Measurement Issues in the Capital Asset Pricing Model & Size Effect and Duration

by

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Abstract

MEASUREMENT ISSUES IN THE CAPITAL
ASSET PRICING MODEL & SIZE
EFFECT AND DURATION

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It has been observed that the value of an asset's beta varies with the frequency of the data used to generate the value, a phenomenon hereafter referred to as "time scale", or simply "scale". If the scale effect is strong enough, then ignoring this phenomenon calls into question studies that rely on comparing beta values. The most notable of these studies is the classification of stocks as "aggressive" or "defensive". I show that such a categorization varies substantially when comparing betas estimated using monthly data versus annual data. Contrary to other studies, betas do not vary monotonically as data scale lengthens. Betas measured on a trailing forty eight month return series with a quarterly time scale

explained expected stock returns better than other month data lengths and frequencies. Between 1926 and 2009, beta was able to explain expected returns in 79.1% of rolling estimation periods.

Firm size is another important scaling factor that can impact beta measurement. I therefore study stock growth rates based on the size of the underlying firm. I show that cumulative growth factors extracted from firm size are related to Macaulay duration. Smaller firms have a higher duration, and this is shown to explain the small firm effect. Duration relates to reinvestment risk. Using a firm's duration, investors are able to generate home-made dividends when they rebalance their portfolios.

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Chapter 1

Measurement Issues in the Capital Asset Pricing Model

This dissertation delves into two related scale issues surrounding beta. First, I explore the hypothesis that beta is sensitive to the frequency of the data used in its estimation, a phenomenon I refer to as “time scale”, or simply “scale”. Any scale is arbitrary and there is no better or worse scale. The second hypothesis I explore is that firm size, a factor that may not be captured by beta, is in reality a duration effect.

1.1 Background and Contribution

The ability of beta to adequately explain the cross section of stock returns has a direct bearing on the validity of the Capital Asset Pricing Model (CAPM), one of the most popular models in modern finance theory. Beta is usually estimated by regressing asset returns on the market returns, and it is interpreted as a measure of the asset’s systematic, or market, risk. A large body of research in the financial economics literature has been devoted to the estimation of, and validity of, beta. Since an asset’s true systematic risk is unobservable, researchers are forced to take an instrumental variables approach to its estimation, such as portfolio beta, lag beta and random sampling beta. Fama-MacBeth (1973) tested the validity of the CAPM using a portfolio approach, i.e., they attempt to determine whether portfolio beta is able to adequately explain portfolio expected returns. Their results were quite comforting. Beta was able to explain portfolio expected returns, and the average risk premium was positive. Moreover, residual risk had no explanatory power in their regression model. Their work validated the notion that the single factor model was adequate to explain capital market equilibrium. Nineteen years later, Fama - French (1992) found that beta alone was unable to adequately explain the cross sectional variation of stock returns. In each size decile in their data, returns actually decrease as beta increases. The explanatory power of beta was sensitive to the sorting sequence. They found that book equity to market equity ratio (B.E/M.E) is the best predictor among the three factors, B.E/M.E, size and market risk. If high B.E/M.E stocks are spotted, investors gradually recognize the "bargain" and invest in the stock whether the portfolio is riskier or not. Their results imply that the market is full of "normal"[1] bargain hunters, not "rational" risk managers. If beta can be better replaced by the size factor or book-to-market equity ratio, then the validity of the Sharp-Lintner-Black model is suspect.

Since Fama-MacBeth (1973), more sophisticated measures have been devised to validate the Capital Asset Pricing Model. It is widely known that the test on any market efficiency model raises the joint hypothesis issue. If the test

[1] 'Normal' means ordinary or of standard intelligence in behavioral context

results reject the CAPM, it can be attributed to an inefficient market, or to misspecified model. Fama (2011) stated that

" *... Specifically, market efficiency can only be tested in the context of an asset pricing model that specifies equilibrium expected returns. In other words, to test whether prices fully reflect available information, we must specify how the market is trying to compensate investors when it sets prices. ..*"

However, I need to point out the reliability[2] issue also. The test on the CAPM must yield consistent results regardless of the time period, or varying methodologies because reliability is a prerequisite for the validity of any model. Even though the market may be efficient and beta correctly represents market risk, the CAPM can be still rejected as unreliable. Prior research has asserted that beta is valid in the CAPM model if the average risk premium[3] is non-zero. However, the focus on simple averages neglects the reliability issue. The average risk premium is computed by averaging the risk premium time series over all the rolling periods. I assert that the significance of the slope in each rolling period must be investigated to ascertain that beta really works in each period. The simple average of insignificant slopes reported in the previous research leads to statistically meaningless conclusions. Negative relations between the expected

[2] Reliability refers to the consistency of a measure. A test is said to be reliable if the test yields consistent results. The CAPM is reliable if beta can explain the expected returns repeatedly.
[3] Risk premium is equivalent to the slope on beta. If risk premium is zero, then beta cannot explain expected returns at all.

returns and beta are frequent. The presence of insignificant and negative risk premiums renders the simple average approach meaningless. Before averaging the time series of 2^{nd} pass slopes, every slope in each rolling period must be studied. Beta must be carefully examined before we can validate market efficiency. If beta predicts expected returns repeatedly, then CAPM shall hold. To perform a reliability test, I use a frequency approach called "Hitting Ratio". I define the hitting ratio as the successful prediction of expected returns of beta variable over rolling periods. If the hitting ratio changes significantly across scale and estimation period, then the CAPM is dependent on data. Contrary to prior researches, we can choose better scale which can explain expected returns better.

Many researchers have investigated the explanatory power of beta over different time periods such as during bull or bear markets. However, we do not know future market conditions and thus such a classification of bull or bear markets are ex-post indicators. I will vary the measurement period, not the time period, to see if the longer-term observation yields better predictability, i.e., higher hitting ratio. Ritter (2003) points out the observation period issue.

" ...*People under weigh long-term averages. People tend to put too much weight on recent experience. This is sometimes known as the ''law of small numbers.'' As an example, when equity returns have been high for many years* *(such as 1982–2000 in the US and Western Europe), many people begin to believe that high equity returns are ‘‘normal...’’*

Investors depend on past information, but we don’t know how far they tend to look back. What is the most representative period by which investors formulate their investment plan? With the emergence of web-based finance, many investors choose financial websites such as Yahoo! Finance or Google Finance to begin examining accounting numbers, or to process financial information. These popular websites typically use 60 months of trailing returns. The basic assumption is that the past 60 months of firm behavior vis-à-vis the market is the most representative indicator of the firm behavior in the next month. Is the 60 months of observation period the most representative period? People use representativeness heuristics when they make a decision. In the frame of prospect theory, heuristics are

“*the rules of thumb which are less than perfectly correlated with the variables that actually determine the event’s probability*”.[4]

With memory of limited capacity, people cannot go over all the long and pertinent variables. To make a reasonable decision quickly, they focus on the most representative features and ignore average features. Statman (1999)

[4] Tversky and Kahneman (1974, p. 1124)

remarked that psychology was never irrelevant to finance. Behavioral finance heavily utilizes psychology while conventional finance depends on mathematical or statistical modeling. Considering that stock prices are determined by human decisions, the ergodicity will be caused by human psychology. The validity of the rolling beta method depends on ergodicity assumption. Examining beta's predictability by altering the estimation period sheds light on the representativeness heuristics issue.

Using the rolling beta technique, I investigate the relation between varying scale beta and expected returns to see if the positive risk-reward relation still holds. The direction and magnitude of the reward is investigated. For example, the risk premium will triple if the return distribution is time-invariant and the scale triples. If the risk premium changes disproportionately as the time scale increases, investor risk tolerances need to be redefined according to their investment horizon. If stocks have different betas and risk-reward schemes by changing scales, then the custom-tailored beta will be more useful because all investors rebalance on different time scales. Further, the possibility of a new type of arbitrage transaction by using the mispricing between time scales could arise.

I investigate the frequency effect on both individual stocks and portfolios. It is common in the financial economics literature to classify a firm's stock based on the value of its estimated beta. The most common of these is the dichotomization of stocks into aggressive (i.e., beta greater than one) and defensive (i.e., beta less than one). Quite simply, if a stock classified as aggressive (defensive) when its beta is estimated from data at one frequency is classified as defensive (aggressive) using data of a different frequency, we must account for this problem when interpreting results from beta and the CAPM. On the other hand, if beta estimates are not sensitive to data frequency, we may then proceed on the assumption that stock prices are fractals. This would be convenient for empirical researchers.

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Table 1 Scale and Market

Beta / Risk Premium	No change in beta on varying scales	Significant change in beta on varying scales
Proportional Change	Market is fractal Market risk is compensated only by beta level	Market is non-fractal Market risk is compensated only by beta level
Disproportional Change	Market is fractal Market risk is compensated by beta level and scale	Market is non-fractal Market risk is compensated by beta level and scale

I investigate the scale effect on portfolio formation by examining the change in membership of beta-sorted portfolios. Many techniques such as sorting or grouping are used to reduce the errors-in-variables and so enhance the precision of predictors. However, the membership of each beta-sorted portfolio can change depending on scale. If a given beta- portfolio member composition shifts significantly, then the membership of subsequent size portfolios or B.E/M.E portfolios would be of different distribution as a result. If the turnover ratio in each portfolio is significantly high, then the traditional portfolio approach must be reconsidered. Reducing the errors-in-variables problem can cause bigger side effect such as unstable membership. If the phenomenon is ubiquitous irrespective of firm size or beta, then stock picking process will demand more cautious analysis.

One of the main benefits of the time scale approach is to neutralize the transitory spikes in returns. The residuals (e_{it}) can be attributed to firm specific factors. To handle the residual issue, researchers use many different measures. Fama-MacBeth (1973) employed a time-lag process to cancel out the abnormal activity of firms in extreme portfolios. Fama- MacBeth (1973) argue that the regression phenomenon will occur to the extreme portfolios and the average beta of the extreme portfolio tends to regress toward the true mean as time goes by. To handle the residual returns issue, they took a double-safety measure. First, they sorted stocks by their beta ranking and assigned stocks to 20 portfolios neutralizing inter-dependent residuals, the covariance (e_{it}, e_{jt}). Second, they re-estimated the betas of the same portfolio after a 3-5 year time lag. They waited 3 - 5 years after the phase of portfolio formation to normalize the abnormalities of extreme beta stocks. The magnitude of the residual returns is supposed to reduce to its true mean, zero. However, the time-lag method creates as many problems as it solves. First, some firms in the portfolios will regress toward the true mean as Fama-MacBeth argued, but other firms in the same portfolio can be more volatile in the next 3-5 years. Fama and French (1992) reported that their post-ranking βs closely reproduce the ordering of the pre-ranking βs. Their results support my statement empirically. There must be little, if any, difference between pre-ranking β order and post-ranking β order. Second, the extreme behavior is also part of the true return distributions. The extreme portfolios are made of extreme stocks. If what they argued is true, then the extreme portfolios will have less extreme stocks after the normalizing phase. The result is a statistical artifact, rather than the true estimate of return distributions. The scale approach, by contrast, absorbs the abnormal behavior in the short term such as month or quarter over the gamut of all ranked portfolios, while the time-lag approach reduces the magnitude of the abnormal behaviors only in extreme portfolios. In this analysis, I will use 3 month, 6 month, 9 month and 12 month time scales to absorb the abnormal behaviors. If extreme stocks change their characteristics to become classified as defensive, time scale approach can display better characteristics than the portfolio approach.

Another benefit of time scale approach lies in reducing the correlated residuals. When we use the two-pass approach to arrive at any conclusion on the CAPM, we assume that the residual returns (e_{it} , e_{jt}) are independent, which is controversial. The portfolio approach can cancel out the interdependency between the residual returns (e_{it}, e_{jt}). The assumption behind this is that the residual returns are negatively related to each other, and there must be trade-off among the residuals. Scale approach will eliminate the short-run correlation between stocks but keep the long-run structural correlation which can be captured by a market factor.

1.2 Literature Review

Many researchers use variance as a proxy for volatility. The variance of market returns has been widely investigated. If the market returns follow a random walk process, the standard deviation of returns will increase by $\sqrt{\mathbf{n}}$ as the measurement interval increases by n times. Lo-MacKinlay (1988) showed that the variance of the differential series increases as the measurement interval increases through 2, 4, 8 and 16 weeks. The one-step difference series and the two-step difference series must have the proportional variance if the level return series follows a random walk process. Lo-MacKinlay's (1988) results did not support the random walk hypothesis. In the long-interval, they estimated a positive autocorrelation of 30%. Poterba-Summers (1988) also used their own variance ratio test using the return level. They used different sampling intervals against yearly measurements: one month, 24 months, 36 months, 48 months, 60 months, 72 months, 84 months, and 96 months. The volatility drops significantly as the duration increases, which implies that there is a negative serial correlation in the long run. Poterba-Summers argue that the negative autocorrelation in the long-term is a proof of the mean reversion of market returns. Fama-French (1986b) also use the volatility ratio test over different investment horizons. Cochrane (2001) shows that the relative variance can be expressed in the summation of successive autocorrelation series. The variance of returns over different intervals can be summarized in an autocorrelation function.

The time scale effect on the covariance matrix and market variance can be captured by investigating beta transition along varying scales. Hawawini(1983) found conflicting results that beta estimates of small cap stocks decline and beta estimates of large cap stocks increase as the interval shortened. He concluded that small firm betas were underestimated. Handa et al (1989) tested the size effect for varying intervals. The small firm effect disappeared as scale varied. Handa et al (1993) rejected the Capital Asset Pricing Model (CAPM) on the monthly interval basis but could not reject the CAPM on the annual interval basis. Frankfurter et al (1994) investigated the distribution of beta. They showed that as the interval lengthened the distribution of beta increased and the intercept α increased. They argued that any results from event study must be reconsidered due to compounding period issues and there was no reason to consider any compounding period as “true” while other periods are “untrue”. Brailsford and Josev (1997) tested the time scale effect on the Australian market and reported similar results. The beta estimates of large (small) stocks fell (rose) as the return interval lengthened. Ramsey and Lampart (1997) perform Granger causality tests between money and income with a wavelet method. Their approach differed from the traditional short-run and long-run analysis in macroeconomics. They decomposed time series into orthogonal time scale components. They found that the direction of causality differed by time scale as well as the degree of causal relationship varied across scales. On the lowest scale, income Granger-caused money supply. However, money supply Granger-caused income on the business cycle scale. It seemed that the Federal Reserve System controlled the money supply and initiated job market response on a macro scale. Bjornson et al(1999) tested a multi factor asset pricing model by Chen, Roll and Ross. They used frequency decomposition with a Chebyshev filter. Low frequency beta better explained the expected returns and high frequency (4 months) beta was negatively priced. Small stocks, sensitive at low frequency, suffered from prolonged earnings depression that bypassed large firms. Levhari and Levy (2001) found that aggressive stocks (betas greater than one) tended to have a higher beta, and defensive stocks (betas less than one) had a lower beta, if a longer term interval was used. Gençay et al(2003) tested the CAPM using a wavelet approach. Dyadic analysis (2^j) was employed and the CAPM had better fit as scale increased.

Chapter 2

Preliminary Test

As a preliminary test, I followed General Electric Co. (G.E.) from January 1926 to December 2008. G.E has been listed on the CRSP (The Center for Research in Security Prices) data since December 1925. Betas by different scales over the same period were measured. Monthly, quarterly and semi-annual returns were measured by non-overlapping calendar months. Annual returns were measured over non-overlapping calendar years starting from every January. Two, three, four and five year returns were also measured over non-overlapping calendar years.

G.E.'s beta on an equally weighted index went flat as the scale increases; monthly beta=0.71, quarterly beta=0.62, semi-annual beta=0.58, annual beta=0.53, two year beta=0.30, three year beta=0.13, 4 year beta=0.08, 5 year beta=0.02. If investors have a 5 year investment horizon or rebalancing schedule, they can choose G.E. as a safe asset because G.E. stock yielded market-irrelevant returns in 5 year scale. G.E. has yielded higher returns than T-Bill for the past 83 years, but G.E's beta is close to zero in 5 year scale. The result shows why long term investors win in the long run over short term traders. Also, the result implies that investors can compose a portfolio utilizing the scale arbitrage. To generalize the result, I will expand the sample size to the whole market in next chapter. All common stocks traded on the New York Stock Exchange (NYSE), the American Stock Exchange (AMEX), and the over-the-counter market (NASDAQ) will be used.

In contrast to the equal-weighted index, the value-weighted beta showed the opposite results; monthly beta=1.16, quarterly beta=1.13, semi-annual beta=1.14, annual beta=1.09, two year beta=1.16, three year beta=1.49, 4 year beta=1.22, 5 year beta=1.66. For all the scales, G.E. stayed in aggressive stock category. In a 5 year scale, G.E. was volatile with a beta 1.66, but G.E. was assigned to defensive category on equally weighted index. G.E.'s beta is scale-dependent on equally weighted index but scale-independent on value weighted index. The discrepancy can be explained by investigating the volatility of the index itself.

Table 2 Transition OF G.E. Beta over Scales

G.E	Monthly	Quarterly	Semi-Annual	Annual	2 year	3 year	4 year	5 year
Beta on equally weighted index	0.71	0.62	0.58	0.53	0.3	0.13	0.08	0.02*[5]
Beta on value weighted index	1.16	1.13	1.14	1.09	1.16	1.49	1.22	1.66

[5] Insignificant under 95% confidence level

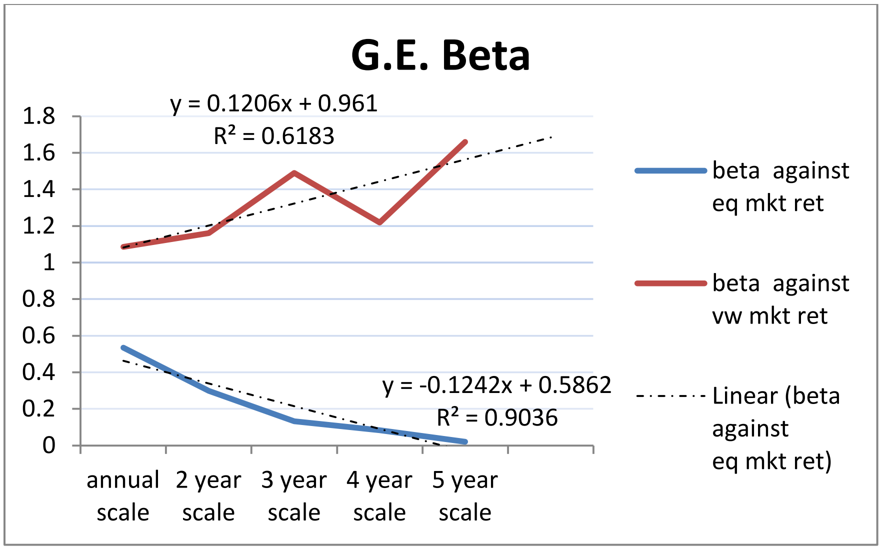


Figure 1 Transition of Beta over Scales

I calculated the standard deviation for G.E., equally weighted market index and value weighted market index on different scales. As scale increased, the standard deviation of all return series increased. The standard deviations of G.E. and market index increased linearly, which violate the random walk hypothesis. If the return series follows a random walk process, then the standard deviation must increase by $\sqrt{\mathbf{n}}$ as scale increases by n times.

The correlation between G.E. and the market index is given below. The correlation between G.E. and the equally weighted market index decreased as the scale increased. In monthly scale, the correlation is 0.67 but the correlation dropped to 0.02 on the 5 year scale. The correlation between G.E and the value weighted index did not have any specific patterns.

