ms. The estimated conditions during a selected run are shown in Table 8.2.

Table 8.2. Typical flow properties during shock tunnel tests.

<table>
<thead>
<tr>
<th>Reservoir</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure</td>
<td>3.67</td>
<td>MPa</td>
</tr>
<tr>
<td>Specific enthalpy</td>
<td>0.67</td>
<td>MJ/kg</td>
</tr>
<tr>
<td>(zero is taken at 298.15 K, 1.0 atm)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Free stream</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure</td>
<td>220</td>
<td>Pa</td>
</tr>
<tr>
<td>Temperature</td>
<td>67.62</td>
<td>K</td>
</tr>
<tr>
<td>Density</td>
<td>0.00648</td>
<td>kg/m³</td>
</tr>
<tr>
<td>Velocity</td>
<td>1423</td>
<td>m/s</td>
</tr>
<tr>
<td>Mach Number</td>
<td>8.6</td>
<td>-</td>
</tr>
<tr>
<td>Reynolds Number</td>
<td>1.94 · 10⁶</td>
<td>1/m</td>
</tr>
<tr>
<td>Test time</td>
<td>300–500</td>
<td>µs</td>
</tr>
<tr>
<td>Wall temperature</td>
<td>294.15</td>
<td>K</td>
</tr>
<tr>
<td>N₂</td>
<td>79%</td>
<td></td>
</tr>
<tr>
<td>O₂</td>
<td>21%</td>
<td></td>
</tr>
<tr>
<td>Air composition</td>
<td>NO 1.38 · 10⁻³%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>N &lt; 10⁻³%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>O &lt; 10⁻³%</td>
<td></td>
</tr>
</tbody>
</table>
8.2 Numerical Simulation

As stated in Chapter 3, numerical simulation of the flow field on the inlet was performed using Ansys Fluent [33], Academic Research, Release 14.0. A thermally perfect gas assumption and the Wilcox $K-\omega$ turbulence model was applied. The solver option was density based and the Green–Gauss cell based method was applied to calculate gradients. The unstructured mesh used tetrahedral elements.

The pressure field is very important for any flow related with propulsion systems. The general structure of the static pressure contours on the inlet with and without sidewall can been seen in Fig. 8.3. The primary contributors of this flow structure are the cross flow due to the lateral spillage, see Fig. 8.4, causing a transverse shock wave pair in the inlet entrance, and the primary shock wave system, formed at the inlet ramp.

In addition to the primary shock wave, the effect of the transverse shock wave is to increase the static pressure and decelerate the flow in near wall regions. The no-sidewall case, configuration A, Fig. 8.3(a), presents significant interaction between the flow processed by the ramp and the fixed walls in the inner inlet stations, see Fig. 8.3. That could be seen by the large values of pressure found in near wall regions. The interaction between the primary and transverse shock denigrates the quality of flow through distortion of the pressure distribution.

As expected, there is a clear relationship between the sidewall length and the structure of the flow. In configuration C, the cross flow is less pronounced so is the transverse shock wave pair, see Fig. 8.3(c). The primary shock wave becomes less curved and the region of high pressure tends to move from the wall region to the center. On an hypothetical case of a full sidewall, one can expect that almost perfectly localized shock waves and expansions structures on $x$-coordinate would be found.
Figure 8.3. Static pressure contours in Pa on lower symmetry plane, for configurations A, B and C.

Figure 8.4. Static pressure contours on selected planes for case A, in Pa.
8.3 Static Pressure Readings

The static pressure transducers allowed us to assess some flow field feature. Due to the large quantity of data, only pressure traces of these sensors from a selected run are presented here, see Fig. 8.5. Examining these figures, one can also see the starting process described later and the formation of the test conditions after the starting process.

Due to the recesses between the model wetted surface and the transducer diaphragms, readings of the PCB type 103B02 transducers, placed in the first three stations, were expected to have a slight time delay on the rising event. However, since they were placed in stations close to the leading edge, the author believes that this delay was compensated with the distance between them and the flushed PCB 111A21 transducers, which were placed in the hindmost stations. These data were also characterized by a high noise level. Compared with the pressure transducers placed at inner stations of the inlet, which were type PCB 111A21, the PCB 103B02 transducers also presented higher noise level. This can be explained by the high sensitivity of the PCB 103B02 when compared with the PCB 111A21. On the other hand, shock tunnel vibrations were not enough to produce significant noise in the pressure readings. This was concluded after comparing the time when the shock tunnel reaches the reflected mode with the time when the starting shock impinges the model. If the shock tunnel vibration were the cause of noise, false readings would have been seen instants after the diaphragm burst, which obviously was not the case.
Figure 8.5. Typical traces of pressure sensors installed in model centerline.
Figure 8.6. Static pressure distribution, obtained from experiments and numerical simulations.

The surface pressure distribution along the centerline is shown in Fig. 8.6 for the three cases. From the figure, one can conclude that the cowl length has a significant effect on the pressure distribution. As can also be seen in Figs. 8.3(a) and 8.4 (simulations for case A), the cross flow, characterized by transverse pressure gradients, is associated with the influence of the ramp leading-edges corners. This interference causes flow distortion capable of producing a pair of crossing shocks in the inlet. This pair of lateral shock waves then interacts with the shock wave from the lower ramp and produces a far more complex flow than in the two-dimensional case. In the no-side wall case (case A), the cross flow is more pronounced and hence
so is the crossing shock pair and the related flow distortion. When the cowl length is increased, the crossing shock pair is weaker. Moreover, we can also see in Fig. 8.6 that the effect of sidewall is present in earlier stations, see station $x = 171.45$ mm, indicated by the pressure rise. This also indicates that an unpredicted separation bubble was formed near that the inlet entrance. The readings also indicate that this separation is more severe as the sidewall length is increased. At the entrance of the flat part of the inlet, station $x = 205.44$ mm, the flow is expanded before the impingement of the opposite shock wave generated at the leading edge of the model. At inner stations, $x = 240.03$ mm and $x = 274.32$ mm, the pressure rise is observed again and a clear distinction between configurations can be noted. From
the numerical results, the maximum pressure ratio varied from 19.7 for the case A (no cowl) to 28.7 to the long cowl case C. In Fig. 8.7, the pressure on one station is compared for several cowl lengths. A general conclusion is that with less distortion, the pressure rise is higher for the longer cow. In this case, the reflected shocks from the ramps become more localized since less attenuation by the expansion was observed so this configuration was able to reach a pressure rise close to the two-dimensional value of 26.3.

It is also of interest to compare both numerical and experimental data presented in Fig. 8.6. One can see good agreement in general. However this agreement is weaker after station \( x = 171.45 \) mm. Although not caught by the simulations, as stated before, the author believes that shock-wave/boundary-layer interactions produce a more complex flow which was not caught due to the limitations of the applied computer program. After the expansion corner, the results are closer, and both results show the general tendency that the pressure decreases with wall length decrease.

8.4 Exit Mach Number

The rake readings were characterized by a time delay between the nozzle starting and the rise of the rake pressure signals of about 5 ms that is due to their recess and to the distance between the probes and the other transducers placed on the model. See graphs on Fig. 8.8 where the traces of the three total probes are shown, each probe has a corresponding normal position \( z \), as depicted in Fig. 7.3. These transducers traces were characterized by a higher noise level than the ones for static measurements. Because they are recessed, damping occurs and therefore an attenuated response to the starting shock wave was observed. Furthermore, the tubes act
Figure 8.8. Typical traces of pressure sensors installed in the inlet exit plane.
as Helmholtz resonators: when air is forced into the tubes, the pressure inside rises and the air column trapped in the tubes starts to flow out, decreasing the pressure, so the air is allowed to be trapped again. This process has an oscillatory behavior and its frequency is dependent on the depth of the passage and the gas speed of sound [89, 90].

These Pitot probes allowed us to calculate the Mach number distribution on the centerline of the exit plane of the inlet. The flow was assumed frozen chemistry with respect to the test section properties. The calculated Mach number distribution is depicted in Fig. 8.9. As one can see, the Mach number showed a mild variation from both experimental and computational analysis. Among the investigated con-
figurations, the Mach number varied from 2.9 to 3.8 (31 percent). The no cowl case (A) provided the highest Mach number while a cowl length of 50.8 mm (B) resulted in the lowest Mach number. From the numerical results we can see that a thicker boundary layer was found also in configuration B. Yet, a clear conclusion of these results is eclipsed by the large error obtained in measurements and the limited computational model to predict shock-wave/boundary-layer interactions. To bolster this argument, as one can also see in Fig. 8.9, numerical and experimental data did not show good agreement in the stations nearest to the wall. Also, the no-cowl case A experienced the best agreement between simulations and tunnel tests.

Finally, one can also conclude that in all cases, a mild Mach number uniformity is achieved. Furthermore, the largest variation was observed in configuration A, with no cowl where the Mach number was from 3.83 at the center and 3.47 at station $z = 12.7$ mm, with a 10.3 percent variation.
CHAPTER 9

CONCLUSIONS AND RECOMMENDATIONS

In this work we presented experimental and numerical results showing how the side cowl length can impact the inlet flow field. Shock tunnel experiments were used to reveal the pressure distribution along the center line of the inlet and at its exit plane. The measurements were further compared with numerical calculations using the Wilcox $K-\omega$ turbulence model.

The convergent inlet was composed of two symmetric ramps with a baseline geometry taken as opposing 12 degree ramps with a 101.6 mm (4 in) entrance height and 50.8 mm (2 in) span. The design velocity was 2 km/s at an altitude of 30 km. The shock wave position and flow properties were calculated using the chemical equilibrium flow assumption and the viscous correction was applied using the Cebeci–Bradshaw algorithm [34], considering an over speed of 3 km/s, at the design altitude. The resulting model had a total length of 304.8 mm with the compression at 11.54 degrees. The throat height was 21.22 mm, thus $CR = 4.79$. The rake probes used 3.175 mm diameter stainless steel tubes in order to minimize perturbations on the inlet exit. The model consisted of three main parts: an instrumented upper ramp, a lower ramp and the rake mechanism. A wedge was chosen to hold the total pressure rake at the exit. The upper ramp was closed by two side walls, two plates at the top and a rear wall that allows the transducers wiring to go out from the model.

Due to the lack of measurement systems on the hypersonic shock tunnel to acquire the stagnation temperature or static temperature inside the test section, the shock tunnel flow had to be modeled so as to provide the free stream conditions.
Thereby a computer code simulated the flow expansion from the driver to the driven 
sections, the shock-wave interactions that occur inside the driven section as well 
as the nozzle expansion. In the last two processes the code accounted for real gas 
behavior, based on the minimization of the Gibbs free energy. Three pressure sensors 
located in the end of the driven section of the HST allowed the direct measurement 
of the incident shock wave speed and the reservoir pressure. This last measurement 
was used as correction-entry for the calculated stagnation pressure.

During the tests, the stagnation conditions were a pressure of 3.67 MPa and a 
specific enthalpy of 0.67 MJ/kg. The free stream flow was characterized by a Mach 
number of 8.6, static pressure of 220 Pa, and Reynolds number of $1.94 \cdot 10^6 \text{ m}^{-1}$. 
The test time varied from 300 to 500 $\mu$s.

Numerical simulations of the flow field on the inlet were performed using AN-
SYS Fluent [33], Academic Research, Release 14.0. A thermally perfect gas as-
sumption and the Wilcox $K-\omega$ turbulence model was applied. The solver option 
was density based and the Green–Gauss cell based method was applied to calculate 
gradients. The unstructured mesh used tetrahedral elements.

The pressure distribution over the inlet center line was measured and the results 
confirmed the theoretical predictions concerning the inlet flow structure. The main 
conclusions taken from numeric and experimental investigations were:

- The static pressure distribution can be severely altered by the cowl length, 
  shown by both experimental and computational studies;
- The cowl length has a mild influence in the Mach number distribution at the 
  exit plane, also shown by both experimental and computational studies;
- The results from the CFD analysis showed that a complex flow field is formed in 
  the presence of the side cowl. In fact the side opening is cause of the presence
of a cross pressure gradient, decreasing flow uniformity. Thus, a transverse shock wave is formed by at the inlet entrance.

