A NEW COMPUTER APPROACH TO THE MODELING
OF CLOSE BINARY STARS

by

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ABSTRACT

A new, n-body approach is developed for the modeling of close binary systems, the most commonly occurring star systems known. Both local and long range forces are included. Computer examples of streaming, mass loss, and self-reorganization processes are described and discussed.

1. Introduction. Binary star systems in which the two rotating stars are visible separately have long been of interest to astronomers and applied mathematicians (see, e.g., (1), (3), (15), (16)). For many such visual binaries, judicious combination of observational data and Kepler's laws allow the determination of the mass of each star of the system (16). Such deductions are not possible for isolated stars or for larger star systems (15), (16).

Interest in binary systems has become cogent since the application of modern telescopic, spectroscopic, and computer techniques have revealed that most stars are binary systems, classified now as visual, spectroscopic, and eclipsing binaries (15), (16). Especially in the case of close binaries, a variety of interesting phenomena have also been discovered, including strong sources of X-rays, evolutionary and dynamical phenomena which occur in very short periods of time (~ 2 days), and the presence of circumscribed gaseous material which emits radio waves and exhibits strong tidal and streaming motions (3), (7), (14), (15), (18). In addition, evidence also indicates that close binaries are often related to light and Nova outbursts (15).

Since, in general, a Keplerian two-body model is insufficient for modeling the complex interactions of close binaries, extensive effort has been made in recent years to model such systems (see, e.g., (4), (14), (15) and the numerous references contained therein). The models range in complexity from Roche models to ones which include viscosity, compressibility, and rotation in a fully three dimensional hydrodynamical setting. These models, invariably, are continuous

and are simplified sufficiently so that the resulting equations are, to some extent, solvable. Each such model is applicable only to special classes of close binary systems.

Our objective in this paper is to introduce and to apply an n-body, computer approach ((9)-(11), (13), (17), (19)) to the modeling and study of close binaries. In the modeling, we will approximate the classical forces which act on, and between, molecules (2), (8), rather than the gross conservation principles which are usually approximated. Like some of the more sophisticated, continuous models, our discrete approach will allow for the inclusion of viscous and rotational effects; but, in addition, we will allow for fully nonlinear effects and for the simulation of streaming and free surface motion. Fundamental to our modeling will be an intrinsic capability for self-reorganization. But, like all types of modeling, it has its shortcomings, which we will try to make clear at the appropriate times.

2. Formulation of the Method. We begin with the widely accepted physical assumption that the dynamical behavior of any solid, liquid, or gas is determined from the dynamical behavior of its constituent molecules (8). Classically, the forces which act on molecules are of two types, the long range and the short range. Long range forces, like gravity and gravitation, are those which act uniformly on every molecule. Short range forces are those which act only between any molecule and each of its immediate neighbors. This local interaction is of the following general nature (8). If two molecules are pushed together they repel each other, if pulled

apart they attract each other, and mutual repulsion is of a greater order of magnitude than is mutual attraction. Mathematically, this behavior is often formulated as follows. The magnitude F of the force $\vec{F}$ between two molecules which are locally r units apart is of the form

$$(2.1) \quad F = - \frac{\alpha}{r^p} + \frac{\beta}{r^q},$$

where, typically,

$$(2.2) \quad \alpha > 0, \quad \beta > 0, \quad q > p > 7.$$

The major problem in the simulation of any physical body is that there are too many component molecules to incorporate into the model. The classical mathematical approach, then, is to replace the large, but finite, number of molecules by an infinite set of points. In so doing, the rich physics of molecular interaction is lost. A viable computer alternative is to replace the large number of molecules by a much smaller number of particles, continue the long range forces without change, but readjust the parameters in (2.1) to compensate (11). It is this latter approach which we will follow.

With regard to our simulation of binary star systems, we will, for consistency with other modelers, assume that the only long range force is gravitation, though we could just as easily include Coulombic effects. To justify the use of local force interactions, note that viscosity and compressibility are considered to be fundamental characteristics of close binary systems (15). These characteristics are those of a dense or heavy gas, and, on the molecular level, such gases cannot be differentiated from liquids (2).

3. The Computer Algorithm. The general idea outlined in Section 2 will be implemented in the following constructive fashion. Consider N particles P_i , each of mass m_i , $i = 1, 2, \dots, N$. For $\Delta t > 0$, let $t_k = k \Delta t$, $k = 0, 1, 2, \dots$. For each of $i = 1, 2, \dots, N$, let particle P_i at time t_k be located at $\vec{r}_{i,k}$, have velocity $\vec{v}_{i,k}$, and have acceleration $\vec{a}_{i,k}$. Let position, velocity and acceleration be related by the recursion formulas

$$(3.1) \quad \vec{v}_{i, \frac{1}{2}} = \vec{v}_{i,0} + \frac{1}{2} (\Delta t) \vec{a}_{i,0}, \quad (\text{starter formula})$$

$$(3.2) \quad \vec{v}_{i, k + \frac{1}{2}} = \vec{v}_{i, k - \frac{1}{2}} + (\Delta t) \vec{a}_{i,k}, \quad k = 1, 2, 3, \dots$$

$$(3.3) \quad \vec{r}_{i, k+1} = \vec{r}_{i,k} + (\Delta t) \vec{v}_{i, k + \frac{1}{2}}, \quad k = 0, 1, 2, 3, \dots$$

At t_k , let the force acting on P_i be $\vec{F}_{i,k}$. We relate force and acceleration by the dynamical equation

$$(3.4) \quad \vec{F}_{i,k} = m_i \vec{a}_{i,k}.$$

As soon as the precise structure of $\vec{F}_{i,k}$ is given, then the motion of each P_i will be determined explicitly and recursively by (3.1)-(3.4) from given initial data. The force $\vec{F}_{i,k}$ is described now as follows.

Let $\vec{r}_{ij,k}$ be the vector from P_i to P_j at time t_k , so that

$r_{ij,k} = |\vec{r}_{i,k} - \vec{r}_{j,k}|$ is the distance between the two particles.

Let D be a positive constant. Then, for simplicity in the present paper, the force effects will be separated as follows. If $r_{ij,k} > D$, the force on P_i exerted by P_j is taken to be the long range force

$$(3.5) \quad \vec{F}_{ij,k} = - \frac{G^* m_i m_j}{(r_{ij,k})^2} \frac{\vec{r}_{ji,k}}{r_{ij,k}},$$

where G^* is a gravitational constant; while, if $r_{ij,k} \leq D$, then the force on P_i exerted by P_j is taken to be the short range force

$$(3.6) \quad \vec{F}_{ij,k} = \left[- \frac{Gm_i m_j}{(r_{ij,k})^p} + \frac{Hm_i m_j}{(r_{ij,k})^q} \right] \frac{\vec{r}_{ji,k}}{r_{ij,k}},$$

where G , H , p , and q are positive constants, establishing consistency with (2.1). The total force $\vec{F}_{i,k}$ on P_i at t_k due to all particles different from P_i is defined by

$$(3.7) \quad \vec{F}_{i,k} = \sum_{\substack{j=1 \\ j \neq i}}^N \vec{F}_{ij,k}.$$

For the convenience of the reader, a typical FORTRAN program of the above algorithm is given in the Appendix of Greenspan (12).

