

STRUCTURAL ANALYSIS OF FIBER REINFORCED
COMPOSITE MATERIALS

by

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ABSTRACT

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This study includes analysis of laminates including calculation of structural section properties and failure prediction and analysis of composite laminated beams.

The analysis of the laminated composite beams covers:

1. beam with a solid rectangular cross-section subjected to a transverse load
2. beam with a I-section under bending
3. beam with a squared tubular section under bending
4. beam with truss section under bending

An ANSYS finite element analysis for each beam is conducted to verify the results obtained by analytical model. Excellent agreements between FEM and analytical method were obtain.

A user friendly computer analysis using MATLAB via Graphical User Interface (GUI) programming in all the process is developed. The application allows users to conduct parametric study in composite beam design faster and with confident accuracy.

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CHAPTER 1

INTRODUCTION

Composite materials are materials constituted for two or more components, which remain independent at the macroscopic level when they become part of a structure. The main advantages for the use composite materials are high strength and high stiffness to weight ratio. Those advantages are why composite materials are used in many fields of industry. It is more common to see composite material products in many forms and applications. Even though composite materials in their form of fiber-reinforced composites offer many design possibilities, its complexity in design increases. The design engineer must be aware of the results and analyze many cases and possible designs; this is possible nowadays with the aid of a computer and with some computer software.

1.1 Structural Applications of Composite Materials

1.1.1 Composite Technology in Structural Applications

Composite material products increase its use in industry of the aviation and construction fields among other areas. In aviation, a transition has been seen of the use of isotropic materials in the past to the use of composite materials. For example, the airplane main frames of the wing and fuselage structures are now made of composites. This is because composite materials offer the same capabilities as isotropic materials

with a significant reduction in weight. Besides this, laminated composites can be tailored to meet the required stiffness and strength in the structural applications because their material property is directional dependence.

Recently, the civil construction industries began the transition to implement composite material structures in designs. Composite rebar, bridge repairing, bridge decks are some of examples that have been used in civil structural applications. In these applications the composite materials not only give possibilities to obtain better bending rigidity and axial stiffness but also offer better environmental resistance than the use of steel.

Because of their directionally dependent property composite structure response under a given load can vary due to change of layer stacking sequence in laminated structures. Moreover, temperature and hygroscopic environmental conditions are also among the factors that will change the structural response. Because of these, analysis of composite structures becomes a cumbersome task in design.

1.1.2 Past Work in Composite Beam Analysis

In structural applications, beam is one of the most common structural members that have been considered in design. In structural analysis, most structures are often analyzed beams as one-dimensional structural members if one dimension (the length) is much larger than the other two dimensions (width and thickness). However, laminated composites are inherent of two dimensional properties. Hence, the structural response of composite beams possesses a unique characteristic in analysis.

The analysis of composite beam has been extensively studied for sometimes. In study of structural response, composite beam is often analyzed by two approaches: one is by finite element method and the other is using analytical method.

In finite element analysis, several finite element codes such as, MSC NASTRAN, ANSYS, ABAQUS, etc. were often used for analysis. To obtain a reasonable accuracy of analysis, a large of elements is needed in the model. Although the computer capability has been tremendously increased in the past decades, analysis by using FEM is still not an efficient method because of structural configuration dependent. Hence, finite element analysis is not an effective method to perform a parametric study, particularly in the preliminary design stage in which final configuration of composite structure is not well determined yet.

Therefore, designing composite beams has a need for analytical methods that not only provide accurate evaluation of sectional property for better prediction of structural response but also can be easily used for parametric study.

In using analytical method for composite beam analysis, several models have been included in the recently published books [1-6]. In his book, Tuttle [5] used the smear property obtained by classical lamination theory to calculate section properties for beams, but his method can only analyze the beams with a rectangular cross section. Barbero [3] in his book analyzed the composite thin-walled beams with arbitrary cross-section by using the smear property of laminate. His approach covers complicated mathematical formulations that make them not amenable for computation. In general,

among analytical methods covered in those books, some of them are too complicated to be used in design practice and some of them are too simple to simulate the characteristics of the structure response. However, among those simple methods, smeared property of composite section was used in their analysis. In doing so, the induced curvature due to unsymmetrical laminate lay-up was neglected. In analysis of composite tubular beams, Chan and his co-workers [7, 8] include ply orientation change due to the beam section contour in formulating their stiffness model. Their results indicated that using smear property for computing bending stiffness of composite tubular section can results in significant error in bending stiffness.

1.2 *Structural Analysis Using MATLAB*

1.2.1 *Introduction to MATLAB [9]*

MATLAB, abbreviated by MATrix LABoratory is a special program for engineering calculations, in particular for matrix calculations. Its program implements the MATLAB programming language and provides extensive libraries of pre-defined functions to make computations efficient. Unlike other computer languages, MATLAB has many integral plotting and imaging commands.

1.2.2 *Advantages of Using MATLAB*

A good characteristic of MATLAB that differentiate it from other mathematical programs like Mathematica is that MATLAB allows to interact easily with object oriented programs and classic programs, as Visual Basic, Visual C++, c++, Fortran, Java.

The process is made by a MATLAB compiler that translates the code written in MATLAB in other languages and once it is made the whole program can be compiled in other languages. Also, the details can be finished in other programs. MATLAB is a powerful mathematical application and has functions that no other program has.

MATLAB also can take advantages of other programs, as an example it is possible to write a program in MATLAB and to export it to visual BASIC. Then in Visual Basic, details can be added to make it more interactive and after this compile it and pack it using Visual Basic for its later distribution.

Even though MATLAB application has its own compiler the programs, it needs to have installed MATLAB in the computer to work. Unlike other programs that once the application is compiled and packed, this can work by themselves in Windows environment.

Another advantage that MATLAB offers is the program to pack functions and objects and to make them available as objects of other programs. This is very useful because programs with non- specialized in mathematics can benefit from these functions.

1.2.3 *Description of the use of GUIs in MATLAB*

The Graphical User Interface in *MATLAB* is new and soon to be implemented in the new versions of *MATLAB*. In fact, the implementation of this interface in *MATLAB* is not finished yet. Some options of the GUI interface that can be seen when programming are in construction and they have a warning message that they will be implemented in future versions. The *MATLAB* GUI programming begins to be an object

oriented programming; that is, each object (text button, push button, option button, among others) is programmed independently and responds to an event or callback in itself. Each object has a main event. For example, the main event or callback of a push button is when the button is pressed. If the user wants that the program responds for that event has to write code inside its event. When the button is pressed the desired result is obtained, but aside from this main event has other events like push, push down, push up.

In this way each object has its own events, and the events in the object are independent like the events in different objects, this allows greater freedom of programming, but in the same way it requires more code depuration since the events can coincide and if they have code inside it can produce an error result.

Object oriented programs are for example Visual Basic, C++ among others. The first is commonly used to work with data bases and the second it is used to work when data acquisition is required. These programs already have the objects and events options already well structured. The main difference between these programs and the classic programs like Fortran and C++ are that in the last programming applications the code follows linear sequence from the begin to end. It is possible encoding events in the classic programs but the process becomes very tedious.

1.3 Objectives of This Thesis

The main objective of this study is to develop a simple method for composite structural analysis by using MATLAB specifically composite beams with various cross-sections under bending and transverse loading are analyzed for their deflection and ply stresses. A user friendly MATLAB program with GUI (graphic user interface)

command is written for conducting this analysis. Besides these, the program was used for conducting parametric studies for understanding:

- a) the stacking sequence effect of interlaminar shear stress due to transverse loading
- b) the effect of ply stresses due to the web angle change for a composite truss beam
- c) the effect of ply stresses due to the flange and web lay-ups of symmetry for a composite I-beam.

1.4 *Outline of This Thesis*

Methodologies that used for composite beam analysis by practicing engineers is briefly described. The description of the MATLAB program and their GUI command are also included in this chapter. The developed MATLAB code for composite laminate analysis is introduced in Chapter 2. Chapter 3 presents analytical models that are used for analysis of composite beams with various cross-sections. A new model of calculating the bending stiffness for a composite truss beam analysis is also included in this chapter. Finite element analysis for composite beams with various cross-sections is conducted Chapter 4. The finite element results used for comparing the results obtained by the developed MATLAB codes are included in this chapter. Chapter 5 contains the parametric studies using the developed code. The effects of the ply stress due to stacking sequence, symmetrical lay-up and truss web angle are investigated in Chapter 5. A conclusion of the entire study is made in Chapter 6.

CHAPTER 2

ANALYSIS OF COMPOSITE LAMINATES USING MATLAB

2.1 Laminate Constitutive Equations

The stresses acting on a lamina (ply of the laminate) are three dimensional state stresses, in general. However, the thickness of a lamina is very thin. Hence, a generalized state of plane stress is assumed. That means, no interlaminar stresses are included in this analysis. Considering x-y-z as the global coordinate and 1-2-3 as the principal material coordinate (Figure 2.1), the stress-strain relationship in 1-2-3 coordinate can be written as equation (2.1).

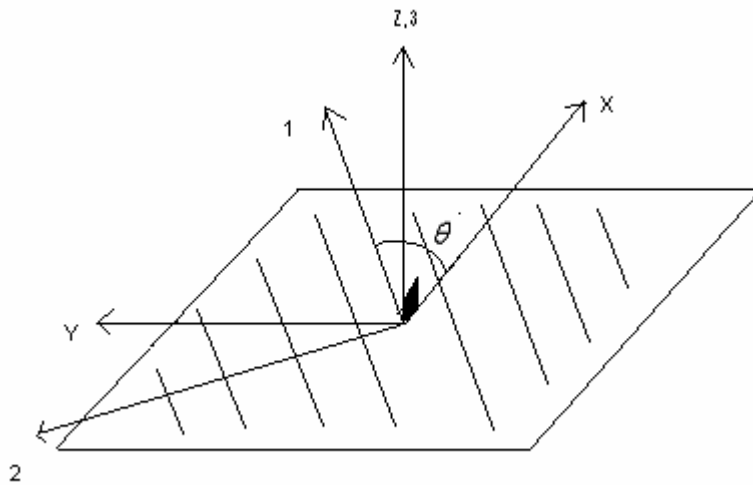


Figure 2.1 Representation of global and individual axis acting on a single lamina

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ & Q_{22} & 0 \\ Sym & & Q_{66} \end{bmatrix} \cdot \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} \quad (2.1)$$

where components of the Q_{1-2} matrix depend only on the material properties of the lamina.

$$Q_{11} = \frac{E_1}{1 - \nu_{12} \nu_{21}} \quad (2.1.1)$$

$$Q_{22} = \frac{E_2}{1 - \nu_{12} \nu_{21}} \quad (2.1.2)$$

$$Q_{12} = \frac{\nu_{12} E_2}{1 - \nu_{12} \nu_{21}} = \frac{\nu_{21} E_1}{1 - \nu_{12} \nu_{21}} \quad (2.1.3)$$

$$Q_{66} = G_{12} \quad (2.1.4)$$

Then if the principal axes of the lamina are rotated to coincide with the global x and y axes, the transformation of stiffness matrix is expressed in equation (2.2).

$$[\bar{Q}]_{xy} = [T_\sigma(-\theta)][Q]_{1-2}[T_\varepsilon(\theta)] \quad (2.2)$$

$$[T_\sigma(\theta)] = \begin{bmatrix} m^2 & n^2 & 2mn \\ n^2 & m^2 & -2mn \\ -mn & mn & m^2 - n^2 \end{bmatrix} \quad (2.2.1)$$

$$[T_\varepsilon(\theta)] = \begin{bmatrix} m^2 & n^2 & mn \\ n^2 & m^2 & -mn \\ -2mn & 2mn & m^2 - n^2 \end{bmatrix} \quad (2.2.2)$$

$$m = \cos \theta \quad ; \quad n = \sin \theta \quad (2.2.3)$$

Hence, the stress/strain relationship for angle ply is obtained as

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ & \overline{Q}_{22} & \overline{Q}_{26} \\ Sym & & \overline{Q}_{66} \end{bmatrix} \cdot \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} \quad (2.3)$$

where:

$$\overline{Q}_{11} = Q_{xx} = m^4 Q_{11} + n^4 Q_{22} + 2(Q_{12} + 2Q_{66})m^2 n^2 \quad (2.3.1)$$

$$\overline{Q}_{22} = Q_{yy} = n^4 Q_{11} + m^4 Q_{22} + 2(Q_{12} + 2Q_{66})m^2 n^2 \quad (2.3.2)$$

$$\overline{Q}_{12} = Q_{xy} = m^2 n^2 (Q_{11} + Q_{22} - 4Q_{66}) + (m^4 + n^4)Q_{12} \quad (2.3.3)$$

$$\overline{Q}_{16} = Q_{xs} = m^3 n (Q_{11} - Q_{12} - 2Q_{66}) - mn^3 (Q_{22} - Q_{12} - 2Q_{66}) \quad (2.3.4)$$

$$\overline{Q}_{26} = Q_{ys} = mn^3 (Q_{11} - Q_{12} - 2Q_{66}) - m^3 n (Q_{22} - Q_{12} - 2Q_{66}) \quad (2.3.5)$$

$$\overline{Q}_{66} = Q_{ss} = m^2 n^2 (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66}) + (m^4 + n^4)Q_{66} \quad (2.3.6)$$

Let consider x-y-z coordinate being located in the mid-plane of the entire laminate, (see Figure 2.2), The state of strain for any given layer is given as .

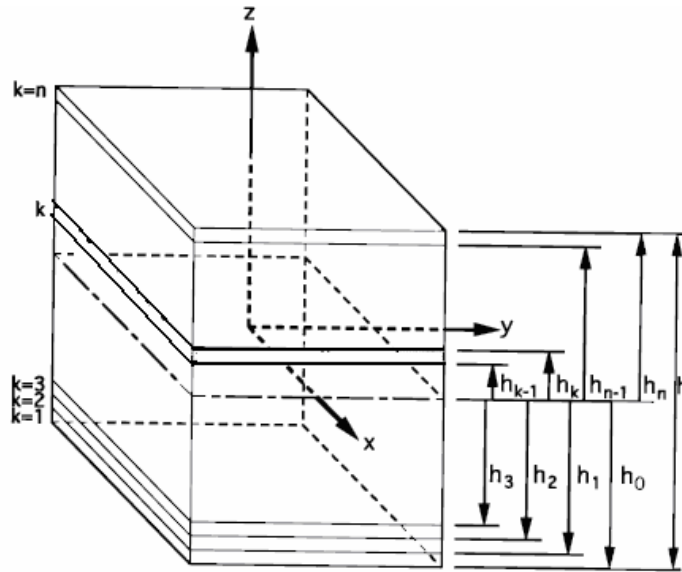


Figure 2.2 Lamina showing position notation for individual plies

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} + z \begin{bmatrix} k_x \\ k_y \\ k_{xy} \end{bmatrix} \quad (2.3)$$

The stress distribution for a given lamina (kth) in the laminate is

$$[\sigma_{x-y}]_{kth} = [\bar{Q}_{x-y}]_{kth} [\varepsilon_{x-y}]_{kth} = [\bar{Q}_{x-y}]_{kth} ([\varepsilon^0] + z_{kth} [k]) \quad (2.4)$$

The sum of forces and moments over the entire laminates gives

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \sum_{k=1}^n \int_{h_{k-1}}^{h_k} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}_k dz \cdot \frac{lb}{in} \quad (2.5)$$

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \sum_{k=1}^n \int_{h_{k-1}}^{h_k} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}_k z dz \cdot \frac{lb \cdot in}{in} \quad (2.6)$$

Combining equations (2.3), (2.4), (2.5), and (2.6), we obtain

$$\begin{bmatrix} N \\ M \end{bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{bmatrix} \varepsilon^0 \\ \kappa \end{bmatrix} \quad (2.7)$$

Where the universal A, B and D matrices are given as :

$$[A] = \sum_{k=1}^n [\bar{Q}_{x-y}]_k (h_k - h_{k-1}) \quad (2.7.1)$$

$$[B] = \frac{1}{2} \sum_{k=1}^n [\bar{Q}_{x-y}]_k (h_k^2 - h_{k-1}^2) \quad (2.7.2)$$

$$[D] = \frac{1}{3} \sum_{k=1}^n [\bar{Q}_{x-y}]_k (h_k^3 - h_{k-1}^3) \quad (2.7.3)$$

Inverting this matrix it is possible to obtain the compliance matrix:

$$\begin{bmatrix} \varepsilon^0 \\ k \end{bmatrix} = \begin{bmatrix} a & b \\ b^t & d \end{bmatrix} \begin{bmatrix} N \\ M \end{bmatrix} \quad (2.8)$$

Equation (2.7) or (2.8) refers to the laminate constitutive equation.