As recommendations, concerning the inlet flow modeling, the author underscores the importance of a better mesh independence study, and also simulations with different turbulence models are necessary. Considerable improvement of the numerical analysis could be attained with a more detailed model of the sidewalls, to account the finite leading edge thickness and, thus, the shock waves formed near those regions. In addition, the author recommends testing of the inlet model with stagnation enthalpies close to those corresponding to the air-breathing corridor. Application of schlieren technique, with the apparatus described in Chapter 7 would also improve experimental analysis.
APPENDIX A

NORMAL AND OBLIQUE SHOCK WAVE RELATIONS
For steady supersonic flow of a perfect gas, subscripts 1 and 2 denotes conditions before and after the shock wave, respectively.

\[ M_2^2 = \frac{1 + \frac{\gamma - 1}{2} M_1^2 }{\gamma M_1^2 - \frac{\gamma - 1}{2}} \]  
\[ p_2 \frac{p_1}{p_1} = \frac{2 \gamma M_1^2 - (\gamma - 1)}{\gamma + 1} \]  
\[ \frac{p_2}{p_1} = \frac{u_1 + \frac{\gamma + 1}{2}}{1 + \frac{\gamma - 1}{2} M_1^2} \]  
\[ \frac{T_2}{T_1} = \frac{h_2}{h_1} = \frac{p_2}{p_1} \frac{p_2}{p_1} \frac{p_2}{p_1} = \left[ 1 + \frac{2\gamma}{\gamma + 1} (M_1^2 - 1) \right] \frac{2 + (\gamma - 1) M_1^2}{(\gamma + 1) M_1^2} \]  
\[ \frac{a_2}{a_1} = \left[ 1 + \frac{2\gamma}{\gamma + 1} (M_1^2 - 1) \right]^{1/2} \left[ \frac{2 + (\gamma - 1) M_1^2}{(\gamma + 1) M_1^2} \right]^{1/2} \]  
\[ \frac{u_1 - u_2}{a_1} = \left[ \frac{2}{\gamma(\gamma - 1)} \right]^{1/2} \left( \frac{p_2}{p_1} - 1 \right) \left[ \left( \frac{\gamma + 1}{\gamma - 1} \right) \frac{p_2}{p_1} + 1 \right]^{-1/2} \]  
\[ \frac{u_1 - u_2}{a_1} = \frac{2}{\gamma + 1} \frac{M_1^2 - 1}{M_1} \]  
\[ M_1^2 = 1 + \frac{\gamma + 1}{2\gamma} \left( \frac{p_2}{p_1} - 1 \right) \]  
\[ \frac{p_{t2}}{p_{t1}} = \left( \frac{\gamma + 1}{2} \right) \frac{2}{\gamma - 1} M_1^2 \left[ 1 + \frac{\gamma - 1}{2} M_1^2 \right]^{-\frac{\gamma - 1}{\gamma + 1}} \left[ \frac{\gamma - 1}{2} + \gamma M_1^2 \right]^{-\frac{\gamma - 1}{\gamma + 1}} \]  
\[ \frac{p_{t2}}{p_1} = \left[ \frac{(\gamma + 1)^2 M_1^2}{4\gamma M_1^2 - 2(\gamma - 1)} \right]^{\gamma/(\gamma - 1)} \frac{1 - \gamma + 2\gamma M_1^2}{\gamma + 1} \]
\[ s_2 - s_1 = -R \ln \left( \frac{p_{t2}}{p_{t1}} \right) \]  \hspace{1cm} (A.11)

For an oblique shock wave:

\[ M_{1n} = M_1 \sin \beta \]  \hspace{1cm} (A.12)

\[ M_{2n} = M_2 \sin(\beta - \theta) \]  \hspace{1cm} (A.13)

\[ \tan \theta = \cot \beta \frac{M_2^2 \sin^2 \beta - 1}{1 + \left( \frac{\gamma + 1}{2} - \sin^2 \beta \right) M_1^2} \]  \hspace{1cm} (A.14)

where \( n \) stands for normal components.
APPENDIX B

ISENTROPIC RELATIONS
\[
\frac{p_2}{p_1} = \left( \frac{\rho_2}{\rho_1} \right)^\gamma \tag{B.1}
\]
\[
\frac{\rho_2}{\rho_1} = \left( \frac{T_2}{T_1} \right)^{\frac{1}{\gamma - 1}} \tag{B.2}
\]
\[
\frac{p_2}{p_1} = \left( \frac{T_2}{T_1} \right)^{\frac{\gamma}{\gamma - 1}} \tag{B.3}
\]
\[
\frac{T_i}{T} = 1 + \frac{\gamma - 1}{2} M^2 \tag{B.4}
\]
\[
\frac{\rho_t}{\rho} = \left( 1 + \frac{\gamma - 1}{2} M^2 \right)^{\frac{1}{\gamma - 1}} \tag{B.5}
\]
\[
\frac{p_t}{p} = \left( 1 + \frac{\gamma - 1}{2} M^2 \right)^{\frac{\gamma}{\gamma - 1}} \tag{B.6}
\]
\[
h_t = h + \frac{V^2}{2} \tag{B.7}
\]
\[
\frac{a_i^2}{\gamma - 1} = \frac{a^2}{\gamma - 1} + \frac{u^2}{2} \tag{B.8}
\]
\[
T^* = 2 \frac{T_i}{\gamma + 1} \tag{B.9}
\]
\[
a_i^2 = \frac{\gamma + 1}{2} a^* \tag{B.10}
\]
\[
a^* = a \left[ \frac{2 + (\gamma - 1) M^2}{\gamma + 1} \right]^{1/2} \tag{B.11}
\]
\[
\left( \frac{A}{A^*} \right)^2 = \frac{1}{M^2} \left[ \frac{2}{\gamma + 1} \left( 1 + \frac{\gamma - 1}{2} M^2 \right) \right]^{(\gamma + 1)/(\gamma - 1)} \tag{B.12}
\]
APPENDIX C

CALCULATION OF CHEMICAL EQUILIBRIUM COMPOSITION OF A GAS MIXTURE
This Appendix provides a basis for the calculation of the chemical composition of a gas mixture based on the Gibbs free energy minimization. We assume that the pressure \( p \) and temperature \( T \) of this mixture are known. Furthermore, we assume that each gas is a thermally perfect gas and each one acts as if it were alone in a container [70, 91].

The total Gibbs free energy of the mixture can be represented as:

\[
G(T, p) = \sum_{i=1}^{NS} n_i \cdot G_i(T, p) \tag{C.1}
\]

where \( n_i \) is the number of moles of species \( i \). And \( G_i \) is the species \( i \) free energy per unit of mole.

The dependence of \( G_i \) upon the pressure is given by:

\[
G_i(T, p) = G_{refi}(T) + \Re T \ln \left( \frac{p_i}{p_{ref}} \right) \tag{C.2}
\]

where,

\[
p_i = \frac{n_i}{\sum_{q=1}^{NS} n_q} \cdot p. \tag{C.3}
\]

The subscript \( ref \) indicates the reference pressure of 1 atm and \( \Re = 8.314 \text{ kJ/(mol}\cdot\text{K}) \) is the universal gas constant. Thus,

\[
G(T, p) = \sum_{i=1}^{NS} n_i \cdot \left\{ G_{refi}(T) + \Re T \ln \left( \frac{p_i}{p_{ref}} \right) \right\} \tag{C.4}
\]

Since the number of atoms of each element is conserved, this leads us to the constraint relation:

\[
C_j = \sum_{k=1}^{NS} n_k \cdot MM_{kj} - N_j^{init} = 0, \quad j = 1, \ldots, NE. \tag{C.5}
\]

where, \( MM_{kj} \) (matrix) is the quantity of the atoms of element \( j \) in species \( k \). And, \( N_j^{init} \) is the initial quantity of atoms of element \( j \) present in the initial mixture. Total number of elements is \( NE \).
The minimization of mixture’s Gibbs free energy with the constraints $C_j$ can be found using the Lagrange’s multipliers method. Thus, we define the functional:

$$F = G(T,p) - \sum_{j=1}^{NE} \lambda_j \cdot C_j$$  \hspace{1cm} (C.6)

where $\lambda_j$’s are the Lagrange multipliers. Using Eqs.(C.4), (C.5), and (C.6), one can define a new functional as:

$$F^\dagger(\vec{n}, \vec{\lambda}) = \sum_{i=1}^{NS} \left\{ n_i \cdot \left[ G_{refi}^{\dagger}(T) + \Re T \ln \left( \frac{p_i}{p_{ref}} \right) \right] \right\} - \sum_{j=1}^{NE} \left\{ \lambda_j \cdot \left[ \sum_{k=1}^{NS} (n_k \cdot MM_{kj}) - N_j^{init} \right] \right\}$$  \hspace{1cm} (C.7)

where the new variables are: $\vec{n} = \{n_1, n_2, ..., n_{NS}\}$ and $\vec{\lambda} = \{\lambda_1, \lambda_2, ..., \lambda_{NE}\}$. Substituting Eq. (C.3) in Eq. (C.7), and defining the dimensionless variables $F^\dagger = \frac{F}{\Re T}$, $G_{refi}^\dagger = \frac{G_{refi}}{\Re T}$ and $\lambda_j^\dagger = \frac{\lambda_j}{\Re T}$ one can get:

$$F^\dagger(\vec{n}, \vec{\lambda}) = \sum_{i=1}^{NS} n_i \cdot \left[ G_{refi}^\dagger(T) + \ln (n_i) - \ln \left( \sum_{q=1}^{NS} n_q \right) + \ln \left( \frac{p}{p_{ref}} \right) \right] - \sum_{j=1}^{NE} \lambda_j^\dagger \left[ \sum_{k=1}^{NS} (n_k \cdot MM_{kj}) - N_j^{init} \right]$$  \hspace{1cm} (C.8)

Lagrange’s conditions for minimization are that:

$$\frac{\partial F^\dagger}{\partial n_i^\dagger} = 0, i = 1, \ldots NS$$  \hspace{1cm} (C.9)

and,

$$\frac{\partial F^\dagger}{\partial \lambda_j^\dagger} = 0, j = 1, \ldots NE$$  \hspace{1cm} (C.10)

Applying the first condition to Eq. (C.8) yields:

$$\frac{\partial F^\dagger}{\partial n_i} = G_{refi}^\dagger(T) + \frac{d}{dn_i} \left[ \sum_{i=1}^{NS} n_i \ln (n_i) - \sum_{i=1}^{NS} n_i \cdot \ln \left( \sum_{q=1}^{NS} n_q \right) \right] + \ln \left( \frac{p}{p_{ref}} \right) - \sum_{j=1}^{NE} \lambda_j^\dagger \cdot MM_{ij}$$  \hspace{1cm} (C.11)
The second term in this equation can be treated as follows:

\[
\frac{d}{dn_i} \left[ \sum_{i=1}^{NS} n_i \ln (n_i) - \sum_{i=1}^{NS} n_i \cdot \ln \left( \sum_{q=1}^{NS} n_q \right) \right] = \ln (n_i) + n_i \cdot \frac{1}{n_i} - \ln \left( \sum_{q=1}^{NS} n_q \right) - \sum_{i=1}^{NS} n_i \cdot \frac{1}{\sum_{q=1}^{NS} n_q} = \ln n_i - \ln \left( \sum_{q=1}^{NS} n_q \right)
\]

(C.12)

Finally,

\[
\frac{\partial F^\dagger}{\partial n_i} = G_{refi}^\dagger (T) + \ln \left( \frac{p}{p_{ref}} \right) + \ln n_i - \ln \left( \sum_{q=1}^{NS} n_q \right) - \sum_{j=1}^{NE} \lambda_j^\dagger \cdot MM_{ij} = 0 \quad \text{(C.13)}
\]

Now, applying the second condition of minimization into (C.8), we find the same constraint of Eq. (C.5):

\[
\frac{\partial F^\dagger}{\partial \lambda_j^\dagger} = \sum_{k=1}^{NS} n_k \cdot MM_{kj} - N_j^{init} = 0 \quad \text{(C.14)}
\]

One can note that Eqs. (C.13) and (C.14) constitute a non-linear system of \((NE+NS) \times (NE+NS)\) equations. To solve this system we can define auxiliary functions:

\[
f_{1i} = \frac{\partial F^\dagger}{\partial n_i} \left( \rightarrow n, \rightarrow \lambda^\dagger \right), \quad i = 1, 2 \ldots NS \quad \text{(C.15)}
\]

\[
f_{2j} = \frac{\partial F^\dagger}{\partial \lambda_j^\dagger} \left( \rightarrow n, \rightarrow \lambda^\dagger \right), \quad j = 1, 2 \ldots NE \quad \text{(C.16)}
\]

and use a secant method to find the solution vector \(\rightarrow n\).

The JANAF’s 15 coefficients [92] are used to evaluate \(g_{refi}^\dagger\). In fact, these coefficients permit us to obtain \(S_{refi}^\dagger = \frac{S_{refi}}{\mathcal{R}}\), \(H_{refi}^\dagger = \frac{H_{refi}}{\mathcal{R} T}\) and \(C_{pi}^\dagger = \frac{C_{pi}}{\mathcal{R}}\) as:

\[
S_{refi}^\dagger (T) = a_{1i} \ln T + a_{2i} T + a_{3i} \frac{T^2}{2} + a_{4i} \frac{T^3}{3} + a_{5i} \frac{T^4}{4} + a_{7i} \quad \text{(C.17)}
\]

\[
H_{refi}^\dagger (T) = a_{1i} + a_{2i} \frac{T}{2} + a_{3i} \frac{T^2}{3} + a_{4i} \frac{T^3}{4} + a_{5i} \frac{T^4}{5} + a_{6i} T \quad \text{(C.18)}
\]

\[
C_{pi}^\dagger (T) = a_{1i} + a_{2i} T + a_{3i} T^2 + a_{4i} T^3 + a_{5i} T^4 \quad \text{(C.19)}
\]
where the first seven coefficients ($a_1$ to $a_7$) are applied for temperatures above 1000 K, and the remaining 7 are used for lower temperatures. The 15$^{th}$ coefficient is the dimensionless molar enthalpy at $T = 298.15$ K.