Table 3 Standard Deviation of G.E. and Market

Scale	Sigma of G.E.	Sigma of VW MKT	Sigma of EQ MKT
1 year scale	26.98%	20.62%	31.31%
2 year scale	44.56%	27.93%	45.74%
3 year scale	66.16%	35.41%	55.35%
4 year scale	84.66%	45.71%	88.19%
5 year scale	111.50%	49.81%	117.37%

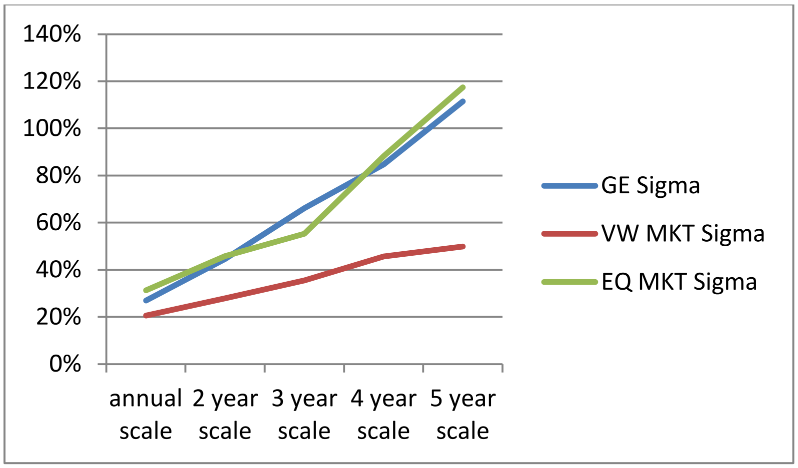


Figure 2 Scale and Volatility

The risk-return trade-off graph is given below. With the longer scale, the frontier shifted up and to the right. The equally weighted index has a higher volatility in any scale than the value weighted index. The standard deviation of G.E. and the value weighted index increased linearly as the scale increased. This is counter to the random walk hypothesis. G.E.'s beta decreased as scale increased when the equally weighted market index was used as the reference return. The main reason for the decreasing beta was the decreasing correlation between G.E and the equally weighted market index. In equally weighted index, small firms have the same weight of 1/n as that of big firms. However, the value-weighted approach yielded different results. G.E.'s beta on the value-weighted index did not change significantly as the scale increased. The main reason for staying in the aggressive category was the stable correlation between G.E. and the value weighted market index. In value weighted market index, small number of big firms holds the largest share of market capitalization. G.E. is mainly compared to its own peer group in value weighted index and so the higher but stable correlation leads to higher beta in any scales.

The results for G.E result showed that the measurement scale plays an important role in beta estimation. I will expand the sample size to the whole market to reach statistically significant conclusions.

Table 4 Scale and Correlation

Scale	Correlation with VW	Correlation with EW
1m scale	0.81	0.67
3m scale	0.86	0.75
6 m scale	0.84	0.70
12m scale	0.83	0.62
2y scale	0.73	0.31
3y scale	0.80	0.11
4y scale	0.66	0.09
5y scale	0.74	0.02

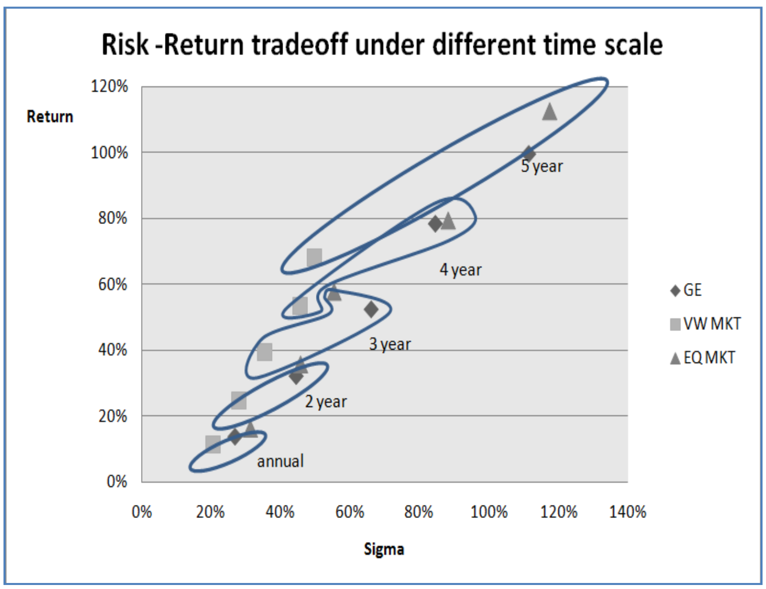


Figure 3 Risk-Return Tradeoff

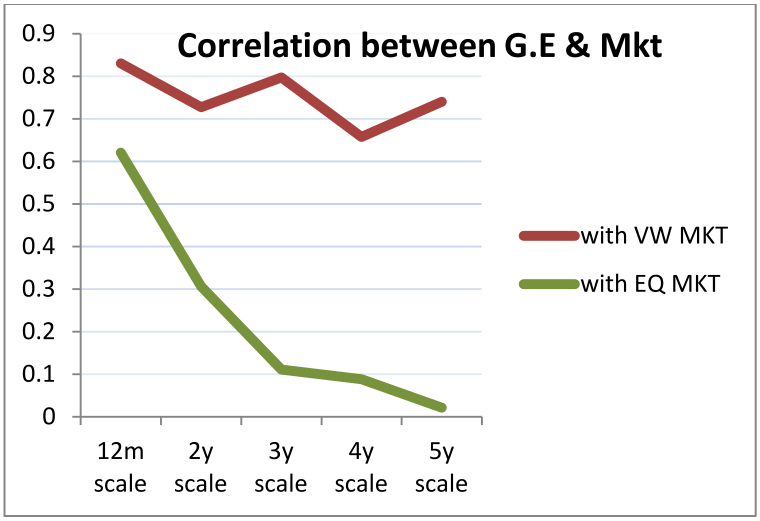


Figure 4 Correlation between G.E. and Market

Chapter 3

Methodology

I use all common stocks traded on the New York Stock Exchange (NYSE), the American Stock Exchange (AMEX), and the over-the-counter market (NASDAQ). From 1926 through 2009, there is data for 23,031 firms. There are 40 active firms which have been listed on the CRSP (The Center for Research in Security Prices) data continuously since 1926.

Utilizing monthly-scale returns with almost a century's worth of data gives me a large but manageable sample. Any chosen time scale is arbitrary and the relative volatility of stock returns could be quite different depending on the time scale. If we use annual returns, the sample size reduces to 1/12. Any outliers—extremely low or high returns—would bear more weight on beta. Consequently, the firm beta can change significantly by the outliers in the regression analysis. Also, the distribution of monthly returns will be quite different from that of quarterly returns or semi-annual returns if the return series is not fractal. The extrapolation of the expected returns will be dependent on the time scale. If the juxtaposition of the two return series gives the same patterns regardless of the time scale, then the beta is scale-invariant or arguably, the market is fractal in the time dimension.

I trace the transition of firm beta over different scales such as one month, three months, six months and twelve months. I use non-overlapping calendar month approach and filtered out firms with less than 10 years of trading history to acquire minimum observations for the regression analysis on the annual returns series. Filtering resulted in total 8,309 firms.

I compute the difference of betas over the differing scales and average out the difference in beta. If the average difference is close to zero, there are two different conclusions. First, firm beta can be time-invariant. Second, the difference of firm betas can be symmetric. Some firms may have a positive difference and others may have a negative difference at the same time.

Using the estimated betas on each scale, I measure the switch ratio, defined as the category change between aggressive and defensive stock classification, in firm level and portfolio level. The traditional dichotomy of risky assets – aggressive or defensive assets– is examined. If scales can change the asset characteristics easily, investors need to use betas estimated according to their investment horizon. The more important ratio is the switch ratio in each beta-sorted portfolio. If the switch is clustered around mid-beta portfolios and rare in extreme portfolios, the pattern will reflects only the random change in the distribution of firm returns over the differing scales, not a structural change following scale shift. The portfolio approach heavily depends on the sequence of sorting and grouping. If stocks tend to have different betas depending on differing scales, then the sequential sorting and grouping by size or book-to-market ratio will cause a chain reaction on the membership of the final portfolios and their expected returns. Comparing the membership of the beta-sorted portfolios gave insight to the time-dependent characteristics of portfolio formation. If the time-scaled portfolio barely changed its members, the portfolio formation is scale-independent. If the membership has similar constituents in our portfolios whether we use monthly beta or longer term beta, traditional beta portfolio approach would work as a reference portfolio in multi-factor models. However, if the switch ratio inside each portfolio is high, then the other factor portfolios such as size or book equity-to-market equity would have different members in each portfolio.

I also investigated risk premiums. If the 2^{nd} pass slopes are insignificant, then the simple mean of risk premium time series would be meaningless. Insignificant premiums mean that the slope is random considering the variation of the explanatory variable, beta. I investigate the significance of beta over each specific estimation period. To see if the variation of portfolio beta caused the insignificant slopes in the 2^{nd} pass regression, I also employ 20-portfolio and single stock approach to see if the wider range of explanatory variables can enhance predictability. However, there is a trade-off between increasing the spread of explanatory variables and the errors- in-variables problem. Errors-in-variables can cause an inaccurate prediction in 2nd pass regression. It is well known that the slope and intercept are biased if the errors-in- variables are significant. To reduce the errors, researchers employ many instrumental variables such as portfolio beta, time-lagged beta and random order sampling. The scale approach can be also interpreted as an instrument variable approach, i.e., correlated with the true variable, but not correlated with the residuals. I vary time-scale to see if the predictability of beta can be enhanced without losing the spread of beta values.

I vary estimation periods from 1 year to 10 years and I investigate which scale period produces the most powerful beta in predicting expected returns. The observation period ranges from 1926 to 2009 (84 years). I assign beta-ranked stocks into 10 portfolios and 20 portfolios. Both the portfolio approach and firm level approach are used in 2nd pass regression. Using the rolling beta technique, I investigate the significance of the slope on the beta over various estimation periods. I use the same method as Fama-MacBeth (1973) except for the absence of a separate estimation period. As I mentioned earlier, lagged estimation loses valuable information. Fama-MacBeth (1973) used a relatively short time period from 1926 to 1968 and, accordingly, a small sample size. The period they investigated included the Great Depression, World War II and the bear market in 1960s, highly negative -biased periods. My sample size is 23,031 and the total period is 84 years from 1926 to 2009. The long range of the testing period and ample sample size will result in a more comprehensive and unbiased analysis.

Chapter 4

Results

4.1 Scale Dependency of Firm Beta

The average difference of betas using the equal-weighted index is close to zero. I asserted that stocks experiencing short term turbulence or seasonality will have higher monthly betas because of the higher volatility in the monthly scale, but, in the larger scale, their instability will lessen as time extends. My results show that this is not true of U.S. stock markets for the past 84 years. The scale does not cause any flatter firm betas on average.

I also compute the average absolute values of beta difference to find out if beta is scale-invariant. The absolute value of beta difference gets bigger as scale increases. The absolute value of beta difference, 0.61, between monthly scale and annual scale is meaningful since beta values have a small variation. According to the two tables below, the net difference is close to zero and the absolute difference has clear patterns. As scale increases, firm beta changes significantly. I conclude that firm beta changes more as scale increases and in a symmetric manner. Some firms' betas increase as scale increases, but other firms' betas decrease. Stock prices appear not to be fractals because the return patterns are not similar across scale.

Levhari and Levy (2001) found that aggressive stocks tend to have higher beta when using a longer term scale, whereas defensive stocks have a beta that declines along the scale. Neutral 101 stocks traded on the New York Stock Exchange from 1948 -1968. They asserted that if returns are independent over time, then the betas of aggressive stocks increase with the length of the horizon. Defensive stocks will have a declining beta as the estimation period gets longer. Stocks around unity beta will have stable beta over different time scale.

Table 5 Difference in Beta over Scales

Net Difference	monthly β – annual β	monthly β – 9 month β	monthly β – 6 month β	monthly β – quarterly β
equally-weight index	-0.003	-0.002	-0.007	-0.005

Table 6 Absolute Difference in Beta over Scales

Absolute Difference	monthly β – annual β	monthly β – 9 month β	monthly β – 6 month β	monthly β – quarterly β
equally-weight index	0.613	0.474	0.363	0.212

In contrast to Levhari and Levy (2001), I find that there is no pattern whether stocks belong to defensive or aggressive category. In each sub-group, i.e., aggressive stocks or defensive stocks, the distribution of differences is again symmetric. The net difference between monthly beta and annual beta is 0.00 for aggressive stocks and -0.00 for defensive stocks. The absolute value of beta difference between monthly beta and annual beta is 0.41 in defensive stocks and 0.83 in aggressive stocks. Aggressive stocks change their beta level twice as much as defensive stocks did, but again there is no pattern in the net differences whether stocks belong to the aggressive category or the defensive category. These findings contradict previous research.

4.2 Scale and Switch Ratio

I analyze the switch ratio between aggressive (betas greater than one) and defensive (betas less than one) using 10 years of observation period. When comparing monthly beta with annual beta, 28.52% of all the CRSP common stocks switch categories. The ratio is 14.13% when comparing monthly scale beta with quarterly scale beta. As the scale gets bigger, the switch ratio increases monotonically. The switch ratio is significant and the traditional dichotomy of aggressive or defensive assets must be applied with caution. If investors aim for long-term returns, they should use annual betas rather than monthly ones to build better portfolios.

Table 7 Switch Ratio over Scales

Scales	1m vs. 3m	1m vs. 6m	1m vs. 9m	1m vs. 1y
Switch Ratio	14.13%	21.30%	25.25%	28.52%

To see if the switch ratio implies a significant change in the return structure, I tested the switch ratio with more conservative thresholds. I divided all the common stocks into 3 groups. The firms in the first group have beta less than 0.8 and the firms in the third group have beta greater than 1.2. The first group accounts for 33.84 % of all the CRSP common stocks and the third group accounts for 33.34% of all the CRSP common stocks. The second group is between the two groups. I checked the switch ratio between the first group and the third group. In the first group, I counted stocks whose beta value got bigger beyond the threshold 1.2 as scale lengthens to annual from monthly. In the third group, I checked stocks whose beta value changed down below 0.8 as scale lengthens to annual from monthly.

The first group has 8.49% of switch ratio and the third group has 21.50% of switch ratio as scale lengthens from monthly to annual. In annual scale, 21.50% of the aggressive stocks turned into defensive stocks and 8.49% of defensive stocks changed their category. Again, aggressive stocks change their characteristics more frequently. Scale approach can invalidate the traditional dichotomy as we can see the switch ratio in the robustness tests. Some of the aggressive stocks are not as risky as when we first estimated. Their characteristics can change along the scales and it may possibly present an arbitrage opportunity for investors.

The distribution of beta is illustrated in two scales. The monthly scale beta has a narrower distribution than the annual scale beta. The mean beta is 1.00 for both scales. The median is 0.94 for monthly beta and 0.80 for annual beta. The wider range of beta in the annual scale may be attributed to the small sample size. The outliers in annual returns have more weight than the outliers in monthly returns.

Fama-MacBeth (1973) argued that the extreme portfolios tend to revert to the mean if we wait for another 3-5 years to normalize the temporary abnormal behaviors. However, Fama-MacBeth method distorts the true return distribution for those 3-5 years. If the abnormal behavior lasts for the short-term, less than a year, it makes sense to delete the year's information. However, 3-5 years are too long to ignore whether the extreme behavior is a market fad or a systematic changes in the volatility structure. The various scale approach proved that the extreme behaviors of stocks can be controlled by using longer term scales without losing valuable information and distorting the distribution of returns.

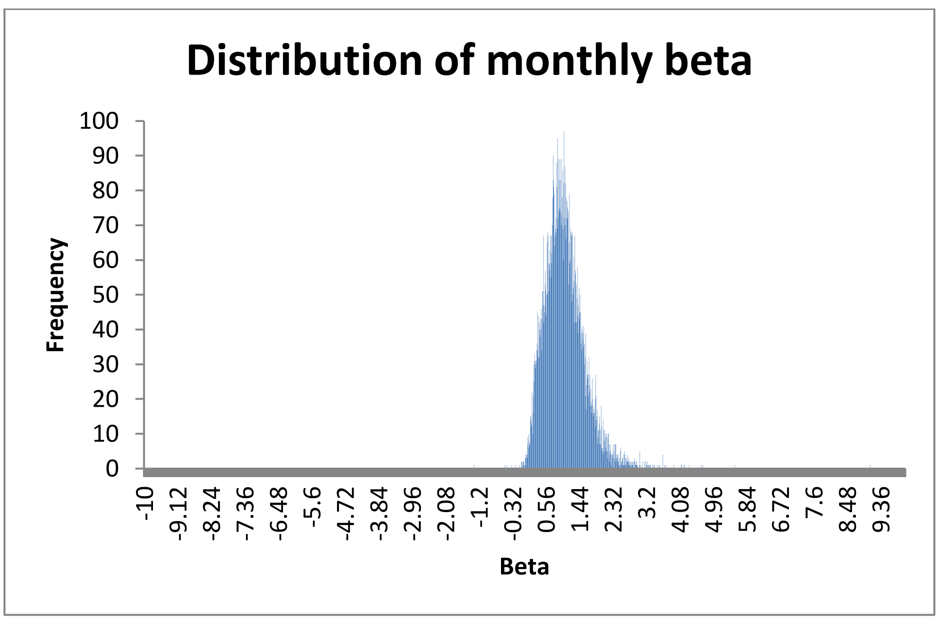


Figure 5 Distribution of Monthly Beta

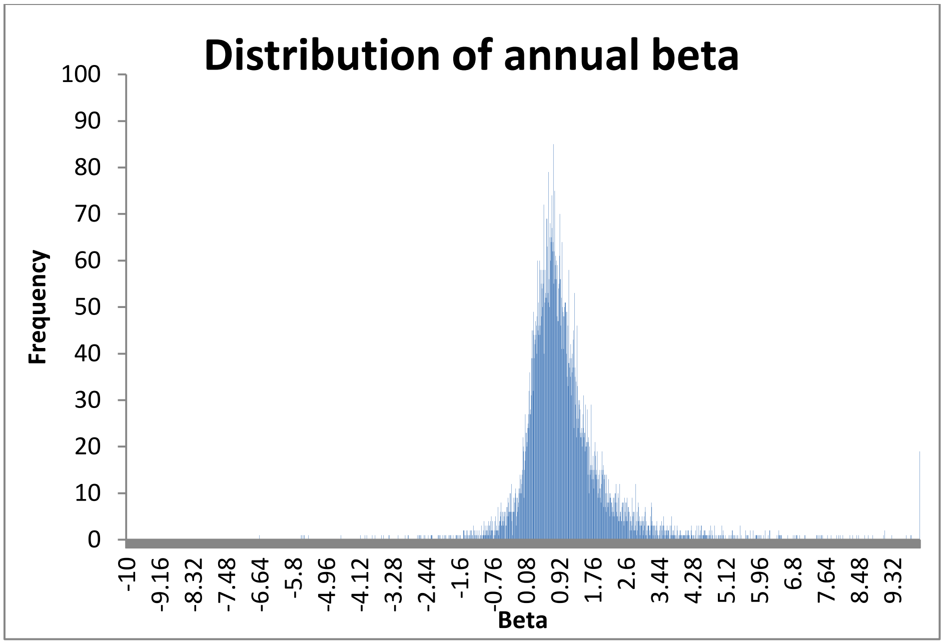


Figure 6 Distribution of Annual Beta

4.3 Scale Independency of Portfolio Beta

I assign all the CRSP common stocks into 10 portfolios by their beta ranking. Any common stocks with less than 10 years of observation period are excluded from the sample. Each portfolio has 922 component stocks. Portfolio beta is equivalent to the arithmetic mean of all firm betas without any weight.