2.2 Failure Analysis of the Laminate

When a laminate is under load, each lamina exhibits a different stress level. If a lamina exhibits an excess deformation, or beyond stress allowable in its specific material principal direction, the lamina is said to be failure. Since the laminate is constituted of various laminas bonded together and orientated in the different direction, failure analysis of the laminate is more complicate than that of its individual lamina. Because only two-dimensional state of stresses are included, no delamination failure is considered in this analysis.

2.2.1 First Ply Failure Analysis

This analysis is focused to determine which lamina in the laminate fails first. In practice, the first ply failure often occurs in the matrix direction which is perpendicular to the loading direction if loading is in tension.

2.2.2 Progressive Failure Analysis

Progressive failure analysis consists in determining the sequence of failure process in laminate. Progressive failure analysis is important because when a lamina of the laminate fails the stiffness of the entire laminate is affected. Hence, in this analysis, A, B, and D matrices are needed to be recalculated at each occurrence of the failure. The modification of A, B, and D matrices depend on the failure mode of the lamina.

The laminate can not carry any load. Hence, th lamina is assumed to be a zero stiffness, in example: $Q_{11} = Q_{12} = Q_{16} = 0$. If the matrix failure occurs in a lamina, this lamina can not carry shear load and the load in the transverse to the fiber direction. If the lamina fails in shear, this lamina can not carry the shear load. Hence, $Q_{12} = Q_{66} = 0$.

The CLT formulas used for progressive failure analysis use the CLT formulas; this formulas were detailed in section 2.1 and the procedures are shown in the flow chart of figure (3.1) The hygrothermal stress analysis can be added for laminates under different environmental conditions.

The procedure to analyze the progressive failure analysis is shown in the flow chart of Figure 2.3.

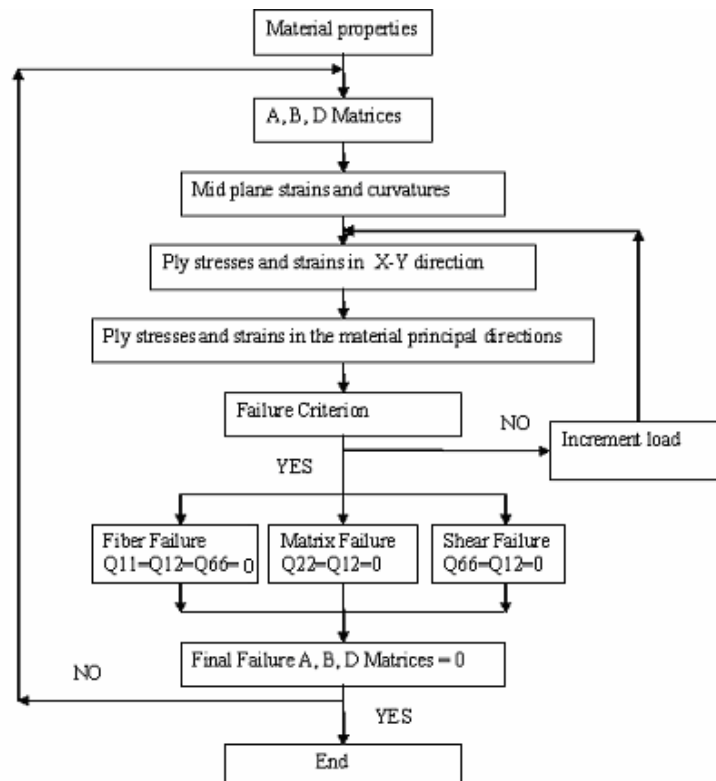


Figure 2.3 Progressive failure flow chart used in the MATLAB application

2.3 Development of the MATLAB Program

The MATLAB program is created in order to automatically perform tedious and repeated calculations of stress and strain of lamina and their stiffness.

The MATLAB application consists of files that are divided into Fig-files and M-Files that make the interface and the calculations possible.

2.3.1 Data Input and Results Presentation in the MATLAB Application

In order to development this application, data entrance and the results presentation are the two key issues for creating an useful interface with easy and fast access to the information, these problems were the entrance, and the presentation of results .

2.3.1.1 Data Input and the Use of GUIs

The first problem to be noticed was the entrance of data, it was slow using a data by data input, due to the many constants that are required in composite materials. For an optimization study, like to change the stacking sequence of the material or to change the material properties, it was tedious to enter all the data again. To solve this problem the implementation of a data base for the materials and a user interface input data was made, which is stored inside the application. The user can change by clicking in the desired material and input the material properties for the selected material. This interactive interface with the user in MATLAB receives the name of GUI, Graphical User Interface.

2.3.1.2 Data Output and the Use of HTML Report

The second problem was the presentation of the results. Since the application can make graphs of the results and show the results in the main screen of MATLAB, there are many results for user to see and these results can be lost. For this reason another tool of MATLAB was used to export the data to an HTML page. MATLAB can export results and equations to other formats like extensions (.doc, word documents). HTML format offers many variants and capabilities, and the location of results is easier.

2.4 Example and Demonstration of the Application

The following example is taken to demonstrate the capability of the program.

Example: Find the minimum axial load, N_x that will cause the first ply failure and final failure of the laminate according to maximum stress failure criteria for a graphite/epoxy $[\pm 45/0/90]_s$ laminate under a temperature of 100^0 F above the room temperature. Assume the room temperature to be the stress free condition.

The material constants and the ply thickness are given in table 1.

Table 1 Material Properties for Failure Analysis Example

E1	20 MSI
E2	2 MSI
G12	1 MSI
ν_{12}	0.3 MSI
Thickness	0.005 inches
F1t	0.3160 MSI
F1c	-0.2060 MSI
F2t	0.0056 MSI
F2c	-0.0375 MSI
F6	0.0117 MSI
α_1	2E-6
α_2	15E-6

2.4.1 Analytical Solution of the Progressive Failure Example

The program gives the $[\bar{Q}]$ matrices and [A], [B] and [D] matrices listed below.

Qbar[45]:

$$1.0 \text{ E6} \begin{bmatrix} 3.5905 & 1.0916 & 0 \\ 1.0916 & 3.5905 & 0 \\ 0 & 0 & 1.2494 \end{bmatrix} \text{ psi}$$

Qbar[0]

$$1.0 \text{ E7} \begin{bmatrix} 2.0182 & 0.0605 & 0 \\ 0.0605 & 0.2018 & 0 \\ 0 & 0 & 0.1 \end{bmatrix} \text{ psi}$$

Qbar[-45]:

$$1.0 \text{ E6} \begin{bmatrix} 6.8527 & 4.8527 & -4.5409 \\ 4.8527 & 6.8527 & -4.5409 \\ -4.5409 & -4.5409 & 5.2472 \end{bmatrix} \text{ psi}$$

Qbar[90]

$$1.0 \text{ E7} \begin{bmatrix} 0.2018 & 0.0605 & 0 \\ 0.0605 & 2.0182 & 0 \\ 0 & 0 & 0.1 \end{bmatrix} \text{ psi}$$

A =

$$1.0 \text{ E5} \begin{bmatrix} 3.5905 & 1.0916 & 0 \\ 1.0916 & 3.5905 & 0 \\ 0 & 0 & 1.2494 \end{bmatrix} \text{ lb/in}$$

D =

$$\begin{bmatrix} 43.9199 & 23.0494 & 6.8113 \\ 23.0494 & 34.8382 & 6.8113 \\ 6.8113 & 6.8113 & 25.1537 \end{bmatrix} \text{ lb}$$

B = 0 (since the laminate is symmetric with respect to its mid-plane)

a =

$$1.0 \text{ E-5} \begin{bmatrix} 0.3069 & -0.0933 & 0 \\ -0.0933 & 0.3069 & 0 \\ 0 & 0 & 0.8004 \end{bmatrix} \text{ in/lb}$$

d =

$$\begin{bmatrix} 0.0352 & -0.0226 & -0.0034 \\ -0.0226 & 0.0448 & -0.0060 \\ -0.0034 & -0.0060 & 0.0423 \end{bmatrix} \text{ 1/lb-in}$$

b = 0

The thermal induced load and moment are given as

$$[N^T] = \begin{pmatrix} 2241 \\ -2241 \\ 0 \end{pmatrix} \text{ Lb/in} \quad [M^T] = 0 \text{ (a symmetrical laminate)}$$

Recalling the laminate constitutive equations, we have:

$$\begin{bmatrix} \varepsilon_0 \\ k \end{bmatrix} = \begin{bmatrix} a & b \\ b^T & d \end{bmatrix} \cdot \begin{bmatrix} N + N^T \\ M + M^T \end{bmatrix} \text{ or } \begin{bmatrix} \varepsilon_0 \\ k \end{bmatrix} = \begin{bmatrix} a & 0 \\ 0 & d \end{bmatrix} \cdot \begin{bmatrix} N + N^T \\ 0 \end{bmatrix}$$

Where $[N] = \begin{bmatrix} N_x \\ 0 \\ 0 \end{bmatrix}$ and N_x is to be found

$$[k] = 0$$

$$[\varepsilon_{x-y}]_k = [\varepsilon^0] + z[k] - [\alpha_{x-y}]_k \cdot \Delta T$$

$$[\sigma_{x-y}]_{45} = [\bar{Q}] \varepsilon_{x-y}$$

The unknown N_x will remain in the equations as the value to be found.

Then,

for $k=1$ ($\theta = 45^\circ$)

If $N_x > 0$

$$\begin{bmatrix} 22.52N_x + 2241 \\ 2.835N_x - 2241 \\ -3.975N_x \end{bmatrix} < \begin{bmatrix} 316000 \\ 5600 \\ 11700 \end{bmatrix}$$

The 45° ply will fail in the transverse direction at $N_x = 2766$ lb/in

Following this procedure it is found that the minimum value for failure is $N_x = 1345$ lb/in in tension in the 90° ply along the transverse direction. After this the

components that not contribute to the stiffness of the lamina in matrix $[Q]$ in the material axis direction are replaced by zero and the new ABD matrix is calculated . Then the procedure is repeated for the next ply failure until all the components in the matrix $[Q]$ for all the plies are equal to zero. In this way it is possible to analytically calculate that the final N_x to cause a final failure of the lamina is 4.243E3 lb/in.

2.4.1 MATLAB Program Solution

Applying an arbitrary load of $N_x=1$ lb/in like initial value for the force, the following results are obtained. Figure (2.4) shows part of the program output with the values of the force and the safety factor, that multiplied together gives the necessary force to cause the first ply failure also it shows the total force to cause the total failure of the laminate.

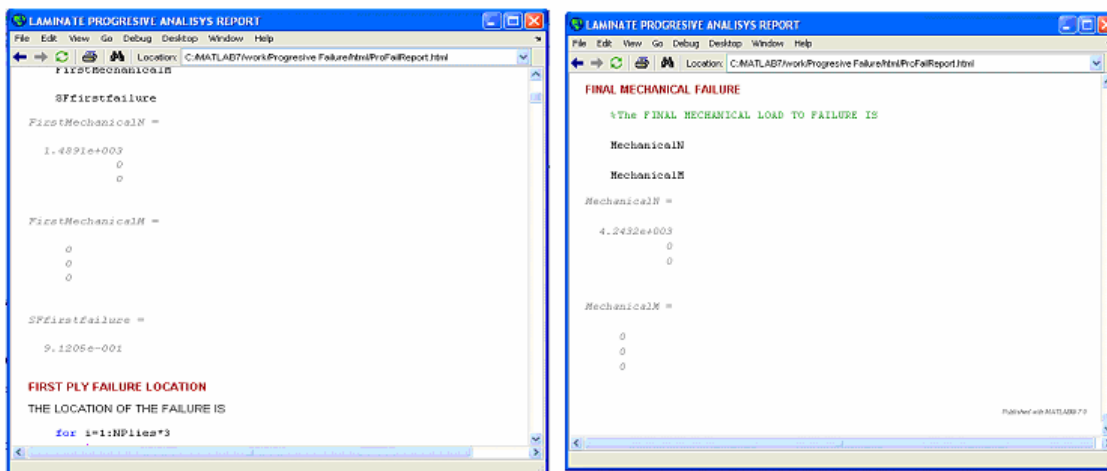


Figure 2.4 MATLAB results for the progressive failure example
 $\text{FirstMechanicalN} * \text{SFfirstfailure} = 1.4891\text{E}3 \text{ lb/in} * 9.1205\text{E}-1 = 1.358\text{E}3 \text{ lb/in}$
 Final Failure = 4.2432E3 lb/in.

As illustrated in the example the results obtained from the MATLAB program are exactly the same as the value obtained from the hand calculations.

CHAPTER 3

ANALYTICAL MODEL OF COMPOSITE LAMINATED BEAMS

3.1 *Fundamentals of Laminated Beam Analysis*

The analysis of beam is considered as one-dimensional problem. The foundation of the beam analysis is based upon the moment-curvature relationship along the longitudinal axis of the beam. This approach used for laminated composite beam is not different from the isotropic beam. It should be noted that composite laminate is inherent of two dimensional properties. Hence, in evaluation of the moment-curvature relationship, so-called the bending stiffness of the beam, laminated composite beam possesses a unique behavior that is different from the isotropic beam. This will be explained in the section of constitutive equation of one-dimensional structural members.

In this chapter, the beams are with either rectangular solid or thin-walled cross-sections. The beams undergo small deformations and behave linear elastic manner. The shear deformations are neglected and the plane cross-section remains plane and perpendicular to the beam axis after deformation.

Several cross-sections of the laminated beams were studied in this chapter. Most of analytical models for structural response of the beams have been published in the literatures except the truss beam. A brief review of those models published in the

literature is given in this chapter. A new developed analytical model for the truss beam is presented in this chapter.

3.2 Composite Beams with Rectangular Solid Cross-Section Under Transverse Loads

An isotropic beam subjected to a transverse load gives a transverse shear. This shear often plays not significant role to failure comparing to bending stress due to the transverse load. However, this is not the case for a composite beam. In composites, this shear is often called as interlaminar shear.

There are two kinds of interlaminar shear occurred in a laminated beam under load. One is due to effect of the free edges. The presence of interlaminar stresses σ_x , τ_{yz} and τ_{xz} (both shear and normal stresses) exists near the free edge is due to both mismatch of Poisson's ratio and in-plane shear between the laminas. This type of interlaminar shear is not the case studied in this chapter.

The other type of interlaminar shear is due to the transverse load. Since a laminated beam is considered in this section, τ_{yz} is considered to be zero.

Now consider a laminated composite beam is subjected to a transverse load as shown in Fig. 3.1.. The interlaminar shear, τ_{xz} reviewed in the following section has been developed in Ref.[13], The expression of this interlaminar shear stresses, τ_{xz} is reviewed in the next page.

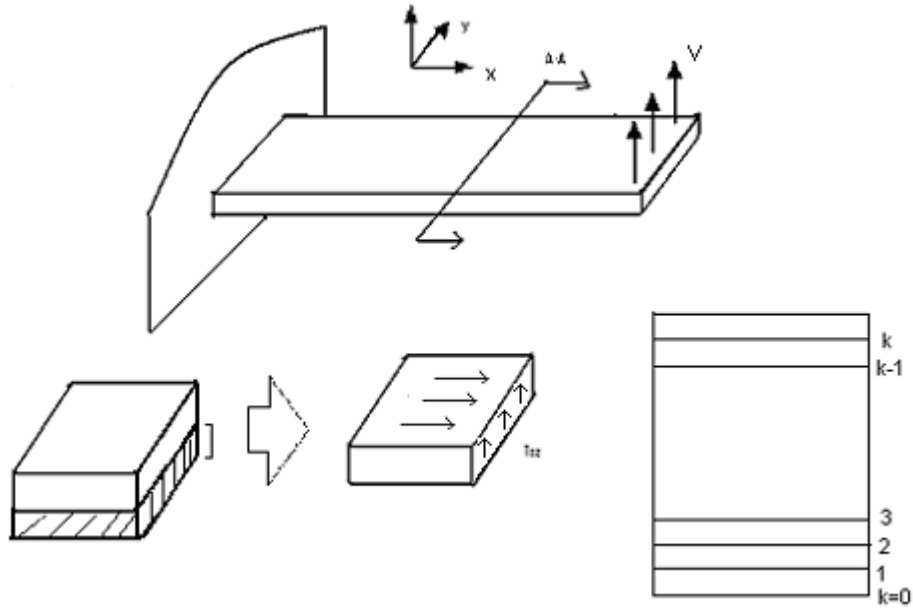


Figure 3.1 Cantilever beam showing the direction of induce stresses

Consider a shear force acting on the edge of the beam. Let V be the shear force per unit width acting on a laminated beam with a narrow width.