From the above, the dimensionless Gibbs free energy is given by:

$$G_{\text{refi}}^\dagger = \frac{G_{\text{refi}}}{RT} = H_{\text{refi}}^\dagger - S_{\text{refi}}^\dagger$$  \hspace{1cm} (C.20)

Once the molar concentration is found, one can obtain the molar fraction of each species:

$$x_{n_i} = \frac{n_i}{\sum_{k=1}^{NS} n_k}$$  \hspace{1cm} (C.21)

Then, the average molecular weight is given by:

$$MW = \sum_{i=1}^{NS} x_{n_i} MW_i$$  \hspace{1cm} (C.22)

where $MW_i$ is the molecular weight of species $i$.

The mixture density can be evaluated with the relationship:

$$\rho = \frac{p}{RT} MW$$  \hspace{1cm} (C.23)

Also, thermodynamic properties of the mixture can be calculated. For the mixture specific enthalpy (enthalpy per unit mass) we have:

$$h = \frac{RT}{MW} \sum_{i=1}^{NS} x_{n_i} h_{\text{refi}}^\dagger$$  \hspace{1cm} (C.24)

The entropy of the mixture per unit mass is:

$$s = \frac{R}{MW} \sum_{i=1}^{NS} x_{n_i} \left[ s_{\text{refi}}^\dagger - \ln \left( \frac{p_i}{p_{\text{ref}}} \right) \right]$$  \hspace{1cm} (C.25)

And the specific internal energy is given by:

$$e = h - \frac{p}{\rho}$$  \hspace{1cm} (C.26)
The specific heats of the mixture are calculated via:

\[ c_p = \left( \frac{\partial h}{\partial T} \right)_p \]  \hspace{1cm} (C.27)

and,

\[ c_v = \left( \frac{\partial e}{\partial T} \right)_v \]  \hspace{1cm} (C.28)

through a finite difference method.

Similarly, the speed of sound is obtained by [20]:

\[ a^2 = \left( \frac{\partial p}{\partial \rho} \right)_s = \frac{\gamma p}{\rho} \left[ 1 - \frac{1}{\gamma \rho} \left( \frac{\partial e}{\partial \rho} \right)_T \right] \]  \hspace{1cm} (C.29)

with \( \gamma = c_p/c_v \).
APPENDIX D

WILCOX $K - \omega$ TURBULENCE MODEL
The following equations represent the Wilcox $K - \omega$ used in combination with the classical Reynolds averaging Navier-Stokes scheme, applied in ANSYS Fluent calculations [33]. Differently from Section 3.3, we dropped the over-bars of the non-fluctuating terms and did not use the mass-weighted variables. The turbulent kinetic energy is obtained from [33]:

$$\frac{\partial (\rho K)}{\partial t} + \frac{\partial (\rho K u_j)}{\partial x_j} = \frac{\partial}{\partial x_j} \left( \Gamma_K \frac{\partial K}{\partial x_j} \right) + G_K - Y_K + S_K \quad (D.1)$$

and the specific dissipation rate from:

$$\frac{\partial (\rho \omega)}{\partial t} + \frac{\partial (\rho \omega u_j)}{\partial x_j} = \frac{\partial}{\partial x_j} \left( \Gamma_\omega \frac{\partial \omega}{\partial x_j} \right) + G_\omega - Y_\omega + S_\omega \quad (D.2)$$

where the terms $G_K$, $Y_K$ and $S_K$ represents the generation, diffusion and dissipation of turbulence, respectively, for K. The correspondent terms for $\omega$ are $G_\omega$, $Y_\omega$ and $S_\omega$.

The effective diffusivity is modeled by:

$$\Gamma_K = \mu + \frac{\mu_t}{\sigma_K} \quad (D.3)$$

where,

$$\mu_t = \alpha^* \frac{\rho K}{\omega} \quad (D.4)$$

A low-Reynolds-number correction is given by:

$$\alpha^* = \alpha^*_\infty \left( \frac{\alpha^*_0 + Re_t/R_K}{1 + Re_t/R_K} \right) \quad (D.5)$$

with,

$$Re_t = \frac{\rho K}{\mu \omega} \quad (D.6)$$

$$R_K = 6 \quad (D.7)$$

$$\alpha^*_0 = \beta_i / 3 \quad (D.8)$$

and,

$$\beta_i = 0.072 \quad (D.9)$$
For high Reynolds numbers:

\[ \alpha^* = \alpha_{\infty}^* = 1 \]  \hspace{1cm} (D.10)

The turbulence generation terms are:

\[ G_K = -\rho u'_i u'_j \frac{\partial u_j}{\partial x_i} \]  \hspace{1cm} (D.11)

and,

\[ G_\omega = a \frac{\omega}{K} G_K \]  \hspace{1cm} (D.12)

where,

\[ a = \frac{\alpha_{\infty}}{\alpha^*} \left( \frac{\alpha_0 + Re_t/R_\omega}{1 + Re_t/R_\omega} \right) \]  \hspace{1cm} (D.13)

with \( Re_\omega = 2.95 \).

The turbulence dissipation terms are given by:

\[ Y_K = \rho \beta^* f_{\beta^*} K \omega \]  \hspace{1cm} (D.14)

with,

\[ f_{\beta^*} = \begin{cases} 
1, & x_K \leq 0 \\
\frac{1 + 680x_K^2}{1 + 400x_K^2}, & x_K > 0 
\end{cases} \]  \hspace{1cm} (D.15)

and,

\[ x_K = \frac{1}{\omega^2} \frac{\partial K \partial \omega}{\partial x_j \partial x_i} \]  \hspace{1cm} (D.16)

and,

\[ \beta^* = \beta_{\infty}^* \left[ 1 + \zeta^* F(M_t) \right] \]  \hspace{1cm} (D.17)

where,

\[ \beta_{\infty}^* = \beta_{\infty}^* \left[ \frac{4/15 + (Re_t/R_\beta)^4}{1 + (Re_t/R_\beta)^4} \right] \]  \hspace{1cm} (D.18)

with,

\[ \zeta^* = 1.5 \]  \hspace{1cm} (D.19)
and,

\[ R_\beta = 8 \]  \hspace{1cm} (D.20)

and,

\[ \beta^*_\infty = 0.09 \]  \hspace{1cm} (D.21)

Moreover, for \( \omega \), the dissipation is obtained via:

\[ Y_\omega = \rho \beta f_\beta \omega^2 \]  \hspace{1cm} (D.22)

where,

\[ f_\beta = \frac{1 + 70 \chi_\omega}{1 + 80 \chi_\omega} \]  \hspace{1cm} (D.23)

with,

\[ \chi_\omega = \left| \frac{\Omega_{ij} \Omega_{ji} S_{ii}}{(\beta^*_\infty \omega)^3} \right| \]  \hspace{1cm} (D.24)

and,

\[ \Omega_{ij} = \frac{1}{2} \left( \frac{\partial \omega_i}{\partial x_j} - \frac{\partial u_i}{\partial x_j} \right) \]  \hspace{1cm} (D.25)

With the mean strain rate tensor given by:

\[ S_{ij} = \frac{1}{2} \left( \frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j} \right) \]  \hspace{1cm} (D.26)

and,

\[ \beta = \beta_t \left[ 1 - \frac{\beta^*_t \zeta F(M_t)}{\beta_t} \right] \]  \hspace{1cm} (D.27)

and with,

\[ F(M_t) = \begin{cases} 
0, & M_t \leq M_{t0} \\
M_t^2 - M_{t0}^2, & M_t > M_{t0}
\end{cases} \]  \hspace{1cm} (D.28)

where,

\[ M_t^2 = 2K/a^2 \]  \hspace{1cm} (D.29)

and,

\[ M_{t0} = 0.25 \]  \hspace{1cm} (D.30)
\[ a = \sqrt{\gamma RT} \quad \text{(D.31)} \]

\[ \beta_i^* = \beta_{\omega}^* \quad \text{(D.32)} \]

Values of the parameters are:

\[ \alpha_{\infty}^* = 1 \quad \text{(D.33)} \]

\[ \alpha_{\infty} = 1 \quad \text{(D.34)} \]

\[ \beta_{\infty}^* = 0.09 \quad \text{(D.35)} \]

\[ \beta_i = 0.072 \quad \text{(D.36)} \]

\[ R_\beta = 8 \quad \text{(D.37)} \]

\[ R_K = 6 \quad \text{(D.38)} \]

\[ R_\omega = 2.95 \quad \text{(D.39)} \]

\[ \zeta^* = 1.5 \quad \text{(D.40)} \]

\[ M_{t0} = 0.25 \quad \text{(D.41)} \]

\[ \sigma_K = 2.0 \quad \text{(D.42)} \]

\[ \sigma_\omega = 2.0 \quad \text{(D.43)} \]
APPENDIX E

MAIN AND AUXILIARY BOARD SETUPS
Table E.1. Check list for filling the HST yellow sphere

**COMPRESSOR BUILDING**

Fill 2100 psi storage tanks (follow specific instructions)
Turn on the 175 compressor

**MAIN PANEL**

OPEN V-01 (2100 psi line)
CLOSE V-02
OPEN V-03 (175 psi for solenoid valves)
OPEN V-05
CLOSE V-06
REGULATE RV-0 UNTIL MAX.
CLOSE V4-B

**AUXILIARY PANEL**

OPEN V-1
CLOSE V-2
OPEN V-3
CLOSE V-4
CLOSE V-A
OPEN V-B
CLOSE V-C
OPEN INLET VALVE
OPEN YELLOW SPHERE VALVE

**CONTROL PANEL**

TURN ON MASTER SWITCH
SWITCH ON SPHE. ISO
TURN ON HASKELL PUMP
WAIT UNTIL 4500 psi ON THE MIDDLE GAUGE, LOCATED AT THE PUMP
Table E.2. HST check list for a low enthalpy test

**GENERAL**
Fill yellow sphere (follow specific instructions)
Turn on the 175 compressor
Make vacuum in test section

**MAIN PANEL**
OPEN V-01 (2100 psi line)
CLOSE V-02
OPEN V-03 (175 psi for solenoid valves)
OPEN V-05
CLOSE V-06
REGULATE RV-0 UNTIL MAX.
CLOSE V4-B

**AUXILIARY PANEL**
OPEN V-1
CLOSE V-2
OPEN V-3
CLOSE V-4
OPEN V-A
CLOSE V-B
OPEN V-C
OPEN INLET VALVE
OPEN YELLOW SPHERE VALVE
CLOSE ALL HAND VALVES ON THE TUNNEL (VACUUM LINES)
START YOUR DAQ

**CONTROL PANEL**
SWITCH ON SPHE. ISO
TURN ON DRIVER, DRIVER TEMP., DRIVEN AND DIAPHRAGM SWITCHES
TURN ON DRIVEN TUBE VACUUM DISPLAY
TURN ON SWITCH 1 RIGHT FOR TEST SECTION AND LEFT FOR DRIVEN
SWITCH ON SPHE. ISO.
FILL THE DRIVER WITH DRVR SWITCH
FILL THE DDS WITH DIAP. CHGE. (KEEP HALF OF DRVR. PRESS.)
TURN ON HASKELL PUMP
TURN OFF HASKELL PUMP WHEN DESIRED PRESSURE IS REACHED
SET TO OFF DRVR. AND DIAP. CHGE. SWITCHES

**MAIN PANEL**
CLOSE V-01 (VERY IMPORTANT TO PRESERVE HASKELL PUMP)

**CONTROL PANEL**
SWITCH OFF VACUUM DISPLAY AND SWITCH 1
TO START THE TEST, SWITCH ON DIAP. VENT
APPENDIX F

MATLAB COMPUTER CODES
F.1 Equilibrium interface mode for perfect gases: equintperf.m

% UTA
% Aerodynamics Research Center
% Hypersonic Shock Tunnel Facility

% Code Description: Simulates a shock tube in the
% Equilibrium Interface mode, perfect gas is assumed. Simple
% reflected shock tunnel properties are calculated for given
% initial temperatures of driver and driven gases, T₄ and T₁,
% and type of driver gas (gamma₄,R₄). Also, the incident Mach
% number Ms is given. In that case, no CS/reflec-SW
% interaction exists (i = 0). After that, if conditions are
% favorable, it calculates the equilibrium interface
% properties (i>0). Last Update: 18/MAI/2013
% Programmer’s Name: Tiago Rolim

% | | | c | x
% | | | c | x
% 4 <-| | 3 c 2 |-> 1 x i = 0
% | | | c | x
% | | | c | x
<table>
<thead>
<tr>
<th>i</th>
<th>c</th>
<th>x</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>7 c 6</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>10 c 9</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>13 c 12</td>
</tr>
</tbody>
</table>
% | | | – expansion wave  
% | – shockwave  
% c – contact surface

function out = equintperf(gas, T4, T1, Ms)

%begin of initialization

%type of gas in driver sec
if gas == 'He'
    R4 = 2077; gamma4 = 1.664;
end

if gas == 'Ar'
    R4 = 287; gamma4 = 1.4;
end

if gas == 'H2'
\[ R_4 = 4124 \; ; \gamma_4 = 1.4; \]
end

\[ R_1 = 287; \gamma_1 = 1.4; \; a_1 = (\gamma_1 R_1 T_1)^{0.5}; \]
\[ c_{p1} = \gamma_1/((\gamma_1 - 1) \cdot R_1); \]
a_4 = (\gamma_4 R_4 T_4)^{0.5}; \; c_{p4} = \gamma_4/((\gamma_4 - 1) \cdot R_4);

%end of initialization -----------------------------------------------
%----------------- Reflected Shock Tunnel (i=0)---------------------