4. Computer Examples. At first, to illustrate the implementation and basic properties of the method, let us consider the simulation of evolutionary phenomena from a planar, rotating, gaseous body whose density is nonuniform. All considerations and parameter choices to be made are dictated by the constraints imposed by the use of a relatively small DEC-20 computer. Modifications for larger conventional computers and for parallel computers will be discussed later.

Consider a set of $N = 239$ particles arranged in a triangular mosaic, as shown in Figure 1. Each particle is one unit from each of its immediate neighbors and the entire configuration is relatively circular. For completeness, the initial positions are listed in Table I.

To simulate the nonuniformity of the density, the particles have been assigned masses, scaled appropriately and distributed at random (13), from the values 10000, 8000, 6000, 4000, and 2000. The number of particles assigned these masses are 2, 4, 12, 26, and 195, respectively. The exact assignment is given in Table I

and is shown in Figure 1 by the use of circles of decreasing radii, the largest corresponding to a mass of 10000 and, in decreasing order, the smallest corresponding to a mass of 2000. For clarity in later discussions, circles representing masses of 2000 will be left unshaded and will be called light particles, while all other circles will be shaded and will be called the heavy particles.

The entire system shown in Figure 1 is then set into counterclockwise rotation about the origin with an angular speed $\theta = 4$, but with various small perturbations superimposed (13). The initial velocity field is given in Table I and is shown in Figure 2. Thus, all initial data have been prescribed.

Since the masses of the particles have been assigned, let us next set the parameter values for the force formulas (3.1)-(3.6). Extensive preliminary studies (11), (13) resulted in the scaled choices $\Delta t = 0.0001$, $G = H = 5$, $p = 4$, $q = 6$. The parameter θ will, at present, be chosen to be 2.3 in order to prevent too many particles from escaping. The choices $G^* = 0.08$, 0.05, and 0.01 for the gravitational constant will all be considered.

One final consideration relates to heat loss from the system. Though more complex techniques can be developed (13), we will simulate the loss of heat energy merely by multiplying the velocity of each particle of mass 2000 by the factor 0.999 every two thousand time steps. The rationale for this simple procedure follows from hindsight, since it will be observed later that light particles always self-reorganize into an outer layer around the system and are then the transmitters of energy from the interior.

Consider now the particular choice $G^* = 0.08$. The resulting

particle motions were determined from (3.1)-(3.7) through t_{32000} . Figure 2 shows at t_{8000} that the heavy particles are self-reorganizing into four groups, while light particles have formed an outer layer of the system. Figure 3 shows clearly at t_{12000} the type of fluid distortion which is characteristic of the system as it rotates. Also, it shows that the four groups of heavy particles are now reorganizing into two larger subgroups. Figures 4-6 show the very slow motion of the two concentrations of heavy particles towards each other at the respective times t_{16000} , t_{24000} , and t_{32000} . At these times the number of light particles which either have escaped or are too far out to be included in the graphs are 15, 31, and 42, respectively. The rapid rotation, loss of mass, and formation of two heavy cores are characteristic of close binary systems and are already present in Figures 4-6.

Consider next the choice $G^* = 0.05$. The motion of the system up to t_{32000} is similar to that of the case $G^* = 0.08$. However, this time the system actually separates, as shown in Figure 7, into two distinct parts, which is reminiscent of the case of visual binaries. However, computer time limitations did not allow sufficiently long runs to determine whether or not the two subsystems would orbit.

Finally, for the choice $G^* = 0.01$, the system first behaved in the same general fashion as described above, but this time it divided into a triple system by t_{32000} .

Figure 1. The initial configuration.

TABLE I

<u>1</u>	<u>m</u>	<u>x</u>	<u>y</u>	<u>v_x</u>	<u>v_y</u>
1	10000.0	-2.000	3.464	-54.00000	-58.00000
2	10000.0	3.000	-1.732	51.00000	39.00000
3	8000.0	-4.500	-2.598	51.00000	-58.00000
4	8000.0	1.500	0.866	-37.00000	34.00000
5	8000.0	-2.500	2.598	-30.00000	-50.00000
6	8000.0	-2.500	-6.062	65.00000	-50.00000
7	6000.0	2.000	-5.196	50.00000	22.00000
8	6000.0	-1.000	-1.732	37.00000	-34.00000
9	6000.0	5.500	-0.866	-33.00000	8.00000
10	6000.0	-5.500	0.866	26.00000	-52.00000
11	6000.0	-1.000	1.732	-23.00000	-33.00000
12	6000.0	1.000	5.196	-9.00000	26.00000
13	6000.0	-2.000	-3.464	44.00000	-48.00000
14	6000.0	4.500	-2.598	40.00000	11.00000
15	6000.0	3.500	4.330	-12.00000	16.00000
16	6000.0	2.000	-1.732	23.00000	37.00000
17	6000.0	-3.000	3.464	-42.00000	-18.00000
18	6000.0	5.000	1.732	-37.00000	50.00000
19	4000.0	4.500	-6.062	44.00000	39.00000
20	4000.0	-5.000	-5.196	41.00000	1.00000
21	4000.0	-3.500	2.598	10.00000	-6.00000
22	4000.0	-2.500	-7.794	52.00000	11.00000
23	4000.0	6.500	2.598	9.00000	46.00000
24	4000.0	-2.000	-6.928	48.00000	13.00000
25	4000.0	-0.500	-4.330	-3.00000	-22.00000
26	4000.0	5.000	0.000	-20.00000	0.00000
27	4000.0	-1.500	-4.330	-3.00000	-260.00000
28	4000.0	-5.500	2.598	-30.00000	-42.00000
29	4000.0	-1.500	7.794	-51.00000	-27.00000
30	4000.0	-3.000	-1.732	-13.00000	-32.00000
31	4000.0	6.000	-5.196	40.00000	5.00000
32	4000.0	0.500	-6.062	45.00000	-18.00000
33	4000.0	-2.000	-5.196	41.00000	-28.00000
34	4000.0	5.000	-3.464	34.00000	0.00000
35	4000.0	1.000	-3.464	34.00000	-16.00000
36	4000.0	0.500	2.598	-27.00000	-18.00000
37	4000.0	-6.500	-0.866	24.00000	-46.00000
38	4000.0	-2.000	0.000	-20.00000	12.00000
39	4000.0	4.500	0.866	-23.00000	38.00000
40	4000.0	1.500	2.598	-30.00000	25.00000
41	4000.0	5.000	3.464	-34.00000	40.00000
42	4000.0	-4.500	4.330	-37.00000	2.00000
43	4000.0	4.000	5.196	-41.00000	35.00000
44	4000.0	-2.000	6.928	-47.00000	11.00000
45	2000.0	-2.500	7.794	-21.00000	0.00000
46	2000.0	3.000	-6.928	37.00000	23.00000
47	2000.0	1.500	-6.062	34.00000	16.00000
48	2000.0	-4.500	-6.062	35.00000	-8.00000
49	2000.0	0.000	-5.196	31.00000	10.00000
50	2000.0	-1.000	-5.196	30.00000	6.00000
51	2000.0	0.500	-4.330	28.00000	12.00000
52	2000.0	1.500	-4.330	27.00000	1.60000
53	2000.0	7.000	-3.464	23.00000	38.00000
54	2000.0	-6.000	-3.464	25.00000	-14.00000
55	2000.0	3.500	-2.598	20.00000	24.00000
56	2000.0	-3.500	-2.598	21.00000	-4.00000
57	2000.0	6.000	-1.732	16.00000	34.00000
58	2000.0	0.000	-1.732	17.00000	10.00000
59	2000.0	-6.000	-1.732	18.00000	-14.00000