The equations of equilibrium of a given k th lamina can be written as:

$$\begin{aligned} \frac{\partial \sigma_x^k}{\partial x} + \frac{\partial \tau_{xy}^k}{\partial y} + \frac{\partial \tau_{xz}^k}{\partial z} &= 0 \\ \frac{\partial \tau_{xy}^k}{\partial x} + \frac{\partial \sigma_y^k}{\partial y} + \frac{\partial \tau_{yz}^k}{\partial z} &= 0 \\ \frac{\partial \tau_{xz}^k}{\partial x} + \frac{\partial \tau_{yz}^k}{\partial y} + \frac{\partial \sigma_z^k}{\partial z} &= 0 \end{aligned} \quad (3.1)$$

At the any section, the beam produces a moment M_x due to the transverse load, V .

$$M_x = V \cdot x \quad (3.2)$$

Equation (3.2) shows that moment is dependent on distance x . Hence, the mid-plane strains and curvatures k of the laminate are x dependent in the presence of a transverse load. Because of a narrow width, the stress of the beam can be considered as not dependent of the y -direction. Hence, the equation of equilibrium of the k th lamina can be written as

$$\frac{\partial \sigma_x^k}{\partial x} + \frac{\partial \tau_{xz}^k}{\partial z} = 0 \quad (3.3)$$

Taking integration on the above equation, we can write

$$\int_{z_{k-1}}^{z_k} \partial \tau_{xy}^k = - \int_{z_{k-1}}^{z_k} \frac{\partial \sigma_x^k}{\partial x} dz$$

we obtain

$$\tau_{xz}^k - \tau_{xz}^{k-1} = - \int_{z_{k-1}}^{z_k} \frac{\partial \sigma_x^k}{\partial x} dz$$

$$\tau_{xz}^{k-1} - \tau_{xz}^{k-2} = - \int_{z_{k-2}}^{z_{k-1}} \frac{\partial \sigma_x^{k-1}}{\partial x} dz$$

$$(\dots) \quad (3.4)$$

$$\tau_{xz}^2 - \tau_{xz}^1 = - \int_{z_1}^{z_2} \frac{\partial \sigma_x^2}{\partial x} dz$$

$$\tau_{xz}^1 - \tau_{xz}^0 = - \int_{z_{01}}^{z_1} \frac{\partial \sigma_x^1}{\partial x} dz$$

and $\tau_{xy}^0 = 0$ (free surface)

Adding equations [3.4] together we obtain

$$\tau_{xz}^k - 0 = - \sum_{i=1}^k \int_{z_{i-1}}^{z_i} \frac{\partial \sigma_x^i}{\partial x} dz \quad (3.5)$$

The equation [3.6] is from shear and bending moments on beams

$$\frac{\partial M_x}{\partial x} = V \quad (3.6)$$

Where V is the shear acting along the wide of the beam. From lamination theory the mid-plane stress and curvatures are related to in-plane loads and moments for the compliance matrix equation (2.15), then it is possible to find lamina strains using equation (2.8), and the lamina stresses are found using equation (2.9).

Recalling that

$$[\sigma^k] = [Q^k] \cdot \{[\varepsilon^0] + [k]\}$$

Then

$$\frac{\partial [\sigma_{x-y}]_{kth}}{\partial x} = [\bar{Q}_{x-y}]_{kth} \left(\frac{\partial [\varepsilon^0]}{\partial x} + z_{kth} \frac{\partial [k]}{\partial x} \right) \quad (3.7)$$

Since $[\bar{Q}]$ matrix and location z_k are independent of x.

If only M_x is applied, we have

$$[\varepsilon_0] = [b][M] ; [k] = [d][M]$$

then

$$\frac{\partial [\varepsilon_0]}{\partial x} = [b] \frac{\partial [M]}{\partial x} ; \frac{\partial [k]}{\partial x} = [d] \frac{\partial [M]}{\partial x}$$

Using equation (3.6), we obtain

$$\frac{\partial [\varepsilon_0]}{\partial x} = [b][V] \quad \text{and} \quad \frac{\partial [k]}{\partial x} = [d][V] \quad (3.8)$$

or in full expression,

$$\begin{bmatrix} \frac{\partial \varepsilon_x^0}{\partial x} \\ \frac{\partial \varepsilon_y^0}{\partial x} \\ \frac{\partial \gamma_{xy}^0}{\partial x} \end{bmatrix} = \begin{bmatrix} b_{11}V \\ b_{21}V \\ b_{61}V \end{bmatrix} \quad ; \quad \begin{bmatrix} \frac{\partial k_x}{\partial k_y} \\ \frac{\partial x}{\partial k_y} \\ \frac{\partial k_{xy}}{\partial x} \end{bmatrix} = \begin{bmatrix} d_{11}V \\ d_{12}V \\ d_{16}V \end{bmatrix} \quad (3.9)$$

It should be noted that $b_{21} \neq b_{12}$, $b_{61} \neq b_{16}$. However, $d_{21} \neq d_{12}$ and $b_{16} \neq b_{61}$

Combining equation (3.9) into equation(3.7) and extracting the first component of Eq. [3.7]

$$\frac{\partial \sigma_x^i}{\partial x} = V \left(\left[\overline{Q}_{11}^i . b_{11} + \overline{Q}_{12}^i . b_{21} + \overline{Q}_{16}^i . b_{61} \right] + z \left[\overline{Q}_{11}^i . d_{11} + \overline{Q}_{12}^i . d_{12} + \overline{Q}_{16}^i . d_{16} \right] \right) \quad (3.10)$$

or

$$\frac{\partial \sigma_x^i}{\partial x} = V \left[\overline{Q}b^i + z_i \overline{Q}d^i \right] \quad (3.11)$$

and z_i is the location of interest to find τ_{xz}

From Eq. [3.5], we have

$$\tau_{xz}^k = - \sum_{i=1}^k \int_{z_{i-1}}^{z_i} V \left[\overline{Q}b^i + z_i \overline{Q}d^i \right] dz$$

Finally integrating respect to z

$$\text{Where} \quad \overline{Q}b^i = \overline{Q}_{11}^i . b_{11} + \overline{Q}_{12}^i . b_{21} + \overline{Q}_{16}^i . b_{61}$$

$$\overline{Q}d^i = \overline{Q}_{11}^i . d_{11} + \overline{Q}_{12}^i . d_{12} + \overline{Q}_{16}^i . d_{16}$$

$$\tau_{xz}^k = -V \left\{ \sum_{i=1}^k \overline{Q}b^i (z_i - z_{i-1}) + \frac{1}{2} \sum_{i=1}^k \overline{Q}d^i (z_i^2 - z_{i-1}^2) \right\} \quad (3.12)$$

3.2.1 Cantilever Beam

In this section, we focus to determine the deflection of the beams. The deflection of a beam can be obtained from the moment-curvature relationship regardless of the material system.

As shown in Fig. 3.2, like in strength of materials, the deflection of the beam can be express as

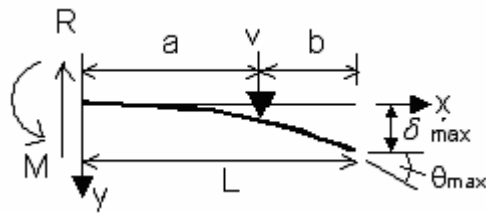


Figure 3.2 Cantilever beam diagram

$$\delta = \frac{V(3x^2a - x^3)}{6EI} \text{ for } x < a \quad ; \quad \delta = \frac{V(3a^2x - a^3)}{6EI} \text{ for } x \geq a \quad (3.13)$$

Where \overline{EI} is the bending rigidity of the beam.

The equivalent modulus of laminate to calculate \overline{EI} is first computed by lamination

theory as $E = \frac{1}{a_{11}t}$, then the moment of inertia is obtained from the geometry which

$$\text{is } I = \frac{1}{12}bh^3.$$

By doing so, Drummond and Chan [10] found that this conventional method for obtaining bending stiffness results in a significant error comparing to both FEM results and experimental test data in their I-beam analysis.

An accurate method to obtain a bending stiffness is given in Ref [14], this method is briefly reviewed below

Let us consider a beam subjected to both N_x and M_x for a narrow beam with small wide to height ratio equation (3.15) can be written in the following way

From equation (2.15)

$$\begin{bmatrix} \varepsilon_x^o \\ \kappa_x \end{bmatrix} = \begin{bmatrix} a_{11} & b_{11} \\ b_{11} & d_{11} \end{bmatrix} \begin{bmatrix} N_x \\ M_x \end{bmatrix}$$

The bending and axial stiffness of the beam are:

$$\overline{EI} = \frac{a_{11}}{a_{11}d_{11} - b_{11}^2} \quad (3.14.1)$$

$$\overline{EA} = \frac{d_{11}}{a_{11}d_{11} - b_{11}^2} \quad (3.14.2)$$

For the case when stacking sequence is symmetric, equations (3.12.1) and (3.12.2) became

$$\overline{EI} = \frac{1}{d_{11}} \quad (3.15.1)$$

$$\overline{EA} = \frac{1}{a_{11}} \quad (3.15.2)$$

After find the bending stiffness this value can be replaced in the formulas for deflection (3.13) and find the deflection at any point.

3.2.2 Simply Supported Beam

For a simply supported beam shown in Fig. 3.3, the equations of deflection are given as

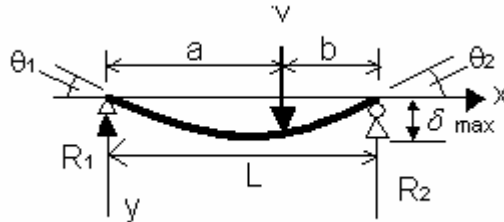


Figure 3.3 Simply supported beam diagram

$$\delta = \frac{Vb(-6x^3 + (L^2 - b^2)x)}{6LEI} \text{ for } x < a \quad (3.16.1)$$

$$\delta = \frac{Vb\left(\frac{L}{b}(x-a)^3 - x^3 + (L^2 - b^2)x\right)}{6LEI} \text{ for } x \geq a \quad (3.16.2)$$

For a simply supported beam the same procedure can be applied. First, we find the bending rigidity EI using equation (3.14.1) and then replace this into equation (3.16.1) and (3.16.2).

3.3 Analysis of Truss Beam

For a truss beam as shown in fig. 3.4, the goal is to find the bending stiffness and the axial stiffness. With these values are found, then we can find deflections and elongations used the equations given in the strength of materials .

For this purpose an approach developed by Syed and Chan [12] will be used. This approach consists in developing ABD stiffness matrix laminate that is not parallel to the x-y plane. This type of laminate is termed as the web laminate.

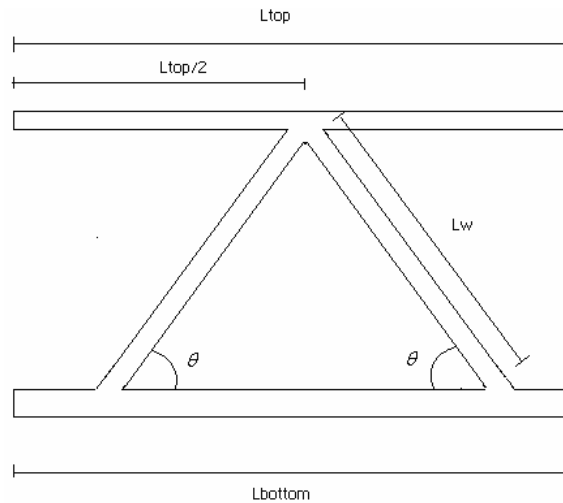


Figure 3.4 Truss beam model showing dimensions

The stiffness matrices for the top flanges laminates are calculated separately.. The flange is divided in two sections. This is because the flange stacking sequence changes as the web stacking sequence change. It can cause that the left side flange has a different stacking sequence from the right side flange. The same reason is applied for the bottom flange, which is divided in three parts left bottom flange, middle bottom flange and right bottom flange.

This stiffness matrix for the flange on top and bottom are calculated using the CLT theory and they are multiplied for the width of each laminate. Then this stiffness matrices are translated to the mid-plane of the cross section. The mid-plane is located at the middle height of the web flange.

For the web laminate, rotation of the stiffness matrix of the laminas [Q] along the x axis and z axis is required.

The following formula is used:

$$[\bar{Q}]'_{x-y} = [T_{\sigma}(-\beta)]_z [T_{\sigma}(+\theta)]_x [Q]_{1-2} [T_{\varepsilon}(-\theta)]_x [T_{\varepsilon}(+\beta)]_z \quad (3.17)$$

Where:

$[Q]_{1-2}$ is the stiffness matrix of each ply, θ is the angle between the web and the bottom laminate, and β is the angle orientation of each ply.

The stress transformation matrices are given in appendix A.

The entire formulas for stiffness matrices of the plies in the laminate using equation (3.17) are:

$$\bar{Q}'_{11} = c_z^4 Q_{11} + 2s_z^2 c_z^2 (c_x^2 Q_{12} + 2c_x^2 Q_{66}) + s_z^4 c_x^4 Q_{22}$$

$$\begin{aligned}
\bar{Q}_{22}' &= s_z^4 Q_{11} + 2s_z^2 c_z^2 (c_x^2 Q_{12} + 2c_x^2 Q_{66}) + c_z^4 c_x^4 Q_{22} \\
\bar{Q}_{12}' &= s_z^2 c_z^2 (Q_{11} + c_x^4 Q_{22} - 4c_x^2 Q_{66}) + (s_z^4 + c_z^4) c_x^2 Q_{12} \\
\bar{Q}_{16}' &= s_z c_z^3 (Q_{11} - c_x^2 Q_{12} - 2c_x^2 Q_{66}) + s_z^3 c_z (c_x^2 Q_{12} - c_x^4 Q_{22} + 2c_x^2 Q_{66}) \\
\bar{Q}_{26}' &= c_z s_z^3 (Q_{11} - c_x^2 Q_{12} - 2c_x^2 Q_{66}) + c_z^3 s_z (c_x^2 Q_{12} - c_x^4 Q_{22} + 2c_x^2 Q_{66}) \\
\bar{Q}_{66}' &= s_z^2 c_z^2 (Q_{11} + c_x^4 Q_{22} - 2c_x^2 Q_{12} - 2c_x^2 Q_{66}) + (s_z^4 + c_z^4) c_x^2 Q_{66}
\end{aligned} \tag{3.18}$$

Where:

$c_x = \cos \theta$; $c_z = \cos \beta$; and $s_z = \sin \beta$.

Then applying equations: (2.9); (2.13.1); (2.13.2) and (2.14)

Where ABD matrix is the stiffness matrix per unit length and it need to be multiplied for the length of each web then

$$\begin{aligned}
[\bar{A}]_{web} &= L_w \cdot [A] \\
[\bar{B}]_{web} &= L_w \cdot [B] \\
[\bar{D}]_{web} &= L_w \cdot [D] + \frac{L_w^3}{12} \cdot \text{Sin}^2 \theta \cdot [A]
\end{aligned} \tag{3.19}$$

The A, B and D matrices of each laminate in which the structure is divided are in the mid plane of the cross section and can be adding together to form the total stiffness matrix of the entire beam cross-section.

The neutral axis can be found using the following formula with the condition that the neutral axis is the axis of zero deformation.

$$\text{Neutral Axis} = \frac{b_{11}}{d_{11}} \tag{3.20}$$

After the neutral axis location is found the total stiffness matrix ABD is translated to this neutral axis location. Finally the bending stiffness and axial stiffness can be found using equations [3.14.1] and [3.14.2].