Average portfolio beta does not change much. Portfolio 9 has monthly beta of 1.55 and annual beta of 1.55. Portfolio 10 has monthly beta of 2.11 and annual beta of 2.19. However, robustness test in the previous chapter tells us that the switch ratio is higher for the aggressive groups whose beta value is bigger than 1.2. Extreme portfolios tend to yield stable average beta regardless of scale, but significant portion of stocks go defensive at the same time. The inactive groups, such as portfolio 2 and portfolio 3, show a similar pattern. Average beta does not show significant change as the scale lengthens. The portfolio approach is not vulnerable to scale effect. The portfolio averaging effect seems to subdue the scale effect. The symmetric pattern is also seen in the portfolio level.

Next I compute the switch ratio in each beta-sorted portfolio. If the switch is clustered around mid-beta portfolios and rare in extreme portfolios, such patterns will not imply any structural change following scale shift. Portfolio 1 and portfolio 2 have a switch ratio of approximately 7% and 12% respectively, while the most active portfolios, 9 and 10, have a higher switch ratio. Twenty nine percent of portfolio 9 stocks and 21.80% of portfolio 10 stocks change categories when their beta is measured using an annual return series. The mid-portfolio (portfolio 6) has the highest switch ratio of 53.36%. Considering the mid-portfolio is on the borderline between the defensive and the aggressive, such a high switch ratio is not surprising. The mid-range portfolios have highest switch ratio. Portfolio 6 has 53.36% of switch ratio and portfolio 7 has 47.35% of switch ratio. However, their average beta is close to one and their switch ratio could be caused by white noise. The scale approach gives a different picture of the beta distribution along the portfolio formation. The stock market shows a different distribution pattern of betas depending on the scale.

Last, I analyze beta-shift by industry level. Using SIC (Standard Industrial Classification) code , I assign firms to each division. The switch ratio[6] is computed in each division as below. Stocks in financial division tend to switch to 'aggressive' or stay aggressive as scale lengthens. The result is consistent with Hamada (1972) model below.

$$B_L = \beta_U [1+(1-t_c) B/S]$$

[6] Switch from aggressive (beta>1.2) to defensive (beta<0.8) between monthly scale and annual scale over 10 year period

Levered beta increases proportionally as debt ratio increases. Firms in financial division have significantly higher debt ratio compared to those of other divisions.

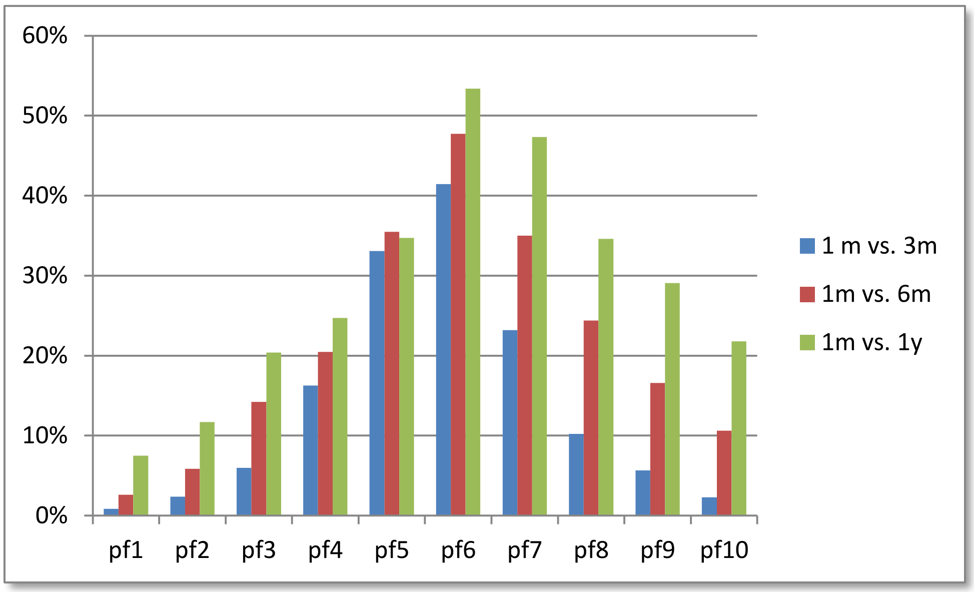


Figure 7 Switch Ratio in Beta-Sorted Portfolios

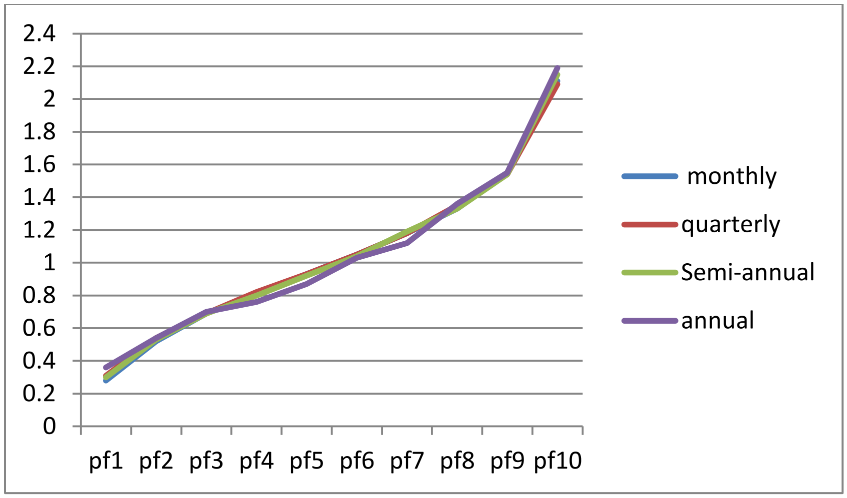


Figure 8 Portfolio Beta over Scales

Table 8 Switch Ratio in Beta Portfolio

scale	pf1	pf2	pf3	pf4	pf5	pf6	pf7	pf8	pf9	pf10
1m vs. 3m	0.87 %	2.39 %	5.97 %	16.25 %	33.08 %	41.43 %	23.19 %	10.20 %	5.64 %	2.28 %
1m vs. 6m	2.60 %	5.86 %	14.21 %	20.48 %	35.47 %	47.72 %	34.99 %	24.40 %	16.59 %	10.63 %
1m vs. 1y	7.48 %	11.71 %	20.39 %	24.70 %	34.71 %	53.36 %	47.35 %	34.60 %	29.07 %	21.80 %

Table 9 Average Portfolio Beta over Scales

scale	pf1	pf2	pf3	pf4	pf5	pf6	pf7	pf8	pf9	pf10
monthly	0.28	0.52	0.69	0.81	0.93	1.05	1.18	1.33	1.55	2.11
quarterly	0.31	0.54	0.69	0.82	0.93	1.05	1.18	1.35	1.54	2.09
Semi-annual	0.30	0.53	0.69	0.80	0.92	1.04	1.19	1.33	1.54	2.15
annual	0.36	0.54	0.70	0.76	0.87	1.03	1.12	1.36	1.55	2.19

Table 10 Switch from 'Aggressive' to 'Defensive' in Industry Division Level

Division	# firms	% in sample	% in population	Difference
Mining Division	31	4.70%	4.62%	0.08%
Construction	4	0.61%	1.05%	-0.44%
Manufacturing	259	39.24%	34.62%	4.62%
Transportation, Communication Electric Gas, Sanitary	44	6.67%	6.89%	-0.22%
Wholesale	35	5.30%	3.65%	1.65%
Retail	36	5.45%	6.11%	-0.66%
Finance, Insurance Real Estate	44	6.67%	17.43%	-10.76%
Services	141	21.36%	16.44%	4.92%
etc	66	10.00%	7.08%	2.92%
Agriculture		0.00%	0.36%	-0.36%
Public Administration		0.00%	1.73%	-1.73%
	660	100.00%	100.0%	

Table 11 Switch from 'Defensive' to 'Aggressive' in Industry Division level

Division	# firms	% in sample	% in population	difference
Mining Division	22	8.33%	4.62%	3.71%
Construction	1	0.38%	1.05%	-0.67%
Manufacturing	71	26.89%	34.62%	-7.73%
Transportation, communication Electric Gas, Sanitary	14	5.30%	6.89%	-1.59%
Wholesale	13	4.92%	3.65%	1.27%
Retail	19	7.20%	6.11%	1.09%
Finance, Insurance Real Estate	64	24.24%	17.43%	6.81%
Services	31	11.74%	16.44%	-4.70%
etc	29	10.98%	7.08%	3.90%
Agriculture		0.00%	0.36%	-0.36%

4.4 Scale and Risk Premium

I measure portfolio betas on various time scales over the past 10 years and run 2nd pass regressions to estimate the slope, i.e., risk premium. I assign beta-ranked stocks into 10 portfolios and 20 portfolios. Under the 20 portfolio approach, monthly-scale beta yields 0.33% of premium per month and quarterly-scale beta gives 0.99% of premium per quarter. Six month scale yields 1.66% of risk premium per six months. As the interval increases, the risk premium is supposed to increase proportionally if the distribution of return series is time-invariant. I annualize the risk premium in each scale to compare the risk premium in the same unit.

The result is that the increment in risk premium is not proportional. According to the graph, either monthly premium appears overvalued or semi-annual premium appears undervalued. Beta is equivalent to the scaled covariance of firm returns over market returns. If the return series follows a random walk process, and the correlation between the given stock and the market is stable, the ratio will not change. Also, the risk premium should be proportional to scales. However, my results show that the risk premium increases with decreasing speed. If stocks have different betas over various intervals and also have different risk-premium schemes, then a specific time-scale beta will be more useful, depending on the investor's time horizon.

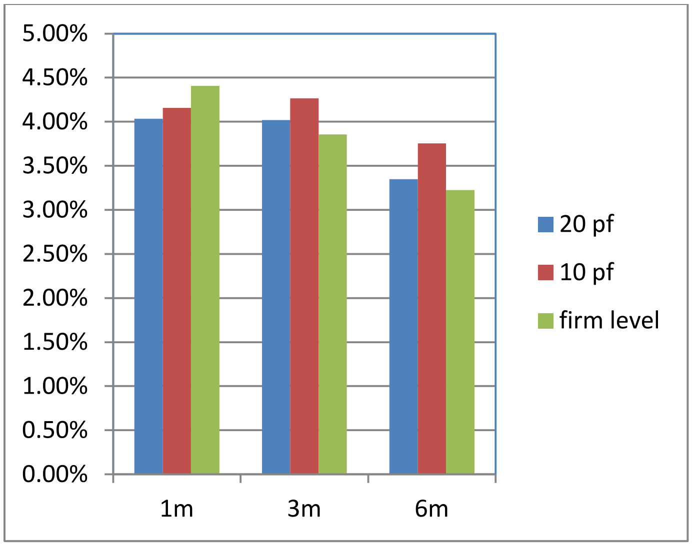


Figure 9 Annualized Risk Premium over Scales

Table 12 Risk Premium over Scales

scale	1m	3m	6m
20 pf	0.33%	0.99%	1.66%
10 pf	0.34%	1.05%	1.86%
Firm Level	0.36%	0.95%	1.60%

Chapter 5

Behavioral Aspect of Firm Beta

This part delves into two related behavioral issues surrounding beta. First, I investigate on the popular estimation period on which investors rely. Stock investors respond to the past market information, but we do not know how further they look back. Second, I focus on the relation between presidential term and the efficacy of beta.

5.1 Representative Estimation Period

The average risk premium is computed by averaging the time series of risk premium over all the rolling periods. However, the significance of the slope as well as the sign of the slopes in each rolling period must be investigated. The presence of insignificant risk premiums and negative risk premiums may render the average premium of limited use because the simple average of the insignificant slopes and negative slopes may lead to statistically meaningless conclusions.

The frequency approach using hitting ratio includes a test of the reliability of the CAPM. I define the hitting ratio as the number of successful prediction of expected returns over the number of rolling sessions.

$$\text{Hitting ratio} = \frac{\textbf{The number of significant 2nd pass slopes}}{\textbf{The number of rolling periods}}$$

I change estimation period and scale to examine their influence on the hitting ratio. If the hitting ratio changes significantly along the scale and estimation period, then the CAPM depends on the data. In all different estimation periods, four year observation gives the best hitting ratio. Firm-level beta can best explain the expected returns with a 76.8 % success rate for the past 84 years. When stocks are assigned to 20 portfolios, the hitting ratio is 75.7%. Ten portfolios yield 74.1% of hitting ratio. The next highest hitting ratio is yielded by three year observation period. In all level, the hitting ratio peaks around the four year observation and the pattern is convex-shaped. The result is the corollary of portfolio's averaging effect which reduces the range of explanatory variables. The hitting ratio in each estimation period is given in the table below.

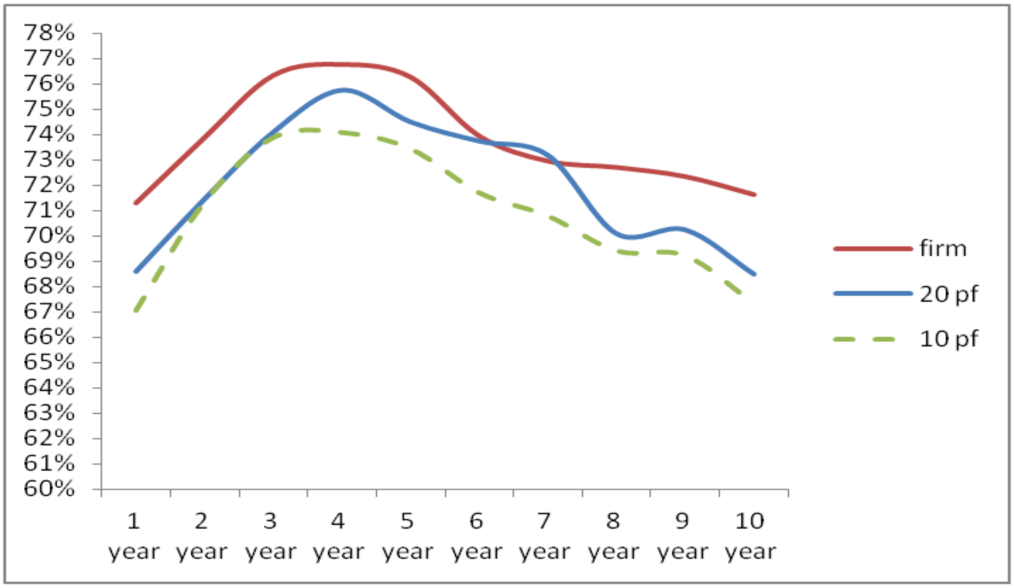


Figure 10 Hitting Ratio by Estimation Periods

Table 13 Hitting Ratio over Estimation Periods

level	1 year	2 year	3 year	4 year	5 year	6 year	7 year	8 year	9 year	10 year
firm	71.3%	73.9%	76.3%	76.8%	76.3%	73.9%	72.9%	72.7%	72.3%	71.6%
20 pf	68.6%	71.4%	74.1%	75.7%	74.5%	73.7%	73.2%	70.1%	70.2%	68.5%
10 pf	67.1%	71.3%	73.9%	74.1%	73.4%	71.7%	70.8%	69.4%	69.2%	67.3%

Table 14 Hitting Ratio by 10 Portfolios

Period	# rolling session	# of significant sessions	# negative premium	Hitting Ratio
1 year	996	668	517	67.07%
2 year	984	702	519	71.34%
3 year	972	718	510	73.87%
4 year	960	711	495	74.06%
5 year	948	696	489	73.42%
6 year	936	671	472	71.69%
7 year	924	654	472	70.78%
8 year	912	633	463	69.41%
9 year	900	623	445	69.22%
10 year	888	598	460	67.34%

Table 15 Hitting Ratio by 20 Portfolios

Period	# rolling session	# of significant sessions	# negative premium	Hitting Ratio
1 year	996	683	517	68.57%
2 year	984	703	515	71.44%
3 year	972	720	510	74.07%
4 year	960	727	498	75.73%
5 year	948	706	491	74.47%
6 year	936	690	476	73.72%
7 year	924	676	476	73.16%
8 year	912	639	466	70.07%
9 year	900	632	447	70.22%
10 year	888	608	468	68.47%

Table 16 Hitting Ratio by Single Stock Level

Period	# rolling session	# of significant sessions	# negative premium	Hitting Ratio
1 year	996	710	524	71.29%
2 year	984	727	519	73.88%
3 year	972	742	508	76.34%
4 year	960	737	499	76.77%
5 year	948	723	487	76.27%
6 year	936	692	476	73.93%
7 year	924	674	478	72.94%
8 year	912	663	466	72.70%
9 year	900	651	456	72.33%
10 year	888	636	449	71.62%

I mix the scale effect with different estimation periods. I vary scales over both 10 years of rolling observation and 4 years of rolling observation. In 4 year estimation (right-hand side table), three month scale yields the best hitting ratio, 79%. Longer term observation is more useful for longer term prediction. In 10 year estimation (left-hand side table), six month scale yields the best hitting ratio, 76%.

Table 17 Hitting Ratio over Scales :

Estimation Period of 10 years vs. Estimation Period 4 years

Level	1m	3m	6m		Level	1 m	3 m	6 m
firm level	71%	76%	79%	vs.	firm level	76%	79%	75%
20 portfolios	68%	74%	79%		20 portfolios	75%	77%	71%
10 portfolios	67%	72%	77%		10 portfolios	74%	78%	72%

In sum, stock investors tend to respond to the most recent three to five years of market information and, particularly, four years of observation yields the best predicting power. Investors seem to base their decisions on beta estimates on the past four years of market information and on the ergodicity of market. The hitting ratio approach enables us to choose the best scale and estimation period. In contrast to prior research, there is a better choice of scale and estimation period.

5.2 Presidential Terms and Beta

The highest hitting ratio of beta is 77% when using 4 year observation and I conclude that CAPM holds in terms of frequency, not in terms of simple average risk premium. One interesting point is that politics affects to the CAPM. During the 84 years in the investigation, democratic presidents were in office for 41 years and republican presidents were in office for 43 years. However, the positive risk premiums are observed more frequently during Democratic administration. During Democratic presidencies, 60% of the positive risk premiums were observed. The remaining 40% were observed during Republican residencies. One of the main assumptions of the CAPM is that investors require the same returns over the same unit of risk regardless of the investment vehicles. Two parties have different economic plans to suit different class of electorate in U.S. Their economic policies seem to affect the investors' behavior and attitude to risk and compensation level. Santa-Clara and Valkanov (2003) found that the excess returns are higher under Democratic presidencies. They examined the average excess returns of equal-weighted CRSP index over the three-month Treasury bill rates from 1927 to 1998. The difference in excess returns between Republican and Democrat was 16 percent. They asserted that the difference came from higher stock returns and lower real interest rates. The difference in returns was not explained by business-cycle variables.

Also, a calendar effect is observed. Beta can explain expected returns in January most frequently. The number of positive risk premiums in January is twice the average of other months. During the investigation period from 1926 to 2009 (84 years), 57 years (67%) show positive risk premium in January. In other months, the average ratio was 32%. I count the number of months per year in which beta can explain expected monthly returns from 1930 to 2009. The annual frequency does not show any particular pattern.