TABLE I (CONTINUED)

60	2000.0	1.500	-0.866	14.00000	24.00000
61	2000.0	-2.500	-0.866	14.00000	0.00000
62	2000.0	0.000	0.000	10.00000	14.00000
63	2000.0	3.000	0.000	10.00000	22.00000
64	2000.0	-6.000	0.000	10.00000	-14.00000
65	2000.0	-1.500	0.866	6.00000	4.00000
66	2000.0	-0.500	0.866	7.00000	8.00000
67	2000.0	-6.500	0.866	7.00000	-16.00000
68	2000.0	-2.500	0.866	7.00000	1.00000
69	2000.0	1.000	1.732	3.00000	14.00000
70	2000.0	-5.000	1.732	3.00000	-10.00000
71	2000.0	-4.000	1.732	3.00000	-6.00000
72	2000.0	-6.500	2.598	0.00000	-16.00000
73	2000.0	0.000	3.464	-4.00000	10.00000
74	2000.0	0.500	4.330	-7.00000	11.00000
75	2000.0	-1.500	4.330	-8.00000	3.00000
76	2000.0	-6.500	4.330	-6.00000	-16.00000
77	2000.0	2.000	5.196	-30.00000	2.00000
78	2000.0	-6.000	5.196	-30.00000	-35.00000
79	2000.0	-0.500	6.062	-34.00000	-12.00000
80	2000.0	2.000	6.928	-38.00000	-3.00000
81	2000.0	1.500	7.794	-41.00000	-5.00000
82	2000.0	2.500	-7.794	21.00000	1.00000
83	2000.0	-1.000	-6.928	17.00000	-13.00000
84	2000.0	2.500	-6.062	14.00000	0.00000
85	2000.0	-3.500	-6.062	15.00000	-2.40000
86	2000.0	1.000	-5.196	11.00000	-6.00000
87	2000.0	-3.000	-5.196	11.00000	-22.00000
88	2000.0	2.500	-4.330	7.00000	0.00000
89	2000.0	6.000	-3.464	3.00000	14.00000
90	2000.0	0.000	-3.464	3.00000	-10.00000
91	2000.0	-7.000	-3.464	5.00000	-38.00000
92	2000.0	-2.500	-2.598	0.00000	-20.00000
93	2000.0	7.000	-1.732	-4.00000	18.00000
94	2000.0	-5.000	-1.732	-3.00000	-30.00000
95	2000.0	4.500	-0.866	-7.00000	8.00000
96	2000.0	-1.500	-0.866	-7.00000	-16.00000
97	2000.0	7.000	0.000	-11.00000	18.00000
98	2000.0	4.000	0.000	-10.00000	6.00000
99	2000.0	-7.000	0.000	-9.00000	-38.00000
100	2000.0	0.500	0.866	-14.00000	-7.00000
101	2000.0	-4.500	0.866	-13.00000	-28.00000
102	2000.0	-3.500	0.866	-13.00000	-24.00000
103	2000.0	0.000	1.732	-16.00000	-10.00000
104	2000.0	-6.000	1.732	-17.00000	-34.00000
105	2000.0	7.500	2.598	-21.00000	20.00000
106	2000.0	-4.500	2.598	-20.00000	-28.00000
107	2000.0	1.000	3.464	-24.00000	-5.00000
108	2000.0	-1.000	3.464	-24.00000	-15.00000
109	2000.0	4.500	4.330	-27.00000	8.00000
110	2000.0	5.500	4.330	-28.00000	12.00000
111	2000.0	1.500	4.330	-27.00000	-5.00000
112	2000.0	-0.500	4.330	-28.00000	-13.00000
113	2000.0	-5.000	5.196	-30.00000	-31.00000
114	2000.0	-1.500	6.062	-34.00000	-16.00000
115	2000.0	3.000	6.928	-38.00000	1.00000
116	2000.0	-1.000	6.928	-38.00000	-14.00000
117	2000.0	1.500	-7.794	21.00000	-3.00000
118	2000.0	-1.500	-7.794	21.00000	-15.00000
119	2000.0	0.000	-6.928	18.00000	-10.00000

TABLE I (CONTINUED)