3.4 Analysis of Tubular Beam

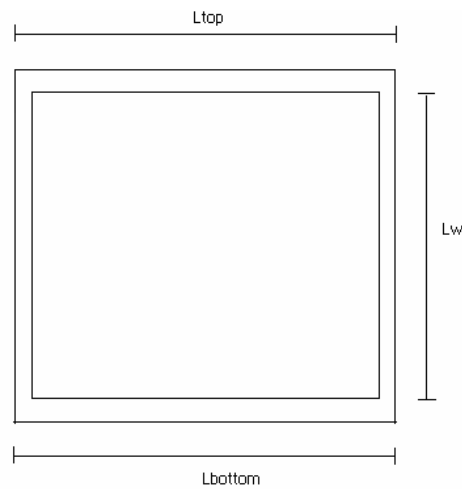


Figure 3.5 Square tube beam model showing dimensions

For square tube beam the same approach as for the truss beam is taken.

First the section is divided in four plies, one top flange, one bottom flange and two web flanges. For the top flange and the bottom flange, the first step is to find the ABD stiffness matrix, this is done using CLT equations developed in chapter 2, then the stiffness matrix per unit length of the top and bottom flanges are multiplied times the width of the top and bottom flanges, respectively. After this, the stiffness matrices are translated to the middle of the cross section. For the web the procedure applied for the truss beam is followed, with the difference that for the square tube beam the angle θ is 90 degrees. First the axis rotation transformation is used and formulas (3.18) are used to find the stiffness of the plies in the global coordinate system it should be noticed that

the left web is rotated clock wise direction for the x-axis rotation and the right web is rotated in a counter-clock-wise direction. Then the ABD stiffness matrix is constructed using CLT formulas (2.14) developed in chapter 2 . After using lamination theory and find the ABD stiffness matrices per unit length for both webs , these are multiplied times the longitude of the web laminate L_w using equations (3.19) . Web lengths are the same as in truss beam.

After have all the stiffness matrices on the middle of the cross section, we can add the A, B and D matrices together to obtain the total stiffness matrix in the global coordinate system. Then it is possible to find the neutral axis location using formula (3.20).

Finally the ABD stiffness matrix is translating to the neutral axis (it should be noticed that if the neutral axis is negative it will be below the mid section by the distance indicated in (3.20) and using equations (3.13.1) and (3.13.2) the bending stiffness and the axial stiffness can be found for a square tube beam.

3.5 Analysis of I-Beam

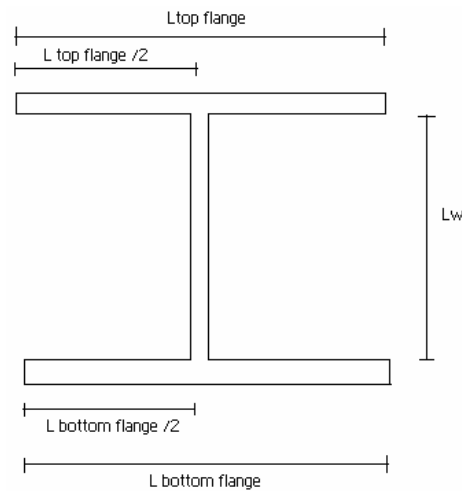


Figure 3.6 I beam model showing dimensions

For I beam, the structure is divided into five segments, two on top flange, two on bottom flange and one ply representing the web. On top and bottom the plies share one common area located at the web. It is necessary because if a symmetric stacking sequence for the web is selected, the flanges on top and bottom do not have the same stacking sequence and they will be one symmetric and one anti-symmetric

The procedure for the laminates on top and bottom flanges is to find the ABD stiffness matrix using CLT theory and then multiply these matrices per unit length times the width of the left flange and the wide of the right flange, respectively. The matrix generated have the form of equation (2.14).

For the web laminate in I-beam, the procedure used for the truss beam is also applied with the web angle, $\theta = 90^0$. For the web laminate equations (3.18) are used to find the lamina stiffness matrices in the global coordinate system, then using CLT to find the ABD stiffness matrix which has the same form as the matrix of equation (2.14). This is the web stiffness matrix per unit length. then this matrix is multiplied by the web length using equations (3.19). After all the matrices are on the mid-plane of the cross section they are adding together to form the total stiffness matrix at the mid plane.

Finally the ABD total stiffness matrix is translated to the neutral axis and equations (3.14.1) and (3.14.2) are used to find the bending stiffness and axial stiffness of I beam

Finally the ABD total stiffness matrix is translated to the neutral axis and equations (3.14.1) and (3.14.2) are used to find the bending stiffness and axial stiffness of and I beam.

CHAPTER 4

COMPOSITE BEAM ANALYSIS USING MATLAB AND FINITE ELEMENT METHODS

This chapter covers analysis of composite beams using both MATLAB and finite element methods. Composite beams with rectangular cross-section, I-section, squared section, and truss section were studied. Analytical models used in the MATLAB are described in the previous chapter. Analysis was focused to determine the bending stiffness and stresses of the beam. This chapter demonstrated accuracy of MATLAB results by comparing with the results by the detailed model of finite element method. The developed MATLAB code was also described in the previous chapter.

4.1 Finite Element Model

For the finite element analysis, all of the beams with various cross-sections have a rigid constraint across the cross-sections at the loading end. The beams are fully constrained on the other end to simulate the symmetric part of loading or built-in end condition. The length of the beam ratio to its width is maintained a minimum ratio of 6:1 or more to minimize the short beam shear effect.

SHELL 91 of ANSYS element is used in the entire model. The bending stiffness of the beam is calculated based upon the relationship of the beam deflected curvature and the applied moment. Since the finite element result does not give the

deflection curve. Instead, we took the deflections of any given three points on the centerline of the flange of the beam. These three points are equally separated each other along the plane of the neutral axis. Based these deflections, the curvature of the deflected beam can be calculated. This method has been successfully used to determine the curvature of the beam [13]. The section of the deflection locations along the neutral axis to determine the curvature is to eliminate the error from the induced twisting effect of the composite beam under bending. A MATLAB code for obtaining the curvature of the beam deflection is written and listed in Appendix B.

ANSYS model is also developed for each different configurations of the beam. These include auto meshing and results plotting are listed in Appendix C.

The material used in this study is E-glass/Epoxy. The material properties of this material listed below is also implemented in the material data bank of the MATLAB code. The ply thickness used here, 0.01 inch is twice of the nominal ply thickness of these materials which is 0.005 inch.

Table 2 Material Properties for Beam Analysis

E_1	5.7E6 MSI
E_2	1.24E6 MSI
G_{12}	0.54E6 MSI
ν_{12}	0.28
Thickness	0.01 inches

4.2 Beams Under Pure Bending

A simple beam under a pure bending is modeled to find the bending stiffness and stress. The beam is chosen to be 20 inches long and 1 inch wide with applying

moment of 1 lb-in on one end. The other side of the beam is fully constrained. The stacking sequence of the beam is $[45/-45/0/0]_s$.

4.2.1 MATLAB Analysis for a Beam under Bending

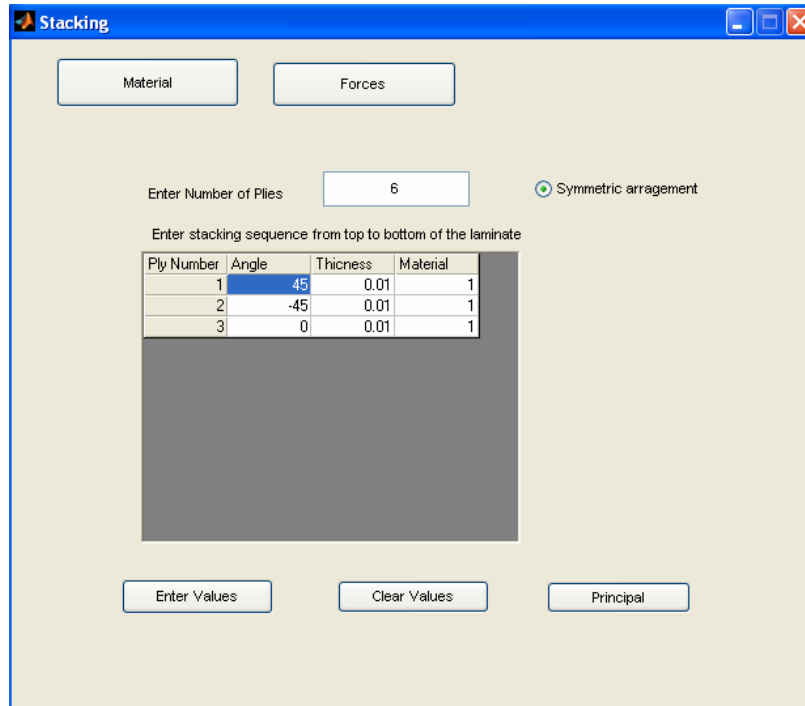


Figure 4.1 Screen with data for the simple beam analysis

ABD Matrix at the neutral axis (As show the result report of the Application):

$$\begin{bmatrix} 2.1525e+005 & 6.3133e+004 & -1.8190e-012 & 0 & 0 & 0 \\ 6.3133e+004 & 1.2450e+005 & 0 & 0 & 0 & 0 \\ -1.8190e-012 & -1.8190e-012 & 7.4340e+004 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4.6882e+001 & 2.4532e+001 & 9.0748e+000 \\ 0 & 0 & 0 & 2.4532e+001 & 4.3857e+001 & 9.0748e+000 \\ 0 & 0 & 0 & 9.0748e+000 & 9.0748e+000 & 2.7894e+001 \end{bmatrix}$$

The bending stiffness obtained from MATLAB is $EI_{MATLAB} = 32.55 \text{ lb-in}^2$.

4.2.2 Finite Element Results

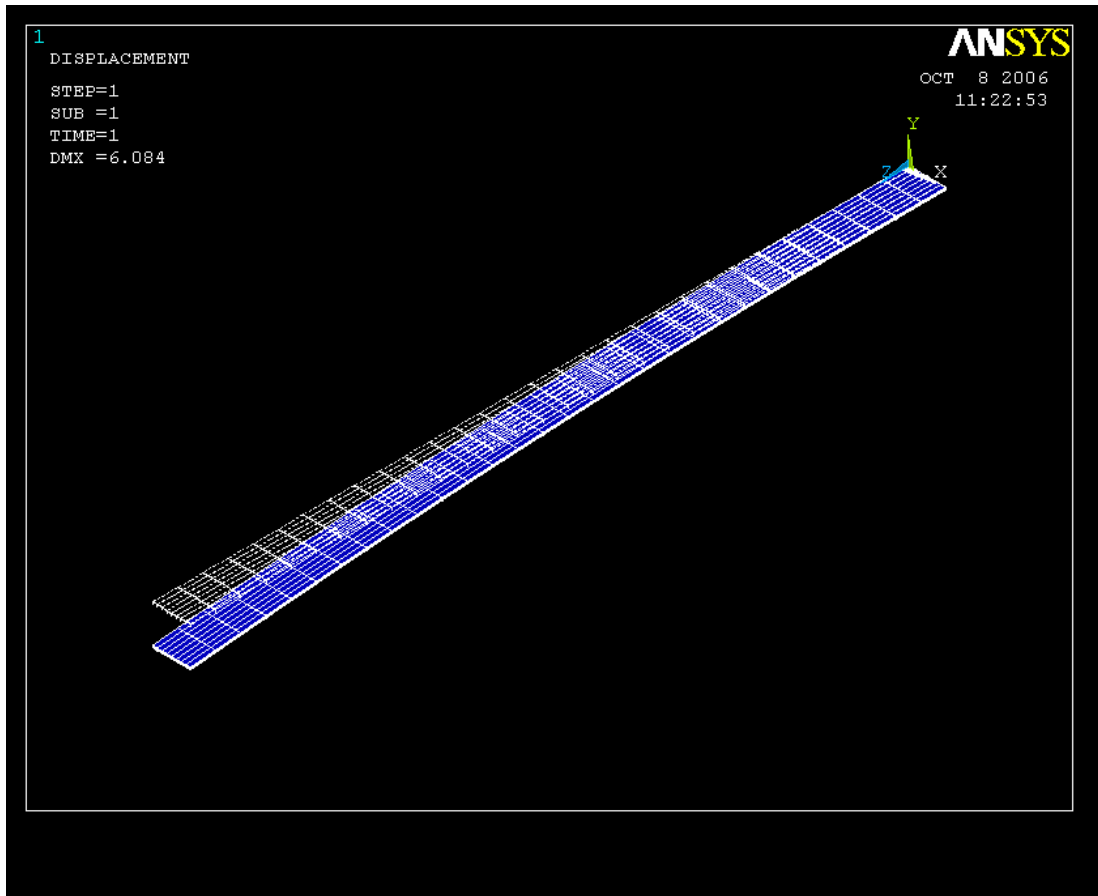


Figure 4.2 Undeformed and deformed shape of the simple beam

The ANSYS model contains 240 elements. The deformed and undeformed beams are shown in Fig. 4.2. For a beam under pure bending, the bending stiffness can be calculated by the expression given as:

$$EI_{FEM} = \frac{(Applied\ Moment)(beam\ length)^2}{2 * (Vertical\ Displacement)} = \frac{(1)(20)^2}{(2) * (6.084)} = 32.87 \text{ lb-in}^2$$

The difference of bending stiffness between two methods gives about 1%.

4.2.3 Stresses σ_x of the Beam

Figure 4.3 shows a stress distribution of the beam under bending. As indicated, the stress is a constant along the beam axis for a given layer. It is also indicated the top and bottom layer of the stress are equal magnitude but opposite sign.

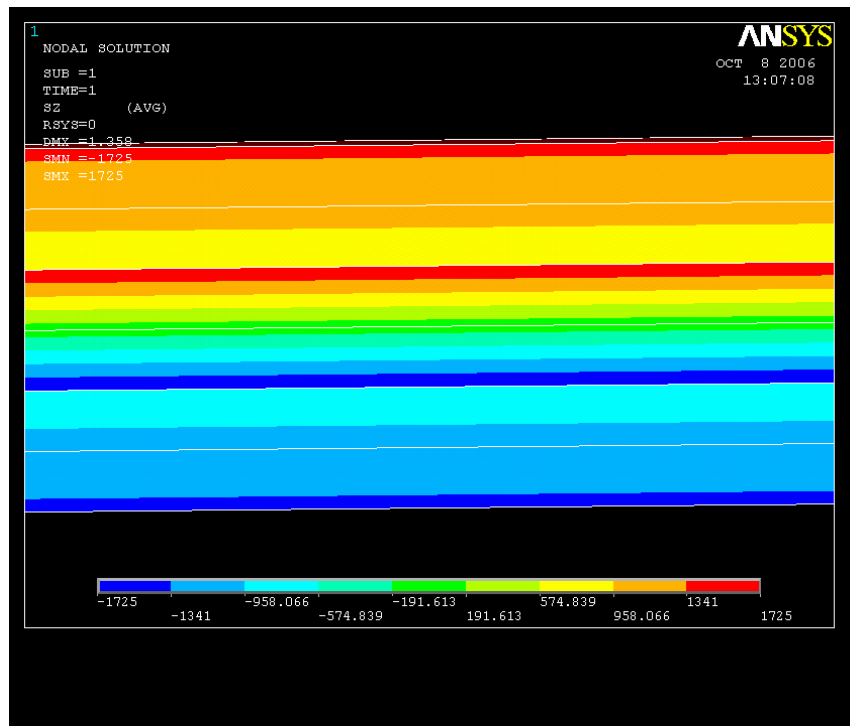


Figure 4.3 Stresses σ_x in the simple beam

The stress, σ_x obtained from the ANSYS model is 1725 psi. in 0^0 ply.

The mid-plane strain and curvature as well as σ_x obtained from the MATLAB are given below.

$$\varepsilon_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \text{ in/in} \quad k = \begin{bmatrix} 30.73E-3 \\ -16.21E-3 \\ -4.723E-3 \end{bmatrix} 1/\text{in}$$

$$\sigma_x^0 = 1725E3 \text{ psi}$$

σ_x obtained from the finite element is exactly the same as the value obtained from MATLAB.

4.3 I Beam

The screenshot shows the 'Ibeam' software interface with the following data:

Material and **Principal** buttons are at the top.

