A DISCRETE MODEL FOR THE RECOVERY
OF OIL FROM A RESERVOIR

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ABSTRACT

In this work we simulate oil recovery by means of a molecular type approach. This means that we consider the solid and liquid materials to be composed of a finite number of particles, which are approximants for molecules. Porous flow is studied qualitatively under the assumption that particles of rock, oil and the flooding flow interact with each other locally by means of a compensating Lennard-Jones type potential and are also under the influence of gravity. Extensive computations are described and discussed in which initial data and parameters are varied. A comparison is made with actual physical experiments.

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1. INTRODUCTION

The flow of fluids through porous media is of importance in many different fields [3,11,25,26,35]. In this paper, we will be interested in porous flow when two different fluids are present. Related problems are of great importance in the study of oil recovery processes and the contamination of ground water [3,9,25,31,32]. At present, our approach will be qualitative. We will assume that rock and fluid particles interact locally through a compensated Lennard-Jones type force [20]. We will also allow for the inclusion of the effects of gravity. The parameter choices studied will depend upon the particular interacting particles and the distances of separation.

2. DISCRETE MATHEMATICAL FORMULATION

The exceptional complexity of porous flow has led to a variety of modelling techniques and approaches. These include laboratory techniques (see refs. [3,25]), classical numerical techniques (see [1,4-5,7-8,10,17-19,27-28,34,37] and the references therein), and analytical approaches [12,23,35-37].

In this work we will use a new approach [33]. We will consider a model which will be called simply a discrete model for the qualitative study of the porous flow. This type of modelling has been applied successfully [20,21,29] to problems other than the one considered here.

In the approach, we assume that our physical system consists of N particles, say, $P_1, \dots, P_N$, with corresponding masses $m_1, m_2, \dots, m_N$. For a fixed, positive time step Δt , let $t_k = k\Delta t$, $k = 0, 1, \dots$. The coordinates of particle P_1 at time t_k will be denoted by $\bar{r}_{1,k}$, the velocity by $\bar{v}_{1,k}$ and the acceleration by $\bar{a}_{1,k}$.

The "leap frog" formulas [1,13,29] will be used to approximate velocities and positions with time:

$$(2.1) \quad \bar{v}_{i,1/2} = \bar{v}_{i,0} + 1/2(\Delta t)\bar{a}_{i,0}$$

$$(2.2) \quad \bar{v}_{i,k+1/2} = \bar{v}_{i,k-1/2} + (\Delta t)\bar{a}_{i,k}, \quad k = 1, 2, 3, \dots$$

$$(2.3) \quad \bar{r}_{i,k+1} = \bar{r}_{i,k} + (\Delta t)\bar{v}_{i,k+1/2}, \quad k = 0, 1, 2, \dots$$

Of course, other formulas for the approximate solution of n-body initial value problems can also be used.

Let us denote $\bar{F}_{i,k}$ the force acting upon P_i at time t_k and assume that the acceleration of P_i is related to the force by

$$(2.4) \quad \bar{F}_{i,k} = m_i \bar{a}_{i,k}.$$

If the expression for the force $\bar{F}_{i,k}$ is known, then by means of equations (2.1)-(2.3) we shall be able to determine the dynamical history of the particles, as long as we are able to prescribe the initial data.

For the force we will emphasize the use of a Lennard-Jones type formula [13,14]. But, first, let us introduce the following notation.