120	2000.0	3.500	-6.062	13.00000	5.00000
121	2000.0	3.000	-5.196	11.00000	2.00000
122	2000.0	-4.000	-5.196	12.00000	-27.00000
123	2000.0	3.500	-4.330	7.00000	4.00000
124	2000.0	-2.500	-4.330	7.00000	-20.00000
125	2000.0	4.000	-3.464	4.00000	6.00000
126	2000.0	-1.000	-3.464	4.00000	-14.00000
127	2000.0	-5.000	-3.464	4.00000	-30.00000
128	2000.0	5.500	-2.598	0.00000	13.00000
129	2000.0	-1.500	-2.598	0.00000	-16.00000
130	2000.0	-7.500	-2.598	21.00000	-40.00000
131	2000.0	1.000	-1.732	17.00000	-6.00000
132	2000.0	-4.000	-1.732	17.00000	-26.00000
133	2000.0	6.500	-0.866	13.00000	16.00000
134	2000.0	-0.500	-0.866	13.00000	-12.00000
135	2000.0	0.500	-0.866	14.00000	-8.00000
136	2000.0	-7.500	-0.866	14.00000	-40.00000
137	2000.0	-5.500	-0.866	14.00000	-3.20000
138	2000.0	2.000	0.000	10.00000	-2.00000
139	2000.0	-4.000	0.000	10.00000	-26.00000
140	2000.0	3.500	0.866	6.00000	4.00000
141	2000.0	5.500	0.866	7.00000	12.00000
142	2000.0	6.000	1.732	3.00000	14.00000
143	2000.0	2.000	1.732	3.00000	-1.00000
144	2000.0	-7.000	1.732	4.00000	-38.00000
145	2000.0	5.500	2.598	0.00000	12.00000
146	2000.0	-1.500	2.598	0.00000	-16.00000
147	2000.0	-7.500	2.598	0.00000	-40.00000
148	2000.0	2.000	3.464	-4.00000	-2.00000
149	2000.0	2.500	4.330	-8.00000	0.00000
150	2000.0	-2.500	4.330	-8.00000	0.00000
151	2000.0	3.000	5.196	-11.00000	2.00000
152	2000.0	-4.000	5.196	-10.00000	-27.00000
153	2000.0	0.500	6.062	-14.00000	-8.00000
154	2000.0	-3.500	6.062	-14.00000	-24.00000
155	2000.0	-3.000	6.928	-17.00000	-23.00000
156	2000.0	0.500	7.794	-21.00000	-0.19000
157	2000.0	-0.500	-7.794	41.00000	-11.00000
158	2000.0	1.000	-6.928	38.00000	-5.00000
159	2000.0	-1.500	-6.062	34.00000	-1.60000
160	2000.0	4.000	-5.196	30.00000	7.00000
161	2000.0	-6.000	-5.196	31.00000	-34.00000
162	2000.0	5.500	-4.330	27.00000	13.00000
163	2000.0	-3.500	-4.330	28.00000	-24.00000
164	2000.0	-6.500	-4.330	28.00000	-36.00000
165	2000.0	2.000	-3.464	24.00000	-7.00000
166	2000.0	-4.000	-3.464	24.00000	-26.00000
167	2000.0	6.500	-2.598	19.00000	16.00000
168	2000.0	-0.500	-2.598	20.00000	-12.00000
169	2000.0	0.500	-2.598	20.00000	-80.00000
170	2000.0	-6.500	-2.598	21.00000	-36.00000
171	2000.0	4.000	-1.732	17.00000	6.00000
172	2000.0	7.500	-0.866	13.00000	20.00000
173	2000.0	1.500	-0.866	14.00000	-3.00000
174	2000.0	-4.500	-0.866	13.00000	-28.00000
175	2000.0	-1.000	0.000	10.00000	-14.00000
176	2000.0	-5.000	0.000	10.00000	-40.00000
177	2000.0	2.500	0.866	7.00000	0.00000
178	2000.0	7.000	1.732	2.00000	18.00000
179	2000.0	-7.500	0.866	8.00000	-40.00000

TABLE I (COMPLETED)

180	2000.0	3.000	1.732	3.00000	2.00000
181	2000.0	-3.000	1.732	4.00000	-21.00000
182	2000.0	3.500	2.598	0.00000	4.00000
183	2000.0	2.500	2.598	-21.00000	20.00000
184	2000.0	-0.500	2.598	-21.00000	8.00000
185	2000.0	6.000	3.464	-24.00000	34.00000
186	2000.0	3.000	3.464	-24.00000	22.00000
187	2000.0	-4.000	3.464	-23.00000	-6.00000
188	2000.0	-7.000	3.464	-23.00000	-18.00000
189	2000.0	-6.000	3.464	-24.00000	-14.00000
190	2000.0	-3.500	4.330	-26.00000	-5.00000
191	2000.0	5.000	5.196	-31.00000	29.00000
192	2000.0	-3.000	5.196	-31.00000	-3.00000
193	2000.0	-2.000	5.196	-31.00000	1.00000
194	2000.0	3.500	6.062	-35.00000	23.00000
195	2000.0	1.500	6.062	-34.00000	16.00000
196	2000.0	-4.500	6.062	-34.00000	-9.00000
197	2000.0	1.000	6.928	-38.00000	13.00000
198	2000.0	-0.500	7.794	-41.00000	7.00000
199	2000.0	0.500	-7.794	21.00000	13.00000
200	2000.0	2.000	-6.928	17.00000	19.00000
201	2000.0	-3.000	-6.928	18.00000	-1.00000
202	2000.0	-0.500	-6.062	14.00000	8.00000
203	2000.0	5.000	-5.196	11.00000	31.00000
204	2000.0	6.500	-4.330	7.00000	37.00000
205	2000.0	4.500	-4.330	7.00000	28.00000
206	2000.0	-5.500	-4.330	8.00000	-11.00000
207	2000.0	-4.500	-4.330	7.00000	-7.00000
208	2000.0	3.000	-3.464	4.00000	22.00000
209	2000.0	-3.000	-3.464	4.00000	-2.00000
210	2000.0	7.500	-2.598	0.00000	40.00000
211	2000.0	1.500	-2.598	1.00000	16.00000
212	2000.0	2.500	-2.598	0.00000	20.00000
213	2000.0	-5.500	-2.598	0.00000	-11.00000
214	2000.0	5.000	-1.732	-3.00000	30.00000
215	2000.0	-2.000	-1.732	3.00000	-2.00000
216	2000.0	-7.000	-1.732	-2.00000	-18.00000
217	2000.0	2.500	-0.866	-7.00000	20.00000
218	2000.0	-3.500	-0.866	-7.00000	-4.00000
219	2000.0	0.000	0.000	-10.00000	10.00000
220	2000.0	1.000	0.000	-10.00000	14.00000
221	2000.0	-3.000	0.000	-10.00000	-2.00000
222	2000.0	6.500	0.866	-15.00000	36.00000
223	2000.0	7.500	0.866	-14.00000	40.00000
224	2000.0	4.000	1.732	-17.00000	26.00000
225	2000.0	-2.000	1.732	-17.00000	3.00000
226	2000.0	4.500	2.598	-20.00000	28.00000
227	2000.0	7.000	3.464	-25.00000	38.00000
228	2000.0	4.000	3.464	-24.00000	26.00000
229	2000.0	-5.000	3.464	-24.00000	-10.00000
230	2000.0	6.500	4.330	-28.00000	35.00000
231	2000.0	-5.500	4.330	-27.00000	-13.00000
232	2000.0	6.000	5.196	-32.00000	33.00000
233	2000.0	-1.000	5.196	-31.00000	6.00000
234	2000.0	0.000	5.196	-31.00000	10.00000
235	2000.0	4.500	6.062	-35.00000	27.00000
236	2000.0	2.500	6.062	-35.00000	19.00000
237	2000.0	-2.500	6.062	-34.00000	0.00000
238	2000.0	0.000	6.928	-38.00000	9.00000
239	2000.0	2.500	7.794	-42.00000	19.00000

Figure 2. $G^* = 0.08$, $T = t_{3000}$

Figure 3. $G^* = 0.05$, $t = t_{12000}$

Figure 4. $G^* = 0.08$, $T = t_{16000}$.

Figure 5. $G^* = 0.08$, $T = t_{24000}$

Figure 6. $G^* = 0.08$, $T = t_{52000}$

Figure 7. $G^* = 0.05$, $T = t_{32000}$

Let us discuss now a different close binary simulation into which are incorporated the following special physical properties. First, recall that the masses of most Main Sequence stars are concentrated at their centers (15). Second, recall that, in general, the axial rotation of each star in a binary system is in the same direction as its orbital motion (15). Consider then the following example.