N. Plies T. L. Flange: 6

Ply Number	Angle	Thickness	Material
1	45	0.01	1
2	-45	0.01	1
3	0	0.01	1
4	0	0.01	1
5	-45	0.01	1
6	45	0.01	1

N. Plies T. R. Flange: 6

Ply Number	Angle	Thickness	Material
1	45	0.01	1
2	-45	0.01	1
3	0	0.01	1
4	0	0.01	1
5	45	0.01	1
6	-45	0.01	1

N. Plies Web: 4

Ply Number	Angle	Thickness	Material
1	-45	0.01	1
2	45	0.01	1
3	45	0.01	1
4	-45	0.01	1

N. Plies B. L. Flange: 6

Ply Number	Angle	Thickness	Material
1	-45	0.01	1
2	45	0.01	1
3	0	0.01	1
4	0	0.01	1
5	45	0.01	1
6	-45	0.01	1

N. Plies B. R. Flange: 6

Ply Number	Angle	Thickness	Material
1	45	0.01	1
2	-45	0.01	1
3	0	0.01	1
4	0	0.01	1
5	45	0.01	1
6	-45	0.01	1

Lengths:

- Lenght TOP LEFT: 1
- Lenght TOP RIGHT: 1
- Lenght BOTTOM LEFT: 1
- Lenght BOTTOM RIGHT: 1
- Lenght WEB: 1

Buttons: Enter Values, Clear Values, Calculate

Figure 4.4 Screen with data for I-beam analysis

As described in Chapter 3, there is not any possible way to fabricate the identical lay up of the top and bottom flanges of laminated I-beam unless the flange contains only 0^0 and 90^0 plies. Hence, in modeling, we divide an I beam into 5 segments of laminates which includes left and right laminates on each flange and one shear web laminate. The laminate stacking sequence is shown below.

Top Left Flange:[45/-45/0]s

Top Right Flange:[45/-45/0]s

Web:[-45/45/-45/45]

Bottom Left Flange:[-45/45/0]s

Bottom Right Flange:[-45/45/0]s

The I-beam is fully constrained at one end and has a rigid bar elements connecting all of the nodes on the cross-section at the other end where a moment of 1 lb-in is applied.

4.3.1 *MATLAB Results*

ABD Matrix at the Neutral Axis (As show the result report of the Application):

$$\begin{bmatrix} 9.1898e+005 & 3.1052e+005 & -1.0914e-011 & -8.5672e-003 & -2.8948e-003 & 4.5374e+002 \\ 3.1052e+005 & 5.5598e+005 & -7.2760e-012 & -2.8948e-003 & -5.1832e-003 & 4.5374e+002 \\ -1.0914e-011 & -1.0914e-011 & 3.5535e+005 & 4.5374e+002 & 4.5374e+002 & -3.3127e-003 \\ -8.5672e-003 & -2.8948e-003 & 4.5374e+002 & 2.2027e+005 & 6.8071e+004 & -5.7989e+000 \\ -2.8948e-003 & -5.1832e-003 & 4.5374e+002 & 6.8071e+004 & 1.2951e+005 & -5.7989e+000 \\ 4.5374e+002 & 4.5374e+002 & -3.3127e-003 & -5.7989e+000 & -5.7989e+000 & 7.9291e+004 \end{bmatrix}$$

$$EI_{MATLAB} = 1.8450e+005 \text{ lb-in}^2$$

4.3.2 *Finite Element Results*

The model for the I-Beam contains 660 elements. The displacements at Nodes 13, 40 and 67 were extracted for calculating the curvature of the beam. Their displacements are list below:

Node 13:

$$X1=1+0.24202E-05 \text{ in, } Y1=1-0.25465E-05 \text{ in}$$

Node 40:

$$X2=10+0.26315E-04 \text{ in, } Y2=1-0.26180E-03 \text{ in}$$

Node 67:

$$X3=19+0.50209E-04 \text{ in, } Y3= 1-0.94987E-03 \text{ in}$$

$$\text{Curvature} = 5.2940e-006$$

$$EI_{FEM} = \text{Moment} / \text{curvature} = 1 / 5.2940\text{e-}006 = 1.88893\text{E+}005 \text{ lb-in}^2$$

Figure 4.5 below shows the deformed and undeformed shape of I-beam.

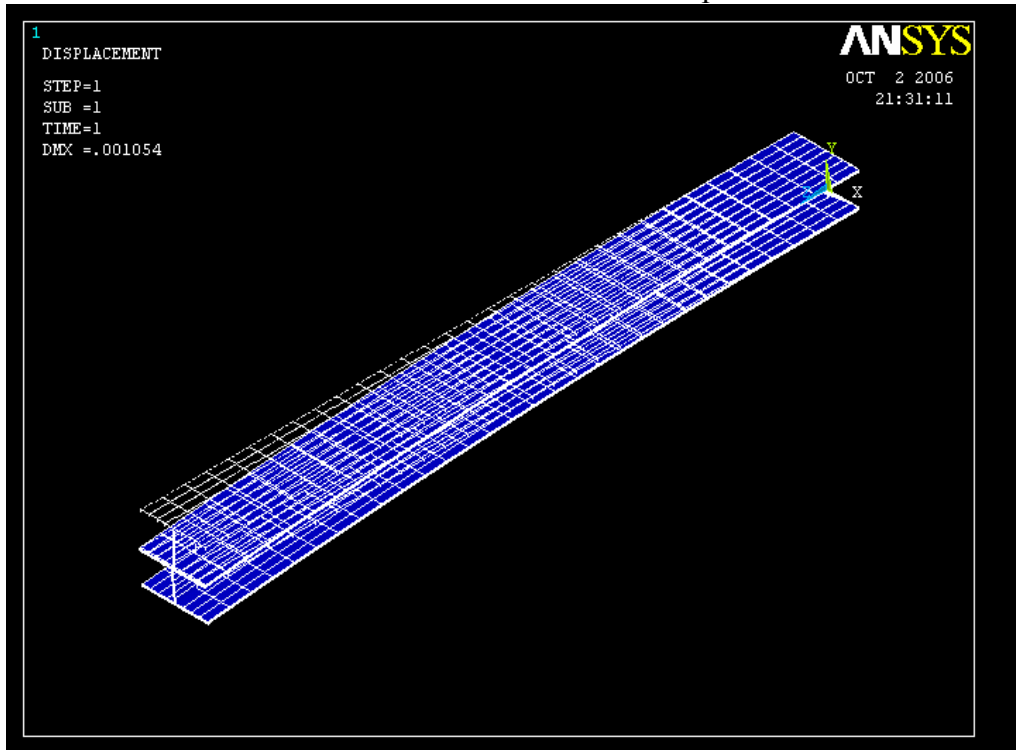


Figure 4.5 Undeformed and deformed shape of the I-Beam

Comparison between the MATLAB and finite element results, a 2.3 % difference is obtained.

4.3.3 Comparison of Stress σ_x

σ_x distribution across the cross-section of I-beam. σ_x in the upper layer of 0^0 ply is given by the finite element as 15.165 psi. The same stress obtained by MATLAB is 15.21 psi, which gives 0.3% difference

Figure 4.6 shows σ_x .

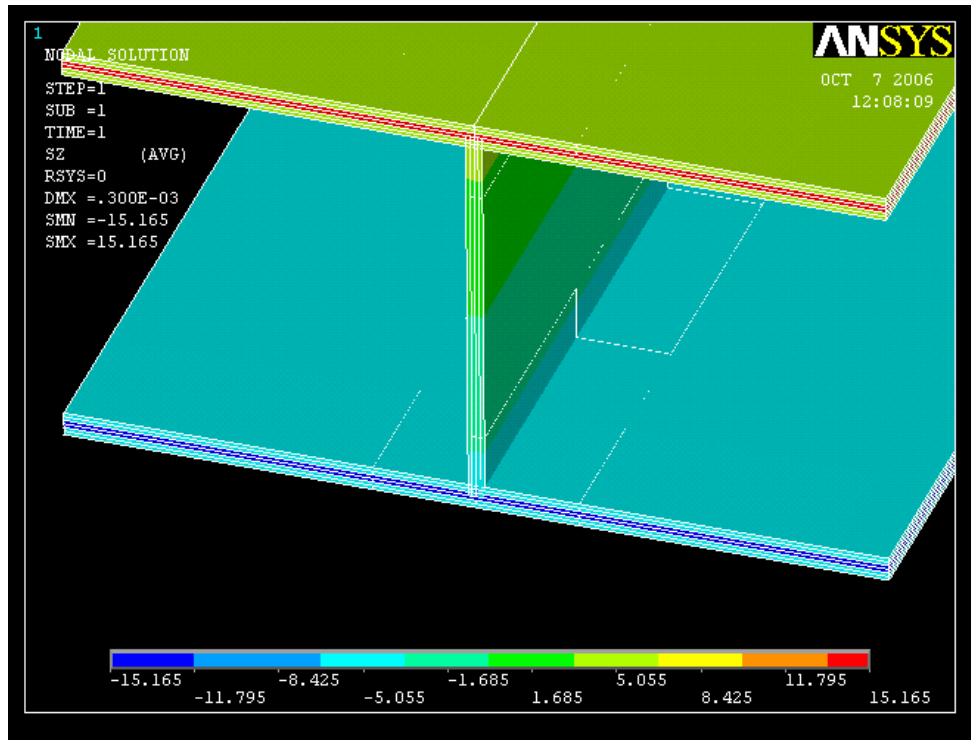


Figure 4.6 Stresses σ_x in the I-Beam

4.4 Square Tubular Beam

A beam with squared cross-section is studied for the bending stiffness as well as ply stresses. The cross-section is divided into 4 segments of laminate which include 2 top and bottom laminates and a vertical laminate at each side of the tube. The square tube is fully constrained at one end and is connected by rigid bar elements to the other end, where a moment of 1 lb-in is applied on the rigid surface. The stacking sequence used in the analysis is shown below:

Top Flange:[45/-45/0]s

Left Web:[-45/45]s

Right Web:[-45/45]s

Bottom Flange:[-45/45/0]s

4.4.1 MATLAB Results

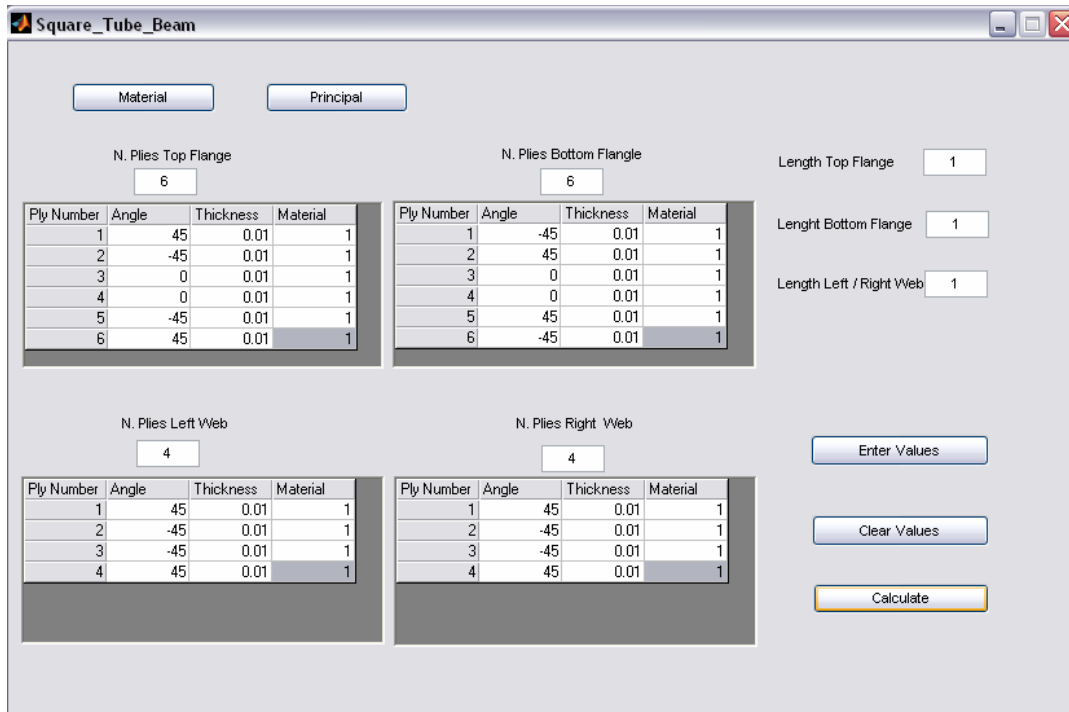


Figure 4.7 Screen with data for the square tube analysis

ABD matrix at the neutral axis (as show the result report of the application):

$$\begin{bmatrix} 5.4647e+005 & 2.4224e+005 & 0 & 2.3417e-014 & 3.5579e-014 & -1.9850e-012 \\ 2.4224e+005 & 3.6498e+005 & 0 & 3.5579e-014 & 5.3604e-014 & 5.6843e-014 \\ 0 & 0 & 2.6466e+005 & -1.9850e-012 & -1.8713e-012 & 3.8870e-014 \\ 2.3417e-014 & 3.5579e-014 & -1.9850e-012 & 1.3070e+005 & 4.5198e+004 & 1.1598e+001 \\ 3.5579e-014 & 5.3604e-014 & 5.6843e-014 & 4.5198e+004 & 7.9712e+004 & 1.1598e+001 \\ -1.9850e-012 & -1.8713e-012 & 3.8870e-014 & 1.1598e+001 & 1.1598e+001 & 5.1500e+004 \end{bmatrix}$$

This gives the bending stiffness of the squared beam is $1.051 \text{ e}+05 \text{ lb-in}^2$.

4.4.2 Finite Element Results

The finite element model of the squared tube beam contains 720 elements.

Figure 4.8 shows the deformed and undeformed shape of the beam. As shown in the cross-section, the beam does not shown any visible twisting deformation. The displacements at Nodes 352, 379 and 406 listed below were extracted for calculating the

curvature of the deflected beam. This gives curvature of the beam is $9.7088e-06$ 1/inch.

With a moment, $M_x=1$ lb-in, we obtain the bending stiffness of $103.0E+3$ lb-in². This gives about 2% difference comparing to the MATLAB result.

Node 352:

$X1=1+ 0.46851E-05$ in, $Y1=1-0.57240E-05$ in

Node 379:

$X2=10+ 0.48417E-04$ in, $Y2=1-0.48400E-03$ in

Node 406:

$X3=19+ 0.92148E-04$ in, $Y3= 1-0.17487E-02$ in

Curvature = $9.7088e-006$

$$EI_{FEM} = \text{Moment} / \text{curvature} = 1 / 9.7088e-006 = 103.0E3 \text{ lb-in}^2$$

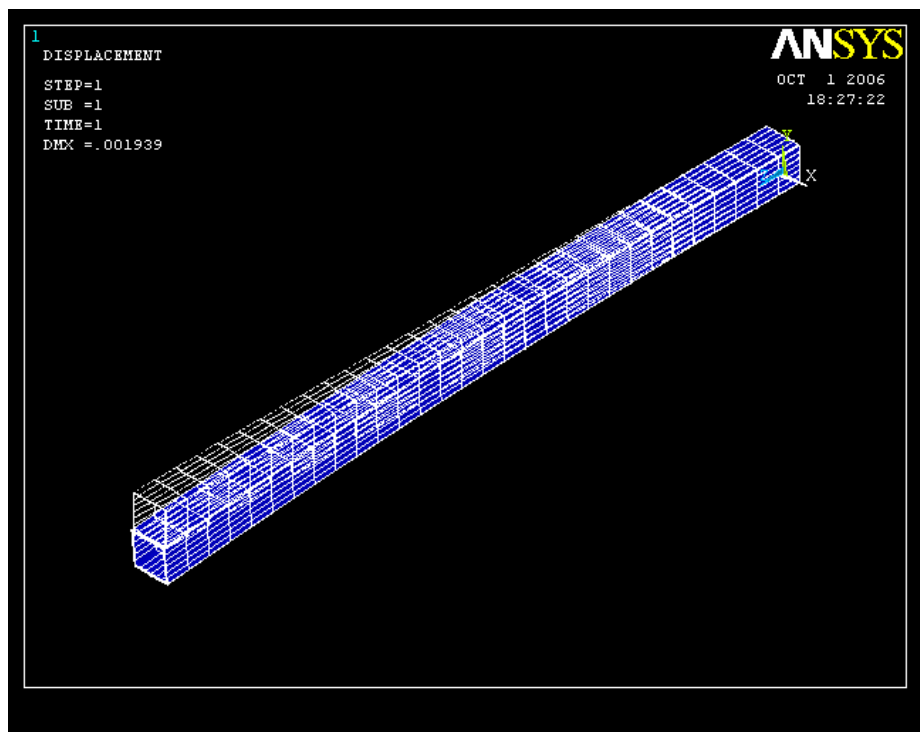


Figure 4.8 Undeformed and deformed shape of the square tube

4.4.3 Comparison of Ply Stress σ_x

Figure 4.9 shows stress distribution across the cross-section of the beam. The stress, σ_x of the upper layer of 0^0 in the top flange is used for comparison. The results of σ_x are 27.86 psi from the finite element method and 26.74 psi from the MATLAB method. The difference between these two is 4.4 %.