%Region 2
\[ a_{21} = ((1+2 \cdot \gamma_1/(\gamma_1+1) \cdot (M_s^2 - 1)) / ((\gamma_1+1)/2 \cdot M_s^2/(1+(\gamma_1 - 1)/2 \cdot M_s^2)))^{1/2}; \]
\[ p_{21} = (1+2 \cdot \gamma_1/(\gamma_1+1) \cdot (M_s^2 - 1)); \]
\[ h_{21} = a_{21}^2; \]
\[ u_{2qa1} = 2/(\gamma_1+1) \cdot (M_s^2 - 1)/M_s; \]
\[ \rho_{21} = p_{21} / h_{21}; \]

%Region 4
\[ a_{41} = a_4/a_1; \]
\[ p_{41} = (2 \cdot \gamma_1 M_s^2 - (\gamma_1-1))/(\gamma_1+1) \cdot (1-(\gamma_4-1)/(\gamma_1+1) \cdot a_1/a_4 \cdot ((M_s^2 - 1)/M_s))^{(-2 \cdot \gamma_4/(\gamma_4-1))}; \]
\[ h_{41} = a_{41}^2 \cdot (\gamma_1 - 1)/((\gamma_4 - 1)); \]
\[ u_{4qa1} = 0; \]
\[ \rho_{41} = \gamma_4/\gamma_1 \cdot p_{41} / a_{41}^2; \]
\textit{Region 3}

\[ p_{31} = p_{21}; \]
\[ p_{43} = p_{41}/p_{31}; \]
\[ a_{43} = (p_{43})^{((\gamma_4-1)/2)/\gamma_4}; \]
\[ a_{31} = a_{41}/a_{43}; \]
\[ h_{31} = a_{31}^2 \times (\gamma_1-1)/(\gamma_4-1); \]
\[ u_{3qa3} = u_{2qa1} \times a_{1}/a_{4} \times a_{43}; \]
\[ u_{3qa1} = u_{3qa3} \times a_{31}; \]
\[ \rho_{31} = \gamma_4/\gamma_1 \times p_{31}/a_{31}^2; \]

\textit{Region 5}

\[ Mr = \left(\left((\gamma_1 \times M_s^2 - (\gamma_1-1)/2)/\left(1 + ((\gamma_1-1)/2 \times M_s^2)\right)\right)^0.5; \right. \]
\[ p_{52} = \left(1 + 2 \times \gamma_1/(\gamma_1+1) \times (M_r^2-1)\right); \]
\[ a_{52} = \left((1 + 2 \times \gamma_1/(\gamma_1+1) \times (M_r^2-1)) / ((\gamma_1+1)/2 \times M_r^2/(1 + ((\gamma_1-1)/2 \times M_r^2))) \right)^{(1/2)}; \]
\[ h_{52} = a_{52}^2; \]
\[ a_{51} = a_{52} \times a_{21}; \]
\[ p_{51} = p_{52} \times p_{21}; \]
\[ h_{51} = a_{51}^2; \]
\[ u_{5qa1} = 0; \]
\[ \rho_{51} = p_{51}/h_{51}; \]
%Store the p, h, a, u ratios in the structure called REGION
REGION(2).p = p21; REGION(2).h = h21; REGION(2).a = a21;
REGION(2).u = u3qa1; REGION(2).rho = rho21;
REGION(3).p = p31; REGION(3).h = h31; REGION(3).a = a31;
REGION(3).u = u3qa1; REGION(3).rho = rho31;
REGION(4).p = p41; REGION(4).h = h41; REGION(4).a = a41;
REGION(4).u = u4qa1; REGION(4).rho = rho41;
REGION(5).p = p51; REGION(5).h = h51; REGION(5).a = a51;
REGION(5).u = u5qa1; REGION(5).rho = rho51;

%test if we are over-tailored
a23 = a21 / a31;
a23t = gamma1/gamma4 *((1+(gamma1+1)/2/gamma1*(p52-1))
/(1+(gamma4+1)/2/gamma4*(p52-1))) ^ 0.5; %tailored mode
        condition

if a23 < a23t
    disp('––––> Under_tailored_condition <––––');
    out = REGION;
    return; %program ends
else
    disp('––––> Over_tailored_condition <––––');

end

%--------------------Equilibrium Interface Shock Tunnel (i=1)
%calculate regions 6 and 7, regions 1 to 5 are known

%solves Equilibrium interface fundamental equation
x0 = 1.5 * p52; %initial guess for p75
[x, fval] = fsolve(@(x)fovtail(x, p52, u3qa3, a53, gamma1, gamma4), x0, optimset('Display', 'off'));
p65 = x;

%6 —— post shock region after SW reflected in CS
Mri = sqrt(1 + (gamma1+1)/2/gamma1*(p65-1));
u6qa5 = 2/(gamma1+1) * (Mri^2-1)/Mri;
a65 = ((1+2*gamma1/(gamma1+1)*(Mri^2-1))/((gamma1+1)/2*Mri^2/ (1+(gamma1-1)/2*Mri^2)) )^(1/2);
h65 = a65^2;
p61 = p65 * p51;
a61 = a65 * a51;
h61 = h65 * h51;
u6qa1 = u6qa5 * a51;
rho61 = p61 / h61;

%7 —— post shock region after SW transmitted in CS
\[ p_{73} = p_{61} / p_{31}; \]

\[ M_{ti} = \sqrt{ (1 + (\gamma + 1)/2/\gamma^4 \times (p_{73} - 1)) }; \]

\[ a_{73} = ((1 + 2\times\gamma/(\gamma + 1) \times (M_{ti}^2 - 1)) \]
\[ /((\gamma + 1)/2 \times M_{ti}^2 / (1 + (\gamma - 1)/2 \times M_{ti}^2)) } \times (1/2); \]

\[ h_{73} = a_{73}^2; \]

\[ a_{71} = a_{73} \times a_{31}; \]
\[ p_{71} = p_{73} \times p_{31}; \]
\[ h_{71} = h_{73} \times h_{31}; \]
\[ u_{7qa1} = u_{6qa1}; \]
\[ \rho_{71} = \gamma / \gamma \times p_{71} / a_{71}^2; \]

\% Store the \( p, h, a, u \) ratios in the structure called REGION
i = 1;

\[ \text{REGION(6).p = p}_{61}; \text{REGION(6).h = h}_{61}; \text{REGION(6).a = a}_{61}; \text{REGION(6).u = u}_{6qa1}; \text{REGION(6).rho = rho}_{61}; \]
\[ \text{REGION(7).p = p}_{71}; \text{REGION(7).h = h}_{71}; \text{REGION(7).a = a}_{71}; \text{REGION(7).u = u}_{7qa1}; \text{REGION(7).rho = rho}_{71}; \]
\[ \text{MTI(1) = M}_{ti}; \]
\[ \text{MRI(1) = M}_{ri}; \]
\[ \text{MR(1) = M}_{r}; \]

\% Equilibrium Interface Shock Tunnel (i>1)
% regions 6 (3n) and 7 (3n+1) are known as well as the strength of SW ref. on CS MRI (MRI(n−1))

i = 2;

while MRI(i−1)>1

%7 —— 3n+1

\[ p71 = \text{REGION}(3\ast i+1).p; \quad h71 = \text{REGION}(3\ast i+1).h; \quad a71 = \text{REGION}(3\ast i+1).a; \quad u7qa1 = \text{REGION}(3\ast i+1).u; \]

%6 —— 3n

\[ p61 = \text{REGION}(3\ast i).p; \quad h61 = \text{REGION}(3\ast i).h; \quad a61 = \text{REGION}(3\ast i).a; \quad u6qa1 = \text{REGION}(3\ast i).u; \]

\[ \text{Mri} = \text{MRI}(i−1); \]

%8 —— 3n + 2 (reflection at the end of driven tube)

\[ \text{Mr} = \left( (\gamma1\ast\text{Mri}^2-(\gamma1−1)/2)/(1+(\gamma1−1)/2\ast\text{Mri}^2) \right)^{0.5}; \]

\[ p86 = (1+2\ast\gamma1/(\gamma1+1)\ast(\text{Mr}^2−1))/(( \gamma1+1)/2 \ast \text{Mr}^2/ (1+(\gamma1−1)/2\ast\text{Mr}^2)); \]

\[ a86 = ( (1+2\ast\gamma1/(\gamma1+1)\ast(\text{Mr}^2−1))/(( \gamma1+1)/2 \ast \text{Mr}^2/ (1+(\gamma1−1)/2\ast\text{Mr}^2)))^{(1/2)}; \]

\[ h86 = a86^2; \]

\[ a81 = a86 \ast a61; \]

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\begin{verbatim}
p81 = p86 * p61;
h81 = h86 * h61;
u8qa1 = 0;
rho81 = p81 / h81;

%Calculates the new Mri, solves the equation for over
tailored condition
u7qa7 = u7qa1 / a71;
a87 = a81/a71;
x0 = 1.5 * p86;

[x, fval] = fsolve(@(x) fovtail(x, p86, u7qa7, a87, gamma1, gamma4), x0, optimset('Display', 'off'));

%9 —— 3n+3 (post shock region after SW reflected in CS)
p98 = x;

Mri = sqrt(1+(gamma1+1)/2/gamma1*(p98-1));
u9qa8 = 2/(gamma1+1) * (Mri^2-1)/Mri;
a98 = ((1+2*gamma1/(gamma1+1)*(Mri^2-1)) / (gamma1+1)/2 * Mri^2 / (1+(gamma1-1)/2*Mri^2))^((1/2);

h98 = a98^2;

a91 = a98 * a81;
p91 = p98 * p81;
\end{verbatim}
h91 = h98 * h81;
u9qa1 = u9qa8 * a81;
rho91 = p91 / h91;

%10 —— 3n+4 (post shock region after SW transmitted in CS)
p101 = p91;
p107 = p101/p71;

Mti = sqrt((1 + (gamma4+1)/2/gamma4*(p107-1)))
a107 = ((1+2*gamma4/(gamma4+1)*(Mti^2-1))/(gamma4+1)/2*Mti^2/((1+(gamma4-1)/2*Mti^2)))^(1/2);
h107 = a107^2;

a101 = a107 * a71;
p101 = p91;
h101 = h107 * h71;
u10qa1 = u9qa1;
rho101 = gamma4/gamma1 * p101 / a101^2;

%Store the p, h, a, u ratios in the structure called REGION
REGION(3*i+2).p = p81; REGION(3*i+2).h = h81; REGION(3*i+2).a = a81; REGION(3*i+2).u = u9qa1; REGION(3*i+2).rho = rho81;
\[ REGION(3i+3).p = p91; \quad REGION(3i+3).h = h91; \quad REGION(3i+3).a = a91; \quad REGION(3i+3).u = u9qa1; \quad REGION(3i+3).\rho = \rho91; \]
\[ REGION(3i+4).p = p101; \quad REGION(3i+4).h = h101; \quad REGION(3i+4).a = a101; \quad REGION(3i+4).u = u10qa1; \quad REGION(3i+4).\rho = \rho101; \]
\[ MTI(i) = Mti; \]
\[ MRI(i) = Mri; \]
\[ MR(i) = Mr; \]

% prepare for the next iteration
i = i + 1;
end %while

out = REGION;
end

function out = fovtail(p75,p52,u3qa3,a53,gamma1,gamma4)
out = \((2/(\text{gamma}4 * (\text{gamma}4 - 1)))^{(1/2)} * (p75 * p52 - 1) * ((\text{gamma}4 + 1)/(\text{gamma}4 - 1) * p75 * p52 + 1)^{(-1/2)} - (u3qa3 - a53 * (2/(\text{gamma}1 - 1)))^{(1/2)} * (p75 - 1) * ((\text{gamma}1 + 1)/(\text{gamma}1 - 1) * p75 + 1)^{(-1/2)})\);
end
F.2 Equilibrium interface mode for real gases: equinteq.m

%xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
% 
% UTA
% Aerodynamics Research Center
% Hypersonic Shock Tunnel Facility
%xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
%Code Description: Simulates a shock tube in the
%Equilibrium Interface mode, equilibrium is assumed. Simple
%reflected shock tunnel properties are calculated for given
%initial temperatures of driver and driven gases, T4 and T1,
%and type of driver gas (gamma4,R4). Also, the incident Mach
%number Ms is given. In that case, no CS/reflec−SW
%interaction exists (i = 0). After that, if conditions are
%favorable, it calculates the equilibrium interface.
%properties (i>0). Last Update: 18/MAI/2013
%Programmer’s Name: Tiago Rolim
%xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
%xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
% |   c  |  x  |
% |   c  |  x  |
% 4 <−| 3  c  2  |→ 1  x  i = 0
% |   c  |  x  |
% |   c  |  x  |
<table>
<thead>
<tr>
<th></th>
<th>c</th>
<th>x</th>
</tr>
</thead>
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<tr>
<td>3 &lt;-</td>
<td>7 c 6</td>
<td>-&gt; 5 x i = 1</td>
</tr>
<tr>
<td></td>
<td>c</td>
<td>x</td>
</tr>
<tr>
<td></td>
<td>c</td>
<td>x</td>
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</tbody>
</table>