Table 18 Presidency and Positive Risk Premium

Party	Months	Percent
Democratic	212	60.40%
Republican	139	39.60%

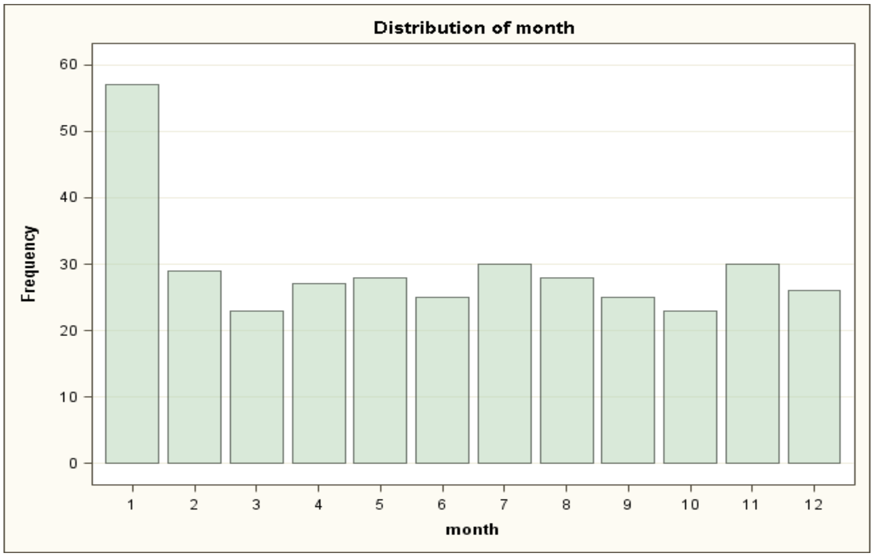


Figure 11 Calendar Effect on Positive Risk Premium

Table 19 Calendar Effect on Positive Risk Premium

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Counts	57	29	23	27	28	25	30	28	25	23	30	26
Percent	68%	35%	27%	32%	33%	30%	36%	33%	30%	27%	36%	31%

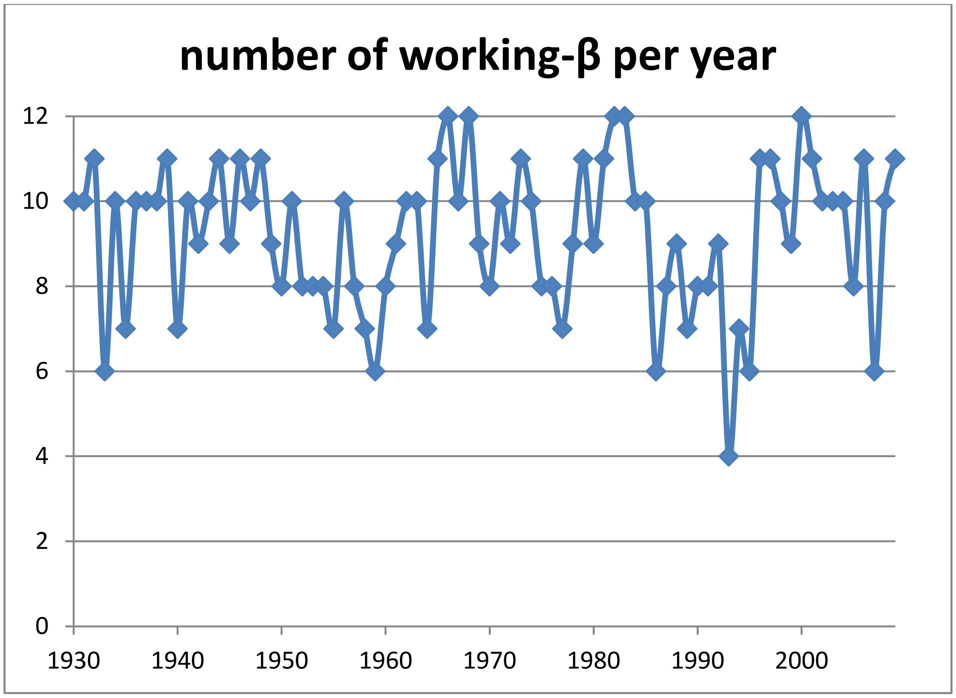


Figure 12 The Number of Months in which Beta Works from 1930 to 2009

Table 20 Beta Working Months from 1930 to 2009

year	β-working months	year	β-working months	year	β-working months
1930	10	1957	8	1984	10
1931	10	1958	7	1985	10
1932	11	1959	6	1986	6
1933	6	1960	8	1987	8
1934	10	1961	9	1988	9
1935	7	1962	10	1989	7
1936	10	1963	10	1990	8
1937	10	1964	7	1991	8
1938	10	1965	11	1992	9
1939	11	1966	12	1993	4
1940	7	1967	10	1994	7
1941	10	1968	12	1995	6
1942	9	1969	9	1996	11
1943	10	1970	8	1997	11
1944	11	1971	10	1998	10
1945	9	1972	9	1999	9
1946	11	1973	11	2000	12
1947	10	1974	10	2001	11
1948	11	1975	8	2002	10
1949	9	1976	8	2003	10
1950	8	1977	7	2004	10
1951	10	1978	9	2005	8
1952	8	1979	11	2006	11
1953	8	1980	9	2007	6
1954	8	1981	11	2008	10
1955	7	1982	12	2009	11
1956	10	1983	12		

Chapter 6

Size Effect and Duration

6.1 Introduction

According to Berk (1995), firm size can explain the variation of expected returns because of the implied returns built in firm size itself. The size anomaly is not an anomaly as long as expected returns and firm size are inversely related. If size effect is not abnormal, another issue rises. Smaller firm size can be the result of higher expected returns, not the cause of higher expected returns. The size effect is confounded with endogeneity. The circular reasoning makes the size loading vulnerable.

A big flaw in the discount model is that the expected returns and the discount rates are the same. Researchers used various techniques to eliminate the endogeneity issue. For example, Campbell (1991) used cash flow factor. He employed a Vector Auto Regression (VAR) approach to filter out cash flow growth level from the implied risk level. Campbell and Vuolteenaho (2004) used two-beta model. They decomposed beta into two components, one reflecting market's future cash flow and the other reflecting market's discount rates. They found value stocks and small stocks have considerably higher cash-flow betas than growth stocks and large stocks. Bad news about future cash flows decreases present value of assets, but future investment opportunities remain stable. In contrast, bad news about discount rates improves future investment opportunities. However, fuzzy cash flow news can cause worse problems in VAR. Dividends track firm's earnings with a lag and many firms do not pay dividends. According to Shiller (1981), stock prices are too volatile to be justified by subsequent changes in future dividend flows. Campbell and Vuolteenaho (2004) remarked that short-run dynamics of dividends is not important and the dynamics of expected returns need to be understood.

We need to extract an exogenous factor from the size variable to avoid the built-in endogeneity in the firm size effect. Firm size is related to the firm’s history as well as future events. Many investors watch firms growing and bet on what will happen to the firm in the future by referring to the past growing patterns. A firm’s market risk, beta, is a kind of history book by which we can see how the firm has behaved in the past 60 months or another period in the market. The beta approach shares similarities to the chart analyst’s. Both the approaches depend heavily on the ergodicity assumption for the estimates of future returns. Samuelson (1969) asserted that ergodicity assumption is necessary for economics to be a science. Once we get the exogenous explanatory variable from the size, then we are qualified to depend on the ergodicity assumption on the future pattern of expected returns. We need to replace the firm size factor with an exogenous variable which is related to the firm size, but less related to error terms which cause the endogeneity issue.

I will apply Macaulay's duration to explain firm size effect. Cash flow beta and discount beta can be combined into one formula, duration. Duration factors in two conflicting components, cash flow news and discount rates news. Firm duration can be interpreted in two ways. Duration measures the elasticity of firm size with respect to a change in market discount rates. If any firm has higher duration, size will fluctuate more than that of a lower duration firm. Also, the duration measures the immunization point when investors can have the fixed future wealth level regardless of the future changes in market discount rates. Investors are always exposed to risk and their wealth level is always uncertain except for the immunization point. Bank managers use the duration approach to reduce the risk from the mismatch between the asset's cash flow and the liability's cash flow.

6.2 Literature Review

Banz (1981) found that a significant amount of excess returns exist in the New York Stock Exchange over the period 1936 -1977. He found that the CAPM cannot capture the firm size effect as one of the explanations for the excess returns. According to his simulation, an arbitrage portfolio - long top 20% big firms and short bottom 20% small firms - could yield 19.8 % every year. According to "*Stocks, Bonds, Bills and Inflation (SBBI) 2007 yearbook*", the excess returns have existed since 1926. Ten size portfolios were created by size rank and 4,252 firms were covered over the period 1926 to 2006. As shown in the table below, smaller portfolios had higher beta and higher excess returns which could not be explained by the systematic risk. If we had employed an arbitrage portfolio over the period 1926 to 2006, the average arbitrage returns would have been 6.63% for those 80 years. The table confirmed Banz findings (1981) that very small firms, not all small firms, have strong small firm effects. Banz found that only the top 20% of the smallest firms produced most of the excess returns while the other 80% of firms didn't show large differentials. Banz illustrated his findings using the graph below. He calculated mean residual returns of portfolios (1936 - 1975) using equally weighted CRSP index as the market proxy. The residual returns were calculated with a two factor model ($\varepsilon_{it} = R_{it} - \gamma_0 - \beta_{it}\gamma_{1t}$). First, five size portfolios were created and again he sorted five beta portfolios within each size group. Second, Fama- MacBeth regression was employed within each size group. Only the first decile, the smallest portfolio, showed significant results compared to 3, 4 and 5 portfolios. The size effect was not linear with the ranked firm size through the whole sample, and so simple linear regression might not be the best way to express the relation between portfolio size and its expected returns.

Chan and Chen (1988) found beta and size were negatively correlated and their coefficient is -0.99. However, Banz (1981) stated that the size effect did not seem to be just a proxy for the unobservable true beta even though the market proportion and the beta of securities were negatively correlated. Keim (1983) found that January has larger abnormal returns than the remaining eleven months and the size-return relation is always negative in January. Dimson and Marsh (1999) declared "Demise of Size" based on their research on U.K. stock market. Once the small firm premium effect was disseminated, the small firm discount of around 6% was observed in the U.K. stock market. Fama - French (1992) argued that variation in beta that was not related to size cannot explain the expected returns. One solution for the multicollinearity problem is to increase sample size. However, researchers typically use two-pass sort to avoid such orthogonal issues because stock market data are limited and thus the sample size cannot be increased beyond the current observation date. Once stocks are assigned to a size portfolio, the stocks in the size deciles are again assigned into one of beta portfolios. The correlation between size and beta disappears through the two-pass sort. Those beta-sorted portfolios in size deciles are not related to size. I will illustrate the efficacy of sequential sorting method using the Fama - French (1992) findings. In each size portfolio, beta and size do not have any correlation and beta cannot explain the variation of expected returns. Fama-French found that market beta cannot explain expected returns on NYSE, AMEX, and NASDAQ stocks for 1963-1990. The other 2 factors, size and book equity-to-market equity captured the cross-sectional variation in average stock returns. The average size loading they found was -0.15%. The correlation between size and beta causes unstable factor loadings.

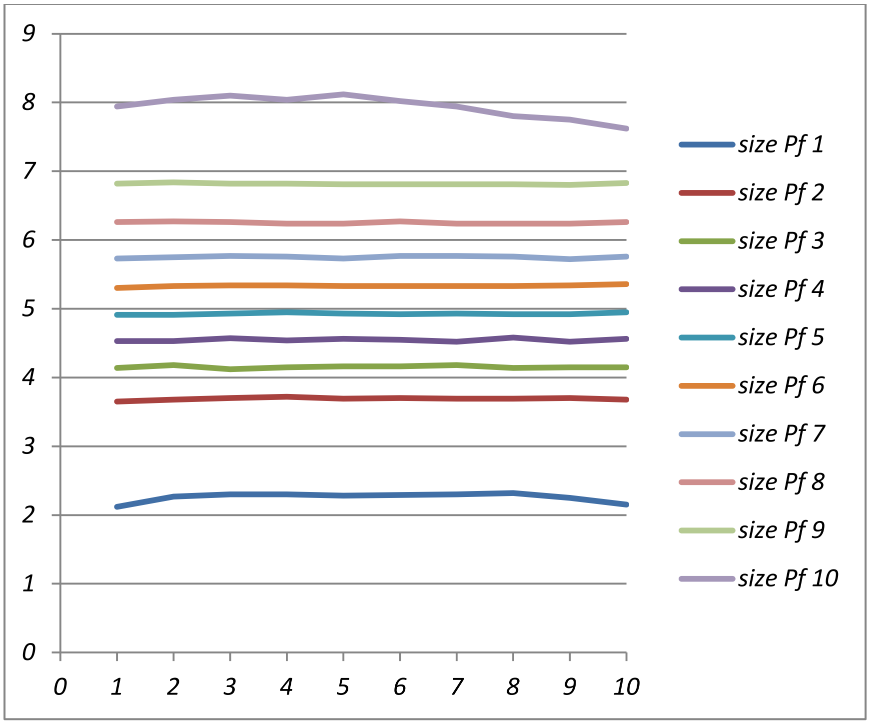
One group of researchers cast doubt on CAPM itself. If CAPM was misspecified, then excess returns were also miscalculated and thus the excess returns will disappear or reduce to insignificant amount. Chan, Chen and Hsieh (1985) used Arbitrage Pricing Model (APT) and compute arbitrage returns which were compute by shorting the largest portfolio (top 5 %) and going long the smallest portfolio (bottom 5%). About one to two percent of arbitrage returns a year were captured by APT while 11.5% of arbitrage returns were captured by CAPM. They showed that the size effect can be reduced to insignificant level.

Reinganum (1981) found that earnings-price effect disappeared after returns were controlled for the firm size. The firm size effect largely subsumed the earnings/price effect. He insisted that CAPM was misspecified due to lack of missing factors. He concluded that those missing factors were more closely associated with firm size and earnings-price ratios.

Table 21 Fama-French (1992) Size vs. Beta in Size Portfolio

sort	β_1 pf	β_2 pf	β_3 pf	β_4 pf	β_5 pf	β_6 pf	β_7 pf	β_8 pf	β_9 pf	β_{10} pf
size1 Pf	2.12	2.27	2.3	2.3	2.28	2.29	2.3	2.32	2.25	2.15
size2 Pf	3.65	3.68	3.7	3.72	3.69	3.7	3.69	3.69	3.7	3.68
size3 Pf	4.14	4.18	4.12	4.15	4.16	4.16	4.18	4.14	4.15	4.15
size4 Pf	4.53	4.53	4.57	4.54	4.56	4.55	4.52	4.58	4.52	4.56
size5 Pf	4.91	4.91	4.93	4.95	4.93	4.92	4.93	4.92	4.92	4.95
size6 Pf	5.3	5.33	5.34	5.34	5.33	5.33	5.33	5.33	5.34	5.36
size7 Pf	5.73	5.75	5.77	5.76	5.73	5.77	5.77	5.76	5.72	5.76
size8 Pf	6.26	6.27	6.26	6.24	6.24	6.27	6.24	6.24	6.24	6.26
size9 Pf	6.82	6.84	6.82	6.82	6.81	6.81	6.81	6.81	6.8	6.83
size10 Pf	7.94	8.04	8.1	8.04	8.12	8.02	7.94	7.8	7.75	7.62

Figure 13 Fama-French (1992) Size –Beta Relation in Size Portfolios



Some researchers attribute small firm effects to market micro-structure which cause biased estimations of beta. Roll (1981) proved that nonsynchronous trading leads to lower beta and small firms are traded less frequently than big firms. The biased beta of small firms will result in higher excessive returns. However, Reinganum (1982) refuted Roll's conjecture. The magnitude of the bias was too small to explain the firm size effect. Amihud and Mendelson (1989) argue that illiquidity of small firms requires higher compensation of the investors. Less liquid assets usually have higher bid-ask spread than hot selling assets do.

Blume and Stambaugh (1983) found that the higher transaction cost of small firms requires higher compensation. Size effect based on daily returns was biased and full- year size effect was half of the size effect based on daily returns. Most of the full-year size effect was due to the January effect. In sum, the size effect does exist on short term scale but disappears on long term scale.

Macaulay first introduced the duration to capture the risk of unexpected interest rate changes. He defines the duration as the weighted average of maturity where C is cash flows at each time and y is the continuously compounded yield-to-maturity.

$$\boldsymbol{Duration} = \frac{\sum \boldsymbol{t C_t e^{-yt}}}{\sum \boldsymbol{C_t e^{-yt}}}$$

Fisher (1966) showed that the derivative of the logarithm of discounted cash flow is equal to Macaulay's duration. Cox, Ingersoll and Ross (1979) assumes that the interest rate differential follows random walk where μ is steady-state mean and β is the speed of returning to mean state.

$$dr = \beta (\mu - r)\, dt + \sigma r^{0.5} dz$$

$$D \cdot R_2 = \Sigma t R_1(t;r) C(t) P(t) / \Sigma C(t) P(t)$$

$$^{*}R = YTM \text{ (yield –to-maturity)}$$

The second term is a heteroskedastic error. Empirical evidence shows that interest rate fluctuations tend to be greater in periods of higher interest rates. Cox, Ingersoll and Ross (1979) derive stochastic duration where a bond with a long duration is not necessarily affected proportionally more than a bond of short duration. YTM of short duration bond may change more as internal growth rates shift.

Lettau and Wachter (2007) explain value premium by using duration effect. They find that growth firms covary more with the discount rate than do value firms. Cash flow are further in time horizon for the growth firms and their price is more sensitive to the change in discount rates. However, they use aggregate dividend flow in the market. Campbell and Vuolteenaho (2004) also find that growth stocks have greater betas with respect to discount rates. Their result are consistent with duration-based explanation of Lettau and Wachter (2007)

Lakonishok, Schleifer and Vishny (1994) found that it takes 11 years for glamour stocks to yield the same amount of cash flow per $1 invested as value stocks do. They assert that glamour portfolio is suboptimal choice for investors because they need to wait for 11 years in the market. However, if duration effect is factored in, the price of glamour stocks is not overvalued because the cash flow is further in the future and the present value is very sensitive to the change in discount rates. Favorable market condition can boost up the price of glamour stocks rapidly.

Table 22 Excess Returns over the CAPM[7]

Deciles	# firms	% of total cap	compound returns	Beta	Excess returns
Largest	168	61.64%	9.60%	0.91	-0.36%
2	179	13.81%	11.00%	1.04	0.65%
3	198	7.24%	11.35%	1.1	0.81%
4	184	4.02%	11.31%	1.13	1.03%
5	209	3.17%	11.69%	1.16	1.45%
6	264	2.76%	11.79%	1.18	1.67%
7	291	2.15%	11.68%	1.23	1.62%
8	355	1.83%	11.88%	1.28	2.28%
9	660	1.92%	12.09%	1.34	2.70%
Smallest	1,744	1.47%	14.03%	1.41	6.27%
Total	4,252	100.00%	10.31%	1	0.00%

[7] Siegel adapted the original data from SBBI 2007 yearbook

Chapter 7

Exogenous Factor in Firm Size

As I pointed in the previous chapter, size itself is a confounding factor. We need an instrumental variable which is related to the firm size, but less related to error terms which cause unstable factor loadings. I will apply "duration" to explain firm size effect.

I assume that a firm's growth pattern follows the Brownian motion with drift. Firm age, not calendar months, will be used in my investigation. I will use a cohort approach rather than time series approach because firm's growth pattern and its expected returns are now being investigated. Once I extract the exogenous factor, I will compare the slope with the original size loadings. If the slope is of the same sign, then we can use the exogenous factor to further analyze the size anomaly. The correlation between firm size and the exogenous factor will be examined. Higher correlation level justifies the use of the new factor as an instrument variable for firm size.