Let $N = 194$ and consider a binary star configuration as shown in Figure 8. The primary star is located initially with its most massive component at the origin. Of the 194 particles, 2, 4, 10, 19 and 159 have been assigned masses 30000, 15000, 7500, 5000 and 2000, respectively. The initial positions and masses are as shown in Figure 8 and, for completeness, are given in Table II. The two stars are then each set into counterclockwise motion with rotational speed $\dot{\theta} = 4$, but with an element of randomness in the actual velocity components and with relatively chaotic motions at their centers. This axial velocity field is shown in Figure 8. Finally, the component of velocity in the Y direction for each particle in the secondary star is decreased by $V_Y = 32.5$, so that the secondary star is set into counterclockwise motion relative to the primary. The resulting initial velocity field is given in Table II. The gravitational parameter G^* is chosen to be 0.1. The heat loss damping factor is set, this time, at 0.9999. Otherwise, all parameters are the same as those chosen previously.

Figures 9-25 show the resulting motion of the system every 2000 time steps through two full orbital rotations. In Figures 9-14, the motion of the system is shown relative to the XY coordinate

axes. The figures reveal the loss of mass by expulsion of light particles only, the gradual motion of the center of mass in, primarily, the negative Y direction, and a very fluid, gaseous type of motion of the outer particles. Figures 15-25 show the continued loss of mass, fluidity of the outer boundary, and a small amount of motion of the heavy mass concentrations towards each other. This small motion is accompanied by an increase in the rotational speed of the heavy mass concentrations. Also, this motion compresses the intervening light particles, resulting in an increase in the potential energy of these particles. Conversion of this potential energy into kinetic energy, a natural event because the particles are free to expand at all boundaries, accounts for the continued expulsion of mass. By the time t_{34000} , shown in Figure 25, 23 particles have escaped the system, all being light particles, while one additional particle is sufficiently far out that it cannot be plotted in the figure.

Let us now show how the velocity field has changed with time by studying in greater detail the configuration at t_{34000} shown in Figure 25. To do this, we must proceed as follows (10). The velocity field defined by (3.1)-(3.2) is the result of both local and long range force interactions. At any given time, it may, therefore, possess some characteristics of Brownian motion. To describe the gross motion of the system, then, we define the average velocity $\vec{v}_{i,k}^*$ of P_i at t_k by

$$\vec{v}_{i,k}^* = \frac{\vec{r}_{i,k+500} - \vec{r}_{i,k}}{0.05}$$

An entirely analogous definition holds for the average velocity of the center of mass. Using, then, average velocities, Figure 26 shows the velocity field of the particle configuration at t_{34000} relative to the center of mass. The entire system rotates in a streaming, counterclockwise, vortex motion about the mass center. Figure 27 shows the velocity field at t_{34000} , but relative to the most massive particle of the primary star, and reveals the counterclockwise motion of the primary system. Figure 28 shows the velocity field relative to the most massive particle of the secondary star and reveals not only the counterclockwise motion of the heavier particles of this system, but also two vortices which lie between the heavy particles and the section of the boundary nearest to them.

With regard to other calculations, which had the same data as shown in Figure 8, note that using VY equal to any one of 30.0, 27.5, or 25.0 yielded analogous results. The particular choice $VY = 30$, however, did eject five neighboring particles in a single unit at time t_{10000} . Additional examples in which D was decreased moderately led to a marked increase in streaming between the stars and to a sufficiently large mass loss that the remaining binary configuration was too small to yield interesting implications. Decreasing G^* not only had a similar mass loss effect, but, in addition, slowed the motion of the heavy cores towards each other. Indeed, G^* could be chosen so that the distance between the primary and secondary components either decreased arbitrarily slowly or increased arbitrarily slowly. Because of the large fluid deformation present, we found it impossible to ascertain a G^* for which the interstellar distance would be constant for all time, the implication being that, eventually, close binaries either unite or separate.

Figure 1. The initial configuration.

TABLE II

<u>i</u>	<u>M</u>	<u>X</u>	<u>Y</u>	<u>VX</u>	<u>VY</u>
1	30000.0	-10.000	0.000	-10.00000	-19.50000
2	30000.0	0.000	0.000	-10.00000	10.00000
3	15000.0	-0.500	0.866	7.00000	8.00000
4	15000.0	1.000	0.000	-10.00000	14.00000
5	15000.0	-10.500	-0.866	3.00000	-38.50000
6	15000.0	-9.000	0.000	-7.00000	-14.50000
7	7500.0	-1.500	0.866	6.00000	4.00000
8	7500.0	-1.000	0.000	10.00000	-14.00000
9	7500.0	0.000	-1.732	17.00000	13.00000
10	7500.0	0.500	0.866	-14.00000	-7.00000
11	7500.0	0.500	-0.866	13.00000	-8.00000
12	7500.0	1.500	0.866	-37.00000	34.00000
13	7500.0	-11.000	0.000	5.00000	-48.50000
14	7500.0	-10.500	0.866	17.00000	-22.50000
15	7500.0	-10.000	-1.732	15.00000	-39.50000
16	7500.0	-9.500	0.866	-16.00000	-37.50000
17	5000.0	-2.000	1.732	-17.00000	-3.00000
18	5000.0	-2.000	0.000	-12.00000	-20.00000
19	5000.0	-1.500	-0.866	-7.00000	-16.00000
20	5000.0	-1.000	1.732	-23.00000	-33.00000
21	5000.0	-0.500	2.598	-21.00000	8.00000
22	5000.0	0.000	1.732	-16.00000	-10.00000
23	5000.0	1.000	1.732	3.00000	14.00000
24	5000.0	1.000	-1.732	17.00000	-6.00000
25	5000.0	1.500	-0.866	14.00000	-3.00000
26	5000.0	2.000	0.000	10.00000	-2.00000
27	5000.0	3.000	0.000	10.00000	21.00000
28	5000.0	-11.000	1.732	-29.00000	-45.50000
29	5000.0	-11.000	-1.732	30.00000	-73.50000
30	5000.0	-10.000	1.732	-20.00000	-39.50000
31	5000.0	-9.500	-0.866	12.00000	-42.50000
32	5000.0	-9.500	-2.598	35.00000	10.50000
33	5000.0	-9.000	1.732	5.00000	-19.50000
34	5000.0	-9.000	-1.732	12.00000	-42.50000
35	5000.0	-8.000	0.000	-9.00000	-29.50000
36	2000.0	-5.000	1.732	-3.00000	-10.00000
37	2000.0	-5.000	0.000	10.00000	-30.00000
38	2000.0	-5.000	-1.732	3.00000	-30.00000
39	2000.0	-4.500	2.598	-20.00000	-28.00000
40	2000.0	-4.500	0.866	-13.00000	-28.00000
41	2000.0	-4.500	-0.866	10.00000	-26.00000
42	2000.0	-4.500	-2.598	51.00000	-58.00000
43	2000.0	-4.000	3.464	-23.00000	-6.00000
44	2000.0	-4.000	1.732	4.00000	-6.00000
45	2000.0	-4.000	0.000	-10.00000	-6.00000
46	2000.0	-4.000	-1.732	17.00000	-26.00000
47	2000.0	-4.000	-3.464	24.00000	-26.00000
48	2000.0	-3.500	2.598	-10.00000	-6.00000
49	2000.0	-3.500	0.866	-13.00000	-24.00000
50	2000.0	-3.500	-0.866	13.00000	-28.00000
51	2000.0	-3.500	-2.598	21.00000	-4.00000
52	2000.0	-3.000	3.464	-42.00000	-18.00000
53	2000.0	-3.000	1.732	4.00000	-21.00000
54	2000.0	-3.000	0.000	10.00000	-2.00000
55	2000.0	-3.000	-1.732	13.00000	-32.00000
56	2000.0	-3.000	-3.464	4.00000	-2.00000
57	2000.0	-2.500	4.330	-8.00000	0.00000
58	2000.0	-2.500	2.598	-10.00000	-50.00000
59	2000.0	-2.500	0.866	7.00000	1.00000