At time t_k , the distance between particles P_i and P_j will be denoted by $r_{i,j,k}$. Then, the local force $\bar{F}_{i,j,k}$ exerted on P_i by P_j is taken to be

$$(2.5) \quad F_{i,j,k} = m_i m_j \left[\frac{H_{i,j}}{(r_{i,j,k})^{q_{i,j}}} - \frac{G_{i,j}}{(r_{i,j,k})^{p_{i,j}}} \right] \frac{\bar{r}_{j,k} - \bar{r}_{i,k}}{r_{i,j,k}}.$$

The values of the parameters $H_{i,j}$, $G_{i,j}$, $q_{i,j}$, $p_{i,j}$ will depend on the particles which are interacting. The total local force, $F_{i,k}^*$, acting on P_i due to all the other particles at time t_k , will be given by

$$(2.6) \quad \bar{F}_{i,k}^* = \sum_{\substack{j=1 \\ j \neq i}}^N \bar{F}_{i,j,k}.$$

Finally denoting by $\bar{F}_{i,k}$ a possible long range force, like gravity, acting on P_i at time t_k , we take the total force on P_i to be

$$\bar{F}_{i,k} = \bar{F}_{i,k}^* + \bar{F}_{i,k}$$

Equipped with these concepts we shall study the displacement of one fluid by another within a porous medium.

3. SIMULTANEOUS FLOW OF MISCIBLE FLUIDS

The problem in which we are interested is described as follows. Consider a rectangular region, R , which is a porous medium. We assume that in this region we have a fluid, which we call the resident fluid or oil.

We shall also introduce a different kind of fluid, which, as a matter of convenience, will be called water, although it is an aqueous solution which could be a polymeric solution, a surfactant solution or a brine.

We consider the flow region R , for simplicity, to be a square of width four units and height four units. Let the particles $P_1, P_2, \dots, P_{N_1}$ (with $N_1 = 25$) represent rocks; particles $P_{N_1+1}, \dots, P_{N_2}$ (with $N_2 = 81$) represent oil; and particles $P_{N_2+1}, \dots, P_{N_3}$ (with $N_3 = 161$) represent the incoming water. The distance of separation between the rocks is taken to be our scale unit length and is assumed to be the same in both directions, vertical and horizontal. The particular rock arrangement to be studied is shown in Figure 3.1.

It is also assumed that this region R is bounded by impermeable walls, which means that no fluid can enter or leave the region without further assumptions.

For purposes of injection of water and production of oil, two wells are now opened, one in the bottom left corner of R for injection, and another in the diagonally opposite corner for production, as shown in Figure 3.2.

The diameter of the production well is taken to be 0.5. The rock particles P_1 and P_{25} have been removed to allow for injection and production.

To simplify the present study, the particles P_2-P_{24} will be kept fixed and only the liquid particles will be allowed to move. This means that the migration problem will not be considered here, though it could be, and that the region R is composed entirely of very heavy rocks, not of sand.

Finally, the mass of each oil particle is taken as unity, the mass of each water particle as 2.5, and the mass of each rock particle as 69.22.

A list of the parameters that were chosen to calculate the local force $F_{i,j,k}$ are given below. The parameters were found after several simulation of different degrees of complexity. The remaining parameters will be specified as the method is being developed. Let $q_{i,j} \equiv 5$, $p_{i,j} \equiv 3$.

With regard to the H and G parameters in (2.5), we will use the notation

- a) HSS is the H parameter to be used in (2.5) between two rock particles,
- b) GSS is the G parameter to be used in (2.5) between two rock particles,
- c) HOO is the H parameter to be used in (2.5) between two oil particles,
- d) GOW is the G parameter to be used in (2.5) between an oil and a water particle,

and so forth, where S denotes solid rock, O denotes oil, W denotes water.

We then assume

$$\left. \begin{array}{l} H_{1j} = HOS = 1 \\ G_{1j} = GOS = 3 \end{array} \right\} \begin{array}{l} i = 26, \dots, 81 \\ j = 2, \dots, 24 \end{array}$$

$$\left. \begin{array}{l} H_{1j} = HOO = 1 \\ G_{1j} = GOO = 1 \end{array} \right\} i, j = 26, \dots, 81$$

$$\left. \begin{array}{l} H_{1j} = HOW = 1 \\ G_{1j} = GOW = 0 \end{array} \right\} \begin{array}{l} i = 26, \dots, 81 \\ j = 82, \dots, 161 \end{array}$$