<table>
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<th>x</th>
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<tr>
<td>7 &lt;-</td>
<td>10 c 9</td>
<td>-&gt; 8 x i = 2</td>
</tr>
<tr>
<td></td>
<td>c</td>
<td>x</td>
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<tr>
<td></td>
<td>c</td>
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<th></th>
<th>c</th>
<th>x</th>
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<tbody>
<tr>
<td>10&lt;-</td>
<td>13 c 12</td>
<td>-&gt; 11 x i = 3</td>
</tr>
<tr>
<td></td>
<td>c</td>
<td>x</td>
</tr>
<tr>
<td></td>
<td>c</td>
<td>x</td>
</tr>
</tbody>
</table>
%expansion wave
% shockwave
% contact surface

function equinteq

%begin of initialization

gas = 'He';

%type of gas in driver sec
if gas == 'He'
    R4 = 2077;gamma4 = 1.664;
end

if gas == 'Ar'
    R4 = 287;gamma4 = 1.4;
end
if gas == 'H2'
    R4 = 4124; gamma4 = 1.4;
end

T1 = 300; R1 = 287; p1 = 101330; gamma1 = 1.4;
cp1 = gamma1/(gamma1-1) * R1;
T4 = T1; a4 = (gamma4*R4*T4)^0.5;
cp4 = gamma4/(gamma4-1) * R4;

%assume an incident shock wave speed, us
us = 1800;%m/s
%end of initialization

%——— Reflected Shock Tunnel (i = 0)———

%———Region 1
aux = equilibrium(p1,T1);
Rmix1 = 8314/aux(7);
rhol = p1/Rmix1/T1;
h1 = aux(8);
    al = a_preq(p1, rhol);
    Ms = us / al;

%Store the p,h,a,u,rho and T in structure called REGION_eq
REGION_eq(1).p = p1; REGION_eq(1).h = h1;
    REGION_eq(1).a = a1; REGION_eq(1).u = us;
    REGION_eq(1).rho = rhol; REGION_eq(1).T = T1;
%——Region 2

aux = nsw_eq(p1, T1, a1, us, Ms);
 rho2 = aux(1); u2 = us - aux(2); p2 = aux(3); h2 = aux(4);
 T2 = aux(5); a2 = a_preq(p2, rho2);

%Store the p,h,a,u,rho and T in structure called REGION_eq
REGION_eq(2).p = p2; REGION_eq(2).h = h2;
 REGION_eq(2).a = a2; REGION_eq(2).u = u2;
 REGION_eq(2).rho = rho2; REGION_eq(2).T = T2;

%——Region 4

p41 = (1+2*gamma1/(gamma1+1) * (Ms^2 - 1) *) * (1 - (gamma4 - 1)/(gamma1 + 1) * a1/a4 * ((Ms^2 - 1)/Ms)) * (-2*gamma4/(gamma4 - 1));
 p4 = p41 * p1;
 h4 = a4^2 / (gamma4 - 1);
 u4 = 0;
 T4 = h4 / cp4;
 rho4 = p4 / R4 / T4;

%Store the p,h,a,u,rho and T in structure called REGION_eq
REGION_eq(4).p = p4; REGION_eq(4).h = h4;
 REGION_eq(4).a = a4; REGION_eq(4).u = u4;
\( g \theta = \rho_4 \); \( \theta = \theta_4 \);

\%——Region 3

\[ p_3 = p_2; \]
\[ a_{43} = (p_4/p_3)^\left(\frac{(\gamma_4-1)/2}{\gamma_4}\right); \]
\[ a_3 = a_4 / a_{43}; \]
\[ u_3 = u_2; \]
\[ h_3 = a_3^2 / (\gamma_4 - 1); \]
\[ T_3 = h_3 / c_p; \]
\[ \rho_3 = p_3 / R_4 / T_3; \]

\%Store the \( p, h, a, u, \rho \) and \( T \) in structure called REGION_eq

\[ \text{REGION}_{eq}(3).p = p_3; \ \text{REGION}_{eq}(3).h = h_3; \]
\[ \text{REGION}_{eq}(3).a = a_3; \ \text{REGION}_{eq}(3).u = u_3; \]
\[ \text{REGION}_{eq}(3).\rho = \rho_3; \ \text{REGION}_{eq}(3).T = T_3; \]

\%——Region 5

\[ \text{aux} = \text{rsw}_{eq}(p_2, T_2, u_2); \]
\[ \rho_5 = \text{aux}(1); \ \text{ur} = \text{aux}(2); \ p_5 = \text{aux}(3); \ h_5 = \text{aux}(4); \]
\[ T_5 = \text{aux}(5); \ a_5 = \text{a}_{\text{preq}}(p_5, \rho_5); \ u_5 = 0; \]

\%Store the \( p, h, a, u, \rho \) and \( T \) in structure called REGION_eq

\[ \text{REGION}_{eq}(5).p = p_5; \ \text{REGION}_{eq}(5).h = h_5; \]

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\[ \text{REGION}_{eq}(5).a = a_5; \text{REGION}_{eq}(5).u = u_5; \]
\[ \text{REGION}_{eq}(5).\rho = \rho_5; \text{REGION}_{eq}(5).T = T_5; \]

\[ \text{--- Equilibrium Interface Shock Tunnel (i=1) ---} \]
\[ \text{--- calculate regions 6 and 7, regions 1 to 5 are known ---} \]

\[ \text{--- test if we are over-tailored ---} \]
\[ a_{23} = a_2 / a_3; \]
\[ a_{23t} = \gamma_1/\gamma_4 * ((1+(\gamma_1+1)/2/\gamma_1*(p_5/p_2-1)) / (1+(\gamma_4+1)/2/\gamma_4*(p_5/p_2-1)))^{0.5}; \]
\[ \text{--- tailored mode condition ---} \]

\[ \text{if } a_{23} < a_{23t} \]
\[ \text{disp('---> Under\_tailored\_condition <---');} \]
\[ \text{out = REGION}_{eq}; \]
\[ \text{return; % program ends} \]
\[ \text{else} \]
\[ \text{disp('---> Over\_tailored\_condition <---');} \]
\[ \text{end} \]

\[ \text{disp('---> Starting\_perfect\_gas\_calculation <---');} \]

\[ \text{--- Perfect gas assumption for first guess ---} \]
solves Equilibrium interface fundamental equation

\[ i = 1; \]
\[ \text{gamma1} = \text{gamma}_{\text{preq}}(p5, \rho5); \]
\[ x0 = 1.5 \times p5/p2; \text{initial guess for p65} \]
\[ [x, \text{fval}] = \text{fsolve}(@(x) \text{fovtail}(x, p5/p2, u3/a3, a5/a3, \)
\[ \text{gamma1, gamma4}), x0, \text{optimset}('\text{Display}', 'off')); \]
\[ p65 = x; \]

\%6 —— post shock region after SW reflected in CS

\[ \text{Mri} = \sqrt{1 + \left(\text{gamma1} + 1\right)/2/\text{gamma1} \times (p65 - 1)}; \]
\[ \text{u6qa5} = 2/(\text{gamma1} + 1) \times (\text{Mri}^2 - 1)/\text{Mri}; \]
\[ \text{a65} = \left((1+2*\text{gamma1}/(\text{gamma1}+1)*(\text{Mri}^2-1)) / ((\right) \times (1/2); \]
\[ \text{h65} = \text{a65}^2; \]
\[ \text{rho65} = (\text{gamma1}+1) \times \text{Mri}^2/(2+(\text{gamma1} \leftarrow 1)*\text{Mri}^2); \]

\[ p6 = p65 \times p5; \]
\[ \text{a6} = \text{a65} \times \text{a5}; \]
\[ \text{h6} = \text{h65} \times \text{h5}; \]
\[ \text{u6} = \text{u6qa5} \times \text{a5}; \]
\[ \text{rho6} = \text{rho65} \times \text{rho5}; \]
\[ \text{uri} = \text{Mri} \times \text{a5}; \]
\%7 — post shock region after SW transmitted in CS

\[ p_{73} = \frac{p_6}{p_3}; \]
\[ M_{ti} = \sqrt{\frac{1 + \frac{\gamma_4+1}{2} \gamma_4 (p_{73}^{-1} - 1)}{\frac{\gamma_4+1}{2} M_{ti}^2 (1 + \frac{\gamma_4-1}{2} M_{ti}^2)}} \]
\[ a_{73} = \left( \frac{1 + 2 \gamma_4/(\gamma_4+1) (M_{ti}^2 - 1)}{2 \gamma_4 (M_{ti}^2 - 1)} \right)^{\frac{1}{2}} \]
\[ h_{73} = a_{73}^2; \]
\[ \rho_{73} = \frac{\gamma_4+1}{2} M_{ti}^2 (1 + \frac{\gamma_4-1}{2} M_{ti}^2) \]
\[ a_7 = a_{73} \cdot a_3; \]
\[ p_7 = p_{73} \cdot p_3; \]
\[ h_7 = h_{73} \cdot h_3; \]
\[ u_7 = u_6; \]
\[ \rho_7 = \rho_{73} \cdot \rho_3; \]
\[ u_{ti} = M_{ti} a_3 - u_3; \]
\%—— Initial guesses
\[ X = []; \]
\[ X(1) = \rho_7; \]
\[ X(2) = u_7; \]
\[ X(3) = p_7; \]
\[ X(4) = h_7; \]
\[ X(5) = \rho_6; \]
\[ X_0(6) = h_6; \]
\[ X_0(7) = u_{ti}; \]
\[ X_0(8) = u_{ri}; \]

%—— Convergence coefficients
\[
K_1 = \rho_3 \cdot (u_3 + u_{ti});
\]
\[
K_2 = \rho_3 \cdot (u_3 + u_{ti})^2 + p_3;
\]
\[
K_3 = h_3 + (u_3 + u_{ti})^2 / 2;
\]
\[ K_4 = X_0(4); \]
\[
K_5 = \rho_5 \cdot (u_{ri} - u_5);
\]
\[
K_6 = p_5 + \rho_5 \cdot (u_{ri} - u_5)^2;
\]
\[
K_7 = h_5 + (u_{ri} - u_5)^2 / 2;
\]
\[ K_8 = X_0(6); \]
\[
K = [K_1, K_2, K_3, K_4, K_5, K_6, K_7, K_8];
\]

%— Real gas calculation —
\[
\text{disp('—> proceeding real gas—')};
\]
\[
[X_{sol}, fval] = \text{fsolve (@(X) gfunction(X, REGION_eq, K), X0, optimset('Display', 'off', 'PlotFcns', @optimplotfval, 'MaxFunEvals', 200, 'MaxIter', 200))};
\]
\[
rho_7 = X_{sol}(1);
\]
\[ u_7 = X_{sol}(2); \]
\[ p_7 = \text{Xsol}(3); \]
\[ h_7 = \text{Xsol}(4); \]
\[ \rho_6 = \text{Xsol}(5); \]
\[ h_6 = \text{Xsol}(6); \]
\[ u_{ti} = \text{Xsol}(7); \]
\[ u_{ri} = \text{Xsol}(8); \]

%Other properties
\[ a_7 = \text{a\_preq}(p_7, \rho_7); \]
\[ T_7 = \text{t\_preq}(p_7, \rho_7); \]

\[ p_6 = p_7; \]
\[ u_6 = u_7; \]
\[ a_6 = \text{a\_preq}(p_6, \rho_6); \]
\[ T_6 = \text{t\_preq}(p_6, \rho_6); \]

%Store the \( p, h, a, u, \rho \) and \( T \) in structure called REGION_eq
\text{REGION}\_eq(6).p = p_6; \text{REGION}\_eq(6).h = h_6;
\text{REGION}\_eq(6).a = a_6; \text{REGION}\_eq(6).u = u_6;
\text{REGION}\_eq(6).\rho = \rho_6; \text{REGION}\_eq(6).T = T_6;

%Store the \( p, h, a, u, \rho \) and \( T \) in structure called REGION_eq
\text{REGION}\_eq(7).p = p_7; \text{REGION}\_eq(7).h = h_7;
\text{REGION}\_eq(7).a = a_7; \text{REGION}\_eq(7).u = u_7;
\text{REGION}\_eq(7).\rho = \rho_7; \text{REGION}\_eq(7).T = T_7;
%Shock wave speeds from the calculations

URI(1) = uri;

UR(1) = ur;

UTI(1) = uti;

MRI(i) = uri / a5;

%------------------Equilibrium Interface Shock Tunnel (i>1)
%regions 6 (3n) and 7 (3n+1) are known as well as the
%strength of SW ref. on CS Mrr (Mrr(n−1))

i = 2;

while MRI(i-1) > 1.05

%Region 7 —— 3n+1

p7 = REGION_eq(3*i+1).p; h7 = REGION_eq(3*i+1).h;

a7 = REGION_eq(3*i+1).a; u7 = REGION_eq(3*i+1).u;

rho7 = REGION_eq(3*i+1).rho; T7 = REGION_eq(3*i+1).T;

%Region 6 —— 3n

p6 = REGION_eq(3*i).p; h6 = REGION_eq(3*i).h;

a6 = REGION_eq(3*i).a; u6 = REGION_eq(3*i).u;

rho6 = REGION_eq(3*i).rho; T6 = REGION_eq(3*i).T;
uri = URI(i-1);