If a firm's growth follows the Brownian motion with drift, then we can write a stochastic differential equation as below.

$$\boldsymbol{\delta} \mathbf{x} = \boldsymbol{\alpha} \mathbf{x} \boldsymbol{\delta} \boldsymbol{t} + \boldsymbol{\sigma} \mathbf{x} \boldsymbol{\delta} \boldsymbol{z}$$

x: firm size

t: firm age

δt: finite interval

α: drift term (continuous growth rate)

σ: standard deviation

δz: increment of a Wiener process

If the percentage changes in **x**, $\frac{\boldsymbol{\delta}\mathbf{x}}{\mathbf{x}}$, are normally distributed, then, $\boldsymbol{\delta}\mathbf{x}$ is log-normally distributed.

F(x) = ln x

$dF = (\alpha - \frac{1}{2}\sigma^2)*\boldsymbol{\delta} t + \sigma*\boldsymbol{\delta} z$

Over the finite interval *t*, dF is normally distributed with mean $(\alpha - \frac{1}{2}\sigma^2) * t$ and variance $(\sigma^2 t)$. The expected stock prices at time *t* can be written as

$E(x_t) = x_{0*} e^{\alpha t}$

The log size, $\ln \mathbf{x_0}\, e^{\alpha t}$, is 'poorly adjusted' for the general price level. Using the logarithm of size solves two issues. One is the issue of linearization between two different scales, % returns and dollar amount. The other is the inflation issue. The loading on log size will not be affected by the inflation level and only the intercept will be changed. However, the initial public offering issue is still left after we use logarithm of size. Some firms are big only because they started big while other firms started small but have grown fast over a long time. The logarithm of size needs to be adjusted for the I.P.O effect. I subtracted initial size from the current size $x_0 e^{\alpha t}$.

$$\ln \mathbf{x_0} e^{\alpha t} - \ln \mathbf{x_0} = \alpha t$$

The initial log size, $\ln \mathbf{x_0}$, plays a part as a constant in the regression model and it will not affect the slope and its significance. The size factor is now transformed to the cumulative continuous growth rates αt. I expect the inverse relation between the expected returns and 'αt' to justify the transformation above. I will also perform a variance ratio test to see if the transformation satisfies the random walk assumptions. I assumed that a firm's growth follows the Brownian motion with drift. If the logarithm of stock price (X) follows random walk process, the variance of $[X_n - X_0]$ is half of the variance of $[X_{2n} - X_0]$. The size difference $[X_n - X_0]$ of firm *i* is 'αt_i' factor as below.

$$X_{t,i} - X_{0,i} = \ln x_{0,i} e^{\alpha t} - \ln x_{0,i} = dF_i = \alpha t_i$$

i: *i* th firm

The loading on 'αt' is expected to be different from the original risk premium in size. In the next chapter, I will run a regression analysis to find out if the transformation can be justified empirically.

Chapter 8

Methodology

I used all the common stocks, 22,904 firms, traded on the New York Stock Exchange (NYSE), the American Stock Exchange (AMEX), and the over-the-counter market (NASDAQ) from 1926 to 2009.

First, I perform a variance ratio test on the exogenous factor to see if the random walk assumption is met. I compute the variance of 'αt' of all the firms at every age status. The variance of $\boldsymbol{\alpha t}$ must increase in a linear way as a firm's age increases. I assumed that a firm's growth follows the Brownian motion with drift. If the logarithm of stock price (X_t) follows a random walk process, the variance of $[X_n - X_0]$ is half of the variance of $[X_{2n} - X_0]$. The size difference $[X_n - X_0]$ of firm i is 'αt_i' .

Second, I regress monthly expected returns on the firm size to find out if the size factor alone can explain the expected returns for the period from 1926 to 2009. I used a pure cross-sectional regression with a firm level approach. The regression model is given below.

$$E(\tilde{\mathbf{r}}_{\mathbf{t+1}}) = \mathrm{c} + \beta * \ln(\mathrm{ME}_{t,i})$$

Third, I compute the cumulative continuous growth rates, αt_i, of each firm every month and used the time series of αt_i in the regression analysis.

$E(\tilde{\mathbf{r}}_{\mathbf{t+1},\boldsymbol{i}}) = \mathrm{c} + \beta * \alpha \mathrm{t}_{\mathrm{t},i}$

Last, I decomposed $\boldsymbol{\alpha}\mathbf{t}$ into the continuous growth rate (α_t) and firm age (t). I ran two separate regressions to find out the relation between the expected returns and the two factors.

$E(\tilde{\mathbf{r}}_{\mathbf{t+1}}) = \mathrm{c} + \beta * \alpha_t$

$E(\tilde{\mathbf{r}}_{\mathbf{t+1}}) = \mathrm{c} + \beta * \mathrm{Age}$

Chapter 9

Results

9.1 Size and Cumulative Growth Rates

Considering $\boldsymbol{\alpha}$ is the drift term, 'αt' (cumulative continuous growth rates) must increase in a linear way as the firm age increases. At every age status, I compute 'αt' for each firm and averaged all 'αt'. $E\,(\boldsymbol{\alpha t})$, average cumulative continuous growth rates, increases in linear way as firm age increases. The regression result is given below. The relation between firm age and average growth rates is linear with R^2 of 0.99.

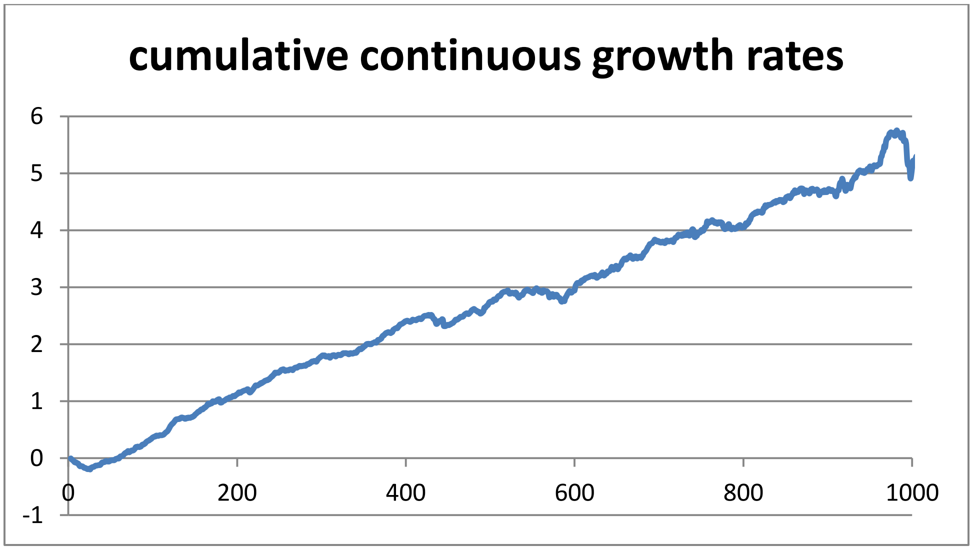
$$E\,(\boldsymbol{\alpha t}) = -0.0440 + 0.0054^{8}\cdot\text{Age}$$

I compute the variance of αt of all the firms at every age status. As firm age increases, the mean 'αt' increases and the variance of 'αt' also increases. The difference between the popular variance ratio test and mine is that 't' is not the time but firm age. I run a regression to see if there is a linear relation between firm age and variance. According to the graph below, there's a long- run positive relation between the age (t) and the variance of $\boldsymbol{\alpha t}$. The regression result is given below.

[8] t- statistics =319

Variance of $\boldsymbol{\alpha t} = \mathbf{1.47} + \mathbf{0.0034} * \mathbf{Age}$ $(\mathbf{t\ stat} = \mathbf{79})$

Figure 14 Firm Age in Month vs. Cumulative Continuous Growth Rates



According to the graph above, the variance ratio satisfies the random walk process in some intervals but fails in other intervals, depending on the measurement intervals. The variance does not change significantly during firm age 200 - 500 months.

The size factor explains the expected returns with a slope of - 0.254 % and t - statistics of - 52.83. As the log firm size increases by a unit, the expected returns decreases by -0.254%.

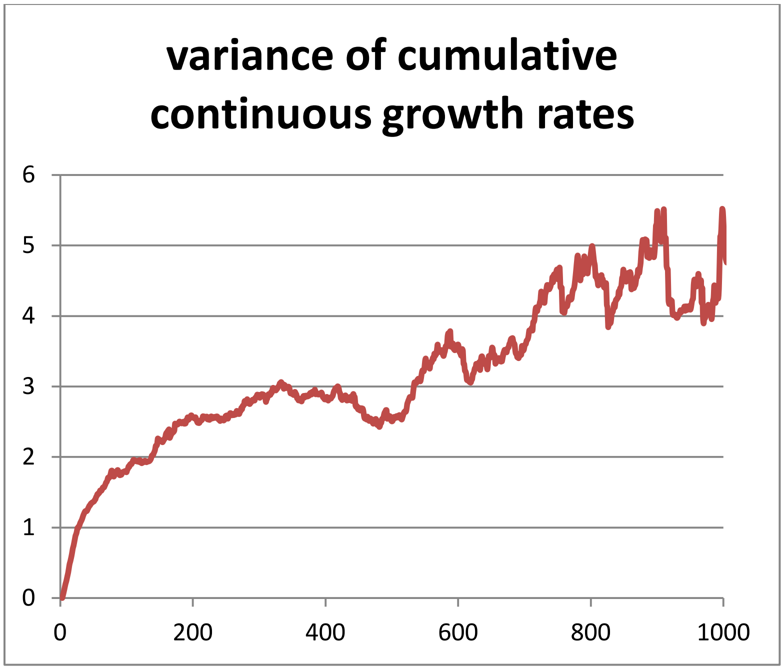
$E(\tilde{\mathbf{r}}_{\mathbf{t+1}})$ = 4.002% - 0.254% * ln ($ME_{t,i}$) (t = -52.83)

Also, the cumulative continuous growth rate, αt, can explain the expected returns with the slope of - 0.331% and the t-statistics of - 53.47.

$$E(\tilde{\mathbf{r}}_{\mathbf{t+1}}) = 1.445\% - 0.331\% * \alpha t \qquad (t = -53.47)$$

The simple regression result empirically justifies the replacement of size with the cumulative continuous growth rates. Both the size factor and the continuous growth rate factor have negative slopes, but the magnitude of the loadings has a monthly difference of 0.077%. Continuous growth rates show a steeper slope. The t-statistics close to each other imply that the distribution of the two slopes has almost the same standard errors. The correlation between firm size and cumulative continuous growth rates is 0.689. The correlation level justifies the use of cumulative continuous growth rates as an instrument variable for firm size.

Figure 15 Firm Age vs. Variance of at



The cumulative continuous growth rates 'αt' is a product of two variables, α and t. I run two separate regressions below to see if either variable has a negative relation with the expected returns.

$$E(\tilde{\mathbf{r}}_{\mathbf{t+1},i}) = c + \beta * \alpha_{t,i} = 1.224\% - 7.351\%^{9} * \alpha_{t,i}$$

$$E(\tilde{\mathbf{r}}_{\mathbf{t+1},i}) = c + \beta * \text{Age} = 1.14\% + 0.000419\%^{10} * \text{Age}_{t,i}$$

The comparison table is given below.

[9] t=-26.35
[10] t=6.46

Table 23 Slope on Size, Cumulative Growth Rates, Growth Rates and Age

Explanatory Variable	Parameter Estimate	t-stat
Firm Size [ln ME]	-0.254%	-52.83
Cumulative Continuous Growth Rate (αt)	-0.331%	-53.47
Continuous Growth Rate(α)	-7.351%	26.35
Firm Age(t)	0.000%	6.46

Firm age has a positive relation with the expected returns but its magnitude is negligible. The cumulative continuous growth rates are negatively related to expected returns. If continuous growth rates increase by 1%, investors can expect -0.07 % of lower expected returns in next month.

9.2 Non-Linearity in Size Effect

Non-linearity in size effect is another criterion which must be met. According to "*Stocks, Bonds, Bills and Inflation (SBBI) 2007 yearbook*", the non-linear excess returns have existed since 1926. The size effect is not linear with the ranked firm size and so simple linear regression might not be the best way to express the relation between portfolio size and its expected returns.

I used all common stocks traded on the New York Stock Exchange (NYSE), the American Stock Exchange (AMEX), and the over-the-counter market (NASDAQ) from 1926 to 2009. I sorted stocks by log size rank and assigned them to 10 size portfolios every month. After calculating the arithmetic average of firm returns in each portfolio, I regress portfolio expected returns on portfolio sizes. The process is repeated for 1,003 months. I also did the same analysis on cumulative growth rates and expected returns. Market risk was not controlled to examine the gross size effect. The main focus of the chapter is to compare the size effect with the internal growth rates of firms. The implication of CAPM is that the risk and the rewards must have a linear relation. If CAPM holds, the non-linear size effect will not be affected whether we add beta or not in the equation. If CAPM does not hold, adding beta will cause more serious errors.

The results show that a non-linear small firm effect has existed for the past 84 years in the U.S market. The small firm effect is most clearly seen on the smallest 10% firms. If we had built an arbitrage portfolio by selling the top 10% of large firms and buying the bottom 10% of small firms, we might have earned a 6.69% of arbitrage returns. As Banz pointed out, the small firm effect is not linear with respect to firm size. Smallest 10% of firms perform better than any other size portfolio and the other 9 portfolios did not show any significant patterns.

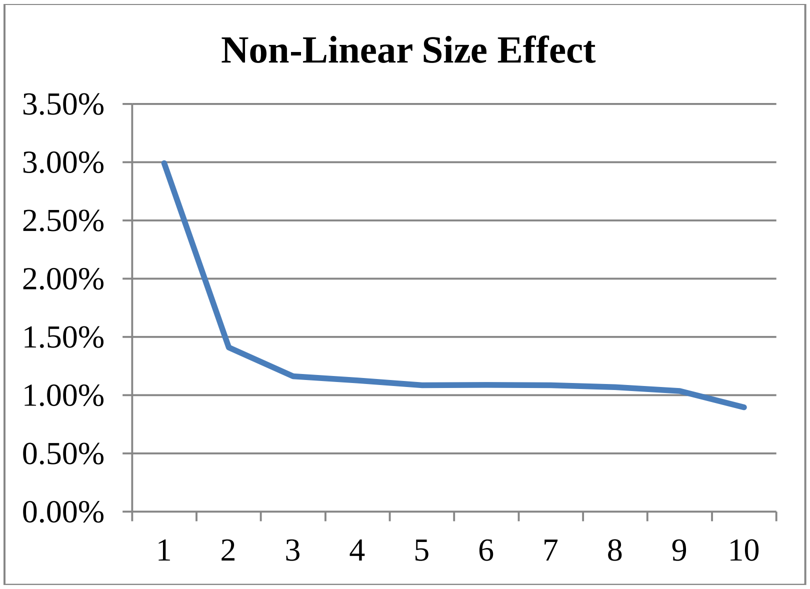
Table 24 Size Portfolios and Average Returns

Size Portfolio	Month	Pf return	Std Dev
Smallest	1003	2.99%	12.16%
2	1003	1.41%	9.57%
3	1003	1.16%	8.78%
4	1003	1.13%	8.09%
5	1003	1.09%	7.69%
6	1003	1.09%	7.39%
7	1003	1.09%	7.14%
8	1003	1.07%	6.70%
9	1003	1.04%	6.22%
largest	1003	0.90%	5.47%

Table 25 Growth Portfolio and Average Returns

Growth Portfolio	Months	Pf Return	Std Dev
1	1002	2.82%	13.16%
2	1002	1.40%	9.92%
3	1002	1.16%	8.68%
4	1002	1.25%	7.83%
5	1002	1.20%	7.19%
6	1002	1.10%	6.94%
7	1002	1.07%	6.69%
8	1002	1.00%	6.36%
9	1002	1.01%	6.11%
10	1002	0.94%	5.88%

Figure 16 Size Portfolio and Expected Returns



Using cumulative growth rates also give similar result. The non-linear pattern is observed in the graph. Only the lowest cumulative continuous group performs shows higher returns for the past 1,002 months. The other groups show a flat pattern. If we had built an arbitrage portfolio by selling the top 10% of cumulative growth rate firms and buying the bottom 10% of cumulative growth rate firms, we might have earned a 1.89% returns.

The size effect occurs only in the smallest group and non-linear pattern appears as a result. The cumulative continuous growth rates meet the requirement of non-linear return patterns.

Figure 17 Growth Portfolio and Expected Returns

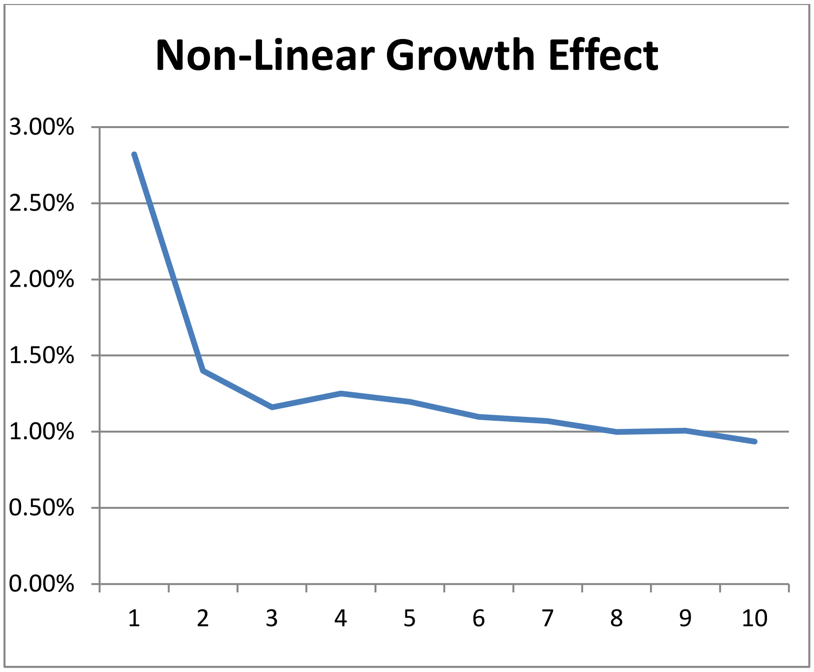
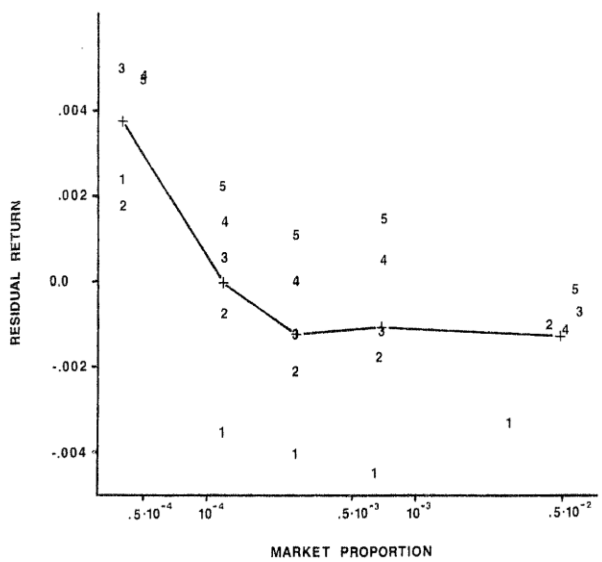


Figure 18 Banz : Non-Linear Small Firm Effect



9.3 Growth Rates and Firm Age

Considering αt has negative loadings, I need to investigate the relation between firm age and continuous growth rates. The graph below shows the relation between the firm life span (X axis) and the firm's life time growth rates (Y axis). Short-lived firms have higher variances. As firms mature, their growth rates converge to a narrower band. The firm average growth rates show strong reversion to mean as firms live longer regardless of firms industry classification.