TABLE II (CONTINUED)

60	2000.0	-2.500	-0.866	14.00000	0.00000
61	2000.0	-2.500	-2.598	0.00000	-20.00000
62	2000.0	-2.500	-4.330	17.00000	-20.00000
63	2000.0	-2.000	3.464	-54.00000	-58.00000
64	2000.0	-2.000	-1.732	3.00000	-2.00000
65	2000.0	-2.000	-3.464	44.00000	-38.00000
66	2000.0	-1.500	4.330	-8.00000	3.00000
67	2000.0	-1.500	2.598	0.00000	-16.00000
68	2000.0	-1.500	-2.598	0.00000	-16.00000
69	2000.0	-1.500	-4.330	23.00000	-6.00000
70	2000.0	-1.000	5.196	-31.00000	6.00000
71	2000.0	-1.000	3.464	-24.00000	-15.00000
72	2000.0	-1.000	-1.732	37.00000	-34.00000
73	2000.0	-1.000	-3.464	4.00000	-14.00000
74	2000.0	-1.000	-5.196	30.00000	6.00000
75	2000.0	-0.500	4.330	-28.00000	-13.00000
76	2000.0	-0.500	-0.866	13.00000	-12.00000
77	2000.0	-0.500	-2.598	20.00000	-12.00000
78	2000.0	-0.500	-4.330	3.00000	22.00000
79	2000.0	0.000	5.196	-31.00000	-14.00000
80	2000.0	0.000	3.464	-4.00000	12.00000
81	2000.0	0.000	-3.464	3.00000	-19.00000
82	2000.0	0.000	-5.196	31.00000	16.00000
83	2000.0	0.500	4.330	-7.00000	11.00000
84	2000.0	0.500	2.598	-27.00000	-18.00000
85	2000.0	0.500	-2.598	60.00000	22.00000
86	2000.0	0.500	-4.330	28.00000	12.00000
87	2000.0	1.000	5.196	-19.00000	6.00000
88	2000.0	1.000	3.464	-24.00000	-5.00000
89	2000.0	1.000	-3.464	34.00000	-16.00000
90	2000.0	1.000	-5.196	11.00000	6.00000
91	2000.0	1.500	4.330	-27.00000	-5.00000
92	2000.0	1.500	2.598	-30.00000	25.00000
93	2000.0	1.500	-2.598	1.00000	16.00000
94	2000.0	1.500	-4.330	27.00000	2.00000
95	2000.0	2.000	3.464	-4.00000	-2.00000
96	2000.0	2.000	1.732	3.00000	-1.00000
97	2000.0	2.000	-1.732	23.00000	-2.00000
98	2000.0	2.000	-3.464	24.00000	37.00000
99	2000.0	2.500	4.330	-8.00000	0.00000
100	2000.0	2.500	2.598	-21.00000	17.00000
101	2000.0	2.500	0.866	7.00000	1.00000
102	2000.0	2.500	-0.866	-7.00000	20.00000
103	2000.0	2.500	-2.598	0.00000	19.00000
104	2000.0	2.500	-4.330	7.00000	10.00000
105	2000.0	3.000	3.464	-24.00000	22.00000
106	2000.0	3.000	1.732	3.00000	2.00000
107	2000.0	3.000	-1.732	51.00000	39.00000
108	2000.0	3.000	-3.464	4.00000	27.00000
109	2000.0	3.500	2.598	-1.00000	4.00000
110	2000.0	3.500	0.866	6.00000	5.00000
111	2000.0	3.500	-0.866	14.00000	24.00000
112	2000.0	3.500	-2.598	20.00000	21.00000
113	2000.0	4.000	3.464	-23.00000	25.00000
114	2000.0	4.000	1.732	-17.00000	26.00000
115	2000.0	4.000	0.000	-10.00000	17.00000
116	2000.0	4.000	-1.732	17.00000	8.00000
117	2000.0	4.000	-3.464	4.00000	6.00000
118	2000.0	4.500	2.598	-20.00000	28.00000
119	2000.0	4.500	0.866	-23.00000	38.00000

TABLE II (CONTINUED)