%Region 8 —— 3n + 2 (reflection at the end of driven tube)
aux = rsw_eq(p6, T6, u6);
rho8 = aux(1); ur = aux(2); p8 = aux(3); h8 = aux(4);
T8 = aux(5); a8 = a_preq(p8, rho8); u8 = 0;

%Store the p, h, a, u, rho and T in structure called REGION_eq
REGION_eq(3*i+2).p = p8; REGION_eq(3*i+2).h = h8;
REGION_eq(3*i+2).a = a8; REGION_eq(3*i+2).u = u8;
REGION_eq(3*i+2).rho = rho8; REGION_eq(3*i+2).T = T8;

%——— Perfect gas assumption for first guess ———
%solves Equilibrium interface fundamental equation

x0 = 1.5 * p8/p6; %initial guess for p98
[x, fval] = fsolve(@(x) fovtail(x, p8/p6, u7/a7, a8/a7, gamma1, gamma4), x0, optimset('Display', 'off'));
p98 = x;

%6 —— post shock region after SW reflected in CS
Mri = sqrt(1 + (gamma1+1)/2/gamma1*(p98-1));
\[ u_{9qa8} = \frac{2}{(\gamma_1+1)} \times \left( \frac{M_{ri}^2 - 1}{M_{ri}} \right) \]

\[ a_{98} = \left( \frac{(1 + 2 \times \gamma_1)/(\gamma_1+1) \times (M_{ri}^2 - 1)}{(\gamma_1+1)/2 \times M_{ri}^2 / (1 + ((\gamma_1 - 1)/2 \times M_{ri}^2))} \right)^{1/2} \]

\[ h_{98} = a_{98}^2 \]

\[ \rho_{98} = \frac{1}{(\gamma_1+1)} \times \frac{M_{ri}^2}{(2 + (\gamma_1 - 1) \times M_{ri}^2)} \]

\[ p_9 = p_{98} \times p_8 \]

\[ a_9 = a_{98} \times a_8 \]

\[ h_9 = h_{98} \times h_8 \]

\[ u_9 = u_{9qa8} \times a_8 \]

\[ \rho_9 = \rho_{98} \times \rho_8 \]

\[ u_{ri} = M_{ri} \times a_8 \]

\%7 —— post shock region after SW transmitted in CS

\[ p_{107} = \frac{p_9}{p_7} \]

\[ M_{ti} = \sqrt{1 + \left( \frac{1}{(\gamma_4+1)/2 + \gamma_4 * (p_{107} - 1)} \right)} \]

\[ a_{107} = \left( \frac{(1 + 2 \times \gamma_4)/(\gamma_4+1) \times (M_{ti}^2 - 1)}{(\gamma_4+1)/2 \times M_{ti}^2 / (1 + ((\gamma_4 - 1)/2 \times M_{ti}^2))} \right)^{1/2} \]

\[ h_{107} = a_{107}^2 \]

\[ \rho_{107} = \frac{1}{(\gamma_4+1)} \times \frac{M_{ti}^2}{(2 + (\gamma_4 - 1) \times M_{ti}^2)} \]
a10 = a107 * a7;
p10 = p107 * p7;
    h10 = h107 * h7;
    u10 = u9;
    rho10 = rho107 * rho7;
    uti = Mti*a7 - u7;

%—— Initial guesses
X = [];

X0(1) = rho10;
    X0(2) = u10;
    X0(3) = p10;
    X0(4) = h10;
    X0(5) = rho10;
    X0(6) = h10;
    X0(7) = uti;
    X0(8) = uri;

%—— Convergence coefficients
K1 = rho7 * (u7 + uti);
    K2 = p7 + rho7 * (u7 + uti)^2;
    K3 = h7 + (u7 + uti)^2 / 2;
    K4 = X0(4);

K5 = rho8 * (uri - u8);
K6 = p8 + rho8 * (uri - u8)^2;
K7 = h8 + (uri - u8)^2 / 2;
K8 = X0(6);

K = [K1, K2, K3, K4, K5, K6, K7, K8];

%—— Real gas calculation

[Xsol, fval] = fsolve(@(X) gNfunction(X, REGION_eq, K, i), X0, optimset('Display','off','PlotFcns',@optimplotfval,'MaxFunEvals', 200,'MaxIter', 200));

rho10 = Xsol(1);
u10 = Xsol(2);
p10 = Xsol(3);
h10 = Xsol(4);
rho9 = Xsol(5);
h9 = Xsol(6);
uti = Xsol(7);
uri = Xsol(8);

%Other properties
a10 = a_prev(p10, rho10);
T10 = t_prev(p10, rho10);

p9 = p10;
u9 = u10;

a9 = a_preq(p9, rho9);

T9 = t_preq(p9, rho9);

%9 —— 3n+3 (post shock region after SW reflected in CS)
%Store the p, h, a, u, rho and T in structure called REGION_eq
REGION_eq(3*i+3).p = p9; REGION_eq(3*i+3).h = h9;
REGION_eq(3*i+3).a = a9; REGION_eq(3*i+3).u = u9;
REGION_eq(3*i+3).rho = rho9; REGION_eq(3*i+3).T = T9;

%10 —— 3n+4 (post shock region after SW transmitted in CS)
%Store the p, h, a, u, rho and T in structure called REGION_eq
REGION_eq(3*i+4).p = p10; REGION_eq(3*i+4).h = h10;
REGION_eq(3*i+4).a = a10; REGION_eq(3*i+4).u = u10;
REGION_eq(3*i+4).rho = rho10; REGION_eq(3*i+4).T = T10;

%Shock wave speeds from the calculations
URI(i) = uri;

UR(i) = ur;

UTI(i) = uti;

MRI(i) = uri / a8;

i = i+1;
function erro = gfunction(X, REGION_eq, K)
    \%for i = 1;
    rho7 = X(1);
    u7 = X(2);
    p7 = X(3);
    h7 = X(4);

    rho6 = X(5);
    h6 = X(6);

    uti = X(7);
    uri = X(8);

    \%other inputs
    rho3 = REGION_eq(3).rho;
    u3 = REGION_eq(3).u;
    p3 = REGION_eq(3).p;
    h3 = REGION_eq(3).h;

    rho5 = REGION_eq(5).rho;
    u5 = REGION_eq(5).u;
\[ p_5 = \text{REGION} \_\text{eq}(5) \_p; \]
\[ h_5 = \text{REGION} \_\text{eq}(5) \_h; \]

%interface condition
\[ p_6 = p_7; \]
\[ u_6 = u_7; \]

%transmitted shock wave
\[ f_1 = (\rho_3 \times (u_3 + uti) - (uti + u_7) \times \rho_7) / K(1); \]
\[ f_2 = (\rho_3 \times (u_3 + uti)^2 + p_3 - p_7 - \rho_7 \times (uti + u_7)^2) / K(2); \]
\[ f_3 = (h_3 + (u_3 + uti)^2 / 2 - h_7 - (uti + u_7)^2 / 2) / K(3); \]
\[ f_4 = (h_7 - \text{h\_preq}(p_7, \rho_7)) / K(4); \]

%reflected shock wave
\[ f_5 = (\rho_6 \times (uri - u_6) - \rho_5 \times (uri - u_5)) / K(5); \]
\[ f_6 = (p_6 + \rho_6 \times (uri - u_6)^2 - p_5 - \rho_5 \times (uri - u_5)^2) / K(6); \]
\[ f_7 = (h_6 + (uri - u_6)^2 / 2 - h_5 - (uri - u_5)^2 / 2) / K(7); \]
\[ f_8 = (h_6 - \text{h\_preq}(p_6, \rho_6)) / K(8); \]
\[
\text{error} = \left[ f_1^2; f_2^2; f_3^2; f_4^2; f_5^2; f_6^2; f_7^2; f_8^2 \right];
\]

end

function error = gNfunction(X, REGION_eq, K, j)

%for i>1
    \rho_{10} = X(1);
    u_{10} = X(2);
    p_{10} = X(3);
    h_{10} = X(4);

\rho_{9} = X(5);
\h_{9} = X(6);

\u_t{i} = X(7);
\u_r{i} = X(8);

%other inputs
\rho_{7} = \text{REGION\_eq}(3*j+1) . \rho;
\u_{7} = \text{REGION\_eq}(3*j+1) . u;
\p_{7} = \text{REGION\_eq}(3*j+1) . p;
\h_{7} = \text{REGION\_eq}(3*j+1) . h;

\rho_{8} = \text{REGION\_eq}(3*j+2) . \rho;
\u_{8} = \text{REGION\_eq}(3*j+2) . u;
\[ p_8 = \text{REGION}_\text{eq}(3 \ast j + 2) \cdot p; \]
\[ h_8 = \text{REGION}_\text{eq}(3 \ast j + 2) \cdot h; \]

\%interface condition
\[ p_9 = p_{10}; \]
\[ u_9 = u_{10}; \]

\%transmitted shock wave
\[ f_1 = (\rho_7 \ast (u_7 + uti) - (uti + u_{10}) \ast \rho_{10}) / K(1); \]
\[ f_2 = (p_7 + \rho_7 \ast (u_7 + uti)^2 - p_{10} - \rho_{10} \ast (uti + u_{10})^2) / K(2); \]
\[ f_3 = (h_7 + (u_7 + uti)^2 / 2 - h_{10} - (uti + u_{10})^2 / 2) / K(3); \]
\[ f_4 = (h_{10} - h_{\text{preq}}(p_{10}, \rho_{10})) / K(4); \]

\%reflected shock wave
\[ f_5 = (\rho_{9} \ast (uri - u_9) - \rho_{8} \ast (uri - u_{8})) / K(5); \]
\[ f_6 = (p_9 + \rho_9 \ast (uri - u_9)^2 - p_{8} - \rho_{8} \ast (uri - u_{8})^2) / K(6); \]
\[ f_7 = (h_9 + (uri - u_9)^2 / 2 - h_{8} - (uri - u_{8})^2 / 2) / K(7); \]
\[ f_8 = (h_{9} - h_{\text{preq}}(p_{9}, \rho_{9})) / K(8); \]
\[
\text{err} = [f_1^2; f_2^2; f_3^2; f_4^2; f_5^2; f_6^2; f_7^2; f_8^2];
\]
\[
\text{end}
\]
\[
\text{function} \quad \text{out} = \text{fovtail}(p_{75}, p_{52}, u_{3qa3}, a_{53}, \gamma_{1}, \gamma_{4})
\]
\[
\text{out} = \left(\frac{2}{(\gamma_{4} \cdot (\gamma_{4} - 1))}\right)^{1/2} \cdot \left(p_{75} \cdot p_{52} - 1\right) \cdot \left(\frac{(\gamma_{4} + 1)}{(\gamma_{4} - 1) \cdot p_{75} \cdot p_{52} + 1}\right)^{-1/2} - (u_{3qa3} - a_{53} \cdot \left(\frac{2}{(\gamma_{1} - 1)}\right)^{1/2} \cdot \left(p_{75} - 1\right) \cdot \left((\gamma_{1} + 1) / (\gamma_{1} - 1) \cdot p_{75} + 1\right)^{-1/2})
\]
\[
\text{end}
\]
F.3 Equilibrium Nozzle Flow: nzfloweq.m

%UTA
%Aerodynamics Research Center
%Hypersonic Shock Tunnel Facility

%Code Description: Calculates the free stream conditions
%for equilibrium air. Inputs: total pressure and
%temperature, pt0 (Pa) and Tt0 (K), Nozzle nominal Mach
%number Mnom and Pitot pressure (psi).
%Last Update: 21/05/2013
%Programmer's Name: Tiago Rolim

function out = nzflow_eq

%sample inputs
pt0 = 3.67e6;%Pa
Tt0 = 920;%K
Mnom = 10; %nominal Mach number, from A/A*
ppitot = 1.8;%Psi
ppitot = ppitot/14.7 * 101330;%Pa

gamma = 1.4;
A0qAstar = (2/(gamma+1))^((gamma+1)/2/(gamma-1))/Mnom*(1+(gamma-1)/2*Mnom^2)^((gamma+1)/2/(gamma-1));

Astar = 1; A0 = Astar * A0qAstar;
%obtaining other properties of stagnation region

aux = equilibrium(pt0, Tt0);
    rhot0 = r_pteq(pt0, Tt0);
    gammat0 = gamma_preq(pt0, rhot0);
    ht0 = aux(8);
    st0 = aux(9);

%----------------------------------
%—— First part, without Pitot pressure reading ———
%----------------------------------

%—— from reservoir (t0) to throat(*star)
%perfect gas is used for initial guess for p* and rho*

    pstar = pt0 * (2/(gammat0+1))^(gammat0/(gammat0-1));
    rhostar = rhot0 * (2/(gammat0+1))^(1/(gammat0-1));

    Xinit = [pstar rhostar];

    [X, Fval] = fsolve(@(X) gfunction1(X, ht0, st0), Xinit,
                    optimset('TolFun', 1e-6, 'TolX', 1e-6, 'MaxFunEvals', 200,
                    'MaxIter', 200, 'Display', 'off'));

    pstar = X(1);
    rhostar = X(2);
aux = equilibrium_pr(pstar, rhostar);
    hstar = aux(8);
    sstar = aux(9);
    Tstar = aux(10);
    astar = a_preq(pstar, rhostar);