I examined the relation between the continuous growth rates and their standard deviation of all the firms during 1926 to 2009. The pattern is close to horizontal line where we cannot find any linear relation. Higher growth rates do not necessarily mean higher standard deviation. However, the relation between the firm's life and the firm's life time standard deviation of continuous growth rates makes a convex curve. As the firm ages, the standard deviation of continuous growth rates decreases with decreasing speed.

The graph shows the relation between the firm's age (X axis) and the firm's life time average monthly returns (Y axis). As the firm matures, their life time mean returns converge to narrow band. Newly listed firms under 60 months have higher variances in returns. A firm's average returns shows strong reversion to mean as the firm lives longer.

The relation between the firm life and firm's life time standard deviation of returns shows a convex curve. As the firm ages, the standard deviation decreases with decreasing speed.

Figure 19 Firm Life Span and Continuous Growth Rates

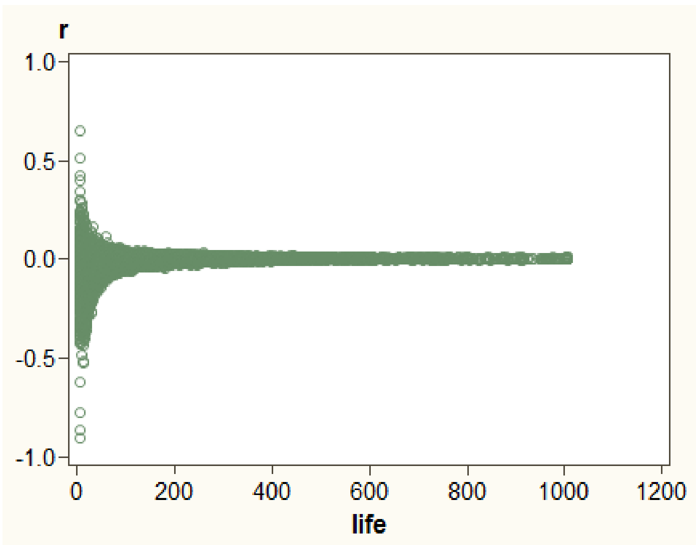


Figure 20 Firm Life Span and Standard Deviation of Growth Rates

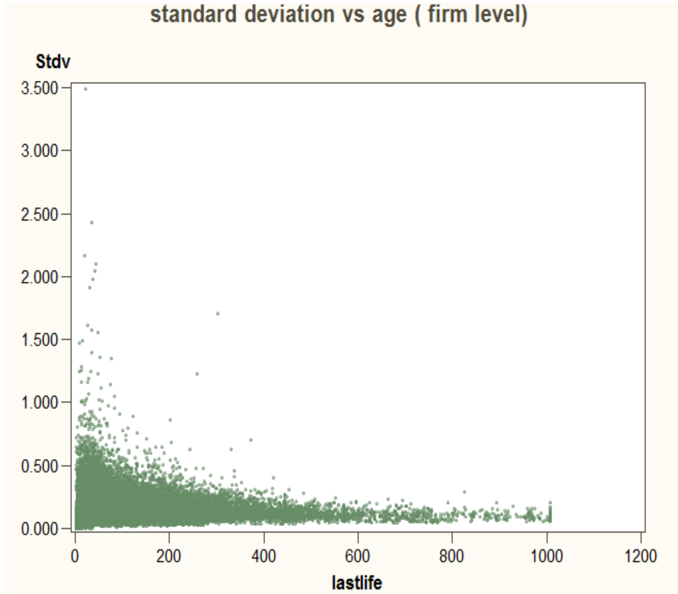


Figure 21 Firm Life Span and Average Monthly Returns

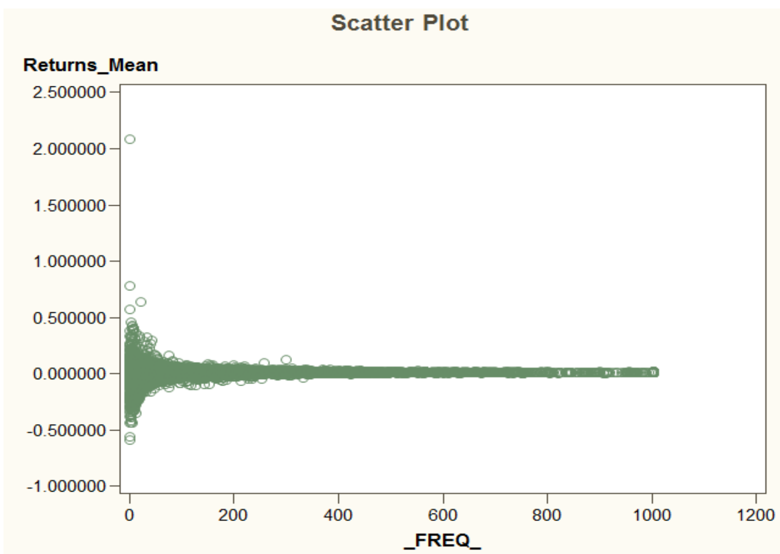
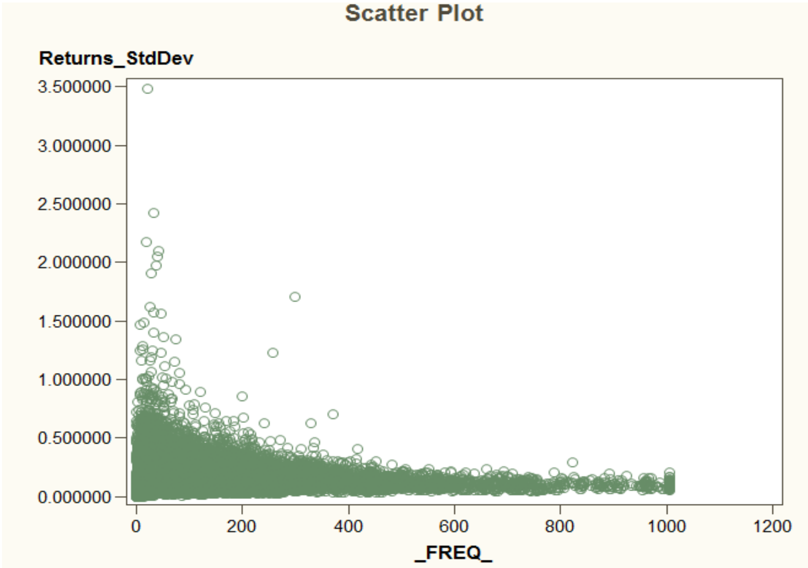


Figure 22 Firm Life Span and Standard Deviation of Returns



9.4 Age Portfolio and Their Characteristics

As researchers used portfolio approach to better measure beta, I also employed a portfolio approach to overcome the errors-in-variable problem. I assigned firms into age portfolios by their age rank. The youngest 10% of firms constitute portfolio 1 and the oldest 10% make up portfolio 10. I eliminated any firm which had less than 12 month of trading history.

I measured a portfolio's monthly beta by averaging all firms' betas in the portfolio without any size weight. Firm beta is measured over the whole life. The number of firms in each age portfolio is evenly distributed.

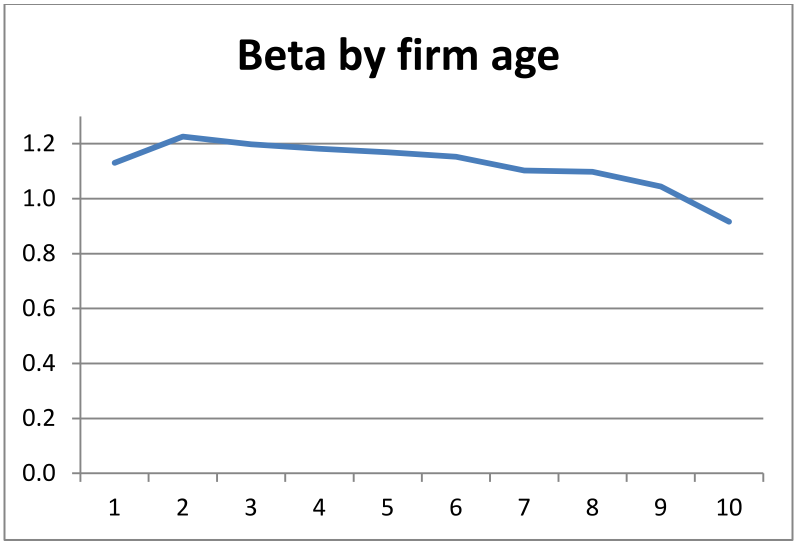
Long-lived firms have smaller betas than short-lived firms. The distribution of beta over the gamut of portfolios is very narrow. The reason for a small range of beta variation is that each portfolio is made to hold around 10% of whole firms. Younger firms constitute most of the portfolios and biggest firms are all sorted to the 10^{th} portfolio. The long-lived firms have higher average returns and continuous growth rates proportionally. The arithmetic average returns and the continuous growth returns converge as the firm ages. Also the firm size increases as the firm age increases. The variance of monthly returns decreases steeply as portfolio age increases. The long-lived firms are less volatile compared to short-lived firms.

I did a cross-sectional analysis on monthly returns and standard deviation. The single stock approach did not give any clear relation between standard deviation and returns, but the portfolio approach yielded a negative curvilinear relation. It is against our basic assumption that the risk and its reward are positively related.

Table 26 Age Portfolios and Their Beta

Age Portfolio	Age Range	Number of Firms	Portfolio Beta	Std Dev
1	12 months ~ 24 months	2176	1.13	1.52
2	25 months ~ 37 months	2248	1.23	1.15
3	38 months ~ 52 months	2248	1.20	0.97
4	53 months ~ 68 months	2113	1.18	0.85
5	69 months ~ 92 months	2242	1.17	0.77
6	93 months ~ 122 months	2195	1.15	0.70
7	123 months ~ 159 months	2235	1.10	0.59
8	160 months ~ 212 months	2210	1.10	0.58
9	213 months ~ 321 months	2207	1.04	0.51
10	322 months ~ 1006 months	2203	0.92	0.37

Figure 23 Age Portfolio and Beta



Chapter 10

Firm Size and Duration

The cumulative continuous growth rate, αt, is inversely related to the expected returns. The continuous growth rate 'α' has a negative relation to the expected returns. The cumulative continuous growth rate, 'αt', is related to growth factor and age factor at the same time, but each has different loadings. I need to investigate the exogenous factor 'αt' as a single inseparable factor.

Macaulay's duration assumes that cash flow level, C_t, is fixed. The formula for the duration is presented below.

$$\text{Macaulay duration D} = \sum_{t=1}^{n} t \cdot \frac{\frac{C_t}{e^{\alpha t}}}{S_0}$$

D: Duration

T: Time

C_t: Cash flow at time t

S_0: Bond price at time zero

α: Internal yield

I make the following assumptions about firm cash flow.

1) Firms are constantly growing entity with limited life, not an eternal going concern, and investors believe that long-lived firms are likely to outlive short-lived firms.

2) The amount of cash flow grows continuously

Firm's cash flow increases continuously and the present size of a firm is given as the function of initial cash flow, discount rates and growth rates.

$$y = f\,[C, \alpha, r] = \frac{C}{e^{r}} + \frac{Ce^{\alpha}}{e^{2r}} + \frac{Ce^{2\alpha}}{e^{3r}} + \frac{Ce^{3\alpha}}{e^{4r}} + \cdots \frac{Ce^{(n-1)\alpha}}{e^{nr}} = \frac{\frac{C}{e^{r}}[1-(\frac{e^{\alpha n}}{e^{r}})}{1-\frac{e^{\alpha}}{e^{r}}}$$

$$= \frac{C[1-\left(\frac{e^{\alpha n}}{e^{r}}\right)]}{e^{r}-e^{\alpha}}$$

r: discount rates at time zero

α: continuous growth rates (internal yield)

C: cash flow

Then duration can be written as below

$$D=\sum_{t=1}^{t=n} t\,\frac{\frac{Ce^{\alpha(t-1)}}{e^{rt}}}{\frac{C\left[1-\left(\frac{e^{\alpha n}}{e^{r}}\right)\right]}{e^{r}-e^{\alpha}}} = \sum_{t=1}^{t=n} t\,\frac{(e^{r}-e^{\alpha})e^{\alpha(t-1)}}{e^{rt}\left[1-\left(\frac{e^{\alpha n}}{e^{r}}\right)\right]} = \frac{(e^{r}-e^{\alpha})}{\left[1-\left(\frac{e^{\alpha n}}{e^{r}}\right)\right]}\sum_{t=1}^{t=n} t\,\frac{e^{\alpha(t-1)}}{e^{rt}}$$

pull out the $\sum_{t=1}^{t=n} t\,\frac{e^{\alpha(t-1)}}{e^{rt}}$ as below

$$\sum_{t=1}^{t=n} \mathbf{t}\frac{e^{\alpha(t-1)}}{e^{rt}} = \frac{1}{e^{r}} + \frac{2e^{\alpha}}{e^{2r}} + \frac{3e^{2\alpha}}{e^{3r}} + \cdots \qquad ①$$

then multiply $\frac{e^{\alpha}}{\frac{(e^{r}-e^{\alpha})}{\left[1-\left(\frac{e^{\alpha n}}{e^{r}}\right)\right]}e^{r}}$ to each side

$$\frac{(e^{r}-e^{\alpha})}{\left[1-\left(\frac{e^{\alpha n}}{e^{r}}\right)\right]}\sum_{t=1}^{t=n} \mathbf{t}\frac{e^{\alpha(t-1)}}{e^{rt}} = \frac{e^{r}-e^{\alpha}}{e^{r}-e^{\alpha n}} + \mathbf{2}\ \frac{(e^{r}-e^{\alpha})}{e^{r}-e^{\alpha n}}\mathbf{e}^{(\alpha-r)} + \mathbf{3}\ \frac{(e^{r}-e^{\alpha})}{e^{r}-e^{\alpha n}}\ \mathbf{e}^{(\alpha-r)} + \ \ldots ②$$

$$\sum_{t=1}^{t=n} \mathbf{t}\frac{(e^{r}-e^{\alpha})}{e^{r}-e^{\alpha n}}\ \mathbf{e}^{(\alpha-r)} = \frac{n(n+1)}{2}\frac{(e^{r}-e^{\alpha})}{e^{r}-e^{\alpha n}}\ \mathbf{e}^{(\alpha-r)}$$

The relative changes in size can be written as a product of duration and the change in interest rates as below.

$$\frac{d\mathrm{P}_0}{\mathrm{P}_0} = -\frac{\mathrm{D}}{1+r}\cdot dr = -\frac{n(n+1)}{2}\frac{\frac{(e^{r}-e^{\alpha})}{e^{r}-e^{\alpha n}}e^{(\alpha-r)}}{1+r}\cdot dr = -\frac{n(n+1)}{2}\frac{(e^{r}-e^{\alpha})e^{(\alpha-r)}}{(1+r)(e^{r}-e^{\alpha n})}\cdot dr$$

The percentage change in stock price, $\frac{d\mathrm{P}_0}{\mathrm{P}_0}$, can be interpreted as an arithmetic return which we can see in CRSP database. Its relation to the duration is negative. Even though we don’t know the true discount rate, r, we know the relation between αt and the expected returns. As α or n increases, the duration gets smaller. Bigger firms have bigger αt and, as a result, a smaller duration is derived. If small firm effect is the compensation for higher risk, then we can interpret the risk in terms of duration. Firm duration can be interpreted in two ways. First, it is the size sensitivity to the change in the discount rates. If a firm has yielded sound and continuous internal growth rates over a longer period, the market believes that the firm will have higher ‘αt’ factor in the future. Higher ‘αt’ carries messages that the firm will survive for long times with higher growth rates in the future. Any change in market environment will influence the discount rates but higher ‘αt’ firms will have smaller change in size than lower ‘αt’ firms. The sensitivity of stock price to unexpected change in discount rates will be smaller. Second, duration is the immunization point. Investors will suffer from uncertainty of wealth level until the immunization point is reached. The change in the discount rates carries two pieces of information. If the discount rate goes up, then the present size will be smaller but at the same time, investors will have better reinvestment opportunities. If the discount rate goes down, then the present size will be bigger but reinvestment will produce inferior returns.

Investors can restore the target wealth level in two ways. First, investors can use the near-future cash flow, dividend, from the smaller duration securities to reinvest in other stocks. Second, they can create home-made dividends by selling smaller duration stocks. The dividend or home-made dividend can be used to buy deep-discounted stocks which have bigger duration. Thus investors can reach the immunization point sooner.

Chapter 11

Separation Theorem and Duration

Tobin (1986) stated that the procedure of mixing risky assets must be separated from the procedure of placing funds into risk-free assets. This was later called the 'Separation Theorem'. Investors can adjust their cash level according to the perceived market risk. Tobin's approach made the investing procedure objective regardless of investor's subjective financial status.

The separation theorem recommends that investors adjust their cash level at every turn of the market. By increasing the portion of risk-free assets, investors can reduce their exposure to the market. Tobin's idea was extended to portfolio insurance in the 1970s. Shorting stocks and buying T-Bills replicates put options. Selling off the delta fraction of a portfolio reduces as much the exposure and keeps the value of a portfolio at a preset level. The synthetic put option can be called an applied separation theorem. However, Tobin's idea ended up as great fiasco due to two major drawbacks. First, delta value changes drastically as the market goes down. As absolute value of delta increases steeply, the naked position increases. To off-set the risk exposure, managers needed to sell off stocks more and more and the market crashed as the herd rushed to the exit at the same time. Positive feedback in the market made portfolio insurance infeasible. Second, mutual funds keep or at best change slightly their original cash level because they cannot tinker with the original prospectus they delivered to individual investors. Also, increasing the portion of risk-free assets in the fund conveys to the market negative messages that fund managers are at a loss for what to do next. The managers will be more concerned about their own evaluation. Tobin's excellent idea hit a dead end at the downturn of the market because of the frictions in real world markets.

During a bear market, investors, including even novice investors, rebalance their risky portfolios. If investors want to take more risk, they can sell less - discounted stocks and switch to deep-discounted stocks. This differs from downsizing a portfolio itself as seen in Tobin's separation theorem. Selling less-discounted stocks generates home-made dividends in a downturn market. Operating earning power is the only legitimate criteria with which investors can judge the true firm value according to Modigliani-Miller (1958). Deep-discounted stocks are believed to revert to their average size eventually except when the duration of the down market is long enough to end up resulting in Great Depression during which many firms will belly up. Many investors choose stocks in view of 'returns on investment', but also they consider another side, 'returns of investment'. The home-made dividend from selling less-discounted stocks offers extra opportunities available in a down market. Investors holding higher 'αt' stocks can be more flexible when they rebalance their portfolios. We can measure the size stability of stocks using duration. If any stocks have smaller durations against the variation of market discount rates, the stocks can be used to generate home - made dividends. As I proved in an earlier chapter, bigger firms have lower durations. Investors can regain their target wealth level sooner if they hold smaller duration assets in their portfolio. Risk means not only higher probability of failure but also higher probability of success at the same time. Duration can capture both sides of risk but beta cannot capture both sides of risk. Understanding the duration side of investing will be of great help at every turn of the market.

Chapter 12

Summary and Conclusion

It is widely known that the test on any market efficiency model raises the joint hypothesis issue. The test on the CAPM must yield consistent results regardless of the time periods or varying interval because reliability is a prerequisite for the validity of any model. However, beta is not time-invariant and dependent on the scales. I reverse-engineered the way people use beta by finding the best hitting ratio of beta. I varied the estimation period and scales to find out the best instrument beta. Four year beta gives highest hitting ratio of 77%. Quarterly scale gives the best hitting ratio in four year estimation period but semi-annual scale gives the best hitting ratio in 10 year estimation period. The hitting ratio satisfies the reliability criteria. Finance is not physics and such a high hitting ratio is useful investment tool in the stock market.