$$\left. \begin{array}{l} H_{1j} = HWW = 1 \\ G_{1j} = GWW = 0 \end{array} \right\} i, j = 82, \dots, 161$$

$$\left. \begin{array}{l} H_{1j} = HWS = 1.5 \\ G_{1j} = GWS = 0 \end{array} \right\} \begin{array}{l} i = 82, \dots, 161 \\ j = 2, \dots, 24 \end{array}$$

$$\left. \begin{array}{l} H_{1j} = HSS = 0 \\ G_{1j} = GSS = 0 \end{array} \right\} i, j = 2, \dots, 24$$

Since we are interested in constraining $\bar{F}_{1,k}^*$ to act locally only, we now introduce the parameters

$$EOO = 1.3 = \xi_{1j}; \quad i, j = 26, \dots, 81$$

$$EWW = 1 = \xi_{1j}; \quad i, j = 82, \dots, 161$$

$$EOW = 1.15 = \xi_{1j}; \quad \begin{cases} i = 26, \dots, 81 \\ i = 82, \dots, 161 \end{cases}$$

$$\begin{aligned} \text{EOS} = \sqrt{13/36} = \epsilon_{ij}; & \quad \begin{cases} i = 26, \dots, 81 \\ j = 2, \dots, 24 \end{cases} \\ \text{EWS} = 13/36 = \epsilon_{ij}; & \quad \begin{cases} i = 82, \dots, 161 \\ j = 2, \dots, 24 \end{cases} \end{aligned}$$

which are used in the following manner: if $r_{i,j,k} > \epsilon_{i,j}$, we replace (2.5) by $\bar{F}_{i,j,k} = 0$.

We also assume that when the particles of the fluids interact with the walls of the region R , a velocity damping factor, δ_i , will be necessary to model the hardness of the wall relative to the reflection of the interacting fluid. This is done by introducing the following parameters.

$$\begin{aligned} \delta_i &= .4 \quad \text{for } i = 26, \dots, 81 \\ \delta_i &= .8 \quad \text{for } i = 82, \dots, 161 \\ \text{WTUBE} &= 3.5 \end{aligned}$$

In implementing such reflections, we will use the following rules:

- (a) If $x_{i,k} < 0$, then $-x_{i,k} \rightarrow x_{i,k}$
 $-\delta_i v_{i,k,x} \rightarrow v_{i,k,x}$
- (b) If $x_{i,k} \geq 4$, then: $2 \cdot 4 - x_{i,k} \rightarrow x_{i,k}$
 $-\delta_i v_{i,k,x} \rightarrow v_{i,k,x}$
- (c) If $y_{i,k} \leq 0$, then $-y_{i,k} \rightarrow y_{i,k}$
 $-\delta_i v_{i,k,y} \rightarrow v_{i,k,y}$
- (d) If $y_{i,k} \geq 4$ and $x_{i,k} < \text{WTUBE}$,
then: $2 \cdot 4 - y_{i,k} \rightarrow y_{i,k}$
 $-\delta_i \cdot v_{i,k,y} \rightarrow v_{i,k,y}$

Finally, before considering actual computations, let us mention that in the Petroleum and Chemical Engineering literature [2,6,16], the relative magnitude of the gravitational force and the interfacial forces is measured by using the concept of a Bond Number, where Bond Number = $\alpha(\text{gravity}/\text{interfacial forces})$. In order to study the effect of a system where the interfacial forces have a greater or lesser magnitude, we shall study several examples where the gravity force has been varied up to three orders of magnitude.