%Store the p,h,a,u,rho and T in structure called NOZZLE_eq
NOZZLE_eq(1).p = pstar; NOZZLE_eq(1).h = hstar;
    NOZZLE_eq(1).a = astar; NOZZLE_eq(1).u = astar;
    NOZZLE_eq(1).rho = rhostar; NOZZLE_eq(1).T = Tstar;

%—— from throat(*star) to test section (0)
%perfect gas is used for initial guess for p0, rho0 and u0
gamma0 = 1.4;
Mnom = 0.8 * Mnom;
p0 = pt0/(1 + (gamma0-1)/2 * Mnom^2)^(gamma0/(gamma0-1));
 rho0 = rhot0/(1 + (gamma0-1)/2 * Mnom^2)^(1/(gamma0-1));
 T0 = Tt0/(1 + (gamma0-1)/2 * Mnom^2);
 a0qastar = ((gamma0+1)/(2+(gamma0-1)*Mnom^2))^(1/2);
    a0 = astar * a0qastar;
    u0 = Mnom * a0;

Xinit = [p0 rho0 u0];
\[ [X, Fval] = \text{fsolve}(@(X) gfunction2(X, A0, Astar, rhostar, hstar, astar, sstar), Xinit, \text{optimset}('Display', 'off', 'TolFun', 1e-4, 'TolX', 1e-4, 'MaxFunEvals', 100, 'MaxIter', 100)) ; \]

\[ p0 = X(1) ; \]
\[ rho0 = X(2) ; \]
\[ u0 = X(3) ; \]

\[ \text{aux} = \text{equilibrium}_\text{-pr}(p0, rho0) ; \]
\[ h0 = \text{aux}(8) ; \]
\[ s0 = \text{aux}(9) ; \]
\[ T0 = \text{aux}(10) ; \]
\[ a0 = a\_preq(p0, rho0) ; \]

\%Store the \( p, h, a, u, rho \) and \( T \) in structure called NOZZLE\_eq

\[ \text{NOZZLE}_\text{eq}(2) . p = p0 ; \text{NOZZLE}_\text{eq}(2) . h = h0 ; \]
\[ \text{NOZZLE}_\text{eq}(2) . a = a0 ; \text{NOZZLE}_\text{eq}(2) . u = a0 ; \]
\[ \text{NOZZLE}_\text{eq}(2) . rho = rho0 ; \text{NOZZLE}_\text{eq}(2) . T = T0 ; \]

\%XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
\%——— Second part, with Pitot pressure reading ———
\%XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
% find Temperature at Pitot, initial guess is perfect gas

[x, fval] = fsolve(@(x)gfunction3(x, ppitot, ht0), Tt0,
    optimset('Display', 'off', 'TolFun', 1e-6, 'TolX', 1e-6,' 
    MaxFunEvals', 100, 'MaxIter', 100));

Tpitot = x;
aux = equilibrium(ppitot, Tpitot);
    hpitot = ht0;
    spitot = aux(9);
        rhopitot = r_preq(ppitot, Tpitot);
            apitot = a_preq(ppitot, rhopitot);

% Store the p, h, a, u, rho and T in structure called NOZZLE_eq
NOZZLE_eq(3).p = ppitot; NOZZLE_eq(3).h = hpitot;
            NOZZLE_eq(3).a = apitot; NOZZLE_eq(3).u = 0;
                NOZZLE_eq(3).rho = rhopitot; NOZZLE_eq(3).T = Tpitot

% find the free stream conditions

% initial guess for the region aft bow shock wave
gamma0 = 1.4;
pds0 = p0 * (1+2*gamma0/(gamma0+1)*Mnom^2-1));
rhods0 = rho0 * (gamma0+1)/2*Mnom^2/(1+(gamma0-1)/2*Mnom^2) ;
uds0 = u0 * rho0/rhods0 ;

Xinit = [p0 rho0 u0 pds0 rhods0 uds0];

%convergence coefficients for shock wave eqs
K = [rhods0*uds0 pds0+rhods0*uds0^2 ht0];

[X,Fval] = fsolve(@(X) gfunction4(X, ht0, st0, spitot, K),
   Xinit, optimset('Display','iter','TolFun',1e-6,'TolX',1e-6,'MaxFunEvals',100,'MaxIter',100));

X(1) = p0;
X(2) = rho0;
X(3) = u0;
X(4) = pds0;
X(5) = rhods0;
X(6) = uds0;

aux = equilibrium_pr(pds0, rhods0);
hrs0 = aux(8);
sds0 = spitot;
Tds0 = aux(10);
ads0 = a_preq(pds0, rhods0);
%Store the p, h, a, u, rho and T in structure called NOZZLE_eq

NOZZLE_eq(4).p = pds0; NOZZLE_eq(4).h = hds0;
   NOZZLE_eq(4).a = ads0; NOZZLE_eq(4).u = uds0;
   NOZZLE_eq(4).rho = rhods0; NOZZLE_eq(4).T = Tds0;

aux = equilibrium_pr(p0, rho0);
   h0 = aux(8);
   s0 = st0;
   T0 = aux(10);
   a0 = a_preq(p0, rho0);

%Store the p, h, a, u, rho and T in structure called NOZZLE_eq

NOZZLE_eq(5).p = p0; NOZZLE_eq(5).h = h0;
   NOZZLE_eq(5).a = a0; NOZZLE_eq(5).u = u0;
   NOZZLE_eq(5).rho = rho0; NOZZLE_eq(5).T = T0;

end

function erro = gfunction1(X, ht0, st0)

pstar = X(1);
rhostar = X(2);

aux = equilibrium_pr(pstar, rhostar);
hstar = aux(8);
sstar = aux(9);

astar = a_preq(pstar, rhostar);

erro1 = (st0 - sstar)/st0;
    erro2 = (ht0 - (hstar + astar^2/2))/ht0;

erro = [erro1; erro2];
end

function erro = gfunction2(X, A0, Astar, rhostar, hstar, ustar, sstar)
p0 = X(1);
rho0 = X(2);
u0 = X(3);

aux = equilibrium_pr(p0, rho0);
    h0 = aux(8);
    s0 = aux(9);

erro1 = (rho0*u0*A0 - rhostar*ustar*Astar)/(rhostar*ustar*Astar);
\[\text{erro2} = \left( h_0 + u_0^2/2 - (h_{\text{star}} + u_{\text{star}}^2/2) \right) / \left( h_{\text{star}} + u_{\text{star}}^2/2 \right);\]

\[\text{erro3} = (s_0 - s_{\text{star}}) / s_{\text{star}};\]

\[\text{erro} = [\text{erro1}; \text{erro2}; \text{erro3}];\]

end

function erro = gfunction3(x, ppitot, ht0)
    Tpitot = x;
    aux = equilibrium(ppitot, Tpitot);
    hpitot = aux(8);
    erro = (hpitot - ht0)/ht0;
end

function erro = gfunction4(X, ht0, st0, spitot, K)
    p0 = X(1);
    rho0 = X(2);
    u0 = X(3);
    pds0 = X(4);
    rhods0 = X(5);
    uds0 = X(6);
aux = equilibrium_pr(p0, rho0);
    h0 = aux(8);
    s0 = aux(9);

aux = equilibrium_pr(pds0, rhods0);
    hds0 = aux(8);
    sds0 = aux(9);

%isentropic expansion
    erro1 = (s0 - st0)/st0;
    erro2 = (h0 + u0^2/2 - ht0)/ht0;

%normal shock wave equations
    erro3 = (rho0*u0 - rhods0*uds0)/K(1);
    erro4 = (p0 + rho0*u0^2 - (pds0 + rhods0*uds0^2)) /K(2);
    erro5 = (h0 + u0^2/2 - (hds0 + uds0^2/2))/K(3);

%isentropic compression from ds0 to pitot
    erro6 = (sds0 - spitot)/spitot;

erro = [erro1; erro2; erro3; erro4; erro5; erro6];
end
F.4 Equilibrium normal shock wave: nswe\_eq.m

\texttt{function out = nswe\_eq(p1, T1, a1, u1, M1)}

\begin{verbatim}
aux = equilibrium(p1,T1);
Rmix1 = 8314/aux(7);
rho1 = p1/Rmix1/T1;
h1 = aux(8);
gamma1 = gamma\_preq(p1, rho1);
\end{verbatim}

\begin{verbatim}
%initial guess, perfect gas
rho21i = (gamma1+1)/2 \ast M1^2/ (1+(gamma1-1)/2\ast M1^2) ;
\end{verbatim}

\begin{verbatim}
% solves the conservation equations
\end{verbatim}
[X,fval] = \texttt{fzero}( @(x) gfunc(x,rhol,u1,p1,h1),rho2i,optimset(}
  'TolFun',1e-8,'TolX',1e-8,'MaxFunEvals',20,'MaxIter',20, '}
  Display', 'off'));

rho2 = X * rho1;

u2 = u1 /X ;

p2 = -rho2*u2^2 + p1 + rho1*u1^2;

h2 = h_{preq}(p2, rho2);

T2 = t_{preq}(p2, rho2);

out = [rho2; u2; p2; h2; T2];

end

%—Goal function for properties after a normal shock wave—

\textbf{function} erro = gfunc(x,rhol,u1,p1,h1)

rho2 = x * rho1;

u2 = u1 /x ; %mass

p2 = -rho2*u2^2 + p1 + rho1*u1^2 ;%momentum

h2 = h1 + u1^2/2 - u2^2/2; %energy

erro = (h2 - h_{preq}(p2, rho2)); %EOS

end
F.5 Equilibrium normal shock wave reflection: rsweq.m

```
function out = rsw_eq(p2, T2, u2)

aux = equilibrium(p2, T2);
    Rmix2 = 8314/aux(7);
    rho2 = p2/Rmix2/T2;
    h2 = aux(8);
    gamma2 = gamma_preq(p2, rho2);

% initial guess for reflected shock wave Mach number and
% density ratio (perfect gas)
Mri = 2;
rho52i = (gamma2+1)/2 * Mri^2/ (1+(gamma2-1)/2*Mri^2);
```

% Code Description : Equilibrium reflected shock wave, given
% properties upstream SW : p2[Pa], T2[K] and u2[m/s]
% calculates postshock region 5
% Last Update : 21/MAI/2013
% Programmer's Name : Tiago Rolim
% solves the conservation equations

[X, fval] = \texttt{fzero}(@(x)gfunc(x, rho2, u2, p2, h2), rho52i, optimset('TolFun', 1e-8, 'TolX', 1e-8, 'Display', 'off', 'MaxFunEvals', 50, 'MaxIter', 50));

rho5 = X \times rho2;
ur = u2 / (X-1);
p5 = -rho5 \times ur^2 + p2 + rho2 \times (u2 + ur)^2;
    aux = equilibrium\_pr(p5, rho5);

h5 = aux(8);
T5 = aux(10);

out = [rho5; ur; p5; h5; T5];
end

%—Goal function for properties after a reflected shock wave

\texttt{function} erro = gfunc(x, rho2, u2, p2, h2)

rho5 = x \times rho2;
ur = u2 / (x-1);

p5 = -rho5 \times ur^2 + p2 + rho2 \times (u2 + ur)^2; \texttt{mass}

h5 = h2 + (u2+ur)^2/2 - ur^2/2; \texttt{momentum}

erro = (h5 - h\_preq(p5, rho5)); \texttt{energy}
end

end
F.6 Equilibrium oblique shock wave: osweq.m

function out = osweq(u1, p1, T1, theta)

%other free stream temperatures
rholl = r_ pteq(p1, T1);
h1 = h_ pteq(p1, T1);