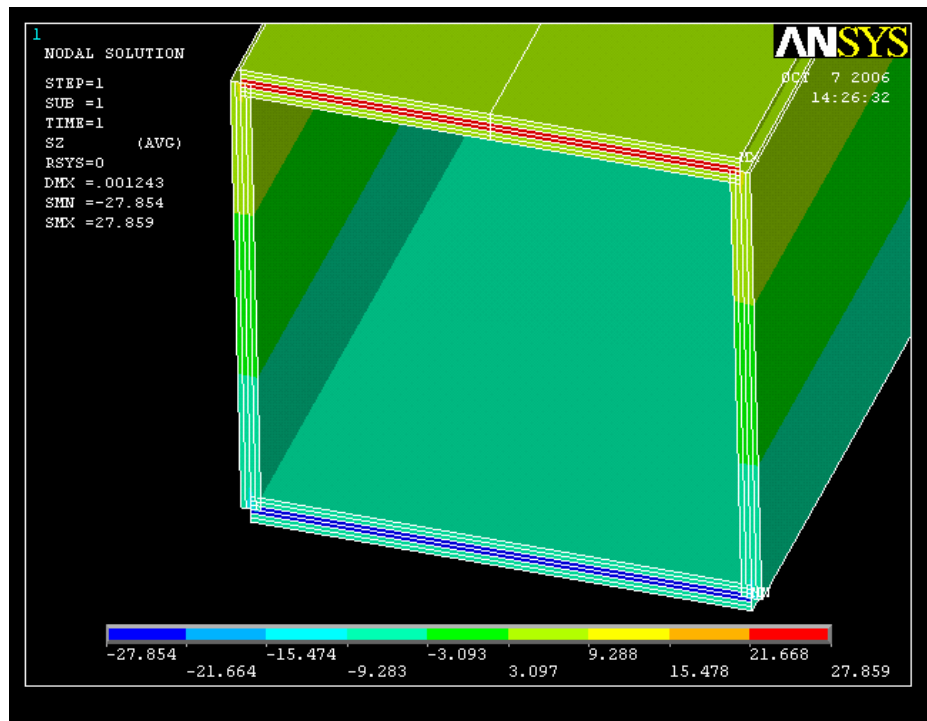


Figure 4.9 Stresses σ_x in the square tube

4.5 Truss Beam

A truss beam is divided into seven segments of laminates. These include two on the top flange ,three on the bottom flange and two of the web laminates, The finite element model of the truss beam contained 1872 elements. The model is fully constrained at one end and has rigid bar elements connected to the cross-section at the

other end of the beam where. a moment of 100 lb-in is applied. The stacking sequence of each laminate is listed below:

Top Flange:[45/-45/0]s

Left Web:[-45/45]s

Right Web:[-45/45]s

Bottom Flange:[-45/45/0]s

4.5.1 MATLAB Results

Figure 4.9 shows the input on screen of the MATLAB code. The $[abb^T d]$ matrices are given below. The bending stiffness of the truss beam is obtained as $3.761E+5 \text{ lb-in}^2$.

The screenshot shows a MATLAB GUI titled "Triangular_Beam" with the following data:

Material (selected) | **Principal**

N. Plies T. L. Flange: 6

Ply Number	Angle	Thicness	Material
1	45	0.01	1
2	-45	0.01	1
3	0	0.01	1
4	0	0.01	1
5	-45	0.01	1
6	45	0.01	1

N. Plies T. R. Flange: 6

Ply Number	Angle	Thicness	Material
1	45	0.01	1
2	-45	0.01	1
3	0	0.01	1
4	0	0.01	1
5	-45	0.01	1
6	45	0.01	1

N. Plies LEFT WEB: 4

Ply Number	Angle	Thicness	Material
1	-45	0.01	1
2	45	0.01	1
3	45	0.01	1
4	-45	0.01	1

N. Plies Right Web: 4

Ply Number	Angle	Thicness	Material
1	-45	0.01	1
2	45	0.01	1
3	45	0.01	1
4	-45	0.01	1

N. Plies B.L.Flange: 6

Ply Number	Angle	Thicness	Material
1	-45	0.01	1
2	45	0.01	1
3	0	0.01	1
4	0	0.01	1
5	45	0.01	1
6	-45	0.01	1

N. Plies B.M.Flange: 6

Ply Number	Angle	Thicness	Material
1	-45	0.01	1
2	45	0.01	1
3	0	0.01	1
4	0	0.01	1
5	45	0.01	1
6	-45	0.01	1

N. Plies B.R. Flange: 6

Ply Number	Angle	Thicness	Material
1	-45	0.01	1
2	45	0.01	1
3	0	0.01	1
4	0	0.01	1
5	45	0.01	1
6	-45	0.01	1

Dimensions and Angles:

- Lenght T. R. Flange: 2
- Lenght T. L. Flange: 2
- Lenght LR Web: 1.414
- Web Angle: 45
- Lenght B.L.Flange: 1
- Lenght B.M.Flange: 2
- Lenght B.R.Flange: 1

Buttons: Enter Values, Clear Values, Calculate

Figure 4.10 Screen with data for the truss beam analysis

ABD Matrix at the Neutral Axis (As show the result report of the Application):

$$\begin{bmatrix} 1.9354e+006 & 6.5742e+005 & 0 & -2.5911e-012 & 2.8124e-012 & -7.1945e-012 \\ 6.5742e+005 & 1.2094e+006 & 0 & -4.4635e-012 & 5.7572e-012 & -8.0377e-014 \\ 0 & 0 & 7.5122e+005 & -7.1945e-012 & -7.3552e-012 & -1.0237e-012 \\ -2.5911e-012 & 2.8124e-012 & -7.1945e-012 & 4.4855e+005 & 1.3914e+005 & -1.5507e+001 \\ -4.4635e-012 & 5.7572e-012 & -8.0377e-014 & 1.3914e+005 & 2.6708e+005 & -1.5507e+001 \\ -7.1945e-012 & -7.3552e-012 & -1.0237e-012 & -1.5507e+001 & -1.5507e+001 & 1.6192e+005 \end{bmatrix}$$

4.5.2 Finite Element Result

Figure 4.11 shows the deformed and undeformed shapes of the truss beam.

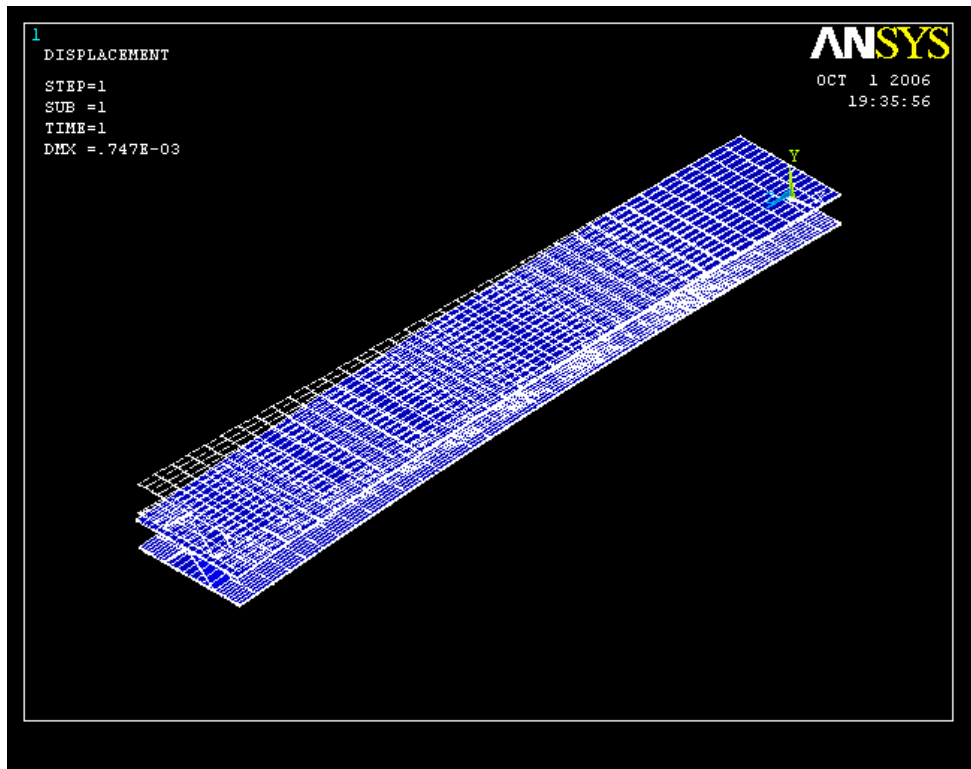


Figure 4.11 Undeformed and deformed shape of the truss beam

The displacements of Nodes 20, 38 and 57 listed below were selected for calculating the curvature of the deflected beam. The curvature obtained is 2.616 e-06.

The bending stiffness of the beam is then obtained by using applying moment dividing

by the curvature. The calculated value of the bending stiffness of the finite element model is $3.822 \text{ e}+5 \text{ lb-in}^2$. The difference in the bending stiffness between these two methods is 1.6%.

Node 20:
 $X1=1.2+0.13698\text{E-}05 \text{ in}$, $Y1=1-0.18237\text{E-}05 \text{ in}$

Node 38:
 $X2=12+0.15543\text{E-}04 \text{ in}$, $Y2=1-0.18501\text{E-}03 \text{ in}$

Node 57:
 $X3=23.4+0.30436\text{E-}04 \text{ in}$, $Y3= 1-0.70945\text{E-}03 \text{ in}$

4.5.3 Comparison of Ply Stresses σ_x

Figure 4.12 shows the stress distribution of σ_x .

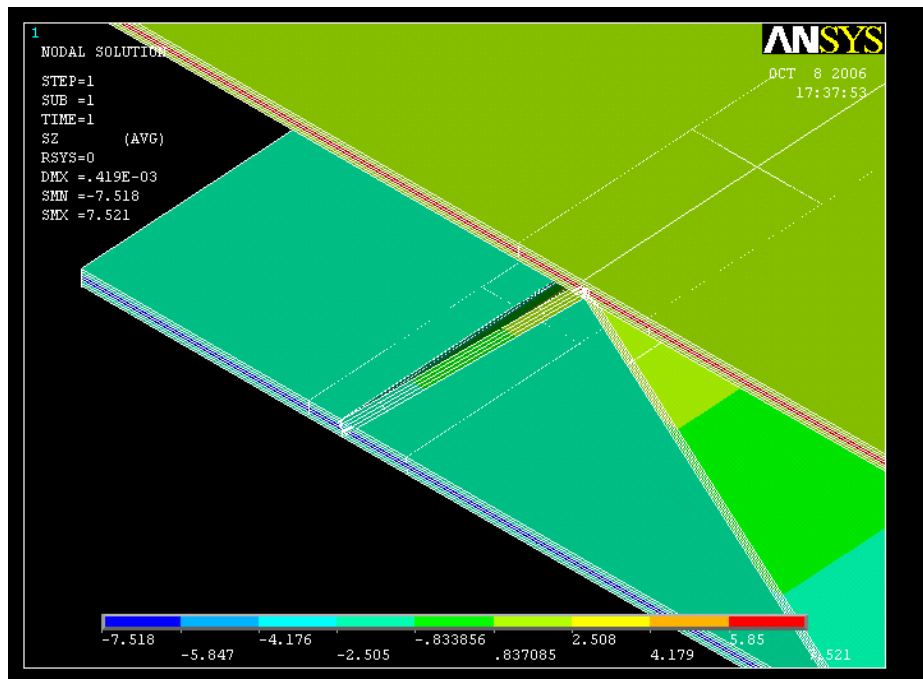


Figure 4.12 Stresses σ_x in the truss beam

As indicated, the higher stress occurs at the upper layer of 0^0 ply in the top flange. The stress at this ply is used for comparison. The MATLAB result gives $\sigma_x=7.465$ psi and the finite element result gives $\sigma_x=7.521$ psi. A comparison between these two methods gives a difference of 0.74%.

CHAPTER 5
PARAMETRIC STUDY OF COMPOSITE BEAMS

This chapter used the previous developed MAT code to conduct the study the following topics:

- 1) the interlaminar shear due to stacking sequence change for a composite beam under a transverse load
- 2) the effect of the bending stiffness due to unsymmetric lay-up of the web of an I-beam
- 3) the effect of bending stiffness due to variation of the web angles in truss beam.

5.1 *Effect of Stacking Sequence of a Laminate Subjected to Transverse Load*

A composite beam with $[+45/-45/0/90]_s$ lay-up is used to study the effect of interlaminar shear due to change of stacking sequence. The beam is subjected to a transverse load, $V=10$ lb/in. The material properties are tabulated in Table 3.

Table 3 Material Properties for Interlaminar Shear Stress Analysis

E_1	5.7E6 MSI
E_2	1.24E6 MSI
G_{12}	0.54E6 MSI
ν_{12}	0.28
Ply Thickness	0.01 inches

5.1.1 *Effect of Interlaminar Shear on Symmetric Stacking Sequence Laminates*

Using the MATLAB code developed in the previous chapter, we obtain the interlaminar shear at each interface. Three different stacking sequences varying the

positions of the 0^0 ply in the laminate were studied. The results are listed in Table 5.2 and plotted in Figure 5.1.

Table 4 Interlaminar Shear τ_{xz} for Laminates with Symmetric Stacking Sequence

Interface location	Stresses τ_{xz} (psi) for different stacking sequences		
	[45/-45/0/90]s	[90/0/-45/45]s	[0/90/45/-45]s
0	0	0	0
1	5.11E+01	1.78E+01	1.29E+02
2	9.79E+01	1.72E+02	1.37E+02
3	2.26E+02	1.97E+02	1.51E+02
4	2.29E+02	2.08E+02	1.57E+02
5	2.26E+02	1.97E+02	1.51E+02
6	9.79E+01	1.72E+02	1.37E+02
7	5.11E+01	1.78E+01	1.29E+02
8	0	0	0

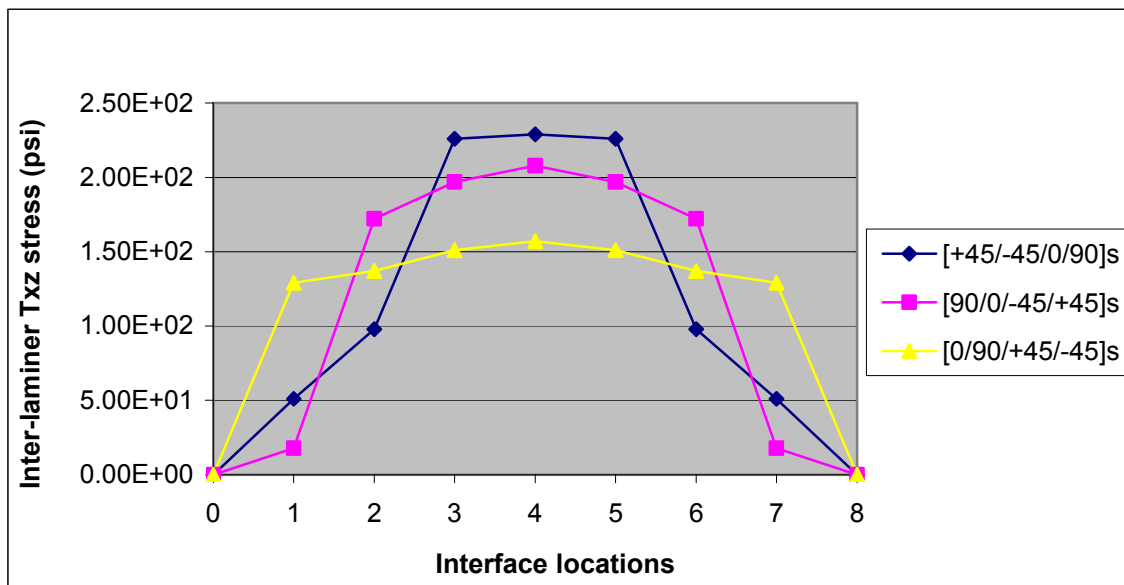


Figure 5.1 Interlaminar shear stresses τ_{xz} for laminates with symmetric stacking sequence

As shown in the figure, the interlaminar shear is symmetric with respect to the mid-plane interface, which is the 4th interface. The maximum interlaminar shear occurs at the mid-plane for all of the three laminates. It is also shown that the laminate with the 0⁰ ply placing near the mid-plane has the largest interlaminar shear among these three laminates.