4. EFFECTS OF THE VELOCITY OF INJECTION

The objective of the examples in this section is to compare the effect of the injection velocity of the water on the displacement of fluids, while the volume of injection and the ratio between the gravity and the intermolecular forces are kept constant. Several examples in addition to those to be described were studied and some of the related data will be presented graphically. All the examples were run with a time step $\Delta t = 10^{-4}$ on the DEC-20 of the University of Texas at Arlington. For each of the examples throughout this work, the initial positions of the resident fluid are given in Table 1. All initial velocities of the resident fluid are taken to be zero.

The parameters to be varied are

- (a) d : the distance of separation between the particles of water before going into the well.
- (b) v : the velocity of the water particles before going into the well.
- (c) g : the gravity constant.

Example 4.1

In this first example we have the following values for the parameters:

$$d = 1, \quad v = 15, \quad g = 9.8 .$$

Figures 4.1.1 - 4.1.12 give the evolution in time for the system.

We observe that at iteration $k = 4500$, i. e., at $T = .45$, (see Figure 4.1.1) the water has displaced most of the resident fluid in the neighborhood of the entrance. Also we see that gravity forces a cluster of oil in the lower side, opposite to the injection point.

For $k = 7800$, Figure 4.1.2 shows that the water finds less resistance to its motion above than to the right. The same observation remains valid for Figure 4.1.3.

In Figures 4.1.4 - 4.1.5, where $k = 10,500$ and $12,300$, respectively, we observe the water finally advancing through the lower layers, although it still has a tendency to go in the upward direction. This can be observed more easily at $k = 13,800$ and $k = 15,000$, (Figures 4.1.6 and 4.1.7, respectively).

This trend is continued, and for $k = 21,000$ (Figure 4.1.9) we can say that the water is beginning to remove the oil from the bottom part of the region. This is also the time when the first particle of water leaves the region.

Figures 4.1.10 - 4.1.12 show the displacement of the resident fluid. It can also be noted that an increasing number of water particles are leaving the region at these times.

Let us look at some typical results indicating the amount of water and oil particles which are in the well and which have left the well as follows:

k	3000	4500	7800	10,500	15,000	19,800	22,500	25,800
Oil out	2	3	5	8	18	24	28	32
Water out	0	0	0	0	0	0	3	7
Water in	5	7	12	16	23	30	34	39

In discussing the actual recovery of the oil, note that, at some time, the water and the oil exit as a mixture which can be used conveniently to define a measure of efficiency. Several such definitions are possible. These will depend on the

amount of water injected, the amount of water coming out, the amount of oil coming out, the time involved and the percentage of oil produced from the estimated amount of oil (in our case, this last number would be known). The measure which we found most satisfactory is one related to that known in Petroleum Engineering as the Water Oil Ratio. The water oil ratio is defined as:

$$\text{WOR} = \frac{\text{amount of water out}}{\text{amount of oil out}} .$$

This number is widely used in oil production. When $\text{WOR} = 0$, we have no water going out, which means we are obtaining only oil in the well outlet. If WOR increases, our production becomes of lesser quality, meaning that we have a mixture with some percentage of water.

For figures 4.1.10 - 4.1.12, the WOR increases, telling us that our procedure is becoming less efficient with time, as is to be expected.

Example 4.2

For this case our parameters are:

$$d = 1, \quad v = 10, \quad g = 9.8.$$

Thus, we have diminished the injection velocity, with respect to the former example, but all the other parameters are kept the same.

Initially, the behavior is qualitatively similar to Example 4.1. However, let us look now at Figure 4.2.1 ($k = 11,400$). In it we notice the water beginning to wet the bottom layer. Also, the information in it tells us that five particles of oil have exited from the well and twelve particles of water have been injected. This contrasts with Figure 4.1.2, where five oil particles have exited while twelve particles have been injected, but at $k = 7800$.

Therefore, up to this time, we can say that decreasing the velocity did not enhance the recovery, since we have slowed down the production as compared with $v = 15$.

However, notice now that Figures 4.2.2 - 4.2.5 resemble the actual porous flow of water, Figure 4.2.6 is typical of photographs from an actual laboratory model [22].