120	2000.0	4.500	-0.866	-7.00000	8.00000
121	2000.0	4.500	-2.598	11.00000	40.00000
122	2000.0	5.000	1.732	-37.00000	50.00000
123	2000.0	5.000	0.000	-20.00000	0.00000
124	2000.0	5.000	-1.732	-3.00000	30.00000
125	2000.0	-14.500	0.866	-12.00000	-62.50000
126	2000.0	-14.500	-0.866	11.00000	-61.50000
127	2000.0	-14.000	1.732	8.00000	-37.50000
128	2000.0	-14.000	0.000	-7.00000	-41.50000
129	2000.0	-14.000	-1.732	14.00000	-57.50000
130	2000.0	-13.500	2.598	-12.00000	-36.50000
131	2000.0	-13.500	0.866	-10.00000	-59.50000
132	2000.0	-13.500	-0.866	15.00000	-55.50000
133	2000.0	-13.500	-2.598	14.00000	-38.50000
134	2000.0	-13.000	3.464	-28.00000	-58.50000
135	2000.0	-13.000	1.732	11.00000	-49.50000
136	2000.0	-13.000	0.000	20.00000	-39.50000
137	2000.0	-13.000	-1.732	10.00000	-57.50000
138	2000.0	-13.000	-3.464	5.00000	-44.50000
139	2000.0	-12.500	2.598	-20.00000	-73.50000
140	2000.0	-12.500	0.866	0.00000	-20.50000
141	2000.0	-12.500	-0.866	12.00000	-40.50000
142	2000.0	-12.500	-2.598	15.00000	-45.50000
143	2000.0	-12.000	3.464	-23.00000	-79.50000
144	2000.0	-12.000	1.732	-30.00000	-42.50000
145	2000.0	-12.000	0.000	-12.00000	-35.50000
146	2000.0	-12.000	-1.732	8.00000	-44.50000
147	2000.0	-12.000	-3.464	23.00000	-59.50000
148	2000.0	-11.500	4.330	-16.00000	-20.50000
149	2000.0	-11.500	2.598	5.00000	-40.50000
150	2000.0	-11.500	0.866	17.00000	-29.50000
151	2000.0	-11.500	-0.866	-5.00000	-49.50000
152	2000.0	-11.500	-2.598	10.00000	-39.50000
153	2000.0	-11.500	-4.330	30.00000	-31.50000
154	2000.0	-11.000	3.464	-24.00000	-56.50000
155	2000.0	-11.000	-3.464	8.00000	-43.50000
156	2000.0	-10.500	4.330	-20.00000	-39.50000
157	2000.0	-10.500	2.598	-27.00000	-21.50000
158	2000.0	-10.500	-2.598	15.00000	-40.50000
159	2000.0	-10.500	-4.330	10.00000	-15.50000
160	2000.0	-10.000	3.464	-6.00000	-38.50000
161	2000.0	-10.000	-3.464	10.00000	-42.50000
162	2000.0	-9.500	4.330	-5.00000	-19.50000
163	2000.0	-9.500	2.598	-15.00000	-57.50000
164	2000.0	-9.500	-4.330	12.00000	-29.50000
165	2000.0	-9.000	3.464	-17.00000	-35.50000
166	2000.0	-9.000	-3.464	40.00000	-31.50000
167	2000.0	-8.500	4.330	-15.00000	-35.50000
168	2000.0	-8.500	2.598	-16.00000	2.50000
169	2000.0	-8.500	0.866	-40.00000	-5.50000
170	2000.0	-8.500	-0.866	11.00000	-32.50000
171	2000.0	-8.500	-2.598	7.00000	-26.50000
172	2000.0	-8.500	-4.330	24.00000	-25.50000
173	2000.0	-8.000	3.464	-5.00000	-25.50000
174	2000.0	-8.000	1.732	7.00000	-34.50000
175	2000.0	-8.000	-1.732	31.00000	-28.50000
176	2000.0	-8.000	-3.464	23.00000	-5.50000
177	2000.0	-7.500	2.598	-17.00000	-17.50000
178	2000.0	-7.500	0.866	14.00000	-28.50000
179	2000.0	-7.500	-0.866	-3.00000	-10.50000

TABLE II (COMPLETED)

180	2000.0	-7.500	-2.598	3.00000	-15.50000
181	2000.0	-7.000	3.464	-19.00000	-11.50000
182	2000.0	-7.000	1.732	-3.00000	-20.50000
183	2000.0	-7.000	0.000	-1.00000	-21.50000
184	2000.0	-7.000	-1.732	32.00000	-5.50000
185	2000.0	-7.000	-3.464	8.00000	2.50000
186	2000.0	-6.500	2.598	7.00000	-28.50000
187	2000.0	-6.500	0.866	13.00000	-30.50000
188	2000.0	-6.500	-0.866	3.00000	-16.50000
189	2000.0	-6.500	-2.598	13.00000	-3.50000
190	2000.0	-6.000	1.732	-20.00000	-7.50000
191	2000.0	-6.000	0.000	-13.00000	-1.50000
192	2000.0	-6.000	-1.732	0.00000	-13.50000
193	2000.0	-5.500	0.866	27.00000	-31.50000
194	2000.0	-5.500	-0.866	4.00000	-15.50000

FIGURE 9. $T = T_{2000}$.

Figure 11, $T = 6000$

-19

-14

Y

2

4

X

-15

Y

10

X

-20

Figure 12, $T = 6000$

Figure 13. $T = t_{10000}$.

Figure 14. $T = t_{12000}$.

Figure 15. $T = t_{14000}^{\circ}$

Figure 16. $T = t_{16000}^{\circ}$

Figure 17. $T = t_{18000}$

Figure 18. $T = t_{20000}$

Figure 19. $T = t_{22000}$

Figure 20. $T = t_{24000}$

Figure 21. $T = t_{26000}$

Figure 22. $T = t_{28000}$

Figure 23. $T = t_{30000}$.

Figure 24. $T = t_{32000}$.

Figure 25. $T = \pm 3400^\circ$

Figure 26. $T=t_{34000}$

Figure 27. $T=t_{34000}$

Figure 28. $T=t_{34000}$

5. Remarks. The method we have developed extends readily to three dimensions and is capable of simulating triple and multiple star systems. Also, it requires only rectangular, cartesian coordinates, thus avoiding the nonphysical singularities introduced by the use of special coordinates.

Next, observe that it would be of great value to increase the number of particles in any simulation of close binaries. This should yield better approximations. It would certainly allow one to decrease the local force distance to a more reasonable value. At present, computer codes do exist for the larger CDC computers which enable one to simulate interactions of 200000 particles, and codes are being prepared to handle 2000000 particles (5). Further, current microtechnology being applied in the development of parallel computers promises to reduce dramatically the computer time for particle modeling by executing the computations in parallel, rather than in sequence.

Observe also that the computer examples of Section 4 yield qualitative behavior only. The determination of quantitative behavior would require, at the least, a curve fitting process to determine the parameters of the force formulas (17).

Finally, note that chemical and nuclear reactions have yet to be included in our models and in those of others. The reasons are that the dynamics of chemical reactions are, at best, poorly understood, and that there is as yet no acceptable model of the nucleus, since all quantum mechanical formulations fail to predict the correct, experimentally determined, energies of nuclei (6).

References

1. R. G. Aitken, THE BINARY STARS, Dover, N.Y., 1964
2. J. A. Barker and D. Henderson, "What is 'liquid'? Understanding the states of matter", Reviews of Modern Physics, 48, 1976, pp. 587-664.
3. A. H. Batten, BINARY AND MULTIPLE SYSTEMS OF STARS, Pergamon, Oxford, 1973.
4. L. M. Beskovic, "Gylden-Meščerskii problems", Celestial Mech., 24, 1981, pp. 407-429.
5. O. Bunemann, C. W. Barnes, J. C. Green, and D. E. Nielsen, "Principles and capabilities of 3-D, E-M particle simulations", Jour. Comp. Phys., 38, 1980, pp. 1-44.
6. C. S. Cook, STRUCTURE OF ATOMIC NUCLEI, Van Nostrand, Princeton, N.J., 1964.
7. P. Eggleston et al (Eds.), STRUCTURE AND EVOLUTION OF CLOSE BINARY SYSTEMS, Reidel, Dordrecht, 1976.
8. R. P. Feynman, R. B. Leighton, and M. Sands, THE FEYNMAN LECTURES ON PHYSICS, Addison-Wesley, Reading, 1963.
9. D. Greenspan, "Computer studies of a von Neumann type fluid", Appl. Math. and Comp., 4, 1978, pp. 15-25.
10. D. Greenspan, "A particle model of the Stefan problem", Comp. Meth. in Appl. Mech. and Eng., 13, 1978, pp. 95-104.
11. D. Greenspan, ARITHMETIC APPLIED MATHEMATICS, Pergamon, Oxford, 1980.
12. D. Greenspan, "A new computer approach to the modeling of close binary stars", TR 195, Math. Dept., Univ. Texas at Arlington, Arlington, Texas, 1983.