5.1.2 Effect of Interlaminar Shear on Symmetric vs. Unsymmetric Stacking Sequence Laminates

[+45/-45/0/90]_s, [+45/0/-45/90]_s, [+45/-45/0/90/+45/-45/0/90]_T, were used for study effect of the interlaminar shear distribution across-cross the laminate thickness. Table 5.3 lists the interlaminar shear at the various interfacial locations. The results are also plotted in Fig. 5.2. As shown in the figure, the peak of the interlaminar shear for the [+45/-45/0/90/+45/-45/0/90]_T laminate is smaller compared to the [+45/-45/0/90]_s laminate, but larger than the [+45/0/-45/90]_s. This is because the [+45/0/-45/90]_s laminate has least 0⁰ ply closer to the mid-plane of the laminate.

Table 5 Interlaminar Shear τ_{xz} for laminates with Symmetric and Unsymmetric Stacking Sequences

Interfacial location	Stresses τ_{xz} (psi) for different stacking sequences		
	[45/-45/0/90] _s	[45/0/-45/90] _s	[45/-45/0/90] _{2T}
0	0	0	0
1	5.11E+01	3.09E+01	1.79E+01
2	9.79E+01	1.70E+02	1.75E+02
3	2.26E+02	1.94E+02	2.02E+02
4	2.29E+02	1.96E+02	2.17E+02
5	2.26E+02	1.94E+02	2.14E+02
6	9.79E+01	1.70E+02	1.02E+02
7	5.11E+01	3.09E+01	5.42E+01
8	0	0	0

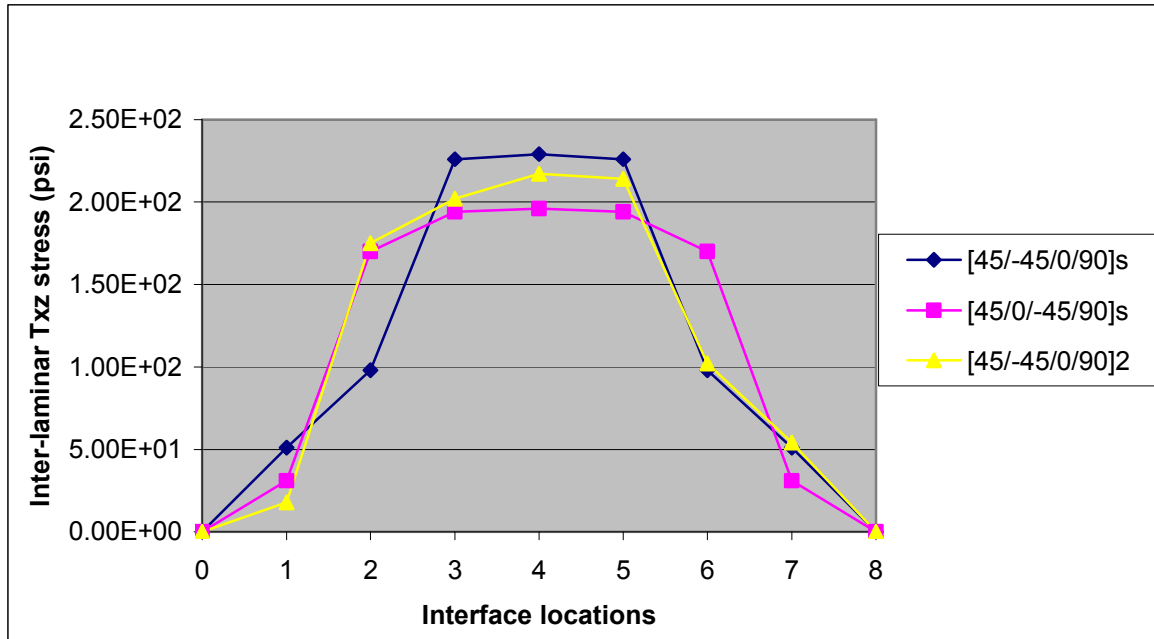


Figure 5.2 Interlaminar stresses τ_{xz} for symmetric and unsymmetric stacking sequence laminates

5.2 Effect of Stacking Sequence in Laminates of I-Beam

Two I- Beams with symmetric and unsymmetric web laminates were studied for the bending stiffness. The I-beam with unsymmetrical web gives unidentical flanges of the top and bottom laminates due to manufacturing process. However, in the engineering practice, the laminates with symmetrical web and identical flanges are often assumed. Hence, this study is to investigate if there is any difference of the bending stiffness between the actual manufacturing stacking sequence and assumed stacking sequence. These two cases are described below:

Case 1:

Symmetric unidentical flange laminates and unsymmetric web laminate

Top Left/Right Flange : [45/-45/0]s

Web : [-45/45]_{2T}

Bottom Left/Right Flange: : [-45/45/0]_s

Case 2:

Identical Flange laminates and symmetric web laminate

Top Left/Right Flange : [45/-45/0]_s

Web : [45/-45/-45/45]_s

Bottom Left/Right Flange: : [45/-45/0]_s

It is found that the bending stiffness for both beams is $1.845E+5$ lb/in². There is not difference in bending stiffness since the bending stiffness of I-beam is contributed most from the flange laminates.

5.3 Effect of Bending Stiffness due to Web Angle in Truss Beams

The geometry of the truss beam with lay-ups of each member is shown in Fig. 5.3. In this study, the beam has 1 inch thick and 2 inches long for length of the top and bottom of the flanges. The angles of the web considered in the study are ranging from 30^0 to 90^0 at an interval of 5^0 . The bending stiffness for the beam with each different web angle is tabulated in Table 6 and is also plotted in Fig. 5.4. The results indicated that the bending stiffness increases as the web angle decreases. The results also reveal that when the web angle is greater than 60^0 , the decrease of the bending stiffness becomes small.

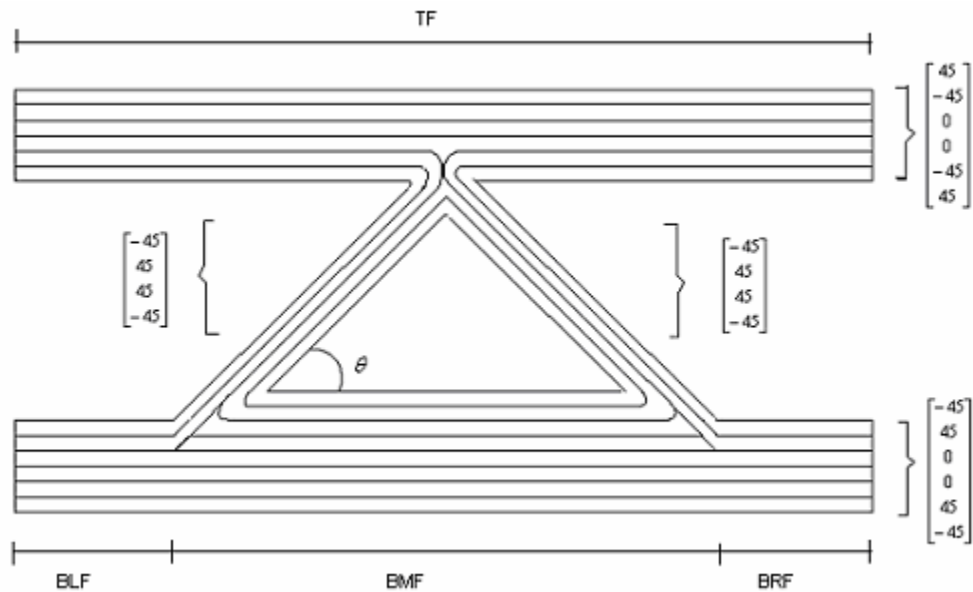


Figure 5.3 Stacking sequence of the truss beam for parametric study

Table 6 Effect of Web Angle in Bending Rigidity for Truss Beams

θ	Length Left/Right laminates webs	BMF (Bottom Middle Flange)	BLF/BRF (Bottom left and right flanges)	Bending Rigidity (lb-in ²)
90	1.00E+00	1.23E-16	2.00E+00	3.69E+05
85	1.00E+00	1.75E-01	1.91E+00	3.69E+05
80	1.02E+00	3.53E-01	1.82E+00	3.69E+05
75	1.04E+00	5.36E-01	1.73E+00	3.70E+05
70	1.06E+00	7.28E-01	1.64E+00	3.70E+05
65	1.10E+00	9.33E-01	1.53E+00	3.71E+05
60	1.15E+00	1.15E+00	1.42E+00	3.72E+05
55	1.22E+00	1.40E+00	1.30E+00	3.73E+05
50	1.31E+00	1.68E+00	1.16E+00	3.74E+05
45	1.41E+00	2.00E+00	1.00E+00	3.76E+05
40	1.56E+00	2.38E+00	8.08E-01	3.78E+05
35	1.74E+00	2.86E+00	5.72E-01	3.81E+05
30	2.00E+00	3.46E+00	2.68E-01	3.85E+05

* The value for TF (Top flange) is a constant value of 2 inches and the height of the beam (distance from the middle of Top Flange to the middle of bottom Flange) is 1 inch constant.

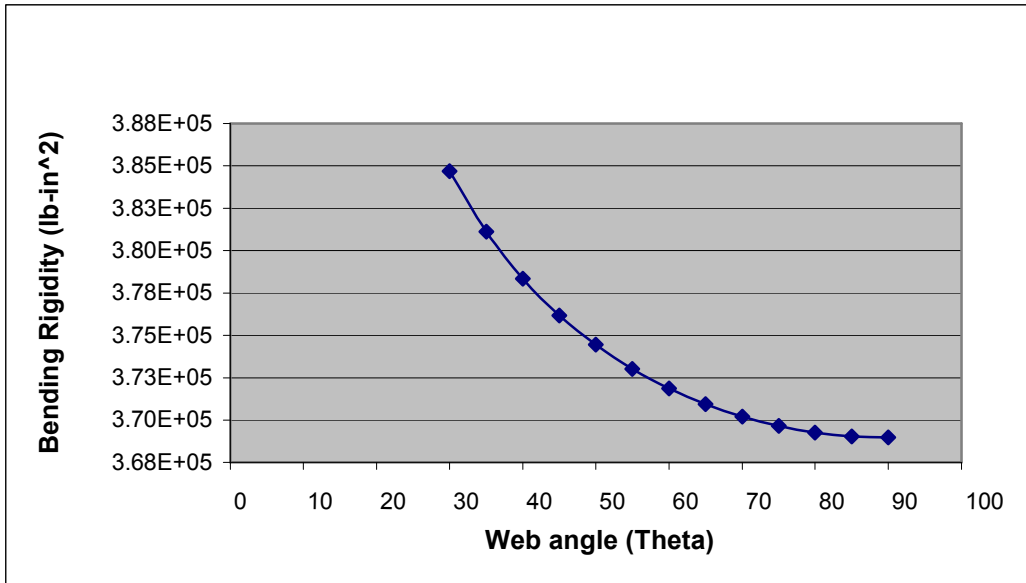


Figure 5.4 Web angle vs. bending rigidity for truss beams

CHAPTER 6

CONCLUSIONS

A user friendly MATLAB program interfaced with GUI command was developed for fiber reinforced composite structural analysis. The program was implemented the previous developed closed-form expressions for evaluating axial and bending stiffness for composite beams analysis. The composite beams studied in this thesis are the beams with solid rectangular cross-section, rectangular tubular section and I-shape cross-sections, respectively. The analytical model for the transverse shear analysis for a composite beam with solid rectangular cross-section subjected to a transverse load is also included in the program. The developed MATLAB code also contains ply stress and stiffness analysis of laminate under mechanical and hygrothermal induced loads. The program also incorporated with ply progressive failure analysis. Demonstration examples of the program are also included in this thesis.

A closed-form expression for bending stiffness of a composite truss beam was developed. In the present method of a composite truss beam analysis, the equivalent stiffness includes the coupling effects among tension, bending and twisting due to unsymmetrical and unbalanced of the cross-section which are ignored in the conventional analysis. A finite element model using ANSYS commercial code was also constructed to compare to the results obtain from the present model. The results indicate less than 3

percentages difference in ply stress and bending stiffness between the present and finite element methods.

A parametric study on transverse shear analysis was performed. A graphite/epoxy laminate with $[\pm 45/0/90]_s$ lay-up was used to investigate the stacking sequence effect on transverse shear. It is found that the laminate with 0° degree placed away from the mid-plane exhibits a smaller transverse shear comparing with the laminate with 0° degree placed closer to the mid-plane of the laminate.

A study was also conducted to investigate the effect of bending stiffness on symmetrical versus unsymmetrical flanges and web laminates in I-beam analysis. It is concluded that effect of bending stiffness due to symmetry of flange and web laminates is insignificant since the bending stiffness of I-beam is dominant by the depth of the I-beam.

Parametric study on the effect of bending stiffness of the truss angle on a truss beam was also investigated. It is found that the bending stiffness increases as the truss angle decreases providing the fixed value of the flange dimensions and the depth of truss beam.

APPENDIX A

STRESS AND STRAIN TRANSFORMATION MATRICES

A.1 3-Dimensional Stress Transformation Matrices about x axis rotation

$$[T_{\sigma}(\theta)]_x^{3D} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & c_x^2 & s_x^2 & 2s_x c_x & 0 & 0 \\ 0 & s_x^2 & c_x^2 & -2s_x c_x & 0 & 0 \\ 0 & -s_x c_x & s_x c_x & c_x^2 - s_x^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & c_x & -s_x \\ 0 & 0 & 0 & 0 & s_x & c_x \end{bmatrix}$$

where $c_x = \cos \theta$ and $s_x = \sin \theta$.

A.2 2-Dimensional Stress Transformation Matrices about x axis rotation

$$[T_{\sigma}(\theta)]_x^{2D} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_x^2 & 0 \\ 0 & 0 & c_x \end{bmatrix}$$

A.3 3-Dimensional Stress Transformation Matrices about z axis rotation

$$[T_{\sigma}(\beta)]_z^{3D} = \begin{bmatrix} c_z^2 & s_z^2 & 0 & 0 & 0 & 2c_z s_z \\ s_z^2 & c_z^2 & 0 & 0 & 0 & -2c_z s_z \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & c_z & -s_z & 0 \\ 0 & 0 & 0 & s_z & c_z & 0 \\ -c_z s_z & c_z s_z & 0 & 0 & 0 & c_z^2 - s_z^2 \end{bmatrix}$$

A.4 2-Dimensional Stress Transformation Matrices about x axis rotation

$$[T_{\sigma}(\theta)]_z^{2D} = \begin{bmatrix} c_z^2 & s_z^2 & 2c_z s_z \\ s_z^2 & c_z^2 & -2c_z s_z \\ -c_z s_z & c_z s_z & c_z^2 - s_z^2 \end{bmatrix}$$

A.5 Strain Transformation Matrices

$$[T_{\varepsilon}(\theta)] = [T_{\sigma}(-\theta)]^T$$

APPENDIX B

MATLAB CODE TO FIND CURVATURE OF BEAMS USING FINITE ELEMENT RESULTS


```

%% The value for x and y change according the local coordinate system
%% For example for this case y remains nodal y displacements
%% but x changes for nodal z displacements in the DOF solution

```

```

%% First Node (at 1 inch of the fixed displacement side of the beam)

```

```

x1 =(ENTER VALUE HERE) ;

```

```

y1 =(ENTER VALUE HERE);

```

```

%% Second Node (at the middle of the beam)

```

```

x2 =(ENTER VALUE HERE);

```

```

y2 =(ENTER VALUE HERE);

```

```

%% Third Node (at 1 inch of the rigid constraint of the beam)

```

```

x3 =(ENTER VALUE HERE);

```

```

y3 =(ENTER VALUE HERE);

```

```

a1 = (x1 + x2)/2;

```

```

b1 = (y1 + y2)/2;

```

```

Sab = (y2 - y1)/(x2 - x1);

```

```

SL1 = -1/Sab;

```

```

a2 = (x2 + x3)/2;

```

```

b2 = (y2 + y3)/2;

```

```

Sbc = (y3 - y2)/(x3 - x2);

```

```

SL2 = -1/Sbc;