Example 4.3

For this case we have increased only the velocity with respect to Example 4.1, while the other parameters are the same.

The parameters are now:

$$d = 1, \quad g = 9.8, \quad v = 50.$$

Comparing Figures 4.3.1 and 4.1.5, we observe that the water has wetted the lower part of the region, but in Example 4.3 it took $k = 5100$ steps, while for Example 4.1 it took $k = 12,300$ steps. For additional detail, see [33].

Next let us compare all three of the examples given above in a more complete fashion by examining graphs 4.1 and 4.2.

In graph 4.1 we have the number of particles of oil out and the number of particles of water out versus time. We can see from the graph that for t small, the rate of oil production is higher when v is higher. But as t increases, the amount of oil obtained with $v = 10$ becomes bigger. We can also observe that water comes out of the production well sooner for $v = 50$ than for the other velocities.

In order to analyze the efficiency of the methods used, we have plotted in Graph 4.2 the Water Oil Ratio versus time. The straight lines for this graph have been determined by the least square method. The numbers which appear in the intersection of the lines with the time axis represent the maximum number of oil particles that are produced before any particle of water exits.

From Graph 4.2, we observe that the slope for $v = 50$ is the largest and the slope for $v = 10$ is the smallest, so that the mixture produced with $v = 10$ will

have the least amount of water in it, although the time to carry out this process will be much longer.

5. EFFECTS OF VARYING THE RATIO BETWEEN GRAVITY AND MOLECULAR FORCES

In this section we study two examples, which together with Example 4.1 allow us to study the effect of changing the ratio of gravity to the intermolecular forces. At present, the velocity and the volume rate of increase of the injected water are kept constant. The values of d and v are 1 and 15, respectively, the same as in Example 4.1.

Example 5.1

For this example we assume $g = .098$. In our comparison, we had $g = 9.8$ (Example 4.1). A drastic difference between $g = 9.8$ and $g = .098$ is observed from figures 5.1.1 and 4.1.8. The case of smaller gravity shows a cluster of oil in the lower right corner of the region, which results because the water is not falling rapidly due to gravity, so that it is overrunning the oil in the lower layer. For additional details, see [33].

The next question is: What happens when the gravity force becomes more important relative to the intermolecular forces?

To answer this question, consider the next example.

Example 5.2

The parameters were chosen as follows: $d = 1$, $v = 15$, $g = 98$. A production comparison is easier if we study graphs 5.1 and 5.2. In Graph 5.1 we have plotted the amount of oil out and water out versus time.

Observe here that the case $g = .098$ is the best up to $k = 21,000$ and later the case $g = 98$ is the best.

In section 3 we introduced the Bond Number as a constant times the quotient of g and the interfacial forces. Therefore, decreasing g to .098 means decreasing the Bond Number, while increasing g to 98 means increasing the Bond Number. This last case will be the case of a surfactant in the injected fluid.

The next graph, 5.2, has the water oil ratio versus time. Considering the amount of pure oil that came out before the breakthrough time, the case $g = 98$ is the best, since for this case oil out is 31. It is also the case with smallest slope. The next better case would be $g = .098$.

We would say that in any case, increasing or decreasing the Bond Number with respect to the case $g = 9.8$ enhances the oil production.

6. EFFECTS OF VARYING THE INJECTION VOLUME

In this section we discuss the effect on displacement and production when we vary the amount of the injected water, while the velocity of injection and the ratio of gravity to intermolecular forces are kept constant. This variation would be due, for example, to the temperature of the water.

Our comparison case will be Example 4.2, in which we had $g = 9.8$, $v = 10$, $d = 1$.

Example 6.1

In this example let $g = 9.8$, $v = 10$, $d = .5$.

Relative to Example 4.2, making $d = .5$ diminishes the distance that the particles injected have to travel before going into the well. This implies a greater number of particles per unit of time entering the well. However, since the velocity was kept constant the momentum of each water particle will be the same as for Example 4.2.