The magnitude of change in beta value gets bigger as the interval lengthens. Betas change their value up to an average of 0.613 when I compared a monthly scale with an annual scale. The magnitude of change is very big considering the small range of original beta value. There is a significant ratio of switch from the aggressive to the defensive or vice versa when I compared betas on various scales. 29% of the stocks switched their label when I compared monthly betas with annual betas. The two most active portfolios, the 9th portfolio and the 10th portfolio, have 29% and 22% of switch ratio inside each portfolio. The traditional dichotomy heavily depends on scale as well as the sensitivity of the assets to market movement. Portfolio beta also changes its value. The risk premium is not proportional along the scales. Annual scale yields smaller risk premium when compared to monthly scale. The market seems to be non-fractal in terms of beta measurement over various scales. Beta index does not satisfy reliability criteria. We need to approach the market with a different set of perspectives – a distant view or a close view. The scales give us totally a different frame of investment.

The size effect is confounded with endogeneity issue because the riskier firms, ceteris paribus, must have a smaller size because the present value of firms depends on the risk level. I extract the exogenous variable from the logarithm of firm size. I assume that a firm's internal growth follows the geometric Brownian motion and compute cumulative continuous growth rates. I test the characteristics of this new factor and find that it can replace the firm size. Contrary to the decomposition of beta into cash flow factor and discount factor, I combine those two risks into one concept, duration. Bigger firms have a bigger cumulative continuous growth rate (αt) which induces a smaller duration. Smaller duration firms can yield bigger home-made dividends and enable the investors to rebalance their portfolios from unexpected changes in the market risk. Small firms have higher duration and vulnerability to the change in discount rates. The size effect is related not only to market risk but also to the reinvestment risk, which cannot be captured by beta. Duration can capture the market risk and reinvestment risk at the same time.

Appendix A

Mean &Variance Test of αt

The table below presents the average cumulative continuous growth rates (at). The data covers all the stocks in N.Y.S.E, AMEX and NASDAQ from 1926 to 2009. Each firm's cumulative continuous growth rates are computed along the firm age. Then the arithmetic average of cohort group are computed. The average cumulative continuous growth rates of cohort group converge as firms stay longer in stock market.

Table A-1 Mean & Variance of at

Age	at	st.dev	sample	Age	at	st.dev	sample
4	-1.8%	19.0%	22,625	506	277.7%	160.4%	720
5	-3.0%	27.5%	22,527	507	278.5%	160.6%	713
6	-4.1%	34.3%	22,431	508	281.6%	160.5%	711
7	-6.1%	40.2%	22,320	509	283.4%	160.0%	709
8	-7.2%	45.2%	22,229	510	285.1%	160.4%	707
9	-6.8%	48.5%	22,099	511	284.5%	161.1%	701
10	-8.1%	52.7%	22,003	512	285.6%	160.8%	698
11	-8.6%	56.7%	21,891	513	288.0%	160.6%	695
12	-9.5%	60.4%	21,746	514	290.4%	159.3%	691
13	-12.4%	65.1%	21,552	515	291.3%	159.1%	686
14	-13.9%	69.3%	21,371	516	292.2%	160.1%	682
15	-13.3%	71.7%	21,228	517	292.5%	161.0%	679
16	-13.7%	74.5%	21,067	518	292.6%	162.1%	676
17	-14.1%	77.4%	20,929	519	293.1%	162.4%	674
18	-15.5%	80.7%	20,743	520	293.3%	162.9%	668
19	-16.8%	84.0%	20,556	521	294.7%	162.5%	664
20	-16.7%	86.0%	20,380	522	291.1%	163.6%	658
21	-17.4%	88.7%	20,181	523	288.8%	165.8%	654
22	-18.3%	91.5%	19,991	524	290.4%	166.7%	653
23	-19.0%	94.0%	19,808	525	289.4%	167.5%	651
24	-18.0%	95.3%	19,594	526	290.8%	168.1%	652

Table A.1—Continued

25	-18.4%	97.3%	19,383	527	289.8%	168.9%	649
26	-19.5%	99.7%	19,175	528	289.6%	168.4%	645
27	-16.7%	99.7%	19,000	529	288.6%	168.7%	642
28	-15.8%	100.9%	18,841	530	291.1%	168.6%	639
29	-15.1%	101.6%	18,680	531	290.1%	168.5%	634
30	-15.0%	103.1%	18,489	532	285.8%	171.0%	633
32	-13.0%	104.9%	18,167	534	281.9%	174.9%	627
33	-12.5%	106.0%	17,985	535	284.0%	174.4%	621
34	-12.6%	107.4%	17,847	536	285.2%	175.0%	619
35	-12.2%	108.6%	17,657	537	286.8%	174.7%	618
36	-11.7%	109.6%	17,497	538	286.4%	175.7%	614
37	-11.9%	110.5%	17,331	539	288.2%	176.1%	611
38	-11.5%	111.1%	17,167	540	291.9%	176.4%	607
39	-9.2%	111.1%	16,984	541	293.8%	175.6%	605
40	-7.2%	111.0%	16,815	542	293.5%	175.4%	603
41	-7.1%	111.9%	16,683	543	295.7%	175.3%	600
42	-6.7%	112.8%	16,526	544	296.3%	176.7%	598
43	-6.5%	113.6%	16,374	545	295.3%	178.5%	594
44	-5.0%	114.5%	16,219	546	294.2%	179.7%	589
45	-4.4%	114.7%	16,103	547	292.9%	179.6%	587
46	-5.3%	115.7%	15,919	548	292.5%	179.3%	586
47	-5.0%	115.9%	15,795	549	292.2%	180.6%	584
48	-5.6%	116.4%	15,681	550	290.2%	183.4%	580
49	-5.0%	116.6%	15,529	551	290.3%	184.3%	579
50	-3.8%	116.8%	15,382	552	294.6%	182.9%	577
51	-3.3%	117.1%	15,240	553	295.7%	182.8%	576
52	-3.4%	118.0%	15,104	554	297.6%	182.0%	576
53	-3.1%	118.3%	14,942	555	298.6%	182.2%	573
54	-3.4%	119.7%	14,804	556	296.6%	182.6%	572
55	-2.8%	120.3%	14,674	557	295.3%	180.3%	566
56	-1.1%	121.4%	14,516	558	292.6%	182.2%	564
57	-0.7%	121.3%	14,380	559	292.4%	183.0%	561
58	-0.3%	122.1%	14,268	560	293.8%	182.8%	555
59	0.5%	122.2%	14,132	561	290.5%	183.8%	556
60	-0.3%	123.5%	14,001	562	290.9%	184.7%	555

Table A.1—Continued

61	2.0%	123.3%	13,861	563	294.9%	184.1%	554
62	3.7%	123.4%	13,720	564	294.0%	186.2%	557
63	3.8%	124.1%	13,601	565	293.5%	185.3%	550
64	4.3%	125.1%	13,480	566	292.8%	185.8%	548
65	5.2%	125.4%	13,361	567	293.4%	186.0%	546
66	7.2%	125.2%	13,216	568	292.0%	187.9%	547
67	8.8%	126.1%	13,089	569	289.5%	189.6%	546
68	9.8%	126.9%	12,958	570	282.6%	187.1%	492
69	9.8%	128.0%	12,843	571	284.3%	187.2%	490
70	11.8%	128.3%	12,745	572	284.1%	187.0%	485
71	13.0%	129.3%	12,636	573	287.5%	186.9%	484
72	10.6%	130.6%	12,525	574	288.2%	186.7%	483
73	11.7%	130.3%	12,405	575	283.7%	187.5%	482
74	12.5%	130.8%	12,287	576	284.1%	187.4%	479
75	13.9%	131.6%	12,201	577	286.8%	185.8%	477
76	13.8%	132.7%	12,106	578	287.0%	185.3%	476
77	13.9%	134.3%	12,015	579	286.8%	186.4%	476
78	15.3%	134.4%	11,929	580	283.7%	187.2%	472
79	17.0%	133.6%	11,844	581	282.4%	189.0%	470
80	19.9%	131.4%	11,743	582	281.7%	188.6%	468
81	19.8%	131.3%	11,667	583	280.1%	189.5%	465
82	20.8%	132.5%	11,570	584	275.8%	192.7%	464
83	20.5%	133.2%	11,484	585	274.9%	193.8%	461
84	19.7%	133.7%	11,406	586	277.9%	192.9%	462
85	20.5%	133.5%	11,326	587	276.8%	194.0%	462
86	20.7%	134.6%	11,214	588	275.9%	194.5%	459
87	22.8%	134.7%	11,143	589	281.6%	188.8%	455
88	24.3%	133.5%	11,068	590	283.6%	189.1%	453
89	24.1%	132.2%	10,993	591	286.7%	189.1%	451
90	25.2%	132.0%	10,908	592	289.4%	190.0%	446
91	26.1%	132.7%	10,843	593	291.7%	188.1%	445
92	28.1%	132.2%	10,752	594	293.7%	187.7%	443
93	29.4%	132.9%	10,691	595	292.6%	187.5%	441
94	30.4%	133.6%	10,622	596	290.6%	188.9%	439
95	31.0%	133.8%	10,545	597	291.7%	189.1%	434

Table A.1—Continued

96	31.8%	133.9%	10,470	598	293.8%	189.1%	432
97	33.3%	134.1%	10,403	599	294.4%	189.6%	432
98	33.7%	134.2%	10,344	600	294.5%	189.6%	432
99	35.3%	133.7%	10,246	601	302.1%	188.2%	426
100	36.6%	133.6%	10,190	602	305.1%	186.7%	424
101	37.4%	134.8%	10,124	603	307.6%	186.6%	420
102	38.2%	135.4%	10,071	604	308.0%	185.6%	415
103	38.2%	136.7%	9,997	605	308.0%	185.3%	415
104	39.8%	136.4%	9,948	606	307.4%	187.9%	412
105	40.1%	137.5%	9,885	607	308.0%	187.2%	410
106	39.5%	137.9%	9,803	608	311.9%	182.2%	404
107	39.6%	137.7%	9,726	609	310.9%	182.2%	407
108	40.5%	138.2%	9,661	610	313.2%	179.6%	401
109	40.9%	139.0%	9,584	611	312.7%	179.3%	403
110	40.5%	139.8%	9,499	612	315.3%	178.4%	401
111	40.7%	140.0%	9,429	613	316.4%	175.9%	400
112	41.1%	139.5%	9,357	614	316.4%	175.7%	400
113	41.7%	139.5%	9,276	615	317.0%	176.0%	399
114	43.3%	139.3%	9,187	616	318.0%	175.5%	398
115	44.7%	139.5%	9,114	617	318.0%	175.0%	397
116	46.1%	139.1%	9,050	618	319.2%	174.9%	395
117	47.0%	139.6%	8,978	619	320.3%	174.8%	393
118	48.7%	139.5%	8,909	620	320.3%	175.8%	392
119	51.1%	138.9%	8,839	621	320.8%	176.2%	392
120	53.7%	139.9%	8,781	622	319.1%	178.1%	392
121	56.7%	138.8%	8,708	623	320.5%	178.5%	392
122	58.5%	138.5%	8,614	624	322.1%	179.4%	388
123	60.6%	138.3%	8,563	625	319.4%	180.5%	386
124	61.3%	139.1%	8,505	626	316.9%	180.4%	384
125	63.3%	138.7%	8,431	627	317.1%	182.2%	383
126	65.1%	138.9%	8,360	628	319.6%	182.1%	381
127	67.3%	139.5%	8,289	629	319.9%	181.9%	379
128	68.6%	139.5%	8,217	630	320.1%	182.4%	377
129	68.7%	139.2%	8,145	631	322.9%	182.6%	375
130	68.8%	138.8%	8,094	632	324.3%	182.9%	376

Table A.1—Continued

131	69.5%	139.2%	8,036	633	326.4%	179.8%	373
132	69.2%	139.2%	7,973	634	321.6%	183.2%	371
133	70.7%	139.2%	7,920	635	320.7%	185.0%	371
134	71.7%	139.3%	7,868	636	322.2%	185.1%	370
135	71.8%	139.6%	7,810	637	324.9%	183.4%	366
136	71.0%	140.0%	7,754	638	324.3%	183.8%	365
137	70.8%	140.8%	7,710	639	327.9%	182.3%	365
138	69.5%	142.1%	7,655	640	328.0%	182.2%	365
139	69.6%	142.4%	7,583	641	328.0%	182.4%	363
140	71.0%	142.4%	7,533	642	329.7%	182.2%	363
141	70.3%	143.4%	7,463	643	332.9%	181.2%	361
142	71.7%	144.0%	7,416	644	336.2%	180.1%	361
143	71.4%	145.2%	7,355	645	333.7%	182.6%	363
144	70.9%	147.0%	7,291	646	330.9%	185.2%	361
145	71.5%	146.9%	7,234	647	333.2%	184.6%	361
146	72.5%	147.5%	7,172	648	335.0%	184.6%	360
147	72.5%	150.4%	7,090	649	337.9%	185.0%	356
148	73.8%	149.3%	7,037	650	336.7%	186.5%	356
149	74.3%	150.0%	6,982	651	331.8%	188.5%	356
150	76.4%	149.1%	6,911	652	333.5%	188.1%	356
151	77.7%	149.4%	6,850	653	337.1%	185.7%	355
152	79.0%	149.4%	6,811	654	338.1%	185.8%	353
153	80.5%	149.5%	6,758	655	339.6%	185.4%	351
154	81.9%	148.7%	6,729	656	344.1%	182.3%	346
155	82.0%	149.0%	6,679	657	346.8%	182.9%	345
156	83.6%	149.6%	6,611	658	348.7%	183.1%	345
157	85.1%	150.3%	6,565	659	350.6%	183.6%	345
158	86.2%	150.5%	6,504	660	349.9%	183.5%	342
159	85.7%	152.2%	6,452	661	348.7%	184.5%	338
160	87.0%	153.0%	6,397	662	349.7%	184.1%	337
161	88.9%	153.2%	6,345	663	352.5%	184.1%	334
162	89.1%	153.9%	6,292	664	352.1%	183.2%	334
163	90.6%	154.2%	6,251	665	353.8%	183.3%	330
164	92.1%	154.6%	6,171	666	356.4%	183.2%	327
165	94.6%	150.8%	6,135	667	354.6%	184.8%	327

Table A.1—Continued

166	96.2%	151.3%	6,085	668	351.8%	186.1%	324
167	94.7%	152.1%	6,048	669	349.9%	187.0%	323
168	95.9%	152.6%	5,999	670	351.1%	187.6%	322
169	96.8%	153.2%	5,935	671	353.8%	187.1%	322
170	96.6%	153.2%	5,878	672	354.1%	187.2%	321
171	100.0%	153.6%	5,769	673	354.2%	186.7%	319
172	100.1%	154.2%	5,796	674	350.8%	188.0%	319
173	99.0%	157.2%	5,750	675	351.5%	188.3%	319
174	99.2%	156.9%	5,727	676	352.3%	188.6%	318
175	99.9%	156.8%	5,694	677	354.0%	189.8%	314
176	102.0%	156.8%	5,644	678	352.8%	190.2%	314
177	102.8%	156.9%	5,623	679	351.6%	191.3%	314
178	103.7%	157.3%	5,579	680	355.1%	191.8%	311
179	103.6%	158.0%	5,545	681	355.9%	191.5%	307
180	98.3%	158.2%	5,501	682	360.4%	192.0%	304
181	97.8%	157.6%	5,467	683	361.3%	190.6%	303
182	98.4%	157.9%	5,437	684	363.1%	190.1%	302
183	99.9%	157.4%	5,407	685	364.9%	189.6%	300
184	100.9%	157.8%	5,351	686	367.9%	187.2%	298
185	101.2%	158.1%	5,314	687	370.7%	186.1%	297
186	102.5%	158.1%	5,283	688	372.4%	186.3%	295
187	103.7%	157.2%	5,249	689	375.8%	184.8%	293
188	104.6%	157.8%	5,208	690	376.2%	185.7%	292
189	105.1%	157.4%	5,176	691	377.0%	184.4%	291
190	104.7%	158.9%	5,132	692	377.6%	185.8%	291
191	106.9%	158.7%	5,080	693	379.7%	185.3%	290
192	106.7%	160.1%	5,048	694	381.1%	185.8%	289
193	107.1%	159.0%	5,003	695	383.9%	185.7%	287
194	108.0%	159.3%	4,947	696	381.8%	189.1%	284
195	109.4%	159.6%	4,911	697	382.3%	187.3%	282
196	109.7%	160.1%	4,880	698	380.2%	187.3%	281
197	108.9%	160.9%	4,853	699	381.5%	187.7%	281
198	110.2%	160.9%	4,821	700	380.6%	189.1%	279
199	111.6%	160.3%	4,778	701	378.8%	189.6%	277
200	112.5%	160.1%	4,742	702	379.7%	189.9%	276

Table A.1—Continued

201	113.7%	160.0%	4,706	703	378.6%	191.0%	273
202	115.7%	159.6%	4,664	704	380.2%	191.3%	272
203	115.8%	159.9%	4,630	705	380.3%	191.8%	272
204	114.9%	160.1%	4,603	706	378.6%	192.9%	270
205	116.4%	159.2%	4,566	707	377.6%	194.8%	270
206	117.2%	158.3%	4,531	708	379.7%	194.9%	268
207	118.1%	157.7%	4,500	709	382.5%	195.0%	268
208	118.7%	157.7%	4,473	710	381.5%	195.2%	266
209	119.2%	157.5%	4,451	711	380.9%	195.8%	266
210	119.5%	157.7%	4,413	712	380.5%	194.8%	261
211	120.0%	158.6%	4,370	713	381.6%	197.8%	260
212	121.3%	158.8%	4,330	714	381.6%	197.9%	260
213	121.0%	158.7%	4,302	715	382.7%	198.7%	259
214	117.2%	159.1%	4,265	716	381.0%	201.3%	253
215	115.6%	160.6%	4,231	717	379.5%	202.9%	253
216	116.2%	160.0%	4,195	718	383.4%	203.0%	249
217	117.9%	160.3%	4,163	719	386.3%	201.7%	247
218	119.6%	160.4%	4,131	720	387.7%	203.1%	245
219	121.5%	160.2%	4,108	721	387.2%	203.4%	243
220	123.8%	160.1%	4,079	722	389.8%	203.5%	242
221	126.0%	160.4%	4,058	723	392.6%	204.8%	240
222	128.1%	159.2%	4,021	724	391.4%	205.7%	239
223	128.1%	159.9%	3,989	725	391.0%	208.4%	236
224	127.7%	160.1%	3,962	726	392.5%	207.5%	232
225	128.6%	158.9%	3,932	727	390.2%	206.0%	230
226	129.6%	160.0%	3,909	728	393.2%	205.2%	227
227	130.3%	160.4%	3,892	729	390.9%	205.7%	224
228	132.0%	160.5%	3,867	730	394.3%	204.6%	222
229	131.8%	159.6%	3,857	731	393.7%	207.3%	221
230	133.1%	160.2%	3,835	732	391.6%	208.2%	221
231	133.8%	160.2%	3,829	733	396.1%	209.5%	221
232	134.8%	159.9%	3,810	734	396.4%	210.7%	220
233	136.1%	159.8%	3,784	735	393.2%	210.5%	218
234	136.5%	160.3%	3,757	736	390.6%	210.6%	216
235	137.2%	160.4%	3,726	737	393.2%	209.3%	215