%initial guess for the post shock density from perfect gas
gamma = 1.4; R = 287;
al = (gamma*R*T1)^0.5;
M1 = u1/a1;

beta = fzero(@(beta)(tan(theta)*(M1^2*(gamma*cos(2*beta))+2)
                    ) - 2*cot(beta)*(M1^2*sin(beta)^2-1),[theta pi /4]);
\[ M_{n1} = M_1 \sin(\beta) ; \]
\[ M_{n2} = \left( \frac{(M_{n1}^2 + 2/(\gamma - 1))}{(2\gamma/(\gamma - 1) \times M_{n1}^2 - 1)} \right)^{0.5} ; \]

\[ M_{2i} = M_{n2} / \sin(\beta - \theta) ; \]
\[ T_{2i} = T_1 \times \left( \frac{1 + (\gamma - 1)/2 \times M_1^2}{1 + (\gamma - 1)/2 \times M_{2i}^2} \right) ; \]
\[ p_{2i} = p_1 \times (1 + 2 \times \gamma / (\gamma + 1) \times (M_{n1}^2 - 1)) ; \]
\[ \rho_{2i} = \frac{p_{2i}}{R/T_{2i}} ; \]

\[ [x, fval] = \text{fzero}(@x \text{osw}(x, \rho_{1}, u_{1}, p_{1}, h_{1}, \theta), \rho_{2i}, \text{optimset(’TolFun’,1e-8,’Display’,’on’,’PlotFcns’,}) ; \]
\[ @\text{optimplotfval}) ; \]

\[ \rho_{2} = x \times \rho_{1} ; \]
\[ \beta_{1} = \frac{\text{atan}((x-1-\sqrt{(x-1)^2 - 4 \times x \times (\tan(\theta))^2}))}{2/\tan(\theta)} ; \]
\[ u_{1n} = u_{1} \times \sin(\beta_{1}) ; \]
\[ u_{2n} = u_{1n} / x ; \]
\[ u_{2t} = u_{1n} / \tan(\beta_{1}) ; \]
\[ u_{2} = \sqrt{u_{2n}^2 + u_{2t}^2} ; \]
\[ p_{2} = -\rho_{2} \times u_{2n}^2 + p_{1} + \rho_{1} \times u_{1n}^2 ; \]
\[ h_{2} = \text{h}_{\text{preq}}(p_{2}, \rho_{2}) ; \]
\[ T_{2} = \text{t}_{\text{preq}}(p_{2}, \rho_{2}) ; \]
out = [rho2 u2 p2 h2 T2 beta1];

end

% calculates the properties after an oblique shock wave

function erro = osw(x,rho1,u1,p1,h1,theta)

rho2 = rho1 * x;

beta = atan((x-1-sqrt((x-1)^2-4*x*(tan(theta))^2))/2/tan(theta));

u1n = u1 * sin(beta);

u2n = u1n / x; % mass

p2 = (p1 + rho1 * u1n^2 - rho2 * u2n^2); % momentum

h2 = (h1 + u1n^2/2 - u2n^2/2); % energy

erro = (h2 - h_preq(p2, rho2));

end
F.7 Equilibrium specific enthalpy: hpreq.m

% Equilibrium specific enthalpy: hpreq.m
% Aerodynamics Research Center
% Hypersonic Shock Tunnel Facility
% Code Description: Gives the enthalpy h [J/kg] for given
% p [Pa] and rho [kg/m3] (air).
% Last Update: 21/05/2013
% Programmer’s Name: Tiago Rolim

function out = hpreq(p,rho)

in = equilibrium_pr(p,rho);
out = in(8);

end
F.8 Equilibrium temperature: treq.m

function out = treq(p, rho)

    aux = equilibrium_pr(p, rho);
    T = aux(10);
    out = T;

end
F.9 Equilibrium ratio of specific heats: gammapreq.m

function out = gamma_preq(p, rho)  

aux = heats_preq(p, rho);  
    cp = aux(1);  
    cv = aux(2);  

    gamma = cp/cv;  

out = gamma;  

end
function out = a_preq(p, rho)

gamma = gamma_preq(p, rho);
T = t_preq(p, rho);

drho = 0.001;
de_T = (e_rteq(rho+drho, T)−e_rteq(rho−drho, T))/2;

expression1 = 1 −1/rho^2/p * de_T/drho;

dp = 0.1;
dh_T = (h_ pteq(p+dp, T)−h_ pteq(p−dp, T))/2;
expression2 = 1 - rho * dh_{T}/dp;

a = \sqrt{gamma*p/rho * (expression1/expression2)};

out = a;

end
function out = heats_preq(p, rho)

dT = 0.001;

%cp = dh/dT at cte p

T = t_preq(p, rho);

dh_p = (h_preq(p,T+dT) - h_preq(p,T));

cp = dh_p/dT;

%cv = de/dT at cte v = 1/rho
\[ de_v = \left( e_{rteq}(\rho, T+dT) - e_{rteq}(\rho, T) \right); \]

\[ cv = de_v /dT; \]

\[ out = [cp; cv]; \]

end
function out = equilibrium(p, T)

global pol_O2 pol_N2 pol_O pol_NO pol_N;

DHf0_O2 = 0;
DHf0_N2 = 0;
DHf0_O = 249.18; %kJ/mol
DHf0_N = 472.68; %kJ/mol
DHf0_NO = 90.29; %kJ/mol

R = 8.314; %J/mol/K
p0 = 101330; %Pa
\begin{align*}
G_0\text{N}_2 &= -\texttt{polyval}(\texttt{pol}_\text{N}_2 \cdot G_0, T) \times T + \text{DHf}_0\text{N}_2 \times 1000; \\
G_0\text{O}_2 &= -\texttt{polyval}(\texttt{pol}_\text{O}_2 \cdot G_0, T) \times T + \text{DHf}_0\text{O}_2 \times 1000; \\
G_0\text{N} &= -\texttt{polyval}(\texttt{pol}_\text{N} \cdot G_0, T) \times T + \text{DHf}_0\text{N} \times 1000; \\
G_0\text{O} &= -\texttt{polyval}(\texttt{pol}_\text{O} \cdot G_0, T) \times T + \text{DHf}_0\text{O} \times 1000; \\
G_0\text{NO} &= -\texttt{polyval}(\texttt{pol}_\text{NO} \cdot G_0, T) \times T + \text{DHf}_0\text{NO} \times 1000; \\
H_0\text{N}_2 &= (\texttt{polyval}(\texttt{pol}_\text{N}_2 \cdot H_0, T) + \text{DHf}_0\text{N}_2) \times 1000; \\
H_0\text{O}_2 &= (\texttt{polyval}(\texttt{pol}_\text{O}_2 \cdot H_0, T) + \text{DHf}_0\text{O}_2) \times 1000; \\
H_0\text{N} &= (\texttt{polyval}(\texttt{pol}_\text{N} \cdot H_0, T) + \text{DHf}_0\text{N}) \times 1000; \\
H_0\text{O} &= (\texttt{polyval}(\texttt{pol}_\text{O} \cdot H_0, T) + \text{DHf}_0\text{O}) \times 1000; \\
H_0\text{NO} &= (\texttt{polyval}(\texttt{pol}_\text{NO} \cdot H_0, T) + \text{DHf}_0\text{NO}) \times 1000; \\
CP\text{N}_2 &= \texttt{polyval}(\texttt{pol}_\text{N}_2 \cdot Cp, T); \\
CP\text{O}_2 &= \texttt{polyval}(\texttt{pol}_\text{O}_2 \cdot Cp, T); \\
CP\text{N} &= \texttt{polyval}(\texttt{pol}_\text{N} \cdot Cp, T); \\
CP\text{O} &= \texttt{polyval}(\texttt{pol}_\text{O} \cdot Cp, T); \\
CP\text{NO} &= \texttt{polyval}(\texttt{pol}_\text{NO} \cdot Cp, T); \end{align*}

\begin{align*}
g_0 &= \left[ G_0\text{N}_2; G_0\text{O}_2; G_0\text{N}; G_0\text{O}; G_0\text{NO} \right]; & \text{\(\text{\%J/mol}\)} \\
h_0 &= \left[ H_0\text{N}_2; H_0\text{O}_2; H_0\text{N}; H_0\text{O}; H_0\text{NO} \right]; & \text{\(\text{\%J/mol}\)} \\
h_{298} &= \left[ \text{DHf}_0\text{N}_2; \text{DHf}_0\text{O}_2; \text{DHf}_0\text{N}; \text{DHf}_0\text{O}; \text{DHf}_0\text{NO} \right] \times 1000; & \text{\(\text{\%J/mol}\)} \\
cp &= \left[ CP\text{N}_2; CP\text{O}_2; CP\text{N}; CP\text{O}; CP\text{NO} \right]; & \text{\(\text{\%J/(mol.K)}\)}
\end{align*}
\[ MV = \begin{bmatrix} 28; & 32; & 14; & 16; & 30 \end{bmatrix}; \text{ molar weight in grams} \]

\[ \text{nelem} = 2; \]

\[ MM = \begin{bmatrix} 2 & 0; & 0 & 2; & 1; & 0; & 1; & 1 \end{bmatrix}; \text{ mass} \]

\[ \text{nelem N}^2, \text{ O}_2, \text{ N}, \text{ O}, \text{ NO} \]

\[ \text{ninit} = \begin{bmatrix} 0.79; & 0.21; & 0.0; & 0.0; & 0.0 \end{bmatrix}; \text{ initial species moles} \]

\[ \text{conditioning data} \]

\[ \text{Ninit} = \text{MM} \times \text{ninit}; \text{ initial element moles} \]

\[ N = 5; \text{ number of reactants} \]

\[ \text{n0} = \begin{bmatrix} 0.759; & 0.15; & 0.05; & 0.2; & 0.02 \end{bmatrix}; \text{ guess for species moles} \]

\[ \text{lambda0} = \text{zeros}(\text{nelem},1); \]

\[ \text{X0} = [\log(\text{n0}); \text{lambda0}]; \]

\[ \text{calculates the gibbs function for each species} \]

\[ G = []; \quad H = []; \quad H298 = []; \quad CP = []; \]

\[ G = g0/R/T + \log(p/p0); \]

\[ H = h0/R/T; \]

\[ H298 = h298/R/T; \]

\[ CP = \text{cp}/R; \]

\[ \text{for } i=1:30 \]

\[ [\text{X}, \text{fval}] = \text{fsolve}(\text{ @(x) lagrange(\text{x}, \text{G}, \text{MM}, \text{Ninit}), \text{X0}, \text{optimset}(\text{'}\left\text{TolFun'}\right\text{,1e-8,}'\text{TolX'}\right\text{,1e-8,}'\text{MaxFunEvals'}\right\text{,100,}'\text{MaxIter'}\right\text{,100,}'\text{Display'}\right\text{,off'}));\%,'\text{PlotFcns'}\right\text{,optimplotfval});\]

\[ \text{X0} = \text{X}; \]

200
%data output
enes = exp(X);
ene_ver = (enes * eye(N+nelem ,N))';
sum_enes = sum(ene_ver);
MWmix = MW * ene_ver / sum_enes;

%g/mol
Hmix = (H* ene_ver) * R * T / (MW * ene_ver / 1000); %J/kg of mixture
Gmix = (G* ene_ver) * R * T / (MW * ene_ver / 1000); %J/kg of mixture
H298mix = (H298* ene_ver) * R * T / (MW * ene_ver / 1000); %J/kg of mixture
Smix = (Hmix - Gmix) / T;

out = [ene_ver / sum_enes; H298mix; MWmix; Hmix; Smix];

end

%-----------------------------------------------------

function erro = lagrange(X,G,MM,Ninit)
[nspec,nelem] = size(MM);

for i = 1:nspec
    x(i,1) = X(i,1);
end

for j = 1:nelem
    lambda(j,1) = X(j+nspec,1);
end
f1 = G + x –log(ones(numspec)*exp(x))–MM*lambda;
f2 = (MM*exp(x)–Ninit);
error = [f1;f2];
end
F.13 Load coefficients for $N$, $O$, $N_2$, $O_2$ and $NO$: loadPols.m

```matlab
% UTA
%Aerodynamics Research Center
%Hypersonic Shock Tunnel Facility

%Code Description: Load polynomials for equil. calculations

% Last Update: 21/05/2012

% Programmer's Name: Tiago Rolim

function loadPols

global pol_O2 pol_N2 pol_O pol_NO pol_N;

% --- Importing species data ---

% $T$ cp $s^0$ $-(g_0-H^0_{\text{ref}})/T$ $H^0-H^0_{\text{ref}}$ NIST format
% K J/mol/K J/mol/K J/mol/K kJ/mol

O2 = dlmread('O2.txt');
N2 = dlmread('N2.txt');
N = dlmread('N.txt');
O = dlmread('O.txt');
NO = dlmread('NO.txt');

warning off all; % disable a polyfit warning (it sucks!)

```
pol\_O2\_G0 = \texttt{polyfit}(O2(:,1),O2(:,4),12);
pol\_N2\_G0 = \texttt{polyfit}(N2(:,1),N2(:,4),12);
pol\_N\_G0 = \texttt{polyfit}(N(:,1),N(:,4),12);
pol\_O\_G0 = \texttt{polyfit}(O(:,1),O(:,4),12);
pol\_NO\_G0 = \texttt{polyfit}(NO(:,1),NO(:,4),12);

pol\_O2\_H0 = \texttt{polyfit}(O2(:,1),O2(:,5),12);
pol\_N2\_H0 = \texttt{polyfit}(N2(:,1),N2(:,5),12);
pol\_N\_H0 = \texttt{polyfit}(N(:,1),N(:,5),12);
pol\_O\_H0 = \texttt{polyfit}(O(:,1),O(:,5),12);
pol\_NO\_H0 = \texttt{polyfit}(NO(:,1),NO(:,5),12);

pol\_O2\_Cp = \texttt{polyfit}(O2(:,1),O2(:,2),12);
pol\_N2\_Cp = \texttt{polyfit}(N2(:,1),N2(:,2),12);
pol\_N\_Cp = \texttt{polyfit}(N(:,1),N(:,2),12);
pol\_O\_Cp = \texttt{polyfit}(O(:,1),O(:,2),12);
pol\_NO\_Cp = \texttt{polyfit}(NO(:,1),NO(:,2),12);
end
REFERENCES


[76] PCB PIEZOTRONICS, “General Piezoelectric Theory.”


BIOGRAPHICAL STATEMENT

Tiago C Rolim was born in Paulo Afonso, Bahia state, in Brazil on November 24th of 1982. He received a Bachelor of Science in Aeronautical Engineering in the Instituto Tecnológico de Aeronáutica (ITA), at São José dos Campos–SP, Brazil, in 2005. After graduating, he started to work as 1st lieutenant in Brazilian Air Force, serving at Instituto de Estudos Avançados (IEAv), also in São José dos Campos. While working with hypersonic shock tunnels as a researcher in this institute, he also got a degree of Master of Science in Mechanical and Aeronautical Engineering at ITA, on April 8th of 2009. He married his wife Vanessa Rolim this same year. After receiving a grant from Brazilian Air Force, he became a Ph.D. candidate at UTA on the Fall semester of 2010.