If we examine Graphs 6.1 and 6.2, we can make a long range comparison easily.

Observe that after $k = 17,400$, the oil produced for $d = 0.5$ is not twice as much for $d = 1$. But, in fact, in the long run the production is greater for $d = 1$.

In Graph 6.2 the water oil ratio is plotted versus time, and, using least squares, we have constructed a line through the corresponding data for each case. The slope for $d = .5$ is greater than for $d = 1$, which tells us that the mixture produced for $d = 1$ will have a greater percentage of oil. For additional details, see [33].

7. SPECIAL ASPECTS OF EXAMPLES

In this section we want to show three sets of figures; 7.1.1 - 7.1.4, 7.2.1 - 7.2.5 and 7.3.1 - 7.3.4, which relate to the cases where $v = 10$, $d = 1$, and gravity is equal to 9.8, 98 and .098 respectively.

The curves in these figures were drawn as follows. First, we draw the trajectories of all the water particles from time T_1 to $T_1 + \Delta t$, then from $T_1 + \Delta t$ to $T_1 + 2\Delta t$, etc., up to time T_2 . Afterwards we draw the resulting envelope.

Each curve so constructed defines a particular area, namely, the area enclosed by it and by the boundary walls. Then, we deleted any subarea which had been traveled by an oil particle between times T_1 and T_2 . In the examples, we took at least 600 iterations in time.

Intuitively, the area which results is the area which has been traveled by water, and not by oil, for the given period of time.

Certainly, the difficulty of this process increases with time, since the number of injected particles increases, and this is something to be expected.

However, we also noticed that the difficulty also increases when we diminish the value of gravity, the reason being that the intermolecular forces become more important relative to gravity.

Since there is not a precise definition of the "fingering phenomenon" studied in the literature, we are tempted to say that our curves show fingers, for the area enclosed by this curves shows the advancement of water from time T_1 to time T_2 .

For the case $g = 9.8$, we observe at different times how the area which has been traveled by water is moving towards the exit. At time $t = 1.74$ the water is moving as a whole, but at time $t = 2.16$ we observe that we have two separated areas in the upper part. Those areas represent water which has been trapped by oil. At time $t = 2.34$ we observe how the water has spread out and how one of the small areas, which was isolated before, has gone through the exit. In the area close to the injection well, now we observe some oil which has been trapped by the water.

We must mention now that these figures are qualitatively similar to the actual porous flow of water [22] shown, typically, in Figure 7.4. These figures in the mentioned references are from an actual laboratory model.

For the case $g = 98$, it is observed that a greater quantity of water separates from the main area, and it remains separated for a relatively long period. For the case $g = .098$, the separation occurs more rapidly than for any other case, but after a short time, the main area reaches one or all of the isolated areas, and the procedure repeats.

8. CONCLUSIONS AND OPEN PROBLEMS

In this work we have approached an old, very complex problem with a relatively new technique. At present, it yields only qualitative results. However,

we believe it will be of greater use when modern technology (parallel computers) can be more readily available.

This first step can be extended in many directions. Of prime importance for quantitative modeling, would be the determination of the actual parameters by experimental scientists. From the many interesting questions that can be posed, we expect to be able to pursue several directions and obtain further results. For example, among the possibilities of extension, we include the following:

- a) Variation of the solid particles, including sand.
- b) Variations of the geometries for the positioning of the solids, which will simulate different kinds of porosities.
- c) Introduction of a third type of particle, to simulate gas in the reservoir.
- d) Variation of injection velocities in the same run, since this is actually done in the field.
- e) Consideration of injecting two different kinds of particles to simulate slugs.
- f) Increasing the number of particles.
- g) Carefully defining and thoroughly studying the fingering phenomenon.