Table A.1—Continued

236	137.1%	160.5%	3,703	738	395.1%	209.6%	214
237	138.5%	160.4%	3,674	739	400.7%	210.1%	210
238	138.7%	159.4%	3,651	740	402.2%	211.0%	210
239	140.6%	159.0%	3,621	741	400.0%	211.5%	208
240	141.8%	159.1%	3,603	742	388.9%	213.4%	206
241	143.8%	158.4%	3,589	743	388.1%	211.6%	205
242	144.2%	159.6%	3,570	744	390.7%	214.0%	205
243	146.5%	159.3%	3,555	745	390.7%	214.4%	203
244	147.5%	159.3%	3,531	746	395.7%	214.0%	200
245	150.5%	159.0%	3,517	747	395.7%	214.2%	198
246	149.9%	158.7%	3,495	748	395.8%	215.4%	197
247	150.1%	158.8%	3,471	749	396.7%	215.8%	195
248	150.8%	159.3%	3,457	750	400.3%	212.7%	194
249	150.1%	160.6%	3,445	751	401.0%	212.7%	193
250	151.4%	160.7%	3,429	752	399.0%	214.5%	193
251	152.3%	160.9%	3,415	753	400.3%	216.3%	193
252	154.9%	159.7%	3,396	754	405.0%	210.0%	191
253	155.6%	159.5%	3,375	755	405.9%	209.8%	188
254	155.9%	160.8%	3,352	756	407.7%	209.8%	187
255	156.4%	161.0%	3,331	757	415.7%	201.5%	184
256	155.0%	161.9%	3,321	758	413.2%	201.7%	182
257	153.3%	161.9%	3,300	759	413.2%	201.4%	180
258	154.4%	161.7%	3,286	760	415.5%	201.1%	179
259	154.7%	161.0%	3,265	761	415.3%	202.6%	179
260	154.4%	161.5%	3,250	762	415.2%	203.4%	177
261	154.4%	161.6%	3,235	763	418.1%	204.0%	177
262	155.8%	161.1%	3,216	764	416.2%	203.4%	175
263	155.8%	161.7%	3,200	765	415.8%	204.4%	174
264	156.4%	161.3%	3,187	766	413.1%	205.6%	174
265	155.6%	161.5%	3,174	767	413.3%	206.6%	174
266	154.8%	162.9%	3,159	768	414.3%	205.6%	173
267	157.1%	162.2%	3,142	769	411.6%	206.5%	171
268	158.2%	162.2%	3,122	770	413.0%	205.7%	170
269	159.3%	161.6%	3,092	771	414.5%	206.1%	170
270	159.2%	161.6%	3,065	772	412.5%	207.9%	170

Table A.1—Continued

271	158.9%	162.7%	3,046	773	414.3%	209.0%	169
272	160.6%	162.9%	3,032	774	413.8%	209.3%	169
273	160.8%	164.3%	3,022	775	412.3%	210.2%	169
274	162.4%	163.6%	3,002	776	406.5%	211.9%	168
275	161.8%	165.1%	2,979	777	404.0%	214.0%	167
276	161.4%	165.1%	2,961	778	402.0%	216.3%	167
277	162.2%	165.6%	2,942	779	402.7%	218.7%	166
278	162.0%	167.2%	2,915	780	403.6%	220.4%	166
279	163.1%	166.1%	2,901	781	405.6%	219.3%	166
280	162.9%	166.4%	2,873	782	410.1%	218.1%	166
281	162.2%	167.4%	2,857	783	410.8%	216.6%	166
282	163.5%	167.9%	2,834	784	404.0%	212.3%	164
283	165.4%	166.7%	2,808	785	404.3%	214.2%	164
284	165.4%	166.4%	2,787	786	401.7%	215.5%	164
285	165.4%	165.9%	2,771	787	404.2%	215.6%	164
286	167.1%	165.9%	2,749	788	404.8%	214.2%	163
287	167.3%	166.9%	2,734	789	403.4%	215.7%	163
288	169.0%	166.7%	2,714	790	402.2%	220.1%	163
289	170.2%	167.3%	2,696	791	403.2%	218.5%	161
290	169.7%	167.7%	2,684	792	403.5%	219.3%	159
291	170.9%	167.6%	2,662	793	406.4%	217.4%	159
292	171.0%	167.8%	2,650	794	407.5%	216.1%	158
293	170.3%	167.4%	2,622	795	408.7%	214.5%	158
294	169.5%	168.5%	2,610	796	409.5%	216.7%	158
295	171.6%	168.8%	2,599	797	407.9%	217.5%	157
296	174.0%	169.0%	2,582	798	404.6%	219.1%	155
297	175.4%	169.9%	2,564	799	407.1%	219.7%	155
298	176.7%	169.1%	2,547	800	405.7%	221.2%	155
299	178.2%	169.8%	2,538	801	405.6%	222.8%	154
300	178.8%	168.7%	2,524	802	407.7%	223.4%	151
301	180.5%	168.5%	2,500	803	412.7%	221.9%	150
302	180.7%	169.6%	2,486	804	410.1%	219.9%	149
303	179.8%	169.8%	2,476	805	413.2%	218.2%	148
304	180.7%	170.1%	2,460	806	413.1%	217.1%	148
305	178.7%	169.6%	2,446	807	417.8%	213.5%	147

Table A.1—Continued

306	178.4%	169.9%	2,424	808	418.4%	214.1%	146
307	178.5%	169.5%	2,401	809	422.9%	212.8%	145
308	179.4%	168.8%	2,386	810	426.0%	212.0%	144
309	179.5%	168.7%	2,377	811	427.3%	210.5%	144
310	176.5%	166.8%	2,354	812	429.0%	211.8%	143
311	177.8%	167.7%	2,335	813	428.9%	212.5%	143
312	179.6%	168.2%	2,321	814	431.2%	213.6%	142
313	180.8%	169.3%	2,298	815	431.3%	213.4%	142
314	180.9%	170.0%	2,286	816	430.5%	214.0%	142
315	181.2%	169.6%	2,263	817	433.2%	212.2%	142
316	179.6%	169.7%	2,244	818	431.8%	212.6%	142
317	178.7%	169.8%	2,225	819	432.5%	210.5%	141
318	180.0%	171.0%	2,216	820	433.0%	209.9%	141
319	181.0%	170.7%	2,200	821	431.9%	210.6%	141
320	181.9%	172.1%	2,186	822	430.7%	209.5%	141
321	181.4%	172.7%	2,177	823	432.5%	211.2%	141
322	181.0%	172.2%	2,160	824	439.0%	204.0%	139
323	181.2%	171.9%	2,151	825	438.9%	202.9%	139
324	182.1%	172.3%	2,141	826	444.1%	196.1%	138
325	183.8%	171.6%	2,127	827	441.6%	197.5%	137
326	184.7%	172.2%	2,117	828	442.9%	197.4%	137
327	184.6%	172.2%	2,099	829	444.7%	197.3%	135
328	184.4%	172.8%	2,092	830	444.4%	197.7%	135
329	184.8%	173.2%	2,082	831	445.0%	199.9%	135
330	184.2%	173.5%	2,075	832	445.4%	201.7%	135
331	184.2%	174.5%	2,065	833	445.9%	201.4%	135
332	182.7%	174.5%	2,056	834	446.6%	202.8%	135
333	182.8%	175.0%	2,042	835	448.2%	204.3%	135
334	184.5%	174.4%	2,024	836	449.3%	203.0%	135
335	184.7%	173.4%	2,015	837	450.2%	204.9%	134
336	183.9%	174.1%	2,006	838	448.4%	205.7%	134
337	183.7%	172.3%	1,992	839	451.8%	205.7%	134
338	184.5%	173.8%	1,979	840	450.2%	206.3%	132
339	185.7%	173.4%	1,965	841	451.3%	206.5%	130
340	185.2%	172.5%	1,953	842	451.1%	208.6%	127

Table A.1—Continued

341	185.1%	172.8%	1,951	843	453.7%	207.3%	127
342	186.8%	172.9%	1,932	844	452.1%	209.6%	126
343	189.3%	173.0%	1,919	845	452.9%	209.9%	126
344	190.3%	173.2%	1,911	846	453.2%	210.6%	123
345	192.1%	172.9%	1,897	847	449.5%	213.0%	122
346	192.1%	171.8%	1,885	848	450.3%	213.6%	122
347	191.4%	170.4%	1,877	849	451.2%	215.8%	122
348	193.7%	170.3%	1,865	850	455.9%	212.4%	121
349	193.7%	170.5%	1,849	851	458.6%	212.1%	121
350	196.0%	170.1%	1,840	852	457.9%	212.3%	121
351	196.6%	170.0%	1,829	853	459.9%	211.8%	121
352	197.7%	169.7%	1,815	854	459.9%	212.3%	121
353	199.8%	170.8%	1,806	855	456.5%	214.0%	121
354	200.8%	171.0%	1,800	856	457.8%	214.7%	121
355	200.9%	170.3%	1,793	857	461.2%	212.6%	121
356	201.1%	169.9%	1,785	858	462.9%	212.7%	120
357	200.1%	168.3%	1,777	859	466.7%	215.0%	119
358	200.5%	168.6%	1,769	860	465.1%	214.3%	119
359	200.1%	167.5%	1,759	861	470.5%	209.3%	119
360	201.6%	167.5%	1,749	862	466.0%	210.6%	119
361	202.4%	167.8%	1,739	863	467.5%	211.0%	118
362	202.9%	167.8%	1,730	864	467.7%	209.8%	118
363	203.8%	167.2%	1,724	865	467.3%	209.7%	118
364	204.1%	167.0%	1,713	866	470.3%	210.5%	118
365	202.8%	168.4%	1,710	867	472.6%	210.7%	118
366	205.3%	168.9%	1,700	868	473.6%	212.1%	117
367	207.8%	169.3%	1,699	869	473.4%	213.5%	116
368	208.0%	168.6%	1,694	870	473.4%	213.4%	116
369	208.1%	169.1%	1,680	871	471.6%	215.8%	114
370	209.6%	169.5%	1,676	872	463.5%	215.0%	114
371	210.4%	169.2%	1,666	873	470.2%	214.8%	112
372	213.7%	169.1%	1,658	874	470.1%	216.5%	110
373	215.4%	169.1%	1,651	875	470.0%	218.0%	109
374	215.8%	169.7%	1,644	876	467.5%	221.6%	107
375	217.8%	169.3%	1,639	877	468.3%	223.3%	107

Table A.1—Continued

376	219.3%	170.1%	1,633	878	464.8%	225.3%	106
377	220.4%	170.6%	1,625	879	466.1%	225.1%	104
378	220.2%	169.9%	1,618	880	472.1%	224.0%	104
379	221.6%	169.9%	1,604	881	472.3%	224.7%	104
380	221.3%	171.0%	1,596	882	473.1%	225.5%	104
381	220.8%	170.3%	1,588	883	469.8%	223.3%	103
382	219.7%	170.8%	1,578	884	469.3%	225.2%	103
383	221.4%	171.2%	1,570	885	470.7%	220.0%	102
384	221.6%	171.9%	1,567	886	470.1%	221.0%	101
385	225.4%	170.0%	1,553	887	471.9%	221.1%	101
386	226.3%	170.7%	1,545	888	470.7%	219.6%	101
387	227.6%	170.0%	1,534	889	466.5%	221.5%	101
388	228.2%	170.1%	1,523	890	461.8%	222.1%	100
389	228.9%	169.9%	1,518	891	465.8%	221.9%	99
390	228.3%	169.9%	1,513	892	466.3%	221.6%	98
391	231.1%	170.1%	1,502	893	467.3%	222.0%	98
392	232.2%	170.3%	1,500	894	469.1%	221.0%	95
393	234.9%	170.6%	1,493	895	466.7%	219.8%	95
394	235.3%	170.7%	1,489	896	470.5%	221.5%	95
395	235.9%	169.9%	1,484	897	467.1%	224.6%	92
396	236.6%	168.7%	1,478	898	468.4%	229.8%	91
397	237.8%	168.7%	1,475	899	467.1%	230.0%	91
398	238.5%	168.0%	1,470	900	470.1%	234.3%	90
399	240.4%	167.8%	1,463	901	472.6%	231.5%	90
400	240.7%	167.9%	1,458	902	471.9%	230.9%	90
401	241.4%	168.3%	1,454	903	469.7%	230.2%	90
402	241.6%	168.9%	1,450	904	468.9%	226.8%	89
403	240.6%	168.1%	1,444	905	471.0%	225.1%	89
404	240.5%	167.3%	1,439	906	469.3%	224.8%	89
405	239.3%	168.3%	1,435	907	468.1%	225.1%	89
406	240.3%	168.2%	1,433	908	467.5%	226.9%	89
407	240.9%	168.6%	1,430	909	460.9%	230.7%	88
408	243.4%	168.2%	1,422	910	459.9%	234.7%	88
409	243.0%	168.8%	1,420	911	468.6%	225.3%	85
410	242.8%	169.0%	1,416	912	470.1%	225.5%	85

Table A.1—Continued

411	242.7%	169.5%	1,413	913	470.3%	225.9%	85
412	242.2%	170.7%	1,407	914	480.1%	217.2%	84
413	243.1%	171.5%	1,397	915	484.0%	216.1%	84
414	244.7%	171.9%	1,394	916	479.8%	215.8%	83
415	244.7%	172.6%	1,388	917	490.6%	205.5%	81
416	245.8%	172.3%	1,380	918	486.7%	204.2%	81
417	244.7%	172.2%	1,371	919	477.3%	204.7%	80
418	244.1%	173.2%	1,366	920	476.3%	205.7%	80
419	245.3%	173.1%	1,359	921	469.3%	205.2%	80
420	247.3%	172.3%	1,352	922	471.4%	205.3%	80
421	248.9%	170.8%	1,348	923	479.9%	201.1%	79
422	250.0%	170.3%	1,346	924	477.8%	200.3%	79
423	250.2%	168.4%	1,339	925	475.6%	200.3%	79
424	250.2%	168.7%	1,335	926	474.2%	200.6%	79
425	250.7%	167.6%	1,328	927	473.9%	200.1%	79
426	250.7%	168.9%	1,326	928	481.5%	200.3%	78
427	251.9%	169.2%	1,322	929	486.4%	199.5%	78
428	250.2%	168.2%	1,322	930	488.7%	199.3%	78
429	250.2%	169.1%	1,317	931	491.6%	199.6%	78
430	251.8%	169.5%	1,314	932	494.1%	200.5%	78
431	250.6%	169.0%	1,310	933	492.3%	200.6%	78
432	247.2%	167.4%	1,303	934	497.3%	201.1%	78
433	244.4%	168.0%	1,297	935	498.9%	202.1%	77
434	244.4%	168.3%	1,293	936	503.0%	201.3%	77
435	240.5%	167.5%	1,288	937	503.3%	202.0%	76
436	236.0%	168.1%	1,283	938	505.4%	202.1%	76
437	236.1%	168.8%	1,277	939	503.8%	202.2%	76
438	237.8%	168.9%	1,273	940	502.7%	202.5%	76
439	240.4%	168.5%	1,269	941	501.8%	203.3%	76
440	241.2%	167.2%	1,266	942	504.4%	201.8%	76
441	240.9%	169.6%	1,263	943	500.9%	203.1%	76
442	240.6%	170.1%	1,257	944	501.3%	203.4%	76
443	244.3%	168.8%	1,248	945	504.2%	202.1%	76
444	243.0%	168.8%	1,240	946	504.3%	203.3%	76
445	232.3%	165.4%	1,015	947	507.8%	202.6%	76

Table A.1—Continued

446	232.0%	164.7%	1,010	948	507.1%	203.5%	76
447	232.0%	165.1%	1,006	949	507.5%	202.5%	76
448	232.9%	164.4%	999	950	512.3%	202.2%	75
449	234.0%	163.5%	995	951	509.3%	203.0%	75
450	234.4%	164.1%	987	952	504.5%	204.6%	75
451	233.7%	164.1%	982	953	510.6%	205.6%	74
452	235.0%	163.1%	977	954	510.9%	206.1%	74
453	235.2%	164.0%	971	955	514.3%	210.7%	72
454	236.4%	164.0%	965	956	512.5%	212.5%	72
455	237.1%	163.8%	958	957	512.3%	211.8%	72
456	237.8%	163.9%	949	958	512.6%	211.1%	71
457	239.6%	162.7%	945	959	514.8%	210.2%	71
458	241.3%	160.6%	937	960	515.6%	211.3%	70
459	243.4%	159.6%	933	961	515.9%	211.6%	69
460	243.3%	159.8%	928	962	517.3%	214.4%	67
461	243.3%	159.9%	923	963	528.5%	212.7%	64
462	244.7%	159.1%	916	964	529.9%	212.4%	64
463	245.5%	160.4%	911	965	537.3%	210.7%	61
464	247.3%	159.8%	905	966	538.7%	212.2%	60
465	248.6%	158.9%	896	967	548.3%	204.6%	59
466	248.3%	158.5%	888	968	545.2%	209.7%	57
467	248.3%	159.4%	886	969	555.7%	199.3%	55
468	249.0%	159.6%	885	970	560.5%	197.3%	55
469	251.3%	159.0%	877	971	562.5%	199.6%	54
470	252.8%	158.8%	869	972	562.8%	199.2%	54
471	253.8%	157.7%	866	973	569.3%	199.9%	52
472	254.6%	157.3%	860	974	571.0%	201.3%	50
473	254.5%	158.3%	858	975	572.2%	202.0%	50
474	253.1%	159.1%	853	976	569.8%	202.1%	49
475	254.0%	159.2%	851	977	570.9%	204.0%	49
476	255.7%	159.3%	846	978	570.6%	202.1%	49
477	257.3%	158.4%	842	979	566.2%	201.1%	49
478	259.9%	156.6%	834	980	566.3%	202.7%	49
479	260.4%	157.4%	832	981	573.7%	200.8%	48
480	261.8%	156.7%	829	982	575.4%	198.9%	48

Table A.1—Continued

481	262.2%	155.8%	825	983	570.0%	201.3%	48
482	260.4%	156.2%	823	984	571.0%	205.2%	47
483	259.3%	157.6%	820	985	567.8%	206.5%	46
484	259.0%	157.6%	814	986	566.2%	210.6%	45
485	257.2%	159.3%	808	987	562.3%	206.2%	45
486	257.0%	159.4%	807	988	566.1%	204.6%	45
487	257.0%	159.8%	802	989	571.1%	206.4%	44
488	254.2%	161.2%	798	990	556.6%	207.8%	45
489	254.2%	162.2%	795	991	556.8%	206.2%	45
490	255.2%	162.6%	790	992	558.0%	206.1%	45
491	256.7%	163.3%	782	993	550.6%	209.3%	44
492	258.3%	163.2%	777	994	525.2%	218.0%	44
493	264.2%	159.5%	774	995	514.8%	226.4%	44
494	264.1%	159.8%	774	996	517.5%	226.8%	43
495	266.2%	160.3%	769	997	508.4%	230.8%	43
496	267.9%	160.1%	767	998	491.2%	234.9%	43
497	268.6%	160.6%	759	999	497.9%	234.0%	43
498	271.3%	159.5%	752	1000	510.6%	230.0%	43
499	274.1%	158.6%	749	1001	522.1%	224.7%	42
500	274.1%	158.3%	744	1002	521.2%	225.3%	42
501	275.5%	158.5%	742	1003	520.1%	219.7%	41
502	274.8%	159.3%	735	1004	525.7%	218.7%	40
503	275.1%	160.3%	730	1005	529.9%	218.2%	40
504	278.4%	159.8%	723	1006	527.3%	221.0%	40
505	278.9%	159.7%	721				

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