```

```

x0 = ((SL1*a1) - (SL2*a2) - b1 + b2)/(SL1 - SL2);

```

```

y0 = ((SL1*SL2*(a1 - a2)) - SL2*b1 + SL1*b2)/(SL1 - SL2);

```

```

R1 = sqrt((x0-x1)^2 + (y0 - y1)^2);

```

```

R2 = sqrt((x0-x2)^2 + (y0 - y2)^2);

```

```

R3 = sqrt((x0-x3)^2 + (y0 - y3)^2);

```

```

%% Curvature

```

```

K1 = 1/R1

```

```

K2 = 1/R2

```

```

K3 = 1/R3

```

APPENDIX C

ANSYS CODE FOR BEAM ANALYSIS

C.1 Simple Beam

```
/PREP7
```

```
! DEFINE ELEMENT  
ET,1,SHELL91,,1
```

```
! DEFINE ELEMENT OPTIONS  
KEYOPT,1,5,1  
KEYOPT,1,6,1  
KEYOPT,1,8,1
```

```
! DEFINE REAL CONSTANTS  
R,1  
RMODIF,1,1,6,1, , , 0  
RMODIF,1,13,1,45,0.01,0.01,0.01,0.01,  
RMODIF,1,19,1,-45,0.01,0.01,0.01,0.01,  
RMODIF,1,25,1,0,0.01,0.01,0.01,0.01,
```

```
! DEFINE MATERIAL CONSTANT  
UIMP,1,EX,EY,EZ,5.7E6 ,1.24E6 ,1.24E6  
UIMP,1,PRXY,,,0.28 ,,,  
UIMP,1,GXY,,,0.54E6,,,
```

```
! CREATE KEYPOINTS  
K,1,0,0,0,  
K,2,1,0,0,  
K,3,0,0,20,  
K,4,1,0,20,
```

```
! CREATE AREAS  
A,1,2,4,3
```

```
! DEFINE LOCAL COORDINATE SYSTEMS  
LOCAL,11,0,0,0,0,90,0,0,1,1,
```

```
! DEFINE MESH ATTRIBUTES FOR AREAS  
ASEL, , , 1  
AATT,1,1,1,11,
```

```

! DEFINE ELEMENT SIZE ON LINES
LSEL,,,2,4,2
LESIZE,ALL,,,50,,1

LSEL,,,1,3,2
LESIZE,ALL,,,8,,1

! SELECT ALL
ALLSEL

! MESH AREAS
AMESH,ALL

! DEFINE GLOBAL COORDINATE SYSTEM
CSYS,0

! APPLY STRUCTURAL DISPLACEMENTS ON ONE SIDE OF THE BEAM
NSEL,,loc,z,0
D,ALL,ALL

! APPLY RIGID CONSTRAINT AND MOMENT ON THE OTHER SIDE
NSEL,,loc,z,20
CERIG,126,ALL,all,
F,126,MX,1

! SELECT ALL ELEMENTS OF THE BEAM
ALLSEL

! DISPLAY ELEMENT BASED ON REAL CONSTANT DESCRIPTIONS
/ESHAPE,1.0
EPLOT

! APPLY ISOMETRIC VIEW
/VIEW,1,1,1,1
/REPLOT

! SOLVE
/SOL
SOLVE

! PLOT DEFORM + UNDEFORM SHAPES
/POST1
PLDISP,1

```

/REPLOT

! FINISH CALCULATIONS
FINISH

C.2 *I beam*

/PREP7

! DEFINE ELEMENT
ET,1,SHELL91,,1

! DEFINE ELEMENT OPTIONS
KEYOPT,1,5,1
KEYOPT,1,6,1
KEYOPT,1,8,1

! DEFINE REAL CONSTANTS
R,1
RMODIF,1,1,6,1, , , 0
RMODIF,1,13,1,45,0.01,0.01,0.01,0.01,
RMODIF,1,19,1,-45,0.01,0.01,0.01,0.01,
RMODIF,1,25,1,0,0.01,0.01,0.01,0.01,
R,2
RMODIF,2,1,4,0, , , 0
RMODIF,2,13,1,-45,0.01,0.01,0.01,0.01,
RMODIF,2,19,0,45,0.01,0.01,0.01,0.01,
RMODIF,2,25,1,-45,0.01,0.01,0.01,0.01,
RMODIF,2,31,1,45,0.01,0.01,0.01,0.01,
R,3
RMODIF,3,1,6,1, , , 0
RMODIF,3,13,1,-45,0.01,0.01,0.01,0.01,
RMODIF,3,19,1,45,0.01,0.01,0.01,0.01,
RMODIF,3,25,1,0,0.01,0.01,0.01,0.01,

! DEFINE MATERIAL CONSTANT
UIMP,1,EX,EY,EZ,5.7E6 ,1.24E6 ,1.24E6
UIMP,1,PRXY,,,0.28 ,,,
UIMP,1,GXY,,,0.54E6,,,

! CREATE KEYPOINTS
K,1,-1,1,0,

```
K,2,0,1,0,  
K,3,1,1,0,  
K,4,-1,0,0,  
K,5,0,0,0,  
K,6,1,0,0,  
K,7,-1,1,20,  
K,8,0,1,20,  
K,9,1,1,20,  
K,10,-1,0,20,  
K,11,0,0,20,  
K,12,1,0,20,
```

```
! CREATE AREAS
```

```
A,1,2,8,7  
A,2,3,9,8  
A,2,5,11,8  
A,4,5,11,10  
A,5,6,12,11
```

```
! DEFINE LOCAL COORDINATE SYSTEMS
```

```
LOCAL,11,0,0,0,0,90,0,90,1,1,  
LOCAL,12,0,0,0,0,0,0,-90,1,1,
```

```
! DEFINE MESH ATTRIBUTES FOR AREAS
```

```
ASEL, , , 1  
AATT,1,1,1,11,
```

```
ASEL, , , 2  
AATT,1,1,1,11,
```

```
ASEL, , , 3  
AATT,1,2,1,12,
```

```
ASEL, , , 4  
AATT,1,3,1,11,
```

```
ASEL, , , 5  
AATT,1,3,1,11,
```

```
! DEFINE ELEMENT SIZE ON LINES
```

```
LSEL,,,2,6,2  
LSEL,A,,,13,15,2  
LSEL,A,,,9,9
```

```
LESIZE,ALL,,30,,1
```

```
LSEL,,,,1,7,2  
LSEL,A,,,11,12,1  
LSEL,A,,,14,16,2  
LESIZE,ALL,,4,,1
```

```
LSEL,,,,8,10,2  
LESIZE,ALL,,6,,1
```

```
! SELECT ALL  
ALLSEL
```

```
! MESH AREAS  
AMESH,ALL
```

```
! DEFINE GLOBAL COORDINATE SYSTEM  
CSYS,0
```

```
! APPLY STRUCTURAL DISPLACEMENTS ON ONE SIDE OF THE BEAM  
NSEL,,loc,z,0  
D,ALL,ALL
```

```
! APPLY RIGID CONSTRAINT AND MOMENT ON THE OTHER SIDE  
NSEL,,loc,z,20  
CERIG,875,ALL,all,  
F,875,MX,1
```

```
! SELECT ALL ELEMENTS OF THE BEAM  
ALLSEL
```

```
! DISPLAY ELEMENT BASED ON REAL CONSTANT DESCRIPTIONS  
/ESHAPE,1.0  
EPLOT
```

```
! APPLY ISOMETRIC VIEW  
/VIEW,1,1,1,1  
/REPLOT
```

```
! SOLVE  
/SOL  
SOLVE
```

```
! PLOT DEFORM + UNDEFORM SHAPES
```

```
/POST1
PLDISP,1
/REPLOT
```

```
! FINISH CALCULATIONS
FINISH
```

C.3 Square Tube

```
/PREP7
```

```
! DEFINE ELEMENT
ET,1,SHELL91,,1
```

```
KEYOPT,1,5,1
KEYOPT,1,6,1
KEYOPT,1,8,1
```

```
! DEFINE REAL CONSTANTS
```

```
R,1
RMODIF,1,1,6,1, , , ,0
RMODIF,1,13,1,45,0.01,0.01,0.01,0.01,
RMODIF,1,19,1,-45,0.01,0.01,0.01,0.01,
RMODIF,1,25,1,0,0.01,0.01,0.01,0.01,
R,2
RMODIF,2,1,4,1, , , ,0
RMODIF,2,13,1,45,0.01,0.01,0.01,0.01,
RMODIF,2,19,1,-45,0.01,0.01,0.01,0.01,
R,3
RMODIF,3,1,6,1, , , ,0
RMODIF,3,13,1,-45,0.01,0.01,0.01,0.01,
RMODIF,3,19,1,45,0.01,0.01,0.01,0.01,
RMODIF,3,25,1,0,0.01,0.01,0.01,0.01,
```

```
! DEFINE MATERIAL CONSTANT
```

```
UIMP,1,EX,EY,EZ,5.7E6 ,1.24E6 ,1.24E6
UIMP,1,PRXY,,,0.28,,,
UIMP,1,GXY,,,0.54E6,,,
```

```
! CREATE KEYPOINTS
```

```
K,1,-0.5,1,0,
K,2,0.5,1,0,
```



```
K,3,-0.5,0,0,  
K,4,0.5,0,0,  
K,5,-0.5,1,20,  
K,6,0.5,1,20,  
K,7,-0.5,0,20,  
K,8,0.5,0,20,
```

```
! CREATE AREAS
```

```
A,1,2,6,5  
A,1,3,7,5  
A,2,4,8,6  
A,3,4,8,7
```

```
! DEFINE LOCAL COORDINATE SYSTEMS
```

```
LOCAL,11,0,0,0,0,90,0,90,1,1,  
LOCAL,12,0,0,0,0,0,0,-90,1,1,  
LOCAL,13,0,0,0,0,0,0,90,1,1,
```

```
! DEFINE MESH ATTRIBUTES FOR AREAS
```

```
ASEL, , , 1  
AATT,1,1,1,11,
```

```
ASEL, , , 2  
AATT,1,2,1,12,
```

```
ASEL, , , 3  
AATT,1,2,1,13,
```

```
ASEL, , , 4  
AATT,1,3,1,11,
```

```
! DEFINE ELEMENT SIZE ON LINES
```

```
LSEL,,,2,6,2  
LSEL,A,,,9,9  
LESIZE,ALL,,,30,,1
```

```
LSEL,,,1,7,2  
LSEL,A,,,8,12,2  
LSEL,A,,,11,11  
LESIZE,ALL,,,6,,1
```

```
! SELECT ALL
```

ALLSEL

! MESH AREAS
AMESH,ALL

! DEFINE GLOBAL COORDINATE SYSTEM
CSYS,0

! APPLY STRUCTURAL DISPLACEMENTS ON ONE SIDE OF THE BEAM
NSEL,,loc,z,0
D,ALL,ALL

! APPLY RIGID CONSTRAINT AND MOMENT ON THE OTHER SIDE
NSEL,,loc,z,20
CERIG,80,ALL,all,
F,80,MX,1

! SELECT ALL ELEMENTS OF THE BEAM
ALLSEL

! DISPLAY ELEMENT BASED ON REAL CONSTANT DESCRIPTIONS
/ESHAPE,1.0
EPLOT

! APPLY ISOMETRIC VIEW
/VIEW,1,1,1,1
/REPLOT

! SOLVE
/SOL
SOLVE

! PLOT DEFORM + UNDEFORM SHAPES
/POST1
PLDISP,1
/REPLOT

! FINISH CALCULATIONS
FINISH

C.4 Truss Beam

/PREP7

```

! DEFINE ELEMENT
ET,1,SHELL91,,1

! DEFINE ELEMENT OPTIONS
KEYOPT,1,5,1
KEYOPT,1,6,1
KEYOPT,1,8,1

! DEFINE REAL CONSTANTS
R,1
RMODIF,1,1,6,1, , , 0
RMODIF,1,13,1,45,0.01,0.01,0.01,0.01,
RMODIF,1,19,1,-45,0.01,0.01,0.01,0.01,
RMODIF,1,25,1,0,0.01,0.01,0.01,0.01,

R,2
RMODIF,2,1,4,1, , , 0
RMODIF,2,13,1,-45,0.01,0.01,0.01,0.01,
RMODIF,2,19,1,45,0.01,0.01,0.01,0.01,

R,3
RMODIF,3,1,6,1, , , 0
RMODIF,3,13,1,-45,0.01,0.01,0.01,0.01,
RMODIF,3,19,1,45,0.01,0.01,0.01,0.01,
RMODIF,3,25,1,0,0.01,0.01,0.01,0.01,

! DEFINE MATERIAL CONSTANT
UIMP,1,EX,EY,,5.7E6 ,1.24E6,1.24E6
UIMP,1,PRXY,,,0.28 ,,,
UIMP,1,GXY,,,0.54E6,,,

! CREATE KEYPOINTS
K,1,-2,1,0,
K,2,0,1,0,
K,3,2,1,0,
K,4,-2,0,0,
K,5,-1,0,0,
K,6,1,0,0,
K,7,2,0,0,
K,8,-2,1,24,
K,9,0,1,24,
K,10,2,1,24,

```

```
K,11,-2,0,24,  
K,12,-1,0,24,  
K,13,1,0,24,  
K,14,2,0,24,
```

```
! CREATE AREAS
```

```
A,1,2,9,8  
A,2,3,10,9  
A,2,5,12,9  
A,2,6,13,9  
A,4,5,12,11  
A,5,6,13,12  
A,6,7,14,13
```

```
! DEFINE LOCAL COORDINATE SYSTEMS
```

```
LOCAL,11,0,0,0,0,90,0,90,1,1,  
LOCAL,12,0,0,0,0,45,0,90,1,1,  
LOCAL,13,0,0,0,0,-45,0,-90,1,1,
```

```
! DEFINE MESH ATTRIBUTES FOR AREAS
```

```
ASEL, , , 1  
AATT,1,1,1,11,
```

```
ASEL, , , 2  
AATT,1,1,1,11,
```

```
ASEL, , , 3  
AATT,1,2,1,12,
```

```
ASEL, , , 4  
AATT,1,2,1,13,
```

```
ASEL, , , 5  
AATT,1,3,1,11,
```

```
ASEL, , , 6  
AATT,1,3,1,11,
```

```
ASEL, , , 7  
AATT,1,3,1,11,
```

```
! DEFINE ELEMENT SIZE ON LINES
```

```
LSEL,,,2,6,2  
LSEL,A,,,9,12,3
```

```
LSEL,A,,,16,20,4
LESIZE,ALL,,,36,,1
```

```
LSEL,,,,1,7,2
LSEL,A,,,14,15,1
LSEL,A,,,19,21,2
LESIZE,ALL,,,4,,1
```

```
LSEL,A,,,17,18,1
LESIZE,ALL,,,8,,1
```

```
LSEL,,,,8,10,2
LSEL,A,,,11,13,2
LESIZE,ALL,,,6,,1
```

```
! SELECT ALL
ALLSEL
```

```
! MESH AREAS
AMESH,ALL
```

```
! DEFINE GLOBAL COORDINATE SYSTEM
CSYS,0
```

```
! APPLY STRUCTURAL DISPLACEMENTS ON ONE SIDE OF THE BEAM
NSEL,,loc,z,0
D,ALL,ALL
```

```
! APPLY RIGID CONSTRAINT AND MOMENT ON THE OTHER SIDE
NSEL,,loc,z,24
CERIG,18,ALL,all,
F,18,MX,1
```

```
! SELECT ALL ELEMENTS OF THE BEAM
ALLSEL
```

```
! DISPLAY ELEMENT BASED ON REAL CONSTANT DESCRIPTIONS
/ESHAPE,1.0
EPLOT
```

```
! APPLY ISOMETRIC VIEW
/VIEW,1,1,1,1
/REPLOT
```

```
! SOLVE  
/SOL  
SOLVE
```

```
! PLOT DEFORM + UNDEFORM SHAPES  
/POST1  
PLDISP,1  
/REPLOT
```

```
! FINISH CALCULATIONS  
FINISH
```

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BIOGRAPHICAL INFORMATION

Cesar Augusto Rojas started his academic studies in San Marcos University of Peru, getting bachelor's degree in Mechanical Engineering, and continued his graduate studies in the University of Texas at Arlington to obtain his Master of Science degree in Mechanical Engineering . His areas of research interests are composite materials, structural design, fracture mechanics, fatigue and failure analysis, finite element analysis and engineering programming.