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TABLE 1

PARTICLE	X	Y
26	0.3000	0.4000
27	1.5000	0.0000
28	2.5000	0.0000
29	3.5000	0.0000
30	0.0000	0.5000
31	0.5000	0.5000
32	1.0000	0.5000
33	1.5000	0.5000
34	2.0000	0.5000
35	2.5000	0.5000
36	3.0000	0.5000
37	3.5000	0.5000
38	4.0000	0.5000
39	0.5000	1.0000
40	1.5000	1.0000
41	2.5000	1.0000
42	3.5000	1.0000
43	0.0000	1.5000
44	0.5000	1.5000
45	1.0000	1.5000
46	1.5000	1.5000
47	2.0000	1.5000
48	2.5000	1.5000
49	3.0000	1.5000
50	3.5000	1.5000
51	4.0000	1.5000
52	0.5000	2.0000
53	1.5000	2.0000
54	2.5000	2.0000
55	3.5000	2.0000
56	0.0000	2.5000
57	0.5000	2.5000
58	1.0000	2.5000
59	1.5000	2.5000
60	2.0000	2.5000
61	2.5000	2.5000
62	3.0000	2.5000
63	3.5000	2.5000
64	4.0000	2.5000
65	0.5000	3.0000
66	1.5000	3.0000
67	2.5000	3.0000
68	3.5000	3.0000
69	0.0000	3.5000
70	0.5000	3.5000
71	1.0000	3.5000
72	1.5000	3.5000
73	2.0000	3.5000
74	2.5000	3.5000
75	3.0000	3.5000
76	3.5000	3.5000
77	4.0000	3.5000
78	0.5000	4.0000
79	1.5000	4.0000
80	2.5000	4.0000
81	3.5000	4.0000

FIGURE 3.1

Original array of the rocks in the region R

INJECTION
WELLPRODUCT
WELL

FIGURE 3.2

Region R with production and injection wells

FIGURE 4.1.1

$g = 9.8$; $v = 15$; $d = 1$; $k = 4500$;
Oil out = 3; Water out = 0; Water in = 7

FIGURE 4.1.2

$g = 9.8$; $v = 15$; $d = 1$; $k = 7800$;
Oil out = 5; Water out = 0; Water in = 12

FIGURE 4.1.3

$g = 9.8$; $v = 15$; $d = 1$; $k = 9000$
 out = 6; Water out = 0; Water in = 14

FIGURE 4.1.4

$g = 9.8$; $v = 15$; $d = 1$; $k = 10,500$
 Oil out = 8; Water out = 0; Water in = 16

FIGURE 4.1.5

$g = 9.8$; $v = 15$; $d = 1$; $k = 12,300$
 out = 12; Water out = 0; Water in = 19

FIGURE 4.1.6

$g = 9.8$; $v = 15$; $d = 1$; $k = 13,800$
 Oil out = 15; Water out = 0; Water in = 21

FIGURE 4.1.7

$g = 9.8$; $v = 15$; $d = 1$; $k = 15,000$
 Oil out = 18; Water out = 0; Water in = 23

FIGURE 4.1.8

$g = 9.8$; $v = 15$; $d = 1$; $k = 16,500$
 Oil out = 19; Water out = 0; Water in = 2

FIGURE 4.1.9

$g = 9.8$; $v = 15$; $d = 1$; $k = 21,000$
 Oil out = 27; Water out = 1; Water in = 32

FIGURE 4.1.10

$g = 9.8$; $v = 15$; $d = 1$; $k = 22,500$
 Oil out = 28; Water out = 3; Water in = 34

FIGURE 4.1.11

$g = 9.8$; $v = 15$; $d = 1$; $k = 24,300$
 Oil out = 29; Water out = 4; Water in = 37

FIGURE 4.1.12

$g = 9.8$; $v = 15$; $d = 1$; $k = 25,800$
 Oil out = 32; Water out = 7; Water in = 39