13. D. Greenspan and J. Collier, "Computer studies of swirling particle fluids and the evolution of planetary type bodies", JIMA, 22, 1978, pp. 235-253.
14. K. Gyldenkerne and R. M. West (Eds.), MASS LOSS AND EVOLUTION IN CLOSE BINARIES, Copenhagen Univ. Publications Fund, Denmark, 1978
15. Z. Kopal, DYNAMICS OF CLOSE BINARY SYSTEMS, Reidel, Dordrecht, 1978
16. W. S. Krogdahl, THE ASTRONOMICAL UNIVERSE, Macmillan, N.Y., 1952
17. W. R. Reeves and D. Greenspan, "An analysis of stress wave propagation in slender bars using a discrete particle approach", Appl. Math. Modelling, 6, 1982, pp. 185-191.
18. J. Sahade and F. B. Woods, INTERACTING BINARY STARS, Pergamon, Oxford, 1978.
19. L. W. Uccellini and R. A. Peterson, "Applying discrete model concepts to the computation of atmospheric trajectories", Int. Jour. General Systems, 6, 1980, pp. 13-23.

APPENDIX

C THIS IS THE 2-D STAR10 PROGRAM. WE WISH TO SIMULATE DOUBLE
 C STAR EVOLUTION. THERE ARE 239 PARTICLES.
 C THE FORCES ARE OF LOCAL MOLECULAR TYPE AND LONG RANGE GRAVITATION.
 C RUN TIME OF 1 HOUR GIVES 10000 ITERATIONS.

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    DOUBLE PRECISION XO(239),YO(239),VXO(239),VYO(239),
    1X(239,2),Y(239,2),VX(239,2),VY(239,2),
    1ACX(239),ACY(239),R2,R,FX,FY,F,PMASS(239),R010
    OPEN (UNIT=7,FILE='STAR10.OUT',DEVICE='DSK')
    OPEN (UNIT=21,FILE='STAR10.DAT')
    OPEN (UNIT=5,DEVICE='LPT',FILE='STAR10.PRT',ACCESS='SEQOUT')
    KPRINT=1000
    NM1=238
    K=1
    DO 4 I=1,239
      X(I,1)=0.
      Y(I,1)=0.
      VX(I,1)=0.
      VY(I,1)=0.
      X(I,2)=0.
      Y(I,2)=0.
      VX(I,2)=0.
      VY(I,2)=0.
      ACX(I)=0.
      ACY(I)=0.
4     CONTINUE
      DO 5 I=1,239
        IF (I.LE.2) GO TO 4001
        IF (I.LE.6) GO TO 4002
        IF (I.LE.18) GO TO 4003
        IF (I.LE.44) GO TO 4004
        IF (I.LE.239) GO TO 4005
4001      PMASS(I)=10000.
          GO TO 5
4002      PMASS(I)=8000.
          GO TO 5
4003      PMASS(I)=6000.
          GO TO 5
4004      PMASS(I)=4000.
          GO TO 5
4005      PMASS(I)=2000.
5     CONTINUE
      READ (21,10) (XO(I),YO(I),VXO(I),VYO(I),I=1,239)
10    FORMAT (2F12.3,2F12.5)
11    DO 20 I=1,239
      WRITE (5,15) I,PMASS(I),XO(I),YO(I),VXO(I),
1VYO(I)
15    FORMAT (5X,15,F10.1,2F12.3,2F12.5)
20    CONTINUE
      DO 30 I=1,239
        X(I,1)=XO(I)
        Y(I,1)=YO(I)
        VX(I,1)=VXO(I)
        VY(I,1)=VYO(I)
30    CONTINUE
      GO TO 71
65    DO 70 I=1,239
        X(I,1)=X(I,2)
        Y(I,1)=Y(I,2)

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      ACX(I)=0.
      ACY(I)=0.
701      CONTINUE
71      DO 77 I=1,238
      IP1=I+1
      DO 76 J=IP1,239
      R2=(X(I,1)-X(J,1))**2+(Y(I,1)-Y(J,1))**2
      R=DSQRT(R2)
C THE LOCAL INTERACTION DISTANCE IS SET NEXT.
      IF (R.LT.2.3) GO TO 721
C THE GRAVITATIONAL CONSTANT IS SET NEXT.
      F=PMASS(I)*PMASS(J)*(-0.1)/(R2*R)
      GO TO 727
721      F=5.0*PMASS(I)*PMASS(J)*(-1.+(1./R2))/(R2*R2)
727      FX=F*(X(I,1)-X(J,1))/R
      FY=F*(Y(I,1)-Y(J,1))/R
75      ACX(I)=ACX(I)+FX
      ACX(J)=ACX(J)-FX
      ACY(I)=ACY(I)+FY
      ACY(J)=ACY(J)-FY
76      CONTINUE
77      CONTINUE
79      DO 80 I=1,239
      VX(I,2)=VX(I,1)+0.0001*ACX(I)/PMASS(I)
      VY(I,2)=VY(I,1)+0.0001*ACY(I)/PMASS(I)
      X(I,2)=X(I,1)+0.0001*VX(I,2)
      Y(I,2)=Y(I,1)+0.0001*VY(I,2)
80      CONTINUE
8043      K=K+1
      IF (MOD(K,KPRINT).GT.0) GO TO 82
      DO 810 I=1,239
      WRITE (5,81) K,I,X(I,2),Y(I,2),VX(I,2),VY(I,2)
81      FORMAT (5X,I12,15,2F18.6,2F18.9)
810      CONTINUE
C WE NEXT CALCULATE THE DISTANCE BETWEEN THE TWO LARGEST MASSES.
      RB10=(X(1,2)-X(2,2))**2+(Y(1,2)-Y(2,2))**2
      RB10=DSQRT(RB10)
      WRITE (5,8619) RB10
8619      FORMAT (2X,F20.8)
82      IF (K.LT.2000) GO TO 65
C DAMP HERE AS A CONSEQUENCE OF HEAT RADIATION.
      DO 9053 I=1,239
      IF (I.LE.44) GO TO 9053
      VX(I,2)=0.999*VX(I,2)
      VY(I,2)=0.999*VY(I,2)
9053      CONTINUE
92      WRITE (7,10) (X(I,2),Y(I,2),VX(I,2),VY(I,2),I=1,
1239)
      STOP
      END

```