FIGURE 4.2.1

$g = 9.8$; $v = 10$; $d = 1$; $k = 11,400$
 Oil out = 5; Water out = 0; Water in = 12

FIGURE 4.2.9

$g = 9.8$; $v = 10$; $d = 1$; $k = 17,400$
 Oil out = 14; Water out = 0; Water in = 18

FIGURE 4.2.3

$g = 9.8$; $v = 10$; $d = 1$; $k = 19,500$
 Oil out = 18; Water out = 0; Water in = 20

FIGURE 4.2.4

$g = 9.8$; $v = 10$; $d = 1$; $k = 21,600$
 Oil out = 21; Water out = 0; Water in = 22

FIGURE 4.2.5

$g = 9.8$; $v = 10$; $d = 1$; $k = 23,400$
 Oil out = 25; Water out = 1; Water in = 24

FIGURE 4.2.6

Displacement front at breakthrough,
 with a mobility ratio of 71.5

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GRAPH 4.1

Comparison of the effect of the velocity on the oil and water production, with $d = 1$ and $g = 9.8$

GRAPH 4.2

Comparison of the effect of the velocity of the Water Oil Ratio, with $d = 1$ and $g = 9.8$

FIGURE 4.3.1

$g = 9.8$; $v = 50$; $d = 1$; $k = 5100$
 Oil out = 10; Water out = 0; Water in = 26

FIGURE 5.1.1

$g = .098$; $v = 15$; $d = 1$; $k = 16,500$
 Oil out = 22; Water out = 0; Water in = 25

GRAPH 5.1

Comparison of the effect of the gravity on the oil and water production, with $d = 1$ and $v = 15$

GRAPH 5.2

Comparison of the effect of the gravity on the Water Oil Ratio, with $d = 1$ and $v = 15$

GRAPH 6.2

Comparison of the effect of the distance of separation between the water particles on the oil and water production, with $g = 9.8$ and $v = 10$

GRAPH 6.1

Comparison of the effect of the distance of separation between the water particles on the Water Oil Ratio, with $g = 9.8$ and $v = 10$

FIGURE 7.1.1

Advancement of water for $k = 13,300$,
with $v = 10$, $d = 1$ and $g = 9.8$.

FIGURE 7.1.2

Advancement of water for $k = 17,400$,
with $v = 10$, $d = 1$ and $g = 9.8$.

FIGURE 7.1.3

Advancement of water for $k = 21,600$,
with $v = 10$, $d = 1$ and $g = 9.8$.

FIGURE 7.1.4

Advancement of water for $k = 23,400$,
with $v = 10$, $d = 1$ and $g = 9.8$.

FIGURE 7.2.1

Advancement of water for $k = 13,500$,
with $v = 10$, $d = 1$ and $g = 98$

FIGURE 7.2.2

Advancement of water for $k = 17,400$,
with $v = 10$, $d = 1$ and $g = 98$

FIGURE 7.2.3

Advancement of water for $k = 19,500$,
with $v = 10$, $d = 1$ and $g = 98$

FIGURE 7.2.4

Advancement of water for $k = 21,600$,
with $v = 10$, $d = 1$ and $g = 98$

FIGURE 7.2.8

Advancement of water for $k = 23,400$,
with $v = 10$, $d = 1$ and $g = 98$

FIGURE 7.3.1

Advancement of water for $k = 13,500$,
with $v = 10$, $d = 1$ and $g = .098$

FIGURE 7.3.2

Advancement of water for $k = 17,400$,
with $v = 10$, $d = 1$ and $g = .098$

FIGURE 7.3.3

Advancement of water for $k = 21,600$,
with $v = 10$, $d = 1$ and $g = .098$

FIGURE 7.3.4

Advancement of water for $k = 23,400$,
with $v = 10$, $d = 1$ and $g = .098$

FIGURE 7.4

Displacement front at breakthrough, with mobility ratios 4.58, 17.3, 71.5

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